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EFFECTIVE SOURCE WAVELET DETERMINATION

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ABSTRACT

For the imaging of ground-penetrating radar (GPR) data, the form of the radar wavelet is usually not taken into account, because the behaviour of actual source and receiver antennas, that have a large influence on the wavelet, is rather complex. Nevertheless, knowledge of the wavelet has the potential to improve the imaging and interpretation of GPR data. An efficient way to deal with the wavelet is proposed by introducing an effective wavelet that incorporates the influence of the finite-length antennas. To obtain this effective wavelet, the impulse response for a point source-receiver antenna system is calculated using the medium properties, that are obtained from the isolated air- and ground-waves observed on the actual CMP data. The deconvolution of this impulse response with the actual CMP data, yields an effective wavelet. Together with the well-known radiation characteristics of dipole antennas in a dielectric halfspace, the propagation of the electromagnetic waves emitted a finite-length source-receiver antenna system can be effectively described. We demonstrate that properties of the shallow subsurface can be extracted from the ground-wave with reasonable accuracy. An effective wavelet determined from numerical data calculated for a finite-length source-receiver antenna system shows an effective wavelet that is less minimum phase than the effective wavelet calculated from the electric field generated by a point source-receiver antenna system.

Key words: Wavelet estimation, 3D imaging, ground penetrating radar, halfspace

INTRODUCTION

Ground-penetrating radar (GPR) has become a widely used geophysical technique for investigating the shallow subsurface. Improved resolution of radar images can be obtained by applying deconvolution and imaging. However, Turner (1993) has shown that georadar signals are generally not minimum phase, an important prerequisite for many predictive deconvolution routines. With de-

tailed knowledge of the emitted source wavelet it should be possible to define alternative approaches to deconvolution.

A GPR system is an exploration system that uses transmitter and receiver antennas, that are placed on or near the ground surface. Physical antennas have a quite complex performance (Roberts and Daniels, 1997). In this paper, an efficient way of describing this complex behaviour is introduced.

In the following, we define various types of wavelet relevant to georadar propagation between a source and receiver antenna:

- The initial wavelet is the current pulse injected into the source antenna. Conceptually, it can be determined from air- and ground-waves observed on common-midpoint (CMP) data recorded using an ideal point source-receiver antenna system; since the propagation effects for such a system are well described by closed-form expressions, once the conductivity and permittivity are known these effects can be eliminated from the data.
- The actual wavelets transmitted from a finite-length source antenna are referred to here as the emitted wavelets. They are complicated functions of the initial wavelet and the size and geometry of the source antenna (e.g. they include the phase and amplitude effects associated with complex-shaped radiation patterns).
- The effective wavelet represents the finite-length characteristics of a standard source-receiver antenna system, such that this relatively complex system can be described by the response of a much simpler point source-receiver antenna system suitably combined with the effective wavelet. To determine the effective wavelet, it is necessary to remove the effects of wavefield propagation from CMP data acquired with the standard source-receiver antenna system. This can be achieved to a good approximation by using closed-form expressions for a point source-receiver antenna system, together with

knowledge of the near-surface conductivity and permittivity.

This idea is tested on numerical data of point source and receiver antennas to investigate robustness. Finally, it is tested on numerical data of finite-length antennas and on physical data.

THE FORWARD SOURCE PROBLEM

In general, the electric field due to a finite antenna can be represented mathematically as the sum of elemental current sources:

$$\hat{E}_k(\mathbf{x}^R, f) = \int_{\mathbf{x}^S \in \mathbb{D}^S} \hat{G}_{kl}(\mathbf{x}^R - \mathbf{x}^S, f, \varepsilon_r, \sigma) \hat{J}_l(\mathbf{x}^S, f) d\mathbf{x}^S, \quad (1)$$

where $\hat{G}_{kl}(\mathbf{x}^R - \mathbf{x}^S, f, \varepsilon_r, \sigma)$ is the Green's tensor function, that describes wave propagation from a point source at location $\mathbf{x}^S$ to an arbitrary location $\mathbf{x}^R$. f is frequency, ε_r is relative permittivity, σ is electrical conductivity, $\mathbb{D}^S$ is the volume of the antenna, J_l is current and k and l are indices 1 to 3. For a broadside configuration where finite-length source and receiver antennas are oriented in the x_1 -direction (Figure 1), $k = l = 1$, $x_3^S = x_3^R = 0$. Eq. (1) can then be rewritten as

$$\hat{E}_1(0, x_2^R, 0, f) = \int_{x_1^S \in \mathbb{D}^S} \hat{G}_{11}(-x_1^S, x_2^R, 0, f, \varepsilon_r, \sigma) \times \hat{J}_1(x_1^S, 0, 0, f) dx_1^S, \quad (2)$$

where we assume that the transmitter pulse is the initial wavelet at the feedpoint in the middle of the antenna:

$$\hat{J}_1(0, 0, 0; f) = \hat{W}(f) \delta_{r_1}. \quad (3)$$

Using the principle of reciprocity, the signal measured at the receiver antenna can also be represented as a sum of elemental currents. For an x_1 -directed point source antenna located at the origin, Eq. (2) reduces to (omitting the subscripts)

$$\hat{E}(0, x_2^R, 0, f) = \hat{G}(0, x_2^R, 0, f, \varepsilon_r, \sigma) \hat{W}(f). \quad (4)$$

Figure 1: The source-receiver configuration where the height of both antennas is indicated by d .

THE AIR- AND GROUND-WAVES

Closed-form expressions may be derived for the air- and ground-waves that travel directly from the point source towards the point receiver antenna. For a point source present at the interface of a halfspace, $\hat{G}(\mathbf{x}, f, \varepsilon_r, \sigma)$ can be derived from Smith (1984):

$$\hat{G}(x_1, x_2, 0, f, \varepsilon_r, \sigma) = \frac{1}{2\pi} \times \int_{\kappa=0}^{\infty} \left(\frac{-\hat{\zeta}_0}{\tilde{\Gamma}_0 + \tilde{\Gamma}_1} + \frac{\partial_1^2}{\hat{\eta}_1 \tilde{\Gamma}_0 + \hat{\eta}_0 \tilde{\Gamma}_1} \right) J_0(\kappa \rho) \kappa d\kappa, \quad (5)$$

$$\zeta_0 = j\omega\mu_0, \quad \eta_0 = j\omega\varepsilon_0, \quad \eta_1 = \sigma_1 + j\omega\varepsilon_0\varepsilon_r,$$

$$\omega = 2\pi f, \quad \tilde{\Gamma}_s = \sqrt{k_1^2 + k_2^2 + \eta_s \zeta_0},$$

$$\rho = \sqrt{x_1^2 + x_2^2}, \quad \phi = \arctan(x_2/x_1),$$

$$\kappa = \sqrt{k_1^2 + k_2^2},$$

where j is the imaginary unit, κ is the wavenumber and J_0 is the zeroth order Bessel function. The subscript $s=0$ indicates properties of the air and $s=1$ indicates properties of the soil. For further analysis, two approximations are made:

$$1 \gg \frac{\sigma_1}{\omega\varepsilon_1}, \quad (6)$$

$$|\gamma_s x_2| \gg 1. \quad (7)$$

which are the high-frequency and large-offset approximations, respectively. Using the zeroth-order term of the Taylor expansion of the second-order derivative of the Bessel function J_0 in Eq. (5), yields

$$\hat{G}(0, x_2, 0, f, \varepsilon_r, \sigma) = \frac{Z_0}{2\pi(1 - \varepsilon_r)x_2^2} \left[\exp\left(-j\omega \frac{|x_2|}{c_0}\right) - \sqrt{\varepsilon_r} \exp\left(-j\omega \frac{|x_2|}{c_1}\right) \exp\left(-\frac{\sigma Z_1}{2}|x_2|\right) \right], \quad (8)$$

where $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$, $Z_1 = \frac{Z_0}{\sqrt{\varepsilon_r}}$, $c_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ and $c_1 = \frac{c_0}{\sqrt{\varepsilon_r}}$.

With the aid of Eq. (8), Eq. (4) can be rewritten as

$$\hat{E}(0, x_2^R, 0, f) = \hat{E}^0(x_2^R; f) + \hat{E}^1(x_2^R; f), \quad (9)$$

in which the air-wave $\hat{E}_1^0$ and ground-wave $\hat{E}_1^1$ are represented by

$$\hat{E}^0(x_2^R; f) = \frac{Z_0 \hat{W}(f)}{2\pi(1 - \varepsilon_r)|x_2^R|^2} \exp\left(-j\omega \frac{|x_2^R|}{c_0}\right), \quad (10)$$

$$\hat{E}_1^1(x_2^R; f) = \frac{-\sqrt{\varepsilon_r} Z_0 \hat{W}(f)}{2\pi(1 - \varepsilon_r)|x_2^R|^2} \exp\left(-j\omega \frac{|x_2^R|}{c_1} - \frac{\sigma Z_1}{2}|x_2^R|\right). \quad (11)$$

Transformation to the time domain yields,

$$E^0(x_2^R; t) = \frac{Z_0}{2\pi(1 - \varepsilon_r)|x_2^R|^2} W\left(t - \frac{|x_2^R|}{c_0}\right), \quad (12)$$

$$E^1(x_2^R; t) = \frac{-\sqrt{\varepsilon_r} Z_0 \exp(-\frac{\sigma Z_1}{2}|x_2^R|)}{2\pi(1 - \varepsilon_r)|x_2^R|^2} W\left(t - \frac{|x_2^R|}{c_1}\right). \quad (13)$$

Several properties of the air- and ground-waves are worth highlighting:

- Both spread geometrically as $|x_2^R|^{-2}$. Although both waves may be regarded as lossy interface waves, they are better described as spherical waves that travel along the air/ground interface.
- The ground-wave has an attenuation factor of $\exp[-\sigma Z_1 |x_2^R|/2]$, that depends on the near-surface electric permittivity and conductivity. Once the velocity of the ground-wave is determined, this amplitude decay relationship can be used in the time and frequency domains to obtain estimates of conductivity.
- The shapes of the wavelets traveling through the air and ground along the interface are identical to that of the initial source wavelet $W(t)$, except the ground wavelet has a 180° polarity reversal.
- The permittivity of the ground may be estimated from the travel-time versus offset curve of the ground-wave.

ESTIMATION OF GROUND PROPERTIES

Permittivity estimation

The permittivity of the ground is most easily determined from the velocity of the ground-wave.

Conductivity estimation

After compensating for geometrical spreading the amplitudes of the ground-wave peaks and troughs should decay as $\exp(-\sigma Z|x_2|/2)$. Fitting an exponential function to the compensated amplitudes results in:

$$|x_2^R|^2 E(x_2^R; \{t_{min}, t_{max}\}) = A \exp(-B_{\{t_{min}, t_{max}\}} |x_2^R|), \quad (14)$$

where t_{min} and t_{max} refer to the picked time ground-wave peaks and troughs, so that

$$\sigma_{\{min, max\}} = \frac{2B_{\{t_{min}, t_{max}\}}}{Z}. \quad (15)$$

A minimum offset must be introduced, such that the air- and ground-waves are clearly separated and the large offset approximation is valid.

An estimate of conductivity can also be carried out in the frequency domain for each frequency component. First, the ground-wave delineated in the time domain is tapered with a Hanning window. After compensating for geometrical spreading, Fourier transformation of the windowed ground-wave is carried out. Fitting an exponential function for each separate frequency yields,

$$|x_2^R|^2 E(x_2^R; f_i) = A \exp(-B(f_i) |x_2^R|), \quad (16)$$

such that the conductivity for each frequency is

$$\sigma(f_i) = \frac{2B(f_i)}{Z}. \quad (17)$$

EFFECTIVE SOURCE WAVELET

We now approximate the electric field $\hat{E}(0, x_2^R, 0; f)$, measured by finite-length antennas by introducing,

$$\hat{E}(0, x_2^R, 0; f) = \hat{G}(0, x_2^R, 0, f, \varepsilon_r, \sigma) \hat{W}^{eff}(f, x_2^R), \quad (18)$$

where $\hat{G}(0, x_2^R, 0, f, \varepsilon_r, \sigma)$ is the Greens function for a point source and receiver antenna system (see Eq. (4)) and $\hat{W}^{eff}$ is defined as the effective wavelet. The Green's function can be estimated from the electric properties of the lower halfspace, ε_r and σ (See Eq. (5)). The influence of the finite length of the source and receiver antennas on the initial wavelet is then simply

$$\hat{W}^{eff}(x_2^R, f) = \frac{\hat{E}_1(0, x_2^R, 0; f)}{\hat{G}(0, x_2^R, 0, f, \varepsilon_r, \sigma) + \epsilon}. \quad (19)$$

Note that an effective wavelet can be determined for each location x_2^R . The quantity ϵ , which has a small value, has been introduced to accomodate zeros in the spectrum of the Green's function. In this way, the performance of finite-length source-receiver antenna system can be described using the effective wavelet and point source descriptions of the source and receiver antennas. Note, that Eq. (19) can be used to estimate the initial wavelet of a point dipole source-receiver antenna system.

NUMERICAL RESULTS

Synthetic common-midpoint data are computed for point and finite-length source-receiver antenna systems. From these data we show how properties of the ground may be extracted, and the effective wavelet determined. The initial wavelet sent to the antenna in our numerical calculations has a center frequency $f_c = 450$ MHz and a

400 MHz bandwidth. For the finite-length antenna, this wavelet represents the incident electric field at the feed-point. The antennas have a length of 0.25 m and a radius of $5 \cdot 10^{-4}$ m. For all numerical experiments, the model consists of two homogeneous halfspaces. The upper halfspace is a vacuum with $\varepsilon_r = 1$ and $\sigma = 0$ S/m and the lower halfspace has a relative permittivity $\varepsilon_r = 9$ and a conductivity $\sigma = 0.01$ S/m. All sources and receivers are located above the interface; $d = 5 \times 10^{-4}$ m for the dipole source-receiver antenna system and $d = 10^{-3}$ m for the linear antennas. Note, that expressions for the air- and ground-waves (Eqs. 12 and 13) are obtained for zero height. Therefore, special attention needs to be paid to the validity of these expressions.

Influence of antenna height

The influence of antenna height on the measured field was investigated for dipole and finite-length antennas. In general, increasing height resulted in lower amplitude ground-waves.

Time domain properties determination

Time domain analysis was carried out for different antenna heights to obtain estimates of permittivity (input $\varepsilon_r = 9$), conductivity (input $\sigma = 0.01$ S/m) and the amplitude ratio of air- to ground-wave (3). The results obtained for a point source and receiver antenna system with a minimum antenna offset of 0.7 m had errors of less than 5% for electric permittivity and 10% for conductivity, as long as the antenna height was less than 1 cm. For the finite-length antenna system, the errors were roughly twice those of the point antennas.

Frequency domain properties determination

Estimates of permittivity based on frequency-domain calculations are similar to those obtained from time-domain calculations. Figure 2 compares conductivity results as functions of frequency $\sigma(f)$ for the point and finite-length antenna systems. It shows that the conductivities determined from finite-length data differ by 20% from the actual value. Note, that the erroneous values for $f > 1$ GHz are due to the limited bandwidth of the source wavelet.

Figure 3 shows estimated conductivities for different heights of the point antennas. The similarity of the results suggest that conductivity estimates based on zero-height expressions for the ground-waves are reasonable approximations for antennas placed up to 1 cm above the ground.

Figure 2: Estimated conductivities for point and finite-length antennas.

Figure 3: Conductivities for different heights for point antennas.

Figure 4: Comparison point and finite-length antenna for $r = 0.8$ m.

Figure 4 compares radargrams for point and finite-length antenna systems with an antenna offset of 0.8 m. The first minimum of the air-wave and the first maximum of the ground-wave are similar for the two types of antenna. Other features are noticeable different. Furthermore, it is observed that the maximum amplitude shifts to a later time for the finite-length antennas, indicating a less minimum-phase wavelet.

Determination of the effective wavelet

Once the electric properties have been determined, the Green's functions and source wavelet may be estimated. For each receiver position, the effective wavelet is computed for point and finite-length antenna systems using Eq. (19). For both types of antenna, stable wavelets are derived. In Figure 5, the mean initial and effective source wavelets are depicted. They show that the average effective wavelet for the point antennas has more energy concentrated at the onset, whereas the effective wavelet for finite antennas has its energy concentrated at the center. This is consistent with the phase spectra, which show that the effective wavelet for point antennas is closer to minimum phase than the wavelet of finite antennas.

Figure 5: Properties of initial and effective source wavelets for point antennas and finite antennas, respectively.

EXPERIMENTAL DATA

Experimental common-midpoint data were obtained using pulseEKKO 1000 antennas with specified center frequencies $f_c = 450$ and 900 MHz. The actual center frequencies for these antennas are 400 and 600 MHz, respectively.

Figure 6: Common-midpoint measurement with 450 MHz antennas.

Figure 7: Common-midpoint measurement with 900 MHz antennas.

Figure 8: Conductivity result as function of frequency for 450 MHz and 900 MHz antennas.

Figure 9: Comparison between the mean air- and ground-waves for $f_c = 450$ MHz.

Figure 10: Comparison between the mean air- and ground-waves for $f_c = 900$ MHz.

The measured fields for $f_c = 450$ and 900 MHz are depicted in Figures 6 and 7. Permittivity results obtained in the frequency domain ($\epsilon_r = 10.7$ and 8.2 , for the 450 and 900 MHz antennas, respectively) are close to those estimated in the time domain; $\epsilon_r = 9.98$ and 11.96 for the 450 MHz antennas and 7.65 and 8.83 for the 900 MHz antennas, respectively (for the peak and trough, respectively). Although, theoretically the amplitude ratio of the ground- to air-wave for zero offset data is $\sqrt{\epsilon_r}$ (See Eq. (8)), the amplitude ratio of the air- to ground-wave is much larger. This is probably caused by the shielding, which reduces the energy emitted into the air.

The conductivity results for the 450 and 900 MHz antennas in the frequency domain are depicted in Figure 8. The conductivity values are reliable within the bandwidth of the used antennas; for the 450 MHz antennas between 200 and 600 MHz and for the 900 MHz antennas between 300 and 900 MHz. A representative effective wavelet could not be obtained. Therefore, the air- and the ground-waves were studied more thoroughly to give a possible explanation. The air- and ground-waves were separated,

aligned, normalised and summed to give mean air- and ground-waves depicted in Figures 9 and 10 for the 450 and 900 MHz antennas (the polarity of the ground-wave has been flipped). Clearly the air- and ground-waves do not resemble each other. Furthermore, for both antenna types the air-wave has an extra local maximum at the beginning of the pulse. A possible explanation for this phenomenon is the presence of shielding, which is not incorporated in the Green's function, resulting in an erroneous effective wavelet. A way to circumvent this problem is to use only the closed-form expression for the ground-wave (Eq. (11)) in Eq. (19) to determine an effective wavelet.

CONCLUSIONS

To determine an effective wavelet, the medium properties need to be determined from the ground-wave. The height of the antennas above the surface may have a large influence on the ground-wave amplitude, but the data can still be used, for fixed heights, as the medium parameters are obtained using relative and not absolute amplitudes. An effective wavelet has been obtained from numerical data for a finite-length antenna system. The effective wavelet of finite-length antenna systems is less minimum-phase than that of the initial wavelet. For real data, no representative effective wavelet could be obtained, which is probably caused by the lack of account for the shielding of the antennas. An analysis of the results for the shielded antennas showed that the air- and ground-waves have some minor differences, which is not described by the theory. A possible solution for this problem may be the use of only the ground wave to determine an effective wavelet.

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