Erratum: Fast simulation of Gaussian random fields[Monte Carlo Methods Appl. 17 (2011), 195–214]

Journal Article**Author(s):**

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Publication date:

2013

Permanent link:

<https://doi.org/10.3929/ethz-b-000422894>

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Originally published in:

Monte Carlo methods and applications 19(1), <https://doi.org/10.1515/mcma-2013-0003>

Erratum

Fast simulation of Gaussian random fields

[Monte Carlo Methods Appl. 17 (2011), 195–214]

Annika Lang and Jürgen Potthoff

Abstract. In the paper “Fast simulation of Gaussian random fields”, a typo occurred. Instead of $(p_{k_1 \dots k_d})_i = k_i / l_i$ it should read $(p_{k_1 \dots k_d})_i = (k_i - N_i/2)/l_i$ in Algorithm 3.1, Remark d). For convenience of the reader we reproduce below the complete corrected algorithms.

Keywords. Gaussian random fields, fast Fourier transform, simulation.

2010 Mathematics Subject Classification. 65C05, 65C50, 60G60, 60H40.

Algorithm 3.1.

Remarks

- a) The functions FFT and FFT^{-1} include all necessary rescaling depending on the used FFT algorithm and the integers N_i .
- b) A is a d -dimensional complex-valued array, B is real-valued.
- c) $x_{k_1 \dots k_d}$ denotes the grid point corresponding to the integers $(k_1, \dots, k_d)$. The grid points are distributed equidistantly in each direction, i.e., the distance of two arbitrary neighbor grid points in direction e_i is given by a constant Δx_i .
- d) The points $p_{k_1 \dots k_d}$ in the Fourier domain are given by

$$(p_{k_1 \dots k_d})_i = (k_i - N_i/2)/l_i \quad \text{for } i = 1, \dots, d.$$

Input

- a) D a d -dimensional rectangular region with $l_1, \dots, l_d$ lengths of the edges,
- b) $N_1, \dots, N_d$ the numbers of discretization points in each direction, all even,
- c) $\gamma^{1/2}$ a symmetric, positive function on $\mathbb{R}^d$,
- d) $R(\cdot)$ a function that generates independent $\mathcal{N}(0, |\Delta^N|^{-1})$ -distributed random numbers.

Output

GRF B on D , where the covariance is given by the Fourier transform of γ .

```

for  $k_i = 0, \dots, N_i - 1, i = 1, \dots, d$  do
   $B(k_1, \dots, k_d) \leftarrow R()$ ;
end for
 $A \leftarrow \text{FFT } B$ ;
for  $k_i = 0, \dots, N_i - 1, i = 1, \dots, d$  do
   $A(k_1, \dots, k_d) \leftarrow A(k_1, \dots, k_d) \cdot \gamma(p_{k_1 \dots k_d})^{1/2} \cdot |D|^{-1}$ ;
end for
 $B \leftarrow \text{FFT}^{-1} A$ ;

```

Algorithm 3.2.

Remarks Same as in Algorithm 3.1.

All calculations have to be done modulo N_i in the i -th direction.

Input Same as in Algorithm 3.1.

Output Same as in Algorithm 3.1.

```

for  $k_i = 0, \dots, N_i - 1, i = 1, \dots, d - 1, k_d = 0, \dots, N_d/2$  do
  if  $k_i \in \{0, N_i/2\}$ , for all  $i = 1, \dots, d$  then
     $\text{Re } A(k_1, \dots, k_d) \leftarrow R() \cdot \gamma(p_{k_1 \dots k_d})^{1/2} \cdot |D|^{-1}$ ;
     $\text{Im } A(k_1, \dots, k_d) \leftarrow 0$ ;
  else
     $\text{Re } A(k_1, \dots, k_d) \leftarrow 2^{-1/2} R() \cdot \gamma(p_{k_1 \dots k_d})^{1/2} \cdot |D|^{-1}$ ;
     $\text{Im } A(k_1, \dots, k_d) \leftarrow 2^{-1/2} R() \cdot \gamma(p_{k_1 \dots k_d})^{1/2} \cdot |D|^{-1}$ ;
     $\text{Re } A(N_1 - k_1, \dots, N_d - k_d) \leftarrow \text{Re } A(k_1, \dots, k_d)$ ;
     $\text{Im } A(N_1 - k_1, \dots, N_d - k_d) \leftarrow -\text{Im } A(k_1, \dots, k_d)$ ;
  end if
end for
 $B \leftarrow \text{FFT}^{-1} A$ ;

```

Algorithm 3.3.

Remarks Same as in Algorithm 3.2.

Input Same as in Algorithm 3.2.

Output Same as in Algorithm 3.2.

```

function  $\text{create\_random\_field}(D, N_1, \dots, N_d)$ 
 $A \leftarrow \text{complex\_array}(N_1, \dots, N_d)$ ;

```

```

set_array({N1, ..., Nd}, { }, A);
B ← FFT-1 A;
end function

function set_array({N1, ..., Nj}, {kj+1, ..., kd}, A)
for ki = 0, ..., Ni, i = 1, ..., j - 1, kj = 1, ..., Nj/2 - 1 do
  Re A(k1, ..., kd) ← 2-1/2 R() · γ(pk1...kd)1/2 · |D|-1;
  Im A(k1, ..., kd) ← 2-1/2 R() · γ(pk1...kd)1/2 · |D|-1;
  Re A(N1 - k1, ..., Nd - kd) ← Re A(k1, ..., kd);
  Im A(N1 - k1, ..., Nd - kd) ← -Im A(k1, ..., kd);
  if (|{N1, ..., Nj}| = 1) then
    Re A(0, ..., kd) ← R() · γ(p0 k2...kd)1/2 · |D|-1;
    Im A(0, ..., kd) ← 0;
    Re A(N1/2, ..., kd) ← R() · γ(pN1/2 k2...kd)1/2 · |D|-1;
    Im A(N1/2, ..., kd) ← 0;
  else
    set_array({N1, ..., Nj-1}, {0, kj+1, ..., kd}, A);
    set_array({N1, ..., Nj-1}, {Nj/2, kj+1, ..., kd}, A);
  end if
end for
end function

```

Received February 2, 2013; accepted February 18, 2013.

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