Doctoral Thesis

Large-scale structures in Rayleigh-Bénard convection and flow over waves

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LARGE-SCALE STRUCTURES IN
RAYLEIGH-BÉNARD CONVECTION AND
FLOW OVER WAVES

A dissertation submitted to the
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presented by
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Abstract

An experimental investigation is carried out on large-scale structures of the fluid velocity and temperature in the turbulent flow over a heated, wavy surface. The study is motivated in part by the findings of Gong et al. (1996), who report a spanwise variation of the mean streamwise velocity in a boundary layer flow over a train of waves; where the width of the wind tunnel was only four times the wavelength. The flow conditions considered herein are similar to the ones of recent pointwise measurements (Hudson et al., 1996) or direct numerical simulations (e.g. Cherukat et al., 1998).

Spatio-temporal information on the fluid temperature is obtained from liquid crystal thermometry (LCT). Using liquid-dispersed liquid crystal particles, the technique is developed and calibrated in a thermally stratified fluid layer. Influences of the fluid properties and the optical configuration are thoroughly assessed and provide the basis for a wide range of potential measurement applications. Digital particle image velocimetry (PIV) is used to examine the spatial variation of the velocity in different planes of the flow. As a reference situation for a transient flow with heat transfer, LCT and PIV measurements are first applied to turbulent Rayleigh-Bénard convection in water. Quantitative structural information is obtained from the two-point correlation function and a proper orthogonal decomposition (POD) of the wall-normal velocity component at a Rayleigh number of $7.8 \times 10^6$, and a Prandtl number of 4.8. Both methods reveal dominant contributions with a characteristic scale of two layer depths in the direction parallel to the wall, where POD analysis proves to be a more effective tool in order to distinguish between the different persistent modes.

At isothermal conditions, streamwise-oriented structures in the developed flow through a channel with a flat top and a wavy bottom wall are obtained from PIV measurements. For the presented results, the wave amplitude is ten times smaller than the wavelength $\Lambda$ and Reynolds numbers between 500 and 7300, defined with the bulk velocity and the half-height of the channel, are considered. The spanwise variation of the velocity fluctuations is assessed in a plane parallel to the top wall, and in one that intersects with the wavy surface at an uphill location. In contrast to the findings of Gong et al. (1996), no significant spanwise variations of the streamwise mean velocity were observed, indicating that the aspect ratio of 12:1 is large enough to assume homogeneity in this direction. A POD analysis of the streamwise velocity fluctuations reveals dominant eigenfunctions with a characteristic spanwise scale of $1.5\Lambda$, in agreement with the scale of the spanwise perturbation
of the streamwise velocity at laminar conditions.

POD analysis of the turbulent velocity field close to the uphill section of the wavy surface enables us to connect the eigenfunctions of the dominant modes (scale 1.5\(\Lambda\)) to smaller scales that are represented by higher POD modes. Extrema of the corresponding eigenfunctions are located in the vicinity of the maximum Reynolds shear stress region. When comparing the results obtained at the Reynolds numbers 3800 and 7300, we find indications that the relative fractional contribution of the eigenfunctions characterized by scale 1.5\(\Lambda\) increases with increasing Reynolds number. We further relate the dominant modes to an instability that is catalized by the wavyness of the bottom wall.

To the knowledge of the author for the first time, structural information is obtained for the flow over heated waves. A constant heat flux condition is imposed at the wavy surface through a resistively heated foil. LCT is used to obtain spatio-temporal temperature fields above an uphill location of the wavy surface. Two conditions at different Reynolds numbers with (mixed convection) and without (forced convection) a buoyancy influence are considered. For a Reynolds number of 3300, this effect is negligible. POD analysis reveals, for the two dominant modes, eigenfunctions with a characteristic spanwise scale of 1.5\(\Lambda\), in agreement with the findings for the velocity field. The 1.5\(\Lambda\) scale is therefore obtained from both, temperature and velocity fields. Together with the extrema of the eigenfunctions for higher POD modes that were observed above the uphill side of the wavy surface, they play an important role with respect to the fluctuation energy of the velocity and temperature (to which the two dominant modes contribute more than 30\%). They also provide a mechanism for the convective transport of heat between the wavy surface and the bulk fluid.
Zusammenfassung


Bei isothermen Bedingungen werden zunächst in Strömungsrichtung ausgerichtete Grobstrukturen für die ausgebildete Strömung durch einen Rechteckkanal mit einem Kantenlängenverhältnis von 12:1 in der Querschnittsfläche mit PIV-Messungen untersucht. Der Kanal besitzt eine gewellte Grund- und eine ebene Deckfläche. Die Wellenamplitude ist zehnmal kleiner als die Wellenlänge $\Lambda$. Reynolds-Zahlen, definiert mit der querschnittsgemittelten Geschwindigkeit und der halben Kanalhöhe, von 500 bis 7300 werden betrachtet. Strömungsstrukturen werden in einer Ebene parallel zur Kanaldeckfläche sowie in einer die wellige Grundfläche an einer strömungs-

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Nomenclature

Roman Symbols

\( a \) half-amplitude of the wave profile (m)
\( a, b, c \) constants in the Basset equation
\( a_i \) coefficient for the proper orthogonal decomposition
\( A \) cross-sectional area (m)
\( AOV \) area of view (m\(^2\))
\( B \) channel width (m)
\( B \) blue light intensity, \( m_{\text{pix}} \times n_{\text{pix}} \) matrix
\( c_p \) specific heat capacity at constant pressure (kJ/kg K)
\( C \) convection term
\( C \) \( M \times N \) covariance matrix
\( \dot{d}_{ij} \) instantaneous deformation tensor, second-order
\( d_e \) particle image diameter (m)
\( d_P \) particle diameter (m)
\( d_r \) size of a square pixel (m)
\( D \) viscous diffusion term, domain of a scalar field
\( D_a \) aperture diameter (m)
\( \bar{D}_{ij} \) mean deformation tensor, second-order
\( E \) energy
\( E \) electric vector
\( f \) focal length (m)
\( f_\# \) f-number of the camera objective, \( f / D_a \) (-)
\( f_{FR} \) camera frame rate (Hz)
\( F_i \) external force
\( g \) gravitational acceleration (m/s\(^2\))
\( G \) green light intensity, \( m_{\text{pix}} \times n_{\text{pix}} \) matrix
\( Go \) Görtler number, \( (x/\mathcal{R})^{0.5} \) (-)
\( h \) half channel height (m)
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\( H \)  full channel height (m)

\( H \)  Hue, matrix \( m_{pix} \times n_{pix} \) (raw data) or \( m \times n \) (locally averaged)

\( i, j, k, l \)  integer counters (-)

\( I \)  intensity, \( \frac{1}{3} (R + G + B) \), \( m_{pix} \times n_{pix} \) matrix (raw data)

\( I^* \)  dimensionless light intensity, \( I^* : [-1, +1] \) (-)

\( J \)  light flux

\( J_E \)  integral time scale of \( \tau_E \)

\( m \)  liquid mass (kg), number of (locally averaged) temperature and velocity locations in the horizontal direction of a recangular 2-D domain (-)

\( m_{pix} \)  number of pixels on CCD chip (horizontal)

\( M \)  magnification factor, number of temperature or velocity fields (-)

\( n \)  number of (locally averaged) temperature and velocity locations in the vertical direction of a recangular 2-D domain (-), POD mode, refractive index (-)

\( n_{pix} \)  number of pixels on CCD chip (vertical)

\( N \)  spatial dimension of dataset for POD analysis, for 2-D analysis: \( n \cdot m \) (-)

\( N_s \)  Stokes number (-)

\( Nu \)  Nusselt number (-)

\( \rho \)  pitch length for LC formulation (m)

\( P \)  turbulent production term, pressure (Pa)

\( Pr \)  Prandtl number, \( \nu/\kappa \) (-)

\( P_w \)  mean wall pressure (Pa)

\( \dot{Q} \)  heat flux (W/m²)

\( R \)  two-point correlation tensor (-)

\( R \)  red light intensity, \( m_{pix} \times n_{pix} \) matrix

\( N \times N \) cross-correlation matrix

\( \mathcal{R} \)  radius of the wall curvature (m)

\( Ra \)  Rayleigh number, \( g\beta \delta TH^3/(\kappa \nu) \) (-)

\( Re \)  Reynolds number (-)

\( Re_h \)  Reynolds number for CF, \( U_b h/\nu \) (-)

\( Re_A \)  Reynolds number for BL flow, \( U_{\infty} \Lambda/\nu \) (-)

\( \mathcal{R}_E \)  coefficient of Eulerian time correlation (-)

\( s \)  density ratio, \( \varrho_p/\varrho \) (-)

\( S \)  saturation, \( m_{pix} \times n_{pix} \) matrix (raw data)

\( t \)  time coordinate (s)

\( T \)  turbulent transport term, fluid temperature (K)

\( u, v, v_1, w \)  components of the instantaneous fluid velocity (m/s)
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\( u_{\text{buoy}}, u_{\text{core}} \)  velocity scales for RB convection (m/s)
\( u_s \)  sedimentation velocity (m/s)
\( U \)  mean streamwise velocity (m/s)
\( U_b \)  bulk velocity (CF) (m/s)
\( U_\infty \)  free stream velocity (BL flow) (m/s)
\( \mathbf{U} \)  set of \( u \)-velocities in the \((x, z)\)-plane, \( N \times M \) matrix
\( \mathbf{V}_1 \)  set of \( v_1 \)-velocities in the \((y_1, z)\)-plane, \( N \times M \) matrix
\( \mathcal{U} \)  voltage (V)
\( V \)  mean wall-normal velocity component (m/s)
\( W \)  mean spanwise velocity component (m/s)
x, y, z  Eulerian Cartesian coordinates (m)
\( \mathbf{X} \)  set of spatio-temporal data, \( N \times M \) matrix
\( y_w \)  profile of the wavy bottom wall (m)

**Greek Symbols**

\( \alpha \)  amplitude to wavelength ratio, \( 2a/\Lambda \) (-)
diffusor angle (°)
heat transfer coefficient (W/(m²K))
\( \beta \)  angle between the \((y_1, z)\)-plane and the \( x \)-axis (°),
thermal expansion coefficient (K⁻¹),
phase angle in the Basset equation (rad)
\( \delta_{ij} \)  Kronecker delta
\( \delta_{LS} \)  light sheet thickness (m)
\( \delta_{tel} \)  telecentric range (m)
\( \delta_z \)  depth of field (m)
\( \epsilon_{ijk} \)  alternating tensor, third-order
\( \varepsilon \)  viscous dissipation term (m²s⁻³)
\( \eta \)  Kolmogorov length scale, \( (\nu^3/\varepsilon)^{1/4} \) (m),
amplitude ratio in the Basset equation
\( \vartheta \)  Celsius temperature (°C)
\( \kappa \)  thermal diffusivity (m²/s)
\( \lambda \)  light wavelength (m), thermal conductivity (W/(Km))
\( \lambda_n \)  eigenvalue of POD mode \( n \) (-)
\( \Lambda \)  wavelength of the sinusoidal profile at the bottom wall (m)
\( \Lambda_z \)  characteristic scale in the spanwise direction (m)
\( \mu \)  dynamic viscosity (kg/(ms))
Kinematic viscosity (m²/s)

Parameter for RGB→HSI conversion of color system, mpix x npix matrix

Parameter for RGB→HSI conversion of color system, mpx x npix matrix

Velocity-pressure gradient term

Eigenfunction, N x 1 matrix

Fluid density (kg/m³)

Particle density (kg/m³)

Standard deviation of {·}

Instantaneous stress tensor (second-order)

Time-averaged stress tensor (second-order)

Shear stress

Eulerian dissipation time scale (s)

Angle between light-sheet plane and direction of illumination (°)

Pressure to density ratio, p/ρ ((m/s)²)

Mean pressure to density ratio, P/ρ ((m/s)²)

Postprocessing paths for the velocity and the temperature field

Mean stream function (-)

Angular velocity (m/s), circular frequency (1/s)

Scalar field of spatio-temporal data (a.u.)

Abbreviations

BG: bright green color of TLC slurry
BL: boundary layer flow
BS: blue start of TLC slurry
CCD: charge coupled device
CL2: Craik-Leibovich type 2 instability
CF: channel flow facility
CT: color temperature
CW: continuous wave
DMA: direct memory access
DNS: direct numerical simulation
ERCOFTAC: European Research Community on Flow, Turbulence and Combustion
FFT{·}: fast Fourier transformation of {·}
\mathcal{F}\{·\}: Fourier transformation of {·}
FV: flow visualization study
IMSL: mathematics and statistics libraries, Visual Numerics, Inc.
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ISO</td>
<td>International Standardization Organisation</td>
</tr>
<tr>
<td>JPEG</td>
<td>Joint Photographic Expert Group</td>
</tr>
<tr>
<td>LC</td>
<td>liquid crystal</td>
</tr>
<tr>
<td>LDV</td>
<td>laser Doppler velocimetry</td>
</tr>
<tr>
<td>LES</td>
<td>large eddy simulation</td>
</tr>
<tr>
<td>LIF</td>
<td>laser induced fluorescence</td>
</tr>
<tr>
<td>LSE</td>
<td>linear stochastic estimation</td>
</tr>
<tr>
<td>MPEG</td>
<td>Moving Picture Expert Group</td>
</tr>
<tr>
<td>Nd:YAG</td>
<td>neodymium: yttrium aluminum garnet ($Y_3Al_5O_{12}$) crystal</td>
</tr>
<tr>
<td>$O{\cdot}$</td>
<td>order of ($\cdot$)</td>
</tr>
<tr>
<td>PIV</td>
<td>particle image velocimetry</td>
</tr>
<tr>
<td>PAL</td>
<td>phase-alternating line (video standard)</td>
</tr>
<tr>
<td>pix</td>
<td>pixel</td>
</tr>
<tr>
<td>PMMA</td>
<td>polymethyl methacrylate</td>
</tr>
<tr>
<td>POD</td>
<td>proper orthogonal decomposition</td>
</tr>
<tr>
<td>PVC</td>
<td>polyvinyl chloride</td>
</tr>
<tr>
<td>PTFE</td>
<td>polytetrafluoroethylene</td>
</tr>
<tr>
<td>RAM</td>
<td>random access memory</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds averaged Navier Stokes equations</td>
</tr>
<tr>
<td>RhB</td>
<td>fluorescent dye Rhodamine B</td>
</tr>
<tr>
<td>RMS</td>
<td>root mean square</td>
</tr>
<tr>
<td>RS</td>
<td>red start of TLC slurry</td>
</tr>
<tr>
<td>SEM</td>
<td>scanning electron microscope</td>
</tr>
<tr>
<td>T</td>
<td>tube flow</td>
</tr>
<tr>
<td>TIFF</td>
<td>Tag Image File Format</td>
</tr>
<tr>
<td>TLC</td>
<td>thermochromic liquid crystals</td>
</tr>
<tr>
<td>TTL</td>
<td>transistor transistor logic</td>
</tr>
<tr>
<td>VS</td>
<td>visible start of TLC slurry</td>
</tr>
<tr>
<td>W</td>
<td>working fluid water</td>
</tr>
<tr>
<td>WB</td>
<td>white balance</td>
</tr>
<tr>
<td>WT</td>
<td>wind tunnel facility</td>
</tr>
<tr>
<td>2-D</td>
<td>two-dimensional</td>
</tr>
<tr>
<td>3-D</td>
<td>three-dimensional</td>
</tr>
<tr>
<td>$\overline{\cdot}$</td>
<td>time average</td>
</tr>
<tr>
<td>$\cdot'$</td>
<td>fluctuation of ($\cdot$), $\cdot - \overline{\cdot}$</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>spatial average</td>
</tr>
<tr>
<td>$\langle\cdot,\cdot\rangle$</td>
<td>Euclidean inner product</td>
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</tbody>
</table>
Subscript and Superscript

- $h$: half channel height used as length scale (CF)
- $L$: lower wall
- $mean$: mean value
- $P$: particle properties
- $rms$: root-mean-square
- $U$: upper wall
- $\Lambda$: wavelength used as length scale (BL flow)
- $w$: value at the wall
Chapter 1

Introduction

Technically or geophysically relevant turbulent flows are often characterized by high Reynolds numbers, complex wall geometries and, in particular for chemical or process engineering applications, they are mostly connected to heat or mass transfer phenomena, or even involve multiple phases. A systematic analysis of such complex flow situations is either obtained through a reduction to well studied reference cases, such as a free-jet, a channel or a pipe flow, or through numerical approximations of the involved velocity and scalar fields. For the latter, numerical codes that solve the Reynolds averaged Navier-Stokes equations (RANS) or more recently large eddy simulations (LES), are commonly used. The quality of their prediction relies on the accuracy with which, depending on the chosen method, turbulence or subgrid-scale models correctly resample the physical characteristics of the considered turbulent transport problem. Such models have been mostly developed for isotropic turbulence and have been applied and validated against wall turbulence, and, more recently, rotating flows. Validation and an improved strategy for modeling complex flows motivates detailed experimental studies of more complex turbulent transport problems, which, at the same time, still have well-defined boundary conditions.

In the following, we start our considerations with the single-phase flow through a pipe of diameter $2h$ or a channel with parallel top and bottom walls that are separated by the same distance. The channel shall be of infinite depth. A fluid that flows through such a pipe or channel with a velocity exceeding a critical value creates turbulence. The mechanisms that produce and sustain wall turbulence are a continued focus of turbulence research. Many of the more recent developments center around the turbulence structure, and its dynamics (e.g. Robinson, 1991; Moin & Moser, 1989). In the last decade, new insights were possible mainly due to the availability of increasingly powerful computing resources.
An isothermal, developed channel flow can be characterized by the Reynolds number:

$$Re_h = \frac{U_b h}{\nu}, \quad (1.1)$$

where $U_b$ is the bulk velocity, and $\nu$ the kinematic viscosity. Since the inflow and outflow sections of the flow show periodicity, direct numerical simulations (DNS) of turbulent pipe or channel flow (Kim, Moin & Moser, 1987; Antonia et al, 1994) and laboratory measurements can be performed at similar conditions for moderate Reynolds numbers, see Fig. 1.1. Since for DNS, all scales of the flow are resolved and no modeling assumptions are invoked, we refer to it as a numerical experiment. Information on the mean and turbulence quantities, and on the turbulence structure can be accessed. For velocity measurements we exclusively consider time-resolving methods and distinguish between pointwise (1-D) techniques, e.g. hot-wire anemometry or laser Doppler velocimetry (LDV), two-dimensional (2-D) techniques, mostly laser sheet techniques, and particle tracking velocimetry as a 3-D method. The choice of the Reynolds number imposes requirements on the spatial and temporal resolution of the measurement techniques with respect to the smallest scales of the flow. On the numerical side, even for the relatively simple wall-geometry of a pipe or channel, the smallest scales of the flow cannot be fully resolved for Reynolds numbers $Re_h > 10^5$. Approximative solutions obtained from the Reynolds averaged Navier-Stokes equations, or from large eddy simulations are therefore required. Results from LES and RANS simulations provide estimates of the mean, and depending on the used turbulence model, higher-order turbulence quantities. Large-scale flow structures can only be obtained from LES, and RANS solutions do, in general, not provide structural information. The flow through a flat-walled channel or pipe is well-defined but, even if large $Re_h$ are considered, this configuration does not resemble certain characteristics of more complex, wall-bounded, and technically relevant flows, namely:

- a curved wall geometry,
- flow separation, and
- combined momentum and scalar (heat or mass) transport, or chemical reaction.

Figure 1.2 schematically connects the turbulent flow through a flat-walled pipe or channel at moderate $Re_h$ to technically relevant single-phase flow problems with chemical engineering applications. Note that only experiments and no numerical approximations are considered in this figure.
Figure 1.1: Schematic of laboratory and numerical capabilities for studying a developed, turbulent flow through a flat-walled channel (without scalar transport).

As previously discussed, when starting our considerations with the turbulent pipe or channel flow at moderate $Re_h$, one step towards more complex flows is to increase $Re_h$ for the same geometry. Measurements in hydrodynamically smooth pipes were obtained at a maximum Reynolds number (defined with the pipe radius) of $6.48 \times 10^6$ from Nikuradze (1932), and at Reynolds numbers of up to $7 \times 10^7$ in the recent Princeton superpipe experiment (e.g. Zagarola, 1996) and provide arguments for scaling considerations. Experimental limits are given by the spatial and temporal resolution of the employed techniques and by the roughness of the bounding walls (Barenblatt & Chorin, 1998). A second step would be to consider additional transport mechanisms. Therefore, the transport of a scalar $\xi$ (heat, $gc_pT$, or a species, $gc$), or chemical reactions would be added to the transport of momentum, $gu_i$; where $g$ is the fluid density, $c_p$, the specific heat capacity, $c$ is the concentration of the considered species, and $T$, and $u_i$ denote the fluid temperature and velocity.

A third step is the choice of more complex wall geometries. All three steps would be necessary to fully describe real-scale, single-phase technical flows around bluff bodies, in mixing devices, and in chemical reactors. However, the large complexity and the wide range of possible technical-scale applications of such flows instead suggest profound studies of less complex, but well defined reference problems.
Figure 1.2: Connection between laboratory and numerical experiments of turbulent, single-phase, wall-bounded flows as a function of the Reynolds number, the complexity of the wall geometry, and the involved transport processes.

The objective of this work is to advance, in a defined step, the existing knowledge on the isothermal flow through a flat-walled channel or pipe towards a more complex flow situation. At the same time, comparability with DNS studies and a connection to the turbulent flow through a flat-walled channel or pipe are sought. We therefore restrict ourselves to moderate Reynolds numbers. A more complex wall geometry is obtained by considering a wavy wall surface. This work focuses on structural information in a developed flow in a rectangular channel with a flat top-wall and a wavy bottom wall. The conditions considered herein are schematically shown in Fig. 1.2. Compared with the turbulent flow through a flat-walled channel or pipe, the wavyness of the wall and heat transfer through it add a defined degree of complexity: partial flow separation occurs, wave-induced large-scale structures and scalar transport of heat are present. Since moderate Reynolds numbers are considered and the boundary conditions at the inflow and outflow sides remain well-defined, detailed comparisons between numerical and laboratory experiments remain...
possible.

The thesis is organized as follows: chapter 2 provides the theoretical framework. From the governing equations for the mass, the momentum and a scalar, we introduce the budget equations for mean and turbulence quantities. Procedures for extracting large-scale structures from spatiotemporal data of a scalar field, namely the proper orthogonal decomposition (POD) method, are being discussed. Experimental techniques that are used for obtaining two-dimensional (2-D) velocity and temperature fields, as well as detailed information on the tracer materials, are summarized in chapter 3. The fluid velocity is obtained from PIV measurements. Liquid crystal thermometry (LCT) is used to determine the fluid temperature, where encapsulated liquid crystals provide the flow tracers. For developing the LCT technique, the image processing procedures and the influence of the optical configuration, i.e. the angle between the directions of illumination and observation, and the fluids refractive index, are systematically determined in a thermally stratified fluid layer. To demonstrate the applicability of combined PIV and LCT measurements to a transient flow situation, chapter 4 describes the reconstruction of instantaneous temperature and velocity fields for turbulent Rayleigh-Bénard convection in water. In chapter 5, we then consider the developed flow in a wide channel with a train of solid waves as the bottom wall. First, the velocity field is examined for an isothermal flow. Mean velocities are validated against literature data. Large sequences of 2-D instantaneous velocity fields are acquired in various planes of the flow. At laminar conditions, the spanwise variation of instantaneous streamwise velocities already reveals information on persistent flow structures. For turbulent flow, however, a statistical analysis is required to document the contributions from the different scales. A POD analysis is performed for the streamwise velocity fluctuations. In a second step, a constant heat-flux boundary condition is established for a section of the wavy bottom wall on which a resistively heated metal foil is appliqued. LCT is facilitated to obtain 2-D information on the temperature field.
Chapter 2

Theory

We herein provide some of the mathematical foundations for the quantitative description of turbulent velocity and scalar fields. In section 2.1, the governing equations for the mean and the turbulence quantities are introduced. Section 2.2 discusses means to describe the structure of turbulent flows. In particular, we focus on extracting large-scale structures from 2-D, spatio-temporal measurement data.

2.1 Governing Equations

In this section we introduce the governing equations for a single-phase fluid flow with scalar transport at constant fluid properties. The equations for the mean flow are given in section 2.1.1. Sections 2.1.2 and 2.1.3 provide the budget equations for the turbulence quantities. For more detailed information we refer to the textbooks of Batchelor (1977), Hinze (1975), Monin & Yaglom (1971), and Schlichting (1979).

2.1.1 Equations for the Mean Quantities

We consider a continuous fluid of density $\rho$, the dynamic viscosity, $\mu$, and the thermal conductivity $\lambda$. Spatial locations in the flow domain are described by the Cartesian coordinates $x$, $y$, and $z$, or, in the symbolic notation, $x_i$, with $i = 1, 2, 3$. The vector of the fluid velocity is denoted as $u = (u, v, w)$, and, synonymous, $u_i$, with $i = 1, 2, 3$. The fluid temperature is $T$. In the Eulerian description of the flow field the conservation of mass, $\rho$, momentum, $\rho u_i$, and thermal energy, $\rho c_p T$, are given by the equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

(2.1)
\[ \frac{\partial}{\partial t} q_{u_i} + \frac{\partial}{\partial x_j} q_{u_j} u_i = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \mu d_{ji} + F_i \]

\[ \frac{\partial}{\partial t} c_p T + \frac{\partial}{\partial x_j} q_{u_j} c_p T = \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T}{\partial x_j} \right) \]

where term \( F_i \) is an external force, and \( d_{ij} \) is the second-order stress tensor. Henceforth, we make the following assumptions regarding the fluid properties:

- incompressibility, i.e. \( \frac{\partial \rho}{\partial p} = 0 \)
- Newtonian fluid, i.e. constant dynamic and kinematic viscosity, \( \mu \), and \( \nu = \frac{\mu}{\rho} \)
- constant thermal conductivity, \( \lambda \)
- constant \( \rho \), except for the external force \( F_i \), where the density changes are proportional to the temperature, \( \rho = \rho_0 (1 - \beta (T - T_{ref})) \), i.e. the Boussinesq approximation; \( \beta \) is the thermal expansion coefficient, \( T \) is the local fluid temperature, and \( T_{ref} \) denotes a reference temperature
- dissipation of mechanical energy into heat is neglected

With the assumption of an incompressible Newtonian fluid, \( d_{ij} \) is symmetric and reads

\[ d_{ji} = d_{ij} = \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (2.2) \]

where \( \mu \) is the dynamic viscosity, \( p \) the static pressure. If Eq. (2.2) is substituted into the first equation of Eq. (2.1) and if the continuity equation is invoked, one obtains

\[ \frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = - \frac{\partial \varphi}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j x_j} + \frac{F_i}{\rho}, \quad (2.3) \]

the Navier-Stokes equation for an incompressible Newtonian fluid with constant properties. Here, \( \varphi = p/\rho \) is the ratio of pressure to density and \( \nu = \mu/\rho \) the kinematic viscosity. In the case that buoyancy is considered, as for Rayleigh Bénard convection in chapter 4, and for the flow over heated waves at low Reynolds numbers in chapter 5, the Boussinesq approximation is used in the external force term:

\[ \frac{F_i}{\rho} = g \left( 1 - \beta (T - T_{ref}) \right). \quad (2.4) \]
Decomposition of an instantaneous scalar quantity, $\xi(x,t)$, into a mean value, $\overline{\xi}(x)$, and a fluctuating part, $\xi'(x,t) = \xi(x,t) - \overline{\xi}(x)$, and taking the time-average leads to the budget equations for the average quantities. For the momentum budget, Eq. (2.3) leads to the Reynolds averaged Navier-Stokes equation (RANS)

$$\frac{DU_i}{Dt} = -\frac{\partial \Phi}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial U_i}{\partial x_j} - \overline{u_i u_j} \right) + \frac{F_i}{\rho}. \quad (2.5)$$

With the time-averaged stress tensor

$$\Sigma_{ij} = -\Phi \delta_{ij} + \nu \tilde{D}_{ij} - \overline{u'_i u'_j} \quad (2.6)$$

and $\Phi = P/\rho$, where $P$ is the mean static pressure, Eq. (2.5) reads

$$\frac{\partial U_i}{\partial t} = -\frac{\partial}{\partial x_j} U_j U_i + \frac{\partial}{\partial x_j} \Sigma_{ji} + \frac{F_i}{\rho}. \quad (2.7)$$

The mean deformation tensor is given by

$$\tilde{D}_{ij} = \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}. \quad (2.8)$$

The second-order tensor $-\overline{u'_i u'_j}$ is called Reynolds stress tensor. The elements $-\overline{u'_i u'_j}$ with $i \neq j$ are the Reynolds shear stresses. The diagonal contains normal stresses $-\overline{u'_i u'_i}$. In a similar fashion, the temperature $T$ is decomposed into a mean, $\overline{T}$, and a fluctuating part, $T'$. The corresponding budget equation is:

$$\frac{DT}{Dt} = -\frac{\partial T'}{\partial x_j} u_j + \kappa \frac{\partial^2 T}{\partial x_j^2}, \quad (2.9)$$

where $\kappa$ denotes the thermal diffusivity.

### 2.1.2 Reynolds Stress Budget

Multiplying Eq. (2.7) by $u_i$ and adding the same with interchanged indices gives the transport equation for the Reynolds stress

$$\frac{\partial \overline{u'_i u'_j}}{\partial t} = P_{ij} + C_{ij} + T_{ij} + \Pi_{ij} + \epsilon_{ij} + D_{ij} \quad (2.10)$$

with

$$P_{ij} = -\left( \overline{u'_i u'_k} \frac{\partial U_j}{\partial x_k} + \overline{u'_j u'_k} \frac{\partial U_i}{\partial x_k} \right)$$

$$C_{ij} = -U_k \frac{\partial}{\partial x_k} \overline{u'_i u'_j}$$
\[ T_{ij} = -\frac{\partial}{\partial x_k} u'_i u'_j u'_k \]

\[ \Pi_{ij} = -(u'_i \frac{\partial \varphi'}{\partial x_j} + u'_j \frac{\partial \varphi'}{\partial x_i}) \]

\[ \epsilon_{ij} = -2\nu \frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k} \]

\[ D_{ij} = \nu \frac{\partial^2}{\partial x_k \partial x_j} u'_i u'_j. \]

\( P_{ij} \) is the production term, \( C_{ij} \) the convection term and \( T_{ij} \) the turbulent transport term for the Reynolds stress. \( \Pi_{ij} \) stands for the velocity-pressure gradient, \( \epsilon_{ij} \) for the viscous dissipation and \( D_{ij} \) for the viscous diffusion term.

In case of \( i = j \) one obtains the budget for the turbulent kinetic energy \( k = \frac{q^2}{2} = \frac{(u'^2 + v'^2 + w'^2)}{2} \) from Eq. (2.10)

\[ \frac{\partial q^2}{\partial t} = P_k + C_k + T_k + \Pi_k + \epsilon_k + D_k \quad (2.11) \]

where

\[ P_k = -u'_i u'_j \frac{\partial U_i}{\partial x_j} \]

\[ C_k = -\frac{1}{2} U_j \frac{\partial}{\partial x_j} u'_i u'_i \]

\[ T_k = -\frac{\partial}{\partial x_j} u'_i u'_j u'_j \]

\[ \Pi_k = -u'_i \frac{\partial \varphi'}{\partial x_i} \]

\[ \epsilon_k = 2\nu \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_i}{\partial x_j} \]

\[ D_k = \nu \frac{\partial^2}{\partial x_j \partial x_j} u'_i u'_i. \]

\( P_k \) is the production, \( C_k \) the convection and \( T_k \) the transport of turbulent kinetic energy. \( \Pi_k \) stands for the pressure-gradient term. \( \epsilon_k \) and \( D_k \) stand for viscous dissipation and viscous diffusion of turbulent kinetic energy.

### 2.1.3 Budget for \( \frac{1}{2} \cdot T'^2 \)

The quantity \( \frac{1}{2} \cdot T'^2 \) is proportional to the (negative) contribution to the mean entropy arising from the presence of temperature fluctuations, and the destruction of fluctuation entropy by molecular conduction of heat is analogous to the dissipation of
fluctuation kinetic energy by viscous stresses (Townsend, 1980). A budget equation for $\frac{1}{2}T'^2$ is obtained by multiplying equation $\partial T/\partial t + u_j \partial T/\partial x_j = \kappa \partial^2 T/\partial x_j^2$ with the temperature fluctuation and taking the mean:

$$\frac{\partial}{\partial t} \left( \frac{1}{2} T'^2 \right) = -T'n'_{j} \frac{\partial T}{\partial x_j} - U_j \frac{\partial}{\partial x_j} \left( \frac{1}{2} T'^2 \right) - \frac{\partial}{\partial x_j} \left[ \frac{1}{2} T'^2 u'_j - \kappa \frac{\partial}{\partial x_j} \left( \frac{1}{2} T'^2 \right) \right]$$

$$- \kappa \left( \frac{\partial T'}{\partial x_j} \right)^2$$

where (i) is the production term, term (ii) denotes convection, terms (iii) are the contributions from turbulent and molecular transport, and (iv) is the dissipation term.

### 2.2 Structure and Transport

Similar to describing a flow or scalar field in the terms of its mean and turbulence quantities, structural information can be used for its characterization. A quantitative description of the flow structure and its dynamics is an active field of research in the turbulence community. Possible engineering applications are, e.g., structure-based turbulence models (Reynolds & Kassinos, 1995). In section 2.2.1, we define two-point correlation functions and distinguish structural information with regard to the length scales. Section 2.2.2 provides an overview of different means to extract large-scale structures. In section 2.3, the proper orthogonal decomposition (POD) method is discussed as one means of obtaining information on large-scale structures from spatio-temporal data.

#### 2.2.1 Scales and Correlation Function

We consider a scalar quantity $\xi(x,t)$, which may represent a velocity component, $u_i(x,t)$, a temperature, $T(x,t)$, or a concentration field, $c(x,t)$. Small-scale and large-scale structures are distinguished. However, we note that the term structure is not a precise definition, neither coherent structure or vortex are. The smallest spatial scales of a flow are connected to turbulent dissipation and are characterized by the Kolmogorov scale:

$$\eta = \left( \frac{\nu^3}{\epsilon_k} \right)^{1/4}$$

(2.13)
or by the Taylor microscale, $\lambda_u$, with:

$$\lambda_u^2 = \frac{\bar{u}_i'^2}{\left( \frac{\partial \bar{u}_i}{\partial x_i} \right)^2}$$

(2.14)

where $\epsilon_k$ is the viscous dissipation and $\nu$ is the kinematic viscosity. The role and dynamics of small-scale flow structures in wall turbulence is an active field of research and, in part, controversial discussions. For more detailed information on this field, we refer to a review article by Robinson et al. (1991) and a paper by Jeong et al. (1997). We note that the study of flow structures, and its dynamics, has always been closely linked to the available experimental and computational capabilities.

Two-point correlation functions were extensively used for connecting results from pointwise measurements, i.e. hot-wire anemometry or laser Doppler velocimetry (LDV), to the flow structure. In the following, we focus on characterizing large-scale structures. For the scalar quantity $\xi_i(x, t)$ we define the double correlation tensor, or a two-point correlation function, as (Townsend, 1980)

$$R_{\xi_i \xi_j}(x, x', t + \Delta t) = \xi_i'(x', t + \Delta t),$$

(2.15)

where $\xi_i'(x, t)$ stands for the instantaneous value of the $i$th component the fluctuating velocity, the temperature or the concentration at positions $x$, $x'$, and times $t$, $t + \Delta t$. We will restrict ourselves to the discussion of spatial variations, i.e. $R_{\xi_j}(x, x', 0)$. For conditions where the velocity field is homogeneous or weakly inhomogeneous in the streamwise direction, $x$, we follow the notation of Moin & Moser (1989) and write:

$$R_{\xi_i \xi_j}(\Delta x, y, y', \Delta z) = \langle \xi_i'(x, y, z, t) \xi_j'(x + \Delta x, y', z + \Delta z, t) \rangle_{xt},$$

(2.16)

where $\xi_i' (i = 1, 2, 3)$ represents instantaneous fluctuations of the velocity, the temperature, or the concentration in a homogeneous or weakly inhomogeneous streamwise, $x$, an inhomogeneous normal, $y$, and a homogeneous spanwise direction, $z$, and $\langle \cdot \rangle_{xt}$ denotes an ensemble average over the homogeneous or weakly inhomogeneous direction, here $x$, and the time coordinate, $t$. Since our measurements are two-dimensional, we can perform the correlations in the individual measuring planes. In the $(x, y)$-plane, where $x$ is homogeneous or weakly inhomogeneous, and $y$ is inhomogeneous, we would for example write:

$$R_{\xi_i \xi_j}(\Delta x, y, y') = \langle \xi_i'(x, y, t) \xi_j'(x + \Delta x, y', t) \rangle.$$  

(2.17)

From an autocorrelation of the velocity fluctuations $u'$, an integral length-scale is commonly defined (Townsend, 1980) as

$$\Lambda_u = \frac{1}{u'^2} \int_0^\infty R_{uu}(r, 0, 0) dr.$$  

(2.18)
2.2.2 Structural Information from Spatiotemporal Data

In the previous section, correlation functions were introduced as one means for obtaining information on the scales of a turbulent velocity or scalar field. Figure 2.1 from a recent paper by Adrian et al. (2000) illustrates the different means to decompose velocity or scalar fields in a space-time, \((\omega, k)\), spectrum. The Reynolds decomposition based on time-averaging, (a), was already introduced in section 2.1.1 and is a projection to \(\omega = 0\). In a similar fashion for spatial Reynolds averaging, (b), only \(k = 0\) is respected in the Fourier domain. Separation of scales that is performed purely in space is the approach of a LES decomposition, (c). The large-eddy field is defined as low-pass filtering the scalar quantity \(\xi\) as

\[
\overline{\xi}(x, t) = \int_D f(x, \hat{x})\xi(\hat{x}, t)d\hat{x}, \tag{2.19}
\]

where \(f\) is the filtering kernel and \(D\) the domain of the scalar field. We distinguish between homogeneous (\(f\) is shift-invariant) and inhomogeneous filters. Homogeneous filtering can be done e.g. by using a Fourier decomposition. One of the problems associated with homogeneous filtering in a turbulent field that has statistically inhomogeneous directions is that the character of the filter should change as a function of the inhomogeneous coordinate. The problem can be addressed by using the method of proper orthogonal decomposition (POD) to construct low-pass filters that are inhomogeneous in a single or multiple directions (Adrian et al., 2000). Since we consider wall-bounded flows, where one or more directions of the flow domain are inhomogeneous, we facilitate the POD method to define an inhomogeneous filter.
see section 2.3. The total field can be written as a sum of the large-scale and the remaining smaller-scale field:

$$\xi_i = \xi_{i,\text{LES}} + \xi_{i,\text{LES}}. \quad (2.20)$$

Relevant to POD techniques, conditional sampling methods (Antonia, 1981) are widely used to identify and describe large-scale structures in turbulent flows. An unconditional extraction technique that is closely related to the POD is linear stochastic estimation (LSE) (Adrian, 1977). LSE, as well as POD, use the cross-correlation tensor to extract information on the flow structure. Brereton (1992) shows that LSE can be treated as a weighted sum of an infinite number of POD modes. Therefore LSE provides a representation of large-scale structures in terms of a single characteristic pattern (Gordeyev, 1999).

The simplest method of decomposition is the Galilean transformation, (d). The scalar is represented as the sum of a constant, $\Xi_{i,c}$, plus the deviation $\xi_{i,c}$:

$$\xi_i = \Xi_{i,c} + \xi_{i,c}. \quad (2.21)$$

### 2.3 Proper Orthogonal Decomposition

The method of proper orthogonal decomposition (POD) for the analysis of spatio-temporal data is based on correlation functions that are decomposed into a set of optimal eigenfunctions. Lumley (1970) was the first to suggest the POD analysis for the quantitative analysis of flow fields. In the following, we give an overview over selected works that use POD analysis for obtaining structural information on turbulent flows:

- Payne (1966), Blakewell & Lumley (1967) were the first to apply the POD technique to the study of turbulent flows.

- Moin (1984) was the first to make use of the full correlation tensor, based on a large eddy simulation (LES).

- Herzog (1986) obtained the correlation tensor $R_{u_iu_j}$ from pointwise (hot film) measurements in a pipe for $i, j = 1, 2, 3$.

- Moin & Moser (1989) evaluated the full correlation tensor based on the DNS database for turbulent channel flow by Kim, Moin & Moser (1987) and identified the most energy-containing modes.
2.3. PROPER ORTHOGONAL DECOMPOSITION

• Liu, Hanratty & Adrian (1994) performed a 1-D POD analysis from autocorrelation PIV measurements in the \((x, y)\)-plane with a high spatial resolution for turbulent channel flow at the Reynolds numbers 5380 and 29900. They conclude that, provided the Reynolds number, the considered domain and the number of samples are sufficiently large, the eigenfunctions and spectra of eigenvalues are independent of \( \text{Re}_h \) at locations outside the viscous wall region.

• Liu, Hanratty & Adrian (2001) extended their analysis to a 2-D POD analysis from PIV measurements in the \((x, y)\)-plane for turbulent channel flow at \( \text{Re}_h = 5380 \) and 29900.

• Gordeyev (1999) and Gordeyev & Thomas (2000) used POD analysis to extract structural information for the velocity field in the similarity region of a planar jet.

2.3.1 Mathematical Background

In order to prevent a confusion of indices, we now denote the individual velocity components as \( u, v, \) and \( w \) instead of using the symbolic notation. Indices \( i, j = 1, \ldots, M \) henceforth characterize an eigenvalue or an individual scalar field, \( \xi(x, t) \).

Again, we consider spatio-temporal information given by the scalar quantity \( \xi(x, t) \) that could represent a velocity component, the instantaneous contribution to the Reynolds shear stress, the fluid temperature, or the concentration of a species. The mathematical background is essentially the Karhunen-Loève (KL) transform (Karhunen, 1946; Loève, 1945) with the idea to describe a given statistical ensemble through a minimal number of deterministic modes. We therefore view \( \xi(x, t) \) as a random generalized process that is represented by members of an ensemble. Maximizing the projection of \( \xi(x, t) \) into \( \Pi(x) \) leads to a homogeneous Fredholm integral equation of the second kind for the basis functions (Holmes, Lumley & Berkooz, 1996):

\[
\int R_{\xi\xi}(x, x') \Pi_i(x') dx' = \lambda_i \Pi_i(x), \quad (2.22)
\]

where \( R_{\xi\xi}(x, x') \) is the cross-correlation function between the points \( x \) and \( x' \) that was defined in section 2.2.1. The solution of Eq. (2.22) forms a complete set of square-integrable eigenfunctions, \( \Pi_i \), with associated eigenvalues \( \lambda_i \). Any ensemble of random generalized functions can be represented by a series of orthogonal functions with random coefficients:

\[
\xi(x, t) = \sum_{n=1}^{\infty} a_n(t) \Pi_i(x), \quad (2.23)
\]
where the coefficients $a_i$ are un-correlated, i.e. $\bar{a}_i\bar{a}_j = \delta_{ij}\lambda_j$, averaging is performed in the time domain, and $\delta_{ij}$ is the Kronecker delta. The cross-correlation tensor can be presented in terms of $\Pi_i$ itself:

$$R_{\xi\xi}(\mathbf{x}, \mathbf{x}') = \sum_{i=1}^{\infty} \lambda_i \Pi_i(\mathbf{x})\Pi_i^T(\mathbf{x}').$$ (2.24)

Finally for the energy follows:

$$E = \langle \langle \xi, \xi \rangle \rangle = \int_I R(\mathbf{x}, \mathbf{x}) d\mathbf{x} = \sum_{i=1}^{\infty} \lambda_i.$$ (2.25)

Thus the eigenvalues provide the energy contributions of the various eigenfunctions. Since the criterion for determining the POD modes was to maximize the energy per mode, the series in Eq. (2.24) converges as rapidly as possible. Equations (2.22) and (2.23) characterize the POD transform. For a more detailed account on the theory we refer to Holmes, Lumley & Berkooz (1996). For discrete measurement data, a vector form of POD is widely used. In this case, the physical signal, $\xi(\mathbf{x}, t)$ is replaced by an ensemble of finite-dimensional vectors, $\mathbf{X}_i$, the correlation function $R$ becomes a correlation matrix, $\mathbf{R}$, and the eigenfunctions are represented by eigenvectors. For practical reasons, Eq. (2.23) usually is represented by a finite set of $K$ functions:

$$\xi(\mathbf{x}, t) \approx \xi_K(\mathbf{x}, t) = \sum_{i=1}^{K} a_i(t) \Pi_i(\mathbf{x}).$$ (2.26)

The properties of optimality, symmetry and homogeneity of the POD are discussed by Berkooz, Holmes & Lumley (1993) in detail and shall not be repeated here. We only stress a particular kind of symmetry and say that a direction of the studied scalar field $\xi(\mathbf{x}, t)$ is homogeneous or translation invariant, if the correlation function only depends on the difference between the two coordinates. POD modes then correspond to Fourier modes.

### 2.3.2 Method of Snapshots

At discrete times $t_i$, with $i = 1, \ldots, M$, and at $1, \ldots, N$ discrete locations within a 2-D plane we now consider a set of spatio-temporal data as the following $N \times M$ matrix:

$$\mathbf{X} = \{\mathbf{X}_i\}_{i=1}^{M} = \begin{bmatrix} \xi_{11}, \xi_{12}, \ldots, \xi_{1M} \\ \xi_{21}, \xi_{22}, \ldots, \xi_{2M} \\ \vdots \\ \xi_{N1}, \xi_{N2}, \ldots, \xi_{NM} \end{bmatrix}$$ (2.27)
with the $N \times 1$ matrix $X_i = [\xi_1, \xi_2, \ldots, \xi_N]^T$. The theoretical calculations are approximated in actual computations from the given set of vectors. We first compute its mean:

$$\bar{X} = \frac{1}{M} \sum_{i=1}^{M} X_i.$$  (2.28)

By subtracting the mean, we obtain the fluctuations (or caricature vectors):

$$X'_i = X_i - \bar{X} \quad i = 1, \ldots, M.$$  (2.29)

For the $N \times N$ correlation matrix follows:

$$R = \frac{1}{M} \sum_{i=1}^{M} X'_i X'_i^T.$$  (2.30)

A direct solution of Eqs. (2.22) and (2.23) would require to invert the large matrix $R$, with $N = 1209 \ldots 4977$ in our case. Sirovich (1987) pointed out that the temporal correlation matrix will yield the same dominant spatial modes, while giving rise to a much smaller and computationally more tractable eigenproblem, the method of snapshots. We follow this approach. Instead of finding a spatial two-point correlation matrix, $R$, one computes a temporal correlation $M \times M$-matrix:

$$C = \frac{1}{M} X'X.$$  (2.31)

The transponse product of the matrix $C$ is evaluated using the IMSL subroutine MXTXF. The dimension of the matrix is therefore reduced from $N \times N$ to $M \times M$, with $M = \mathcal{O}(250)$ for our measurements. Since the covariance matrix is symmetric, its eigenvalues, $\lambda_i$, are nonnegative and its eigenvectors, $\phi_i, i = 1, \ldots, M$, form a complete orthogonal set. The eigenfunctions of the data are defined as:

$$\Pi^{[k]} = \sum_{i=1}^{M} \phi_i^{[k]} X'_i \quad k = 1, \ldots, M,$$  (2.32)

where $\phi_i^{[k]}$, the $i$-th component of the $k$-th eigenvector, is obtained by solving the eigenvalue problem $C[\phi^{[k]}] = \lambda[\phi^{[k]}]$ using the IMSL subroutine EVCRG that returns the eigenvalues and eigenvectors.

With Eq. (2.25), the total energy can be obtained through summation of the eigenvalues:

$$E = \sum_{i=1}^{M} \lambda_i,$$  (2.33)
and is, if the velocity component $u$ is considered, an approximation of the turbulence intensity $\langle \overline{u'^2} \rangle_{xy}$. Thus, the energy fraction based on the eigenfunction’s associated eigenvalue follows to:

$$\frac{E_k}{E} = \frac{\lambda_k}{E}. \tag{2.34}$$

Since the used IMSL subroutine returns the eigenvalues in a decreasing order of magnitude, the eigenfunctions are ordered from the most to the least energetic. Any sample field $X_j$ can finally be reconstructed using the eigenfunctions $\Pi_i$:

$$X_j = \overline{X} + \sum_{i=1}^{M} a_i \Pi_i, \tag{2.35}$$

where the coefficients are computed from a projection of the sample vector $X'_j$ onto the eigenfunction:

$$a_i = \left( \frac{X'_j \cdot \Pi_i}{\Pi_i \cdot \Pi_i} \right). \tag{2.36}$$

Using only the first $K$ ($K < M$) most energetic eigenfunctions, we can construct an approximation to the data

$$X_j \approx \overline{X} + \sum_{i=1}^{K} a_i \Pi_i. \tag{2.37}$$

The described approach is applicable to 1-D and 2-D spatio-temporal data. For the latter, field $\xi(x, t)$ can be inhomogeneous in both directions, and the two Cartesian coordinates in the 2-D domain are replaced by one single coordinate in the $N \times 1$ vector $X_i$. In chapters 4 and 5, a POD analysis will be used to obtain structural information in turbulent Rayleigh-Bénard convection and for the turbulent flow over a wavy wall.

### 2.3.3 Analysis of 2-D Data with one Homogeneous Direction

In the case that 2-D data with one homogeneous direction are considered for $\xi(x, t)$, the dimension of the $N \times N$ matrix in Eq. (2.22) can be reduced by taking advantage of the homogeneity condition. A Fourier expansion is therefore used in the homogeneous direction. We mention this approach for the sake of completeness and refer to a paper by Liu, Adrian & Hanratty (2001) for more detailed information.
Chapter 3
Experimental Techniques

In this chapter, we introduce the techniques for obtaining the fluid velocity and temperature in 2-D sections of the flow field. Digital particle image velocimetry (PIV) is used for the velocity measurements and discussed in section 3.1. Tracer particles that are suitable for qualitative or quantitative measurements in water flows are summarized with their physical properties. In section 3.2, we specify working conditions, measurement accuracy, and the range of applicability for liquid crystal thermometry (LCT) with regard to 2-D temperature measurements in liquids.

3.1 Velocity Measurements

The fluid velocity in a 2-D section through the flow field is transiently obtained by the PIV method. Comprehensive introductions to laser sheet techniques in general and PIV in particular can be found in Adrian (1991), Westerweel (1993), and Raffel et al. (1998).

Tracer particles are added to a continuous fluid. Ideally, they are of spherical shape, the same density as the working fluid, i.e. they are equally buoyant, and have a mean diameter that is small enough in order not to affect the smallest scales of the flow. The particles passing a sheet of laser light are illuminated and a camera records the light scattered at the particle surfaces as an intensity information. More detailed information on seeding particles for water flows can be found in section 3.1.1. In order to extract velocity information, images from two laser pulses that are separated by the temporal difference $\Delta t$ are recorded, either both on the same image or at two separate images. The first case is commonly used for analog recording techniques (photographic films), where the velocities (however not the velocity directions) can be obtained by locally auto-correlating the obtained and digitized images. The
latter is a popular method where digital cameras are applied. The 2-D velocity field (without an directional ambiguity) follows from locally cross-correlating two subsequent images. For details on the digital PIV technique it is referred to section 3.1.2.

3.1.1 Tracer Particles

Melling (1997) provides a comprehensive summary of different seeding particles for PIV applications. Here, we restrict ourselves to liquid working fluids, in particular to water. Important properties are the particle density, $g_P$, the diameter distribution, and (for transparent particles) the refractive index, $n_P$. Desirable properties are a particle density equal to the one of the continuous fluid, a narrow diameter distribution, and a large relative refractive index $n_{rel} = n_P/n$, where the refractive index of water for a wavelength of 589 nm is 1.333 at 20°C (Wagner et al., 1998). In contrast to several qualitative light sheet techniques (e.g. Thoroddsen & Bauer, 1999), isotropic particles are used in PIV. Table 3.1 provides a summary of popular tracer particles for PIV measurements in water concerning the particle density, $g_P$, the refractive index, $n_P$, the range of particle diameters, color, shape, and manufacturers. Figure 3.1 shows scanning electron microscope (SEM) images of isotropic or nearly isotropic tracer particles with mean diameters in the range 5...80 μm that are commonly used for water flows: Conifer pollens, glass beads, polyamide granulate, and hollow glass spheres. Figures 3.1 (e-f) and 3.2 show SEM images and the volume cumulative distribution of tracer particles that were used in this study. They are latex particles product source 15ETH ($d_{p,50} = 15.6$ μm) and source 30ETH ($d_{p,50} = 29.0$ μm) from Pharmacia Biotech and polyamide particles products PSP-20 ($d_{p,50} = 21.0$ μm) and product PSP-50 ($d_{p,50} = 49.9$ μm) from Dantec. The diameter distribution is obtained from a laser diffraction measurement (Sympatec Helos). Polyamide particles are advantageous since their density is very close to the one of water. However, their diameter distribution is – in comparison to the one of latex particles – broader.

Reflective flakes (Savas, 1985, Matisse & Gorman, 1984) are a prominent example for non-isotropic tracer particles. We used a rheoscopic fluid from Kalliroscope. The product contains mica flakes that are coated with titanium dioxide. The particle density is specified with 3.1 g/cm³. Figure 3.3 shows a SEM image of one reflective flake. When dispersed in a fluid and illuminated with a light sheet, flakes provide information on the structure of the flow. Two remarks need to be made: reflective flakes mainly reveal information on small-scale flow structures. Secondly,
Figure 3.1: Scanning electron microscope (SEM) images of seeding particles for water flows: (a) Conifer pollens, (b) glass beads, (c) broken polyamide, (d) hollow glass spheres, (e) latex particles (Pharmacia Biotech product source 30ETH), (f) polyamide particles (product Dantec PSP-50).
<table>
<thead>
<tr>
<th>Material</th>
<th>$\varrho_P$ (g/cm³)</th>
<th>$n_P$ (-)</th>
<th>$d_P$ (µm)</th>
<th>Shape</th>
<th>Color</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hollow glass spheres</td>
<td>1.1 ± 0.05</td>
<td>-</td>
<td>1 – 20</td>
<td>SP</td>
<td>T</td>
<td>Sphericell PQ 110P8CP01, Potter Industries</td>
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<tr>
<td></td>
<td>1.1 ± 0.05</td>
<td>-</td>
<td>1 – 20</td>
<td>SP</td>
<td>T</td>
<td>Topas PQ 110P8CP00</td>
</tr>
<tr>
<td></td>
<td>1.03</td>
<td>1.532</td>
<td>0 – 800</td>
<td>SP</td>
<td>W</td>
<td>Pliolite AC5, Goodyear</td>
</tr>
<tr>
<td></td>
<td>0.45 – 1.5</td>
<td>-</td>
<td>10 – 65</td>
<td>SP</td>
<td>T</td>
<td>Scotchlite, 3M/Canada</td>
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<tr>
<td>Pollens</td>
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<td>-</td>
<td>40 – 100</td>
<td>pollens</td>
<td>Y</td>
<td>Conifer, OFS, Edinburgh/UK</td>
</tr>
<tr>
<td>Glaspearls</td>
<td>2.5</td>
<td>1.48</td>
<td>0 – 60,</td>
<td>SP</td>
<td>T</td>
<td>Microbeads/CH</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>50 – 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Latex</td>
<td>-</td>
<td>-</td>
<td>15</td>
<td>SP, MD</td>
<td>W</td>
<td>Source 15ETH, Pharmacia Biotech/Sweden</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>30</td>
<td>SP, MD</td>
<td>W</td>
<td>Source 30ETH, Pharmacia Biotech/Sweden</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td></td>
<td>SP, MD</td>
<td>W/F</td>
<td>Microparticles GmbH, Berlin/D</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td></td>
<td>SP, MD</td>
<td>W/F</td>
<td>J. Katz, JHU, Baltimore/USA</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td></td>
<td>SP, MD</td>
<td>W/F</td>
<td>Bangs Laboratories, Fishers/USA</td>
</tr>
<tr>
<td>Polyamide</td>
<td>1.01</td>
<td>1.52</td>
<td>0 – 200</td>
<td>broken</td>
<td>T/W</td>
<td>Grilamid L20, EMS Chemie/CH</td>
</tr>
<tr>
<td>Reflective flakes</td>
<td>1.01</td>
<td>1.52</td>
<td>30 × 6 × 0.07</td>
<td>platelets</td>
<td>R</td>
<td>Kalliroscope Rheoscropic Fluid, Kalliroscope USA</td>
</tr>
</tbody>
</table>

F=fluorescent, MD=mono disperse, R=reflective, SP=spherical shape, T=transparent, W=white.
the obtained information is of qualitative nature.

Since our interest is in single-phase flows, the tracer particles are supposed to closely follow the motion of the continuous fluid they are dispersed in. We distinguish between two limits. At small fluid velocities, particle settling plays a role. For highly turbulent flows, the response of the particle to the velocity fluctuations of the fluid becomes important.

**Particle Settling**

Assuming stationary Stokes flow between the particle and the surrounding fluid, the influence of particle settling due to the gravitational acceleration $g$ results in a relative velocity:

$$ u_{rel} = u_P - u = \frac{d_{p}^2 g}{18 \nu} \cdot (s - 1), $$

(3.1)
where $s$ is the ratio of the particle density, $\varrho_P$, to the fluid density, $\varrho$, $d_p$ is the mean particle diameter, and $\nu$ the kinematic viscosity of the fluid.

**Relative Motion of Particles in a Turbulent Flow**

For a discussion of the relative motion of seeding particles in a continuous fluid we refer to Hjemfelt & Mockros (1966). They summarized studies by Lumley (1957), Friedlander (1957), and others, and made the following assumptions:

- homogeneous and stationary turbulence,
- spherical particles with a mean diameter $d_{p,50}$ smaller than the smallest eddy,
- Stokes flow, and
- a suspension sufficiently dilute, thus particle-particle interactions are neglected.

For these assumptions, Basset (1888) gives the equation of motion:

$$\frac{\pi d_p^3}{6} \left( \varrho_P + 0.5 \varrho \right) \frac{du_{rel}}{dt} = -3\pi \varrho \nu u_{rel} - 1.5 d_P^2 \varrho \sqrt{\pi \nu} \int_{t_0}^{t} \frac{du_{rel}/d\xi}{\sqrt{t - \xi}} d\xi$$

\text{(3.2)}
where $t$ is the time. Following Hinze (1971) the further assumptions $\partial u_i/\partial x_i \ll \nu/d_p^2$ and $u_i/d_P^2 \gg \partial^2 u_i/\partial x_i^2$ are used. Equation (3.2) then follows to

$$\frac{du_P}{dt} + a \cdot u_P + c \cdot \int_0^t \frac{du_P}{\sqrt{t-\xi}} d\xi = a \cdot u + b \frac{du}{dt} + c \cdot \int_0^t \frac{du}{\sqrt{t-\xi}} d\xi$$

with the abbreviations

$$a = \frac{18 \nu}{(s+0.5)d_P^2}$$

$$b = \frac{3}{2(s+0.5)}$$

$$c = \frac{9}{(s+0.5)^2 \sqrt{\nu/\pi}}.$$

For describing the relative motion between particle and fluid, two parameters, the amplitude ratio, $\eta$, and the phase angle, $\beta$, are used (Hjelmfelt & Mockros, 1966):

$$\eta = \sqrt{(1+f_1)^2 + f_2^2}$$

$$\beta = \tan^{-1} \left\{ \frac{f_2}{1 + f_1} \right\}.$$ 

Analytic expressions for terms $f_1(s,N_s)$ and $f_2(s,N_s)$ can be found in Hjelmfelt & Mockros (1966), with the circular frequency of the fluid/particle motion, $\omega$, the definition for the Stokes number is

$$N_s = \sqrt{\frac{\nu}{\omega d_P^2}}.$$ 

Figure 3.4 shows the amplitude ratio, $\eta$, and the phase angle, $\beta$, versus the Stokes number, $N_s$, for the considered spherical seeding particles of diameter $d_P$ and density $\rho_P$ in a turbulent water flow that is characterized by frequency $\omega$. For the flow through a flat-walled channel at conditions that are similar to the ones used in chapter 5, the highest frequency of velocity fluctuations from laser Doppler velocimetry measurements (e.g. Günther et al., 1998) is approximately 50 – 100 Hz. For the considered seeding particles with a maximum mean diameter, $d_P$, of 50 $\mu$m, Stokes numbers between $N_s = 2.8$ (50 Hz) and 2.0 (100 Hz) are obtained. The maximum density ratio for the considered seeding material is 1.2, see Table 3.1. At these conditions, the obtained amplitude ratio of 0.02% and phase angle of 0.1% are negligible. Note that this would even be true for considerably larger $s$.

### 3.1.2 Digital Particle Image Velocimetry

Digital particle image velocimetry (PIV) is used to determine the velocity fields in a 2-D plane of the flow. Comprehensive introductions to the digital PIV technique
Figure 3.4: Solution of the Basset equation. Amplitude ratio, $\eta$ (left axis), and phase angle, $\beta$ (right axis), versus the Stokes number, $N_s$, for spherical particles with the density ratios $s = 1.05, 1.1, 1.2, 1.5, 2,$ and $3$.


Figure 3.5 gives a schematic of the imaging set-up for digital PIV. We consider the flow field of interest to be seeded with tracer particles. Particles passing the light sheet of thickness $\delta_{LS}$ are illuminated. Particles located within the volume $AOV \times \delta_z$, where $AOV$ refers to the area of view and $\delta_z$ is the depth of field, are projected by a lens system to a CCD chip. Table 3.2 summarizes the cameras that are used in this work with regard to their dynamic ranges, the pixel resolution, and the frame rates. The optical system can be represented by an aberration-free, thin spherical lens with focal length $f$ and diameter $D_e$. Object distance $Z_0$ and image distance $z_0$ satisfy the geometrical law:

$$\frac{1}{Z_0} + \frac{1}{z_0} = \frac{1}{f} \quad (3.6)$$

where

$$M = \frac{z_0}{Z_0} \quad (3.7)$$

defines the image magnification. The particles in the object plane are illuminated by monochromatic or polychromatic light with a sheet of thickness $\delta_{LS}$. A laser provides
monochromatic light of wavelength $\lambda$, where a white light (halogen or metal-halid) source emits its intensity over a frequency band. Passive and active tracers are considered. The first have, given a uniform size and shape, homogeneous reflective properties in all regions of the flow. The latter, namely encapsulated thermochromic liquid crystals (TLC, see section 3.2.2) and fluorescent tracer particles, change their reflective properties in different regions of the flow, depending on the temperature, the concentration, or the pH value. We assume all particles to be in focus. For a given aperture, $f_\# = f/D_a$, the focal depth of the camera objective, $\delta_z$, is given by
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Megaplus ES 1.0, Kodak</th>
<th>SensiCam, PCO</th>
<th>DXC-9100P, Sony</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic range (bit)</td>
<td>8</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>Chip size</td>
<td>1&quot;</td>
<td>2/3&quot;</td>
<td>1/2&quot;</td>
</tr>
<tr>
<td>Number of CCD chips</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Pixel resol. ((m_{pix} \times n_{pix}))</td>
<td>1008 \times 1016</td>
<td>1376 \times 1040</td>
<td>782 \times 582</td>
</tr>
<tr>
<td>Max. frame rate (Hz)</td>
<td>30</td>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>Pixel size ((\mu m)^2)</td>
<td>9 \times 9</td>
<td>6.45 \times 6.45</td>
<td>8.2 \times 8.2</td>
</tr>
</tbody>
</table>

Table 3.2: Specifications of the employed progressive-scan CCD cameras.

(Adrian, 1991):

\[
\delta_z \doteq 4.88\lambda \left[ f_\#(1 + \frac{1}{M}) \right]^2 , \tag{3.8}
\]

where \(M\) is the magnification factor defined in Eq. (3.7). For diffraction limited imaging, the particle image diameter can be obtained (Adrian, 1991) from:

\[
d_e \doteq \sqrt{(Md_P)^2 + (2.44f_\#(M + 1)\lambda)^2} \tag{3.9}
\]

where \(d_P\) denotes the average particle diameter.

In Fig. 3.6, the different processing steps are shown schematically. So far, we mainly focused on image acquisition and formation at the camera chip. In the frame straddling mode, a sequence of dual image pairs is acquired with a temporal separation \(\Delta t\) at a given frame rate, \(f_{FR}\). Quantization of the image to an 8-bit or 12-bit format is done either by the camera or by the framegrabber board connecting the CCD camera with a PC. Image enhancement refers to techniques that correct for, e.g., non-uniform illuminantion of images depending on the location in the light sheet. We perform high-pass filtering with a window width of 128 pixels. Sub-image extraction, correlation, and interpolation are performed with the commercial software package Insight provided by TSI. The image plane consisting of \(m_{pix} \times n_{pix}\) square pixels is subdivided into \(m \times n\) square spots with an edge length of \(n_{spot}\) pixels. For each of the spots, a local cross-correlation is carried out. At a subpixel level, the displacement is obtained by fitting the peak of the correlation function with a Gaussian three-point estimator (Willert, 1989; Raffel et al. 1998). With the displacement and the known temporal separation of the two image frames, two velocity components are locally obtained as averages over the spot size.
3.2 Temperature Measurements

In this section we discuss the application of the measurement technique liquid crystal thermometry (LCT) to temperature measurements in transparent liquids, which is also described in Günther and Rudolf von Rohr (2002). An accurate temperature calibration of fluid-dispersed, encapsulated thermochromic liquid crystals\(^1\), is an important prerequisite for quantitative LCT measurements in fluid flows. TLC particles are subjected to uniform and linear temperature fields and illuminated with a sheet of white light. A digital camera records the color distribution reflected

\(^1\)We use the term “TLC particles” in the following.
by the particles. Within the image plane, the angular dependency of the color is eliminated by using a telecentric camera lens.

Section 3.2.1 compares LCT with other measuring techniques. Mechanical and optical properties of TLC particles are provided in section 3.2.2. Section 3.2.3 and 3.2.4 introduce the experimental set-up and the image processing procedures. The calibration and the influence of such optical parameters as the angle between the directions of illumination and observation, and the fluid refractive index are being discussed in sections 3.2.5 and 3.2.6. They provide design criteria for quantitative LCT measurements for a wide range of different flow situations, and for combining the technique with PIV for simultaneous temperature and velocity measurements.

3.2.1 Introduction

Figure 3.7: Operating conditions for the temperature measurement techniques liquid crystal thermometry (LCT) and light induced fluorescence (LIF) using an Rhodamine B (RhB) dye and the 488 nm Ar line.

Unlike for measuring 2-D velocity fields, few techniques are readily available that provide whole field information for the fluid temperature. Commonly used are LCT
and laser induced fluorescence (LIF). Figure 3.7 schematically illustrates the operating conditions for the two techniques. The effective temperature sensitive range, $\Delta T_{\text{meas}}$, is plotted versus the start temperature, $T_{\text{Start}}$. For the majority of LIF temperature measurements in transparent liquids, the dye Rhodamine B (RhB) is excited with the blue wavelength of an Ar laser (488 nm). Thus a single operating point is obtained with a start temperature of 15°C and a temperature sensitive range of 25°C (Sakakibara & Adrian, 1999). For LCT, formulations with starting temperatures between $-10^\circ\text{C}$ and $120^\circ\text{C}$ are commercially available. The largest available nominal temperature sensitive range is approximately $20^\circ\text{C}$. In this section, we will see that the effective temperature sensitive range, $\Delta T_{\text{meas}}$, depends on the angle $\phi$ between the directions of illumination and observation. For light-sheet applications, i.e. for $\phi = 90^\circ$, $\Delta T_{\text{meas}}$ is significantly smaller than the nominal value. The vertical line represents the conditions for the TLC formulation Hallcrest product BM/R35C20W that we use for our measurements. The flexibility in choosing the temperature sensitive range of the measurement system according to the temperature variations in the considered flow field is an advantage of the LCT technique since the measurement uncertainty of both methods, LIF and LCT, is a function of the total effective temperature sensitive range, $\Delta T_{\text{meas}}$.

In particular for problems of forced convection in water flows, similar to the one we will consider in chapter 5, the temperature variations in the fluid are moderate and LCT is therefore advantageous. LCT allows to select the temperature sensitive range as well as its starting temperature over a wide range through modifying the LC formulation. An additional benefit is the temperature calibrations insensitivity with regard to the overall intensity of the illuminating white light. Disadvantages are that the tracer material for LCT is much more expensive than standard fluorescent dyes, such as RhB, and that white light sheets are required for illumination, resulting in a larger light sheet thickness in comparison with laser light sheets that are commonly used for LIF or PIV. A two-color LIF technique, which overcomes the limitation of single-color LIF measurements, where local variations in the intensity of the illuminating laser light lead to ill-predicted temperatures, was recently suggested by Sakakibara & Adrian (1999). They use two fluorescent dyes with emission peaks at different wavelengths and with different temperature sensitivities and relate the fluid temperature to an ratio of the intensities observed close to the two emission peaks.
3.2.2 Encapsulated TLCs

![Diagram of cholesteric liquid crystal structure]

Figure 3.8: Cholesteric (or chiral nematic) liquid crystal structure. The temperature dependent pitch length, \( p(T) \), denotes the distance for which the rotation of vector \( \hat{r} \) is equal 360° (Hallcrest)

Liquid crystals describe an intermediate state between that of a crystalline solid and an isotropic liquid. When a solid is heated, it transforms, at its melting point, into an isotropic liquid. However, certain organic substances do not directly pass from a crystalline solid to an isotropic liquid, and vice versa, but adopt an intermediate structure in terms of their molecular order. They possess the mechanical properties of liquids (viscosity, surface tension) and the optical properties of crystalline solids (anisotropy to light, birefringence). Their ordering properties can be controlled by electric and magnetic fields. Of particular interest here are the color change and light-modifying properties of thermochromic cholesteric liquid crystals. The molecules are uniformly oriented within one plane and form a helical structure perpendicular to it, see Fig. 3.8. A characteristic parameter is the pitch length, \( p \), defining the vertical distance between the two closest planes with molecules of the same orientation. When the cholesteric mesophase is in the planar texture and \( p \) is of the order of the wavelength of visible light, 400 - 700 nm, due to interference, a
dominant color is reflected when the liquid crystals are subjected to white light. The pitch length, \( p \), is temperature dependent, and thus the dominant reflected color is. This mechanism is used as the measurement principle. The color is not only a function of the temperature, but also of the applied shear stress, a dependency that is eliminated by encapsulating the LC formulation. When suspended in a transparent liquid, the color change can therefore be connected to the local fluid temperature.

![Figure 3.9: Schematic for the illumination of encapsulated TLCs with white light.](image)

We consider an experimental configuration for the LCT technique, which allows the combination with PIV in order to provide simultaneous temperature and velocity measurements in transient flow situations. Figure 3.11 illustrates the two paths for reconstructing the velocity, \( \Phi_1 \), and the temperature field, \( \Phi_2 \). Previous applications of the LCT technique are reviewed, e.g., by Smith et al. (2001), Dabiri & Gharib (1991), and Kasagi et al. (1989). In most previous studies the particle image diameters were smaller than the size of one CCD pixel, which does not imply disadvantages for measuring fluid temperatures, but restricts the accuracy of simultaneously obtaining the velocity prediction through local crosscorrelation of subsequent digital images (Adrian, 1995).

For the measurements presented in this section, the digital particle images are represented by more than one pixel and therefore provide the spatial resolution
required to prevent peak locking. The section exclusively focuses on the temperature calibration in a thermally stratified fluid layer, a prerequisite for path $\Phi_2$. In section 3.2.3, we present an optical set-up, in which the angle between light sheet and camera axis, the working fluid, and the influence of the white light balance (WB) of the light source and the camera can be varied. Image processing and calibration procedures are described in sections 3.2.4 and 3.2.5. The influence of the optical configuration on the temperature calibration is discussed in section 3.2.6.

### 3.2.3 Experimental

Figure 3.12 shows a schematic of the experimental set-up. Measurements are conducted in a cylindrical, fluid-filled cavity. The rectangular bottom and top walls have the dimensions $300 \text{ mm} \times 150 \text{ mm}$. The horizontal walls are separated by 24 mm in the vertical direction and consist of two parallel, black-anodized aluminum bodies. The wall temperatures – $T_L$ at the bottom, and $T_U$ at the top wall – are adjusted by two independent, constant-temperature, water baths. The transparent cylinder is located between the walls, has an inner diameter of 100 mm, a wall thickness of 5 mm, and is made of PMMA (polymethyl methacrylate, $\lambda = 0.21 \text{ W/mK}$) material.
A rectangular section of the cylinder, located adjacent to the camera, is replaced by a flat window of the same material, see Fig. 3.13. The entire box is insulated by a 50 mm thermal shield. Only during image acquisition, the insulation is partially removed in order to provide optical access. A fraction of approximately 0.03 vol.% of TLC slurry, Hallcrest product BM/R35C20W, is added to the transparent working fluid in the cylindrical cavity, either de-ionized water, or glycerol. In order to apply the technique to measurements in turbulent flow situations, encapsulated TLCs with a relatively small particle diameter of 19.75 μm and a standard deviation of 7.18 μm were used. The particle size was measured in aqueous suspension using laser light diffraction (Sympatec Helios). The thermal diffusivity, $\alpha_P$, of encapsulated TLCs is $5.8 \cdot 10^{-8}$ m$^2$/s (Dabiri & Gharib, 1991). According to Hallcrest (1999), a typical value of the overall particle density, $\rho_P$, is 1020 kg/m$^3$, where the density of the TLC core varies depending on the composition between 970 kg/m$^3$
and 1010 kg/m³. The wall material consisting of gelatin, gum arabic, and water, has a density of approximately 1100 kg/m³. From these quantities a thermal response time of 1 ms can be estimated. In Table 3.3, the area of view (AOV), mean particle diameter, response time, magnification factor, and the working fluid for the present configuration are summarized and compared with previous heat transfer studies in an impinging jet (Dabiri & Gharib, 1991), around a heated cylinder (Park, 1998), and in a cubic cavity that is heated from below (Lutjen et al., 1999).

A magnetic stirrer is located inside the cavity and is used to mix the fluid during measurements in uniform temperature fields, and prior to measurements in a thermally stratified layer to prevent the particles from settling. Inside the cavity, the temperature is measured with a reference resistance thermometer (Pt 100 Ω), which was calibrated according to ITS-90 and has a measurement uncertainty of ±0.03°C for the considered temperature range. In addition, a thermocouple allows the local temperature to be measured at variable distances y from the bottom wall.

Figure 3.12: Experimental configuration consisting of the fluid-filled cavity between an isothermal bottom ($T_L$) and top surface ($T_U$), the light sheet plane, and the CCD camera with a telecentric lens. The temperature profile is shown for a thermally stratified layer.
Figure 3.13: Optical configuration with an insulated cylindrical cavity. The angle $\phi$ between the light sheet plane and the axis of the telecentric objective is variable (distances in mm).

A 90 W metal halid light source (Volpi model Intralux IWL500e) with the color temperature of 5500 K ± 900 K is used. The light is guided through a fiberoptic lineconverter (Volpi model VVLC), where a continuous sheet of white light is produced. A cylindrical lens and an aperture adjust the light sheet thickness, to
approximately \( \delta_{LS} = 1 \text{ mm} \). The light sheet illuminates the TLC particles in a vertical plane between the two walls. A progressive scan 3-chip video camera (Sony model DXC-9100P) is positioned with its optical axis perpendicular \((\phi = 90^\circ)\) to the light sheet plane. The white balance (WB) of the camera can be altered manually. To exclude the dependency of the recorded color information on the viewing angle within the AOV, the camera is equipped with a telecentric objective from Sill Imaging/Germany, which is focused on the light sheet, see Fig. 3.13. The objective has a magnification \( M \) of 0.19, an AOV of 33 mm \( \times \) 25 mm, and a working distance of 300 mm.

Spherical TLC particles, located in the light sheet plane, are illuminated with white light and a fraction of it is scattered at particle surfaces. In contrast to a standard PIV configuration, the influence of scattered light, which - for given particle properties - depends on the fluids refractive index, \( n \), is unwanted. The desired information is a band of dominant wavelengths, which is, as a function of the local fluid temperature, reflected from the TLC particles. The three CCD chips of the camera \((768 \times 572 \text{ pixels})\) transiently record separate color bands of the reflected light as analog RGB information. The size of one square pixel is \(8.3 \times 8.3 \text{ \(\mu\)m}^2\). With a 4 MB dualported memory framegrabber board (ITI model IC4-RGB), the images are digitized and downloaded to the RAM \((128 \text{ MB})\) of a personal computer, where the commercial software package Optimas 6.5 (Media Cybernetics, Inc.) is used for image acquisition.

### 3.2.4 Image Processing and Optical Resolution

The downloaded images are subsequently stored as RGB files \((3 \times 8 \text{ bit})\) in the .tif format at a local harddisk. Even though postprocessing starts with the digitized images, it is important to note that the recorded RGB information not only depends on the angle of illumination but also on the WB between the three chips.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LCT Dabiri &amp; Gharib</th>
<th>PIV/T Park</th>
<th>LCT Lutjen et al.</th>
<th>Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>( AOV ) (mm(^2))</td>
<td>( 163 \times 120 )</td>
<td>( 60 \times 50 )</td>
<td>( \approx 76 \times 57 )</td>
<td>( 35 \times 25 )</td>
</tr>
<tr>
<td>( d_p ) ((\mu\m))</td>
<td>150</td>
<td>40</td>
<td>40</td>
<td>19.75</td>
</tr>
<tr>
<td>( t_{resp} ) (ms)</td>
<td>60</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>( M ) (-)</td>
<td>( \approx 0.04 )</td>
<td>0.11</td>
<td>( \approx 0.084 )</td>
<td>0.19</td>
</tr>
<tr>
<td>Fluid</td>
<td>water</td>
<td>water</td>
<td>glycerol</td>
<td>water/glycerol</td>
</tr>
</tbody>
</table>

Table 3.3: Comparison of experimental parameters for different LCT measurements.
Only a fixed WB enables to obtain quantitative results from the recorded RGB intensities. For a known temperature calibration, such information can then be used for simultaneously obtaining temperature and velocity in transient flows. Temperature and velocity are extracted with two post-processing steps, $\Phi_1$ and $\Phi_2$, from the transiently recorded RGB images, see Fig. 3.11.

Provided that the time-separation between two subsequent RGB images is sufficiently small, which is the case at small fluid velocities, e.g. for Rayleigh-Bénard convection at moderate Rayleigh numbers (chapter 4), the reconstruction of the velocity fields, path $\Phi_1$ in Fig. 3.11, is done with the cross-correlation PIV technique as described in section 3.1.2. For higher velocities, e.g. for the flow over heated waves (chapter 5), the camera frame rate becomes insufficient and a pulsed light source is required for obtaining the velocities. We will use a conventional PIV configuration with a pulsed laser for velocity measurements in this case.

We briefly demonstrate that the particle image diameter in the considered configuration is sufficient for combined LCT and PIV measurements in transient flows. The intensities, $I_{ij}$, of two subsequent images, which are recorded at the times $t_\xi$ and $t_{\xi+1}$, are locally cross-correlated. Standard PIV algorithms use three-point estimators to fit the discrete correlation data, and to predict the displacement at a sub-pixel level. The uncertainty of the velocity prediction depends on the particle image diameter, $d_e$ (Raffel et al., 1998), and can be reduced if a particle image is represented by more than one pixel on the camera chip. In the following, we estimate the average number of pixels, by which a TLC particle is represented on the chip. For diffraction limited imaging, the particle image diameter is obtained from Eq. (3.9) for a given magnification $M$, average particle diameter, $d_p$, and aperture of the camera objective, $f_\#$. In Table 3.4, the calculated particle image diameters as well as the depth of field, that is obtained from Eq. (3.8), are listed for wavelengths between $\lambda_1 = 470$ nm (blue), and $\lambda_2 = 630$ nm (red) as a function of $f_\#$. Satisfying $d_e/d_r > 1$ and using the full telecentric range, $\delta_{tel}$, of the objective, 15 mm, therefore implies a minimum $f_\#$ of 11. This is not a restriction regarding the spatial resolution since we consider uniform or linear temperature fields, which are homogeneous in the direction of the camera axis, in this section. For transient flows however, higher energy (pulsed) light sources might be required, depending on the characteristic velocity of the flow.

We now discuss how temperature fields are related to the RGB images, path $\Phi_2$ in Fig. 3.11. As commonly done in the literature we perform a linear transformation for the red, $R_{ij}$, the green, $G_{ij}$, and the blue intensities, $B_{ij}$, in the image plane with
Table 3.4: Particle image diameter and depth of field for diffraction limited imaging.

<table>
<thead>
<tr>
<th>$f_#$</th>
<th>$d_e/d_r$ (470 – 630 nm)</th>
<th>$\delta_z$ (470 – 630 nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>0.52 – 0.57</td>
<td>0.23 – 0.31 mm</td>
</tr>
<tr>
<td>2.8</td>
<td>0.64 – 0.77</td>
<td>0.72 – 0.96 mm</td>
</tr>
<tr>
<td>4.0</td>
<td>0.80 – 0.99</td>
<td>1.5 – 2.0 mm</td>
</tr>
<tr>
<td>5.6</td>
<td>1.0 – 1.3</td>
<td>2.9 – 3.9 mm</td>
</tr>
<tr>
<td>8.0</td>
<td>1.4 – 1.8</td>
<td>5.9 – 7.9 mm</td>
</tr>
<tr>
<td>11</td>
<td>1.9 – 2.5</td>
<td>11 – 15 mm</td>
</tr>
<tr>
<td>16</td>
<td>2.7 – 3.6</td>
<td>23 – 31 mm</td>
</tr>
</tbody>
</table>

$i : 1, \ldots, 768$ and $j : 1, \ldots, 572$ (Dabiri & Gharib, 1991):

$$
\begin{bmatrix}
\xi_{1,ij} \\
\xi_{2,ij} \\
I_{ij}
\end{bmatrix} =
\begin{bmatrix}
\frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\
0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\
\frac{1}{3} & 1/3 & 1/3
\end{bmatrix} \cdot
\begin{bmatrix}
R_{ij} \\
G_{ij} \\
B_{ij}
\end{bmatrix}
$$

(3.10)

where the third component of the vector on the left represents the local intensity $I_{ij}$. The local hue, $H_{ij}$, and the saturation, $S_{ij}$, are obtained as follows:

$$
H_{ij} = \begin{cases}
\frac{\pi}{2} - \tan^{-1} (\frac{\xi_{1,ij}}{\xi_{2,ij}}) & : B_{ij} < G_{ij} \\
\frac{\pi}{2} + \tan^{-1} (\frac{\xi_{1,ij}}{\xi_{2,ij}}) & : B_{ij} \geq G_{ij}
\end{cases}
$$

(3.11)

$$
S_{ij} = \sqrt{\xi_{1,ij}^2 + \xi_{2,ij}^2}
$$

(3.12)

The performed 8-bit RGB→HSI conversion is schematically illustrated in Fig. 3.14.

In the considered color system, $H_{ij}$ defines an angle in the intervall $[0,2\pi]$. Three criteria are defined:

- $I_{ij} > \langle I \rangle_{ij}$, $\langle I \rangle_{ij}$ denotes an average over all pixels
- $R_{ij} \lor G_{ij} \lor B_{ij} < 255$
- $S_{ij} > S_{\text{min}}$

The first criterion excludes low-intensity background light (underflow) and ensures that the color information is obtained from TLC particles only. The second criterion disregards saturated pixels (overflow). The purpose of the third criterion is to further minimize the contribution of scattered white light. Only pixels which fulfill all criteria are considered. From the validated hues, $H_{ij}$, local averages, $H_{kl}$, are calculated. Herein, we consider $k : 1, \ldots, m$ and $l : 1, \ldots, n$ with $m = n = 40$, and a spot size of $32 \times 32$ pixels. Provided a relation, $T_{kl} = f (H_{kl})$, the fluid temperatures can be locally obtained. The calibration curve, the measurement accuracy, and the
3.2. Temperature Measurements

3.2.5 Calibration

A number of papers describe the temperature calibration for encapsulated TLCs (e.g. Hay & Hollingsworth, 1998). However, mostly encapsulated TLCs are applied on a constant temperature surface, where the location of the camera axis is close to the direction of illumination (small $\phi$). Behle et al. (1996) studied the dependency of the calibration of liquid crystal sheets on the angle between the direction of illumination and observation. For the calibration of fluid-dispersed TLCs with $\phi = 90^\circ$, a thermally stratified fluid layer can be used (Günther & Rudolf von Rohr, 2002a) where the bottom wall of the cavity is cooled, the top wall is heated, and the side walls are thermally insulated.

The TLC slurry is dispersed in the working fluid. To ensure that particles are homogeneously distributed, to avoid particle agglomeration and the formation of air bubbles at wall surfaces, the suspension of TLC particles in the working fluid is heated up to 70°C and then subjected to an ultrasound bath prior to filling it into the cavity.

At the low end of the temperature sensitive range, TLC particles reflect a red color. With increasing temperatures green, followed by blue, become the domi-
nant colors. Since the color response depends on the illumination angle (Kasagi et al., 1989) and the directions of illumination and observation do not coincide for light sheet applications, the “effective” temperature sensitive range is significantly reduced compared to its nominal value ($\phi = 0^\circ$) of approximately 20°C. In this section, and for the application of the technique in chapters 4 and 5, we use the angle $\phi = 90^\circ$ for the calibration, the configuration most relevant to light sheet measurements. The angular dependency of the results is discussed in section 3.2.6. The metal halid light source is used and the calibration is carried out in a uniform and a linear (thermally stratified layer) temperature field.

**Thermally Stratified Layer**

A thermally stratified layer is desired, since the calibration for the whole temperature range can be reduced to one single measurement. However, this is only feasible, if (a) the vertical temperature distribution in the fluid is linear, i.e. the side walls are well insulated, and, (b), the TLC particles have not settled before the linear profile is developed.

a. **Linearity.** After the horizontal aluminum walls reach the constant temperatures $T_L$ and $T_U$, the suspension of TLC particles and de-ionized water is homogeneously distributed in the cavity with a magnetic stirrer. After having operated the stirrer, the linear temperature profile develops through thermal conduction. Assuming constant fluid properties, the necessary time $t$ to develop the linear profile can be estimated by solving the conduction equation,

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2}$$

for the initial condition $T(t = 0, y/h) = 0.5 \cdot (T_L + T_U)$ and the boundary conditions $T(t, y/h = 0) = T_L$ and $T(t, y/h = 1) = T_U$. At different instants in time, the numerically obtained temperature profiles are plotted in Fig. 3.15 for a 25 mm layer of water. Only the lower half is shown, since the deviation of the obtained profiles from a straight line is a point-symmetric function with respect to the layer center. Figure 3.16 shows the relative deviation between the transient fluid temperature, $T(t, y/h = 0.25)$, and the developed linear profile, $T(t \to \infty, y/h = 0.25)$, versus time. The thermal diffusivities, $\kappa$, are $0.150 \cdot 10^{-6}$ m$^2$/s for water, and $0.914 \cdot 10^{-7}$ m$^2$/s for glycerol. Figure 3.17 shows the temperature distribution that is measured with a thermocouple at steady state conditions at different vertical positions in the insulated apparatus in comparison with the ideal profile (dashed line).
A relatively large temperature difference of 10 K is chosen to exclude uncertainties that are connected to the measurement uncertainty of the calibrated thermocouple. The agreement between the measurement points and the ideal profile in Fig. 3.17 confirms a linear temperature distribution across the fluid layer.

b. **Particle settling.** The influence of particle sedimentation is shown in the same diagram. Assuming Stokes flow, the effect of settling can be obtained from Eq. (3.1) and gives the linear relation

$$\frac{\Delta y(i)}{h} = \frac{u_{rel} \cdot \Delta t}{h} = \frac{d_p^2 g \cdot t}{18 \nu h} \cdot \left( \frac{\rho_p}{\rho} - 1 \right),$$

(3.14)

where $\rho_p$ is the overall particle density, 1020 kg/m$^3$ (Hallcrest 1999), $d_p$ is the mean particle diameter, and $g$ denotes the gravitational acceleration. The density, $\rho$, and the kinematic viscosity, $\nu$, of the fluids are summarized in Table 4.1. Positive $\Delta y/h$ indicate a downward, negative values an upward motion of the particles. For the
Figure 3.16: Left axis: relative difference between the developing temperature profile and the linear limit versus time at \( y/h = 0.25 \) for the fluids water (solid line) and glycerol (dashed line). Right axis: influence of sedimentation of particles with the overall density of 1020 kg/m\(^3\) in the two fluids.

Water layer (solid lines), it takes about 6 minutes until the deviation from the linear profile is smaller than 1\% of the temperature drop across the layer, resulting in a distance \( \Delta y/h \) equal to 11\% of the layer depth. The corresponding time for glycerol (dashed lines) is 10 minutes, during which the TLC particles only pass 0.2\% of the layer depth for the considered particle density.

In Figure 3.18 the line averages of the hue, \( \langle H_{kl} \rangle_k \), the saturation, \( \langle S_{kl} \rangle_k \), and the intensity, \( \langle I_{kl} \rangle_k \), are plotted versus the linear temperature profile. The temperature measurements at the bottom wall, 33.61°C, and at the top wall, 35.37°C,
3.2. TEMPERATURE MEASUREMENTS

Figure 3.17: Dimensionless temperature profile measured with a thermocouple in a thermally stratified layer with $\Delta T = 10$ K.

<table>
<thead>
<tr>
<th>Fluid property</th>
<th>Water</th>
<th>Glycerol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematic viscosity $\nu$ (m$^2$/s)</td>
<td>$0.724 \cdot 10^{-6}$</td>
<td>$0.371 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>Density $\rho$ (kg/m$^3$)</td>
<td>994</td>
<td>1250</td>
</tr>
<tr>
<td>Thermal conductivity $\lambda$ (W/K m)</td>
<td>0.623</td>
<td>0.281</td>
</tr>
<tr>
<td>Thermal diffusivity $\kappa$ (m$^2$/s)</td>
<td>$0.150 \cdot 10^{-6}$</td>
<td>$0.914 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>Prandtl number $Pr$</td>
<td>4.82</td>
<td>4060</td>
</tr>
<tr>
<td>Vol. thermal expansion coefficient $\beta$ (1/K)</td>
<td>$3.46 \cdot 10^{-4}$</td>
<td>$6.10 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>Heat capacity $c_p$ (kJ/kg K)</td>
<td>4.18</td>
<td>2.45</td>
</tr>
<tr>
<td>Refractive index $n$ (-)</td>
<td>1.33</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Table 3.5: Fluid properties of water and glycerol at $T_b = 35^\circ$C (Wagner & Kruse 1998, D’Ans et al., 1977).

are obtained from resistance thermometer readings. The uncertainties for $\langle H_{kl}\rangle_k$ are plotted for a confidence interval of 95.4% (twice the standard deviation). Small hues correspond to low fluid temperatures. The nonlinear character of the calibration should be noted. Calibration in a linear temperature field allows to accurately resolve the steep gradient at the beginning of the temperature sensitive range. The saturation is large at low fluid temperatures and decreases for increasing hues. One important result is that for hue values within the temperature sensitive range, the relation $T \leftrightarrow H$ is unequivocal. Hue values outside the effective temperature sensitive
Figure 3.18: Hue, saturation, and intensity vs. the fluid temperature from a measurement in a thermally stratified layer of de-ionized water for $\phi = 90^\circ$ (confidence interval: 95.4%). Measurements from constant temperature fields are included.

range are almost constant and do not impose ill-predicted temperatures. Temperature regions above or below the sensitive range can be reliably detected because of the significantly reduced intensities. The data points in Fig 3.18 can be represented by the polynomial:

$$T_{poly} = \sum_{\eta=0}^{8} C_{\eta} \cdot (H_{kl})^\eta_k.$$  \hspace{1cm} (3.15)

Using this explicit expression, the fluid temperature can be calculated from locally obtained hues. Coefficients are summarized in Table 3.6. Conditions of applicability are:

- the TLC formulation BM/R35C20W is considered,
- a digital camera with NTSC color filters and a WB of 5600K is used,
- the color temperature of the light source matches the one of the cameras WB,


Table 3.6: Coefficients $C_n$ of the eighth-order polynomial.

<table>
<thead>
<tr>
<th>$C_0$</th>
<th>31.9194</th>
<th>$C_3$</th>
<th>33.6841</th>
<th>$C_6$</th>
<th>-2.6206</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>11.1639</td>
<td>$C_4$</td>
<td>-24.4564</td>
<td>$C_7$</td>
<td>0.350746</td>
</tr>
<tr>
<td>$C_2$</td>
<td>-26.6679</td>
<td>$C_5$</td>
<td>10.4994</td>
<td>$C_8$</td>
<td>-0.0194357</td>
</tr>
</tbody>
</table>

- the angle $\phi$ is equal to 90°C,
- the temperature interval $T$ : $[T_{1,W} = 33.77^\circ C, T_{2,W} = 35.09^\circ C]$.

For a confidence interval of 95.4%, the uncertainty of the hue angle, $\delta H_t$, can be calculated from the data plotted in Fig. 3.18 (Moffat, 1988):

$$\delta H_t = 2 \left[ \frac{1}{m-1} \sum_{k=1}^{m} \left( H_{kl} - \langle H_{kl} \rangle_k \right) \right]^{0.5},$$

where $\langle H_{kl} \rangle_k$ denotes a line-average ($x$-direction). If the polynomial in Eq. (3.15) is used, the uncertainty of the temperature prediction follows to:

$$\delta T = \frac{dT_{poly}}{dH} \delta H_t.$$ (3.17)

The result is shown in Fig. 3.19, an inverse plot of Fig. 3.18. Between the temperature levels $T_{1,W}$ and $T_{2,W}$, Eq. (3.15) is included in the plot, together with the residuals for the polynomial regression.

We now re-arrange Eq. (3.1) and use the measurements in a thermally stratified layer of de-ionized water to estimate the particle density, $\rho_P$, from transient images of the settling particles:

$$\rho_P = \rho \cdot \left( 1 + \frac{18 u_s \nu}{d_P g} \right).$$ (3.18)

Images with a temporal separation of 4 s are locally cross-correlated (spot size $32 \times 32$, $m = n = 40$). From a cross-correlation, an average of the settling velocity, $u_s$, of $2.13 \cdot 10^{-5}$ m/s $\pm 1.54 \cdot 10^{-6}$ m/s is obtained within the AOV. For the mean temperature in the image plane, 34.46°C, the kinematic viscosity of water is $0.7473 \cdot 10^{-6} \pm 0.23 \cdot 10^{-8}$ m$^2$/s. With Eq. (3.18) and the gravitational acceleration, $g$, the particle density can be estimated to 1072 kg/m$^3$. The different influences are:

- the standard deviation of the measured settling velocity, $\Delta \rho_P/\rho_P |_u = 0.02\%$,
- the viscosity variation in the fluid layer, $\Delta \rho_P/\rho_P |_\nu = 0.50\%$,
- the non-uniform particle size distribution, $\Delta \rho_P/\rho_P |_{d_p} = 5.51\%$. 
For the considered sample it can be concluded that the measured average particle density was found to be larger than the value of 1020 kg/m³, which is commonly referred to. The relatively broad diameter distribution was found to be the most significant influence for estimating the particle density from sedimentation measurements.

**Uniform Temperature Fields**

For the measurements in uniform temperature fields, only one constant-temperature water bath is used and connected to both heat exchangers at the bottom and the top wall. The magnetic stirrer inside the cavity is now constantly operated to ensure homogeneous temperature fields within the image plane. Images are taken after steady state conditions are reached, for water typically after 20 minutes. Following
the procedures explained in section 3.2.4, $H_{kl}$ can be locally calculated from the RGB intensities. In Fig. 3.18, the measured hues obtained in uniform temperature fields are compared with the results from the linear profile. The measurements are consistent with the results obtained in a thermally stratified layer and confirm the validity of the expression in Eq. (3.15). Note that accurately resolving the steep gradient of the calibration curve in Fig. 3.18 from measurements in uniform temperature fields is much more challenging than it is for a linear temperature profile. A well mixed fluid is required in the AOV, since minor temperature differences result in significant variations of the locally evaluated hues.

### 3.2.6 Role of Experimental Parameters

To obtain quantitative temperature information from fluid-dispersed, encapsulated TLC particles, not only a sufficient optical resolution, described in the previous section, needs to be achieved. The effective temperature sensitive range of the liquid crystal formulation is an important experimental parameter as well. First, we discuss the influence of the working fluid, and compare the results obtained for glycerol to the ones for de-ionized water. Whereas a fixed angle $\phi$ of 90° was considered in the previous section, we here examine the influence of the angle $\phi$ on the temperature calibration.

#### Fluid Properties

To study the effect of different fluid refractive indices on the calibration, measurements with water and glycerol are compared at a fixed angular position $\phi$ of 90°. Figure 3.20 suggests that the effective sensitive range starts at significantly different temperatures for the two fluids. The low end of the temperature sensitive range is $T_{1,W} = 33.77°C$ for water, and $T_{1,G} = 34.70°C$ for glycerol. In addition, the shapes of the two calibration curves are different. For glycerol, a more linear behavior is obtained than for de-ionized water. Calibration is therefore strongly dependent on the optical properties of the working fluid.

A variation of the fluid properties is of potential interest for measurements in turbulent flows with large temporal scales, namely for natural convection problems. Since the particle density is slightly larger than the fluid density of water, settling can be prevented at the considered mean temperature of 35°C, in an aqueous solution with approximately 11 wt% glycerol. Figure 3.21 shows the fluid density, $\rho$, the kinematic viscosity, $\nu$, and refractive index, $n$, of aqueous glycerol solutions with different mass fractions of glycerol (D’Ans et al., 1977). However, for homoge-
neous mixtures of fluids with different optical properties it is important that density stratifications can be neglected for the considered flow fields, since they would result in local color changes and make an accurate reconstruction of the temperature impossible. This implies that calibration for homogeneous mixtures of working fluids with different refractive indices and densities cannot be done in a stratified layer. Furthermore, quantitative LCT measurements in double-diffusive convection are not meaningful.

**Angle of Observation**

The experimental set-up, shown in Fig. 3.13, allows the angle between the light-sheet plane and the camera axis, $\phi$, to be varied. The light sheet crosses the volume spanned by the cameras area of view (AOV) and the telecentric range of the objective, $\delta_{tel}$. Since a telecentric objective is used, the obtained images are not only
sharp but also distortion free. Glycerol is considered as the working fluid. For the considered liquid crystal formulation, product BM/R35C20, Hallcrest specifies the following characteristic temperatures: visible start, $T_{VS} = 34.1\, ^\circ C$, the red start, $T_{RS} = 34.9\, ^\circ C$, the green start, $T_{GS} = 37.8\, ^\circ C$, the bright green, $T_{BG} = 42.8\, ^\circ C$, the blue start, $T_{BS} = 55.8\, ^\circ C$, and the upper colorless limit, $T_{CL} = 68.0\, ^\circ C$. Three angular positions, $\phi$, equal to 50°, 70°, and 90° are considered. The obtained results are shown in Figure 3.22 and indicate a strong dependency of the color angle, $H$, on $\phi$. For decreasing $\phi$, a slight increase in the start and a significant increase in the width of the larger temperature range is observed. Most importantly, the obtained results connect the calibration conducted in a 90° configuration with the nominal tempera-
Figure 3.22: Influence of the angle $\phi$ between the camera axis and the light sheet plane on the color response obtained from isothermal measurements in glycerol (confidence interval: 95.4%). Visible start, $T_{VS}$, red start, $T_{RS}$, green start, $T_{GS}$, and bright green, $T_{BG}$, temperatures reported by Hallcrest (1999) are included for comparison.

Color Temperatures

In this section we demonstrate the dependency of the calibration on the color temperature. One motivation for using the concept of a color angle, $H$, instead of the individual intensities, $R$, $G$, and $B$, is that parameter $H$ is intended to be independent of the overall intensity, $I$. However, this requirement is only fulfilled, if the color temperature of the light source coincides with the WB that is adjusted at the camera. Figure 3.23 shows a polar plot (color wheel representation) of the
Figure 3.23: Polar plot of the saturation $S$ and the color angle, $H$, for different color temperatures, white balance settings at the camera, and intensity thresholds. Halogen light source, color temperature (CT) 3200 K: (a) white balance: WB=3200 K, (b) WB=5600 K. Metal halid source, CT=5500±900 K: (c) WB=3200 K, (d) WB=5600 K.

saturation, $S$, and the color angle, $H$, for two light sources, a halogen source with a color temperature (CT) of 3200 K (top), and a metal halid source with a CT of 5500 ± 900 K (bottom). The plots at the left side are for a WB setting at the cam-
era of 3200 K, and the ones at the right are for WB = 5600 K. The three different
curves are obtained when the particle detection algorithm uses threshold values of
$I = 100/255, 150/255, \text{ and } 200/255$ for the 8-bit intensity signal (the first criterion
in section 3.2.4). For cases, where the CT and the WB are matched, i.e. for the
plots in Fig. 3.23 (a) and (d), the lines for constant intensity thresholds are almost
symmetric to the origin of the diagram. When CT and WB are not matched, as in
Fig. 3.23 (b) and (c), this is not the case and the resulting calibration curves are
strongly dependent on the intensity. As a consequence, the desired behavior is only
obtained in cases (a) and (d). For the measurements in Rayleigh-Bénard convection,
chapter 4, and flow over heated waves, chapter 5, we will use configuration (d).

3.2.7 Summary

The accurate relation of the color response from TLC particles, which are suspended
in a working fluid, has been systematically studied for glycerol and de-ionized wa¬
ter. A 3-chip progressive scan RGB camera equipped with a telecentric measuring
objective is used for image formation.

Calibration is conducted for both, a uniform temperature field, and a linear tem¬
perature profile. Care is taken that particle images are represented by more than one
pixel and that saturated pixels (pixel overflow) are avoided. Consistent with litera¬
ture findings, the calibration curve obtained for $\phi = 90^\circ$ in de-ionized water shows an
effective temperature sensitive range that is significantly smaller (approximately 7%)
than its nominal value. Since the temperature sensitive range of commercially avail¬
able TLC formulations is usually specified for $\phi \to 0$, such knowledge is necessary
to properly select a TLC formulation for the considered measurement application,
i.e. the temperature field. The results obtained for constant temperature fields and
a thermally stratified layer show very good agreement. From measurements in a
thermally stratified layer of water, the average particle density was estimated to
1072 kg/m$^3$, a value larger than the one used in most references. When comparing
the results that are obtained for glycerol and de-ionized water, a similar width of
the effective temperature sensitive range is found. However, the start temperature
for glycerol is approximately 0.9°C higher than the one for water. The dependency
of the calibration on the fluids refractive index suggests, that LCT is not applicable
to double-diffusive convection for homogeneous mixtures of fluids with different op¬
tical properties. The width of the effective temperature sensitive range is found to
increase significantly with an decreasing angle $\phi$ between the light sheet plane and
the optical axis of the camera.
The obtained information allows to design and conduct reliable, simultaneous temperature and velocity measurements for a wider range of transient flow applications and experimental configurations than considered in most present works. The combined LCT/PIV technique provides an experimental tool for detailed studies of transport processes in, e.g. in natural convection, for heat transfer in porous media, as well as for local heat transfer studies in separated flows.
Chapter 4

Rayleigh-Bénard Convection

To demonstrate the applicability of the technique to transient problems, turbulent natural convection between two differentially heated horizontal walls is considered, a flow that is commonly referred to as Rayleigh-Bénard (RB) convection. We use digital PIV in connection with LCT to transiently record turbulent velocity and temperature fields in a 2-D domain of the flow. This chapter demonstrates how the technique can be used to connect structural information with integral quantities for local heat transfer studies in turbulent flows. In section 4.1 we provide a short introduction to RB convection, in section 4.2, the experimental set-up is discussed. Sections 4.3 and 4.4, present the obtained temperature and velocity fields.

4.1 Introduction

We now focus on how measurements of the fluid temperature and velocity can be used to locally determine temperature and velocity fields in steady, turbulent flows. Turbulent RB convection in a horizontal water layer that is located between two differentially heated (ΔT) walls, is used as a reference flow. Note that, consistent with the notation commonly used in the literature, the depth of the layer, 25.5 mm, is denoted as h in this chapter, whereas the height of the channel considered in chapter 5, 30 mm, we denote as \( H = 2h \).

For details on the physical characteristics of RB convection we refer to a recent review paper by Bodenschatz et al. (2000) and the original contributions of Krishnamurti (1968a, 1968b, 1970a, 1970b, 1973). Table 4.1 summarizes the fluid properties of water for a core temperature, \( T_b \), of 35.0°C and the flow conditions.
We define the Rayleigh number

\[ \text{Ra} = \frac{g \beta \Delta T h^3}{\kappa \nu} \]  

(4.1)

and the Prandtl number

\[ \text{Pr} = \frac{\nu}{\kappa} \]  

(4.2)

where \( g \) denotes the gravitational acceleration, \( \beta \) is the thermal expansion coefficient, \( \kappa \) is the thermal diffusivity, \( \nu \) is the kinematic viscosity, and \( \Delta T = T_L - T_U \) denotes the temperature drop across the layer. With the experimental set-up shown in Fig. 3.12, turbulent convective fields are obtained if – in contrast to the thermally stratified layer that was used for calibration – the temperature of the bottom wall is higher than the one of the top wall. As suggested by Krishnamurti (1970b), the different regimes for natural convection between two horizontal walls can be characterized if the Rayleigh number is plotted versus the Prandtl number (see Fig. 4.1). Different regimes are bounded by the limiting curves (I-V). The line labelled (I) denotes the critical Rayleigh number, \( \text{Ra}_c \), that is independent of \( \text{Pr} \). Below \( \text{Ra}_c \), the fluid is stagnant, and above it the flow is two-dimensional. Curve (II) marks the transition from a steady two-dimensional to a steady three-dimensional flow. Above curve (III), a time-dependent flow is obtained. Curve (IV) marks the next higher change of slope at which higher frequencies were observed. The transition to turbulent RB convection has been extensively studied. In contrast to recent LCT results reported by Lutjen et al. (1999) for steady convection at the conditions \( \text{Ra} = 6 \times 10^4 \) and \( \text{Pr} = 6 \times 10^3 \), this study focuses on quantitative visualizations in the turbulent regime.

With the fluid properties of water at the considered core temperature of 35°C and the (fixed) depth of the fluid layer, \( h = 25.5 \) mm, the Rayleigh number follows as:

\[ \text{Ra} = 5.2 \times 10^5 \frac{\Delta T}{K} \]  

(4.3)

With Eq. (2.9), the heat flux per unit area exchanged between the two walls can be decomposed into a mean and a turbulence contribution:

\[ \dot{q}_0 = \rho \cdot c_p \left( \frac{T}{v} - \kappa \frac{\partial T}{\partial y} \right) \]  

(4.4)

and one obtains for the Nusselt number:

\[ \text{Nu} = \frac{\dot{q}_0}{\rho c_p} \cdot \frac{h}{\kappa \Delta T} = \frac{h}{\kappa \Delta T} \frac{T}{v} - \frac{h}{\Delta T} \frac{\partial T}{\partial y}. \]  

(4.5)
In general, \( \text{Nu} \) is expressed in terms of \( \alpha_1 \text{Ra}^{\alpha_2} \), with \( \alpha_1 \) and \( \alpha_2 \) being constants. With a correlation that Rossby (1969) obtained from measurements in water

\[
\text{Nu}_R = 0.131 \text{Ra}^{0.3 \pm 0.005} \quad \text{for} \quad \text{Ra} > 34000
\]  

(4.6)

follows \( \text{Nu}_R = 6.67(\Delta T/K)^{0.3} \). The total heat flux exchanged between the two walls, \( \dot{q}_0 A \), is therefore

\[
\dot{Q}_0 = \frac{\varrho c_p \kappa A \Delta T}{h} \text{Nu} \propto (\Delta T)^{\alpha_2 + 1},
\]

(4.7)

where \( A \) denotes the area of the horizontal cross-section, 0.0204 m\(^2\). With Rossby’s correlation follows \( \dot{Q}_{0,R}/W = 3.39(\Delta T/K)^{1.3} \).
Table 4.1: Properties for a fixed depth of the fluid layer $h = 25.5$ mm at $R_\text{a}_1 = 1.2 \times 10^6$ ($\Delta T_1 = 2.2^\circ\text{C}$), and $R_\text{a}_2 = 7.8 \times 10^6$ ($\Delta T_2 = 14.9^\circ\text{C}$) in water, $Pr = 4.8$.

<table>
<thead>
<tr>
<th>$\Delta T$ (K)</th>
<th>2.2</th>
<th>14.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_\text{a}_{1,2}$ (-)</td>
<td>$1.2 \times 10^6$</td>
<td>$7.8 \times 10^6$</td>
</tr>
<tr>
<td>$Nu_R$ (-)</td>
<td>8.60</td>
<td>15.26</td>
</tr>
<tr>
<td>$\dot{Q}_{0,R}$ (W)</td>
<td>9.43</td>
<td>113.4</td>
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4.2 Experimental

We consider a fluid-filled cavity with a rectangular cross-section and the inner dimensions $270 \times 120 \times 25.5$ mm$^3$ being located between two constant-temperature walls, that are separated by the distance $h$, see Fig. 4.2. Four flat side-walls are made of 5 mm thick plexiglas (PMMA). The entire facility is thermally insulated, except for an optical viewport at its front and two slots for entering light sheets at

![Figure 4.2: Experimental configuration with the fluid-filled cavity and the used coordinate system. For clarity, the thermal insulation covering the cavity is omitted.](image-url)
4.3 VELOCITY FIELDS

the left and the right side-wall. Except for the flat side-walls and the exchanged temperature levels at the horizontal walls, the configuration is identical to the one we used for the calibration, see Fig. 3.13. A section of the fluid is illuminated with a sheet of white light. A 3-chip, RGB camera, model Sony DXC-9100P, is focused on the light sheet and captures the color information reflected from particles in it. De-ionized water is used as the working fluid. To provide a satisfactory spatial resolution, TLC-particles with a diameter of $19.8 \pm 7.2 \mu m$ and a particle density of $1.07 \pm 0.06 \, g/cm^3$ were used.

The reconstruction of the velocity fields from the stored image files, path $\Phi_2$ in Fig. 3.11, is done with a standard two-frame cross-correlation PIV algorithm. The reconstruction of the temperature fields from the stored image files, path $\Phi_2$ in Fig. 3.11, is performed as described in section 3.2.4. Equation (3.15) is used to connect the locally calculated hue values, $H_{ki}$, to the corresponding fluid temperatures. The data-set consists of $\xi : 0, \ldots, 49$ frames, which are recorded at the maximum framerate, 25 Hz. Considering the low fluid velocities, this temporal resolution is sufficient in order to obtain velocity fields through local cross-correlation for the lower Rayleigh number, $Ra_1$. As a result, the image processing steps, $\Phi_1$ and $\Phi_2$, produce a 2-D field of instantaneous fluid temperatures, $T_{ki}$, and velocities, $u_{ki}$, and $v_{ki}$, at the same discrete locations $k : 1, \ldots, m$ and $l : 1, \ldots, n$ with $m = n = 40$. Velocity fields at the higher Rayleigh number require a shorter temporal separation of the image frames. In this case, a sheet from a double-pulsed Nd:YAG laser and a monochrome CCD camera are employed, so that the information on the velocity and temperature fields cannot be obtained for the same instant in time. Sequences of instantaneous velocity fields and structural information on the fluid velocity are discussed in section 4.3. Section 4.4 presents the equivalent quantities for the temperature field.

4.3 Velocity Fields

To study the temporal scales of the flow, and the flow structure, we first examine the velocity field. In general, integral time-scales for natural convection experiments in water are rather large, i.e. in the order of several minutes. This implies that relatively long averaging times are required to obtain satisfactory statistics for mean and turbulence quantities (Adrian & Fernandes, 1999). For the presented data, the facility was operated several hours before the beginning of one measurement to ensure steady-state conditions. The measuring time for obtaining average quantities of the flow was 40 min. For an impression on the temporal flow behavior, we first consider
Figure 4.3: Series of subsequent velocity fields with a temporal separation of 1 s for the two Rayleigh numbers $Ra_1 = 1.2 \times 10^6$ (a), and $Ra_2 = 7.8 \times 10^6$ (b) in water ($Pr = 4.8$), $AOV = 1.3h(x) \times h(y)$. 
sequences of instantaneous image pairs that were acquired at a rate of 1 Hz. To be able to obtain velocities at both Rayleigh numbers, a double-pulsed Nd:YAG laser is facilitated as a light source. A PIV cross-correlation algorithm is used to extract the velocity fields from the intensity distributions $I_{ij}$ of two subsequent images. The AOV is $1.3h \times h$. A Gaussian three-point estimator and a spot size of 32 pixels were used. Figure 4.3 shows a sequence of velocity fields with a temporal separation of 1 s for the two conditions, $Ra_1 = 1.2 \times 10^6 (\Delta T_1 = 2.2^\circ C)$, and $Ra_2 = 7.8 \times 10^6 (\Delta T_2 = 14.9^\circ C)$ in water, $Pr = 4.8$. Note that the used acquisition rate does not exceed the limiting time-resolution of the used PIV system (0.07 s). A series of 100 image pairs is acquired for each condition. In the left column, the slow motion of a structure with a characteristic scale that is comparable or even larger than the AOV can be followed with the given temporal resolution. A characteristic scale of the dominant structure could be qualitatively obtained from the instantaneous images. In the right column, where the Rayleigh number is almost one order of magnitude larger, the same time difference is used to allow a comparison. The increased velocities reduce the temporal as well as the spatial scales of individual flow structures. The number of characteristic modes increases, and it becomes impossible to judge on their role based on instantaneous velocity fields. Note that for the considered flow field, $x$ is the homogeneous direction and $y$ denotes an inhomogeneous direction.

For determining the statistical properties we consider $Ra_2$, where multiple scales are involved, and use a second series of 200 individual vector fields that was obtained at a low sampling rate of 0.12 Hz. In the following, we consider the wall-normal velocity component, $v(x, y, t)$. For a quantitative assessment, we present the spatial two-point correlation function, and then decompose the velocity field using the method of snapshots for the POD analysis.

**Two-point correlation function.** The correlation functions for one homogeneous flow direction were defined in Eq. (2.17) of section 2.2.1. The function $R_{xv}(\Delta x, y, y')$ is evaluated between the two locations $(x, y)$ and $(x', y')$ in the measuring plane, with $\Delta x = |x - x'|$, since for a neglected influence of the side-walls, the flow is homogeneous in $x$. In Fig. 4.4, the correlation function $R_{xv}$ is plotted, for constant $y'/h$, in a $(y, \Delta x)$-plane. The correlation function is a temporal average over the number of $M = 200$ frames and spatially averaged over the coordinate $x$. It is evaluated at the following distances from the bottom wall: $y/h = 0.9$ (Fig. 4.4 a), 0.75 (b), 0.5 (c), 0.25 (d), and 0.1 (e). Because of the considered problems symmetry in the $y$-direction, a comparison between Figs. 4.4 (a) and (e) for wall distances $\Delta y/h = 0.1$,
as well as between Figs. 4.4 (b) and (d) for wall distances $\Delta y/h = 0.25$ provides information on the experimental uncertainty of the measurement with regard to determining large-scale structures in the velocity field. Good agreement is obtained between the two corresponding correlation functions, which provides a consistency test for the measurement data. Furthermore, we note that a characteristic distance
between maximum and minimum locations in the x-direction is approximately equal to h. If we define a characteristic scale in x as the distance between two maximum (or two minimum) locations, it would be twice the depth of the fluid layer, \( \Lambda_x = 2h \). For a Rayleigh number of \( 2 \times 10^7 \), Prasad & Gonuguntla (1996) assessed the flow structure of the velocity field that was obtained from PIV measurements in turbulent, nonpenetrative convection. The authors calculated two-point correlation functions and applied the linear stochastic estimation (LSE) technique. The characteristic scale retrieved from their two-point correlation functions \( R_{vv} \) agrees well with our findings.

**POD analysis.** The POD method is discussed in section 2.3. Two important properties should be stressed: (a) the obtained eigenfunctions are dependent on the size of the observed flow domain, i.e. the considered AOV, and (b) the considered ensemble of M velocity fields should provide a sufficient representation of the relevant large-scale structures of the flow. Again, we consider the velocity component \( v(x, y, t) \) in an ensemble of \( M = 200 \) velocity fields at \( \text{Ra}_2 \) and follow the procedures described in section 2.3 for its decomposition into eigenfunctions. The method of snapshots is used for the POD analysis. The AOV remained unchanged.

Figure 4.5 shows the eigenfunctions \( \Pi_{1,v}, \ldots, \Pi_{6,v} \) with the corresponding eigenvalues \( \lambda_1, \ldots, \lambda_6 \), sorted after their fractional energy contributions. Solid lines denote positive, and dashed lines, negative values. Note that the x-position of the obtained eigenfunctions is arbitrary since it describes a homogeneous direction of the flow.

The eigenvalues of the dominant six POD modes contribute 19.8\% (\( \lambda_1 \)), 16.0\% (\( \lambda_2 \)), 11.4\% (\( \lambda_3 \)), 7.2\% (\( \lambda_4 \)), 4.7\% (\( \lambda_5 \)), and 4.0\% (\( \lambda_6 \)) to the kinetic energy

\[
\frac{E_v}{\varrho} = \left< v'^2 \right>_{xy} = \sum_{i=1}^{M} \lambda_i. \tag{4.8}
\]

Eigenfunctions \( \Pi_{1,v}, \ldots, \Pi_{3,v} \) have characteristic scales in the x-direction of \( \Lambda_x = 2h \), in agreement with previous findings for the two-point correlation function \( R_{vv} \). The characteristic scales of the higher, less energy containing modes, are smaller. Furthermore, for the considered Rayleigh number, eigenfunctions \( \Pi_{1,v}, \ldots, \Pi_{5,v} \) extend over the full height between the two walls. Note that the extrema of the dominant eigenfunctions are much more pronounced than extrema in the cross-correlation function are. This observation can be explained with Eq. (2.24), the relation between the two functions, where the information contained in the two-point correlation function is found to correspond to an integral over all individual eigenfunctions weighted by their eigenvalues. Figure 4.6 shows the fractional and cumulative en-
Figure 4.5: Eigenfunctions $\Pi_{1,v}, \ldots, \Pi_{6,v}$ in the $(x,y)$-plane from a POD analysis of the velocity component, $v(x,y)$. $Ra_2 = 7.8 \times 10^6$ in water ($Pr = 4.8$), $AOV = 1.3h(x) \times h(y)$. The energy contributions of the first 30 POD modes for $Ra_2 = 7.8 \times 10^6$. The cumulative contribution of modes $1, \ldots, 6$ is 63%. We therefore state that – provided that a dominant eigenfunction can be extracted from the considered flow field – POD analysis is a more effective tool for obtaining the characteristic scale $\Lambda_x$ than the cross-correlation function, $R_{vv}$. 
4.4 Temperature Fields

For the temperature measurements, TLCs of the formulation BM/R35C20W are used. The RGB→HSI conversion is done with the calibration curve shown in Fig. 3.18. Note that in this case the temperature difference across the layer, ΔT, is required to be similar to the effective temperature sensitive range of the liquid crystal formulation in the φ = 90° configuration, T2 − T1. The relatively small effective temperature sensitive range of the LCT technique in the 90° configuration—which we will take advantage of in chapter 5 for the study of a forced convection problem—limits its applicability for natural convection problems to moderate Rayleigh numbers. For measuring the fluid temperature within the entire AOV between the two wall surfaces, we are therefore restricted to the lower Rayleigh number, Ra1 = 1.2 × 10⁶ with ΔT = 2.2°C.
Figure 4.7: Example for extracting the fluid temperature and velocity: (a) shows the raw-data files, (b) the vector field with the velocities $u(x, y)$, $v(x, y)$, (c) temperature field, $T(x, y)$, where blue denotes low and red high temperatures, and (d) a LIC representation of velocity and temperature field. $Ra_1 = 1.2 \times 10^6$, $Pr = 4.8$, $AOV=1.3h(x) \times h(y)$. 
For this Rayleigh number, Fig. 4.7 illustrates how both, the instantaneous velocity and the temperature information are obtained from raw RGB images. Commonly, temperature and velocity distributions are shown as contour and vector plots, see Fig. 4.7 (b) and (c). However, information on the flow topology can be connected with the temperature field if a line integral convolution (LIC) procedure is followed from the measured vector field. The temperature is then mapped on it as a color distribution. The LIC images were obtained with an extension module of the visualization package AVS Express that was implemented at the Swiss Center for Scientific Computing (CSCS, Dr. Jean M. Favre). Combining the velocity and temperature information in one image is useful since it allows to identify up-/downward flow regions with high/low normal velocities, which are large contributors to the turbulent heat transfer between the two horizontal walls, i.e. to the first term in in Eq. (4.4). For a series of temperature fields that was obtained at $Ra_1 = 1.2 \times 10^6$, Fig. 4.8 shows the temporal evolution of the fluid temperature. The downward motion of a cold plume that departs from the top wall can be followed in the sequence. For consistency, we use the same temporal separation (1 s) as for the velocity field, see Fig. 4.3a.

4.5 Summary

Particle image velocimetry and liquid crystal thermometry are applied to turbulent Rayleigh-Bénard convection as a first reference situation for transient measurements in water ($Pr = 4.8$) at two Rayleigh numbers, $1.2 \times 10^6$ and $7.8 \times 10^6$. Whereas the temporal scales of the flow are significantly larger than for the turbulent flow over heated waves that is considered in chapter 5, a similar AOV is used.

From PIV measurements with long averaging times and a frame rate of 0.12 Hz, a characteristic scale of the flow structures in the direction parallel to the horizontal walls, $\Lambda_x$, is obtained. The velocity component in the normal direction, $v(x, y, t)$, is considered, and the two-point correlation function $R_{vv}(\Delta x, y, y')$ is determined at different distances $y'$. In addition, the dominant eigenfunctions $\Pi_{i,y}(x, y)$ are obtained from a POD analysis. From both methods, $\Lambda_x \approx 2h$ is determined as a characteristic spatial scale for $Ra_2$. However, POD analysis is seen to provide a more effective tool for extracting structural information.

Due to the relatively small effective temperature sensitive range of the used LC formulation that is selected for measurements in a turbulent water flow over heated waves (chapter 5), temperature measurements in Rayleigh-Bénard convection are
Figure 4.8: Departure of a cold plume from the top wall as observed in a sequence of temperature fields with a temporal separation of 1 s for Rayleigh-Bénard convection at $\text{Ra}_1 = 1.2 \times 10^6$ in water ($\text{Pr} = 4.8$), $\text{AOV} = 1.3h(x) \times h(y)$. 
restricted to moderate Rayleigh numbers. We demonstrate the reconstruction of instantaneous temperature fields, $T(x, y, t)$, for $Ra = 1.2 \times 10^6$ and extract structural information from spatiotemporal temperature fields – in a similar way than it is done here for the fluid velocity – in chapter 5.
Chapter 5
Flow over Wavy Walls

The mean and turbulence quantities of a developed turbulent flow in a channel with a sinusoidal bottom wall (wavelength \( \lambda \)) and a flat top wall have been a focus of numerous studies, based on laboratory as well as numerical experiments\(^1\). The interest in this flow configuration is motivated by its technical relevance, and, even more importantly, by its applicability as a reference for complex flows. The wavyness of the bottom wall adds a degree of complexity to the flow through a flat-walled channel (e.g. Niederschulte, et al., 1990; Antonia, et al., 1992; Günther et al., 1998). The flow conditions remain well-defined by the no-slip boundary conditions at the wall surfaces, and by periodic boundary conditions (except for the pressure) at the inflow and outflow sections of the channel. This fact is important in order to allow numerical experiments to be performed at similar boundary conditions. Three characteristic regions of the flow have been identified from two-dimensional information on the mean and turbulence quantities. On the other hand, stability analysis suggests three-dimensional (3-D), longitudinal structures to persist due to the wavyness of the surface.

The present work is concerned with the structure and three-dimensionality of the flow. The objective is to connect the different regions in a 2-D plane with spatially resolving information on the scales of longitudinal, 3-D, structures. In a further step, scalar transport over a wavy wall is examined in a hydrodynamically developed flow over a train of heated waves. Structural information on the temperature field is obtained and connected to the structures that were identified in the velocity field. In order to allow detailed comparisons with direct numerical simulations (DNS), we restrict ourselves to moderate Reynolds and Rayleigh numbers.

\(^1\)We exclusively refer to direct numerical simulations, but not to large eddy simulations or RANS closures, as numerical experiments.
The chapter is organized as follows: A description of the flow problem and a detailed literature survey are presented in section 5.1. Section 5.2 describes the experimental facility. Results on the isothermal flow over solid waves are presented in section 5.3. For laminar flow, information with regard to longitudinal structures is obtained from instantaneous flow fields. At turbulent flow conditions, characteristic modes of large-scale, streamwise-oriented structures are identified from a POD analysis of the PIV measurement data. Section 5.4 presents results from the flow over heated waves, where the effect of longitudinal structures on the transport of a heat is assessed. LCT is used to obtain quantitative information on the fluid temperature. A POD analysis reveals information on the structure of the temperature field.

5.1 Introduction

Figure 5.1: Characteristic regions for a developed turbulent flow in a channel with a wavy bottom wall: (1) the separation region, and the regions (2) of maximum positive and (3) negative Reynolds shear stress. \(2a/\Lambda = 0.1, \Lambda/H = 1, B/H = 12:1, \text{wrms} = \sqrt{\overline{w'^2}}\).

Figure 5.1 shows the coordinate system for the considered flow configuration, where \(x\) is the direction of the mean flow (parallel to the top wall), \(y\) is perpendicular to the top wall, and \(z\) is the spanwise coordinate. The corresponding velocity components are denoted as \(u, v,\) and \(w\). Channel flow (CF) and boundary layer
(BL) flow measurements are considered. The developed flow is characterized by the ratio of the amplitude $2a$ to the wavelength

$$\alpha = \frac{2a}{\lambda}$$  \hspace{1cm} (5.1)

and the Reynolds number definitions for a channel flow

$$\text{Re}_b = \frac{U_b \cdot h}{\nu}$$  \hspace{1cm} (5.2)

and for a boundary layer flow

$$\text{Re}_A = \frac{U_\infty \cdot \Lambda}{\nu},$$  \hspace{1cm} (5.3)

where $\nu$ is the kinematic viscosity of the fluid, $h$ is the half-height of the channel, and $U_\infty$ denotes the free stream velocity. The bulk velocity $U_b$ is obtained as

$$U_b = \frac{\int_{y_w}^h U(x_\xi, y) dy}{\int_{y_w}^h dy},$$  \hspace{1cm} (5.4)

where $x_\xi$ denotes an arbitrary $x$-location and

$$y_w(x) = a \cos(2\pi \frac{x}{\Lambda})$$  \hspace{1cm} (5.5)

describes the profile of the wavy surface with $a = 0.05\Lambda$. Tables 5.1 and 5.2 provide an overview of laboratory experiments, computer simulations and theoretical work on the internal or BL flow over waves.

**Mean flow.** Early works described the non-separated flow over small amplitude waves by a linear stability analysis. With increasing ratio $\alpha$, linear analysis eventually becomes insufficient. Following the original contributions of Motzfeld (1937), Miles (1957), Benjamin (1959), and Hanratty (e.g. Buckles, et al., 1984), a number of laboratory and recent numerical experiments were conducted to describe the flow field in terms of its mean and turbulence quantities, and the mechanisms for turbulence production. For large $\alpha$, the flow partially separates and can be characterized by the following features:

- periodicity of the mean flow in the streamwise direction, except for the pressure,
- partial separation behind the wave crests, and
- Reynolds numbers in the range where detailed comparisons with direct numerical simulations (DNS) are possible.
<table>
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<td>( U, V, \sqrt{u'^2}, \sqrt{v'^2}, \sqrt{w'^2}, -\bar{u}'w' )</td>
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<td>( P_w, \tau_w )</td>
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<tr>
<td>Russ &amp; Beer (1997)</td>
<td>( 0.3 - 13d )</td>
<td>0.106</td>
<td>T, A</td>
<td>( \text{FV and heat transfer measurements} )</td>
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<tr>
<td>Nakagawa et al. (2001)</td>
<td>( 46h )</td>
<td>0.10 (0.1)</td>
<td>CF, W</td>
<td>( \text{turb. quantities, connection to fully rough surface} )</td>
</tr>
<tr>
<td>Nakagawa &amp; Hanratty (2001)</td>
<td>( 3.2h, 11h )</td>
<td>0.10 (0.1)</td>
<td>CF, W</td>
<td>( \text{turb. quantities, connection to intermediate rough and hydraulically smooth surface} )</td>
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<th>References</th>
<th>Description of Investigation</th>
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<tr>
<td>Benjamin (1958)</td>
<td>ST. Developed linear disturbance theory.</td>
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<tr>
<td>Markatos (1978)</td>
<td>RANS. Heat and mass transfer across a wavy boundary two equation closure.</td>
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<tr>
<td>Balasubramanian et al. (1981)</td>
<td>RANS. Zero and two-equation closures.</td>
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<td>Patel et al. (1991)</td>
<td>( \text{Re}_h = 6400, \alpha = 0.3125 ) (sep. flow), (ref. measurements by Frederick (1986)), and ( \text{Re}_h = 4080, \alpha = 0.2 ) non-separated flow (ref. measurements by Kuzan (1986)) RANS, two-layer ( k, \epsilon )-closure.</td>
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<tr>
<td>Krettenhauer &amp; Schumann (1991)</td>
<td>DNS and LES simulation of turbulent flow over wavy boundary, transport of passive scalar.</td>
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<tr>
<td>Hino et al. (1993)</td>
<td>( \text{Re}_h = 3400, \alpha = 0.0184 ), DNS, quasi stationary streaky pattern at wavy surface observed.</td>
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<td>Maas &amp; Schumann (1994)</td>
<td>( \text{Re}_h = 3380, \alpha = 0.1, \Lambda/H = 1, B/\Lambda = 2 ), DNS.</td>
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<td>De Angelis et al. (1997)</td>
<td>( \alpha = 0.1, \Lambda/H = 1.04, B/\Lambda = 1 ), DNS.</td>
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<td>Cherukat et al. (1998)</td>
<td>( \text{Re}_h = 3460, \alpha = 0.1, \Lambda/H = 1, B/\Lambda = 2 ), DNS.</td>
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<tr>
<td>Henn &amp; Sykes (1999)</td>
<td>( \text{Re}_h = 6560 - 20060, \alpha = 0.031 - 0.2, \Lambda/H = 1, B/\Lambda = 1 ), LES</td>
</tr>
<tr>
<td>Boersma (2000)</td>
<td>( \text{Re}_h = 1750, \alpha = 0.1, \Lambda/H = 1, B/\Lambda = 3 ) DNS of particle laden flow over waves. Indications for streamwise-oriented structures found.</td>
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RANS=Reynolds averaged Navier-Stokes equations, ST=Stability analysis.
Characteristic regions of the separated flow are shown in Fig. 5.1. A separation zone (1), sometimes referred to as a separation bubble, is located in the wave troughs bounded by the isosurface for $\Psi = 0$, where

$$\Psi(x, y) = \frac{\int_y U(x, \tilde{y})d\tilde{y}}{U_0 2h}$$

(5.6)
denotes the mean streamfunction that is obtained by integrating the mean streamwise velocity in the $y$-direction. In the vicinity of the separated region, scalings that are commonly applied to free shear layers were used to describe the flow (Hudson, Dykhno & Hanratty, 1996). At the uphill side of the wave, two regions of maximum positive (2) and negative (3) Reynolds shear stress, $-\overline{\omega'v'}$, are found. From the DNS results of Cherukat et al. (1998) for $Re_h = 3460$, the locations of regions (2) and (3) are found approximately 0.08$\Lambda$, and 0.01$\Lambda$ above the wall at the uphill side. Furthermore, the data of Cherukat et al. (1998) and Henn & Sykes (1999) identify the energy of traverse velocity fluctuations, $\overline{\omega''^2}$, to be maximal at a location that is close to region (3).

**Flow structure.** Longitudinal structures play a dominant role in a number of transport processes, e.g. as streaky structures for turbulence generation in the vicinity of a solid wall, or as Langmuir circulations at gas-liquid interfaces. Even though such structures are of three-dimensional nature, the mostly qualitative visualizations were so far restricted to observations in the $(x, y)$-plane. Only recently, the attention was drawn to the effect of the wavy wall with respect to the formation of three-dimensional, large-scale structures (Calhoun & Street, 2001). The literature on the stability of a sheared flow over rigid waves suggests different mechanisms to produce, or catalyse, spanwise-periodic longitudinal vortices: the Görtler (Görtler 1940, Saric 1994) and the Craik-Leibovich type 2 instability (Phillips & Wu 1994).

**Görtler mechanism.** In the inviscid description given by Rayleigh (1916), the sheared flow over a concave surface is subject to a centrifugal instability. For a BL flow over a train of sinusoidal waves, Saric (1997) performed a stability analysis. For the considered wall geometry, given by Eq. (5.5), the Görtler mechanism is therefore locally restricted to the intervals with a concave wall curvature, $x/\Lambda : [0 + 0.5 \cdot i, 0.25 + 0.5 \cdot i]$. These locations are denoted with (+) in Fig. 5.2. With the radius of the wall curvature,

$$R = \frac{(1 + y'^2)^{3/2}}{y''}$$

(5.7)
the Görtler number follows as
\[ Go = \left( \frac{x - x_0}{R} \right)^{0.5}, \] (5.8)
where \( x_0 \) denotes the closest location \( 0.5\Lambda \cdot i \). Figure 5.2 shows Go for the interval \( x/\Lambda : [0.5, 0.75] \). Note that an alternative route for defining an Görtler number is based on the curvature of the mean streamlines, see e.g. Fig. 5.12.

**Langmuir type mechanism.** The works of Phillips & Wu (1994) on the development of longitudinal vortex modes in inviscid linear shear and of Phillips et al. (1996) on the inviscid and viscous instability of power-law and logarithmic velocity profiles to spanwise-periodic longitudinal vortex modes, explain the presence of longitudinal large-scale structures with a Craik-Leibovich type 2 (CL2) instability. The main difference to the Görtler mechanism is that the instability is caused by
the periodicity of the bottom wall rather than by the local curvature. Gong et al. (1996) were the first to conclude the presence of such structures from single-point measurements in a turbulent BL flow over waves, and to connect them to a CL2 instability. However, the small aspect ratio of the used facility of 4 : 1 is likely to have affected the lateral motion of the structures. On the other hand, most numerical studies were performed for relatively small spanwise domain sizes. For a DNS with a spanwise domain of 3A, Boersma (2000) recently obtained indications of such large-scale structures in instantaneous fields of the streamwise velocity in the (y, z)-plane. We can state that there is a lack of detailed laboratory or computer experiments on the BL or internal flow over 2-D waves with larger spanwise domains to clarify the role and lateral motion of such flow structures.

Before characterizing the experimental details of the present work in section 5.2, we briefly summarize the motivations of previous studies.

a. Reference flow with partial separation. From a flow physics standpoint, the described geometry adds a defined degree of complexity to the turbulent flow in a flat-walled channel (e.g. Niederschulte, et al., 1990; Antonia, et al., 1992; Günther et al., 1998). The flow partially separates behind the wave crests, thus multiple time-scales persist. Since the flow has periodic mean quantities in the streamwise direction (with the exception of the pressure), its boundary conditions are well defined. This is a major advantage in comparison with other test cases for separated flows, namely the flow over a backward-facing or a forward-facing step (Stüer, 1999), where the proper definition of the inflow conditions remains a challenge for laboratory and numerical experiments. At moderate Reynolds numbers, comparisons with DNS results are possible. Detailed information on such a reference flow field can then be used to systematically evaluate and improve different means for numerically predicting turbulent flows in complex geometries. The laser Doppler velocimetry (LDV) data of Hudson (1993) provided a test case for the second ERCOFTAC workshop on Direct and Large-Eddy Simulation. The present data provide, for the first time, quantitative information on the flow structure that is based on spatially resolving measurements.

b. Hydrodynamically rough wall. Above a certain velocity, a fluid flowing over a solid boundary creates turbulence. Turbulence production at smooth walls has been and is excessively studied by numerical and experimental means. The formation of streamwise vortices in the wall region has been identified as a key for understanding how turbulence is sustained. Altering those structures is seen as an
Figure 5.3: Top: Mass-transfer study of Gschwind et al. (1995). The local intensity change observed at the coated wavy surface (depicted) was related to the local mass-transfer for the absorbed species. Bottom: spanwise variation of the streamwise mean velocity in a low aspect-ratio ($B/A = 4 : 1$) wind tunnel measurement of a boundary layer flow over sinusoidal waves by Gong et al. (1996).

An effective tool for changing mean and turbulence properties of the flow, most notably the drag. The mechanisms for turbulence production and sustainment, and thus the turbulence structure, in the near-wall region are likely to be different from a smooth surface. However, a similarity of the flow structures in the outer flow has been proposed, e.g. by Raupach et al. (1991). By using small wavelengths, Nakagawa, Na & Hanratty (2001) and Nakagawa & Hanratty (2001) systematically connect the flow over wavy walls to a hydrodynamically fully rough surface.
c. **Flow over topology.** Many technically, geophysically or environmentally relevant flows occur over rough or structured surfaces. To improve existing models of such flows, DNS and large eddy simulations (LES) of the flow over sinusoidal waves were conducted, by Dörnbrack & Schumann (1993), and Krettenhauer & Schumann (1992), Calhoun & Street (2001), and Sullivan & McWilliams (2002). Boersma (2000) studies the particle laden flow over waves by means of DNS. His work is motivated by a civil engineering application since the prediction of sediment transport over a wavy boundary provides useful information for the maintenance of coastal structures. Since DNS is limited to moderate Reynolds numbers, LES involves modeling of the smallest scales, and real-scale flows are characterized by very high Reynolds numbers, spatially resolving laboratory measurements are required in all of these applications.

d. **Wavy gas-liquid flows.** The developed flow over solid waves can be connected to non-breaking wavy gas-liquid flows. De Angelis, Lombardi & Banerjee (1997) argue that, since the ratio between the gas and the liquid density is of $\mathcal{O}(0.001)$, the flow can be approximated by a single-phase flow over a solid wave. Note that the 2-D, sinusoidal waves considered herein are a simplification of real 3-D surface waves. Secondly, we consider a standing wave, whereas a moving solid wave would be a better approximation of a wavy two-phase flow.

### 5.2 Experimental

Measurements are carried out in a channel facility with de-ionized and filtered water as the working fluid, see Fig. 5.4. The flow loop is designed for low Reynolds number turbulence measurements with light sheet techniques. Care has been taken in order to minimize the volume of working fluid, which is approximately $0.280 \text{ m}^3$. The entire facility is made of black anodized aluminum, PVC, and Schott BK-7 glass, a hard crown glass. The recirculation system, the channel and its test section are described in sections 5.2.1-5.2.3. All parts are positioned in a frame of welded stainless-steel profile with a cross section of $45 \text{ mm} \times 45 \text{ mm}$.

#### 5.2.1 Recirculation System

After passing the channel (pos. 5 in Fig. 5.4), the water flows into a reservoir, (9). A frequency controlled stainless-steel pump (Emile Egger, type 21-50 HF4),
Figure 5.4: Facility with the channel sections and the recirculation system: (1) turning elbows, (2) honeycomb, (3) flat-walled entrance section, (5) section with wavy bottom wall (partially heated), (4,6-8) optical viewports, (9) reservoir, (10) frequency controlled pump, (11) pipe, and (12) diffusor.
5.2. EXPERIMENTAL

Figure 5.5: Flat-walled entrance section of the channel (pos. 3 in Fig. 5.4) and recirculation section: (12) the diffusor, (1) the turning elbows, (2) the honeycomb, a contraction section (nozzle), and the boundary layer trip.

(10), draws fluid from an intake installed at the bottom of the reservoir tank and feeds it through a transparent PVC tube with an inner diameter of 50 mm (10). To reduce transmission of vibrations, the pump has flexible connections at the inflow and the outflow sides. At the outflow side, a 2.25 m long diffusor (12) with a maximum opening angle of 1.9°, (12), is located. Gradual expansion from the circular cross-section of the inflow tube to a cross-section of 360 mm x 200 mm is necessary to avoid boundary layer separation which may affect the flow characteristics in the test section (Niederschulte, 1988, Pankhurst & Holder, 1952, Pope, 1954). The direction of the flow is changed with the turning vanes, pos. (1) in Fig. 5.4. Both, the diffusor and the section with the turning elbows are entirely made of black anodized aluminum and connected through flanges. The flow then passes a honeycomb, (2), with a hexagonal cell structure of carbon fiber reinforced plastics. Honeycomb length-to-cell-size-ratios of four to seven have been recommended
by Prandtl (1933), and a ratio of seven is chosen in the described facility. Longer honeycombs reduce the traverse velocity fluctuations. A contraction immediately upstream of the channel section produces a flow of uniform velocity distribution and low turbulence intensity (Goldstein, 1938). The larger the contraction-area-ratio, the greater the beneficial effects. However, this increases the volume of the channel for a given channel geometry. The present facility has a contraction-area-ratio of 6.7. Abrupt contractions are obviously undesirable. Instead, a smooth and gradual transition is necessary to achieve the beneficial effects (Niederschulte, 1988). Figure 5.5 shows the nozzle. At the entrance to the rectangular channel, a boundary layer trip, which extends 1 mm into the flow from all four walls, is positioned. It was installed to ensure fully developed turbulent channel flow by uniformly disturbing the boundary layer of the flow at the entrance of the channel. Figure 5.5 shows a photograph with the diffusor, the turning elbows, the contraction section and the entrance to the channel. All sections are interconnected with flanges and 1 mm silicone gaskets.

5.2.2 Channel

With positioning screws, the two channel sections are aligned horizontally and connected to the stainless-steel frame. The full height of the channel, $H$, is 30 mm, and its aspect ratio, $B/H$, is 12:1. The inflow section of the channel, (3), has a length of 67 channel heights and consists of flat top and bottom walls. Both walls are made of black anodized aluminum material with a wall thickness of 6 mm. The parallelism of the two walls is achieved by avoiding thermal deformations through welding and using a screw connection instead.

A second section with a flat top and a wavy bottom wall, (5), is 72 channel heights long and provides the possibility of heating the sinusoidal bottom wall resistively in a 34 channel heights long section. Both channel sections are connected through flanges. Positioning screws and a thin liquid sealing layer quaranty precise horizontal alignment. The wavelength $\Lambda$ of the sinusoidal wall profile is equal to the full height of the channel, $H = 2h$, and the ratio of twice the amplitude $a$ to the wavelength, $\alpha$, is 0.1. The sinusoidal bottom wall is removable and made of PVC material. Care has been taken in manufacturing a sinusoidal shape of the surface. A cutting tool with the sinusoidal profile was machined and gauge measurements confirmed that the machined PVC surface deviated less than 0.3% from the desired shape, see Fig. 5.6.
Figure 5.6: Gauge measurement of the used wave profile (left axis) and relative deviation from the sinusoidal shape (right axis) in comparison with Hudson (1993).

5.2.3 Test Section

Optical access is provided at four steamwise locations of the wavy channel section through viewing-ports (positions 4, 6, 7, 8 in Fig. 5.4) at both side walls (thickness $t = 5\text{mm}$) and at the flat top wall ($t = 7\text{mm}$), all made of optical grade Schott BK-7 glass, see Fig. 5.8. Measurements are conducted at the positions 4 (developing flow) and 7 (developed flow) after the 4th and the 50th wave crest. With o-ring sealed aluminum frames, the viewing-ports are flush-mounted to the channel inner walls. Side windows provide optical access with a maximum AOV of $3 \times \Lambda_{\text{streamwise}} \times 1.2 \times \Lambda_{\text{normal}}$, where the maximum AOV for the top windows is $3.3 \times \Lambda_{\text{streamwise}} \times 3.3 \times \Lambda_{\text{spanwise}}$. For a thickness of 5 mm, and the visible range of light wavelengths, the transmittance $\tau$ and the refractive index $n_{\text{glass}}$ of the used Schott BK-7 glass are shown in Fig. 5.7. Figure 5.9 shows the Cartesian coordinate system, which is used to describe the flow in the test section: $x$ describes the direction of the mean flow (parallel to the top wall), $y$ is perpendicular to the top wall, and $z$ is the spanwise coordinate. The Reynolds number is adjusted through the pump frequency. In order to determine the fluid viscosity, the water temperature is monitored. The facility can be operated starting at laminar flow conditions up to a maximum Reynolds number of approximately $1.0 \times 10^4$. Pressure drop
measurements were obtained using a piezoresistive differential transducer Fisher Rosemount model 3051 with a range of $0 - 7.5$ mbar.

We use digital particle image velocimetry (PIV) to experimentally assess the role of large-scale structures in the velocity field. The flow is seeded with 30 micron spherical Latex particles (Pharmacia Biotech product Source 30 ETH), and with 50 micron polyamide particles (Dantec product PSP-50) with the properties discussed in section 3.1.1. Mainly because of the uniform particle diameter this seeding material was found to be advantageous compared with, e.g., hollow glass spheres, and Conifer pollens. The measurement system consisting of the laser, the laser optics, and two cameras, is mounted on a traverse to alter the vertical measurement position. The accuracy of adjusting the $y$-position with the traverse is approximately 10 microns. A flashlamp-pumped dual Nd:YAG laser ($\lambda = 532$ nm, New Wave, Inc., Minilase II) provides the pulse light source and two CCD cameras, a 8 bit Kodak Megaplus ES 1.0 with a spatial resolution of $1008 \times 1016$ (vertical) square pixels, and a 12 bit PCO Sensicam with a spatial resolution of $1280 \times 1024$ (vertical) square pixels are used for image acquisition. Both are components of a commercial PIV system of TSI, Inc. Measurements are taken in the $(x, y)$-plane, in the $(y_1, z)$-plane that intersects with the wavy surface at an uphill location, and in the $(x, z)$-plane at the vertical distances $y/H = 0.26$, and 0.74, see Fig. 5.9. The individual PIV results for isothermal flow over wavy walls are being presented and

Figure 5.7: Relative transmittance and refractive index of Schott BK-7 glass (thickness $t = 5$ mm).
5.2. EXPERIMENTAL

Figure 5.8: Optical access through (a) a pair of two side-windows and (b) one top window at different streamwise positions of the channel.

discussed in section 5.3.

5.2.4 Resistively Heated Wall Section

For studying the role of large-scale structures on momentum and scalar transport over solid waves we decide to study heat transfer through examining temperature fields. We favourize heat transfer measurements since a well-defined boundary condition at the wall surface can be experimentally established for the wall temperature or the wall heat flux. From the two options of a constant wall temperature and a constant wall heat flux boundary condition, the latter is chosen. The second-order boundary condition for the temperature field is provided by a resistively heated foil. Similar designs have been used by Hetsroni et al. (1994, 1996, 1997a, and 1997b) for the measurement of wall temperatures with infrared thermography in a flume. Detailed information on the design and instrumentation of resistively heated channels can be found in, e.g., Žukauskas & Šlančiauskas (1987).
Figure 5.9: Side-view of the flow field with a schematic of the separated region (streamline $\Psi = 0$) and the measurement locations in the $(x, z)$-plane, and in the $(y_1, z)$-plane with $\beta = 53^\circ$.

A resistively heated manganin foil of length $L_M = 1024$ mm and width $B_M = 300$ mm with an uniform thickness of 45 $\mu$m is used. Manganin is a copper-manganese-nickel (CuMnNi) alloy and has the property that its specific electric resistance, 0.482 $\Omega \text{mm}^2 \text{m}^{-1}$ at 20°C, is almost independent of the temperature. The physical properties of manganin are summarized in section 5.4. At both ends, the foil is soldered to copper electrodes. The electrodes are flush-mounted to the surface of the PVC bottom wall. The two electrodes are connected to a direct cur-
rent welding transformer with a maximum current of 380 A, see Fig 5.10. The

Figure 5.10: Flush-mounted copper electrode for resistively heating the manganin foil (shown at the channel bottom at the exit of the test section).

manganin foil is attached to a milled PVC wavy wall segment. Several structural adhesives (epoxy and acrylic products) were evaluated. The viscosity was found to be the critical parameter. A two component adhesive with a methyl methacrylate basis, Degussa product Agomet F310, was found to provide a satisfactory adhesion between PVC and manganin, and have a relatively low viscosity. However, its reaction time of 5 – 8 minutes is relatively short.

To preserve the sinusoidal shape, the acrylic adhesive was first uniformly distributed at the wavy PVC segment. The metal foil was positioned above it. A second wave segment, covered with a 1 mm polytetrafluoroethylene (PTFE) sheet, was used as a negative part and pressed the manganin foil to the wavy shape of the PVC segment. The rapid reaction times of the adhesive required to quickly absolve all steps of the described manufacturing procedure.
5.3 Results for Isothermal Flow

For data validation, the streamwise mean velocity, obtained from DPIV measurements in the \((x, y)\)-plane, is first compared with the laser Doppler velocimetry data of Hudson (1993), and a DNS by Cherukat et al. (1996), see section 5.3.1. In section 5.3.2, longitudinal structures are identified based on their instantaneous streamwise velocity in the \((x, z)\)-plane. Since the spanwise extension of the flow domain is relatively large, longitudinal structures are expected to meander laterally. In section 5.3.3, a measurement plane that intersects with the wavy surface at the uphill side of the flow, is considered.

5.3.1 Mean Flow Field: \((x, y)\)-Plane

To connect to previous work, flow conditions close to those of a laser Doppler velocimetry study by Hudson (1996), and a DNS by Cherukat et al. (1998) are chosen. Measurements in the \((x, y)\)-plane are performed at a Reynolds number of 3350, with two areas of view, \(0.99\Lambda \times 0.79\Lambda\), and \(1.1\Lambda \times 1.1\Lambda\). The first allows a more detailed assessment of the separated region in the wave troughs, where the latter includes the entire flow field between the wavy bottom and the flat top wall, and therefore allows to determine the bulk velocity (Eq. 5.4) and the Reynolds number of the flow. It should however be noted that the large AOV limits the spatial resolution in characteristic regions of the flow, namely in the vicinity of the walls. For the used spot size, the spatial resolution for one obtained vector is \(0.03\Lambda \times 0.03\Lambda\).

For a Reynolds number of 3350, Fig. 5.11 provides qualitative information on the structure of the flow in the \((x, y)\)-plane from six instantaneous realizations with the dimensionless velocity fluctuations (outer scales) in the streamwise and normal directions of the mean flow. The Kodak camera is used with the larger AOV. Even though judgement from instantaneous information is subjective, since nothing is said about the significance of one particular realization with regard to the entire flow field, it provides useful information on the scales of the flow. Consistent with previous findings, large eruptions are observed in the wave trough, which can reach far into the outer flow, almost to the top wall. Figure 5.12 (a-c) shows contour plots of the mean streamwise velocity, \(U(x, y)/U_b\), the mean normal velocity, \(V(x, y)/U_b\), and the mean stream function, \(\Psi(x, y)\). The averages are obtained from 250 instantaneous velocity fields that were acquired in the frame-straddle mode at a rate of 1 Hz. The PCO Sensicam camera is used with the smaller AOV. Outer scaling is used for the velocities, i.e. they are made dimensionless with the bulk velocity, \(U_b = 0.22 \text{ m/s}\). In
Figure 5.11: Fluctuations in the streamwise and normal directions, $u'/U_b$ and $v'/U_b$, in the $(x,y)$-plane at $Re_b = 3350$. AOV $= 1.1 \Lambda \times 1.1 \Lambda$. 
Figure 5.12: Contours of (a) the dimensionless mean streamwise velocity, $U/U_b$, (b) the mean spanwise velocity, $V/U_b$, and (c) the mean stream function in the $(x,y)$-plane in outer scales for the flow over waves at $Re_h = 3350$. Solid lines represent positive and broken lines denote negative velocities.
5.3. RESULTS FOR ISOTHERMAL FLOW

Fig. 5.13, the streamwise mean velocities that are obtained from PIV measurements in the \((x, y)\)-plane are compared with the laser Doppler velocimetry (LDV) data of Hudson (1996). Since Hudson’s data where only obtained for the lower channel half, the common definition for the bulk velocity, Eq. (5.4), is not applicable. In order to compare the data, our velocities are made dimensionless with the streamwise mean velocity that is obtained from integrating \(U(x, y)\) over the lower channel half:

\[
U_{b,Hudson} = \frac{1}{hA} \int_0^A \int_{g_w}^h U(x, y) \, dy \, dx.
\]  (5.9)

In Fig. 5.13, the profiles of the streamwise mean velocity are compared at the two locations, \(x/A = 0.0\) (crest), and \(x/A = 0.5\) (trough). Considering the limited resolution of the PIV measurements of 0.04\(\Lambda\), a good agreement is obtained. Figure 5.12 (c) shows the mean stream function at the Reynolds number of 3350. Streamline \(\Psi = 0\) bounds the separated region.

Figure 5.13: Comparison of the dimensionless mean streamwise velocity, \(U/U_{b,Hudson}\), at the two locations, \(x/A = 0.0\) (crest), and \(x/A = 0.5\) (trough), at \(Re_h = 3350\) with the LDV data of Hudson et al. (1996).
Figure 5.14: Contours of (a) $\sqrt{\overline{u'^2}}/U_b$, (b) $\sqrt{\overline{v'^2}}/U_b$, and (c) $-u'v'/U_b^2$ in the $(x, y)$-plane, where $u'$ and $v'$ are the instantaneous fluctuations. $Re_h = 3350$. 
Contours of the turbulence intensities in the streamwise and normal directions and the Reynolds shear stress are shown in Fig. 5.14. Consistent with the conceptual sketch in Fig. 5.1, three distinct regions are found. First, a surface of vanishing $\Psi$ bounds the separated region. The two intersections of that surface with the wavy wall represent the separation and reattachment lines, which are perpendicular to the $(x, y)$-plane. Note that, in comparison with the instantaneous flow fields in Fig. 5.11, the locations of separation and reattachment are not spatially fixed but they change transiently. Secondly, the streamwise turbulence intensity has a maximum at a location above the wave crest that coincides with a maximum of the Reynolds shear stress, region (2). It is consistent with the experimental results of Hudson et al. (1996), and with the numerical prediction of Cherukat et al. (1998). Third, a region (3) of minimum Reynolds shear stress is located in the vicinity of the wavy wall at its uphill side. This region has previously been connected to structural information in recent DNS and LES studies (e.g. Henn & Sykes, 1999). Note that the limited dynamic and spatial resolution of the PIV measurements limits the accessibility of region (3), see Fig. 5.1. Furthermore, the limited spatial resolution invokes low-pass filtering of the instantaneous velocities which is, if Gaussian behavior is assumed, of limited influence on the mean velocities. However, we are aware that it limits the accuracy of determining the turbulent quantities. By examining the $(y_1, z)$-plane, section 5.3.3, we collect quantitative information on the scale of longitudinal structures and connect it to the characteristic region (2) of the flow.

**POD analysis.** The method of snapshots is used for a proper orthogonal decomposition (POD) of the dimensionless streamwise velocity fluctuations, $u'/U_b$, in the $(x, y)$ plane. The flow field is inhomogeneous in the $x$ and $y$-directions. A single spatial coordinate $1, \ldots, N$ with $N = n \cdot m$ distinguishes between the different positions in the $(x, y)$-plane. We therefore write for the spatio-temporal set of $M = 250$ velocity fields:

$$U = \{U_i\}_{i=1}^{M} = \frac{1}{U_b} \begin{bmatrix} u_{11}, u_{12}, \ldots, u_{1M} \\ u_{21}, u_{22}, \ldots, u_{2M} \\ \vdots \\ u_{N1}, u_{N2}, \ldots, u_{NM} \end{bmatrix} \quad (5.10)$$

with $U_i = 1/U_b[u_1, u_2, \ldots, u_N]^T$, and calculate the fluctuation matrix. We then follow the numerical procedure discussed in section 2.3.2, and obtain the eigenfunctions of the dominating eigenvalues. Eigenfunctions $\Pi_{1, u}, \ldots, \Pi_{6, u}$ are shown in Fig. 5.15. The same dataset as for Figs. 5.12 and 5.14 is used. However, in
Figure 5.15: Contours of the eigenfunctions $\Pi_{1,u}, \ldots, \Pi_{6,u}$ from for a decomposition of $u'/U_b$ from 250 velocity fields of the developed flow (after the 50th crest) in the $(x, y)$-plane. Solid lines represent positive, and broken lines denote negative values, $Re_h = 3350$. 
difference to the mean and second-order turbulence quantities, we recognize that characteristic regions of the obtained eigenfunctions are not restricted to the vicinity of the wavy surface. A larger normal extension of the domain is therefore chosen. Figure 5.16 shows the distribution of the fractional energy amongst the thirty dominant POD modes. Note that $\lambda_1/E$ already contributes 33% of the kinetic energy of streamwise velocity fluctuations that is averaged over the AOV. Furthermore, we note that eigenfunction $\Pi_{1,u}$ has its extrema at a location in the lower channel half, approximately $y/\Lambda = 0.3$ above the wavy wall. In the streamwise direction, the characteristic scale described by $\Pi_{1,u}$ is larger than one wavelength. We explain this significant deterministic contribution as caused by a streamwise-oriented large-scale structure meandering in the spanwise direction, i.e. with a spanwise position that is not fixed. Note that $\Pi_{4,u}, \ldots, \Pi_{6,u}$ have extrema close to the Reynolds shear stress maximum, region (2), and resemble certain characteristics of the $\sqrt{u'^2}$ contours in Fig. 5.14. In the following sections, we will continue to assess the structure of the velocity field in different planes of the flow. One interest is to determine large-scale flow structures in the spanwise direction, $z$, and to learn about their characteristic scale in $z$.

Figure 5.16: Fractional energy $E_k/E$ (solid line) of the thirty dominant POD modes and the cumulative energy (broken line) from a decomposition of $u'/U_b$ for the developed flow in the $(x, y)$-plane at the Reynolds number, $Re_h = 3350$. 
5.3.2 \((x, z)\)-Plane

The instantaneous velocities in the \((x, z)\)-plane are obtained at the wall-normal distance \(y/\Lambda = 0.26\), see Fig. 5.9. In this plane above the wave crests, the perturbations in the velocity field were found to be the largest. Due to the wavyness of the bottom wall, the mean flow is weakly inhomogeneous in the \(x\)-direction in this plane, where the spanwise direction, \(z\), can be considered homogeneous at the measurement location in the channel center. Measurements in the \((x, z)\)-plane are expected to reveal information that is related to large-scale, longitudinal flow structures. In section 5.3.3, these structures are then connected to smaller scales that can be observed at locations closer to the wavy wall surface.

For qualitative information, the flow is first seeded with Kalliroscope reflective flakes. Figure 3.3 in section 3.1.1 shows a SEM image of the tracers. The shape of the titanium dioxide coated flakes is flat, with a length of \(5 - 10 \mu m\), and a thickness of approximately \(0.05 \mu m\). When illuminated with a laser sheet, the reflected light strongly depends on the particle orientation in the flow. Under laminar conditions at \(Re_h = 600\), large fluid columns with a characteristic distance of \(1.5\Lambda\) in the spanwise direction can be observed through the view port at the top wall. For an AOV of \(2.23\Lambda \times 1.79\Lambda\), Fig. 5.17 (a) shows the distribution of the intensity \(I^*(x, z)\) that is obtained with the PCO camera, where \(I^*(x, z)\) represents a local average over \(10 \times 10\) \(\text{pix}^2\) and is made dimensionless so that the extrema of the intensity are equal to \(\pm 1\). Figures 5.17 (b) and (c) correspond to the Reynolds numbers of 2000 and 3800. The obtained images provide information on the scales of the flow. However, they also demonstrate the limitations of the visualization method with regard to quantifying large-scale flow structures.

To quantitatively address the role of large-scales in the \((x, z)\)-plane, we first consider the flow at a Reynolds number of 700. Figure 5.18 shows a sequence of nine contour plots of the instantaneous streamwise velocity that is obtained from transient PIV measurements with a time separation of 1 s and a large AOV of \(2.23\Lambda \times 1.79\Lambda\) being obtained with the PCO camera. The streamwise positions \(x/\Lambda = 0.0, 1.0\) denote wave crests, and \(x/\Lambda = 0.5, 1.5\) are troughs. Initially, the flow is laminar and large variations of the streamwise velocity are found in the spanwise direction.

In frames (a-f) of Fig. 5.18, the spanwise extension of the fluid columns is fixed. However, a slow lateral motion is observed. In frames (g-i), the traverse motion increases. Even though the Reynolds number is kept constant, transition to turbulence eventually occurs in frames (k-m). This process periodically repeats itself at
Figure 5.17: Instantaneous contours of the dimensionless light intensity $I^*(x, z) : [-1, \ldots, 1]$ that is reflected from flakes in the $(x, z)$-plane at $y/A = 0.26$ for $Re_h = 600$ (a), 2000 (b), and 3800 (c). Streamwise locations $x/A = 0.0, 1.0$ denote wave crests, and $x/A = 0.5, 1.5$ are troughs. AOV=$2.23A \times 1.79A$. 

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Figure 5.18: Sequence of contours for the instantaneous streamwise velocity, $u'/U_b$, in the $(x, z)$-plane at $y/\Lambda = 0.26$ during transition to turbulence at $Re_h = 700$. Streamwise locations $x/\Lambda = 0.0, 1.0$ denote wave crests, and $x/\Lambda = 0.5, 1.5$ are troughs. The individual velocity fields (a-m) are separated by a temporal difference of 1.0 s. AOV= $2.23\Lambda \times 1.79\Lambda$. 
the fixed Reynolds number. In a further step we examine, whether similar structures are present at turbulent flow conditions. The considered Reynolds number is 7300. Figure 5.19 shows a sequence of nine contours for the instantaneous streamwise velocity that is acquired with the same AOV and the maximum frame rate of the PCO camera in the frame straddle mode, 3.75 Hz. Subsequent realizations are therefore separated by 0.27 s. Already from the instantaneous plots, the existence of large-scale, longitudinal structures becomes obvious. However, quantitative contributions of the different scales, or the dominant scale cannot be found by such means. One observation however is, that the observed longitudinal structures do not have fixed spanwise locations but they meander laterally. Figure 5.20 shows the spanwise variation of the wave-averaged mean streamwise velocity for Re_h = 3800 (dotted line) and 7300 (dashed line). The averages are obtained from sequences of 250 vector fields that were acquired with a frame rate of 1 Hz. The curves are made dimensionless with their averages in the spanwise direction

\[
(U)_{xz} = \frac{1}{\Delta z \Lambda} \int_0^{\Lambda} \int_{z_1}^{z_2} U(z) dz dx.
\]

For the two turbulent flow cases, the spanwise variation of the mean streamwise velocity is smaller than ±2%. In this context we relate the large spanwise variation of the mean streamwise velocity of up to ±7% that was reported from the wind-tunnel measurements of Gong et al. (1996) to the relatively low aspect ratio of their facility. The graph that corresponds to Fig. 5.18 (a) is plotted as a solid line, where spanwise variations of up to up to ±20% are found.

**POD analysis.** From PIV measurements in the \((x, z)\)-plane, the streamwise and the spanwise velocity component, \(u(x, z, t)\) and \(w(x, z, t)\), are obtained at discrete times \(t_i\), with \(i = 1, \ldots, M\), at \(1, \ldots, m\) discrete \(x\)-locations, and \(1, \ldots, n\) discrete \(y\)-locations. For characterizing large-scale flow structures at turbulent conditions, we use the method of snapshots that was described in section 2.3.2 and perform a POD analysis for the streamwise velocity component. A single coordinate \(1, \ldots, N\) with \(N = n \cdot m\) is used to distinguish between the different positions in the \((x, z)\)-plane and we write the set of spatio-temporal velocity data as:

\[
U = \{U_i\}_{i=1}^{M} = \frac{1}{U_b} \begin{bmatrix}
  u_{11}, u_{12}, \ldots, u_{1M} \\
  u_{21}, u_{22}, \ldots, u_{2M} \\
  \vdots \\
  u_{N1}, u_{N2}, \ldots, u_{NM}
\end{bmatrix}
\]

with \(U_i = 1/U_b[u_1, u_2, \ldots, u_N]^T\). Note that the \(N \times M\)-matrix \(U\) is already normal-
Figure 5.19: Sequence of contours for the instantaneous streamwise velocity, $u' / U_b$, in the $(x, z)$-plane at $y / \Lambda = 0.26$. The streamwise locations $x / \Lambda = 0.0$, 1.0 denote wave crests, and $x / \Lambda = 0.5$, 1.5 are troughs. The individual velocity fields (a-i) are separated by a temporal difference of 0.27 s. $AOV = 2.23 \Lambda \times 1.79 \Lambda$, $Re_b = 7300$.

ized with $U_b$. We obtain the mean velocity by averaging over the columns:

$$\bar{U} = \frac{1}{M} \sum_{i=1}^{M} U_i, \quad i = 1, \ldots, M. \quad (5.13)$$
Figure 5.20: Spanwise variation of the streamwise-averaged streamwise mean velocity (normalized) for the turbulent Reynolds numbers $Re_\theta = 3800$ (dotted line), 7300 (broken line), and for $Re_\theta = 700$ (solid line with circles) before the transition.

For the velocity fluctuations then follows:

$$U'_i = U_i - \bar{U}, \quad i = 1, \ldots, M.$$  \hspace{1cm} (5.14)

Using the method of snapshots, the $M \times M$ covariance matrix becomes:

$$C_{ij} = \langle U'_i U'_j \rangle, \quad i, j = 1, \ldots, M,$$  \hspace{1cm} (5.15)

where $\langle \cdot, \cdot \rangle$ is the Euclidean inner product. Since the matrix is symmetric its eigenvalues, $\lambda_i$, are nonnegative, and its eigenvectors, $\phi_i$, $i = 1, \ldots, M$, form a complete orthogonal set. The orthogonal eigenfunctions are:

$$\Pi^{[k]} = \sum_{i=1}^{M} \phi_i^{[k]} U'_i, \quad k = 1, \ldots, M,$$  \hspace{1cm} (5.16)

where $\phi_i^{[k]}$ is the $i$-th component of the $k$-th eigenvector. Index $i$ distinguishes between different velocity fields, not between different velocity components. For the contribution of the streamwise fluctuations to the turbulence energy we can write:

$$\frac{E}{\theta} = \left\langle \left( \frac{u'}{U_b} \right)^2 \right\rangle_{xz} U_b^2 = \sum_{i=1}^{M} \lambda_i$$  \hspace{1cm} (5.17)
and the fractional contribution of one eigenfunctions associated eigenvalue is

\[
\frac{E_k}{E} = \frac{\lambda_k}{E/\varrho}.
\]  

(5.18)

Since the spanwise flow direction is homogeneous, note that the POD analysis is identical to a Fourier decomposition in this direction. However, this is not true in the weakly inhomogeneous \(x\)-direction, where POD analysis is the correct way of decomposing the velocity field. Note that, somewhat similar to the approach chosen by Liu et al. (2000), our attempt is not to filter characteristic eddies. Figure 5.21 shows the eigenfunctions \(\Pi_{1,u}, \ldots, \Pi_{6,u}\) that correspond to the six dominant eigenvalues, \(\lambda_1, \ldots, \lambda_6\), being ranked in a decreasing order of their fractional contribution to the turbulent kinetic energy in the streamwise direction, \(E\). Figure 5.22 shows the fractional contributions \(E_k/E\) and the cumulative values for \(\lambda_1, \ldots, \lambda_{30}\). Solid circles correspond to \(Re_h = 7300\), and the open circles to 3800. The results suggest that, for the considered Reynolds numbers, the eigenfunctions \(\Pi_{1,u}\) and \(\Pi_{2,u}\) have a characteristic scale \(\Lambda_z = 1.5\Lambda\) in the spanwise direction. This value is identical with the scale that was obtained for the spanwise variation of the mean streamwise velocity at laminar conditions. The cumulative contribution of \((E_1 + E_2)/E\) at \(Re_h = 7300\) is 31% for the lower and 47% for the higher Reynolds number and therefore increases with increasing Reynolds number. POD modes 1, \ldots, 6 have cumulative contributions of 55% and 72% respectively. The open triangles in Fig. 5.22 correspond to results that are obtained in a \((x, z)\)-plane being separated from the top wall by the same distance than the two other measurement locations from the bottom wall, \(\Delta y/\Lambda = 0.26\). The Reynolds number is 3800. From a comparison between the two curves for \(Re_h = 3800\), we conclude that the dominance of modes 1 and 2 is limited to the lower channel half.

Since a relatively large AOV is considered for the measurements in the \((x, z)\)-plane, the influence of the spatial measurement resolution on the results is a possible point of concern. Figure 5.23 shows the spanwise variation for the streamwise-averaged eigenfunctions \(\langle \Pi_{1,u}(z) \rangle_x\) and \(\langle \Pi_{2,u}(z) \rangle_x\) that are obtained for a decomposition of \(u'/U_b\) in the \((x, z)\)-plane at \(Re_h = 7300\). From the same sequence of image data, velocity fields were extracted through local cross-correlation with the spot sizes of \(32 \times 32\) pixel\(^2\) and \(64 \times 64\) pixel\(^2\). Very good agreement can be found if, for the eigenfunctions \(\langle \Pi_{1,u}(z) \rangle_x\) and \(\langle \Pi_{2,u}(z) \rangle_x\), the results for the two spot sizes are compared with each other. The result suggests that the large spanwise scales determined by modes 1 and 2 are not effected by the spatial resolution of the measurements. In the upper half of Fig. 5.24, the spanwise variations of the streamwise-averaged eigenfunctions \(\langle \Pi_{1,u}(z) \rangle_x\) and \(\langle \Pi_{2,u}(z) \rangle_x\) at \(Re_h = 7300\) (Fig. 5.21) are compared to
Figure 5.21: Eigenfunctions $\Pi_{1,u}, \ldots, \Pi_{6,u}$ in the $(x, z)$-plane at $y/\Lambda = 0.26$ from a POD analysis of the velocity component, $u'/U_b$. AOV=2.23$\lambda \times 1.79\lambda$, $Re_b = 7300$. 
Figure 5.22: Fractional energy $E_k/E$ (solid lines) of the thirty dominant POD modes and the cumulative energy (broken lines) from a decomposition of $u'/U_b$ for the developed flow in the $(x, z)$-plane $y/\Lambda = 0.26$ at $Re_h = 3800$ ($\circ$), and $7300$ ($\bullet$). Curve $\triangle$ shows the corresponding decomposition of the velocity fields that is obtained for the same distance from the flat top wall, $Re_h = 3800$, AOV$=2.23\Lambda \times 1.79\Lambda$. 
Figure 5.23: Comparison between the spanwise variation of the streamwise-averaged eigenfunctions \( \langle \Pi_{1,u}(z) \rangle_x \) and \( \langle \Pi_{2,u}(z) \rangle_x \) obtained for a decomposition of \( u'/U_b \) in the \((x,z)\)-plane at \( Re_h = 7300 \) for spot sizes of \( 32 \times 32 \) pix\(^2\) (dash-dotted line) and \( 64 \times 64 \) pix\(^2\) (solid line). AOV=2.23\(\Lambda \times 1.79\Lambda\).

Each other. The curves are shifted in the homogeneous spanwise direction so that their maxima coincide at \( z/\Lambda = 0 \). In the lower half of Fig. 5.24, the streamwise-averaged streamwise mean velocity is plotted for a laminar flow field at \( Re_h = 700 \), see Fig. 5.18 (a). The mean streamwise velocity Gong et al. (1996), obtained at a location \( y/\Lambda = 0.16 \) above the 12th crest in a BL flow over waves is included into the plot. Again, the traverse position of both curves is adjusted. In the study of Gong et al. (1996) a wind tunnel with the relatively low aspect ratio of 4 : 1 was used. The comparison between the curves that were obtained at different flow configurations confirm a characteristic scale of \( \Lambda_z = 1.5\Lambda \) in the spanwise direction. In that context we note that for the measurements of Gong et al. (1996), the spanwise variation of the mean flow was not symmetric to the channel center, suggesting that \( \Lambda_z \) is not
a multiple integer of $\Lambda$. We can summarize that the dominant eigenfunctions are characterized by a spanwise scale of $1.5\Lambda$. Their influence is restricted to the lower half of the channel and increases with increasing Reynolds number.

### 5.3.3 $(y_1, z)$-Plane

In this section we connect the structural information that we obtained in the $(x, z)$-plane at $y/\Lambda = 0.26$ to structures in the vicinity of the uphill side of the wavy surface. Since the optical access for laser-sheet measurements in the $(y, z)$-plane is limited in a water channel facility, we instead consider a plane that is tilted by an angle of $\beta = 53^\circ$ to the $x$-axis, and refer to it as the $(y_1, z)$-plane, see Fig. 5.9. The considered plane intersects with the wavy wall at an uphill location, where we
Figure 5.25: Optical configuration for measurements in the \((y_1, z)\)-plane: (a) schematic of the optical path, and (b) photograph of the set-up with the water prism.
expect structural information that is connected to the local curvature of the wall – i.e. the Görtler mechanism – and to the Reynolds shear stress maximum, region (2) in Fig. 5.1. In order to allow distortion-free imaging through the optical viewport at the top-wall of the channel, we position a second window parallel to the light sheet plane and fill the volume between the two glass windows with water, see Fig. 5.25. Such a configuration is sometimes referred to as a water prism.

To obtain quantitative information, PIV measurements are performed using monodisperse latex particles. Such measurements are challenging since the mean flow passes through the measurement plane, and velocities in the spanwise direction are expected to be small compared with the mean streamwise velocity. Measurements are performed with the Kodak Megaplus camera. In order to provide an acceptable spatial resolution in the near-wall region, a smaller AOV of $0.67\Lambda \times 0.67\Lambda$ is used. We first examine the flow at laminar conditions. For a Reynolds number of 600, corresponding to a bulk velocity of 0.033 m/s, Figure 5.26 shows the velocity variation in the $y_1$-direction, $\left(V_1 - \langle V_1 \rangle_z \right)/U_b$, and in the spanwise direction, $W/U_b$, where $\langle V_1 \rangle_z$ denotes an average in the spanwise direction (constant $y_1$). Line $y_1 = 0$ describes the intersection between the measurement plane and the uphill side of the wall ($x = 0.818\Lambda, y = 0.021\Lambda$) and $y_1 = 0.67\Lambda$ corresponds to a location close to the channel center, $(x = 0.415\Lambda, y = 0.556\Lambda)$. The $(x, z)$-plane with $y/\Lambda = 0.26$ that was studied in the previous section, intersects with the $(y_1, z)$-plane at $y_1 = 0.30\Lambda$.

At turbulent flow conditions, Fig. 5.27 shows velocities (left) and the corresponding velocity fluctuations (right) that are obtained in the $(y_1, z)$-plane at three
Figure 5.27: Instantaneous velocities (left) and velocity fluctuations (right) in the $(y_1, z)$-plane. The horizontal broken line marks region (2), where the Reynolds shear stress has a maximum. $AOV=0.67 \Lambda \times 0.67 \Lambda$, $Re_h = 3800$. 

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instances in time. The Reynolds number is 3800, and velocities are made dimensionless with the corresponding bulk velocity, $U_b = 0.26$ m/s. The broken horizontal lines mark regions (2) and (3) in Fig. 5.1, where the Reynolds shear stress has a local maximum or minimum.

![Diagram](image1)

(a)  

![Diagram](image2)

(b)  

![Diagram](image3)

(c)  

![Diagram](image4)

(d)  

Figure 5.28: Contours of the two-point correlation functions $R_{uv}(y_1, y_1', \Delta z)$ (a, c) and $R_{ww}(y_1, y_1', \Delta z)$ (b, d) in the $(y_1, z)$-plane that are evaluated for $y_1' / \Lambda = 0.3$ (a,b), and 0.1 (c,d). The horizontal broken line marks region (2), where the Reynolds shear stress has a maximum. AOV=0.67$\Lambda \times 0.67\Lambda$, $Re_h = 3800$.

Two-point correlation function. Similar to the approach that was used in chapter 4, where we assessed large-scale flow structures in a turbulent Rayleigh-Bénard
flow, we first calculate the two-point correlation functions in the \((y_1, z)\)-plane. As for the Rayleigh-Bénard flow field, the considered turbulent flow field is characterized by a homogeneous direction, \(z\), and an inhomogeneous direction, \(y_1\), in the \((y_1, z)\)-plane. The ensemble contains 189 velocity fields that were acquired at a frame rate of 1 Hz. Figure 5.28 shows the obtained two-point correlation functions \(R_{v_1,v_1}(y_1,y'_1,\Delta z)\) (a, c) and \(R_{uv}(y_1,y'_1,\Delta z)\) (b, d) that are evaluated at two wall distances, \(y'_1/\Lambda\). Location \(y'_1/\Lambda = 0.3\) (a, b) corresponds to \(y/\Lambda = 0.26\), where we previously obtained velocity measurements in the \((x, z)\)-plane (section 5.3.2), and location \(y'_1/\Lambda = 0.1\) (c, d) characterizes the maximum Reynolds shear stress region (2). Based on the findings in section 5.3.2 and on the results of Gong et al. (1996) we expect structural information from a spatio-temporal decomposition of the velocity fluctuations in the inhomogeneous flow direction, i.e. for the velocity component \(v'_1\). The analysis of two-point correlations in chapter 4 and Eq. (2.24) led to the conclusion that mainly the dominant large-scale contributions appear in the two-point correlation function, and POD analysis provides a more effective tool for extracting structural information for different scales. A similar observation is made here. We first recognize that the functions \(R_{v_1,v_1}\) are better suited for obtaining structural information than \(R_{uv}\). The dominant flow structures, which we identified to have a characteristic spanwise scale of \(1.5\Lambda\) in section 5.3.2, are well represented by \(R_{v_1,v_1}(y_1,y'_1/\Lambda = 0.3,\Delta z)\) in Fig. 5.28 (a). However, the limitations of this approach become obvious in part (c), for \(R_{v_1,v_1}(y_1,y'_1/\Lambda = 0.1,\Delta z)\). Here, we expect to obtain information on the smaller structures that we found from the instantaneous realizations close to region (2) in Fig. 5.27, where the Reynolds shear stress has a maximum. However, since the information is dominated by the large-scale flow structure, we only obtain a very weak maximum in the two-point correlation function, denoted with “A” in Fig. 5.28 (c).

**POD analysis.** In order to better distinguish between the large-scale structures of spanwise scale \(1.5\Lambda\) and the structures that, suggested by Fig. 5.27, seem to be locally present at the uphill locations of the wavy wall, we perform a POD analysis. We use the method of snapshots and, similar to Eq. (5.12), we now consider a set

\[
V_1 = \{V_1\}_{i=1}^{M}.
\]  

(5.19)

The analysis is performed for the \(M = 189\) velocity fluctuations

\[
V_i' = V_i - \overline{V}_1, \quad i = 1, \ldots, M
\]

(5.20)
with the mean

$$\bar{v}_1 = \frac{1}{M} \sum_{i=1}^{M} v_{1i}. \quad (5.21)$$

Figures 5.29 and 5.30 show contours of the eigenfunctions $\Pi_{1,v_1}, \ldots, \Pi_{7,v_1}$ that correspond to the eigenvalues $\lambda_1, \ldots, \lambda_7$. The POD modes are ranked in a decreasing order of their individual contribution to the kinetic energy in the $y_1$-direction. The characteristic length scale in the spanwise direction, $\Lambda_z = 1.5\Lambda$ can be connected to the measurements in the $(x,z)$-plane for the dominant two POD modes. Extrema of $\Pi_{1,v_1}$ and $\Pi_{2,v_1}$ are obtained at a location $y_1 = 0.3\Lambda$, corresponding to
Figure 5.30: Contours of the eigenfunctions $\Pi_{5,v_1}, \ldots, \Pi_{7,v_1}$ from POD modes 5, \ldots, 7 for a decomposition of $v'_1/U_b$ from 189 velocity fields of the developed flow (after the 50th crest) in the $(y_1, z)$-plane. Solid lines represent positive, broken lines negative values, and the horizontal broken line marks region (2), where the Reynolds shear stress has a maximum. AOV=0.67$\Lambda \times 0.67$,$\Lambda$, $Re_b = 3800$.

$y = 0.26\Lambda$, where measurements were previously taken in the $(x, z)$-plane. This information is consistent with the one retrieved from the two-point correlation function $R_{v_1,v_1}(y_1, y'_1/\Lambda = 0.3, \Delta z)$ in Fig. 5.28 (c).

Eigenfunctions of higher modes are characterized by smaller scales $\Lambda_z$, $0.2 - 0.3\Lambda$ for modes 6 and 7. For these two modes we note that the extrema of the eigenfunctions coincide with the Reynolds shear stress maximum location. In comparison with the previously obtained two-point correlation function we conclude that such structural information can solely be accessed from a POD analysis, i.e. the decomposition of the entire tensor $R_{v_1,v_1}$ into optimal eigenfunctions, but is only
Figure 5.31: Fractional energy $E_k/E$ (dotted lines) of the thirty dominant modes, and cumulative value (solid lines) from a decomposition of the velocity component $v_1/U_b$ for the developed flow in the $(y_1, z)$-plane at the two Reynolds numbers, $Re_b = 3800 (\circ)$, and 7300 (●), and for the non-developed flow at $Re_b = 3800 (\Delta)$. 

very weakly contained in individual representations of $R_{v_1,v_1}(y_1, y_1'/\Lambda = 0.1, \Delta z)$.

Figure 5.31 shows the fractional (broken line) and cumulative (solid line) con-
tributions to the turbulent kinetic energy in the \( y_1 \) direction amongst the dominant eigenvalues, \( \lambda_1, \ldots, \lambda_{30} \). Results for the two turbulent Reynolds numbers, 3800 (open circles), and 7300 (solid circles), are given. Consistent with the findings in the \((x, z)\)-plane, the relative contribution of POD modes 1 and 2 increases for increasing Reynolds number. For \( Re_h = 3800 \) and 7300, the cumulative contribution of these modes is 29% and 39%, where modes 1, \ldots, 7 contribute 50% and 61% respectively. However, the dominant eigenfunctions \( \Pi_{1,x1} \) that are obtained at the two Reynolds numbers, are similar. It is important to note the difference regarding the energy spectra for the two Reynolds numbers. From a POD analysis of PIV measurement data in the \((x, y)\)-plane of a developed, turbulent flow through a flat-walled channel at the two Reynolds numbers \( Re_h = 5380 \) and 29900, Liu et al. (1994) found both, the eigenfunction, and the energy spectra to be independent of Reynolds number when scaled with outer variables. Applied to Fig. 5.31, we would therefore expect the curves for plain channel flow at different Reynolds numbers to be parallel in the double-logarithmic plot, independent of the applied scaling. However, this is not the case here. Instead, the relative contributions from the first two modes increase. This observation is consistent with the findings in the \((x, z)\) plane, see Fig. 5.22, and we relate it to the influence of a wave-induced instability that increases with Reynolds number. However, since the range of Reynolds numbers considered so far is relatively small, this issue provides a motivation for future studies at an extended range of Reynolds numbers, see chapter 6.

The triangles in Fig. 5.31 show the situation at measurement location (4) in Fig. 5.4, where the non-developed flow is assessed at \( Re_h = 3800 \) in the \((y_1, z)\)-plane after the 4th crest. A comparison with the results for the developed flow shows that a dominance of POD modes 1 and 2 is not pronounced. Figures 5.32 and 5.33 show the eigenfunctions \( \Pi_{1,y1}, \ldots, \Pi_{7,y1} \) for the non-developed case. Characteristic scales that are obtained from the eigenfunctions of the first modes are smaller than \( \Lambda_z = 1.5 \Lambda \), which was observed for the developed flow. Extrema of the eigenfunctions are located closer to the wavy surface. From these observations we conclude that the non-developed flow does not already contain the structures we observe for the developed flow situation, but they develop and grow when passing the periodic train of waves in the streamwise direction.
Figure 5.32: Contours of the eigenfunctions $\Pi_{1,v_1}, \ldots, \Pi_{4,v_1}$ from the dominant POD modes $1, \ldots, 4$ for a decomposition of $v'_i/U_b$ from 214 velocity fields of the non-developed flow (after the 4th crest) in the $(y_1, z)$-plane. Solid lines represent positive, broken lines negative values. $AOV=0.82\Lambda \times 0.82\Lambda$, $Re_b = 3800$.

5.4 Results for Flow over Heated Waves

The studies reported so far have been for isothermal flow. Krettenhauer & Schumann (1992) performed a DNS and LES computations for $\alpha = 0; 0.05; 0.1; 0.2$, $Ra = 4.9 \times 10^5$, and $Pr = 0.7$. Kenjereš & Hanjalić (2001) used Reynolds-averaged Navier-Stokes equations (T-RANS) for studying natural convection above a heated, sinusoidal wall with $\alpha = 0.1$, $Ra = 10^7 - 10^9$, and $Pr = 0.71$. For turbulent flow over waves, the effect of large-scale structures on scalar transport is experimentally investigated for the first time. In chapter 3, a combined PIV and LCT technique for measuring the fluid velocity and temperature was developed. As a transient ref-
Figure 5.33: Contours of the eigenfunctions $\Pi_{5,v_1}, \ldots, \Pi_{7,v_1}$ from POD modes 5, $\ldots$, 7 for a decomposition of $v'_i/U_b$ from 214 velocity fields of the non-developed flow (after the 4th crest) in the $(y_1,z)$-plane. Solid lines represent positive, broken lines negative values. AOV=0.82$\Lambda$ $\times$ 0.82$\Lambda$, $Re_b = 3800$.

In the reference flow case, it was applied to turbulent Rayleigh-Bénard convection in chapter 4. We are now interested in facilitating LCT in a flow over resistively heated waves, see also Günther & Rudolf von Rohr (2002c). In particular, we assess the effect of the large-scale longitudinal structures, similar to the ones obtained in the velocity field of the isothermal flow in section 5.3, with respect to scalar transport.

Section 5.4.1 characterizes the properties of the resistively heated wall. For two different Reynolds numbers, instantaneous temperature fields in the $(y_1,z)$-plane are presented in section 5.4.3, and section 5.4.4 provides structural information based on two-point correlation functions and on POD analysis.
Composition | 4.0%Ni, 11.0%Mn, 85.0%Cu
---|---
Density, $\rho_M$ | 8.4 g/cm³
Specific electric resistance (20°C), $\rho_{el,M}$ | 0.482 Ωmm²m⁻¹
Temp. coefficient of $\rho_{el,M}$ (15 – 35°C), $a_M$ | $(0 \pm 0.15) \times 10^{-6}$ K⁻¹
Thermal conductivity of CuMnNi (20°C), $\lambda_M$ | 22 Wm⁻¹K⁻¹
Spec. heat capacity of CuMnNi (20°C), $c_M$ | 0.410 kJkg⁻¹K⁻¹
Thermal expansion of CuMnNi (20 – 100°C), $\beta_M$ | $18 \times 10^{-6}$ K⁻¹

Table 5.3: Mechanical and electrical properties of manganin (CuMnNi).

### 5.4.1 Properties of the Resistively Heated Wall

The manganin covered wall (length $L_M$) of the test section (see Fig. 5.10) is now resistively heated in order to provide a constant heat flux boundary condition. Manganin (CuMnNi) has a moderate specific electric resistance and, in the range between 15°C and 35°C, a low temperature coefficient, see Table 5.3. Its resistivity is given by

$$R_M = \frac{\rho_{el,M} L_M}{A_M}, \quad (5.22)$$

where $\rho_{el,M}$ denotes the specific electric resistance. The length, $L_M$, of the manganin layer being measured between the two electrodes along the wavy profile $y_w(x)$, Eq. (5.5), is obtained as:

$$L_M = \int_0^{1m} \sqrt{1 + y_w'^2(x)} \, dx = 1.024 \, \text{m}, \quad (5.23)$$

where 1 m is the horizontal distance between the two electrodes, and $A_M$ is the cross-section of the metal foil, equal to $0.045 \times 300$ mm². The resistance of the foil follows to $36.6 \, \text{mΩ}$.

A direct current (DC) welding transformer with a maximum current of 380 A provides the power supply. The electric circuit for measuring the voltage and current is shown in Fig. 5.34. The majority of the current $I$, heats the wave section, where a fraction of it travels as a blind current through the working fluid. To decrease this effect, de-ionized water is used and the water conductivity is monitored during the experiments. For a neglected blind current, the resulting heat flux can be obtained with the resistance, Eq. (5.22), and the measured current follows to:

$$Q_w = R_M I^2. \quad (5.24)$$

Figure 5.35 illustrates the absolute wall heat flux, $Q_w$, and the specific heat flux, $q_w$, provided by the manganin foil as a function of $I$. The vertical dashed line marks
the upper limit for the current that can be provided by the used DC supply. With the specific heat capacity of water at 35°C, \( c_p = 4.179 \text{ kJ/(kgK)} \) and the total mass of working fluid, \( m = 280 \text{ kg} \), the temporal change of the working fluids mean temperature without cooling can be estimated:

\[
\frac{\Delta T}{\Delta t} = \frac{R_M I^2}{mc_p}. \tag{5.25}
\]

The corresponding axis is included at the left side of Fig. 5.35 and represents an upper limit since it neglects blind currents and heat losses to the environment. At the right axis, the corresponding voltage across the foil is shown. Note that for all considered operating conditions, the voltage is below the safety limit for DC supplies of 75 V (e.g., European community low voltage directive, 1973), and even below the very low voltage limit of 24 V. In the following we estimate the effects of blind currents and conductive losses.

**Effect of blind currents.** Blind currents flow between the manganin foil and the grounded aluminum top wall and can be estimated for a known specific resistance of water. For the used de-ionized water, \( \rho_{\text{H}_2\text{O}} = 200 \Omega\text{m} \) is a typical value.
measured with a resistivity probe; for tap water it is approximately 20 Ωm. Since the PVC bottom wall is characterized by a very high specific electric resistance of $\rho_{PVC} = 10^{13} \, \Omega m$, the electric losses through it can be neglected. With the voltage $\pm U$ applied at both ends of the foil, a foil width $B_M$ of 0.3 m, the channel height, $H = 0.03$ m, and the length of the foil, $L_M$, we obtain for the ratio between the power loss due to the blind current, $Q_{\text{blind}}$, and the heating power at the wall surface, $Q_w$:

$$\frac{Q_{\text{blind}}}{Q_w} = \frac{U^2 B_M L_M}{3 \rho_{el,h2o} H^2}.$$  

(5.26)

At a direct current of 200 A, one obtains a relative loss of heating power of 0.014% for the de-ionized water, and 0.14% for tap water, which is smaller than the uncertainty of the temperature and velocity measurements. For the known resistance $R_M$, we therefore conclude to the heating power $Q_w$ by measuring the direct current.

**Effect of conductive heat losses.** Conductive losses through the PVC bottom wall are relatively small. With an average thickness of the supporting PVC wall of 20 mm, and a thermal conductivity of 0.09 W/(m · K), and a maximum temperature difference of 10°C across it, the heat loss due to conduction through the
supporting PVC material is, for a current of 200 A, approximately 1% of the heat that is provided by the manganin foil.

5.4.2 Influence of Natural Convection

![Graph showing relative deviation of fluid properties vs. temperature difference](image_url)

Figure 5.36: Relative deviation of the fluid properties: kinematic viscosity, $\nu$, density, $\rho$, and thermal conductivity, $\lambda$, for water with respect to the low-end of the temperature sensitive range, $T_1 = 33.77^\circ C$.

For most numerical experiments, where turbulent flow studies are extended towards scalar transport, a passive scalar is considered, i.e. the temperature dependency of the fluid properties is neglected. Even in chapter 2 the validity of the Boussinesq-approximation – i.e. a linear dependency of the fluid density on the temperature, whereas all other fluid properties are constant – was assumed. However, for real fluid systems, a temperature dependency of the fluid properties exists. Figure 5.36 illustrates the relative deviation of the kinematic viscosity, $\Delta \nu$, the density, $\Delta \rho$, and the thermal conductivity, $\Delta \lambda$, for water with respect to the low-end of the effective temperature sensitive range, $T_1 = 33.77^\circ C$, for the used LC formulation.
BM/R35C20W. The light-sheet plane and the camera axis are perpendicular. Since a constant heat flux boundary condition is imposed at the wavy surface, the wall and the bulk temperatures increase in the streamwise direction. Considering the given dynamic range of the measuring technique, $\Delta T_{\text{meas}} = 1.36^\circ \text{C}$, see Fig. 3.18, the maximum deviation of the fluid properties is indicated by the vertical dotted line in Fig. 5.36.

**Velocity Scales**

We determine the buoyancy influence on the flow over heated, solid waves, i.e. we study a mixed convection problem. However, in difference to classical mixed convection, the considered problem is somewhat more complicated, since the directions of the buoyancy force and the meanflow are perpendicular.

$\text{Re}_h \rightarrow 0$. We start our considerations for the limit $\text{Re}_h \rightarrow 0$ at finite Rayleigh numbers. For approximating velocity scales, we neglect the wavyness of the bottom wall and connect to Rayleigh-Bénard convection. Note that the limit $\text{Re}_h \rightarrow 0$ cannot be reached in the channel flow facility, since it would require to constantly remove a defined heat flux at the top wall. Flow regimes could be described by a modified regime diagram, similar to the one suggested by Krishnamurti, Fig. 4.1. Kenjereš & Hanjalić (2001) used Reynolds-averaged Navier-Stokes equations (RANS) to study natural convection between two differentially heated, constant-temperature walls, where the lower wall has a wavy surface. In this case, the constant heat-flux boundary condition at the bottom wall is identical to a constant-temperature one. Through Eq. (4.1), the buoyancy influence is commonly quantified by a Rayleigh number. The driving temperature gradient is obtained between the temperatures at the top and the bottom walls. Since a constant heat-flux condition is applied, a characteristic temperature difference $\Delta T \propto q_w H / \lambda$ is often used, where $q_w$ is the specific wall heat flux and $\lambda$ the thermal conductivity of the working fluid. To ensure comparability with chapter 4, we instead use the correlation of Rossby (1969), Eq. (4.6), and obtain for the temperature difference

$$\Delta T_R = \left[ \frac{1}{0.131} \frac{q_w H^{0.1}}{\lambda} \left( \frac{\nu \kappa}{g \beta} \right)^{0.3} \right]^{1/1.3},$$

where $\kappa$ denotes the thermal diffusivity, $\beta$ is the thermal expansion coefficient, and $\lambda$ is the thermal conductivity. Figure 5.37 shows the obtained temperature difference, $\Delta T_R$, and Rayleigh number, $\text{Ra}_R$, as functions of the applied direct current. For
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Figure 5.37: Rayleigh number $Ra_R$ (left axis) and driving temperature difference $\Delta T_R$ (right axis) as a function of the applied direct current, $I$, according to Rossby’s (1969) correlation for Rayleigh-Bénard between two differentially heated, horizontal walls.

the considered current of $O(250 \, \text{A})$, the obtained Rayleigh number is comparable to $Ra_2$ in chapter 4. The simplest means to obtain a characteristic velocity in RB convection, sometimes referred to as the buoyant velocity, is from an energy balance:

$$U_{buoy} = \sqrt{g H \beta \Delta T_R}.$$  \hfill (5.28)

According to Castaing et al. (1989), for the characteristic velocities in the core the following scaling applies:

$$u_{core} = \left( \frac{\beta g q_w H}{\rho c} \right)^{1/3}.$$  \hfill (5.29)

Figure 5.38 shows the two velocity scales versus the applied current. Significant differences can be observed. The largest values are obtained from Eq. (5.4.2): $u_{buoy} = 0.043 \, \text{m/s}$ for a current of 235 A. One criterion for equally important roles of natural and forced convection would therefore be, that the characteristic velocity $u_{buoy}$ is matched by the bulk velocity, $U_b$. By such means, we obtain a Reynolds number of $Re_h = 1100$.

Finite $Re_h$. For finite Reynolds numbers, i.e. forced convection present, the situation is more complex. The driving temperature difference, as well as a characteristic
scale in the normal direction become less influential. A Rayleigh number definition can only formally be applied and a Richardson number is often used instead to determine the influence of natural convection. As a result, the estimated upper limit $Re_h = 1100$ for the influence of natural convection is a very conservative estimate and the dominance of natural convection is likely to be limited to Reynolds numbers much smaller than this value. To study this influence on the velocity field, we consider the two cases of $Re_h = 725$, where we expect natural convection to still be influential, and 3300, where forced convection dominates and buoyancy effects are expected to be negligible.

Decomposition of Velocity Fields.

In contrast to evaluating the influence of natural convection based upon the temporal scales of the flow, we perform a POD analysis of the velocity field in the $(x, y)$-plane. Since the directions of the mean velocity of the channel flow, and the flow direction caused by the convective instability are perpendicular, a POD analysis was carried out for the velocity fluctuations in the normal direction, $v'$. The bulk temperature is $35^\circ C$ ($Pr = 4.8$). Since the flow is laminar for low Reynolds numbers and isothermal conditions, we prefer to compare flow fields that are obtained for non-isothermal flow at different Reynolds numbers. Two relatively low Reynolds numbers of 725, and
Figure 5.39: Contours of the eigenfunctions $\Pi_{1,v}, \ldots, \Pi_{4,v}$ from the decomposition of the normal fluid velocity fluctuations, $v'/U_h$, for $Re_h = 725$ (left row) and 3300 (right row), after the 50th crest in the $(x,y)$-plane. Solid lines represent positive, broken lines negative values. AOV=$1.34\Lambda \times 1.07\Lambda$, Pr = 4.82.
3300 are considered. For the first one, the influence of natural convection is expected to be the strongest. Without heating, we would expect a transitional flow. In both cases, a heating current of 235 A is applied to the manganin foil. For the velocity component $u'/U_b$, eigenfunctions similar to the ones for isothermal flow, see Fig. 5.15, are obtained. We therefore assess the effect of natural convection by comparing the eigenfunctions from an orthogonal decomposition of the correlation matrix for the normal velocity component, i.e. for $\Pi_{i,v}$. Figure 5.39 shows the eigenfunctions corresponding to the four dominant POD modes at the two Reynolds numbers. Where $\Pi_{1,v}$ and $\Pi_{2,v}$ are similar, the two higher eigenfunctions show for the lower Reynolds number a segmentation similar to the one we obtained in chapter 4 for eigenfunctions $\Pi_{3,v}, \ldots, \Pi_{6,v}$ for Rayleigh-Bénard convection at $Ra_2 = 7.8 \times 10^6$.

Keeping in mind the operating conditions for the techniques that are commonly used for quantitative, 2-D temperature measurements, LCT, and LIF (Fig. 3.7), the small effective temperature sensitive range of $\Delta T_{meas} = 1.32^\circ C$ for LCT in the $\phi = 90^\circ$ configuration with the considered LC formulation, is clearly advantageous. Using the LIF technique would require the temperature difference between the wall and the bulk fluid to be one order of magnitude larger. Establishing such large temperature differences in turbulent water flows has significant implications on the experimental facility. Large heat fluxes would be required from the resistance heater, and, in the recirculation system, to be removed from the working fluid. In addition, these large differences in the temperature and consequently in the density would increase the buoyancy influence by one order of magnitude. This is not desirable, since it would limit the comparability of the results with direct numerical simulations, where mostly scalar transport is considered.

5.4.3 Instantaneous Temperature Fields

In this section, instantaneous temperature fields in a flow over heated waves are presented. The liquid crystal thermometry technique that was described in section 3.2 is facilitated for obtaining the fluid temperature in the $(y_1, z)$-plane, see Fig. 5.9. In difference to section 5.3.3, where the sheet of a Nd:YAG laser is introduced through a side window, we now consider a white light sheet from a metal halid source (Volpi model Intralux IWL500e) with a fiberoptic line converter. Through an aperture, the light sheet thickness is adjusted to approximately 1 mm. The light sheet is coupled in from the optical viewport at the top of the channel. Figure 5.40 schematically shows the optical set-up for the temperature measurements in the $(y_1, z)$-plane over a heated wavy wall that consists of the white-light sheet from
5.4. RESULTS FOR FLOW OVER HEATED WAVES

Figure 5.40: Optical configuration consisting of the white-light sheet from a fiberoptic line converter, a color video camera with a telecentric lens, and a water prism, for the temperature measurements in the \((y_1, z)\)-plane over a heated wavy wall.

Experimental procedures. Measurements are performed at the uphill section of the heated wavy surface, see Fig. 5.9. Through the resistance heater, the temperature of the working fluid in the channel, de-ionized water, is adjusted to a value that corresponds to the lower end of the temperature sensitive range, \(T_1 = 33.77^\circ\text{C}\), for the LC formulation BM/R35C20W. LCT measurements are performed at the downstream locations (7), and (8) in Fig. 5.4. For location (7) the flow has passed 50 wavelengths, where the last 16 are heated with the constant specific heat flux, \(q_w\). For location (8), the flow has passed 63 wavelengths, 29 of them heated. Through a 0.5 mm diameter hole that is located 1.5 m upstream of the measuring position (7) in the channel top wall (position (4)), a suspension of TLC particles and de-ionized water is continuously fed to the working fluid. A syringe pump ensures a constant feeding rate. At the beginning of an experiment, a 60 ml syringe is filled...
with de-ionized water and a TLC particle contents of approximately 10 g.

Note the difference between the (bulk) kinematic viscosity of the working fluid between the measurements presented in section 5.3, where the temperature of the bulk fluid was approximately 20°C, and the measurements with heat transfer at a bulk temperature of 35°C. For the same Reynolds numbers we obtain for the ratio of bulk velocities at the two temperatures:

$$\frac{U_{b,v_{35}}}{U_{b,v_{20}}} = \frac{v_{35}}{v_{20}} = 0.721. \quad (5.30)$$

At the considered Reynolds numbers, 725 ($U_{b,v_{35}} = 0.035$ m/s) and 3300 ($U_{b,v_{35}} = 0.16$ m/s), and for the given working fluid volume in the channel flow facility, the residence times are 14.9 min and 2.9 min, respectively. After the water temperature in the non-heated channel has adjusted to a value slightly above the start temperature of the TLCs effective temperature sensitive range, $T_{W_1} = 33.77°C$, the resistance heater is turned on. At sampling rates of 10 Hz for the lower and 2 Hz for the higher Reynolds number, a sequence of images is acquired with a measuring duration that corresponds to the residence time. In order to obtain satisfactory statistics, the resistance heater is switched off after one measurement series. After the water temperature in the channel has decreased to the initial start temperature, the procedure is repeated for additional image sequences.

**RGB raw images.** Figures 5.41 and 5.42 present instantaneous realizations of the instantaneous RGB intensity distributions that were obtained for an AOV of $1.11\Lambda \times 0.83\Lambda$ at the Reynolds numbers 725, and 3300 respectively. The flow is seeded with encapsulated LC particles of the formulation BM/R35C20W. The lower image boundary is located 1 mm below the line, where the image plane intersects with the wavy surface at an uphill location.

### 5.4.4 Large-Scale Structures in the Temperature Field

At a Reynolds number of approximately 725, comparable with the case where stable fluid columns were observed for laminar flow under isothermal conditions (section 5.3.3), the situation is different. The temperature variation of the fluid density, $\rho$, and kinematic viscosity, $\nu$, become important. As a consequence, the transport of heat cannot be approximated as the transport of a passive scalar. Driven by the influence of buoyancy, turbulent mixed convection is obtained.
Figure 5.41: Instantaneous RGB intensities for a flow over heated waves that is seeded with encapsulated TLCs of Hallcrest product BM/R35C20W (after 50 wavelengths, 16 of which heated) in the $(y_1, z)$-plane. AOV=$1.11\Lambda \times 0.83\Lambda$, $Re_h = 725$ (mixed convection), $Pr = 4.82$, $I = 205$ A.
Figure 5.42: Instantaneous RGB intensities for a flow over heated waves that is seeded with encapsulated TLCs of Hallcrest product BM/R35C20W (after 50 wavelengths, 16 of which heated) in the $(y_1, z)$-plane. $AOV = 1.11\Delta \times 0.83\Delta$, $Re_h = 3300$ (forced convection), $Pr = 4.82$, $I = 255$ A.
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Two-point correlation function for $Re_h = 725$. As previously done for the velocity data, we first calculate the two-point correlation function for the obtained temperature fluctuations,

$$R_{TT}(y_1, y'_1, \Delta z) = \frac{1}{T^2} \langle T'(y_1, z, t)T'(y'_1, z + \Delta z, t) \rangle$$ (5.31)

in the $(y_1, z)$ measuring plane. Figure 5.43 shows the obtained result for $Re_h = 725$ being calculated from 204 instantaneous temperature fields for $y'_1/\Lambda = 0.3$ (a), and 0.1 (b). The measured current is 205 A.

![Figure 5.43: Contours of the two-point correlation function $R_{TT}(y_1, y'_1, \Delta z)$ in the $(y_1, z)$-plane that is evaluated for $y'_1/\Lambda = 0.3$ (a), and 0.1 (b). Solid lines represent positive, broken lines negative values. AOV=1.11A x 0.83A, Re$_h$ = 725 (mixed convection), Pr = 4.82, $I$ = 205 A.](image)

POD analysis for $Re_h = 725$. Figure 5.44 shows the eigenfunctions $\Pi_{1,T}, \ldots, \Pi_{6,T}$ from POD modes 1, $\ldots$, 6 for a decomposition of the fluid temperature fluctuations, $T'_{1}$ from 204 instantaneous realizations in the $(y_1, z)$-plane. The applied current was 205 A. The fractional contribution $E_1/E$ of mode 1 is 16.0%, 15.3% are contributed by mode 2, 8.7% by mode 3, 6.4% by mode 4, 5.3% by mode 5, and 4.8% by mode 6. The cumulative contribution of the dominant six modes is 56.5%. Even though the characteristic scale in the spanwise direction is similar to the $Re_h = 3300$ case, the extension in the $y_1$-direction is significantly different. Figure 5.45 shows the profile of the mean temperature $\langle T \rangle_z(y_1)$ that is evaluated as an average over 140 instantaneous temperature fields (solid line) or over half of this ensemble size (dash-dotted line). Good agreement is found, indicating a sufficient sample size. The horizontal dashed curve indicates the start temperature of the effective temperature sensitive range, $T_{W,1}$, of the used LC formulation. Note that the near-wall resolution of the
CHAPTER 5. FLOW OVER WAVY WALLS

Figure 5.44: Contours of the eigenfunctions \( \Pi_{1,T}, \ldots, \Pi_{6,T} \) from the dominant POD modes 1, \ldots, 6 for a decomposition of the fluid temperature fluctuations, \( T' \), from 204 temperature fields (after the 50th crest) in the \((y,z)\)-plane. Solid lines represent positive, broken lines negative values. AOV=1.11\( \Lambda \times 0.83\Lambda \), \( Re_h = 725 \) (mixed convection), \( Pr = 4.82 \), \( I = 205 \ A \).

temperature measurements is limited, e.g. by reflections at the wall surface.

Two-point correlation function for \( Re_h = 3300 \). Figure 5.46 shows the two-point correlation function \( R_{TT}(y_1, y_1', \Delta z) \) for the temperature field in the \((y_1, z)\)-
Figure 5.45: Mean temperature profile obtained from averaging over a sample with 70 vs. 140 temperature fields (after the 50th crest) in the \((y_l, z)\)-plane. Solid lines represent positive, broken lines negative values, \(T_{W,1}\) marks the start of the effective temperature sensitive range. \(AOV=1.11\Lambda \times 0.83\Lambda, Re_h = 725\) (mixed convection), \(Pr = 4.82, I = 205\) A.

plane that is evaluated for \(y'_l/\Lambda = 0.3\) (a), and 0.1 (b) for the forced convection case. The ensemble contained 317 realizations of temperature fields. The applied current was 255 A. Note the similarity of the obtained contours with Fig. 5.43 for the mixed convection case.

**POD analysis for \(Re_h = 3300\).** For a Reynolds number of 3300, Fig. 5.47 shows the eigenfunctions \(\Pi_{1,T}, \ldots, \Pi_{6,T}\) from POD modes 1, \ldots, 6 for a decomposition of the fluid temperature fluctuations, \(T'_f\) from 317 realizations in the \((y_l, z)\)-plane. Again, the obtained eigenfunctions reveal more insights on the flow structures than the two-point correlation does. Even though, functions \(R_{TT}\) for the mixed and forced convection cases reveal similar results, differences are obtained for the individual eigenfunctions. In comparison with Fig. 5.44, the eigenfunctions of POD modes 1, \ldots, 6 in Fig. 5.47 all point to structures that are elongated columns in the streamwise direction. They do not indicate an influence of natural convection.
Figure 5.46: Contours of the two-point correlation function $R_{TT}(y_1, y'_1, \Delta z)$ in the $(y_1, z)$-plane that is evaluated for $y'_1/\Lambda = 0.3$ (a), and 0.1 (b). Solid lines represent positive, broken lines negative values. $AOV=1.11\Lambda \times 0.83\Lambda$, $Re_h = 3300$ (forced convection), Pr = 4.82.

(e.g. eigenfunctions similar to $\Pi_{3,T}$, $\Pi_{5,T}$, and $\Pi_{6,T}$ in Fig. 5.44). The fractional contribution $E_1/E$ of POD mode 1 is 17.5%, 13.4% are contributed by mode 2, 10.2% by mode 3, 6.6% by mode 4, 4.2% by mode 5, and 3.2% by mode 6. The cumulative contribution of the shown six modes is 58.6%. At this Reynolds number, the influence of buoyancy, i.e. natural convection, on heat transfer can be neglected.

Note that the eigenfunctions of the dominant POD modes have their extrema close to the location of maximum Reynolds shear stress. The dominant spanwise scale, the distance between two maximum or minimum locations, is $1.5\Lambda$. This observation is consistent with the results previously obtained from a decomposition of the velocity field.
Figure 5.47: Contours of the eigenfunctions $\Pi_{1,T}, \ldots, \Pi_{6,T}$ from the dominant POD modes 1, ..., 6 for a decomposition of the fluid temperature fluctuations, $T_1^f$, from 317 temperature fields (after the 50th crest) in the $(y_1, z)$-plane. Solid lines represent positive, broken lines negative values. $AOV = 1.11\Lambda \times 0.83\Lambda$, $Re_h = 3300$ (forced convection), $Pr = 4.82$.

5.5 Summary and Discussion

5.5.1 Isothermal Flow

Based on spatially resolving measurements, we reported on the structure of the developed flow between a wavy bottom and a flat top wall in a water channel facility.
of aspect ratio of 12 : 1. Conditions between laminar flow and the turbulent flow at a Reynolds number of 7300 are considered. Three characteristic regions of the mean flow that were previously identified could be confirmed from PIV measurements in the (x, y)-plane. To the authors knowledge for the first time, longitudinal structures with a characteristic scale \( \Lambda_z = 1.5\Lambda \) in the spanwise direction are identified from spatially resolving measurements in the (x, z)-plane at the laminar and the two turbulent flow conditions, \( Re_h = 3800 \), and 7300. For laminar flow, the

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**Figure 5.48:** Comparison of the fractional energy \( E_k/E \) for the POD modes 1, \ldots, 30 from a decomposition of the velocity component \( v_1(y_1, z) \) at \( Re_h = 3800 \) and 7300 (AOV=0.67\( \Lambda \times 0.67\Lambda \)), and from a decomposition of the temperature field \( T(y_1, z) \) at \( Re_h = 3300 \) (AOV=1.11\( \Lambda \times 0.83\Lambda \)).
observed structures have spatially fixed positions and $\Lambda_z$ is obtained from instantaneous images. At turbulent conditions, a POD analysis is used to extract $\Lambda_z$ from the dominant modes. We find the two dominant modes to contribute almost 50% of the energy contained in the streamwise velocity fluctuations. Considering the spanwise domain sizes of present DNS and LES studies, the large $\Lambda_z$ is seen to have considerable implications on computational studies, since it imposes requirements on the spanwise domain size required to accurately resolve all scales of the flow. Scale $\Lambda_z$ is consistent with the spanwise variation of the streamwise mean velocity reported by Gong et al. (1996).

**Comparison of two-point correlation functions with plain channel flow.** In the previous sections, we obtained a characteristic spanwise scale of streamwise velocity fluctuations from a POD analysis in the $(x, z)$ and the $(y_i, z)$-planes. In section 5.3.2 we compared measurements in $(x, z)$-planes located $\Delta y/A = 0.26$ above the (wavy) bottom wall and below the (flat) top wall. Based on the comparison of the energy spectrum of the POD modes, the observed characteristic spanwise distance $\Lambda_z$ of $1.5A$ was related to the wavyness of the bottom wall. In this section, we discuss $\Lambda_z$ for the outer region of a turbulent flow through a flat-walled channel in the light of DNS literature data on the two-point streamwise correlation, $R_{uu}(z) = \langle R_{uu} \rangle_x$.

In a similar way, such a correlation provides statistical information about the spacing between high and low streamwise velocities. The relation between the POD eigenfunctions, eigenvalues, and the cross-correlation matrix is given by Eq. (2.24). Figure 5.49 shows $R_{uu}(z)$ from the data of Kim, Moin & Moser (1987), Moser, Kim & Mansour (1999), Gilbert & Kleiser (1993), and Kristoffersen & Andersson (1993). The normal positions, $y/H$, are chosen to be similar to 0.26 that was considered in section 5.3.2. Table 5.4 shows the corresponding experimental parameters, namely the spanwise extension of the computational domain, $B/H$. Note that $B/H$ lies between 1.5 and $\pi$ for all simulations. Results for $R_{uu}(z/2h = 1.5)$ are therefore, if available, very likely to be affected by the finite box size. Considering the moderate spanwise extensions of the computational domains, a comparison of the present findings of $R_{uu}(z/H = 1, \ldots, 1.5)$ for the wavy-wall flow with present DNS data for the flow through a flat-walled channel is therefore not conclusive.

**Structural information at an uphill location.** In the $(y_i, z)$-plane, an experimental set-up with a water prism enabled us, for the first time, to quantitatively connect the $1.5A$-structures to those in the vicinity of the wall surface. Based on instantaneous realizations from numerical studies, first evidence of such structures is
supplied by a number of investigators. However, to date, none of the works quantitatively assessed their spanwise extension and their energy contribution. Two-point correlation functions, and a POD analysis of the $v_1$-velocity component in the $(y_1, z)$-plane were performed. As already found for the RB flow, the POD analysis proves to be the more effective tool with regard to detecting structural information from dominant modes that are characterized by different spatial scales. The eigenfunctions of the POD modes 1 and 2 with a characteristic scale of $1.5\Lambda$ were already identified in the $(x, y)$-plane. The eigenfunctions of POD modes 4, $\ldots$, 7 characterize smaller structures that show local extrema at region (2), the region of maximum Reynolds shear stress.
5.5. SUMMARY AND DISCUSSION

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<tr>
<th>Reference</th>
<th>B/H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kim, Moin &amp; Moser (1987), $Re^+ = 180, y/H = 0.2752$</td>
<td>2.09</td>
</tr>
<tr>
<td>Gilbert &amp; Kleiser (1993), $Re^+ = 211, Re_h = 1333, y/H = 0.222$</td>
<td>2.98</td>
</tr>
<tr>
<td>Kristoffersen &amp; Andersson (1993), $Re^+ = 194, Re_h = 2900, y/H = 0.2264$</td>
<td>3.14</td>
</tr>
<tr>
<td>Moser, Kim &amp; Mansour (1999), $Re^+ = 395, y/H = 0.3252$</td>
<td>1.57</td>
</tr>
<tr>
<td>Moser, Kim &amp; Mansour (1999), $Re^+ = 590, y/H = 0.2536$</td>
<td>1.57</td>
</tr>
</tbody>
</table>

Table 5.4: Conditions for DNS literature data on the turbulent flow through a flat-walled channel.

5.5.2 Flow over Heated Waves

For the first time, the temperature distribution in the $(y_1, z)$-plane of a flow over resistively heated waves was experimentally assessed at conditions of mixed and forced convection. LCT is used to determine the temperature fields. The technique was found advantageous for measurements at flow fields that are characterized by small spatial variations of the fluid temperature in the image plane, i.e. for forced convection. Structural information on the flow is obtained from two-point correlation functions, and from a POD analysis of the temperature fields. From the dominant POD eigenfunctions for both mixed and forced convection, the characteristic spanwise scale of approximately $1.5A$ was obtained from the temperature data in agreement with the decomposition of the velocity field. This observation is considered important, since it documents the effect of the large scales on scalar transport. In the forced convection case, the eigenfunctions that correspond to the higher POD modes, show characteristic spanwise scales that are, also in agreement with the velocity data, smaller and likely to be connected to the local wall curvature, i.e. the Görtler mechanism.
Chapter 6

Concluding Remarks

At moderate Reynolds or Rayleigh numbers, e.g. for RB convection at Ra_1 in chapter 4, the fluid velocity can be obtained from locally cross-correlating single image frames acquired with the color video camera. However, for Ra_2 or for the channel flow measurements, this approach is not applicable any more. Temperature and velocity measurements were therefore de-coupled for the study of the turbulent flow over a wavy bottom wall that is described in chapter 5. For the temperature measurements, a white light sheet and a color video camera (Sony DXC-9100P) were facilitated. For the velocity measurements, a Nd:YAG laser sheet, and a monochrome CCD camera (Kodak Megaplus or PCO Sensicam) were used. In the (x, z) and the (y_1, z)-planes, the dominant eigenfunctions of the velocity v_1(y_1, z), u(x, z), and the fluid temperature, T(y_1, z), could be quantified. At Reynolds numbers of approximately 3000, the dominant eigenfunctions, observed for a POD analysis of both the velocity and the temperature field, show a characteristic spanwise scale of 1.5A. We explain this observation with the amplification of the small positive correlation of the streamwise velocity in the spanwise direction that is observed for turbulent flow through a flat-walled channel through a Langmuir-type mechanism. Future research should address the dynamics of the 1.5A scales since it relates to the required streamwise extension of the computational domain and to the required averaging times.

For a more detailed analysis of turbulent heat transfer in a channel with a wavy bottom wall, we consider the budget Eq. (2.12) in section 2.1.3. Since the flow is homogeneous in the z-direction and only moderately changes in the x-direction, term transport (iii) in Eq. (2.12) is small. Since the flow is steady, the dominant
contribution to the production term \((i)\),

\[-T' u' \frac{\partial T}{\partial y}\]  \(6.1\)

is therefore equal to the molecular dissipation, term \((iv)\). Figure 6.1 proposes

Figure 6.1: Proposed experimental set-up for simultaneous temperature and velocity measurements in the \((x, z)\) or the \((y_1, z)\)-plane. Two monochrome CCD cameras (PCO Sensicam) are positioned in the Scheimpflug configuration. They are used for the velocity measurement and focused on the same AOV as a 3-CCD color camera (Sony DXC-9100P).

an optical configuration for combined temperature and velocity measurements in the \((x, z)\), and the \((y_1, z)\)-planes. For the velocity measurements, a Nd:YAG laser sheet is suggested and, taking advantage of the so-called Scheimflug configuration, two monochrome CCD cameras (e.g., PCO Sensicam) are focused on the AOV. With such a configuration, sometimes referred to as stereographic PIV, the three velocity components can be obtained within the 2-D measurement plane. Note that the measurement uncertainty for the velocity component normal to the plane is relatively large, notably if the direction of the mean flow is almost perpendicular to the measurement plane, i.e. for measurements in the \((y_1, z)\) plane. A short time interval after the two Nd:YAG pulses are fired, a 3-CCD color camera (e.g. Sony DXC-9100P) acquires, within the same AOV, the spatial color distribution of liquid crystal particles that are illuminated with a pulsed sheet of white light. The RGB information can then be used for extracting instantaneous temperature fields. From
a set of instantaneous $v'_T$ fields, a POD analysis can be performed and is expected to reveal information on the scales of the flow in the $(y_1, z)$ plane, i.e. at the uphill side of the wavy wall, that are the largest contributors to the convective heat transfer between the heated wavy wall and the bulk flow.

As it was indicated in Fig. 1.2, one motivation of this work is to study of turbulent flow over wall surfaces with a defined degree of complexity at moderate Reynolds numbers. A restriction to moderate Reynolds numbers has been made, since it allows direct numerical simulations to performed at similar conditions. On the experimental side, moderate Reynolds numbers are advantageous with respect of the limited spatial (or temporal) resolution of the measurement systems.

However, as a motivation for a separate research project and extension of this work, it would be highly interesting to extend the range of considered Reynolds numbers for the same wall geometry. One strong motivation for such an effort is that most technically relevant turbulent flows are characterized by Reynolds numbers that are several orders of magnitude larger than the ones considered herein. Therefore, provided the required resolution is achieved and the Reynolds number is increased by at least one order of magnitude, scaling arguments for such flows can be addressed. Namely the necessary spatial resolution provides a challenge for applying light-sheet techniques in connection with digital imaging techniques (digital PIV, LCT, LIF) to such flow conditions. The experimental implications of measurements at increased Reynolds numbers, given the size of the water channel, are a pump with a higher volume flux, and larger cross-sections in the recirculation loop. For the velocity measurements, we propose to operate two PCO cameras in tandem as shown in Fig. 6.2, rather than to use a commercially available CCD camera with a relative high spatial of 2 K x 2 K but a reduced temporal resolution. Both cameras are mounted on one optical bench with the same optical axis. Through a system of two mirrors, the number of pixels in the available AOV is twice as large (assuming no overlap) as for a single-camera configuration, corresponding to 2560 x 1040 pix$^2$ or 1280 x 2080 pix$^2$, depending on the orientation of the cameras.

Given the effective temperature sensitive range of the considered LC formulation BM/R35C20W, either a higher current for the resistive heater, or the use of a LC formulation with an even smaller effective temperature sensitive range is required. Using a moderate heating power and lowering the effective temperature sensitive range by choosing an appropriate LC formulation provides two advantages:

- The effect of fluid motion due to buoyancy, i.e. natural convection, is negligible. The problem can therefore be closely approximated by the transport of a
passive scalar.

- The required DC currents can still be provided by one single welding transformer. For steady state operation, a commercial heat exchanger can be used for removing the heat in the recirculation loop.
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Appendix A

Isothermal Flow over Waves

In the following, the parameters of the PIV measurements at isothermal conditions are specified, where the 8-bit Kodak Megaplus (Kodak), and the 12-bit PCO Sensicam (PCO) cameras are used. Both are equipped with an 60 mm macro objective.

A.1 \((x, y)\)-Plane, Developed Flow

<table>
<thead>
<tr>
<th>(Re_h)</th>
<th>(B/\Lambda)</th>
<th>AOV</th>
<th>FR Hz</th>
<th>N</th>
<th>(f#)</th>
<th>Camera</th>
</tr>
</thead>
<tbody>
<tr>
<td>3350</td>
<td>12</td>
<td>1.10(\Lambda \times 1.10\Lambda)</td>
<td>1</td>
<td>234</td>
<td>4.0</td>
<td>Kodak</td>
</tr>
<tr>
<td>3350</td>
<td>12</td>
<td>0.83(\Lambda \times 0.83\Lambda)</td>
<td>1</td>
<td>210</td>
<td>4.0</td>
<td>Kodak</td>
</tr>
<tr>
<td>3350</td>
<td>12</td>
<td>0.99(\Lambda \times 0.79\Lambda)</td>
<td>1</td>
<td>250</td>
<td>4.0</td>
<td>PCO</td>
</tr>
</tbody>
</table>

AOV=area of view.

A.2 \((x, z)\)-Plane, Developed Flow

<table>
<thead>
<tr>
<th>(Re_h)</th>
<th>(y/\Lambda)</th>
<th>(B/\Lambda)</th>
<th>AOV</th>
<th>FR Hz</th>
<th>N</th>
<th>(f#)</th>
<th>Camera</th>
</tr>
</thead>
<tbody>
<tr>
<td>7300</td>
<td>0.26</td>
<td>12</td>
<td>2.23(\Lambda \times 1.79\Lambda)</td>
<td>15</td>
<td>40</td>
<td>5.6</td>
<td>PCO</td>
</tr>
<tr>
<td>700</td>
<td>0.26</td>
<td>12</td>
<td>2.23(\Lambda \times 1.79\Lambda)</td>
<td>1</td>
<td>100</td>
<td>4.0</td>
<td>PCO</td>
</tr>
<tr>
<td>7300</td>
<td>0.26</td>
<td>12</td>
<td>2.23(\Lambda \times 1.79\Lambda)</td>
<td>1</td>
<td>250</td>
<td>5.6</td>
<td>PCO</td>
</tr>
<tr>
<td>3800</td>
<td>0.74</td>
<td>12</td>
<td>2.23(\Lambda \times 1.79\Lambda)</td>
<td>1</td>
<td>250</td>
<td>4.0</td>
<td>PCO</td>
</tr>
</tbody>
</table>

AOV=area of view.
### A.3 \((y_1, z)\)-Plane, Developed Flow

<table>
<thead>
<tr>
<th>(Re_h)</th>
<th>(B/\Lambda)</th>
<th>AOV ((-)</th>
<th>FR (\text{Hz})</th>
<th>(N)</th>
<th>(f_#)</th>
<th>Camera</th>
</tr>
</thead>
<tbody>
<tr>
<td>7300</td>
<td>12</td>
<td>0.67(\Lambda) (\times) 0.67(\Lambda)</td>
<td>1</td>
<td>191</td>
<td>4.0</td>
<td>Kodak</td>
</tr>
<tr>
<td>3800</td>
<td>12</td>
<td>0.67(\Lambda) (\times) 0.67(\Lambda)</td>
<td>1</td>
<td>189</td>
<td>4.0</td>
<td>Kodak</td>
</tr>
<tr>
<td>lam.</td>
<td>12</td>
<td>0.67(\Lambda) (\times) 0.67(\Lambda)</td>
<td>1</td>
<td>40</td>
<td>4.0</td>
<td>Kodak</td>
</tr>
</tbody>
</table>

AOV = area of view.

### A.4 \((y_1, z)\)-Plane, Non-Developed Flow

<table>
<thead>
<tr>
<th>(Re_h)</th>
<th>(B/\Lambda)</th>
<th>AOV ((-)</th>
<th>FR (\text{Hz})</th>
<th>(N)</th>
<th>(f_#)</th>
<th>Camera</th>
</tr>
</thead>
<tbody>
<tr>
<td>3350</td>
<td>12</td>
<td>0.82(\Lambda) (\times) 0.82(\Lambda)</td>
<td>1</td>
<td>214</td>
<td>5.6</td>
<td>Kodak</td>
</tr>
<tr>
<td>lam.</td>
<td>12</td>
<td>0.82(\Lambda) (\times) 0.82(\Lambda)</td>
<td>1</td>
<td>43</td>
<td>5.6</td>
<td>Kodak</td>
</tr>
</tbody>
</table>

AOV = area of view.
Appendix B

Flow over Heated Waves

In the following, the parameters of the LCT measurements of the flow over heated waves are specified, where the 3-chip, RGB camera (Sony D XC-9100P) equipped with a telecentric objective was used.

B.1 \((y_1, z)\)-Plane, \(Re_h = 725\)

Reynolds number: \(Re_h = 725\)
Heating current (DC): \(I = 205 \text{ A}\)
Prandtl number: \(Pr = 4.82\)
AOV = 1.11A x 0.83A
TLC-formulation: Hallcrest product BM/R35C20W
Angle between light-sheet and camera axis: \(\phi = 90^\circ\) configuration
Light source: Metal halide
Number of frames: 204

B.2 \((y_1, z)\)-Plane, \(Re_h = 3300\)

Reynolds number: \(Re_h = 3300\)
Heating current (DC): \(I = 255 \text{ A}\)
Prandtl number: \(Pr = 4.82\)
AOV = 1.11A x 0.83A
TLC-formulation: Hallcrest product BM/R35C20W
Angle between light-sheet and camera axis: \(\phi = 90^\circ\) configuration
Light source: Metal halide
Number of frames: 317
Acknowledgement

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