Adhesive Bonding of Structural Hardwood Elements

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Abstract

Wood as the most important natural and renewable building material plays an important role in the construction sector. Recently, the application of engineered wood, especially in the form of adhesively bonded timbers has remarkably increased. Novel progress in plywood made of high strength and high stiffness hardwoods, like European beech, provides designers, in general, with more flexibility and efficiency by enhanced mechanical performance. Nevertheless, the higher tendency of hardwood for moisture sorption as well as for swelling and shrinkage causes significant residual stresses in glued-laminated configurations, particularly in cross-laminated ones. These moisture-induced stress fields lead to the initiation and propagation of both interfacial delamination and solid wood cracking which lessen the integrity and load-bearing capacity of such layered structural elements. Moreover, the strong hygroscopic character of hardwood basically affects all related mechanical properties resulting in degradation of material stiffness and strength over the service life. Additionally, long-term creep and mechano-sorption under changing environmental conditions cause further loss of stiffness and can amplify delamination growth over the lifetime of a timber structure.

Accordingly, the current research investigated the delamination process of adhesively bonded hardwood (European beech) elements subject to changing climatic conditions. For the study of the long-term fracture mechanical behavior of glued-laminated components under varying moisture content, the role of moisture development, time- and moisture-dependent responses are absolutely crucial. For this purpose, a 3D orthotropic hygro-elastic, plastic, visco-elastic, mechano-sorptive wood constitutive model with moisture-dependent material constants was presented in this work. Such a comprehensive material model is capable to capture the true history-dependent stress states and deformations which are essential to achieve reliable design of timber structures. Besides the solid wood substrates, the adhesive material also influences the interface performance considerably. Hence, to gain further insight into the stresses and deformations generated in the bond-line, a general hygro-elastic,
plastic, visco-elastic creep material model for adhesive was introduced as well. The associated numerical algorithms developed on the basis of additive decomposition of the total strain were formulated and implemented within the Abaqus Finite Element (FE) package. Functionality and performance of the proposed approach were evaluated by performing multiple verification simulations of wood components, under different combinations of mechanical loading and moisture variation. Moreover, the generality and efficiency of the presented approach was further demonstrated by conducting an application example of a hybrid wood element.

Following validation of the developed rheological models, damage onset and propagation in three-layered uni-directional and cross-laminated samples out of European beech due to climatic changes were studied based on a combined numerical and experimental approach. The inter- and intra-laminar damage evolution was characterized for various configurations adhesively bonded by three structural adhesive systems. Typical situations were simulated by means of comprehensive moisture-dependent non-linear rheological FE models for wood and adhesive with the capability to capture delaminations. The simulations gave insight into the role of different strain components such as visco-elastic, mechano-sorptive, plastic, and hygro-elastic deformations under changing moisture content in progressive damage and delamination.

Subsequently, the experimental observations on various configurations were evaluated after the first two moisture cycles. Afterwards, the damage evolution was characterized by typical properties common to intra- and inter-laminar fracture studies. Thereafter, FEM simulations on the macroscopic behavior of a set of experiments were verified before the models were applied to gain a deeper insight on the mechanical situations that lead to the observed damage evolution. Hence, cracks were artificially introduced in the middle lamella and then the changes in the strain fields and stresses were calculated for the onset of delamination as well as its propagation using fracture mechanics. These simulations are heavily detailed with respect to the material behavior. To be able to make more general statements on the damage evolution and scaling, a variational mechanical approach, originally developed for cross-ply laminates, was adopted to the moisture-induced damage evolution investigated in this study. As the experimental time frame was limited to two moisture cycles only, it was extended by simulations to 10 cycles (3 years) to be able to quantify the stress buildup under cyclic hygric loading resulting in hygro-fatigue.
Sommario

Il legno è il più importante materiale naturale e rinnovabile per la costruzione, e come tale riveste un ruolo centrale nel settore edile. Recentemente, l’uso del legno lamellare, composto da strati di legname legati da un collante, è aumentato in modo considerevole, e i progressi nella costruzione di pannelli di compensato composti da legname ad alta resistenza e rigidità come il faggio, hanno reso possibili nuove soluzioni progettuali, allo stesso tempo flessibili ed efficienti. Ciò nonostante, il legno duro è caratterizzato da una elevata tendenza all’assorbimento di umidità, che può causare fratture e stati di tensione residua in pannelli lamellari, specialmente se a strati incrociati. Questi stati di tensione possono indurre l’iniziazione e la propagazione di fratture sia nelle interfacce tra le lamelle che all’interno del legno stesso, riducendo la capacità portante degli elementi strutturali di cui fanno parte. In più, le caratteristiche igroscopiche del legno duro influiscono anche su tutte le sue caratteristiche strutturali, che tendono a impoverirsi durante la vita utile delle strutture.

Questa tesi si pone in quest’ambito, e ha come obbiettivo lo studio delle caratteristiche di pannelli lamellari di faggio sotto variabili condizioni ambientali. La risposta a lungo termine degli elementi strutturali in legno è strettamente dipendente dal ruolo giocato dall’influenza dell’umidità sulle caratteristiche meccaniche. Per questa ragione, un modello 3D ortotropico, visco-elastico, plastico e igroscopico per il legno legno, comprendente la variazione delle caratteristiche meccaniche con l’umidità, è stato sviluppato. Il modello è sufficientemente completo da rendere possibile la determinazione degli stati di tensione e deformazione residui, i quali sono elementi essenziali per la progettazione di strutture lignee. In aggiunta, anche il collante è stato simulato, usando per questo un modello igro-elastico, plastico, visco-elastico e di creep, rendendo possibile lo studio dello stato tensionale nelle giunture tra le lamelle. Il modello accoppiato finale è stato formulato e implementato in un algoritmo che si basa sulla decomposizione del tensore di deformazione, ed è stato risolto attraverso l’uso del codice agli elementi finiti Abaqus. Funzionalità
e performance dell’algoritmo sono state testate attraverso molteplici simulazioni di componenti lignee sotto diverse combinazioni di carico strutturale e condizioni ambientali, mentre l’affidabilità del metodo è stata provata attraverso il confronto con un esperimento su di un provino di legno ibrido.

Attraverso una combinazione di test di laboratorio e di simulazioni, l’iniziazione delle fratture e la loro propagazione in un pannello di legno a tre strati, monodirezionali e incrociati è stata studiata sotto diverse condizioni di umidità ambientale. L’evoluzione del danno su varie configurazioni è stata studiata per tre diversi sistemi di collaggio. Alcune situazioni tipiche sono state riprodotte utilizzando modelli meccanici non-lineari sia per il legno che per il collante. La soluzione agli elementi finite ha permesso di riprodurre i fenomeni di delaminazione e propagazione del danno, evidenziando il ruolo delle diverse componenti dello stato di deformazione (viscoelastica, meccanico-assorbente, plastica e idro-elastica).

Successivamente, dati sperimentali sono stati raccolti per due cicli di umidità su diverse configurazioni, e l’evoluzione del danno è stata caratterizzata tramite l’individuazione di proprietà tipiche delle fratture intra e interlaminari. Le simulazioni agli elementi finiti, validate osservando il comportamento macroscopico di un set di esperimenti, hanno permesso di comprendere più a fondo il processo meccanico alla base dello stato di danneggiamento osservato. Per riprodurre questo meccanismo, le fratture sono state iniziate forzatamente nella lamella di mezzeria, mentre l’evoluzione degli stati di tensione e deformazione è stata calcolata sia per lo stato di iniziazione della delaminazione che per la sua propagazione. Le simulazioni che ne conseguono sono altamente dettagliate nel descrivere il comportamento del materiale, e permettono di giungere a conclusioni di validità generale riguardo all’evoluzione del danno dovuto all’umidità. Per questo, un approccio variazionale, originalmente proposto per pannelli laminati a strati incrociati, è stato adottato, permettendo l’estensione dei risultati sperimentali, originariamente limitati a due cicli di umidità, fino a 10 cicli, corrispondenti a tre anni. Durante questa finestra temporale, lo studio delle tensioni residue dovute al carico igroscopico ha permesso di valutare quantitativamente gli sforzi di fatica.
Contents

1 Introduction .................................................. 1
   1.1 Objective and motivation ..................................... 1
   1.2 Strategy and methods ........................................ 2
   1.3 Outline of the thesis ........................................ 4

2 Wood physics and adhesive bonding ............................... 5
   2.1 Wood as a hierarchical material ............................... 5
   2.2 Wood mechanics ............................................... 7
   2.3 Hygric behavior of wood ...................................... 10
   2.4 Hygro-mechanical behavior of adhesives ....................... 14
   2.5 Engineered wood components .................................. 18
   2.6 Damage and fracture of engineered wood ..................... 21

3 Rheological model for wood ...................................... 25
   3.1 Previous studies on wood constitutive models ................. 25
   3.2 Hygro-mechanical constitutive model ........................ 30
      3.2.1 Elastic deformation ...................................... 32
      3.2.2 Plastic deformation ...................................... 33
      3.2.3 Hygro-expansion ......................................... 38
      3.2.4 Visco-elastic creep ...................................... 39
      3.2.5 Mechano-sorptive creep .................................. 41
   3.3 Numerical implementation of the comprehensive moisture-dependent rheological model ............................................. 42
   3.4 Moisture-stress analysis ...................................... 48
   3.5 Modeling of interfacial damage ............................... 51

4 Verification of the material model .............................. 57
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Simple stress state under uni-axial mechanical loading and moisture variation</td>
<td>57</td>
</tr>
<tr>
<td>4.2 Stress state under multi-axial mechanical and hygric loading</td>
<td>62</td>
</tr>
<tr>
<td>4.3 Stress analysis under restrained swelling</td>
<td>63</td>
</tr>
<tr>
<td>4.4 Analysis of moisture-induced residual stresses in a glulam beam during the manufacturing process</td>
<td>66</td>
</tr>
<tr>
<td>4.5 Multi-species glulam beam</td>
<td>70</td>
</tr>
<tr>
<td>4.6 Numerical simulation of Mode I delamination</td>
<td>73</td>
</tr>
<tr>
<td>4.7 Remarks on convergence of the developed material model</td>
<td>78</td>
</tr>
<tr>
<td>5 Damage evolution in adhesively-bonded hardwood elements</td>
<td>81</td>
</tr>
<tr>
<td>5.1 Experimental investigation of fracture propagation in glued-laminated beech elements under hygric loading</td>
<td>83</td>
</tr>
<tr>
<td>5.1.1 Samples with coaxial L-directions (P1)</td>
<td>86</td>
</tr>
<tr>
<td>5.1.2 Samples with crossed L-directions (P2)</td>
<td>88</td>
</tr>
<tr>
<td>5.1.3 Observations on bond-line performance</td>
<td>91</td>
</tr>
<tr>
<td>5.2 Numerical analysis</td>
<td>92</td>
</tr>
<tr>
<td>5.2.1 Stress analysis of glued-laminated panels under hygric loading</td>
<td>94</td>
</tr>
<tr>
<td>5.2.2 Non-linear fracture mechanical simulations of damage in cross-laminated samples</td>
<td>103</td>
</tr>
<tr>
<td>5.2.3 Analytical study of micro-cracking in cross-laminated samples</td>
<td>105</td>
</tr>
<tr>
<td>6 Summary and outlook</td>
<td>109</td>
</tr>
<tr>
<td>6.1 Summary of the thesis</td>
<td>109</td>
</tr>
<tr>
<td>6.2 Concluding remarks</td>
<td>112</td>
</tr>
<tr>
<td>6.3 Ideas for future research</td>
<td>113</td>
</tr>
<tr>
<td>A Total algorithmic tangent operator</td>
<td>115</td>
</tr>
<tr>
<td>B Material properties for MUF, PRF, PUR</td>
<td>119</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Hierarchical structure of wood: a) A transverse cross-section of the trunk illustrating the macroscopic structure of wood [203], b) Cellular structure of a typical hardwood [104], c) The layered structure of the cell wall [37], d) Schematic illustration of the arrangement of cellulose, hemicellulose and lignin as the main constituents of the cell walls [19].

2.2 Different stages of the creep phenomenon as a function of time [56, 135].

2.3 All possibly emerging deformation modes, namely elastic, plastic, and visco-elastic creep observed in wood under a constant load level and unvarying environmental conditions [38].

2.4 Typical sorption isotherms for European beech and Norway spruce and the hysteresis between adsorption and desorption [72].

2.5 Distinct shrinkage behavior of the cut-sections with different orientation of the anatomical directions in the plane perpendicular to the grain [103].

2.6 Schematic interpretation of the mechano-sorptive creep. $\Delta L_{Me}$, $\Delta L_{\omega}$, and $\Delta L_{MS}$ denote increase in the length caused by external mechanical loading, swelling, and mechano-sorptive effect, respectively.

2.7 a) Schematic illustration of the simplified layered model representing the different regions of an adhesive-bonded joint [167, 201], b) The electron-microscope picture of a typical wood-adhesive bond with adherends made of softwood [167].

2.8 Sorption isotherms of the three most common commercial wood adhesives, namely PRF, MUF, and 1C PUR during both moisture adsorption and desorption [202].

2.9 Wood-based structural composites: a) Glued laminated timber (glulam), b) Cross-laminated timber panel.
### LIST OF FIGURES

#### 2.10 Moisture profiles in the transverse section of a glulam element subjected to climate variations changing from RH1=65% \((\omega_1 \approx 12\%)\) to RH2=50% \((\omega_2 \approx 9\%)\) at T=20\(^\circ\). a) Adhesive layer with zero permeability, b) adhesive layer with the same permeability as solid wood.

#### 2.11 a) All principal crack growth systems in wood defined based on the orientation of the crack plane (first letter) and the corresponding propagation direction (second letter) [148], b) Schematic representation of different modes of fracture [148, 208].

#### 2.12 Three basic failure mechanisms responsible for the damage of the adhesive bond-line [8, 34].

#### 3.1 Schematic representation of the a) *Kelvin-Voigt* and b) *Maxwell* elements. Subscripts S and D denote spring and dashpot, respectively.

#### 3.2 Schematic illustration of the constitutive material model.

#### 3.3 a) 2D illustration of the yield surfaces of European beech in the R-L plane and for \(\sigma_{RL} = 0\). The color bar shows the value of the yield function Eq. 3.7 for \(l = R\) and \(L\), b) 3D visualization of the boundaries between elastic and plastic domains in the principal stress space. Both are illustrated for \(q_l = 0\) at \(\omega = 12\%\).

#### 3.4 Schematic representation of the concept of elastic predictor/plastic corrector in the context of the closest point projection iteration algorithm, based on Ref. [173].

#### 3.5 Schematic illustration of the boundary layer representing the surface resistance (surface-emission coefficient) to the 1D diffusion [56].

#### 3.6 Schematic depiction of the cohesive interface element: a) 4-noded two-dimensional, b) 8-noded three-dimensional [21]. \(n, s,\) and \(t\) denote the normal, the first and the second shear directions, respectively.

#### 3.7 Various types of traction-separation laws: a) Exponential, b) Bi-linear, and c) Trapezoidal [162]. \(t^0\) and \(\delta^0\) symbolize the peak value of the nominal traction and the corresponding nominal separation, respectively, and \(\delta^f\) represents the nominal displacement at complete failure.
3.8 Schematic presentation of a typical bi-linear traction-separation relationship: a) Pure normal deformation, b) Pure shear deformations [22, 36]. $G_n^c$, $G_s^c$, and $G_t^c$ represent the amount of energy needed for a complete failure in the single modes (normal and two shear directions).

4.1 a) Geometry and finite element model of the wood cubic specimen, b) Amplitudes of the exerted radial compressive stress and uniform moisture variation on the wood block sample.

4.2 a) Time evolution of all constituents of the total strain along the radial direction, b) Development of the radial plastic and mechano-sorptive creep strains.

4.3 Evolution of the swelling/shrinkage-modified strains along the T- and L-direction ($\varepsilon_{TT}^t/LL^t$). The superscript $pl$ refers to the plastic strain.

4.4 a) Illustration of the tri-axial state of stress, b) Time evolution of the irreversible plastic strains along all three anatomical directions. (p) and (q) denote the iso-hygric loading and the moistening phase of the 1st moisture cycle, respectively.

4.5 a) Finite Element discretization of the rectangular prism specimen, b) Time evolution of the swelling pressure. $R\sigma_{L2}$ and $R\sigma_{L3}$ refer to the residual longitudinal stresses based on the Cases 2 and 3, respectively after $t = 1400h$.

4.6 a) The geometry and FE-model of the five-layered glulam beam as well as the applied boundary conditions, alignment of the orthotropy directions, and the initial MC of every wood lamella. The same color refers to the layer number and its conjugate material coordinate system, b) The distribution of moisture in the glulam beam at equilibrium state ($\omega_{eq} \approx 11\%$) after $t = 750$ days.

4.7 The distribution of moisture-induced stresses perpendicular to the grain (top: radial and bottom: tangential) in the equilibrated glulam at $\omega_{eq} \approx 11\%$.

4.8 The time evolution of the MC and related induced cross-grain stress at point $r$: at the interface of layers 4 and 5 with the highest value of the radial stress and at point $t$: on the vertical edge of layer 4 in the proximity of the interface of layers 4 and 5 with the largest amount of the tangential stress.
4.9  a) Geometrical dimensions and finite element discretization of the multi-species beam, b) Distribution of the tangential plastic strain after one year ($t = 365$ days). (B) and (S) designate beech and spruce, respectively.

4.10  Geometry and dimensions of the DCB specimen [50]. All dimensions are in mm.

4.11  Finite Element discretization of the DCB specimen with the applied opening mode loading, Cartesian material coordinate systems, and boundary conditions. Because of the symmetry with respect to the YZ-plane just one half of the model is simulated.

4.12  Vertical reaction force-displacement curve of the upper reference point (see Fig. 4.11) and the development of damage onset and growth with respect to the applied opening displacement. (1): initiation of the material stiffness degradation, (2): commencement of the separation, (3) and (4): propagation of delamination. All deformed shapes are scaled by a factor of 20 for better visualization.

4.13  The vertical prescribed displacement on the upper RF (a) and the respective calculated reaction force (b) vs. the total crack length. The initial crack length ($a_0$) measured as the distance between the center of the pin holes and the beginning of the interface (see Fig. 4.10) is 10mm.

4.14  The effect of the viscosity parameter value $\mu$ on the vertical force-displacement response of the DCB sample (see Fig. 4.12 top).

5.1  a) The arrangement of glued-laminated beech elements inside a climate box, b) and c): An internally ventilated climate box filled with a layer of Silica gel (b) and with vaporized distilled water (c), d) A climate box with sealed cover and a RH recorder.

5.2  A cross laminated sample with dimensions in mm and different kinds of failure mechanisms [132].
5.3 Dependence of the maximum warping deflection on the adhesive type and middle lamella thickness $d$ averaged over three samples and side view of the deformed shape of the P1-PUR-T1-4 sample at the end of the second de-moistening step. In the following, the central box indicates the central 50% of a data set (limited by the lower and upper boundaries representing the 25% and 75% quantile of data). The central point shows the median and the dashed lines connect the two lower and upper markers that quantify the remaining data lying outside the central box.

5.4 End grain surface views of the P1-PUR-T4-1 sample after the second drying, illustrating the deflected and penetrating cracks. Arrows mark the propagation trajectory after intersection with the interface.

5.5 Side warping of all P2 samples after the first de-moistening step. Inset: sample P2-MUF-T1-2 at the end of the second de-moistening stage.

5.6 Side views of the fractured P2-MUF-T2-2 sample after the first (left) and the second (right) de-moistening steps, displaying the formation of equally distanced curved micro-cracks that span the entire sample width and microcrack-induced de-bonding.

5.7 Side views of the fractured P2-PRF-T3-2 sample after the first (left) and the second (right) de-moistening steps with a pair of relatively straight transverse micro-cracks formed as well as a top layer crack with delamination.

5.8 Side views of the fractured P2-PUR-T4-1 sample after the first (left) and the second (right) de-moistening steps with a single straight transverse micro-crack and severe intra-laminar TL-cracking.

5.9 P2-X-T3 samples ($d=40\text{mm}$) with different adhesives after the first de-moistening exhibiting adhesive failure in the MUF and interphase failure in the PRF and PUR bond-lines.

5.10 Damage states after the second drying for different thicknesses and adhesive types. The inset gives the unit cell of damage used for the analytical stress calculation (see Subsection 5.2.3).

5.11 Exemplary FEM model with the applied boundary conditions and local material coordinate system consisting of twenty-node quadratic brick elements with reduced integration (C3D20R) and linear cohesive elements (COH3D8).
5.12 The principles of the general approach utilized to analyze the fracture behavior of glued-laminated panels...

5.13 Moisture evolution at specific points and (inset) moisture field in the P2-T3-3 sample after 1.7 days.

5.14 Strain evolution in quarter-models upon formation of a transverse crack with suppressed delamination for the P2-PUR-T3-3 sample with identical legends for the time evolution.

5.15 a) Peeling and shear stresses in the adhesive bond-line along the Path 1 and Path 2 for different types of adhesive, b) The equivalent plastic strain in the lower PUR adhesive layer of the P2-T3 sample after the 2nd de-moistening phase.

5.16 The dependence of the interfacial stresses $\sigma_{yy}$ and $\sigma_{yz}$ along the Path 1 (see Fig. 5.15) on the thickness of the middle layer ($d$) of cross-laminated samples (P2) after the 1st drying step.

5.17 Radial stress evolution in 10 consecutive drying-moistening (case I, top) and moistening-drying (case II, bottom) cycles at the center of the P2-T3 sample (Point 2 in Fig. 5.13).

5.18 The evolution of different components of the total strain in drying-moistening (case I, top) and moistening-drying (case II, bottom) cycles (with identical legends). Subscript $X$ represents $ms$, $ve$, $pl$, or $el$ for the respective strain components.

5.19 Time evolution of delamination initiation and propagation and distribution of degradation scalar parameter (blue intact, red almost delaminated, while delaminated elements are deleted) with delaminations for a multiplier value of 2.65 compared to the P2-PUR-T3-3 sample.

5.20 Relative delaminations ($(A_D/A_{Tot})$ and $(L_D/L)$) versus the energy multiplier in the adhesive bond-lines of the P2-PUR-T3-3 sample.

5.21 Side views of the fractured and delaminated P2-PUR-T2-1/T3-3 samples after the 1st and the 2nd drying. The detail shows the microcrack-induced delaminations emanating from the micro-crack tips after the 2nd de-moistening. $\sigma_{xx}$ is the average tensile stress along the x-direction in the middle lamella, from the analytical solution [132].
5.22 Dimensionless ERR vs. micro-crack density fitted to T2 and applied to T3 samples, giving excellent agreement with the experimental observations. Inset: Micro-crack density as a function of moisture gradient for P2-PUR-T1/T2/T3 samples and the limit line for micro-cracking initiation. ...
### List of Tables

<table>
<thead>
<tr>
<th>Number</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Coefficients for moisture-dependent engineering constants for European beech [83] and Norway spruce [56, 134]</td>
<td>33</td>
</tr>
<tr>
<td>3.2</td>
<td>Coefficients for calculation of moisture-dependent hardening stress for European beech [55, 85]</td>
<td>34</td>
</tr>
<tr>
<td>3.3</td>
<td>Coefficients for calculation of moisture-dependent strength values for European beech [82] and Norway spruce [157, 163]</td>
<td>36</td>
</tr>
<tr>
<td>3.4</td>
<td>Swelling/shrinkage coefficients for European beech [82] and Norway spruce [56, 134]</td>
<td>39</td>
</tr>
<tr>
<td>3.5</td>
<td>Coefficients for calculation of moisture- and time-dependent entry of the visco-elastic compliance tensor parallel to grain for European beech [84] and dimensionless scalar parameters for Norway spruce [49]</td>
<td>40</td>
</tr>
<tr>
<td>3.6</td>
<td>Longitudinal and tangential components of the mechano-sorptive compliance tensor for European beech and Norway spruce [49]</td>
<td>42</td>
</tr>
<tr>
<td>3.7</td>
<td>Parameters for calculation of the diffusion coefficients as an exponential function of the moisture content for European beech [82] and Norway spruce [157]</td>
<td>50</td>
</tr>
<tr>
<td>3.8</td>
<td>The directional surface-emission coefficients of European beech [82]</td>
<td>50</td>
</tr>
<tr>
<td>3.9</td>
<td>The normal specific fracture energy of European beech bonding under changing climate for the PRF and PUR adhesives [5]</td>
<td>55</td>
</tr>
<tr>
<td>4.1</td>
<td>Cohesive parameters describing the bi-linear traction-separation response of the cohesive layer in the DCB representing the MUF adhesive system [50]. $K_{nn}$, $T_{max}$, and $G_{IC}$ denote normal penalty stiffness, maximum attainable traction before damage initiation, and Mode I critical fracture energy, respectively.</td>
<td>75</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.2</td>
<td>Residual norm values of iterations of the Newton solution procedure for some typical increments for the time between 62.5h and 65h in the first verification example (see Section 4.1). The control parameter for the convergence criterion is taken as $R^\alpha = 5 \times 10^{-3}$.</td>
<td>79</td>
</tr>
<tr>
<td>5.1</td>
<td>Test specimen catalogue</td>
<td>83</td>
</tr>
<tr>
<td>5.2</td>
<td>Processing parameters of the three adhesive systems utilized to assemble the laminated panels</td>
<td>85</td>
</tr>
<tr>
<td>5.3</td>
<td>Parameters for the non-linear traction-separation behavior of the cohesive element for PUR</td>
<td>94</td>
</tr>
<tr>
<td>B.1</td>
<td>Coefficients for calculation of moisture-dependent Young’s modulus for different adhesive systems fitted to the data published in Ref. [98]. The value of the Poisson’s ratio for all adhesive types is taken as $\nu_{adh} = 0.3$.</td>
<td>119</td>
</tr>
<tr>
<td>B.2</td>
<td>Moisture expansion coefficients (CME) of different generic adhesive types given in Ref. [210]</td>
<td>119</td>
</tr>
<tr>
<td>B.3</td>
<td>Normal entries of the visco-elastic compliance tensor $J_i$ pertaining to a serial association of six Kelvin-Voigt elements for different adhesive systems identified in Refs. [194] and [193]. Similar to Table 3.5, $\tau_i$ designates the characteristic retardation time relevant to the $i^{th}$ Kelvin-Voigt element.</td>
<td>120</td>
</tr>
<tr>
<td>B.4</td>
<td>Coefficients for calculation of moisture-dependent strength value and non-linear hardening stress function for PUR adhesive fitted to the data published in Ref. [98]. $\omega$ and $\alpha$ denote the MC and strain-type internal state variable, respectively.</td>
<td>120</td>
</tr>
<tr>
<td>B.5</td>
<td>Parameters for calculation of moisture-dependent diffusion coefficients for different adhesive systems following Refs. [198] and [202].</td>
<td>120</td>
</tr>
</tbody>
</table>
# Nomenclature

## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1C PUR</td>
<td>One-component Polyurethane</td>
</tr>
<tr>
<td>BK</td>
<td>Benzeggagh-Kenane</td>
</tr>
<tr>
<td>CDM</td>
<td>Continuum Damage Mechanics</td>
</tr>
<tr>
<td>CLT</td>
<td>Cross-Laminated Timber</td>
</tr>
<tr>
<td>COD</td>
<td>Crack Opening Displacement</td>
</tr>
<tr>
<td>CZM</td>
<td>Cohesive Zone Model</td>
</tr>
<tr>
<td>DCB</td>
<td>Double Cantilever Beam</td>
</tr>
<tr>
<td>EMC</td>
<td>Equilibrium Moisture Content</td>
</tr>
<tr>
<td>ERR</td>
<td>Energy Release Rate</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FPZ</td>
<td>Fracture Process Zone</td>
</tr>
<tr>
<td>FSP</td>
<td>Fiber Saturation Point</td>
</tr>
<tr>
<td>FWC</td>
<td>Fractional Weight Change</td>
</tr>
<tr>
<td>Glulam</td>
<td>Glued-Laminated Timber</td>
</tr>
<tr>
<td>L</td>
<td>Longitudinal direction</td>
</tr>
<tr>
<td>LVL</td>
<td>Laminated Veneer Lumber</td>
</tr>
<tr>
<td>MC</td>
<td>Moisture Content</td>
</tr>
<tr>
<td>MEC</td>
<td>Moisture Expansion Coefficient</td>
</tr>
<tr>
<td>MF</td>
<td>Melamine Formaldehyde</td>
</tr>
<tr>
<td>MFA</td>
<td>Micro-Fibril Angle</td>
</tr>
<tr>
<td>MUF</td>
<td>Melamine Urea Formaldehyde</td>
</tr>
<tr>
<td>OSB</td>
<td>Oriented Strand Board</td>
</tr>
<tr>
<td>PB</td>
<td>Particleboard</td>
</tr>
<tr>
<td>PF</td>
<td>Phenol Formaldehyde</td>
</tr>
<tr>
<td>PRF</td>
<td>Phenol Resorcinol Formaldehyde</td>
</tr>
<tr>
<td>PVAc</td>
<td>Polyvinyl Acetate</td>
</tr>
</tbody>
</table>
Nomenclature

R  Radial direction
RF  Resorcinol Formaldehyde
RH  Relative Humidity
RP  Reference Point
T  Tangential direction
TSL  Traction Separation Law
UF  Urea Formaldehyde

Greek letters

\( \alpha \)  Power law criterion exponent
\( \alpha_{(p)} \)  Internal hardening variable after \( p \)th iteration (at \( t_{n+1} \), \( l=R,T,L \))
\( \alpha_{i,n+1} \)  Internal hardening variable after \( p+1 \)th iteration (at \( t_{n+1} \), \( l=R,T,L \))
\( \alpha_{(\text{Trial})} \)  Trial internal hardening variable (at \( t_{n+1} \), \( l=R,T,L \))
\( \alpha_{l} \)  Strain type internal variable, \( l=R,T,L \)
\( \alpha_{\omega} \)  Hygro-expansion coefficients tensor

\( \sigma \)  Total stress tensor
\( \sigma_{ms} \)  \( j \)th mechano-sorptive stress tensor
\( \sigma_{(p)} \)  Total stress tensor (at \( t_{n+1} \) and \( t_{n} \))
\( \sigma_{(p+1)} \)  Total stress tensor after \( p \)th iteration (at \( t_{n+1} \))
\( \sigma_{(\text{Trial})} \)  Trial total stress tensor (at \( t_{n+1} \))
\( \varepsilon \)  Total strain tensor
\( \varepsilon_{\omega} \)  Hygro-expansion strain tensor
\( \varepsilon_{(p)} \)  Elastic strain tensor after \( p \)th iteration (at \( t_{n+1} \))
\( \varepsilon_{(\text{Trial})} \)  Trial elastic strain tensor (at \( t_{n+1} \))
\( \varepsilon_{i} \)  Elastic strain tensor
\( \varepsilon_{pl(k)} \)  Plastic strain tensor after \( k \)th strain decomposition algorithm
\( \varepsilon_{pl(p)} \)  Plastic strain tensor after \( p \)th iteration (at \( t_{n+1} \))
\( \varepsilon_{pl(p+1)} \)  Plastic strain tensor after \( p+1 \)th iteration (at \( t_{n+1} \))
\( \varepsilon_{(\text{Trial})} \)  Trial plastic strain tensor (at \( t_{n+1} \))
\( \varepsilon_{pl} \)  Plastic strain tensor
\( \varepsilon_{ve(k)} \)  \( i \)th visco-elastic strain tensor after \( k \)th iteration (at \( t_{n+1} \))
\( \varepsilon_{ve} \)  \( i \)th visco-elastic strain tensor (at \( t_{n+1} \) and \( t_{n} \))
\( \varepsilon_{ve} \)  \( i \)th visco-elastic strain tensor
\( \varepsilon_{ms(k)} \)  \( j \)th mechano-sorptive strain tensor after \( k \)th iteration (at \( t_{n+1} \))
\( \varepsilon_{ms(j,n+1)} \)  \( j \)th mechano-sorptive strain tensor (at \( t_{n+1} \) and \( t_{n} \))
\( \varepsilon_{ms(j)} \)  \( j \)th mechano-sorptive strain tensor
\( \Xi_{n+1} \)  Algorithmic moduli
Nomenclature

δ Nominal relative separation

\( \Delta \sigma_{n+1}^{(k)} \) Change of total stress tensor after \( k^{th} \) iteration (at \( t_{n+1} \))

\( \Delta \varepsilon_{ve}^{(k)} \) Change of \( i^{th} \) visco-elastic strain tensor after \( k^{th} \) iteration

\( \Delta \varepsilon_{ms}^{(k)} \) Change of \( j^{th} \) mechano-sorptive strain tensor after \( k^{th} \) iteration

\( \delta \Delta \gamma_{l}^{(p)} \) Increment of consistency parameter after \( p^{th} \) iteration (at \( t_{n+1} \)), \( l=R,T,L \)

\( \Delta \omega \) Moisture gradient

\( \delta_{m}^{0} \) Effective displacement at failure initiation

\( \delta_{f}^{max} \) Maximum separation at fully damaged state

\( \delta_{m}^{f} \) Effective displacement at complete damage

\( \delta_{n}, \delta_{s}, \delta_{t} \) Relative separations along the normal and two shear directions

\( \Delta \alpha_{l,n+1}^{(p)} \) Change of internal hardening variable after \( p^{th} \) iteration (at \( t_{n+1} \)), \( l=R,T,L \)

\( \Delta \alpha_{l,n+1} \) Increment of internal hardening variable from \( t_{n} \) to \( t_{n+1} \), \( l=R,T,L \)

\( \Delta \varepsilon_{pl}^{(p)} \) Change of plastic strain tensor after \( p^{th} \) iteration (at \( t_{n+1} \))

\( \Delta \varepsilon_{pl} \) Increment of plastic strain tensor from \( t_{n} \) to \( t_{n+1} \)

\( \Delta \gamma_{l}^{(p)} \) Non-rate form of the consistency parameter after \( p^{th} \) iteration (at \( t_{n+1} \)), \( l=R,T,L \)

\( \Delta \gamma_{l}^{(p+1)} \) Non-rate form of the consistency parameter after \( p + 1^{th} \) iteration (at \( t_{n+1} \)), \( l=R,T,L \)

\( \Delta t \) Time step

\( \dot{\gamma}_{l} \) Rate form of plastic consistency parameter, \( l=R,T,L \)

\( \gamma_{ve}^{i} \) \( i^{th} \) scalar visco-elastic compliance fraction

\( \gamma_{ms}^{j} \) \( j^{th} \) scalar mechano-sorptive compliance fraction

\( \mu \) Viscosity parameter

\( \mu_{j} \) \( j^{th} \) characteristic moisture change

\( \nu_{RT}, \nu_{RL}, \nu_{TL} \) Wood Poissons ratios

\( \omega \) Moisture content

\( \omega_{\infty} \) Equilibrium moisture content

\( \omega_{0} \) Initial reference moisture content

\( \omega_{n+1}, \omega_{n} \) Moisture values \( n \) and \( n + 1 \)

\( \omega_{s} \) Current value of surface moisture

\( \partial \sigma_{f_{l}} \) Plastic flow direction tensor, \( l=R,T,L \)

\( \partial \varepsilon_{f_{l}} \) Hardening strain flow direction

\( \phi(T, \omega) \) Thermal energy function
Nomenclature

\( \Psi \) Chemical or water potential
\( \psi \) Free energy function
\( \psi^{el} \) Elastic strain energy
\( \psi^{ms} \) Mechano-sorptive strain energy
\( \psi^{ve} \) Visco-elastic strain energy
\( \rho_0 \) Oven-dry wood density
\( \sigma_D \) Surface emission coefficient
\( \tau_i \) \( i^{th} \) characteristic retardation time
\( \omega_{FS} \) Fiber saturation moisture level

**Roman symbols**

\( n \) surface normal vector
\( a_i \) Strength tensor
\( b_i \) Strength tensor
\( C_{el}^{n+1} \) Elastic tangent operator
\( C_{ep}^{n+1} \) Elasto-plastic tangent operator
\( R^{el} \) Elastic residual vector
\( R^{ve} \) \( i^{th} \) visco-elastic residual vector
\( R_j^{ms} \) \( j^{th} \) mechano-sorptive residual vector
\( R^{pl(p)}_{n+1} \) Plastic residual vector after \( p^{th} \) iteration (at \( t_{n+1} \))
\( \dot{D}_v \) Rate form of viscous damage variable
\( S_{act} \) Set of active yield conditions
\( S_{adm} \) Set of admissible yield constraints
\( T^{ms}_{n+1}, T^{ms}_n \) Mechano-sorption moisture functions
\( T^{ve}_{n+1}, T^{ve}_n \) Visco-elastic time functions
\( d \) Middle lamella thickness
\( c \) Water concentration
\( D \) Tensor of diffusion coefficients
\( K \) Tensor of thermal conductivity coefficients
\( a_0 \) Initial crack length
\( c_\infty \) Equilibrium concentration
\( c_T \) Specific heat
\( c_{\text{wood}} \) Wood surface concentration
\( D \) Damage scalar variable
\( D_v \) Viscous damage variable
\( E_{RR}, E_{TT}, E_{LL} \) Wood moduli of elasticity along the anatomical directions
\( f_{c,l} \) Normal compressive strength of the material, \( l=R,T,L \)
Nomenclature

\( f_{l,n+1}^{(p)} \) Yield function value after \( p^{th} \) iteration (at \( t_{n+1} \)), \( l=R,T,L \)
\( f_{l,n+1}^{(\text{Trial})} \) Trial yield function value (at \( t_{n+1} \)), \( l=R,T,L \)
\( f_l \) Directional yield function, \( l=R,T,L \)
\( f_{s,RL} \) Shear strength in RL plane
\( f_{s,RT} \) Shear strength in RT plane
\( f_{s,TL} \) Shear strength in TL plane
\( f_{l,L} \) Normal longitudinal tensile strength
\( G^c \) Total critical fracture energy
\( G_{RT}, G_{RL}, G_{TL} \) Shear moduli of wood in the orthotropic planes
\( H_l \) Hardening modulus, \( l=R,T,L \)
\( J_{i,L}^{i} \) \( i^{th} \) longitudinal component of visco-elastic compliance tensor
\( J_{j,L}^{ms} \) Longitudinal component of \( j^{th} \) mechano-sorptive compliance tensor
\( J_{j,T}^{ms} \) Tangential entry of \( j^{th} \) mechano-sorptive compliance tensor at standard climatic condition
\( J_{j,T}^{TR} \) Tangential entry of \( j^{th} \) mechano-sorptive compliance tensor
\( K_n, K_s, K_t \) Penalty stiffness
\( m \) Number of mechano-sorptive Kelvin-Voigt elements
\( m(t) \) The current mass at time \( t \)
\( m_0 \) The mass of the oven-dry sample
\( m_1, m_2, m_3 \) Energy based mode-mix ratios
\( m_\infty \) The mass at the equilibrium state
\( m_\omega \) The mass of moist wood
\( n \) Number of visco-elastic Kelvin-Voigt elements
\( p \) Vapor partial pressure
\( P_b \) European beech property
\( P_s \) Norway spruce property
\( q_{l,n+1}^{(p)} \) Hardening function after \( p^{th} \) iteration (at \( t_{n+1} \)), \( l=R,T,L \)
\( q_{l,n+1}^{(\text{Trial})} \) Trial hardening function (at \( t_{n+1} \)), \( l=R,T,L \)
\( q_l \) Plastic hardening function
\( r \) Number of active yield mechanisms
\( R^c \) Residual control parameter
\( r_{adm} \) Number of active yield constraints
\( T \) Temperature
\( t \) Nominal traction
\( t^0 \) Traction at damage initiation
\( t_{n+1}, t_n \) Time moments \( n \) and \( n+1 \)
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_0$</td>
<td>Wood oven-dry volume</td>
</tr>
<tr>
<td>$C_0$</td>
<td>Elastic stiffness tensor</td>
</tr>
<tr>
<td>$C_0^{-1}$</td>
<td>Elastic compliance tensor</td>
</tr>
<tr>
<td>$C_{i,n+1}^{-1}, C_{i,n}^{-1}$</td>
<td>$i^{th}$ visco-elastic compliance tensor (at $t_{n+1}$ and $t_n$)</td>
</tr>
<tr>
<td>$C_{i,n+1}^{ve}$</td>
<td>$i^{th}$ algorithmic visco-elastic operator</td>
</tr>
<tr>
<td>$C_i$</td>
<td>$i^{th}$ visco-elastic stiffness tensor</td>
</tr>
<tr>
<td>$C_i^{-1}$</td>
<td>$i^{th}$ visco-elastic compliance tensor</td>
</tr>
<tr>
<td>$C_{j,n+1}^{-1}, C_{j,n}^{-1}$</td>
<td>$j^{th}$ mechano-sorptive compliance tensor (at $t_{n+1}$ and $t_n$)</td>
</tr>
<tr>
<td>$C_j$</td>
<td>$j^{th}$ mechano-sorptive stiffness tensor</td>
</tr>
<tr>
<td>$C_j^{-1}$</td>
<td>$j^{th}$ mechano-sorptive compliance tensor</td>
</tr>
<tr>
<td>$J^b$</td>
<td>Body moisture flux vector</td>
</tr>
<tr>
<td>$J_s^m$</td>
<td>Surface moisture flow</td>
</tr>
<tr>
<td>$J_T$</td>
<td>Heat flux</td>
</tr>
<tr>
<td>$C_{j,n+1}^{ms}$</td>
<td>$j^{th}$ algorithmic mechano-sorptive operator</td>
</tr>
<tr>
<td>$C_T^{n+1}$</td>
<td>Total algorithmic tangent operator</td>
</tr>
<tr>
<td>$R_{n+1}^{(k+1)}$</td>
<td>Generalized residual vector after $k + 1^{th}$ iteration</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Objective and motivation

Wood application, in particular, in the forms of engineered or composite elements, has significantly increased as a structural building material. Newly engineered glued-laminated wood products, entirely or partially out of hardwood, are recently developed to overcome restrictions of timber constructions made of softwood. The advantages of composite products out of hardwood such as beech, oak, or ash are obvious: high strength and stiffness allow for smaller cross-section or span width compared to softwood, resulting in increased dimensional stability, load-carrying capacity and finally more freedom in design.

Despite the aforementioned advantages, unfortunately the strong hygric dependence of basically all mechanical properties of hardwood renders many innovative ideas futile. Besides the outstanding mechanical performance of hardwood engineered products, their propensity to absorb moisture during the manufacturing process - because of the difference in the moisture content (MC) of two adjacent layers and also the application of waterborne adhesive systems - causes an amount of residual stresses in adhesively bonded elements. These moisture-induced shear or tensile stresses which are substantially amplified during service by larger swelling coefficients, the higher rigidity and moisture sorption of hardwood substrates act on the solid wood and the bonding together with the history-dependent responses of wood adherends, long-term visco-elastic creep and mechano-sorption under changing environmental conditions. They initiate and evolve cracks in both adhesive bondline and solid wood substrate, resulting in: intra- and inter-laminar fracture, loss of structural integrity, and the diminishing of load-bearing capacity. In addition, time- and history-dependent phenomena (visco-elastic and mechano-sorptive creep)
can accelerate the degradation of stiffness and load-carrying capacity due to the accumulation of the built-up stress and deformation fields and result in the further loss of the mechanical performance even after being in use for decades.

Accordingly, cracking and delamination of glued-laminated hardwood timbers can be considered the dominant failure mechanisms in such composite products. Realistic long-term predictions of the mechanical performance of hardwood or hybrid elements under external mechanical and hygric loading should be a central concern for assuring both - serviceability and safety of timber structures. Consequently, to attain reliable and optimized design criteria as well as durable and long-lasting engineered wood structural elements, prediction of the fracture mechanical response of glued-laminated components out of hardwood subjected to changing moisture content is of great significance.

Therefore, the current study was intended to investigate the delamination process in adhesively bonded hardwood (European beech) elements under varying environmental conditions. The general purpose of this research was to develop appropriate numerical approaches to calculate history-dependent moisture-induced stress and deformation fields and to predict the long-term fracture mechanical response of glued-laminated hardwood elements under climatic loading.

1.2 Strategy and methods

The general strategy for the mechanical analysis and study of the fracture and delamination of glued-laminated hardwood panels is founded on the following main constituents:

- comprehensive moisture-dependent rheological models for wood substrate and adhesive layer,
- moisture transfer analysis and calculation of moisture-induced stress and deformation fields,
- fracture mechanical simulations,
- comparison with experimental observations.

To gain further insight into the long-term mechanical performance of adhesively bonded hardwood elements and to attain reliable design of the timber structures, the influence of moisture evolution and the role of time- and moisture-dependent
behavior have to be taken into account. Hence, the development and verification of authentic moisture-dependent constitutive models are of great importance. Since up to now, a comprehensive moisture-dependent rheological model comprising all possibly emerging deformation mechanisms has been missing, a 3D orthotropic elasto-plastic, visco-elastic, mechano-sorptive material model for wood, with all material constants being defined as a function of moisture content, was developed.

Apart from the solid wood adherends, the adhesive layer also plays a crucial role in the generation and distribution of the interfacial stresses. Adhesive substance can be treated as a continuum layer constructed from a homogeneous and isotropic material. To obtain a pragmatic assessment from the mechanical behavior of the adhesive layer and also a detailed look at the distribution of interfacial stresses, a generic constitutive model including all potentially activated deformation modes, namely hygro-elastic, plastic, and visco-elastic creep was developed. The current study focused on the three most common adhesive systems for structural timber engineering: one-component polyurethane (1C PUR), melamine-urea-formaldehyde (MUF), and phenol-resorcinol-formaldehyde (PRF). The corresponding numerical integration approaches, with additive decomposition of the total strain were implemented within the ABAQUS Finite Element (FE) program by means of material user subroutine UMAT.

To evaluate the true stress states and time-dependent deformations, a sequentially three-dimensional history-dependent moisture-stress analysis was performed using the developed material models for both wood substrate and adhesive layer. For this purpose, moisture distribution profiles in solid wood and adhesive layer were calculated by a transient moisture analysis in advance. Analytical studies and Finite Element numerical simulations were employed to predict the damage initiation and evolution inside adhesively bonded elements. Furthermore, the modeling of interfacial de-bonding was carried out based on the fracture mechanical properties of the bond-line and application of the cohesive interface elements within the framework of non-linear fracture mechanics. Moreover, the influence of stacking sequence along with the thickness of individual layers and the adhesive type on the fracture mechanical behavior and the developed modes of failure were also investigated. Eventually, the numerical evaluations were compared with the experimental observations of cracking and delamination in glued-laminated samples under changing relative humidity (RH) of the ambient air.
1.3 Outline of the thesis

The general outline of the thesis can be summarized as follows:

Chapter 2 focuses on the characteristics of wood and adhesive systems and functionality of engineered wood products. The first two sections describe the hierarchical structure of wood and its macroscopic mechanical behavior. The subsequent sections cover wood hygroscopic character along with hygro-mechanical response of adhesive systems. Finally, the ending of the chapter addresses enhanced mechanical performance of composite wood products and their vulnerability to damage and fracture under the varying moisture content.

Chapter 3 deals with constitutive material models for wood. The first section provides a literature review of previous works describing different rheological models for wood. Afterwards, the following section describes the details of the comprehensive hygro-mechanical constitutive model, developed in the framework of the current research. The next sections explain the general algorithm for the numerical implementation of the material model together with the principles of the numerical simulation of moisture transport and moisture-induced stresses. Eventually, the remaining of the chapter gives descriptions concerning numerical modeling of interfacial damage based on the application of the cohesive interface elements.

In Chapter 4 the capability and proficiency of the proposed material model are verified by several numerical examples. In addition, one application example demonstrates the flexibility and universality of the developed wood rheological model to predict the mechanical behavior of different species by the analysis of a hybrid wood element, i.e., a multi-layered beam constructed from two different wood types. Finally, the last part of the chapter states some remarks regarding the convergence and robustness of the implemented algorithm.

Damage initiation and progression in glued-laminated hardwood elements under varying climatic conditions is the topic of Chapter 5. The first section of this chapter gives the experimental observations of fracture propagation in adhesively bonded elements. The last section describes the corresponding numerical analyses including analytical studies, Finite Element moisture-stress simulations, and non-linear fracture mechanical modelings in the following of the chapter.

Chapter 6 summarizes the main results of the thesis and contains reflections on some ideas for future research in this field.
Chapter 2

Wood physics and adhesive bonding

2.1 Wood as a hierarchical material

Wood is a natural and renewable composite material with a multi-level structural hierarchy. From the macroscopic perspective, the cross-section of a stem consists of pith, heartwood, sapwood, and bark from the center of the trunk to the outer part. Typically, in comparison with the darker-colored heartwood, sapwood consisting of alive cells has a lighter color and serves as the actively conducting part of the stem. Wood, typically, grows in discrete time periods. It means, that portion of the stem produced earlier at the beginning of the growth interval, called as earlywood, has distinct physical properties compared with the latewood formed at the end of the growth increment \[104\]. The higher density and the darker color of the latewood lead to the appearance of the so-called concentric annual rings or growth rings representing the yearly growth of the trunk. The outermost layer of the stem is bark and underneath that is the cambium which produces bark cells on the outer side and wood cells in the inner part (see Fig. 2.1).

In the mesoscopic scale wood can be described as a honeycomb structure extended along the growth direction of the stem. Construction and arrangement of wood cells differ between softwood (conifer) and hardwood (deciduous) \[95\]. Softwood cellular structures are constructed from tracheid and parenchyma cells. The tracheid cells are oriented longitudinally along the axial direction of the trunk and their main job is to transport nutrients and to provide mechanical stability for the top part of the stem. The parenchyma cells are primarily aligned radially from the pith - the center of the stem - to the bark and are the basic structural components
of the wood rays that serve as channels to transport fluid to the cambium and also to store nutrient. The axial structural system of a hardwood is more complex, and consists of additional elements like fibers and vessels compared with a typical softwood. Fibrous cells solely give mechanical support to the trunk and the large-diameter pores (vessels) are responsible to carry water. Fig. 2.1b shows the cellular structure of a typical hardwood.

The cell wall of the honeycomb structure is a composite laminate that is made up of the primary wall (P) and the multi-layered secondary wall (S). The compound secondary wall comprises of three layers, namely S1 the outer layer, S2 the middle layer, and S3 the inner layer. Fig. 2.1c displays the laminated structure of the cell wall. As it is depicted schematically, the orientation of the reinforcing fibers (microfibrils) is different for every layer; they show no particular pattern in the primary wall and are oriented randomly, while in the secondary wall, they are aligned at a certain angle with reference to the axial direction termed as the microfibril angle (MFA). The outer layer S1 can be considered as an angle-ply laminate with an MFA ranging from 50° to 70° depending on the wood type. The middle layer S2 forms about 85% of the secondary wall thickness and its MFA changes from 10° to 30° based on the species and also the age of the wood cells. The S2 layer significantly affects the properties of the cellular structure such as swelling and shrinkage, strength, stiffness, and fracture mechanical behavior. The innermost layer S3 has a structure similar to the outermost layer S1 with a different MFA varying from 60° to 90°. The middle lamella with no reinforcement acts as an interconnecting layer and joins the individual cells together (see Fig. 2.1c).

On the microscopic ultra-scale, the cell walls consist of three primary components: cellulose, hemicelluloses, and lignin. Cellulose is a long-chain polymer and is formed as a result of polymerization of glucose during the growth period. Cellulose chains act as reinforcing microfibers in the laminated walls of the wood cells and are embedded in an uncrystallized matrix comprising of hemicelluloses and lignin. Hemicelluloses are branched polymers with irregular structure and are composed of additional monomers compared with the cellulose. Lignin as an amorphous polymeric material functions as glue between the cellulose microfibrils chains and the surrounding hemicellulose layer and also among the walls of the cellular structure (see Fig. 2.1c).

Further details and information on the anatomy and structure of wood are given in Kollmann and Côté, Dinwoodie, Bodig and Jayne, and Niemz.
2.2. WOOD MECHANICS

Owing to the cellular nature of wood and its cylindrical growth, wood in general is anisotropic. Nevertheless, for cut-sections distant from the center of the stem, it is usually considered as an orthotropic material with three major axes, namely
longitudinal (L) along the fiber (grain) direction, radial (R) and tangential (T), respectively normal and tangent to the growth rings in the plane perpendicular to the grain (see Fig. 2.1a). Accordingly, wood displays distinct material properties and independent mechanical behavior along the three anatomical directions. The longitudinal arrangement of the fibers in the cellular structure results in larger values of stiffness and strength in the grain direction compared with the values perpendicular to the grain. Moreover, material properties differ remarkably in the cross-grain directions and generally the radial stiffness is twice that of the tangential one \[56\]. Furthermore, due to the cellular structure, wood shows the so-called "bimodular behavior" implying the elastic (stiffness) and strength parameters are different under tension and compression in all orthotropy directions \[142\].

At low stress levels wood behaves elastically which represents the scleronomous linear material behavior and is fully recoverable after load removal. The elastic deformation (strain) is linearly proportional to the applied load (stress) and Hooke’s law of elasticity describes this relationship \[14\]. Nine independent material constants, including three Young’s moduli \((E_{RR}/E_{TT}/E_{LL})\), three shear moduli \((G_{RT}/G_{RL}/G_{TL})\), and three Poisson’s ratios \((\nu_{RT}/\nu_{RL}/\nu_{TL})\) characterize the elastic behavior of wood material \[104\].

When the applied load exceeds the strength of the material, depending on the type and direction of load, either plastic deformation or failure occurs. A plastic deformation is irreversible and even after the removal of the applied stress, the material does not return to its initial configuration. In the last decade, significant efforts were made to describe the elasto-plastic mechanical behavior of wood based on the progress in metal plasticity. Experiments were conducted for uni- and bi-axial loading under tension and compression \[43, 47, 48, 114\] for different species that lead to the following striking observations:

(a) Failure of wood under tensile or shear loading exhibits localized brittle fracture, however under compression, pronounced inelastic behavior (ductile failure) is witnessed.

(b) Under compression, two consecutive regimes are observed \[113, 163\], namely cellular collapse, and for larger inelastic strains densification or compaction of the collapsed cells \[47, 48\].

(c) Plastic hardening in different anatomical directions is only weakly coupled, since different orientation-dependent micro-mechanical mechanisms act on the cellular scale.
2.2. WOOD MECHANICS

Wood in constant environmental conditions and under sustained loading exhibits time-dependent deformation generally termed as visco-elastic creep. Many studies have investigated this long-term phenomenon to find the link between the invariable applied load and the increase in the deformation (see e.g., Toratti [189], Hanhijärvi [64], Morlier [124], and Hartnack [73]). Unlike the elastic strain, the visco-elastic deformation is partly linearly proportionate to the applied stress and partly to the viscosity of the material and the exposure time to the constant acting stress [95]. Generally, the visco-elastic deformation is divided into three main stages. The first stage called primary creep is characterized with comparatively high strain rate which slows down by passing time. In the subsequent step (secondary stage) the strain rate decreases until it becomes stable about a constant value. Eventually, in the third stage known as tertiary creep which occurs just before the final failure, the deformation increases exponentially [120]. (see Fig. 2.2)

![Figure 2.2: Different stages of the creep phenomenon as a function of time][1]

As Fig. 2.3 depicts schematically, application of a constant stress level over an extended interval causes an immediate deformation (elastic and possibly plastic) at the time $t_0$ accompanied by a continuous increase in the deformation (visco-elastic creep). In contrast to the elastic and possibly plastic deformations, the visco-elastic creep is a gradual process and accumulates during the load application period. Following the load removal at the time $t_1$, the elastic part of the deformation recovers instantaneously and the remaining deformation decreases gradually to a certain amount of residual strain. However, this time-dependent strain recovery is only a part of the total visco-elastic strain because a part of the creep deformation, termed as plastic creep, is irreversible [56, 95].
2.3 Hygric behavior of wood

Wood as a hygroscopic and capillary-porous material exchanges water with the ambient environment as either water vapor through the sorption from the surrounding air or as liquid water by capillary forces \[56, 95\]. The moisture exchange process continues until reaching to an equilibrium moisture content (EMC). Its direction, in the form of adsorption or desorption, is dependent on the RH and temperature of the ambient air. Adsorption means absorbing water from the ambient air with higher MC, while desorption denotes an inverse process. Moisture in wood appears as bound water in the walls of the cellular structure or as free water in the cavity of the cells (lumen) beyond fiber saturation point (FSP), a saturated state in which the surface of the cell wall is no longer able to take up more moisture \[56, 95\].

The fraction of the weight of moisture in a wood sample to the oven-dry mass \(m_0\) of that sample expresses the moisture content \(\omega\) [%]:

\[
\omega = \frac{m_\omega - m_0}{m_0} \times 100, \tag{2.1}
\]

with the mass of moist wood \(m_\omega\) [Kg]. Furthermore, the water concentration, denoted as \(c\) [kgm\(^{-3}\)], can be calculated with the dry volume \(V_0\) and the dry den-
2.3. HYGRIC BEHAVIOR OF WOOD

Moisture content $\rho_0$ through the following relationship:

$$c = \frac{m_\omega - m_0}{V_0} = \omega \rho_0.$$  \hspace{1cm} (2.2)

EMC changes with the RH of the ambient air basically from 0% in a totally dry environment to the FSP varying between 24% and 35\% \cite{100,161} at nearly 98\% RH to a fully saturated state of the lumen at 100\% RH \cite{56,95}. Sorption isotherms characterize the relationship between the RH and the corresponding EMC for moistening and de-moistening at a constant temperature (see Fig. 2.4). In general, the EMC in desorption is higher than that in adsorption for the same RH which is termed as sorption hysteresis. Therefore, the EMC for a given RH depends not only on the wood species, but also on the temperature level \cite{15,170} and the direction of the moisture exchange \cite{184}. The hygric behavior of wood has been investigated in a wide range of researches and studies where further details and information can be found for example in Refs. \cite{100,170,178,182,183}.

![Figure 2.4: Typical sorption isotherms for European beech and Norway spruce and the hysteresis between adsorption and desorption \cite{72}.](image)

The motion of water inside the porous material wood is dominated by diffusive...
transport and the flow through the lumens of the cells below and above the FSP, respectively. The diffusion process takes place as bound-water diffusion through the cell walls or water-vapor diffusion along the lumens \[56, 95, 100, 170\]. The flow of water obeys the Darcian law where the volumetric flow rate \( [m^3/s] \) is directly proportional to the pressure gradient by a permeability constant \( [m^2] \) \[95\]. The transport of moisture due to the gradient in the concentration of the diffusing fluid is referred to as Fick’s law of diffusion \[46\] where the diffusion flux \( [Kgm^{-2}s^{-1}] \) is proportionate to the concentration gradient by the diffusion coefficient \( D [m^2s^{-1}] \) \[56, 82\]. The longitudinal direction has the lowest resistance against the flow of water and the permeability along the fiber direction is much larger than that in the transverse directions. For instance, the longitudinal permeability of European beech is basically several orders of magnitude greater than the transverse ones. In the plane perpendicular to the grain, the radial direction is more permeable than the tangential one because of the radial arrangement of the ray cells.

The diffusion coefficient is a direction- and concentration-dependent quantity and is generally measured in two different ways: the steady-state cup method and the unsteady-state (transient) sorption method \[1, 30, 153\]. In the steady-state method, a wood sample is placed on the cover of a sealed cup filled with silica gel, saturated salt solutions, or water. Hence, the sample will be subjected to a moisture gradient resulting from the difference between the RH of the cup environment and the ambient air. If the RH inside the cup is higher than that of the outside air, the moisture flows through the sample from the cup towards the surrounding air and vice versa. Subsequently, the diffusion coefficient is calculated based on the time variation of the cup weight \[170\]. In the transient approach, a wood sample is climatized in a relative humidity of RH1 until equilibrium and then is rapidly exposed to a new condition with a relative humidity of RH2. Depending on the values of RH1 and RH2 the wood sample absorbs or desorbs moisture and accordingly the fractional weight change (FWC) [-] of the sample is defined as

\[
FWC = \frac{m(t) - m_0}{m_\infty - m_0}, \tag{2.3}
\]

with \( m(t) [Kg] \) as the current mass at time \( t [s] \) and \( m_\infty \) the mass at the equilibrium state. The FWC ratios are then plotted versus the square root of the exposure time and the diffusion coefficient is realized from the initial slope of a line fitted to the measured FWC data \[58, 199\].

As mentioned above, wood exchanges moisture with the ambient air which influ-
ences its behavior with respect to the environmental conditions. Hygro-expansion describes swelling or shrinkage of the material under varying moisture content. Moisture absorption below the FSP results in an expansion of the cell walls, thereby swelling takes place, whereas volume contraction termed shrinkage happens by moisture loss during desorption. The moisture-induced swelling/shrinkage deformation is linearly proportional to the moisture gradient, $\Delta \omega \%$, by swelling/shrinkage coefficient. The swelling/shrinkage coefficient is a direction-dependent property where the tangential coefficient is nearly twice the radial one while the one along the fiber direction is remarkably smaller compared with the cross-grain directions. Fig. 2.5 schematically illustrates the effect of direction-dependency of the hygro-expansion coefficients on the distortion of wood pieces from different cut-sections in the trunk.

![Figure 2.5: Distinct shrinkage behavior of the cut-sections with different orientation of the anatomical directions in the plane perpendicular to the grain](103)

The effect that the observed deformation of a loaded specimen under changing moisture content is significantly higher than the deformation of a loaded specimen under constant climatic conditions superimposed on the deformation of an unloaded specimen under varying moisture level, is known as mechano-sorptive creep (see Fig. 2.6). The mechano-sorption was first observed by Hearmon and Paton [80] and has been further investigated, for example, by Grossman [59], Bažant [10], Hunt [88, 89], Hunt and Shelton [92], Hoffmeyer and Davidson [86], Ranta-Maunus [149, 150], Mårtensson [126], Toratti [189], Liu [110], Mohager and Toratti [123], Svensson [185], Toratti and Svensson [186, 192], Entwistle [44], and Olsson [137].

Mechano-sorption occurs under different types of loading, i.e., normal tension or
Chapter 2. WOOD PHYSICS AND ADHESIVE BONDING

compression, bending, or shear and also in all anatomical directions. Nonetheless, the magnitude of the mechano-sorptive deformation differs depending on the moisture level referring that the mechano-sorptive effect is more pronounced at higher moisture contents [110, 189]. Besides the moisture level, the rate of the change in the moisture content, $\dot{\omega}$, also influences the amount of the mechano-sorptive strain [90, 191]. In addition, the mechano-sorptive effect accumulates under cyclic moisture variation which is known as hygro-fatigue. The entire phenomenon occurring under combined mechanical and hygric loading, however, is not yet fully understood. This is partly because of the highly complex test methodology, the time-consuming character, and often very large scatter of the measured data in determining mechano-sorptive characteristic quantities. Experimental studies regarding the mechano-sorption for spruce can be found in Refs. [67, 127, 151], while for European beech such studies are still missing.

Both the moisture content and temperature of wood affect its material properties. Nevertheless, since 1% change in the MC has an effect ten times more significant than 1 [K] change in the temperature [135], in most of the cases the temperature dependency of the wood properties is neglected. Likewise, the current study assumes that the temperature to be constant and only takes into account the moisture-dependency of material properties used to define the mechanical behavior of wood samples. Refs. [83, 142] and Refs. [56, 134] provide all material constants required to describe the elastic behavior of European beech and Norway spruce at different moisture levels, respectively. The moisture-dependent normal and shear strength values characterizing the plastic behavior are taken from Refs. [82, 142] for European beech and Refs. [157, 163] for Norway spruce. The properties describing the long-term visco-elastic behavior are calculated based on the experimental measurements of European beech reported in Ref. [84] and the data presented in Ref. [49] for Norway spruce.

2.4 Hygro-mechanical behavior of adhesives

Adhesives are the most important joining system in wood construction and are widely used in all areas of timber engineering as well as in interior design and furniture industry. Adhesive joints generally transfer and distribute loads from one component to another and their performance depends on the efficiency of the adhesion controlled by the strength of the mechanisms contributing to the adhesive-bonded joint. Three principal mechanisms may participate in the formation of a
2.4. HYDRO-MECHANICAL BEHAVIOR OF ADHESIVES

Figure 2.6: Schematic interpretation of the mechano-sorptive creep. $\Delta L_{Me}$, $\Delta L_{\omega}$, and $\Delta L_{MS}$ denote increase in the length caused by external mechanical loading, swelling, and mechano-sorptive effect, respectively.

wood-adhesive bond [97, 118, 146, 167]:

1. A mechanical entanglement arising from the physical interlocking of the adhesive material into the pores and irregularities of the wood substrate’s surface,

2. Primary bonds including interatomic ionic and covalent bonds resulting by
electrons transfer from one atom to another (ionic) or electrons sharing (covalent) upon the contact of two surfaces,

(3) Secondary bonds as hydrogen and Van der Waal’s bonds originating from the intermolecular and interatomic attractive forces formed between the atoms and molecules of the adhesive material and the wood adherend.

The wood-adhesive bond can be considered an imaginary symmetrical chain across the bond-line with nine different links (zones) \[118\] which in practice is simplified to a model with five distinct regions: two wood substrates, two inter-phase regions of wood and adhesive, and one adhesive bulk material (see Fig. 2.7a) \[104, 167\]. The inter-phase zone is regarded as a layer with an infinitesimal thickness nearly equal to the penetration depth of the adhesive into the cell wall micro-structure and both wood adherend and adhesive material influence its properties. Fig. 2.7b displays the electron-microscope picture of a wood-adhesive bond where the thickness of the inter-phase region is restricted to the first row of wood cells on each side of the bond-line.

Since wood adhesives are employed in many various applications, a broad variety of adhesive types are available \[53, 209\]. Wood adhesives are generally classified as either natural or synthetic organic. Furthermore, based on their structural performance under different levels of exposure to changing RH are categorized into: structural or fully exterior, semi-structural or limited exterior, and non-structural or interior \[31\]. Phenol-formaldehyde (PF), resorcinol-formaldehyde (RF), melamine-formaldehyde (MF), urea-formaldehyde (UF), isocyanate, epoxy, polyurethane (PUR), and polyvinyl acetate (PVAc) are examples of wood adhesive systems \[31, 104\].

The adhesive bonding process in general consists of three main steps: surface preparation, spreading, and curing. Proper surface preparation enhances the interaction of the adhesive with the adherends and spreading the liquid adhesive all over the surfaces develops an intermolecular contact with the wood substrates. The final stage, curing, refers to the conversion of the adhesive into a solidified or hardened state through physical or chemical actions \[53\]. The curing process differs depending on the type of the adhesive and is basically governed by one or more of the following mechanisms: (a) cooling of a hot melt adhesive, (b) the loss of solvent (mostly water) by evaporation into the air and diffusion into the wood substrate, (c) polymerization and formation of cross-linked structures. For most wood adhesives, the solidification happens because of the simultaneous occurrence of the loss of water and chemical
2.4. HYGRO-MECHANICAL BEHAVIOR OF ADHESIVES

Wood adherend
Interphase region
Adhesive
Interphase region
Wood adherend

Adherend 1  Adhesive  Adherend 2

50 µm

Figure 2.7: a) Schematic illustration of the simplified layered model representing the different regions of an adhesive-bonded joint [167, 201], b) The electron-microscope picture of a typical wood-adhesive bond with adherends made of softwood [167].

polymerization which leads to the development of the bond-line strength [31, 104].

Wood adhesives similar to the solid wood show hygric behavior and can exchange moisture with both the ambient air and the adjacent wood. Two primary mechanisms are supposed to dominate the process of water uptake in wood adhesives [45, 125]: (1) diffusion of water through the free volume available in the polymer and (2) binding of water with ionic groups of the polymer chain. Water absorption potentially leads to undesirable changes in the structure and properties of adhesives [179] which may influence the mechanical performance of the adhesive-bonded joint and glued wood products as well [62]. Therefore, the influence of water on adhesive material is of remarkable significance and has to be considered for deeper investigation of the safety and reliability of the adhesive bond-line.

As it was mentioned earlier in Section 1.2, the current research concentrates on 1C PUR, MUF, and PRF as three cold-setting adhesives which are among the most
employed adhesive systems for semi-exterior and exterior wood products. Several studies have investigated the hygro-mechanical behavior of 1C PUR, MUF, and PRF. Ref. [202] presents the characteristic sorption isotherms of these adhesives reflecting the correlation between the RH and the MC for both adsorption and desorption (see Fig. 2.8). The experimental measurements revealed that PRF reached a high moisture content of 18% at 98% RH under a slow uptake speed, whereas MUF showed the highest weight gain (22% MC) and PUR the lowest one (3.5% MC) achieved at 98% RH, respectively with an intermediate and a high diffusion speed.

Kläusler et al. [98] have studied the mechanical properties of the PRF, MUF, and 1C PUR under various environmental conditions ranging from 5-95% RH. The stiffness (Young’s modulus) and the tensile strength of all the tested adhesives were dependent on the MC and notably decreased with simultaneous increase in the RH. Additionally, the PRF and MUF adhesives indicated remarkably higher stiffness and extremely lower ductility compared with the 1C PUR in all MC levels. Moreover, the 1C PUR and MUF showed the tendency to flow and increase in the deflection under sustained load (visco-elastic creep), while this trend was negligible in the PRF.

### 2.5 Engineered wood components

Wood-based composites, also named as engineered woods, comprise a broad range of adhesively bonded products which are constructed by binding the timbers, veneers, strands, or particles of wood together. A variety of engineered wood products such as oriented strand board (OSB), particleboard (PB), laminated veneer lumber (LVL), I-joist, glued laminated timber (glulam), and cross-laminated timber (CLT) of various thicknesses, sizes, and grades are available. Engineered woods are generally employed for non-structural and structural purposes ranging from products with applications in interior covering and furniture manufacturing to components for exterior uses and construction of buildings [104]. Such reconstituted elements feature higher homogeneity and uniformity with lower variability in the mechanical properties like stiffness and strength compared with solid wood, while they can be made from small-diameter timber or even recovered wood obtained from other manufacturing processes [13]. Wood composites are produced in different shapes in compliance with various applications and present greater structural integrity and dimensional stability than the original clear wood. Moreover, they can be tailored
2.5. ENGINEERED WOOD COMPONENTS

Figure 2.8: Sorption isotherms of the three most common commercial wood adhesives, namely PRF, MUF, and 1C PUR during both moisture adsorption and desorption [202].

Among all structural wood-based composite products, glulam and CLT are of remarkable importance in wood construction and are basically preferred to solid wood counterparts because of comparative advantages. Glulams consist of two or more adhesively bonded-layers of timber where the fiber direction of all laminations runs parallel to the length. These structural wood composites generally act as load-bearing elements like rafters and beams in residential or industrial buildings (see Fig. 2.9a). They are produced in a wide variety of cross-sections, lengths, and curvatures and offer more flexibility and freedom in construction for example through spanning large distances with no necessity for intermediate supports, optimal thickness with the possibility of local reinforcement only at intersections with columns or other beams, and enhanced shape stability under changing environmental conditions. Furthermore, to reduce construction costs, timbers of lower quality
can be combined with high-grade lumbers based on the expected stress level of laminations. While the inner section of the beam can be of low-grade timbers, the outermost layers should be of high quality ones \[104, 167\].

CLT is a load-transferring component which can serve as straight or curved panelized floor and wall systems that replace steel or concrete in many applications \[181\]. CLT comprises odd numbers of veneer sheets or timber layers (lamellae) which are glued together, while the fiber direction of every layer is perpendicular to the adjacent one (see Fig. 2.9b). The higher strength and stiffness of wood parallel to the grain compared to the cross-grain directions in combination with crosswise lamination pattern result in a much stronger and more uniform product with enhanced mechanical performance in both directions. In other words, CLT panels exhibit a level of isotropy, improved structural integrity, and higher dimensional stability than solid wood. Similarly, since the hygro-expansion coefficient of timber in the longitudinal direction is considerably less than the radial and tangential ones, the crosswise alignment ensures that each layer will try to move in a different direction than the neighbor layer; consequently, restraining each other in place which gives rise to a restrictive effect with respect to the hygroscopic behavior. This leads to a more consistent engineered product with minimal edge-swelling and high resistance to splitting under exposure to varying MC \[104\].

Figure 2.9: Wood-based structural composites: a) Glued laminated timber (glulam), b) Cross-laminated timber panel.
2.6 Damage and fracture of engineered wood

In spite of the aforementioned benefits of structural wood composites, these products have some shortcomings, as well. The most important disadvantage to the engineered woods is that they are more susceptible to absorb moisture either during their manufacturing process or within their service life. Subsequently, they are more prone to moisture-induced stresses (Eigen-stresses) than equivalent solid woods, which is especially true for laminated configurations and cross-laminated panels in particular. Laminated composite woods differ in the number of layers and the applied adhesive system as two main characteristics essential for attaining more consistent properties, superior durability, and higher resistance to variable humidity. Generally, two possibilities for making thicker laminated panels are: increasing the thickness of individual layers or using more lamellae. The latter method has a priority over the former one since it leads to a more homogeneous and dimensionally stable product; on the other hand, attaching more laminations together increases the number of adhesive layers and the amount of waterborne adhesives which add to the total moisture content of the composite element \[53\]. This rise in the moisture level combined with the likely difference in the initial moisture content of two adjacent lamellae and their differential swelling/shrinkage behavior cause an amount of residual stresses in adhesively bonded elements during fabrication. The magnitude of the produced residual stresses is remarkably higher in cross-laminated patterns due to the noticeable mismatch of the hygro-expansion coefficients between the grain and the cross-grain directions of two adjoining layers.

Moreover, further moisture-related stresses develop in glued-laminated configurations under varying environmental conditions during their service period, which are superimposed on the above-mentioned residual stresses. Wood orthotropy and hygroscopicity along with non-uniform moisture distributions and spatial variation of the swelling/shrinkage in the wood substrates generate a complex state of moisture-induced stresses, specifically in cross-laminated elements \[57\]. In addition, as water permeates inside the wood, adhesive layer may behave as a barrier against moisture transfer because of the distinctive hygric properties and lower permeability of the adhesive material \[180\]. It potentially induces abrupt changes in the moisture profile below and above the bond-line (see Fig. 2.10) and also leads to the localized moistening of the adhesive layer \[202\]. The possible local swelling of the bond-line and the perturbations in the moisture distribution create extra internal stresses in the vicinity of the wood-adhesive interface and lead to the evolution of higher level of
interfacial stresses \[180\]. Furthermore, the occurrence of some inherently time- and moisture-dependent phenomena like long-term visco-elastic creep (see Section 2.2) and mechano-sorption (see Section 2.3) accelerate the degradation of material stiffness and strength. Accordingly, deterioration of the material properties in addition to development of moisture-induced tensile or shear stresses result in initiation and propagation of cracks in both the wood substrate and adhesive bond-line.

![Figure 2.10: Moisture profiles in the transverse section of a glulam element subjected to climate variations changing from RH1=65% (\(\omega_1 \approx 12\%\)) to RH2=50% (\(\omega_2 \approx 9\%\)) at T=20\(^\circ\). a) Adhesive layer with zero permeability, b) adhesive layer with the same permeability as solid wood.](image)

On the basis of wood orthotropy, solid wood shows six different crack propagation systems in total. Each crack system is designated by a pair of letters representing the normal to the crack plane (first letter) and the crack propagation direction (second letter), respectively (see Fig. 2.11a). Among these propagation systems, RL, TL, TR, and RT are the most common ones in engineered wood structures \[104\]; since cracks are typically more prone to be formed in the planes with the least tensile strength (perpendicular to the grain) and basically tend to propagate along the grain direction, RL and TL growth systems predominate and are of practical significance \[50, 148, 172\]. Furthermore, from a fracture mechanical point of view, there are three different modes of fracture defined based on the direction of the applied load and the consequent motions of the crack surfaces, namely Mode I, Mode II, and Mode III \[93\]. Mode I or opening mode occurs under a tensile load perpendicular to the crack faces. A shear load parallel to the crack planes and acting
orthogonal to the crack front causes Mode II or in-plane sliding mode, while Mode III or out of plane tearing mode happens because of a shear load applied parallel to both the crack surfaces and the crack front. Fig. 2.11 displays a schematic illustration of various fracture modes. Wood composites barely experience pure-mode loading in practical applications and are basically subjected to a combination of modes called mixed-mode loading condition \[148, 166\]. Subsequently, each of the aforementioned growth systems potentially may undergo 2-3 fracture modes simultaneously \[63\].

(a) RL RT LR
(b) TL TR LT

Figure 2.11: a) All principal crack growth systems in wood defined based on the orientation of the crack plane (first letter) and the corresponding propagation direction (second letter) \[148\]. b) Schematic representation of different modes of fracture \[148, 208\].

In addition, the failure of the bond-line, depending on the fracture mechanical properties of the interface and the mechanical performance of the adhesive system and the solid wood adherend, can be categorized into three different types: 1) adhesive failure in which a crack initiates and propagates at the interface of adhesive layer and solid wood substrate; 2) cohesive failure in which a crack forms and develops within the thickness of the bulk adhesive layer; 3) interphase fracture in which a crack starts and runs in a very thin layer inside the solid wood in the vicinity of
Chapter 2. WOOD PHYSICS AND ADHESIVE BONDING

Adhesive failure  Cohesive failure  Interphase failure

Figure 2.12: Three basic failure mechanisms responsible for the damage of the adhesive bond-line [8, 34].

the bond-line interface [50, 167, 176]. (see Fig. 2.12)

To summarize, solid wood fracture and bond-line delamination influence unfavorably the service life and mechanical performance of adhesively bonded wood elements and reduce the safety, serviceability, and efficiency of these types of wood products dramatically. Thus, it is highly desirable to develop efficient design criteria which guarantee the durability and consistency of glued-laminated hardwood products under exposure to varying moisture content.
Chapter 3

Rheological model for wood

3.1 Previous studies on wood constitutive models

Besides the primary influential factors such as wood species, moisture content, and fiber-load angle, the occurrence of the time- and moisture-dependent behavior affect markedly the serviceability and durability of timber structures under external mechanical and hygric loading. Subsequently, constitutive material models are highly desired to gain further insight into the mechanical performance of wood components. However, the accuracy of prediction of the wood behavior extremely depends on the reliability and validity of the applied material models since simplified ones are not able to completely and correctly capture the long-term mechanical responses under the combination of moisture variation and mechanical loading.

In general, wood rheological models are formulated in terms of a continuum approach and describe the global mechanical behavior of wood by integrating the influences of the stress level, moisture history, and the rate of the moisture content change. Constitutive equations characterize the relations among stress, strain, and moisture level and are generally developed based on this assumption that every mechanism contributes separately to the overall mechanical response. It means, depending on the comprehensiveness of the material model, the total strain may consist of all or some of the following phenomena: short-term elastic, plastic, hygro-expansion, visco-elastic, and mechano-sorptive deformations. Nowadays, it is a common practice to disregard moisture, plasticity, creep, and mechano-sorption for technical approvals of new products, not because they are insignificant, but because of the difficulties in their experimental assessment that is challenging due to a high degree of coupling, time-consuming due to low diffusivity, and in some cases simply impossible.
A review on the general use of FE in wood analysis was published by Mack-erle [117], while other authors focused on reviewing proposed rheological models [16, 64, 165, 197]. Hanhijärvi [69] gave an overview about the various types of approaches introduced to characterize the mechano-sorpptive effect. The first category presented by Takemura [188] and Leicester [106] and improved later by Ranta-Maunus [149, 151] and Santaoja et al. [160] correlates the rate of the mechano-sorptive deforma-
tion linearly to both the stress level and the absolute value of the rate of moisture variation. The second classification developed by van der Put [147] is defined in terms of Eringen’s activation energy theory and the third variant is described on the basis of "creep limit" concept suggested by Hunt and Shelton [91].

In addition to the aforementioned models, Hanhijärvi [64] proposed another method based on the relaxation and recoverability of the mechano-sorption and its interaction with the visco-elastic creep. In this rheological approach, the mechano-
sorptive deformation is characterized by the application of one or more Newto-
nian elements (dashpot) with other Hookean elements (spring). The rheological
dashpot is activated under moisture variations and the amount of recoverability
of the mechano-sorption principally depends on the number and arrangement of
the dashpot elements with respect to other rheological units. In a similar theo-
retical approach, the visco-elastic creep is mainly formulated by rheological spring
and dashpot elements. The spring element describes the elastic properties while
the dashpot represents the viscous character of the material. Fig. 3.1 shows two
possible connections of the spring and dashpot, either in a series or in a parallel
association called as Maxwell element and Kelvin-Voigt model, respectively [15, 112,
136]. In general, Kelvin type elements are preferred to the Maxwell ones because
from a thermodynamical point of view the Kelvin model is more consistent [197]
and additionally recovery under load removal cannot be captured by the Maxwell
description [6].

Due to the wood orthotropy and strong direction dependency of its material
properties, wood structural elements are basically designed to be preferably loaded
along the grain direction. Because of this, the majority of the existing wood material
models were developed based on a 1D formulation [197]. Nevertheless, in the last two
decades several experimental and theoretical 2D and 3D constitutive models with
different ambient relative humidity (RH) and mechanical loading were proposed in
which the sources for non-linearity like plasticity, creep, or mechano-sorption are
covered to quite different extents. Generally, non-linear numerical models for wood,
in particular with long-term character are very rare due to the need of species- and
3.1. PREVIOUS STUDIES ON WOOD CONSTITUTIVE MODELS

Figure 3.1: Schematic representation of the a) Kelvin-Voigt and b) Maxwell elements. Subscripts S and D denote spring and dashpot, respectively.

moisture-dependent mechanical parameters.

Leicester [106] proposed a 1D constitutive formulation - also called first approximation model - to measure the deflection of drying beams. This unidimensional model was rheologically similar to a Maxwell cell and consisted of two serial elements denoting an elastic and a mechano-sorptive deformation, while did not consider the long-term visco-elastic creep. A uni-axial model was developed by Bažant [10] to investigate the experimentally observed increase in the creep rate under simultaneous changes in the moisture content and temperature. The corresponding formulations were derived based on the application of the Maxwell elements with moisture- and temperature-dependent viscosity. The model introduced by Liu [111] was another unidirectional law providing satisfactory predictions regarding the creep behavior of wood under a large variety of load levels and changing environmental conditions. In this model, the visco-elastic creep was formulated by six Kelvin-Voigt elements whereas the constitutive equation for the mechano-sorption was described according to a slightly modified version of the model proposed in Refs. [106, 151, 188]. Liu [111] altered this model by using a non-linear stress function instead of a linear one to account for the experimental observations in Ref. [110] concerning the non-linear character of the mechano-sorption at the load levels above 30% of the ultimate strength. Further 1D constitutive relations originally developed for the numerical analysis of timber drying or the investigation of timber long-term behavior under variable humidity are given in Ranta-Maunus [149], Salin [158],
Chapter 3. RHEOLOGICAL MODEL FOR WOOD

Mårtensson [127, 128], Mårtensson and Svensson [129, 130], Liu and Schaffer [108], Mauget and Perré [119], and Häglund [61].

The material model presented by Dubois et al. [39] is a 2D numerical approach proposed to investigate the time-dependent crack growth in timber structures under constant environmental conditions. The linear visco-elastic orthotropic response was formulated through the combination of one single Maxwell unit and multiple Kelvin-Voigt elements for the plane state of stress. A more recent study from the same author based on the generalized Kelvin-Voigt model incorporated a linearly visco-elastic behavior into mechano-sorptive creep and swelling/shrinkage to include the influences of both external loading and moisture variation histories [40]. Another 2D constitutive model for the simulation of wood under drying conditions in the plane perpendicular to the grain (RT) has been introduced by Hanhijärvi et al. [70, 71, 115]. This model is a 2D extension of the previously developed uni-axial formulations presented in Refs. [65–68] which considers the hygro-elastic, visco-elastic, and mechano-sorptive responses. The proposed constitutive relations employ a series of Kelvin-type elements for the mathematical description of both the visco-elastic and mechano-sorptive creep. Additionally, to cover the evolution of the irrecoverable deformations occurring at elevated temperature, a plastic element is integrated with the mechano-sorptive portion. Srpčič et al. [180] proposed a 2D approach to assess the mechanical behavior of glulam beams under simultaneous exposure to the mechanical loading and varying RH. Their work formulated the time-dependent visco-elastic creep by a serial association of the Kelvin-Voigt elements, but expressed the increment of the mechano-sorptive deformation as an exponential function of the absolute value of moisture content change. Additionally, the effect of the plastic deformation was neglected. More 2D wood material models can be found in Refs. [2, 51, 68, 130, 186].

For multi-axial stress states, Eberhardsteiner et al. [42] presented a material model which was a reformulation of the uni-axial model in Ref. [64]. The 3D rheological model describes the coupled visco-elastic (stress relaxation) and mechano-sorptive creep of wood and uses a parallel association of the Maxwell elements while every chain is enhanced by a hygro-expansion unit. Likewise, Vidal-Sallé and Chassagne [197] and Chassagne et al. [25] proposed a 3D nonlinear material law by using a generalized Maxwell model comprising of two elements whose viscosities depend on the stress level and also on the moisture variation. This model is able to capture creep and recovery phenomena under constant and changing environmental conditions. Another 3D visco-elastic mechano-sorptive material law comprising a parallel
association of several *Maxwell* cells with non-linear dashpot is explained in Ref. [18]. In this constitutive model, the viscosity of the dashpots representing the mechanical creep behavior is defined on the basis of the stress level, whereas the remaining mechano-sorptive dashpots are described depending on the moisture level and the rate of moisture content change. The constitutive model introduced by Fortino et al. [49] is an extension of the general 2D computational approach in Refs. [70, 71, 115] to a 3D generalization for numerical analysis of timber structures. Similar to the 2D variant, the 3D model consists of five deformation mechanisms while these models differ in the description of the irrecoverable mechano-sorption. From a rheological point of view, the 2D approach uses a plastic element in combination with the mechano-sorptive part; whereas in the 3D version the plastic element is omitted and an independent mechano-sorptive dashpot represents the irreversible deformation instead [186]. Both the 3D visco-elastic and recoverable mechano-sorptive creep are formulated by means of *Kelvin* type elements and are characterized based on the associated 1D models proposed by Toratti [189] and Toratti and Svensson [186], respectively. Santaoja et al. [160], Ormarsson et al. [138–140], Gereke [56] are further examples of the 3D wood material laws.

In contrast to the above-mentioned constitutive models dealing with the long-term mechanical response of wood, others concentrated on the short-term elasto-plastic behavior, disregarding time dependency [60, 116, 141]. Experimental observations of wood failure under tension, compression, and shear (refer to Section 2.2) have consequences for the elasto-plastic models, shape and type of yield surfaces and their evolution. Simple models used a single deformable 3D ellipsoidal yield surface [115, 116], however, despite the advantages of having a closed surface with C2-consistency, occurrence of the brittle fracture under tension or shear as well as the weak coupling of the plastic hardening among different anatomical directions pointed at the limited validity of such an approach. The other extreme was given by a multi-surface formulation, where seven distinct failure mechanisms and subsequently seven yield flow criteria comprising three tensile and three compressive failures along orthotropy directions together with one shear failure were considered [163]. The constructed surface is only C0-consistent, requiring procedures for stress states that lie on the intersections, since the evolution of the individual surfaces is not coupled, leading to seven internal state variables. The model is basically a generalization of the presented plane stress model [113] to 3D. In principle, the brittle tensile or shear responses can be treated in the framework of plasticity as a smeared crack, nevertheless other approaches e.g., using cohesive elements are more
stable \cite{154,164}. In later versions, therefore, only three compressive surfaces were used by the authors \cite{164}. A further reduction was proposed by Refs. \cite{85,154,157} with smooth corners (C1-continuity) and a combined evolution of all surfaces being described by a single strain-type internal state variable. Refs. \cite{81,143,187} give additional constitutive models characterizing wood elasto-plastic behavior.

A comprehensive constitutive model comprising all potentially activated mechanical responses as a function of moisture content for different species is still missing. However, it is the basis for numerical simulations that grant insight into the long-term behavior of structural elements of hardwoods, softwoods and hybrid ones. Therefore, to attain effective design criteria for new products, the development and characterization of an authentic moisture-dependent constitutive material model for different wood species and the robust implementation of its corresponding rheological model in broadly used non-linear FE environments is of great importance.

### 3.2 Hygro-mechanical constitutive model

To capture the consequences of the hygric behavior, all mechanical properties along the anatomical directions have to be consequentially expressed as a function of moisture content. Moreover, all potentially participating deformation mechanisms should be considered when moisture content changes during the use of wood structures. To predict the true deformation field and for the subsequent stress analysis, the respective material model consists of deformation modes originating from

- elastic deformation $\varepsilon^{el}$ (Section 3.2.1),
- irrecoverable plastic deformation $\varepsilon^{pl}$ (Section 3.2.2),
- swelling/shrinkage called hygro-expansion in this context $\varepsilon^{\omega}$ (Section 3.2.3),
- visco-elastic creep $\varepsilon^{ve}_i$ (Kelvin-Voigt element-wise visco-elastic strain tensor) (Section 3.2.4),
- mechano-sorption $\varepsilon^{ms}_j$ (Kelvin-Voigt element-wise mechano-sorptive strain tensor) (Section 3.2.5).

In general, a total strain tensor $\varepsilon$ is defined in a way that it accumulates the respective contributions: (see Refs. \cite{49,71,115} and Fig. 3.2)

$$\varepsilon = \varepsilon^{el} + \varepsilon^{pl} + \varepsilon^{\omega} + \sum_{i=1}^{n} \varepsilon^{ve}_i + \sum_{j=1}^{m} \varepsilon^{ms}_j.$$  \hspace{1cm} (3.1)
From a thermodynamical perspective \[49, 71, 115\], the free energy function, \( \psi \), is described as:

\[
\psi = \psi(T, \omega, \varepsilon, \varepsilon^{el}, \varepsilon^{ve}, \varepsilon^{ms}, \alpha_l) = \phi(T, \omega) + \psi^{el} + \psi^{ve} + \psi^{ms} + \frac{1}{2} \sum_{l=1}^{r} q_l \alpha_l. \tag{3.2}
\]

Here \( \phi(T, \omega) \) is a general expression for the thermal energy and since in the present study the effect of temperature on the mechanical response of the material is ignored, it is not further considered. \( \psi^{el} \) specifies elastic strain energy, \( \psi^{ve} \) and \( \psi^{ms} \) represent energy accumulated in the visco-elastic and mechano-sorptive elements:

\[
\psi^{el} = \frac{1}{2} e^{el} : C_0 : e^{el}, \quad \psi^{ve} = \frac{1}{2} \sum_{i=1}^{n} e^{ve}_i : C_i : e^{ve}_i, \quad \psi^{ms} = \frac{1}{2} \sum_{j=1}^{m} e^{ms}_j : C_j : e^{ms}_j,
\]

with the elastic stiffness tensor \( C_0 \), and the element-wise visco-elastic and mechano-sorptive stiffness tensors \( C_i \) and \( C_j \). Finally, the last term of the right-hand-side of Eq. (3.2) designates isotropic hardening energy, which appears during the evolution of the irrecoverable plastic deformations. In the following, all partial strains based on an additive decomposition of the total strain in addition to their associated thermodynamic driving stresses are described following Refs. \[71, 115\]. It should be noted the proposed hygro-mechanical constitutive model is formulated and implemented for infinitesimal strains. It means the application of the material model in
the case of large deformations, non-linear visco-elasticity, or damage phenomenon is not relevant.

### 3.2.1 Elastic deformation

After differentiating the free energy function Eq. (3.2) with respect to the total strain, the corresponding rheological relation is obtained as:

$$\sigma = \frac{\partial \psi}{\partial \varepsilon} = C_0 : \left( \varepsilon - \varepsilon^{pl} - \omega - \sum_{i=1}^{n} \varepsilon_i^{ve} - \sum_{j=1}^{m} \varepsilon_j^{ms} \right) = C_0 : \varepsilon^{el}. \quad (3.4)$$

Consequently, one can write $$\varepsilon^{el} = C_0^{-1} : \sigma$$, with the orthotropic elastic compliance tensor $$C_0^{-1}$$ constituted by nine independent material engineering constants given as

$$C_0^{-1} = \begin{pmatrix}
\frac{1}{E_R} & \frac{-\nu_{TR}}{E_T} & \frac{-\nu_{LR}}{E_L} & 0 & 0 & 0 \\
\frac{-\nu_{RT}}{E_R} & \frac{1}{E_T} & \frac{-\nu_{LT}}{E_L} & 0 & 0 & 0 \\
\frac{-\nu_{RL}}{E_R} & \frac{-\nu_{TL}}{E_T} & \frac{1}{E_L} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{RT}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{RL}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{TL}}
\end{pmatrix}. \quad (3.5)$$

The non-zero off-diagonal terms of the elastic compliance matrix are mutually equal which is referred to as reciprocal dependencies and can be written by the following relations:

$$\frac{\nu_{RT}}{E_R} = \frac{\nu_{TR}}{E_T}, \quad \frac{\nu_{RL}}{E_R} = \frac{\nu_{LR}}{E_L}, \quad \frac{\nu_{TL}}{E_T} = \frac{\nu_{LT}}{E_L}. \quad (3.6)$$

Note that all engineering constants depend on the moisture level. They can be fitted with linear and third degree polynomial functions of moisture content $$\omega$$ for the properties $$P_b$$ and $$P_s$$ associated with the species European beech and Norway spruce, respectively via parameters $$b_x, s_y$$ (x = 0,1 and y = 0,...,3): $$P_b = b_0 + b_1 \omega$$ and $$P_s = s_0 + s_1 \omega + s_2 \omega^2 + s_3 \omega^3$$ valid from the oven-dry condition to the fiber saturated state. The relevant coefficients for describing material constants utilized in the definition of the compliance tensors are summarized in Table 3.1.
Table 3.1: Coefficients for moisture-dependent engineering constants for European beech [83] and Norway spruce [56, 134].

<table>
<thead>
<tr>
<th></th>
<th>( E_R ) [MPa]</th>
<th>( E_T ) [MPa]</th>
<th>( E_L ) [MPa]</th>
<th>( G_{RT} ) [MPa]</th>
<th>( G_{RL} ) [MPa]</th>
<th>( \nu_{TR} ) [( \times 10^{-3} )]</th>
<th>( \nu_{LR} ) [( \times 10^{-3} )]</th>
<th>( \nu_{LT} ) [( \times 10^{-3} )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beech</td>
<td>2565.6</td>
<td>855.4</td>
<td>17136.7</td>
<td>667.8</td>
<td>1482</td>
<td>1100</td>
<td>293.3</td>
<td>336.8</td>
</tr>
<tr>
<td></td>
<td>-59.7</td>
<td>-23.4</td>
<td>-282.4</td>
<td>-15.19</td>
<td>-15.26</td>
<td>-17.72</td>
<td>-1.012</td>
<td>-8.722</td>
</tr>
<tr>
<td>Spruce</td>
<td>999.64</td>
<td>506.08</td>
<td>12791.75</td>
<td>762.8</td>
<td>880.75</td>
<td>153.4</td>
<td>232.0</td>
<td>285.7</td>
</tr>
<tr>
<td></td>
<td>3.61</td>
<td>5.0</td>
<td>15.22</td>
<td>-1.07</td>
<td>5.93</td>
<td>1.39</td>
<td>10.8</td>
<td>-8.6</td>
</tr>
<tr>
<td></td>
<td>-2.09</td>
<td>-1.35</td>
<td>-9.01</td>
<td>-0.06</td>
<td>-1.99</td>
<td>-1.39</td>
<td>0.398</td>
<td>2.8784</td>
</tr>
<tr>
<td></td>
<td>0.0477</td>
<td>0.0297</td>
<td>0.1885</td>
<td>0.0017</td>
<td>0.0477</td>
<td>0.0277</td>
<td>-0.0191</td>
<td>-0.07862</td>
</tr>
</tbody>
</table>

3.2.2 Plastic deformation

Wood is a hygroscopic material with strong dependence of stiffness and strength on the moisture content. It is, therefore, prone to accumulate irrecoverable deformations even under combinations of moderate load with simultaneous moisture increase. The tendency of important hardwood species, such as European beech for high moisture sorption amplifies the drop of mechanical properties - strength in particular - what can result in the excessive plastic deformations. Hence, for reliable and meaningful predictions of the behavior of wood in constructions, it is advisable to consider the irrecoverable constituent of the total strain.

The current work uses a yield surface similar to Ref. [164], namely a three-dimensional orthotropic non-smooth multi-surface plasticity model (C0-continuity) consisting of three independent failure mechanisms along the anatomical directions for compressive loading. To include the role of changing moisture content on the development of plasticity, all relevant strength values and the respective hardening parameters are defined to be moisture dependent. The transition between the elastic and inelastic domains in the stress space is characterized by three yield functions in the same form as the second-order polynomial failure criterion proposed by Ref. [195] as follows:

\[
f_l(\sigma, \alpha_l, \omega) = a_l(\omega) : \sigma + \sigma : b_l(\omega) : \sigma + q_l(\alpha_l, \omega) - 1, \quad l = R, T, L. \quad (3.7)
\]

\( \alpha_l \) denotes the strain-type internal state variable related to every anatomical direction, \( a_l \) and \( b_l \) resemble the strength tensors to be defined in the following, and eventually \( q_l \) is a scalar value for the plastic hardening.

Quantitative experimental data for the moisture dependence of the plastic hardening behavior is rather sparse. Therefore, to characterize the hardening behavior, this work adopts a mathematical approach proposed by Ref. [85] with a modified
form of Ramberg-Osgood equations \cite{173} that was successfully applied to the compressive behavior of European beech at various moisture contents. Uni-directional moisture-dependent isotropic hardening laws are applied which describe measured constitutive behavior under uni-axial compression along the three anatomical orientations. The term "uni-directional" emphasizes the independence of the hardening phenomenon of different failure mechanisms from each other, described above (see Section \ref{2.2}). The moisture-dependent hardening responses derived from the modified Ramberg-Osgood curves (see Ref. \cite{85}) are approximated by exponential functions as follows:

\[
q_l(\alpha, \omega) = \left(\beta_0 \omega + \beta_1\right)\left(1 - e^{-\beta_2 \alpha}\right) / f_{c,l}(\omega),
\]

in accordance with the available experimental results. Here \(l\) denotes the orientation \(R, T, L\), \(\beta_0, \beta_1, \beta_2\) are material constants, and \(f_{c,l}(\omega)\) is the compressive strength of the material at the current level of moisture content. Since the last term of the right-hand-side of Eq. (3.7) is equal to unity, the current value of the hardening, i.e., \(q_l(\alpha, \omega)\) must be normalized by the compressive strengths to make it consistent with the dimension of the yield criterion expression. All parameters needed for the calculation of the hardening functions for spruce and beech are summarized in Table 3.2. Note that this study contrary to Refs. \cite{113,163} omits the densification regime (see Section \ref{2.2}). The strength values are described by a linear dependence

Table 3.2: Coefficients for calculation of moisture-dependent hardening stress for European beech \cite{55,85}.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Beech</th>
<th>(\beta_0)</th>
<th>(\beta_1)</th>
<th>(\beta_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(l)</td>
<td></td>
<td>[MPa/%]</td>
<td>[MPa]</td>
<td>[-]</td>
</tr>
<tr>
<td>R</td>
<td>-0.1123</td>
<td>2.8765</td>
<td>55.29</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>-0.1411</td>
<td>3.2736</td>
<td>65.56</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>0.0</td>
<td>2.5180</td>
<td>95.3</td>
<td></td>
</tr>
</tbody>
</table>

on the moisture content \(\omega\). Analogous to Table 3.1, all corresponding properties for beech and spruce are calculated as \(P_b = (z_{b0}\omega + z_{b1})\) and \(P_s = (z_{s0}\omega + z_{s1})\), respectively where coefficients \(z_{b0}, z_{b1}, z_{s0},\) and \(z_{s1}\) are given in Table 3.3. The first index in the symbolic presentations of the strength values \((c, t, s)\) implies compressive, tensile, and shear, while the second one indicates either one of the anatomical directions or the corresponding plane. In the following, all strength tensors, i.e., \(a_l\) and \(b_l\) required for the formulation of the yield criterion belonging to each failure mechanism based on the approach introduced in Ref. \cite{163} and for the RTL alignment of the
3.2. HYDRO-MECHANICAL CONSTITUTIVE MODEL

orthotropic material coordinate system are given. \( \mathbf{b}_t \) are diagonal matrices with entries outside the main diagonal being zero. Note that all compressive yield stresses in the definition of the strength tensors are accompanied by a minus sign.

**Compression in radial direction**

\[
a_R = \begin{bmatrix} \frac{1}{f_{c,R}} & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad \text{for European beech and Norway spruce,}
\]

\[
b_R = \text{diag} \left[ 0, \frac{0.0805}{f_{c,T}}, -\frac{0.1490}{f_{c,L} f_{c,L}}, \frac{0.1080}{f_{s,RT}}, \frac{0.1125}{f_{s,RL}}, \frac{0.0705}{f_{s,TL}} \right], \quad \text{for European beech,}
\]

\[
b_R = \text{diag} \left[ 0, \frac{0.4000}{f_{c,T}}, -\frac{0.2500}{f_{c,L} f_{c,L}}, \frac{0.4000}{f_{s,RT}}, \frac{0.3300}{f_{s,RL}}, \frac{0.3300}{f_{s,TL}} \right], \quad \text{for Norway spruce.}
\]

(3.9a)

(3.9b)

**Compression in tangential direction**

\[
a_T = \begin{bmatrix} \frac{1}{f_{c,T}} & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad \text{for European beech and Norway spruce,}
\]

\[
b_T = \text{diag} \left[ \frac{0.0805}{f_{c,R}}, 0, -\frac{0.1490}{f_{c,L} f_{c,L}}, \frac{0.1080}{f_{s,RT}}, \frac{0.1125}{f_{s,RL}}, \frac{0.0705}{f_{s,TL}} \right], \quad \text{for European beech,}
\]

\[
b_T = \text{diag} \left[ \frac{0.4000}{f_{c,R}}, 0, -\frac{0.2500}{f_{c,L} f_{c,L}}, \frac{0.4000}{f_{s,RT}}, \frac{0.3300}{f_{s,RL}}, \frac{0.3300}{f_{s,TL}} \right], \quad \text{for Norway spruce.}
\]

(3.10a)

(3.10b)

**Compression in longitudinal direction**

\[
a_L = \begin{bmatrix} 0 & \frac{1}{f_{c,L}} & 0 & 0 & 0 \end{bmatrix}^T, \quad \text{for European beech and Norway spruce,}
\]

\[
b_L = \text{diag} \left[ \frac{0.0665}{f_{c,R}}, \frac{0.0665}{f_{c,T}}, 0, \frac{0.0675}{f_{s,RT}}, \frac{0.0855}{f_{s,RL}}, \frac{0.0530}{f_{s,TL}} \right], \quad \text{for European beech,}
\]

\[
b_L = \text{diag} \left[ \frac{0.3300}{f_{c,R}}, \frac{0.3300}{f_{c,T}}, 0, \frac{0.2500}{f_{s,RT}}, \frac{0.2500}{f_{s,RL}}, \frac{0.2500}{f_{s,TL}} \right], \quad \text{for Norway spruce.}
\]

(3.11a)

(3.11b)
The scalar coefficients in the numerator of the diagonal components of the strength

Table 3.3: Coefficients for calculation of moisture-dependent strength values for European beech [82] and Norway spruce [157, 163].

<table>
<thead>
<tr>
<th>Strength</th>
<th>Beech</th>
<th>Spruce</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z_{b0}$ [MPa/%]</td>
<td>$z_{s0}$ [MPa/%]</td>
</tr>
<tr>
<td>$f_{c,R}$</td>
<td>-0.5789</td>
<td>20.32</td>
</tr>
<tr>
<td>$f_{c,T}$</td>
<td>-0.2084</td>
<td>7.965</td>
</tr>
<tr>
<td>$f_{c,L}$</td>
<td>-3.5040</td>
<td>103.9</td>
</tr>
<tr>
<td>$f_{t,L}$</td>
<td>-1.4650</td>
<td>106.9</td>
</tr>
<tr>
<td>$f_{s,RT}$</td>
<td>-0.1213</td>
<td>4.982</td>
</tr>
<tr>
<td>$f_{s,RL}$</td>
<td>-0.3884</td>
<td>15.51</td>
</tr>
<tr>
<td>$f_{s,TL}$</td>
<td>-0.3861</td>
<td>16.21</td>
</tr>
</tbody>
</table>

matrices $b_l$ are weighting factors which following Ref. [163] can be adjusted to the respective species. To make an adaptation from spruce (s) to beech (b), the related strength value of the denominator is replaced with $f^s = f^b \cdot \left( \frac{f_{12}^b}{f_{12}^s} \right)$ which corresponds to a scaling of the scalar values with strength value ratios at $\omega = 12\%$. Until today, bi-axial tests for beech have not been published in literature, hence a final justification for this adaptation assumption is not possible. Fig. 3.3 shows the boundary between the linear elastic domain and the non-linear behavior of the material, based on the strength tensors $a_l$ and $b_l$ at $\omega = 12\%$, and for ($q_l = 0$). Because of the three-dimensional representation of the stress tensor, the demonstration of all yield surfaces for any arbitrary stress state through one individual image is not feasible. Accordingly, Fig. 3.3a depicts a two-dimensional illustration of the yield conditions under the planar state of stress in the R-L plane, i.e., ($\sigma_T = \sigma_{RT} = \sigma_{TL} = 0$) and for principal normal stresses ($\sigma_{RL} = 0$). Fig. 3.3b displays a three-dimensional visualization of the failure surfaces for the situation in which the principal directions of stress and axes of the local material coordinate system are coincident ($\sigma_{RT} = \sigma_{RL} = \sigma_{TL} = 0$).

By now, all moisture-dependent parameters needed for the description of the rate-independent multi-surface plasticity material model are introduced. The general algorithm for the numerical implementation of the above-mentioned model is explained in detail in Refs. [94, 173]. Now, a concise description of the principles of the evolution equations for the irrecoverable deformations is presented in the context of non-smooth multi-surface plasticity. The constitutive relation of the plastic deformation under the assumption of a standard associative plasticity is described
by the following formula known as flow rule, usually referred to as Koiter’s rule \[99\]:

\[
\dot{\varepsilon}^{pl} = \sum_{l=1}^{r} \dot{\gamma}^l \partial_{\sigma} f_l(\sigma, \alpha_l, \omega) = \sum_{l=1}^{r} \dot{\gamma}^l (a_l(\omega) + 2b_l(\omega) : \sigma).
\] (3.12)

Here \( r \) denotes the number of active yield mechanisms. Similarly, with associative hardening, the evolution equation corresponding to the hardening law takes the form

\[
\dot{\alpha} = \sum_{l=1}^{r} \dot{\gamma}^l \partial_{q} f_l(\sigma, \alpha_l, \omega) = \sum_{l=1}^{r} \dot{\gamma}^l.
\] (3.13)

In Eqs. (3.12) and (3.13), \( \dot{\gamma}^l \) are plastic consistency parameters, which fulfill the
Kuhn-Tucker complementary requirements:

\[ \dot{\gamma}_l^I \geq 0, \quad f_l(\sigma, \alpha_l, \omega) \leq 0, \quad \dot{\gamma}_l^I f_l(\sigma, \alpha_l, \omega) = 0, \quad (3.14) \]

together with the consistency condition:

\[ \dot{\gamma}_l^I \dot{f}_l(\sigma, \alpha_l, \omega) = 0. \quad (3.15) \]

For an assumed number of active yield constraints \( r_{adm} \) and a given state of the stress and internal hardening variable(s)

\[ S_{adm} := \{ l \in \{ R, T, L \} \mid f_l(\sigma, \alpha_l, \omega) = 0 \}, \quad (3.16) \]

the Kuhn-Tucker complementary conditions also known as loading/unloading requirements can be restated for multi-surface plasticity as the following [173]:

**condition 1.** If \( f_l(\sigma, \alpha_l, \omega) < 0 \) or \( f_l(\sigma, \alpha_l, \omega) = 0 \) and \( \dot{f}_l(\sigma, \alpha_l, \omega) < 0 \), subsequently \( \dot{\gamma}_l^I = 0 \), which defines the case of elastic loading or unloading, where the plastic strains along with the hardening variable(s) do not change;

**condition 2.** If \( f_l(\sigma, \alpha_l, \omega) = 0 \) and \( \dot{f}_l(\sigma, \alpha_l, \omega) = 0 \), then \( \dot{\gamma}_l^I \geq 0 \), which specifies the plastic loading and subsequently the evolution of the plastic deformation and respective internal variable(s).

Accordingly, based on Eq. (3.16) and an expanded form of the loading/unloading conditions, \( 1 \leq r \leq r_{adm} \) representing the number of active yield conditions reads as:

\[ S_{act} := \{ l \in S_{adm} \mid \dot{f}_l(\sigma, \alpha_l, \omega) = 0 \}. \quad (3.17) \]

To summarize, the flow role Eq. (3.12) and the hardening law Eq. (3.13) with the Kuhn-Tucker complementary requirements form a set of non-linear equations with \( \dot{\gamma}_l^I \) as unknown variables. The corresponding solution can be realized through an iterative numerical procedure like a Newton-Raphson scheme [71, 115, 173].

### 3.2.3 Hygro-expansion

Analogous to thermal expansion with respect to the temperature gradients, it is assumed hygro-expansion to be proportional to the increments of the moisture content as

\[ \varepsilon^{\omega} = \alpha_\omega (\text{Min}(\omega, \omega_{FS}) - \omega_0), \quad (3.18) \]
3.2. HYGRO-MECHANICAL CONSTITUTIVE MODEL

with the current moisture content $\omega$ and the fiber saturation moisture level $\omega_{FS}$ above which no hygro-expansion occurs. $\omega_0$ signifies the initial reference moisture content. The vector $\alpha_\omega$ contains hygro-expansion coefficients along the orthotropy directions in an RTL material coordinate system defined by $\alpha_\omega = \{\alpha_R, \alpha_T, \alpha_L, 0, 0, 0\}^T$. It is assumed that the hygro-expansion coefficients are constant and independent from moisture variations. Table 3.4 gives the corresponding values for European beech [82] and Norway spruce [56, 134].

Table 3.4: Swelling/shrinkage coefficients for European beech [82] and Norway spruce [56, 134].

<table>
<thead>
<tr>
<th>Hygro-expansion coefficient</th>
<th>$\alpha_R$ [1/%]</th>
<th>$\alpha_T$ [1/%]</th>
<th>$\alpha_L$ [1/%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beech</td>
<td>0.00191</td>
<td>0.00462</td>
<td>0.00011</td>
</tr>
<tr>
<td>Spruce</td>
<td>0.00170</td>
<td>0.00330</td>
<td>0.00005</td>
</tr>
</tbody>
</table>

3.2.4 Visco-elastic creep

The current study describes visco-elastic behavior based on a fully recoverable approach by a serial association of the Kelvin-Voigt elements. It is noteworthy to mention that the following formulations are given within the framework of linear visco-elasticity and are valid for the first stage known as primary creep (see Section 2.2).

By taking the derivative of the free energy function Eq. (3.2) with respect to the element-wise visco-elastic strain, the driving stress for the $i^{th}$ visco-elastic Kelvin-Voigt element is obtained as

$$\sigma_{ve}^i = -\frac{\partial \psi}{\partial \epsilon_{ve}^i} = C_0 : \left( \epsilon - \epsilon^{pl} - \epsilon^{w} - \sum_{i=1}^{n} \epsilon_{ve}^i - \sum_{j=1}^{m} \epsilon_{ms}^j \right) - C_i : \epsilon_{ve}^i = \sigma - C_i : \epsilon_{ve}^i. \tag{3.19}$$

In this relation, $C_i$ stands for the visco-elastic stiffness matrix. The visco-elastic strain rate $\dot{\epsilon}_{ve}^i$ is considered to be a linear function of the visco-elastic driving stress and reads:

$$\dot{\epsilon}_{ve}^i = \frac{1}{\tau_i} C_i^{-1} : \sigma_{ve}^i. \tag{3.20}$$

Subsequently, the governing rate equation for an individual visco-elastic Kelvin-Voigt element is

$$\dot{\epsilon}_{ve}^i + \frac{1}{\tau_i} \epsilon_{ve}^i = \frac{1}{\tau_i} C_i^{-1} : \sigma(t), \tag{3.21}$$

where $C_i^{-1}$ and $\tau_i$ denote the visco-elastic compliance tensor and the characteristic
retardation time relevant to the $i^{th}$ Kelvin-Voigt element, respectively. Following Refs. [49, 71, 115], the visco-elastic compliance tensor is assumed to be proportional to the elastic compliance matrix with a unitless scalar $\gamma_{ve}^i$, namely

$$\gamma_{ve}^i = C_0^{-1} / C_i^{-1}. \quad (3.22)$$

For spruce, the dimensionless fractions Eq. (3.22) are taken from Ref. [49], while for beech they are calculated based on creep measurements of component of the visco-elastic compliance tensor in the grain $J_{iL}^{ve}$ at different moisture levels with linear moisture dependence. For a serial combination of four Kelvin-Voigt elements, Table 3.5 provides the parameters that describe the moisture-dependent longitudinal component of the creep compliance tensor for the $i^{th}$ Kelvin-Voigt element:

$$J_{iL}^{ve} = (J_{i1} + J_{i0})(1 - e^{-t/\tau_i}). \quad (3.23)$$

The ratio of the longitudinal component of the elastic compliance tensor $\frac{1}{E_L}$ with this value gives the fraction $\gamma_{ve}^i$ relevant to each Kelvin-Voigt element. Note that due to sparse experimental data, it is a common practice to apply the fraction $\gamma_{ve}^i$ measured in the grain also to the cross-grain directions. Additionally, the same value of the characteristic time (viscosity) along the grain is utilized for other anatomical directions as well. Thus, the retardation times are defined isotropically.

Following Refs. [49, 71, 115] by integrating Eq. (3.21), the element-wise visco-elastic strain response of the $i^{th}$ Kelvin-Voigt element reads

$$\varepsilon_{i,n+1}^{ve} = \varepsilon_{i,n}^{ve} \exp(-\Delta t / \tau_i) + \int_{t_n}^{t_{n+1}} \frac{C_i^{-1} : \sigma(t)}{\tau_i} \exp(-t_{n+1} - t / \tau_i) \, dt, \quad (3.24)$$

for a stress driven problem, with the time step $\Delta t = t_{n+1} - t_n$ and the visco-elastic strain tensors $\varepsilon_{i,n+1}^{ve}, \varepsilon_{i,n}^{ve}$ at the time $t_{n+1}$ and $t_n$, respectively.

Table 3.5: Coefficients for calculation of moisture- and time-dependent entry of the visco-elastic compliance tensor parallel to grain for European beech [84] and dimensionless scalar parameters for Norway spruce [49].

<table>
<thead>
<tr>
<th>$i$ [-]</th>
<th>$J_{i1}$ [MPa$^{-1}$]</th>
<th>$J_{i0}$ [MPa$^{-1}$]</th>
<th>$\tau_i$ [h]</th>
<th>$\gamma_{ve}^i$</th>
<th>$\tau_i$ [h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.11e-6</td>
<td>-4.99e-6</td>
<td>0.82</td>
<td>1</td>
<td>0.085</td>
</tr>
<tr>
<td>2</td>
<td>9.84e-7</td>
<td>-5.22e-6</td>
<td>60.86</td>
<td>2</td>
<td>0.035</td>
</tr>
<tr>
<td>3</td>
<td>8.51e-7</td>
<td>-5.97e-6</td>
<td>8.90</td>
<td>3</td>
<td>0.070</td>
</tr>
<tr>
<td>4</td>
<td>2.82e-6</td>
<td>1.03e-5</td>
<td>3427.65</td>
<td>4</td>
<td>0.200</td>
</tr>
</tbody>
</table>
3.2.5 Mechano-sorptive creep

For the numerical description of the mechano-sorption, in principle Kelvin-Voigt type elements are used [49, 71, 115]. In Refs. [71, 115] plasticity is a part of the mechano-sorption, while in the formulation by Ref. [49] followed in the current study, it is an independent strain contribution. After differentiating the free energy function Eq. (3.2) with respect to the element-wise mechano-sorptive creep strain, the corresponding driving stress becomes

\[ \sigma_{jn}^{ms} = C^0 : \left( \varepsilon - \varepsilon^pl - \varepsilon^\omega - \sum_{i=1}^{n} \varepsilon_{iv}^{ve} - \sum_{j=1}^{m} \varepsilon_{j}^{ms} \right) - C_j : \varepsilon_{jn}^{ms} = \sigma - C_j : \varepsilon_{jn}^{ms}, \]

(3.25)

where \( C_j \) stands for the mechano-sorptive stiffness tensor. For characterizing the mechano-sorption, a rate equation quite similar to the one of the visco-elastic creep Eq. (3.20) is applied:

\[ \dot{\varepsilon}_{jn}^{ms} = \frac{\mid \dot{\omega} \mid}{\mu_j} C_j^{-1} : \sigma_{jn}^{ms}. \]

(3.26)

Here \( \mu_j \) is called characteristic moisture analogous to the characteristic retardation time of the visco-elasticity and \( C_j^{-1} \) designates the tensor of the mechano-sorptive compliance respective to the \( j^{th} \) Kelvin-Voigt element. By inserting Eq. (3.25) in Eq. (3.26) and rearranging, the constitutive relation of a single \( (j^{th}) \) mechano-sorptive Kelvin-Voigt type element can be written as

\[ \dot{\varepsilon}_{jn}^{ms} + \frac{\mid \dot{\omega} \mid}{\mu_j} \varepsilon_{jn}^{ms} = \frac{\mid \dot{\omega} \mid}{\mu_j} C_j^{-1} : \sigma(t). \]

(3.27)

Note that the solution of Eq. (3.27) is identical to the visco-elasticity Eq. (3.21), but instead of a time increment the value of the absolute moisture content change is taken. Consequently, like in Eq. (3.24), the mechano-sorptive strain is

\[ \varepsilon_{jn+1}^{ms} = \varepsilon_{jn}^{ms} \exp \left( -\frac{\mid \Delta \omega \mid}{\mu_j} \right) + \int_{\omega_n}^{\omega_{n+1}} \frac{C_j^{-1} : \sigma(t)}{\mu_j} \exp \left( -\frac{\mid \omega_{n+1} - \omega(t) \mid}{\mu_j} \right) d \mid \omega(t) \mid. \]

(3.28)

Here \( \mid \Delta \omega \mid = \mid \omega_{n+1} - \omega_n \mid \) is the absolute value of the moisture increment, \( \varepsilon_{jn+1}^{ms} \) and \( \varepsilon_{jn}^{ms} \) denote the mechano-sorptive strains evaluated at the times \( t_{n+1} \) and \( t_n \), respectively. In order to calculate the element-wise mechano-sorptive compliance, a similar approach as for the visco-elastic counterpart is utilized. The corresponding scalar fractions, i.e., \( \gamma_j^{ms} \) are obtained based on the tangential and longitudinal components of the mechano-sorptive compliance tensor presented in Ref. [49] for...
spruce as
\[ \gamma_{ms}^{\gamma} = C_0^{-1} / C_j^{-1}. \] (3.29)

Following the work of Ref. [49], who calibrated the values for three serial Kelvin-Voigt elements on experimental data from Refs. [186, 189], the element-wise mechano-sorptive compliance tensor takes the following form:

\[
C_j^{-1} = \frac{J_{ms}^{RT} E_{T0}}{E_R} \begin{bmatrix}
J_{ms}^{RT} E_{T0}^\nu & -J_{ms}^{RT} E_{T0}^\nu & 0 & 0 & 0 \\
-J_{ms}^{RT} E_{T0}^\nu & J_{ms}^{RT} E_{T0}^\nu & 0 & 0 & 0 \\
-J_{ms}^{RT} E_{T0}^\nu & -J_{ms}^{RT} E_{T0}^\nu & J_{ms}^{L} & 0 & 0 \\
0 & 0 & 0 & J_{ms}^{RT} E_{T0}^\nu G_{RT} & 0 \\
0 & 0 & 0 & 0 & J_{ms}^{RT} E_{T0}^\nu G_{RT} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & J_{ms}^{RT} E_{T0}^\nu G_{RL} \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\] (3.30)

The values corresponding to \( J_{ms}^{RT} \) and \( J_{ms}^{L} \) together with the characteristic moistures \( \mu_j \) are summarized in Table 3.6. Due to lack of data on the mechano-sorptive behavior of beech, the corresponding values of the mechano-sorptive compliance tensor are calculated through scaling the spruce values by the ratio of the densities of two species (470/720) [135].

<table>
<thead>
<tr>
<th>Beech</th>
<th>Spruce</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_{ms}^{RT} ) [MPa(^{-1})]</td>
<td>( J_{ms}^{L} ) [MPa(^{-1})]</td>
</tr>
<tr>
<td>1</td>
<td>0.0004</td>
</tr>
<tr>
<td>2</td>
<td>0.0004</td>
</tr>
<tr>
<td>3</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

### 3.3 Numerical implementation of the comprehensive moisture-dependent rheological model

To implement the material model with the presented rheological behavior within the framework of a FE simulation, an incremental, iterative numerical approach is needed [49, 71, 115] with time increment (\( \Delta t = t_{n+1} - t_n \)). In the following,
subscript \((\bullet)_n\) indicates a state at the beginning of a time step, whereas the subscript \((\bullet)_{n+1}\) refers to the end of the time increment. The stress update algorithm is based on the assumption that at the time \(t = t_n\), the state of the material is available to be able to calculate the corresponding solution at the time \(t = t_{n+1}\) by means of an incremental update procedure. In detail, the state variables are the moisture distribution, the total strain including all five corresponding partial constituents, the internal plastic hardening variables and the total stress tensor. The individual algorithmic tangent operators associated with each deformation mode are computed separately and then, by incorporating all single Jacobians, the tangent operator for the entire model is obtained following Refs. [49, 71, 115].

Additionally, the total strain increment and the amount of moisture content change are needed for the iteration. The hygro-expansion strain tensor \(\left(\varepsilon^\omega_{n+1}\right)\) and the total strain tensor \(\left(\varepsilon_{n+1}\right)\) at the end of the time increment are estimated and needed for the incrementation as well, using the old tangent operator. At each integration point the total stress tensor and total Jacobian matrix, meaning the algorithmic tangent operator for the whole constitutive model, are updated at the end of the time increment. Moreover, the values of the state variables in terms of the elastic strain \(\left(\varepsilon^{el}\right)\), the irrecoverable plastic deformation \(\left(\varepsilon^{pl}\right)\) along with the related strain-type hardening variable(s) \(\left(\alpha_l\right)\), the viscoelastic strain tensor respective to every Kelvin-Voigt element \(\left(\varepsilon^{vei}\right)\), and all element-wise mechano-sorptive strain tensors \(\left(\varepsilon^{msj}\right)\) have to be updated for the next increment. A brief overview of the iterative algorithm utilized for the decomposition of the total strain \(\left(\varepsilon\right)\) and the incremental procedure for the update of the total stress and all state variables can be summarized as below:

1. **Stage 1. Data initialization** Based on the iterative algorithm for the stress update, all state variables are set to their corresponding values from the last converged iteration of the previous increment as:

\[
\varepsilon^{ve(k=0)}_{i,n+1} = \varepsilon^{ve}_{i,n}, \quad \varepsilon^{ms(k=0)}_{j,n+1} = \varepsilon^{ms}_{j,n}.
\]

The superscript \(k\) refers to the \((k)\text{th}\) iteration of the considered increment. The first one \((k = 0)\), therefore, is identical to the converged solution of the former increment.

2. **Stage 2. Plastic deformation** In the next step, the possible development of the irrecoverable deformation by plastic strain is examined. For this pur-
pose, a two-step return-mapping algorithm known as elastic predictor/plastic corrector based on the general multi-surface closest point projection approach is used [94, 173]. According to the trial state of the elastic strain at the end of the time increment $\left(\varepsilon_{\text{el}}^{\text{Trial}}\right)$ the position of the stress state relative to the evolved yield surfaces is checked. If the plastic loading is the case, the trial state of the stress calculated from the trial elastic strain is projected onto the current yield surface (see Fig. 3.4).

![Figure 3.4: Schematic representation of the concept of elastic predictor/plastic corrector in the context of the closest point projection iteration algorithm, based on Ref. 173.](image)

The plastic strain and the respective internal variable(s) also need to be initialized as follows:

$$
\varepsilon_{\text{pl}}^{\text{Trial}} = \varepsilon_{\text{pl}}^{(k=0)} = \varepsilon_{\text{pl}}^{(k=0)} = \alpha_{\text{pl},n+1}^{(k=0)} = \alpha_{\text{pl},n+1} = \alpha_{\text{pl},n}.
$$

(3.32)

From a theoretical point of view, the trial state of the elastic strain is achieved by suppressing the evolution of the plastic flow within the time increment. As a consequence, $\varepsilon_{\text{el}}^{\text{Trial}}$ and all corresponding state variables in the trial state
3.3. NUMERICAL IMPLEMENTATION OF THE MODEL

read as [173]:

\[ \varepsilon^{el(Trial)}_{n+1} = \left( \varepsilon_{n+1} - \varepsilon^{pl(Trial)}_{n+1} - \varepsilon^{\omega}_{n+1} - \sum_{i=1}^{n} \varepsilon^{ve(k)}_{i,n+1} - \sum_{j=1}^{m} \varepsilon^{ms(k)}_{j,n+1} \right), \]  
(3.33)

\[ \sigma^{(Trial)}_{n+1} = \left( C_{0,n+1} : \varepsilon^{el(Trial)}_{n+1} \right), \]  
(3.34)

\[ q^{(Trial)}_{l,n+1} = q_l \left( \alpha^{(Trial)}_{l,n+1}, \omega_{n+1} \right) = q_l (\alpha_{l,n}, \omega_{n+1}), \]  
(3.35)

\[ f^{(Trial)}_{l,n+1} = f_l \left( \sigma^{(Trial)}_{n+1}, q^{(Trial)}_{l,n+1}, \omega_{n+1} \right), \]  
(3.36)

subsequently, based on the last expression for the yield functions, if all \( f^{(Trial)}_{l,n+1} \leq 0 \) the deformation is purely elastic and the consistency parameters of all yield mechanisms do not change (\( \Delta \gamma_l = 0 \) for \( l = R, T, L \)), but if at least for one of them \( f^{(Trial)}_{l,n+1} > 0 \), the time step is plastic and the plastic strain increment along with changes of the strain hardening variable(s) and changes in the consistency parameters have to be evaluated. Therefore, by defining

\[ \varepsilon^{el(p)}_{n+1} = \left( \varepsilon_{n+1} - \varepsilon^{pl(p)}_{n+1} - \varepsilon^{\omega}_{n+1} - \sum_{i=1}^{n} \varepsilon^{ve(k)}_{i,n+1} - \sum_{j=1}^{m} \varepsilon^{ms(k)}_{j,n+1} \right), \]  
(3.37)

\[ \sigma^{(p)}_{n+1} = \left( C_{0,n+1} : \varepsilon^{el(p)}_{n+1} \right), \]  
(3.38)

\[ q^{(p)}_{l,n+1} = q_l \left( \alpha^{(p)}_{l,n+1}, \omega_{n+1} \right), \]  
(3.39)

\[ f^{(p)}_{l,n+1} = f_l \left( \sigma^{(p)}_{n+1}, q^{(p)}_{l,n+1}, \omega_{n+1} \right), \]  
(3.40)

and by means of an iterative return-mapping algorithm (i.e., plastic corrector) incremental plastic deformations, hardening variable(s), and consistency parameter(s) associated with the active yield surface(s) are obtained. Note that superscripts \( p \) in Eq. (3.37) refer to the \( p^{th} \) iteration of the return mapping algorithm. Consequently, the iterative formulations concerning update of the state variables and consistency parameters are

\[ \varepsilon^{pl(p+1)}_{n+1} = \varepsilon^{pl(p)}_{n+1} + \Delta \varepsilon^{pl(p)}_{n+1}, \]  
(3.41)

\[ \alpha^{(p+1)}_{l,n+1} = \alpha^{(p)}_{l,n+1} + \Delta \alpha^{(p)}_{l,n+1}, \]  
(3.42)

\[ \Delta \gamma^{l(p+1)}_{n+1} = \Delta \gamma^{l(p)}_{n+1} + \delta \Delta \gamma^{l(p)}_{n+1}, \]  
(3.43)

\( \Delta \gamma^{l}_{n+1} \) denotes the non-rate form of the consistency parameter, whereas the term \( \delta \Delta \gamma^{l}_{n+1} \) signifies the corresponding increment. At the end of the return
mapping iteration, \((p + 1)\) is set to \((p)\) and the convergence of the iteration is checked. A plastic residuum vector consisting of components related to the iterative evolution of the plastic strain tensor and hardening variable(s):

\[
\begin{aligned}
R^{pl}_{n+1} := & \left\{ -\varepsilon^{pl}_{n+1} + \varepsilon^{pl}_n \right\} \right. \\
& \left. + \sum_{l \in S^{act}} \Delta \gamma^{l(p)}_{n+1} \left\{ \partial_{\sigma} f_l(\sigma^{(p)}_{n+1}, \alpha^{(p)}_{l,n+1}, \omega^{(p)}_{n+1}) \right\} \right. \\
& \left. \right. \left\{ \partial_{\sigma} f_l(\sigma^{(p)}_{n+1}, \alpha^{(p)}_{l,n+1}, \omega^{(p)}_{n+1}) \right\} \\
\end{aligned}
\]

is computed for the convergence check. Now, the values of all active yield functions are recalculated using the updated stress tensor and hardening variable(s) via Eq. (3.7). If

\[
f^{(p)}_{l,n+1} := f_l(\sigma^{(p)}_{n+1}, \alpha^{(p)}_{l,n+1}, \omega^{(p)}_{n+1}) < TOL_1 \text{ for all } l \in S^{act}, \text{ and } \left\| R^{pl}_{n+1} \right\| < TOL_2,
\]

is not fully fulfilled, the procedure loops to the next iteration. Otherwise, the current iteration has converged and consequently, the definite increments of the plastic strain tensor as well as the hardening variable(s) are obtained. Now, all state variables relevant to the plastic part of the total strain are updated to the new values at the end of the time step, namely

\[
\varepsilon^{pl}_{n+1} = \varepsilon^{pl}_n + \Delta \varepsilon^{pl}_{n+1}, \quad \alpha_{l,n+1} = \alpha_{l,n} + \Delta \alpha_{l,n+1}, \quad \text{for all } l \in S^{act}.
\]

3. Stage 3. Stress calculation In this stage, the driving stress in the current iteration of the general additive decomposition scheme can be calculated. Taking the strain components and the total strain tensor at the end of the increment, the elastic contribution is

\[
\varepsilon^{el}_{n+1} = \begin{cases} 
\varepsilon^{el}_{n+1} - \varepsilon^{pl}_{n+1} - \varepsilon^{\omega}_{n+1} - \sum_{i=1}^{n} \varepsilon^{ve}_{i,n+1} - \sum_{j=1}^{m} \varepsilon^{ms}_{j,n+1}, & \text{if } \varepsilon^{pl}_{n+1} \neq 0, \\
\varepsilon^{el}_{n+1} - \varepsilon^{\omega}_{n+1} - \sum_{i=1}^{n} \varepsilon^{ve}_{i,n+1} - \sum_{j=1}^{m} \varepsilon^{ms}_{j,n+1}, & \text{if } \varepsilon^{pl}_{n+1} = 0.
\end{cases}
\]

With the calculated elastic strain, the stress tensor at the \((k)\)th iteration of the stress update increment is

\[
\sigma^{(k)}_{n+1} = \left( C_{0,n+1} : \varepsilon^{el}_{n+1} \right).
\]
3.3. NUMERICAL IMPLEMENTATION OF THE MODEL

In the next step, the tangent operator for the whole serial model has to be given. For this reason, all individual Jacobians from the 3-4 deformation mechanisms are evaluated separately and assembled to a total algorithmic tangent operator \([49, 71, 115]\). Since this step is crucial for the convergence of the entire implementation, a detailed explanation is given in the Appendix A.

With the total tangent operator \(C_{n+1}^T\), the iterative change of the total stress is calculated by the expression

\[
\Delta\sigma_{n+1}^{(k)} = -C_{n+1}^T \left( R^{el} + \sum_{i=1}^{n} R_i^{ve} + \sum_{j=1}^{m} R_j^{ms} \right)_{n+1}^{(k)}. \tag{3.49}
\]

The residual vectors belonging to each deformation mechanism are defined as:

\[
R^{el(k)} = \begin{cases} \sum_{j=1}^{m} C^{-1}_{0,n+1}: \sigma_{n+1}^{(k)} - \left( \epsilon_{n+1}^{ve} - \epsilon_{n+1}^{me} - \sum_{j=1}^{m} \epsilon_{i,n+1}^{ve(k)} - \sum_{j=1}^{m} \epsilon_{j,n+1}^{ms(k)} \right), \text{if } \epsilon_{n+1}^{ve(k)} \neq 0, \\
\sum_{j=1}^{m} C^{-1}_{0,n+1}: \sigma_{n+1}^{(k)} - \left( \epsilon_{n+1}^{ve} - \sum_{j=1}^{m} \epsilon_{i,n+1}^{ve(k)} - \sum_{j=1}^{m} \epsilon_{j,n+1}^{ms(k)} \right), \text{if } \epsilon_{n+1}^{ve(k)} = 0, \end{cases} \tag{3.50}
\]

\[
R_i^{ve(k)} = \left\{ \epsilon_{i,n}^{ve} \exp(-\xi_i) + T_{n}^{ve}(\xi_i) C_{i,n}^{-1}: \sigma_{n} + T_{n+1}^{ve}(\xi_i) C_{i,n+1}^{-1}: \sigma_{n+1}^{(k)} - \epsilon_{i,n+1}^{ve(k)} \right\}, \tag{3.51}
\]

\[
R_j^{ms(k)} = \left\{ \epsilon_{j,n}^{ms} \exp(-\xi_j) + T_{n}^{ms}(\xi_j) C_{j,n}^{-1}: \sigma_{n} + T_{n+1}^{ms}(\xi_j) C_{j,n+1}^{-1}: \sigma_{n+1}^{(k)} - \epsilon_{j,n+1}^{ms(k)} \right\}. \tag{3.52}
\]

4. Stage 4. Visco-elastic and mechano-sorptive deformations Based on the pre-calculated incremental stress tensor Eq. (3.49) and all decoupled residual vectors related to all individual deformation modes Eqs. (3.50, 3.52), the change of the visco-elastic strains in conjunction with the variation of the mechano-sorptive deformations for the current iteration of the stress update algorithm are determined. Accordingly, the visco-elastic and mechano-sorptive creep strain increments \(\Delta\epsilon_{i,n+1}^{ve(k)}\) and \(\Delta\epsilon_{j,n+1}^{ms(k)}\) are evaluated via the following relationships

\[
\Delta\epsilon_{i,n+1}^{ve(k)} = R_i^{ve(k)} + T_{n+1}^{ve}(\xi_i) C_{i,n+1}^{-1}: \Delta\sigma_{n+1}^{(k)}, \text{ for } i = (1, ..., n), \tag{3.53}
\]

\[
\Delta\epsilon_{j,n+1}^{ms(k)} = R_j^{ms(k)} + T_{n+1}^{ms}(\xi_j) C_{j,n+1}^{-1}: \Delta\sigma_{n+1}^{(k)}, \text{ for } j = (1, ..., m),
\]

that are derived in accordance with the definition of the respective algorithmic operators (see Appendix A). Consequentially, all the state variables for all
element-wise visco-elastic and mechano-sorptive creep deformations are updated at the end of the ongoing iteration using the expression

$$
\epsilon_{ve}^{(k+1)}_{i,n} = \epsilon_{ve}^{(k)}_{i,n} + \Delta \epsilon_{ve}^{(k)}_{i,n+1}, \quad \epsilon_{ms}^{(k+1)}_{j,n} = \epsilon_{ms}^{(k)}_{j,n} + \Delta \epsilon_{ms}^{(k)}_{j,n+1}.
$$

(3.54)

The iterative procedure is completed by comparing a generalized residual vector $R_{n+1}^{(k+1)}$

$$
R_{n+1}^{(k+1)} = \begin{pmatrix} R_{ed}^{(k+1)} & R_{ve}^{(k+1)} & \cdots & R_{ms}^{(k+1)} & \cdots \end{pmatrix}^T,
$$

(3.55)

composed of all recomputed deformation-based residual vectors Eqs. (3.50)-(3.52), or more precisely its norm up to a tolerance level. If $\|R_{n+1}^{(k+1)}\| \leq \text{TOL}_3$ the iterative scheme is converged and the solution obtained in the $(k+1)^{th}$ iteration is regarded as the final response of the current increment. Otherwise, if $\|R_{n+1}^{(k+1)}\| > \text{TOL}_3$, $(k+1)$ is changed to $(k)$ and stages 2-4 are repeated.

After convergence, all values of the state variables related to the visco-elastic and mechano-sorptive strains as well as the elastic deformation Eq. (3.47) are substituted by the updated ones:

$$
\epsilon_{ve}^{(k+1)}_{i,n} = \epsilon_{ve}^{(k)}_{i,n} + \Delta \epsilon_{ve}^{(k)}_{i,n+1}, \quad \epsilon_{ms}^{(k+1)}_{j,n} = \epsilon_{ms}^{(k)}_{j,n} + \Delta \epsilon_{ms}^{(k)}_{j,n+1}.
$$

(3.56)

These updated state values will be considered as the initial values in the next time step.

### 3.4 Moisture-stress analysis

Due to the importance of the moisture fields and gradients for the behavior of wood, moisture transport is essential for simulations. In this study, the mechanical and thermal fields do not influence moisture transport, so the problem is only partially coupled. Furthermore, it is assumed moisture transport inside the porous material wood to be dominated by diffusive transport [56] (see Section 2.3). Experimental observations of moisture transport below the fiber saturation show more correspondence with non-Fickian behavior [103, 200], or multi-Fickian diffusion [52]. In the present work, for the sake of simplicity and also because of insufficient details regarding either non-Fickian or multi-Fickian formulations, Fick’s law for moisture transfer is employed.

In accordance with the Fick’s first law of diffusion, the body moisture flux under
steady-state conditions \((\partial c/\partial t = 0)\) is given by

\[
J^b_\omega = -D \nabla c, \quad (3.57)
\]

where \(J^b_\omega\) is the body moisture flux vector, \(D\) denotes the matrix of diffusion coefficients, \(c = \rho_0 \omega\) is the water concentration, \(\rho_0\) the oven-dry wood density, \(\nabla\) the spatial gradient operator, and \(\omega\) is again the moisture content in \([\%]\). Moreover, other alternative potentials besides the concentration and moisture content gradients are also presented, for example based on vapor partial pressure \(p\) and chemical or water potential \(\Psi\) [27–29]. Fick’s second law expresses the change of the concentration with respect to time. Following the substitution of Eq. (3.57) into the continuity equation (mass conservation) for time-varying processes

\[
\nabla \cdot (J^b_\omega) = -\frac{\partial c}{\partial t}, \quad (3.58)
\]

the time variation of the concentration, in a general form and for changing diffusion coefficients, is written as:

\[
\frac{\partial c}{\partial t} = \nabla \cdot (D \nabla c). \quad (3.59)
\]

For constant density Eq. (3.59) simplifies to

\[
\frac{\partial \omega}{\partial t} = \nabla \cdot (D \nabla \omega). \quad (3.60)
\]

Basically, mathematical formulations of the moisture diffusion and heat transfer are similar. The Fourier equations of heat transfer are

\[
J_T = -K \nabla T, \quad \text{and} \quad (\rho c_T) \frac{\partial T}{\partial t} = \nabla \cdot (K \nabla T), \quad (3.61)
\]

with the heat flux \(J_T\) \([\text{Jm}^{-2}\text{s}^{-1}]\), density \(\rho\), specific heat \(c_T\) \([\text{JKg}^{-1}\text{K}^{-1}]\) and temperature \(T\), as well as the matrix of thermal conductivity coefficients \(K\) \([\text{Jm}^{-1}\text{K}^{-1}\text{s}^{-1}]\). The analogy between Eqs. (3.60) and (3.61) is preserved when \((\rho c_T) = 1\), rendering thermal analysis capabilities of FE packages valid for moisture transport simulations. In the current work, the diffusion process is assumed to be uncoupled among the anatomical directions and subsequently, the matrix of orthotropic diffusion coefficients is defined as:

\[
D = diag \begin{bmatrix} D_R & D_T & D_L \end{bmatrix}. \quad (3.62)
\]

The diffusion coefficients are considered to be moisture-dependent and can be de-
scribed by exponential laws as a function of moisture, namely

\[ D_l(\omega) = D_{0l} \left(e^{\alpha_{0l} \omega}\right), \]  

(3.63)

where \( l = R, T, \) and \( L. \) All parameters required to calculate the moisture-dependent diffusion coefficients for spruce and beech \([82, 157]\) are summarized in Table 3.7.

Table 3.7: Parameters for calculation of the diffusion coefficients as an exponential function of the moisture content for European beech \([82]\) and Norway spruce \([157]\).

<table>
<thead>
<tr>
<th>Direction</th>
<th>( D_{0l} ) [( \text{mm}^2 \text{h}^{-1} )]</th>
<th>( \alpha_{0l} )</th>
<th>Direction</th>
<th>( D_{0l} ) [( \text{mm}^2 \text{h}^{-1} )]</th>
<th>( \alpha_{0l} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial</td>
<td>0.02630</td>
<td>0.199724</td>
<td>Radial</td>
<td>0.288</td>
<td>0.04</td>
</tr>
<tr>
<td>Tangential</td>
<td>0.00370</td>
<td>0.265280</td>
<td>Tangential</td>
<td>0.288</td>
<td>0.04</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>21.8999</td>
<td>-0.038545</td>
<td>Longitudinal</td>
<td>0.720</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Furthermore, convective boundary conditions describing the surface resistance against diffusion are required for calculation of the moisture transport inside wood. For this purpose, boundary conditions of the following form are generally applied \([56, 82]\): (see Fig. 3.5)

\[ n J_s^m = \sigma_D (c_\infty - c_{\text{wood}}^s), \]  

(3.64)

where \( \sigma_D \) [\( \text{ms}^{-1} \)] is surface-emission coefficient \([169]\), \( c_\infty \) and \( c_{\text{wood}}^s \) are the equilibrium concentration based on the RH of the ambient air and the concentration of the wood surface, respectively, and \( J_s^m \) [\( \text{Kgm}^{-2}\text{s}^{-1} \)] and \( n \) designate the surface mass flow and the normal vector of the associated surface. However, under high ambient air velocities the surface resistance or identically the influence of the \( \sigma_D \) diminishes and the concentrations of the wood surface and the ambient air can be assumed the same from the beginning \((c_\infty = c_{\text{wood}}^s)\) \([156]\).

Table 3.8 provides the surface-emission coefficients of European beech along three anatomical directions as constant values neglecting the effects of the moisture concentration and the ambient air velocity \([82]\).

Table 3.8: The directional surface-emission coefficients of European beech \([82]\).

<table>
<thead>
<tr>
<th>Surface-emission coefficient ( \sigma_D ) [( \text{mm}^2 \text{h}^{-1} )]</th>
<th>( \alpha_R )</th>
<th>( \alpha_T )</th>
<th>( \alpha_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beech</td>
<td>0.349</td>
<td>0.348</td>
<td>3.270</td>
</tr>
</tbody>
</table>
3.5 Modeling of interfacial damage

As it was stated earlier, prediction of the interfacial cracking is a key issue in the assessment of the long-term serviceability and durability of adhesively bonded wood composites under exposure to changing climatic conditions [177]. Delamination of a bond-line (de-bonding) can be considered as a progressive damage of the cohesion between two adjoining substrates, resulting in a cumulative degradation of the interfacial bonding strength. Typically, the formation and growth of inter-laminar de-bonding is numerically simulated based on the principles of cohesive zone model (CZM) within the frameworks of non-linear fracture and damage mechanics. The CZM provides an ideal failure-based computational tool for prediction of damage initiation and propagation in an integrated procedure (see e.g., Allix and Corigliano [4]; Chen et al. [26]; Alfano and Crisfield [3]; de Borst [17]; de Moura et al. [35]; da Silva and Campilho [171]; Fortino et al. [50]; Danielsson [33]). The concept of the CZM dates back to the work presented by Barenblatt [9] for fracture analysis of perfectly brittle materials. Later on, Dugdale [41] extended this model for investigation of elasto-plastic fracture response of ductile metals by hypothesizing the development of a localized fracture process zone (FPZ) ahead of the crack tip [207], where damage onset and growth occur.

The CZM approach is incorporated in conventional FE packages basically in terms of spring [32, 105] or mostly cohesive interface elements [122, 145], which
join 2D or 3D substrates elements on both sides of the interface. In the numerical implementation of the CZM, it is assumed that the crack growth path is known a-priori and damage propagation occurs as a consequence of a discontinuity in the displacement field \[21, 171\]. The cohesive interface elements can be considered as two lines or two surfaces, respectively in 2D and 3D, which are separated by the thickness of the element (see Fig. 3.6). The displacement of the lower part with respect to the upper one in the thickness direction designates the interface opening or closure, whereas the relative motion of the top and bottom lines or surfaces in the plane perpendicular to the thickness represents the shear deformations across the interface.

![Figure 3.6: Schematic depiction of the cohesive interface element: a) 4-noded two-dimensional, b) 8-noded three-dimensional \[21\]. \(n\), \(s\), and \(t\) denote the normal, the first and the second shear directions, respectively.](image)

In the case of an infinitesimally thin cohesive layer, the constitutive response of the cohesive elements in the FPZ is characterized by the so-called traction-separation law (TSL). The TSL describes the correlation between the nominal traction across the surface of decohesion \((t)\) and the nominal relative separations of the interface \((\delta)\). Additionally, the TSL defines damage initiation together with its evolution law as the progressive degradation of the interface stiffness (softening behavior). Different types of TSLs have been proposed such as bi- or tri-linear, trapezoidal, and exponential (see for example Refs. \[7, 24, 206\]), which have two common characteristics: the maximum sustainable traction by the interface representing the stress at damage initiation \(t^0\) and the maximum displacement at fully damaged state \(\delta^f\). These two characteristic parameters along with the shape of TSL determine the area under
3.5. MODELING OF INTERFACIAL DAMAGE

traction-separation curve which quantitatively specifies the total work needed for a complete separation, or equivalently the amount of energy required to generate a unit of newly fractured area (fracture energy) \[11, 50\] (see Fig. 3.7).

Figure 3.7: Various types of traction-separation laws: a) Exponential, b) Bi-linear, and c) Trapezoidal \[162\]. \(t^0\) and \(\delta^0\) symbolize the peak value of the nominal traction and the corresponding nominal separation, respectively, and \(\delta^f\) represents the nominal displacement at complete failure.

All various types of TSLs assume a linear elastic relationship in the ascending part of the curve (point 1 in Fig. 3.8) preceding damage initiation. This linear elastic response for the uncoupled traction-separation behavior is described through three penalty stiffness namely, \(K_n\) and \(K_s/K_t\) along the normal and two shear directions (see Fig. 3.6), respectively, which relate the nominal stresses to the nominal strains (see Fig. 3.8). The starting point of the descending branch of TSL (point 2 in Fig. 3.8) defines the initiation of damage referring to the onset of material response degradation (softening behavior). Maximum nominal stress, maximum nominal strain, quadratic nominal stress, and quadratic nominal strain are examples of damage initiation criteria available in FE packages \[196\]. When a damage initiation criterion is reached, material degradation evolves (point 3 in Fig. 3.8) based on a damage evolution law characterizing the rate at which the interface stiffness deteriorates. Typically, a scalar variable \(D\) expresses the extent of the progressive damage occurring due to failure mechanisms, which develops from 0 (damage initiation) to 1 at completely damaged state (point 4 in Fig. 3.8).

A damage evolution law describes two constituents: the first one specifies the condition when the damage variable attains its maximum value and is defined either as the fraction of the effective displacement at complete damage \(\delta^f_m\) to the effective displacement at failure initiation \(\delta^0_m\), or as the total fracture energy \(G^c\). The second
component determines the character of the gradual development of the damage variable and shows how it varies between 0 and 1 by means of either assigning a linear or exponential softening behavior, or tabulating the effective displacement $\delta_m$ versus the effective displacement at the initiation of failure $\delta_{0m}$. It should be noted to describe the progression of decohesion under mixed-mode loading, the effective displacement is introduced

$$\delta_m = \sqrt{\langle \delta_n \rangle^2 + \delta_s^2 + \delta_t^2},$$

(3.65)

where $\delta_n$, $\delta_s$, and $\delta_t$ stand for the relative separations along the normal and two shear directions in pure-mode loading (see Fig. 3.8) and the symbol $\langle \cdot \rangle$ is the Macaulay bracket with the common definition. Based on a similar concept, in energy based evolution laws the contribution of each single-mode loading into the total critical fracture energy $G^c$ can be determined either by a power law or Benzeggagh-Kenane (BK) criterion [12], suitable for isotropic shear behavior, under mixed-mode conditions. For example, the power law criterion states the dependency of the total
3.5. MODELING OF INTERFACIAL DAMAGE

Critical fracture energy on the mode-mix as

$$G^c = 1 \left( \frac{m_1}{G_n^c} \right)^{\alpha} + \left( \frac{m_2}{G_s^c} \right)^{\alpha} + \left( \frac{m_3}{G_t^c} \right)^{\alpha/\alpha} \right)^{1/\alpha}, \quad (3.66)$$

with $m_1 = \frac{G_n}{G_T^n}$, $m_2 = \frac{G_s}{G_T}$, $m_3 = \frac{G_t}{G_T}$ energy based mode-mix ratios and $G_T = G_n + G_s + G_t$. In Eq. (3.66), $G_n, G_s,$ and $G_t$ are the amount of work done by the traction and their associated separations along the normal and two shear directions [196].

Table 3.9 gives the normal critical fracture energy $G_n^c$ of adhesive joints consisting of European beech substrates and bonded with the PRF and PUR adhesive systems in different climatic conditions [5].

Table 3.9: The normal specific fracture energy of European beech bonding under changing climate for the PRF and PUR adhesives [5].

<table>
<thead>
<tr>
<th>Adhesive type</th>
<th>RH (%)</th>
<th>Wood substrate MC (%)</th>
<th>$G_n^c$ [N/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRF</td>
<td>50</td>
<td>11.6</td>
<td>0.829 ± 0.228</td>
</tr>
<tr>
<td></td>
<td>65</td>
<td>14.7</td>
<td>0.888 ± 0.188</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>21.5</td>
<td>1.295 ± 0.333</td>
</tr>
<tr>
<td>PUR</td>
<td>50</td>
<td>11.6</td>
<td>0.235 ± 0.042</td>
</tr>
<tr>
<td></td>
<td>65</td>
<td>14.7</td>
<td>0.236 ± 0.029</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>21.5</td>
<td>0.237 ± 0.051</td>
</tr>
</tbody>
</table>

Ref. [196] details further information about the cohesive interface elements.
Chapter 4

Verification of the material model

In this Chapter, a set of numerical examples is calculated that verifies the capability and efficiency of the developed 3D constitutive model in the prediction of realistic behavior for short-term and long-term responses under combined moisture and mechanical loading. The examples are selected in such a way that different deformation components act in an isolated and combined way under uni- and multi-axial loading as well as for restrained swelling. All examples use the material properties given in Chapter 3 for European beech (Fagus sylvatica L.). Furthermore, this Chapter is complemented with two application examples. The first example addresses the moisture-induced residual stresses in a glulam beam during the fabrication process and due to bonding of lamellae with various initial MC, and the second one represents the numerical simulation of interfacial delamination in a wood double cantilever beam specimen.

4.1 Simple stress state under uni-axial mechanical loading and moisture variation

The first simple example studies a cubic sample with edge length of 40mm. The model uses three confining symmetry planes, reducing the edge length to 20mm (see Fig. 4.1a). Hence, it is allowed to swell or shrink freely during moisture variations and hygro-expansions do not lead to any residual stresses. Since the dimensions are quite small compared to the ones of a stem, the curvature of the growth rings is ignored by assigning an orthotropic material behavior to the system that is defined in a Cartesian coordinate system with axes aligned along the cube edges. The geometry is discretized with 512 quadratic brick elements (C3D20) and loaded by a
uniform compression in the radial direction. Simultaneously a homogeneous moisture distribution can be applied (see Fig. 4.1b). The pressure and the moisture content $\omega$ are chosen in a way that all potential components of the total strain are addressed and can be distinguished clearly during the following seven stages:

- **Stage 1** (0-5h): the sample is at standard climate, i.e., 65% RH resulting in $\omega_1 = 12\%$. The pressure is ramped up to 10MPa during 5 hours,

- **Stage 2** (5-55h): for the next 50 hours all conditions are kept constant,

- **Stage 3** (55-60h): within the next 5 hours, the pressure is increased from 10 to 16MPa, while $\omega_1 = 12\% = \text{const.}$,

- **Stage 4** (60-135h): at constant load, $\omega$ is going through five cycles of wetting and drying, each lasting 15 hours. For 2.5h $\omega_1 = 12\%$, then ramped up to $\omega_2 = 18\%$ within 2.5h, held constant for 5 hours, ramped down from $\omega_2$ to $\omega_1$ for 2.5h, held constant for 2.5h, a.s.o.,

- **Stage 5** (135-140h): at $\omega_1$ the load is completely removed during 5h and will remain this way for the rest of the simulation,

- **Stage 6** (140-200h): for the next 60 hours all conditions are kept constant,

- **Stage 7** (200-290h): finally six more moisture cycles like in Stage 4, but without mechanical loading are imposed.

The resulting strain along the radial direction for the loading in the seven stages is illustrated in Fig. 4.2. In order to observe the development of all partial strains in an easy to interpret way, the hygro-expansion strain is subtracted from the total strain and the remaining value, called *swelling/shrinkage-modified* strain, is shown in Fig. 4.2a. For the different stages are observed:

- **Stage 1** (0-5h): the linear elastic behavior dominates,

- **Stage 2** (5-55h): an increase in the visco-elastic creep is noticed,

- **Stage 3** (55-60h): the plastic deformation starts as the compressive strength of the material along the radial direction (i.e., -13.4MPa) is reached. The radial yield criterion is activated, the material starts to flow, and the irrecoverable plastic deformation evolves as the material simultaneously hardens until the yield surface reaches 16MPa.
4.1. SIMPLE UNI-AXIAL STRESS STATE

- **Stage 4** (60-135h): during the first moisture increase from \( \omega_1 = 12(\%) \) to \( \omega_2 = 18(\%) \), the material strength values degrade and consequently further plastic deformation, even under fixed external loading, evolves (see Fig. 4.2b). The repetition of moistening in the following cycles leads only to a comparably
small increase in the irreversible part of the total strain. The increase of the strain during the moisture cycles is, therefore, dominated by the mechano-sorptive creep. Note that the general response of the material model representing the simultaneous interaction of a fixed level of loading and varying moisture content shows a good agreement with experimental observations of the mechano-sorptive creep reported by Refs. [87, 186, 190]. It can be noticed that both moistening and de-moistening lead to the increase of the mechano-sorptive deformation and generally, the material behavior demonstrates an ascending trend which implies that the mechano-sorptive creep is accumulated over time.

- **Stage 5** (135-140h): during unloading, the instantaneous elastic response is immediately compensated,

- **Stage 6** (140-200h): the visco-elastic creep is partly recovered,

- **Stage 7** (200-290h): partial recovery of the mechano-sorptive creep.

It can be concluded that retrieval of the visco-elastic deformation requires adequate time, while the recovery of the mechano-sorption is achieved by moisture variation. As can be observed in Fig. 4.2a, even though the external pressure is removed, a remarkable amount of strain stays in the material. A part of this remaining deformation consists of time-dependent responses and another part is due to the occurrence of the irreversible plastic deformation.

In addition, mechanical responses of the wood block along two other directions, i.e., tangential and longitudinal are also given in Fig. 4.3. Although no mechanical loading is applied along these two directions, significant lateral strains are generated. Contrary to the radial direction and as a consequence of the implemented multi-surface plasticity, where all failure mechanisms in each anatomic direction are independent, no plastic deformations can occur in the T- and L-direction. Therefore, all deformations are fully recoverable after time and moisture variations under zero external loading. The offset between the two directions can be explained by the fact that beech is much stiffer parallel than perpendicular to the grain. Additionally, since the longitudinal entry of the mechano-sorptive compliance tensor was calculated differently than the other components (see Section 3.2.5), the general mechano-sorptive response of the material model parallel to the growth direction differs qualitatively from the corresponding behavior perpendicular to the grain.
4.1. SIMPLE UNI-AXIAL STRESS STATE

<table>
<thead>
<tr>
<th>Stage</th>
<th>Time (h)</th>
<th>0</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
<th>225</th>
<th>250</th>
<th>275</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain (%)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Moisture Content (ω) (%) | 55 | 75 | 95 | 115 | 135 |

Figure 4.2: a) Time evolution of all constituents of the total strain along the radial direction, b) Development of the radial plastic and mechano-sorptive creep strains.
Chapter 4. VERIFICATION OF THE MATERIAL MODEL

4.2 Stress state under multi-axial mechanical and hygric loading

To evaluate the model behavior under arbitrary combinations of load with multiple yield surfaces activated, a tri-axial state of stress is imposed on the same specimen (see Fig. 4.1a), but now with a load in all anatomical directions (see Fig. 4.4a). Boundary conditions as well as loading history are identical to the first example (see Fig. 4.1) only that the load in the tangential direction is scaled by a factor of 0.4 and the one in the longitudinal orientation by a factor of 4.

For the second simulation, the same interpretations as for the first verification example apply concerning the evolution of all instant and time-dependent mechanical behavior. However, in contrast to the previous example (Section 4.1), all three yield mechanisms will be activated and consequently, the irrecoverable plastic deformations evolve along all three anatomic directions simultaneously. The resulting plastic deformations are illustrated in Fig. 4.4b. As it can be seen, the tangential direction deforms more plastically, since it is the least stiff orientation. Furthermore, it should be noted that due to the application of a consistent tangent operator, the quadratic rate of convergence is preserved, even when multiple failure mechanisms are active (see Section 4.7).

Figure 4.3: Evolution of the swelling/shrinkage-modified strains along the T- and L-direction (ε_{TT/LL}). The superscript \textit{pl} refers to the plastic strain.
4.3. STRESS ANALYSIS UNDER RESTRAINED SWELLING

As another verification example, restrained swelling is simulated since this is an experimentally studied loading case of applied relevance. The evolution of the swelling pressure is observed as the rectangular sample of size 45mm x 15mm x 15mm.
(L×R×T) is exposed to a cycling moisture change (see Fig. 4.5). This transient numerical simulation aims to assess the practical behavior of wood components in terms of moisture distribution, all possibly emerging deformation mechanisms, moisture-dependent plastification in particular, and associated stress fields generated during moistening processes. Hence, moisture transport is calculated based on the procedure outlined in Section 3.4. Like the previous examples, advantages are taken of the symmetric nature of the problem and in addition to the three symmetry planes a non-moving boundary condition in the L-direction at the top surface is imposed. The moisture content cycles from the oven-dry condition (ω₁ = 0.5%) in 62.5h to a state with 95% RH resulting in an MC of ω₂ ≈ 22.5% close to the fiber saturation, only that moisture diffuses into the system from the exposed surfaces. Then, to observe the permanent deformations, the moisture content again returns back to ω₁ = 0.5% for the next 62.5(h). To identify the long-term effects of the history- and moisture-dependent deformation modes on the resulting strain and stress fields, the moistening/de-moistening cycle is repeated ten times. Additionally, to provide more time for the recovery of the time-dependent deformation mechanisms, the analysis is run for further 150 hours.

In order to investigate the role of the instantaneous and long-term responses on the mechanical behavior of wood specimen, this simulation is carried out for three different cases: (Case 1:) purely elastic material model with instantaneous responses, (Case 2:) material behavior with a rheological model consisting of the history-dependent modes, i.e., visco-elasticity and mechnano-sorption with no plasticity, and (Case 3:) like case 2, but including plasticity. Fig. 4.5b shows the resulting time evolution of the swelling pressures as the mean value of the longitudinal stress in the symmetry plane (see Fig. 4.5a).

As it is expected, increasing moisture content and swelling constraints result in the compressive stresses. Their maximum is significantly higher for the Case 1 than for the other two cases, convincingly demonstrating that a realistic stress analysis of wood should not be of purely elastic nature. On the basis of the material models with time- and moisture-dependent behavior as well as plastification, i.e., Cases 2,3 and in accordance with (Eqs. (3.47) and (3.48)), more terms are subtracted from the total strain value and subsequently, the elastic strain and equivalently the total stress are lower. In the Case 3, due to the occurrence of the plastic deformation, this reduction in the elastic strain is even more pronounced. Here, even significant tensile stresses can build up due to de-moistening to overcome compressive time-dependent or plastic deformations, which are the highest for the Case 3. It is interesting to
4.3. STRESS ANALYSIS UNDER RESTRAINED SWELLING

Figure 4.5: a) Finite Element discretization of the rectangular prism specimen, b) Time evolution of the swelling pressure. $R\sigma_{LL2}$ and $R\sigma_{LL3}$ refer to the residual longitudinal stresses based on the Cases 2 and 3, respectively after $t = 1400\text{h}$.

observe that due to repetition of moisture cycles, for the full model opposite to the fully recoverable time-dependent mechanisms, these stresses even increase - a
Chapter 4. VERIFICATION OF THE MATERIAL MODEL

long-term effect that is not taken into account in any calculation for construction elements. By considering all three numerical examples, it can be observed that the developed material model can be applied to any arbitrary combination of mechanical loading, inhomogeneous moisture distribution, and boundary condition. In what follows, the numerical simulations of moisture-related stresses induced during the manufacturing of a glulam as well as delamination of a double cantilever beam under opening load in terms of two application examples are presented.

4.4 Analysis of moisture-induced residual stresses in a glulam beam during the manufacturing process

As it was mentioned before (see Section 2.4), adhesive bonding can add to the total MC of glued-laminated elements. In timber laminates, however, the amount of the adhesive employed for gluing layers compared to wood is negligible and application of waterborne adhesives in general results in only 1% to 2% increase in the total MC \[54\]. On the other hand, if lumbers with non-equal initial MC are bonded to each other, due to the porosity character of wood, moisture permeates from the layer with higher MC to the neighboring one with a lower moisture level. The process of moisture transfer continues until the entire element reaches to an EMC, which basically differs from the initial MC of all individual timbers. This variation in the MC along with mismatched orthotropic material directions give rise into distinctive swelling/shrinkage behavior in every layer. Note that unlike the numerical example discussed in Section 4.3 where the wood sample was externally restrained, here the adhesive bond-line imposes internal boundary conditions between two adjacent layers. Therefore, the dissimilar hygro-expansion of multiple internally constrained lamellae causes residual stresses, especially in the bond-lines.

Accordingly, this application example investigates the role of the difference in the initial MCs of a glulam beam lamellae on the development of the moisture-induced residual stresses during the manufacturing process. Moisture transfer and evolution of the induced residual stresses are evaluated in a glulam beam consisting of five layers with the dimension of 475mm×145mm×30mm each. Fig. 4.6a shows the geometry and FE-model of the five-layered glulam along with the applied boundary conditions and the values of the initial MC of every wood layer. Each lamella is discretized by 30×10×8 20-node reduced integration quadratic brick elements
4.4. MOISTURE-INDUCED STRESSES IN A GLULAM BEAM

Figure 4.6: a) The geometry and FE-model of the five-layered glulam beam as well as the applied boundary conditions, alignment of the orthotropy directions, and the initial MC of every wood lamella. The same color refers to the layer number and its conjugate material coordinate system, b) The distribution of moisture in the glulam beam at equilibrium state ($\omega_{eq} \approx 11\%$) after $t = 750$ days.

(C3D20R). In practice, timber MC before gluing can vary from 6\% to 14\% [54], but in this simulation the initial MCs range from 10\% to 12\% to achieve the equilib-
Chapter 4. VERIFICATION OF THE MATERIAL MODEL

rium state in a reasonable time. Note that in the FE-model, the adhesive material among adjacent layers is not considered and two wood substrates are directly tied together. Moreover, the material orthotropy directions are defined by assigning randomly oriented cylindrical coordinate systems to wood lamellae. It is assumed that the whole laminate is completely isolated from the surrounding environment. This implies that moisture can only diffuse through the interface of the wood layers inside the glulam. By performing a transient analysis in accordance with the approach described in Section 3.4, the evolution of moisture transfer throughout the glulam element is calculated. Fig. 4.6 displays the distribution of moisture in the glulam at equilibrium condition ($\omega_{eq} \approx 11\%$), when no further moisture transport occurs.

![Figure 4.6: The distribution of moisture in the glulam at equilibrium condition.](image)

In the next step, the evolution of moisture-induced stresses are computed by a sequential moisture-stress analysis following the calculated moisture history and application of the developed wood material model. Fig. 4.7 indicates the profiles of residual stresses in the cross-grain directions corresponding to the equilibrium

![Figure 4.7: The distribution of moisture-induced stresses.](image)
situation. It should be noted the stress distribution in every lamella is given on the basis of the associated local material coordinate system (see Fig. 4.6a). As expected, the maximum values of moisture-related stresses occur in the vicinity of the interface of two adjoining layers, referring to the role of the difference in the orthotropy directions of two neighboring lamellae as well as the significant influence of the bond-line on the local concentration of stresses in glued-laminated configurations. Fig. 4.8 illustrates the time evolution of the stress and MC of two points with the highest value of the radial (point r) and tangential (point t) tensile stresses in the glulam. As aforementioned, the interval of the initial MC distribution within the lamellae was restricted to 10%-12%. When the MC of the glulam structure equilibrates, even with this narrow initial MC distribution, the resultant residual stresses are within one order of magnitude of the solid wood ultimate tensile stress values. At \( \omega_{eq} \), these ultimate tensile values are 20.1MPa and 9.0MPa \([142]\) in the radial and tangential directions, respectively. These stresses are subsequently superimposed on the ones occurring during service conditions under mechanical and hygric loading and can exceed the strength of the material and lead to inter- and intra-laminar fracture.

Figure 4.8: The time evolution of the MC and related induced cross-grain stress at point r: at the interface of layers 4 and 5 with the highest value of the radial stress and at point t: on the vertical edge of layer 4 in the proximity of the interface of layers 4 and 5 with the largest amount of the tangential stress.
Consequently, the introduced moisture-dependent wood material model provides an efficient computational approach for the numerical simulation of moisture transfer and stress analysis in glulam beams. Moreover, it allows obtaining a good insight for the selection of an optimal range of the initial moisture contents of lumbers before bonding to enhance the dimensional stability and to reduce the residual stresses generated during the fabrication in the whole laminated component, particularly at interfaces.

In what follows, the flexibility of the implemented material model for predicting the mechanical behavior of different wood species is demonstrated on an advanced application example.

### 4.5 Multi-species glulam beam

Combinations of different materials and wood species are a promising approach to overcome design limitations which result in extreme cross-sections of structural elements made of softwood or in expensive solutions for load transfer from other elements perpendicular to the grain, just to give two examples. The combination of beech and spruce has recently attracted significant attention due to a general technical approval for such a hybrid glulam element. It is interesting to note that neither plastic behavior nor long-term effects did play a role in the approval procedure.

To demonstrate the capability of the introduced computational approach, a hybrid glulam beam made of European beech and Norway spruce is considered. Both were adhesively joined at initial moisture content of 10%. The multi-species beam is subjected to changing environmental conditions varying between 50% to 90% RH. Although European beech and Norway spruce show different moisture sorption isotherms, due to the small difference between the equivalent moisture contents at mentioned RH a single value representing the moisture content of the whole beam is applied. The initial moisture content at 50% RH is estimated as 10% and the new equilibrium moisture level at 90% RH is approximated as 21%. They are taken as surface conditions for the moisture transport simulation. Note that these moisture levels are taken as the mean values of the adsorption and desorption curves for simplicity. The time variation of RH or equivalently moisture content is expressed by means of the positive part of the sine function, which varies between 10% to 21% for the duration of one year (365 days).

In the simulation, two distinct constitutive models consisting of five deformation mechanisms, each related to every wood type, are considered. In practice, two ma-
Figure 4.9: a) Geometrical dimensions and finite element discretization of the multi-species beam, b) Distribution of the tangential plastic strain after one year \((t = 365\) days). (B) and (S) designate beech and spruce, respectively.
terial (UMAT) subroutines are defined simultaneously. The adhesive layer between two lamellae is neglected and lamellae are regarded as different sections of one part, but with distinct material type and individual cylindrical local material coordinate systems, each. Shared nodes, therefore, have only one value for the moisture content and adhesive bond lines are fully permeable.

As illustrated in Fig. 4.9a, the simulated system represents a part of a hybrid glulam beam consisting of ten 30mm thick and 150mm wide lamellae, all aligned in the longitudinal direction with the two top and bottom made of beech, while the core is made of six spruce layers. In order to avoid modeling the entire length of the beam, a section with the length of 50mm under the assumption of generalized plane strain state has been considered. For this purpose, at the top right corner of the cross-section a reference point is defined and the movements of the end plane with red boundaries, i.e., translation along the Z-direction and rotation around the X and Y axes are kinematically coupled to the reference point (Fig. 4.9a).

Fig. 4.9b gives the distribution of the plastic deformations along the tangential direction. It should be noted that the plastic strains in each lamella are illustrated based on the corresponding local material coordinate system. Depending on the position of the center and the orientation of the assigned local cylindrical coordinate systems, the distribution of the plastic strain varies. As it is expected and can also be observed in the calculations, the magnitudes of the plastic strain in the beech layers are smaller than the counterpart values in the spruce lamellae. Additionally, the plastic deformations are mostly generated near the adhesive bond-lines. This is due to the occurrence of higher stress concentration at the interface of two adjacent lamellae, resulting from different material properties and material orientations.

These stress concentrations, in forms of tension or shear in particular, can lead to the initiation of interfacial cracks that propagate under applied service mechanical loadings and changing environmental conditions. Subsequently, structural integrity and load-carrying capacity are diminished. So, plasticity-induced de-bonding can be considered as a critical failure mode of such laminated structural components, the consequences of which have to be taken into account during the design stage of hybrid elements. It is interesting to note that no plastic deformations are observed in the counterpart single-species glulam made of Norway spruce under identical hygric loading. This indicates the higher propensity of hybrid laminated elements, consisting of both hardwood and softwood, to plasticity-induced delamination compared to the ones made completely out of softwood.

By application of the annual moisture profile for some consecutive years, the role
of both short- and long-term responses in the mechanical performance of the hybrid element in addition to the influence of moisture cycles on the development of time- and history-dependent stress states are captured.

4.6 Numerical simulation of Mode I delamination

DCB is an appropriate specimen to evaluate the fracture mechanical behavior as well as to measure Mode I critical fracture energy of both solid wood and adhesive bond-line experimentally. Therefore, the second application example investigates Mode I crack initiation and propagation in a DCB sample under short-term monotonic loading. The objective of this numerical analysis is to present modeling procedures for the prediction of delamination onset and growth within the context of the interface cohesive elements (see Section 3.5).

Fig. 4.10 gives the geometry and dimensions of the DCB specimen analyzed in this simulation. In general, to force delamination to grow inside the interface, the specimen geometry is modified by introducing a Chevron notch. It is assumed that the two arms of the DCB test sample are made of European beech acclimated and glued with the MUF adhesive at standard climatic conditions (20°C and 65% RH resulting in MC of 12.4%) with the RL crack propagation system (see Fig. 4.10).

![Figure 4.10: Geometry and dimensions of the DCB specimen. All dimensions are in mm.](image)

In the FE-model of the DCB sample (see Fig. 4.11), the adhesive bulk material is neglected and two wood substrates are connected directly together by a layer of cohesive elements with a thickness of 0.001mm. Hexahedral quadratic brick elements with reduced integration (C3D20R) and three-dimensional linear cohesive elements (COH3D8) are utilized to mesh the wood adherends and cohesive layer, respectively.
Additionally, the material directions of each arm are defined by assigning a rectangular coordinate system (RTL) with axes parallel to the edges. Moreover, due to the symmetry of the DCB geometry with respect to the YZ-plane only one half of the specimen with properly applied symmetric boundary conditions (XSymm) is modeled. As other boundary conditions, the movement of a horizontal edge in the back side of the lower and upper substrates in the vicinity of the interface is fixed in all directions. Furthermore, to load the DCB sample under pure opening mode, normal displacements (along the Y-axis) in the opposite directions are exerted on two reference points (RP) which are defined at the center of pin holes and kinematically coupled to all internal surfaces of the respective cylindrical holes (see Fig. 4.11). The applied vertical displacements ramp up from zero to 0.75mm at the end of the simulation. Note that the proposed wood rheological model defines the material behavior of the DCB adherends.

The cohesive layer is connected to the wood adherends by "Tie" constraint which provides certain advantages compared with shared node approach. In this way, a similar FPZ can be utilized for a sample containing multiple cracks or for various single cracked geometries, yielding an efficient modeling and enhanced comparability.
of results. In addition, the influences of environmental conditions like temperature or moisture content and possibly other field variables on the response of CZM can also be taken into account. To describe the behavior of the cohesive zone, joining two arms of the DCB a bi-linear traction-separation curve with a damage onset criterion of maximum nominal stress and an energy-based damage propagation law for single Mode I loading is chosen.

Table 4.1 lists all cohesive parameters employed to characterize the FPZ in the DCB sample. Note that Abaqus by default uses the constitutive thickness equal to unity in the calculations. To consider the influence of the real thickness, the normal stiffness of the cohesive elements, representing the slope of the linear part of the traction-separation law (see Fig. 3.8), is divided by the geometrical thickness. Accordingly, the magnitude of the MUF Young’s modulus at 65%RH ($\approx$ 6.6% MC) divided by 0.001 is taken as the normal stiffness value of the cohesive elements.

Table 4.1: Cohesive parameters describing the bi-linear traction-separation response of the cohesive layer in the DCB representing the MUF adhesive system. $K_{nn}$, $T_{max}$, and $G_{IC}$ denote normal penalty stiffness, maximum attainable traction before damage initiation, and Mode I critical fracture energy, respectively.

<table>
<thead>
<tr>
<th></th>
<th>$K_{nn}$ [MPa]</th>
<th>$T_{max}$ [MPa]</th>
<th>$G_{IC}$ [N/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUF</td>
<td>2.626116e+06</td>
<td>1.75</td>
<td>0.1414</td>
</tr>
</tbody>
</table>

Because of the short-term character of the applied loading under constant environmental conditions, the visco-elastic effects are trivial and hygro-expansion as well as mechano-sorption have no influence on the delamination process. Fig. 4.12 represents the vertical reaction force versus the prescribed displacement on the upper RP together with the evolution of delamination propagation in the DCB specimen. Furthermore, Fig. 4.13a and b show the vertical displacement of the upper RP and computed reaction force versus the total crack length, respectively. The horizontal dashed line in Fig. 4.13a indicates the critical vertical displacement at which the delamination of the interface initiates.

Occurrence of softening behavior and degradation of material stiffness typically result in numerical instabilities and convergence difficulties. Accordingly, to improve the convergence behavior and to attain a more robust solution, the constitutive model of the softening material is regularized by introducing a viscous parameter in the TSL. Application of the viscous stabilization ensures the positivity of the Jacobian matrix of the degrading material for small time increments and allows
stresses to deviate from the path determined by the TSL. The stabilization is basically characterized using a viscous damage variable $D_v$, expressed in the rate form.
4.6. NUMERICAL SIMULATION OF MODE I DELAMINATION

Figure 4.13: The vertical prescribed displacement on the upper RF (a) and the respective calculated reaction force (b) vs. the total crack length. The initial crack length \( (a_0) \) measured as the distance between the center of the pin holes and the beginning of the interface (see Fig. 4.10) is 10mm.

as \( \dot{D}_v = \frac{1}{\mu}(D - D_v) \), with viscosity parameter \( \mu \) and the damage variable without regularization \( D \). The viscosity parameter specifies the rate at which the viscous response approaches the inviscid one.

Fig. 4.14 depicts the comparison of the vertical force-displacement response for different values of the viscosity. As it can be observed, in contrast to the regularized responses, the solution with \( (\mu = 0) \) representing the inviscid behavior is terminated prior to reaching the maximum opening displacement (0.75mm) due to convergence problem. Moreover, the influence of the viscosity value on the interface performance is obvious as \( (\mu = 1e - 3) \) causes a remarkable deviation from the original curve with \( (\mu = 0) \), whereas \( (\mu = 1e - 5) \) leads to a negligible discrepancy and a more stable solution. Note that the stabilization process is basically employed to minimize the numerical difficulties. On the other hand, to judge about the reliability and authenticity of the solution, the amount of energy dissipated due to the viscous regularization over the whole model must be checked to verify that the stabilization is not affecting the results notably.

The described numerical method can be utilized to analyze crack growth under pure Mode II and mixed-mode conditions as well. Additionally, it can be extended for the analysis of cohesive crack propagation under the combination of long-
Figure 4.14: The effect of the viscosity parameter value $\mu$ on the vertical force-displacement response of the DCB sample (see Fig. 4.12 top).

term mechanical and hygric loading to assess the role of the history- and moisture-dependent behavior of the wood substrates and adhesive bond-lines on the initiation and propagation of damage.

4.7 Remarks on convergence of the developed material model

The classical Newton-Raphson scheme is employed to solve non-linear problems within the framework of the FE environment. Therefore, regarding the performance and the convergence rate of the proposed computational algorithm, the application of a consistent tangent operator is of great significance. Convergence is evaluated in terms of a residual control parameter ($R^a$), which is calculated as the fraction of the largest residual in the equilibrium equation for the field displacement to the mean value of the conjugate force flux. Here, the convergence criterion is taken as $5 \times 10^{-3}$ which would appear rather stringent for engineering applications, but to achieve precise solutions to non-linear problems such a small tolerance is inevitable.

Table 4.2 shows the computational effort for some typical increments during the
moistening phase of the first moisture cycle for \( t = 62.5\text{h} \) to 65h (see 4.1). As it can be observed clearly, the convergence rate is quadratic. At most three iterations are sufficient to meet the convergence tolerance in spite of the increase in the moisture content resulting in the variation of the whole model Jacobian in every increment and the appearance of further non-linear behavior. Although it was supposed that such a strict convergence tolerance imposes a very difficult condition to fulfill, via this numerical example, it is illustrated that the convergence is satisfied after a few number of iterations. Due to the application of the consistent tangent operator, the general robustness of the computational model is preserved.

Table 4.2: Residual norm values of iterations of the *Newton* solution procedure for some typical increments for the time between 62.5h and 65h in the first verification example (see Section 4.1). The control parameter for the convergence criterion is taken as \( R^\alpha = 5 \times 10^{-3} \).

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Residual norm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inc. 429</td>
</tr>
<tr>
<td>1</td>
<td>2.9128e-02</td>
</tr>
<tr>
<td>2</td>
<td>2.3883e-04</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>
Chapter 5

Damage evolution in adhesively-bonded hardwood elements

As it was stated in Section 2.5, new innovative timber engineering applications harness the improved mechanical performance of hardwood. High stiffness and strength of beech, for example, make it possible to achieve comparably slender, wide spanning structures with respect to softwood. However, the disadvantageous low dimensional stability and inhomogeneity of beech make it necessary to adhesively bond smaller sized pieces to form large laminated structural members. After several years of intense research on adhesive bonding of hardwood, it becomes evident that the traditional framework, manifested in codes for adhesive bonding of softwood, is hardly applicable to hardwood. Typically, for beam or plate structures the longitudinal axes (L) of lamellae are oriented coaxially or perpendicularly to each other. Climatic changes or moisture gradients can result in high residual stresses that often exceed the values of the yield stress or strength of bulk wood and adhesive bonding alike (see Section 2.6). Moreover, in real structures differential moisture contents of components prior to gluing can significantly contribute to the development of residual stresses (refer to Section 4.4). Consequently, various types of inter- and intra-laminar cracks form (see Fig. 2.11a), which result in the diminishing of structural integrity and load-carrying capacity.

Adhesive bond-lines can be considered as internal interfaces with distinct properties from adherends and adhesives and strong gradients in material behavior. Cracks can penetrate adhesive bond-lines or can be deflected into it what results in delaminations. This is determined by mechanical performance of the bond-line due
to adhesive, cohesive, or interphase fracture \cite{50, 74, 167, 176} (see Fig. 2.12). The conditions that lead to damage and its propagation, however, cannot be understood if the available energy for crack formation, hence the stress state and elastic energy are not known. Unfortunately, experiments alone will not be able to provide this insight as the observed deformations are a combination of reversible hygro-elastic, irreversible plastic and history-dependent visco-elastic and mechano-sorptive strain components. To make matter worse, most constitutive parameters of wood and adhesives exhibit significant non-linear dependence on the moisture content rendering intuitive explanations questionable \cite{82}.

However, numerical simulations, e.g., with Finite Element Methods (FEM), can cope with the arising complexity of the coupled hygro-mechanical problem. Since they are based on continuum assumptions, effects of disorder - present on all hierarchical scales of wood - are smeared out. One obtains so to say the average answer over many components or samples. Even though it might be off with respect to a very single specific sample, the much more important general behavior as well as physical insight are gained.

In this Chapter, the damage evolution in small, three-layered beech wood panels of various thicknesses, adhesively laminated in aligned or crosswise way by three structural adhesives with distinct mechanical behavior is studied based on a combined experimental and numerical approach. Intra- and inter-laminar damage develop and evolve above a certain thickness as freely supported samples cycle through drying and moistening conditions. By making an analogy to the well-studied problem of progressive transverse ply cracking in cross-ply composite laminates \cite{205} experimental observations for cross-laminated wood elements are interpreted in the framework of the energy based micro-mechanics of damage approach \cite{109, 131, 133}. Unfortunately, this approach is limited to linear hygro(thermo)-elastic, transversely isotropic material bodies.

To capture the complicated moisture-dependent rheology of wood, however, advanced material models are required, like the model introduced in Chapter 3 that combines moisture-mechanical simulations with a hygro-elastic material, including multi-surface plasticity, linear visco-elasticity and mechano-sorption, all considering directional and moisture-dependent parameters \cite{78}. Only if one does not neglect the relevant rheonoumous and scleronomous strain portions that contribute to the total strain, local stresses and strain energy densities can be calculated correctly and failure criteria can be used to make correct predictions. With this approach, the important problem of hygro-fatigue that is puzzling timber engineers in the form of
spontaneous structural failures, even decades after erection, naturally emerges by the sequential stress buildup.

5.1 Experimental investigation of fracture propagation in glued-laminated beech elements under hygric loading

Three-layered European beech samples of various configurations were produced to undergo cyclic climatic changes. All samples had dimensions of 100mm × 100mm, but different thickness of 2d with d=4, 10, 20, 30mm being named T1 to T4, respectively. They were either glued with parallel grain (P1) or crosswise (P2) with the L-direction of the middle lamella of thickness d being perpendicular to the ones of the outer layers with thickness d/2 each (see Fig. 5.2). For adhesive bonding, three commonly used adhesive systems in timber construction, namely MUF, PRF, and 1C PUR were applied. The wood was conditioned and glued at 20° C/95% RH (resulting in a MC of approximately 22.5%), following the specifications given in Table 5.2. To avoid drying during cold curing in the press machine under 1.2MPa, samples were sealed from the environment by wrapping them in foil. For each configuration T1-T4, P1-P2 and adhesive type MUF-PRF-PUR, 4 samples were produced, resulting in 90 samples for testing (see Table 5.1). Just to give an example, sample P2-PUR-T3-2 refers to the second cross-laminated sample with 20mm thick middle lamella glued by PUR adhesive. Note that bonding with MUF and PRF at 20° C/95% RH can result in starved bond-lines, while PUR performs outstandingly when bonded under wet conditions. Nevertheless, in the current study this approach for the model experiments was chosen to obtain strong tensile stresses upon drying.

Table 5.1: Test specimen catalogue

<table>
<thead>
<tr>
<th>Thickness d (mm)</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>Adhesive type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uni-directional (P1)/ Cross-laminated (P2)</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>MUF/PRF/PUR</td>
</tr>
</tbody>
</table>

All samples were placed in internally ventilated climate boxes for drying and re-moistening (see Fig. 5.1). A moisture cycle comprised the following steps: (a) de-moistening from a nearly fiber saturated condition (95% RH) to 2% RH (resulting in a wood MC of about 2.4%), (b) re-moistening to 95% RH. Drying was established
using a layer of Silica gel (see Fig. 5.1b) and re-moistening by vaporized distilled water (see Fig. 5.1c). The RH was recorded by means of a RH recorder located in each box (see Fig. 5.1d). When the mass change was below 0.1%/day, the samples were considered to be equilibrated with the climate inside the box. At the end of every drying stage, the moisture-induced dimensional changes and deformations were recorded using a dial gauge. The climatic changes resulted in a change of moisture
Table 5.2: Processing parameters of the three adhesive systems utilized to assemble the laminated panels

<table>
<thead>
<tr>
<th>Adhesive type</th>
<th>MUF</th>
<th>PRF</th>
<th>PUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adhesive name</td>
<td>Kauramin 683</td>
<td>Aerodux 185 RL</td>
<td>Purbond HBS 709</td>
</tr>
<tr>
<td>Hardener</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Amount of adhesive application (g m⁻²)</td>
<td>340-440 (one side)</td>
<td>225 (per side)</td>
<td>180 (one side)</td>
</tr>
<tr>
<td>Mixing ratio</td>
<td>100 g adhesive + 20 g hardener</td>
<td>100 g adhesive + 20 g hardener</td>
<td>-</td>
</tr>
<tr>
<td>Pressing time (h)</td>
<td>18.5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

content of approximately 20%. Due to differences in the shrinkage behavior of adjacent layers, micro-cracks and delaminations were expected to form (see Fig. 5.2). The initiation and evolution of the inter- and intra-laminar cracks were recorded by scanning each side of every laminated sample with a flatbed scanner for further evaluation.

Figure 5.2: A cross laminated sample with dimensions in mm and different kinds of failure mechanisms. 

On a micro-mechanical scale, crack formation can be interpreted either as a fiber
cell wall failure or as fiber-debonding \cite{204}. In principle, crack propagation by de-
bonding consumes less energy, but is bound to fixed degradation planes, originating
from the cellular arrangements. The large content of ray tissue of beech and the
huge porosity due to the intra-ring vessel network \cite{75} result in additional micro-
mechanical damage mechanisms. In this study, however, cracks are addressed as
if they would grow in a homogeneous, anisotropic medium. The expected types of
intra-laminar failure are straight or curved RT or TL transverse micro-cracks that
partially or entirely split the middle or outer lamellae. At the bond-line, cracks
become inter-laminar by penetrating the bond-line or by being deflected along the
interface resulting in delaminations, whatever mechanism is energetically favorable.
A summary of the expected fracture types was added to Fig. \ref{fig:5.2}.

5.1.1 Samples with coaxial L-directions (P1)

Lamellae with aligned grain orientations are typically found in structural glulam
beams. Transverse cracks are usually not problematic, but micro delaminations
from crack deflections into the interface significantly disturb the shear coupling of
adjacent lamellae, weakening the entire element in terms of strength and stiffness.
To reduce cracking, lamella thicknesses are limited by design codes and longitudinal
stress relief cuts are often introduced.

Thin samples with a middle lamella thickness of \(d=4\)mm (P1-MUF/PRF/PUR-
T1) showed no sign of damage, but an excessive warping deformation that even
increased in consecutive cycles. The dependence of the maximum warping deflection
on the adhesive type and middle lamella thickness \(d\) is shown in Fig. \ref{fig:5.3}. The
moisture-induced deformations accumulate when cycling, resulting in hygro-fatigue.
Also the accumulated magnitude of the moisture-induced warping depends on the
mechanical performance of the adhesive type in the order of MUF-PRF-PUR, what
corresponds to the respective order of stiffness (see Table \ref{table:B.1}). The stiffest bond-
line, therefore, has more hindering ability and lessens the deformation.

For all aligned samples with the middle lamella thickness \(d=10\)mm (denoted as
P1-MUF/PRF/PUR-T2), no damage was recorded at the end of the first drying step.
P1-MUF/PUR-T2 samples remained flawless, even after the second de-moistening
while few TL-cracks appeared in the middle lamella of T2 samples laminated with
PRF adhesive, pointing at the importance of the relative orientation of the longitudi-
dinally aligned lamellae material coordinate systems.

For thicker samples with \(d=20\)mm and \(d=30\)mm (P1-T3/T4), mainly all middle
5.1. EXPERIMENTAL INVESTIGATION OF FRACTURE

Figure 5.3: Dependence of the maximum warping deflection on the adhesive type and middle lamella thickness $d$ averaged over three samples and side view of the deformed shape of the P1-PUR-T1-4 sample at the end of the second de-moistening step. In the following, the central box indicates the central 50% of a data set (limited by the lower and upper boundaries representing the 25% and 75% quantile of data). The central point shows the median and the dashed lines connect the two lower and upper markers that quantify the remaining data lying outside the central box.

lamellae cracked with the TL crack system, primarily during the first de-moistening. In the second drying step, those cracks mostly propagated towards the bond-lines (TR growth system). When reaching it, three different scenarios were observed: (a) crack arrest at the interface, (b) crack penetration through the bond-line and further propagation, (c) deflection into the interface (see Fig. 5.4). In principle, cracks propagate along the path that minimizes the energy. Herein, the anatomic orientations of wood lamellae are a key issue, if the stress enhancement at the crack tip is not weakened by a very soft interface. From Fig. 5.4 it can be realized that when the weak TL planes of adjacent laminates are similar, cracks simply penetrate the interface, whereas for strongly different T-axes, it can be more favorable for cracks to be deflected and to grow along the bond-line, depending on the adhesive type. This results in micro delaminations or - if bond-lines are tough - in bond-line penetration, followed by crack growth along paths that are more energy consuming.
with respect to the growth in the fixed degradation plane of the TL-cracks. Note that also changes of the growth direction into the weak TL growth system were observed at later stages.

Figure 5.4: End grain surface views of the P1-PUR-T4-1 sample after the second drying, illustrating the deflected and penetrating cracks. Arrows mark the propagation trajectory after intersection with the interface.

5.1.2 Samples with crossed L-directions (P2)

Similar to the aligned ones, none of the T1 samples with d=4mm showed any observable damage. As expected, the warping was significantly reduced in the crosswise configurations; however, the increased dimensional stability resulted in stronger edge face deformations (see Fig. 5.5) due to larger residual stresses. One can observe a similar dependence on the adhesive stiffness as for the warping deflection of the aligned samples.

With increasing thickness d, stresses due to drying exceeded strength values. For d=4mm (T1) no damage was observed, while in d=10mm (T2) samples, inter- and intra-laminar failure developed regardless of the material orientation and adhesive type. Fig. 5.6 shows the edge views of a sample (P2-MUF-T2-2) for two successive de-moistening stages. After the first de-moistening (left), an approximately evenly spaced set of inclined micro-cracks is observed. The quasi-periodicity is the result of a process where cracks form sequentially between existing cracks in accordance to transverse ply cracking in cross-ply laminates [109, 131, 133]. As stresses in the
middle layer are introduced by shear at the layer interfaces, they exhibit shear lag in a spatially limited zone. If the shear lag zone is smaller than the half crack spacing, an unperturbed zone exists where new cracks can initiate at the weakest location. If the shear lag zones interfere, cracks primarily form between existing cracks where the stress is maximal. In the second de-moistening no further transverse cracks formed, but microcrack-induced delaminations emanated from all micro-crack tips. Since no shear stress can be transmitted through the delaminations, the shear lag zones became closer and consequently the saturation crack density was reached [132].

The damage of the outer layers is best visible when the sample is rotated. It is characterized by staggered or anti-symmetric pattern of micro-cracks and adhesive failure (see Fig. 5.6). In the second drying phase, a new micro-crack initiated in a non-symmetric manner and additionally further de-bonding, originating from the micro-crack tips and edges formed. Samples with PUR and PRF adhesive behaved similar to MUF glued samples, but exhibited a smaller crack density in general.

These observations are in agreement with typical crack formation processes in cross-ply laminates driven by a contrast in the Poisson’s ratios and expansion coefficients of adjacent layers [131, 132]. Like for MUF samples, the crack opening increased in the second moisture cycle, pointing at the importance of the irreversible strains. It is interesting to note that cracks can be inclined since the formation of a longer crack along the weak TL plane can be energetically advantageous compared to a
perpendicular crack which is shorter but requires more energy to grow.

![Figure 5.6: Side views of the fractured P2-MUF-T2-2 sample after the first (left) and the second (right) de-moistening steps, displaying the formation of equally distanced curved micro-cracks that span the entire sample width and microcrack-induced debonding.](image)

The general fracture behavior of P2-T3 samples (d=20mm) was identical with their P2-T2 counterparts with the same adhesive system. The damage evolution is shown for the P2-PRF-T3-2 sample in Fig. 5.7. Note that regardless of the TL degradation plane, two more or less regularly spaced partially straight micro-cracks were formed.

![Figure 5.7: Side views of the fractured P2-PRF-T3-2 sample after the first (left) and the second (right) de-moistening steps with a pair of relatively straight transverse micro-cracks formed as well as a top layer crack with delamination.](image)

As the middle lamella thickness increased to d=30mm (P2-T4) the analogy to transverse ply cracking breaks down, as the crack spacing reached sample dimensions. Eventually, growth in the weak TL plane dominated, entirely splitting the
middle lamella and thus, avoiding further transverse crack formation (see Fig. 5.8). The curvature of the growth rings, hence the location of the pith, plays a crucial role for intra- and inter-laminar damage as it dominates the overall stress state from the restrained shrinkage.

![Image of fracture sample](image)

Figure 5.8: Side views of the fractured P2-PUR-T4-1 sample after the first (left) and the second (right) de-moistening steps with a single straight transverse micro-crack and severe intra-laminar TL-cracking.

### 5.1.3 Observations on bond-line performance

The failure modes of the MUF and PRF/PUR bond-lines were characterized differently by adhesive failure and interphase delamination, respectively. MUF bond-lines predominantly exhibited adhesive failure for all thicknesses \(d\). This is due to the high stiffness of the MUF resulting in a hard layer of the bulk adhesive incapable of reducing interfacial stresses. Moreover, the penetration of the MUF adhesive into the interphase layer increases the brittleness of the cellular structure and decreases the rigidity and the strength of the interface [50]. The superior performance of the PUR bond-line with a fracture toughness higher than adjoining hard substrates can be associated with the occurrence of the plastic deformations within the adhesive
material leading to a more flexible interface with the ability of reducing stress concentrations \[155\]. Note that, differences also originate from a distinct liquid adhesive penetration behavior into the vessel network \[76, 121\]. Hence, under identical situations, the delamination is less pronounced (see Fig. 5.9). Similarly, the rigidity and the endurance of the PRF bonded-joints compared to the solid wood adherends can be attributed to the moderate stiffness and also the tendency of the PRF to receive more water resulting in a more plasticized and softer interface able to lower the stress concentrations all over the bond-line \[155\]. Note that the present study shows similarities to delamination testing (DIN EN 14080) and analogously one could measure the wood fracture portion by injecting ink and tearing the sample mechanically apart.

![MUF PRF PUR](image)

Figure 5.9: P2-X-T3 samples (d=40mm) with different adhesives after the first de-moistening exhibiting adhesive failure in the MUF and interphase failure in the PRF and PUR bond-lines.

The damage state is best quantified by the micro-crack density and the delamination ratio defined as \((d_1 + d_2)/2a\) (see Fig. 5.10). One can observe the tendency of increasing delamination ratios but decreasing crack density with increasing thickness \(d\).

### 5.2 Numerical analysis

For a better interpretation, the 3D orthotropic moisture-dependent constitutive material model for wood previously developed (see Chapter 3) and validated (see Chapter 4) is used. The model gives the possibility to attain a better quantitative understanding of the true distribution and evolution of the moisture-induced stresses
Figure 5.10: Damage states after the second drying for different thicknesses and adhesive types. The inset gives the unit cell of damage used for the analytical stress calculation (see Subsection 5.2.3).

as the main driving forces for the formation of inter- and intra-laminar fracture. As it was mentioned earlier, the moisture analysis is only sequentially coupled with the non-linear mechanical analysis. The presence of adhesive bond-lines requires special treatment with respect to moisture transport and mechanical behavior.

In principle, a simplified moisture-dependent material model compared to the one used for wood was employed for adhesives. In case of the PRF, it was a simple isotropic, hygro-elastic model; for the MUF viscoelasticity was added, and for the PUR additionally plasticity, represented by the simple J2-plasticity with isotropic hardening [173]. A complete set of moisture-dependent adhesive properties for the MUF, PRF, and PUR are summarized in Table B.1-B.5 in the Appendix B. Hence, in combination with the full model description in Chapter 3, all required parameters are defined.

The five-layered FEM model consists of three solid wood lamellae and two adhesive layers. Cohesive elements connect the adhesives with the middle layer to allow for the simulation of the onset and evolution of delamination within the framework of non-linear fracture mechanics [3, 17]. The cohesive zone models were characterized in terms of the so-called traction-separation relationships describing the damage
Chapter 5. DAMAGE OF ADHESIVELY-BONDED ELEMENTS

initiation and evolution laws. In the current study, the COH3D8 8-node 3D cohesive elements of ABAQUS \[190\] with an uncoupled traction-separation approach were used. Delaminations were initiated due to a maximum nominal stress criterion followed by an energy-based damage evolution law with exponential softening. Mixed-mode cases were considered by a power law interaction criterion with the exponent of 1.6 taken from Ref. \[50\]. Table 5.3 summarizes all cohesive parameters utilized to characterize the fracture process zone.

Table 5.3: Parameters for the non-linear traction-separation behavior of the cohesive element for PUR.

| Damage initiation parameters \[167\] |  |
|------------------|------------------|------------------|
| $T_{nn}$ [Mpa]   | 6.55             |  |
| $T_{ss}$ [Mpa]   | 14.2             |  |
| $T_{tt}$ [Mpa]   | 14.2             |  |

| Damage evolution parameters: Fracture energies \[168\] |  |
|------------------|------------------|------------------|
| $G_I$ [N/mm]     | 0.75             |  |
| $G_{II}$ [N/mm]  | 1.45             |  |
| $G_{III}$ [N/mm] | 1.45             |  |

<table>
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<th>Stiffness values</th>
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<td>$K_{nn}$ [Mpa]</td>
<td>1157900</td>
</tr>
<tr>
<td>$K_{ss}$ [Mpa]</td>
<td>445400</td>
</tr>
<tr>
<td>$K_{tt}$ [Mpa]</td>
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The adhesive bond-line was modeled by volume elements of thickness 0.1mm, while a thickness of 0.001mm was assigned to the cohesive layer. To capture the influence of the geometrical thickness, the normal and shear stiffness of the cohesive elements were normalized by the actual thickness (see Table 5.3). Note that only average values of the adhesive moisture-dependent Young’s and shear moduli in a range from 0-5% moisture content were considered, since the dry state is the critical one with respect to the delamination growth. For wood layers with significant curvature of the growth rings ($d > 10$mm), the material orthotropy was assigned in a cylindrical coordinate system by estimating the location of the center (pith) from the local curvatures of each layer obtained from the side scans.

Fig. 5.12 depicts the different steps of the computational approach employed to predict the damage onset and growth in adhesively bonded hardwood elements.

5.2.1 Stress analysis of glued-laminated panels under hygric loading

As the moisture controls all hygro-mechanical properties, its gradients due to diffusive moisture transport as well as differential swelling are the main driving forces for
5.2. NUMERICAL ANALYSIS

Figure 5.11: Exemplary FEM model with the applied boundary conditions and local material coordinate system consisting of twenty-node quadratic brick elements with reduced integration (C3D20R) and linear cohesive elements (COH3D8).

the damage evolution in laminated wood. In the first step the moisture fields were calculated, exhibiting large gradients across the lamellae interfaces (see Fig. 5.13). The simulation was repeated for all adhesives using sorptive transport properties from Refs. [198, 202] fitted to the equation and parameters summarized in Table B.5. The resulting moisture profiles for different adhesive systems are basically identical, showing the irrelevance of the diffusive transport through adhesives compared to the one in the solid wood.

The deflection of glued-laminated panels under changing MC was computed with the material models using the previously determined evolution of the moisture field. This calculation was done for all coaxial T1/2 samples with their respective growth ring orientations. Good agreement is found with respect to the experimental values at the end of the second de-moistening step (see Fig. 5.3), even though material parameters are identical for all calculations which is clearly not the case for the experiments. In the following, the material models are applied to predict the influence of the total strain constituents on the failure of cross-laminated configurations.

To understand the damage process, it is important to look at the situation for delamination initiation at a single crack. This case is simulated by artificial opening
Chapter 5. DAMAGE OF ADHESIVELY-BONDED ELEMENTS

Variation in the RH of the ambient air
Change in the moisture distribution
Panel configuration and structure
Moisture-dependent material properties
Comprehensive material models for Solid wood substrate & Adhesive layer
Simulation of mechanical behavior
Prediction of the fracture mechanical response
Comparison with the experimental observations

Figure 5.12: The principles of the general approach utilized to analyze the fracture behavior of glued-laminated panels.
5.2. NUMERICAL ANALYSIS

Figure 5.13: Moisture evolution at specific points and (inset) moisture field in the P2-T3-3 sample after 1.7 days.

...a crack at the dry state and then calculating a moistening and another de-moistening phase. It is striking to observe how the crack opening displacement (COD) of the transverse crack, that forms in the first drying phase, increases in the next step (see Fig. 5.14), driving micro delaminations. When the crack forms, the elastic strains are immediately relaxed and the relaxation of the visco-elastic strains starts. This is true for the cracked middle layer as well as for the compressively loaded outer plies. However, this does not explain the increased COD in the consecutive cycles. Provided that no delaminations occur, the COD increases as visible in Fig. 5.14 by a decrease of the tensile mechano-sorptive strain during re-moistening, resulting in an increased COD of about 25%. Additionally, as soon as the crack forms, the compressive plastic strains in the thickness directions develop (Fig. 5.14 top left in terms of the plastic hardening variable).

Delaminations can initiate and grow under different fracture modes like mode I opening and mode II shearing mode, most likely, however, as a mixture of both. To have a better understanding of how the middle layer cracking and adhesive type influence the distribution and value of interfacial stresses, the stress profiles with the components for mode I, the peeling stress ($\sigma_{yy}$) and for mode II, the $yz$ shear component ($\sigma_{yz}$) are calculated along the edge (Path 1) and the center line (Path 2) of the lower adhesive bond-line in the z-direction (see Fig. 5.15) after the second de-moistening step.

The results show the effect of adhesive stiffness on the stress state (see Fig. 5.15a). As it can be noticed, the distribution and amount of the shear components are more...
Figure 5.14: Strain evolution in quarter-models upon formation of a transverse crack with suppressed delamination for the P2-PUR-T3-3 sample with identical legends for the time evolution.

or less identical for all adhesive systems, while the magnitude of the normal stresses with similar distributions is inversely correlated with the order of adhesives stiffness (see Table B.1).

Following the results presented in Fig. 5.15a, exhibiting the relationship between adhesive type and stresses generated in the adhesive bond-line of the P2-T3 sample,
5.2. NUMERICAL ANALYSIS

Figure 5.15: a) Peeling and shear stresses in the adhesive bond-line along the Path 1 and Path 2 for different types of adhesive, b) The equivalent plastic strain in the lower PUR adhesive layer of the P2-T3 sample after the 2nd de-moistening phase.

A parametric study is conducted to investigate the dependency of interfacial stresses on the lamellae thickness in cross-laminated specimens. In this set of simulations, to have a better comparability a Cartesian coordinate system with axes parallel to the edges (R-T-L along the width, thickness, and length, respectively) defines the orthotropy directions of every wood layer.

Fig. 5.16 shows the distributions of the normal $\sigma_{yy}$ and shear $\sigma_{yz}$ stresses for different values of $d$ along the Path 1 at the end of the first de-moistening stage.
It is obvious that the interfacial stress profiles alter with respect to the sample thickness. For $d \leq 10\text{mm}$ the distributions of the normal stress show two peaks in the vicinity of the edges with a plateau in the middle, which for $d=4\text{mm}$ the peak values are closer to the edges. For $d > 10\text{mm}$ the maximum peeling stresses occur...
exactly at the both ends of the Path 1 with a local maxima at the midspan between the two ends. The profiles representing the shear component are approximately similar for various thicknesses. The highest values of the shear stresses lie at the both endpoints (with opposite signs), where the transition from negative to positive happens at the middle point of the considered path. Moreover, for thinner samples \( (d \leq 10 \text{mm}) \) the shear stresses are zero in the neighborhood of the midspan. It is visible that the maximum of the interfacial stresses corresponding to the both opening and shearing modes occurs near the edges, subsequently, provided that no solid wood cracking takes place, edge delaminations are the dominant failure mechanism in cross-laminated configurations under drying. Furthermore, increasing the middle layer thickness causes larger peeling and shear stresses under identical hygric loading for all adhesive types. This indicates the higher propensity of thicker samples to failure in terms of delamination initiation and propagation, which is also in agreement with the experimental observations (see Fig. 5.10).

Moisture gradients and swelling anisotropy both contribute to rather complicated stress states that relax by the buildup of the visco-elastic, mechano-sorptive or plastic strains. As a result, by inversion of the moisture content to the initial value, the sample does not return to the stress-free state, but can build up inverse stresses by the plastic deformation and mechano-sorption. To investigate the evolution of the stresses and different constituents of the total strain under cyclic hygric loading, the experimentally studied two cycles is extrapolated in time up to 10 cycles computationally, corresponding to three years in practice. This is demonstrated in two typical scenarios that both start at the stress-free condition, but case (I) is initially wet and then dries, while case (II) is initially dry and goes first through a moistening cycle. For clarity only stresses and strains in the center point (Point 2 in Fig. 5.13) in the radial direction with suppressed damage are visualized in Fig. 5.17.

**Case I** first dries, resulting in large tensile stresses that relax by the evolution of the visco-elastic and mechano-sorptive strains. When the sample is re-moistened, the stress state inverts, leading to the compressive stresses and dissipation by plastic strains, as the moisture-dependent yield body shrinks due to moisture increase. In the consecutive moisture cycles, the added tensile stress due to the plastic strain increment is over-compensated by the increased visco-elastic strain and more importantly by the mechano-sorptive one. During every re-moistening phase an additional but decreasing plastic strain increment is added (see Fig. 5.18). In total, this causes a cyclic decrease of the maximum tensile stress that is becoming less significant after
Figure 5.17: Radial stress evolution in 10 consecutive drying-moistening (case I, top) and moistening-drying (case II, bottom) cycles at the center of the P2-T3 sample (Point 2 in Fig. 5.13).

5-6 cycles (see Fig. 5.17).

**Case II** corresponds to adhesive bonding at rather dry states. When the sample is moistened for the first time, a significant plastic strain increment is formed while the central layer is in a compressive state. In re-drying, due to the expansion of yield surfaces, the inverted stresses cannot relax, resulting in significant tensile stresses. During consecutive cycles, the plastic strains remain constant, but mechano-sorptive and visco-elastic ones evolve (see Fig. 5.18), leading to cyclic tensile stress accumulation upon re-drying that does exceed the strength values. This can be considered as the driving mechanism behind *hygro-fatigue* (see Fig. 5.17).
5.2. NUMERICAL ANALYSIS

5.2.2 Non-linear fracture mechanical simulations of damage in cross-laminated samples

To gain further insight into the damage onset and evolution, fracture mechanical simulations using the cohesive interface elements are performed. Delaminations emanate from existing transverse cracks, before they initiate at free edges (see Fig. 5.19). This, however, does not mean that transverse cracks necessarily have to form before edge delaminations can occur.

Delaminations are much more severe than transverse cracks in terms of loss of stiffness of the laminate, as they result in large stress-free regions and significantly reduce the shear lag zones. Additionally, at the delamination fronts, the loading on the adhesive bond-lines becomes maximal. Literature values for the critical energy release rates $G_{IC}$ and $G_{IIC}$ of adhesive bond-lines in beech wood unfortunately are given with a substantial bandwidth. Therefore, in the current study a typical value used in Ref. 168 is picked and both modes are multiplied by a scalar multiplier.

The calculated delamination ratio ($A_D/A_{Total}$) and edge delamination length fraction ($L_D/L$) for two successive cycles are shown as function of the multiplier in Fig. 5.20. It is evident that delamination evolves in consecutive cycles with a con-
Figure 5.19: Time evolution of delamination initiation and propagation and distribution of degradation scalar parameter (blue intact, red almost delaminated, while delaminated elements are deleted) with delaminations for a multiplier value of 2.65 compared to the P2-PUR-T3-3 sample.
5.2. NUMERICAL ANALYSIS

cave delamination front (see Fig. 5.19). Note that due to the curvature, pure fracture modes are not present, pointing at the importance of mixed mode criteria. In this case, the best agreement between the edge delamination length and the corresponding experimental observation (see Fig. 5.10) is obtained by setting the multiplier between 2.65 and 2.75 with respect to the values used in the linear elastic calculations (see Ref. [168]), exhibiting the role of non-linear energy dissipation at the delamination front.

Figure 5.20: Relative delaminations \((A_D/A_{tot})\) and \((L_D/L)\) versus the energy multiplier in the adhesive bond-lines of the P2-PUR-T3-3 sample.

5.2.3 Analytical study of micro-cracking in cross-laminated samples

Up to this point, cracks were artificially introduced in the middle lamella at experimentally observed positions and subsequently, the evolution of the strain fields as well as delamination initiation and propagation were calculated. In principle, one could extend the material model with capabilities for softening under tension or shear to localize micro-cracks [159]. However, this subsection tries to adopt a much more general fracture mechanical approach that is not limited by system sizes and is capable of predicting size effects.

The most suitable analytical solution to the damage evolution in cross-laminated samples is given by a micro-mechanics of damage approach, originally developed for
transverse ply cracking and micro-delamination in cross-ply composites \[109, 131, 133\]. Since, this approach has been considered as a standard in composite design for at least two decades, the current study refers to the review articles in Ref. \[132\] for a detailed description. It consists of two steps: (a) the use of micro-mechanics to analyze the stress field in a composite in the presence of damage, determining the unit cell by its limiting cracks (see Fig. 5.10 inset) and (b) the use of a failure criterion to predict the evolution of damage. The equations for calculating the stress field in the two-dimensional (2D) unit cell are derived from variational mechanics. For high stiffness contrast of adjacent lamellae, like in the case of beech \(E_{LL}/E_{RR}\), the sample edge can limit the unit cell.

The process is driven under drying in analogy to thermal cycling by differences in the hygro-expansion of the lamellae. Energy can be dissipated either by the formation of new micro-cracks, increasing the crack density, or by the initiation of delaminations at the tips of existing micro-cracks. This is expressed by the competition between the respective energy release rates (ERR) \(G_{mc}\) and \(G_{dc}\) for intra- and inter-laminar failure. Once \(G_{dc}\) dominates \(G_{mc}\), micro-delaminations form and the saturation crack density is reached only allowing further energy dissipation by delamination growth. The model predicts the stress states in the middle as well as the ones in the outer lamellae, resulting in their cracking and delamination from the middle lamella under drying as well (Fig. 5.21). For simplification, the material is considered as transversely orthotropic, linear elastic with properties that correspond to the dry state.

A first micro-crack forms during the first drying in a region of more or less constant tensile stress \(\sigma_{xx}\). As one can see in the stress profiles (see Fig. 5.21), the material assumptions lead to stresses above known strength values. However, verification calculation with the FEM approach that only considers the hygro-elastic part of the wood material model gives rise to similar stress values. Hence, the analytical approach will have mainly qualitative character. Nevertheless, the framework of micro-mechanics of damage helps in rationalizing the damage evolution. It is only during the second drying step that the strength of the middle lamella is overcome and segment II (in Fig. 5.21 T2) fractures, preferably in the mid span, in the region of maximum tensile stress as predicted by the model resulting in \(N=3\) segments. Since, also micro-delaminations form, \(\rho^{10}=(N/L)=(3/100)=0.03\text{mm}^{-1}\) is taken as the critical micro-crack density for this sample. Once \(\rho^{10}\) is known, the critical intra-laminar ERR \((G_{mc})\) is determined and its relation with the inter-laminar one \((G_{dc})\) can be tuned such that \(\rho^{10}\) emerges leading to \(G_{dc}^{10}=0.45G_{mc}^{10}\) (see Fig. 5.22).
As it can be seen, during the first de-moistening micro-cracking is the preferred damage mechanism. Since now all parameters are determined, the behavior of other thicknesses can be calculated (Fig. 5.22). The predicted evolution of the micro-crack density is given in Fig. 5.22 showing that for an identical moisture change of $\Delta \omega = 20\%$ the resulting crack density for $d=20\text{mm}$ is $\rho^{20} = 0.02\text{mm}^{-1}$, which is in agreement with the experiments (see 5.10). As a matter of fact, the accurate equilibrium critical inter-laminar ERR can be obtained as $G^{20}_{dc} = 0.71G^{10}_{mc}$, which is close to $G^{10}_{dc}$ considering the finite size of the samples. Also, it becomes evident that the threshold for micro-cracking is not reached for T1 samples. When all critical moisture changes for the onset of micro-cracking are plotted as a function of thickness, the critical thickness for $\Delta \omega = 20\%$ is extracted to be about $d=6\text{mm}$, presented in Fig. 5.22 inset as the limit line (bold dashed line).

Micro-mechanics of damage is a simple 2D approach and cannot capture the effect of the third direction. Considering the simplified material behavior and the limited dimensions of the samples, the predictability of the model and its quantitative agreement is surprisingly good. However, the obtained critical energy release
Figure 5.22: Dimensionless ERR vs. micro-crack density fitted to T2 and applied to T3 samples, giving excellent agreement with the experimental observations. Inset: Micro-crack density as a function of moisture gradient for P2-PUR-T1/T2/T3 samples and the limit line for micro-cracking initiation.

rates are strongly dependent on the anatomic orientations which should be considered in a study with broader experimental basis.
Chapter 6

Summary and outlook

6.1 Summary of the thesis

Wood as the most important natural and renewable construction material plays a significant role in the building arena. Besides many outstanding advantages, it shows some complex characteristics like strong orthotropic character, hygroscopic nature with moisture-dependency of more or less all material properties, aging and decay of stiffness and strength over the service period. To mitigate the consequences of such inherent attributes and to attain safe and efficient load bearing wood structures, capable of fulfilling the mechanical and environmental requirements, the application of engineered elements in the forms of glued-laminated configurations is of fundamental importance. However, wood hygroscopicity together with differential swelling/shrinkage along the anatomical directions cause moisture-induced stresses and deformations in adhesively bonded components - cross-laminated patterns in particular - under changing climatic conditions. It should be noted that the magnitude of the induced stress and deformation fields is more pronounced in laminated elements out of hardwood compared to the softwood counterparts.

The aforementioned moisture related tensile or shear stresses, augmented by the occurrence of time- and moisture-dependent mechanical responses, result in inter- and intra-laminar fracture. Subsequently, delamination of adhesive bond-lines and cracking of solid wood substrates diminish the structural integrity and lessen the performance and efficiency of such structural components. Hence, to evaluate the fracture mechanical behavior of adhesively bonded elements, appropriate computational approaches based on moisture-dependent material data as well as thorough assessments of the relevant determinative factors are key issues. Therefore, prediction of the true history-dependent stress states as the primary driving forces for the
onset and growth of damage is of great significance.

Accordingly, in the current study a comprehensive rheological model for the mechanical behavior of wood under simultaneous mechanical loadings and varying environmental conditions was introduced. The considered deformation modes were both short- and long-term responses, namely elastic, plastic, hygro-expansion, visco-elastic creep, and mechano-sorption. To characterize the irrecoverable part of the material response, described as the plastic deformation, a general multi-surface plasticity model with three independent failure mechanisms - each one belonging to an anatomical direction under compression - was employed. Moreover, the time- and moisture-dependent behavior like visco-elastic and mechano-sorptive creep were simulated through a serial association of the Kelvin-Voigt elements. To consider the intrinsic hygroscopic nature of wood in the assessment of timber structures, all material input parameters were defined as functions of the moisture level. The numerical integration algorithm along with the utilized approach for the stress update scheme were established on an iteratively additive decomposition of the total strain. All corresponding constitutive formulations were developed and implemented into a user-material subroutine (UMAT) within the environment of the FEM.

The material model was applied to two important wood species, namely European beech and Norwegian spruce. However, due to the scarce availability of stress-strain relationships under different loading cases and moisture contents, for beech in particular, some property adaptation procedures had to be implemented to obtain a full set of material data. The functionality and performance of the developed rheological model and the robustness of its implementation were evaluated through several numerical simulations for uni- and multi-axial loading and for restrained swelling as well as by two application examples - simulating the moisture-induced stresses during the manufacturing of a glulam beam and interfacial de-bonding of a wood DCB sample. The numerical examples demonstrated the applicability of the presented constitutive model under any optional mechanical and hygric loading combination, specifically when more than one failure mechanism is activated.

Following the verification of the proposed wood material model, delamination of hardwood bonding as well as fracture of hardwood adherends under changing climate were studied on configurations and also under conditions that are much closer to real applications than typical delamination tests like the DIN EN 14080 for hardwood. In a combined experimental and numerical investigation, three-layered European beech panels of aligned and crosswise configurations adhesively bonded with the MUF/PRF/PUR were subjected to cyclic climatic variations. In this study,
6.1. SUMMARY OF THE THESIS

the aim was set on covering a large parameter space, which led to a larger scatter of results by nature. Similar to the tensile-shear tests, the scatter can be drastically reduced by a careful selection of samples. Note that all adhesive bonds were produced in a moist condition, outside the specifications of the respective adhesives, to obtain more severe crack and delamination states. However, calculations showed (see Fig. 5.17) that, in principle, samples bonded at the dry states or lamellae with moisture differences at bonding (see Section 4.4) also can be covered to extend the reach of this study from a phenomenological one to a pre-study for alternative delamination testing of adhesive bond-lines in hardwood, that are closer to applications than today’s strongly criticized codes.

Furthermore, it was indicated that due to high moisture gradients across the bond-line (see Fig. 5.13), resulting from the noticeable vapor resistance of the used adhesive systems, significant inelastic deformations and consequently local stresses build up under moisture cycling that lead to inter- and intra-laminar damage. By performing 3D moisture-stress FE calculations with elaborated wood and adhesive material models, changes in the strain field in the region of the crack tips that are important for delamination initiation were captured. Moreover, it was demonstrated how the visco-elastic and mechano-sorptive strains relax upon formation of transverse cracks in the middle layer, how the plastic strains build up, and how the strain evolution in the moist states determines the stress situation upon drying, where crack initiation is observed in practice (see Fig. 5.14). The potential of stress dissipation within bond-lines to avoid delamination initiation was illustrated by calculating identical situations with different adhesive material models (see Fig. 5.15). Numerical simulations of interfacial delamination under successive de-moistening steps were also carried out based on the concept of the CZM and the use of the cohesive interface elements within the context of non-linear fracture mechanics (see Fig. 5.19).

In addition, it was shown within a certain range of the middle layer thickness $d \leq 20\text{mm}$, how an adopted micro-mechanics of damage approach can be applied, for comparable growth ring orientations. This provides limit thickness values for the onset of micro-cracking and the values of adhesive bond-line delamination ratios under conditions similar to practical situations. This approach can be directly utilized by producers of CLT and veneer ply wood to calculate the range of application of specific laminates for climatic changes. It is believed that the numerical model has a large potential for optimizing even hybrid CLT configurations with respect to the composition (lamella orientation and thickness) and other process parameters such
as moisture differences among lamellae.

The introduced material model is numerically implemented in a general fashion and can also be used for other wood species. By application of moisture-dependent material properties, the rheological model can easily be adapted to predict the mechanical response of different wood types. In this respect, the flexibility and efficiency of the developed material model were proven by simulating a hybrid beech/spruce glulam beam under a changing climate (see Fig. 4.9).

Following the application of the developed computational approach, the goal of attaining more realistic and reliable predictions on mechanical performance of engineered wood elements is achieved. This comprehensive evaluation is essential to increase the safety and long-term reliability of composite wood structures under any arbitrary combinations of loading and moisture content.

6.2 Concluding remarks

Based on the results of the experimental investigations and the corresponding numerical analyses, the following principal conclusions can be drawn:

- Small diffusive transport through bond-lines results in large moisture gradients, while the resulting gradients are irrelevant to the adhesive system applied.

- The plastic deformation, time-dependent visco-elastic creep, and moisture-dependent mechano-sorption have central roles in the stress buildup under cyclic hygric loading starting from dry state called as hygro-fatigue.

- The MUF bond-lines show different failure mode compared with the PRF/PUR adhesive joints with predominant adhesive failure of the MUF bond-lines and interphase failure of the PRF/PUR ones. This distinction refers to the superior performance of the PRF/PUR adhesive systems exhibiting the fracture toughness more than solid wood adherends with less noticeable delaminations.

- The thickness and the orientation of the cross-grain directions of every wood layer together with the adhesive system used for gluing are the influential factors on the initiation of damage and its propagation in adhesively bonded hardwood elements under varying environmental conditions.

- Arrest at the bond-line, penetration through the interface, or deflection into the bond-line (interfacial de-bonding) are three possible scenarios after inter-
6.3. IDEAS FOR FUTURE RESEARCH

section of cracks with the interface in aligned panels subjected to consecutive de-moistening phases.

• The TL crack system as well as the development of micro-cracks based on an approximately equally distanced arrangement and the micro-crack-induced delaminations emanating from the micro-crack tips are the prevailing failure modes in cross-laminated panels under successive drying steps.

• Visco-elastic and mechano-sorptive relaxations after transverse cracking of the middle lamella result in cyclic increase of the crack opening displacement in three-layered adhesively bonded panels.

• Models with linear character simply lead to wrong results which are off by a factor of 2-3 compared with realistic ones. Subsequently, for the analysis of complicated situations and capturing the long-term responses, the application of non-linear models is of fundamental significance.

• Delamination tests with changing moisture content could be based on cross-ply cracking under situations which are much closer to service conditions.

6.3 Ideas for future research

As it was mentioned in Section 3.4 moisture diffusion inside wood shows more compliance with the non-Fickian or multi-Fickian behavior. Therefore, to gain a more realistic prediction on moisture evolution, the non-Fickian or multi-Fickian moisture transport formulations can be implemented within the FE environment through UMATHT subroutine.

In Section 2.2 it was pointed out that there is a pronounced asymmetry between wood elastic properties measured under tension and compression, known as the ”bimodular behavior”. To attain a more reliable and accurate assessment of wood mechanical performance, the load direction dependency of all elastic material constants can be added to the developed wood constitutive model.

In the framework of this thesis, the effects of temperature variation on wood moisture sorption as well as its properties were ignored. Since the change of temperature affects wood mechanical behavior, the role of temperature should also be taken into account. In this respect, a coupled temperature-moisture-stress analysis with moisture- and temperature-dependent material properties would be of significant interest.
Since the current study primarily focused on the moisture-induced failure (delamination and fracture) of adhesively bonded hardwood elements, their performance under simultaneous application of changing MC and external mechanical loads was not addressed. Hence, experimental investigations of engineered woods subjected to static or dynamic loading and changing climate along with the corresponding numerical simulations are required. These investigations are essential to gain more knowledge about the physical response and damage process of glued-laminated elements under practical service conditions.

As it is known, wood exhibits ductile and brittle failure modes, which can occur simultaneously along different anatomical directions (see Section 2.2). In the proposed wood material model, the occurrence of the plastic deformations (ductile failure) is only activated under compressive loads, while no failure mode is detected under tension. Accordingly, based on the theory of continuum damage mechanics (CDM), the developed rheological model can be extended to cover softening behavior under tension and shear (brittle failure) as well. In this case, both ductile and brittle failure modes can be captured with one single material model.

Application of the cohesive interface elements based on the concept of the CZM is a well-known computational approach for the numerical simulation of delamination. Nevertheless, to describe the material behavior in the FPZ with a more complicated and pragmatic response than the linear elastic relationship and also to consider moisture transfer across the interface, a user defined hybrid interface element can be implemented. This can be realized either by using UEL subroutine to define a multi-physical cohesive element, or by UMAT subroutine to characterize moisture-dependent non-linear mechanical behavior of the interface.
Appendix A

Total algorithmic tangent operator

Purely elastic or elasto-plastic tangent operator: From a rheological point of view, the tangent operator corresponding to the first two elements of the constitutive material model (Fig. 3.2), namely the elastic spring and the plastic Kelvin element are correlated. In the absence of plastic strain, the plastic Kelvin element will be inactive without giving a contribution to the tangent operator. Trivially, the single Jacobian is equal to the elastic stiffness tensor $C_{n+1}^{el} = C_{0,n+1}$. However, in the presence of plastic deformation, both elastic and plastic elements contribute to the definition of the elasto-plastic algorithmic tangent modulus [173]:

$$C_{n+1}^{ep} = \Xi_{n+1} - \sum_{\beta \in S_{act}} \sum_{\alpha \in S_{act}} \left( G_{n+1}^{\beta \alpha} N_{\beta,n+1} \otimes N_{\alpha,n+1} \right).$$  \hspace{1cm} (A.1)

Note that $\otimes$ symbolizes the dyadic product of two vectors and $\Xi_{n+1}$ is the algorithmic modulus and is given by the following relationship:

$$\Xi_{n+1} = \left[ C_{0,n+1}^{-1} + \sum_{l=1}^{r} \Delta \gamma_{n+1}^{l} \partial_{\sigma} f_{l} (\sigma_{n+1}, \alpha_{l,n+1}, \omega_{n+1}) \right]^{-1} =$$

$$\left[ C_{0,n+1}^{-1} + 2 \sum_{l=1}^{r} \Delta \gamma_{n+1}^{l} b_{l,n+1} \right]^{-1}, \hspace{1cm} (A.2)$$

and

$$N_{\alpha,n+1} = \Xi_{n+1} : \partial_{\sigma} f_{\alpha,n+1}, \hspace{1cm} (A.3)$$

115
where \( r \) is the number of active yield surfaces and \( G_{n+1}^{\beta \alpha} \) is defined by the succeeding expression:

\[
G_{n+1}^{\beta \alpha} = \left[ \partial_\sigma f_{\beta,n+1} : \Xi_{n+1} + \partial_\sigma f_{\alpha,n+1} \ H_{n+1}^{\beta \alpha} \partial_q f_{\alpha,n+1} \right]^{-1}. \tag{A.4}
\]

Here \( H_{n+1}^{\beta \alpha} \) denotes the \( \beta \alpha \) component of the matrix of hardening moduli \( H_{n+1} \) filled with the respective values for every active yield criterion (\( H_l = \partial q_l / \partial \alpha_l, \ l = R, T, L \)).

Eq. (A.1) represents the desired description for the algorithmic or equivalently the consistent tangent operator which ensures the quadratic rate of asymptotic convergence of the global iteration. Note that the application of the continuum elastoplastic moduli yields at most a linear rate of convergence \[115, 173, 174\].

**Visco-elastic tangent operator:** The analytical solution of Eq. (3.24) under assumption of linear variation of stress during time increment (\( \Delta t = t_{n+1} - t_n \)) holds as: \[115\]

\[
\varepsilon_{i,n+1}^{ve} = \varepsilon_{i,n}^{ve} \exp \left( -\frac{\Delta t}{\tau_i} \right) + T_{n+1}^{ve} \left( \frac{\Delta t}{\tau_i} \right) C_{i,n}^{-1} \sigma_n + T_{n+1}^{ve} \left( \frac{\Delta t}{\tau_i} \right) C_{i,n+1}^{-1} : \sigma_{n+1}. \tag{A.5}
\]

The time functions \( T_{n+1}^{ve} (\xi_i) \) and \( T_n^{ve} (\xi_i) \) in Eq. (A.5), where \( \xi_i = \Delta t / \tau_i \), are described as:

\[
T_{n+1}^{ve} (\xi_i) = 1 - \frac{1}{\xi_i} (1 - \exp (-\xi_i)) , \quad T_n^{ve} (\xi_i) = 1 - \exp (-\xi_i) - T_{n+1}^{ve} (\xi_i). \tag{A.6}
\]

The differential form of Eq. (A.5) after taking the derivative from both sides reads as

\[
d\varepsilon_{i,n+1}^{ve} = T_{n+1}^{ve} \left( \frac{\Delta t}{\tau_i} \right) C_{i,n+1}^{-1} : d\sigma_{n+1}, \tag{A.7}
\]

that can be rearranged to give the general description of the algorithmic visco-elastic tangent operator \( C_{i,n+1}^{ve} \):

\[
C_{i,n+1}^{ve} = C_{i,n+1} / T_{n+1}^{ve} (\xi_i). \tag{A.8}
\]

**Mechano-sorptive tangent operator:** Based on an approach similar to the one outlined for visco-elasticity, the response of Eq. (3.28) can be stated by the following
relationship:

\[ \varepsilon_{j,n+1}^{ms} = \varepsilon_{j,n}^{ms} \exp \left( -\frac{|\Delta \omega|}{\mu_j} \right) + T_n^{ms} \left( \frac{|\Delta \omega|}{\mu_j} \right) C_j^{-1} : \sigma_n + T_{n+1}^{ms} \left( \frac{|\Delta \omega|}{\mu_j} \right) C_{j,n+1}^{-1} : \sigma_{n+1}. \]  

(A.9)

Analogous to Eq. (A.6) the moisture functions \( T_n^{ms} (\xi_j) \) and \( T_{n+1}^{ms} (\xi_j) \), where \( \xi_j = |\Delta \omega| / \mu_j \), can be given by the following expressions:

\[ T_n^{ms} (\xi_j) = 1 - \frac{1}{\xi_j} (1 - \exp (-\xi_j)), \quad T_{n+1}^{ms} (\xi_j) = 1 - \exp (-\xi_j) - T_{n+1}^{ms} (\xi_j). \]  

(A.10)

Similar to the approach taken for the visco-elastic Jacobian, differentiating both sides of Eq. (A.9) and comparing to the standard definition of the tangent operator, leads to the algorithmic mechano-sorptive tangent

\[ C_{j,n+1}^{ms} = C_{j,n+1} / T_{n+1}^{ms} (\xi_j). \]  

(A.11)

It should be noted that hygro-expansion has no contribution to the definition of the total operator. Therefore, after specifying all above-mentioned individual algorithmic operators, the mathematical description of the total Jacobian, corresponding to the entire material model \( C_{n+1}^{T} \) takes the following form:

\[ C_{n+1}^{T} = \begin{cases} 
\left( C_{n+1}^{ep} \right)^{-1} + \sum_{i=1}^{n} C_{i,n+1}^{ve} \left( C_{j,n+1}^{ms} \right)^{-1}, & \text{if } \varepsilon_{n+1}^{pl(k)} \neq 0, \\
\left( C_{n+1}^{el} \right)^{-1} + \sum_{i=1}^{n} C_{i,n+1}^{ve} \left( C_{j,n+1}^{ms} \right)^{-1}, & \text{if } \varepsilon_{n+1}^{pl(k)} = 0.
\end{cases} \]  

(A.12)
Appendix B

Material properties for MUF, PRF, PUR

Table B.1: Coefficients for calculation of moisture-dependent Young’s modulus for different adhesive systems fitted to the data published in Ref. [98]. The value of the Poisson’s ratio for all adhesive types is taken as $\nu_{adh} = 0.3$.

$$E_{adh}(\omega) = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3$$

<table>
<thead>
<tr>
<th>Adhesive Type</th>
<th>$a_0$ [MPa]</th>
<th>$a_1$ [MPa/%MC]</th>
<th>$a_2$ [MPa/%MC$^2$]</th>
<th>$a_3$ [MPa/%MC$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUF</td>
<td>5355.0</td>
<td>-604.10</td>
<td>33.270</td>
<td>-0.6805</td>
</tr>
<tr>
<td>PRF</td>
<td>4176.0</td>
<td>-176.90</td>
<td>19.380</td>
<td>-0.8521</td>
</tr>
<tr>
<td>PUR</td>
<td>1242.0</td>
<td>-158.30</td>
<td>26.250</td>
<td>-6.4430</td>
</tr>
</tbody>
</table>

Table B.2: Moisture expansion coefficients (CME) of different generic adhesive types given in Ref. [210].

<table>
<thead>
<tr>
<th>Adhesive Type</th>
<th>CME ($\beta$) [1/%MC]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUF</td>
<td>0.00198063</td>
</tr>
<tr>
<td>PRF</td>
<td>0.00172869</td>
</tr>
<tr>
<td>PUR</td>
<td>0.00171628</td>
</tr>
</tbody>
</table>
Table B.3: Normal entries of the visco-elastic compliance tensor \( J_i \) pertaining to a serial association of six Kelvin-Voigt elements for different adhesive systems identified in Refs. \[194\] and \[193\]. Similar to Table 3.5, \( \tau_i \) designates the characteristic retardation time relevant to the \( i \)th Kelvin-Voigt element.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( J_i ) [MPa(^{-1})]</th>
<th>( \tau_i ) [h]</th>
<th>( i )</th>
<th>( J_i ) [MPa(^{-1})]</th>
<th>( \tau_i ) [h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.946580e-06</td>
<td>1e-4</td>
<td>1</td>
<td>4.63471e-06</td>
<td>1e-4</td>
</tr>
<tr>
<td>2</td>
<td>0.755417e-06</td>
<td>1e-3</td>
<td>2</td>
<td>1.79861e-06</td>
<td>1e-3</td>
</tr>
<tr>
<td>3</td>
<td>1.133170e-06</td>
<td>1e-2</td>
<td>3</td>
<td>2.69802e-06</td>
<td>1e-2</td>
</tr>
<tr>
<td>4</td>
<td>1.406120e-06</td>
<td>1e-1</td>
<td>4</td>
<td>3.34791e-06</td>
<td>1e-1</td>
</tr>
<tr>
<td>5</td>
<td>2.506390e-06</td>
<td>1e0</td>
<td>5</td>
<td>6.18187e-06</td>
<td>1e0</td>
</tr>
<tr>
<td>6</td>
<td>1.006450e-06</td>
<td>1e1</td>
<td>6</td>
<td>2.39631e-06</td>
<td>1e1</td>
</tr>
</tbody>
</table>

Table B.4: Coefficients for calculation of moisture-dependent strength value and non-linear hardening stress function for PUR adhesive fitted to the data published in Ref. \[98\]. \( \omega \) and \( \alpha \) denote the MC and strain-type internal state variable, respectively.

\[
S_y(\omega) = b_0 + b_1 \omega + b_2 \omega^2 + b_3 \omega^3
\]

<table>
<thead>
<tr>
<th>( b_0 ) [MPa]</th>
<th>( b_1 ) [MPa/%MC]</th>
<th>( b_2 ) [MPa/%MC(^2)]</th>
<th>( b_3 ) [MPa/%MC(^3)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.21</td>
<td>-7.734</td>
<td>4.696</td>
<td>-0.9505</td>
</tr>
</tbody>
</table>

\[
g(\alpha, \omega) = (c_0 + c_1 \omega)\alpha^{c_2}
\]

<table>
<thead>
<tr>
<th>( c_0 ) [MPa]</th>
<th>( c_1 ) [MPa/%MC]</th>
<th>( c_2 ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.3030</td>
<td>-12.1352</td>
<td>0.5326</td>
</tr>
</tbody>
</table>

Table B.5: Parameters for calculation of moisture-dependent diffusion coefficients for different adhesive systems following Refs. \[198\] and \[202\].

\[
D(\omega) = D_0 e^{\omega \alpha_0}
\]

<table>
<thead>
<tr>
<th>( D_0 ) [mm(^2)/h]</th>
<th>( \alpha_0 ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUF</td>
<td>9.792e-04</td>
</tr>
<tr>
<td>PRF</td>
<td>4.047e-04</td>
</tr>
<tr>
<td>PUR</td>
<td>3.067e-03</td>
</tr>
</tbody>
</table>
References


[6] V. ANGST and K. A. MALO. “Moisture induced stresses perpendicular to the grain in glulam: Review and evaluation of the relative importance of models and parameters”. In: Wood Research (Holzforschung) 64.5 (2010), pp. 609–617.


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