Value of Information Analysis in Structural Safety

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ABSTRACT

Pre-posterior analysis can be used to assess the potential of an experiment to enhance decision making by providing information on parameters characterized by uncertainty. The present paper describes a framework for pre-posterior analysis for support of decisions related to maintenance of structural systems. In this context, experiments may refer to inspections or techniques of structural health monitoring. The Value of Information concept provides a powerful tool for determining whether the experimental cost is justified by the expected benefit and for identifying the optimal among different possible experimental schemes. This concept is elaborated through principal examples for structural components and system models. Sensitivity analyses are performed to investigate how the decision problem is influenced by the level of uncertainty that characterizes the structural properties, the amount and quality of information and the probabilistic dependencies between components of a system.

INTRODUCTION

The societal importance of safe and well-functioning structures in conjunction with the large amount of resources allocated to the construction of new facilities and maintenance of deteriorating ones indicate the criticality of decisions related to management of structural systems. A proper decision-analysis framework must incorporate the epistemic and aleatory uncertainties that characterize the physical properties, loading and functionality of a structural system, accounting for their evolution in time.

Traditional inspection methods as well as a growing number of structural health monitoring techniques can be used to reduce the uncertainty in system properties and thus, enhance decisions on maintenance actions. However, the benefit gained from reduction of uncertainty must be weighed against the associated cost; according to Howard (1966): "Placing a value on the reduction of uncertainty is the first step in experimental design, for only when we know what is worth to reduce uncertainty do we have a basis for allocating our resources in experimentation designed to reduce uncertainty." Pre-posterior decision analysis provides the formal framework for assessing potential benefits from different inspection and monitoring schemes.

A powerful notion in pre-posterior analysis is the Value of Information (VoI), which represents the difference between expected benefits evaluated with and without a certain piece of information. The VoI concept was introduced by Raiffa and Schlaifer (1961) and has been employed in several decision-analysis studies in the field of Informatics (e.g., Dearden et al. 1998), Economics (e.g., Howard 1966), Health Care (e.g., Claxton et al. 2001) and Environmental Risk Management (e.g., Yokota and Thompson 2004). Applications in the field of Structural Safety remain limited (e.g., Pozzi and Der Kiureghian 2011, Thöns and Faber 2013, Straub 2013). The present paper describes a general framework for application of VoI analysis in Structural Safety and presents example applications for structural components and system models. Sensitivity analyses demonstrate the dependence of VoI on the degree of uncertainty in structural properties, the amount and quality of information and the probabilistic dependencies between components of a system.

VALUE OF INFORMATION

We consider a structural system defined through a set of parameters that describe the physical properties, loading and functionality of the system, accounting for their evolution in time. We use $\boldsymbol{\Theta}$ to denote a vector of random variables that represent parameters characterized by uncertainty, and A to denote a set of possible maintenance strategies over the life cycle of the system. In an expected benefit maximization framework, the benefit, B, gained over the life cycle of the system is determined as

$$B = \max_{\alpha \in \mathrm{A}} E_{\boldsymbol{\Theta}}\left[b(\alpha, \boldsymbol{\theta})\right], \tag{1}$$

where $b(\alpha, \boldsymbol{\theta})$ denotes the benefit corresponding to maintenance strategy $\alpha \in \mathrm{A}$ and a realization $\boldsymbol{\theta}$ of the random vector $\boldsymbol{\Theta}$.

In *prior* decision analysis, the optimal maintenance strategy is identified on the basis of our prior probabilistic model for $\boldsymbol{\Theta}$, which may combine knowledge from code specifications, expert judgment, laboratory tests and computer simulations, consistent with the selected configuration, materials, location and intended function of the structural system. Hereafter, we use B' to denote the life-cycle benefit evaluated on the basis of prior probabilistic knowledge. Consistently with Equation 1:

$$B' = \max_{\alpha \in \mathrm{A}} \int_{\Omega_{\boldsymbol{\Theta}}} b(\alpha, \boldsymbol{\theta}) f'_{\boldsymbol{\Theta}}\left(\boldsymbol{\theta}\right) d\boldsymbol{\theta}, \tag{2}$$

where $\Omega_{\boldsymbol{\Theta}}$ and $f'_{\boldsymbol{\Theta}}\left(\boldsymbol{\theta}\right)$ respectively denote the outcome space and prior joint Probability Distribution Function (PDF) of $\boldsymbol{\Theta}$.

In *posterior* decision analysis, the optimal maintenance strategy is identified on the basis of the posterior probabilistic model for $\boldsymbol{\Theta}$, obtained by updating our prior model after new information has become available. Hereafter, we use $B''\left(\mathbf{x}\right)$ to denote the updated life-cycle benefit in light of information $\mathbf{x}$. Consistently with Equation 1:

$$B''(\mathbf{x}) = \max_{\alpha \in A} \int_{\Omega_{\boldsymbol{\Theta}}} b(\alpha, \boldsymbol{\theta})\, f''_{\boldsymbol{\Theta}|\mathbf{x}}(\boldsymbol{\theta} \mid \mathbf{x})\, d\boldsymbol{\theta}, \tag{3}$$

where $f''_{\boldsymbol{\Theta}|\mathbf{x}}(\boldsymbol{\theta} \mid \mathbf{x})$ denotes the posterior PDF of $\boldsymbol{\Theta}$ given $\mathbf{x}$. The Conditional Value of Information, $CVI(\mathbf{x})$, is determined by the difference between the life-cycle benefits with and without information $\mathbf{x}$, i.e. $CVI(\mathbf{x}) = B''(\mathbf{x}) - B'$.

Pre-posterior analysis is used to assess the value of information that an experiment can provide before the experimental outcome becomes available. Herein, an experiment denotes an inspection method or structural health monitoring scheme. In pre-posterior analysis, $\mathbf{x}$ represents a possible realization of the random vector $\mathbf{X}$. The value of experiment e is assessed by the Expected Value of Information, $EVI(e)$, determined as $E_{\mathbf{X}}\left[CVI(\mathbf{x})\right] = E_{\mathbf{X}}\left[B''(\mathbf{x})\right] - B'$, or analytically:

$$EVI(e) = E_{\mathbf{X}}\left\{\max_{\alpha \in A} E_{\boldsymbol{\Theta}|\mathbf{X}}\left[b(\alpha, \boldsymbol{\theta})\right]\right\} - \max_{\alpha \in A} E_{\boldsymbol{\Theta}}\left[b(\alpha, \boldsymbol{\theta})\right]. \tag{4}$$

It can be seen that for any experiment, $EVI(e) \geq 0$, since

$$E_{\mathbf{X}}\left\{\max_{\alpha \in A} E_{\boldsymbol{\Theta}|\mathbf{X}}\left[b(\alpha, \boldsymbol{\theta})\right]\right\} \geq \max_{\alpha \in A} E_{\mathbf{X}}\left\{E_{\boldsymbol{\Theta}|\mathbf{X}}\left[b(\alpha, \boldsymbol{\theta})\right]\right\} = \max_{\alpha \in A} E_{\boldsymbol{\Theta}}\left[b(\alpha, \boldsymbol{\theta})\right]. \tag{5}$$

A rational decision maker will undertake an experiment with cost $C(e)$ as long as $EVI(e) \geq C(e)$. Among a set of experiments that satisfy this condition, the optimal experiment is determined as the one yielding the maximum value of $EVI(e) - C(e)$.

The main computational effort in the evaluation of EVI lies in the evaluation of $E_{\mathbf{X}}\left[B''(\mathbf{x})\right]$. In a general case, an experiment provides information on a sub-vector $\boldsymbol{\Theta}_1$ of $\boldsymbol{\Theta}$, in the form $\mathbf{X} = h(\boldsymbol{\Theta}_1) + \boldsymbol{\varepsilon}$, where the function h relates the measured quantities with the uncertain parameters $\boldsymbol{\Theta}_1$ and $\boldsymbol{\varepsilon}$ denotes the measurement error. We characterize this type of information *partial imperfect information*. In this case:

$$E_{\mathbf{X}}\left[B''(\mathbf{x})\right] = \int_{\Omega_{\mathbf{X}}} \left[\max_{\alpha \in A} \int_{\Omega_{\boldsymbol{\Theta}}} b(\alpha, \boldsymbol{\theta})\, f''_{\boldsymbol{\Theta}|\mathbf{X}}(\boldsymbol{\theta} \mid \mathbf{x})\, d\boldsymbol{\theta}\right] f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}, \tag{6}$$

where $\Omega_{\mathbf{X}}$ and $f_{\mathbf{X}}(\mathbf{x})$ respectively denote the outcome space and joint PDF of $\mathbf{X}$. The PDF of $\mathbf{X}$ can be obtained in terms of the prior distribution of $\boldsymbol{\Theta}$ and the likelihood of the experiment, $L(\boldsymbol{\theta} \mid \mathbf{x}) = f_{\mathbf{X}|\boldsymbol{\Theta}}(\mathbf{x} \mid \boldsymbol{\theta})$, according to equation

$$f_{\mathbf{X}}(\mathbf{x}) = \int_{\Omega_{\boldsymbol{\Theta}}} L(\boldsymbol{\theta} \mid \mathbf{x})\, f'_{\boldsymbol{\Theta}}(\boldsymbol{\theta})\, d\boldsymbol{\theta}. \tag{7}$$

A special case is an error-free experiment, which reveals the true values of $\boldsymbol{\Theta}_1$, i.e. $\mathbf{X} = \boldsymbol{\Theta}_1$. This is the case of *partial perfect information*. Let $\boldsymbol{\Theta}_2$ denote the sub-vector of $\boldsymbol{\Theta}$ including the elements not belonging in $\boldsymbol{\Theta}_1$, i.e. $\boldsymbol{\Theta}_2 = \left\{\Theta_i \in \boldsymbol{\Theta} : \Theta_i \notin \mathbf{X}\right\}$. Then:

$$E_{\mathbf{X}}\left[B''(\mathbf{x})\right] = \int_{\Omega_{\boldsymbol{\Theta}_1}} \left[\max_{\alpha \in A} \int_{\Omega_{\boldsymbol{\Theta}_2}} b(\alpha, \boldsymbol{\theta})\, f'_{\boldsymbol{\Theta}_2|\boldsymbol{\Theta}_1}(\boldsymbol{\theta}_2 \mid \boldsymbol{\theta}_1)\, d\boldsymbol{\theta}_2\right] f'_{\boldsymbol{\Theta}_1}(\boldsymbol{\theta}_1)\, d\boldsymbol{\theta}_1. \tag{8}$$

Finally, the experiment $\mathbf{X} = \mathbf{\Theta}$, leading to complete elimination of uncertainty, represents the case of *complete perfect information*. In this case:

$$E_{\mathbf{X}}\left[B''(\mathbf{x})\right] = \int_{\Omega_{\mathbf{\Theta}}} \left[\max_{\alpha \in A} b(\alpha, \mathbf{\theta})\right] f'_{\mathbf{\Theta}}(\mathbf{\theta})\, d\mathbf{\theta}. \tag{9}$$

We note that EVI for complete perfect information is different from the sum of EVI for partial perfect information for all elements of $\mathbf{\Theta}$. EVI for complete perfect information provides an upper bound of EVI for partial perfect information.

CASE STUDIES FOR SRUCTURAL COMPONENTS

We consider a structural component with a service life of $T_l\ yrs$. The component capacity is described by a linearly decreasing function of time, $R_t = R(1 - k\,t)$, where R is the initial capacity at $t_0 = 0$ and k is the degradation rate. The demand, S, on the component is assumed independent of time. R and S are modeled as random variables, whereas k is considered deterministic. The component is in a safe state when $R_t > S$.

We assume that in year T_d, $0 < T_d < T_l$, an agent needs to decide on the optimal maintenance strategy between α_0, representing *no action* and α_1, representing *maintenance*. In this example application, maintenance involves replacement with a new component of similar type to the original one. (Note that in a more general case, possible maintenance strategies may involve sequences of actions; for a simple yet instructive illustration of VoI analysis, case studies herein are confined to maintenance strategies that involve single actions.) Let $C_f(T_i)$ and $C_r(T_i)$ respectively denote the costs of failure and replacement in year T_i. Considering an annual interest rate r, the expected costs $c(\alpha_i) = -E_{\mathbf{\Theta}}\left[b(\alpha_i, \mathbf{\theta})\right]$ associated with actions α_0 and α_1 are given by

$$c(\alpha_0) = C_f(T_d) \sum_{T_i = T_d}^{T_l} \frac{1}{(1+r)^{T_i - T_d}} f_T^o(T_i), \tag{10}$$

$$c(\alpha_1) = C_r(T_d) + C_f(T_d) \sum_{T_i = T_d}^{T_l} \frac{1}{(1+r)^{T_i - T_d}} f_T^n(T_i), \tag{11}$$

where $f_T^o(T_i)$ denotes the probability that the original component fails in year T_i and $f_T^n(T_i)$ denotes the probability that a new component fails in year T_i.

In the following numerical investigations, we assume the service life is $T_l = 30\, yrs$ and the degradation rate is $k = 0.01$. The failure and replacement costs are given in monetary units as $C_f(T_d) = 1000$ and $C_r(T_d) = 10$, respectively. The annual interest rate is assumed $r = 0.02/yr$. Under prior information, R is modeled as a lognormal variable with mean and standard deviation $\mu_R = 2.5$ and $\sigma_R = 0.25$, respectively, and S is modeled as a lognormal variable with mean and standard deviation $\mu_S = 1$ and $\sigma_S = 0.3$, respectively. For the above distributions, the probability of failure at $t_0 = 0$ is $P_f(t_0) = 10^{-3}$.

In prior analysis, we evaluate the expected costs $c(\alpha_i)$ using the prior probabilistic models for R and S in computing $f_T(T_i)$. These costs are shown in the left graph of Figure 1, as a function of T_d. According to this graph, a rational agent will choose to replace the component if the decision is made between the 8th and 24th year of its service life. For newer components, the cost of replacement is not justified by the relatively small reduction in the annual failure probability; for older components, the cost of replacement is not justified by the relatively short remaining service life. The right graph of Figure 1 shows the corresponding life-cycle cost, $C' = -B' = \min[c(\alpha_0), c(\alpha_1)]$, i.e. the expected cost for selection of optimal strategy.

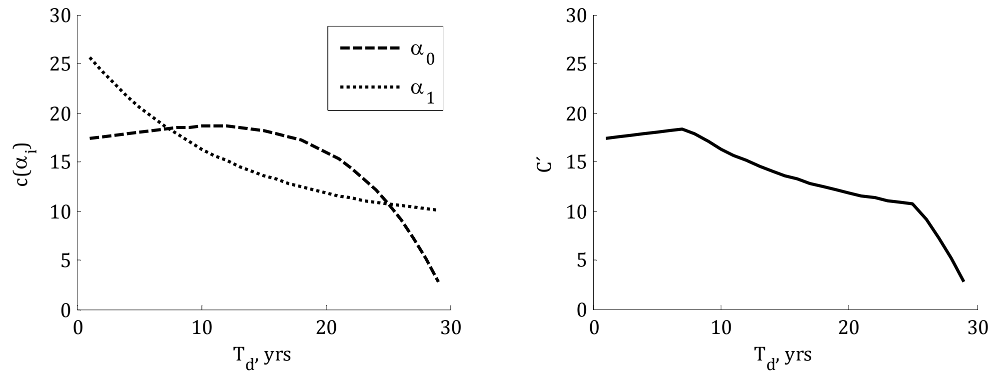

Figure 1. Expected costs in prior analysis.

Partial perfect information. Let us now consider the possibility of undertaking an experiment to aid our selection of maintenance strategy. The experiment considered herein is an inspection that reveals the actual value of the capacity, i.e. $\mathbf{X} = \{R\}$. Note that in this particular case of partial perfect information, $\mathbf{\Theta}_1 = \{R\}$ is independent of $\mathbf{\Theta}_2 = \{S\}$. In posterior analysis, we evaluate the expected costs $c(\alpha_i)$ in terms of the probability $f_T(T_i)$ conditioned on the known value of R. The value of the experiment is equal to the reduction of the life-cycle cost achieved by updating our choice of action after revelation of the value of R. Thus, the experiment has a non-zero value only if the optimal decision when the value of R is known is different from the optimal decision indicated by prior analysis.

Before conducting the inspection, pre-posterior analysis can be used to determine whether the cost of inspection is justified by its potential to enhance our decision on maintenance. Because at this stage the inspection outcome is unknown, the value of the experiment is assessed by considering the entire outcome space of R. In the present example where the outcome space is continuous, we employ a Monte Carlo (MC) approach to evaluate EVI: values of R are sampled from the prior distribution and for each sample, posterior analysis is performed to identify CVI; EVI is approximated by the average CVI for all sampled values of R.

After preliminary investigations, a number of 10,000 simulations is selected for the MC evaluations. The left graph of Figure 2 compares the prior life-cycle cost, $C' = -B'$, with the expected posterior life-cycle cost, $E[C''] = E[-B'']$. The difference between C' and $E[C'']$ is equal to EVI, which is shown in the right graph of the same figure. Although the behavior of EVI versus T_d is not monotonic, values of EVI tend to be lower for older components.

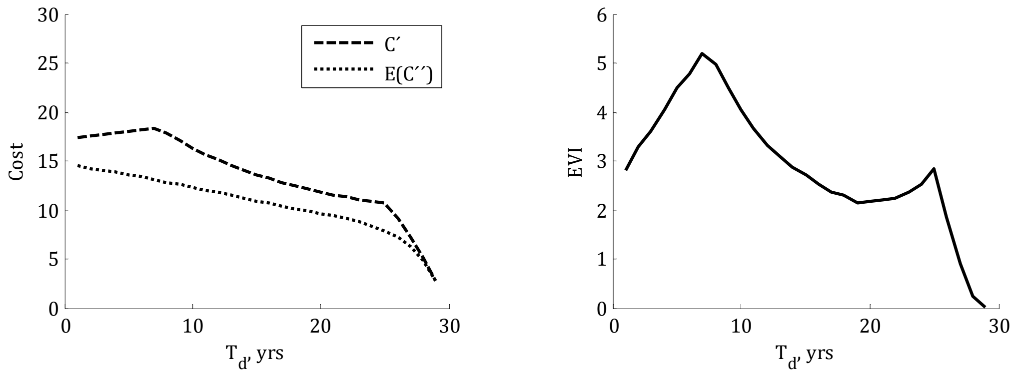

Figure 2. Life-cycle costs in prior and pre-posterior analysis (left) and *EVI* (right) for perfect measurement of capacity.

In order to examine how the degree of uncertainty in our prior knowledge influences EVI, we perform sensitivity analysis by varying the coefficient of variation of R, denoted δ_R. We vary μ_R and σ_R simultaneously, so that the probability of failure at $t_0 = 0$ remains constant, i.e. $P_f\left(t_0\right) = 10^{-3}$. The left graph of Figure 3 shows EVI versus δ_R for $T_d = 20\,yrs$; the right graph of the same figure shows EVI normalized with C'. We note that although EVI does not exhibit monotonic behavior with increasing δ_R, the normalized EVI almost monotonically increases at a decaying rate with increasing δ_R.

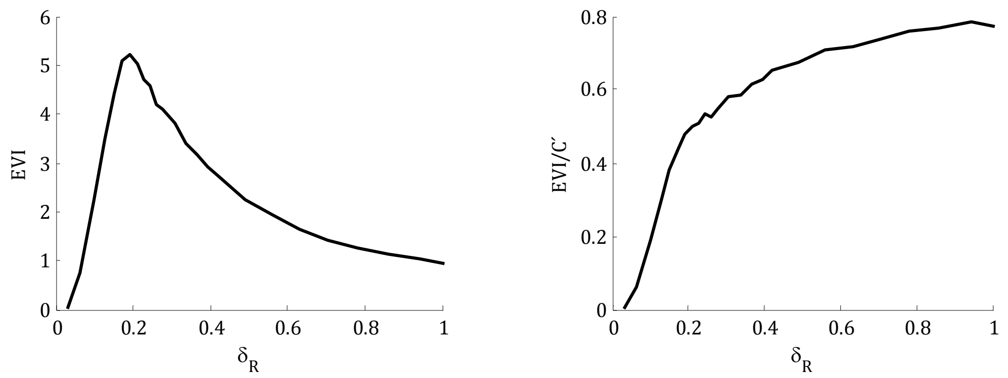

Figure 3. *EVI* (left) and normalized *EVI* (right) versus the coefficient of variation of capacity.

Partial imperfect information. Let us now consider an experiment where the log-capacity is measured with a random error ε, i.e. $\mathbf{X} = \left\{\log R + \varepsilon\right\}$. Assuming the experiment provides unbiased information, the measurement error is modeled by a zero-mean normal variable with standard deviation σ_ε. The error standard deviation defines the quality of information. For an experiment comprising n measurements of the log-capacity, we have $\mathbf{X} = \log R + \boldsymbol{\varepsilon}$, with $\mathbf{X} = \lfloor X_1 \; \dots \; X_n \rfloor^{\mathrm{T}}$ and $\boldsymbol{\varepsilon} = \lfloor \varepsilon_1 \; \dots \; \varepsilon_n \rfloor^{\mathrm{T}}$. Assuming the errors are random and uncorrelated, the elements of $\boldsymbol{\varepsilon}$ are modeled as independent zero-mean normal variables with standard deviation σ_ε. In this case, the assumption of a log-normal prior distribution for R, leads to a log-normal posterior distribution for R given $\mathbf{x}$. The parameters of the posterior distribution are obtained in closed form in terms of σ_ε, n and the parameters of the prior distribution.

In the following, we investigate the dependence of EVI on σ_ε for cases with $n=1$ and $n=2$ measurements. Realizations of $\mathbf{X}$ are obtained by first, sampling from the prior distribution of R and subsequently, sampling from $f_{\mathbf{X}|\log R}(\mathbf{X} \,|\, \log R) = f_\varepsilon(\boldsymbol{\varepsilon})$. In the following, a MC approach with 10,000 simulations is employed, where a single realization of $\mathbf{X}$ is obtained for each R sample. Numerical results for the case $T_d = 20\,yrs$ are shown in Figure 4, in which EVI is plotted versus $\sigma_\varepsilon / \sigma_{\log R}$, i.e. the error standard deviation normalized with the standard deviation of the log-capacity. As $\sigma_\varepsilon / \sigma_{\log R}$ tends to zero, EVI for imperfect information approaches the respective EVI for perfect information (see Figure 2). As $\sigma_\varepsilon / \sigma_{\log R}$ attains large values, EVI tends to zero. For intermediate values of $\sigma_\varepsilon / \sigma_{\log R}$, EVI depends on the precision and number of measurements. As expected, for certain n, experiments with higher precision, i.e. smaller values of σ_ε, are characterized by larger EVI; however, experiments with higher precision are typically more expensive. Identification of the optimal experiment must account for the dependence of the experimental cost, C_e, on both the amount and precision of information. Thus, the optimal experiment is the one yielding the maximum value of $EVI(\sigma_\varepsilon, n) - C_e(\sigma_\varepsilon, n)$.

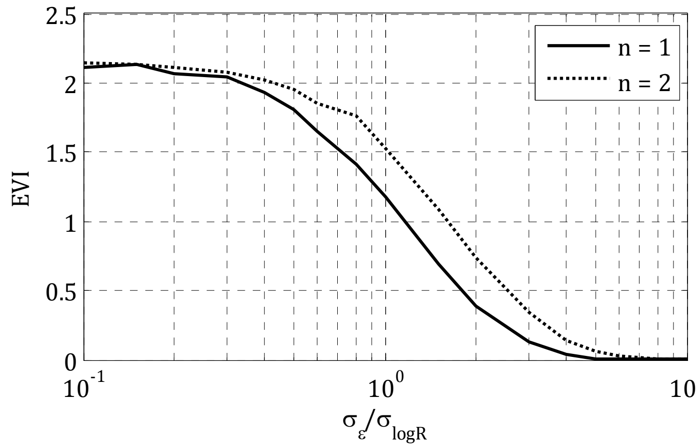

Figure 4. Dependence of *EVI* on precision and number of measurements.

Evaluation of EVI for the case of imperfect information requires a higher computational effort than for the case of perfect information. Since computational efficiency can be a critical aspect in VoI analysis, the value of EVI for perfect information can be used as an upper bound for the value of EVI for imperfect information.

For cases when the degradation rate is modeled as a random variable, its probability distribution can be updated by use of measurements obtained at different points in time. In this case, VoI analysis can be used to identify the optimal combination of precision and the time interval between measurements. Dependence of EVI versus measurement precision in long-term monitoring was investigated by Pozzi and Der Kiureghian (2011) by use of Bayesian regression analysis.

CASE STUDIES FOR SYSTEM MODELS

We consider a structural system, with a service life of $T_l = 30\,yrs$, consisting of N components. All components are of similar type to the component examined in the previous section. The correlation coefficient between the log-capacities of different components is denoted $\rho_{\log R}$. The total load on the system, $N \times S$, is assumed equally distributed between the N components. The demand variable, S, is modeled as in the previous section.

We assume that in year T_d of the service life of the system, an agent must select the optimal maintenance strategy between α_0 representing *no action*, and α_1, representing *maintenance*. In this case, maintenance involves replacement of all components with new ones of similar type. The failure and maintenance costs at the beginning of year T_d are respectively given by $1000 \times C_f\left(T_d\right)$ and $N \times C_r\left(T_d\right)$, where $C_f\left(T_d\right)$ and $C_r\left(T_d\right)$ are the failure and replacement costs for a single component and are assumed equal to the respective costs in the previous section. Note that the maintenance cost for the system is equal to the sum of replacement costs for all components; however, the failure cost of the system is significantly higher than the sum of failure costs for all the components due to additional *indirect* consequences induced by loss of the system functionality (Baker et al. 2006).

In the following, we investigate EVI for perfect inspections that reveal the capacity of one or more of the components. Due to probabilistic interdependencies, this case of partial perfect information is distinctly different from that in the example for a single component. A given experimental outcome allows updating not only the failure probabilities of the inspected components, but also the failure probabilities of the non-inspected components due to the assumed correlation between the capacities.

Figures 5 and 6 show EVI versus $\rho_{\log R}$ for $T_d = 20\,yrs$ and $k = 1, 2$ inspected components for a parallel and a series system, respectively, both consisting of $N = 3$ components. Numerical evaluations have been performed with a MC approach with 10,000 simulations. The right graphs of the same figures show the difference between EVI and the inspection cost, $C_e\left(k\right)$, in monetary units, assuming that the inspection costs for one and two components are $C_e\left(1\right) = 1$ and $C_e\left(2\right) = 2$, respectively. Figures 5 and 6 indicate strong dependence of EVI on the type of system and the degree of correlation between the component capacities. In the present example, EVI outweighs the inspection cost only for cases with high correlation; in these cases, inspection of one component is optimal for the parallel system, whereas inspection of two components is optimal for the series system.

CONCLUSION AND PERSPECTIVES

Value of Information (VoI) analysis provides a formal framework for assessing the benefit from inspections or monitoring of structural systems. VoI analysis can be used to identify optimal methods of inspection and/or monitoring including the option of undertaking none. In the present paper, a framework for VoI analysis was presented and elaborated through example applications on structural components and system models. Effects on the Expected Value of Information of the level of uncertainty in prior knowledge, the amount and quality of information, and the probabilistic dependencies between components of a system were investigated. The main limitation of VoI analysis is identified in the computational demand, which can be particularly high for large structural systems with interdependencies and cases with sequential decision making.

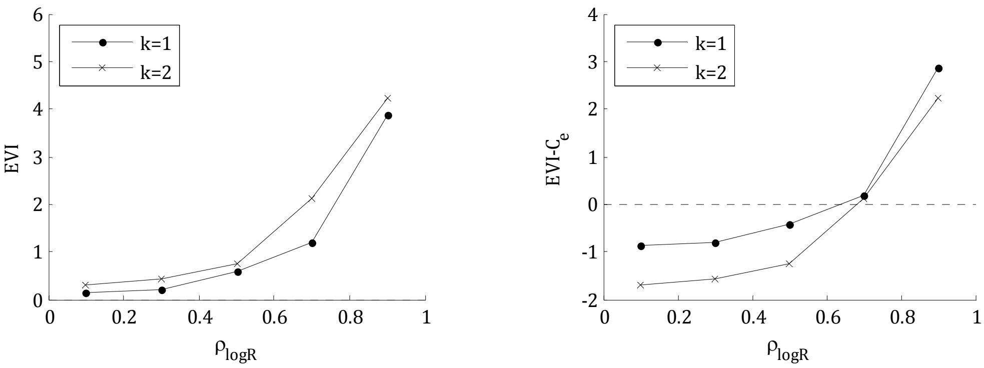

Figure 5. Parallel system: *EVI* for perfect inspection of one or two components.

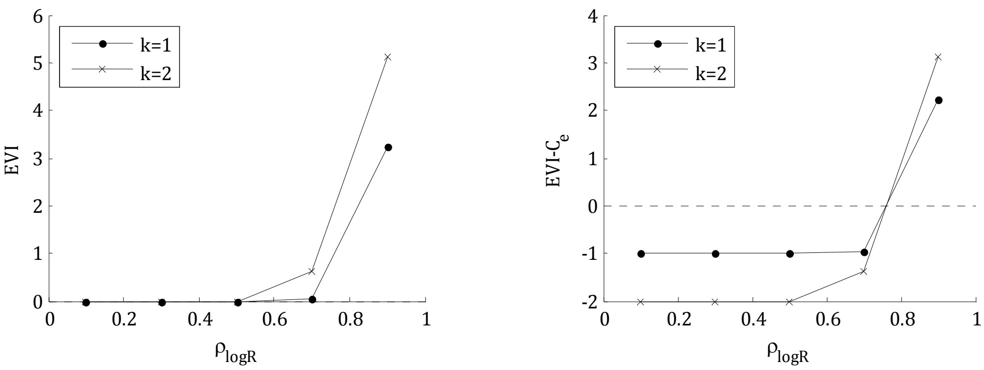

Figure 6. Series system: *EVI* for perfect inspection of one or two components.

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