**Problem Statement & Context**

A computational model is defined as a mapping:

\[ x \in \mathbb{D}_X \subset \mathbb{R}^n \rightarrow y = \mathcal{M}(x) \in \mathbb{R} \]

- \( x \) is modelled by an imprecise random vector, \( X \), which accounts for both aleatory uncertainty (natural variability) and epistemic uncertainty (lack of knowledge).
- The elements of \( X \) are assumed statistically independent.
- The computational model is considered as a black-box.

**Goal:** Propagate uncertainty in \( X \) through an expensive-to-evaluate model to the random response \( Y = \mathcal{M}(X) \).

**Nested Monte Carlo Algorithm**

The uncertainty in parametric p-boxes can be propagated by nested MC loops.

- **Outer loop** samples the distribution parameters \( \theta \in [\theta, \bar{\theta}] \).
- **Inner loop** samples \( x \) from the distribution \( F_X(x|\theta) \).

\( \Rightarrow \) Requires a huge number of model evaluations.

\( \Rightarrow \) Speed-up achieved by using a surrogate model.

**Augmented Polynomial Input Vector**

**Definition:** \( Z \) is the vector of all parameters of all marginal distributions.

\[ Z = \{ X | \Theta_X \} \]

**Example: Simply Supported Truss**

**Problem:** assess deflection \( u_i(p) \) of truss (Hurtado, 2013):

- loads \( P_i \), \( i = 1, \ldots, 7 \) independent,
- \( P_i \in [0,100] \), \( kN \), \( \sigma_{P} \in [13,17] \), \( kN \).

**Augmented PCE:**

- \( N = 100 \) latin-hypercube samples.
- \( N_{ph} = \{1,2,5,10\} \) phantom points.

**Results:** \( N_{ph} \) := convergence to true response p-box.

**Conclusions**

- The augmented input space allows for a distinction between aleatory and epistemic uncertainty in \( X \).
- Augmented PCE makes nested Monte Carlo simulations tractable for expensive-to-evaluate models with random input described by parametric p-boxes.
- The increased dimensionality of the augmented input space is handled with phantom points at no additional cost.
- Due to computational speed up, more advanced analyses (e.g. sensitivity analysis) become tractable.