Yield Functions taking into account Anisotropic Hardening Effects for an Improved Virtual Representation of Deep Drawing Processes

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Yield functions taking into account anisotropic hardening effects for an improved virtual representation of deep drawing processes

A thesis submitted to attain the degree of

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(Dr. sc. ETH Zurich)

presented by

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Zurich, August 2015
Philip Peters
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Abstract

The present study aims at an improved virtual representation of sheet metal forming processes by the use of material models taking into account anisotropic hardening effects. The anisotropic hardening behavior of two different materials, namely a DC05 deep drawing steel and a 5018-based aluminum alloy, and its influence on real forming processes are investigated. In a series of laboratory experiments, the direction- and deformation-dependent mechanical properties of the two materials are identified. The results of these tests are subsequently used to determine parameters of different plasticity models. The common models Hill48, Barlat89 and Yld2000-2d are compared to an extended version of the Yld2000-2d model, where a dependence of the model parameters on the equivalent plastic strain was introduced. In addition, the latter is combined with the recently proposed HAH (homogenous function based anisotropic hardening). For validation purposes, the enhanced models have been implemented into the commercial FE program LS-DYNA. Following the mathematical formulation of the applied material models, the integration algorithm for the stress update is explained in detail and the numerical problems are briefly discussed. Finally, the different models are evaluated based on simulations of real parts. The simulation results are compared to the results of strain measurements of the parts from real tests. The thesis concludes with a discussion of the results and possible follow-up work.
Kurzfassung

Nomenclature

Latin characters

\( a \)  Acceleration vector
\( A \)  Area
\( A_g \)  Uniform elongation
\( B \)  Shape function
\( C \)  Elastic stiffness tensor
\( C \)  Damping tensor
\( c \)  Strain rate scale factor
\( D \)  Rate of deformation tensor
\( \hat{e}_i \)  Unit vectors of element coordinate system
\( E \)  Young’s modulus
\( f \)  Force vector
\( F \)  Force
\( \hat{h} \)  Microstructure deviator
\( J_2 \)  Second invariant of the stress tensor
\( l \)  Length
\( l_0 \)  Initial Length
\( L \)  Velocity gradient
\( M \)  Mass matrix
\( m \)  Vector of derivatives of yield function with respect to stress
\( N \)  Shape function
\( P \)  Material point in the actual configuration
\( P_0 \)  Material point in the initial configuration
\( \partial P^{ext} \)  External virtual power
\( \partial P^{int} \)  Internal virtual power
\( \partial P^{kin} \)  Virtual kinetic power
\( R_m \)  Ultimate tensile strength
\( R_{p0.2} \)  Yield stress (at 0.02\% eng. strain)
\( r_\alpha \)  Lankford parameter in \( \alpha \)-direction
\( s \)  Deviatoric stress tensor
Nomenclature

\( t \) \hspace{1em} Time
\( \mathbf{T} \) \hspace{1em} Transformation matrix
\( \mathbf{u} \) \hspace{1em} Displacement vector
\( \mathbf{\hat{u}} \) \hspace{1em} Node displacements
\( \mathbf{v} \) \hspace{1em} Velocity vector
\( \mathbf{\hat{v}} \) \hspace{1em} Node velocities
\( V \) \hspace{1em} Volume
\( \mathbf{W} \) \hspace{1em} Spin tensor
\( \mathbf{x} \) \hspace{1em} Position vector of the actual configuration
\( \mathbf{X} \) \hspace{1em} Position vector of the reference configuration

Greek Characters

\( \Gamma \) \hspace{1em} Boundary
\( \varepsilon \) \hspace{1em} Strain
\( \varepsilon^p \) \hspace{1em} Plastic strain tensor
\( \varepsilon_{pl} \) \hspace{1em} Equivalent plastic strain
\( \eta \) \hspace{1em} Isoparametric coordinate
\( \theta \) \hspace{1em} Angular velocity vector
\( \kappa \) \hspace{1em} Shear factor
\( \lambda \) \hspace{1em} Plastic multiplier
\( \mu \) \hspace{1em} Friction coefficient
\( \nu \) \hspace{1em} Poisson’s ratio
\( \xi \) \hspace{1em} Isoparametric coordinate
\( \rho \) \hspace{1em} Density
\( \rho \) \hspace{1em} Radius of curvature
\( \boldsymbol{\sigma} \) \hspace{1em} Cauchy stress tensor
\( \boldsymbol{\sigma}^T \) \hspace{1em} Trial stress
\( \bar{\sigma} \) \hspace{1em} Effective stress
\( \sigma_y \) \hspace{1em} Yield stress
\( \phi \) \hspace{1em} Stable yield part of HAH yield function
\( \Phi \) \hspace{1em} Yield condition
\( \chi \) \hspace{1em} Mapping function
\( \vartheta \psi \) \hspace{1em} Virtual velocity distribution
\( \Omega \) \hspace{1em} Actual configuration
\( \Omega \) \hspace{1em} Body
\( \Omega_0 \) \hspace{1em} Reference configuration

XIV
Operators

\[ \Delta \quad \text{Increment} \]
\[ A : B \quad \text{double contraction} \]
\[ \dot{x} \quad \text{Time derivative of } x \]
\[ \text{div } x \quad \text{Divergence of } x \]
\[ \text{grad } x \quad \text{Gradient of } x \]
\[ \ln x \quad \text{Natural logarithm of } x \]

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>BEM</td>
<td>Boundary element method</td>
</tr>
<tr>
<td>CCPA</td>
<td>Convex cutting plane algorithm</td>
</tr>
<tr>
<td>CPPA</td>
<td>Closest point projection algorithm</td>
</tr>
<tr>
<td>DD</td>
<td>Diagonal direction</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite difference method</td>
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<tr>
<td>FE</td>
<td>Finite element</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element method</td>
</tr>
<tr>
<td>fcc</td>
<td>Face-centered cubic</td>
</tr>
<tr>
<td>bcc</td>
<td>Body-centered cubic</td>
</tr>
<tr>
<td>PDE</td>
<td>Partial differential equation</td>
</tr>
<tr>
<td>RD</td>
<td>Rolling direction</td>
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<tr>
<td>TD</td>
<td>Transverse direction</td>
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1 Introduction

1.1 Numerical modeling of sheet metal forming processes

Because of the high added value at relatively low costs, sheet metal forming and especially stamping has developed from an ancient handcraft to a highly automated high-tech process. In the course of this development also the products have evolved from fairly simple to very complex parts. Nowadays, various industries produce parts with complex geometries, sophisticated designs and high requirements in terms of geometrical accuracy and mechanical properties.

A milestone in this evolution was the introduction of finite element tools in the planning process of stamped parts. While the feasibility of a part could only be judged based on experience before, the engineers now have a tool at their disposal which allows them to identify critical areas of a part and to make feasibility studies without the need for try-out tools. This led to a drastic reduction of the time needed for the planning phase for stamped parts.

Although the usage of finite element simulations for the process layout of deep drawn and stamped sheet metal parts can meanwhile be considered as an industrial standard, the accuracy of the process models suffers from numerous simplifications. The main challenges lie in an accurate representation of the tribological conditions and in the complex constitutive behavior of sheet metal material, which are the key to correctly predict strain and stress distributions and subsequently assess failure in numerical simulations.

Considering constitutive modeling, considerable progress has been made in the last few decades. However, finding a good compromise between model accuracy and low characterization effort remains a challenging task. This is the point at which the present study starts.
1 Introduction

1.2 Scope of the thesis

The present work deals with material modeling for metals under plane stress conditions with a special focus on anisotropic hardening effects. Several publications as well as in-house experiments at the Institute of Virtual Manufacturing at the ETH Zurich have shown, that for certain materials the concept of isotropic hardening, meaning a uniform expansion of the yield surface, does not hold, even for linear strain paths. Therefore, the goal of this study is to investigate several materials concerning their anisotropic hardening behavior as well as to find ways to numerically model it. A 5182 based aluminum alloy called Formalex™-5x produced by Constellium as well as a DC05 deep drawing steel from Thyssen Krupp Steel were pulled up for this purpose. An extensive series of mechanical experiments is carried out to analyze the deformation behavior of these materials. Different material models, established ones as well as newer developments, are used to model deep drawing processes that were also carried out in reality. It is assumed that the hardening behavior of the material influences the strain distribution in final parts. Therefore, it is meaningful to validate models based on comparisons with strain measurements of real part experiments.

The scope of this work is limited to the mathematical description of the deformation behavior of sheet metals. Other issues related to FEM process modeling like e.g. contact, friction or failure models are not investigated.

1.3 Organization of the thesis

Besides 'Introduction' and 'Discussion and outlook', the present work is basically organized in six chapters. Chapter 2 provides the physical fundamentals of plastic deformations in metallic materials. It briefly describes the atomic structure of metals and the phenomena that cause plastic deformation, hardening, anisotropy and anisotropic hardening effects. Chapter 3 gives a short introduction to the finite element method. Although not all aspects can be discussed in detail, the reader should at least have the possibility to understand the general idea of the method and the concepts that lie behind as well as the approximations that are made. Chapter 4 gives an overview of the most important developments in the field of plasticity models for sheet metal materials in the last century. It should also
help to understand the most important steps that eventually led to today’s established models. Also the newest models that have been implemented and investigated in this study are described in this chapter. Finally, the integration procedures that are needed in order to implement a plasticity model into a finite element code are given. Chapter 5 documents the laboratory experiments that were carried out in order to investigate the material behavior and to identify model parameters. The experiments are all described in detail and the results for the different materials are presented. Chapter 6 explains the identification of the model parameters based on the experimental results. Chapter 7 describes the experiments that were carried out in order to validate the implemented models. The setup for the experiments is given and experimental results are compared to numerical ones in order to compare the quality of the different models. Eventually, chapter 8 discusses the results presented in chapter 7 and gives an outlook of which topics are currently under investigations and what could and should be the research goals in the upcoming years.
2 Plasticity

The word plasticity has its origin in the Greek verb πλασσεῖν which means "to shape". It refers to a non-reversible deformation of materials, also called plastic deformation, under the influence of external forces. In contrary to elastic deformation, a plastically deformed material retains its new shape upon removal of such forces. There exists a great variety of materials that can be plastically deformed. This work however is limited to sheet metal materials.

This chapter will give a brief overview of the physical phenomena, which occur on the microscopic level. The deformation characteristics of metals strongly depend on the underlying microstructure. The lattice structure, preferred orientations and grain size all play an important role. This chapter provides the physical fundamentals for the deformation of metallic materials. The information given in this chapter is based on (Lange, 1984; Banabic, 2000; Lubliner, 2008)

2.1 Crystal structure of metals

Metals usually exist as so-called polycrystalline aggregates. This means that the material is composed of large numbers of grains, each of which has the structure of a single crystal. Depending on the alloy and process history, the grain size can vary between less than a micrometer and several centimeters. A typical micrograph of a polycrystalline C35E steel is given in Fig. 2.1a. The single grains as well as the grain boundaries can easily be seen. Different grains represent regions of different crystallographic orientation. A schematic figure of different grains and grain boundaries is shown in Fig. 2.1b. The size and shape of the grains as well as the lattice orientation in the grain depend on the history of the material. Although the grain orientations can be statistically random, there are generally some preferred orientations.

Within every grain, the atoms are stacked in a regular array forming
Figure 2.1: Polycrystalline structure, micrograph and schematic figure

Figure 2.2: Unit cells for different crystallographic systems
2.2 Plastic deformation

Plastic deformation only occurs because the initial lattice is not perfect, but has defects. A distinction is made between three groups of defects: **Zero-dimensional** defects, which are either vacancies or foreign atoms. The foreign atoms can be either substitutional or interstitial atoms. Meaning they take a place on normal lattice positions or between them. **One-dimensional** defects are dislocations. The basic types of dislocations are edge dislocations, where an atomic plane ends in the lattice, or screw dislocations, where the lattice planes spiral the dislocation line as the axis. At last there are **Two-dimensional** defects, the most important of which are grain boundaries and phase boundaries. Grain boundaries are divided into two groups: The difference of the orientation of the lattice of two grains that are separated by a small-angle grain boundary is less than 5° while larger differences cause large-angle grain boundaries. A special case are twin boundaries for which the two separate crystals share some of the same crystal lattice points in a symmetrical manner.

2.2 Plastic deformation

Plastic deformation is caused by two basic mechanisms. The most important one is gliding of dislocations. If a force is applied, the interatomic bonds are stretched while every atom keeps its place in the lattice. This reversible deformation is elastic. After reaching a critical force, crystallographic slip occurs, which means that an atomic plane is sheared against its adjacent plane. For this deformation, the atomic bonds need to be broken and reformed, which is an irreversible process and thus leads to a plastic deformation of the material. A schematic representation of the movement of an edge dislocation is given in Fig. 2.3. The arrows represent the direction of the shear stress. Experiments on single crystal show that crystal slip occurs on specific crystallographic planes and in a certain direction. The combination of a
slip plane and a slip direction is called slip system. The slip planes tend to be the most close-packed planes and the directions the ones with the highest atom density. The number of possible slip systems therefore depends on the lattice structure of the material (see Fig. 2.2) and has a strong influence on its forming behavior.

Evidence for crystal slip being the origin of plastic deformation can be obtained from mechanical tests carried out on single crystal material. Several researchers have carried out such experiments (e.g (Kiener et al., 2008; Bei et al., 2008; Dimiduk et al., 2005)). Fig. 2.4 shows a SEM image of a single crystal deformed under compression (Dimiduk et al., 2005) with obvious slip planes.

Plastic slip is not associated with a change of volume. This is referred to as the incompressibility condition of plasticity and is essential for macro scale plasticity models. Furthermore, since slip is a shearing process, it is independent of hydrostatic pressure. However, this statement is not true for porous material where, under hydrostatic pressure, the pore volume is decreased and thus the material volume changes. Nonetheless, also for porous material, the change in volume is not related to plastic slip.

Another phenomenon that leads to plastic deformation is mechanical twinning. Caused by shear stresses, part of the crystal lattice is transferred to a mirror-imaged position. The symmetry plane is called twinning plane.

Twinning mainly occurs in bcc and hcp metals. Since the stress required for twinning is high compared to sliding, especially hcp materials exhibit twinning caused by the few slip systems for certain directions and thus high flow stresses.
2.2 Plastic deformation

Figure 2.4: SEM images of a 2 μm diameter microcrystal sample after shear (Dimiduk et al., 2005)

Figure 2.5: Scheme of twinning in crystal lattice

(a) Before twinning  (b) After twinning
2 Plasticity

2.3 Hardening

A crucial effect for the forming and the final mechanical properties of metals is hardening. It means that, at low temperatures, the stress needed for plastic deformation increases with increasing strain. Due to the increasing number of dislocations, zones of higher dislocation density, so-called dislocation forests, emerge, which act as barriers for moving dislocations. Only at higher stresses, they can pass or cut each other. The stress fields of the dislocations, which act against building and movement of further dislocations must be considered as the main cause of strain hardening. For polycrystalline materials also grain boundaries and the transition of slip systems between grains act as such barriers.

2.4 Anisotropy

For many metallic materials, the assumption of isotropic behavior does not hold, but the mechanical properties are dependent on the loading direction. This anisotropy has a significant influence on the material flow and therefore on the strain distribution. A prominent consequence of the anisotropy is the earing after drawing a circular cup. The origin of anisotropy lies in the crystal anisotropy in combination with the texture and in the anisotropy of the microstructure, which is caused by the orientation of certain microstructure elements such as grain boundaries or phases.

In a single crystal metal, the properties are the same in directions of equal atomic arrangement. In directions of other atomic distances, other properties are measured. Therefore, single crystals are highly anisotropic. The crystal anisotropy is the most important crystal property. With decreasing symmetry of the crystal structure, the anisotropy becomes more pronounced. Therefore, a hcp material exhibits a more distinctive anisotropy than a cubic structure (see. Fig. 2.2). In polycrystals, the crystal anisotropy is only noticeable if preferred directions exist. Such preferred grain orientations are also called a texture.

Another reason for direction dependent properties are size and shape of the grains and the structural arrangement. A typical example are the stretched grains in a steel sheet, the so called pan-cake-structure.
2.5 Bauschinger effect

The Bauschinger effect, named after Johann Bauschinger (Bauschinger, 1886), is the reyielding at a lower stress level after load reversal. On top of the Bauschinger effect, such load reversal processes often go along with a permanent softening effect, meaning the stress does not reach its isotropic hardening level anymore. The stress strain answer of a material subjected to loading and reverse loading under presence of a Bauschinger effect and permanent softening is illustrated in Fig. 2.6.

According to (Mollica et al., 2001), the microscopic reasons for the Bauschinger effect are "far from clear". (Hasegawa et al., 1986) explained the effect with an untangling of loosely tangled dislocations. However, this conclusion was doubted by (Rauch, 1997). He conducted reverse shear experiments, prestraining the material at different temperatures and perform the reverse shear at room temperature. Although almost no structuration of the cells could be detected if the material was prestrained at low temperatures (125 K), he obtained similar results for the subsequent loading at room temperatures. Thus, he concluded that the transition cannot be due to the annihilation of the old structures.

![Figure 2.6: Stress-strain response for reverse loading under presence of Bauschinger effect and permanent softening](image-url)
2.6 Latent effects

Besides the Bauschinger effect described in the last chapter, many metallic materials also exhibit latent effects. These effects are usually the most distinctive after an orthogonal strain path change. An orthogonal strain path change means that the double contraction of the stress tensor before and after the change of the strain path reduces to 0 ($\boldsymbol{\sigma}_{\text{bef.}} : \boldsymbol{\sigma}_{\text{aft.}} = 0$). In the remainder, these effects are denoted as latent hardening (for a stress overshoot after orthogonal loading) or as latent softening (for a reyielding at a lower stress level after orthogonal loading). Amongst others, latent hardening was shown and explained by (Schmitt et al., 1991), who performed sequential tensile tests on polycrystalline copper sheets. They concluded that the latent hardening effect is caused by the activation of slip systems that were inactive before. If the material was prestrained in a certain direction and subsequently loaded under an angle $\Phi$ to the initial tensile direction, the stress overshoot reached a maximum for $\Phi = 45^\circ$. At this angle, the strain path change is very close to orthogonal and the number of newly activated slip systems reaches a maximum.

Other materials, like e.g. the aluminum alloy that was tested in this work (see sec. 5.5) exhibit a Bauschinger-like effect when subjected to orthogonal loading. (Barlat et al., 2003b) explained this behavior with an evolution of the dislocation microstructure.
3 The finite element method

Mathematical models of physical processes are often based on partial differential equations. Since in most cases it is impossible to find analytical solutions for these equations, numerical methods are used to approximate the solution. Several methods such as the boundary element method (BEM) (e.g. (Banerjee and Butterfield, 1981)), the finite differences method (FDM) (e.g. (Courant et al., 1952)) or the finite element method (FEM) are established today. Moreover, increasing attention is paid on so-called meshless methods like e.g. the smooth particle hydrodynamics (SPH) (Belytschko et al., 1996). However, for forming applications, the finite element method still is the one with the highest industrial relevance.

In the scope of this work, the finite element method is used in order to assess the accuracy of the newly introduced or modified constitutive models described in chapter 4. Therefore, this section gives a brief introduction to continuum mechanics, from which the basic equations of the finite element method are derived, before it explains the spatial discretization, which is needed to compute strains and, using constitutive evaluations, stresses and eventually nodal forces and moments. Finally the time discretization that leads to the new nodal accelerations, velocities and displacements is explained. The methods described in this chapter are the ones used by the commercial finite element software LS-DYNA, which is mainly used for the validation examples of this work.

3.1 Continuum mechanical fundamentals

In the following, a few continuum mechanical basics are reviewed. The explanations will be limited to the points which are essential for the understanding of the principle of the finite element method and the determination of strains which are used as input for the stress integration (see sec. 4.4). See (Altenbach and Altenbach, 1994; Holzapfel, 2000; Parisch, 2003; Chadwick, 2012; Belytschko et al., 2013) for further reference. The
references just mentioned also build the basis for the explanations in the following sections.

3.1.1 Kinematics

In general, there are two different formulations for the description of the kinematics. These are the Lagrangian and the Eulerian formulation. In the case of the Eulerian (spatial) formulation, a fixed point in space is observed. The properties of the material which are at or pass this particular point are described. This way of describing the properties is particularly suited for fluid dynamic problems if a fixed control volume is considered. For solid mechanical problems, the considered volume is mostly bounded by the surface of the continuum body which may vary its shape with ongoing deformation, hence the Lagrangian formulation is more suitable. For the Lagrangian (material) formulation, a fixed point in the material, which is usually not fixed in space is observed. The properties of this particular point are described over time. Thus, after the discretization of a material continuum into a finite element mesh, the material and the mesh are directly coupled. This gives the possibility to describe complex geometries as well as the history of material points with a high accuracy. Besides the Eulerian and the Lagrangian formulation, also mixed formulations, so called ALE (Arbitrary Lagrangian Eulerian) formulations exist. They are e.g. applied for special forming applications with very large deformations like extrusion (Tong, 1995) or fineblanking (Manopulo et al., 2011). In the remainder of this chapter, only the Lagrangian formulation is explained.

In order to describe movements and displacements of bodies, an undeformed (reference) $\Omega_0$ and a deformed (current) $\Omega$ configuration are considered. It is assumed that the material undergoes combined stretch, rigid body rotations, and translations. All quantities are measured in a global coordinate system. A mapping function $\chi$ describes the relation between the actual position $x$ and the reference position $X$ of a material point $P$.

$$x = \chi(X, t)$$  \hspace{1cm} (3.1)

Fig. 3.1 illustrates the two configurations $\Omega_0$ and $\Omega$ as well as the position vector $X$ and $x$ that point to the position of $P_0$ and $P$. Thus, the displacement $u$ of a material point is equal to the difference of those two position vectors.
3.1 Continuum mechanical fundamentals

![Figure 3.1: Reference configuration \( \Omega_0 \) and current configuration \( \Omega \)](image)

\[ u(X,t) = \chi(x,t) - \chi(x,0) = x - X \]  

(3.2)

3.1.2 Strain and stress measures

From Eqs. (3.1) and (3.2), the velocity and the acceleration of the material point can be determined taking the first and the second time derivative of the displacement or the mapping function respectively.

\[ v(X,t) = \frac{d\chi(X,t)}{dt} = \frac{du(X,t)}{dt} = \dot{u} \]  

(3.3a)

\[ a(X,t) = \frac{dv(X,t)}{dt} = \frac{d^2u(X,t)}{dt^2} = \ddot{v} = \ddot{u} \]  

(3.3b)

A distinction is made between the **total** and the **updated** Lagrangian formulation. For the total Lagrangian formulation, the initial configuration \( \Omega_0 \) always is the reference, while for the updated Lagrange formulation, the configuration \( \Omega \) from the last deformation increment is used as reference. In sheet metal forming processes, as discussed in this work, the deformations are typically in a range, which is well described by the Lagrangian formulation as outlined in the following. In order to capture strain histories and be able to deal with nonlinearities due to large deformations, an updated Lagrange formulation is usually applied.

In order to compute strains (in LS-DYNA), the so-called **rate of deformation** tensor is used. To determine this tensor, first, the gradient of the
velocity field has to be defined as
\[ \mathbf{L} = \frac{\partial \mathbf{v}}{\partial \mathbf{x}} \quad (3.4) \]

By decomposing \( \mathbf{L} \) into a symmetric and a skew-symmetric part,
\[ \mathbf{L} = \frac{1}{2} (\mathbf{L} + \mathbf{L}^T) + \frac{1}{2} (\mathbf{L} - \mathbf{L}^T) = \mathbf{D} + \mathbf{W} \quad (3.5) \]
the rate of deformation \( \mathbf{D} \), i.e. the first term of the right-hand side of Eq. (3.5) and the spin \( \mathbf{W} \), i.e. the second term on the right-hand side of Eq. (3.5) is obtained.

\[ \mathbf{D} = \frac{1}{2} \left( \mathbf{L} + \mathbf{L}^T \right) \quad (3.6a) \]
\[ \mathbf{W} = \frac{1}{2} \left( \mathbf{L} - \mathbf{L}^T \right) \quad (3.6b) \]

The rate of deformation tensor is a measure of the rate of change of the square of the length of infinitesimal material line segments while the spin tensor \( \mathbf{W} \) can be shown to provide a measure of the average angular velocity of all material fibers passing through a material point (Bower, 2009). The integration of \( \mathbf{D} \) over time leads to the Hencky strain tensor, that is also called logarithmic strain or true strain and is the most widely used strain measure in the field of metal forming. The stress, which is the work conjugate to the rate of deformation is the Cauchy stress, also called true stress. It completely defines the stress state at a material point in the deformed configuration. The Cauchy stress is defined through Cauchy’s stress theorem:
\[ \mathbf{t}(\mathbf{x}, t, \mathbf{n}) = \mathbf{\sigma}(\mathbf{x}, t)\mathbf{n} \quad (3.7) \]

### 3.1.3 Balance principles

Classical continuum mechanics is based on equations that express the balance of mass and momentum. These equations, also known as balance principles, are stated in this section.

The law of conservation of mass is the most simple balance principle and gives information about the mass of a body. It basically states that mass cannot be vanished. This means, that in order to change the density in
a control volume, either material has to flow in or out of that volume. If $\rho(x,t)$ is the density (mass per unit volume) in a continuum, then the following holds

$$\frac{d\rho}{dt} + \rho \text{div}(v) = 0$$

(3.8)

Since for forming applications, usually a Lagrangian formulation is used and the plastic deformation is assumed to take place at constant volume\(^1\), the mass within an element is conserved and this condition is automatically satisfied.

The conservation of the linear momentum states, that the change of the momentum of a body is equal to the sum of the forces acting on that body. Using Eq. (3.8), the local form of the momentum equation, which is valid in every point $x$ of a continuum is mathematically expressed as:

$$\rho \frac{dv}{dt} = \text{div} \sigma + \rho b$$

(3.9)

The left hand side of Eq. (3.9), which describes the change of momentum, is referred to as the inertia term. For static problems, this term vanishes and Eq. (3.9) is called the equilibrium equation.

The divergence of the Cauchy stress tensor on the right hand side gives the internal forces per volume and the term $\rho b$ expresses the volumetric forces. The conservation of momentum yields a hyperbolic partial differential equation. Eq. (3.9) is also called the balance of linear momentum.

By taking the cross-product of each term in the corresponding linear momentum equation with the the position vector $x$, the integral form of the conservation of angular momentum is obtained. It can be derived, that in order to fulfill this condition, the Cauchy stress tensor needs to be symmetric.

$$\sigma = \sigma^T \quad \text{or} \quad \sigma_{ij} = \sigma_{ji}$$

(3.10)

Therefore, the Cauchy stress space is three-dimensional for plane stress problems and six-dimensional for three-dimensional problems. By using the symmetric Cauchy stress tensor, the conservation of angular momentum is automatically satisfied.

\(^1\)Due to the elastic deformation, which is not incompressible, the volume slightly changes, but this is usually neglected
### 3.2 Weak form of the momentum equation

Based on the balance principle (sec. 3.1.3), the weak form of the momentum equation is derived in the following. Starting point is the strong form of the momentum equation (3.9) with the velocity boundary conditions

\[ \mathbf{v} = \bar{\mathbf{v}} \quad \text{on} \quad \Gamma_v \quad (3.11) \]

and stress boundary conditions

\[ \sigma \mathbf{n} = \bar{\mathbf{t}} \quad \text{on} \quad \Gamma_t \quad (3.12) \]

The weak form of the momentum equation is obtained using the principle of virtual power. Eq. (3.9) is multiplied with a virtual velocity distribution \( \partial \psi \) and integrated over \( \Omega \)

\[ \int_{\Omega} \partial \psi \rho \dot{\mathbf{v}} d\Omega = \int_{\Omega} \partial \psi \text{div} \sigma d\Omega + \int_{\Omega} \partial \psi \rho \mathbf{b} d\Omega \quad (3.13) \]

The divergence term on the right-hand-side of Eq. (3.13) can be substituted using partial integration

\[ \int_{\Omega} \partial \psi \text{div} \sigma d\Omega = \int_{\Gamma_t} \partial \psi \sigma \mathbf{n} d\Gamma_t - \int_{\Omega} (\text{grad} \partial \psi) \sigma d\Omega \quad (3.14) \]

leading to

\[ \int_{\Omega} \partial \psi \rho \dot{\mathbf{v}} d\Omega + \int_{\Omega} (\text{grad} \partial \psi) \sigma d\Omega = \int_{\Gamma_t} \partial \psi \bar{\mathbf{t}} d\Gamma_t + \int_{\Omega} \partial \psi \rho \mathbf{b} d\Omega \quad (3.15) \]

Eq. (3.15) represents the weak form of the momentum equation. The individual terms represent expressions of virtual power. The first term represents the virtual kinetic power

\[ \partial P^{\text{kin}} = \int_{\Omega} \partial \psi \rho \dot{\mathbf{v}} d\Omega \quad (3.16) \]

The second term of Eq. (3.15) represents the internal virtual power, which also can be expressed as the double contraction of the rate of deformation tensor \( \mathbf{D} \) and the Cauchy stress \( \sigma \)

\[ \partial P^{\text{int}} = \int_{\Omega} (\text{grad} \partial \psi) \sigma d\Omega = \int_{\Omega} \partial \mathbf{D} : \sigma d\Omega \quad (3.17) \]
3.3 Spatial discretization with finite elements

The terms on the right hand side of Eq. (3.15) are due to external forces like gravity and forces acting on $\Gamma_t$ and thus represent the external virtual power

$$\partial P^{ext} = \int_{\Gamma_t} \partial \psi \bar{t} d\Gamma_t + \int_{\Omega} \partial \psi \rho b d\Omega$$  \hspace{1cm} (3.18)

Using Eqs. (3.16) to (3.18), the weak form of the momentum equation can as well be expressed as

$$\partial P^{kin} + \partial P^{int} = \partial P^{ext}$$  \hspace{1cm} (3.19)

### 3.3 Spatial discretization with finite elements

To spatially discretize a continuous body, it is separated into a finite number of subregions, so-called finite elements. Neighboring elements are connected by nodes. The velocities and accelerations which need to be known to compute the displacements and strains and consequently forces and moments are solely computed in these nodes. The distributions of these field quantities within an element are interpolated with so-called shape functions $N$ and thus can be computed from the nodal displacements $\hat{u}$ or the nodal velocities $\hat{v}$. Several methods for the interpolation of the field quantities within an element have been proposed. Due to the general applicability, the isoparametric concept, for which the geometry as well as the field quantities within the element are approximated with the same shape functions, is the most commonly used (Wriggers, 2001). There are numerous possibilities for the element formulation that differ in the type of the shape function and the number of integration (quadrature) points. When dealing with thin materials like sheet metals, it is reasonable to assume plane stress conditions and therefore use three-dimensional shell elements. The most common shell element approach is discretization of the shell midplane, e.g. with quadrilateral elements with four nodes. Rotational degrees of freedom are added to the the translative ones to represent locations and deformations outside the shell plane. In the following, the Belytschko-Lin-Tsay shell, which was used for the validation simulations with LS-Dyna in chapter 7, is discussed in detail. Other prominent examples of shell element formulations are e.g. the Hughes-Liu shell (Hughes and Liu, 1981a,b) or the fully integrated shell element with assumed strain interpolants used to alleviate locking and enhance in-plane bending behavior (Simo and Hughes, 1986; Engelmann et al., 1989).
For further information about finite element formulations refer to (Onate et al., 1991), (Belytschko et al., 2013) or (Bathe, 2006).

3.3.1 The Belytschko-Lin-Tsay shell

The Belytschko-Lin-Tsay element proposed by (Belytschko et al., 1984) is a computationally efficient shell formulation and thus, usually the shell element formulation of choice in LS-Dyna (Hallquist et al., 2006). It is a bilinear four-node quadrilateral element with single-point quadrature, which uses a co-rotational element coordinate system that rotates with the material to account for rigid body motion and thus satisfies frame invariance. This coordinate system deforms with the element and is defined in terms of the nodal coordinates. The definition of unit vectors which define the local (element) coordinate \( \hat{e}_1, \hat{e}_2 \) and \( \hat{e}_3 \) is given in the following according to (Hallquist et al., 2006)

\[
\hat{e}_3 = \frac{s_3}{||s_3||} \tag{3.20}
\]

where \( s_3 \) is the cross product of the two diagonals \( r_{31} \) and \( r_{42} \) that connect nodes 1 and 3, or 4 and 2 respectively. Furthermore

\[
s_1 = r_{21} - (r_{21} \cdot \hat{e}_3)\hat{e}_3 \tag{3.21a}
\]

\[
\hat{e}_1 = \frac{s_1}{||s_1||} \tag{3.21b}
\]

The vector \( s_1 \) is approximately along \( r_{21} \), which is the vector from node 1 to node 2. Eventually, the third unit vector \( \hat{e}_2 \) is defined as

\[
\hat{e}_2 = \hat{e}_3 \times \hat{e}_1 \tag{3.22}
\]

The construction of the co-rotational coordinate system is visualized in Fig. 3.2.

The global coordinates of the triad defined above can be used to transform global to local vectors and vice-versa.

\[
a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} e_{1x} & e_{2x} & e_{3x} \\ e_{1y} & e_{2y} & e_{3y} \\ e_{3x} & e_{3y} & e_{3z} \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = T \hat{a} \tag{3.23}
\]
3.3 Spatial discretization with finite elements

For the inverse transformation, the transpose of the orthogonal matrix $\mathbf{T}$ is used

$$\hat{\mathbf{a}} = \mathbf{T}^T \mathbf{a}$$  \hfill (3.24)

The relation between velocities, strains and displacements as computed in the Belytschko-Lin-Tsay element is described in the following based on (Hallquist et al., 2006; Belytschko et al., 1984). For shell elements, several integration points along the fiber length (thickness) are used in order to account for the bending stiffness of the material. The nodes not only have translational degrees of freedom but also rotational ones. According to the Mindlin theory of plates and shells (Mindlin, 1951), the velocity for every point in the shell is determined by

$$\mathbf{v} = \mathbf{v}^m - \hat{z} \mathbf{e}_3 \times \boldsymbol{\theta}$$  \hfill (3.25)

In Eq. (3.25), $\mathbf{v}^m$ is the velocity of the mid-surface, $\boldsymbol{\theta}$ is the angular velocity vector and $\hat{z}$ is the distance from the mid-surface along the fiber (thickness) direction. The components of the rate of deformation tensor in the element coordinate system $\mathbf{D}_{ij}$ are given by

$$\mathbf{D}_{ij} = \frac{1}{2} \left( \frac{\partial \hat{v}_i}{\partial \hat{x}_j} + \frac{\partial \hat{v}_j}{\partial \hat{x}_i} \right)$$  \hfill (3.26)

By substituting (3.25) in (3.26), the following expressions are obtained for
the components of the velocity strain (rate of deformation):

\[
\hat{D}_x = \frac{\partial \hat{v}_x^m}{\partial \hat{x}} + \hat{z} \frac{\partial \hat{\theta}_y}{\partial \hat{x}}, \quad \hat{D}_y = \frac{\partial \hat{v}_y^m}{\partial \hat{y}} + \hat{z} \frac{\partial \hat{\theta}_x}{\partial \hat{y}},
\]

\[
2\hat{D}_{xy} = \frac{\partial \hat{v}_x^m}{\partial \hat{y}} + \frac{\partial \hat{v}_y^m}{\partial \hat{x}} + \hat{z} \left( \frac{\partial \hat{\theta}_y}{\partial \hat{y}} - \frac{\partial \hat{\theta}_x}{\partial \hat{x}} \right),
\]

\[
2\hat{D}_{yz} = \frac{\partial \hat{v}_z^m}{\partial \hat{y}} - \hat{\theta}_x, \quad 2\hat{D}_{xz} = \frac{\partial \hat{v}_z^m}{\partial \hat{x}} - \hat{\theta}_y
\] (3.27)

The component \( \hat{D}_z \) is computed based on the assumption \( \hat{\sigma}_z = 0 \). Although a plane stress condition is assumed, the transverse stress components \( \hat{\sigma}_{xy} \) and \( \hat{\sigma}_{zx} \) are computed which act as a penalty factor against warping of the element.

Both, the nodal and the angular velocity are now approximated within the element by assuming a bilinear distribution using the same shape functions denoted in Eq. (3.28)

\[
N_1 = \frac{1}{4}(1 - \xi)(1 - \eta), \quad N_2 = \frac{1}{4}(1 + \xi)(1 - \eta)
\]

\[
N_3 = \frac{1}{4}(1 + \xi)(1 + \eta), \quad N_4 = \frac{1}{4}(1 - \xi)(1 + \eta)
\] (3.28)

The mid-surface and angular velocities can then be approximated with

\[
\mathbf{v}^m = N_I(\xi, \eta) \mathbf{v}_I \quad (3.29a)
\]

\[
\mathbf{\theta}^m = N_I(\xi, \eta) \mathbf{\theta}_I \quad (3.29b)
\]

In Eqs. (3.29) the subscript \( I \) refers to the node number and is summed over. By substituting Eqs. (3.29) into Eqs. (3.27), the following expressions for the components of the rate of deformation tensor are obtained

\[
\hat{D}_x = B_{1I} \hat{v}_{xi} + \hat{z} B_{1I} \hat{\theta}_{yi}, \quad \hat{D}_y = B_{2I} \hat{v}_{yi} + \hat{z} B_{2I} \hat{\theta}_{xi},
\]

\[
2\hat{D}_{xy} = B_{2I} \hat{v}_{xi} + B_{1I} \hat{v}_{yi} + \hat{z} (B_{2I} \hat{\theta}_{yi} - B_{1I} \hat{\theta}_{xi}),
\]

\[
2\hat{D}_{yz} = B_{1I} \hat{v}_{zi} + N_I \hat{\theta}_{yi}, \quad 2\hat{D}_{xz} = B_{2I} \hat{v}_{zi} + N_I \hat{\theta}_{xi}
\] (3.30)

where

\[
B_{1I} = \frac{\partial N_I}{\partial \hat{x}}, \quad B_{2I} = \frac{\partial N_I}{\partial \hat{y}}
\] (3.31)
3.3 Spatial discretization with finite elements

Because a single-point quadrature is used, the expressions in Eq. (3.30) are only evaluated in the integration point in the center of the element where $\xi = 0$ and $\eta = 0$ and in the through thickness integration points. For the through-thickness integration of the stresses, usually a Gauss quadrature rule is applied and the integration points are distributed accordingly. After the constitutive evaluations that make use of the above velocity-strains and are described in sec. 4, the resulting stresses are integrated through the thickness to obtain forces and moments. The integration is carried out with

$$\hat{f}_{\alpha\beta}^R = \int \hat{\sigma}_{\alpha\beta} d\hat{z} \quad (3.32a)$$

$$\hat{m}_{\alpha\beta}^R = -\int \hat{z} \hat{\sigma}_{\alpha\beta} d\hat{z} \quad (3.32b)$$

In Eqs. (3.32), the superscript $R$ designates resultant quantities and the Greek subscripts indicate the limited range of the indices for plane stress conditions. By applying the principal of virtual power and integrating with a one-point quadrature, the element-centered force and moment resultants are related to the nodal ones in the following manner

$$\hat{f}_{xI} = A \left( B_{1I} \hat{f}_{xx}^R + B_{2I} \hat{f}_{xy}^R \right) \quad (3.33a)$$

$$\hat{f}_{yI} = A \left( B_{2I} \hat{f}_{yy}^R + B_{1I} \hat{f}_{xy}^R \right) \quad (3.33b)$$

$$\hat{f}_{zI} = A \kappa \left( B_{1I} \hat{f}_{xz}^R + B_{2I} \hat{f}_{yz}^R \right) \quad (3.33c)$$

$$\hat{m}_{xI} = A \left( B_{2I} \hat{m}_{yy}^R + B_{1I} \hat{m}_{xy}^R - \frac{\kappa}{4} \hat{f}_{yz}^R \right) \quad (3.33d)$$

$$\hat{m}_{yI} = -A \left( B_{1I} \hat{m}_{xx}^R + B_{2I} \hat{m}_{xy}^R - \frac{\kappa}{4} \hat{f}_{xz}^R \right) \quad (3.33e)$$

$$\hat{m}_{zI} = 0 \quad (3.33f)$$

In Eqs. (3.33) $A$ designates the area of the element and $\kappa$ is the shear factor according to the Mindlin theory. In the Belytschko-Lin-Tsay formulation, $\kappa$ is used as a penalty parameter to enforce the Kirchoff normality condition as the shell becomes thin.

Using Eq. (3.23), the nodal forces are transformed back from the local element coordinate system to the global one. These global forces and moments are then approximately summed over all nodes and inserted into the momentum equation.
The usage of a single integration point makes hourglass control necessary, because zero-energy modes exist. To suppress these so-called hourglass deformation modes, hourglass viscosity stresses are added to the physical stresses at the local element level. The hourglass control used for the the Belytschko-Lin-Tsay element formulation is based on (Flanagan and Belytschko, 1981) and not further discussed here.

### 3.4 Time discretization

For the integration of the displacement between the actual time step \( t_n \) and the next time step \( t_{n+1} \), the central difference method (see e.g. (Bathe and Wilson, 1976)) is used. The central difference method is one of the most popular explicit methods methods in computational mechanics and physics (Belytschko et al., 2013).

The time increment for the central difference method is defined as

\[
\begin{align*}
\Delta t_{n+\frac{1}{2}} &= t_{n+1} - t_n \\
t_{n+\frac{1}{2}} &= \frac{1}{2} (t_{n+1} + t_n) \\
\Delta t_n &= t_{n+\frac{1}{2}} - t_{n-\frac{1}{2}}
\end{align*}
\]

In the equations above, the subscripts represent the indices of the timestep. For the velocity, the central difference gives the following expression

\[
\dot{u}_{n+\frac{1}{2}} \equiv v_{n+\frac{1}{2}} = \frac{u_{n+1} - u_n}{t_{n+1} - t_n} = \frac{1}{\Delta t_{n+\frac{1}{2}}} (u_{n+1} - u_n)
\]

(3.35)

where \( u_n \) designates the displacement at time \( t_n \). By rearranging Eq. (3.35), the integration formula is obtained as

\[
u_{n+1} = u_n + \Delta t_{n+\frac{1}{2}} v_{n+\frac{1}{2}}
\]

(3.36)

Similarly, the acceleration and the integration formula is

\[
\dot{u}_n \equiv a_n = \frac{v_{n+\frac{1}{2}} - v_{n-\frac{1}{2}}}{t_{n+\frac{1}{2}} - t_{n-\frac{1}{2}}}, \quad v_{n+\frac{1}{2}} = v_{n-\frac{1}{2}} + \Delta t_n a_n
\]

(3.37)
3.4 Time discretization

By substituting (3.35) in (3.37), the acceleration can be expressed in terms of the displacements

$$\ddot{u}_n \equiv a_n = \frac{t_{n-\frac{1}{2}} u_{n+1} - u_n - t_{n+\frac{1}{2}} u_n - u_{n-1}}{\Delta t_{n+\frac{1}{2}} \Delta t_n \Delta t_{n-\frac{1}{2}}}$$  \hspace{1cm} (3.38)

A graphical illustration showing the discrete times and time increments used in the equations above is shown in Fig. 3.3. The momentum equation at time $t_n$ can now be solved for the acceleration

$$a_n = M^{-1} \left[ f_{ext}^n - f_{int}^n - Cv_n \right]$$  \hspace{1cm} (3.39)

where $M$ is the Mass matrix. In (3.39), the velocity $v_n$ at time $t_n$ is needed to evaluate the damping component. Since this quantity is unknown, usually an asynchronous damping is assumed and the velocity at time $t_{n-\frac{1}{2}}$ is used (Crisfield, 1997).

$$Cv_n \approx Cv_{n-\frac{1}{2}}$$  \hspace{1cm} (3.40)
4 Yield criteria for metal sheets

A yield criterion mathematically describes the condition, i.e. the stress state, at which a material reaches the limit of elastic deformation and starts to deform plastically. Furthermore, a rule for the relationship between stresses and strains is necessary to fully describe the deformation of a metallic material. At last, a hardening law is needed, which describes the evolution of the initial yield stress under isotropic hardening conditions, or the evolution of the yield criterion if the material hardens anisotropically. A material passes from an elastic to a plastic state of deformation when the stress within reaches a critical level, the so-called yield point $\sigma_y$. For a uniaxial stress state, this critical stress and its evolution (i.e. the flow curve) can be determined by performing a tensile test. The exact point of yield is determined as described in sec. 5.2. For a multiaxial stress state, it is necessary to find a relationship between the stress components that reduces to a scalar value which can be compared with the yield stress. This relationship is usually defined as a scalar function of the stress components and is referred to as the yield function.

$$ F(\sigma_1, \sigma_2, \sigma_3, \bar{\varepsilon}_p) = \bar{\sigma}(\sigma_1, \sigma_2, \sigma_3) - \sigma_y(\bar{\varepsilon}_p) $$

(4.1)

where $\sigma_1$, $\sigma_2$ and $\sigma_3$ are the principal stress components and $\sigma_y$ is the yield stress. Eq. (4.1) is representing a surface in the principal stress space. For anisotropic criteria, the yield function depends on a material reference frame. Thus the function depends on all six stress components of the symmetric stress tensor $\sigma$ and represents a surface in this six-dimensional stress space.

In order to determine the plastic flow, i.e. the strains caused by an infinitesimal increase of the stress, usually the associative flow rule is applied. The associative flow rule, also known as the normality condition, states that the plastic strain rate $\dot{\varepsilon}_p$ is proportional to the gradient of the yield function $\bar{\sigma}$.

$$ \dot{\varepsilon}_p = \lambda \cdot \frac{\partial F}{\partial \sigma} $$

(4.2)
4 Yield criteria for metal sheets

where $\lambda$ is called the plastic multiplier. For non-associative flow rule models, two functions are considered, one of which is used for computing the equivalent stress $\bar{\sigma}$ (yield function) and the other one to compute the gradient in order to find the strain increment (flow potential). A non-associative yield model was e.g. published by (Stoughton, 2002).

Anisotropy is usually characterized by the Lankford parameters ($r$-values) which are defined as the ratio of width to thickness strain under uniaxial tension (see 5.2). For simple anisotropic criteria, the function parameters can be obtained either directly from the Lankford parameters, from the yield stresses in different directions or from a combination of the two. For more advanced models, the parameters need to be fitted by minimizing the errors between model prediction and measurements for both, the yield stress as well as the Lankford parameters. In case of an associative flow rule, this means that both, the value of the function as well as its derivative with respect to the stress components need to deliver the correct values at the corresponding stress states.

The following sections give an overview of the yield criteria and constitutive model developments for sheet metals that eventually led to the models used as reference in this thesis. For the models that have been implemented in the framework of this thesis, all the mathematics including the needed derivatives are given. The explanations in this section are derived from (Banabic, 2000; Raabe et al., 2006; Banabic et al., 2009).

4.1 Isotropic yield criteria

4.1.1 Tresca yield criterion

The Tresca yield criterion (Tresca, 1869) is the oldest yield criterion for full stress states (Banabic, 2000). It is also known as the maximum shear stress theory and based on Trescas observations that plastic strain appears to be a crystallographic gliding caused by shear stresses. From this, he derived that the maximum shear stress is responsible for yielding in a material. Thus, the criterion is expressed as

$$\max(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) - \sigma_y = 0 \quad (4.3)$$

In the expression above, $\sigma_1$, $\sigma_2$, $\sigma_3$ are the principal stresses and $\sigma_y$ is the uniaxial yield stress in tension. The yield surface defined by (4.3)
is shown in Fig. 4.1. It represents a six sided prism of infinite length along the space diagonal of the principal stress space. For plane stress conditions, the condition can be rewritten as

\[
(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2 = \sigma_y^2
\]  

(4.4)

A drawback of the Tresca yield criterion are the sharp edges and the straight segments of the yield surface in stress space. The normal vector does not change along a straight segment, but has a singularity at the edges. While being very useful for dimensioning technical parts, especially for brittle materials, this criterion is not suitable in combination with the associated flow rule because of the reasons stated above.

### 4.1.2 Von Mises yield criterion

In 1904, (Huber, 1904) and later in 1913 (Mises, 1913) independently proposed the Maxwell-Huber-Hencky-von Mises yield criterion, today mostly just called the von Mises yield criterion. The observations that led to this model were that a material does not deform under pure hydrostatic pressure. Thus, the basic assumption of this model is, that a material starts to deform plastically if a critical amount of distortion energy, that is independent of the stress state, is reached within the material. Hence, the criterion is also known as the distortion energy theory. The elastic potential energy of distortion in terms of principal stresses can be written as

\[
W_f = \frac{1 + \mu}{6E} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] 
\]  

(4.5)

For a uniaxial stress state ($\sigma_2 = \sigma_3 = 0$), plastic yielding occurs if $\sigma_1 = \sigma_y$ where $\sigma_y$ is the uniaxial yield stress. The elastic potential of distortion at this point can be expressed as

\[
W_{f,\text{uniax}} = \frac{1 + \mu}{6E} 2\sigma_y^2 
\]  

(4.6)

Setting Eq. (4.5) equal to Eq. (4.6) and solving for $\sigma_y$ leads to

\[
\sigma_y = \sqrt{\frac{1}{2} ((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)} 
\]  

(4.7)

If the principal directions of the stress state don’t coincide with the axes of the chosen coordinate system, all six stress components of the stress
4 Yield criteria for metal sheets

Figure 4.1: Tresca and von Mises yield locus in the principal stress space

tensor need to be taken into account. In this case, the function for the equivalent von Mises stress is expressed as

\[ \bar{\sigma} = \sqrt{\frac{1}{2} \left[ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{223} + \sigma_{231} + \sigma_{212}) \right]} \]  

(4.8)

Since the function given in Eq. (4.8) represents \( \sqrt{3J_2} \), where \( J_2 \) is the second invariant of the deviatoric stress tensor, the von Mises criterion is often referred to as \( J_2 \)-plasticity or \( J_2 \) flow theory. For plane stress conditions \( (\sigma_{33} = \sigma_{31} = \sigma_{23} = 0) \), the criterion reduces to

\[ F = \sqrt{\sigma_{11}^2 - \sigma_{11}\sigma_{22} + \sigma_{22}^2 + 3\sigma_{12}^2 - \sigma_y} = 0 \]  

(4.9)

In the principal stress space, the von Mises criterion represents a cylinder with the space diagonal as axes, the yield stress as radius and an infinite length (see Fig. 4.1). For plane stress conditions, it represents a spheroid in the \( \sigma_{11}-\sigma_{22}-\sigma_{12} \)-space.
4.2 Anisotropic yield criteria

4.1.3 Hershey Yield criterion

Applying the laws proposed by (Norton, 1929) and (Bailey, 1930) for non-linear creep, (Hershey, 1954) proposed a criterion of the form

\[(\sigma_1 - \sigma_2)^a + (\sigma_2 - \sigma_3)^a + (\sigma_3 - \sigma_1)^a = 2\bar{\sigma}^a\]  \hspace{1cm} (4.10)

In the equation above, \(\bar{\sigma}\) is the uniaxial yield stress and \(a\) is an exponent which depends on the crystallographic structure of the material. The yield criterion reduces to the von Mises criterion for \(a=2\) and to the Tresca criterion for \(a=1\) or \(a \to \infty\). For \(1 < a < 2\) and for \(a > 4\), the corresponding yield surface lies between the von Mises and the Tresca yield surface while it lies outside the Von Mises cylinder for \(2 < a < 4\). Hershey found a particularly good agreement with experimental data by using a value of 6 for \(a\). The Hershey criterion was later used by (Hosford, 1972) who added the corresponding flow rules, Lode variables and effective strain functions.

4.2 Anisotropic yield criteria

4.2.1 Hill 1948 Yield criterion

(Hill, 1948) generalized the von Mises criterion by assuming three orthogonal symmetry planes. The quadratic yield surface that he proposed is expressed as

\[2f(\sigma_{ij}) = F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 = 1\]  \hspace{1cm} (4.11)

In (4.11), \(f\) is the yield function and the parameters \(F, G, H, L, M\) and \(N\) are the function parameters, which are determined based on the measured anisotropy of the material. Since there is a direction dependency of the mechanical properties of the material, the yield function has to be defined in a coordinate system fixed in the material. For sheet metal material, direction 1 usually is the rolling direction (RD), 2 is the transverse direction (TD) and 3 is the through thickness direction (normal to the sheet plane). For plane stress conditions (\(\sigma_{33} = \sigma_{31} = \sigma_{32} = 0\)), the yield
function reduces to
\[(G + H)\sigma_x^2 - 2H\sigma_x\sigma_y + (F + H)\sigma_y^2 + 2N\sigma_{xy}^2 - (G + H)\sigma_f^2 = 0 \quad (4.12)\]

Under the assumption of an associated flow rule, the parameters in Eq. (4.12) can be determined on the basis of the Lankford parameters measured in RD, DD and TD as follows:
\[F = r_0, \quad G = r_{90}, \quad H = r_0r_{90}, \quad N = (r_{45} + 0.5)(r_0 + r_{90}) \quad (4.13)\]

The parameters might as well be determined on the basis of yield stresses in different directions. (Volk et al., 2013) proposed a weighted combination of the two determination methods to minimize the error of the model prediction.

Later, Hill extended his criterion to non-quadratic functions (Hill, 1979, 1990, 1993) to overcome certain limitations of the original formulation such as the disability to describe the Woodthorpe-Pearce ‘anomalous’ behavior (Woodthorpe and Pearce, 1970) which is \(r_0 = r_{90} = r < 1\) while the biaxial yield stress \(\sigma_b\) is larger than the uniaxial one, or the ‘second order anomalous behavior’ \((r_0 < r_{90}\) while \(\sigma_0 > \sigma_{90}\) and vice-versa).

### 4.2.2 Barlat Lian ‘89

(Barlat and Richmond, 1987) reformulated Hershey’s yield criterion (see sec. 4.1.3) for plane stress conditions in an arbitrary reference frame \(x, y\) and \(z\) by making use of the stress invariants \(k_1\) and \(k_2\):
\[\Phi = |k_1 - k_2|^m + |k_1 - k_2|^m + c|2k_2|^m = 2\bar{\sigma}^m \quad (4.14)\]

where
\[k_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} \quad (4.15a)\]
\[k_2 = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} \quad (4.15b)\]

are the invariants of the stress tensor while \(m\) is an the material exponent with the same significance as the exponent \(a\) used by Hosford. This model has been generalized later by (Barlat and Lian, 1989) to planar anisotropy by introducing the following yield function:
\[\Phi = a|k_1 + k_2|^m + a|k_1 - k_2|^m + c|2k_2|^m = 2\bar{\sigma}^m \quad (4.16)\]
4.2 Anisotropic yield criteria

with a modified definition of the stress invariants \( k_1 \) and \( k_2 \)

\[
k_1 = \frac{\sigma_{xx} + h\sigma_{yy}}{2} \quad (4.17a)
\]
\[
k_2 = \sqrt{\left(\frac{\sigma_{xx} - h\sigma_{yy}}{2}\right)^2 + p^2\sigma_{xy}^2} \quad (4.17b)
\]

\( a, c, h \) and \( p \) are the model parameters which describe the anisotropy of the material. There are two common ways to identify these parameters, based on yield stresses or based on the Lankford parameters. The more common identification procedure is based on the Lankford parameters and is denoted as

\[
c = 2\sqrt{\frac{r_0}{1 + r_0} \frac{r_{90}}{1 + r_{90}}} \quad (4.18a)
\]
\[
a = 2 - c = 2 - 2\sqrt{\frac{r_0}{1 + r_0} \frac{r_{90}}{1 + r_{90}}} \quad (4.18b)
\]
\[
h = \sqrt{\frac{r_0}{1 + r_0} \frac{1 + r_{90}}{r_{90}}} \quad (4.18c)
\]

The parameter \( p \) cannot be expressed explicitly in terms of \( r_0, r_{45} \) and \( r_{90} \). It has to be iteratively calculated. For example \( p \) can be determined by minimizing the difference between the measured \( r_{45} \) value and the model prediction. Besides the identification procedures given above, one can always identify the parameters by minimizing an error function which computes the model errors compared to experimental values.

4.2.3 Karafillis and Boyce

Based on the work of (Mendelson, 1968), (Karafillis and Boyce, 1993) proposed a generalization of Hershey and Hosford’s yield criterion by assuming an upper and a lower bound surface. The lower bound surface is represented by the Tresca criterion whereas the upper bound *corresponds to a limiting value the sum of the two greater diameters of Mohr’s circles* (Karafillis and Boyce, 1993) (see Fig. 4.2). The criterion is given as

\[
\phi(\sigma) = (1 - c)\phi_1(\sigma) + c\phi_2(\sigma) = 2\sigma_y^a \quad (4.19)
\]
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\[ \pi \text{-plane} \]

Figure 4.2: The lower bound, the upper bound and the von Mises yield surface in the \( \pi \)-plane

where \( \phi_1 \) is the Hershey yield function, \( c \) is a material coefficient (weighting factor) and

\[
\phi_2(\sigma) = \frac{3^a}{2^{a-1} + 1} (|s_1|^a + |s_2|^a + |s_3|^a) \tag{4.20}
\]

, where \( s_1 \) to \( s_3 \) are the principal components of the deviatoric stress tensor, represents the upper bound as mentioned by (Hosford, 1972). The idea is a weighted combination of two surfaces, one lying between the Tresca and the von Mises and the other one lying between von Mises and the upper bound. (Karafillis and Boyce, 1993) already provided ideas about how to extend their criterion to anisotropy by using linear transformations of the deviatoric stress tensor. These ideas are further discussed in sec. 4.2.4.

4.2.4 Yld2000-2d model

In 2003 (Barlat et al., 2003a) published the so-called Yld2000-2d yield criteria. It was mainly developed to overcome issues of the Yld96 (Barlat et al., 1997) yield criterion, namely the lack of a proof of convexity as well as the difficulty to obtain analytical derivatives. The Yld2000-2d criterion is an extension of the criterion of (Hershey, 1954) and (Hosford, 1972) to orthotropic anisotropy based on two linear transformations of the deviatoric stress tensor. The idea of the sum of two yield functions and the usage of linear transformation was adopted from (Karafillis and
4.2 Anisotropic yield criteria

Boyce, 1993). The proposed yield function has eight parameters that are determined based on eight mechanical properties. These are the uniaxial yield stress in RD, TD and DD ($\sigma_0$, $\sigma_{45}$, $\sigma_{90}$) and the yield stress under equibiaxial condition $\sigma_b$ as well as the Lankford parameters in the three directions ($r_0$, $r_{45}$, $r_{90}$) and the biaxial r-value $r_b$. The criterion is given as

$$\phi = \phi' + \phi'' = 2\sigma^m$$

where the exponent m is a material coefficient (which is typically set to 6 for bcc and to 8 for bcc materials) and

$$\phi' = |X_1' - X_2'|^m$$
$$\phi'' = [2X_2'' + X_1'']^m + |2X_1'' + X_2'|^m$$

where $X_i'$ and $X_j''$ are the principal values of the matrices $X'$ and $X''$

$$X_i' = \frac{1}{2} \left( X_{11}' + X_{22}' \pm \sqrt{(X_{11}' - X_{22}')^2 + 4X_{12}'^2} \right)$$
$$X_j'' = \frac{1}{2} \left( X_{11}'' + X_{22}'' \pm \sqrt{(X_{11}'' - X_{22}'')^2 + 4X_{12}''^2} \right)$$

whose components are obtained from the following linear transformation

$$X' = L'\sigma$$
$$X'' = L''\sigma$$

where

$$\begin{bmatrix}
L_{11}' \\
L_{12}' \\
L_{21}' \\
L_{22}' \\
L_{66}'
\end{bmatrix} = \begin{bmatrix}
2/3 & 0 & 0 \\
-1/3 & 0 & 0 \\
0 & -1/3 & 0 \\
0 & 2/3 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_7 \\
\end{bmatrix}$$

and

$$\begin{bmatrix}
L_{11}'' \\
L_{12}'' \\
L_{21}'' \\
L_{22}'' \\
L_{66}''
\end{bmatrix} = \frac{1}{9} \begin{bmatrix}
-2 & 2 & 8 & -2 & 0 \\
1 & -4 & -4 & 4 & 0 \\
4 & -4 & -4 & 1 & 0 \\
-2 & 8 & 2 & -2 & 0 \\
0 & 0 & 0 & 0 & 9
\end{bmatrix} \begin{bmatrix}
\alpha_3 \\
\alpha_4 \\
\alpha_5 \\
\alpha_6 \\
\alpha_8 
\end{bmatrix}$$
In the equations above, $\sigma$ is the Cauchy stress and $\alpha_1$ to $\alpha_8$ are the eight anisotropy coefficients. To determine the coefficients (Barlat et al., 2003a) propose the minimization of eight functions representing the difference between measured and predicted yield stresses or Lankford parameters respectively. The exponent $m$ is assumed to be a real number between 2 and $\infty$. By setting all $\alpha$ parameters equal to 1, and the exponent $m$ equal to 2, the yield criterion is reduced to the von Mises criterion while an exponent $m$ of $\infty$ leads to the Tresca criterion. By comparing their phenomenological model with yield loci calculated by a crystal plasticity model, (Logan and Hosford, 1980) stated, that an exponent of $m = 6$ is best suited for a body centered cubic (BCC) material, while for a material face centered cubic (FCC) material, an exponent $m = 8$ should be chosen. (Kuwabara et al., 2011) recommend determining the exponent $m$ based on yield stresses under biaxial loading with different ratios $\sigma_1/\sigma_2$. For this purpose they designed a testing apparatus for the biaxial testing of cruciform specimen (Kuwabara et al., 1998). Since a biaxial testing machine was not available, the exponent $m$ is set to 6 for the steel and to 8 for aluminum material. The derivatives for the equivalent stress according to the Yld2000-2d model with respect to the stress components are given in sec. 4.3.1.

### 4.3 Anisotropic hardening models

In order to model anisotropic hardening effects, different concepts have been used in the past. Most of the established models focus on a mathematical description of the Bauschinger effect (see sec. 2.5) by using kinematic hardening with a so-called backstress. The backstress is a torsorial state variable and evolves with progressing deformation leading to a shift of the center of the yield surface in the stress space. The most prominent models to describe the Bauschinger effect are Prager’s model (Prager, 1955), who assumed that the evolution of the backstress is in the same direction as the plastic strain increment. (Armstrong et al., 1966) enhanced the evolution law for the backstress by adding a recovery term, which helped them to improve the agreement with experimental observations. While the models just named describe the hardening only by the translation of the yield surface, (Chaboche, 1989) introduced a mixed hardening rule by superposing a nonlinear isotropic with a kinematic hardening rule,
but still used the Armostrong-Ferederick type of backstress evolution but with several backstresses that can evolve differently and are summed up to build the total backstress. (Yoshida and Uemori, 2002) presented a further improvement of the Chaboche model to also describe early reyielding and work-hardening stagnation. The yield surface evolves within a bounding surface, which expands based on a mixed isotropic kinematic hardening. Besides the models that solely describe the Bauschinger effect and work hardening stagnation, models have been proposed to capture latent effects as discussed in sec. 2.6. A model that has drawn great interest is the one proposed by (Teodosiu and Hu, 1995). They introduced state variables to account for the strength and polarity of dislocation structures as well as rapid changes in stress after load reversal due to dislocation pile-ups. This way, they were able to capture both, the Bauschinger and latent effects. Recently (Barlat et al., 2011) proposed an alternative approach to model the Bauschinger effect and extended it to latent effects in (Barlat et al., 2013a). Because of its relative simplicity and the possibility to combine it with any other first order homogenous yield function, its abilities and accuracy in combination with the modified Yld2000-2d is further investigated. Therefore, in the remainder of this section the modified Yld2000-2d model and the HAH model are described in detail including all derivatives needed for the implementation.

4.3.1 The equivalent strain dependent Yld2000-2d model

Basic idea

This modifications of the Yld2000 model was first proposed by (Hora et al., 2009) and later by (Wang et al., 2009). A more detailed analysis of the model with a description of the implementation as well as its application on deep drawing simulations for steel and aluminum material was published in (Peters et al., 2012). The basic idea of the model is to introduce a dependency of the model parameters $\alpha_{1-8}$ on the equivalent plastic strain $\bar{\varepsilon}$. This enables modeling direction dependent hardening for the four different load cases: uniaxial loading in RD, TD and DD as well as for equibiaxial loading.
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Modifications on Yld2000-2d

Constant parameters $\alpha_1$ to $\alpha_8$ lead to constant ratios $\sigma_{ii}/\sigma_{jj}$ for every level of equivalent plastic strain, when $\sigma_{ii}$ and $\sigma_{jj}$ are the yield stresses under two arbitrary stress states, i.e. an homogenous expansion of the yield surface. To overcome this limitation, the parameters $\alpha_1$ to $\alpha_8$ are expressed as a function of equivalent plastic strain (which is proportional to the equivalent plastic work). In order to obtain the parameters, the flow stress in RD ($\sigma_0$) at different levels of equivalent plastic strain $\bar{\varepsilon}_p^i$ is taken. In a second step, the corresponding plastic work $W_{pl}$ is computed according to Eq. (4.27).

$$W_{pl} = \int_{0}^{\bar{\varepsilon}_p^i} \sigma_y(\bar{\varepsilon}_p) d\bar{\varepsilon}_p$$  \hspace{1cm} (4.27)

The remaining yield stresses ($\sigma_{45}$, $\sigma_{90}$ and $\sigma_b$) as well as the Lankford parameters ($r_0$, $r_{45}$, $r_{90}$, $r_b$) are taken at the same amount of plastic work. The four yield stresses in combination with the four $r$-values build the input for the parameter fitting of the Yld2000-2d function. Now the fitting is done at different levels of equivalent plastic strain leading to the eight parameters being a function of the equivalent plastic strain. (Wang et al., 2009) uses a sixth order polynomial function to fit the parameters in the region between 0 and uniform elongation. Basically, an arbitrary function can be used to describe the slope of the parameters. From a numerical point of view, it is more stable to use a low order function for which the second derivative does not vary considerably (see sec. 4.4).

Normals on the yield surface

In the following section, the derivatives with respect to $\sigma$ are given according to (Yoon et al., 2004). In order to derive the expression for the equivalent stress $\bar{\sigma}$ with respect to the stress tensor $\sigma$, it is rewritten as:

$$\bar{\sigma} = \left\{ \frac{1}{2} \psi \right\}^{\frac{1}{2}} = \left\{ \frac{1}{2} \left( |X'_1 - X'_2|^a + |2X''_2 + X''_1|^a + |2X''_1 + X''_2|^a \right) \right\}^{\frac{1}{a}} \hspace{1cm} (4.28)$$
The derivatives of the Yld2000-2d yield function with respect to the stress component $\sigma_k$ is obtained by using the chain rule as follows:

$$\frac{\partial \tilde{\sigma}}{\partial \sigma_k} = \{2a\bar{\sigma}^{a-1}\}^{-1} \frac{\partial \psi}{\partial \sigma_k}$$

$$= \{2a\bar{\sigma}^{a-1}\}^{-1} \sum_{a=1}^{3} \sum_{b=1}^{3} \left( \frac{\partial \psi}{\partial \bar{\eta}_a} \frac{\partial \tilde{X}_b}{\partial \sigma_k} + \frac{\partial \psi}{\partial \bar{\eta}_a} \frac{\partial \tilde{X}_b}{\partial \sigma_k} \right)$$

(for $k=1$ to $3$) \hspace{1cm} (4.29)

Now

$$\left[ \frac{\partial \psi}{\partial \bar{\eta}_i} \right] = \begin{bmatrix} a \left\{ \left( X_1' - X_2' \right) \right| X_1' - X_2' \right|^{a-2} \right\} \\ -a \left\{ \left( X_1' - X_2' \right) \right| X_1' - X_2' \right|^{a-2} \right\} \right]$$

(4.30)

and

$$\left[ \frac{\partial \bar{\eta}_i}{\partial X_j'} \right] = \begin{bmatrix} \frac{1}{2} + \frac{(\tilde{X}_i - \tilde{X}_j)}{r'} \frac{1}{2} - \frac{(\tilde{X}_i - \tilde{X}_j)}{4r'} \frac{\tilde{X}_i}{r'} \\ \frac{1}{2} - \frac{(\tilde{X}_i - \tilde{X}_j)}{4r'} \frac{1}{2} + \frac{(\tilde{X}_i - \tilde{X}_j)}{4r'} \frac{\tilde{X}_i}{r'} \end{bmatrix}$$

(4.32)

$$\left[ \frac{\partial \bar{\eta}_i}{\partial X_j''} \right] = \begin{bmatrix} \frac{1}{2} + \frac{(\tilde{X}_i' + \tilde{X}_j)}{r''} \frac{1}{2} - \frac{(\tilde{X}_i' + \tilde{X}_j)}{4r''} \frac{\tilde{X}_i'}{r''} \\ \frac{1}{2} - \frac{(\tilde{X}_i' + \tilde{X}_j)}{4r''} \frac{1}{2} + \frac{(\tilde{X}_i' + \tilde{X}_j)}{4r''} \frac{\tilde{X}_i'}{r''} \end{bmatrix}$$

(4.33)

where $r' = \sqrt{\left( \frac{\tilde{X}_1 - \tilde{X}_2}{2} \right)^2 + \tilde{X}_3^2}$ and $r'' = \sqrt{\left( \frac{\tilde{X}_1' - \tilde{X}_2'}{2} \right)^2 + \tilde{X}_3'^2}$.

Furthermore

$$\left[ \frac{\partial \tilde{X}_i'}{\partial \sigma_j} \right] = \begin{bmatrix} L_{11}' & L_{12}' & 0 \\ L_{21}' & L_{22}' & 0 \\ 0 & 0 & L_{33}' \end{bmatrix}$$

(4.34)

$$\left[ \frac{\partial \tilde{X}_i''}{\partial \sigma_j} \right] = \begin{bmatrix} L_{11}'' & L_{12}'' & 0 \\ L_{21}'' & L_{22}'' & 0 \\ 0 & 0 & L_{33}'' \end{bmatrix}$$

(4.35)
Hessian matrix

The Hessian Matrix of the Yld2000-2d yield function representing \( \frac{\partial^2 \bar{\sigma}}{\partial \sigma \partial \sigma} \) is obtained using the chain rule:

\[
\frac{\partial^2 \bar{\sigma}}{\partial \sigma_j \partial \sigma_j} = \frac{\bar{\sigma}^{(1-a)}}{2a} \frac{\partial^2 \psi}{\partial \sigma_i \partial \sigma_j} - \frac{(a-1)}{\bar{\sigma}} \frac{\partial \bar{\sigma}}{\partial \sigma_i} \frac{\partial \bar{\sigma}}{\partial \sigma_j} \quad \text{for } i,j = 1 \text{ to } 3 \quad (4.36)
\]

Now

\[
\frac{\partial^2 \psi}{\partial \sigma_i \partial \sigma_j} = \sum_{a=1}^{2} \sum_{r=1}^{3} \sum_{s=1}^{2} \sum_{b=1}^{3} \left\{ \frac{\partial^2 \psi}{\partial \eta_{a}^r \partial \eta_{a}^s} \left( \frac{\partial \eta_{a}^r}{\partial X_{i}^r} \frac{\partial X_{i}^r}{\partial \sigma_i} \right) \left( \frac{\partial \eta_{a}^s}{\partial X_{j}^s} \frac{\partial X_{j}^s}{\partial \sigma_j} \right) \right\} + \sum_{a=1}^{2} \sum_{r=1}^{3} \sum_{b=1}^{3} \left\{ \frac{\partial \psi}{\partial \eta_{a}^r} \frac{\partial^2 \eta_{a}^s}{\partial X_{i}^r \partial X_{j}^s} \left( \frac{\partial X_{i}^r}{\partial \sigma_i} \right) \left( \frac{\partial X_{j}^s}{\partial \sigma_j} \right) \right\} + \sum_{a=1}^{2} \sum_{b=1}^{3} \left\{ \frac{\partial \psi}{\partial \eta_{a}^r} \frac{\partial^2 \eta_{a}^s}{\partial X_{i}^r \partial \sigma_j} \left( \frac{\partial X_{i}^r}{\partial \sigma_i} \right) + \frac{\partial \psi}{\partial \eta_{a}^s} \frac{\partial^2 \eta_{a}^s}{\partial X_{j}^s \partial \sigma_j} \left( \frac{\partial^2 \eta_{a}^s}{\partial \sigma_i \partial \sigma_j} \right) \right\} \quad (4.37)
\]

Note that \( \left( \frac{\partial^2 \bar{X}_{i}^r}{\partial \sigma_j \partial \sigma_i} \right) \) and \( \left( \frac{\partial^2 \bar{X}_{i}^r}{\partial \sigma_j \partial \sigma_i} \right) \) vanish.

\[
\left[ \frac{\partial^2 \psi}{\partial \eta_{a}^r \partial \eta_{a}^s} \right] = a(a-1) \begin{bmatrix} |X_{1}^r - X_{2}^r|^{a-2} & -|X_{1}^r - X_{2}^r|^{a-2} \\ -|X_{1}^r - X_{2}^r|^{a-2} & |X_{1}^r - X_{2}^r|^{a-2} \end{bmatrix} \quad (4.38)
\]

\[
\left[ \frac{\partial^2 \psi}{\partial \eta_{a}^r \partial \eta_{a}^s} \right] = a(a-1) \begin{bmatrix} 2|X_2^r|^a - 2|X_1^r|^a + 4|X_0^r|^a & 2|X_2^r|^a - 2|X_1^r|^a + 4|X_0^r|^a \\ 2|X_2^r|^a - 2|X_1^r|^a + 4|X_0^r|^a & 2|X_2^r|^a - 2|X_1^r|^a + 4|X_0^r|^a \end{bmatrix} \quad (4.39)
\]

\[
\left[ \frac{\partial^2 \eta_{a}^r}{\partial \bar{X}_i^s \partial \bar{X}_j^s} \right] = \begin{bmatrix} -\frac{1}{4} \frac{(X_1^r - X_2^r)^2}{16r^3} - \frac{1}{4} \frac{(X_1^r - X_2^r)^2}{16r^3} - \frac{(X_1^r - X_2^r)X_3^s}{4r^3} & \frac{1}{4r^3} - \frac{X_2^s}{r^3} \\ -\frac{1}{4} \frac{(X_1^r - X_2^r)^2}{16r^3} - \frac{1}{4} \frac{(X_1^r - X_2^r)^2}{16r^3} - \frac{(X_1^r - X_2^r)X_3^s}{4r^3} & \frac{1}{4r^3} - \frac{X_2^s}{r^3} \end{bmatrix} \quad (4.40)
\]
and the derivative with respect to \( \bar{\varepsilon} \)

\[
\frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}} = \left\{ 2a\bar{\sigma}^{-1} \right\} \sum_{a=1}^{2} \sum_{b=1}^{3} \sum_{i=1}^{8} \left( \frac{\partial \psi}{\partial \bar{\sigma}^a_{ab}} \frac{\partial \bar{\sigma}^a_{ab}}{\partial \bar{X}^i} \frac{\partial \bar{X}^i}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial \bar{\varepsilon}} + \frac{\partial \psi}{\partial \bar{\alpha}^a_{ab}} \frac{\partial \bar{\sigma}^a_{ab}}{\partial \bar{X}^i} \frac{\partial \bar{X}^i}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial \bar{\varepsilon}} \right)
\]

(4.44)

where

\[
\left[ \frac{\partial \bar{X}^i}{\partial \alpha} \right] = \left[ \begin{array}{cccccccc}
\frac{2}{3} \sigma_{xx} - \frac{1}{3} \sigma_{yy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{3} \sigma_{xx} + \frac{2}{3} \sigma_{yy} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_{xy} & 0 \\
\end{array} \right]
\]

(4.45)

and

\[
\left[ \frac{\partial \bar{X}^m}{\partial \alpha} \right] = \left[ \begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array} \right]
\]

(4.46)
Mixed derivative with respect to \( \sigma \) and \( \bar{\varepsilon}^p \)

To obtain the mixed derivative of the Yld2000-2d yield function with respect to stress and to strain, (4.28) has to be derived with respect to the effective plastic strain \( \bar{\varepsilon} \).

\[
\frac{\partial^2 \bar{\sigma}}{\partial \sigma \partial \bar{\varepsilon}} = \frac{\bar{\sigma}^{(1-a)}}{2a} \frac{\partial^2 \psi}{\partial \sigma \partial \bar{\sigma}} - \frac{(a-1)}{\bar{\sigma}} \frac{\partial \bar{\sigma}}{\partial \sigma} \frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}}
\]  

(4.47)

The only thing missing is the mixed derivative of \( \Psi \) with respect to stress and strain. This derivative can be obtained analogously to the hessian matrix

\[
\frac{\partial^2 \psi}{\partial \sigma_i \partial \bar{\varepsilon}} = \sum_{r=1}^{2} \sum_{s=1}^{3} \sum_{a=1}^{2} \sum_{b=1}^{3} \left\{ \frac{\partial^2 \psi}{\partial \eta_a' \partial \bar{\eta}_a'} \left( \frac{\partial \bar{\eta}_a'}{\partial \sigma_i} \frac{\partial \bar{\eta}_a'}{\partial \bar{\varepsilon}} \right) \left( \frac{\partial \bar{X}_b'}{\partial \bar{\varepsilon}} \right) + \frac{\partial \psi}{\partial \bar{\eta}_a'} \frac{\partial^2 \eta_a''}{\partial \bar{\varepsilon} \partial \sigma_i} \left( \frac{\partial \bar{X}_b''}{\partial \bar{\varepsilon}} \right) \right\}
\]

(4.48)

Note that \( \left( \frac{\partial^2 \bar{X}_r'}{\partial \sigma_i \partial \sigma_k} \right) \) and \( \left( \frac{\partial^2 \bar{X}_b''}{\partial \sigma_i \partial \sigma_k} \right) \) vanish. The remaining terms of Eq. (4.48) are given in Eqs. (4.38) to (4.43) and Eqs. (4.45) to (4.46).

4.3.2 The homogenous function based anisotropic hardening (HAH) model

Basic idea

The basic idea of this model, published by (Barlat et al., 2011) is to describe the Bauschinger effect not by means of a kinematic hardening model involving a backstress, but with a distortion of the yield locus controlled by state variables. In other words, this means that the yield locus does not shift in stress space, but is deformed. For this purpose, the near deformation history of a material point is stored by means of the so-called
4.3 Anisotropic hardening models

The microstructure deviator \( \hat{h}^* \), which is initialized to the normalized stress deviator \( \hat{s} \) during the first plastic strain increment. The normalization for a tensor \( \mathbf{x} \) in this case is defined as

\[
\hat{\mathbf{x}} = \frac{\mathbf{x}}{\sqrt{\frac{8}{3} \mathbf{x} : \mathbf{x}}}
\]  

(4.49)

The factor \( \frac{8}{3} \) in the equation above is used for matters of convenience. As soon as the stress deviator \( \mathbf{s} \) changes, the microstructure deviator \( \hat{h}^* \) rotates in the direction of \( \mathbf{s} \) or \( -\hat{h}^* \) respectively. Like this, a load reversal can be detected by a change of the sign of the quantity \( \hat{h}^* : \mathbf{s} \) which in turn determines the evolution of the state variables.

**Basic HAH model**

The yield function as proposed in (Barlat et al., 2011) is formulated as

\[
\bar{\sigma}(\mathbf{s}) = \left[ \phi^q(\mathbf{s}) + f_1^q \left| \hat{h}^* : \mathbf{s} \right|^q + f_2^q \left| \hat{h}^* : \mathbf{s} + \hat{h}^* : \mathbf{s} \right|^q \right]^\frac{1}{q}
\]

(4.50)

In the yield function above \( \phi \) is the stable yield function component which can be every first order homogeneous yield function like e.g. von Mises, Hill48 or Yld2000. The two parameters \( f_1 \) and \( f_2 \) are related to the state variables \( g_1 \) and \( g_2 \) in the following manner:

\[
f_k = \left( \frac{1}{g_k} - 1 \right)^\frac{1}{q} \text{ for } k = 1 \text{ and } 2
\]

(4.51)

\( g_k \) are the flow stress ratios of the distorted yield surface to the isotropic hardening case for \( \hat{h}^* : \mathbf{s} \) being negative or positive respectively. The state variables \( g_k \) evolve with ongoing deformation according to the following evolution laws:

1. If \( \hat{h}^* : \mathbf{s} \geq 0 \)
   \[
   \frac{d g_1}{d \varepsilon} = k_2 \left( k_3 \frac{\bar{\sigma}_0}{\bar{\sigma}} - g_1 \right)
   \]
   \[
   \frac{d g_2}{d \varepsilon} = k_1 \left( g_3 - g_2 \right)
   \]
   \[
   \frac{d g_4}{d \varepsilon} = k_5 \left( k_4 - g_4 \right)
   \]
   \[
   \frac{d \hat{h}}{d \varepsilon} = k \left[ \hat{s} - \frac{8}{3} \hat{h} \left( \hat{h}^* : \hat{s} \right) \right]
   \]

2. If \( \hat{h}^* : \mathbf{s} < 0 \)
   \[
   \frac{d g_1}{d \varepsilon} = k_1 \left( g_4 - g_1 \right)
   \]
   \[
   \frac{d g_2}{d \varepsilon} = k_2 \left( k_3 \frac{\bar{\sigma}_0}{\bar{\sigma}} - g_2 \right)
   \]
   \[
   \frac{d g_3}{d \varepsilon} = k_5 \left( k_4 - g_3 \right)
   \]
   \[
   \frac{d \hat{h}}{d \varepsilon} = k \left[ -\hat{s} + \frac{8}{3} \hat{h} \left( \hat{h}^* : \hat{s} \right) \right]
   \]

(4.52)
where $\hat{s}$ is the stress deviator normalized as in Eq. (4.49). Two more state variables were introduced in the equations above, namely $g_3$ and $g_4$. These variables control the permanent softening behavior after load reversal. They quantify the ratio of permanent softening, i.e. the ratio of the flow stress after stress reversal with and without permanent softening. The effect of the parameters $k$ and $k_{1-5}$ can all be clearly explained. $k$ is the rate of the rotation of $\hat{h}$ towards $s$. $k_1$ is the rate, at which the deformed part of the yield locus recovers its original shape or the permanent softening shape respectively. $k_2$ is the rate at which the side of the yield locus opposite to the actual stress state flattens. It saturates at a level $k_3 \bar{\sigma}_0$. The permanent softening ratios $g_3$ and $g_4$ saturate at a level of $k_4$ while $k_5$ scales the rate of saturation. A schematic plot of the evolution of the state variables for forward loading and load reversal at $\bar{\epsilon}^*$ and the influence of the parameters is illustrated in Fig. 4.3. Note that that in these illustrations, forward loading as well as reverse loading is carried out until the state variables have reached their saturation values. Fig. 4.4 shows the deformation of the corresponding yield locus for forward loading (4.4a) and reverse loading (4.4b). The red dot indicates the stress state of the actual load case.

**Extension to latent hardening**

For some materials, especially mild steel, a stress overshoot can be measured after an orthogonal strain path change as explained in sec. 2.6 and reported e.g. by (Rauch and Schmitt, 1989; Bouvier et al., 2005) and in sec. 5.5. The closer the strain path change is to orthogonal loading, the more distinctive is the stress overshoot (see e.g. (Butuc et al., 2013)). A quantity to characterize the strain path change was introduced by (Schmitt et al., 1991), although he proposed this equation with strain increments instead of $\hat{s}$ and $\hat{h}$.

$$
cos\chi = \frac{\hat{s} : \hat{h}}{\sqrt{\hat{s} : \hat{s}} \sqrt{\hat{h} : \hat{h}}} = \frac{8}{3} \hat{s} : \hat{h} \quad (4.53)
$$

(Barlat et al., 2013a) proposed the factor $g_L$ which scales the monotonic flow stress $\sigma_f$ and evolves according to (4.54).

$$
\frac{dg_L}{d\bar{\epsilon}} = k_L \left[ \frac{\sigma_f - \sigma_y}{\sigma_f} \left( \sqrt{L (1 - \cos^2 \chi)} + \cos^2 \chi - 1 \right) + 1 - g_L \right] \quad (4.54)
$$
4.3 Anisotropic hardening models

(a) Evolution of $g_1$

(b) Evolution of $g_2$

(c) Evolution of $g_3$

(d) Evolution of $g_4$

Figure 4.3: Evolution of HAH state variables after load reversal at $\varepsilon^*$

(a) Forward loading

(b) Reverse loading

Figure 4.4: Deformation of HAH yield locus for forward-reverse loading
For monotonic loading, \( g_L \) stays at a constant level of 1, while it increases as soon as the strain path changes with a value \( \cos^2 \chi \) different from 1. The closer to 0 \( \cos^2 \chi \) is, the higher is the rate of \( g_L \), which increases. As the deformation continues, \( \hat{\mathbf{h}} \) "rotates" towards \( \hat{s} \), \( \cos^2 \chi \) reaches 0 again and therefore also \( g_L \) becomes 1 again. The parameters \( k_L \) and \( L \) are material parameters and need to be determined by means of appropriate experiments.

**Extension to latent contraction**

As explained in sec 2.6, for other materials like e.g. high strength steels (see e.g. (Barlat et al., 2013a)) or certain aluminum alloys (see sec. 5.5), the material reyields at a lower stress level after prestrain in a certain direction and subsequent orthogonal loading. In order to capture this kind of behavior, a factor \( g_C \) is proposed, which, again, scales the actual flow curve, this time to a lower level (Peters et al., 2013). Since \( g_C \) should only influence the yield stress if the strain path is different from a linear one, but has to evolve even before because of an earlier re-yielding, another variable \( f_C \) is introduced in the scope of this thesis. The state variable \( f_C \) evolves according to

\[
\frac{df_C}{d\bar{\varepsilon}} = k_C ((C - 1) \cos^2 \chi + 1 - f_C) \tag{4.55}
\]

To make sure, the scaling is only active if the strain path changes, \( g_C \) is defined as

\[
g_C = \frac{1}{f_C + \cos^2 \chi (1 - f_C)} \tag{4.56}
\]

A similar method was proposed in (Barlat et al., 2013b). Instead of scaling the flow curve though, the authors use a factor which evolves in the same way as \( f_C \) to a linear transformation of the orthogonal component of the actual stress deviator. The parameters \( k_C \) and \( C \) are material parameters and need to be determined by means of appropriate experiments.

**Normals on yield surface**

The normal on the yield surface, which is determines the strain increment under the assumption of an associative flow rule is known to be the derivative of the yield function with respect to the actual stress. The
4.3 Anisotropic hardening models

The computation of this derivative is lengthy but straightforward. The following equations show how the derivative can be computed. Note that the state parameters \( f_1 \) and \( f_2 \) as well as the microstructure deviator \( \hat{h} \) are frozen for the computation assuming that there is no change of these quantities with increasing stress. This ensures the normality condition is valid, since these values can iteratively be updated during a return algorithm which guarantees the normality rule being fulfilled at the final iteration. The derivatives given in the following are partly given in (Lee et al., 2012b) and (Lee et al., 2012a) although a wrong assumption was made leading to much more complicated derivatives.

The yield function is written as

\[
\bar{\sigma} = \left[ \phi^q + f_1^q \right] \sum_{\hat{h}} \mathbf{s} - \left| \hat{\mathbf{h}} : \mathbf{s} \right|^q + f_2^q \left| \hat{\mathbf{h}} : \mathbf{s} \right|^q \right]^{1/q}
\] (4.57)

The derivative of the effective stress \( \bar{\sigma} \) with respect to stress \( \sigma \) is computed as follows:

For \( s : \hat{h} < 0 \) the yield function can be written as

\[
\bar{\sigma}(\sigma) = \left( \phi^q + f_1^q \right) \left| 2s : \hat{h} \right|^q \right)^{1/q}
\] (4.58)

Therefore

\[
\frac{\partial \bar{\sigma}}{\partial \sigma} = \bar{\sigma}^{1-q} \left( \frac{\partial \phi}{\partial \sigma} \phi^{q-1} + f_1^q \left| 2\hat{h} : \mathbf{s} \right|^{q-1} \frac{\partial \left( 2\hat{h} : \mathbf{s} \right)}{\partial \sigma} \right)
\] (4.59)

where

\[
\frac{\partial \left| 2\hat{h} : \mathbf{s} \right|}{\partial \sigma} = -2\hat{h} : \frac{ds}{d\sigma}
\] (4.60)

Substituting (4.60) in (4.59) gives

\[
\frac{\partial \bar{\sigma}}{\partial \sigma} = \bar{\sigma}^{1-q} \left( \frac{\partial \phi}{\partial \sigma} \phi^{q-1} - 2^q f_1^q \left| \hat{h} : \mathbf{s} \right|^{q-1} \hat{h} : \frac{ds}{d\sigma} \right)
\] (4.61)

Similarly, the derivative for \( s : \hat{h} \geq 0 \) can be found to be

\[
\frac{\partial \bar{\sigma}}{\partial \sigma} = \bar{\sigma}^{1-q} \left( \frac{\partial \phi}{\partial \sigma} \phi^{q-1} + 2^q f_2^q \left| \hat{h} : \mathbf{s} \right|^{q-1} \hat{h} : \frac{ds}{d\sigma} \right)
\] (4.62)
Hessian Matrix

The hessian matrix for the HAH model is derived from equation (4.62):

\[
\begin{bmatrix}
\frac{\partial^2 \bar{\sigma}}{\partial \sigma \partial \sigma}
\end{bmatrix} = \frac{\partial}{\partial \sigma} \left( \bar{\sigma}^{1-q} \left( \frac{\partial \phi}{\partial \sigma} \phi^{q-1} - 2^q f_1^q \hat{h} : s \right) \hat{h} : \frac{ds}{d\sigma} \right) \\
= \frac{\partial \bar{\sigma}^{-1-q}}{\partial \sigma} \cdot \Psi + \bar{\sigma}^{1-q} \cdot \frac{\partial \Psi}{\partial \sigma}
\]

(4.63)

where

\[
\Psi = \left( \frac{\partial \phi}{\partial \sigma} \phi^{q-1} - 2^q f_1^q \hat{h} : s \right) \hat{h} : \frac{ds}{d\sigma}
\]

and

\[
\frac{\partial \Psi}{\partial \sigma} = \frac{\partial}{\partial \sigma} \left( \frac{\partial \phi}{\partial \sigma} \phi^{q-1} - 2^q f_1^q \hat{h} : s \right) \hat{h} : \frac{ds}{d\sigma} \\
= \frac{\partial}{\partial \sigma} \left( \frac{\partial \phi}{\partial \sigma} \phi^{q-1} \right) - \frac{\partial}{\partial \sigma} \left( 2^q f_1^q \hat{h} : s \hat{h} : \frac{ds}{d\sigma} \right)
\]

(4.64)

In the equations above the following relations hold:

\[
\frac{\partial}{\partial \sigma} \left( \frac{\partial \phi}{\partial \sigma} \phi^{q-1} \right) = \frac{\partial^2 \phi}{\partial \sigma^2} \cdot \phi^{q-1} + \frac{\partial \phi}{\partial \sigma} \cdot (q-1) \phi^{q-2} \frac{\partial \phi}{\partial \sigma}
\]

(4.65)

\[
\frac{\partial}{\partial \sigma} \left( -2^q f_1^q \hat{h} : s \hat{h} : \frac{ds}{d\sigma} \right) = -2^q f_1^q \left( \frac{\partial}{\partial \sigma} \left( \hat{h} : s \right) \hat{h} : \frac{ds}{d\sigma} \right) \\
- \hat{h} : s \hat{h} : \frac{ds}{d\sigma} \left( \hat{h} : \frac{ds}{d\sigma} \right)
\]

(4.66)

Since \( \frac{\partial}{\partial \sigma} \left( \hat{h} : \frac{ds}{d\sigma} \right) \) vanishes, (4.66) reduces to

\[
\frac{\partial}{\partial \sigma} \left( -2^q f_1^q \hat{h} : s \hat{h} : \frac{ds}{d\sigma} \right) = -2^q f_1^q \left( \frac{\partial}{\partial \sigma} \left( \hat{h} : s \right) \hat{h} : \frac{ds}{d\sigma} \right)
\]

(4.67)

Furthermore

\[
\frac{\partial}{\partial \sigma} \left( \hat{h} : s \right) = (q-1) \hat{h} : s \hat{h} : \frac{ds}{d\sigma}
\]

(4.68)
4.3 Anisotropic hardening models

Derivative of the yield function with respect to the equivalent plastic strain \( \bar{\varepsilon} \)

The following equations show the computation of derivative of the HAH yield function with respect to the equivalent plastic strain \( \bar{\varepsilon} \):

Again, for the case \( s : \hat{h} < 0 \) the yield function can be written as

\[
\bar{\sigma}(\sigma) = \left( \phi^q + f_1^q \left| \hat{s} : \hat{h} \right|^q \right)^{1/q} \tag{4.69}
\]

Therefore

\[
\frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}} = \frac{1}{q} \left( \phi^q + f_1^q \left| \hat{s} : \hat{h} \right|^q \right)^{1-q/2} \left( q \phi^{q-1} \frac{\partial \phi}{\partial \bar{\varepsilon}} + \frac{\partial f_1^q}{\partial \bar{\varepsilon}} \left| \hat{s} : \hat{h} \right|^q \right) \tag{4.70}
\]

where

\[
\frac{\partial f_1^q}{\partial \bar{\varepsilon}} = q f_1^{q-1} \frac{\partial f_1}{\partial g_1} \frac{\partial g_1}{\partial \bar{\varepsilon}} \tag{4.71}
\]

\[
\frac{\partial f_1}{\partial g_1} = -f_1^{1-q} g^{-(q+1)} k_1 \frac{g_4 - g_1}{g_1} \tag{4.72}
\]

Using (4.73), (4.72) reduces to

\[
\frac{\partial f_1^q}{\partial \bar{\varepsilon}} = q f_1^{q-1} \frac{\partial f_1}{\partial g_1} \frac{\partial g_1}{\partial \bar{\varepsilon}} = -q f_1^{q-1} f_1^{1-q} g^{-(q+1)} k_1 \frac{g_4 - g_1}{g_1} \tag{4.74}
\]

and

\[
\frac{2 \hat{h} : s}{\partial \bar{\varepsilon}} = q \left| 2 \hat{h} : s \right|^{q-1} \frac{\partial \left| 2 \hat{h} : s \right|}{\partial \bar{\varepsilon}} \tag{4.75}
\]

\[
= q \left| 2 \hat{h} : s \right|^{q-1} - 2 \frac{\partial \hat{h}}{\partial \bar{\varepsilon}} : s
\]

\[
= -2^q q \left| \hat{h} : s \right|^{q-1} \left[ k \left( -\hat{s} + \frac{8}{3} \hat{h} \left( \hat{h} : \hat{s} \right) \right) \right] : s
\]
since $s$ is independent of $\varepsilon$. Now substituting (4.72) to (4.75) in (4.70), the following term is obtained

$$\frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}} = \bar{\sigma}^{1-q} \left( \phi^{q-1} \frac{\partial \phi}{\partial \bar{\varepsilon}} - g_1^{-(q+1)} k_1 \left( \frac{g_4 - g_1}{g_1} \right) \right) |2\hat{h} : s|^q$$

$$- f_1^q 2^q \hat{h} : s \left[ k \left( -\hat{s} + \frac{8}{3} \hat{h} (\hat{h} : \hat{s}) \right) \right] : s$$

(4.76)

Similarly, for the case $s : \hat{h} \geq 0$, the derivative is obtained as

$$\frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}} = \bar{\sigma}^{1-q} \left( \phi^{q-1} \frac{\partial \phi}{\partial \bar{\varepsilon}} - g_2^{-(q+1)} k_1 \left( \frac{g_3 - g_2}{g_2} \right) \right) |2\hat{h} : s|^q$$

$$+ f_2^q 2^q \hat{h} : s \left[ k \left( \hat{s} - \frac{8}{3} \hat{h} (\hat{h} : \hat{s}) \right) \right] : s$$

(4.77)

Mixed derivative with respect to stress and strain

Starting at the derivative with respect to stress (Eq. (4.61)), the mixed derivative of the HAH equivalent stress with respect to the stress $\sigma$ and equivalent strain $\bar{\varepsilon}$ for $s : \hat{h} < 0$ can be written as

$$\frac{\partial^2 \bar{\sigma}}{\partial \sigma \partial \bar{\varepsilon}} = (q - 1) \bar{\sigma}^{-q} \frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}} \cdot \chi + \bar{\sigma}^{1-q} \frac{\partial \chi}{\partial \bar{\varepsilon}}$$

(4.78)

where

$$\chi = \frac{\partial \phi}{\partial \sigma} \phi^{q-1} - 2^q f_1^q |\hat{h} : s|^{q-1} \hat{h} : \frac{ds}{\sigma}$$

(4.79)

for $s : \hat{h} < 0$ and

$$\chi = \frac{\partial \phi}{\partial \sigma} \phi^{q-1} + 2^q f_2^q |\hat{h} : s|^{q-1} \hat{h} : \frac{ds}{\sigma}$$

(4.80)

for $s : \hat{h} \geq 0$. The derivative $\frac{\partial \chi}{\partial \bar{\varepsilon}}$ is for $s : \hat{h} < 0$ is

$$\frac{\partial \chi}{\partial \bar{\varepsilon}} = \frac{\partial \phi}{\partial \sigma} \phi^{q-1} + \frac{\partial \phi}{\partial \sigma} (q - 1) \phi^{q-2} \frac{\partial \phi}{\partial \bar{\varepsilon}} - 2^q \left( \frac{\partial f_1^q}{\partial \bar{\varepsilon}} |\hat{h} : s|^{q-1} \hat{h} : \frac{ds}{d\sigma} \right.$$

$$\left. + f_1^q \hat{h} : s |^{q-1} \hat{h} : \frac{ds}{d\sigma} \right)$$

(4.81)
4.4 Integration algorithms for constitutive models

The expression for $\frac{\partial f}{\partial \bar{\varepsilon}}$ is given in Eq.(4.74). Analogously to Eq. (4.75)

$$\frac{\partial |\hat{h} : s|^{q-1}}{\partial \bar{\varepsilon}} = (q - 1)|\hat{h} : s|^{q-2}\frac{d \hat{h}}{d \bar{\varepsilon}} : s$$

$$= (q - 1)|\hat{h} : s|^{q-2}\left(k \left[-\hat{s} + \frac{8}{3} \hat{h} \left(\hat{h}^s : \hat{s}\right)\right] : s\right)$$

(4.82)

Finally

$$\frac{\partial \hat{h} : \frac{ds}{d\sigma}}{\partial \bar{\varepsilon}} = \frac{d \hat{h}}{d \bar{\varepsilon}} : \frac{ds}{d\sigma}$$

(4.83)

The derivatives for $s : \hat{h} \geq 0$ are computed analogously.

### 4.4 Integration algorithms for constitutive models

#### 4.4.1 Convex cutting plane algorithm

The convex cutting plane algorithm (CCPA), described in (Simo and Hughes, 1998) can be used for the implementation of the described model. The CCPA is an semi-implicit algorithm which enforces normality at the beginning of each increment. Therefore, it is only suitable for small strain increments. According to this algorithm, the yield condition is linearized around the actual stress state. The yield condition is expressed as

$$\Phi (\sigma, \bar{\varepsilon}^p) = \bar{\sigma} (\sigma, \bar{\varepsilon}^p) - \sigma_y (\bar{\varepsilon}^p, \bar{\varepsilon}^p) \leq 0$$

(4.84)

In the equation above, $\Phi$ is the yield condition, $\bar{\sigma}$ the equivalent stress based on an arbitrary constitutive model, and $H$ is the uniaxial stress-strain curve, which might be also dependent on the strain rate $\dot{\bar{\varepsilon}}^p$. The stress increment $\Delta \sigma$ due to a total strain increment $\Delta \varepsilon$ is given as

$$\Delta \sigma = C (\Delta \varepsilon - \Delta \varepsilon^p)$$

(4.85)

where $\Delta \varepsilon^p$ is the plastic strain increment, which is known to be

$$\Delta \varepsilon^p = \Delta \bar{\varepsilon}^p \frac{\partial \bar{\sigma}}{\partial \sigma}$$

(4.86)
according to the associative flow rule (see Eq. 4.2). For a first order homogenous function, the plastic multiplier is equal to the increment in equivalent plastic strain (Barlat et al., 2003a). Substituting (4.86) in (4.85) gives

$$\Delta \sigma = C \left( \Delta \varepsilon - \Delta \bar{\varepsilon}^p \frac{\partial \bar{\sigma}}{\partial \sigma} \right)$$ \hspace{1cm} (4.87)

Now, if we start at the so-called trial stress state \( \sigma^T = C \Delta \varepsilon \), the stress difference \( \Delta \sigma \) due to an increase of the effective plastic strain \( \Delta \bar{\varepsilon}^p \) can be solely expressed by

$$\Delta \sigma = -C \Delta \bar{\varepsilon}^p \frac{\partial \bar{\sigma}}{\partial \sigma}$$ \hspace{1cm} (4.88)

Starting at the trial stress \( \sigma^T \) the linearization of the yield condition, the so-called consistency condition can be expressed as

$$\Phi \left( \sigma + \Delta \sigma, \bar{\varepsilon}^p + \Delta \bar{\varepsilon}^p, \bar{\varepsilon}^p \right) = \bar{\sigma} \left( \sigma + \Delta \sigma, \bar{\varepsilon}^p + \Delta \bar{\varepsilon}^p \right) - \sigma_y \left( \bar{\varepsilon}^p + \Delta \bar{\varepsilon}^p, \bar{\varepsilon}^p \right)$$

$$= \bar{\sigma} \left( \sigma, \bar{\varepsilon}^p \right) - \sigma_y \left( \bar{\varepsilon}^p \right) + \frac{\partial \bar{\sigma}}{\partial \sigma} \Delta \sigma + \frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}^p} \Delta \bar{\varepsilon}^p - \left( \frac{\partial \sigma_y}{\partial \bar{\varepsilon}^p} + \frac{1}{\Delta t} \sigma_y \frac{\partial \sigma_y}{\partial \bar{\varepsilon}^p} \right) \Delta \bar{\varepsilon}^p$$

$$= 0$$ \hspace{1cm} (4.89)

The last term in the equation above appears due to the dependency of the strain rate on the strain increment \( \Delta \bar{\varepsilon} \). Since the actual time step size \( \Delta t \) is known, the strain rate during this increment can be computed as \( \frac{\Delta \bar{\varepsilon}^p}{\Delta t} \). Thus, a direct dependency of the strain rate on the effective plastic strain increment exists, which must be taken into account. The derivative of the yield stress \( \sigma_y \) with respect to the equivalent strain \( \bar{\varepsilon} \) therefore amounts to

$$\frac{\partial \sigma_y}{\partial \bar{\varepsilon}^p} = \frac{\partial \sigma_y}{\partial \bar{\varepsilon}} + \frac{1}{\Delta t} \sigma_y \frac{\partial c}{\partial \bar{\varepsilon}^p}$$ \hspace{1cm} (4.90)

and Eq. (4.89) can be rewritten as

$$\Phi \left( \sigma + \Delta \sigma, \bar{\varepsilon}^p + \Delta \bar{\varepsilon}^p, \bar{\varepsilon}^p \right)$$

$$= \bar{\sigma} \left( \sigma, \bar{\varepsilon}^p \right) - \sigma_y \left( \bar{\varepsilon}^p \right) - \Delta \bar{\varepsilon}^p \left( \frac{\partial \bar{\sigma}}{\partial \sigma} C \frac{\partial \sigma}{\partial \bar{\varepsilon}^p} - \frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}^p} + \frac{\partial \sigma_y}{\partial \bar{\varepsilon}^p} + \frac{1}{\Delta t} \sigma_y \frac{\partial c}{\partial \bar{\varepsilon}^p} \right)$$

$$= 0$$ \hspace{1cm} (4.91)

Thus, the effective plastic strain increment is as

$$\Delta \bar{\varepsilon}^p = \frac{\bar{\sigma} \left( \sigma, \bar{\varepsilon}^p \right) - \sigma_y \left( \bar{\varepsilon}^p \right)}{\frac{\partial \bar{\sigma}}{\partial \sigma} C \frac{\partial \sigma}{\partial \bar{\varepsilon}^p} - \frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}^p} + \frac{\partial \sigma_y}{\partial \bar{\varepsilon}^p} + \frac{1}{\Delta t} \sigma_y \frac{\partial c}{\partial \bar{\varepsilon}^p}}$$ \hspace{1cm} (4.92)
Eventually, for a given strain increment $\Delta \varepsilon$, the stress is initialized to the trial stress and the effective plastic strain increment $\Delta \bar{\varepsilon}^p$ to 0. After each iteration, the plastic strain increment is increased about the quantity given in Eq. 4.92, the stress is updated according to Eq. (4.88). The iteration continues until the consistency condition (4.89) is satisfied within a certain limit (e.g. $10^{-5}$). A graphical interpretation of the CPA is given in Fig. 4.5. Note that for the HAH extensions to latent effects (sec.4.3.2), also the derivatives of the scaling factors need to be considered.

### 4.4.2 Closest point projection algorithm

Unlike the CCP, the closest point projection algorithm (CPPA) described in (Simo and Hughes, 1998) is a fully implicit algorithm which enforces normality at the end of the iteration. The advantage is a stable algorithm also for larger increments of the effective plastic strain $\bar{\varepsilon}^p$. By applying a multi-stage return algorithm as proposed in (Yoon et al., 1999), a solution is found even for relatively large strain increments (up to 0.1) without the need of a line search algorithm. The procedure is similar to the one described in (Yoon et al., 2004). For this purpose, the consistency condition is modified to

$$ F(\gamma(k)) = \bar{\sigma} (\sigma^T - \gamma(k) C m(k), \bar{\varepsilon}_n^p + \gamma(k)) - \sigma_y (\bar{\varepsilon}_n^p + \gamma(k), \dot{\varepsilon}^p) = F_k \quad (4.93) $$
In (4.93) $F_k$ is a a prescribed value which is initialized at $\sigma^T$ and decreased in every substep by the yield stress $\sigma_y$ until $F_k$ is equal to 0 in the last substep. The normal on the yield surface for the $k$-th substep $m(k) = \frac{\partial \tilde{\sigma}}{\partial \sigma} |_{\sigma = \bar{\sigma}(k)}$ is estimated with the normal found in sub-step k-1. In order to find $\gamma(k)$, the plastic multiplier or the effective plastic strain increment for substep $k$. The following relationship is used.

$$\Phi(\gamma(k)) = \tilde{\sigma}(\sigma^T - \gamma(k) C m(k)) - \sigma_y (\tilde{\varepsilon}^p + \gamma(k), \tilde{\varepsilon}^p) - F(k) = 0 \quad (4.94)$$

where

$$\sigma(k) = \sigma^T - \gamma(k) C m(k) \quad (4.95)$$

In order to enforce normality in the final state, the criterion in Eq. (4.95) has to be satisfied in the final state. It can be rewritten as

$$g_2(\gamma(k), \sigma(k)) = C^{-1} (\sigma(k) - \sigma^T) + \gamma(k) m(k) = 0 \quad (4.96)$$

A linearization of Eq. (4.96) gives

$$g_2 + \Delta g_2 = g_2 + \frac{\partial g_2}{\partial \sigma} \Delta \sigma + \frac{\partial g_2}{\partial \varepsilon_p} \Delta \gamma = 0 \quad (4.97)$$

Solving Eq. (4.97) for $\Delta \sigma$ gives

$$\Delta \sigma = -E^{-1} \left( g_2 + \left( m + \gamma \frac{\partial m}{\partial \varepsilon_p} \right) \Delta \gamma \right) \quad (4.98)$$

where $E = C^{-1} + \gamma \frac{\partial m}{\partial \sigma}$. By linearizing the relationship (4.94), the following term is derived

$$\Phi + \Delta \Phi = \Phi + \frac{\partial \tilde{\sigma}}{\partial \sigma} \Delta \sigma + \frac{\partial \tilde{\sigma}}{\partial \varepsilon_p} \Delta \gamma - \frac{\partial \sigma_y}{\partial \sigma} \Delta \sigma - \frac{\partial \sigma_y}{\partial \varepsilon_p} \Delta \gamma - \frac{1}{\Delta \tau} \sigma_y \frac{\partial c}{\partial \varepsilon} \Delta \gamma \quad (4.99)$$

By substituting $\Delta \sigma$ with the expression in Eq. (4.98) and solving for $\Delta \gamma$ we come to the expression

$$\Delta \gamma = \frac{\Phi + \left( \frac{\partial \sigma_y}{\partial \sigma} - m \right) E^{-1} g_2}{\left( m - \frac{\partial \sigma_y}{\partial \sigma} \right) E^{-1} \left( m + \gamma \frac{\partial m}{\partial \varepsilon_p} \right) - \frac{\partial \tilde{\sigma}}{\partial \varepsilon_p} + \frac{\partial \sigma_y}{\partial \varepsilon_p} + \frac{1}{\Delta \tau} \sigma_y \frac{\partial c}{\partial \varepsilon}} \quad (4.100)$$

Using Eq. (4.100), a new estimation for the strain increment is made in every iteration. From this increment, a new stress state and the new
4.4 Integration algorithms for constitutive models

Figure 4.6: Graphical interpretation of the multi-stage CPPA (Yoon et al., 2004)

normal to the yield surface is computed. If the consistency condition is satisfied within a certain limit, the stress state is accepted. Otherwise, another iteration is made using the normal from the last iteration as an estimation for the new normal. A graphical interpretation of the CPPA is illustrated in Fig. 4.6.

4.4.3 Implementation issues

In the framework of this thesis, a new material model has been implemented in LS-DYNA. It includes the standard version of the Yld2000-2d model (sec. 4.2.4), the equivalent strain dependent Yld2000-2d model (sec. 4.3.1) as well as the HAH model with extension to latent hardening and contraction (sec. 4.3.2). All features can be turned off or on if needed by setting the parameters to the according values (e.g. all k’s of the HAH model equal to 0 to turn off the HAH part). The CPPA was used for the implementation to guarantee the highest possible accuracy. There were some points in the implementation which had to be treated with special care to avoid numerical instabilities.

- The strain dependent $\alpha$ parameters of the modified Yld2000 model are given as curves ($\alpha_i$ as a function of the equivalent plastic strain $\bar{\varepsilon}$). It is crucial for the implementation algorithm that these curves are smooth since the derivatives have a significant influence on the stability of the integration algorithm.
For the integration of the state variables the Euler forward method was applied, which leads to an error for large strain increments (above 1e-4). Thus, the increment for the equivalent plastic strain is limited to this value for every iteration of the CPPA, although the CPPA algorithm would actually be able to handle larger strain increments in a single iteration.

The state variables should not fall below or exceed the saturation levels, which cannot be guaranteed when using Euler forward integration. Therefore, these limits are hard coded and the values of the state variables are corrected to the limit value as soon as they exceed the limits.

Another critical point is the derivative of $\cos \chi$ with respect to stress. The evolution laws for $g_L$ and $g_c$, which control the latent hardening and latent contraction parts respectively, are both functions of $\cos \chi$, which in turn is a function of the actual stress state $\sigma$. Especially the derivative of $g_C$ with respect to stress is lengthy and not explicitly given in the text. It has been determined using the symbolic toolbox of MATLAB. Another option would have been to numerically derive $\frac{\partial g_C}{\partial \sigma}$. But since this results in a worse performance, the analytical derivative found with MATLAB was used.

The derivative $\frac{\partial g_L}{\partial \bar{\varepsilon}_p}$ can take a high negative value (around -300) in certain cases. This can cause estimations for the increment of plastic strain that lead to an instability of the algorithm and to divergence. A limitation of the size of the strain increment per iteration also helps to avoid this problem.
5 Laboratory experiments

5.1 Tested Materials

5.1.1 DC05

The first material tested in the framework of this thesis is an unalloyed electrogalvanized DC05 deep drawing steel produced by ThyssenKrupp steel. The technical conditions of delivery and its chemical composition as given in Tab. 5.1 are standardized in (DIN EN 10152). The phosphor, sulfur and manganese are added unintentionally during the manufacturing process. The low carbon content is causes the high ductility and the good formability. The DC05 steel sheet was cold rolled and delivered as a 1mm fine blank. This material is mainly used in the automotive industry for outer panels of car bodies or other parts with high surface requirements (ThyssenKrupp, 2000).

5.1.2 Formalex™-5x

The second material investigated in this work is a high-formable 5182-based mono-alloy aluminum called Formalex™-5x produced by Constelium. The chemical composition as given in Tab. 5.2 and form of manufacture of the 5182 aluminum alloy are standardized in (DIN EN 573-3). This alloy was originally designed for the manufacturing of beverage can easy-opening ends, but is also used for other parts that require high strength in complex formed geometries such as automotive bodies. Since stretcher-strain marks can occur on forming, its usage is limited to invisible parts such as inner door panels.

<table>
<thead>
<tr>
<th>C</th>
<th>P</th>
<th>S</th>
<th>Mn</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.025</td>
<td>0.025</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 5.1: Chemical composition of DC05 (max. weight percent)
## 5 Laboratory experiments

<table>
<thead>
<tr>
<th>Si</th>
<th>Fe</th>
<th>Cu</th>
<th>Mn</th>
<th>Mg</th>
<th>Cr</th>
<th>Zn</th>
<th>Ti</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.35</td>
<td>0.15</td>
<td>0.2-0.5</td>
<td>4-5</td>
<td>0.1</td>
<td>0.25</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 5.2: Chemical composition of 5182 aluminum alloy

### 5.2 Tensile test

**Test description**

The tensile test is the most widely used test to measure mechanical properties of metals. In this test, a sample with a standardized geometry is subjected to uniaxial tension until failure. The explanation in this section is limited to tensile tests of metallic sheet materials using specimen shape H standardized in (DIN 50125). During the test, the elongation $\Delta l$ in the specimen's longitudinal direction, the shortening in the specimen width direction $w(t)$ as well as the applied force $F(t)$ is monitored. Besides characteristic material quantities (a selection is given in Tab. 5.3), the hardening curve of a material is obtained with this test. The hardening curve describes the relation between the Cauchy stress and the logarithmic (true) plastic strain. The computation of the logarithmic plastic strain and Cauchy stress is done in the following manner:

In contrast to the engineering strain defined as

$$\varepsilon_{\text{eng}} = \frac{\Delta l}{l_0} \quad (5.1)$$

the total logarithmic strain $\varepsilon_{\text{tot}}$, also called Hencky strain, is defined as

$$\varepsilon_{\text{tot}} = \ln \left( \frac{l_0 + \Delta l}{l_0} \right) \quad (5.2)$$

The initial measurement length $l_0$ depends on the size of the specimen (see Appendix B of (DIN EN ISO 6892-1))

Since a constant volume of the specimen is assumed $(\varepsilon_1(t) + \varepsilon_2(t) + \varepsilon_3(t) = 0)$, the corresponding Cauchy stress is determined by

$$\sigma(t) = \frac{F(t)}{A(t)} = \frac{F(t)}{A_0 \exp(-\varepsilon(t))} = \frac{F(t)}{A_0} \cdot \exp(\varepsilon(t)) \quad (5.3)$$

Afterward, the plastic logarithmic strain can be computed by subtracting the elastic Part which corresponds to $\varepsilon_{\text{el}} = \frac{\sigma}{E}$, where $E$ is the young's
5.2 Tensile test

modulus of the material.

\[ \varepsilon_{pl} = \varepsilon_{tot} - \varepsilon_{el} = \ln \left( \frac{l}{l_0} \right) - \frac{\sigma}{E} \]  
(5.4)

In the remainder, \( \varepsilon \) always refers to the logarithmic plastic strain.

Furthermore, another characteristic value of sheet metal material, the so-called Lankford parameter, also called \( r \)-value, is determined in the tensile test. This parameter characterizes the anisotropy of the material (see sec. 2.4) and is defined as:

\[ r_\alpha = \frac{\Delta \varepsilon_2}{\Delta \varepsilon_3} \]  
(5.5)

where \( \alpha \) is the angle to RD, in which the specimen is cut out of the material, \( \varepsilon_2 = \ln \left( \frac{w}{w_0} \right) \) is the logarithmic strain in width direction of the specimen and \( \varepsilon_3 = \ln \left( \frac{t}{t_0} \right) \) is the logarithmic strain in thickness direction of the specimen. \( \varepsilon_3 \) is again determined based on the assumption of a constant specimen volume. The \( r \)-value is usually an average value in a certain strain range. In the remainder, the \( r \)-value is always determined between 2\% and 20\% (engineering) strain.

Another characteristic of some metallic materials, especially of low carbon steel grades, is a dependency of the flow stress on the strain rate \( \dot{\varepsilon} = \frac{d\varepsilon}{dt} \). To measure the strain rate sensitivity of a material, tensile tests at different strain rates are carried out. The tensile test explained above including its procedure and evaluation is standardized in (DIN EN ISO 6892-1)

**Results for DC05**

Tensile tests were carried out for the DC05 material investigated in this work. Between RD and TD, every 15° the direction was tested at a strain rate of \( \approx 0.002 s^{-1} \) to obtain the quasi-static material response for tension. Five tests were carried out for every testing direction in order to eliminate measurement errors and to account for material variation. The hardening curves and mechanical quantities shown in the following are averaged over the five tests. The hardening curves for RD, DD and TD are shown in Fig. 5.1. Furthermore, the yield stress \( R_{p0.2} \) (Fig. 5.2a),

\footnote{A constant log. strain rate is difficult to realize because the velocity of the fixation should vary over time, which may lead to unwanted effects, especially for strain rate sensitive materials.}
Table 5.3: characteristic material values determined in tensile test

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus $E$</td>
<td>Stress to strain ratio in the elastic region $E = \frac{\Delta \sigma}{\Delta \varepsilon}$</td>
</tr>
<tr>
<td>Offset yield point $R_{p0.2}$</td>
<td>Stress $\sigma$ after a plastic (permanent) strain of 0.2%</td>
</tr>
<tr>
<td>Ultimate strength $R_m$</td>
<td>Maximum (engineering) stress a material can withstand while being stretched before necking</td>
</tr>
<tr>
<td>Uniform elongation $A_g$</td>
<td>Engineering strain corresponding to the ultimate strength</td>
</tr>
</tbody>
</table>

the $r$-value (Fig. 5.2b), the uniform elongation $A_g$ (Fig. 5.2c) and the ultimate stress $R_m$ (Fig. 5.2d) are plotted as a function of the testing angle $\alpha$. In addition, tensile tests with different strain rates were carried out in RD. Although only strain rates up to $0.01 \text{s}^{-1}$ could be tested due to limitations of the testing equipment, the influence on the resulting hardening curve is significant, as illustrated in Fig. 5.3. The strain rates given in the legend are averaged over the tested strain range.

![Figure 5.1: Hardening curves for DC05](image)

The tests presented show typical behavior for low-carbon steels. Note
5.2 Tensile test

that the hardening in different directions and under biaxial stress is not proportional. The yield curves in TD and RD are even crossing each other.

Results for Formalex™-5x

As for the DC05 material, the Formalex™-5x material was tested five times for every 15° at a strain rate of ≈0.002s⁻¹ and the results were averaged. A series of tests was carried out at a five times higher velocity to check the strain rate sensitivity. No significant differences to the quasi-static response could be observed though, why the material was assumed to be strain rate insensitive. Fig. 5.4 shows the measured and averaged hardening curves in RD, DD and TD. Note the serrated shape of the hardening
5 Laboratory experiments

curves at low strains. This so-called Portevin–Le Chatelier (PLC) effect, named after (Portevin and Le Chatelier, 1923) is a dynamic strain aging effect and related to diffusing solutes pinning dislocations and dislocation breaking free of this stoppage (Abbadi et al., 2002). However, this phenomenon is not subject of the present thesis and was smoothed out for further considerations (see. sec. 6.2.1).

Figure 5.3: Hardening curves at different strain rates for DC05

Figure 5.4: Hardening curves for Formalex™-5x
5.3 Hydraulic bulge test

Test description

During a hydraulic bulge test, a sheet sample is clamped between two circular tools, a die and a binder. A fluid is pressed against the specimen, which results in a bulging of the sample. The pressure inside the fluid is increased until fracture. During the test, the fluid pressure is constantly measured. The surface deformation of the sheet sample is monitored by an optical measurement system. Based on these measurements, the strains on the blanks surface as well as the local curvature in the region around the pole of the resulting bulge can be determined. Knowing
5 Laboratory experiments

these quantities, the stress state in the material is computed based on the spherical pressure vessel formula (5.6). The evaluation method used and described in the sections below is according to (Peters et al., 2011). However, there meanwhile exists a standard, which slightly differs from the described method, but is leading to similar results (DIN EN ISO 16808).

\[
\sigma_b = \frac{1}{2} \rho \frac{p}{t} \quad (5.6)
\]

A principle setup of the test is shown in Fig. 5.6. For the tests carried out in this study, a punch diameter of 100 mm, a die diameter of 102.5 mm and a die radius of 8 mm was used.

\textbf{strain computation}

During the bulge test, only displacements of material points on the upper surface of the specimen are measured based on which the strains are computed. Since at the pole of the bulge, also bending occurs, the strains can be corrected by assuming the bending component of the strain to be

\[
\varepsilon_b = - \ln \left(1 - 0.5 \left( \frac{t}{\rho_u} \right) \right) \quad (5.7)
\]

where \(t\) is the actual blank thickness at the pole and \(\rho_u\) is the upper surface curvature. The thickness strains, which are computed from the measured
in-plane strains under the assumption of constant volume, are averaged in a radius \( r_1 (= 5 \text{ mm}) \) around the pole of the bulge. (Volk et al., 2011) proposed to fit the thickness distribution using a full quadratic approach and evaluate it at the pole. However, since this is not part of (DIN EN ISO 16808), the average value was used.

The thickness strain, which is sought-after to ensure a work conjugate strain measure, is finally computed in the neutral axis of the specimen with

\[
\varepsilon_{\text{thick},m} = \varepsilon_{\text{thick},u} - 2\varepsilon_b \quad (5.8)
\]

where \( \varepsilon_{\text{thick},m} \) is the thickness strain at the mid surface and \( \varepsilon_{\text{thick},u} \) is the thickness strain at the upper surface at the pole of the bulged specimen. After having computed the stress at the pole (see Eq. (5.11)), the elastic strain component in the mid surface is computed

\[
\varepsilon_{\text{thick},m,\text{pl}} = \varepsilon_{\text{thick},m} + \frac{2(1 - 2\nu)}{E}\sigma_b \quad (5.9)
\]

**stress computation**

The computation of the biaxial stress components at the pole of the bulge are based on the well-known vessel formula

\[
\sigma_b = \frac{p\rho}{2t} \quad (5.10)
\]

where \( p \) is the actual fluid pressure, \( \rho \) is the actual radius of curvature at the pole of the bulge and \( t \) is the actual blank thickness at the pole of the bulge. The radius of curvature \( \rho \) is determined by fitting a quadratic function through the measured surface points in space within the radius \( r_2 (= 12.5 \text{ mm}) \) around the pole and evaluate its curvature at the pole. The radius of curvature in Eq. 5.10 is slightly modified because of the situation, that in the case of the hydraulic bulge test, the pressure is actually acting on the lower surface of the blank where the radius is assumed to be \( \rho_l = \rho_u - t \). The membrane stress is assumed the average stress over the thickness of the blank. Potential bending moments caused by a through thickness stress gradient due to the bending strains are assumed to be negligible. Assuming the mean diameter to be \( \rho_m = \rho_u - t/2 \), the equilibrium equation is written as

\[
p \cdot \rho_m^2 \cdot \pi = \sigma \cdot 2\rho_m \cdot \pi \cdot t \quad (5.11)
\]
Results for DC05

From the hydraulic bulge test, the equibiaxial hardening curve is obtained, which describes the hardening behavior of a material subjected to equibiaxial stress. The result for the DC05 material is shown in Fig. 5.7. Again, five specimen were tested and the average curve was computed. It can easily be seen, that under a biaxial stress state much higher strains can be reached than in a regular tensile test (see Fig. 5.1). Note that due to the almost linear increase of the bulge volume, the strain in the material does not increase linearly with time, but passes a wide range of strain rates. Controlling the fluid volume increase in order to have a constant strain rate in the pole of the bulge is technically feasible, but the available equipment did not come with such a control system. Since a strain rate sensitivity of the material was observed (see sec. 5.2), a compensation for the strain rate effect is necessary, which is explained in sec. 6.1. To guarantee comparability, the biaxial hardening curve is plotted as the biaxial stress component as a function of the thickness stress, so the volume specific plastic work amounts to the area under the curve.

![Figure 5.7: Measured biaxial hardening curve for DC05](image)

Results for Formalex™-5x

Also for Formalex™-5X, hydraulic bulge tests were carried out. Four different specimen were tested. While the curves for DC05 were very similar,
the scatter for Formalex-5x was bigger. Fig. 5.8, shows the four measurements as well as a smoothed average curve. For the Formalex-5X material, no strain rate sensitivity was detected so no compensation is necessary. In addition, the stress level of the biaxial curve is lower than the one of the uniaxial, which is expected for a material with an $r$-value lower than 1.0.

Figure 5.8: Measured biaxial hardening curve for Formalex-5x

5.4 Stack compression test

Test description

Stack compression tests are carried out in order to determine the biaxial $r$-value. The fact that the deviatoric stress state is the same for compression in thickness direction and in-plane equibiaxial stress offers the possibility to determine this parameter from a compression test. A stack of circular plates is compressed to a certain level of plastic strain. The resulting strains in RD and TD are measured at the end of the test. Preferably, a continuous monitoring of the strains during the whole test as described in (Merklein and Kuppert, 2009) would be done, but is not possible with the used test equipment.

Due to the anisotropy of the material, the strains in RD and TD are different. The ratio of $\varepsilon_{TD} = \ln \left( \frac{D_{RD}}{D_0} \right)$ to $\varepsilon_{RD} = \ln \left( \frac{D_{TD}}{D_0} \right)$ is defined as the biaxial $r$-value (Barlat et al., 2003a). A schematic illustration of the test and the specimen geometries before and after compression are given in Fig.
5.9 and Fig. 5.10. A picture of a compressed stack of DC05 steel plates is shown in Fig. 5.11. Because high friction leads to barreling of the stack when subjected to compression and thus to a non-homogenous stress state, it should be minimized. This is usually realized by using teflon foil and lubrication between the tools and the most upper or the most lower plates, respectively. The central plate is used to measure the strains because it has the most homogenous deformation because of symmetry reasons.

![Figure 5.9: Schematic illustration of the stack compression test](image)

![Figure 5.10: Deformation of disk in stack compression test](image)

**Results for DC05**

For DC05, seven different tests were carried out. Nine plates with an initial diameter of 10mm and a blank thickness of 1mm were used for all the tests. The plate in the middle of the stack was used to determine the ratio of the major and minor axis of the resulting ellipse, since the
5.4 Stack compression test

Figure 5.11: Stack of 9 plates of DC05 with an initial diameter of 10mm and an initial thickness of 1mm compressed to ≈ 30%

<table>
<thead>
<tr>
<th>Test-Nr.</th>
<th>final stack height [mm]</th>
<th>$\varepsilon_{RD}$ mid plate []</th>
<th>$\varepsilon_{TD}$ mid plate []</th>
<th>rb []</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.57</td>
<td>0.200</td>
<td>0.167</td>
<td>0.834</td>
</tr>
<tr>
<td>2</td>
<td>7.00</td>
<td>0.152</td>
<td>0.132</td>
<td>0.869</td>
</tr>
<tr>
<td>3</td>
<td>6.46</td>
<td>0.200</td>
<td>0.167</td>
<td>0.834</td>
</tr>
<tr>
<td>4</td>
<td>6.50</td>
<td>0.199</td>
<td>0.166</td>
<td>0.837</td>
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<td>5</td>
<td>6.42</td>
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<td>0.173</td>
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<tr>
<td>6</td>
<td>6.47</td>
<td>0.203</td>
<td>0.176</td>
<td>0.862</td>
</tr>
<tr>
<td>7</td>
<td>6.43</td>
<td>0.207</td>
<td>0.176</td>
<td>0.848</td>
</tr>
</tbody>
</table>

Table 5.4: Stack compression test results for DC05

Influence of contact friction is the least in the center of the stack. All tests were carried out with a constant punch velocity of 0.02 mm/s. Initial and measured values for the different tests are given in Tab. 5.4. Note that the values in Tab. 5.4 are rounded. The average biaxial $r$-value was found to be 0.85.

**Results for Formalex™-5x**

The same values for Formalex™-5x are given in Tab. 5.5. Again, the tests were carried out with a stack of 9 plates with an initial diameter of
5 Laboratory experiments

<table>
<thead>
<tr>
<th>Test-Nr.</th>
<th>final stack height [mm]</th>
<th>$\varepsilon_{RD}$ mid plate</th>
<th>$\varepsilon_{TD}$ mid plate</th>
<th>$r_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.27</td>
<td>0.169</td>
<td>0.181</td>
<td>1.075</td>
</tr>
<tr>
<td>2</td>
<td>6.33</td>
<td>0.160</td>
<td>0.184</td>
<td>1.149</td>
</tr>
<tr>
<td>3</td>
<td>7.16</td>
<td>0.091</td>
<td>0.102</td>
<td>1.119</td>
</tr>
<tr>
<td>4</td>
<td>7.17</td>
<td>0.091</td>
<td>0.104</td>
<td>1.144</td>
</tr>
<tr>
<td>5</td>
<td>7.14</td>
<td>0.091</td>
<td>0.105</td>
<td>1.159</td>
</tr>
<tr>
<td>6</td>
<td>7.20</td>
<td>0.090</td>
<td>0.104</td>
<td>1.159</td>
</tr>
</tbody>
</table>

Table 5.5: Stack compression test results for Formalex™-5x

10mm and a initial blank thickness of 1mm at a constant punch velocity of 0.02mm/s. Six tests were carried out in total. The average biaxial $r$-value was 1.13. Note that rounded values are given in Tab. 5.5.

5.5 Two stage tensile test

Test description

In order to investigate the hardening behavior of the materials under non-proportional loading conditions, two stage tensile tests were carried out. The tests conducted in the framework of this thesis were carried out at Constellium CRV in France. Rectangular sheet strips were cut from the blanks and drawn to strains of 0.05, 0.10 and 0.15 in RD, DD and TD. The central third of the drawn strip, which can be assumed to deform under uniaxial stress due to its almost straight sides, is afterward used to cut out tensile specimen with an angle of 45° and 90° to the prestrain direction. Like this, nonlinear strain paths with a sharp change can be tested. A schematic illustration of the test is shown in Fig. 5.12

Results for DC05

The DC05 material was supplied as sheets with the dimensions 1700mm x 1300mm (RD x TD). To maximize the area of the stretched strip from which the specimen for the subsequent tests could be cut out, the following
5.5 Two stage tensile test

Figure 5.12: Schematic illustration of Two stage tensile tests

Dimensions for the strips were chosen:

- **RD**: strips of 375mm x 1700mm
- **DD**: strips of 300mm x 1300mm
- **TD**: strips of 250mm x 1130mm

Simulations in advance have shown a sufficient uniformity of the stress in the center of the specimen (non-uniaxial stress components below 2MPa). As mentioned before, the strips were elongated to strains of approximately 0.05, 0.10 and 0.15 in the center of the strips. Afterward, tensile specimens were cut out in directions 45° and 90° to the prestrain direction and tested in a regular tensile test like described in sec. 5.2. Fig. 5.13 shows the results for prestrain in DD and RD with a subsequent loading in TD together with the monotonic hardening curve in TD. The curves are plotted as a function of plastic work, since this value is proportional to the equivalent plastic strain. Note that the prestrain in DD and TD to 0.1 do not result in the same amount of plastic work. This is due to the different hardening behavior on one hand, and due to the difficulties to exactly reach a certain strain level because of the prestraining device on the other. However, the main phenomenon, a stress overshoot before the flow stress comes back to the monotonic one (see sec. 2.6), which is more distinctive after a rotating the strain path about 45°, but also present after a rotation of the strain path about 90°, can easily be seen. After a prestrain of 0.1, diffuse necking of the specimen starts after just a few percentage of strain. Thus, the displayed curve ends abruptly since only the uniformly elongated range is shown. Still, the stress overshoot is observable.
5 Laboratory experiments

Figure 5.13: DC05: TD hardening after prestrain in RD and DD

Results for Formalex™-5x

The same two stage tensile tests were carried out for the Formalex™-5x material. The dimensions of the initial blanks were 1500mm x 1500mm, which is why also dimensions of the strips differ from the ones used for the DC05 material:

- RD: strips of 350mm x 1500mm
- DD: strips of 275mm x 1250mm
- TD: strips of 350mm x 1500mm

The remaining procedure was performed the same way as described for DC05. Fig. 5.14 shows the monotonic hardening curve for RD together with the hardening curves for prestrained material in DD and TD to 0.05 and 0.1 respectively and subsequently loaded in RD. Obviously, the behavior differs drastically from the one observed for the DC05. Instead of a stress overshoot, the material reyields at a lower stress level and approaches the monotonic curve from below (see sec. 2.6).
5.6 Simple shear test with load reversal

Test description

Metallic materials tend to exhibit a Bauschinger effect, which is a reyielding at a lower stress level after load reversal (see sec. 2.5). Since tension-compression tests are difficult to realize for thin sheets because of buckling under compression, simple shear tests with load reversal were carried out to investigate the behavior. For this purpose, plates with a dimension of 16mm x 50mm were clamped on both sides in such a way that the material could deform in a zone of 4mm x 50 mm. The DIC measurement shows that this rather large ratio of length to width guarantees a relatively small boundary zone, in which the strain conditions differ from a simple shear strain. Thus, this zone is assumed negligible and the shear strain measured with a DIC system is averaged in the sheared zone. The shear stress is computed by dividing the applied force through the cross section area, which is assumed constant and amounts to the blank thickness times the length of the deformation area. The specimen is sheared in one direction to a certain strain level of shear strain before the load is reversed and the material is sheared in the inverse direction. Fig. 5.15 shows a schematic illustration of the experiment. These tests were carried with the MML (Materials Mechanics Laboratory) at Postech Pohang, South Korea. More information about the device used for the test and the test setup itself are e.g. given in (Kim et al., 2012).
Results for DC05

The DC05 was sheared to a strain of 0.05, 0.09 and 0.125 before the load was reversed. Each different test was carried out three times and the results for these tests were averaged. The results are presented in Fig. 5.16. Fig. 5.16a shows the shear stress as a function of the shear strain and in Fig. 5.16b, the absolute stress is plotted as a function of the accumulated shear strain. The Bauschinger effect (early re-yielding after stress reversal) and a permanent softening after stress reversal can easily be seen. Note that for high strains (above 0.2), the results are not as reliable anymore because the strains in the specimen start to deviate from simple shear and tension and compression respectively are superimposed as can be observed in the DIC measurement.

Figure 5.16: Simple shear test with load reversal for DC05
5.7 Nakajima test

Results for Formalex-5x

The same tests were carried out for Formalex-5x. Again, the material was sheared to levels of approximately 0.05, 0.1 and 0.14 before the load was reversed. The resulting stress strain response is shown in Fig. 5.17. Again, Fig. 5.17a shows the shear stress as a function of the shear strain and in Fig. 5.17b, the absolute stress is plotted as a function of the accumulated shear strain. An even more distinct Bauschinger effect and a permanent softening is evident.

![Graphs showing stress-strain response](image)

Figure 5.17: Simple shear test with load reversal for Formalex™-5x

5.7 Nakajima test

Test description

Nakajima tests, named after (Nakazima et al., 1968), are carried out to experimentally determine the Forming Limit Diagram (FLD). The FLD, whose concept goes back to (Keeler and Backofen, 1963), is a curve in the major-minor in-plane strain space which represents, as the name implies, the forming limit for a material and is nowadays the established criterion to assess failure in industrial sheet metal forming simulations. Because the critical strain at which metallic materials become unstable depends on the strain path (i.e. the ratio of the in-plane principal strains), different specimen geometries with varying widths are drawn with a hemispherical punch. This way, strain ratios between uniaxial and biaxial tension can be realized and the most important range of the possible strain states is covered. The different specimen are shown in Fig. 5.18. During the
test, the strains are monitored using a DIC system. The limit strains are determined using the so-called section method, which is standardized in (EN ISO 12004-2), or the lately proposed time-dependent method as described by (Volk and Hora, 2011).

![Figure 5.18: Different specimen for Nakajima test](image)

**Results for DC05**

The DC05 material was tested in the Lab of IVP at ETH Zurich. A punch velocity of 1mm/s was used to conduct the tests. Eventually, the test was evaluated using the time dependent method. Each geometry was tested five times. The resulting strain limits as well as the FLC, which connects the mean limit strains for each geometry are shown in Fig. 5.19. Note that for the biaxial case, a Bulge test was used instead of the full Nakajima specimen geometry.

**Results for Formalex™-5x**

The Formalex™-5x material was tested by Suisse Technology Partners in Neuhausen. The tests were carried out in the same way as the ones for the DC05, but evaluated using the cross-section method. The resulting limit strains as well as the final FLC are shown in Fig. 5.20.
Figure 5.19: FLC for DC05 measured with Nakajima tests
5 Laboratory experiments

Figure 5.20: FLC for Formalex™-5x measured with Nakajima tests
6 Material characterization

This section summarizes the identification of the parameters of the models that were used to mathematically characterize the deformation behavior of the materials investigated. For both materials, it starts with the flow curve model before the parameters identification for different yield functions that are described in chapter 4 are given. The whole parameter identification process is based on the experimental results that were presented in chapter 5.

6.1 DC05

6.1.1 Flow curve

The characterization of the hardening of the DC05 material is based on the quasi-static tensile tests presented in sec. 5.2 as well as on the results of the hydraulic bulge tests shown in sec. 5.3. If isotropic hardening is assumed, the biaxial hardening curve obtained from the hydraulic bulge test can be transformed to the uniaxial curve. As already mentioned in sec. 5.3, the material passes a wide range of strain rates during a hydraulic bulge test. Since a dependency of the flow stress on the strain rate was found for the DC05 material, the hydraulic bulge test has to be compensated for this strain rate effect before it can be used to extrapolate the uniaxial hardening curve, which was carried out at an almost constant strain rate. For this purpose, the tensile tests with different strain rates, also presented in sec 5.2, were used to find a strain rate dependent hardening model.

A modified Cowper-Symonds (Cowper and Symonds, 1957) description was found to capture the strain rate effects with a reasonable accuracy. It multiplies the quasi-static flow curve (the one tested ate the lowest strain rate), with a factor that depends on the strain rate and is equal to 0 for a strain rate of 0. Since the tested strain rate range was limited to a maximal rate of 0.01, it cannot be guaranteed that the model still provides reasonable values for higher rates. However, the following formula
was used:

\[
\sigma_y (\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p) = \sigma_{y0} (\bar{\varepsilon}^p) \cdot c (\dot{\bar{\varepsilon}}^p) = \sigma_{y0} (\bar{\varepsilon}^p) \cdot \left(1 + \left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\bar{\varepsilon}}_0^p}\right)^{\frac{1}{m}}\right) \quad (6.1)
\]

In Eq. (6.1), \(\sigma_{y0} (\bar{\varepsilon}^p)\) denotes the static flow curve (i.e. the flow curve with the minimal strain rate) and \(\dot{\bar{\varepsilon}}_0^p\) and \(m\) are model parameters that were identified as \(\dot{\bar{\varepsilon}}_0^p = 1.48 \times 10^{-4}\) and \(m = 54.99\). The difference to the original Copwer-Symonds is the exponent \(\frac{1}{m}\), which exponentiates the whole term and not only the part involving the strain rate fraction. The curves that result from this model are plotted in Fig. 6.1 together with the curves measured in the tensile tests. Although, the flow stresses are slightly underestimated for strains below 0.025, the model shows good agreement with the experimental data.

![Figure 6.1: Strain rate model for DC05: solid lines: measured, dots: model prediction](image)

In a second step, this strain rate model was used to transform the biaxial hardening curve obtained from the hydraulic bulge test to the uniaxial one in order to extrapolate the uniaxial hardening curve to strains beyond uniform elongation. To do this, the principal of equivalent plastic work
is applied. The plastic work of the biaxial hardening curve is computed piecewise. For the uniaxial hardening curve, the strain rate for which the same amount of plastic work in the same time is sought-after. In other words, the curve with the same work-rate. This way, an equivalent plastic strain rate can be found for every section of the biaxial hardening curve. This strain rate is inserted in the multiplication factor \( c \) in Eq. (6.1), which the stress is eventually divided through to find the compensated stress value. The described procedure was also used in (Peters et al., 2012) and is schematically illustrated in Fig. 6.2. The strain rates \( \dot{\varepsilon}_1 \), \( \dot{\varepsilon}_2 \) and \( \dot{\varepsilon}_3 \) in Fig. 6.2 correspond to the parts of the hardening curve where the plastic work \( W_1 \), \( W_2 \) and \( W_3 \) in a time \( \Delta t_1 \), \( \Delta t_2 \) and \( \Delta t_3 \) is done, respectively. Of course, the time increments, during which the plastic work is computed and compared with the quasi-static one, are smaller than illustrated in Fig. 6.2. In the present case, they correspond to the frequency at which the strain was measured with the DIC, i.e 10 Hz.

Figure 6.2: Schematic illustration of the procedure for the strain rate correction of the hydraulic bulge experiment

After having computed the equivalent strain rate and accordingly the scale factor for each time increment, the curve can be corrected. The originally measured biaxial hardening curve and the compensated one for a strain rate of \( \approx 0.002 \text{ s}^{-1} \) (equal to the strain rate of the tensile tests in different direction for DC05, see sec. 5.2) are shown in Fig. 6.3. Note that the difference is the bigger, the higher the strains are because the strain rate is continuously increasing during the hydraulic bulge test.

In a last step, the corrected biaxial hardening curve is transformed to the uniaxial stress space, again, based on the principle of equivalent plastic work. According to Appendix D of (DIN EN ISO 16808), the equivalent
6 Material characterization

Figure 6.3: Measured and strain rate compensated biaxial hardening curve for DC05

plastic work for the tensile test is computed up to the uniform elongation strain. The biaxial flow stress at the same amount of plastic work $\sigma_{B-ref}$ is determined for the biaxial hardening curve. Now the ratio $f_{bi}$ of $\sigma_{B-ref}$ to the uniaxial flow stress at UE $\sigma_{f-ref}$ is computed, which is eventually used to transform the curve in the following manner

$$ f_{bi} = \frac{\sigma_{B-ref}}{\sigma_{f-ref}}, \quad \sigma_{trans} = \frac{\sigma_B}{f_{bi}}, \quad \varepsilon_{trans} = \varepsilon_B \cdot f_{bi} \quad (6.2) $$

where $\sigma_B$ and $\varepsilon_B$ denote the stress and strain of the biaxial flow curve and $\sigma_{trans}$ and $\varepsilon_{trans}$ the ones of the transformed one.

The combination of the uniaxial hardening curve from the tensile test up to UE and the transformed biaxial hardening curve beyond UE was eventually used to model the hardening behavior. A Hockett-Sherby approach as in Eq. (6.3) was used to fit the hardening curve.

$$ \sigma_f(\ddot{\varepsilon}) = B - (B - A) \cdot \exp \left( -m\ddot{\varepsilon}^n \right) \quad (6.3) $$

The Hockett-Sherby parameters are given in Tab. 6.1 The compensated and transformed biaxial flow curve together with the uniaxial one in RD and the Hockett-Sherby fit are shown in Fig. 6.2.

Using the strain rate model in Eq. (6.1), this curve is scaled back to the quasi-static one ($\dot{\varepsilon} = 0$). This way, the strain rate dependent isotropic hardening behavior is fully described.
6.1 DC05

![Graph showing uniaxial, biaxial, and transformed biaxial hardening curves and Hockett-Sherby fit for DC05.](image)

Figure 6.4: Uniaxial, biaxial and transformed biaxial hardening curve and Hockett Sherby fit for combination

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>m</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hockett-Sherby</td>
<td>154.95</td>
<td>451.89</td>
<td>2.62</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 6.1: Hockett-Sherby parameters for DC05

### 6.1.2 Yield functions

Again, based on the experimental results presented in chapter 5, the parameters for different yield functions were determined. The determination was done according to the explanations in chapter 4. Only the models that were used for the comparisons in chapter 7 are presented. First, the Hill’48 and the Yld2000-2d yield functions with isotropic hardening, that only capture the initial anisotropy are fitted, followed by the HAH parameters, that also take into account anisotropic hardening effects like the Bauschinger effect or latent hardening and softening.

**Hill ’48**

The Hill ’48 yield model is probably the most-widely used model for the simulation of sheet forming processes of steel sheets in the industry. This is why this model was as well chosen as a reference. The parameters for the Hill’48 yield locus are determined based on Eq. (4.13) and are given in Tab. 6.2.
Table 6.2: Hill’48 parameters for DC05

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>G</td>
<td>H</td>
<td>N</td>
</tr>
<tr>
<td>2.01</td>
<td>2.52</td>
<td>5.07</td>
<td>8.92</td>
</tr>
</tbody>
</table>

Fig. 6.5 shows the distribution of the yield stress (6.5a) and the $r$-values (6.5b) predicted by the Hill ’48 model along the angle to RD together with the measured values. Note that these are the initial values and the ratio between the yield stresses in two different directions as well as the $r$-values stay constant for every level of plastic strain.

Figure 6.5: Comparison of Hill’48 predicted and measured yield stress and $r$-value along the angle to RD for DC05

**Yld2000-2d**

The parameters for the Yld2000-2d model are fitted according to (Barlat et al., 2003a). To fit the parameters $\alpha_1$ to $\alpha_6$, the following two equations are used:

\[
F = \phi - 2\left(\bar{\sigma}/\sigma\right)^a = 0 \quad (6.4a)
\]

\[
G = q_x \frac{\partial \phi}{\partial s_{xx}} - q_y \frac{\partial \phi}{\partial s_{yy}} = 0 \quad (6.4b)
\]
where $\phi$ can be written as

$$\phi = |\alpha_1 \gamma - \alpha_2 \delta|^a + |\alpha_3 \gamma - 2\alpha_4 \delta|^a + |\alpha_5 \gamma - \alpha_6 \delta|^a$$

(6.5)

The values for $\gamma$, $\delta$, $q_x$ and $q_y$ in Eq. (6.5) can be taken from Tab. 6.3 depending on the correspondent test.

<table>
<thead>
<tr>
<th>test</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$q_x$</th>
<th>$q_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ° tension</td>
<td>2/3</td>
<td>-1/3</td>
<td>1 - $r_0$</td>
<td>2 + $r_0$</td>
</tr>
<tr>
<td>45 ° tension</td>
<td>-1/3</td>
<td>2/3</td>
<td>2 + $r_{90}$</td>
<td>1 - $r_{90}$</td>
</tr>
<tr>
<td>balanced biaxial tension</td>
<td>-1/3</td>
<td>-1/3</td>
<td>1 + 2$r_b$</td>
<td>2 + $r_b$</td>
</tr>
</tbody>
</table>

Table 6.3: $\gamma$, $\delta$, $q_x$ and $q_y$ according to tests

To determine the coefficients $\alpha_7$ and $\alpha_8$, the following two equations have to be solved.
6 Material characterization

\[ F = \left| \frac{\sqrt{k_2' + 4\alpha_1^2}}{2} \right|^a + \left| \frac{3k_1'' - \sqrt{k_2''^2 + 4\alpha_3^2}}{4} \right| + \left| \frac{3k_1'' - \sqrt{k_2'^2 + 4\alpha_5^2}}{4} \right| \]  

(6.6a)

\[ G = \frac{\partial \phi}{\partial \sigma_{xx}} - \frac{\partial \phi}{\partial \sigma_{yy}} - \frac{2a \bar{\sigma}^a}{\sigma (1 + r_{45})} = 0 \]  

(6.6b)

The equations (6.4) and (6.6) were solved using the built-in MATLAB function `fsolve` which is based on a trust-region-dogleg algorithm. Tab. 6.4 show the input values, i.e. experimental data determined in the mechanical tests (see chapter 5) as well as the resulting Yld2000-2d parameters. An exponent of \( a = 6 \) was used for the fitting. The distribution of the yield stress and the \( r \)-value together with the experimental data is shown in Fig. 6.7 and the resulting yield locus is presented in Fig. 6.8. Again, the ratios of yield stress in two different directions as well as the \( r \)-values stay constant for every level of plastic strain.

<table>
<thead>
<tr>
<th>Input</th>
<th>( \sigma_0 )</th>
<th>( \sigma_{45} )</th>
<th>( \sigma_{90} )</th>
<th>( \sigma_b )</th>
<th>( r_0 )</th>
<th>( r_{45} )</th>
<th>( r_{90} )</th>
<th>( r_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>171.0</td>
<td>179.9</td>
<td>177.3</td>
<td>195</td>
<td>2.00</td>
<td>1.47</td>
<td>2.52</td>
<td>0.85</td>
</tr>
<tr>
<td>Output</td>
<td>( \alpha_1 )</td>
<td>( \alpha_2 )</td>
<td>( \alpha_3 )</td>
<td>( \alpha_4 )</td>
<td>( \alpha_5 )</td>
<td>( \alpha_6 )</td>
<td>( \alpha_7 )</td>
<td>( \alpha_8 )</td>
</tr>
<tr>
<td></td>
<td>1.084</td>
<td>0.982</td>
<td>0.846</td>
<td>0.881</td>
<td>0.908</td>
<td>0.836</td>
<td>0.972</td>
<td>0.975</td>
</tr>
</tbody>
</table>

Table 6.4: Yld2000-2d input and fitted parameters for DC05

**Equivalent strain dependent Yld2000-2d**

As already mentioned in sec. 4.3.1, a dependency of the Yld2000-2d parameters was introduced to overcome the limitation of proportional hardening for monotonic strain paths. The principal of equivalent plastic work is used to determine the strain dependent Yld2000-2d parameters. The procedure for the determination is also explained in sec. 4.3.1. The hardening curves in RD, \( 45^\circ \) and TD as well as the biaxial hardening curve was used for this purpose. The \( r \)-values \( r_0 \), \( r_{45} \) and \( r_{90} \) varied in a very small range during the measurement and were therefore assumed constant. By applying the described fitting procedure, the curves shown in Fig. 6.9 were
6.1 DC05

Figure 6.7: Comparison of Yld2000-2d predicted and measured yield stress and $r$-value along the angle to RD for DC05

Figure 6.8: Yld2000-2d yield locus for DC05

obtained. Since it appeared that the parameters converge to constant values after a certain amount of plastic strain, a Hockett-Sherby approach
was used to smooth the slope of the varying parameters. The Hockett-Sherby fitted curves are displayed as dots in Fig. 6.9. The according parameters are given in Tab. 6.4.

Figure 6.9: Variable Yld2000-2d parameters for DC05, solid: fitted, dots: smoothed with Hockett-Sherby

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$A$</th>
<th>$B$</th>
<th>$m$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>1.090</td>
<td>0.958</td>
<td>11.332</td>
<td>0.738</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.990</td>
<td>1.143</td>
<td>9.748</td>
<td>0.672</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.940</td>
<td>0.720</td>
<td>42.107</td>
<td>1.162</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.905</td>
<td>0.868</td>
<td>36.467</td>
<td>1.124</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>0.930</td>
<td>0.899</td>
<td>73.558</td>
<td>0.7384</td>
</tr>
<tr>
<td>$\alpha_6$</td>
<td>0.945</td>
<td>0.665</td>
<td>35.527</td>
<td>1.102</td>
</tr>
<tr>
<td>$\alpha_7$</td>
<td>0.980</td>
<td>0.980</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_8$</td>
<td>0.92</td>
<td>1.115</td>
<td>20.019</td>
<td>1.031</td>
</tr>
</tbody>
</table>

Table 6.5: Hockett Sherby parameters for smoothed variable Yld2000-2d parameters for DC05

The resulting yield loci at different strain levels is compared to the one with constant parameters in Fig. 6.10. Figure 6.11 shows the measured hardening curves in RD, 45 °, TD and the biaxial one. In Fig. 6.11a,
the Yld2000-2d response using the constant parameters given in Tab. 6.4 for the same curves is displayed as well, while in Fig. 6.11b, the varying parameters shown in Fig. 6.9 were used. It can be clearly seen, that the hardening behavior for these monotonic load paths is significantly better approximated by using the variable parameters. Especially the biaxial hardening behavior, which strongly differs from the uniaxial one, is in good agreement with the experimental data. The slight deviations arise from the Hockett-Sherby fit on the one hand and the strain-rate model on the other.

![Figure 6.10: Yld2000-2d yield locus with constant and variable parameters for DC05 at an equivalent strain of 0, 0.05 and 0.15](image)

**Homogenous function based anisotropic hardening**

In order to fit the HAH parameters for DC05, the simple shear tests with load reversal (see sec. 5.6) and the two stage tensile tests (see sec. 5.5) were used. The parameters were varied by hand to find the ones for which the model shows the best agreement with the experiments. The shear
Figure 6.11: Yield curve prediction of original and modified Yld2000-2d model for DC05

tests with load reversal were used to identify the parameters $k_1$ to $k_5$. The parameter $k$ as well as the parameters $L$ and $k_L$ or $C$ and $k_C$ do not influence the model response for load reversal, i.e. the prediction of the Bauschinger effect. Therefore, these parameters can be identified independently.

If the Yld2000-2d model parameters are fitted according to the standard procedure as described above, the shear stress response of the model shows poor agreement with experiments. However, it was assumed that the Bauschinger effect shows the same tendency in tension and compression. In order to fit the parameters, the measured monotonic shear curve was transferred to the RD flow curve based on the fitted Yld2000-2d model. This curve then was used to identify the HAH parameters. The parameters determined like this are given in Tab. 6.6. The resulting curves are displayed in Fig. 6.12 together with the measured ones. Although there are deviations right after the re-yielding, the overall agreement between the model and the measurement is satisfying.

<table>
<thead>
<tr>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
<th>$k_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>158.37</td>
<td>6.33</td>
<td>0.10</td>
<td>0.92</td>
<td>10.0</td>
</tr>
</tbody>
</table>

Table 6.6: HAH parameters $k_1$ to $k_5$ for DC05

In a second step, the parameters $k$, $k_L$, and $L$, which control the rate of 'rotation' of the microstructure deviator as well as the height of the stress overshoot after orthogonal loading were identified (see sec. 4.3.2). This
Figure 6.12: Shear with load reversal for DC05: Measured and predicted with HAH

parameters were fitted by means of the two stage tensile tests presented in sec. 5.5. The prestrained in 45° and subsequently strained in RD were used for this purpose. The identified parameters are given in Tab. 6.7. The measured curves as well as the model predicted are plotted in Fig. 6.13. Although the hardening curves after a prestrain of 0.1 and 0.15 are only given in a very short strain range since diffuse necking set in after less than one percent plastic strain, they can still be used to tune the height of the stress overshoot after the given amount of prestrain. The model and the measurement shows good agreement for all three cases as shown in Fig. 6.13, especially for a prestrain up to 0.1.

<table>
<thead>
<tr>
<th>(k)</th>
<th>(k_L)</th>
<th>(L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.00</td>
<td>1600.0</td>
<td>1.86</td>
</tr>
</tbody>
</table>

Table 6.7: HAH parameters \(k\), \(k_L\) and \(L\) for DC05

### 6.2 Formalex™-5x

#### 6.2.1 Flow curve

The isotropic behavior of the Formalex™-5x material was again fitted based on the uniaxial tensile tests and the hydraulic bulge test (see sec.
5.2 and 5.3). Since a negligible strain rate influence was ascertained, there is no need to do any compensations for the results of the hydraulic bulge test, but it can directly be transformed to the uniaxial stress space, as explained in sec. 6.1.1, and be used to find the parameters for the flow curve approximation. Again, a Hockett-Sherby approach was used. The measured (smoothed) uniaxial hardening curve, the measured (smoothed) and the transformed biaxial hardening curve as well was the Hockett-Sherby (Eq. (6.3)) approximation are shown in Fig. 6.14.
6.2 Formalex™-5x

### Table 6.8: Hockett-Sherby parameters for Formalex™-5x

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>m</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>127.63</td>
<td>434.28</td>
<td>5.26</td>
<td>0.84</td>
</tr>
</tbody>
</table>

#### 6.2.2 Yield functions

This section summarizes the parameter identification for different yield locus models that were fitted in order to compare their accuracy for the investigated Formalex™-5x material. Starting with the Barlat-Lian '89 model (see. sec. 4.2.2), which is well-established for aluminum materials in sheet metal forming simulations, the Yld2000-2d model is used before, again, the parameters for the strain dependent Yld2000-2d model and its combination with the HAH model are identified. The Formalex™-5x results in chapter 5 are used to find the parameters for the different models.

**Barlat-Lian ’89**

The Barlat-Lian ’89 model, explained in detail in sec. 4.2.2 is as already mentioned one of the most-widely used model for the simulation of sheet forming processes with aluminum materials. This is why it was also used in this thesis in order to have the state of the art reference. The parameters were identified according to the second identification procedure given in sec. 4.2.2 which is based on the Lankford parameters. Tab. 6.9 lists the mechanical properties used as input for the parameter fitting process as well as the resulting Barlat-Lian ’89 parameters. Fig. 6.15 shows the measured as well as the model predicted yield stresses 6.15a and r-values 6.15b. An exponent $M$ of 8 was chosen according to the recommendation in (Barlat and Lian, 1989). Note that the yield stress as well as the r-values vary in a very small range. The variation and therefore the prediction error of the model lies within the measurement uncertainty of the experiments.

**Yld2000-2d**

The Yld2000-2d model parameters for Formalex™-5x were identified analogously to the ones for the DC05 material. Eqs. (6.4) and (6.4) were solved to find the parameters. Tab. 6.10 lists the mechanical properties, which served as input and the resulting Yld2000-2d parameters.
6 Material characterization

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<th>(r_{90})</th>
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<td>1.162</td>
<td>0.838</td>
<td>0.998</td>
<td>1.003</td>
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</table>

Table 6.9: Mechanical properties and fitted Barlat-Lian ’89 parameters for Formalex\textsuperscript{TM} -5x

![Graph](image1)

(a) Distribution of the yield stress along the angle to RD predicted by Barlat-Lian ’89

![Graph](image2)

(b) Distribution of the \(r\)-value along the angle to RD predicted by Barlat-Lian ’89

Figure 6.15: Comparison of Barlat-Lian ’89 predicted and measured yield stress and \(r\)-value along the angle to RD for Formalex\textsuperscript{TM} -5x

<table>
<thead>
<tr>
<th>Input</th>
<th>(\sigma_0)</th>
<th>(\sigma_{45})</th>
<th>(\sigma_{90})</th>
<th>(\sigma_b)</th>
<th>(r_0)</th>
<th>(r_{45})</th>
<th>(r_{90})</th>
<th>(r_b)</th>
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<td>147.7</td>
<td>144.9</td>
<td>146.4</td>
<td>130.0</td>
<td>0.72</td>
<td>0.74</td>
<td>0.71</td>
<td>1.13</td>
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<table>
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<th>(\alpha_2)</th>
<th>(\alpha_3)</th>
<th>(\alpha_4)</th>
<th>(\alpha_5)</th>
<th>(\alpha_6)</th>
<th>(\alpha_7)</th>
<th>(\alpha_8)</th>
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<td></td>
<td>0.978</td>
<td>1.020</td>
<td>1.290</td>
<td>1.078</td>
<td>1.063</td>
<td>1.227</td>
<td>1.012</td>
<td>0.911</td>
</tr>
</tbody>
</table>

Table 6.10: Yld2000-2d input and fitted parameters for Formalex\textsuperscript{TM} -5x

Fig. 6.17 shows the distribution of the yield stress (Fig. 6.17a) and the \(r\)-value (6.17b) along the angle to RD. Note that the distribution of the \(r\)-value is similar to the one predicted with Barlat-Lian’89 while the yield stress can be captured more accurately. However, as already mentioned, the range of variation is very small and therefore, only little effects are
Figure 6.16: Barlat89 yield locus for Formalex™-5x expected.

Figure 6.17: Comparison of Yld2000-2d predicted and measured yield stress and $r$-value along the angle to RD for Formalex™-5x
Equivalent strain dependent Yld2000-2d

As for the DC05 material, strain dependent parameters were determined for the Formalex™-5x material. The procedure for the parameter identification is the same. However, by applying this method, the slope of the parameters as a function of the equivalent plastic strain was quite different than for the DC05 material. Fig. 6.19 shows the varying parameters as they were obtained from the fitting procedure. The dots show the smoothed parameters. Note that the measured (unfiltered) data was used to perform the fitting. Hence, the fitted curve show a certain scattering. Furthermore, the fit is very unstable up to strains of about 0.04. This is due to the PVLC effect, which is very distinct in this strain region. In order not to consider this effect, a constant-linear-constant approach was chosen to approximate the parameters. Since no data is available and a linear extrapolation would lead to unreasonable values, the parameters were kept constant beyond uniform elongation.

\[ \alpha_i = p_{1i} \cdot \bar{\varepsilon}^{p} + p_{2i} \]  

(6.7)

The resulting yield loci using the parameters as given in Tab. 6.11 are shown in Fig. 6.20. Fig. 6.21 shows the measured hardening curves for
Table 6.11: Yld2000-2d input and fitted parameters for Formalex™-5x

<table>
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<tr>
<th></th>
<th>$\alpha_1$</th>
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<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
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<td>0.7343</td>
<td>0.0941</td>
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<tr>
<td>$p_2$</td>
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<td>1.028</td>
<td>1.045</td>
<td>1.025</td>
<td>1.034</td>
<td>-1.011</td>
<td>1.081</td>
</tr>
</tbody>
</table>

Figure 6.19: Varying Yld2000-2d parameters for Formalex-5x: thick: fitted, thin: smoothed with constant-linear-constant

Formalex-5x in RD, 45°, TD and biaxial together with the ones predicted with the original Yld2000-2d model (6.21a) and the modified one (6.21b). It can easily be seen that the prediction of the modified model is exact, while the original model shows deviations.

**Homogenous function based anisotropic hardening model**

As for the DC05 material, the first parameters, $k_{1-5}$, were fitted based on the results of the simple shear test with load reversal (see sec. 5.6). Again, the Yld2000-2d model with the parameters given in Tab. 6.10 is not able to predict the measured simple shear hardening correctly. Therefore, the measured monotonic simple shear curve was extrapolated and transferred to the uniaxial case based on the identified Yld2000-2D model. The Bauschinger effect was assumed similar for simple shear with load reversal and uniaxial tension compression. The measured and model predicted curves (based on the monotonic simple shear test) are shown in Fig. 6.22.
Figure 6.20: Yld2000-2d yield locus with constant and variable parameters for Formalex™-5x at an equivalent strain of 0, 0.05 and 0.15

Figure 6.21: Yield curve prediction of original and modified Yld2000-2d model for Formalex-5x

and the according parameters are given in Tab. 6.12.

Since the Formalex™-5x material exhibits latent contraction (reyielding at lower stress after an orthogonal strain path change), the parameter $k_L$ is set to 0 and the parameters $k$, $k_C$ and $C$ as used in Eq. (4.55) in sec. 4.3.2 were identified in the second step. The two stage tensile tests for
Figure 6.22: Shear with load reversal for Formalex-5x: Measured and predicted with HAH

\[
\begin{array}{ccccc}
  k_1 & k_2 & k_3 & k_4 & k_5 \\
  250 & 10.0 & 0.10 & 0.9 & 10.0 \\
\end{array}
\]

Table 6.12: HAH parameters \( k_1 \) to \( k_5 \) for Formalex-5x

Formalex™-5x described in sec. 5.5 were used for this purpose. For the same reasons as before, the tensile tests in RD after a prestrain in \( 45^\circ \) were used. The measured and model predicted curves are shown in Fig. 6.23 while the identified parameters are given in Tab. 6.13

\[
\begin{array}{ccc}
  k & k_C & C \\
  30.00 & 100.0 & 1.35 \\
\end{array}
\]

Table 6.13: HAH parameters \( k \), \( k_C \) and \( C \) for Formalex™-5x
Figure 6.23: Tenile tests in RD with material prestrained in 45° to 0.05, 0.1 and 0.15 for Formalex™-5x
7 Model validation

In this chapter, the models described in chapter 4 and fitted in sec. 6.1 and sec. 6.2 are validated by means of comparisons of deep drawing experiments with the corresponding simulations. By comparing the major and minor strains measured in the experiments and obtained from the simulations, the models can be qualitatively and quantitatively assessed. The experimental procedure, the simulation models and their calibration are explained in detail before the results for the two different materials investigated are presented.

7.1 Cross Die

The main geometry that served as a validation part is the well known cross-die, which has been reported in several publications before (e.g. (Neukamm et al., 2008; Butz et al., 2010; Govindarajan et al., 2011)) and is also used by major automotive companies for material model approval purposes. The following chapters show the procedure of the calibration of the simulations with the experiments as well as the experimental results and the ones of the simulations obtained with different material models. The dimensions (Fig. 7.1a) as well as a section cut of the three-dimensional shape (Fig. 7.1b) are shown in Fig. 7.1. What is not evident from the figure is the die radius of 5 mm as well as the drawing clearance of 2.5 mm. In order to be able to measure the strains of the final part, the optical strain measurement system ARGUS was used, which computes strains based on digital image correlation (DIC) techniques. A pattern of equidistant dots of a defined diameter (usually 1 mm diameter and 2 mm distance) is applied on the undeformed blank. Several pictures from different angles are taken to measure the initial configuration. After having drawn the part, another series of pictures is taken to measure the positions of the dots in the final state. Using a total Lagrange approach, the deformation tensor and thus the strains can be computed. A picture of the initial blank (Fig. 7.2a), the deformed blank (Fig. 7.2b) as well as
the strain measurement (Fig. 7.2c) is shown in Fig. 7.2. Eventually, the

software SView from GOM offers the possibility to compare simulation results from different FE codes, LS-DYNA and AutoForm among others, with the results from the measurement. Using DIC, the three-dimensional location of the pattern can be computed and a virtual grid is generated. The FE-mesh from the simulations and the one from the measurement are relatively positioned space using a Best fit algorithm which minimizes the geometric deviations between the two. Afterward, the results from the FE simulation are mapped on the measurement grid. By subtracting the FE-results from the measurement data, the deviation between measurement and FE computation can be illustrated in a fringe plot.

7.1.1 DC05

Test configurations and model calibration

For the DC05 material, the tests were carried out at utg TU Munich. The tool was provided by Daimler. Four different configurations were experi-
mentally tested. These were two different drawing depths at two different drawing velocities. The parameters for every configuration are given in Tab. 7.1 The idea behind these configurations was on the one hand to test whether also intermediate states (between initial and final state) can be represented with a higher accuracy due to the dependency of the model parameters on the plastic strain (see sec. 4.3.1), and, on the other hand, to also investigate the strain rate influence, since the material showed a significant strain rate dependency (see sec. 5.2).

The simulations were performed using LS-DYNA. In the simulation, a velocity of 2000 mm/s was used, which results in a time scale factor of 66.6 for the tests with a punch velocity of 30 mm/s and 400 for the ones with 5 mm/s. Since 500 time increments per millimeter tool displacement were defined, this resulted in a timestep of 1e-6 s for all simulations. Quadrilateral Belytschko-Lin-Tsay shells with a initial side length of 1mm and 7 through thickness integration points were used for the spatial discretization of the blank. A rectangular square with cut off edges (isosceles triangles with a height of 70 mm). In Fig. 7.3, the blank geometry and dimensions are shown.

In a first step, the friction coefficient was determined. The binder force was assumed to be given. A value of 200 kN, which was set at the press for these experiments was used. The friction was determined by comparing the measured draw-in from the real part with the draw-in in the simulation. At the same time, the initial blank position was adjusted, since the blank was not exactly positioned in the center of the press, but with a deviation between 0mm and 2mm. In the simulation, the different drawing velocities did not have a significant influence on the result, although a strain rate dependent hardening model (see sec.5.2) was chosen. The differences from the experiments could be captured by assuming different

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Drawing Depth [mm]</th>
<th>Punch velocity [mm/s]</th>
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<tr>
<td>V1</td>
<td>75</td>
<td>30</td>
</tr>
<tr>
<td>V2</td>
<td>50</td>
<td>30</td>
</tr>
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<tr>
<td>V4</td>
<td>35</td>
<td>5</td>
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Table 7.1: Configurations for cross die tests with DC05
friction values for the different velocities. The best agreement between the simulated and the measured draw-in was found with a friction coefficient of 0.10 for the experiments with a punch velocity of 5 mm/s and 0.08 for a punch velocity of 30 mm/s. These values were fixed in the following. Fig.7.4 shows the comparison of the draw-in for the four configurations in Tab. 7.1. The simulated part is displayed in gray and the red curves represent the measured boundary draw in from the real parts.

After fixing the friction coefficients, the simulation parameters were not changed anymore. The same simulation models (V1-V4) were used in combination with different hardening models. Results obtained with the Hill48 model (see sec. 4.2.1), the standard Yld2000-2d model (see sec. 4.2.4), the Yld2000-2d model with varying parameters (see sec. 4.3.1), and the combination of the latter two with the HAH model (see sec. 4.3.2) were compared. In order to assess the different models, the distribution of the major and minor in-plane strains on the outer surface of the part were compared. In the following, the simulation results and the differences between simulation and measurement are compared for the different models. Note that the friction coefficient was determined based on the standard Yld2000 model. The results obtained with this model may benefit from this.

**Strain rate influence**

As outlined in sec. 5.2, the DC05 material exhibits a strain rate dependency that was modeled using a modified Cowper-Symonds approach. Af-
ter the comparison of simulations with and without strain rate dependency with the strain measurement of the cross die experiment, it turned out that neglecting the strain rate dependency does not substantially change the strain distribution. A comparison for configuration V1 is shown in Fig.7.5. Fig.7.5a shows the difference in major strain of V1 simulated with the standard Yld2000-2d model with strain rate dependency. In Fig.7.5b, the strain rate dependency is neglected. It can be clearly seen that the results are almost identical. Similar results were obtained for the remaining configurations and plasticity models. Thus, in the remainder, only results obtained without taking into account the strain rate dependency are presented.

Figure 7.4: Comparison of measured and simulated draw-in for DC05
Simulation results

In the following, the simulation results for the different configurations are presented. Note that there is a single experiment for every configuration. Figs. 7.6 and 7.7 show the difference between simulation and experiment for the in-plane major and minor strains (i.e. the measured strain minus the simulated strain). Because similar tendencies were obtained for all configurations, only configuration V1 is presented. A quantitative comparison of the strains along critical sections is given in Figs. 7.8 and 7.9. The locations of the according sections are indicated in the small picture of the cross die in the plot.

The results show that the variable parameters improve the results compared to the standard Yld2000-2d model. The improvement is basically observed in the bottom corners of the part. This according to expectations since theses corners are biaxially stretched as pointed out in sec. 6.1.2, the variable parameters have the highest influence in the equibiaxial stress state. The HAH however does not significantly change the results. Further discussions is following in sec. 8.1.
Figure 7.6: Major strain differences for configuration V1 with different material models (DC05)
Figure 7.7: Minor strain differences for configuration V1 with different material models (DC05)
7.1 Cross Die

Figure 7.8: Major and minor strain along section S1 for Cross Die DC05 (solid: major strain, dashed: minor strain)

Figure 7.9: Major and minor strain along section S2 for Cross Die DC05 (solid: major strain, dashed: minor strain)
### 7 Model validation

<table>
<thead>
<tr>
<th>Model</th>
<th>Min.</th>
<th>Max.</th>
<th>Avg.</th>
<th>Std. dev.</th>
</tr>
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<tr>
<td>Major strain deviation</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
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<td>0.115</td>
<td>0.032</td>
<td>0.044</td>
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<td>0.053</td>
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<td>0.185</td>
<td>0.046</td>
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<td>0.131</td>
<td>0.018</td>
<td>0.027</td>
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Table 7.2: Statistical values

#### 7.1.2 Formalex™-5x

The cross die was also drawn for Formalex™-5x with Constellium at their research center in Voreppe. Constellium has the same tool geometry available. Again, four different configurations were chosen. Since the material did not show a noteworthy strain rate dependency, the same drawing speed was chosen for all the experiments. Instead, the initial blanks were cut in two different directions, long side in RD and TD. Again, two different drawing depths were used which results in a total of four tests. Unlike

<table>
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<tr>
<th>Configuration</th>
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<th>Blank orientation</th>
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<td>L5</td>
<td>30</td>
<td>long side in RD</td>
</tr>
<tr>
<td>L13</td>
<td>54</td>
<td>long side in RD</td>
</tr>
<tr>
<td>Q3</td>
<td>30</td>
<td>long side in TD</td>
</tr>
<tr>
<td>Q5</td>
<td>54</td>
<td>long side in TD</td>
</tr>
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</table>

Table 7.3: Configurations for cross die tests with Formalex™-5x

for the DC05 experiments, the press used for these experiments has a die cushion which is bedded on gas springs. Therefore, the binder force is a linear function of the punch stroke and was measured during the tests.
7.1 Cross Die

The force measurement as well as the linear approximation is shown in Fig. 7.17. The evaluation of the press forces led to the following linear dependency of the binder force on the punch displacement.

\[ F_{Binder} = 64.318 + 0.603 \cdot x_{Punch} \]  

(7.1)

where \( F_{Binder} \) is the binder force in kN and \( x_{Punch} \) (\( x = 0 \) is the position where the punch touches the initial blank).

The LS-DYNA simulations were performed with the same parameters that have already been used for the simulations with DC05 (see sec. 7.1.1), namely a time scaling factor of 66.6, a timestep of 1e-6 s and quadrilateral Belytschko-Lin-Tsay shells with an initial side length of 1 mm and 7 through thickness integration points. Fig. 7.10 shows the blank geometry and dimensions.

Again, the friction coefficient used in the simulations was calibrated on the basis of the measured draw in. Because the blank was not perfectly centrically positioned in the press, slightly asymmetric parts were drawn. In order to take that into consideration, not only the friction coefficient, but also the x- and y-coordinate of the blanks initial position was varied until a good agreement between the simulated and measured draw-in was found. With a friction coefficient of 0.07, the best agreement was obtained. Fig. 7.11 shows a comparison of the simulated and measured draw in for the four configurations. Also here, the calibration was performed while
using the YLd2000-2d model and slightly different draw-ins were obtained when using the remaining models. This was consciously accepted to make sure no other influence than the one of the used plasticity model is investigated.

The major and minor strains of the simulations and the optical measurements of the real parts were compared to assess the predictive capability of the different models. For the Formalex™-5x material, the strain of the whole part was measured. Fig. 7.12 and 7.13 show the differences between simulations results obtained with different material models and the optical strain measurements. Furthermore, in Figs. 7.14 and 7.15, the major and minor strain distribution along two different sections is plotted.

Figure 7.11: Comparison of measured and simulated draw-in for Formalex™-5x
Figure 7.12: Differences of simulated and measured major strain of configuration L13 for different plasticity models (Formalex™-5x)
Figure 7.13: Differences of simulated and measured minor strain of configuration L13 for different plasticity models (Formalex™-5x)
## 7.1 Cross Die

<table>
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<th>Model</th>
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<th>Max.</th>
<th>Avg.</th>
<th>Std. dev.</th>
</tr>
</thead>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Barlat89</td>
<td>-0.069</td>
<td>0.073</td>
<td>0.007</td>
<td>0.034</td>
</tr>
<tr>
<td>Yld2000</td>
<td>-0.060</td>
<td>0.081</td>
<td>0.011</td>
<td>0.038</td>
</tr>
<tr>
<td>Yld2000 v.</td>
<td>-0.029</td>
<td>0.060</td>
<td>0.015</td>
<td>0.023</td>
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<tr>
<td>Yld2000 HAH</td>
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<td>0.073</td>
<td>0.007</td>
<td>0.037</td>
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<td>0.060</td>
<td>0.008</td>
<td>0.022</td>
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<td><strong>Minor strain deviation</strong></td>
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<td>0.054</td>
<td>0.007</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Table 7.4: Statistical values

![Diagram of Cross Die](image)

**Figure 7.14:** Major and minor strain along section S1 for Cross Die Formalex™-5x
(solid: major strain, dashed: minor strain)
Figure 7.15: Major and minor strain along section S2 for Cross Die Formalex™-5x
(solid: major strain, dashed: minor strain)

Figure 7.16: Major and minor strain along section S3 for Cross Die Formalex™-5x
(solid: major strain, dashed: minor strain)
Figure 7.17: Measured and approximated binder force
7 Model validation

7.2 Lackfrosch

The second part that was used for the model validation is the so-called Lackfrosch of the AUDI AG. This part, mimicking the geometry of a car body and, as the name implies originally used to test car finishes, was used to investigate actuating elements and sensor systems in order to enable to self-adaptively controlled deep drawing processes (see e.g. Annen et al. (2010)). However, in this study, this part was chosen to further validate the material modes, that have already been used to simulate the cross-die experiments. Again, the ARGUS measurement system was used to measure strain distributions on the outer surface of the part and SVi2ew was used to compare simulation results with the measurement of the experiments.

Because of the high number of elements and the high drawing depth, only 300 timesteps per mm tool displacement were used in the LS-DYNA simulations for the Lackfrosch. The initially rectangular blank with size 600x900 mm\(^2\) was discretized using quadrilateral Belytschko-Lin-Tsay shells with a side length of 1.25 mm and 5 through thickness integrations points.

The friction value in the simulation was again determined by a comparison of the experimental and the simulated draw-in. For this purpose, geometrical drawbeads instead of virtual drawbead models had to be used in the simulation. The geometrical drawbeads also guarantee a correct strain history in the material that passes the drawbeads. Because the binder force could not be measured, it has been varied with the friction. A friction coefficient of 0.07 in combination with a binder force of 600 kN led to the best agreement between simulative computed and measured draw-in. A comparison between the draw-in in simulation and measurement is shown in Fig. 7.18.

Figs. 7.19 and 7.20 show the major and minor strain differences between simulated and measured strains, again for the different yield locus models that have already been used for the Cross-Die simulations. In Fig. 7.21 the major and minor strains along a short section through the critical area of the part, is plotted. The section is marked in the small image of the Lackfrosch at the upper left corner of the plot window.

In order to investigate the results more in detailed, statistical figures for the area that lies within a radius of 30 mm around the critical point at the rear mudguard were considered. The statistical figures are given in
Tab. 7.5 while the area considered is marked in red in the figure next to it.
The figures in Tab. 7.5 as well as the strain distributions in Fig. 7.21 show that in the critical area, the best results are obtained by using the variable Yld2000-2d model in combination with HAH. While the standard Yld2000-2d model shows actually worse results than the Hill 48 model, both the variable parameters and the HAH extension by themselves and especially the combination of the two improve the results. The interpretation and discussion of the results is following in sec. 8.1.
Figure 7.19: Differences of simulated and measured major strain for the Lackfrosch for different plasticity models (DC05)
Figure 7.20: Differences of simulated and measured minor strain for the Lackfrosch for different plasticity models (DC05)
Figure 7.21: Major and minor strain along section for Lackfrosch (DC05) (solid: major strain, dashed: minor strain)

<table>
<thead>
<tr>
<th>Model</th>
<th>Min.</th>
<th>Max.</th>
<th>Avg.</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Major strain deviation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hill48</td>
<td>-0.031</td>
<td>0.081</td>
<td>0.021</td>
<td>0.033</td>
</tr>
<tr>
<td>Yld2000</td>
<td>-0.078</td>
<td>0.124</td>
<td>-0.010</td>
<td>0.052</td>
</tr>
<tr>
<td>Yld2000 v.</td>
<td>-0.039</td>
<td>0.074</td>
<td>-0.004</td>
<td>0.021</td>
</tr>
<tr>
<td>Yld2000 HAH</td>
<td>-0.077</td>
<td>0.101</td>
<td>-0.011</td>
<td>0.044</td>
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<tr>
<td>Yld2000 v. HAH</td>
<td>-0.056</td>
<td>0.045</td>
<td>-0.007</td>
<td>0.019</td>
</tr>
<tr>
<td><strong>Minor strain deviation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hill48</td>
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<td>0.008</td>
<td>-0.024</td>
<td>0.012</td>
</tr>
<tr>
<td>Yld2000</td>
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<td>-0.009</td>
<td>-0.041</td>
<td>0.016</td>
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<tr>
<td>Yld2000 v.</td>
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<td>0.017</td>
<td>-0.015</td>
<td>0.012</td>
</tr>
<tr>
<td>Yld2000 HAH</td>
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<td>-0.002</td>
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<td>-0.034</td>
<td>0.024</td>
<td>-0.005</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Table 7.5: Statistical values
8 Discussion and outlook

8.1 Discussion

The goal of this thesis was the consideration of anisotropic hardening effects in sheet metal forming simulations. By introducing a dependency of the Yld2000-2d parameters in the equivalent plastic strain and using the newly developed HAH model with the parameters identified in sec. 6, the anisotropic effects measured in the laboratory experiments (sec. 5) could be well approximated.

The Yld2000-2d with varying parameters is particularly interesting since the experimental effort to determine the parameters is not increased compared to the standard formulation. However, the identification of the parameters is not as straightforward and an extra effort is necessary to find a good approximation for the parameters. The biggest advantage of the HAH model over other anisotropic hardening models is its relative simplicity. It can be combined with every other first-order homogenous yield criterion. The parameters describing the different effects (initial anisotropy, Bauschinger effect, latent effects) can be identified separately and their meaning is understandable.

The new models were successfully implemented in the commercial FE-Code LS-DYNA using the stress update algorithms explained in sec. 4.4 and applied to simulate the stamping process of two different validation experiments. The comparison of the simulation results with the optical strain measurements were presented in sec. 7. Although the differences of the different Yld2000-2d models are only in the range of a few percent in the better part of the geometry, the variable Yld2000-2d parameters trend to lead to a better prediction of the strain field for the investigated parts than the regular Yld2000 model.

Considering the average deviations of the strains obtained in the ex-
Discussion and outlook

Experiments for the DC05 cross die, unexpectedly the best results were obtained by using the well-known Hill 48 model. A possible explanation is the smoother shape of this yield locus. However, the strains were highly overestimated in some regions, which would lead to a wrong failure prediction.

The simulation of the Formalex cross die significantly benefited from variable yield parameters. Especially in regions of biaxial stress, a variation of the parameters leads to a better prediction of the strain field. This is what was expected since the variation of the parameters mainly influences the yield locus in the biaxial region.

In contrast, the HAH delivers results that are comparable with the ones obtained with the base model that was used as the stable component. Only the Lackfrosch showed slightly better agreement when using HAH.

Possible explanations for the obtained results are the following:

- The strain path nonlinearities are not pronounced and/or sharp enough to clearly show the effects.

- In most of the non-linear strain paths that emerge in the parts, the path change happens before the end of the process and the last strain percentages happen to be linear again. The anisotropic hardening effects are the most distinct right after strain path changes and fade after just a few percent of linear strain.

- For the DC05 material the latent hardening, i.e. the stress overshoot after an orthogonal strain path change and the Bauschinger effect, i.e. the early re-yielding after load reversal compensate each other to a certain extend.

- Using the HAH approach in combination with an associated flow rule,

In addition to the reasons just named, only a few strain path changes were investigated and even fewer were used to calibrate the model. It cannot be guaranteed that the model closely approximates stress responses for arbitrary strain path changes.

The general deviations between simulative computed and measured strains can have several origins. The complex tribological conditions are
not probably captured, but simplified using a Coulomb friction model with a constant friction coefficient. Although this is state of the art in industrial process modeling, it is well known that the friction is influenced by contact pressure and temperature. Furthermore, thermal effects were completely neglected although a temperature increase in the material due to plastic work and a temperature dependency of the yield stress must be expected. However, aware of these inaccuracies of the process model, the negligence of the effects just named was consciously accepted. On the one hand, because using more complicated friction models or temperature dependent material models brings even more complexity to the simulation and separating the effects becomes more difficult, on the other hand, because due to their complexity, such models might lead to even larger errors because they cannot be validated under all circumstances.

8.2 Outlook

In (Barlat et al., 2013b), several enhancements of the HAH model are proposed. Basically, the discontinuity of the evolution of the microstructure deviator is avoided by introducing a function $G$ which continuously increases and decreases the rate of "rotation" of the microstructure deviator $\hat{h}$ respectively. Furthermore, the latent effects are modeled by decomposing the stress deviator into a component collinear a component orthogonal to $\hat{h}$. This way, it is possible to expand or shrink the yield locus faster in directions orthogonal to $\hat{h}$, but not in collinear ones. The effects of these enhancements on the strain distributions should be investigated, although no significant differences are expected.

Furthermore, (Manopulo et al., 2015) showed that the mathematical formulation of the HAH model can be significantly simplifies and the constant $H$ in the original formulation is obsolete. They also discussed discontinuities of the derivatives under load path changes. This point should be subject of further model enhancements.

The cross die and Lackfrosch parts used for validation in this study did not show sharp and pronounced strain path changes. For further studies on the performance of the HAH model in real part simulations, other geometries or processes should be used for further validations. A deep
drawing process with two stages for example could lead to sharp strain path changes and the effects could be analyzed more in detail. Besides, also the springback behavior of drawn parts should be investigated in order to not only check simulated strain, but also stress distributions. For this purpose, it would be advantageous to work with high strength materials that might show effects that are more pronounced and higher residual stresses, which leads to more springback. The bigger these effects are and the more the material behavior deviates from anisotropic hardening and the easier it should be to show the effects and the model differences.
Bibliography


Bibliography


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