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Time-delayed Feedback Control of Electron Beam Energy Using H_∞ -Optimal Fractional Delay

Amin Rezaeizadeh and Roy S. Smith

Abstract—In the two-bunch operation mode of the Swiss Free Electron Laser (SwissFEL) machine, two electron bunches, separated by 28 ns, are fired and accelerated every 10 milliseconds. The electron beam is passed through several accelerating cavities which are energized by high power Radio Frequency (RF) pulses. The RF voltage generates a specific time-dependant accelerating gradient profile which results in different energy gains for the two bunches. It is often required that the two electron beams experience the same accelerating gradient through the cavities. This can be achieved by shifting the RF pulse in time to change the relative energy of the two beams. However, in the digital RF system used here, the minimum shift is limited by the sampling time of the Digital-to-Analog Converters (DACs). In this paper, we develop a control scheme for the energy difference control which finely tunes the RF pulse timing by a fractional delay filter. The method has been successfully tested at the SwissFEL injector test facility.

Index Terms—Convex optimization, Time-delayed feedback control, Fractional delay filter, Free electron laser, Linear accelerator.

I. INTRODUCTION

THE SwissFEL project is currently being developed and constructed at Paul Scherrer Institute [1]. It provides a source of very bright and short X-ray pulses enabling scientific research in a wide range of domains, from fundamental research to applied science [2]. The SwissFEL linear accelerator is composed of a series of accelerating structures placed along the line of an electron beam. Each accelerating module is energized by a radio frequency (RF) station that delivers high power RF, within a short pulse, to the structures to form a large accelerating gradient inside the cavities. The SwissFEL machine works in the following way: Every 10 milliseconds, an electron bunch is injected and accelerated to the energy of 5.8 GeV. The electron beam is then passed through several magnet arrays, so called “undulators”, to be transversally accelerated to produce an X-ray pulse. In the two-bunch operation mode, two electron bunches, separated by 28 ns, are accelerated through the machine. It is often required that the two bunches have the same energy gain as they pass through the accelerating structures. However, the energy gain profile of structures exhibits a peak with respect to time (see Fig. 1). Therefore, one approach to achieve this goal is to precisely control the RF pulse timing with respect to the

Fig. 1. The simulated energy gain profile inside the accelerating cavities for the peak power of 60 MW at the pulse compressor output. In the two-bunch operation mode of the SwissFEL, two electron bunches, with the bunch spacing of 28 ns, are injected and accelerated within one RF pulse. Ideally the energy gain should be the same for the two bunches. This can be achieved by precise tuning the RF pulse time with respect to the beam, so that the energy gain curve will be moved back and forth until the energy gains are balanced.

bunches so that the two beam pulses occur on opposite sides of the peak but with the same energy gain. This is the concept of time delay (or lag) control of the RF input waveforms [3]. The amount of the delay (or lag) is given by an integral controller which tries to keep the energy difference at zero. Therefore, the control system uses the measured energy error and acts on the timing of the input voltage.

Since the RF control system is digitally implemented, the minimum shift in time is limited by the sampling time of the DACs. This problem can be overcome by applying a fractional delay filter to the RF pulse. The digital fractional delay filter approximates an ideal continuous delay filter which can take any delay parameter, i.e. not necessarily an integer multiple of the sampling time [4]. In this work, the fractional delay filter is designed in the frequency domain using an H_∞ optimality criterion [5]. In this scheme, the filter coefficients are determined through a convex optimization problem to best approximate the ideal filter response. The designed filter is then applied to the RF pulse to fine tune the accelerating gradient profile inside the cavities.

To the best of our knowledge, this is the first time that such a control method has been proposed and experimentally tested on accelerating machines. The idea of the fractional delay filter design is taken from [5] with some modifications in the optimization formulation. An integral feedback loop is introduced to remove the error offset. The control method is successfully implemented on a high power RF system and the results are presented.

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Fig. 2. The configuration of an RF station in the SwissFEL beamline.

The rest of the paper is organized as follows. Section II describes an RF station system. The accelerating structure model is discussed in Section III. Section IV develops a fractional delay filter based on convex optimization. The experimental results are given in Section V, and Section VI concludes the paper with an overview of the future work.

II. RF SYSTEM DESCRIPTION

The SwissFEL C-band (5.712 GHz) RF station layout is depicted in Fig. 2. The discrete sequences of the reference amplitude and phase waveforms, each 2048 samples long, are converted to in-phase (I) and quadrature (Q) components. The sampling time of the DACs and ADCs is 4.2 ns. The analogue I and Q waveforms are used to modulate the amplitude and phase of the RF C-band signal source generated by the master oscillator. The RF power is then amplified by the preamplifier and klystron to up to approximately 40 MW. This high power is delivered in a very short amount of time, approximately 3 μ s, which implies that the measured discrete amplitude sequence is a pulse with a length of approximately 700 samples. This RF pulse is repeated at the rate of 100 Hz. More details about the C-band RF design can be found in [6].

In C-band stations, an RF pulse compressor, labeled BOC in Fig. 2, is placed after the klystron. The pulse compressor is a passive RF component that can store and release the RF energy thereby changing the RF pulse shape. Flipping the RF phase by 180° at the right time results in a higher peak power at the BOC output than of the klystron output and this is the main purpose of using the pulse compressors [7], [8]. The generated high power RF is transmitted through waveguides to the accelerating structures to build up a strong electromagnetic waves inside the cavities. The SwissFEL pulse compressor is designed based on a single Barrel Open Cavity (BOC) which has a high quality factor with a relatively long filling time [9].

Figure 3 illustrates typical RF amplitude and phase discrete reference waveforms. The input amplitude is constant over the pulse length of approximately 3 μ s. The phase is reversed by 180° at time t_0 , which gives a large peak at the BOC output amplitude followed by a steep decay. This large peak in the voltage signal results in a peak in the energy gain profile (see Fig. 1). In order to achieve the same energy gain for the two bunches, we propose a control method to precisely shift the RF pulse in time. The alternative way is to shape the RF pulse so that the voltage peak is lowered and the pulse is flattened [10]. However, this approach suffers from lower energy gain—as the

Fig. 3. Typical input RF amplitude and phase waveforms to the DAC (in blue) and the corresponding waveforms measured at the BOC output (in red). The sampling time of the DAC and ADC is $T_s = 4.2$ ns. At $t = t_0$, the input phase is flipped by 180 degrees which causes the BOC to release the stored energy and therefore generates a high peak in the output amplitude.

voltage peak is lowered. Hence, in this paper, we proceed with adjusting the RF pulse timing instead.

III. ACCELERATING STRUCTURE SYSTEM

The accelerating structure is composed of several cell cavities stacked together. An RF cavity is a metallic chamber, usually made of copper, designed with a specific geometry to make the electromagnetic waves resonate inside the cavity. In traveling-wave accelerating structures, particles are in phase with the RF wave as they pass through the cavities. This is because the phase velocity of the RF wave is tuned to the velocity of the accelerating particles which is close to the speed of light for electrons [11]. However, the RF power propagates along the structure with a group velocity which determines the filling time of the structure. The RF power reaches at the cell number j with a delay time of D_j , i.e.,

$$P_j(t) = V^2(t - D_j), \quad (1)$$

where V denotes the RF voltage amplitude. Furthermore, we assume that the RF phase is zero, or so called the “on-crest” phase.

The delay, D_j , is determined as follows,

$$D_j = \sum_{i=1}^j d_i, \quad d_i = \frac{l_{cell}}{v_{g_i}}, \quad (2)$$

with v_{g_i} denoting the group velocity of cell number i , and l_{cell} denoting the cell length which is equal for all cells [12].

Hence, the power at cell j is given by

$$P_j = \prod_{i=1}^j \left(1 - \frac{2\pi f l_{cell}}{v_{g_i} Q_i}\right) V^2(t - D_j), \quad (3)$$

where Q_i is the quality factor of cell i and f is the RF frequency.

Fig. 4. The frequency response of the accelerating gradient transfer function $H(s)$ (normalized).

The corresponding electric field at each cell, with resistance R_j , is calculated as follows,

$$E_{zj} = \sqrt{\frac{2\pi f(R_j/Q_j)}{v_{gj}}} P_j. \quad (4)$$

Thus, the energy gain of the structure, including N cells, is simply the summation over all cells,

$$E = l_{\text{cell}} \sum_{i=1}^N E_{zi}. \quad (5)$$

Taking the Laplace transform of (5), gives the following transfer function from the input voltage to the total gradient,

$$H(s) = \frac{E(s)}{V(s)} = \sum_{j=1}^N \gamma_j e^{-D_j s}, \quad (6)$$

where γ_j 's are constant coefficients. Figure 4 illustrates the normalized frequency response of the transfer function $H(s)$ for a C-band accelerating module at the SwissFEL facility [13], [14].

Since the total accelerating gradient, E , is not directly measured, we can apply the H transfer function to the BOC output voltage to estimate the gradient of the cavities:

$$E(s) = H(s) \mathcal{L}_t(v_{\text{BOC}}(t)), \quad (7)$$

where $\mathcal{L}_t$ denotes the Laplace operator, and $v_{\text{BOC}}(t)$ is the BOC output voltage (envelope), calculated by

$$v_{\text{BOC}}(t) = a(t) \cos \varphi(t), \quad (8)$$

where $a(t)$ and $\varphi(t)$ are, respectively, the amplitude and phase waveforms of the RF signal (see Fig. 3).

Figure 1 plots the simulated energy gain profile of the accelerating structure to 60 MW BOC output power. The RF timing and the beam arrival time are synchronized, however, the RF pulse shape can be shifted in time with respect to the beam. Since the sampling time of the DAC is $T_s = 4.2$ ns, which is comparable to the bunch spacing of 28 ns, we need to tune the RF delay well below the sampling time. This leads us to the fractional delay filter design that is a digital approximation of a continuous time delay filter.

IV. FRACTIONAL DELAY FILTER DESIGN

A fractional delay filter is a digital filter designed to delay a discrete signal by a fraction of a unit delay. To achieve this function, the filter must fulfill two main specifications in frequency domain. The filter magnitude should be unitary over a wide frequency range (all-pass behavior), and the phase response should be linear with a fixed slope through the bandwidth [4].

There are several algorithms in the literature for designing such a filter. Some approaches are based on the frequency domain while others are studied in the time domain. Regardless of what method is employed, the problem reduces to finding the impulse response coefficients of the filter, either in FIR or IIR form. In some related work, the filter coefficients are derived in closed form expressions [15], and in some methods the coefficients are determined through an optimization problem [16]. In the current work, the fractional delay filter is designed in the frequency domain, based on a convex optimization problem in which minimizes the maximum norm of the complex error [5].

The transfer function of an all-pass filter with constant group delay of $\tau \in \mathbb{R}$ is

$$H_d(j\Omega) = e^{-j\Omega\tau}. \quad (9)$$

The objective is to design the discrete filter, $H(j\Omega)$, such that its frequency response best approximates the desired response $H_d(j\Omega)$. Here, we choose the ℓ_∞ norm on $\mathbb{C}^L$, which is defined by $\|x\|_\infty := \max_{i=1}^L |x_i|$, to express the discrepancy between $H(j\Omega)$ and $H_d(j\Omega)$ at the frequency samples. The objective will be to minimize the worst case norm of the complex error over the frequency range. In other words, the filter design is based on the following optimization problem:

$$\text{minimize}_{H(j\Omega)} \max_{\Omega \in [0, \Omega_{\max}]} |H(j\Omega) - H_d(j\Omega)| \quad (10)$$

where $H(j\Omega)$ is the frequency response of the approximated fractional delay filter with N coefficients and defined as

$$H(j\Omega) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} h(n) e^{-j\Omega n} = \left[e^{j\Omega \frac{N}{2}}, e^{-j\Omega(1-\frac{N}{2})}, \dots, e^{-j\Omega(\frac{N}{2}-1)} \right] h, \quad (11)$$

where $h = [h(-\frac{N}{2}), \dots, h(\frac{N}{2}-1)]^T$ is the impulse response of the filter. Since the filter is applied offline to the stored RF pulse waveform, it is not required to be causal.

Any time delay $T_d \in \mathbb{R}$ can be expressed as

$$T_d = [T_d] + \tau, \quad (12)$$

where $[\cdot]$ denotes the nearest integer function.

Therefore, the fractional delay filter is designed only for delays (or advances) between $\tau = -\frac{1}{2}$ and $\tau = \frac{1}{2}$, and the integer part of the delay can be simply implemented by a discrete delay function of $z^{-[T_d]}$.

The filter designed according to the optimization problem (10) cannot fulfill the desired frequency response over the whole frequency range of $[0, \pi]$. In fact, there will be an inevitable error at the Nyquist frequency which makes the

designed filter behave differently at high frequencies [17]. Hence, we limit the design frequency band to $\Omega_{\max} < \pi$. To evaluate the optimality criterion defined in (10), we take L linearly spaced samples of the frequency response in the frequency range of $[0, \pi)$, with L_1 samples falling in the sub-range $[0, \Omega_{\max}]$, and $L_2 = L - L_1$ samples in the remaining high frequency zone.

The following optimization (previously proposed in [5]) minimizes the worst case norm of the complex error:

$$\begin{aligned} & \underset{(\delta, h)}{\text{minimize}} \quad \delta \\ & \text{subject to} \quad |a_i^T h - H_d(j\Omega_i)| < \delta, \quad i = 1, \dots, L_1, \end{aligned} \quad (13)$$

where a_i^T is defined as,

$$a_i^T = \left[e^{j\Omega_i \frac{N}{2}}, e^{-j\Omega_i(1 - \frac{N}{2})}, \dots, e^{-j\Omega_i(\frac{N}{2} - 1)} \right]. \quad (14)$$

We add two more constraints to restrict the magnitude and phase frequency response in high frequency zone of $\Omega > \Omega_{\max}$. The magnitude should be less than one, and the group delay should remain constant over the whole frequency range. This leads to the following optimization problem:

$$\begin{aligned} & \underset{\delta, \eta \in \mathbb{R}, h}{\text{minimize}} \quad \delta + w\eta \\ & \text{subject to} \quad \|a_i^T h\|_2^2 < 1, \quad i = L_1 + 1, \dots, L. \end{aligned} \quad (15a)$$

$$|\text{Imag}(a_i^T h) + \tan(\Omega_i \tau) \text{Real}(a_i^T h)| < \eta, \quad (15b)$$

$$|a_i^T h - H_d(j\Omega_i)| < \delta, \quad i = 1, \dots, L_1, \quad (15c)$$

where w is a constant weight.

The constraints (15b) and (15a) are both applied in the frequency range of $\Omega > \Omega_{\max}$, i.e., for Ω_i 's where $i > L_1$. The linear inequality (15b) imposes a constraint on the high frequency phase response to have the same group delay of τ as in the low frequencies. The constraint (15a) is introduced to avoid high-pass behavior and suppress high frequency components.

Figure 5 illustrates the frequency response of the designed fractional delay filter for the delay of $\tau = 0.25$ sample. The filter parameters are given in Table I. As we can see, the magnitude remains unitary over a wide range of frequency which demonstrates the all-pass behavior of the filter. The bandwidth is approximately 2.6 rad/s. The filter group delay also remains constant at 0.25 sample over the design frequency range. Figure 6 shows the impulse response of the fractional delay filter. As already stated, the filter is chosen to be non-causal since it only applies offline on the stored amplitude and phase waveforms.

V. DUAL BUNCH ENERGY CONTROL EXPERIMENTAL RESULTS

The amount of the delay to be applied to the amplitude and phase waveforms is determined by a feedback system which is aimed at keeping the energy difference at zero. Figure 7 schematically shows the experimental setup of the dual bunch control at the SwissFEL injector test facility. The energy of the two electron bunches are subtracted to give the error signal which is then used by the controller to provide the group

Fig. 5. The frequency response of the designed filter, $H(j\Omega)$ for the group delay of one quarter a sample, i.e. $\tau = 0.25$. The bandwidth is approximately 2.6 rad/s. The filter parameters are given in Table I.

Fig. 6. The impulse response of the fractional delay filter for the group delay of $\tau = 0.25$ sample. The filter is non-causal as it is generally designed for group delays of $-0.5 < \tau < 0.5$.

TABLE I
DESIGN PARAMETERS

Parameter	Value
τ	+0.25 sample
$\Omega_{\max}$	0.7π
N	30
L	2000
L_1	1400
L_2	600
w	0.02

Fig. 7. The schematic of the dual bunch energy control at the SwissFEL test facility. The two beam energies are measured and subtracted to give the error which is then used in the optimization problem to design the filter coefficients. The designed filter is applied on the reference amplitude and phase waveforms (each contains 1024 samples). The waveforms are then converted to I and Q signals.

delay for the convex optimization problem. The group delay is updated according to an integral feedback control:

$$\tau^+ = \tau + Ge, \quad (16)$$

where G is a constant controller gain, and where e is the energy error calculated as

$$e = \varepsilon_2 - \varepsilon_1, \quad (17)$$

with ε_j denoting the energy of bunch j .

Once τ is updated, the convex optimization is solved to determine the coefficients of the fractional delay filter. The filter is then separately applied on the amplitude and phase discrete reference waveforms to generate the input waveforms for the DAC. This process is repeated for several iterations until the error of the energy difference is below a predefined tolerance. It is important to mention that the filter is applied on the constant reference waveforms and not on the last modified waveforms, since in the latter case, the RF pulse shape becomes more deformed in every iteration.

The communication with the machine is network-based and through EPICS (Experimental Physics and Industrial Control System) process variables, and since this communication is not in real-time, the CVX Matlab Toolbox [18], [19] can be used to solve the optimization problem.

The experimental results are illustrated in Fig. 8. The top plot shows the energy of the two bunches as the iteration proceeds. Initially, the electron bunches are fired before the energy gain reaches the peak, i.e. on the left side of the gain profile depicted in Fig. 1. The two beams are initially different in energy by 1.5 MeV, with zero sample delay in the RF waveforms. The middle plot shows the error signal (energy difference). In order to achieve the same energy, the gain profile is shifted to the left by applying a non-causal fractional delay filter to the RF waveform. Therefore, as the gain profile moves to the left, the second bunch experiences the maximum energy gain, and this can be seen from the top plot in Fig. 8. As the iteration proceeds, the energy difference is brought to zero and the total delay converges to approximately -25.3 samples (see the bottom plot in Fig. 8). With this lag in time, the two electron bunches—separated by 28 ns—are placed on the two opposite sides of the energy profile and thus

Fig. 8. Closed loop control of the two bunches energy with a fractional delay filter with a variable delay determined by the energy difference of the two beams. The top plot shows the energy of bunch 1 and 2 versus the iteration index. Initially the two bunches are on the left side (before the peak) of the gain profile depicted in Fig. 1. The middle and bottom plots show respectively the energy difference and the corresponding delay versus the number of iterations.

experience the same energy gain. The fractional part of the delay is handled by the fractional delay filter. The integrator gain, G , is lowered to reduce high frequency jitter and also to demonstrate the convergence more clearly. In this particular experiment, the control gain is $G = 2$.

Figure 9 shows the images of the two electron bunches captured at the spectrometer monitor. In this monitor, the horizontal axis is an indicator of energy. The right direction indicates higher energy side. The images are taken at three different iteration indices of 0th, 10th and 40th. As we can see, the two bunches converge to the same energy. Recalling Fig. 1, in this situation, the two bunches are fired on the two sides of the peak.

This is the first time that the proposed control method has been applied on this type of accelerating machine. With the use of a FD filter the shape of the input RF waveforms can be continuously modified and thus the energy difference can be precisely controlled. The alternative way is to flatten the energy profile—by lowering the peak—so that the bunches see the same energy. However, this approach comes at a cost of losing energy as the peak is lowered.

VI. CONCLUSION AND FUTURE WORK

In the two bunch operation mode of Swiss Free Electron Laser machine, two electron bunches, separated by 28 ns, are accelerated within one Radio Frequency (RF) pulse. The control objective is to precisely control the energy difference of the two bunches by manipulating the RF pulse. The sampling time of the RF waveforms is 4.2 ns, and since it

Fig. 9. The two electron beam images captured at the spectrometer camera at the end of the beamline. The horizontal axis, which displays the position of the beams, corresponds to the beam energy. The orange trace denotes the charge density. The two bunches are initially (the most left plot) different in energy. As the iteration proceeds, they move towards one another to achieve the same energy.

is comparable to the bunch spacing of 28 ns, the control objective is challenging. Due to the simple beam energy gain profile of Fig. 1, we studied the energy difference control by shifting the RF pulse in time. To have more precise control over the energy difference, a non-causal fractional delay filter is exploited. The filter coefficients are derived from a convex optimization problem which minimizes the worst case complex norm of the error of the designed filter and the ideal fractional delay filter in the frequency domain. In this work, the delay value is provided by an integral controller which uses the energy difference of the beams as the input. The designed filter is then applied to the RF pulse waveforms and the new beam energies are measured. This process is repeated until the energy difference converges to a value below a tolerance.

The experimental results at the SwissFEL test facility have proved the concept of dual bunch energy control based on delaying the RF pulse with a fractional delay filter. As a second approach to controlling the energy difference, it is planned to develop and test a control scheme in which the RF pulse is not shifted in time, but instead, the waveform shape is modified to generate different RF amplitude and phase for each beam.

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