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A Limited Information Sharing Strategy for the Taxi-Customer Searching Problem in the Non-Booking Taxi Service

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ABSTRACT

One of the issues existed in the current taxi service is the imbalance between supply and demand. In response to this issue, the automatic taxi dispatching approach in which customers can book taxis through phones or mobile devices is widely used in many large cities worldwide. However, the utilization of this approach is not satisfied, since most customers still prefer the Non-Booking Taxi Service (NBTS) that taking the taxi by either waiting at taxi stand or hailing on the street. One important reason for this phenomenon is that the customer takes lower risk in NBTS: they are free from complicated booking procedures and have no commitment to any yet-arrived taxis. In order to facilitate the taxi-customer matching process in NBTS, this paper has proposed a novel control strategy namely the Limited Information Sharing Strategy (LISS) for the Taxi-Customer Searching Problem (TCSP) in NBTS, in which both the taxi and the customer are equipped with mobile devices that can communicate with each other within limited searching ranges. The proposed LISS is based on the game-theoretical formulation in which a learning algorithm is developed to find the pure Nash-Equilibrium (NE). A microscopic traffic simulation model is developed for the evaluation of the LISS. The simulation results show that the proposed LISS is an effective control strategy when taxi supply is low, and it will not increase the risk of taxi in terms of losing the total occupied time.

Keywords: Game theory, Limited Information Sharing Strategy (LISS), Microscopic traffic simulation, Non-Booking Taxi Service (NBTS), Taxi-Customer Searching Problem (TCSP)
1. INTRODUCTION

One issue existed in the taxi service market of today is the imbalance between taxi supply and demand \((I)\), which may cause two negative impacts to the demand side and supply side of the taxi: one is the longer waiting time of customers who are waiting at taxi stands or on the streets; the other is the longer empty taxi cruising time. The two negative impacts cause not only the waste of social resources for both customers and taxi drivers but also environmental problems such as the emissions generated by taxis when they are searching and waiting for customers on the congested road network.

To alleviate the aforementioned issue, automatic taxi dispatching approaches have been widely used in many large cities worldwide, in which customers can book taxis directly through phones or mobile devices \((2, 3)\). Compared with the traditional ways that hailing on the street or waiting at taxi stand, booking taxis through the dispatching system has more advantages: it provides an alternative way for customers and taxi drivers to find each other easily \((4)\). However, one practical problem in adopting the dispatching system is that customers may not have strong willingness to take taxis by booking. For example in Singapore, the largest taxi company ComfortDelGro has received an average of around 65,000 booking calls daily in the year of 2010; however it only accounted for about 17\% of the total daily trips made by its taxi fleet \((5)\). In Beijing, the taxi booking rate is even lower as compared with the case in Singapore \((6)\). In other words, waiting at taxi stand or hailing on the street for incoming available taxis may still be the major and popular ways of getting taxis.

To differentiate between taxi service with bookings to that without bookings, the following two terms are defined in this paper:

- Booking Taxi Service (BTS): taking taxis by booking through phones or mobile devices;
- Non-Booking Taxi Service (NBTS): taking taxis by either waiting at taxi stand or hailing on the street.

One obvious difference between these two types of taxi service is that customers need to reserve taxis in BTS but do not need in NBTS. Furthermore, another important difference is that both taxis and customers who are searching for each other actually bear different levels of risks accordingly. For example, in BTS, the taxi takes lower risk while the customer takes higher one: once a taxi has confirmed with a booking request, the customer who has made the booking should wait until the arrival of the taxi; however, in NBTS, the taxi takes higher risk while the customer takes lower one, this is because the taxi-customer searching (or matching) process in NBTS is not bounded to any agreement so that a customer can take any available taxis coming to his/her location, but a taxi receives no guarantee to find a customer when heading to any taxi stand.

Thus, the problem to be studied in this paper can be described as the Taxi-Customer Searching Problem (TCSP) in Non-Booking Taxi Service (NBTS), TCSP-NBTS in short. The objective of this paper aims to seek for an efficient control strategy for the TCSP-NBTS, which improves not only the Level Of Service (LOS) in terms of reducing the Customer Waiting Time (CWT) but also reduce (or mediate) the risk of taxis in a certain level when they are searching for customers.

Many modeling approaches for the taxi service have been developed in recent years of which the representatives were mainly in the form of mathematical models and simulation models. Yang et al. \((7-9)\) proposed a mathematical model for the taxi service and performed a series of analysis. Cheng and Nguyen \((10)\) proposed a macroscopic simulation model for the taxi service and studied the taxi fleet optimization problem. Both Lee et al. \((11)\) and Seow and Lee \((4)\) proposed microscopic simulation models to explore efficient dispatching
approaches in BTS: the former presented a shortest travel time dispatching rule based on current traffic conditions while the latter proposed an agent-based dispatching policy that has enabled taxis to negotiate and cooperate with each other to achieve group objectives. There are still other studies (12-15) on the topic of taxi service which are more or less based on or related to the aforementioned models.

However, the existing taxi modeling approaches are inadequate to study the control strategies for the TCSP-NBTS. On one hand, the mathematical and macro-simulation models have been formulated in highly aggregated forms which is difficult to capture the micro-level details such as the dynamic customer behaviors (e.g. booking, cancellation, etc.) and the processes of control strategies (e.g. automatic dispatching, information sharing, etc.); on the other hand, even though the microscopic simulation models could be used for studying the taxi service and corresponding control strategies in a detailed level, the dynamic customer behaviors were not considered. Moreover, the simulation-based models mostly focused on the dispatching strategies for BTS but not the control strategies for NBTS. For these reasons, this paper continues to model the taxi service using the microscopic simulation approach and is integrated with more functions to meet the following research objectives:

- Model the TCSP-NBTS;
- Consider dynamic customer behaviors;
- Propose and test a novel control strategy for TCSP-NBTS, namely the Limited Information Sharing Strategy (LISS).

The Limited Information Sharing Strategy (LISS) is a decentralised control strategy that requires both the taxi and the customer to be equipped with mobile devices that can form ad-hoc networks among them. Unlike other decentralised control strategies such as the agent-based dispatching approaches proposed by Seow and Lee (4) in which taxis could communicate with each other (but not with customers) to find the optimal solution, the proposed LISS will enable customers to communicate with taxis via mobile devices directly so as to reduce the taxi-to-taxi communication costs. The LISS proposed in this paper will adopt the game theoretical formulation, which has been applied in few other related areas such as the Vehicle-Target Assignment Problem (VTAP) (16-19). It has to be noted that the TCSP-NBTS is not simply a variant of VTAP but has the following distinctive characteristics:

- Dynamic behaviors of both the taxi and the customer need to be considered;
- Customers may wait at the same geographical locations, i.e. queuing in the same taxi stand;
- The travel time between the taxi and the customer may be affected by the road traffic conditions;
- The definitions for the global utility of the game and the individual utility of the player (the taxi or the customer) in the game theoretical formulation may consider a number of theoretical and practical problems.

In summary, the ultimate goal of this paper is to develop and test the devised LISS for the TCSP-NBTS. The microscopic traffic simulation is adopted in this research as the approach for modeling and analysis of taxi operations. A plugin based on the APIs (Application Programming Interfaces) of the traffic simulator is designed to simulate the dynamic customer behaviors and the control strategies. The performance of the LISS will be evaluated and compared with the strategy without LISS in terms of two performance indicators: 1) the taxi Occupancy Rate (OR): the ratio between the total occupied time and
the total operating time of all taxis; 2) the Customer Waiting Time (CWT): the average waiting time of all customers.

The rest of the paper will be organized as follows: the problem formulation will be presented in Section 2; the solution algorithm will be introduced in Section 3; the simulation experiments will be presented in Section 4; followed by the conclusion in Section 5.

2. PROBLEM FORMULATION

The objective of this paper is to develop a LISS for the TCSP-NBTS which is expected to reduce the CWT of the customer and reduce (or mediate) the risk of the taxi (e.g., the probability of losing the total occupied time) in a certain level. Moreover, the Taxi-Customer Negotiation Process (TCNP) which is the core process in LISS will also be introduced.

2.1. The Problem Assumptions

Assumptions for taxi operations/customer behaviors in the TCSP-NBTS and the limited information sharing mechanism in LISS are presented in the following:

- **Taxi operations:** taxis are assumed to be running freely on a road network $G = (V, E)$ where $V$ is the set of nodes (junctions) and $E$ is the set of links (road segments). A vacant taxi $VT_i$ can pick up customer(s) at a taxi stand $TS_j \in TS \subset V$ where $TS$ is the set of all taxi stands (for simplification but without loss of generality, the case of picking up customers on a road segment $V_i \in V$ will not be considered in this problem). If a taxi has no customer occupied during the operating time, it will randomly choose a destination (e.g., a taxi stand) to look for a new customer; otherwise, it will be heading to the destination of the customer currently occupied on it.

- **Customer behaviors:** the arrival of customers to a taxi stand $TS_j \in TS$ is modeled as a Poisson point process which is similar to the modeling of service requests in Arsie et al. (18). Arrived customers will be then queued at taxi stands and waiting for taxis to arrive. If a customer has been waiting at a taxi stand for more than a certain period of time but no taxi arrives, the customer may decide not to wait any longer. This period of time is defined as the Maximum Customer Waiting Time (MCWT).

- **Limited Information Sharing:** both the taxi and the customer are assumed to be equipped with mobile devices that can communicate with each other. These mobile devices can form ad-hoc networks (e.g., IEEE 802.11 wireless networks) as decentralized control systems. The customers’ devices are detectable during their waiting periods. One constraint should be considered is that each mobile device has only limited searching range (e.g. 500 - 1000m), i.e., a mobile device can only communicate with others that are located within its searching range but not those who are outside the range.

2.2. The Taxi-Customer Negotiation Process (TCNP)

The Taxi-Customer Negotiation Process (TCNP) is designed as the core process in LISS which will be performed periodically in the system: assume at time $t$, a new round of TCNP which can be denoted as TCNP$(t)$ is about to start, there are $N_{VT}(t)$ numbers of vacant taxis running on different locations of the road network $G$. At the same time, there are $N_{WC}(t)$ numbers of customers waiting at $N_{WTS}(t)$ number of taxi stands. It is possible that $N_{WC}(t) \geq N_{WTS}(t)$ which means customers can be queuing at the same taxi stand. Then the TCNP$(t)$ will be performed to provide a solution on which taxi goes to which stand so that a global objective can be achieved.

It has to be noted that this strategy requires no commitment from the customer which is an important difference from other strategies such as the automatic/agent-based dispatching...
(4, 11). In this problem, a customer can at any time leave the taxi stand or choose another taxi even if a yet arrived vacant taxi have already decided to pick up him/her. The TCNP(t) only concerns the negotiation process in time t but not consider any future scenarios.

A number of methods can be adopted for performing and solving the TCNP, e.g., the decentralized agent-based approach proposed by Seow and Lee (4) where taxis could communicate directly with each other (but not with customers) to find the optimal solution in BTS. However, due to the huge amount of demand for the NBTS, the taxi-to-taxi communication will be mounted to a considerably high level in TCNP. Thus, a game-theoretical formulation is adopted to perform and solve the TCNP, which is also a decentralized system. In this type of formulation, only the taxi-to-customer communication is allowed so that the cost of taxi-to-taxi direct communication is saved. Therefore, the potential computational resource of the mobile device of the customer could be utilized.

2.2.1. Game-Theoretical Formulation for TCNP

The game-theoretical formulation for the TCNP can be described as follows: at time t, let the \( N_{VT}(t) \) vacant taxis are denoted as \( VT(t) = \{VT_1(t), \ldots, VT_{N_{VT}(t)}(t)\} \) and the \( N_{WC}(t) \) waiting customers are denoted as \( WC(t) = \{WC_1(t), \ldots, WC_{N_{WC}(t)}(t)\} \). A vacant taxi \( VT_i(t) \) can only communicate with limited number of waiting customers (due to the constraint of limited searching range), namely the candidate customers denoted as set \( CC_i(t) \subseteq WC(t) \) where:

\[
|CC_i(t)| = N_{CC_i(t)} \quad \text{and} \quad WC(t) = \bigcup_{1 \leq i \leq N_{VT}(t)} CC_i(t)
\]

The vacant taxi \( VT_i(t) \) can decide to choose any waiting customer in \( CC_i(t) \) to head to, and the decision of \( VT_i(t) \) can be denoted as \( a_i(t) \). If \( VT_i(t) \) has decided to choose \( WC_j(t) \) of \( CC_i(t) \) to head to, we can say that \( VT_i(t) \) has engaged to \( WC_j(t) \), or \( a_i(t) = WC_j(t) \). It has to be noted that \( VT_i(t) \) can also have no engagement to any waiting customer. The set of decisions of vacant taxis \( VT_i(t) \subseteq VT(t) \) for all \( i \in [1, \ldots, |VT(t)|] \), namely the decision profile, can be denoted by \( a(t) = \{a_1(t), \ldots, a_{N_{VT}(t)}(t)\} \) where \( a(t) \in A(t) \) and \( A(t) \) is the set of all possible decision profiles. Let \( a_i(t) \) be the set of decisions of all vacant taxis except \( VT_i(t) \), so that \( \{a_1(t), a_{i}(t)\} = a(t) \). Let \( A_i(t) \) be the set of all possible \( a_i(t) \) so that \( a_i(t) \subseteq A_i(t) \). Each decision profile can return a global utility \( U_{\tilde{g}}(a(t)) \) which is the objective of the TCNP to maximize, and each vacant taxi \( VT_i(t) \) has a utility function \( U_{VT_i}(a(t)) \).

The TCNP is formulated as a multi-player game in which taxis behave as non-cooperative agents that can make independent decisions. To get the solution of the game or the agreement among all taxis, the concept Nash Equilibrium (NE) is introduced: at time t, an NE is a decision profile \( a^* (t) = \{a_1^*(t), \ldots, a^*_{N_{VT}(t)}(t)\} \) satisfying that no vacant taxi \( VT_i(t) \) can do better to improve its own utility \( U_{VT_i}(a(t)) \) by engaging to another waiting customer different from \( a_i^*(t) \) (20).

From the definition of NE we can see that each vacant taxi \( VT_i(t) \) will try to maximize the utility \( U_{VT_i}(a(t)) \) for achieving an NE in the TCNP(t); however, the ultimate objective of TCNP(t) is to maximize the global utility \( U_{\tilde{g}}(a(t)) \). Thus, for linking the two irrelevant utility functions, the idea of ordinal potential game used for formulating and solving the Vehicle-Target Assignment Problem (VTAP) in Arslan et al. (19) is adopted in this paper.

2.2.2. Constructing An Ordinal Potential Game for TCNP

Definition 2.1 ordinal potential games for TCNP: an ordinal potential game for TCNP has a
potential function $\phi(a): A \mapsto R$ such that for every vacant taxi $VT_i(t) \in VT(t)$ with the utility

$U_{VT}(a(t))$, for every $a_i(t) \in A_i(t)$ and for every $a_i'(t), a_i''(t) \in CC_i(t)$:

$$U_{VT}(a_i'(t), a_i''(t)) - U_{VT}(a_i(t), a_i''(t)) > 0 \iff \phi(a_i(t), a_i''(t)) - \phi(a_i'(t), a_i''(t)) > 0$$

If we substitute the potential function $\phi(a(t))$ with the global utility $U_g(a(t))$:

$$U_{VT}(a_i'(t), a_i''(t)) - U_{VT}(a_i(t), a_i''(t)) > 0 \iff U_g(a_i'(t), a_i''(t)) - U_g(a_i(t), a_i''(t)) > 0$$

The motivation of introducing the concept of ordinal potential games in TCNP is to forge a tight link between the utility function of taxi $U_{VT}(a(t))$ and the global utility function $U_g(a(t))$, i.e., taxis will maximize their own utilities which also improve the global utility at the same time. So the next step is to choose the utility function of taxi and the global utility function properly so that an ordinal potential game can be formed.

**Definition 2.2 the global utility function**: The global utility function $U_g(a(t))$ will be defined as the summation of all waiting customers’ utilities, which is from the perspective of the customers.

$$\text{Max}: U_g(a(t)) = \sum_{t \in \Delta N_{WC}(t)} U_{WC}(a(t))$$

In Equation (1), $U_{WC}(a(t))$ is the waiting customer $WC_j(t)$’s utility function. The difference between the two types of utility functions $U_{WC}(a(t))$ and $U_{VT}(a(t))$ is that $U_{WC}(a(t))$ is the benefit a waiting customer $WC_j(t)$ can get when more than one vacant taxis may engage to him/her. Further, $U_{VT}(a(t))$ is the benefit a vacant taxi $VT_i(t)$ can get when it has engaged to a waiting customer. The following factors will be considered when calculating $U_{WC}(a(t))$:

- A waiting customer $WC_j(t)$ may have a MCWT denoted as $MCWT_j$ for waiting at the taxi stand;
- $WC_j(t)$ has been waiting for a time $At = t - t_{j,0}$, where $t_{j,0}$ is the customer’s arrival time to the taxi stand;
- $WC_j(t)$ can be engaged by more than one vacant taxis. Denote $ET_{j}(t)$ as the set of vacant taxis engaged to $WC_j(t)$, and $\mid ET_{j}(t)\mid = N_{ET_{j}(t)}$;
- The travel time between the current locations of $VT_i(t)$ and $WC_j(t)$ is denoted by $TT(i, j, t)$, and the estimated arrival time of that taxi is denoted as $t_j = t + TT(i, j, t)$.

Then, the waiting customer’s utility function can be defined by Equation (2):

$$U_{WC}(a(t)) = \text{Max}\{0, MCWT_j - (t - t_{j,0}) - \min_{VT_i(t) \in ET_{j}(t)} [TT(i, j, t)]\}$$

Thus, the waiting customer’s utility can be interpreted as the “opportunity cost” the customer can save during his/her waiting period at the taxi stand.

**Definition 2.3 the utility function of taxi**: if the vacant taxi $VT_i(t)$ has engaged to the waiting customer $WC_j(t)$, i.e., $a_i(t) = WC_j(t)$, the utility function of taxi $VT_i(t)$ is defined as the difference between $WC_j(t)$’s utilities when $VT_i(t)$ has hasn’t engaged to $WC_j(t)$.

$$U_{VT}(a_i(t), a_i''(t)) = U_{WC}(a_i(t), a_i''(t)) - U_{WC}(\phi, a_i''(t)) \text{ if } a_i(t) = WC_j(t)$$
This type of definition is named Wonderful Life Utility (WLU) \((21)\) in which the utility of the taxi is defined as the marginal contribution to the utility of the customer engaged by the taxi. It turns out that Definitions 2.2 and 2.3 ensure that an ordinal potential game can be formed.

3. THE SOLUTION ALGORITHM

Based on the game-theoretical formulation for TCNP introduced in Section 2, the solution algorithm for periodically performing the TCNP is proposed in this section. Assume at time \(t\) when the new round TCNP\((t)\) is to be performed, the pseudo code for the TCNP\((t)\) is shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1: The Pseudo Code for TCNP((t))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> set of vacant taxis (VT(t)) and set of waiting customers (WC(t))</td>
</tr>
<tr>
<td><strong>Output:</strong> (VT(t)) allocated to (WC(t))</td>
</tr>
<tr>
<td>Phase 1: Initialization</td>
</tr>
<tr>
<td>For Each vacant taxi (VT(t) \in VT(t))</td>
</tr>
<tr>
<td>(VT(t)) constructs (CC(t)) which is the set of all waiting customers within (VT(t))’s searching range</td>
</tr>
<tr>
<td>Loop</td>
</tr>
<tr>
<td>Phase 2: taxi-customer negotiation</td>
</tr>
<tr>
<td>For (k = 1:N), where (N) is the maximum number of negotiation rounds in TCNP((t))</td>
</tr>
<tr>
<td>For Each vacant taxi (VT(t) \in VT(t))</td>
</tr>
<tr>
<td>(VT(t)) performs G-RM-FM-I(_j)((k)) which may return a proposed waiting customer (WC_m(t) \in CC(t))</td>
</tr>
<tr>
<td>If G-RM-FM-I(_j)((k)) returns null</td>
</tr>
<tr>
<td>Continue;</td>
</tr>
<tr>
<td>Else</td>
</tr>
<tr>
<td>(VT(t)) engages to (WC_m(t)), i.e., (a(t, k) = WC_m(t))</td>
</tr>
<tr>
<td>Insert (VT(t)) to (ET_m(t, k)) which is the set of vacant taxis engaged to (WC_m(t)) at time (t), round (k)</td>
</tr>
<tr>
<td>End If</td>
</tr>
<tr>
<td>Loop</td>
</tr>
<tr>
<td>Phase 3: Finalization</td>
</tr>
<tr>
<td>For Each vacant taxi (VT(t) \in VT(t))</td>
</tr>
<tr>
<td>(VT(t)) changes its direction based on the negotiation results from Phase 2</td>
</tr>
<tr>
<td>Loop</td>
</tr>
</tbody>
</table>

There are two important sub-modules in TCNP\((t)\): one is the Calculation of Customer Utility CCU\(_j\)(\(k\)) performed by the waiting customer \(WC_j(t)\); the other is the Generalized Regret Monitoring with Fading Memory and Inertia G-RM-FM-I\(_j\)(\(k\)) performed by the vacant taxi \(VT_j(t)\), where \(k\) is the sequence number of the negotiation rounds of TCNP\((t)\). These two sub-modules are introduced in detail in Section 3.1 and Section 3.2.

3.1. The Calculation of Customer Utility: CCU\(_j\)(\(k\))

At the \(k^{th}\) round of TCNP\((t)\), each waiting customer \(WC_j(t) \in WC(t)\) needs to calculate two
different utilities, namely the primary utility \( U_{WC_j}(a(t,k)) \) and the secondary utility \( U_{WC_j}^*(a(t,k)) \), and then send them to all vacant taxis engaged to the customer, i.e., \( VT_n(t) \in ET_j(t,k) \), for further negotiation purposes.

**Definition 3.1** the primary utility \( U_{WC_j}(a(t,k)) \): at the \( k^{th} \) round of TCNP(t), each waiting customer \( WC_j(t) \) has a set of engaged taxis \( ET_j(t,k) \) where \( |ET_j(t,k)| = N_{ET_j(t,k)} \), then:

\[
U_{WC_j}(a(t,k)) = \begin{cases} 
\text{Max}[0, MCWT_j - (t-t_{j,0}) - TT(i^*, j, t)], & \text{if } N_{ET_j(t,k)} > 1 \\
0, & \text{if } N_{ET_j(t,k)} = 0 
\end{cases}
\]  

(4)

Where \( TT(i^*, j, t) = \min_{VT_k(t) \in ET_j(t)} [TT(i, j, t)] \)

In other words, the waiting customer’s primary utility \( U_{WC_j}(a(t,k)) \) is the utility that the waiting customer \( WC_j(t) \) can get when choosing the taxi in \( ET_j(t,k) \) with the shortest travel time to him/her, and zero when no vacant taxi is engaged to \( WC_j(t) \).

**Definition 3.2** the secondary utility \( U_{WC_j}^*(a(t,k)) \): at the \( k^{th} \) round of TCNP(t), each waiting customer \( WC_j(t) \) has a set of engaged taxis \( ET_j(t,k) \) where \( |ET_j(t,k)| = N_{ET_j(t,k)} \), then:

\[
U_{WC_j}^*(a(t,k)) = \begin{cases} 
\text{Max}[0, MCWT_j - (t-t_{j,0}) - TT(i^{**}, j, t)], & \text{if } N_{ET_j(t,k)} > 1 \\
U_{WC_j}(a(t,k)), & \text{if } N_{ET_j(t,k)} = 1 \\
0, & \text{if } N_{ET_j(t,k)} = 0 
\end{cases}
\]  

(5)

Where \( TT(i^{**}, j, t) = \min_{VT_k(t) \in ET_j(t), i \neq t} [TT(i, j, t)] \)

The waiting customer’s secondary utility \( U_{WC_j}^*(a(t,k)) \) can be interpreted as follows:

if there are more than one vacant taxis engaged to \( WC_j(t) \), and \( VT_i(t) \) is the one with the shortest travel time to \( WC_j(t) \), then \( U_{WC_j}^*(a(t,k)) \) is the utility the waiting customer \( WC_j(t) \) can get when choosing the taxi in \( ET_j(t,k) \) with the second shortest travel time to him/her; if there is only one taxi engaged to \( WC_j(t) \), then \( U_{WC_j}^*(a(t,k)) \) just equals to the primary utility \( U_{WC_j}(a(t,k)) \); if there is no taxi engaged to \( WC_j(t) \) then \( U_{WC_j}^*(a(t,k)) \) equals to zero.

The Calculation of Customer Utility \( CCU_j(k) \) can facilitate the process of calculating the utilities in the TCNP(t) which also ensure that the problem is an ordinal potential game throughout the entire TCNP(t).

3.2. The Generalized Regret Monitoring with Fading Memory and Inertia: G-RM-FM-I\((k)\)

At the \( k^{th} \) round of the TCNP(t), each vacant taxi \( VT_i(t) \) can choose to propose a waiting customer \( WC_m(t) \) to engage, i.e., \( a(t,k) = WC_m(t) \). Since it is expected that the global utility \( U_p(a(t)) \) could be converged after certain rounds of negotiations, the approach called the Generalized Regret Monitoring with Fading Memory and Inertia or G-RM-FM-I is employed as the negotiation method. It has to be noted that: G-RM-FM-I\((k)\) needs feedbacks from the CCU\(_j(k-1)\) if \( a(t,k-1) = WC_j(t) \), and G-RM-FM-I\((k)\) is performed prior to CCU\(_j(k)\), so that G-RM-FM-I\((k)\) will not be performed for any vacant taxi \( VT(t) \) in
the 1st round \((k=1)\) of the negotiation, and \(VT_i(t)\) can make its proposal randomly in this round. The following steps elaborate how G-RM-FM-I\(_{(k)}\) works at the negotiation round as and when \(k > 1\).

- **Step 1:** calculate \(U_{VT_i}(a(t,k-1))\)
  
  \(U_{VT_i}(a(t,k-1))\) is the utility of the vacant taxi \(VT_i(t)\) with the World Life Utility (WLU) type definition:

  \[
  U_{VT_i}(a(t,k-1)) = \begin{cases} 
  U_{WC_j}(a(t,k-1)) - U'_{WC_j}(a(t,k-1)), & \text{if } a_i(t,k-1) = WC_j(t), \ k > 1 \\
  0, & \text{if } a_i(t,k-1) = \phi, \ \ \ k > 1
  \end{cases} \tag{6}
  \]

- **Step 2:** calculate \(U_{VT_i}(CC^l_i(t),a_{l_i}(t,k-1))\) for all \(l \in \{1,...,|CC^l_i(t)|\}\)

  \(U_{VT_i}(CC^l_i(t),a_{l_i}(t,k-1))\) is the utility the vacant taxi \(VT_i(t)\) can get when it changes the choice to \(CC^l_i(t) \in CC_i(t)\) while the choices of all other vacant taxis remain the same. Suppose that \(CC_i(t) \neq \phi\), and \(VT_i(t)\) has engaged to \(WC_j(t)\) in the round \(k-1\), i.e., \(a_i(t,k-1) = WC_j(t)\), there are four different cases that need to be considered in the calculation:

  **Case 1:** If \(CC^l_i(t) = WC_j(t)\), then:

  \[
  U_{VT_i}(CC^l_i(t),a_{l_i}(t,k-1)) = U_{VT_i}(a(t,k-1)) \tag{7}
  \]

  **Case 2:** If \(CC^l_i(t) = WC_j(t)\) where \(j' \neq j\), and \(N_{ET_j(t,k-1)} = 0\), then:

  \[
  U_{VT_i}(CC^l_i(t),a_{l_i}(t,k-1)) = U_{WC_j}(CC^l_i(t),a_{l_i}(t,k-1)) - U'_{WC_j}(a(t,k-1)) \\
  = U_{WC_j}(CC^l_i(t),a_{l_i}(t,k-1)) - 0 \\
  = \text{Max}[0, MCWT_{j' - (t-t_{j',0})} - TT(i,j',t)] \tag{8}
  \]

  **Case 3:** If \(CC^l_i(t) = WC_j(t)\) where \(j' \neq j\), and

  \[TT(i,j',t) \geq \min_{VT_i(t) \neq ET_{j,k-1}} [TT(i',j',t)], N_{ET_j(t,k-1)} > 0\), then:

  \[
  U_{VT_i}(CC^l_i(t),a_{l_i}(t,k-1)) = 0 \tag{9}
  \]

  **Case 4:** If \(CC^l_i(t) = WC_j(t)\) where \(j' \neq j\), and

  \[TT(i,j',t) < \min_{VT_i(t) \neq ET_{j,k-1}} [TT(i',j',t)], N_{ET_j(t,k-1)} > 0\), then:
\[
U_{VT}(CC_i(t), a_i(t,k-1)) = U_{WC_j}(CC_j(t), a_j(t,k-1)) - U_{WC_j}(a(t,k-1))
\]

\[
= [MCWT_j - (t - t_{T_i})] - TT(i, j', t)
\]

\[
= \min_{VT_i(t) \in ET_i(t,k-1)} \{ TT(i', j', t) \}
\]

\[
= \min_{VT_i(t) \in ET_i(t,k-1)} [TT(i', j', t)] - TT(i, j', t)
\]

1. **Step 3:** calculate \( R_{VT_i}(t,k) \) for all \( l \in \{1, \ldots, |CC_i(t)| \} \)

\[
R_{VT_i}(t,k) = \rho R_{VT_i}(t,k-1) + (1 - \rho) R_{VT_i}(t,k), \text{ for all } l \in \{1, \ldots, |CC_i(t)| \}
\]

Where: \( R_{VT_i}(t,k) = U_{VT_i}(CC_i(t), a_i(t,k-1)) - U_{VT_i}(a(t,k-1)) \)

\( R_{VT_i}(t,k) \) is the regret of the vacant taxi \( VT_i(t) \) for not engaging to \( CC_i(t) \) in the \( k \)-th round of negotiation. \( R_{VT_i}(t,k) \) can be interpreted as the accumulated regret of \( VT_i(t) \) for not engaging to \( CC_i(t) \) in its historical rounds of negotiations. \( \rho \in (0,1] \) is the discount factor that enable each vacant taxi has a fading memory, i.e., to discount the influences of its past regrets to \( R_{VT_i}(t,k) \).

2. **Step 4:** calculate the probability distribution vector \( P_i(k) \)

\[
P_i(k) = \alpha RM_i(R_{VT_i}(t,k)) + (1 - \alpha) v_i^{(t,k-1)}
\]

Where:

\[
RM_i(x) = \frac{[x]^+}{1^T [x]^+} \quad \text{(When } 1^T [x]^+ > 0 \text{)}
\]

Notes:

1) \( \alpha \in (0,1] \) is the willingness to propose a different waiting customer of \( VT_i(t) \) at each round of the negotiation, so that \( 1 - \alpha \) represents the \( VT_i(t) \)'s inertia on the proposal.

2) \( v_i^{(t,k-1)} \) is a \(|CC_i(t)|\) dimensional vector and \( v_i^{(t,k-1)} \) is the \( i \)-th element of \( v_i^{(t,k-1)} \). If \( a_i(t,k-1) = CC_i(t) \) then \( v_i^{(t,k-1)} = 1 \); otherwise, \( v_i^{(t,k-1)} = 0 \), for all \( l \in \{1, \ldots, |CC_i(t)| \} \).

3) \([x]^+ \) is an \( n \) dimensional vector of which the \( i \)-th element equals to \( \max(x_i, 0) \), if \( x \) is also an \( n \) dimensional vector.

- **Step 5:** \( VT_i(t) \) engages to a waiting customer \( WC_m(t) \in CC_i(t) \)

Based on the probability distribution vector \( P_i(k) \) calculated in Step 4, the vacant taxi \( VT_i(t) \) will engage to a waiting customer \( WC_m(t) \in CC_i(t) \) as the return value of G-RM-FM-I(k). It has to be noted that \( VT_i(t) \) can also propose nothing and then G-RM-FM-I(k) just returns a value null.

It has been proved in Arslan et al. (19) that if a Vehicle-Target Assignment Problem (VTAP) can form an *ordinal potential game* and each vehicle has no indifferent utilities response to different strategies (or decisions), The G-RM-FM-I will enable the negotiation process to converge to a pure Nash Equilibrium (NE) almost surely. The TCNP proposed in this paper has the same properties with the case in Arslan et al. (19), so that it also has the same convergence ability when the G-RM-FM-I is applied.

It has to be noted that the NE obtained by G-RM-FM-I may be a sub-optimal solution
in terms of maximizing $U_g(a(t))$ for TCNP($t$), which is a trade-off between the operational efficiency and the theoretical optimality. On one hand, even though there are negotiation mechanisms such as the Spatial Adaptive Play (SAP) that can lead to an optimal or near optimal solution (16), those mechanisms are too time consuming to be implemented in the TCNP where states of both the taxi and the customer are changed quickly; on the other hand, the convergence tests by the simulation experiments in Section 4.2.1 show that G-RM-FM-I has a good convergence performance which results in a better operational performance.

4. SIMULATION EXPERIMENTS

4.1. The Microscopic Simulation Model

A microscopic simulation model is developed based on the concept of customized simulation environment. It includes a microscopic traffic simulation software - PARAMICS (22) and a plugin designed by programming with the Application Program Interfaces (APIs) which enable the software to simulate the customer dynamic behaviors and the taxi operations as well as the LISS. A fictitious road network covering an area of around $3\text{km} \times 3\text{km}$ is created in PARAMICS as shown in Figure 1 which includes two types of taxi stand:

- Taxi stands located within the study area (the boundary of the study area is shown in Figure 1);
- Taxi stands located outside the study area (in fact, these taxi stands can be considered as located in the fringe areas adjacent to the study area).

![The road network for simulation.](image)

In the simulation, the customer behaviors and taxi operations as well as the limited information sharing mechanism are simulated strictly following the assumptions presented in Section 2.1. The customer demand is set to 400 arrivals/hour for the taxi stands located outside the study area and 520 arrivals/hour (30% higher than 400 arrivals/hour) for the taxi stands located inside the study area. The purpose of doing this is to mimic the boom in customer demand of a specific area during a specific period of time, e.g., the Central Business District (CBD) during the peak-hour.

A sensitivity analysis is performed by varying the taxi fleet size from 100 to 250 at the increment of 50 taxis, in which the performance of LISS is evaluated and compared with the strategy without any control (i.e., the free search strategy) in terms of OR and CWT for each...
taxi fleet size.

Other parameters of the simulation are set as following: In LISS, the TCNP will be performed in every 100 seconds in which \( \rho \) and \( \alpha \) are set to 0.1 and 0.5 for the sub-module G-RM-FM-I(\( k \)). The searching range of the taxi is set to 500m. The MCWT of the customer is arbitrarily set to 1 hour which is purposely to test the maximum CWT. The total simulation period is 2 hours with 20 minutes warm-up time.

4.2. Simulation Results

4.2.1. The Convergence Tests

The convergence tests for the TCNP in the case of taxi fleet size=100 is shown in Figure 2. Figure 2(a) shows that the global utility is converged in the negotiation round \( k=40 \) at \( t=1200 \) seconds while Figure 2(b) shows that the global utility is converged in the negotiation round \( k=33 \) at \( t=6000 \) seconds. The convergence tests show that TCNP has a good convergence performance.
4.2.2. The Sensitivity Analysis

The overall performance of the two strategies (free search strategy and LISS) in terms of OR and CWT for different fleet size are shown in Figure 3.

(a) Average Customer Waiting Time (CWT) at all taxi stands

(b) Average Customer Waiting Time (CWT) at taxi stands located within the study area

(c) Occupancy Rate (OR)

**FIGURE 3** The overall performance of control strategies.
For all taxi stands as shown in Figure 3(a), when taxi supply is low (taxi fleet size < 175), LISS can effectively reduce the CWT up to around 50% (taxi fleet size = 100) compared with the free search strategy. However, when taxi supply is high (taxi fleet size > 175), LISS is no better than the free search strategy in terms of reducing the CWT. This is because the number of available taxis is much higher in such situation so that the customer can quickly find the taxi arrives to the stand (even under the free search strategy) which makes the LISS less attractive.

For taxi stands located within the study area as shown in Figure 3(b), when taxi supply is low (taxi fleet size < 150), LISS can effectively reduce the CWT up to around 80% (taxi fleet size = 100) compared with the free search strategy. When taxi supply is high (taxi fleet size > 150), LISS is still (slightly) better than the free search strategy in terms of reducing the CWT.

As shown in Figure 3(c), the OR of taxi under LISS is no lower than that under the free search strategy when taxi supply is high (taxi fleet size > 150), and the OR of taxi under LISS is slightly higher than that under the free search strategy when taxi supply is low (taxi fleet size < 150). This indicates that LISS will not increase the risk of taxi, i.e., the probability of losing the total occupied time.

In all, the simulation results show that LISS is an effective control strategy to reduce the CWT when taxi supply is low, especially for the situation of boom in customer demand of a specific area during a specific period of time, e.g., the Central Business District (CBD) during the peak-hour; moreover, LISS will not increase the risk of taxi even though it requires no commitment from the customer side as stated in Section 2.2.

5. CONCLUSIONS AND FUTURE WORKS

This paper has proposed a novel control strategy, namely the Limited Information Sharing Strategy (LISS) for the Taxi-Customer Searching Problem (TCSP) in the Non-Booking Taxi Service (NBTS), or TCSP-NBTS. The contributions of this paper are highlighted in the followings:

- Game theory has been adopted to formulate the LISS; the global utility of the game and the individual utilities of the players (taxi and customer) are specifically defined by considering a number of theoretical and practical problems;
- A negotiation mechanism namely the Generalized Regret Monitoring with Fading Memory and Inertia (G-RM-FM-I) has been adopted in LISS to find the Nash Equilibrium (NE);
- The operational performance of LISS has been evaluated in this paper by comparing with the strategy without any control (i.e., the free search strategy).

The microscopic traffic simulation is adopted as the modeling approach in this paper. A sensitivity analysis by varying the taxi supply is conducted by the simulation in which the Occupancy Rate (OR) and the Customer Waiting Time (CWT) are calculated for all scenarios in each control strategy. Some implications have been obtained from the results of the simulation experiments:

- The LISS is an effective control strategy when taxi supply is low, especially for the situation of boom in customer demand of a specific area during a specific period of time, e.g., the Central Business District (CBD) during the peak-hour;
- LISS will not increase the risk of taxi even though it requires no commitment from the customer side.
Meanwhile, the microscopic simulation model developed for this research needs further improvements if it is employed for a large study area (e.g., the entire island of Singapore) and for a long period of operation time (e.g., 24 hours of a typical day). Those future works include:

- Taxis’ searching behaviors in a large study area will be considered and modeled, such as the choice of destination for picking passengers;
- The customers’ elasticity in a long period of operating time will be considered and modeled.

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