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Author(s):

Westerhuis, Stephanie ; Fuhrer, Oliver ; Bhattacharya, Ritthik; Schmidli, Jürg; Bretherton, Christopher

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Effects of terrain-following vertical coordinates on simulation of stratus clouds in numerical weather prediction models

Stephanie Westerhuis^{1,2}, Oliver Fuhrer^{2,3}, Ritthik Bhattacharya⁴, Jürg
Schmidli⁴, and Christopher Bretherton^{3,5}

¹ETH Zurich, Switzerland

²Federal Institute of Meteorology and Climatology MeteoSwiss,
Switzerland

³Vulcan Inc., USA

⁴University of Frankfurt, Germany

⁵University of Washington, USA

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Abstract

Many numerical weather prediction models employ terrain-following vertical coordinates. As a consequence, over orography, flat tops of stratus clouds are intersected by the vertical coordinate surfaces. We conduct idealised two-dimensional simulations of a stratus cloud with the COSMO model to study the effect of such sloping vertical coordinate surfaces. The evolution of the stratus cloud above a flat surface within a horizontally homogeneous atmosphere serves as a reference. During night-time, the cloud thickens, driven by radiative cooling at the cloud top. Adding a sinusoidal perturbation to the vertical coordinate surfaces reduces the growth of the stratus cloud. With strong perturbations, the cloud starts to dissipate. The physical processes in the two simulations are identical, hence this behaviour is purely driven by numerical diffusion. The cloud is similarly thinned when sinusoidal orographic features are introduced. The reduction depends on the amplitude and wavelength of the perturbations and hills. Increasing the horizontal resolution partly mitigates the numerical diffusion. However, this is a very costly measure for an operational weather model. We suggest conducting further research on a new vertical coordinate with additional local smoothing of the orographic signal.

1 Introduction

In numerical weather prediction (NWP) models, the time and space evolution of meteorological quantities is represented on a discrete grid. Orography can be introduced as a step-function (Mesinger et al., 1988), using shaved cells (Adcroft et al., 1997), employing an immersed boundary method (Kirkpatrick et al., 2003; Tseng and Ferziger, 2003; Lundquist et al., 2012), or by transforming the equations of motion to a terrain-following coordinate system (Kasahara, 1974; Gal-Chen and Somerville, 1975; Zhu et al., 1992; Schär et al., 2002; Leuenberger et al., 2010). The latter approach is widely used as it offers a simple treatment of the lower boundary condition without introducing sharp discontinuities. On the down-side, spurious pressure-gradient force errors can develop in the

vicinity of very steep topography (Mellor and Yamada, 1974; Mesinger and Janjić, 1985; Haney, 1991; Zängl, 2012), consistent treatment of the metric terms poses a challenge even at small terrain slopes (Klemp et al., 2003), and topography requires low-pass filtering to control numerical errors.

Schär et al. (2002) and Leuenberger et al. (2010) introduced the smooth level vertical (SLEVE) coordinates featuring a scale-dependent vertical decay of the orographic signal to partly eliminate these issues. However, in a model domain comprising orography of strongly differing altitude, the parameters defining this vertical decay have to be chosen such that significant grid deformation still occurs at elevations far above the surface.

Several studies have highlighted the detrimental impact of explicit horizontal diffusion schemes acting on model levels which are employed as numerical filters to damp short waves (Zängl, 2002; Tudor, 2010; Wilson and Fovell, 2018). In a previous study (Westervhuis et al., 2020, hereafter W20), we have shown that also implicit numerical diffusion can heavily affect simulations of fog and low stratus clouds, even if the explicit numerical diffusion is completely turned off. Comparison between forecasts issued by the COSMO (COnsortium for Small-scale MOdeling) model and satellite images has revealed that the model often dissipates these clouds too quickly. Based on a detailed analysis of a case study we have deduced that sloping vertical coordinate surfaces promote excessive dissipation of fog and low stratus for typical situations over the Swiss Plateau due to the spurious numerical diffusion associated with the advection scheme.

Previous studies have aimed at reducing numerical diffusion in simulations of clouds by employing weighted non-oscillatory schemes which capture sharp gradients well (Baba and Takahashi, 2013; Lunet et al., 2017). The current study aims to enhance our understanding about the relationship between sloping vertical coordinate surfaces and the associated implicit diffusion related to the advection of the prognostic quantities.

Possibilities to study this behaviour in a systematic way are limited using a fully three-dimensional NWP model and with real case simulations, due to various co-acting numerical and physical factors. We instead conduct idealised two-dimensional simulations

of a low stratus cloud (Section 2). We analyse how the evolution of the stratus cloud during night-time is affected by increasingly sloping vertical coordinate surfaces, either by artificially perturbing the model grid above a flat surface or by using idealised hilly orography. We compare these simulations to a flat, horizontally homogeneous reference simulation (Section 3). These simplifications allow us to isolate the effect of sloping vertical coordinate surfaces while still working with a close-to-real case setup. In Section 4, we discuss the implications of our findings for operational NWP models and ways to improve forecasts. Our findings are also relevant for other meteorological phenomena with non-negligible horizontal wind speed and exhibiting sharp vertical gradients in the lower part of the atmosphere: e.g. fog, cold air pooling, air pollution and smog.

2 Model setup

2.1 Description of the COSMO model

COSMO¹ is a regional weather prediction and climate model used at numerous national weather services and universities. It solves the non-hydrostatic, fully compressible, hydro-thermodynamical equations using finite difference methods on an Arakawa C-grid (Arakawa and Lamb, 1977; Steppeler et al., 2003; Förstner and Doms, 2004). The model configuration used in this study closely follows the operational configuration with a horizontal grid spacing of 1.1 km (hence called COSMO-1) employed at MeteoSwiss, the Swiss Federal Office of Meteorology and Climatology.

The vertical coordinate transformation is the terrain-following, smooth level vertical (SLEVE) coordinate (Schär et al., 2002; Leuenberger et al., 2010). The corresponding decay constants (named $s1$ and $s2$ in these publications) for large-scale and small-scale orographic features are set at 10,000 m above sea level (asl) and 3,300 m asl, respectively. Above 11,357 m asl the topographic signature in the coordinate surfaces decays completely and model levels become flat.

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¹<http://www.cosmo-model.org/>

A 5th-order upstream advection scheme is used for wind velocity, temperature and pressure perturbations (w.r.t a reference profile) in the horizontal, turbulent kinetic energy (TKE) is not advected. Vertical transport is represented by a 2nd-order implicit Crank-Nicholson scheme (Baldauf et al., 2011). A two-time-level, 3rd-order Runge-Kutta discretisation with a time-step of $\Delta t = 10$ s (unless denoted otherwise) is used for the time integration (Klemp and Wilhelmson, 1978; Wicker and Skamarock, 2002).

Six moist quantities are simulated: Water vapour, cloud water, cloud ice, rain, snow, and graupel. They require a different advection scheme, as only positive values are allowed. They are transported using a one-dimensional positive-definite advection scheme (Bott, 1989; Schneider and Bott, 2014) with Strang splitting (Strang, 1968). A single-moment bulk microphysics scheme describes the conversion between these moist quantities (Reinhardt and Seifert, 2006). The convection parameterisation is turned off.

It is important to note that these are two different advection schemes, which are however intertwined. The advection of the moist tracers is dictated by the wind field which itself is transported with the upstream advection scheme. As this study is about low stratus clouds the focus lies on the advection of the tracers. We have conducted additional tests combining the tracer-advection scheme with a 3rd-order upstream advection scheme, a 4th-order centred scheme requiring minimal explicit horizontal diffusion of the wind velocities, and a symmetric 4th-order centred scheme which conserves kinetic energy (Ogaja and Will, 2016) – all combinations result in qualitatively similar findings as when using the 5th-order upstream scheme.

The turbulence closure is used for subgrid vertical mixing only. The turbulent flux of a scalar quantity is estimated via K-theory, relating the subgrid-scale flux to the product of the vertical gradient of the quantity and a turbulent diffusion coefficient. The vertical turbulent diffusion coefficients are calculated using a prognostic TKE-based 1.5-order scheme (Raschendorfer, 2001) with a level 2.5 closure (following Mellor and Yamada, 1974, 1982). A minimum vertical turbulent diffusion coefficient of $0.4 \text{ m}^2/\text{s}$ is applied (Buzzi, 2008).

Radiative processes are described with a delta-two-stream approach after Ritter and Geleyn (1992). The contributions of each model process to the total tendencies of temperature and moisture are retrieved using the COSMO budget-diagnosis tool (Langhans et al., 2012).

The COSMO model offers an option for fourth-order horizontal diffusion to remove shortwave grid-scale noise (Doms and Baldauf, 2011). This explicit diffusion is turned off completely within the domain for all simulations.

2.2 Experimental setup for idealised simulations

We perform idealised simulations of a stratus cloud in two dimensions (x and z). The balloon radiosonde measurements obtained at Payerne (Switzerland) on December 25, 2017, 00 UTC serve to initialise the simulations. Case details were described in W20.

The liquid water content (LWC) is not measured directly by the radiosounding, so a spin-up simulation serves to initialise all other simulations. It is initialised with vertical profiles of pressure, temperature, wind speed, and relative humidity from the radiosonde. After 3 hours, the cloud extent, i.e. elevation of cloud base and cloud top, matched the observations (see W20); the maximum liquid water content is around 0.3 g/kg and located near the cloud top, similar to simulations by Bergot (2016). The vertical profiles of pressure, temperature, wind speed, specific humidity, and LWC obtained from the spin-up simulation at 3 h are used to initialise the other experiments. These are all restricted to night-time.

The soil (represented with the multi-layer soil model TERRA-ML, Heise et al., 2006) is initialised with a constant temperature matching the surface temperature and zero soil moisture (additional experiments have shown small sensitivities to prescribed latent and sensible heat fluxes; not shown). No Coriolis forcing is applied.

The reference simulation (REF) uses a horizontal grid spacing of 1.1 km, a flat surface at 491 m asl, and 80 flat vertical coordinate surfaces. The level thickness amounts to 19 m at the surface and 44 m at the cloud top altitude.

Sloping vertical coordinate surfaces are generated in two different ways: 1) the surface is kept flat but the vertical coordinate surfaces are artificially perturbed (hereafter referred to as PERT), and 2) the flat surface is replaced by hills (hereafter referred to as HILL). For both configurations, the perturbations and orography, respectively, consist of a periodic sinusoidal wave, y , defined by its amplitude, h , and wavelength, λ :

$$\lambda = \frac{L_x}{N_{\text{waves}}} \quad (1)$$

$$y(x) = h \sin\left(\frac{2\pi x}{\lambda}\right). \quad (2)$$

N_{waves} is the number of waves contained in the periodic domain with $L_x = 110$ km. $\Delta x = 1.1$ km, unless otherwise specified. The sinusoidal signal can also be characterised by α , which describes the angle of the slope directly connecting the lowest with the highest point (see Figures 5 and 7):

$$\alpha = \arctan\left(\frac{4h}{\lambda}\right) \quad (3)$$

For PERT, the perturbation is strongest at a specific altitude, z_{max} , above the surface. Between the surface and $2z_{\text{max}}$, the sinusoidal signal is made to gradually decay by applying a weighting factor, w_{pert} . So the altitude of the perturbed levels (L_{pert}) is calculated from the original levels (L_{orig}) as:

$$L_{\text{pert}} = L_{\text{orig}} + w_{\text{pert}}y; L_{\text{orig}} < 2z_{\text{max}}, \quad (4)$$

$$w_{\text{pert}} = \cos\left(\frac{\pi}{2} \left| \frac{L_{\text{orig}} - z_{\text{max}}}{z_{\text{max}}} \right| \right). \quad (5)$$

$z_{\text{max}} = 400$ m in our simulations, which roughly corresponds to the cloud top at 900 m asl.

Figures 1 and 2 illustrate the distribution of the vertical levels for $h = 100$ m and $\lambda = 11.1 \Delta x$ for PERT and HILL.

For the advection of the humidity tracers, either the 2nd-order or the 4th-order Bott

scheme is employed (Bott, 1989).

3 Results

3.1 Reference simulation: Flat surface, flat vertical coordinate surfaces

Figure 3 shows the 6-hour COSMO model simulation of the stratus cloud during night-time for REF. The cloud gradually becomes deeper and thicker; the maximum LWC increases from 0.3 g/kg to 0.57 g/kg, and the cloud base descends from 620 m asl to the surface (500 m asl). The cloud top altitude remains constant (the location of the maximum LWC shifts upwards by one model level).

Near the cloud top, the temperature profile exhibits a strong inversion. This inversion is maintained throughout the simulation and the whole lower atmosphere experiences a continuous cooling of approximately 2 °C in total. The total humidity profile (sum of LWC and specific humidity) is well-mixed below the inversion, where the total humidity decreases slightly from 3.81 g/kg to 3.66 g/kg over 6 hours. The wind speed at the beginning of the simulation is 3-4 m/s below the inversion and 7 m/s at 1100 m asl. Below this altitude, the wind speed decreases by about 1-2 m/s during the simulation, as can be expected for such an idealised simulation which lacks a pressure gradient forcing.

Figure 4 shows the budget analyses (Langhans et al., 2012) for temperature (a) and LWC (b and c): The cloud evolution is driven by longwave radiative cooling at the cloud top (dash-dotted line, panel a). This leads to condensation of cloud droplets (dashed line, panel b). Turbulent mixing acts to maintain a well-mixed cloud layer (dotted line, panel b). As vertical velocities are negligible and the setup is completely homogeneous in the horizontal dimension, the advective tendencies are insignificant (full lines, panel a and b).

The fully three-dimensional simulation with the COSMO model over the complex orography of Switzerland for this case differs significantly from REF: The stratus cloud dissipated within a few hours (W20). The goal of this study is not to directly evaluate

the performance of REF in comparison with measurements but to use this simulation as a reference to explore the effects of sloping vertical coordinate surfaces.

3.2 Impact of sloping vertical coordinate surfaces above flat surface

This section explores how sloping vertical coordinate surfaces above a flat surface affect the evolution of the idealised stratus cloud. These experiments are designed to isolate the purely numerical effects as a result of changes applied to the computational grid; the physical processes affecting the cloud remain the same as in REF.

Figure 5 compares the zonally averaged liquid water path (LWP; i.e. vertically integrated LWC) of the stratus cloud for setups with sinusoidally perturbed vertical coordinate surfaces with varying wavelength, λ , and amplitude, h . For easier representability, the x -axis is chosen to display the wavenumber (inverse wavelength) $1/\lambda$; $1/\lambda = 0.0$ thus represents REF. E.g. $1/\lambda = 0.09/\Delta x$ (vertical line) refers to the setup with $\lambda = 11.1 \Delta x$. When $h = 100$ m and $\Delta x = 1.1$ km, α is 1.87° – which would be experienced as a very gently rising slope by a walking human.

The LWP is shown 3 hours into the simulation. The setups with sloping vertical coordinate surfaces clearly differ from REF: At the beginning, the cloud in all setups has a LWP of 0.064 kg/m^2 , indicated by the horizontal line. After 3 hours, with $\lambda = 11.1 \Delta x$ and $h = 100$ m, the LWP is only 0.065 kg/m^2 as opposed to 0.118 kg/m^2 for REF. With sloping vertical coordinate surfaces, the cloud at first becomes thinner and then only slowly grows again, or continues decreasing until its full dissipation (e.g. with $\lambda = 4.8 \Delta x$ and $h = 200$ m).

The associated budget analysis (Figure 4c) shows that the presumably numerical contribution from the advection scheme (full line, panel c) is associated with the decrease of the LWC in the uppermost part of the cloud and contributes to its increase above the cloud top. This is in contrast to REF, where advective tendencies are negligible and have no impact on the cloud evolution. The negative microphysics tendency (dashed line, panel

c) above the cloud top indicates that the displaced cloud droplets are mostly evaporated. Over time, the accumulation of LWC above the cloud top is sufficient to make the cloud grow upwards.

This upward growth is illustrated in Figure 6: It shows the zonally averaged LWC-profile after 3 hours. For REF (dash-dotted line), the cloud top is located at 930 m asl, while it is at around 1000 m asl when $1/\lambda = 0.09/\Delta x$ and $h = 100$ m. However, the maximum LWC is considerably reduced (0.29 g/kg instead of 0.48 g/kg).

As the temperature inversion at the cloud top strongly inhibits vertical advection across the cloud top and overall vertical winds remain very small, this behaviour is a result of spurious numerical diffusion from horizontal advection through sloping coordinate surfaces. (See Appendix A for an analysis of mass-conservation of the advection scheme.)

Overall, for shorter wavelengths the impact of numerical diffusion becomes more severe and the LWP 3 h into the simulation is further reduced. Furthermore, the stratus cloud decrease is more severe for larger amplitudes ($h = 200$ m) and less severe for smaller amplitudes ($h = 50$ m), as shown in Figures 5 and 6).

To summarise, the LWP correlates with the wavelength of the sinusoidal perturbation and this correlation is somewhat stronger than its inverse correlation with the amplitude of the perturbation; cutting λ by half reduces the LWP slightly more than doubling h .

3.3 Impact of sloping vertical coordinate surfaces due to hills

Experiments in the previous section with a flat surface isolated the impact of sloping vertical coordinate surfaces since the cloud layer should in principle be horizontally homogeneous. In real-case model forecasts, topography causes the vertical coordinate surfaces to be sloped in a way similar to what has been presented in the previous section. In the following, we take an additional step towards reality by introducing sinusoidal orography (see Eq. 2). As the orographic signal only slowly decays with height, the vertical level distribution near the elevation of the cloud top (between 900 m asl and 1000 m asl) looks nearly identical for PERT and HILL (compare dots in Figures 1 and 2).

Figure 7 shows the LWP as a function of $1/\lambda$ where λ refers to the wavelength of the sinusoidal orography. Overall, it looks similar to Figure 5: The LWP continuously decreases with decreasing λ . Also the horizontal-mean vertical LWC profiles (dotted lines in Figure 6) for $h = 50$ m and 100 m are similar.

The differences between PERT and HILL are largest for $h = 200$ m. As the hills become taller, their effect on the flow-dynamics in the boundary layer becomes more pronounced and starts to interact with the stratus cloud. This leads to additional heterogeneity in the stratus cloud, as can be seen in Figure 8: As long as the hill crests are clearly below the stratus cloud (b), the LWC distribution looks homogeneous in the horizontal dimension, similar to PERT (a). When the hill crests are higher (c), the LWC is much more unevenly distributed which enhances the dissipation of the stratus cloud (see also dotted line for $h=200$ m in Figure 6).

Findings with the HILL-cases corroborate what has been identified in the previous section: Horizontal advection in the presence of sloping coordinate surfaces can introduce spurious numerical mixing. Even small-scale terrain features with amplitudes of 50-100 m can have a strong adverse effect on the evolution of a stratus cloud topped by a strong temperature inversion.

3.4 Order of the tracer advection scheme

In the operational setup of the COSMO model at MeteoSwiss, a 2nd-order scheme is used for the advection of the humidity tracers. Optionally, a 4th-order scheme can be used instead. Bott (1989) showed that the numerical diffusion is reduced and sharp gradients better maintained when using higher-order schemes in simplistic experiments of a cone advected in rotational or deformational flow fields.

The dashed lines in Figures 5, 6, and 7 show the results for simulations employing the 4th- instead of 2nd-order scheme. Both schemes generate the same LWP for REF (at $1/\lambda = 0.0$ in Figure 5). For both, PERT and HILL, the higher-order scheme generates higher LWP values, e.g. with $h = 100$ m and $\lambda = 11.1\Delta x$, the 2nd-order scheme reaches 55% of

the LWP produced in REF while the 4th-order scheme reaches 61%. However, the overall LWP loss with decreasing λ is similar and dominates the evolution of the stratus cloud.

We conclude that while a higher-order advection scheme can indeed decrease the spurious mixing in the presence of sloping coordinate surfaces, it is insufficient to resolve the strong detrimental effect on the evolution of a stratus cloud topped by a temperature inversion.

3.5 Sensitivity to grid resolution

The solution a numerical weather prediction model provides should converge as we decrease grid spacing and time step while keeping their ratio constant. In line with Figure 5, Figure 9 shows the LWP for PERT with $h = 100$ m after 3 hours – but as a function of α (see Eq. 3) instead of $1/\lambda$. α relates to the "steepness" of the sinusoidal perturbation, regardless of Δx , hence setups with unequal horizontal grid spacing can be compared appropriately.

As in Figure 5, the simulations using the 2nd-order (full line) and 4th-order (dashed line) advection scheme are displayed. Additionally, setups with twice the amount of grid points in the horizontal ($\Delta x = 0.55$ km and $\Delta t = 5$ s, dotted line) or vertical (160 levels, dash-dotted line) are shown.

At time $t = 0$, all setups exhibit a LWP of 0.064 kg/m^2 (indicated by horizontal line). After 3 hours, REF (i.e. $\alpha = 0^\circ$) with 80 vertical levels exhibits a LWP of 0.118 kg/m^2 . Employing 160 (flat) vertical levels results in a lower LWP: 0.104 kg/m^2 (full line vs dashed-dotted line). The decrease in LWP with increasing α when employing 160 (sloping) vertical levels is comparable.

Doubling the number of grid points in the horizontal ($\Delta x = 0.55$ km), reduces the decrease in LWP with increasing α – even more than using the 4th-order scheme (dotted compared to full or dashed line). However, this reduction is not linear: When $\alpha = 2^\circ$ and $\Delta x = 0.55$ km, a LWP of 0.076 kg/m^2 is obtained. Doubling α from 2° to 4° decreases the LWP to 0.050 kg/m^2 (-35 %) while, on the other hand, doubling Δx from 0.55 km to

1.1 km merely decreases the LWP to 0.062 kg/m^2 (-18 %).

At approximately 400 m above ground, the cells of a numerical weather prediction model typically have a very large aspect ratio being tens of meters thick but hundreds of meters wide. While increasing the resolution help somewhat mitigate the implicit numerical diffusion of the stratus cloud simulation in terrain-following coordinates over hilly terrain, the steepness of the vertical coordinate surfaces appears to be a critical parameter which can be only partially mitigated by increasing the number of grid points in the horizontal dimension. For HILL, the findings are qualitatively the same (not shown).

4 Implications for real case applications

In the following, we discuss the implications of our idealised experiments for fully three-dimensional operational NWP models. The orography of the COSMO model with a horizontal resolution of 1.1 km is smoothed using a low-pass filter (Raymond, 1988) with a cutoff of about $4\text{-}5\Delta x$. Despite the smoothing of the orography, it features small-scale topographic structures of small enough wavelength and large enough amplitude to cause spurious implicit numerical diffusion in the idealised simulations, see Appendix B.

The use of a higher-order advection scheme shows modest improvements for the idealised simulations. A simulation of the case study presented in W20 using the 4th-order instead of the 2nd-order scheme for the advection of the humidity tracers exhibits only small differences for the clouds (not shown). Therefore we conclude that, for a fully three-dimensional NWP model, the additional computational costs are likely not worth the improvement.

Likewise, considering the limited improvements attained by increasing the number of grid cells in the horizontal dimension of the idealised as well as real-case (see W20) simulations, this seems like a too costly measure, at least for large domains: Cutting the grid spacing by half (two dimensions) and a consequent reduction of the timestep is

roughly $2^3 = 8$ times more expensive. Moreover, increasing the model resolution in real-case simulations goes hand in hand with introducing additional small-scale orographic features which may partly cancel the benefits of having more grid cells.

We have shown in W20 that smoothing the orography while maintaining the grid spacing shows a greater beneficial impact on the simulated fog and low stratus extent than increasing the resolution. Similarly, for the idealised simulations, the smoothness of the vertical coordinate surfaces (i.e. small h and large λ , or small α) has proven to be more determining than the number of grid points.

Additional orographic smoothing comes with no additional computational cost and partly mitigates the diffusion at the cloud top as the vertical coordinate surfaces become less steep. However, excessive smoothing may deteriorate several other aspects of a NWP model. First, e.g. for data assimilation purposes, it is desirable that the altitude of a specific grid point in the model corresponds as closely as possible to its true altitude, as far as the variables at the surface are concerned. Furthermore, processes such as orographically-induced convection and valley winds (e.g. Weusthoff et al., 2010; Schmidli et al., 2018) require detailed orography to represent small-scale physical drivers correctly.

The question remains whether the advection schemes of the COSMO model are too diffusive or if the described problems with sloping vertical coordinate surfaces above complex orography could apply to other models as well. Kinetic energy spectra calculated from the horizontal and vertical wind velocities reveal the "effective" resolution of a NWP model. A coarse effective resolution indicates that the model is too diffusive, even though the grid spacing may be very fine (e.g. Ricard et al., 2013). The effective resolution of the COSMO model ($4-5 \Delta x$, Bierdel et al., 2012) is similar to other high-resolution NWP models, such as WRF ($7 \Delta x$, Skamarock, 2004), Meso-NH ($4-6 \Delta x$), and AROME ($9-10 \Delta x$, Ricard et al., 2013). We therefore suppose that the encountered issues with sloping vertical coordinates could occur similarly.

A last comment concerns other types of clouds which are simulated with a NWP model. Characteristic for low stratus clouds (and fog) are the weak physical drivers, and

their homogeneity in the horizontal dimension. E.g. cumuliform clouds show larger variability in the horizontal, develop faster, and exhibit stronger vertical motions (Lohmann et al., 2016). We therefore hypothesize that the numerical diffusion associated with sloping vertical coordinates is less dominant compared to other processes driving the cloud evolution, but might nevertheless be an issue and should be explored in future studies.

5 Summary

The impact of a terrain-following vertical coordinate and the associated sloping coordinate surfaces aloft on the simulation of a stratus cloud has been explored. The evolution of the cloud during night-time has been compared for three different configurations of the COSMO model in two dimensions: 1) Flat surface, perfectly flat vertical levels, 2) Flat surface, vertical levels perturbed with a sinusoidal signal, and, 3) sinusoidal orography.

In completely flat conditions, the stratus cloud grows, driven by radiative cloud top cooling. With perturbed model levels, this growth is diminished or even reversed. The liquid water path (LWP) of the stratus cloud correlates (anti-correlates) with the wavelength (amplitude) of the sinusoidal perturbations. The effect of small-amplitude hills effectively results in the same behaviour as perturbations above a flat surface as long as the hills do not significantly reach into the boundary layer. A detailed analysis of the temperature and moisture budgets reveals vertical mixing across the cloud top due to advection despite vertical wind speeds being negligible. Spurious numerical diffusion associated with the horizontal advection scheme of the moisture variables stands out as the root cause for the unphysical dissipation of the stratus cloud.

Tests with a higher-order version of the advection scheme show limited improvements. Additional vertical levels do not significantly impact the decrease in LWP attributed to the sloping vertical coordinate surfaces. On the other hand, increasing the horizontal resolution moderately increases the resulting LWP of the perturbed as well as hilly simulations. However, for a fully three-dimensional numerical weather prediction model, increasing

the resolution is very costly in terms of computational effort and cannot be done easily. Also, as resolution is increased, additional small-scale topography is introduced which may partially diminish the benefit.

Stronger smoothing of the orography would clearly improve the stratus cloud simulations. However, this measure would decrease the quality of forecasts of other meteorological phenomena such as valley winds and orographically induced convection. A way to approach this trade-off problem could be to run different versions of the model, one of them with smoothed orography, as an ensemble. Running a small ensemble would be computationally cheaper than increasing the resolution for a single deterministic model run.

So what can be done to improve the representation of stratus clouds for meteorological situations with non-negligible horizontal wind speeds? We see multiple avenues that will be the topic of future research: First, advection schemes may be designed to specifically decrease the spurious numerical diffusion at steep gradients or discontinuities, and in strongly deformed coordinate systems. Second, immersed boundary methods (Lundquist et al., 2012) circumvent sloping coordinate surfaces, but require substantial modifications to existing models. Third, a vertical coordinate transformation that allows for a locally faster decay of small-scale orographic features as compared to the linear decay of the Gal-Chen coordinate transform or the globally specified decay functions of the SLEVE vertical coordinate, may help reduce the deformation of the computational grid aloft.

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Appendices

A Mass conservation

Figure A1 shows the evolution of the mass of a tracer for a PERT setup with $h = 100$ m and $\lambda = 11.1 \Delta x$. The tracer is initialised as 1 for the lowest levels up to the cloud top and 0 above. It is affected by advection and turbulent mixing. The full line represents the evolution of the mass of the tracer summed up over the full domain and the dotted line the sum over the grid cells up to the cloud top. The total mass of the tracer increases during the simulation by about 2 %. This indicates that the model is not perfectly mass-conserving. However, the effects of advection and turbulent mixing are substantially more dominant: The tracer mass in the lower levels decreases to 66 % as it is mixed upwards.

The mass of the humidity tracers is additionally affected by the saturation adjustment which is an isobaric process and therefore not perfectly mass-conserving either. The dashed line shows the evolution of the total water mass. The increase in total mass is less than 1 %. We conclude that mass conservation is not an issue for the interpretation of our simulations.

B Spectral analysis of orography

To assess whether the amplitude and wavelength of the sinusoidal hills of the idealised simulations are representative for the 1.1 km-grid-spacing orography used in the operational COSMO model, we conduct a spectral analysis of the orography of 64x64 grid cells surrounding Zürich airport:

1. Calculate the 2-dimensional Fast Fourier Transform, FFT_{2dim} , of the orography, O , standardised by its mean, $\bar{O}$, which has been tapered with a Hann window, H , to derive the FFT-coefficients, X :

$$X = FFT_{2dim} \left(H(O - \bar{O}) \right). \quad (6)$$

2. Calculate the magnitudes, M , as the absolute values of X divided by the square of the total number of grid points, $N = 64 \times 64$:

$$M = \frac{|X|}{N^2}. \quad (7)$$

The first coefficient of M contains the mean orography, the sum of the remaining coefficients squared equals the variance of O (Parseval's Theorem).

$$M[1, 1] = \bar{O} \quad (8)$$

$$\sum M^2 - \bar{O}^2 = \text{Var}(O) \quad (9)$$

3. Calculate the inverse Fast Fourier Transform of X for frequencies larger than a specific cut-off frequency to derive the high-pass orography, O_{HP} . The standard deviation of $O_{HP} \times 8/3$ (to account for the Hann taper) can be compared with the standard deviation of the sinusoidal signal of the perturbations, and hills of the idealised simulations.

E.g., when the cut-off frequency = 6, the standard deviation of $O_{HP} = 37.6$ m. FFT-coefficients for frequencies ≥ 6 correspond to signals with $\lambda \leq 11.7$ km. In the idealised simulations numerical diffusion clearly is an issue at this λ for the selected amplitudes. The standard deviation of a sine-wave is given by $h/\sqrt{2}$; when $h=50$ m, this results in a standard deviation of 35.4 m, which is comparable to the standard deviation of O_{HP} .

References

Figure 1: Smooth level vertical coordinates (Schär et al., 2002) which have been artificially perturbed with a sinusoidal wave ($N_{waves}=9$) characterised by $h = 100$ m and $\lambda = 11.1 \Delta x$. $\alpha = 1.87^\circ$ when $\Delta x = 1.1$ km.

Figure 2: Smooth level vertical coordinates (Schär et al., 2002) above sinusoidal orography: $h = 100$ m and $\lambda = 11.1 \Delta x$. $\alpha = 1.87^\circ$ when $\Delta x = 1.1$ km.

Figure 3: Night-time evolution of idealised stratus cloud simulated with the COSMO model: 6-hour simulation of liquid water content (a), temperature (b), total humidity (sum of specific humidity and liquid water content, c), and wind speed (d). Initialisation based on 3 hour spin-up simulation started with measurements from balloon radiosounding launched at Payerne (Switzerland) on December 25, 2017, 00 UTC.

Figure 4: Budget analysis of idealised stratus cloud simulated with the COSMO model for temperature (a) and liquid water content (b and c) from +2 hours to +3 hours: Total tendency consisting of advection (full), microphysics (dashed), turbulence (dotted), and radiation (dash-dotted). (a) and (b): completely flat setup (REF). c): sloping vertical coordinate surfaces (PERT). The zonal mean for the whole domain is displayed. Initialisation based on 3 hour spin-up simulation started with measurements from balloon radiosounding launched at Payerne (Switzerland) on December 25, 2017, 00 UTC.

Figure 5: Zonal mean liquid water path of idealised stratus cloud after 3 hours simulated with the COSMO model as a function of the wavenumber, $1/\lambda$, of the sinusoidal perturbation (PERT) generating sloping vertical coordinate surfaces above a flat surface with $h = 50$ m, 100 m, and 200 m. Either the 2nd-order (solid lines) or 4th-order scheme (dashed lines) is employed for the advection of humidity-tracers. The liquid water path is the same for all setups at initialisation time (horizontal line). The vertical profiles of the liquid water content for $\lambda=11.1\Delta x$ (vertical line) are displayed in Figure 6.

Figure 6: Zonal mean vertical profile of liquid water content of idealised stratus cloud simulated with the COSMO model after 3 hours. Completely flat setup (REF, dash-dotted line) is compared with setups exhibiting sinusoidally perturbed vertical coordinate surfaces above flat surface (PERT, full and dashed lines) and setups comprising sinusoidal orography (HILL, dotted lines) with $\lambda=11.1\Delta x$ and $h = 50$ m, $h = 100$ m, and $h = 200$ m. Further, for PERT, sensitivity to using a 4th-order (dashed lines) instead of a 2nd-order scheme (full lines) is shown (dashed lines)

Figure 7: Same as for Figure 5, but $1/\lambda$ refers to sinusoidal orography (HILL).

Figure 8: Vertical cross-section of the liquid water content of an idealised stratus cloud simulated with the COSMO model after 3 hours. All setups exhibit sloping vertical coordinate surfaces: Sinusoidally perturbed coordinates above flat surface with $h = 100$ m (a), sinusoidal orography with $h = 100$ m (b), and $h = 200$ m (c).

Figure 9: Same as for Figure 5, but the liquid water path is shown as a function of α (see Eq. 3) which indicates the steepness of the sloping vertical coordinate surfaces. In addition to the setups employing the 2nd- and 4th-order scheme with $\Delta x=1.1$ km and 80 vertical levels (full and dashed line), additionally setups with $\Delta x=0.55$ km or 160 vertical levels are shown (dotted and dash-dotted line).

Figure A1: Evolution of mass relative to initialisation for a setup with sinusoidally perturbed vertical coordinate surfaces (PERT; $h = 100$ m and $\lambda = 11.1 \Delta x$) for a tracer which undergoes advection and turbulent mixing (full and dotted lines) and for total humidity (sum of specific humidity and liquid water content, dashed line). The initial humidity profile is derived from a balloon radiosounding launched at Payerne (Switzerland) on December 25, 2017, 00 UTC. The tracer is initialised as 1 from the surface up to the initial cloud top and 0 above. Its mass is integrated over the full domain (full line), or only over the grid cells where the tracer occurs at initialisation time (dotted line).