Parallel magnetic resonance imaging: potential and limitations at high fields

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PARALLEL MAGNETIC RESONANCE IMAGING:
POTENTIAL AND LIMITATIONS AT HIGH FIELDS

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presented by

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Summary

Over the past decades Magnetic Resonance Imaging (MRI) has been established as a vital tool for routine clinical diagnostics and basic medical research. In comparison to alternative imaging modalities, MRI stands out by its non-invasiveness and its exceptional versatility. In particular, by appropriate adjustment of sequence parameters, MRI can be used either for basic morphological imaging or for collecting a variety of subsidiary information, such as about blood flow, brain and muscle fiber architecture, and even brain activation. Besides these advantages MRI also faces major limitations, most notably an intrinsically low signal-to-noise ratio (SNR) and comparatively long scan times. These restrictions have been addressed quite effectively by means of sequence optimization and advances in gradient hardware. However, the associated incremental improvements have declined in the meantime, calling for alternative approaches for enhancing the sensitivity and speed of MRI.

Two lines of technological developments have entered the scene during the past years: Firstly, parallel detection with multiple receiver coils has been explored as a means of performing multiple projections of the imaged object at once. With so-called parallel imaging methods the resulting information redundancy can be used to enable significant scan time shortening. Secondly, overcoming technical restrictions and initial skepticism, whole-body systems with very high field strengths ($B_0$) ranging up to 8 T have been successfully installed as a means of boosting the baseline SNR yield. The present thesis is dedicated to analyzing the potential of these recent approaches from a conceptual perspective. Particular emphasis is laid upon the combination of parallel imaging and high field and the promising synergies of the joint approach.

In order for parallel imaging to permit full reconstruction of an image dataset, the factor of data acquisition reduction ($R$) must be smaller than the factor of maximum data redundancy, which is given by the number of receiver coils. In the very beginning of parallel imaging it was hoped that the number of receiver coils may be the only restriction to scan acceleration. However, in the meantime it has been realized that the level of independence of the different image projections is also of major importance. It governs the conditioning of the parallel imaging reconstruction, and hence determines noise amplification in the final image. In
Summary

terms of physics, the sensitivity of a receiver array for MRI signal reception is governed by the radiofrequency (RF) fields of its coil elements. While at low B₀ and/or small object sizes (L), these RF fields are generally near-field dominated, with increasing B₀ and/or L, RF wave behavior and associated interference effects increasingly come into play. Primarily due to these additional RF wave effects the potential performance of parallel imaging changes significantly with increasing B₀.

As a basic step towards identifying general performance characteristics of parallel imaging the idealized case of a complete coil array with infinitely many elements has been considered. In this way, the ultimately achievable SNR performance for both conventional and parallel imaging has been analyzed. Most importantly, the following fundamental performance characteristics have been identified: I.) Irrespective of the specific receiver coil array, the SNR yield in MRI faces fundamental physical restrictions. In particular, for parallel imaging there exists a certain critical reduction factor (Rₖᵣᵦᵣ) beyond which SNR performance deteriorates exponentially. II.) This critical reduction factor depends on the RF wave regime. For common near-field conditions Rₖᵣᵦᵣ is constant and lies in the range between three and four. With onsetting wave behavior Rₖᵣᵦᵣ starts to increase continuously with B₀ and L. By viewing these limitations from a wave-optical perspective, it was found that in the wave regime Rₖᵣᵦᵣ reflects the fundamental optical resolution barrier as established by Abbe and Rayleigh. The aliasing distance that can be readily resolved by parallel imaging is limited to half the wavelength of the coil's RF fields, thus determining Rₖᵣᵦᵣ. This is a consequence of the fact that, like in optics, in parallel imaging spatial encoding is achieved by exploiting spatial variation in the individual coils' signal sensitivities. In order to investigate whether or not the ultimate SNR limits can actually be reached, the SNR yield has also been studied for finite coil arrays. Doing so, it was found that with increasing number of coil elements the ultimate SNR can indeed be closely approximated with an imaging object of typical dimensions. However, the number of coil elements required varies significantly with the spatial location and the sort of RF wave behavior encountered. While for deep parts of the object and near-field conditions the ultimate SNR can be approached with relatively few coils, for superficial regions and wave behavior considerably more coils are required.

The theoretically predicted increase of parallel imaging performance with increasing B₀ is expected to add considerably to the potential of high field MRI. In order to validate this promising result, the B₀ dependence of parallel imaging has also been investigated experimentally. For studying the B₀-dependent behavior of the RF fields the framework of electrodynamic scaling has been developed. Based on adjusting electrodynamic object properties, this concept permits mimicking the effect of varying B₀ without changing the scanner hardware. Using electrodynamic scaling, parallel imaging performance was investigated for B₀ in the range
between 1 T and 12 T using only a single magnet with a $B_0$ of 7 T. The results of these experiments were in excellent agreement with the previous numerical findings, hence confirming the theoretical analysis. With respect to the combination of high field strengths and parallel imaging an often voiced concern regards the potentially adverse effects of dielectric resonance. In the presence of dielectric resonance it may be expected that the RF fields of the receiver coils become predominantly determined by the object and hence less independent. As a result, dielectric resonance could drastically degrade parallel imaging performance. For testing this hypothesis, electrodynamic scaling has been applied to deliberately generate dielectric resonance at the relevant NMR frequency. With a lossless object the described detrimental effect was indeed observed. However, by introducing conductivity similar to that of human tissues the performance of parallel imaging was found to recover to regular values. From these findings it is concluded that for typical in-vivo conditions dielectric resonance is no major obstacle to parallel imaging.

When considering the individual characteristics of parallel detection and high-field effects it turns out that they are highly complementary in numerous regards. Jointly with the aforementioned intrinsic improvement of parallel imaging performance with increasing $B_0$, this complementarity constitutes a unique degree of synergy between the two approaches. While it has been shown that there are no fundamental obstacles to the combination, several technical challenges remain. In particular, the coil array concepts commonly used at lower $B_0$ are not directly transferable to very high fields. Instead, so-called transmit/receive microstrip arrays have been devised to meet the specific high-field requirements and issues. In combination with an extended sensitivity calibration scheme the practical feasibility of parallel SENSE imaging was successfully demonstrated at a field strength as high as 7 T.
Zusammenfassung


Eine grundlegende Voraussetzung für parallele Bildgebung besteht darin, dass die Verminderung der Datenakquisition, beschrieben durch den sogenannten Reduktionsfaktor, geringer sein muss als die Anzahl vorhandener Empfangsspalten. Diese Bedingung drückt aus, dass die Akquisitionsredundanz des Spulenarrays ausrei-
Zusammenfassung

... muss, um die ausgesparten Akquisitionsschritte prinzipiell rekonstruieren zu können. Während ursprünglich angenommen wurde, dass die Anzahl Spulen die einzige Limitation für parallele Bildgebung ist, hat man mittlerweile erkannt, dass auch die gegenseitige Orthogonalität der Einzelspulenbilder von zentraler Bedeutung ist. Schlussendlich bestimmt diese Orthogonalität die Konditionierung der Bildrekonstruktion und quantifiziert so die Rauschverstärkung während dem Rekonstruktionsschritt. Physikalisch ist die Ausleuchtung der einzelnen Empfangsspulen bestimmt durch die elektromagnetischen Radiofrequenz (RF) Felder im Sendebetrieb. Während für niedrige B₀ und/oder kleine Objektabmessungen (L) typischerweise Nahfeldanteile überwiegen, tritt für große Werte von B₀ und/oder L zunehmend Wellenverhalten mit entsprechenden Interferenzerscheinungen auf. Es ist dieser Wechsel im RF Verhalten welcher zu Änderungen von paralleler Bildgebungsperformance mit zunehmender Feldstärke führt.


Chapter 1

Introduction

In the course of the past century numerous technological breakthroughs have resulted in an abundance of non-invasive medical imaging modalities. While the various techniques differ in many respects, the vast majority of them have in common that they are based on the following general prerequisites:

1. In order for the object to be imaged it has to either emit, absorb, or reflect some sort of detectable signal. As the object is usually not active by itself, this is typically achieved via irradiation or excitation by an outside source.

2. Different structures need to vary with respect to their emission, absorption or reflection characteristics in order to permit some sort of contrast behavior.

3. For the purpose of spatial signal encoding the detected signals must carry information specific to their spatial origins.

In addition, when considering the attenuation of electromagnetic and ultrasound radiation by human tissues, as schematically indicated in Fig. 1.1, nature provides us with basically three main windows, which permit us to look inside the human body: The X-ray window was first exploited for basic human imaging by Röntgen in 1895 and has revolutionized medical diagnostics since then. Despite involving non-negligible radiation doses, X-ray and X-ray computed tomography (also known as CT) continue to play a key role in medical diagnostics. The second window is that for low-frequency ultrasonic radiation which is taken advantage of in ultrasound imaging. Finally the radiofrequency (RF) window is used by Magnetic Resonance Imaging (MRI). In the following this method will be introduced with respect to the three aforementioned requirements:

Ad 1: In MRI the object under investigation is activated via Nuclear Magnetic Resonance (NMR) of $^1$H nuclei. To this end, the object is brought into a strong static magnetic field ($B_0$), typically in the range of several Tesla. As
Introduction

Electromagnetic radiation

X-ray UV IR MW Radio frequency

1 Å 100 Å 1 μm 100 μm 1 cm 1 m 100 m

Ultrasound

1 μm 100 μm 1 cm 1 m 100 m

Figure 1.1: Scheme of the absorption spectrum for human tissue by electromagnetic and acoustic radiation. All electromagnetic radiation is strongly absorbed except in the X-ray and radiofrequency ranges. Acoustic radiation is strongly absorbed for wavelengths below 1 mm (adapted from reference 1).

a consequence of the electromagnetic coupling between $B_0$ and the nuclear magnetic moments the energy state of the hydrogen nuclei splits up into two levels with opposite spin orientation. At room temperature the populations of the two nuclear spin states differ slightly, accordingly the whole subject gets slightly polarized. Nuclear magnetic resonance is caused by irradiating electromagnetic fields at a certain radiofrequency (RF), the Larmor frequency, that matches the energy difference between the two spin states. Under this condition, the nuclear magnetic moments absorb RF energy and partly change their orientation with respect to $B_0$, hence shifting the relative population of the two spin states. This process is referred to as spin excitation. Most importantly, it creates coherence of the transverse spin components among the spin ensemble. This coherence at the quantum level manifests itself as macroscopic transverse magnetization that precesses at the Larmor frequency. After switching off the RF excitation, the nuclear magnetic moments gradually return to their equilibrium statistics. During this relaxation process, the precessing magnetization generates oscillating electromagnetic fields. Based on Faraday induction, these fields are then detected using an appropriate RF receiver coil apparatus.

Ad 2: The diagnostic value of NMR is based on the fact that tissues and structures in live organisms vary with respect to their $^1H$ proton densities and relaxation times. In MRI these variations are utilized in order to variably adjust
the image contrast between different tissues. In addition to these basic dependencies, the NMR effect is also susceptible to a number of physical interactions and biological changes. By appropriately tuning the NMR sequence such as to enhance these effects, a variety of subsidiary information, e.g., about blood flow, brain and muscle fiber architecture, and even brain activation, becomes available. This ability of flexibly adjusting the image contrast for specific purposes is among the most important advantages of MRI.

As an example Fig. 1.2 demonstrates the superior image quality and contrast achievable with MRI by comparing a photographed, dyed anatomical cross section of a post-mortem human brain (left) with an in-vivo MRI measurement of a similar brain region (right).

Ad 3: The method that extends $^1$H NMR toward an imaging modality was introduced by Paul C. Lauterbur in 1972.\textsuperscript{5} Termed “induced local interaction” in the original work, Lauterbur’s invention is today mostly referred to as gradient or frequency encoding. According to the Larmor relation, the frequency of the emitted RF signal is proportional to the magnetic field strength at the position of the precessing magnetization. Hence, the signal’s origin in space can be actively encoded in its phase and frequency by superimposing magnetic fields that vary in space. By sequential MR excitation and subsequent application of these so-
called gradient fields along different directions, multiple projections of the object are observed. Using mathematical methods, these projections are then suitably combined to reconstruct an image of the magnetization distribution.

In the most common case of frequency encoding the gradient fields are switched in such a way that the acquired data correspond to the Fourier representation of the imaging object sampled along a Cartesian grid. In the context of MRI, the Fourier domain of data acquisition is commonly referred to as k-space. Accordingly, MR image reconstruction can simply be achieved by inverse Fourier transforming the k-space data. Fig. 1.3 illustrates this k-space concept for the simplified case of using a homogeneous volume coil for both spin excitation and signal acquisition. According to the general properties of the discrete Fourier transformation, resolution and field-of-view (FOV) of the reconstructed image are determined by the extent and density of k-space sampling, respectively.

Figure 1.4 shows a sketch of a modern MRI scanner and thereby explains the basic functioning of such an instrument. The main magnetic field coil (b.), generating a static $B_0$ in the order of one Tesla, is used for spin polarization. During an MRI measurement the gradient coils (c.) are flexibly switched for superimposing a linearly dependent gradient field. In this fashion the spatial origin of each signal contribution is actively encoded in its phase and frequency. Finally, RF coils are used both for spin excitation and signal reception. For this purpose either a system-integrated volume coil (g.), or an anatomically tailored coil array can be utilized.
MRI at High Field Strength (> 2 Tesla)

In general, nuclear magnetic resonance is a rather weak phenomenon: At room temperature and for common $B_0$ of 1.5 Tesla only $\sim 0.0005\%$ of the available nuclear $^1\text{H}$ spins do actually contribute to the total signal. Only the tremendous abundance of $\text{H}_2\text{O}$ in the human body on the order of $\sim 50 \text{ mol/l}$ enables signal-to-noise ratios (SNR) high enough for medical imaging. Nevertheless, low intrinsic SNR is a key limitation of MRI. As shown by Hoult and Lauterbur, the SNR increases approximately linearly with increasing $B_0$. This promise is, and ever has been, the major driving force for the development of costly superconducting high-field MRI systems. Within approximately the past decade both theoretical studies and technological advances have established the feasibility of in vivo MRI in humans at field strengths up to 8 Tesla. Besides increasing the SNR yield, high fields also incur a range of serious problems, such as increased $B_0$ inhomogeneity, reduced $T_2$ and $T_2^*$ relaxation times, increased RF energy absorption and higher acoustic noise levels. Each of these problems by itself forms a major challenge for MRI at high field.
Table 1.1: Radiofrequency wavelength $\lambda$ in human brain tissue as a function of the main magnetic field strength ($B_0$) and the corresponding NMR Larmor frequency ($\nu$) (cf. Fig. 1.1).

<table>
<thead>
<tr>
<th>$B_0$ [T]</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$ [MHz]</td>
<td>42.6</td>
<td>127.7</td>
<td>212.9</td>
<td>298.1</td>
<td>383.2</td>
<td>468.4</td>
</tr>
<tr>
<td>$\lambda$ [cm]</td>
<td>58.8</td>
<td>26.8</td>
<td>17.8</td>
<td>13.3</td>
<td>10.7</td>
<td>8.9</td>
</tr>
</tbody>
</table>

Furthermore, increasing $B_0$ also changes the reception characteristics of the RF detector coils and, equivalently, the fields that these coils generate in the transmission mode. These fields essentially determine the sensitivities of the individual coils for NMR excitation and signal reception. For common field strengths, the RF wavelength $\lambda$ is large compared to typical object dimensions (cf. Table 1.1). Accordingly MRI signal detection is generally near-field dominated and the coils’ RF fields are approximately independent of the object (i.e. to a good approximation they can be calculated using quasi-static Biot-Savart integration). Conversely, with increasing $B_0$, $\lambda$ becomes similarly large as and even smaller than the object. Under these conditions RF wave behavior gradually comes into play and the coils’ RF fields become dependent on the object’s shape and dielectric properties. As a consequence, at high field strengths image homogeneity generally degrades, resulting in partly severe shading artifacts. In addition to increased RF energy deposition, these effects form a key difficulty at high field, with major implications for both MRI sequence layout and RF coil design.

Parallel Imaging (Coil Sensitivity Encoding)

Most medical imaging modalities by far achieve spatial signal encoding by wave-optical principles such as focusing and collimation. In contrast, conventional MRI relies exclusively on the fundamentally different frequency encoding approach. This is a consequence of the fact that the desired image resolution in MRI is far beyond the diffraction limit of wave optics on the order of half a wavelength (cf. Figure 1.1 and Table 1.1). Frequency encoding with gradient coils circumvents this fundamental restriction and enables MR image resolutions in the sub-millimeter range. However, in contrast to wave-optical concepts, frequency encoding is inherently a sequential method, requiring multiple encoding operations for obtaining a complete dataset. The sequential nature of data acquisition translates into long scan times, which form one of the major downsides of MRI.

In 1990 Roemer et al introduced the phased array coil concept, where the imaging region is covered with multiple, local receiver coils. In combination with
appropriate image combination schemes, array acquisition combines the high SNR efficiency of local surface coils with the large object coverage of global volume coils. In addition, the parallel nature of array detection leads to some degree of information redundancy in the acquired data. So-called parallel imaging methods\textsuperscript{19-25} exploit this redundancy in favor of scan acceleration. With parallel techniques the spatially varying reception sensitivities of the individual coils are utilized as a complementary image encoding mechanism, permitting significant reduction of gradient encoding.

Figure 1.5 illustrates the concept of parallel imaging for Cartesian k-space sampling with four coil elements equally distributed around a human head. The Cartesian grid on the left shows the intended full-FOV k-space coverage, whereas the filled circles indicate the actually sampled k-space locations. The two-fold undersampling along the horizontal direction translates into a measurement time reduction by a factor of two, but also leads to two-fold aliased individual coil images. However, provided the signal reception profiles for the individual coils have been determined based on appropriate coil calibration methods, the four two-fold aliased single coil images can be combined into one homogeneous unaliased image.

In addition to shortening the scan time, parallel imaging methods also translate into a variety of alternative advantages, such as improved image resolution, larger object coverage, reduced artifacts due to $B_0$ inhomogeneity and motion, reduced RF energy absorption, and even reduced acoustic noise levels. With several implementations available, such as SMASH,\textsuperscript{23} SENSE,\textsuperscript{24} or GRAPPA,\textsuperscript{25} parallel imaging is increasingly introduced into clinical environments.\textsuperscript{26, 27}

**Potential and Limitations of Parallel Imaging at High Field**

With the ongoing development of parallel imaging and the advent of dedicated coil arrays one of the key questions that arise concerns the inherent limitations of the parallel approach. In terms of mathematics, the degree of encoding reduction is bound by the amount of information redundancy provided by the receiver coil array. At first sight this redundancy could be expected to scale with the number of independent coil elements used simultaneously for signal reception. Accordingly, at the beginning of parallel imaging it was hoped that sufficiently large arrays might permit arbitrarily high reduction factors. However, as pointed out in reference\textsuperscript{24} it is rather the conditioning of the image reconstruction problem than the number of coil elements that limits parallel imaging performance. Bad conditioning causes noise amplification during image reconstruction, which can critically degrade the quality of the resulting image.

This is illustrated in Figure 1.6, which shows parallel imaging results for different amounts of k-space undersampling. With increasing undersampling the degree
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Signal reception using four independent, inhomogeneous receiver coils

**Figure 1.5:** Illustration of parallel imaging for an array with four coils and Cartesian k-space acquisition with an undersampling factor of two.

of aliasing increases and the conditioning of the unfolding reconstruction degrades, which in turn translates into increased local noise amplification. For high k-space undersampling factors this results in severe image degradation.

Generally, the conditioning of parallel imaging reconstruction is determined by the level of independence of the individual coil sensitivities. Therefore the potential performance of parallel imaging is fundamentally governed by the RF electrodynamics of the receiver coil array and the object to be imaged. This observation directly leads to the important question what happens when the RF field characteristics change, for instance as a result of increasing $B_0$. According to the aforementioned considerations, increasing $B_0$ reduces the RF wavelength and the receiver coils’ sensitivities vary more rapidly in space. Broadly speaking, this trend renders the individual coil sensitivities more independent, hence adding to parallel imaging performance. Conversely, if the NMR frequency coincides with one of the object’s natural dielectric eigenfrequencies, dielectric resonance occurs. In this case, the RF electrodynamical fields may become predominantly determined by the object and accordingly parallel imaging performance might degrade.
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Figure 1.6: Illustration of local noise amplification in parallel imaging. With increasing amount of k-space undersampling noise amplification significantly increases, leading to severe image degradation.

Outline

The aim of this thesis was to study fundamental performance characteristics of parallel imaging, high field strengths, and their combination. As a first step, this task is approached from a theoretical perspective by jointly exploring parallel imaging and high field strengths from an RF electrodynamics perspective. This yields a conceptual understanding of the underlying physics as well as valuable guidelines for practical RF coil array design. In a second step the key findings of the theoretical study are verified experimentally. For all performed investigations the model configurations have been kept simple in order to limit the number of relevant parameters and to simplify the physical interpretation. In addition to these conceptual investigations practical methods are presented for applying the parallel SENSE approach at the ultra-high field strength of 7 Tesla.

The thesis builds upon journal publications. Its sub-chapters correspond to scientific publications that either have already been published in peer-reviewed journals or are currently in publishing process. The content of this work is struc-
tured as follows:

In chapter 2, entitled *Theoretical Investigations*, the physical limitations of the SNR performance in conventional and parallel imaging are explored based on the concept of ultimate SNR.\(^{28}\) In this context, the crucial role of RF electrodynamics on parallel imaging performance is studied in detail. Subsequently, the limits of parallel imaging are reviewed with respect to the fundamental resolution barrier of wave optics. Furthermore it is explored whether and to which extent ultimate SNR limitations can actually be approached by means of finite coil arrays.

In chapter 3, entitled *Experimental Investigations*, the \(B_0\) dependence of parallel imaging performance is investigated experimentally. For this purpose, an electrodynamic scaling approach is proposed, which allows flexibly mimicking the effect of different field strengths by adjusting dielectric object properties. In addition, the impact of dielectric resonance on parallel imaging is studied by electrodynamic scaling experiments.

Chapter 4, entitled *Feasibility Aspects at Ultra-High Fields*, summarizes the potential and limitations of parallel imaging at high field strength. Using a novel coil array design in conjunction with an extended sensitivity calibration scheme, the practical feasibility of parallel SENSE imaging at 7 Tesla is demonstrated.
Chapter 2

Theoretical Investigations

2.1 Electrodynamics and Ultimate SNR in Parallel MR Imaging

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Abstract

The purpose of this paper is to elucidate inherent limitations to the performance of parallel MR imaging. The study focuses on the ultimate signal-to-noise ratio (SNR), which refers to the maximum SNR permitted by the electrodynamics of the signal detection process. Using a spherical model object, it is shown that the behavior of the ultimate SNR imposes distinct limits upon the acceleration rate in parallel imaging. For low and moderate acceleration, the ultimate SNR performance is nearly optimal with geometry factors close to 1. However, for high reduction factors beyond a critical value, the ultimate performance deteriorates rapidly, corresponding to exponential growth of the geometry factor. The transition from optimal to deteriorating performance depends on the electrodynamic characteristics of the detected radiofrequency fields. In the near-field regime, i.e. for low B_0 and small object size, the critical reduction factor is constant and approximately equal to four for 1D acceleration in the sphere. In the far-field wave regime the critical reduction factor is larger and increases both with B_0 and the object size. Therefore it is concluded that parallel techniques hold particular promise for human MR imaging at very high field.

Introduction

In recent years, parallel imaging with receiver coil arrays has evolved into a practical and widely used variant of MRI. Exploiting the intrinsic encoding effect of coil sensitivity as a complement to gradient encoding, parallel acquisition permits reducing the density of k-space sampling. This advantage can be utilized to enhance MRI exams in manifold ways, such as for reducing scan time, improving resolution and coverage, suppressing artifacts, lowering RF power deposition, and even mitigating acoustic noise.

The main limiting factor in parallel imaging is the signal-to-noise ratio (SNR) of the resulting image data. For instance, for the parallel Sensitivity Encoding (SENSE) technique, the local SNR yield has been quantified as:

\[
\text{SNR}^{\text{PI}} = \frac{\text{SNR}^{\text{full}}}{\sqrt{R}g},
\]

where SNR^{PI} denotes the SNR obtained with parallel imaging and SNR^{full} denotes the SNR that would be obtained with conventional, full gradient encoding, using the same sequence and the same coil array. According to this relation, the SNR yield of parallel imaging is subject to two types of losses. The factor \(\sqrt{R}\) accounts for reduced intrinsic signal averaging when the k-space density is reduced by the factor R. The second loss term, g, is the local so-called geometry factor, which reflects the ability of the coil array to complement the reduced gradient encoding.
This ability depends crucially on the number and geometrical configuration of the array elements.

As a consequence, the SNR yield of parallel imaging can be significantly improved by coil optimization. Typically, dedicated coil designs yield favourable $g$ values up to reduction factors between three and four in one dimension. However, beyond this threshold, dramatic increases in $g$ are observed even with optimized arrays. These findings suggest that the reduction factor in parallel imaging is limited not only by array imperfection but also in a further, more fundamental fashion. This hypothesis has been the subject of several recent investigations.

The present work builds upon reference. Its goal is to establish the inherent performance limits of parallel imaging and to put them into the perspective of the underlying physics. To this end, the concept of ultimate intrinsic SNR is adopted and extended towards parallel acquisition. With this approach, the highest achievable relative SNR is determined for parallel imaging with varying $B_0$, object size, and reduction factor $R$. The resulting SNR data and the corresponding geometry factors reveal performance limits with a characteristic structure, which directly corresponds to the electrodynamics of MR signal detection.

Theory

In this section, the problem of determining the ultimate intrinsic SNR in parallel MRI is cast into the form of a constrained linear minimization problem. First, the signal and noise amplitudes detected with an MR receiver coil are expressed in terms of the coil's hypothetical transmit fields, applying the principle of reciprocity. In the following part, the impact of parallel imaging reconstruction on the image SNR is incorporated using the concept of net sensitivity, which translates into a constraint on SNR optimization. A second constraint is then imposed according to the underlying physics, which require that the signal and noise sensitivity functions fulfill the Maxwell equations. The Maxwell constraint is incorporated by expanding the sensitivity functions in terms of a basis of Maxwell solutions. This expansion then permits expressing and solving the optimization problem with matrix notation, as described in the last part of the section.

Signal and Noise Sensitivity

The basic determinants of the SNR in MRI are the signal voltage amplitude, $v_s$, and the root-mean-square (RMS) noise voltage, $v_N$, at the output of the detector. As the signal source we consider the nuclear magnetic moment $M_0$ at the position $r_0$, precessing about the $z$-axis with the Larmor frequency $\omega$. Assuming steady-state oscillation and Faraday detection with a receiver coil, the principle of
Figure 2.1: Spatial response and net sensitivity in Cartesian parallel imaging with $R=3$. Rows 1-4 illustrate signal aliasing in a central pixel for the four different elements of a receiver coil array. The left column shows each coil’s spatial sensitivity in the image plane. The sensitivities introduce a spatial weighting in the multiple peaks of the individual spatial response functions (SRF, middle column). Correspondingly, the aliasing contributions in the single coil images are individually weighted (right column). Parallel imaging reconstruction of the central pixel amounts to a linear combination ($\Sigma$) of the aliased pixel values. The same linear combination can equivalently be performed with the coil sensitivities, yielding the net sensitivity for reconstructing this pixel. The net sensitivity has zeros where aliasing contributions need to be suppressed. Consequently, the net SRF exhibits only the desired single peak. In the context of ultimate SNR, the net sensitivity is of special significance because it is subject to the same electrodynamic limitations as actual coil sensitivities.
reciprocity\textsuperscript{37–39} yields the signal voltage:
\[ v_s = M_0 \mu(r_0) \omega |H_x(r_0) - iH_y(r_0)|, \] (2.2)
where \( \mu \) denotes the local magnetic permeability and \( i \) the imaginary unit. \( H_x, H_y \) denote the complex amplitudes of the x- and y-components of the radiofrequency (RF) magnetic field that the coil generates when operated with unit input current of frequency \( \omega \).

Among the various mechanisms that underlie noise in MR detection we consider only the component caused by thermal agitation of charged particles in the object,\textsuperscript{40–42} following the reasoning previously given in reference.\textsuperscript{28} In the reciprocal view, this type of noise corresponds to losses associated with currents induced in the object. Other mechanisms, such as radiation loss or ohmic losses in the coil conductor, are neglected in the following as they can be diminished in principle by enhancing the coil technology. For a given receiver bandwidth \( BW \), the RMS noise voltage is then given by:\textsuperscript{42}
\[ v_N = \sqrt{4BWk_BT \int_{\text{object}} \sigma(r) |E(r)|^2 d^3r} \] (2.3)
where \( k_B \) denotes the Boltzmann constant, \( T \) the absolute temperature, \( \sigma \) the conductivity of the sample, and \( E(r) \) the complex amplitude vector of the RF electric field generated by the receiver coil when driven with unit input current of frequency \( \omega \).

The Eqs. (2.2,2.3) describe the reception behavior of a given detector coil in terms of its hypothetical transmit fields. The transverse components of the magnetic transmit field \( \mathbf{H}(r) \) characterize MR signal reception from the position \( r \). The expression \( H_x(r) - iH_y(r) \) is therefore referred to as the coil’s signal sensitivity in the scope of the present work. Similarly, the electric transmit field, \( \mathbf{E}(r) \), is viewed as the coil’s noise sensitivity.

Image Formation and Spatial Response

In each sample of the induced voltage, the resonance signal from \( r_0 \) is entangled with signal contributions from the entire object, requiring spatial encoding and a reconstruction step for the purpose of imaging. For parallel imaging, the spatial encoding is performed by combining gradient switching with the encoding effect of multiple receiver coil sensitivities. In the scope of the present work, acquisition and reconstruction are modeled according to Cartesian SENSE with weak reconstruction, as described in reference.\textsuperscript{24} According to this method, gradient encoding is assumed to cover a Cartesian k-space grid with reduced density relative to the nominal FOV.
The first step in this type of SENSE reconstruction is a discrete Fourier transform for each coil element, yielding single-coil images with reduced FOV and aliasing artifact. The aliasing effect can be described comprehensively by the so-called spatial response function (SRF), which reflects the spatial weighting of signal contributions in a given pixel value. In each of the single-coil images, the spatial response depends on the coil’s signal sensitivity and the point-spread function (PSF) of Fourier encoding and reconstruction. For the coil element with index $\gamma$ and a pixel at the position $\mathbf{r}_0$, the SRF reads:

$$SRF^\gamma(\mathbf{r}) = (H^\gamma_x(\mathbf{r}) - iH^\gamma_y(\mathbf{r})) PSF_{\text{Fourier}}(\mathbf{r}_0 - \mathbf{r}).$$ \hspace{1cm} (2.4)

The concept of spatial response is illustrated in Fig. 2.1, showing SRFs of a central pixel for four coils, assuming k-space undersampling in one dimension by a factor of $R=3$. According to Cartesian scanning, the PSF is formed by a series of sinc peaks, with a spacing of FOV/$R$. Due to three-fold undersampling, the PSF exhibits three main peaks within the object. One of these peaks is centered at the position $\mathbf{r}_0$, reflecting the desired response, while the others reflect aliasing contributions. Weighting by each coil’s signal sensitivity according to Eq. (2.4) results in individually varying peak heights of the SRF. Hence, each pixel in the aliased single-coil images represents the sum of three individually weighted contributions. Note that signal sensitivity, SRF, and single coil images are complex-valued, Fig. 2.1 showing only moduli. Note also that the same view holds for undersampling in multiple k-space dimensions, where the aliasing peaks form a higher-dimensional grid.\textsuperscript{43}

**Net Coils**

For any given $\mathbf{r}_0$ and undersampling scheme, let the aliasing positions be given by $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N$ in arbitrary order. In each single-coil image, the signal from these positions is entangled in a single aliased pixel. In SENSE reconstruction, an unaltered pixel value for each of the involved positions is obtained by suitable linear combination of the aliased single-coil pixel values. The spatial response of the reconstructed pixel is obtained by likewise linearly combining the corresponding single coil SRFs. For the pixel with index $\rho = 0 \ldots N$, the resulting net SRF reads:

$$SRF^{\text{net},\rho}(\mathbf{r}) = \sum_\gamma c_{\rho,\gamma} SRF^\gamma(\mathbf{r}),$$ \hspace{1cm} (2.5)

where $c_{\rho,\gamma}$ denotes the coefficient used for weighting the coil $\gamma$ in SENSE reconstruction. Inserting Eq. (2.4) and defining the net magnetic transmit field:

$$H^{\text{net},\rho}(\mathbf{r}) = \sum_\gamma c_{\rho,\gamma} H^\gamma(\mathbf{r}),$$ \hspace{1cm} (2.6)
one obtains:

\[ SRF_{\text{net}, \rho}^{\rho}(\mathbf{r}) = (H_{x}^{\text{net}, \rho}(\mathbf{r}) - iH_{y}^{\text{net}, \rho}(\mathbf{r}))PSF_{\text{Fourier}}(\mathbf{r}_{\rho} - \mathbf{r}). \] (2.7)

In the net SRFs, unaliasing is reflected by the suppression of aliasing PSF peaks. Weak SENSE reconstruction ensures that the net signal sensitivity is equal to one at the reconstructed position \( \mathbf{r}_{\rho} \), and equal to zero at the aliasing positions (see Fig. 2.1). Formally, this is expressed as:

\[ H_{x}^{\text{net}, \rho}(\mathbf{r}_{\rho'}) - iH_{y}^{\text{net}, \rho}(\mathbf{r}_{\rho'}) = \delta_{\rho, \rho'} \quad \text{for } \rho, \rho' = 0 \ldots N, \] (2.8)

where \( \delta \) denotes the Kronecker delta.

The strength of the concept of net sensitivity is that it casts the problem of quantifying noise propagation in SENSE reconstruction into the realm of electrodynamics. In a thought experiment, replace the coil array by a single net coil, which is obtained by linear combination of the original array elements, using the coefficients \( c_{\rho, \gamma} \). Due to the linearity of electrodynamics, such a coil will have the magnetic transmit field given in Eq. (2.6) and the electric transmit field:

\[ \mathbf{E}^{\text{net}, \rho}(\mathbf{r}) = \sum_{\gamma} c_{\rho, \gamma} \mathbf{E}^{\gamma}(\mathbf{r}). \] (2.9)

The net coil’s signal sensitivity conforms to Eq. (2.8), hence mere Fourier transform from net coil data will yield a non-aliased pixel value for \( \mathbf{r}_{\rho} \), despite under-sampling. Furthermore, since Fourier transform is likewise linear, this pixel value will have the exact same signal and noise levels as would have been obtained with the original procedure. Hence, for assessing the maximum SNR achievable with SENSE imaging, it is sufficient to identify the net transmit fields that maximize the SNR in standard Fourier reconstruction, while fulfilling the condition (2.8).

**Signal-to-Noise Ratio**

Combining Eqs. (2.2,2,3,2.8) for the net coil and dividing the resulting signal and noise levels yields:

\[ SNR(\mathbf{r}_{\rho}) = \frac{M_{0}\mu(\mathbf{r}_{\rho})\omega n_{K}}{\sqrt{4BWk_{B}Tn_{K} \int_{\text{object}} \sigma(\mathbf{r}) |\mathbf{E}^{\text{net}, \rho}(\mathbf{r})|^{2} d^{3}\mathbf{r}}}. \] (2.10)

Here, the number of k-space samples, \( n_{K} \), is included to account for the signal summation performed by the Fourier transform. When varying the object size, it shall be assumed that the FOV and the slice thickness scale along with the object. Hence, the relevant magnetic moment \( M_{0} \) in Eq. (2.10) is proportional to FOV\(^3\).
Furthermore, both $M_0$ and the Larmor frequency $\omega$ are proportional to $B_0$ under typical in vivo MRI conditions. The scaling factors are neglected in the following, along with further terms which are held constant in the scope of this work. This leads to the relative SNR:

$$
\zeta(r_0) = \frac{\mu(r_0)B_0^2 FOV^3 \sqrt{\eta K}}{\sqrt{\int_{\text{object}} \sigma(r) |E_{\text{net},\rho}(r)|^2 d^3r}},
$$

(2.11)

which is an accurate comparative SNR measure for varying $B_0$, object size, transmit fields, material properties, and SENSE factors. Note that the signal and noise sensitivities are implicit functions of the Larmor frequency and the material properties. The latter are themselves frequency dependent, thus adding multiple implicit dependencies on $B_0$.

According to the previous considerations, ultimate local SNR in SENSE imaging is characterized by a maximum of $\zeta(r)$ under the unfolding constraint (2.8). For a given object, $B_0$, and k-space grid, maximizing $\zeta(r)$ is hence equivalent to the constrained minimization of the denominator in Eq. (2.11):

$$
\int_{\text{object}} \sigma(r) |E_{\text{net},\rho}(r)|^2 d^3r \rightarrow \min.
$$

(2.12)

Maxwell Equations

Without additional constraints, the noise sensitivity could simply be set to zero, yielding infinite SNR. However, this is not an option because signal and noise sensitivity are coupled by the laws of electrodynamics. In order to simplify the latter, two assumptions are made in the following. First, the object to be imaged is assumed to have scalar, homogeneous dielectric constant $\epsilon$, magnetic permeability $\mu$, and conductivity $\sigma$. Second, all receiver coils are assumed to be placed outside the object, which holds for most diagnostic MRI procedures with the exception of interventional or endocavitary studies. Under these conditions, the signal and noise sensitivities of each coil are governed by the source-free Maxwell equations:\textsuperscript{44}

$$
\nabla \cdot \mathbf{E}(r,t) = 0, \quad \nabla \times \mathbf{E}(r,t) = -\mu \frac{\partial \mathbf{H}(r,t)}{\partial t},
$$

$$
\nabla \cdot \mathbf{H}(r,t) = 0, \quad \nabla \times \mathbf{H}(r,t) = \epsilon \frac{\partial \mathbf{E}(r,t)}{\partial t} + \sigma \mathbf{E}(r,t),
$$

(2.13)

where $\nabla$ denotes the gradient operator. Assuming steady-state oscillation at the Larmor frequency with the time dependency $e^{-i\omega t}$, the Maxwell equations permit separating spatial and temporal evolution. For $\omega \neq 0$, the spatial dependence is governed by the reduced, equivalent set of equations:\textsuperscript{44}

$$
(\Delta + k_0^2)\mathbf{H}(r) = 0,
$$

(2.14)
2.1 Electrodynamics and Ultimate SNR in Parallel MRI

\[ \nabla \cdot \mathbf{H}(r) = 0, \quad (2.15) \]
\[ \mathbf{E}(r) = \frac{1}{\sigma - i\omega \epsilon} \nabla \times \mathbf{H}(r), \quad (2.16) \]

where \( \triangle \) denotes the Laplace operator and \( k_0 \) the complex wave number given by:

\[ k_0^2 = \omega \mu (\omega \epsilon + i\sigma). \quad (2.17) \]

As such, these conditions form an awkward constraint in SNR optimization. However, note that the Maxwell equations are linear and homogeneous in this form. Hence, their solutions form a linear space and each linear combination of solutions is again a solution. Therefore, the constraints (2.14–2.16) translate directly to the hypothetical net coil, which is a linear combination of actual coils. Furthermore, these constraints can be enforced by expanding the net coil in terms of a complete basis of the solution space:

\[ \mathbf{E}^{\text{net},\rho}(r) = \sum_m w_{\rho,m} \alpha_m(r), \quad \mathbf{H}^{\text{net},\rho}(r) = \sum_m w_{\rho,m} \beta_m(r), \quad (2.18) \]

where \( \alpha_m(r), \beta_m(r) \) are electric and magnetic vector basis functions, counted by the index \( m \), and \( w_{\rho,m} \) are complex weighting coefficients. Using the expansion (2.18), the problem of fulfilling the Maxwell constraints is transformed into the unconstrained choice of the weighting coefficients \( w_{\rho,m} \).

Matrix Description and Solution

Based on the expansion (2.18), the remaining constraint (2.8) is restated as:

\[ \sum_m w_{\rho,m} (\beta_{m,x}(r_{\rho'}) - i\beta_{m,y}(r_{\rho'})) = \delta_{\rho,\rho'}. \quad (2.19) \]

Assembling the \( w_{\rho,m} \) in the weighting matrix \( W \) and introducing the signal sensitivity matrix \( S \):

\[ S_{m,\rho} = \beta_{m,x}(r_{\rho}) - i\beta_{m,y}(r_{\rho}), \quad (2.20) \]

the constraining Eq. (2.8) can be expressed more compactly as:

\[ WS = I_{d}, \quad (2.21) \]

where \( I_{d} \) denotes the \((N+1)\times(N+1)\) identity matrix. Similarly, the functional to be minimized (Eq. (2.12)) can be expressed in matrix notation. Defining the noise covariance matrix \( \Psi \) as:

\[ \Psi_{m,m'} = \sigma \int_{\text{object}} \alpha_m(r) \cdot \alpha^{*}_{m'}(r) d^3r, \quad (2.22) \]
the functional reads:

$$\int_{\text{object}} \sigma |E_{\text{net},\rho}(r)|^2 \, d^3r = (W\Psi W^H)_{\rho,\rho}.$$  \hfill (2.23)

where the superscript $H$ denotes the complex conjugate transpose. SNR optimization is thus reduced to identifying optimal entries for the weighting matrix $W$ that minimize $(W\Psi W^H)_{\rho,\rho}$ for each pixel $\rho$, while fulfilling Eq. (2.21). Note that for each pixel the optimization involves only the corresponding row of $W$ (and column of $W^H$), making the problem pixel-wise independent. The minimization can therefore be performed simultaneously for all pixels involved by minimizing the trace of $W\Psi W^H$.

This can be done using Lagrange multipliers, like similarly done in reference.24 Alternatively, it can be cast into the form of a minimum-norm problem as described in Appendix A. Either approach yields:

$$W_{\text{min}} = (S^H (\Psi^{-1} S)^{-1} S^H \Psi^{-1}).$$  \hfill (2.24)

This weighting matrix defines an optimal net coil for each of the pixel positions $r_0, r_1, \ldots, r_N$. Inserting it into Eq. (2.23) and substituting into Eq. (2.11) yields a feasible expression for the ultimate SNR:

$$\zeta(r_0) = \frac{\mu(r_0) B_0^2 \text{FOV}^3 \sqrt{m_K}}{\sqrt{[(S^H \Psi^{-1} S)^{-1}]_{0,0}}}.$$  \hfill (2.25)

where the pixel index was set to $\rho = 0$, which corresponds to the original target position $r_0$ according to the definition preceding Eq. (2.5).

For a given object, key parameters that influence the SNR are $r_0$ and $B_0$, as well as the mode and degree of k-space undersampling. $B_0$ determines $\omega$ and thus implicitly the electrodynamic material properties and the basis set $\alpha_m(r), \beta_m(r)$. Thereby, $B_0$ does also co-determine the matrices $\Psi$ and $S$. The target position $r_0$ and the undersampling scheme determine the aliasing positions, hence co-determining the matrix $S$. Based on the ultimate SNR, the ultimate geometry factor is defined in analogy to Eq. (2.1) as:

$$g(r_0) = \frac{1}{\sqrt{R}} \frac{\zeta^{\text{full}}(r_0)}{\zeta^{\text{PI}}(r_0)} \geq 1.$$  \hfill (2.26)

For this expression, $\zeta^{\text{full}}$ and $\zeta^{\text{PI}}$ are both calculated using Eq. (2.25), however with different k-space sampling. $\zeta^{\text{full}}$ is obtained with full sampling. This leads to no aliasing, hence the sensitivity matrix $S$ has only one column in this case. $\zeta^{\text{PI}}$ is obtained with reduced k-space sampling with the undersampling factor $R$. The resulting aliasing is reflected in a matrix $S$ with multiple columns. Note that different degrees of reduction affect only the matrix $S$, while the noise covariance matrix $\Psi$ does not change.
Complete Coil Arrays

Note the close correspondence between Eqs. (2.6,2.9) and Eq. (2.18). It illustrates that creating a net coil follows the same rules, be it either from actual coils, as done in SENSE reconstruction, or from a complete basis set. This prompts the concept of hypothetical "complete" coil arrays, whose elements' transmit fields span the entire solution space of the Maxwell equations. According to the considerations in this section, a complete array would always yield the ultimate SNR, both in parallel and conventional MRI. This is also illustrated by the formal identity of Eq. (2.24) and the formula for Cartesian SENSE reconstruction given in reference.24 According to this finding, calculating the ultimate SNR for SENSE imaging is essentially equivalent to SENSE reconstruction from data obtained with a hypothetical complete coil array. This is important because it translates to any linear reconstruction algorithm, including, e.g., reconstruction from non-Cartesian k-space data.

Materials and Methods

The numerical complexity of evaluating Eqs. (2.25,2.26) depends strongly on the modeled object geometry and the specific choice of the Maxwell basis functions (2.18). For the practical part of the present work, the object was assumed to be a sphere, favouring a multipole expansion into spherical harmonics. This expansion and its numerical implications are detailed in Appendix B. Briefly, within a spherical object the chosen spherical harmonics are mutually orthogonal, making the calculation and inversion of the noise covariance matrix $\Psi$ very efficient and robust. Nevertheless, numerical integration is necessary for determining its diagonal elements. Very importantly, the multipole expansion matches the spherical object also in the sense that it permits a relatively compact representation of the optimal net detection fields. That is, good approximations of the ultimate SNR can be obtained with a moderate number of basis functions.

Based on these choices, the calculation of ultimate SNR and $g$ factors in SENSE imaging was implemented on a standard PC, using MATLAB (MathWorks, Natick, MA).

Model Specifications

Ultimate SNR calculations were performed for 2D Cartesian parallel imaging with undersampling only in the phase encoding direction. Within the MATLAB program, a specific calculation is prescribed as follows. The basic geometric specifications are the size of the sphere and the angulation of the FOV relative to the $B_0$ axis. The FOV is generally assumed to be a concentric square with side length
equal to the sphere's diameter. K-space sampling is determined by choosing the phase encoding direction and the undersampling factor R. The z-axis of the coordinate system is generally parallel to $B_0$. A typical configuration with $R=3.8$ is sketched in Fig. 2.2. Based on the choice of $B_0$, the Larmor frequency $\omega$ is determined, using the gyromagnetic ratio of water protons ($\gamma = 42.576$ MHz/T).

**Figure 2.2:** Model setup for studies of ultimate SNR and $g$ factor in Cartesian parallel imaging of a spherical object. The FOV is a circumscribed, concentric square with side length equal to the diameter of the sphere. The $k$-space sampling density was reduced in phase encoding direction by the reduction factor $R$. In the shown case, $R = 3.8$ results in threefold aliasing of the central pixel.

Unless stated otherwise, the frequency dependent values of the relative dielectric constant $\varepsilon_r$ and the conductivity $\sigma$ were set according to average in vivo brain conditions. Figure 2.3 shows plots of the used $\varepsilon_r$ and $\sigma$ values versus $B_0$, along with average in vivo values for muscle tissue for comparison. According to common in vivo situations, the relative magnetic permeability $\mu_r$ was generally set to unity. For several representative field strengths, the corresponding frequencies, material constants, and wavelengths are listed in Tab. 2.1. Finally, the spherical harmonic basis functions are specified by choosing the expansion order $l_{\text{max}}$. The resulting number of vector basis functions is $2^*(l_{\text{max}}+1)^2$ (see Appendix B).

**Results**

**Numerical Convergence**

For the subsequent numerical study it was crucial that the finite multipole expansion be sufficiently accurate. This was ensured by an initial convergence analysis,
2.1 Electrodynamics and Ultimate SNR in Parallel MRI

establishing how fast the ultimate SNR approaches a limit as the order of the expansion, $l_{\text{max}}$, increases. Figure 2.4 shows the results of this investigation for a few representative situations. In order to facilitate comparisons between different parameter sets, the ultimate SNR (Eq. (2.25)) was normalized by its value for $l_{\text{max}} = 10$. Then the inverse of the normalized SNR was plotted versus the number of basis functions $2^*(l_{\text{max}}+1)^2$. Data are shown for each combination of three different target positions, two extreme field strengths ($B_0 = 1$ T, $B_0 = 10$ T) and two extreme reduction factors ($R = 1$, $R = 10$). The image plane was transverse with a FOV of 0.3 m. In all cases, the SNR clearly converges within the covered range of up to 20,000 basis functions ($l_{\text{max}} \approx 100$).

As may be expected, the required expansion order increases as the reduction factor grows and as the reconstruction point approaches the surface of the sphere. These results show that a multipole expansion of order in the range of 80 is sufficient for determining the ultimate SNR for $r_0$, $B_0$, and $R$ within the ranges spanned by this analysis. The expansion order was hence set to $l_{\text{max}} = 80$ for all following calculations. The results do also suggest that higher order expansion is necessary.

Figure 2.3: Frequency dependence of material properties according to Gabriel et al.\textsuperscript{15} Left: Relative dielectric constant $\varepsilon_r$. Right: conductivity $\sigma$. Solid lines: average in vivo brain tissue. Dashed lines: average in vivo muscle tissue for comparison.
Table 2.1: Relative dielectric constant $\varepsilon_r$, conductivity $\sigma$, resulting wavelength $\lambda$ and skin depth $\delta$ in average brain tissue (according to Gabriel et al.45).

<table>
<thead>
<tr>
<th>$B_{0,T}$ [$T$]</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega/2\pi$ [MHz]</td>
<td>42.6</td>
<td>127.7</td>
<td>212.9</td>
<td>298.1</td>
<td>383.2</td>
<td>468.4</td>
</tr>
<tr>
<td>$\varepsilon_r^{\text{brain}}$</td>
<td>102.5</td>
<td>63.1</td>
<td>55.3</td>
<td>52</td>
<td>50</td>
<td>48.8</td>
</tr>
<tr>
<td>$\sigma^{\text{brain}}$ [1/(Ω m)]</td>
<td>0.36</td>
<td>0.46</td>
<td>0.51</td>
<td>0.55</td>
<td>0.59</td>
<td>0.62</td>
</tr>
<tr>
<td>$\lambda^{\text{brain}}$ [cm]</td>
<td>58.8</td>
<td>26.8</td>
<td>17.8</td>
<td>13.3</td>
<td>10.7</td>
<td>8.9</td>
</tr>
<tr>
<td>$\delta^{\text{brain}}$ [cm]</td>
<td>17.5</td>
<td>10.0</td>
<td>8.2</td>
<td>7.2</td>
<td>6.6</td>
<td>6.2</td>
</tr>
</tbody>
</table>

Figure 2.4: Convergence of the calculated ultimate SNR as a function of the number of basis functions $2^l(l_{\max}+1)^2$, where $l_{\max}$ denotes the order of the spherical harmonic expansion. The ultimate SNR $\zeta$ was calculated for three different positions in the sphere (center, intermediate radius, beneath the surface), two extreme reduction factors ($R=1$, $R=10$) and two extreme field strengths ($B_0=1$ T, $B_0=10$ T). The plots show the inverse of $\zeta$, after normalization by its value for $l_{\max}=10$. At $R=1$ the convergence behavior is virtually equal for 1 T and 10 T. Therefore the solid and dash-dotted lines coincide. In all cases, the SNR clearly converges within the covered range of up to 20,000 basis functions ($l_{\max} \approx 100$). Based on these results, the expansion order was generally set to $l_{\max}=80$. 

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for \( r_0 \) very close to the surface. Therefore, all subsequent SNR calculations were restricted to a spherical volume slightly smaller than the actual object (diameter = 0.95 * FOV).

**Ultimate SNR for \( R=1 \)**

As suggested by Eq. (2.26), the ultimate SNR in parallel imaging may be characterized by the SNR that can be achieved with full Fourier encoding, \( \zeta_{\text{full}} \), and by the geometry factor. First, \( \zeta_{\text{full}} \) is studied here by setting \( R=1 \). Given the spherical symmetry of the object, one may expect that \( \zeta_{\text{full}} \) should exhibit spherical symmetry as well. However, the spherical symmetry is broken by the direction of \( B_0 \), which determines how the magnetic RF field components combine in forming the signal sensitivity (Eq. (2.2)). Therefore, the only symmetry of \( \zeta_{\text{full}} \) is rotation symmetry about the \( B_0 \) direction. As a consequence, the radial dependence of \( \zeta_{\text{full}} \) along the \( z \)-axis differs slightly from that in the transverse plane.

Figure 2.5 shows the radial dependence of the ultimate SNR for \( R=1 \) (\( \zeta_{\text{full}} \)) for sphere sizes between 0.1 m and 0.5 m and \( B_0 \) between 1 T and 10 T. The ultimate SNR in the transverse plane, \( \zeta_{\text{full}} \), increases with increasing field strength and towards the surface of the sphere. In particular, the minimum of the ultimate SNR is always located in the center of the sphere.

The bottom row of Fig. 2.5 shows the relative deviation of the ultimate SNR along the \( z \)-axis, \( \zeta_{\text{ful}} / \zeta_{\text{full transverse}} \), from that in the transverse plane for the same combinations of FOV and \( B_0 \). Starting from 1 in the center of the sphere, the ratio appears to always converge towards approximately 0.82 at the surface. That is, the ultimate SNR at the sphere’s poles is about 82% of that at its equator. For intermediate radii, the behavior of \( \zeta_{\text{ful}} / \zeta_{\text{full transverse}} \) is generally less systematic, showing values larger and smaller than 1 and partly assuming local extrema. At low values of FOV and \( B_0 \), the ratio is almost independent of the two parameters and decreases continuously towards the surface.

The next part of the study focused on the dependence of the ultimate SNR upon the field strength \( B_0 \). In previous studies, the SNR was found to be approximately linear in \( B_0 \) for low \( B_0 \), while changing to higher power growth for higher \( B_0 \).\(^{10,28,46}\) That is, in each \( B_0 \) regime the SNR approximately obeys a law of the form:

\[
\zeta_{\text{full}} = c B_0^n \iff \log (\zeta_{\text{full}}) = \log(c) + n \log(B_0),
\]

with \( n=1 \) for low \( B_0 \) and \( n > 1 \) for high \( B_0 \), \( c \) denoting an individual constant. As suggested by the logarithmic form of Eq. (2.27), such behavior is best revealed in a double-logarithmic representation, in which the slope is equal to the exponent \( n \). Hence, in order to verify a similar model for the ultimate SNR, Fig. 2.6 shows double-logarithmic plots of \( \zeta_{\text{full}}(B_0) \), for \( B_0 \) ranging from 0.5 T to 12 T. Three different positions in the central transverse plane were investigated for several
Figure 2.5: Ultimate SNR in conventional MRI as a function of the position in the sphere. The top row shows the ultimate SNR along an arbitrary radius in the transverse plane ($\zeta_{\text{full}}^r$), for varying object size (=FOV) and $B_0$. The ultimate SNR is different along the z axis, which is aligned with $B_0$. This is illustrated in the bottom row, showing the relative asymmetry $\zeta_{\text{full}}^z / \zeta_{\text{full}}^r$. Note that this ratio always converges to approximately 0.82, independent of the FOV and $B_0$.

For all of these situations, the corresponding plots have in common that they are approximately linear for low $B_0$, with slopes $n$ close to 1. Hence, the ultimate SNR is indeed an approximately linear function of $B_0$ in that regime. For the peripheral position ($r_0 = 0.95*\text{FOV}/2$), this behavior persists throughout the entire $B_0$ range. For the other, deeper positions, a transition to a second regime with greater slope is observed. The transition generally occurs sooner for larger objects/FOVs and deeper target positions. This illustrates that the two types of SNR behavior in fact correspond to the two fundamental distance regimes of RF fields. The near-field is dominated by evanescent field components, which are of
high amplitude but decay rapidly with the distance from the source. The extent of the near-field zone is roughly equal to the RF wavelength, which is inversely proportional to $B_0$. Hence, MR detection is near-field dominated at low $B_0$ or from positions close to the object's surface. The far-field consists of propagating field components, which hence are the carriers of MR detection at high $B_0$ or from positions far away from the surface.

For a more quantitative analysis, the near- and far-field exponents, $n_{NF}$ and $n_{FF}$, were determined from the data plotted in Fig. 2.6. This analysis was performed by linear regression of the individual curve segments corresponding to either
Table 2.2: Exponents describing the growth of the ultimate SNR as a power of $B_0$, as obtained by linear regression analysis of the results shown in Fig. 2.6. The three values in each bracket correspond to object sizes and corresponding FOVs of 0.1 m, 0.3 m, and 0.5 m. The exponents differ significantly between the near-field ($n_{NF}$) and the far-field ($n_{FF}$) regimes. Missing values indicate that the corresponding regime could not be analyzed within the covered range of $B_0$. First section: average brain conductivity. Second section: lossless sample ($\sigma \approx 0$).

<table>
<thead>
<tr>
<th></th>
<th>$r_0 = 0.0^{\text{FOV}}$</th>
<th>$r_0 = 0.5^{\text{FOV}}$</th>
<th>$r_0 = 0.95^{\text{FOV}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{NF}$ (avg. brain)</td>
<td>[0.89, 0.99, -]</td>
<td>[0.87, 0.91, -]</td>
<td>[0.89, 0.90, 0.93]</td>
</tr>
<tr>
<td>$n_{FF}$ (avg. brain)</td>
<td>[-, 1.92, 1.45]</td>
<td>[-, 2.05, 1.85]</td>
<td>[-, -]</td>
</tr>
<tr>
<td>$n_{NF}$ ($\sigma \approx 0$)</td>
<td>[1.03, 1.16, -]</td>
<td>[1.02, 1.07, -]</td>
<td>[1.00, 1.01, 1.04]</td>
</tr>
<tr>
<td>$n_{FF}$ ($\sigma \approx 0$)</td>
<td>[-, 2.82, 2.78]</td>
<td>[-, 2.78, 2.80]</td>
<td>[-, -]</td>
</tr>
</tbody>
</table>

near- or far-field behavior (Tab. 2.2). The linear segments for regression analysis were identified visually and are indicated in Fig. 2.6. In several cases either the near- or the far-field regression could not be performed because the sampled $B_0$ range did not sufficiently cover the respective regime. For instance, the superficial position shows only near-field behavior throughout, hence not yielding a value for $n_{FF}$. Entries missing for this reason are indicated by a dash in Table 2.2.

The numbers show that the near-field exponent is almost exactly equal to 1 for the lossless case. It is only somewhat smaller than 1 for the conductive object, reflecting increasing losses as the Larmor frequency increases (cf. Fig. 2.3). As opposed to that, the far-field exponent was found to strongly depend on the object's conductivity. In the lossless case $n_{FF}$ was approximately equal to 2.8, irrespective of the object size and the depth of the target position, marking a nearly cubic dependence of ultimate SNR on $B_0$. In the lossy sample, the values of $n_{FF}$ were significantly reduced and varied between 1.45 and 2.05, depending on the absolute depth of the target position. Again, the monotonic increase of the tissue conductivity with frequency (cf. Fig. 2.3) results in reduced SNR gains compared to the lossless case.

**Ultimate Geometry Factor**

Figure 2.7 shows maps of the ultimate geometry factor in the transverse plane. It covers each combination of four reduction factors ($R = 3$ to 6), three object sizes (FOV = 0.1 m to 0.5 m), and four different field strengths ($B_0 = 1$ T to 10 T). These results reveal that the performance of parallel imaging is inherently limited.
2.1 Electrodynamics and Ultimate SNR in Parallel MRI

Figure 2.7: Maps of the ultimate geometry factor in the transverse plane. The gray scale was individually adjusted to the actual range of $g$ values. Above each map, the mean and the maximum of $g$ are indicated in brackets. The geometry factor is a measure of noise enhancement specific to parallel imaging reconstruction. Hence these results demonstrate that the performance of parallel imaging is inherently limited by electrodynamics. Even with a hypothetical optimal coil array, noise enhancement will occur.
They show that, even with a hypothetical complete coil array, noise enhancement as reflected by the geometry factor cannot be avoided. While the ultimate $g$ is generally near-optimal ($g \approx 1$) for $R=3$ and still fairly low for $R=4$, unfavorably high $g$ factors occur at $R=5$ and $R=6$. At these reduction factors the SNR efficiency begins to deteriorate. The strongest increases in $g$ factor are observed for low field strength and small FOV, whereas at high field and with large FOV the situation remains favorable.

The maps of the ultimate $g$-factor tend to have an ovoid structure with a longer extension in the phase encoding direction. This reflects the fact that the number of aliasing positions within the sphere decreases gradually when $r_0$ is shifted away from the center in the frequency encoding direction. With favorably large $B_0$ and FOV, the g maps exhibit characteristic patterns reminiscent of the aliasing that results from Fourier reconstruction alone. In the unfavorable cases of strong reduction at low $B_0$ and small FOV, the structure of the g maps is simpler, exhibiting more or less a single blob. As opposed to $\zeta^{\text{full}}$, whose minimum was always in the sphere's center, the highest ultimate $g$ factor does not generally occur in the center. Moreover, equally unlike $\zeta^{\text{full}}$, the ultimate $g$ factor does not exhibit rotational symmetry about the $z$ axis. This symmetry is broken by the choice of the phase encoding direction. The only symmetry remaining is with respect to $z$-rotation about 180 degrees. Note in particular that the g maps exhibit slight asymmetries with respect to both the phase and the frequency encoding axes.

According to the previous results, the center region of the sphere is a sensitive marker for the overall feasibility of parallel imaging. First, $\zeta^{\text{full}}$ did always assume its minimum in the center. Second, $g$ did also assume its maximum in the center when this maximum was critically high. Therefore, the center position was selected for a more detailed analysis of $g$ as a function of the field strength and the object size. Figure 2.8 shows logarithmic plots of the ultimate $g$ factor versus $B_0$ (0.5 T to 12 T) and versus the reduction factor $R$ (1 to 8) for object sizes and corresponding FOVs of 0.1 m to 0.4 m. These graphs further elucidate the transitions previously observed in the g maps. We can generally distinguish two domains with opposite parallel imaging characteristics. The first, favorable domain is characterized by ultimate $g$ close to 1 ($\log g \approx 0$), reflecting optimal ultimate parallel imaging performance. The second, unfavorable domain is characterized by exponential growth of the $g$ factor as a function of $R$, which is highly limiting in view of generally limited baseline SNR. In fact, for most applications the onset of the exponential regime will mark a practical upper bound to the feasible reduction factor. The transition between the two domains depends on $B_0$ and the FOV, indicating that it is again governed by the near- and far-field behavior of the involved RF fields. In near-field conditions, i.e. for low $B_0$ / small FOV, the critical transition is approximately at $R = 4$ and nearly independent of $B_0$ and the
Figure 2.8: The ultimate geometry factor in the center of the sphere as a function of the reduction factor $R$ and $B_0$, for various object sizes with corresponding FOV. Two clearly distinct regimes are observed. For low reduction factors or sufficiently large $B_0$ and FOV the ultimate $g$ factor is benign with values very close to the optimum of 1. For high reduction factors beyond a critical value, the ultimate $g$ increases exponentially as a function of $R$. This behavior characterizes an unfavorable regime where parallel imaging is strongly hampered by deteriorating SNR efficiency. The transition is approximately at $R=4$ in near-field conditions (low $B_0$ /small FOV) and shifts towards higher reduction factors in far-field conditions (high $B_0$ /large FOV).
Discussion

Model Assumptions

In this work, practical procedures have been developed for studying the electrodynamic limits of the SNR in parallel imaging. In order to keep these procedures numerically feasible, the underlying model was subject to several simplifications. Firstly, the SNR model is based on the common assumptions of steady-state oscillation and linear response of the detector. Assuming steady-state oscillation is warranted when the signal bandwidth is significantly smaller than the intrinsic bandwidth of the detector circuit. This is usually the case for MR imaging situations. Linear response is required for using the reciprocity principle as well as for the subsequent linear-algebraic reasoning. This is also a safe assumption for the coils and lumped components typically used in MR detection. Note that the common mechanisms of coil-to-coil coupling, be it inductive, capacitive, or resistive, are likewise linear. The detector electronics often do involve components with potentially nonlinear behavior, such as switching diodes or transistors for pre-amplification. However, the circuitry is generally designed to use these components in a linear regime of operation to avoid distortion and intermodulation. After all, linear overall detector response is a prerequisite for applying the common Fourier principles at the data processing stage.

In setting up the Maxwell equations, it was assumed that the imaged object was source-free, homogeneous, and spherical in shape. The absence of sources corresponds to all RF coils being placed outside the object, which is realistic for most MRI procedures. Electromagnetic homogeneity is an accurate assumption for simple phantoms, while it represents only a first approximation to in vivo situations. In order to approximate the conditions of head imaging, the electromagnetic constants were modeled as frequency-dependent and set to average in vivo brain values. With such a description, the main phenomena such as near-field and far-field wave behavior as well as dissipation can be realistically modeled. The local tissue structure becomes most critical only at very high frequencies, where it may give rise to specific RF resonances. In terms of shape, too, the spherical model, with appropriate radius, may be regarded as a first approximation of a human head. Furthermore, the sphere may be regarded as a worst-case situation for parallel imaging, as it maximizes the mean distance to the surface for a given volume.

Clearly, the model results cannot be expected to directly apply to in vivo situations. However, two points suggest that the main findings may qualitatively translate to practical imaging. First, the results obtained in the homogeneous
sphere are highly regular, i.e. they do not change abruptly as a function of $B_0$, FOV, or material properties. Hence it may be expected that they will also change smoothly as the object approaches reality. Second, the model results are qualitatively in line with many in vivo experiments, which showed that the $g$ factor tends to grow critically at acceleration factors beyond three to four.

Concerning the Maxwell equations, a further, more subtle simplification needs to be pointed out. Note that proper electrodynamic behavior was enforced only inside the object, where the transmit fields $E$ and $H$ affect the SNR immediately. Strictly speaking, this requirement is too weak as any physically feasible fields will solve the Maxwell equations also outside the object and fulfill boundary conditions at the object’s surface. However, these additional conditions effectively do not constrain the solution space inside the object. Intuitively this can be understood by considering that the outside space may host an arbitrary configuration of coil conductors, which act as current sources in the transmission picture. So there are infinitely many degrees of freedom available for continuing a given Maxwell solution towards the outside in a physically meaningful fashion. This can also be established more formally with methods of inverse source identification, as described, e.g., in reference.47 However, due to lack of space a detailed formal treatment of this aspect is left to separate publication. So, strictly speaking, it is an assumption of this work that every physical Maxwell solution inside the object is consistent with a corresponding current distribution on the outside. This implies that the calculated SNR bounds are tight, i.e. that they can be accomplished, at least theoretically, with actual coil conductor distributions. Even if the aforementioned assumption were wrong the calculated limits would still hold as upper, yet not tight, bounds.

The study focused on the most widely used parallel imaging mode, namely on Cartesian k-space sampling with undersampling in one dimension. However, the model and algorithms apply without modification to the Cartesian case with undersampling in two or three dimensions. Based on experimental findings and straightforward geometrical considerations,43 it is expected that the critical reduction factors will be significantly higher for 2D and 3D undersampling than for the elementary 1D case studied in this work. The transition to non-Cartesian sampling with, e.g., spirals, is less simple. With general k-space patterns, the point spread function can no longer be viewed as a set of discrete peaks. As a consequence, the concept of net sensitivity no longer applies in the simple form used for the Cartesian case. Hence, in order to analyze non-Cartesian parallel imaging, more advanced reconstruction approaches must be incorporated.48,49 In analogy to the findings of the present work, the ultimate SNR can then be assessed by applying these reconstruction methods to a hypothetical complete coil array.
Basis Functions and Different Types of Wave Behavior

The choice of an appropriate Maxwell basis is crucial for the accurate calculation of ultimate SNR. On the one hand, each basis element must be a Maxwell solution and the basis as a whole must be approximately complete. On the other hand the number of basis elements is constrained by limited computing resources. In combination with the spherical object, vector spherical harmonics have proven numerically efficient in the present work. A more straightforward choice is a plane wave expansion as previously used in references. It is formally simpler than the spherical expansion, yet it has the drawback of requiring a particularly large number of basis functions, as briefly discussed in the following.

Standard plane wave approaches are based on a Fourier expansion of the electromagnetic field in the x-y plane, constructing a plane wave from each element of 2D k-space. For any pair of real numbers, \((k_x, k_y)\), a plane wave with wave vector \(k\) is defined by choosing the remaining component, \(k_z\), such as to fulfill the Helmholtz equation (2.14):

\[
k_x^2 + k_y^2 + k_z^2 = k_0^2.
\]  

(2.28)

This equation ensures that the corresponding wave oscillates and decays according to \(k_0\), the material's wave number at the respective frequency. When all components of \(k\) have the same phase the situation simplifies in that the magnitude of \(k\) becomes equal to that of \(k_0\). This assumption was used in the original paper on ultimate SNR, as well as in reference, reducing the size of the plane wave basis in favor of more efficient computation. However, it is important to note that \(k_z\) does not need to be in phase with \(k_x\) and \(k_y\) and Eq. (2.28) has solutions for all pairs \((k_x, k_y)\). In particular, the magnitude of the wave vector is not limited by the magnitude of \(k_0\). Note also that the plane waves constructed in this fashion are linearly independent, which is reflected by the fact that they form a common Fourier basis in the x-y plane. As a consequence, for creating a complete plane wave basis the pairs \((k_x, k_y)\) must cover all of 2D k-space. In practice the relevant range of wave vectors is limited by the desired spatial resolution of the study. However, for typical MRI situations with k-space sampling up to, say, \(k_{max} = 1\) mm\(^{-1}\), this limit is still orders of magnitude larger than \(k_0\), leading to basis sets of daunting size. This problem is inherent to plane wave bases and not different when starting from \(k_x-k_z\)-, or \(k_y-k_z\)-space and solving Eq. (2.28) for \(k_y\) or \(k_z\), respectively.

Equation (2.28) is also useful for illustrating near- and far-field behavior as observed in the present work. After calculating \(k_z\) for a pair \((k_x, k_y)\), the real and imaginary parts of \(k_z\) reflect oscillation and decay, respectively, along the z direction. This distinction is clearest for a lossless sample, in which the wave number \(k_0\) is purely real. According to Eq. (2.28), \(k_z\) then is either purely real or purely
Figure 2.9: The components of a plane wave expansion can be categorized into evanescent and propagating waves. The distinction is clearest in the case of a lossless sample with real-valued wave number $k_0$. In this case, the evanescent components are characterized by wave vectors $k$ with $k_x^2 + k_y^2 > k_0^2$ and imaginary $k_z$, causing evanescence, i.e. exponential decay in the $z$ direction. Plane waves with $k_x^2 + k_y^2 < k_0^2$ have a real $k_z$-component and extend indefinitely in all directions.

imaginary. As depicted in Fig. 2.9, these two situations correspond to two distinct domains in the $k_x$-$k_y$-plane, which are separated by a circle with radius $k_0$. Within the circle $k_z$ is real; the corresponding plane waves extend indefinitely in all directions, contributing to the far field. Outside the circle $k_z$ is imaginary, marking so-called evanescent waves that decay exponentially in the $z$ direction. Evanescent waves form the near field of an oscillating source, which reaches up to a distance of approximately one wavelength.\textsuperscript{51} Hence the signal detection in MRI is usually near-field-dominated,\textsuperscript{42} in particular when using surface coils for SNR optimization. In fact, Biot-Savart’s law, which is frequently used for calculating receiver coil sensitivities, accounts exclusively for near-field components.\textsuperscript{39} Nevertheless, as shown in the present work, far-field behavior can be relevant in large objects and at high $B_0$. This illustrates that ultimate SNR calculations should fully capture both near- and far-field behavior, requiring a complete Maxwell basis such as full plane waves or the vector spherical harmonics used in the present work.
Coil Optimality for Parallel and Conventional Imaging

An important question in coil design is whether an array that is optimized for parallel imaging will also perform well in conventional imaging with full Fourier encoding and vice versa. In reference,35 Rejkowski previously suggested that an array should be optimal for SENSE imaging of a given pixel if it contains optimal coils for conventional imaging of each pixel involved in the aliasing process. Based on the theoretical considerations in the present work, this can be formally confirmed and somewhat generalized. For an arbitrary set of aliasing positions, let the vector $s_\rho$ denote the $\rho$-th column of the corresponding sensitivity matrix (Eq. (2.20)) and $w_\rho^\text{SENSE}$ the $\rho$-th row of the optimal weighting matrix according to Eq. (2.24). The latter can then be written as:

$$
\begin{pmatrix}
  w_0^\text{SENSE} \\
  w_1^\text{SENSE} \\
  \vdots \\
  w_N^\text{SENSE}
\end{pmatrix} =
\begin{pmatrix}
  * & * & \cdots & * \\
  * & * & \cdots & * \\
  \vdots & \vdots & \ddots & \vdots \\
  * & * & \cdots & *
\end{pmatrix}
\begin{pmatrix}
  s_0^H \Psi^{-1} \\
  s_1^H \Psi^{-1} \\
  \vdots \\
  s_N^H \Psi^{-1}
\end{pmatrix},
$$

(2.29)

where the asterisks indicate that the entries of the matrix in the middle are not relevant for the following argument. For comparison, the SNR at the same positions can also be optimized for conventional imaging without aliasing. For the pixel with index $\rho$, Eq. (2.24) then simplifies to:

$$w^\text{CONV}_\rho = (\ast)s_\rho^H \Psi^{-1},$$

(2.30)

where the asterisk again replaces an irrelevant factor. Combining Eqs. (2.29,2.30) yields:

$$
\begin{pmatrix}
  w_0^\text{SENSE} \\
  w_1^\text{SENSE} \\
  \vdots \\
  w_N^\text{SENSE}
\end{pmatrix} =
\begin{pmatrix}
  * & * & \cdots & * \\
  * & * & \cdots & * \\
  \vdots & \vdots & \ddots & \vdots \\
  * & * & \cdots & *
\end{pmatrix}
\begin{pmatrix}
  w_0^\text{CONV} \\
  w_1^\text{CONV} \\
  \vdots \\
  w_N^\text{CONV}
\end{pmatrix}.
$$

(2.31)

This equation confirms that the ideal coils for conventional imaging and the ideal net coils for SENSE reconstruction are connected by a simple linear mapping. By this mapping and its inverse, it is possible to convert data obtained with either coil set into virtual data that are exactly equal to what would have been obtained with the other coil set. In other words, both coil sets span the same space of feasible net coils and are thus equivalent at the level of SENSE reconstruction. Hence, a coil array that is optimal for the separation of given aliasing positions in SENSE imaging is also optimal at each of these positions for conventional imaging and vice versa.
Conclusions

In the present work, inherent limits of parallel MR imaging were studied by analyzing the impact of the underlying electrodynamics on the achievable SNR. The key findings are the following:

1. The range of reasonable reduction factors in parallel imaging is fundamentally limited by electrodynamics. For low and moderate reduction factors, the ultimate geometry factor is close to its optimal value of 1. However, for high reduction factors beyond a critical value, the ultimate g factor tends to increase exponentially, marking a regime where parallel imaging is strongly hampered by deteriorating SNR efficiency. This limitation cannot be circumvented by enhanced coil design, as long as the coils are bound to stay outside of the object.

2. The critical reduction factor behaves differently in the near- and far-field regimes of the detected RF fields. In the near-field regime, i.e. for low $B_0$ / small objects, the critical reduction factor is independent of $B_0$ and the object size and approximately equal to four for 1D reduction in a sphere. In the far-field regime, i.e. for high $B_0$ / large objects, the critical reduction factor is generally larger and increases with both $B_0$ and the object size. For a head-sized object with in-vivo properties, the transition between the two regimes is at approximately 5 Tesla. Similar transitions were observed for conventional MRI with full k-space sampling. Hence, at very high field strengths such as 7 Tesla and beyond, both parallel and conventional imaging in humans is expected to benefit significantly from far-field behavior.

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Appendix A: Minimum-Norm Formulation of SNR Optimization

In the Theory section, the problem of optimizing the SNR for a set of aliasing pixels was expressed as that of minimizing the cost functional:

$$\Delta = \text{Tr} (W\Psi W^H)$$

(2.32)
by variation of the weighting matrix \( W \) under the linear constraint:

\[
WS = \text{Id}, \quad (2.33)
\]

where \( \text{Tr} \) denotes the trace operator and \( \text{Id} \) denotes the \((N+1)\times(N+1)\) identity matrix. In order to cast the minimization of \( \triangle \) into the form of a minimum-norm problem, the noise covariance matrix is eliminated by transforming the weighting matrix. Note that, for linearly independent electrical basis functions \( \alpha_m(r) \), \( \Psi \) is positive-definite, thus permitting the Cholesky factorization:\(^\text{53}\)

\[
\Psi = LL^H. \quad (2.34)
\]

Transforming the weighting vector and the sensitivity matrix by the root \( L \):

\[
\tilde{W} = WL \quad (2.35)
\]

\[
\tilde{S} = L^{-1}S, \quad (2.36)
\]

the cost functional and the constraint in Eqs. (2.32,2.33) can be rewritten as:

\[
\Delta = \text{Tr}(\tilde{W}\tilde{W}^H) \quad (2.37)
\]

\[
\tilde{W}\tilde{S} = \text{Id}. \quad (2.38)
\]

According to this formulation, the optimal \( \tilde{W} \) is the minimum-norm solution of Eq. (2.38), which can be calculated by means of the Moore-Penrose pseudoinverse of \( \tilde{S} \):\(^\text{53}\)

\[
\tilde{W}_{\text{min}} = \tilde{S}^+ = (\tilde{S}^H\tilde{S})^{-1}\tilde{S}^H. \quad (2.39)
\]

The latter equality holds if the rank of \( \tilde{S} \) is equal to its width. This will generally be the case with a number of Maxwell basis functions much larger than the number of aliasing pixels. Transforming \( \tilde{W}_{\text{min}}, \tilde{S} \) back to the original coordinates yields the optimal weighting vector:

\[
W_{\text{min}} = (S^H\Psi^{-1}S)^{-1}S^H\Psi^{-1}. \quad (2.40)
\]

**Appendix B: Multipole Expansion**

The multipole expansion used in this work was constructed as follows. Each solution of the homogeneous and source-free Maxwell equations (2.14) can be expressed in spherical coordinates as:\(^\text{44,46,54}\)

\[
H(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( w_{l,m}^E j_l(k_0 r) X_{l,m}(\theta, \varphi) \right.
\]

\[
- \frac{i}{k_0} w_{l,m}^M \nabla \times [j_l(k_0 r) X_{l,m}(\theta, \varphi)] \left. \right) \]
Here, $w_{l,m}^E$ and $w_{l,m}^M$ denote expansion coefficients of electric-source and magnetic-source multipole basis functions, respectively. $j_l$ denotes the $l$-th spherical Bessel function of the first kind and $X_{l,m}$ the vector spherical harmonics, which can be expressed as:

$$X_{l,m}(\theta, \varphi) = \frac{-i}{\sqrt{l(l+1)}}(\hat{r} \times \nabla)Y_{l,m}(\theta, \varphi),$$

where $\hat{r}$ denotes a unit vector in the radial direction and $Y_{l,m}$ are the common scalar spherical harmonics. Expanding the rotation terms in Eq. (2.41),

$$\nabla \times [j_l(k_0r)X_{l,m}(\theta, \varphi)] = \frac{1}{r} \frac{\partial}{\partial r}[rj_l(k_0r)]\hat{r} \times X_{l,m}(\theta, \varphi)$$

$$+ i\sqrt{l(l+1)}\frac{j_l(k_0r)}{r} Y_{l,m}(\theta, \varphi)\hat{r}$$

yields a favorable representation of the multipole expansion (2.41) as a sum of three terms:

$$\mathbf{E}(r) = \frac{\omega \mu}{k_0} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( w_{l,m}^E j_l(k_0r)X_{l,m} + w_{l,m}^M j_l(k_0r)X_{l,m} \right)$$

$$\mathbf{H}(r) = \frac{\omega \mu}{k_0} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( w_{l,m}^E \frac{i}{k_0r} \frac{\partial}{\partial r}[rj_l(k_0r)]\hat{r} \times X_{l,m} - \frac{\sqrt{l(l+1)}}{k_0r} j_l(k_0r)Y_{l,m}\hat{r} \right)$$

$$\mathbf{H}(r) = \frac{\omega \mu}{k_0} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( w_{l,m}^E \frac{i}{k_0r} \frac{\partial}{\partial r}[rj_l(k_0r)]\hat{r} \times X_{l,m} - \frac{\sqrt{l(l+1)}}{k_0r} j_l(k_0r)Y_{l,m}\hat{r} \right)$$

$$+ w_{l,m}^M j_l(k_0r)X_{l,m}.$$
The individual terms obey the following orthogonality relations:

\[ \int X_{l,m} \cdot \hat{r} Y_{l',m'}^* d\Omega = 0, \]
\[ \int (\hat{r} \times X_{l,m}) \cdot \hat{r} Y_{l',m'}^* d\Omega = 0, \]
\[ \int (\hat{r} \times X_{l,m}) \cdot X_{l',m'}^* d\Omega = 0, \]
\[ \int Y_{l,m} Y_{l',m'}^* d\Omega = \delta_{l,l'} \delta_{m,m'}, \]
\[ \int X_{l,m} \cdot X_{l',m'}^* d\Omega = \delta_{l,l'} \delta_{m,m'}, \]
\[ \int (\hat{r} \times X_{l,m}) \cdot (\hat{r} \times X_{l',m'}) d\Omega = \delta_{l,l'} \delta_{m,m'} \]  \( (2.45) \)

where \( \int d\Omega \) indicates integration over the surface of the unit sphere.

For numerical calculations, the order of the expansion must be limited to \( l \leq l_{\text{max}} \), yielding \( 2^*(l_{\text{max}}+1)^2 \) basis functions in total. The corresponding set of weighting coefficients \( w_{l,m}^E \) and \( w_{l,m}^M \) were assembled in the weighting vector \( \mathbf{w} \), using the following convention:

\[
\mathbf{w} = (w_{0,0}^E, w_{0,-1}^E, w_{0,1}^E, \ldots, w_{l_{\text{max}},l_{\text{max}}}^E, w_{l_{\text{max}},l_{\text{max}}}^E, w_{0,0}^M, w_{0,-1}^M, w_{0,1}^M, \ldots, w_{l_{\text{max}},l_{\text{max}}}^M, w_{l_{\text{max}},l_{\text{max}}}^M).
\] \( (2.46) \)

Using the same convention and exploiting the orthogonality relations \( (2.45) \), the noise covariance matrix \( \Psi \) (see Eq. \( (2.22) \)) assumes a convenient diagonal form:

\[
\Psi = \left[ \begin{array}{cc} \Psi^E & 0 \\ 0 & \Psi^M \end{array} \right], \quad \text{where}
\]
\[
\Psi^E_{(l,m),(l',m')} = \delta_{l,l'} \delta_{m,m'} \left( \frac{1}{k_0} \right)^2 \int_{r=0}^{FOV/2} \left( \left| \frac{\partial (r j_l(k_0 r))}{\partial r} \right| + l(l+1) |j_l(k_0 r)|^2 \right) dr
\]
\[
\Psi^M_{(l,m),(l',m')} = \delta_{l,l'} \delta_{m,m'} \int_{r=0}^{FOV/2} r^2 |j_l(k_0 r)|^2 dr.
\] \( (2.47) \)

\( \Psi^E \) involves analytically unknown integrals, which were solved using Gauss quadrature.

The corresponding signal sensitivity matrix (see Eq. \( (2.20) \)) is given by

\[
S = \left( \begin{array}{cc} S^E_{x} - i S^E_{y} \\ S^M_{x} - i S^M_{y} \end{array} \right), \quad \text{where}
\]
\[ S^E_{(l,m),\rho} = j_l(k_0 r_{\rho})[X_{l,m}(\theta_{\rho}, \varphi_{\rho})]_x \]

\[ S^M_{(l,m),\rho} = -\frac{i}{k_0 r_{\rho}} \frac{\partial [r j_l(k_0 r)]}{\partial r} \bigg|_{r=r_{\rho}} \left[ \hat{r}_{\rho} \times X_{l,m}(\theta_{\rho}, \varphi_{\rho}) \right]_x + \frac{\sqrt{l(l+1)}}{k_0 r_{\rho}} j_l(k_0 r_{\rho}) Y_{l,m}(\theta_{\rho}, \varphi_{\rho}) \left[ \hat{r}_{\rho} \right]_x. \] (2.48)

\( S^E_y \) and \( S^M_y \) are defined analogously, using the respective \( y \)-components. \( r_{\rho}, \theta_{\rho}, \varphi_{\rho} \) denote the spherical representation of the \( \rho \)-th aliasing position \( r_{\rho} \).
2.2 Wave Optical Aspects in Conventional and Parallel MR Imaging

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Motivation

When compared with most other imaging modalities, magnetic resonance imaging (MRI) stands out by its unique mechanism for achieving spatial resolution. Most imaging techniques that involve some sort of radiation rely on focusing or collimation for assigning signals to their origin in space. As opposed to that, in conventional MRI spatial resolution is exclusively based on the frequency encoding approach.\(^5\) With this method the position of nuclear moments is encoded in their Larmor frequency by superimposed gradient fields. The necessity for using this inherently spectroscopic approach instead of optical means is based on the fact that the wavelength (\(\lambda\)) of the involved radiofrequency (RF) fields is way too large. With \(\lambda\) on the order of tens of cm the diffraction limit\(^16,17\) of approximately \(\lambda/2\) is much larger than the desired spatial resolution in the mm range.

The wavelength of the radiofrequency (RF) signals in MRI is determined by the Larmor frequency (\(\omega\)) and the object’s electrodynamic properties according to:\(^44\)

\[
\lambda = \frac{2\pi}{\mathfrak{R}(k_0)}, \quad \text{with: } \quad k_0^2 = \mu \omega (\epsilon \omega + i\sigma) \\
\omega = \gamma B_0
\]

(2.49)

where \(k_0\) denotes the wave number, \(B_0\) the main magnetic field strength, \(\epsilon\), \(\sigma\), \(\mu\) the object’s permittivity, conductivity, permeability, respectively, and \(\gamma\) the gyromagnetic ratio of protons. According to Tab. 2.1 the encountered wavelengths are comparable to both typical object sizes and receiver coil dimensions. Hence, MR signal detection is near-field dominated for low values of \(B_0\) and/or \(L\), whereas far-field wave behavior comes into play for high values of \(B_0\) and/or \(L\).\(^42,55\)

In the realm of optical imaging, near-field techniques and, in particular, scanning near-field optical microscopy (SNOM) have been established as a means of obtaining images with sub-wavelength resolution.\(^56-58,58\) In SNOM local near-field distributions are serially scanned along the surface of the imaged object. Unlike wave-optical imaging techniques, in this case the achievable image resolution is limited by technical factors such as the size of the measurement aperture, the distance between the light sources and the detector, and the intrinsic sensitivity of the measurement.\(^59\) Interestingly, the first near-field optical experiments were performed with electromagnetic waves in the RF and microwave regime, achieving resolutions in the range of \(\lambda/60\).\(^57\) In principle, the SNOM idea could be easily adapted for MRI by scanning across the object with a small RF probe. In this way an image could be generated entirely without frequency encoding. However, the achievable image resolution would be limited to the dimension of the receiver coil and degrade rapidly with distance from the object’s surface.

According to these considerations a purely "optical" MRI technique, be it either near- or far-field based, is unrealistic and frequency encoding remains the only
known means of achieving mm resolution in MRI of volume objects. However, as argued in this work, parallel MRI with receiver arrays$^{23-25}$ can be regarded as relying on near-field and even wave optics for complementary spatial encoding. As a consequence, for parallel MRI the spatial structure of the relevant RF fields does play a key role with respect to spatial encoding. In addition, even in conventional MRI with full frequency encoding the SNR performance depends greatly on spatial field variations and RF characteristics.

Elucidating these conceptual connections between optical imaging and both conventional and parallel MRI is the purpose of the present paper.

Materials and Methods

In the previous section the inherent performance limitations of conventional and parallel MRI have been explored based on the concept of ultimate intrinsic SNR. The latter will be used in the following as a general measure of imaging performance.

Ultimate SNR calculations were performed for spherical objects with homogeneous material properties $\epsilon$, $\sigma$, and $\mu$ using the methods described in reference.$^{60}$ Sphere diameters between 0.1 m and 0.6 m and field strengths in the range between 1 T and 12 T have been analyzed. All calculations were performed for a central, transverse imaging plane with regular Cartesian k-space sampling (cf. Fig. 2.2).

From the concept of electrodynamic scaling the adimensional wave number $\kappa$ is known as a means of characterizing RF wave behavior inside an object.$^{61,62}$ It relates the object size $L$ to the wavelength and the skin depth ($\delta$) of the RF fields, according to:

$$\kappa = 2\pi \frac{L}{\lambda} + i \frac{L}{\delta}.$$  \hspace{1cm} (2.50)

Similarly, in the scope of this work the impact of the two independent variables $B_0$ and $L$ is jointly accounted for through the relative object size $L/\lambda$. The relative object size acts as an approximate RF regime indicator, where $L/\lambda < 1$ corresponds to approximately quasi-stationary, near-field conditions throughout the object and $L/\lambda > 1$ indicates significant wave behavior. For a more stringent differentiation between the two kinds of RF wave behavior, the distance between the individual signal source and the respective detector would have also to be taken into account.

SNR and g-Factor in Terms of $L/\lambda$

Figure 2.10 illustrates ultimate SNR performance as a function of $L/\lambda$. Three different pixel positions (center $r=0$, intermediate $r=0.5*L/2$ and surface $r=0.95*L/2$)
2.2 Wave Optical MR Image Encoding

Figure 2.10: Ultimate SNR normalized by the near-field SNR dependency $SNR_{NF}$ (2.51) as a function of $L/\lambda$. Three transverse imaging locations are indicated by different markers. Left: average brain conditions; right: lossless case for comparison.

Figure 2.11: Ultimate $g$ factor as a function of $R$ and $L/\lambda$ at three different positions in the central transverse plane.
are indicated by different markers. Here \( r \) denotes the radial distance with respect to the center of the sphere. In addition to the results obtained assuming average in vivo brain material properties is shown for comparison (right). In order to specifically point out the effect of wave behavior the SNR has been normalized by the trivial near-field dependency of: \( \frac{L}{\sigma} \).

\[
\text{SNR}_{\text{NF}} \propto B_0 \sqrt{\frac{L}{\sigma}}. \quad (2.51)
\]

Apparently, this equation reflects the relevant dependencies accurately for \( L/\lambda > 1 \) and generally for superficial regions. However, for \( L/\lambda < 1 \) the SNR at the depth of the object shows a marked transition into a regime where it increases significantly faster than described by \( \text{SNR}_{\text{NF}} \). This increased SNR rise is related to onsetting field focusing capabilities in the RF wave regime.

Figure 2.11 illustrates the ultimate g factor for three different imaging locations (center \( r=0 \), intermediate \( r=0.5*L/2 \) and surface \( r=0.95*L/2 \)) as a function of the reduction factor (R) and \( L/\lambda \). The shown central g factor map reveals the same characteristics that have been described in detail in the previous section; i.e. an inherent limitation of the feasible reduction factor and an improvement of g with the onset of far-field behavior. Furthermore and similar to the behavior of the ultimate SNR, the ultimate g factor improves drastically for locations closer to the surface of the sphere where highly structured near-field sensitivities can be implemented with nearby coils.

Figures 2.10 and 2.11 bear strong resemblance with Figs. 2.6 and 2.8 shown in the preceding section. However, note that in the present description the parameters \( B_0 \) and L were fused into the single effective variable \( L/\lambda \). The shown results illustrate that this relative object size is a key parameter in systemizing SNR and g performance, showing only slight ambiguities in the conductive case for \( L/\lambda > 1 \).

Wave Optical Resolution Barrier in Parallel MRI

The exponential increase of the g-factor for high R values marks a practical bound for the feasible amount of acquisition speed-up in parallel imaging. For analyzing this limitation more closely the critical reduction factor (\( R_{\text{crit}} \)) is introduced according to:

\[
g(R_{\text{crit}}) = g_{\text{crit}}, \quad (2.52)
\]

with \( g_{\text{crit}} \) a selectable threshold for the g factor. Figure 2.12 shows \( R_{\text{crit}} \) as a function of \( L/\lambda \) for brain conditions as well as for approximate lossless conditions with
2.2 Wave Optical MR Image Encoding

Figure 2.12: The critical reduction factor as a function of $L/\lambda$. The additionally plotted lines indicate regressions according to Eq. (2.53). The linear increase of $R_{\text{crit}}$ in the wave regime (i.e., for $L/\lambda > 1$) corresponds to the diffraction limit of wave optics (2.54).$^{16,17}$

g_{\text{crit}} set to 1.2 and 1.05, respectively. Again both curves reveal the two fundamental types of RF behavior. While $R_{\text{crit}}$ is essentially constant for near-field conditions, it starts to increase exactly linearly with the onset of wave behavior. These characteristics are summarized in the following simplified representations:

$$R_{\text{crit}} = \begin{cases} R^0_{\text{crit}} & \text{... near-field} \\ c_1 L/\lambda + c_2 & \text{... far-field} \end{cases}$$

(2.53)

The parameters $R^0_{\text{crit}}$, $c_1$, and $c_2$ were determined by fitting and are listed in the legend of Fig. 2.12. Note that the great difference in $R^0_{\text{crit}}$ reflects different $g_{\text{crit}}$ rather than effects of conductivity.

In parallel imaging with Cartesian k-space sampling the aliasing distance which has to be resolved is given by $L/R$ (cf. Fig. 2.1). In this sense, $R_{\text{crit}}$ corresponds to a minimum aliasing distance that can be resolved by means of parallel detection. This minimal aliasing distance represents the highest spatial resolution that could be achieved with optical means alone without incurring prohibitive SNR loss. Note that in the lossless case the slope $c_1$ of $R_{\text{crit}}$ was assessed as almost exactly equal
Figure 2.13: Model coil-object arrangement consisting of an RF coil placed at the surface of a lossless (i.e. $\sigma = 0$), infinite half-space.

Figure 2.14: Parallel imaging reconstruction in terms of net sensitivity. In order to reconstruct a pixel in the center $S_{Net}$ must be equal to 1 at this position and equal to 0 at all associated aliasing locations, which occur equidistantly with a spacing of $L/R$ (cf. Fig. 2.1). A basic mathematical function that fulfills this requirement is the sinc function $\text{sinc}(x \pi R/L)$. As shown on the right its spatial frequency range increases linearly with $R/L$.

to 2, while its intercept $c_1$ was negligible. Hence for the wave regime the observed resolution limit can be expressed equivalently as:

$$\frac{L}{R_{crit}} \approx \frac{\lambda}{2}.$$  

(2.54)

In this form, the performance limit of parallel MRI is identical to the well-known diffraction limit of wave optics,$^{16,17}$ underscoring the optical nature of sensitivity encoding.
Near-Field versus Far-Field Wave Propagation

For the purpose of physically interpreting the obtained results a simple coil-object configuration is considered. This arrangement is illustrated in Fig. 2.13 and consists of a lossless (i.e. $\sigma = 0$), infinite half-space with an RF coil placed at its surface. Without actually solving the corresponding Maxwell equations, the coil sensitivity just beneath the surface of the object (i.e. at $z=0$) shall be expressed in terms of a Fourier expansion according:\textsuperscript{51}

$$s(x, y, z = 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x dk_y w(k_x, k_y) e^{i(k_x x + k_y y)}. \quad (2.55)$$

In order to describe the propagation of $s(x, y, z=0)$ towards the inside of the half-space each Fourier term in Eq. (2.55) is conceptually regarded as a wave component of the form $e^{i(k_x x + k_y y + k_z z)}$. Consistency with the Maxwell equations can then be ensured by requiring $k_z$ to fulfill the condition:\textsuperscript{44,51}

$$k_x^2 + k_y^2 + k_z^2 = k_0^2, \quad (2.56)$$

with $k_0$ the material’s wave number, which is purely real for the lossless sample (2.49). Therefore, depending on the specific values of $k_x$, $k_y$ and $k_0$, $k_z$ can either be imaginary (i.e. for $k_x^2 + k_y^2 > k_0^2$) or real (i.e. for $k_x^2 + k_y^2 < k_0^2$). From a physical perspective this corresponds to exponentially damped near-field components and infinitely propagating far-field waves, respectively:

$$s(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x dk_y w(k_x, k_y) e^{i(k_x x + k_y y)} \left\{ \begin{array}{ll}
    e^{-\sqrt{k_0^2+k_x^2+k_y^2}} & \ldots \text{near-field} \\
    e^{\pm \sqrt{k_0^2-k_x^2-k_y^2}} & \ldots \text{far-field} 
    \end{array} \right. \quad (2.57)$$

Hence coil sensitivities can readily include high spatial frequency contents at shallow imaging locations. However, with increasing imaging depth the near-field components beyond $k_x^2 + k_y^2 > k_0^2$ decay exponentially.

Based on these considerations, the wave-optical resolution limit described in Eq. (2.54) becomes immediately comprehensible by exploring parallel imaging reconstruction in terms of the net sensitivity ($s_{\text{Net}}$). From this perspective, the unfolding operation is understood as linearly combining the individual coil sensitivities such that the resulting net sensitivity is equal to 1 at the position of the target pixel, while being equal to 0 at all aliasing locations. Accordingly, with the corresponding virtual net coil the unwanted aliasing signals would be automatically suppressed (cf. Fig. 2.1).

For a certain $L$ and $R$ the requirements for the net sensitivity along a certain aliasing direction $x$ can be achieved canonically with a sinc function of the form
This function implements the required one and zeros with a nearly minimal range of spatial frequencies required. As shown in Fig. 2.14 the Fourier transform of the sinc function involves frequencies up to $\pi R/L$. Hence, for constructing such a net coil sensitivity from propagating field components alone $k_0$ needs to be at least equal to $\pi R/L$. For the lossless sample considered here this translates into the following requirement (2.49):

$$\frac{2\pi}{\lambda} > \frac{R}{L},$$

which is again identical to the previously derived performance limit of parallel MRI (2.54).

On the other hand, for pure near-field conditions (i.e. $k_0 = 0$) the net sensitivity consists exclusively of evanescent contributions (2.57). In this regime, coil sensitivities can vary rapidly in space only near the object’s surface, while for deeper positions they are bound to grow increasingly flat. It is due to this exponential damping of high spatial frequencies that in near-field conditions parallel imaging with large reduction factors is feasible only for shallow imaging depths.

Recently it has been practically demonstrated that very high reduction factors can indeed be achieved when limiting the scope to the surface. In a very thin object MRI was performed with a 64-coil array and skipping phase encoding altogether, hence implementing 64-fold scan acceleration.
2.3 Approaching Ultimate SNR with Finite Coil Arrays

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Submitted to: Magnetic Resonance in Medicine.
Abstract

Recently the inherent signal-to-noise ratio (SNR) limits of both conventional and parallel MR imaging have been studied based on theoretical considerations regarding the physics of MR signal detection. This article addresses the question to which degree these SNR limits can actually be approached with a finite number of common receiver coils. For this purpose, the case of imaging a spherical object with a variable number of coil elements was numerically analyzed, incorporating the SNR effects of imperfect coil conductors and preamplifier circuitry. For both conventional and parallel imaging it was found that the SNR performance indeed approaches its theoretical limit as the number of coil elements increases. The number of coil elements required varies significantly with the pixel position within the imaged object and with radiofrequency (RF) regime characteristics. For parallel imaging it was consistently found that the number of elements required increases with the reduction factor. In certain conditions, noise contributions from coil conductors and electronics were found to considerably degrade the SNR efficiency. Coil overlap was found to generally increase the SNR performance of large arrays, while being subject to a trade-off between baseline SNR and geometry factor for smaller arrays.

Introduction

In the late 1980s the options for signal reception in MRI were dramatically expanded by the advent of receiver coil arrays. In the array approach the imaging region is covered with multiple coils having individual, spatially varying reception characteristics. Complemented by appropriate measures at the level of image reconstruction, array reception combines the high signal-to-noise ratio (SNR) efficiency of surface coils with the large field-of-view (FOV) coverage of volume coils. Parallel imaging techniques, such as SMASH, SENSE, and GRAPPA, additionally use the spatial variation of the coil characteristics for supplementary image encoding.

Recent theoretical investigations have aimed to determine the maximally achievable SNR performance both for conventional and parallel imaging. By studying the radiofrequency (RF) electrodynamics of signal and noise detection in MR, fundamental characteristics of ultimate SNR performance have been identified. These investigations focused on the so-called intrinsic SNR, which is obtained by accounting for noise only from within the imaged sample. The two main results were the following: Firstly, the performance of MRI is inherently limited, i.e., the intrinsic SNR as well as the range of feasible acceleration with parallel imaging have theoretical bounds. Secondly, both conventional and parallel imaging benefit significantly from the onset of wave behavior at high field strengths ($B_0$) and with
2.3 Efficiency of Finite Coil Arrays

large object sizes (L). Accordingly, whole-body (ultra-)high field systems promise greatly enhanced MR imaging capabilities due to both higher baseline SNR and higher feasible reduction factors.

The predicted B0 dependence of parallel imaging performance has recently been confirmed by experiments using the concept of electrodynamic scaling. However, it is still unclear to which extent the ultimate intrinsic SNR can be approached in practice. Several open questions and associated speculations have arisen, such as whether the ultimate intrinsic SNR can be approached with a finite number of coils arranged on a single surface, or if three-dimensional coil structures are required. It might be expected that with an increasing number of coils the intrinsic SNR increases continuously and ultimately converges towards some fixed value. However, it is unclear how fast this convergence will be and how close this limit will be to the theoretical one.

These questions are of significant practical importance given that MR receiver hardware is quite costly, commercial systems currently offering up to 32 independent channels. Another practical limitation consists in the fact that all RF hardware introduces electronic noise, which works against SNR optimization. Due to these additional noise contributions the overall SNR may actually decrease beyond some critical number of coils.

This article addresses the general question to which extent and how quickly ultimate SNR can be approached with finite coil arrays. In order to do so, a spherical imaging object is considered, surrounded uniformly by identical, circular receiver coil elements. This configuration combines two advantages. It permits semi-analytical expressions for the full-wave RF electrodynamic fields and allows comparisons with previous calculations of the ultimate intrinsic SNR. The role of the array size is investigated for both conventional and parallel imaging, incorporating noise contributions from coil conductors and preamplifier circuitry.

Theory

Signal-to-Noise Ratio for Phased Array Imaging

With respect to image reconstruction and SNR considerations, a receiver coil array is characterized by the spatially varying signal sensitivities of its elements, $s_\gamma(r)$, and its noise covariance matrix. The latter reflects the statistics of the noise voltage $V_{N,\gamma}$ at the receiver outputs:

$$\Psi_{\gamma,\gamma'} = V_{N,\gamma}^* V_{N,\gamma'},$$

(2.59)

where $\gamma$ and $\gamma'$ are indices counting the coil elements, the asterisk denotes complex conjugation, and the bar indicates averaging over time. Knowledge of $s_\gamma(r)$ and $\Psi$ permits combining single-coil images into a final image with pixel-wise optimal
SNR. In standard Fourier imaging with full-density k-space sampling the image combination involves only one pixel position at a time. In Cartesian parallel imaging with unfolding reconstruction, each step in the image combination involves multiple pixels, which form an equidistant aliasing clique. For both of these situations it is convenient to assemble the relevant sensitivity values in a sensitivity matrix:

\[ S_{\gamma,\rho} = s_{\gamma}(r_{\rho}), \quad (2.60) \]

where the index \( \rho \) counts the pixels involved and \( r_{\rho} \) denotes their positions. For non-parallel imaging this matrix has only one column, hence reducing to a vector. Based on these notions the SNR that can be achieved for a given pixel can be expressed as:

\[ \text{SNR}_{\rho} = \frac{M_{0,\rho} \omega}{\sqrt{\text{Red} \left[ \left( S^{H} \Psi^{-1} S \right)^{-1} \right]_{\rho,\rho}}}, \quad (2.61) \]

where \( M_{0,\rho} \) denotes the magnetic moment of the corresponding volume element, rotating at the Larmor frequency, \( \omega \), and \( \text{Red} \) denotes the reduction factor, describing the degree of k-space undersampling in parallel imaging. Based on Eq. (2.61), the loss of SNR efficiency suffered with parallel imaging vs. full-density k-space sampling is expressed by the geometry factor:

\[ g_{\rho} = \sqrt{\left( S^{H} \Psi^{-1} S \right)_{\rho,\rho} \left[ \left( S^{H} \Psi^{-1} S \right)^{-1} \right]_{\rho,\rho}}, \quad (2.62) \]

While each coil sensitivity \( s_{\gamma}(r) \) is essentially determined by the coil geometry and the sample to be imaged, several further mechanisms affect the noise statistics \( \Psi \). In the following, these various dependencies are worked into explicit formulas for the net \( s_{\gamma}(r) \) and \( \Psi \) at the analog output of a coil array. Typically, the SNR of raw MR signals changes little beyond the first stage of amplification. Therefore the derivation is limited to a simplified receiver pathway extending from the imaged sample to the first amplifier stage.

### Signal Sensitivity and Noise Modeling

Figure 2.15 shows an equivalent circuit diagram for the major components in the RF receiver pathway of a single coil element. Within this sketch we distinguish three stages at which the signal sensitivity and the noise statistics are considered: (I) beyond the intrinsic signal voltage source \( (V_{\text{sample}}) \) and the equivalent resistance \( (R_{\text{sample}}) \) of the sample, (II) beyond the resistance of the coil wire \( (R_{\text{coil}}) \), and (III) beyond the low-noise amplifier (LNA) with its associated impedance transformation network.
2.3 Efficiency of Finite Coil Arrays

Figure 2.15: Equivalent circuit diagram of a single-coil RF receiver pathway. This model accounts for the intrinsic signal voltage source ($V_{\text{sample}}$), the equivalent resistance of the sample ($R_{\text{sample}}$), the resistance of the coil wire ($R_{\text{coil}}$), as well as the low noise amplifier (LNA) with its associated impedance transformation network. The SNR efficiency has been evaluated at the three stages indicated as I, II, and III (cf. Tab. 2.3).

<table>
<thead>
<tr>
<th>Stage</th>
<th>Sensitivity</th>
<th>Noise Covariance</th>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$s'<em>I = s</em>{\gamma}^{\text{intr}}$</td>
<td>$\Psi^I = \Psi_{\gamma,\gamma'}^{\text{intr}} + \Psi_{\gamma,\gamma'}^{\text{coil}}$</td>
<td>SNR$^I$, $g^I$</td>
</tr>
<tr>
<td>II</td>
<td>$s''<em>I = s</em>{\gamma}^{\text{intr}}$</td>
<td>$\Psi''<em>{\gamma,\gamma'} = G</em>{\gamma,\gamma'} (k_{\gamma,\gamma'} \Psi_{\gamma,\gamma'}^{\text{LNA}} + \Psi_{\gamma,\gamma'}^{\text{LNA}})$</td>
<td>SNR$^{II}$, $g^{II}$</td>
</tr>
<tr>
<td>III</td>
<td>$s''''<em>I = G</em>{\gamma,\gamma'} s_{\gamma}^{\text{intr}}$</td>
<td>$\Psi'''<em>{\gamma,\gamma'} = G</em>{\gamma,\gamma'} (k_{\gamma,\gamma'} \Psi_{\gamma,\gamma'}^{\text{LNA}} + \Psi_{\gamma,\gamma'}^{\text{LNA}})$</td>
<td>SNR$^{III}$, $g^{III}$</td>
</tr>
</tbody>
</table>

Table 2.3: Signal sensitivity, $s$, and noise covariance, $\Psi$, evaluated at the three different stages of signal processing in the RF receiver pathway (cf. Fig. 2.15).

The first stage corresponds to the assumption of idealized, infinitely conductive coil elements. In this approximation only sample noise is taken into account; hence the SNR at this stage, SNR$^I$, is equal to the intrinsic SNR.69 Based on the principle of reciprocity, the corresponding intrinsic coil sensitivities $s_{\gamma}^{\text{intr}}(r)$ and the noise covariance $\Psi^{\text{intr}}$ can be expressed in terms of the RF electric $E$ and magnetic $H$ fields that the coils generate when operated individually with unit input current $I_0$ at the Larmor frequency $\omega$:

\[
s'_I(r) = s_{\gamma}^{\text{intr}}(r) = \mu^{\text{sample}}(r) \left[ \frac{H_{\omega,\gamma}(r)}{I_0,\gamma} - i \frac{H_{\omega,\gamma}(r)}{I_0,\gamma} \right], \tag{2.63}
\]

\[
\Psi^I = \Psi^{\text{intr}} = 4k_B T^{\text{sample}} B W R_{\text{sample}}, \tag{2.64}
\]

\[
R_{\gamma,\gamma'}^{\text{sample}} = \int_{\text{sample}} \sigma^{\text{sample}}(r) \frac{E_{\gamma}(r)}{I_{0,\gamma}} \cdot \frac{E^*(r)}{I_{0,\gamma}} d^3r, \tag{2.65}
\]
where the noise covariance was expressed through the noise resistance matrix of the sample, using Nyquist's formula.\(^{40,41}\) \(k_B\) denotes the Boltzmann constant, \(T_{\text{sample}}\) the temperature of the sample, and \(BW\) the signal bandwidth. \(\mu_{\text{sample}}(r)\) and \(\sigma_{\text{sample}}(r)\) denote the magnetic permeability and the conductivity of the sample, respectively.

At the second stage the finite conductivity of the coil elements is additionally accounted for. The individual coils are modeled as wires of length \(l\) with a circular cross-section of diameter \(d\). For common coil materials at RF frequencies the RF penetration depth is much smaller than typical coil wire diameters. Consequently, the current flows only in a thin layer beneath the surface and \(R_{\text{coil}}^c\) is given by the diagonal matrix:\(^{37}\)

\[
R_{\gamma,\gamma}^c = \frac{l}{\pi d \delta \sigma^c},
\]  
(2.66)

where \(\delta = \sqrt{\frac{2}{\mu^c_0 \sigma^c_0 \omega}}\) denotes the RF penetration or skin depth.\(^{44}\) \(\mu^c_0(r)\) and \(\sigma^c(r)\) denote the magnetic permeability and the conductivity of the coil, respectively. Similar to Eq. (2.64) the covariance of the coil noise is given by

\[
\Psi_{\gamma}^c = 4k_B T^c BW R_{\gamma}^c.
\]

(2.67)

Note that within this description proximity effects, related to additional current displacements arising from magnetic field coupling between successive coil windings, are not included since they are of significant importance only for coils with multiple turns.\(^{71,72}\)

Finally, at the third stage the amplifier is incorporated. In array coils, low-noise preamplifiers are commonly used for decoupling purposes\(^ {18}\) as well as to boost the signal voltages so that the subsequent cables and circuitry cause negligible SNR degradation. For optimal noise performance, each preamplifier must see an optimal source resistance \(R_{opt}\) at its input. To achieve this, each coil is typically connected to its corresponding LNA through an impedance transformation network whose voltage transformation ratio, \(k_\gamma\), is such that the coil’s output resistance is transformed to \(R_{opt}\) according to:\(^ {42}\)

\[
k_\gamma^2 \left( R_{\gamma,\gamma}^I + R_{\gamma,\gamma}^c \right) = R_{opt}.
\]

(2.68)

Note that in general this condition results in a different value of \(k_\gamma\) for each coil. With respect to the signal voltage, the effect of the impedance transformation network can be accounted for by multiplying the corresponding coil sensitivity with \(k_\gamma\).
Impedance matching minimizes SNR loss through the LNA. Nevertheless the noise added by the amplifiers can still be significant. Noise contributions from different LNAs are mutually independent, hence their covariance matrix is diagonal:

$$\psi_{\gamma,\gamma'}^{LNA} = k_\gamma^2 \left( \psi_{\gamma,\gamma}^I + \psi_{\gamma,\gamma}^{coil} \right) (F - 1), \quad (2.69)$$

where $F$ is the amplifier's noise figure. Let $G_\gamma$ denote the gain of the LNA of coil $\gamma$, then the coil sensitivities and the noise covariance at stage III read

$$s_\gamma^{III} = G_\gamma k_\gamma s_\gamma^I(r), \quad (2.70)$$

$$\psi_{\gamma,\gamma'}^{III} = G_\gamma G_{\gamma'} \left[ k_\gamma k_{\gamma'} \left( \psi_{\gamma,\gamma'}^I + \psi_{\gamma,\gamma'}^{coil} \right) + \psi_{\gamma,\gamma'}^{LNA} \right]. \quad (2.71)$$

Substituted into Eqs. (2.60-2.62), these expressions permit studying the SNR performance of conventional and parallel MRI with real coil arrays. Table 2.3 summarizes the evolution of $s_\gamma^I(r)$ and $\psi$ throughout the three stages.

Radiation losses have not been taken into account in the above formalism for the following reasons. At low and moderate $B_0$ radiation losses are generally negligible. At very high $B_0$ where radiation losses become potentially significant they can be effectively reduced by suitable RF shielding or ground planes.\textsuperscript{73}
Table 2.4: Normalized coil radius $b/c$ and packing efficiency $\Omega/(4\pi)$ for various numbers of coil elements (cf. Fig. 2.16).

<table>
<thead>
<tr>
<th>Number Coils</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>32</th>
<th>40</th>
<th>52</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b / c [%]$</td>
<td>81.6</td>
<td>57.9</td>
<td>52.5</td>
<td>41.0</td>
<td>38.4</td>
<td>34.1</td>
<td>31.8</td>
<td>26.1</td>
<td>22.6</td>
<td>20.8</td>
</tr>
<tr>
<td>$\Omega / (4\pi) [%]$</td>
<td>84.5</td>
<td>74.0</td>
<td>89.4</td>
<td>70.2</td>
<td>76.7</td>
<td>72.1</td>
<td>83.2</td>
<td>69.3</td>
<td>67.0</td>
<td>70.1</td>
</tr>
</tbody>
</table>

Methods

Geometrical Arrangement

The SNR performance of finite coil arrays was investigated for a standard coil and object configuration consisting of a homogeneous, spherical model of the human head surrounded by $N$ identical, circular coil elements. The $N$ coils were equally distributed on a spherical shell that concentrically encloses the spherical head model. The left plot in Fig. 2.16 shows the geometrical arrangement for a single element. Here $L$ refers to the diameter of the sphere, $b$ to the coil radius and $c$ to the distance between the center of the sphere and the coil conductors. An approximately even distribution of $N$ such coils was generated by numerical optimization. In this procedure the coils’ center coordinates $r_i$ were treated as point charges on the spherical shell with radius $c$ and their repulsive potential was minimized according to:

$$\min_{r_1,\ldots,r_N} \sum_{i,j:i\neq j}^N \frac{1}{|r_i - r_j|}$$  \hspace{1cm} (2.72)

Unless stated otherwise, the coil radius was chosen constant for all coils such that the closest neighbors just touched but did not overlap. In the middle and right plots of Fig. 2.16 sample coil arrangements are shown for $N = 12$ and $N = 32$, respectively. The coil packing efficiency for a given $N$ can be expressed as the solid angle, $\Omega$, enclosed by all coil elements, divided by the full solid angle of $4\pi$. Table 2.4 shows the relative coil radius, $b/c$, as well as the packing efficiency, as a function of $N$ for $c = 0.55*L$. The packing efficiency varies significantly with $N$, reaching a maximum of almost 90% for $N = 12$ where the coils are packed in a regular pentagon-dodecahedral pattern with five closest neighbors for each coil (cf. Fig. 2.16).

RF Field Calculations

As a consequence of the highly symmetric arrangement with a spherical sample and circular coil elements, the RF electrodynamic fields are identical for all coils when
2.3 Efficiency of Finite Coil Arrays

viewed in individual coil-based coordinates. The full-wave RF electrodynamic fields in the single-coil arrangement (cf. Fig. 2.16 left) can be expressed in the form of a semi-analytical multipole expansion as shown by Keltner et al., and others. The RF fields of the entire coil array in laboratory coordinates can then be readily determined through appropriate rotations of the single-coil solution. Based on the full-wave RF fields of the complete array the intrinsic coil sensitivities and noise covariance were determined according to Eqs. (2.63, 2.64).

Dimensions and Physical Properties

The frequency-dependent conductivity and permittivity of the spherical sample were chosen equal to average in-vivo brain values as given in reference. The coil elements were modeled as circularly bent copper wires (\( \sigma_{\text{coil}}^{-1} = 1.69 \times 10^{-8} \, \text{Qm} \)) of circular cross-section with a diameter of \( d = 2 \, \text{mm} \). The radius of the spherical shell that contained the coil conductors was set to \( c = 0.55 \times L \), corresponding to a distance between the object and the coils of \( 0.05 \times L \). Identical LNAs were assumed for all coils with a noise figure of \( NF = 10 \log_{10} F = 1.1 \, \text{dB} \) throughout the relevant frequency range. Due to spherical symmetry and since the coil elements were identical, the diagonal elements of \( \Psi_{\text{intr}} \) and \( \Psi_{\text{coil}} \) were the same for each coil. The temperature of the sample, the coils and the LNAs was set to room temperature of 293 K.

Numerical Study

SNR and g factors were calculated for the three consecutive stages of the receiver pathway as illustrated in Fig. 2.15, using the Eqs. summarized in Tab. 2.3. Ultimate intrinsic SNR (\( \mu_{\text{SNR}} \)) and ultimate intrinsic g factors (\( \mu_{g} \)) were calculated using the methods previously described in reference.

Results

Noise Components

Figure 2.17 shows the individual noise contributions, i.e. the diagonal element of the noise resistance matrices \( R_{\text{sample}} \), \( R_{\text{coil}} \), and \( R_{\text{LNA}} \) as well as their sum, \( R_{\text{H}} \), as a function of \( B_0 \). Three different coil radii of \( b = 9.0 \, \text{cm} \) (left), 4.5 cm (middle) and 2.3 cm (right) are considered. For an object size of \( L = 0.2 \, \text{m} \) these are the coil radii for array sizes of \( N = 4 \) (left), 16 (middle) and 64 (right), respectively. In order to identify power laws of the form \( R_{\gamma,\gamma} = \alpha B_0^n \), a double logarithmic representation was chosen since such behavior is then revealed by a linear increase: \( \log R_{\gamma,\gamma} = \log \alpha + n \log B_0 \). \( R_{\text{sample}} \) increases approximately proportional to \( B_0^{3.3} \), whereas \( R_{\text{coil}} \) increases comparatively slower (\( \propto B_0^{0.5} \)) as a consequence of the skin
Figure 2.17: Individual noise resistances ($R_{\text{sample}}$, $R_{\text{coil}}$, $R_{\text{LNA}}$, as well as their sum $R_{\text{III}}$) as functions of $B_0$ for coil radii of $b = 9.0$ cm (left), 4.5 cm (middle) and 2.3 cm (right). For an object size of $L = 0.2$ m this corresponds to $N = 4$, 16 and 64 coil elements, respectively. Over a wide range the total noise resistance, $R_{\text{III}}$, is sample-dominated. Only for very small values of $B_0$ and $b$ the coil resistance $R_{\text{coil}}$ is dominant.

The $B_0$ dependence of $R_{\text{LNA}}$ is determined by the previous two according to Eq. (2.69). The total noise resistance, $R_{\text{III}}$, is generally sample-dominated at high $B_0$ and coil-dominated for small values of $B_0$. Additional parameters which tend to increase $R_{\text{coil}}$ relative to $R_{\text{sample}}$ are small values of $L$, $\sigma_{\text{sample}}$, $\sigma_{\text{coil}}$, and $d$, as well as large conductor lengths $l$ (cf. Eqs. (2.65,2.66)). It is interesting to note that after a steep increase $R_{\text{sample}}$ levels off at very high $B_0$ (Fig. 2.17 left), which may be related to destructive interferences associated with RF wave behavior.
2.3 Efficiency of Finite Coil Arrays

SNR

Figure 2.18 illustrates how the intrinsic SNR\(^I\) approaches its ultimate counterpart uSNR\(^I\) as the number of coil elements increases, for a transverse equatorial image plane and field strengths of \(B_0 = 1.5\) T (top row) and 7.5 T (bottom row). The maps show SNR\(^I\) normalized by the corresponding uSNR\(^I\). According to these results, with increasing number of coil elements SNR\(^I\) appears to generally approach the uSNR\(^I\). With few coils, SNR\(^I\) is closer to its ultimate limit at \(B_0 = 1.5\) T than at 7.5 T, suggesting that larger arrays are required for approaching optimal SNR performance at high field. With the maximal \(N\) of 64, SNR\(^I\) approximates the corresponding uSNR\(^I\) quite closely, both in terms of symmetry and with respect to the actual SNR values. Nevertheless, the finiteness of the coil array still limits the SNR at the surface of the object and causes characteristic indentations in the maps that indicate the positions of coil conductors. In the center of the object, the uSNR\(^I\) is nearly reached with few coils already, whereas significantly more coils would be required for accomplishing the same for pixel positions close to the surface.

The impact of the three major noise sources (i.e. the sample, the coils, and the amplifiers) on the SNR efficiency at the sphere's center is studied in Fig. 2.19. It shows plots of the SNR\(^I−III\), normalized by the corresponding uSNR\(^I\), as a function of \(N\) and for various combinations of \(B_0\) and \(L\). These plots again illustrate that in the center the intrinsic SNR indeed converges to its theoretical limit as \(N\) increases.
Figure 2.19: SNR efficiency, $\text{SNR} / u\text{SNR}^I$, in the center as a function of the number of coil elements $N$. Top: $\text{SNR}^I$, middle: $\text{SNR}^II$, bottom: $\text{SNR}^III$. Three different object sizes $L$ and four different field strengths $B_0$ are considered. While SNR convergence is rather fast for near-field RF conditions (i.e. small values of $B_0$ and/or $L$) it is significantly delayed with the onset of RF wave behavior. For coil-noise dominated conditions (i.e. $B_0 = 1.5$ T and $L = 0.1$ m), $\text{SNR}^II$ decreases beyond some critical $N$ (cf. Fig. 4.3). The impact of $\Psi^{LNA}$ on the $\text{SNR}^III$ is most significant for high values of $B_0$ and/or $L$ where it impairs the $\text{SNR}^III$ considerably.

(top row). However, the rate of convergence depends significantly on $B_0$ and $L$, i.e., on the object size relative to the RF wavelength. In the near-field regime, i.e. at small values of $B_0$ and/or $L$ the $\text{SNR}^I$ efficiency is relatively high and roughly independent of $B_0$ and $L$. In these situations a $\text{SNR}^I$ efficiency larger than 94% is achieved with as few as 8 coils. Conversely, as wave behavior becomes more accentuated by increasing $B_0$ and/or $L$, the $\text{SNR}^I$ efficiency decreases and the convergence of $\text{SNR}^I$ to its ultimate value is increasingly delayed and potentially incomplete.

When coil conductor noise is included the corresponding $\text{SNR}^II$ is significantly
2.3 Efficiency of Finite Coil Arrays

Figure 2.20: Maps of the intrinsic geometry factor ($g^I$) in an equatorial transverse plane for $B_0 = 1.5$ T (top row) and 7.5 T (bottom row), assuming $L = 0.2$ m and $\text{Red} = 3.6$. Four different numbers of coil elements are considered together with the ultimate analogue on the right. The numbers in brackets above each map indicate the mean and the maximum $g$ factor. For each map the linear color scale was individually adjusted to the actual range of $g$ values.

Degraded only for small $B_0$ and/or $L$; i.e. $B_0 = 1.5$ T and $L = 0.1$ m. It appears that, in these coil noise dominated conditions, there exists a critical number of coil elements beyond which the SNR$^H$ efficiency actually begins to decrease. This effect can be attributed to the fact that the total length of coil conductor increases with $N$, hence taking an increasing toll on the SNR (2.66).

When LNA noise contributions are also included, the corresponding SNR$^H$ behavior degrades further, both in terms of SNR values and convergence speed. This is most significant for high values of $B_0$ and/or $L$ where the SNR still improves significantly with $N$ even for very large arrays. Note that the presence of slight oscillations in the plots of Fig. 2.19 is due to the different packing efficiencies obtained for different $N$ (cf. Tab. 2.4).

Geometry Factor

Figure 2.20 shows maps of the $g$ factor at stage I ($g^I$) for a reduction factor of $\text{Red} = 3.6$, an object size of $L = 0.2$ m, several different $N$, and $B_0 = 1.5$ T (top row) and 7.5 T (bottom row). For comparison the corresponding ultimate $g$ factor ($ug^I$) is also shown on the right. Above each map the mean and the maximum $g$ factor.
are given in brackets. The color map was adjusted individually to the actual range of g values. Apparently, with increasing N the g factor improves considerably. At small N, $g^I$ exhibits some distinct, local maxima for locations close to the surface of the object and corresponding aliasing positions. However, as the number of coils increases the significance of these peaks diminishes and the largest g factors occur around the center. For the maximal N of 64, $g^I$ already reveals some similarity to the corresponding ultimate analogue, both with respect to the structure of the map and to the range of g values.

For investigating the impact of non-intrinsic noise on the g factor, Fig. 2.21 compares $g^I$ with $g^{III}$ at the center. Calculations are shown for four different $B_0$ and four different N, assuming a constant L of 0.2 m. For comparison, corresponding $ug^I$ factors are also drawn. Again, with increasing N the g factor continuously improves toward $ug^I$. However, the speed of convergence decreases with increasing Red. Generally, $g^I$ and $g^{III}$ deteriorate drastically as soon as Red reaches the range where $ug^I$ also deviates from the optimum value of 1. According to these results, the impact of coil and amplifier noise on the g factor is generally adverse but small. Only for low $B_0$, $g^{III}$ is appreciably worse than $g^I$. This observation is in accordance with increasing sample-noise domination at high $B_0$ (cf. Fig. 2.17).
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Figure 2.22: Impact of coil overlap on SNR performance for conventional imaging (left) and parallel imaging with Red = 4 (right), calculated for $B_0 = 1.5$ T and $L = 0.2$ m. In each case $SNR^{III}$ is evaluated for different amounts of coil overlap, i.e. $b = 0.8*b_0$ (with gap), $b = 1.0*b_0$ (coils touching), $b = 1.2*b_0$ (coils overlapping), and for different $N$. The numbers in brackets indicate the $SNR^{III}$ in the center (left) and the overall mean $SNR^{III}$ (right) as percentages relative to the case of $N = 8$ and $b = b_0$. In each half of the figure a consistent linear color scale was used ranging from $\min[SNR(N = 8, b = b_0)]$ to $2*\max[SNR(N = 8, b = b_0)]$. Interestingly, large arrays appear to generally favor coil overlap, both for conventional and parallel imaging.

Coil Overlap for Conventional and Parallel Imaging

The impact of coil overlap on the $SNR^{III}$ performance is illustrated in Fig. 2.22, considering three different coil radii, $b=0.8*b_0$ (with gap), $b=1.0*b_0$ (coils touching), $b=1.2*b_0$ (coils overlapping) and four different $N$. The calculation was performed for $B_0 = 1.5$ T and $L = 0.2$ m. In the left half of the figure $SNR^{III}$ is
plotted for conventional imaging, whereas the right half shows SNR$_{III}$ for parallel imaging with Red = 4. The numbers in brackets above each plot indicate the central SNR$_{III}$ (left) and the overall mean SNR$_{III}$ (right). These numbers are given as percentages relative to the case of N = 8 and b = b$_0$, which served as a reference.

For conventional imaging with Red = 1, both the central and the mean SNR$_{III}$ increase up to N = 32. However increasing N further to N = 64 causes only the mean SNR$_{III}$ to also increase, whereas central SNR$_{III}$ slightly decreases (cf. Fig. 2.19). The impact of coil overlap on SNR$_{III}$ was generally small and different for the central and more peripheral regions. While the central SNR$_{III}$ favors coil overlap, the periphery benefits from the use of smaller coils, which result from introducing coil gaps.

The SNR$_{III}$ performance is somewhat different for parallel imaging with Red = 4. In this case both the central and the mean SNR$_{III}$ increase monotonically with N without reaching a maximum. Furthermore, in the parallel case the role of coil overlap on SNR$_{III}$ was found to be more ambiguous and dependent on the number of coil elements. For the smallest N of 8, both the central and the mean SNR$_{III}$ are highest with coil gap. This observation is consistent with previous findings suggesting that gapped coils are more favorable for parallel imaging. However, for large N of 32 and 64 the SNR$_{III}$ in the center was highest with coil overlap. At N = 64 even the mean SNR$_{III}$ no longer favors coil gaps but was highest for the case where adjacent coils just touched.

It is important to note that with large arrays coil overlap yielded the highest SNR$_{III}$ in the center, both for conventional and parallel imaging. The center region is usually most critical in terms of SNR, suggesting that large arrays should generally be overlapping.

Discussion

SNR Performance of Finite Coil Arrays

Most importantly, this investigation indicates that with increasing number of coil elements the intrinsic SNR indeed approaches its theoretical limit. For conventional imaging the ultimate SNR$^I$ in the center was almost reached with few coils already, at least in near-field conditions (cf. Figs. 2.18, 2.19). For such conditions, four coils already yield $\geq 84$ % and 12 even $\geq 98$ % of the uSNR$^I$ in the center (cf. Fig. 2.19 top, left subplot). However, when entering the RF wave regime at high $B_0$ and/or large L, the number of coils required to achieve the same SNR efficiency is much larger. Wave conditions permit field focusing, and therefore an SNR increase which is stronger than linear with $B_0$. Apparently, for taking full advantage of this benefit significantly more coil elements are needed. While for central locations the SNR$^I$ convergence is rather quick, the speed of convergence
significantly decreases for locations closer to the surface. This behavior reflects the fact that in order to maximize SNR at the surface many small coils with highly localized sensitivities are required. In other words, increasing N predominantly improves the SNR for superficial regions, while the SNR in the center improves only marginally.

In addition to intrinsic thermal object noise, noise contributions arising from finite coil conductivity and LNA electronics have also been modeled. Over a wide range of $B_0$ and L sample noise was found to be the most significant noise contribution. As expected, only for low values of $B_0$ and/or L coil conductor noise was significant as well (cf. Fig. 2.17). In this situation a significant deviation in SNR convergence behavior was found, namely that beyond a certain critical number of coil elements $SNR^II$ actually decreases with increasing N (cf. Fig. 2.19 middle, left subplot). In this case the benefits of increased coil sensitivity are offset by even greater increases in noise originating from the coil conductors. Note however that unlike typical imaging objects, the coil is generally accessible for technical improvements, such as increasing the coil diameter, choosing a more conductive material, or cooling the coil elements. When LNA noise is also taken into account the corresponding $SNR^{III}$ was found to degrade significantly. This effect is most apparent in pronounced wave conditions such as $L = 0.3$ m and $B_0 = 10.5$ T, where many coils are needed for approaching the ultimate SNR limit (cf. Fig. 2.19 bottom, right subplot).

**Geometry Factor Performance of Finite Coil Arrays**

With increasing number of coil elements also the g factor for parallel imaging was found to improve significantly. In accordance with intuition, Fig. 2.21 indicates that for high amounts of data reduction the convergence is delayed toward higher N. Hence, the SNR benefits of large coil arrays are higher for parallel imaging than for conventional imaging (cf. Fig. 2.22). Furthermore, for smaller N the g factor shows certain specific peaks for locations close to the surface and corresponding aliasing locations. With increasing N these local peaks disappear and the g factor reaches the corresponding ultimate values up to a few percent. Nevertheless, even for the maximum N of 64, the g factor still shows the characteristic aliasing structure (cf. Fig. 2.20). With onset of wave behavior the ultimate geometry factor begins to improve significantly, hence permitting higher feasible reduction factors. However, as opposed to the SNR, this g-factor improvement does not require more coil elements. The impact of coil and LNA noise on the g factor is comparatively small (cf. Fig. 2.21). While at small $B_0$, $g^{III}$ tends to be slightly larger than $g^I$, for higher $B_0$ this difference is almost negligible.
Coil Overlap versus Coil Gap

The strategies used so far for the arrangement of individual array elements differ somewhat between conventional and parallel imaging. While for conventional imaging coil overlap is usually applied to minimize inductive coil coupling, for parallel imaging it was found that gaps between neighboring coil conductors can be beneficial. In the present study these common coil array design paradigms were tested by calculating SNR for Red = 1 as well as Red = 4 (cf. Fig. 2.22). For no reduction, the generally critical central locations experience an SNR increase, while outer regions are affected by a slight SNR reduction. Furthermore, SNR was found to be more homogeneous with a coil overlap.

Similarly, the favored use of a coil gap for parallel imaging with Red = 4 has been confirmed for small N. However, when increasing the number of coil elements this situation changes. For large N coil overlap becomes more SNR-efficient, especially for the SNR-critical central locations. This may reflect the enhanced redundancy of sensitivity encoding provided by large coil arrays. While for small N neighboring coils generally have difficulty distinguishing signals from regions beneath a coil overlap, for larger N signals from such locations can be separated effectively with the aid of further coil elements in the near neighborhood. For large arrays, coil overlap is hence the preferred choice for both conventional and parallel imaging. This significantly simplifies array construction, permitting inductive decoupling of nearest neighbors, thus reducing the necessity for more complex decoupling schemes.

Conclusions

In the present work, the SNR performance of finite coil arrays has been studied theoretically and compared to ultimate, intrinsic SNR limits as determined by the physics of MR signal detection. For this purpose a model configuration consisting of a homogeneous, spherical object surrounded by identical, circular coil elements has been analyzed. Investigations have been performed both for conventional and parallel imaging. The obtained results suggest the following main conclusions:

1. With increasing number of coil elements both the SNR and the g factor approach their ultimate values. However, the number of coils required to do so varies significantly for different parts of the imaged object and different regimes of RF wave behavior. Generally, ultimate SNR performance was achieved significantly faster for near-field conditions (i.e. small values of B_0 and/or L) and central locations.

2. The impact of noise contributions from coil conductors and preamplifiers was found to be significant only for the SNR, while hardly affecting the g factor.
2.3 Efficiency of Finite Coil Arrays

Under certain conditions these contributions considerably degraded the SNR efficiency. However, unlike sample noise, coil noise and amplifier noise can generally be reduced by technical improvements.

3. With increasing number of coil elements the impact of coil overlap on parallel imaging performance changes. While for small $N$ coil overlap is problematic, for large $N$ it is actually beneficial. This suggests that overlapping neighboring coils should be the preferred strategy for both conventional and parallel imaging as long as the number of coil elements is large enough.
Chapter 3

Experimental Investigations

3.1 Parallel Imaging Performance as a Function of Field Strength - An Experimental Investigation Using Electrodynamic Scaling

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Abstract

In this work, the dependence of parallel MRI performance upon main magnetic field strength is experimentally investigated. Using the general framework of electrodynamic scaling, the $B_0$-dependent behavior of the relevant radiofrequency fields is studied at a single physical field strength of 7 Tesla. In the chosen implementation this is accomplished by adjusting the permittivity and conductivity of a homogeneous spherical phantom. With different mixing ratios of Decane, Ethanol, Purified Water, N-Methylformamide, and Sodium Chloride, field strengths in the range of 1.5 to 11.5 Tesla are mimicked. Based on sensitivity maps of an 8-coil receiver array, the field-dependent performance of parallel imaging is assessed in terms of the geometry factor $g$, which reflects noise enhancement in parallel imaging reconstruction. At low field strengths the SNR penalty was nearly independent of $B_0$ and favorably low for 1D reduction factors up to between 3 and 4. At higher field strengths the transition between favorable and prohibitive parallel imaging conditions was found to shift towards higher feasible reduction factors. These findings are in good agreement with previous theoretical predictions. From this agreement it is concluded that parallel MRI at high $B_0$ benefits specifically from onsetting far-field behavior of the involved radiofrequency fields.

Introduction

In recent years, parallel imaging with receiver coil arrays has been established as a flexible means of enhancing MRI exams.\(^1\)\(^,\)\(^2\)\(^,\)\(^3\)\(^,\)\(^4\)\(^,\)\(^5\)\(^,\)\(^6\)\(^,\)\(^7\)\(^,\)\(^8\)\(^,\)\(^9\)\(^,\)\(^10\)\(^,\)\(^11\)\(^,\)\(^12\)\(^,\)\(^13\)\(^,\)\(^14\)\(^,\)\(^15\)\(^,\)\(^16\)\(^,\)\(^17\)\(^,\)\(^18\)\(^,\)\(^19\)\(^,\)\(^20\)\(^,\)\(^21\)\(^,\)\(^22\)\(^,\)\(^23\)\(^,\)\(^24\)\(^,\)\(^25\)\(^,\)\(^26\)\(^,\)\(^27\)\(^,\)\(^28\)\(^,\)\(^29\)\(^,\)\(^30\) Permitting significant reduction in the density of k-space sampling, parallel techniques enable faster scanning, improved resolution and coverage, artifact suppression, and even the mitigation of acoustic noise. With various implementations available, such as SMASH (SiMultaneous Acquisition of Spatial Harmonics), SENSE (SENSitivity Encoding), and GRAPPA (GeneRalized Autocalibrating Partially Parallel Acquisitions), parallel imaging is now increasingly introduced into clinical practice. Another recent development in MRI is the increasing use of very high field strength.\(^1\)\(^,\)\(^2\)\(^,\)\(^3\)\(^,\)\(^4\)\(^,\)\(^5\)\(^,\)\(^6\) With the advent of 7 Tesla and 8 Tesla whole-body magnets, the vast potential and the technical challenges of ultra-high-field MRI continue to drive large-scale research efforts.

Parallel and high-field MRI are particularly promising when combined with one another. This is because the two approaches exhibit a high level of complementarity with respect to their favorable and less favorable characteristics. For instance, the major assets of imaging at ultra-high field are increased baseline signal-to-noise ratio (SNR) and enhanced $T_2^*$ contrast. These benefits come at the expense of stronger field inhomogeneity and higher radiofrequency (RF) power deposition. Field inhomogeneity causes artifacts and blurring in sequences with
long acquisition intervals, e.g., in echo-planar techniques. Power deposition is limiting when too many RF pulses are required for completing a scan in a given time. Both problems can be tackled effectively by parallel acquisition, enabling sequences with faster readouts and fewer excitation pulses. Likewise, enhanced acoustic noise at high field can be mitigated with sparser k-space trajectories.

The major downside of parallel imaging, in turn, is its SNR behavior. Except for a few special situations, parallelizing a given MRI method reduces its SNR yield due to reduced intrinsic data averaging and non-unitary image reconstruction. One important approach for counteracting these SNR losses is coil array optimization. Significant improvements in overall SNR performance can be achieved both by improving the geometric coil configuration and by increasing the number of coil elements. However, the SNR gains that can be achieved by changing the coil design are limited. Therefore, the substantial SNR benefits of higher field strength add significantly to the potential of parallel techniques.

In addition to these basic arguments there are indications that high-field conditions favor the concept of parallel acquisition in yet another, more sophisticated way. Recently, several theoretical investigations have aimed to explore the limitations of parallel techniques in terms of the achievable factors of k-space density reduction. In order to study these limitations beyond the imperfections of coil arrays, the concept of ultimate SNR was extended towards parallel imaging. The key findings of this approach can be readily summarized by means of Fig. 3.1, which was obtained in the fashion described in reference. This figure shows the ultimate performance of the parallel SENSE technique in a spherical object of 18 cm in diameter. The performance measure here is the so-called geometry factor, which quantifies noise enhancement in SENSE reconstruction. Plotted versus the reduction factor and \( B_0 \), the ultimate geometry factor reveals two distinct operating regimes. The first, favorable regime is characterized by low g factors close to the optimal value of 1. The second, unfavorable regime is characterized by exponential growth of the ultimate g factor, which sets in beyond some critical degree of reduction. For low field strength, the critical reduction factor lies between 3 and 4. However, at higher field strength the transition between the two regimes shifts towards higher reduction factors. According to theory the reason for this favorable effect lies in the transition between the near- and far-field regimes of the detected RF signals.

If practically achievable, the predicted increase in parallel imaging performance would add significantly to the potential of high field MRI. However, the prediction is purely theoretical so far. In particular, it is not clear whether the benefit, which was derived for an idealized coil array, will also be appreciable with a limited number of real coils. Therefore an experimental verification of the described effect is called for.
Figure 3.1: Ultimate parallel imaging performance in terms of the geometry factor, according to reference.\textsuperscript{60} $R$ denotes the factor of k-space undersampling. Calculated for a homogeneous spherical object with a diameter of 18 cm, assuming average in vivo brain permittivity and conductivity. The dashed line indicates the transition from favorable to prohibitive geometry factors, with the threshold set arbitrarily at $g = 1.2$. According to this analysis, high field is expected to enhance the performance range of parallel imaging.

Ideally, such a verification should be performed at various field strengths, spanning at least the $B_0$ range indicated in Fig. 3.1. Furthermore, for a fair comparison a single coil array geometry should be identically reproduced for each field strength. The technical demands of such a study are daunting. In particular, sufficiently large magnets with field strengths of 10 Tesla and beyond are not available yet.

In the present work the required experimental verification is performed with an alternative approach. It is based on mimicking the effects of varying $B_0$ by modifying the electrodynamic characteristics of the experimental setup. The underlying
3.1 Parallel Imaging Performance as a Function of Field Strength

phenomenon is known as electrodynamic similitude in electrical engineering; the related method will be referred to as electrodynamic (ED) scaling in this work. It enabled studying the performance of parallel imaging at mimicked field strengths between 1.5 T and 11.5 T, using a single coil array setup in a 7 T magnet.

Theory

Parallel Imaging Performance and Electrodynamics

In this section, the links between parallel imaging performance and the electrodynamics of RF fields shall be briefly reviewed. Relative to conventional Fourier imaging, parallel techniques generally yield reduced SNR. For the SENSE technique the loss in SNR has been quantified as:

$$SNR_{\text{SENSE}} = \frac{SNR_{\text{full}}}{\sqrt{Rg}}$$

(3.1)

where $SNR_{\text{SENSE}}$ and $SNR_{\text{full}}$ denote the SNR obtained with parallel imaging and the SNR that would be obtained with full gradient encoding, respectively. R denotes the factor by which the k-space sampling density is reduced, and g is the local geometry factor. The geometry factor reflects the suitability of the coil array for complementing the reduced gradient encoding and is hence a measure of parallel imaging performance. For Cartesian k-space sampling, g can be formally expressed as follows:

$$\rho = \sqrt{[(S^H \Psi^{-1} S)^{-1}]_{\rho,\rho} (S^H \Psi^{-1} S)_{\rho,\rho}} \geq 1$$

(3.2)

where the index $\rho$ counts a set of pixels that alias in conventional reconstruction, S denotes a matrix of coil sensitivity values, and $\Psi$ denotes the noise covariance matrix of the receiver array. Using the principle of reciprocity, the coil sensitivities and the noise covariance can be expressed in terms of each coil’s hypothetical electromagnetic transmit fields when driven with unit input current of the respective Larmor frequency. The coil sensitivity corresponds to the transverse components of the magnetic transmit field:

$$s_c(r) = \mu(r)(H_x^c(r) - iH_y^c(r))$$

(3.3)

where r denotes the position in 3D space, $\mu$ denotes the magnetic permeability of the sample, $c$ is the coil index, and $H_x^c$, $H_y^c$ denote the x- and y-components of coil c’s magnetic transmit field. Assuming that noise arises predominantly from thermal motion of charges in the sample, the noise matrix $\Psi$ can be expressed in terms of the electric transmit fields:

$$\Psi_{c,c'} = \int_{\text{sample}} \sigma(r) E^c(r) \cdot \overline{E^{c'}(r)} d^3r$$

(3.4)
where $\sigma$ denotes the conductivity of the sample, $\mathbf{E}^c$ is coil $c$'s electric transmit field, and the bar indicates complex conjugation.

**Electrodynamic Scaling**

According to the previous considerations, parallel imaging performance, when quantified through the geometry factor, depends solely on the electrodynamics of the receiver coils and the sample. Hence, changes in $B_0$ can be mimicked by adjusting other parameters such that the electrodynamics change in the same way. Note that changes in $B_0$ affect the RF electrodynamics only via the Larmor frequency $\omega$. Other effects, such as on $B_0$ inhomogeneity or baseline magnetization do not alter coil sensitivities or noise covariance.

For instance, with increasing $B_0$ the Larmor frequency increases, hence the RF wavelength and skin depth decrease. The resulting enhanced interference effects can equally be created by leaving $B_0$ and $\omega$ unchanged and rather increasing the sample size or decreasing the speed of light in the phantom. Similarly, reduced skin depth, corresponding to reduced RF penetration, can be mimicked by enlarging the sample or enhancing the sample's conductivity. Recently, Yang et al. introduced a related method for the design of brain phantoms. For mimicking high-field RF effects they proposed adjusting the phantom size and its conductivity. In the following, this concept is generalized such as to permit ED scaling also in samples of constant size.

The target configuration to be mimicked is characterized by the shape and size of the target sample, its material properties $\mu_t$ (permeability), $\epsilon_t$ (permittivity), and $\sigma_t$ (conductivity), the coil array, and the target field strength, $B_{0,t}$. The target frequency is then determined by the target nucleus with the gyromagnetic ratio $\gamma_t$. The model configuration must be geometrically identical to the target configuration, except for an isotropic length scaling. This relationship is illustrated in Fig. 3.2. Defining some characteristic length in the target configuration, $L_t$ (e.g. the diameter for a sphere), the relative length scaling is determined by the corresponding characteristic length in the model configuration, $L_m$. Then the goal of the ED scaling operation is to choose the remaining model parameters, $\mu_m$, $\epsilon_m$, $\sigma_m$, $\gamma_m$, $B_{0,m}$ such that for each coil the resulting magnetic and electric fields in the two configurations fulfill the following relations:

$$
\mathbf{H}_m \left( \frac{L_m}{L_t} \mathbf{r} \right) = c_H \mathbf{H}_t(\mathbf{r}), \quad \mathbf{E}_m \left( \frac{L_m}{L_t} \mathbf{r} \right) = c_E \mathbf{E}_t(\mathbf{r}),
$$

(3.5)

The constants $c_H$ and $c_E$ reflect the fact that an arbitrary scaling of the electromagnetic fields does not affect the geometry factor. This can be verified by introducing such scaling in Eqs. (3.3,3.4) and then substituting $S$ and $\Psi$ in Eq.
3.1 Parallel Imaging Performance as a Function of Field Strength

![Diagram showing target and model setups with magnetic fields and lengths labeled](image)

Figure 3.2: General electrodynamic scaling. The electromagnetic RF fields in a virtual target setup (left) are mimicked by a rescaled model setup (right). Each setup is described by the characteristic length $L$, the field strength $B_0$, the gyromagnetic ratio $\gamma$, and the material properties electric permittivity $\epsilon$, magnetic permeability $\mu$, and conductivity $\sigma$. The indices $t$ and $m$ refer to target and model, respectively. For electrodynamic scaling, these quantities must fulfill Eq. (3.12).

(3.2). Note that $c_H$ and $c_E$ may be mutually different but must each be the same for all coils.

To derive parameter conditions that warrant the relations in Eq. (3.5), the underlying electrodynamics must be considered. To simplify the treatment, it is assumed that all samples are homogeneous and that the receiver coils are entirely placed outside the samples. Then the electromagnetic fields inside a sample are governed by the source-free, homogeneous Maxwell equations. Assuming steady-state oscillation at the Larmor frequency $\omega = \gamma B_0$, the Maxwell equations can be stated in the Helmholtz form as follows:

$$\nabla \cdot \mathbf{H}(\mathbf{r}) = 0, \quad (\Delta + k_0^2)\mathbf{H}(\mathbf{r}) = 0$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{\sigma - i\omega\epsilon} \nabla \times \mathbf{H}(\mathbf{r}),$$

where $\nabla$ and $\Delta$ denote the gradient and Laplace operators, respectively, and $k_0$ is the complex wave number:

$$k_0 = \sqrt{\gamma \mu B_0 (\gamma \epsilon B_0 + i\sigma)}.$$  

Note that the Eqs. (3.6,3.7) hold independently for each coil in both the target and the model setup. Therefore the corresponding indices were dropped.
In order to make configurations with different length scaling comparable, the Helmholtz equations are now subject to an isotropic spatial scaling. This is done by introducing the dimensionless position vector \( \mathbf{r} = \mathbf{r}/L \) with the characteristic length \( L \) equal to \( L_t \) for the target configuration and equal to \( L_m \) for the model. The transition to dimensionless coordinates preserves the structure of the Helmholtz equations as follows:

\[
\nabla \cdot \mathbf{H}(\mathbf{u}) = 0, \quad (\Delta + \kappa^2)\mathbf{H}(\mathbf{u}) = 0 \tag{3.9}
\]

\[
\mathbf{E}(\mathbf{u}) = \frac{1}{L(\sigma - i\omega \epsilon)} \nabla \times \mathbf{H}(\mathbf{u}). \tag{3.10}
\]

However, the wave number \( k_0 \) was replaced by the adimensional wave number

\[
\kappa = Lk_0 = L\sqrt{\gamma \mu B_0(\gamma \epsilon B_0 + i\sigma)} \tag{3.11}
\]

and the electric field is now additionally scaled by the inverse of the characteristic length.

Note that the geometries of the target and model configurations are identical in the dimensionless space. Hence the modified Helmholtz equations for the magnetic field (3.9) have the same solutions for both setups if the respective adimensional wave numbers are the same:

\[
\kappa_t = \kappa_m
\]

\[
L_t\sqrt{\gamma_t \mu_t B_{0,t}(\gamma_t \epsilon_t B_{0,t} + i\sigma_t)} = L_m\sqrt{\gamma_m \mu_m B_{0,m}(\gamma_m \epsilon_m B_{0,m} + i\sigma_m)}. \tag{3.12}
\]

In this case the electric fields will likewise be the same in both setups, except for a scaling factor (cf. Eq. (3.10)). Hence, after transforming the fields back into the original spatial coordinates, they will differ only by the individual length scaling and some individual amplitude scaling. Therefore they fulfill the scaling condition (3.5) and will consequently yield identical geometry factors.

In summary, mimicking the electrodynamic situation at a given target \( B_0 \) is generally accomplished by choosing the six model parameters \( B_{0,m}, L_M, \mu_m, \epsilon_m, \sigma_m, \) and \( \gamma_m \) such that Eq. (3.12) is fulfilled. Adjusting the magnetic permeability is mainly of conceptual interest, because varying the magnetic susceptibility at \( \omega \) will usually cause field inhomogeneity also in the static field. The model field strength, \( B_{0,m} \), and the characteristic length of the model setup, \( L_m \), also offer only limited flexibility as varying them is technically demanding. The gyromagnetic ratio, \( \gamma_m \), offers a certain range of variation on systems that can be tuned to different frequencies. The most flexible model parameters are the permittivity \( \epsilon_m \) and the conductivity \( \sigma_m \), which are relatively easy to adjust and permit continuous ED scaling if miscible materials are used.
Additional Remarks

Boundary Conditions: ED scaling, as described above, is based on geometric similarity and matching of the adimensional wave number $\kappa$ inside the sample. This derivation is not quite exact in that it neglects boundary conditions at the interface between the object and the ambient medium. A rigorous realization of ED scaling would additionally require adjusting the properties of the ambient medium in the model arrangement, which is difficult to do. In practice, this limitation concerns only the permittivity, $\varepsilon$, as the conductivity, $\sigma$, is generally equal to 0 outside the sample, both for target and model configurations. Mismatches in $\varepsilon$ outside the sample do cause deviations in the resulting Maxwell solutions inside the sample. However, such mismatches are critical only if they are significant relative to the change in $\varepsilon$ across the boundary. Hence, for target configurations with, e.g., water (relative permittivity $\varepsilon \approx 80$) inside the sample and air ($\varepsilon \approx 1$) outside, the mismatch is often negligible. Typically, the boundary issue is significant only when $B_0$ is considerably downscaled by strongly reducing the sample permittivity.

Interpretation of the Adimensional Wave Number $\kappa$: The physical meaning of the adimensional wave number $\kappa$ can be elucidated by considering the definitions of the RF wavelength, $\lambda$, and the skin depth, $\delta$, in physical units:

$$\lambda = \frac{2\pi}{\Re(k_0)}, \quad \delta = \frac{1}{\Im(k_0)},$$

where $\Re$ and $\Im$ denote the real and imaginary part, respectively. Combining with Eq. (3.11) yields

$$\kappa = 2\pi \frac{L}{\lambda} + i \frac{L}{\delta}.$$  \hspace{1cm} (3.14)

According to this equation, a small real part of $\kappa$ corresponds to negligible wavelength effects (near-field regime), whereas a large value indicates far-field wave behavior. Similarly, a small imaginary part of $\kappa$ corresponds to nearly unhindered RF penetration into the object, whereas a large value signifies poor RF penetration. Based on Eq. (3.14), ED scaling may be viewed as reproducing the relative wavelength $\lambda/L$ and the relative skin depth $\delta/L$ of a target configuration with a model setup.

Methods

Electrodynamic Scaling

A basic variant of ED scaling was implemented as follows. The target sample was a sphere with a diameter of 18 cm and a volume of 3 liters. The target material
properties were set to average in-vivo brain values. The target field strength covered the range of

\[ B_{0,t} = 1.5T, 3T, 4.5T, 6T, 7.5T, 9T, 10.5T, 11.5T. \] (3.15)

In order to perform the entire study with one magnet, the model \( B_0 \) was kept constant at

\[ B_{0,m} = 7T. \] (3.16)

For minimizing hardware requirements, the study was designed to use only a single phantom container (a glass sphere) and a single coil array. Hence the characteristic lengths were constant and equal throughout:

\[ L_m = L_t = 18cm. \] (3.17)

The actual setup of the coil array and the phantom is shown in Fig. 3.3. For the reasons outlined in the theory section, only non-magnetic object materials were considered:

\[ \mu_m = \mu_t = \mu_0, \] (3.18)

where \( \mu_0 = 4\pi10^{-7} \text{Vs/(Am)} \) denotes the magnetic permeability of free space. Furthermore, imaging was exclusively based on \(^1\text{H}\) magnetic resonance:

\[ \gamma_m = \gamma_t = \gamma_0, \] (3.19)
where $\gamma_0 = 42.58$ MHz/T is the gyromagnetic ratio of protons. With these parameters set, only two degrees of freedom, that is, $\epsilon_m$ and $\sigma_m$ remain for fulfilling the scaling condition (3.12). The condition yields

$$
\epsilon_m = \frac{B_{0,t}^2}{B_{0,m}^2} \epsilon_t, \quad \sigma_m = \frac{B_{0,t}}{B_{0,m}} \sigma_t.
$$

(3.20)

Table 3.1 shows the required values of $\epsilon_m$ and $\sigma_m$ along with the underlying target values. For low target $B_0$ the ideally required conductivity values are put in parentheses because they were not quite achieved in practice. Note that the targeted in-vivo material properties are frequency-dependent and hence depend on $B_{0,t}$. As discussed in the following section, the required model permittivities cover much of the range accessible under the given experimental constraints. This illustrates that the chosen model field strength of 7 Tesla is essential for covering the full range of target field strengths.

<table>
<thead>
<tr>
<th>$B_{0,t}$ [T]</th>
<th>$\epsilon_{r,t}$</th>
<th>$\sigma_t$ [1/(Ω m)]</th>
<th>$\epsilon_{r,m}$</th>
<th>$\sigma_m$ [1/(Ω m)]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>82.9</td>
<td>0.40</td>
<td>3.81</td>
<td>0.008 (0.09)</td>
<td>13 (11.7)</td>
</tr>
<tr>
<td>3.0</td>
<td>63.1</td>
<td>0.46</td>
<td>11.6</td>
<td>0.031 (0.20)</td>
<td>41 (9.6)</td>
</tr>
<tr>
<td>4.5</td>
<td>56.6</td>
<td>0.50</td>
<td>23.4</td>
<td>0.100 (0.32)</td>
<td>43 (4.1)</td>
</tr>
<tr>
<td>6.0</td>
<td>53.4</td>
<td>0.53</td>
<td>39.2</td>
<td>0.46</td>
<td>1.2</td>
</tr>
<tr>
<td>7.5</td>
<td>51.4</td>
<td>0.56</td>
<td>59.0</td>
<td>0.60</td>
<td>0.5</td>
</tr>
<tr>
<td>9.0</td>
<td>50.0</td>
<td>0.59</td>
<td>82.7</td>
<td>0.75</td>
<td>1.9</td>
</tr>
<tr>
<td>10.5</td>
<td>49.1</td>
<td>0.61</td>
<td>110</td>
<td>0.91</td>
<td>3.0</td>
</tr>
<tr>
<td>11.5</td>
<td>48.5</td>
<td>0.62</td>
<td>131</td>
<td>1.01</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Table 3.1: Parameters used for electrodynamic scaling with a model field strength of 7 Tesla. The model permittivity and conductivity were adjusted to the values shown in columns 4, 5. In this fashion, the target situations described in columns 1-3 were mimicked. In some cases the adjustment of the conductivity remained incomplete due to the inability to dissolve Sodium Chloride in Decane and Ethanol. For these cases the mismatch is indicated by the ideally required conductivity values in parentheses.

Substances and Mixing Ratios

The choice of suitable substances for the model setups is governed by various considerations. First of all, the major constituents should be liquid, yield sufficient $^1$H NMR signal for imaging, and exhibit relaxation times in a reasonable range.
### Table 3.2: Substances used for mixing phantom liquids with desired relative permittivity $\varepsilon_r$ and conductivity $\sigma$. For each of the liquids the intrinsic values of $\varepsilon_r$, $\sigma$ at the model frequency of 300 MHz were assessed with a dielectric probe system. Decane, Ethanol, Water, and N-Methylformamide readily span the required permittivity range (cf. Tab. 3.1). Sodium Chloride was used for adjusting the conductivity.

<table>
<thead>
<tr>
<th>Substances</th>
<th>Structure</th>
<th>Density [g/cm$^3$]</th>
<th>$\varepsilon_r$</th>
<th>$\sigma$ [1/Ωm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decane</td>
<td>C$<em>{10}$H$</em>{22}$</td>
<td>0.73</td>
<td>2.1</td>
<td>0</td>
</tr>
<tr>
<td>Ethanol</td>
<td>C$_2$H$_5$OH</td>
<td>0.79</td>
<td>23.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Purified Water</td>
<td>H$_2$O</td>
<td>1.00</td>
<td>79</td>
<td>0.02</td>
</tr>
<tr>
<td>N-Methylformamide</td>
<td>C$_2$H$_5$NO</td>
<td>1.01</td>
<td>174</td>
<td>0.73</td>
</tr>
<tr>
<td>Sodium Chloride</td>
<td>NaCl</td>
<td>2.17</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Furthermore, the full set of substances should permit spanning the required range of material properties as given in Tab. 3.1. Generally, liquids are well suited for tuning the permittivity, while dissolving an electrolyte is an effective means of adjusting the conductivity. For each value of $\varepsilon$, two miscible liquids should be available, one with higher and one with lower permittivity, such that the precise value of $\varepsilon$ can be accomplished by mixing. For the simultaneous adjustment of the conductivity it is crucial that some electrolyte does actually dissolve in the mixed liquids.

Observing the rules listed above, four liquids were identified for spanning the required range of permittivity: Decane, Ethanol, Purified Water, and N-Methylformamide. For adjusting the conductivity, Sodium Chloride was chosen. Table 3.2 summarizes the relevant properties of these substances, including the chemical structure and the mass density. For the liquids, the table also lists the intrinsic values of $\varepsilon_r$ and $\sigma$ at the model Larmor frequency of 300 MHz. These were assessed using a commercial dielectric probe (HP 85070M Dielectric Probe Measurement System, Hewlett-Packard, USA), connected to a network analyzer (HP 8753, Hewlett-Packard, USA). In the same fashion, the material properties at 300 MHz were charted for graded mixtures of Decane and Ethanol, Ethanol and Purified Water, as well as Purified Water and N-Methylformamide, each without the addition of Sodium Chloride (see Fig. 3.4).

The actual mixing ratios, as required according to Tab. 3.1, were then assessed iteratively. Starting from a pure liquid mixture with accurate $\varepsilon_r$, Sodium Chloride and the two respective liquids were gradually added in small quantities until $\varepsilon_r$ and $\sigma$ were either both accurately adjusted or converged without permitting perfect adjustment. The resulting mixing ratios are summarized in Tab. 3.3. Ac-
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![Figure 3.4: Relative permittivity $\varepsilon_r$ (left axis) and conductivity $\sigma$ (right axis) at 300 MHz, obtained by mixing Decane and Ethanol, Ethanol and Purified Water, as well as Purified Water and N-Methylformamide. Within each of the three mixing intervals, the volume percentages of the two constituent liquids were varied linearly.]

Accurate model properties were accomplished for all intermediate and high target field strengths, i.e. $B_{0,t} = 6$ T, 7.5 T, 9 T, 10.5 T, 11.5 T. For the lower target field strengths $B_{0,t} = 1.5$ T, 3 T, 4.5 T, the permittivity was likewise exactly adjusted. However, the adjustment of the conductivity remained somewhat incomplete for low target fields because neither Decane nor Ethanol dissolve Sodium Chloride. The achieved conductivity values are listed as actual model parameters in Tab. 3.1. The ideally required values are juxtaposed in parentheses.

**Scanner Hardware Setup**

All measurements were performed in a 7 Tesla magnet (Magnex Scientific, UK) equipped with a Varian Inova console (Palo Alto, CA). For RF excitation and reception, a transmit-receive stripline TEM coil array was used, incorporating principles of microstrip coil design. The array consisted of 8 straight coil...
segments equally spaced around a cylindrical former of 16 cm in length and with an inner diameter of 22 cm (Fig. 3.3). RF pulses were applied via all 8 channels, splitting the total power equally among the coils. Approximately homogeneous RF excitation was obtained by incrementing the transmit phase by 45 °per coil element along the perimeter. In signal reception, each coil segment was operated independently, resulting in 8 individual data sets. Tuning and matching was repeated for each experiment.

**MR Protocol**

Eight ED scaling experiments with target field strengths as specified in Tab. 3.1 were performed, using the previously tailored phantom liquids. According to composition, each phantom liquid yielded NMR spectra with multiple significant peaks. In order to avoid chemical shift problems, imaging was performed only with the largest peak, suppressing the smaller ones by chemical-shift selective saturation (CHESS). In each experiment two data sets were acquired using a gradient-recalled echo (GRE) imaging sequence (TR = 150 ms, TE = 5.6 ms, 6 mm slice thickness, matrix = 256 x 256, FOV = 256 x 256 mm²). The first data set served for assessing the noise covariance matrix Ψ and was obtained with all RF pulses suppressed. The second data set served for determining coil sensitivity maps for each coil element and was obtained with regular RF excitation.

**Sensitivity Mapping and Determination of g Factors**

A basic prerequisite for accurate g-factor determination are accurate coil sensitivity maps. However, at 7 Tesla sensitivity mapping is hampered by two major difficulties: First, due to enhanced difficulty in designing volume RF coils, reference images with homogeneous signal weighting are typically not available. Second,
3.1 Parallel Imaging Performance as a Function of Field Strength

at 7 Tesla even regular gradient echo images are typically compromised by severe phase modulations related to $B_0$ inhomogeneity, chemical shifts and phase inhomogeneity of the transmit RF. These phase modulations prevent the use of a sum-of-squares denominator in calculating sensitivity maps. To circumvent these problems, a denominator image was obtained by refining the regular sum-of-squares image by an estimate of the object phase, using a method proposed by de Zwart et al. Raw sensitivity maps were then obtained by dividing the image obtained from each individual coil element by the denominator image. The raw maps were denoised using a variational fitting approach, similar to that previously described by Bammer et al. Note that these procedures do not yield maps of absolute coil sensitivity. Rather, the resulting maps show the coil sensitivity multiplied by some smooth weighting function, which reflects how the denominator image deviated from a reference image with actually homogeneous signal weighting. However, this perturbation cancels out in the geometry factor because the weighting function is the same for all coils. Based on the refined sensitivity information and the noise covariance matrix, geometry factor maps were calculated according to Eq. (3.2) for various reduction factors $R$.

Numerical Verification

For an independent verification of the ED scaling approach, the Maxwell equations were solved numerically for corresponding target and model configurations. In order to permit efficient computation, the setups were simplified in that only a single circular coil was assumed, placed concentrically at the sphere’s surface. The diameter of the assumed coil was 9 cm, its distance from the sphere was 2 cm. This simplified picture permitted using a semi-analytic approach for solving the Maxwell equations, as previously described by Keltner et al. With this method, the spatial sensitivity of the circular coil was calculated for each pair of target and model configurations.

The discrepancy between model and target sensitivities was quantified by calculating the root-mean-square (RMS) difference in a plane that centrally intersected both the sphere and the coil. The RMS error was then obtained through dividing the RMS difference by the RMS of the sensitivity obtained for the target setup. Potential errors are expected to arise from two types of imperfections. First, not adjusting the permittivity outside the phantom violates the strict scaling condition, as discussed in the Theory section. Second, further error arises when the material properties inside the sample cannot be completely adjusted. At low target $B_0$, this was the case for the conductivity. In these cases the RMS error was calculated twice, once for optimal adjustment inside the sample and once for the incomplete adjustment that was actually accomplished.
Results

Numerical Verification

Sample results of the numerical verification study are shown in Fig. 3.5, covering the target field strengths $B_0 = 1.5$ T, $4.5$ T, $7.5$ T, and $10.5$ T. The figure compares the magnitude of the calculated coil sensitivities for the corresponding target and model setups. Where the model conductivities were not exactly accomplished ($B_{0,t} \leq 4.5$ T), sensitivity maps are shown both for the ideal and the
3.1 Parallel Imaging Performance as a Function of Field Strength

actually achieved conductivity values. The shown plane is the same as was used for determining the deviations between the target and model sensitivities. These deviations are listed as RMS errors in Tab. 3.1. The RMS error is quite low, in the range of a few percent, for intermediate and high target field strengths of 6 T and higher. Correspondingly, the calculated model and target sensitivities for 7.5 T and 10.5 T look virtually identical in Fig. 3.5. As may be expected, the lowest error is obtained for a target field of 7.5 T, which was the closest to the model field strength of 7 T. However, for target field strengths of 4.5 T and below, the incomplete adjustment of the conductivity resulted in significant deviations in RF behavior. The greatest RMS error is obtained for $B_{0,t} = 4.5$ T, whereas for $B_{0,t} = 1.5$ T the error is lower again. These two findings are also evident in the corresponding calculated sensitivity maps in Fig. 3.5. For hypothetical, accurate adjustment of the conductivity in these cases, the RMS error of the calculated sensitivities is significantly lower (parentheses in Tab. 3.1).

Parallel Imaging Performance at 1.5 - 11.5 Tesla

Figure 3.6 shows experimental sensitivity maps obtained at various target field strengths. In agreement with the calculated sensitivities (Fig. 3.5) and previous experimental evidence, two specific high-field effects can be appreciated: First, the coil sensitivities grow progressively asymmetric as the target field strength increases.93,94 Second, due to shortening RF wavelength and related interference effects, the coil sensitivity maps become increasingly structured.10,55 Note that the calculated and experimental sensitivity maps (Figs. 3.5, 3.6) are not expected to be actually similar, for two reasons. First, the experimental data were obtained with straight coil elements, while the calculation assumed a circular coil. Second, the experimental sensitivity maps are modulated by an unknown smooth weighting factor. This factor is related to the absence of a homogeneous reference coil, as detailed in the Methods section.

Figure 3.7 shows maps of the resulting g factors, plotted for SENSE reduction by the rates R=3, 4, 5 and 6 and target field strengths between 1.5 T and 11.5 T. Each of these maps shows the periodic structure characteristic of the underlying aliasing relationships at the various reduction factors. More importantly, the figure as a whole does also exhibit distinct structure. At 1.5 T the geometry factor shows the behavior that is typically observed in parallel imaging at this field strength. At threefold reduction (R=3), the corresponding g values are overall favorable, i.e. near the optimal value of 1. However, they deteriorate increasingly at R=4 and beyond, which is also observed at 3 and 4.5 T. The situation begins to change at a target $B_0$ of 6 T, where the g map for R=4 is generally more favorable than for lower $B_0$. At R=5, too, the pronounced plateaus in the g map are mitigated relative to lower $B_0$. This trend persists as the target field strength increases further. For
each value of $B_0$ there appears to exist a critical reduction factor which marks the transition from relatively benign to unfavorable $g$ factor behavior. The critical reduction factor increases with the field strength, reaching values of 6 and higher for field strengths beyond 10 T.

Another somewhat systematic effect is most pronounced at $R=3$ and high field strength. In these cases the geometry factor is relatively low overall, yet tends to exhibit several distinct peaks. To a lower extent, this effect does also occur at $R=4$ and $R=5$, likewise only at high $B_0$. 

Figure 3.6: Experimental signal sensitivity maps obtained with electrodynamic scaling to mimic various target field strengths. a) Sensitivities of 4 out of 8 coil elements at target $B_0$ of 1.5 and 11.5 Tesla (only magnitude shown). b) Sensitivity of one coil element at selected target field strengths. All maps are individually scaled to a maximum of 1.
In summary, the structure of Fig. 3.7 is characterized by two distinct performance regimes, a generally favorable one and one of prohibitive noise enhancement, i.e. prohibitively large $g$ factors. The transitions between these regimes occur at critical reduction factors that increase with the field strength, except at low $B_0$ where the critical reduction factor is approximately constant.

Showing these characteristics, the results given in Fig. 3.7 are strongly reminiscent of Fig. 3.1. In the theoretically obtained plot, the ultimate geometry factor is shown for the center of a sphere where the ultimate SNR is the lowest. For comparison, an equivalent plot of the experimental results is shown in Fig. 3.8. This representation reveals excellent agreement between theory and experiment with respect to the qualitative behavior of the geometry factor. The experimental data show the same structure with a favorable and an unfavorable performance regime, as well as the essentially same dependence of the regime transition upon $B_0$. The experiments confirm that the critical reduction factor is approximately constant for low $B_0$ and grows with field strength beyond some $B_0$ threshold. Nevertheless, the theoretical and experimental data differ in the scaling of the geometry factor. In the favorable regime, $g$ is approximately equal to 1 in both cases. However, in the unfavorable situations $g$ grows substantially faster in the experimental data.
Figure 3.8: Geometry factor as obtained experimentally for the center of the sphere, plotted versus the reduction factor $R$ and $B_0$. Note the close correspondence between this plot and the theoretical prediction of the ultimate geometry factor (Fig. 3.1). These results show that high field facilitates higher reduction factors, leading to significantly enhanced parallel imaging performance. According to theory the performance increase is due to the onset of far-field behavior of the radiofrequency fields at high $B_0$.

Discussion

Experiment versus Theory

The experimental data in Fig. 3.8 are in excellent agreement with previous theoretical results (Fig. 3.1). This is remarkable in several respects. First, this finding indicates that the theoretical approach does correctly account for the relevant underlying physical phenomena. Under this premise, the highly characteristic behavior of SENSE performance as a function of $B_0$ and $R$ may indeed be interpreted
as reflecting the distinct near- and far-field behavior of the detected radiofrequency fields. In the far-field regime both the experimental g factor and its theoretical counterpart generally decrease with increasing B₀, while the baseline SNR grows. This illustrates a specific and highly promising synergy between parallel imaging and high B₀.

The qualitative agreement between experiment and theory is remarkable also in that it was achieved with an array of only 8 coils, while the theoretical analysis was performed for a hypothetical complete coil array. According to the findings the limited number and specific geometry of the array hardly affected the characteristics and regime transitions of parallel imaging performance. However, these limitations apparently did enhance the geometry factor in situations where theory predicts difficulties even with a perfect array. This suggests that optimizing coil arrays in terms of their geometry and number of elements may be most important for imaging situations where optimal performance is inherently impossible. This would imply that advanced array design with a large number of elements may significantly boost the usefulness of parallel imaging. For instance, at low B₀ 5-fold reduction incurred prohibitive noise enhancement with g in the range of 10. According to theory, an optimal array would achieve geometry factors of just over 2, which would be acceptable in many applications.

Optimized coil arrays with a large number of elements may also solve the problem of distinct, scattered peaks of the g factor, which were observed in certain situations at high target field strengths. No such behavior was found in numerical studies assuming a complete coil array. This suggests that the sharp peaks are specific to the particular geometry and limited diversity of the 8-coil array used in the present study. This hypothesis can be readily tested by increasing the number of receiver coil elements in upcoming experimental and numerical investigations.

**Electrodynamic Scaling**

The present study was conducted with a spherical phantom of similar size as a human head. The target and model length scales were identical and the target electrodynamic properties were set to in-vivo brain values. Hence, the target configuration represents a reasonable approximation to the electrodynamic situation in the head and permits extrapolating the results towards in-vivo applications. Alternatively, a head-shaped phantom could have been used. However, in addition to the practical considerations given in the Methods section, the sphere is also the most generic object for the purpose of studying SNR limitations in general. Both the magnitude and, at least in the near-field range, the spatial variation of surface coil sensitivities decrease with distance from the surface. In this respect, the sphere may be regarded as a worst case situation, as it maximizes the mean distance from the surface for a given volume. Hence limitations to parallel imaging
performance that one observes in a sphere can serve as conservative estimates for other object shapes.

In the experiments, ED scaling was limited to adjusting the permittivity and the conductivity in the sample. In doing so, two types of non-idealities were incurred. First, the material properties were not adjusted outside the sample. For the conductivity this is appropriate because the model conductivity on the outside would have been zero anyway. Yet the outside permittivity would require some adjustment for exact ED scaling. However, the related mismatches incurred in the presented experiments were moderate. Numerical verification with an idealized circular coil yielded RMS errors in the resulting sensitivity functions in the range of a few up to about 10 percent. As may be expected, the errors were smaller for target field strengths closer to the model field strength of 7 T. One reason for not adjusting the material properties on the outside are the practical difficulties of doing so. Another reason is that the more significant mismatches occurred for target field strengths smaller than 7 T. In these cases, the outside permittivity would have needed to be reduced below that of air, which is not an option with common materials.

A second compromise had to be made with respect to the conductivity of the sample itself, again for mimicking low field strengths. For target $B_0$ of 1.5 T, 3 T, and 4.5 T, the conductivity could not be raised sufficiently. The underlying difficulty consists in the fact that Decane and Ethanol are too unpolar to dissolve Sodium Chloride. This problem is intrinsic in that low permittivity, as required for the low-field range, and low polarity are closely related at the molecular level. A potential solution for this issue is the use of detergents, which permit dissolving polar or charged constituents in unpolar liquids. For instance, a detergent could be used for forming micelles or reverse micelles containing lossy material.95

Due to the partial inability to exactly adjust the conductivity, low field strengths were effectively mimicked for target materials of lower conductivity than brain tissue. However, the geometry of near-field RF components, which dominate at low $B_0$, depends much less on material properties than that of far-field components.44 This may explain why the mismatches in conductivity did not result in appreciable discontinuities in Fig. 3.8. Rather, the agreement between theory and experiment is equally good at low and high $B_0$. Furthermore, it should be noted that mimicking high field situations, which was achieved rather accurately, is of greater practical importance. For studying low target fields with high accuracy, lower model field strengths are advisable or involving other scaling parameters, such as the characteristic length or the gyromagnetic ratio.

Electrodynamic scaling is a general means of investigating the RF behavior in MRI for variable hardware and object configurations. Varying the target $B_0$ at fixed physical field strength is particularly interesting, with further potential
3.1 Parallel Imaging Performance as a Function of Field Strength

Applications. For instance, the same method could be used for studying effects of dielectric resonance and the field dependence of SNR in conventional Fourier MRI. Coil design and studies of RF energy deposition may also benefit from the scaling approach.

Analogies

Electrodynamic scaling has interesting analogies in other sciences. For instance, in hydrodynamics the corresponding concept of hydrodynamic scaling is widely used. Similar to the adimensional wave number $\kappa$, hydrodynamic configurations are often characterized by the adimensional Reynolds number $Re = L \rho v / \eta$, where $L$ again denotes a characteristic length, $v$ the flow velocity, $\rho$ the mass density and $\eta$ the dynamic viscosity. By matching the Reynolds number, similar flow characteristics can be created on different scales. This is used, for instance, in optimizing water turbines by tests on miniaturized models. In another analogy between hydrodynamic and electrodynamic scaling, the Reynolds number is an indicator of different flow regimes, namely laminar and turbulent flow. While related to completely different physics, the adimensional wave number is likewise a regime indicator, distinguishing near- and far-field behavior.

Conclusions

In the present work, the dependence of parallel MRI performance upon main magnetic field strength has been experimentally investigated for the field range of 1.5 to 11.5 Tesla. This was accomplished using the concept of electrodynamic scaling, which permitted mimicking RF behavior at multiple field strengths with a single MR instrument. The results of this study suggest the following main conclusions:

1. The experimental findings are in remarkable qualitative agreement with theoretical predictions. They confirm the existence of two performance regimes, which are characterized by favorable and prohibitive geometry factors, respectively. In a spherical object, the favorable regime reaches up to reduction factors between 3 and 4 at low field strengths. This limit is initially independent of field strength. However, as the field strength increases, the critical transition increasingly shifts towards higher feasible reduction factors. This is consistent with theoretical predictions that ultimate parallel imaging performance should be independent of $B_0$ in near-field RF conditions, yet increase with $B_0$ as the RF enters the far-field wave regime.

2. Higher feasible reduction rates in far-field conditions are a benefit specific to parallel imaging at high field strength. The demonstrated effect represents
a genuine synergy between the two approaches, which adds to their complemen-
tarity and hence to their joint potential. In this context it is important
to note that the present study did not reveal any specific complications of
parallel imaging in high field conditions. In particular, concerns that dielec-
tric resonance may prevent efficient parallel imaging could not be confirmed
in a head-sized sphere and for field strengths up to 11.5 Tesla.

3. Electrodynamic scaling is a feasible approach for investigating RF behavior
at varying $B_0$, without the need to change the physical field strength. Among
the various scaling parameters, the permittivity and the conductivity of the
sample are the most flexible ones. They permit mimicking changes in param-
eters that are much more demanding to adjust, such as $B_0$ and the length
scale of the receiver coils. The adimensional wave number is a characteristic
measure of RF behavior, relating the size of an object to the wavelength
and the skin depth of RF fields. Promising further applications of electro-
dynamic scaling in the realm of MRI include coil design and studies of RF
power deposition.

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3.2 On the Role of Dielectric Resonance in Parallel MR Imaging

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Introduction

Ultra-high field strengths ($B_0$) and parallel imaging (PI) are expected to form a considerable synergy. This has recently been emphasized both by theoretical investigations $^{36,60,81}$ and practical experiments $^{61,73,96}$. However, a key concern raised by these studies is whether or not dielectric resonance interferes with the efficient use of PI at ultra-high $B_0$. This concern is typically based on the following hypotheses: In the presence of a significant dielectric resonance, coil sensitivity profiles have significant contributions from resonant eigenmodes of the object. These are specific for the geometry of the object rather than for the positioning and geometry of the coil $^{97}$. Accordingly, coil sensitivities become more similar, and hence potentially preventing the efficient use of PI. In order to study these effects, parallel SENSE imaging $^{24}$ has been performed in the presence of dielectric resonance on a 7 Tesla MRI system.

Methods

Two phantoms of different symmetry were chosen: a 3 liter glass flask of spherical symmetry and a 1 gallon (3.785 liters), approximately cuboid plastic container of mirror-symmetry. The dielectric resonance spectra of the water-filled phantoms were measured using a small search coil connected to a calibrated HP 4396A network analyzer as suggested in reference $^{38}$. By replacing small amounts of distilled water ($\varepsilon_r=79$ at 300MHz) by ethanol ($\varepsilon_r=24$ at 300MHz), the frequency of a nearby resonant mode was adjusted to the 7 Tesla Larmor frequency $^{61}$. The effect of conductivity on resonance damping was investigated by introducing small amounts of sodium chloride to achieve saline concentrations of 20mM and 50mM; a 50mM saline water solution corresponds approximately to average in-vivo brain conductivity at 300 MHz $^{55}$. All imaging experiments were performed on a 7 Tesla magnet (Magnex Scientific, UK), equipped with a Varian Inova console (Palo Alto, CA) and Siemens gradient amplifiers (Erlangen, Germany). Data acquisition was performed with an 8-element, straight-line, microstrip TEM transceive coil array $^{73,86}$. Tuning and matching could be performed robustly in all situations. Images were acquired using a gradient-recalled echo (GRE) sequence (TR=50ms, TE=5ms, 3mm slice thickness, matrix=256x256). PI performance was quantified by calculating the local g-factor ($g$) for reduction factors (R) in the range between 1 and 5 with noise correlation taken into account $^{24}$.

Results

Figure 3.9 shows the dielectric resonance patterns in the two phantom geometries, as revealed by single-coil images in the case of 0mM and 20mM salinity: For the
3.2 Dielectric Resonance in Parallel MR Imaging

Figure 3.9: Single coil images in the presence of dielectric resonance.

- Spherical Symmetry:
- 0 mM NaCl
- 20 mM NaCl

- Mirror Symmetry:
- 0 mM NaCl
- 20 mM NaCl

spherical flask (upper part) data from four different coil elements are shown in a central transverse slice. For 0mM a two-lobe mode can be clearly appreciated. Small additional modulations near the surface are due to destructive interference during RF excitation, reflecting non-ideal transmit phase adjustments. For the cuboid container (lower part), a coronal slice is shown. Again, for 0mM a clear higher-order resonance pattern can be appreciated. However, unlike the spherical phantom the resonance does not change significantly with coil position.

Figure 3.10 compares calculated mean g-factors between the spherical (left) and the cuboid phantom (right) in a central transverse slice. Three different saline concentrations are shown: 0mM (black solid line), 20mM (blue dashed line) and 50mM (red dotted line). For the spherically symmetric phantom, the calculated g-factors are all in the same range. However, for the mirror-symmetric container the g-factor improves significantly from 0mM to 20mM and remains approximately constant for higher salinity.

Discussion

The discrepancy in PI performance between the two phantom geometries (Fig. 3.10) can be related to characteristics of the specific dielectric resonance patterns
For the spherical symmetric flask, the excited dielectric mode appears to be spherically degenerate, with the consequence that the resonance pattern in the coil sensitivities still depends on the coil position. In particular, in the 0mM images shown in Fig. 3.9 (upper part) it can be recognized that the corresponding coil elements are consecutively rotated by 45°. Accordingly, in this degenerate eigenmode, the individual coil sensitivities are similarly orthogonal for 0mM, 20mM and 50mM salinity, resulting in almost identical, favorable g-factors (Fig. 3.10 left). Conversely, in the mirror-symmetric container a non-degenerate eigenmode was excited. In the case of 0mM salinity, the single-coil images differ only by small sensitivity variations, while the underlying mode structure is the same for all coil elements (Fig. 3.9 lower part). Correspondingly, the coil sensitivity profiles are less orthogonal, resulting in still moderate but enhanced g-factors for the lossless case. However, inducing conductivity by adding 20mM sodium chloride significantly improves the g-factor. Further increasing the saline concentration to 50mM leaves the g-factor nearly unchanged (Fig. 3.10 right).

The human head exhibits only approximate mirror symmetry. Therefore, non-degenerate resonance modes are expected for brain imaging, indicating the less favorable g factor behavior. However, according to the right plot in Fig. 3.10, the intrinsic tissue conductivity may be expected to sufficiently damp dielectric resonance modes in the head to a sub-critical level for PI.

Conclusions

In summary, it can be concluded that:
1. The impact of dielectric resonance on PI performance appears to depend on the degeneracy of the resonant mode. Generally, a high level of mode degeneracy was observed to be favorable for PI.

2. Independent of the dielectric mode degeneracy, minor amounts of sodium chloride stabilize the g-factor at usual values. This suggests that under in-vivo conditions dielectric resonance may not be a major, specific problem for PI.

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Chapter 4

Feasibility Aspects at Ultra-High Field

4.1 Potential and Feasibility of Parallel MR Imaging at High Field

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Abstract

This survey focuses on the fusion of two major lines of recent progress in MRI methodology: parallel imaging with receiver coil arrays and the transition to high and ultra-high field strength for human applications. As discussed in this paper, combining the two developments has vast potential due to multiple specific synergies. First, parallel acquisition and high field are highly complementary in terms of their individual advantages and downsides. As a consequence, the joint approach generally offers enhanced flexibility in the design of scanning strategies. Second, increasing resonance frequency changes the electrodynamics of the MR signal in such a way that parallel imaging becomes more effective in large objects. The underlying conceptual and theoretical considerations are reviewed in detail. In further sections, technical challenges and practical aspects are discussed. The feasibility of parallel MRI at ultra-high field is illustrated by current results of parallel human MRI at 7 T.

Introduction

This survey addresses the combination of parallel imaging (PI) with another important line of recent development in MRI methodology: the transition to high and ultra-high field strength for human applications. Large-scale research into ultra-high field in vivo MR is driven by the promises of increased intrinsic signal-to-noise ratio (SNR) and enhanced susceptibility contrast, as typically used for functional brain MRI. These benefits come at the expense of serious fundamental issues such as increased tissue heating, limited RF penetration and potential dielectric resonance, which prompted some hesitation in the early days of ultra-high field MR. However, within approximately the past decade both theoretical and experimental studies have established the feasibility of in vivo MR in humans at field strengths as high as 7 T and beyond. Concurrently, parallel MRI technology has entered the scene. Enhancing the efficiency of spatial encoding in a fundamental fashion, parallel acquisition with coil arrays is widely recognized as holding vast potential for a broad range of applications. With several implementations available, such as SMASH, SENSE and GRAPPA, PI is increasingly used in clinical practice.

The importance of these two developments is underscored by their impact on the MRI industry. To date, most major MRI vendors have expanded their range of human whole-body MRI systems toward 3 T and even 7 T. Most of the currently available commercial systems offer embedded PI capability, with numbers of independent receiver channels now ranging up to 32. For recent reviews of high field and parallel in vivo MR see, e.g., references. High field and parallel MRI are conceptually independent developments. How-
ever, it has recently been recognized that their combination holds particular promise and entails vast and partly surprising synergies on two levels. First, it turns out that the two approaches are highly complementary in terms of their individual advantages and downsides, hence adding significant flexibility in the design of scanning strategies. Second, it has been found that high field conditions favor parallel imaging in an even more fundamental way, which is rooted in the physics of MR signal propagation and detection. As it turns out, the transition to ultra-high field changes these physics in such a way that spatial encoding by parallel acquisition becomes more effective.

This article aims to explore these considerations in detail. We will first discuss the individual strengths and weaknesses of the two approaches and how they combine. Special emphasis is then laid upon the crucial role of electrodynamics and their impact on the inherent potential and limitations of PI as a function of $B_0$. Finally, we will discuss high field PI from a practical perspective, highlighting technical issues and initial results.

**Basic Synergies**

Compared with any known gradient encoding scheme, spatial encoding by means of distinct coil sensitivities stands out by the fact that it does not interfere with the nuclear spin magnetization that it encodes. This important property underlines the remarkable versatility of the mechanism, which permits parallelizing virtually any MR imaging sequence. The enhanced encoding efficiency available in PI translates into numerous advantages. Most notably these are: higher scan efficiency (reduced acquisition time, higher resolution and coverage), reduction of artifacts due to $B_0$ inhomogeneity\(^{92,102-104}\) and motion,\(^{105,107}\) reduction of the specific absorption rate (SAR),\(^{108}\) and mitigation of acoustic gradient noise.\(^{109}\) In the majority of applications these benefits come at the expense of reduced SNR efficiency, which forms the major downside of PI. As a consequence, PI is most useful in situations where some spare SNR yield of a conventional technique can be traded for one or multiple of the advantages listed above.

High field (> 1.5 T) and ultra-high field (> 7 T)\(^1\), conversely, provide two main benefits: higher baseline SNR and enhanced susceptibility contrast (e.g. $T_2^*$, BOLD contrast).\(^{11,15}\) However, increasing $B_0$ also enhances a range of specific problems, such as increased $B_0$ inhomogeneity, reduced $T_2$ and $T_2^*$ relaxation times, increased RF energy absorption in the tissue (SAR) and higher acoustic noise levels. The $B_0$ homogeneity in biological samples generally degrades as $B_0$ increases because the interfering effects of magnetic tissue susceptibility scale with $B_0$. For fast imaging techniques, such as EPI,\(^{110}\) spiral scanning,\(^{111}\) and balanced

\(^1\)Ultra-High Field, since the associated $^1$H frequencies are in the Ultra-High Frequency range (300 MHz - 1 GHz).
steady-state methods, B₀ inhomogeneity results in enhanced blurring, distortions, ghosting, or dark band artifacts. Therefore effective higher-order shimming is vital at 3 T and above. Scanning at ultra-high field typically requires further compromises for addressing field inhomogeneity and faster signal decay, such as k-space segmentation and restricting the FOV. RF energy deposition in the tissue increases approximately as the square of the resonance frequency and is a serious safety issue at field strengths of 3 T and beyond. Strict SAR limitations impose significant restrictions on the design of RF-intensive MR sequences at high field. Finally, Lorentz forces on the gradient coils also scale with B₀, causing enhanced acoustic noise levels at high field.

<table>
<thead>
<tr>
<th></th>
<th>High Field Strength</th>
<th>Parallel Imaging</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>INcreased</td>
<td>DEcreased</td>
<td>+</td>
</tr>
<tr>
<td>Scan Efficiency (Time, Resolution, Coverage)</td>
<td>xxx</td>
<td>INcreased</td>
<td>XXX</td>
</tr>
<tr>
<td>Δ B₀ Artifacts (Blurring, Ghosting, Distortions)</td>
<td>INcreased</td>
<td>DEcreased</td>
<td>+</td>
</tr>
<tr>
<td>Motion Artifacts</td>
<td>xxx</td>
<td>DEcreased</td>
<td>XXX</td>
</tr>
<tr>
<td>RF Energy Absorption</td>
<td>INcreased</td>
<td>DEcreased</td>
<td>+</td>
</tr>
<tr>
<td>Acoustic Gradient Noise</td>
<td>INcreased</td>
<td>DEcreased</td>
<td>+</td>
</tr>
<tr>
<td>Intrinsic PI Performance</td>
<td>INcreased</td>
<td>xxx</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 4.1: Intrinsic characteristics of (ultra-) high fields and parallel imaging. Hereby, the entry “xxx” means that there is no immediate effect. Apparently the two approaches are highly complementary. Accordingly and as indicated by the entry “+” the combination of (ultra-) high fields and PI results in a significant synergy regarding numerous aspects. In this context, PI can be regarded as a converter, which puts the SNR benefits of high field to work in favor of alternative advantages.

Table 4.1 summarizes these strengths and limitations. Given the listed characteristics, the two approaches evidently exhibit a high level of complementarity. For example, high field strength affords higher intrinsic SNR but suffers from B₀ inhomogeneity and SAR limitations. In turn, PI is an effective means of tackling B₀ inhomogeneity and mitigating the SAR issue at the expense of SNR. By combining the two options one can effectively address the shortcomings of both concepts and frequently secure a significant net benefit compared to non-parallel imaging at lower field. In this combination, the parallelization plays the crucial role of a converter, which puts the SNR benefit of high field to work in favor of
alternative advantages.

These basic arguments illustrate the more noticeable synergies between PI and high field. Additionally, as indicated in the last row of Tab. 4.1, there is another, more sophisticated kind of synergy, involving the inherent limits of PI performance and the physics of MR signal detection. This aspect is discussed in the following.

Parallel Imaging in terms of RF Electrodynamics

With respect to image reconstruction and SNR considerations, the relevant characteristics of a receiver coil array are the individual, spatially varying MR signal sensitivities of the array elements and the noise statistics of the entire array. Based on the reciprocal nature of the underlying electrodynamics\(^{37,38}\) these properties can be expressed in terms of the electric (\(E\)) and magnetic (\(H\)) fields generated by the array when driving each individual coil with unit input current at the appropriate Larmor frequency. The coil sensitivities correspond to the transverse components of the magnetic fields:

\[ s_c(r) = \mu(r)[H^\text{e}_z(r) - iH^\text{e}_y(r)], \quad (4.1) \]

where \(c\) counts the array elements, \(r\) denotes position in 3D space, \(\mu\) denotes the magnetic permeability of the scanned object, and \(B_0\) is assumed to be aligned with the \(z\) axis. Owing to the negative sign in combining the transverse field components, expression (4.1) is frequently referred to as \(B^\text{c}\) of the respective coil element. The noise statistics is conveniently described by the noise covariance matrix \(\Psi\), whose entries are related to the electric fields:

\[ \Psi_{c,c'} = \int_{\text{sample}} \sigma(r)E_c(r) \cdot \bar{E}_{c'}(r)d^3r, \quad (4.2) \]

where \(\sigma\) denotes the electric conductivity of the scanned object and the bar indicates complex conjugation.

In the common case of Cartesian k-space sampling, PI reconstruction can be performed by image domain unfolding.\(^{24}\) This is done essentially by inverting a matrix of sensitivity values for each set of pixels that alias in single-coil reconstruction. Let the index \(\rho\) count the pixels in any such set and \(r_\rho\) denote their positions. Then the sensitivity matrix for the set is given by:

\[ S_{c,\rho} = s_c(r_\rho). \quad (4.3) \]

Similar to the mathematical description of phased-array imaging with full k-space density,\(^{18}\) the SNR of each of the reconstructed pixels values can now be expressed as:\(^{24,60}\)

\[ \text{SNR}_{\rho}^{\text{PI}} \propto \frac{B_0^2}{\sqrt{R [S^H \Psi^{-1} S]^{-1}}}, \quad (4.4) \]
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where the reduction factor $R$ indicates the factor by which k-space was undersampled and the superscript $H$ denotes the complex conjugate transpose.

Equations (4.1-4.4) illustrate the close relationship between the SNR yield in PI and the electrodynamics of the receiver coil array. In practice the SNR yield in PI is often stated relative to the SNR that would be achieved with full-density k-space sampling:

$$\frac{\text{SNR}_{\text{full}}^P}{\sqrt{Rg_P}}$$

where $g_P$ is the so-called geometry factor. This equation emphasizes that the characteristic SNR loss in PI may be viewed as due to two independent mechanisms. The square root of $R$ reflects reduced overall data acquisition and hence reduced intrinsic signal averaging. Conversely, the $g_P$ factor describes noise amplification related to the conditioning of the unfolding operation. Hence it reflects the suitability of the coil configuration for the specific PI task, which is characterized by the size, shape, and dielectric properties of the object, the imaged slice or volume, and the undersampling scheme. In a broad sense, the $g_P$ factor may as well be regarded as a measure of distinctness among the set of coil sensitivities. Highly distinct coil sensitivities yield low, i.e. favorable $g_P$ factors, which however can never drop below the optimal value of 1.

It is important to note that Eqs. (4.4,4.5) are based on the assumption that all considered scans are performed with the same pulse sequence and differ only in the number of repetitions. This leads to conservative SNR estimates because the application of PI frequently permits optimizing the acquisition strategy in terms of its baseline SNR yield. Higher baseline SNR can result, for instance, from adjusting the sequence timing, the length of echo trains, or the rate of contrast agent administration. Most notably, in some circumstances the SNR benefits of PI-driven sequence optimization actually outweigh the inherent SNR losses.

Based on the aforementioned considerations, PI performance has recently been studied by means of both theoretical models and practical experiments. In such studies the SNR yield of a specific imaging setup is typically quantified using formulas similar to Eq. (4.4), which require the determination of coil sensitivities and noise statistics of the array under investigation. In actual imaging experiments, $s_c(r)$ and $\Psi$ can be determined relatively easily by calibration measurements. For the theoretical assessment of PI performance via Eqs. (4.1,4.2) the RF coil’s reciprocal fields must be calculated by solving Maxwell’s equations.

Calculating the reciprocal fields is fairly straightforward in so-called near-field conditions, where the RF wavelength $\lambda$ is much larger than the object under investigation and the coil array. In this regime the geometry of the RF fields is
4.1 Potential and Feasibility of Parallel MR Imaging at High Field

not affected by the presence of the scanned object and can be calculated by Biot-Savart integration.\textsuperscript{18,122} This approach has been taken in several recent numerical studies exploring PI performance in near-field situations.\textsuperscript{31,33,63,80}

However, in proton MRI at high $B_0$ the near-field assumption is frequently violated. A convenient formal way of relating the object size $L$ to the electrodynamic length scale is the adimensional wave number:\textsuperscript{61}

$$\kappa = 2\pi \frac{L}{\lambda} + i\frac{L}{\delta}$$

where $\delta$ denotes the RF skin depth. For the near-field approach to be valid the real part of the adimensional wave number should be well below $2\pi$, hence $L/\lambda < 1$. Typical values of $\lambda$ and $\delta$ encountered in brain tissue are shown in Tab. 4.2. As seen there, for human head imaging the near-field picture holds reasonably well at most up to 3 T.

<table>
<thead>
<tr>
<th>$B_0$ [T]</th>
<th>1.5</th>
<th>3.0</th>
<th>4.7</th>
<th>7.0</th>
<th>9.4</th>
<th>11.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ [cm]</td>
<td>44</td>
<td>27</td>
<td>19</td>
<td>13</td>
<td>10</td>
<td>8.6</td>
</tr>
<tr>
<td>$\delta$ [cm]</td>
<td>14</td>
<td>10</td>
<td>8.4</td>
<td>7.3</td>
<td>6.5</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Table 4.2: RF wavelength ($\lambda$) and RF skin depth ($\delta$) as a function of $B_0$.

Note, these values are specific to the Proton $^1H$ MR Larmor frequency.

At higher fields or with larger objects the RF electrodynamics grow considerably more complex due to the onset of wave propagation and diffraction within the object and due to the increasing influence of the object’s characteristics.\textsuperscript{9,10,13,46} This regime shall be referred to as the ‘wave regime’ in the following. A simple case of wave regime onset is illustrated in Fig. 4.1, showing simulated sensitivity maps for a loop coil receiving signal from a sphere of 20 cm in diameter. The sphere’s dielectric properties were chosen similar to those of the human brain and $B_0$ was varied between 1.5 T and 11.5 T. The sensitivity maps were calculated with a semi-analytical full-wave method similar to that described by Keltner et al in reference.\textsuperscript{46} In agreement with the near-field picture, the sensitivity profiles are quite similar for the two lowest field strengths. However, as $\lambda$ shrinks to the range of the object size, which in this case occurs between 4 and 5 T, the sensitivity maps become increasingly structured and asymmetric.\textsuperscript{10,46,55,61,93}

With respect to PI this last observation illustrates an important advantage of the wave regime over near-field conditions. Wave behavior within the object, including propagation, reflection, and interference, generally translates into more structured and hence more distinct coil sensitivities. Therefore it may be expected
Figure 4.1: Calculated absolute receive sensitivity in dependence of $B_0$ using methods similar to as described in reference.\textsuperscript{46} For the calculation a circular coil (diameter = 10 cm) was arranged next to a spherical object (diameter = 20 cm). The dielectric properties were chosen to be similar to average in vivo brain conditions. While in the near-field regime (i.e. $L/\lambda < 1$) the coil sensitivities are almost independent of $B_0$, in the wave regime (i.e. $L/\lambda > 1$) the coil sensitivities become increasingly structured due to enhanced wave interference effects. Because the geometry factor is determined by the orthogonality of the coil sensitivities, this indicates improved PI performance for the wave regime.

that wave regime conditions permit more efficient spatial encoding by coil sensitivities, which would be reflected in lower g factors and hence higher SNR yield.

**Fundamental Limitations and High Field Potential of Parallel Imaging**

Several recent reports have addressed the issue of coil array optimization for PI, targeting both the geometrical coil configuration and the number of coil elements.\textsuperscript{31-33, 64, 80, 123, 124} Continued design efforts will further enhance the available array performance, inevitably raising the question of what are the inherent limits of PI performance. In fact, one might speculate whether a hypothetical perfect coil array could yield arbitrarily high baseline SNR in conjunction with unlimited PI reduction factors. In order to address this intriguing question, the concept of ultimate intrinsic SNR as originally introduced by Ocali and Atalar\textsuperscript{28} has been extended toward PI.\textsuperscript{36, 60} The key step in this approach is to abandon the concept of
specific coil arrays with specific characteristics. Instead, one exploits the fact that the reciprocal fields of any coil in any array are solutions of Maxwell's equations. Maximal SNR can thus be achieved with a hypothetical array whose reciprocal fields span the entire solution space of Maxwell’s equations. This prompts the conceptual transition from the reciprocal fields of a specific coil array to a complete basis of Maxwell solutions:

\[ \{ \mathbf{E}_c(\mathbf{r}), \mathbf{H}_c(\mathbf{r}) \} \rightarrow \{ \alpha_c(\mathbf{r}), \beta_c(\mathbf{r}) \}. \]  (4.7)

Each element of this basis can be regarded as one building block of a virtual "complete" coil array. Evaluating Eqs. (4.1-4.5) with the full basis \( \{ \alpha_c(\mathbf{r}), \beta_c(\mathbf{r}) \} \) yields maps of the ultimate SNR for conventional Fourier imaging, and of the ultimate g factor for any specific PI experiment.

To date, ultimate PI performance has been studied in two independent investigations,\(^{36,60}\) assuming different specifications for the object geometry and material properties and using different Maxwell bases. Ohliger et al.\(^{36}\) chose the same specifications as previously used in reference,\(^{28}\) i.e. an elliptical cylinder with dimensions and material properties similar to the human torso in conjunction with a plane wave basis. In reference\(^{60}\) spherical objects were studied using a multipole basis. The key findings of these studies are similar and shall be summarized here along selected results from reference.\(^{60}\) For brevity, the discussion is limited to PI performance in the center of the sphere when imaging the central transverse plane with k-space undersampling in one dimension.

Figure 4.2 shows the ultimate SNR at the sphere's center versus \( B_0 \) for three different sphere sizes, both for average in vivo brain conditions and for hypothetical virtually lossless conditions (\( \sigma = 10^{-5} (\Omega \text{ m})^{-1} \)). Note that due to the double-logarithmic scale the slope of the curve corresponds to the \( B_0 \) exponent that characterizes the local SNR increase with \( B_0 \). The calculated behavior of the ultimate SNR reflects the two aforementioned, fundamentally different regimes of RF wave behavior. In the near-field regime, i.e. for low \( B_0 \) / small object size, the SNR generally increases about linearly with \( B_0 \), corresponding to an exponent of 1 and hence a unit slope in the double-logarithmic plot. In particular, the exponent is independent of the material properties in the near field. Conversely, in the wave regime, i.e. for high \( B_0 \) / large FOV, the SNR increases much faster with \( B_0 \), with an exponent that is significantly larger than 1 and depends on the objects' characteristics. In the lossless case the SNR increase is almost cubic, but reduces to between linear and quadratic for the conductive case.

Figure 4.3 shows the ultimate geometry factor versus \( R \) and \( B_0 \) for four different sphere sizes. Most importantly, these results illustrate that even with an optimal coil array, noise enhancement as reflected by the geometry factor cannot be avoided. In general, two domains with opposite PI characteristics can be distinguished: The first, favorable domain is characterized by an ultimate g factor
Figure 4.2: Ultimate SNR as a function of $B_0$ for three different object sizes (i.e. FOV = 0.1 m, 0.3 m, 0.5 m) evaluated for the center of the spherical object (reproduced from reference60). Left calculated using average in vivo brain conditions and right for approximately lossless conditions ($\sigma = 10^{-5}$ $(\Omega \text{ m})^{-1}$) in comparison. Linear segments in the double-logarithmic plots are emphasized by regression lines. The slope of this segments characterizes the growth of the ultimate SNR as a power of $B_0$.

close to 1, reflecting optimal PI performance. The second, unfavorable domain is characterized by prohibitive exponential growth of $g$ as a function of $R$. The figure also reveals that the transition between the two domains again depends on the nature of the RF electrodynamics. In near-field conditions the critical transition occurs at an approximately constant reduction factor between $R=3$ and $R=4$. In the wave regime the critical transition occurs at a larger $R$ value, which increases both with $B_0$ and the sphere size. The shown graphs were obtained for Cartesian undersampling along one dimension. However, the described calculation scheme can be used equally for considering two- and even three-dimensional undersampling. Higher-dimensional PI enables significantly higher net reduction factors by distributing the aliasing along multiple dimensions.36,43

These results illustrate that the theoretical potential of parallel MRI increases dramatically when entering the wave regime at ultra-high field. From the described analysis the underlying transition in the RF physics is expected to both further
4.1 Potential and Feasibility of Parallel MR Imaging at High Field

Figure 4.3: The ultimate geometry factor in the center of the sphere as a function of the reduction factor \( R \) and \( B_0 \), for various object sizes (i.e. \( \text{FOV} = 0.1 \text{ m}, 0.2 \text{ m}, 0.3 \text{ m}, 0.4 \text{ m} \) ) (reproduced from reference\(^{60}\)). Two clearly distinct regimes are observed: For low reduction factors or sufficiently large \( B_0 \) and \( \text{FOV} \) the ultimate \( g \) factor is benign with values very close to the optimum of 1. For high reduction factors beyond a critical value, \( \text{PI performance starts to exponentially deteriorate as a function of } \) \( R \). This transition is approximately at \( R=4 \) in near-field regime and shifts towards higher reduction factors in the wave regime.

Enhance the achievable baseline SNR and extend the range of feasible reduction factors. This means that along with significant extra SNR the wave effects may also provide the enhanced means of converting it into any of the alternative benefits listed in Tab. 4.1. This observation constitutes the second level of promising synergy between parallel acquisition and high fields.

In addition to theoretical and numerical studies,\(^{36,60,81,125}\) the \( B_0 \) dependence of the geometry factor has recently also been investigated experimentally.\(^{91}\) Using the concept of electrodynamic (ED) scaling\(^{61,62}\) the \( B_0 \) dependence of PI performance
was investigated for effective field strengths ranging from 1.5 T to 11.5 T, using a single hardware setup at 7 T. The results were in remarkable agreement with the described theoretical studies, supporting the prediction that PI will indeed significantly benefit from the onset of wave behavior at high field.

Practical Aspects of Parallel Imaging at High Fields

The previous sections have focused on conceptual and theoretical considerations concerning the synergies between PI and high field and the inherent limitations of parallelization. However, as will be discussed in this section, the parallel approach also involves overcoming numerous technical challenges, especially at ultra-high field. Nevertheless, the technical issues are gradually being resolved and ultra-high field PI is becoming increasingly feasible, as demonstrated by preliminary imaging results obtained at 7 T.

Technical Challenges

To take full advantage of intrinsically limited SNR and parallel encoding capabilities, optimization of the coil array hardware is a key requirement. Steady advances in array and receiver hardware have resulted in the recent availability of MRI systems with up to 32 independent channels.80,124,126 A prototype array and receiver configuration with as many as 64 receiver channels has also been described in the literature.64,79,127 To construct and use very large arrays a multitude of challenges, such as mutual coil decoupling,76,77 high data throughput, and cable handling and safety, must be addressed. At ultra-high field these difficulties are even greater than at 1.5 or 3 T. With increasing frequency electromagnetic interactions between coil elements, both direct and through the scanned object, become more significant. Frequently such interactions compromise the coil tuning or introduce noise correlation.46,76,77,128,129 In addition to sample losses, radiation losses increasingly come into play at high field.46 While sample losses are for the most part unavoidable, radiation losses can be reduced by RF shielding. High field MRI systems are often restricted in terms of space available for coil arrays due to either small bore sizes or to the use of dedicated gradient inserts. This limitation may favor the use of a single so-called transceive array for both RF power transmission and signal reception.

A promising solution to some of these challenges was recently introduced in the form of transceive transmission-line arrays.85,86,88 This design concept is specifically suited for high field application, because each transmission line element is intrinsically shielded and the array as a whole exhibits convenient broadband decoupling characteristics. Furthermore, if the transceive coil array setup permits independent phase and amplitude control, it also enables RF shimming.130 i.e. the
optimization of transmit field uniformity. Even more promising are the prospects of operating each transmit channel with an individual RF waveform. This approach permits accelerating multidimensional selective RF excitation (transmit SENSE,$^{131,132}$) and is a promising means of addressing $B_1$ inhomogeneity and SAR issues.$^{133}$

Partly associated with the coil design issue is the problem of sensitivity calibration. Accurate sensitivity mapping is a prerequisite for robust image-domain unfolding, specifically for high reduction factors. Standard sensitivity mapping relies on eliminating object information based on additional data acquired with a homogeneously sensitive reference coil, such as a body coil. However, such coils are typically not available at ultra-high fields. The common way of replacing the body coil image is by a root-sum-of-squares (RSOS) image from the entire array. However, this approach is limited because it removes only the object’s amplitude modulations, while leaving the object’s phase modulation unaffected. The latter
can be quite substantial at high field due to $B_0$ inhomogeneity, chemical shift effects, and spatial phase variation in the transmit RF. This is illustrated in Fig. 4.4 (top row). Strong remaining phase modulations, as seen there, hamper the refinement of the sensitivity map by smoothing and extrapolation. However, such measures are required to denoise the sensitivity information and to permit small movements of the subject between the calibration and the actual scan.

One solution to this problem, as suggested by de Zwart et al.,91 is modifying the RSOS image such as to include an estimate of the object phase ($\varphi^{\text{obj}}$):

$$\varphi^{\text{obj}} = \text{phase} \left( \sum_{c=1}^{N} |P_c| P_c e^{-i \varphi_{c, \text{ref}}} \right),$$  

(4.8)

where $P_c$ denotes the reference image obtained with coil $c$ and $\varphi_{c, \text{ref}}$ is the phase of this image at some fixed reference location, e.g. at the center of the object. As shown in Fig. 4.4 (bottom row) this results in raw sensitivities with only moderate residual phase modulations, which now readily permit further processing.

Another area of concern is the effect of dielectric resonance on the efficiency of spatial encoding with coil arrays at ultra-high field strengths. This concern is based on the observation that in the presence of dielectric resonance the coil sensitivities become less specific to the coil configuration and more determined by the size, shape, and dielectric properties of the imaged object. Accordingly it may be expected that the individual coil sensitivities will grow more similar, resulting in an enhanced g factor and noise amplification. In a recent study, this effect has indeed been observed experimentally in a lossless phantom.134 However it was also found that the g factor returns to usual values when the phantom is made slightly conductive. These results are consistent with previous investigations concluding that dielectric resonance phenomena are strongly attenuated by dissipation under in vivo conditions.10,55,100

Gradually, computer hardware resources may also become a limiting factor. Highly parallel acquisition with large coil arrays not only yields higher encoding efficiency but also larger amounts of data to be managed and processed. With current reconstruction algorithms the computational load is particularly high for PI with non-Cartesian k-space trajectories.48,135,136 Even higher are the computational demands of advanced algorithms for dealing with increased $B_0$ inhomogeneity at high field.137-139

**Initial Experimental Results**

Using a 4-element transceive transmission line array and the aforementioned extended sensitivity calibration scheme, the feasibility of SENSE imaging at 7 T has recently been demonstrated.85,96 Figure 4.5 shows sample T1-weighted images (top
4.1 Potential and Feasibility of Parallel MR Imaging at High Field

Figure 4.5: Feasibility of parallel SENSE imaging at 7 T using a 4 element transceive coil array and applying the enhanced sensitivity calibration scheme. The top row shows the SENSE reconstructed images for reduction factors $R=1,2,3,4$, while the bottom row illustrates the corresponding g-factors. Data were acquired using a high contrast T1-weighted, inversion recovery sequence (FOV = 210 x 170 mm$^2$, Cartesian matrix = 256 x 256, full acquisition time = 102.4 s).

row together with corresponding g-factor maps (bottom row) for reduction factors of $R = 1$, 2, 3, and 4. The data were acquired with an inversion-recovery turbo-FLASH sequence (FOV = 210 x 170 mm$^2$, Cartesian 256 x 256 matrix acquisition, full acquisition time = 102.4 sec). The observed mean geometry factors are lower than in comparable setups at 1.5 and 3 T. This finding supports the prediction that PI benefits from the onset of wave behavior. From a practical perspective PI at ultra-high $B_0$ is especially appealing for EPI and spiral k-space acquisitions which are commonly used for functional and diffusion-weighted MRI. fMRI with parallel single-shot EPI has recently been demonstrated, enabling whole-brain coverage at exceptional temporal resolution. Due to its susceptibility to image distortion, EPI requires special measures for ensuring geometric consistency between the coil sensitivity maps and the undersampled data. In the aforementioned study, this was achieved by designing the sensitivity reference scan such that it exactly mimics the distortions suffered in the undersampled, parallel acquisition.
Conclusion and Outlook

This thesis has aimed to investigate theoretical and practical aspects of MRI with parallel detection and at high field strengths ($B_0$). Most importantly, the combination of both approaches was shown to be highly synergetic in two respects (cf. Tab. 4.1). Increasing $B_0$ changes the electrodynamics of the resonance signal in favor of parallel imaging. Higher resonance frequency results in reduced radiofrequency (RF) wavelength, giving rise to enhanced RF propagation and interference effects. These translate into more distinct coil sensitivities and hence into more effective sensitivity encoding by parallel acquisition. Added synergy stems from the fact that parallel imaging and high field strengths are per se largely complementary in terms of their basic strengths and weaknesses.

Based on theoretical, ultimate SNR calculations the inherent performance limits of conventional and parallel imaging have been established. For an elementary model configuration it was shown that these limits can be approached with finite coil arrays. Relatively few coil elements are sufficient when the SNR limits are comparatively tight, i.e. for deep regions of the object to be imaged and when the relevant RF fields are near-field dominated. Significantly larger coil arrays are required for achieving high reduction factors and implementing the SNR benefits inherent to RF wave conditions and short-range detection. Furthermore it has been shown that with a sufficiently large number of coil elements gaps between neighboring coils become dispensable. This suggests that large arrays may generally rely on deliberate coil overlaps for inductive decoupling, without limiting their usefulness for either conventional or parallel imaging.

Besides the advantages listed in Tab. 4.1, parallel imaging at high field strength also poses technical challenges. Increased image distortion with fast scanning strategies makes parallel imaging susceptible to inconsistency in sensitivity calibration. This serious problem may be addressed either by auto-calibration$^{25,29,141,142}$ or by $B_0$ mapping and monitoring$^{143,144}$ in combination with enhanced reconstruction schemes.$^{48,137-139}$ Furthermore, the greater distinctness of coil sensitivities at high $B_0$ goes along with enhanced spatial frequency content. Accordingly, coil calibration must rely on more reference information and advanced methods for denoising and extrapolating sensitivity maps. In addition, altered RF characteristics
and further specific requirements of high-field systems favor different coil array configurations. In the scope of this work the transmit/receive transmission line architecture\textsuperscript{88} has proven particularly suited for parallel imaging at high $B_0$.\textsuperscript{73}

Besides parallel acquisition, parallel excitation with a coil array\textsuperscript{131,132} is a promising concept in light of the challenges of high fields. Parallel excitation methods have been shown to enhance multidimensional selective RF excitation and also to enable the reduction of RF energy deposition.\textsuperscript{133} However, from a technical perspective this approach is rather demanding. Besides a transmit coil array it requires separate waveform generators and RF amplifiers for each coil element. Furthermore and similar to the receive analogue, accurate parallel excitation requires reliable knowledge of the transmit characteristics. In this context it is important to note that the transmit characteristics are generally not equal to the receive sensitivities, especially at high field.\textsuperscript{38,84} Hence specialized schemes for transmit calibration are called for.

High field MRI in humans has been pioneered for several years. At the time of writing this thesis a considerable number of whole-body systems of 7 T and beyond are being installed or projected in laboratories all over the world, reflecting high expectations from this technology. The years to come will show how and to which extent the considerable challenges of routine ultra-high field MR can be addressed. It also remains to be seen for which human applications 7 T and higher will become the field range of choice. Based on the results of the present work it seems fair to predict that parallel reception and transmission will play a vital role in this continuing development.
References


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Curriculum Vitae

I was born on April 16th, 1975 in Ried im Innkreis, Austria, as son of Karoline and Adolf Wiesinger. After primary school in Bruckmühl and secondary school in Wolfsegg, I attended the technical school for mechanical engineering in Vöcklabruck, from which I received the Matura in 1994.

From 1994 to 2000, I studied physics at the Johannes Kepler University in Linz, spending one exchange year at the Swiss Federal School of Technology (ETH) Zurich. From 1999 to 2000, I worked on my diploma thesis entitled “Magnetic Resonance Sequence Optimization for T1 Shortening Contrast Agents in Coronary Magnetic Resonance Angiography” at the Cardiac Magnetic Resonance Center of the Beth Israel Deaconess Medical Center and Harvard Medical School in Boston. This work was supervised by Matthias Stuber, PhD in Boston and Prof. Norbert Müller in Linz. In summer 2000 I received my physics diploma from the Johannes Kepler University in Linz.

In October 2000 I started working as a research and teaching assistant at the Institute for Biomedical Engineering (IBT) of the University and ETH Zurich in the group of Prof. Peter Boesiger and Prof. Klaas P. Pruessmann. In the scope of a scientific collaboration on parallel imaging at ultra-high fields I spent several months at the Center for MR Research (CMRR) of Prof. Kamil Ugurbil at the University of Minnesota in Minneapolis.

In 1999 I obtained the Wilhelm Macke Physic Prize from the Johannes Kepler University in Linz, Austria. In 2004 at the 12th annual meeting of the International Society for Magnetic Resonance in Medicine (ISMRM) in Kyoto I was awarded the I. I. Rabi Award based on my work on ”Parallel Imaging Performance as a Function of Field Strength - An Experimental Investigation Using Electrodynamic Scaling”. In 2005 my team-mate Martin Geidl and I were among the winning teams for the Intellectric Competition 2004 of the Departement of Information Technology and Electrical Engineering of the ETH Zurich.

In January 2003, my girlfriend Judith Höög and I became parents of our daughter Klara. By the time this thesis will be finished we expect the birth of our second child.