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DISPLACEMENT BASED ANALYSIS AND DESIGN OF ROCKING BRIDGES

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Abstract. *The response of a rigid rocking block is traditionally described by its tilt angle. This is a correct description, but this paper suggests that describing rocking via displacements is more meaningful, because it uncovers that two geometrically similar blocks of different size will experience the same top displacement, provided that they are not close to overturn. The above is illustrated for both analytical pulse excitations and for recorded ground motions. Thus the displacement demand of a ground motion on a rocking block is only a function of its slenderness; not of its size. This reduces the dimensionality of the problem and allows for the construction of size-independent rocking demand spectra.*

1 INTRODUCTION

The systematic study of the rocking oscillator started with Housner's seminal paper in 1963 [1]. Motivated by the surprising stability that tall slender "golf-ball-on-a-tee" structures presented in the 1960 Chilean earthquake, he showed that (a) out of 2 geometrically similar planar rigid objects, the larger one is harder to overturn dynamically, and (b) the overturning potential of a ground motion increases with its dominant period.

The interest on the rocking oscillator [2-6] sources from its ability to describe systems that cannot be described adequately by the classical elastic oscillator [7]. Indeed, the rocking oscillator can be used to understand the behavior of masonry structures [8-14], the seismic behavior of non-anchored equipment [15-22], as well as to explain the stability of ancient Greco-Roman and Chinese temples that have been standing for more than 2,500 years in earthquake prone regions [23-27]. Rocking motion has also inspired researchers to use inerters as seismic protection devices [28-29]. What is not widely known in the western world, is that rocking has been used since more than 40 years as a seismic isolation method in the USSR (and now in former USSR countries) [30, 31]. The Soviet system comprises an intentionally designed soft rocking story. The uplift of the rocking columns works as a mechanical fuse and limits the forces transmitted to the superstructure.

Rocking walls have been suggested as a resilient design approach for buildings [32, 33 and references therein].

Moreover, a 33-m-tall chimney at the Christchurch airport has been designed to uplift [34], and three 30 to 38-m-tall chimneys in Piraeus, Greece, have been retrofitted by allowing them to uplift in case of an earthquake.

Rocking systems are perfectly compatible with Accelerated Bridge Construction as they comprise prefabricated elements with dry connections. In fact, restrained rocking systems [35-42] have already found their way to practice, with the Wigram-Magdala restrained rocking bridge in New Zealand [43] being the first restrained rocking bridge constructed.

However, several researchers have suggested that the restraining tendon in rocking bridges is not only obsolete, but might unnecessarily decrease the design forces of both the superstructure and the foundation, maybe requiring a pile foundation when it could have been avoided [44-60]. Makris and Vassiliou [61] and Vassiliou and Makris [62] have suggested that as the size of a rocking bridge increases, the restraining system can become obsolete and merely increases the design forces of both the superstructure and the foundation.

A main drawback of unrestrained rocking bridges stems from their response being absolutely uncorrelated to any elastic system. Therefore, the elastic-based research results are not applicable: e.g. intensity measures, response spectra, motion-to-motion variability, design ground motions need to be re-determined. To this end, the rocking oscillator should be described with the minimum parameters needed.

This paper suggests that the current state of the art of using the tilt angle θ as the DOF of a rocking system is, of course, correct, but it is not the optimal. Using the top displacement of the oscillator, u , reduces the dimensionality of the problem. Then, the displacement *demand* on a rocking block becomes only slightly dependent on its size and is a function only of its slenderness [63].

2 ROTATION BASED DIMENSIONAL ANALYSIS OF THE ROCKING OSCILLATOR

The equation of in-plane motion for a rigid rectangular rocking column (Figure 1) with slenderness α and a semi-diagonal of length R (Figure 1) is:

$$\ddot{\theta} = -p^2 \cdot \left(\sin(\pm\alpha - \theta) + \frac{\ddot{u}_g}{g} \cos(\pm\alpha - \theta) \right) \quad (1)$$

where

$$p = \sqrt{(3g)/(4R)} \quad (2)$$

is the frequency parameter of the rocking column. The upper sign in front of α corresponds to a positive, and the lower to a negative rocking angle θ with respect to the defined coordinate system (Figure 1).

It is assumed that energy is only dissipated during impact. Housner [1] assumed that (a) the impact is instantaneous and (b) that the impact forces are concentrated on the impacting corner. Under these assumptions the ratio of post to pre impact rotational velocities is

$$r = \frac{\dot{\theta}_{after}}{\dot{\theta}_{before}} = 1 - \frac{3}{2} \sin^2 \alpha \quad (3)$$

Figure 1: Geometric characteristics of the rigid rocking block.

Researchers (including the senior author of this paper) have critically evaluated the Housner model - especially its damping assumptions [64-68]. Indeed, while assuming the impact to be instantaneous seems a reasonable assumption, there is no evident reason to assume that the impact forces act on the impacting corner. Given the large sensitivity of the time history response of the rocking oscillator to all the parameters that define it, Housner's model might seem simplistic. However, experimental testing shows that even though it cannot predict the response to an individual ground motion, it can predict the statistics of the response to a set of ground motions [69]. Therefore, we consider it adequate within the scope of earthquake engineering [70].

By inspecting Equation (1) and (2) one can conclude that the rotational response of a rocking block to a ground motion is a function of

$$\theta_{max} = f_1(R, \alpha, g, \ddot{u}_g(t)) \quad (4)$$

As the gravity acceleration, g , is constant, the rotational response to a given ground motion is a function of 2 parameters α and R , similarly to the elastic oscillator, in which the response is a function of the eigenperiod, T , and damping ratio ζ . Therefore, by keeping one parameter constant (R or α) one can construct rotational spectra for rocking structures. However, unlike

the elastic oscillator, where, for usual structures, one parameter (T) is more influential than the other (ζ), in the case of rocking structures, both R and α strongly influence the rotational response.

Since ground motions containing distinguishable acceleration and/or velocity pulses are particularly destructive [71 and references therein], Zhang and Makris [72] have studied the response of a planar rocking block to acceleration pulses given by analytical expressions. A pulse of a given waveform can be described by two parameters. Zhang and Makris [69] chose the acceleration amplitude a_p and the dominant cyclic frequency ω_p . Then, the response will be a function of

$$\theta_{\max} = f_2(R, \alpha, g, a_p, \omega_p) \quad (5)$$

Equation (5) involves 6 quantities with 2 reference dimensions (Time and Length). Therefore, according to Vaschy - Buckingham's Π -Theorem of Dimensional Analysis ([73, 74]), the number of dimensionless parameters describing the problem is $6 - 2 = 4$. There is not a unique solution for choosing these four parameters. Zhang and Makris [72] suggested describing the problem as

$$\theta_{\max} = \varphi_1\left(\alpha, \frac{\omega_p}{p}, \frac{a_p}{g \tan \alpha}\right) \quad (6)$$

ω_p/p is often called size-frequency parameter and depends on the frequency of the excitation and on the size of the block. $a_p/(g \tan \alpha)$ is usually called non-dimensional acceleration but it can also be perceived as a non-dimensional strength parameter, since $mgR \sin \alpha$ is the moment that withstands uplift ("strength") and is $ma_p R \cos \alpha$ the overturning moment.

Therefore, dimensional analysis reduces the dimensionality of the problem from 6 to 4. Hence, by keeping the slenderness parameter α constant, one can produce contour plots of the maximum tilt angle θ as a function of ω_p/p and $a_p/(g \tan \alpha)$, the so called "rocking spectra". It is worth mentioning that Dimitrakopoulos and DeJong [75] have shown that for small values of α one can drop it as an independent parameter from Equation (6) as long as the coefficient of restitution, r , is treated as an extra independent parameter – however in this section r is not treated independently.

Figure 2 shows the rocking spectra of symmetric and antisymmetric Ricker wavelets. Ricker wavelets are defined as the 2nd and 3rd derivative of the Gaussian:

$$\ddot{u}_g = a_p \left(1 - \frac{2\pi^2 t^2}{T_p^2}\right) e^{-\frac{1}{2} \frac{2\pi^2 t^2}{T_p^2}} \quad (7)$$

$$\ddot{u}_g = \frac{a_p}{\beta_r} \left(\frac{4\pi^2 t^2}{3T_p^2} - 3\right) \frac{2\pi t}{\sqrt{3}T_p} e^{-\frac{1}{2} \frac{4\pi^2 t^2}{3T_p^2}} \quad (8)$$

where

$$T_p = \frac{2\pi}{\omega_p} \quad (9)$$

and $\beta_r = 1.3801$ to enforce that the function maximum is equal to a_p .

The spectra confirm the remarkable observation that larger structures are harder to overturn dynamically and that higher frequency pulses have a lower overturning potential. Interestingly, they show a heavy dependence of the response on both ω_p/p and $a_p/g \tan \alpha$.

Figure 2: Non-dimensional rocking spectra based on rotations. $\alpha = 0.1$

3 DISPLACEMENT BASED DIMENSIONAL ANALYSIS OF A ROCKING OSCILLATOR EXCITED BY ANALYTICAL PULSES

3.1 Analysis based on the frequency parameter of the block p

The dimensional analysis of the previous section is one of the many correct solutions to describe the problem. It is based on rotations. This section, however, suggests that there is another, displacement based basis of describing the problem, which is also mathematically correct and more convenient. The convenience does not lie only on the fact that earthquake engineers are more used to displacements than rotations: A displacement based analysis further reduces the dimensionality of the problem allowing the construction of 2D rocking spectra.

Indeed, the rotation based analysis of the problem is based on the “recipe for similarity analysis” described in Chapter 5 of the well-known Dimensional Analysis textbook of Barenblatt [76]: “If the problem has an explicit mathematical formulation, the independent variables in the problem and the constant parameters that appear in the equations, boundary conditions and initial conditions, etc., are adopted as the governing parameters.” As this section shows, choosing the parameters that appear in the analytical equation might not be the most convenient way of describing this particular problem.

The top displacement of the rocking block can be obtained by a one-to-one mapping on the rotations:

$$u = 2R \sin(\pm \alpha) - 2R \sin(\pm \alpha - \theta) \quad (10)$$

The upper sign in front of α corresponds to a positive, and the lower sign to a negative tilt angle θ with respect to the defined coordinate system. If we use the top displacement as the single DOF of the problem, then the maximum response can be described as:

$$u_{\max} = f_3(R, \alpha, g, a_p, \omega_p) \quad (11)$$

To numerically compute the response of the block, we will resort to Equation (1), which is given in terms of rotation θ . Then, using Equation (10) we compute the displacement response.

Applying Buckingham's Π -theorem on Equation (11), one possible non-dimensionalization is

$$\frac{u_{\max} \omega_p^2}{a_p} = \varphi_2 \left(\alpha, \frac{\omega_p}{p}, \frac{a_p}{g \tan \alpha} \right) \quad (12)$$

Figure 3 shows the contour plots of $\frac{u_{\max} \omega_p^2}{a_p}$ as a function of ω_p/p and $a_p/(g \tan \alpha)$ for a given $\alpha = 0.1$. The remarkable observation is that within the non-overturning region the non-dimensional displacement depends heavily (and strongly non-linear) on the non-dimensional strength parameter $a_p/(g \tan \alpha)$ but only loosely on the size-frequency parameter ω_p/p . When the block is not close to overturning, the influence of ω_p/p is practically negligible.

Figure 4 plots $\frac{u_{\max} \omega_p^2}{a_p}$ as a function of $a_p/(g \tan \alpha)$ for different values of ω_p/p (and a constant slenderness $\alpha = 0.1$). For reasons of figure clarity, only non-overturning values of $\frac{u_{\max} \omega_p^2}{a_p}$ are plot, i.e. not plotting $\frac{u_{\max} \omega_p^2}{a_p}$ means that the block has overturned. Figure 5 plots $\frac{u_{\max} \omega_p^2}{a_p}$ as a function of ω_p/p for different values of $a_p/(g \tan \alpha)$ (and $\alpha = 0.1$).

Again, it is observed that, as long as the system is away from overturning, the dominant factor that influences $\frac{u_{\max} \omega_p^2}{a_p}$ is $a_p/(g \tan \alpha)$; not ω_p/p . In fact for small values of non-dimensional acceleration $a_p/(g \tan \alpha)$, the response for all values of size-frequency parameter ω_p/p is practically the same. The response starts to deviate only when the system is close to overturning – or has overturned.

In other words, a small and a large block, geometrically similar to each other and excited by analytical pulses, will have roughly equal top displacement, provided that the displacement is not enough to bring them close to overturn. A given pulse will induce the same *displacement demand*. The larger block is more stable simply because its displacement capacity (i.e. the displacement needed to cause overturn, i.e. its width) is larger.

Therefore, using a displacement basis to describe the problem further decreases the number of parameters needed to define it. Practically, the displacement demand on a rocking oscillator excited by a pulse is only a function of its non-dimensional strength parameter $a_p/(g \tan \alpha)$; not of its size.

The strongly nonlinear nature of rocking motion is also evident in Figures 4 and 5.

$\frac{u_{\max} \omega_p^2}{a_p}$, which expresses the relation of the rocking displacement to the ground motion

Figure 3: Non-dimensional rocking spectra based on displacements.

Figure 4: $\frac{u_{\max} \omega_p^2}{a_p}$ vs $a_p / (g \tan \alpha)$ plots for constant ω_p / p . $\alpha = 0.1$

Figure 5: $\frac{u_{\max} \omega_p^2}{a_p}$ vs ω_p / p plots for constant $a_p / (g \tan \alpha)$ and $\alpha = 0.1$

displacement, does not depend monotonically on the strength parameter $a_p / (g \tan \alpha)$. In fact, the discontinuities of the $\omega_p / p = 2$ line of Figure 4 convey that a block can survive a stronger pulse and overturn in a weaker one.

Going back to dimensional quantities, Figure 6 plots the displacement response to a symmetric Ricker pulse with $a_p = 1g$ and $T_p = 0.5s$ and to an antisymmetric Ricker pulse with $a_p = 1g$ and $T_p = 1s$. The plots confirm that the displacement demand only loosely depends on the size, if the block is not close to overturning. The dominant factor is the slenderness. Therefore we can define the displacement demand rocking spectrum of a ground motion as a unary function

$$u_{\text{demand}} = f(\alpha) \text{ if } u_{\text{demand}} \ll 2b \quad (13)$$

that is computed via Equations (1) and (10) for a large enough block size. To check the stability of a block, one has to compute the maximum displacement demand via Equation (13) and compare it with the displacement capacity (i.e. the block width).

Therefore the reduction of the dimension of the problem follows two steps: a) Applying Buckingham's theorem and b) Observing that the displacement demand is roughly independent of the size. The first step is exact and follows from dimensional analysis. The second step is approximate and in this section illustrated for analytical pulses. Blöchliger [77] gave a first indication that the approximation also works for recorded ground motions. Further evidence supporting this approximation and highlighting its limitations are given in a next section of this paper.

Figure 6: Maximum block displacement as a function of block size for Symmetric and Anti-Symmetric Ricker excitation.

3.2 Analysis based on the base width of the block b

The previous section chooses the frequency parameter p and the slenderness of the block, α , as the two parameters to define it. However, p has a physical meaning that is totally unrelated to rocking. It is the natural frequency that the block would have had *if* it was hanging from its corner [75]. But this is merely a coincidence, rocking blocks have no natural frequency [1], and the use of p often creates misunderstandings. In this section we propose to describe the block with two physical parameters that have a clear physical meaning, directly related to the rocking problem. The slenderness α is retained, as it controls the uplift of the structure (and could be parallelized with the strength of a system), but p is replaced by b , which is the half-width of the base and exactly equal to one half of its displacement capacity. Then the displacement response will be:

$$u_{\max} = f_4(b, \alpha, g, a_p, \omega_p) \quad (14)$$

Using Buckingham's Π theorem we get:

$$\frac{u_{\max} \omega_p^2}{a_p} = \varphi_3 \left(\frac{a_p}{g \tan \alpha}, \frac{b \omega_p^2}{a_p}, \alpha \right) \quad (15)$$

The term $a_p/(g \tan \alpha)$ would be the reciprocal of the non-dimensional strength, $(b \omega_p^2)/a_p$ would be the non-dimensional displacement capacity, and α (taken as an independent parameter) controls damping, because it controls the coefficient of restitution.

Figure 7 plots displacement spectra according to the suggested non-dimensionalization. One can observe that for both pulses a base (i.e a displacement capacity = $2b$) of roughly 9

Figure 7: Displacement based non-dimensional rocking spectra using the width of the block to characterize its size.

times the length scale of the pulse $L_e = a_p / \omega_p^2$ is enough to keep the block stable, no matter what the non-dimensional strength parameter is.

4 DISPLACEMENT BASED ANALYSIS OF A ROCKING OSCILLATOR EXCITED BY RECORDED GROUND MOTIONS

Analytical pulses can be used to qualitatively study the rocking oscillator. However, as the rocking problem is very sensitive to all of its parameters, pulses would not suffice to prove that the displacement demand on a rocking structure depends only on its slenderness and not on its size. Therefore, this section examines the displacement response of a rocking block excited by recorded ground motions.

4.1 FEMA P695 Ground motions

There is no consensus in the engineering community on what ground motions should be used in time history analysis. Several approaches exist including using recorded (scaled or unscaled), artificial, or synthetic ground motions. In this paper we choose to use the 3 sets of ground motions proposed by FEMA P695 [79] (far field, near field pulse-like, and near field non-pulse-like) only as a means to illustrate our rocking-related argument, without taking stance on the debate around ground motions. It is evident that any ground motion selection method based on the response of an elastic system is in principle not applicable in the case of the rocking oscillator, as the elastic and rocking oscillator are uncorrelated. More information on the FEMA P695 ground motions can be found in FEMA [79].

4.2 Equal displacement rule for rocking structures and displacements demand spectra

Vassiliou et al. [80] have proven that rigid rocking oscillators of equal height attached to massless foundations of the same size behave identically, no matter what their actual column width is (Figure 8). Therefore, the design question of a rocking structure would be: Find the size, $2B'$, of the foundation for a given oscillator height $2H$. Hence, it is more meaningful to

Figure 8: Rocking oscillators of equal height.

use H as a size parameter instead of R , even if the former does not explicitly appear in the equation of motion.

Figure 9 offers the displacement of a rocking oscillator as function of its slenderness α , and for $2H=2, 4, 10, 20, 80$, and $1000m$, for a selection of the FEMA P695 ground motions. The $2H=1000m$ is offered only for reasons of mathematical completeness, to study the limit case of $H \rightarrow \infty$. For reasons of plot clarity, each line is plotted only for $\alpha > \alpha_{crit}$, where α_{crit} is the minimum slenderness angle for which the block overturns. We observe that all blocks of same slenderness angle present roughly the same displacement, as long as they are not close to overturning. The same observation holds for all the ground motions tested.

As analysis and design of a rocking structure would not involve a single ground motion, but a set of design motions, it makes sense to study the problem by applying sets of multiple excitations and comparing the statistics of the results (e.g. the median displacement among all the ground motions of the excitation set). Figure 10 plots displacement spectra of the median of the displacement for 7 variations of the near-field pulse-like FEMA P695 set: a) Unscaled ground motions, b) scaled so that their $\overline{PGA}$ is equal to $0.5\overline{PGA}$, or $\overline{PGA}$, or $2\overline{PGA}$, c) scaled so that their $\overline{PGV}$ is equal to $0.5\overline{PGV}$, or $\overline{PGV}$, or $2\overline{PGV}$, where $\overline{PGA}$, $\overline{PGV}$ are defined as

$$\overline{PGA} = \text{median}_{i=1 \dots N} \left(\sqrt{PGA_{i_x} \times PGA_{i_y}} \right) \quad (16)$$

$$\overline{PGV} = \text{median}_{i=1 \dots N} \left(\sqrt{PGV_{i_x} \times PGV_{i_y}} \right) \quad (17)$$

where N is the number of the ground motions and x and y are the two components of each ground motion. Note that each horizontal component of each ground motion is treated as an independent motion. Figures 11 and 12 plot the same spectra for the far field and near field non-pulse-like ground motions.

The following observations can be made:

- The median spectra are smoother, likewise design elastic spectra that were derived by statistical processing of elastic spectra of single ground motions are smoother than single ground motion spectra.
- As long as the system is not close to overturning, the displacement does not depend on the size of the block. For this part of the spectrum, instead of computing a different spectrum for each block size, one can compute the design spectrum for $2H \rightarrow \infty$ ($2H =$

 Figure 9: Displacement of a rocking oscillator as function of its slenderness α .

Figure 10: Median Displacement Spectra for Near-Field Pulse-Like Record Set.

1000m seems an adequate value) and use it to calculate the displacement *demand* on any rocking structure (i.e. $u_{\max}=f(\alpha)$). We name the above finding “equal displacement rule” for rocking structures.

- c) As the system gets closer to overturning the equal displacement rule does not apply: smaller systems present larger displacements than larger ones. Moreover, as the system approaches overturning, the slope of the spectrum increases dramatically i.e. a small decrease in $\tan\alpha$ leads to very large increase in displacement. This trend dictates that a rational design of a rocking structure would require that this steep part of the spectrum be avoided. because an earthquake slightly stronger than the design one would cause a tremendous increase in displacement. Therefore, the equal displacement rule applies to the rational design region.
- d) The form of the spectrum for all 3 sets of ground motions presents some repetitive pattern:
 - i. As α tends to zero, $u_{\max}$ tends to a finite value. For spectra of individual ground motions, this value is $\frac{3}{2}PGD$. An explanation for this is offered in the next section.
 - ii. As α increases from zero, the displacement demand amplifies 2-2.5 times and reaches a plateau.
 - iii. Further increase of α leads to a monotonic decrease of the displacement demand.
 - iv. Naturally, when $\tan\alpha$ reaches PGA/g , the displacement demand becomes zero, as there is no uplift.

4.3 Preliminary design based on the equal displacement rule

If not for a final design, the equal displacement rule can be used for preliminary calculations. Indeed it is not an exact method, but a preliminary design method that does not aim at being exact, but at providing a tool for initial calculations, that for certain cases and required degree of accuracy can be enough. The same holds for yielding structures, where the “equal displacement rule” is used for many structural systems, while for more complicated systems it is used only for preliminary design and then more refined methods are applied. It could be stated that the findings of this paper constitute the generalization of equal displacement rule from yielding to rocking systems. This section proposes a methodology to design a rocking structure based on the equal displacement rule:

- a) On the $u_{\max} - \tan\alpha$ curve, we plot the capacity line $u_C = 2H\tan\alpha$.
- b) We determine the intersection of the capacity line and the $2H = \infty$ line. We define the abscissa of this point as $\tan\alpha_k$.
- c) We use a multiplier of 2.5 to determine the design slenderness: $\tan\alpha_D = 2.5\tan\alpha_k$. The multiplier serves as a safety factor to move the design point away from the steep part of the spectrum.

Figure 13 outlines the design procedure applied for a rocking bridge with columns of 6.7m height ($2H = 6.7m$). Based on Makris and Vassiliou [46] the response of the frame is equal to the response of a solitary block of $2H = 10m$. For this bridge, twenty one design scenarios are explored, corresponding to the 21 spectra of Figures 10-12. Tables 1-3 and Figure 14 summarize the findings for the 21 design scenarios and compare the displacement predicted

Figure 11: Median Displacement Spectra for Near-Field No Pulse-Like Record Set.

Figure 12: Median Displacement Spectra for Far-Field Record Set.

by the demand spectrum ($2H = 1000\text{m}$) to the displacement predicted by the $2H=10\text{m}$ spectrum. We observe that in all but two cases (near fault pulse-like scaled to $0.5\overline{PGA}$ and near fault non-pulse-like scaled to $2\overline{PGA}$) the error in predicting the median displacement is less than 20%. In all cases, the error is smaller than 40%, and no system overturned.

	Unscaled	$0.5\overline{PGA}$	$\overline{PGA}$	$2\overline{PGA}$	$0.5\overline{PGV}$	$\overline{PGV}$	$2\overline{PGV}$
$\tan(\alpha_D)$	0.2839	0.1378	0.2671	0.4618	0.0596	0.1094	0.1909
$2H = 1000\text{m}$	0.05	0.11	0.08	0.12	0.10	0.07	0.08
$2H = 10\text{m}$	0.06	0.15	0.07	0.13	0.10	0.10	0.08

Table 1: Near field pulse like FS=2.5.

	Unscaled	$0.5\overline{PGA}$	$\overline{PGA}$	$2\overline{PGA}$	$0.5\overline{PGV}$	$\overline{PGV}$	$2\overline{PGV}$
$\tan(\alpha_D)$	0.1524	0.0708	0.1356	0.2446	0.0916	0.1892	0.3405
$2H = 1000\text{m}$	0.34	0.22	0.44	0.44	0.19	0.27	0.36
$2H = 10\text{m}$	0.41	0.26	0.41	0.69	0.21	0.30	0.38

Table 2: Near field non pulse like FS=2.5.

	Unscaled	$0.5\overline{PGA}$	$\overline{PGA}$	$2\overline{PGA}$	$0.5\overline{PGV}$	$\overline{PGV}$	$2\overline{PGV}$
$\tan(\alpha_D)$	0.1228	0.0620	0.1215	0.2411	0.0675	0.1320	0.2249
$2H = 1000\text{m}$	0.30	0.21	0.38	0.52	0.17	0.28	0.52
$2H = 10\text{m}$	0.33	0.20	0.37	0.50	0.19	0.29	0.49

Table 3: Far field FS=2.5.

Figure 13: Design procedure.

independent rocking spectrum is suggested. This should be taken into account when intensity measures for rocking structures [81-84] designed not to get close to overturning are explored.

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REFERENCES

- [1] GW. Housner, The behavior of inverted pendulum structures during earthquakes, *Bulletin of the seismological society of America* 53 (2), 403-417, 1963.
- [2] AK. Chopra, SCS. Yim, Simplified earthquake analysis of structures with foundation uplift, *Journal of Structural Engineering* 111 (4), 906-930, 1985.
- [3] IN. Psycharis, Effect of base uplift on dynamic response of SDOF structures, *Journal of Structural Engineering* 117 (3), 733-754, 1991.
- [4] G. Oliveto, I. Calì, A. Greco, Large displacement behaviour of a structural model with foundation uplift under impulsive and earthquake excitations, *Earthquake Engineering & Structural Dynamics* 32 (3), 369-393, 2003.
- [5] QTM. Ma, The mechanics of rocking structures subjected to ground motion, Doctoral dissertation, ResearchSpace, Auckland, 2010.
- [6] S. Acikgoz, MJ. DeJong, The interaction of elasticity and rocking in flexible structures allowed to uplift, *Earthquake Engineering & Structural Dynamics*, 41 (15), 2177-2194, 2012.
- [7] N. Makris, D. Konstantinidis, The rocking spectrum and the limitations of practical design methodologies, *Earthquake Engineering & Structural Dynamics* 32(2), 265-289, 2003.
- [8] I. Stefanou, I. Psycharis, IO. Georgopoulos, Dynamic response of reinforced masonry columns in classical monuments, *Construction and Building Materials* 25 (12), 4325-4337, 2011.
- [9] MJ. DeJong, Seismic response of stone masonry spires: Analytical modeling. *Engineering Structures* 40, 556-565, 2012.
- [10] M. Tondelli, K. Beyer, M. DeJong, M., Influence of boundary conditions on the out-of-plane response of brick masonry walls in buildings with RC slabs, *Earthquake Engineering & Structural Dynamics* 45 (8), 1337-1356, 2016.
- [11] C. Casapulla, L. Giresini, PB. Lourenço, Rocking and Kinematic Approaches for Rigid Block Analysis of Masonry Walls: State of the Art and Recent Developments, *Buildings* 7 (3), 69, 2017.
- [12] D. Kalliontzis, AE. Schultz, Characterizing the In-Plane Rocking Response of Masonry Walls with Unbonded Posttensioning, *Journal of Structural Engineering* 143 (9), 04017110, 2017.

- [13] A. Mehrotra, MJ. DeJong, The influence of interface geometry, stiffness, and crushing on the dynamic response of masonry collapse mechanisms, *Earthquake Engineering & Structural Dynamics* 47 (13), 2661-2681, 2018.
- [14] Giresini, L., Sassu, M., & Sorrentino, L. (2018). In situ free-vibration tests on unrestrained and restrained rocking masonry walls. *Earthquake Engineering & Structural Dynamics*, 47(15), 3006-3025.
- [15] D. Konstantinidis, N. Makris, Experimental and analytical studies on the response of 1/4-scale models of freestanding laboratory equipment subjected to strong earthquake shaking, *Bulletin of earthquake engineering* 8 (6), 1457-1477, 2010.
- [16] A. Di Egidio, R. Alaggio, A. Contento, M. Tursini, E. Della Loggia, Experimental characterization of the overturning of three-dimensional square based rigid block, *International Journal of Non-Linear Mechanics* 69, 137-145, 2015.
- [17] CE. Wittich, TC. Hutchinson, Shake table tests of stiff, unattached, asymmetric structures, *Earthquake Engineering & Structural Dynamics* 44 (14), 2425-2443, 2015.
- [18] A. Dar, D. Konstantinidis, WW. El-Dakhakhni, Evaluation of ASCE 43-05 seismic design criteria for rocking objects in nuclear facilities, *Journal of Structural Engineering* 142 (11), 04016110, 2016.
- [19] AG. Sextos, GD. Manolis, N. Ioannidis, A. Athanasiou, Seismically induced uplift effects on nuclear power plants. Part 2: Demand on internal equipment, *Nuclear Engineering and Design* 318, 288-296, 2017.
- [20] A. Dar, D. Konstantinidis, W. El-Dakhakhni, Seismic response of rocking frames with top support eccentricity, *Earthquake Engineering & Structural Dynamics* 47 (12), 2496-2518, 2018.
- [21] E. Voyagaki, P. Kloukinas, M. Dietz, L. Dihoru, T. Horseman, O. Oddbjornsson, A. Steer, Earthquake response of a multiblock nuclear reactor graphite core: Experimental model vs simulations, *Earthquake Engineering & Structural Dynamics* 47(13), 2601-2626, 2018.
- [22] L. Di Sarno, G. Magliulo, D. D'Angela, E. Cosenza, Experimental assessment of the seismic performance of hospital cabinets using shake table testing, *Earthquake Engineering & Structural Dynamics* 48 (1), 103-123, 2019.
- [23] HP. Mouzakis, IN. Psycharis, DY. Papastamatiou, PG. Carydis, C. Papantonopoulos, C. Zambas, Experimental investigation of the earthquake response of a model of a marble classical column, *Earthquake Engineering & Structural Dynamics* 31 (9), 1681-1698, 2002.
- [24] C. Papantonopoulos, IN. Psycharis, DY. Papastamatiou, JV. Lemos, HP. Mouzakis, Numerical prediction of the earthquake response of classical columns using the distinct element method, *Earthquake Engineering & Structural Dynamics* 31 (9), 1699-1717, 2002.
- [25] L. Papaloizou, P. Komodromos, Planar investigation of the seismic response of ancient columns and colonnades with epistyles using a custom-made software, *Soil Dynamics and Earthquake Engineering* 29 (11), 1437-1454, 2009.

- [26] MF. Vassiliou, N. Makris, Analysis of the rocking response of rigid blocks standing free on a seismically isolated base, *Earthquake Engineering & Structural Dynamics* 41(2), 177-196, 2012.
- [27] Ž. Nikolić, L. Krstevska, P. Marović, H. Smoljanović, Experimental investigation of seismic behaviour of the ancient Protiron monument model, *Earthquake Engineering & Structural Dynamics* 48 (6), 573-593, 2019.
- [28] N. Makris, G. Kampas, Seismic protection of structures with supplemental rotational inertia, *Journal of Engineering Mechanics* 142 (11), 04016089, 2016.
- [29] R. Thiers-Moggia, C. Málaga-Chuquitaype, Seismic protection of rocking structures with inerters. *Earthquake Engineering & Structural Dynamics* 48 (5), 528-547, 2019.
- [30] JA. Bachmann, MF. Vassiliou, B. Stojadinović, Dynamics of rocking podium structures, *Earthquake Engineering & Structural Dynamics* 46 (14), 2499-2517, 2018.
- [31] Bachmann, J. A., Vassiliou, M. F., & Stojadinovic, B. (2019). Rolling and rocking of rigid uplifting structures. *Earthquake Engineering & Structural Dynamics*, 48(14), 1556-1574.
- [32] N. Makris, M. Aghagholizadeh, The dynamics of an elastic structure coupled with a rocking wall, *Earthquake Engineering & Structural Dynamics* 46 (6), 945-962, 2017.
- [33] M. Aghagholizadeh, N. Makris, Earthquake response analysis of yielding structures coupled with vertically restrained rocking walls, *Earthquake Engineering & Structural Dynamics* 47(15), 2965-2984, 2018.
- [34] RD. Sharpe, RI. Skinner, The seismic design of an industrial chimney with rocking base. *Bulletin of the New Zealand National Society for Earthquake Engineering* 16 (2), 98-106, 1983.
- [35] JB. Mander, CT. Cheng, Seismic resistance of bridge piers based on damage avoidance design, Technical Report NCEER-97-0014, State University of New York, Buffalo, NY, 1997.
- [36] Palermo, A., Pampanin, S., & Calvi, G. M. (2004). The use of controlled rocking in the seismic design of bridges. *Doctate Thesis, Technical Institute of Milan, Milan*, (1), 1-5.
- [37] Marriott, D., Pampanin, S., & Palermo, A. (2009). Quasi-static and pseudo-dynamic testing of unbonded post-tensioned rocking bridge piers with external replaceable dissipaters. *Earthquake engineering & structural dynamics*, 38(3), 331-354.
- [38] Motaref, S., Saiidi, M. S., and Sanders, D. (2010). "Experimental study of precast bridge columns with built-in elastomer." *Transportation Research Record*, 2202, 109–116.
- [39] Sideris, P., Aref, A. J., & Filiatrault, A. (2014). Large-scale seismic testing of a hybrid sliding-rocking posttensioned segmental bridge
- [40] Sideris, P., Aref, A. J., & Filiatrault, A. (2015). Experimental seismic performance of a hybrid sliding–rocking bridge for various specimen configurations and seismic loading conditions. *Journal of Bridge Engineering*, 20(11), 04015009.
- [41] T. Thonstad, IM. Mantawy, JF. Stanton, MO. Eberhard, DH. Sanders, Shaking table performance of a new bridge system with pretensioned rocking columns, *Journal of Bridge Engineering* 21 (4), 04015079, 2016.

- [42] AI. Giouvanidis, EG. Dimitrakopoulos, Seismic performance of rocking frames with flag-shaped hysteretic behavior, *Journal of Engineering Mechanics* 143 (5), 04017008, 2017.
- [43] PJ. Routledge, MJ. Cowan, A. Palermo, Low-damage detailing for bridges - a case study of Wigram-Magdala bridge, *Proceedings of New Zealand society for earthquake engineering 2016 Conference*, 2016.
- [44] I. Anastasopoulos, G. Gazetas, M. Loli, M. Apostolou, N. Gerolymos, Soil failure can be used for seismic protection of structures, *Bulletin of Earthquake Engineering* 8 (2), 309-326, 2010.
- [45] Makris N and Vassiliou MF (2012), Sizing the slenderness of free-standing rocking columns to withstand earthquake shaking, *Archive of Applied Mechanics*, 1-15.
- [46] N. Makris, MF. Vassiliou, Planar rocking response and stability analysis of an array of free standing columns capped with a freely supported rigid beam, *Earthquake Engineering & Structural Dynamics* 42 (3), 431-449, 2013.
- [47] N. Makris, MF. Vassiliou, Are Some Top-Heavy Structures More Stable?, *Journal of Structural Engineering* 140 (5), 2014.
- [48] EG. Dimitrakopoulos, AI. Giouvanidis, Seismic Response Analysis of the Planar Rocking Frame, *Journal of Engineering Mechanics* 141 (7), 04015003, 2015.
- [49] A. Agalianos, A. Psychari, MF. Vassiliou, B. Stojadinovic, I. Anastasopoulos, Comparative Assessment of Two Rocking Isolation Techniques for a Motorway Overpass Bridge, *Frontiers in Built Environment* 3, 47, 2017.
- [50] MF. Vassiliou, KR. Mackie, B. Stojadinović, A finite element model for seismic response analysis of deformable rocking frames, *Earthquake Engineering & Structural Dynamics* 46, 447–466, 2017.
- [51] MF. Vassiliou, S. Burger, M. Egger, JA. Bachmann, M. Broccardo, B. Stojadinovic, The three-dimensional behavior of inverted pendulum cylindrical structures during earthquakes, *Earthquake Engineering & Structural Dynamics* 46 (14), 2261-2280, 2018.
- [52] MF. Vassiliou, Seismic response of a wobbling 3D frame, *Earthquake Engineering & Structural Dynamics* 47 (5), 1212-1228, 2018.
- [53] Y. Xie, J. Zhang, R. DesRoches, JE. Padgett, Seismic fragilities of single-column highway bridges with rocking column-footing, *Earthquake Engineering & Structural Dynamics* 48, 843–864, 2019.
- [54] Sieber, M., Klar, S., Vassiliou, M. F., & Anastasopoulos, I. Robustness of simplified analysis methods for rocking structures on compliant soil (2020). *Earthquake Engineering & Structural Dynamics* DOI: 10.1002/eqe.3294
- [55] Thomaidis, I. M., Kappos, A. J., & Camara, A. (2020). Dynamics and seismic performance of rocking bridges accounting for the abutment-backfill contribution. *Earthquake Engineering & Structural Dynamics*. DOI: 10.1002/eqe.3283
- [56] Giouvanidis, A., & Dong, Y. (2020). Seismic loss and resilience assessment of single-column rocking bridges. *Bulletin of Earthquake Engineering*, <https://doi.org/10.1007/s10518-020-00865-5>

- [57] Aghagholizadeh, M. (2020). A finite element model for seismic response analysis of vertically-damped rocking-columns. *Engineering Structures*, 219, 110894.
- [58] Avgenakis, E., & Psycharis, I. N. (2019). Determination of the nonlinear displacement distribution of the semi-infinite strip—application to deformable rocking bodies. *International Journal of Solids and Structures*, 170, 22-37.
- [59] Avgenakis, E., & Psycharis, I. N. An integrated macroelement formulation for the dynamic response of inelastic deformable rocking bodies. *Earthquake Engineering & Structural Dynamics*.
- [60] Avgenakis, E., & Psycharis, I. N. (2020). Modeling of inelastic rocking bodies under cyclic loading. *Journal of Engineering Mechanics*, 146(4), 04020020.
- [61] N. Makris, MF. Vassiliou, Dynamics of the rocking frame with vertical restrainers, *Journal of Structural Engineering* 141 (10), 04014245, 2014.
- [62] MF. Vassiliou, N. Makris, Dynamics of the Vertically Restrained Rocking Column. *Journal of Engineering Mechanics* 141(12), 04015049, 2015.
- [63] Reggiani Manzo, N., & Vassiliou, M. F. (2019). Displacement-based analysis and design of rocking structures. *Earthquake Engineering & Structural Dynamics*, 48(14), 1613-1629.
- [64] MF. Vassiliou, R. Truniger, B. Stojadinović, An analytical model of a deformable cantilever structure rocking on a rigid surface: development and verification, *Earthquake Engineering & Structural Dynamics* 44 (15), 2775-2794, 2015.
- [65] R. Truniger, MF. Vassiliou, B. Stojadinović, An analytical model of a deformable cantilever structure rocking on a rigid surface: experimental validation, *Earthquake Engineering & Structural Dynamics* 44 (15), 2795-2815, 2015.
- [66] AI. Giouvanidis, EG. Dimitrakopoulos, Nonsmooth dynamic analysis of sticking impacts in rocking structures, *Bulletin of Earthquake Engineering* 15 (5), 2273-2304, 2017.
- [67] MN. Chatzis, MG. Espinosa, AW. Smyth, Examining the energy loss in the inverted pendulum model for rocking bodies, *Journal of Engineering Mechanics* 143 (5), 04017013, 2017.
- [68] D. Kalliontzis, S. Sritharan, Characterizing Dynamic Decay of Motion of Free-Standing Rocking Members, *Earthquake Spectra* 34 (2), 843-866, 2018.
- [69] JA. Bachmann, M. Strand, MF. Vassiliou, M. Broccardo, B. Stojadinović, Is rocking motion predictable?, *Earthquake Engineering & Structural Dynamics* 47 (2), 535-552, 2018.
- [70] Del Giudice, L., Wrobel, R., Leinenbach, C., & Vassiliou, M. F. (2020). Static Testing of Additively Manufactured Microreinforced Concrete Specimens for Statistical Structural Model Validation at a Small Scale. In *8th International Conference on Advances in Experimental Structural Engineering (8AESE)*.
- [71] MF. Vassiliou, N. Makris, Estimating time scales and length scales in pulse-like earthquake acceleration records with wavelet analysis. *Bulletin of Seismological Society of America* 101 (2), 596-618, 2011.
- [72] J. Zhang, N. Makris, Rocking response of free-standing blocks under cycloidal pulses, *Journal of Engineering Mechanics*, 127(5), 473-483.

- [73] A. Vaschy, Sur les lois de similitude en physique, *Annales Télégraphiques* 19, 25–28, 1892.
- [74] E. Buckingham, On physically similar systems; illustrations of the use of dimensional equations, *Physical Review* 4 (4), 345–376, 1914.
- [75] EG. Dimitrakopoulos, MJ. DeJong, Revisiting the rocking block: closed-form solutions and similarity laws, *Proceedings of Royal Society A* 468 (2144), 2294–2318, 2012.
- [76] GI. Barenblatt, Scaling, self-similarity, and intermediate asymptotics: dimensional analysis and intermediate asymptotics, *Cambridge University Press*, 1996.
- [77] Blöchlinger, Rigid body rocking spectra for recorded earthquake ground motions, Master's Thesis, ETH Zurich, 2016.
- [78] MJ. DeJong, EG. Dimitrakopoulos, Dynamically equivalent rocking structures, *Earthquake Engineering & Structural Dynamics* 43 (10), 1543–1563, 2014.
- [79] FEMA P695, Quantification of Building Seismic Performance Factors, Washington, D.C: Rep. FEMA P695, Federal Emergency Management Agency, 2009.
- [80] MF. Vassiliou, KR. Mackie, B. Stojadinović, Dynamic response analysis of solitary flexible rocking bodies: modeling and behavior under pulse-like ground excitation, *Earthquake Engineering & Structural Dynamics* 43, 1463–1481, 2014.
- [81] EG. Dimitrakopoulos, TS. Paraskeva, Dimensionless fragility curves for rocking response to near-fault excitations, *Earthquake Engineering & Structural Dynamics* 44 (12), 2015–2033, 2015.
- [82] A. Pappas, A. Sextos, F. Da Porto, C. Modena, Efficiency of alternative intensity measures for the seismic assessment of monolithic free-standing columns, *Bulletin of Earthquake Engineering* 15 (4), 1635–1659, 2017.
- [83] AI. Giouvanidis, EG. Dimitrakopoulos, Rocking amplification and strong-motion duration, *Earthquake Engineering & Structural Dynamics* 47 (10), 2094–2116, 2018.
- [84] IE. Kavvadias, GA. Papachatzakis, KE. Bantilas, LK. Vasiliadis, A. Elenas, Rocking spectrum intensity measures for seismic assessment of rocking rigid blocks, *Soil Dynamics and Earthquake Engineering* 101, 116–124, 2017.