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Manufacturing and Morphing Behavior of High-Amplitude Corrugated Laminates

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Extreme anisotropy of high-amplitude corrugated laminates

Arguments for use of corrugated laminates as flexible skin in morphing-wing design

- High in-plane stiffness A_{11} contributes to wing-skin span-wise stiffness
- Low in-plane stiffness A_{22} allows for large chord-wise stretch
- Very high bending stiffness D_{11} adds the function of local stringer

Viable method for realizing high-amplitude corrugated circular-sections-shape laminates

- Because of undercuts it is difficult to imagine that use of molds could lead to economic manufacturing process
- Forming of laminates by thermal effects has already been pointed out in the literature [3,4]
- The suggestion is applied to manufacture of corrugated laminates

Circular-segment corrugation shape conventions

Periodic length P and corrugation amplitude c

determine curvature radius R :

$$R = \frac{16c^2 + P^2}{32c}$$

Dimensionless geometry values:

$$\bar{c} = \frac{c}{P}, \quad \bar{t} = \frac{t}{P}, \quad \bar{R} = \frac{R}{P}, \quad \bar{P} = 1$$

Characteristic opening angle: $\psi_0 = \arccos\left(1 - \frac{c}{R}\right)$

Dimensionless curved length: $\bar{L}_s = 4\psi_0 \bar{R}$

Values range of shape characteristic:

$$0 \leq \bar{c} \leq \frac{1 - \bar{t}}{2} + \frac{1}{2} \sqrt{(1 - \bar{t})^2 - \frac{1}{4}}$$

Manufacturing and morphing-performance arguments for high-amplitude shape

Shape curvature is smaller for highest than for intermediate amplitudes:

Stretch values by unbending action increase significantly with corrugation amplitude:

Recall of classical theory of laminated plates

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix}^{equ} = \sum_{k=1}^N \bar{\mathbf{Q}}_k (\bar{\boldsymbol{\alpha}}_k \Delta T + \bar{\boldsymbol{\beta}}_k \Delta H) \begin{Bmatrix} (z_k - z_{k-1}) \\ \frac{1}{2}(z_k^2 - z_{k-1}^2) \end{Bmatrix}$$

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix}^{fict} = \begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix}^{equ} + \begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix}^{mec} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa} \end{Bmatrix}$$

Realistic reduced system for estimating thermal laminate curvature under constraint of **cylindrical bending**:

cross-ply

$$\begin{array}{lcl} \varepsilon_x^0 & \neq & 0 \\ \varepsilon_y^0 & \neq & 0 \\ \gamma_{xy}^0 & = & 0 \\ \kappa_{xx} & = & 0 \\ \kappa_{yy} & \neq & 0 \\ \text{cross-ply } \kappa_{xy} & = & 0 \end{array} \longrightarrow \begin{Bmatrix} N_x \\ N_y \\ M_x \end{Bmatrix}^{fict} = \begin{bmatrix} A_{11} & A_{12} & B_{12} \\ A_{12} & A_{22} & B_{22} \\ B_{12} & B_{22} & D_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \kappa_y \end{Bmatrix}$$

Material data and thermally induced curvatures for cylindrical bending

Table 1: Relevant material properties of unidirectional composites with fiber-volume fraction $v_f = 0.6$. Source: DORNIER SYSTEM GmbH

fiber	E_1	E_2	ν_{12}	α_1	α_2
	[MPa]		[-]	$[10^{-6}/K]$	
T 300	135000	10000	0.27	-0.6	30
M 40	220000	7000	0.35	-0.8	30
GY 70	290000	5000	0.41	-1.0	30

Conclusion:

Material system with moderately stiff fibers provides highest thermal curvatures in $[0/90_2]$ or $[90_2/0]$ laminates

Table 2: Temperature-curvature coefficients $\kappa_y^T [10^{-4}/mmK]$

fiber	[90/0]	[90/0 ₂]	[90/0 ₃]	[90/0 ₄]
T 300	1.0233	1.0863	0.8863	0.7149
M 40	0.6348	0.9081	0.8395	0.7148
GY 70	0.4066	0.7214	0.7571	0.6848

Manufacturing: Prototype

Conclusion and outlook manufacturing part:

- circular-sections corrugated laminates can be realized in good quality and moderate manufacturing effort by using
 - unsymmetric cross-ply laminates
 - elevated curing temperatures
- however, laminate-thickness to curvature-radius ratios remain small
- More details on mathematical estimate of curvature radius and other aspects are found in [5]
- **Morphing models must take into account unsymmetric laminate design !**

Conclusion and outlook manufacturing part: Aerodynamically smooth surface

Nonlinear morphing model: Geometric and mechanical unit cells

Geometric periodic unit cell:

- statically determinate support
- periodicity conditions

Mechanical unit cell:

existing symmetries justify quarter unit-cell model

- simply supported at origin
- rotation: homogeneous EBC
- displacement: finite values specified

Nonlinear morphing model: Coordinate transformation and reference configuration

$$\begin{aligned} y^{(0)}(\xi_2) &= R \sin \Delta\psi \\ z^{(0)}(\xi_2) &= R(1 - \cos \Delta\psi) - c \end{aligned}$$

$$\Delta\psi = \psi - \psi_0 = \frac{\xi_2}{R} - \psi_0 = \kappa^0 \xi_2 - \psi_0$$

coordinate transformation

$$\begin{Bmatrix} x \\ \xi_2 \\ \xi_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

Nonlinear morphing model: Cylindrical bending, constitutive equations, external equilibrium

reference configuration curvature : κ bending curvature : ε^1

$$\begin{aligned}
 N_x &= A_{12} \varepsilon_2^0 \\
 N_2 &= A_{22} \varepsilon_2^0 + B_{22} \varepsilon_2^1 \\
 M_x &= D_{12} \varepsilon_2^1 \\
 M_2 &= B_{22} \varepsilon_2^0 + D_{22} \varepsilon_2^1
 \end{aligned}$$

$$\begin{Bmatrix} N \cos \theta \\ N z \end{Bmatrix} = \begin{bmatrix} A_{22} & B_{22} \\ B_{22} & D_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_2^0 \\ \varepsilon_2^1 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_2^0 \\ \varepsilon_2^1 \end{Bmatrix} = \frac{1}{\det} \begin{bmatrix} D_{22} & -B_{22} \\ -B_{22} & A_{22} \end{bmatrix} \begin{Bmatrix} N \cos \theta \\ N z \end{Bmatrix}; \quad \det = A_{22} D_{22} - B_{22}^2$$

Nonlinear morphing model: Deformed configuration

Total curvature is the sum of
 shape and bending curvatures:

$$\kappa^{(t)}(\xi_2) = \boxed{\kappa^{(0)}} + \boxed{\varepsilon_2^1(\xi_2)}$$

Cross-sectional **rotation** follows from
 integrating the total curvature:

$$\theta^{(t)}(\xi_2) = \int_0^{\xi_2} \kappa^{(t)}(\sigma) d\sigma + C_1$$

Stretch of line elements [7]:

$$d\xi_2^{(t)} = \lambda d\xi_2^{(0)} = (1 + \varepsilon_2^0) d\xi_2^{(0)}$$

Incremental displacements due to stretch [7]:

$$du_y^0 = \cos(\theta) \varepsilon_2^0 d\xi_2^{(0)}; \quad du_z^0 = -\sin(\theta) \varepsilon_2^0 d\xi_2^{(0)}$$

Incremental displacements due to stretch:

$$du_y = du_y^0 + du_y^1; \quad du_z = du_z^0 + du_z^1$$

Deformed configuration:

$$\begin{aligned} y^{(t)} &= \int_0^{\xi_2} \cos(\theta) \lambda d\sigma & z^{(t)} &= -\int_0^{\xi_2} \sin(\theta) \lambda d\sigma \\ y_B^{(t)} &= \int_0^{L_s} \cos(\theta) \lambda d\sigma & z_B^{(t)} &= -\int_0^{L_s} \sin(\theta) \lambda d\sigma \end{aligned}$$

Nonlinear morphing model: Solution algorithm

It is desired to specify global morphing stretch

→ outer process

However, deformed configurations depend most directly on applied line force N

→ inner process

Loop on load steps creates data points for force-displacement diagrams

Outer process then finds force corresponding with specified maximum stretch

Nonlinear morphing model: Solution algorithm inner process

Minimize scalar error measure f :

$$f = \int_0^{L_s} \begin{Bmatrix} \Delta N_2 \\ \Delta M_2 \end{Bmatrix}^T \begin{Bmatrix} \Delta N_2 \\ \Delta M_2 \end{Bmatrix} ds$$

$$\begin{Bmatrix} \Delta N_2 \\ \Delta M_2 \end{Bmatrix} = \begin{Bmatrix} \Delta N_2^{\text{int}} \\ \Delta M_2^{\text{int}} \end{Bmatrix} - \begin{Bmatrix} \Delta N_2^{\text{ext}} \\ \Delta M_2^{\text{ext}} \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta N_2^{\text{int}} \\ \Delta M_2^{\text{int}} \end{Bmatrix} = \begin{bmatrix} A_{22} & B_{22} \\ B_{22} & D_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_2^0 \\ \varepsilon_2^1 \end{Bmatrix}$$

Loads from external equilibrium

Numerical version:

$$f = \sum_{k=1}^{n+1} f_k = \sum_{k=1}^{n+1} \begin{Bmatrix} \Delta N_2 \\ \Delta M_2 \end{Bmatrix}_k^T \begin{Bmatrix} \Delta N_2 \\ \Delta M_2 \end{Bmatrix}_k$$

The laminate deformations ε_{2k}^0 and ε_{2k}^1 control the deformed configuration and are solution parameters.

Error minimization after Newton optimization method:

$$\begin{aligned} \boldsymbol{\varepsilon}_k^{(t+1)} &= \boldsymbol{\varepsilon}_k^{(t)} + \Delta \boldsymbol{\varepsilon}_k^{(t)} & \boldsymbol{\varepsilon}_k^{(t)} &= \begin{Bmatrix} \varepsilon_2^0 \\ \varepsilon_2^1 \end{Bmatrix}_k^{(t)} & \Delta \boldsymbol{\varepsilon}_k^{(t)} &= -(\nabla \nabla f)^{-1} \nabla f_k^{(t)} \\ \nabla f_k^{(t)} &= 2 \left(\begin{Bmatrix} \Delta N_2 \\ \Delta M_2 \end{Bmatrix}_k^{(t)} \right)^T \begin{bmatrix} A_{22} & B_{22} \\ B_{22} & D_{22} \end{bmatrix} & \nabla \nabla f_k^{(t)} &= 2 \begin{bmatrix} A_{22} & B_{22} \\ B_{22} & D_{22} \end{bmatrix} \begin{bmatrix} A_{22} & B_{22} \\ B_{22} & D_{22} \end{bmatrix} ds \end{aligned}$$

Nonlinear morphing model: Solution algorithm inner process

Newton-method Up-date rule:

$$\boldsymbol{\varepsilon}_k^{(t+1)} = -\frac{1}{\det \begin{bmatrix} D_{22} & -B_{22} \\ -B_{22} & A_{22} \end{bmatrix}} \begin{Bmatrix} \Delta N_2^{ext} \\ \Delta M_2^{ext} \end{Bmatrix}_k^t$$

converges quickly for moderate stretches
however diverges at high stretches.

Divergence is caused by
the objective function deviating too much
from the quadratic polynomial
estimated with Taylor-series expansion
at reference point

Retain convergence over full stretch
range with **modified Newton method**,
which combines minimum-point
estimate with line search.

Nonlinear morphing model: Material and laminates

Table 1: Relevant material properties of unidirectional T300/epoxy with fiber-volume fraction $v_f = 0.6$. Source: DORNIER SYSTEM GmbH

fiber	E_1	E_2	ν_{12}	α_1	α_2
	[MPa]		[-]	$[10^{-6}/K]$	
T 300	135000	10000	0.27	-0.6	30

symmetric laminate: $[90_3]$, $t=1\text{mm}$

unsymmetric laminate: $[90/0_2]$, $t=1\text{mm}$

Nonlinear morphing model: Simulations

Very small amplitude reduction
at stretch $\lambda=2$

Laminate influence significant
at very high stretch only

$[90_3]$

$[90/0_2]$

Nonlinear morphing model: Verification of deformation result by comparison with FEM

$$\frac{z_B^{ana} - z_B^{FEM}}{z_B^{FEM}} \approx 0.7\%$$

$$\frac{u_z^{ana} - u_z^{FEM}}{u_z^{FEM}} \approx 0.07\%$$

Nonlinear morphing model: Influence of laminate thickness

Influence of laminate thickness

Influence of corrugation amplitude

Summary

- Manufacturing of high-amplitude circular-sections corrugated laminates:
 - is possible with exploiting thermal deformation effects in unsymmetric laminates at high curing temperatures
 - thickness-to-curvature-radius ratios tend to be small
 - higher ratios could be obtained with matrix systems consolidating at high temperature (PEEK: 340° C)
- The moldless manufacturing method naturally produces circular-sections corrugation shapes
- An analytical nonlinear morphing model has been explained.
 - term «analytic» is justified with analytic form of gradient and Hesse matrix in the Newton iteration method
 - Its simplicity relies on linear relation between line loads and local laminate deformations
- Results have been shown and large-deformation results have been verified by comparison with FEM

Conclusion and outlook

- Morphing model functions up to very high stretches
- Morphing simulations show that stringer function is retained up to stretches of $\lambda=2$ (100% strain)
- In practical application the thickness-to-radius ratio should be higher than obtained with the demonstrator

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End of presentation

Thank you for your attention – Questions ?