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Bimetric gravity is cosmologically viable

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ABSTRACT

Bimetric theory describes gravitational interactions in the presence of an extra spin-2 field. Previous work has suggested that its cosmological solutions are generically plagued by instabilities. We show that by taking the Planck mass for the second metric, M_f, to be small, these instabilities can be pushed back to unobservably early times. In this limit, the theory approaches general relativity with an effective cosmological constant which is, remarkably, determined by the spin-2 interaction scale. This provides a late-time expansion history which is extremely close to ΛCDM, but with a technically-natural value for the cosmological constant. We find M_f should be no larger than the electroweak scale in order for cosmological perturbations to be stable by big-bang nucleosynthesis. We further show that in this limit the helicity-0 mode is no longer strongly-coupled at low energy scales.

"The reports of my death have been greatly exaggerated."
—*Metrics Twain*

1. Introduction

The Standard Model of particle physics contains fields with spins 0, 1/2, and 1, describing matter as well as the strong and electroweak forces. General relativity (GR) extends this to the gravitational interactions by introducing a massless spin-2 field. There is theoretical and observational motivation to seek physics beyond the Standard Model and GR. In particular, GR is nonrenormalizable and is associated with the cosmological constant, dark energy, and dark matter problems. To compound the puzzle, the GR-based Λ-cold dark matter (ΛCDM) model provides a very good fit to observational data, despite its theoretical problems. In order to be observationally viable, any modified theory of gravity must be able to mimic GR over a wide range of distances.

A natural possibility for extending the set of known classical field theories is to include additional spin-2 fields and interactions. While "massive" and "bimetric" theories of gravity have a long history [1,2], nonlinear theories of interacting spin-2 fields were found, in general, to suffer from the Boulware–Deser (BD) ghost instability [3]. Recently a particular bimetric theory (or bigravity) has been shown to avoid this ghost instability [4,5]. This theory describes nonlinear interactions of the gravitational metric with an additional spin-2 field. It is an extension of an earlier ghost-free theory of massive gravity (a massive spin-2 field on a nondynamical flat background) [6–8] for which the absence of the BD ghost at the nonlinear level was established in Refs. [5,9–11].

Including spin-2 interactions modifies GR, *inter alia*, at large distances. Bimetric theory is therefore a candidate to explain the accelerated expansion of the Universe [12,13]. Indeed, bigravity has been shown to possess Friedmann–Lemaître–Robertson–Walker (FLRW) solutions which can match observations of the cosmic

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expansion history, even in the absence of vacuum energy [14–20].[1] Linear perturbations around these cosmological backgrounds have also been studied extensively [28–41]. The epoch of acceleration is set by the mass scale m of the spin-2 interactions. Unlike a small vacuum energy, m is protected from large quantum corrections due to an extra diffeomorphism symmetry that is recovered in the limit $m \to 0$, just as fermion masses are protected by chiral symmetry in the Standard Model (see Ref. [42] for an explicit analysis in the massive gravity setup). This makes interacting spin-2 fields especially attractive from a theoretical point of view.

Cosmological solutions lie on one of two branches, called the finite and infinite branches.[2] The infinite-branch models can have sensible backgrounds [19,32], but the perturbations have been found to contain ghosts in both the scalar and tensor sectors [33,34,41]. Most viable background solutions lie on the finite branch [16–19]. While these avoid the aforementioned ghosts, they contain a scalar instability at early times [29,32,33] that invalidates the use of linear perturbation theory and could potentially rule these models out. For parameter values thought to be favored by data, this instability was found to be present until recent times (i.e., a similar time to the onset of cosmic acceleration) and thus seemed to spoil the predictivity of bimetric cosmology.

In this Letter we study a physically well-motivated region in the parameter space of bimetric theory that has been missed in earlier work due to a ubiquitous choice of parameter rescaling. We demonstrate how in this region the instability problem in the finite branch can be resolved while the model still provides late-time acceleration in agreement with observations.

Our search for viable bimetric cosmologies will be guided by the precise agreement of GR with data on all scales, which motivates us to study models of modified gravity which are close to their GR limit. Often this limit is dismayingly trivial; if a theory of modified gravity is meant to produce late-time self-acceleration in the absence of a cosmological constant degenerate with vacuum energy, then we would expect that self-acceleration to disappear as the theory approaches GR. We will see, however, that there exists a GR limit of bigravity which retains its self-acceleration, leading to a GR-like universe with an effective cosmological constant produced purely by the spin-2 interactions.

2. Bimetric gravity

The ghost-free action for bigravity containing metrics $g_{\mu\nu}$ and $f_{\mu\nu}$ is given by [4,43]

$$S = \int \mathrm{d}^4x \Big[-\frac{M_{\mathrm{Pl}}^2}{2}\sqrt{g}R(g) - \frac{M_f^2}{2}\sqrt{f}R(f) + m^2 M_{\mathrm{Pl}}^2 \sqrt{g}\, V(\mathbb{X}) + \sqrt{g}\mathcal{L}_m(g,\Phi_i)\Big]. \tag{1}$$

Here M_{Pl} and M_f are the Planck masses for $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively, and we will frequently refer to their ratio,

$$\alpha \equiv \frac{M_f}{M_{\mathrm{Pl}}}. \tag{2}$$

The potential $V(\mathbb{X})$ is constructed from the elementary symmetric polynomials $e_n(\mathbb{X})$ of the eigenvalues of the matrix $\mathbb{X} \equiv \sqrt{g^{-1}f}$, defined by

$$\mathbb{X}^\mu{}_\alpha \mathbb{X}^\alpha{}_\nu \equiv g^{\mu\alpha} f_{\alpha\nu}, \tag{3}$$

and has the form [8,43],[3]

$$\sqrt{g}\, V(\mathbb{X}) = \sqrt{g}\beta_0 + \sqrt{g}\sum_{n=1}^{3}\beta_n e_n(\mathbb{X}) + \sqrt{f}\beta_4. \tag{4}$$

In the above, m is a mass scale and β_n are dimensionless interaction parameters. β_0 and β_4 parameterize the vacuum energies in the two sectors. Guided by the absence of ghosts and the weak equivalence principle, we take the matter sector to be coupled to $g_{\mu\nu}$.[4] Then the vacuum-energy contributions from the matter sector $\mathcal{L}_{\mathrm{m}}$ are captured in β_0. We can interpret $g_{\mu\nu}$ as the spacetime metric used for measuring distance and time, while $f_{\mu\nu}$ is an additional symmetric tensor that mixes nontrivially with gravity. As we discuss further below, the two metrics do not correspond to the spin-2 mass eigenstates but each contain both massive and massless components. Even before fitting to observational data, the parameters in the bimetric action are subject to several theoretical constraints. For instance, the squared mass of the massive spin-2 field needs to be positive, it must not violate the Higuchi bound [59,60], and ghost modes should be absent.

In terms of the Einstein tensor, $G_{\mu\nu}$, the equations of motion for the two metrics take the form

$$G_{\mu\nu}(g) + m^2 V^g_{\mu\nu} = \frac{1}{M_{\mathrm{Pl}}^2}T_{\mu\nu}, \tag{5}$$

$$\alpha^2 G_{\mu\nu}(f) + m^2 V^f_{\mu\nu} = 0, \tag{6}$$

where $V^{(g,f)}_{\mu\nu}$ are determined by varying the interaction potential, V. Taking the divergence of eq. (5) and using the Bianchi identity leads to the *Bianchi constraint*,

$$\nabla^\mu_{(g)} V^g_{\mu\nu} = 0. \tag{7}$$

The analogous equation for $f_{\mu\nu}$ carries no additional information due to the general covariance of the action.

Finally, note that the action (1) has a status similar to Proca theory on curved backgrounds. It is therefore expected to require an analogue of the Higgs mechanism, with new degrees of freedom, in order to have improved quantum behavior. The search for a ghost-free Higgs mechanism for gravity is still in progress [61].

3. The GR limit

When bigravity is linearized around proportional backgrounds $\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$ with constant c,[5]

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_{\mathrm{Pl}}}\delta g_{\mu\nu}, \tag{8}$$

$$f_{\mu\nu} = c^2\bar{g}_{\mu\nu} + \frac{c}{M_f}\delta f_{\mu\nu}, \tag{9}$$

the canonically-normalized perturbations can be diagonalized into massless modes $\delta G_{\mu\nu}$ and massive modes $\delta M_{\mu\nu}$ as [4,62]

$$\delta G_{\mu\nu} \propto \left(\delta g_{\mu\nu} + c\alpha\, \delta f_{\mu\nu}\right), \tag{10}$$

$$\delta M_{\mu\nu} \propto \left(\delta f_{\mu\nu} - c\alpha\, \delta g_{\mu\nu}\right). \tag{11}$$

[1] Stable FLRW solutions do not exist in massive gravity [21–27].

[2] There is a third branch containing bouncing solutions, but these tend to have pathologies [41].

[3] This is a generalization of the massive-gravity potential [8] (to which it reduces for $f_{\mu\nu} = \eta_{\mu\nu}$ and a restricted set of β_n) given in Ref. [43].

[4] More general matter couplings not constrained by these requirements have been studied in Refs. [20,44–58].

[5] These correspond to Einstein spaces and, for nonvanishing α, solve the field equations only in vacuum. A quartic equation determines $c = c(\beta_n, \alpha)$.

Notice that when $\alpha \to 0$ (or $M_{\rm Pl} \gg M_f$), the massless state aligns with $\delta g_{\mu\nu}$, i.e., up to normalization,

$$\delta G_{\mu\nu} \to \delta g_{\mu\nu} + \mathcal{O}(\alpha^2). \tag{12}$$

Because $g_{\mu\nu}$ is the physical metric, this suggests that $\alpha \to 0$ is the general-relativity limit of bigravity.[6] We will see below that the nonlinear field equations indeed reduce to Einstein's equations for $\alpha = 0$ and that the limit is continuous. Thus $g_{\mu\nu}$ is close to a GR solution for sufficiently small values of α. We therefore identify $M_{\rm Pl}$ with the measured physical Planck mass whenever $\alpha \ll 1$, holding it fixed while making M_f smaller. Interestingly, in the bimetric setup a large physical Planck mass is correlated with the fact that gravity is approximated well by a massless field. In other words, when bimetric theory is close to GR, the gravitational force is naturally weak.

The GR limit can be directly realized at the nonlinear level [64,65]. The metric potentials satisfy the identity

$$\sqrt{g} g^{\mu\alpha} V^g_{\alpha\nu} + \sqrt{f} f^{\mu\alpha} V^f_{\alpha\nu} = \sqrt{g} V \delta^\mu{}_\nu, \tag{13}$$

where V is the potential in the action (1). For $M_f = 0$, the $f_{\mu\nu}$ equation (6) gives $V^f_{\mu\nu} = 0$, an algebraic constraint on $f_{\mu\nu}$. Then, using the above identity, the $g_{\mu\nu}$ equation (5) becomes

$$G_{\mu\nu}(g) + m^2 V g_{\mu\nu} = \frac{1}{M^2_{\rm Pl}} T_{\mu\nu}. \tag{14}$$

Since $T_{\mu\nu}$ is conserved, taking the divergence gives

$$\partial_\mu V = 0. \tag{15}$$

We see that eq. (14) is the Einstein equation for $g_{\mu\nu}$ with cosmological constant $m^2 V$. Remarkably, because V depends on $f_{\mu\nu}$ and all the β_n, this effective cosmological constant is generically *not* simply the vacuum energy from matter loops (which is parameterized by β_0). Even in the GR limit, the impact of the spin-2 interactions remains and bigravity's self-acceleration survives.

It is straightforward to see that, unlike the $m \to 0$ limit, the $\alpha \to 0$ limit is not affected by the van Dam–Veltman–Zakharov (vDVZ) discontinuity [66,67]. The cause of this discontinuity is the Bianchi constraint (7) which constrains the solutions even when $m = 0$. On the contrary, when $\alpha \to 0$, the Bianchi constraint simply reduces to eq. (15) and is automatically satisfied.

The conditions $V^f_{\mu\nu} = 0$ and $\partial_\mu V = 0$ determine $f_{\mu\nu}$ algebraically in terms of $g_{\mu\nu}$, generically as $f_{\mu\nu} = c^2 g_{\mu\nu}$. In the limit $M_f = 0$, the f sector is infinitely strongly coupled.[7] Due to the nontrivial potential, this causes the f metric to exactly follow the g metric (both at the background and perturbative levels), while the g sector remains weakly coupled.

4. Strong-coupling scales

We now argue that at energy scales relevant to cosmology, this model avoids known strong-coupling issues, sometimes contrary to intuition gained from massive gravity.

There are several strong-coupling scales one might expect to arise. At an energy scale k, the f sector has an effective coupling k/M_f, as can be seen from expanding the Einstein–Hilbert action in $\delta f_{\mu\nu}/M_f$, just as in GR. Then, for small but nonzero α, which is the case of interest here, one might worry that perturbations of $f_{\mu\nu}$ with momentum k become strongly coupled at low scales $k \sim M_f$. However, we have seen that in the limit of infinite strong coupling, $M_f = 0$, $f_{\mu\nu}$ becomes nondynamical and is entirely determined in terms of $g_{\mu\nu}$, while the $g_{\mu\nu}$ equation is degenerate with GR and its perturbations remain weakly coupled. Due to the continuity of the limit, we expect that, for small enough α, strong-coupling effects will continue to not affect the g sector, even when perturbations of $f_{\mu\nu}$ are strongly coupled at relatively small energy scales. In practice, however, since the measured value of $M_{\rm Pl}$ is very large, even reasonably high values of M_f can still lead to small α. In cosmological applications, all observable perturbations satisfy $k/M_f \ll 1$ for $M_f \gg 100 H_0 \sim 10^{-31}$ eV, roughly the scale at which linear cosmological perturbation theory breaks down at recent times, so that perturbations of $f_{\mu\nu}$ remain weakly coupled in any case.

Another potentially-problematic scale is associated with the helicity-0 mode of the massive graviton. In massive gravity, this mode becomes strongly coupled at the scale [72,73]

$$\Lambda_3 \equiv \left(m^2 M_{\rm Pl}\right)^{1/3}, \tag{16}$$

where m is defined to coincide with the Fierz–Pauli mass [1] on flat backgrounds. This scale is rather small, $\Lambda_3 \sim 10^{-13}$ eV $\sim$ (1000 km)$^{-1}$ for $m \sim H_0 \sim 10^{-33}$ eV, and severely restricts the applicability of massive gravity [74]. The same scale also appears in the decoupling-limit analysis of bimetric theory [37], where m is now the parameter in front of the potential in the action (1). In the limit $\alpha \to 0$, the f sector approaches massive gravity [65] and one might worry that the strong-coupling problem persists or becomes worse with the emergence of an even lower scale $(m^2 M_f)^{1/3}$. This is not the case. In the bimetric context, the scale defined in eq. (16) is not physical, since m^2 is degenerate with the β_n. The physically relevant strong-coupling scale must be defined with respect to the bimetric Fierz–Pauli mass [62],

$$m^2_{\rm FP} = m^2 \left(\frac{1}{c^2\alpha^2} + 1\right)\left(c\beta_1 + 2c^2\beta_2 + c^3\beta_3\right), \tag{17}$$

which is only defined around proportional backgrounds, $f_{\mu\nu} = c^2 g_{\mu\nu}$. In the massive-gravity limit, $\alpha \to \infty$, the helicity-0 mode is mostly contained in g with a strong-coupling scale

$$\Lambda_3 \equiv \left(m^2_{\rm FP} M_{\rm Pl}\right)^{1/3}, \tag{18}$$

consistent with eq. (16) for appropriately restricted parameters. However, in the GR limit, $\alpha \to 0$, the helicity-0 mode resides mostly in f, where the strong-coupling scale is

$$\tilde{\Lambda}_3 \equiv \left(m^2_{\rm FP} M_f\right)^{1/3} \to \left(\frac{m^2 M_{\rm Pl}}{\alpha}\,\mathcal{O}(\beta_n)\right)^{1/3}, \tag{19}$$

which is no longer small. Note that for solutions that admit this limit, c becomes independent of α. We can also consider the $\alpha \to 0$ limit of eq. (18), to verify that the small part of the helicity-0 mode in g is not strongly coupled,

$$\Lambda_3 \to \left(\frac{m^2 M_{\rm Pl}}{\alpha^2}\,\mathcal{O}(\beta_n)\right)^{1/3}. \tag{20}$$

This is even higher than $\tilde{\Lambda}_3$. Therefore the strong-coupling issues with the helicity-0 mode are alleviated, rather than exacerbated, when $\alpha \to 0$.

[6] See Ref. [63] for an early discussion of such a limit.

[7] Strongly-coupled gravity in the context of GR has been studied, for instance, in Refs. [68–71] and has been argued to allow for a simplified quantum-mechanical treatment.

5. Cosmology

We now proceed to apply the above arguments to the particular example of a homogeneous and isotropic universe. We will take both metrics to be of the diagonal FLRW form [14–16],[8]

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{21}$$

$$f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -X^2(t)\mathrm{d}t^2 + Y^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{22}$$

where we can freely choose the cosmic-time coordinate for $g_{\mu\nu}$ ($g_{00} = -1$) because of general covariance. Because matter couples minimally to $g_{\mu\nu}$, this choice is physical, and $a(t)$ corresponds to the scale factor inferred from observations. We furthermore take the matter source to be a perfect fluid, $T^\mu_{\ \nu} = \mathrm{diag}(-\rho, p, p, p)$. The g-metric equation (5) leads to the Friedmann equation,

$$3H^2 = \frac{\rho}{M_{\mathrm{Pl}}^2} + m^2\left(\beta_0 + 3\beta_1 y + 3\beta_2 y^2 + \beta_3 y^3\right), \tag{23}$$

where the Hubble rate is defined as $H \equiv \dot{a}/a$ and the ratio of the scale factors is

$$y \equiv \frac{Y}{a}. \tag{24}$$

The analogous equation for the f metric is

$$3K^2 = \frac{m^2}{\alpha^2}X^2\left(\frac{\beta_1}{y^3} + 3\frac{\beta_2}{y^2} + 3\frac{\beta_3}{y} + \beta_4\right), \tag{25}$$

with $K \equiv \dot{Y}/Y$. The final ingredient is the Bianchi constraint (7), which yields

$$(HX - Ky)\left(\beta_1 + 2\beta_2 y + \beta_3 y^2\right) = 0. \tag{26}$$

Taking the first or second term of eq. (26) to vanish selects the so-called dynamical or algebraic branches, respectively. Perturbations in the algebraic branch are pathological [29], so we will consider the dynamical branch in which the f-metric lapse is fixed,

$$X = \frac{Ky}{H}. \tag{27}$$

Inserting this into the $f_{\mu\nu}$ equation (25) transforms it into an "alternate" Friedmann equation,

$$3\alpha^2 H^2 = m^2\left(\frac{\beta_1}{y} + 3\beta_2 + 3\beta_3 y + \beta_4 y^2\right). \tag{28}$$

We take at least two of the β_n for $n \geq 1$ to be nonzero in order to ensure the existence of interesting solutions in the GR limit $\alpha \to 0$. The solutions to eq. (28) in the GR limit are always on the "finite" branch, i.e., y evolves from 0 to a finite late-time value. The perturbations on this branch are healthy *except* for a scalar instability, which we discuss below.

Equation (28) has two features which are useful for our purposes. First, in the limit $\alpha \to 0$ it tends to a polynomial constraint that leads to a constant solution for y, so that the potential term in the Friedmann equation (23) becomes a cosmological constant. This provides an explicit example of the statement above that as $\alpha \to 0$, the theory approaches general relativity with an effective cosmological constant (even with $\beta_0 = 0$). Recall that even though the theory approaches GR in this limit, the bigravity interactions survive in the form of this constant. The other useful feature is that, because eq. (28) does not involve ρ, it can be used to rephrase the potential term in eq. (23) in terms of the Hubble rate. This will allow us to determine the time-dependence of the potential term order by order in α.[9]

6. The effective cosmological constant

Let us illustrate the new viable bimetric cosmologies qualitatively by selecting the model with $\beta_0 = \beta_3 = \beta_4 = 0$,[10] which we will refer to as the $\beta_1\beta_2$ model. The Friedmann and "alternate" Friedmann equations (23) and (28) are

$$3H^2 = \frac{\rho}{M_{\mathrm{Pl}}^2} + 3m^2\left(\beta_1 y + \beta_2 y^2\right), \tag{29}$$

$$3\alpha^2 H^2 = m^2\left(\frac{\beta_1}{y} + 3\beta_2\right). \tag{30}$$

We can use eq. (30) to eliminate y in eq. (29). It is instructive to work in the GR limit where eq. (30) gives

$$y \xrightarrow{\alpha\to 0} -\frac{1}{3}\frac{\beta_1}{\beta_2}. \tag{31}$$

The $\alpha \to 0$ limit is nonsingular only if both β_1 and β_2 are nonzero. Plugging this into eq. (29) we obtain

$$3H^2 = \frac{\rho}{M_{\mathrm{Pl}}^2} - \frac{2}{3}\frac{\beta_1^2}{\beta_2}m^2. \tag{32}$$

The effective cosmological constant is

$$\Lambda_{\mathrm{eff}} = -\frac{2}{3}\frac{\beta_1^2}{\beta_2}m^2. \tag{33}$$

Late-time acceleration requires $\beta_2 < 0$.

When we are not exactly in the GR limit, we should consider corrections to eq. (32),

$$\begin{aligned} 3H^2 &= \frac{\rho}{M_{\mathrm{Pl}}^2} + \frac{\beta_1^2 m^4}{3\left(H^2\alpha^2 - \beta_2 m^2\right)^2}\left(3\alpha^2 H^2 - 2\beta_2 m^2\right) \\ &= \frac{\rho}{M_{\mathrm{Pl}}^2} - \frac{2}{3}\frac{\beta_1^2}{\beta_2}m^2 - \frac{\alpha^2\beta_1^2}{3\beta_2^2}H^2 + \mathcal{O}(\alpha^4). \end{aligned} \tag{34}$$

This expansion is valid as long as

$$H^2 \lesssim \frac{\beta_2 m^2}{\alpha^2}. \tag{35}$$

Rearranging and again keeping terms up to $\mathcal{O}(\alpha^2)$, we find a standard Friedmann equation with a time-varying effective cosmological constant given by

$$\Lambda_{\mathrm{eff}} = -\frac{2}{3}\frac{\beta_1^2}{\beta_2}m^2 - \frac{2}{9}\frac{\beta_1^2}{\beta_2^2}\alpha^2\left(\frac{\rho}{2M_{\mathrm{Pl}}^2} - \frac{\beta_1^2}{3\beta_2}m^2\right) + \mathcal{O}(\alpha^4). \tag{36}$$

Because matter is coupled minimally to $g_{\mu\nu}$, it will have the standard behavior $\rho \sim a^{-3(1+w)}$, where $w = p/\rho$ is the equation-of-state parameter, allowing ρ to stand in for time. This captures the first hint of the dynamical dark energy that is typical of bigravity [16–20].

These results generalize easily to other parameter combinations. We list the effective cosmological constant up to $\mathcal{O}(\alpha^2)$ for all the

[8] See Ref. [75] and the references therein for other possible metrics in bimetric cosmology.

[9] One can also combine eqs. (23) and (28) to obtain a quartic equation for y involving ρ [14–17,31], but this is more cumbersome as it involves higher powers of y than eq. (28) does.

[10] Since we are interested in finding self-accelerating solutions in the absence of vacuum energy, we will set $\beta_0 = 0$ herein, but emphasize that this is not necessary.

Table 1
The effective cosmological constant and lowest-order corrections (which are time-dependent through ρ) for a variety of two-parameter models. We have chosen solution branches which lead to positive $\Lambda_{\rm eff}$ for appropriate signs of the β_n, and generally take $\beta_1 \geq 0$ based on viability conditions [19]. The $\beta_3, \beta_4 \neq 0$ model does not possess a finite-branch solution [19].

Model	$\Lambda_{\rm eff}$ $(\alpha \to 0)$	$\mathcal{O}(\alpha^2)$ correction
$\beta_1, \beta_2 \neq 0$	$-\frac{2}{3}\frac{\beta_1^2}{\beta_2}m^2$	$-\frac{2}{9}\frac{\beta_1^2}{\beta_2^2}\alpha^2\left(\frac{\rho}{2M_{\rm Pl}^2} - \frac{\beta_1^2}{3\beta_2}m^2\right)$
$\beta_1, \beta_3 \neq 0$	$\frac{8}{3\sqrt{3}}\frac{\beta_1^{3/2}}{\sqrt{-\beta_3}}m^2$	$\frac{\beta_1}{\beta_3}\alpha^2\left(\frac{\rho}{3M_{\rm Pl}^2} - \frac{8\beta_1^{3/2}}{9\sqrt{-3\beta_3}}m^2\right)$
$\beta_1, \beta_4 \neq 0$	$3\frac{\beta_1^{4/3}}{\sqrt[3]{-\beta_4}}m^2$	$-\left(-\frac{\beta_1}{\beta_4}\right)^{\frac{2}{3}}\alpha^2\left(\frac{\rho}{M_{\rm Pl}^2} + 3\frac{\beta_1^{4/3}}{\sqrt[3]{-\beta_4}}m^2\right)$
$\beta_2, \beta_3 \neq 0$	$2\frac{\beta_2^3}{\beta_3^2}m^2$	$-\frac{\beta_2^2}{\beta_3^2}\alpha^2\left(\frac{\rho}{M_{\rm Pl}^2} + \frac{2\beta_2^3}{\beta_3^2}m^2\right)$
$\beta_2, \beta_4 \neq 0$	$-9\frac{\beta_2^2}{\beta_4}m^2$	$3\frac{\beta_2}{\beta_4}\alpha^2\left(\frac{\rho}{M_{\rm Pl}^2} - \frac{9\beta_2^2}{\beta_4}m^2\right)$

two-parameter models (setting $\beta_0 = 0$) in Table 1. We remind the reader that, in order for the $\alpha \to 0$ limit to be well-behaved, at least two of the β_n parameters (excluding the vacuum energy contribution, β_0) must be nonzero.

7. Exorcising the instability

The stability of cosmological perturbations in bigravity was investigated in Ref. [32] by determining the full solutions to the linearized Einstein equations in the subhorizon régime. The perturbations were shown to obey a WKB solution given by

$$\Phi \sim e^{i\omega N}, \tag{37}$$

where Φ represents any of the scalar perturbation variables, $N \equiv \ln a$, and we have taken the limit $k \gg aH$ where k is the comoving wavenumber. The eigenfrequencies ω were presented for particular models in Ref. [32], where it was found that all models with viable backgrounds have $\omega^2 < 0$ at early times, revealing a gradient instability that only ends at a very low redshift. Using the formulation of the linearized equations of motion presented in Ref. [33], we can write the eigenfrequencies for general β_n and α in the compact form [41]

$$\left(\frac{aH}{k}\right)^2\omega^2 = 1 + \frac{\left(\beta_1 + 4\beta_2 y + 3\beta_3 y^2\right)y'}{3y\left(\beta_1 + 2\beta_2 y + \beta_3 y^2\right)} - \frac{\left(1+\alpha^2 y^2\right)\left(\beta_1 - \beta_3 y^2\right)y'^2}{3\alpha^2 y^3 \tilde{\rho}(1+w)}, \tag{38}$$

where $\tilde{\rho} \equiv \rho/m^2 M_{\rm Pl}^2$ and primes denote $\mathrm{d}/\mathrm{d}\ln a$.

We apply this to the $\beta_1\beta_2$ model. Assuming a universe dominated by dust ($w = 0$), ω^2 crosses zero when[11]

$$18\alpha^2\beta_2\left(\alpha^2\beta_1^2 + 4\beta_2^2\right)y^5 + 9\alpha^2\beta_1\left(\alpha^2\beta_1^2 + 10\beta_2^2\right)y^4 + 48\alpha^2\beta_1^2\beta_2 y^3 + 6\beta_2\left(2\alpha^2\beta_1^2 - \beta_2^2\right)y^2 - 6\beta_1^2\beta_2 y - \beta_1^3 = 0. \tag{39}$$

Solving this for y, we can then use eq. (30) to determine the value of Hubble rate at the *transition era*, before which the gradient instability is present and after which it vanishes. While this solution is too complicated to write down explicitly, in the limit $\alpha \to 0$ the leading-order term is remarkably simple,[12]

$$H_\star^2 = \pm\frac{\beta_2 m^2}{\sqrt{3}\alpha^2} + \mathcal{O}(\alpha^0), \tag{40}$$

where $H_\star$ is defined as the Hubble rate at the time when $\omega^2 = 0$, i.e., after which the gradient instability is absent. We pick the negative branch of eq. (40) for physical reasons, i.e., so that $H_\star^2 > 0$ given that $\beta_2 < 0$. We have checked explicitly that by solving for y with this value of H and plugging it into ω^2, all terms up to $\mathcal{O}(\alpha^2)$ vanish.

Interestingly, eq. (40) is the same as the condition (35) for the small-α expansion of the background solution to be valid. Therefore, simply by pushing the instability back to early times, one gets late-time bimetric dynamics that can be described as perturbative corrections to GR, except for the effective cosmological constant which remains nonperturbative. This is nontrivial; while we expect everything to reduce to GR at late times when we can expand in $\alpha H/\sqrt{\beta_n}m$, there could in principle have been earlier times during which perturbations were stable but still fundamentally different than in GR.

We can rewrite eq. (40) in more physical terms as

$$H_\star^2 = -\frac{3\sqrt{3}}{2\alpha^2}\left(\frac{\beta_2}{\beta_1}\right)^2 H_\Lambda^2, \tag{41}$$

where H_Λ is the far-future value of H and should be comparable to the present Hubble rate, H_0. For $|\beta_1| \sim |\beta_2|$, this implies simply

$$H_\star \sim \frac{H_0}{\alpha}. \tag{42}$$

We see that as we approach the GR limit, the smaller one takes the f-metric Planck mass, the earlier in time bigravity's gradient instability is cured. Our goal is to make this era so early as to be effectively unobservable. One has a variety of choices for the scale where the instability sets in; the values of α and M_f for various choices are summarized in Table 2.

Table 2
The values of α and M_f for a few choices of the era at which perturbations become stable.

Era of transition to stability	$H_\star$	α	M_f
BBN	10^{-16} eV	10^{-17}	100 GeV
$\tilde{\Lambda}_3 = (m^2 M_{\rm Pl}/\alpha)^{1/3}$	10^{-3} eV	10^{-31}	10^{-3} eV
GUT-scale inflation	10^{13} GeV	10^{-55}	10^{-27} eV
$M_{\rm Pl}$	10^{19} GeV	10^{-61}	10^{-33} eV

A natural requirement would be to push the instability outside the range of the effective field theory, i.e., above either the cut-off scale where new physics must enter, or the strong-coupling scale where tree-level unitary breaks down.[13] The cut-off scale in massive and bimetric gravity is not known. The strong-coupling scale, to the extent it is understood, was discussed above. Here we focus on observational constraints. It is natural to demand that the instability lie beyond some important cosmic era which we can indirectly probe, such as big-bang nucleosynthesis (BBN) or inflation. Both of these possibilities are then likely to be observationally safe as long as the Universe is decelerating (e.g., is

[11] We have used eqs. (29) and (30) and their derivatives to solve for y' and ρ in eq. (38) in terms of β_n and y [31]. Note that $\omega^2 = 0$ does not imply strong coupling because, while the gradient terms vanish, the kinetic terms remain nonzero.

[12] While eq. (40) only holds exactly in the presence of dust, $w = 0$, for other reasonable equations of state, such as radiation ($w = 1/3$), it will only be modified by an $\mathcal{O}(1)$ factor. Since we will be using this analysis only to make order-of-magnitude estimates, the exact factors are unimportant.

[13] These two are not always the same, and may not be in massive and bimetric gravity [76,77].

radiation-dominated) after inflation, because the instability is only a problem for subhorizon modes with large k/aH, and during a decelerating epoch modes with fixed comoving wavelength always become smaller with respect to the horizon. Consider, as an example, that the transition to stability occurs between inflation and BBN. During that period, modes will grow rapidly on small scales, but those will be far, far smaller than the modes relevant for the cosmic microwave background or large-scale structure. One might worry that inflation's ability to set initial conditions is spoiled in this scenario (assuming that the linear theory is even valid during inflation, which is not guaranteed due to the arguments above). However, the instability should be absent during inflation; notice from eq. (38) that ω^2 generically becomes large and positive for w close to -1.[14] Therefore the instability would not affect the generation of primordial perturbations during inflation. If the instability later appears with the onset of radiation domination, it would only affect small scales which are irrelevant for present-day cosmology.

If the instability ends at the time of BBN, M_f can be as high as about 100 GeV, far larger than the wavenumbers probed by cosmological observations. We remind the reader that for such a "large" M_f, perturbations in the Einstein–Hilbert term for $f_{\mu\nu}$ remain weakly-coupled for all observationally-relevant k.

While analytic results like eq. (40) cannot be obtained for most of the other two-parameter models, we have checked that in each case the relevant behavior, $H_\star \sim H_\Lambda/\alpha$, holds.[15] The values given in Table 2 are therefore fairly model-independent.

The other pathology that is typical of massive and bimetric gravity, the Higuchi ghost, is not present in these models. There is a simple condition for the absence of this ghost, $\mathrm{d}\rho/\mathrm{d}y < 0$ [35,36] (see also Refs. [33,37]). Because for normal matter ρ is always decreasing with time, this amounts to demanding that y be increasing. In the "finite-branch" solutions which we are considering, y evolves monotonically from 0 at early times to a fixed positive value at late times, and so the Higuchi bound is always satisfied [41].

8. Parameter rescalings

We have presented a physically well-motivated region of bimetric parameter space, near the GR limit, in which observable cosmological perturbations are stable and yet self-acceleration remains. One is naturally led to ask how this has been missed by the many previous studies of bimetric cosmology. The issue lies in a rescaling which leaves the action (1) invariant [28,62],

$$f_{\mu\nu} \to \Omega^2 f_{\mu\nu}, \qquad \beta_n \to \frac{1}{\Omega^n}\beta_n, \qquad M_f \to \Omega M_f, \tag{43}$$

and hence gives rise to a redundant parameter. It has become common to let α play this role and perform the rescaling $\Omega = 1/\alpha$ such that α is set to unity. While our results do not invalidate this rescaling, they do show that it picks out a particular region of parameter space which may not capture all physically-meaningful situations. In particular, the $\alpha \to 0$ limit, in which the theory approaches GR—the behavior at the heart of our removing the gradient instability—would look extremely odd after this rescaling: the β_n would not only be very large, but each β_{n+1} would be *parametrically* larger than β_n.[16] Therefore, studies which set α to unity could in principle have found the GR-like solutions which we study here, but only by looking at what would have appeared to be a highly unnatural and tuned set of parameters, even though they have a simple and sensible physical explanation. Without performing this rescaling, we can simply take the nonzero β_n to be $\mathcal{O}(1)$ and consider that we are in the small-M_f régime.

It is clear that in phenomenological studies of bigravity, α must not automatically be set to unity. When working with a two-β_n model, perhaps a more sensible rescaling would be one such that the two β_n are equal to each other (up to a possible sign). They can further be absorbed into m^2. In this case, the free parameters are effectively the spin-2 interaction scale, m^2, and the f-metric Planck mass, M_f. Their effects decouple nicely: M_f controls the earliness of the instability, while m sets the acceleration scale. Alternatively, one may consider that the rescaling (43) simply tells us that rather different regions of parameter space happen to have the same solutions, and therefore not perform any rescaling *a priori* at all.

9. Summary and discussion

We have shown that a well-motivated but heretofore underexplored region of parameter space in bimetric gravity can lead to cosmological solutions which are observationally viable and close to general relativity, with an effective cosmological constant that is set by the spin-2 interaction scale m. In this limit, obtained by taking a small f-metric Planck mass, the gradient instability that seems to generically plague bimetric models at late times is relegated to the very early Universe, where it can be either made unobservable or pushed outside the régime of validity of the effective theory. This instability had been considered in previous work to make bimetric cosmologies nonpredictive even at late times. Furthermore, in this limit the theory avoids the usual low-scale strong-coupling issue that affects the helicity-0 sector in the massive-gravity limit.

What is encouraging is that the one property of bigravity which survives in the small-α limit is its cosmologically most useful feature, the technically-natural dark energy scale. In other words, the effective cosmological constant of bigravity in a region close to GR is not just the vacuum-energy contribution and can give rise to self-acceleration in its absence.

The model we have presented is expected to be extremely close to GR at all but very high energy scales. In particular the Newtonian limit is well-behaved; unlike $m^2 \to 0$, which suffers from the vDVZ discontinuity, the GR limit $\alpha \to 0$ is completely smooth because all the helicity states of the massive spin-2 mode decouple from matter. Note also that massive gravity does not possess such a continuous GR limit.

It is worth emphasizing that the $\alpha \to 0$ limit brings bimetric theory arbitrarily close to GR even for a large value of the spin-2 mass scale, $m \gg H_0$. The presence of heavy spin-2 fields in the Universe is therefore not excluded as long as their self-interaction scale (set by M_f) is sufficiently small compared to M_{Pl}. In this case, however, the β_n parameters need to be highly tuned for the effective cosmological constant small enough to be compatible with observations.[17] Note however that, since the β_n are

[14] This depends on the exact β_n parameters and the evolution of y. Background viability requires $\beta_1 > 0$ [19], so as long as $\beta_3 \leq 0$, at the very least the last term in eq. (38) is large and manifestly positive.

[15] Specifically, this holds in the models with $\beta_1 \neq 0$. The gradient instability is absent from the $\beta_2\beta_3$ and $\beta_2\beta_4$ models at early times [32]. These were shown in Ref. [19] to have problematic background behavior at early times, but these again can be made unobservably early in the GR limit.

[16] We can recast this as a large m^2, but there would remain a specific tuning among the β_n of the form $\beta_n/\beta_{n+1} \sim \epsilon$, where ϵ is the value of α before the rescaling.

[17] Indeed, without this tuning of the β_n, the interaction term would lead to acceleration at an unacceptably early epoch. This scenario is related to the findings of Ref. [36], where it was shown that the instability becomes negligible for large values of m.

protected against loop corrections [42,78,79], this tuning does not violate *technical* naturalness.

Finally we comment on the potential observable signatures of this theory. While at low energies, corresponding to recent cosmological epochs, this limit of bigravity is extremely close to GR, there may be observable effects at early times when the effects of strong coupling become important. In this case, given by $H > H_\star$, the small-α approximation breaks down and modified-gravity effects must be taken into account. This may be particularly important for inflation, which will see such effects unless M_f is extraordinarily small. A better understanding of strong coupling in the $f_{\mu\nu}$ sector will therefore point the way towards tests of this important region of bimetric parameter space, since at this point it is not clear how to perform computations in the strong-coupling regime. There may also be effects related to the Vainshtein mechanism [80,81]. We conclude that the closeness of this theory to GR is both a blessing and a curse: while it is behind the exorcism of the gradient instability and brings the theory in excellent agreement with experiments, it presents a serious observational challenge if it is to be compared *against* GR. It is nevertheless encouraging that this "GR-adjacent" bigravity naturally explains cosmic acceleration while avoiding the instabilities that plague other bimetric models, and therefore merits serious consideration.

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References

[1] M. Fierz, W. Pauli, On relativistic wave equations for particles of arbitrary spin in an electromagnetic field, Proc. R. Soc. Lond. Ser. A, Math. Phys. Sci. 173 (1939) 211–232.
[2] C.J. Isham, Abdus Salam, J.A. Strathdee, F-dominance of gravity, Phys. Rev. D 3 (1971) 867–873.
[3] D.G. Boulware, Stanley Deser, Can gravitation have a finite range?, Phys. Rev. D 6 (1972) 3368–3382.
[4] S.F. Hassan, Rachel A. Rosen, Bimetric gravity from ghost-free massive gravity, J. High Energy Phys. 1202 (2012) 126, arXiv:1109.3515 [hep-th].
[5] S.F. Hassan, Rachel A. Rosen, Confirmation of the secondary constraint and absence of ghost in massive gravity and bimetric gravity, J. High Energy Phys. 1204 (2012) 123, arXiv:1111.2070 [hep-th].
[6] Paolo Creminelli, Alberto Nicolis, Michele Papucci, Enrico Trincherini, Ghosts in massive gravity, J. High Energy Phys. 0509 (2005) 003, arXiv:hep-th/0505147.
[7] Claudia de Rham, Gregory Gabadadze, Generalization of the Fierz–Pauli action, Phys. Rev. D 82 (2010) 044020, arXiv:1007.0443 [hep-th].
[8] Claudia de Rham, Gregory Gabadadze, Andrew J. Tolley, Resummation of massive gravity, Phys. Rev. Lett. 106 (2011) 231101, arXiv:1011.1232 [hep-th].
[9] S.F. Hassan, Rachel A. Rosen, Resolving the ghost problem in non-linear massive gravity, Phys. Rev. Lett. 108 (2012) 041101, arXiv:1106.3344 [hep-th].
[10] S.F. Hassan, Rachel A. Rosen, Angnis Schmidt-May, Ghost-free massive gravity with a general reference metric, J. High Energy Phys. 1202 (2012) 026, arXiv:1109.3230 [hep-th].
[11] S.F. Hassan, Angnis Schmidt-May, Mikael von Strauss, Proof of consistency of nonlinear massive gravity in the Stúckelberg formulation, Phys. Lett. B 715 (2012) 335–339, arXiv:1203.5283 [hep-th].
[12] Adam G. Riess, et al., Supernova Search Team, Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. 116 (1998) 1009–1038, arXiv:astro-ph/9805201.
[13] S. Perlmutter, et al., Supernova Cosmology Project, Measurements of Omega and Lambda from 42 high redshift supernovae, Astrophys. J. 517 (1999) 565–586, arXiv:astro-ph/9812133.
[14] Mikhail S. Volkov, Cosmological solutions with massive gravitons in the bigravity theory, J. High Energy Phys. 1201 (2012) 035, arXiv:1110.6153 [hep-th].
[15] D. Comelli, M. Crisostomi, F. Nesti, L. Pilo, FRW cosmology in ghost free massive gravity, J. High Energy Phys. 1203 (2012) 067, arXiv:1111.1983 [hep-th].
[16] Mikael von Strauss, Angnis Schmidt-May, Jonas Enander, Edvard Mörtsell, S.F. Hassan, Cosmological solutions in bimetric gravity and their observational tests, J. Cosmol. Astropart. Phys. 1203 (2012) 042, arXiv:1111.1655 [gr-qc].
[17] Yashar Akrami, Tomi S. Koivisto, Marit Sandstad, Accelerated expansion from ghost-free bigravity: a statistical analysis with improved generality, J. High Energy Phys. 1303 (2013) 099, arXiv:1209.0457 [astro-ph.CO].
[18] Yashar Akrami, Tomi S. Koivisto, Marit Sandstad, Cosmological constraints on ghost-free bigravity: background dynamics and late-time acceleration, arXiv:1302.5268 [astro-ph.CO], 2013.
[19] Frank Könnig, Aashay Patil, Luca Amendola, Viable cosmological solutions in massive bimetric gravity, J. Cosmol. Astropart. Phys. 1403 (2014) 029, arXiv:1312.3208 [astro-ph.CO].
[20] Jonas Enander, Adam R. Solomon, Yashar Akrami, Edvard Mortsell, Cosmic expansion histories in massive bigravity with symmetric matter coupling, J. Cosmol. Astropart. Phys. 01 (2015) 006, arXiv:1409.2860 [astro-ph.CO].
[21] G. D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, et al., Massive cosmologies, Phys. Rev. D 84 (2011) 124046, arXiv:1108.5231 [hep-th].
[22] A. Emir Gümrükçüoğlu, Chunshan Lin, Shinji Mukohyama, Open FRW universes and self-acceleration from nonlinear massive gravity, J. Cosmol. Astropart. Phys. 1111 (2011) 030, arXiv:1109.3845 [hep-th].
[23] A. Emir Gümrükçüoğlu, Chunshan Lin, Shinji Mukohyama, Cosmological perturbations of self-accelerating universe in nonlinear massive gravity, J. Cosmol. Astropart. Phys. 1203 (2012) 006, arXiv:1111.4107 [hep-th].
[24] Babak Vakili, Nima Khosravi, Classical and quantum massive cosmology for the open FRW universe, Phys. Rev. D 85 (2012) 083529, arXiv:1204.1456 [gr-qc].
[25] Antonio De Felice, A. Emir Gümrükçüoğlu, Shinji Mukohyama, Massive gravity: nonlinear instability of the homogeneous and isotropic universe, Phys. Rev. Lett. 109 (2012) 171101, arXiv:1206.2080 [hep-th].
[26] Matteo Fasiello, Andrew J. Tolley, Cosmological perturbations in massive gravity and the Higuchi bound, J. Cosmol. Astropart. Phys. 1211 (2012) 035, arXiv:1206.3852 [hep-th].
[27] Antonio De Felice, A. Emir Gümrükçüoğlu, Chunshan Lin, Shinji Mukohyama, Nonlinear stability of cosmological solutions in massive gravity, J. Cosmol. Astropart. Phys. 1305 (2013) 035, arXiv:1303.4154 [hep-th].
[28] Marcus Berg, Igor Buchberger, Jonas Enander, Edvard Mörtsell, Stefan Sjörs, Growth histories in bimetric massive gravity, J. Cosmol. Astropart. Phys. 1212 (2012) 021, arXiv:1206.3496 [gr-qc].
[29] D. Comelli, M. Crisostomi, L. Pilo, Perturbations in massive gravity cosmology, J. High Energy Phys. 1206 (2012) 085, arXiv:1202.1986 [hep-th].
[30] Frank Könnig, Luca Amendola, Instability in a minimal bimetric gravity model, Phys. Rev. D 90 (2014) 044030, arXiv:1402.1988 [astro-ph.CO].
[31] Adam R. Solomon, Yashar Akrami, Tomi S. Koivisto, Linear growth of structure in massive bigravity, J. Cosmol. Astropart. Phys. 1410 (2014) 066, arXiv:1404.4061 [astro-ph.CO].
[32] Frank Könnig, Yashar Akrami, Luca Amendola, Mariele Motta, Adam R. Solomon, Stable and unstable cosmological models in bimetric massive gravity, Phys. Rev. D 90 (2014) 124014, arXiv:1407.4331 [astro-ph.CO].
[33] Macarena Lagos, Pedro G. Ferreira, Cosmological perturbations in massive bigravity, J. Cosmol. Astropart. Phys. 1412 (2014) 026, arXiv:1410.0207 [gr-qc].
[34] Giulia Cusin, Ruth Durrer, Pietro Guarato, Mariele Motta, Gravitational waves in bigravity cosmology, J. Cosmol. Astropart. Phys. 1505 (2015) 030, arXiv:1412.5979 [astro-ph.CO].
[35] Yasuho Yamashita, Takahiro Tanaka, Mapping the ghost free bigravity into braneworld setup, J. Cosmol. Astropart. Phys. 1406 (2014) 004, arXiv:1401.4336 [hep-th].
[36] Antonio De Felice, A. Emir Gümrükçüoğlu, Shinji Mukohyama, Norihiro Tanahashi, Takahiro Tanaka, Viable cosmology in bimetric theory, J. Cosmol. Astropart. Phys. 1406 (2014) 037, arXiv:1404.0008 [hep-th].
[37] Matteo Fasiello, Andrew J. Tolley, Cosmological stability bound in massive gravity and bigravity, J. Cosmol. Astropart. Phys. 1312 (2013) 002, arXiv:1308.1647 [hep-th].
[38] Jonas Enander, Yashar Akrami, Edvard Mörtsell, Malin Renneby, Adam R. Solomon, Integrated Sachs–Wolfe effect in massive bigravity, Phys. Rev. D 91 (2015) 084046, arXiv:1501.02140 [astro-ph.CO].
[39] Luca Amendola, Frank Könnig, Matteo Martinelli, Valeria Pettorino, Miguel Zumalacarregui, Surfing gravitational waves: can bigravity survive growing tensor modes?, J. Cosmol. Astropart. Phys. 1505 (2015) 052, arXiv:1503.02490 [astro-ph.CO].
[40] Matthew Johnson, Alexandra Terrana, Tensor modes in bigravity: primordial to present, arXiv:1503.05560 [astro-ph.CO], 2015.
[41] Frank Könnig, Higuchi ghosts and gradient instabilities in bimetric gravity, Phys. Rev. D 91 (2015) 104019, arXiv:1503.07436 [astro-ph.CO].

[42] Claudia de Rham, Lavinia Heisenberg, Raquel H. Ribeiro, Quantum corrections in massive gravity, Phys. Rev. D 88 (2013) 084058, arXiv:1307.7169 [hep-th].
[43] S.F. Hassan, Rachel A. Rosen, On non-linear actions for massive gravity, J. High Energy Phys. 1107 (2011) 009, arXiv:1103.6055 [hep-th].
[44] Yashar Akrami, Tomi S. Koivisto, David F. Mota, Marit Sandstad, Bimetric gravity doubly coupled to matter: theory and cosmological implications, J. Cosmol. Astropart. Phys. 1310 (2013) 046, arXiv:1306.0004 [hep-th].
[45] Nicola Tamanini, Emmanuel N. Saridakis, Tomi S. Koivisto, The cosmology of interacting spin-2 fields, J. Cosmol. Astropart. Phys. 1402 (2014) 015, arXiv:1307.5984 [hep-th].
[46] Yashar Akrami, Tomi S. Koivisto, Adam R. Solomon, The nature of spacetime in bigravity: two metrics or none?, Gen. Relativ. Gravit. 47 (2014) 1838, arXiv:1404.0006 [gr-qc].
[47] Yasuho Yamashita, Antonio De Felice, Takahiro Tanaka, Appearance of Boulware–Deser ghost in bigravity with doubly coupled matter, Int. J. Mod. Phys. D 23 (2014) 3003, arXiv:1408.0487 [hep-th].
[48] Claudia de Rham, Lavinia Heisenberg, Raquel H. Ribeiro, On couplings to matter in massive (bi-)gravity, Class. Quantum Gravity 32 (2015) 035022, arXiv:1408.1678 [hep-th].
[49] Claudia de Rham, Lavinia Heisenberg, Raquel H. Ribeiro, Ghosts & matter couplings in massive (bi-&multi-)gravity, Phys. Rev. D 90 (2014) 124042, arXiv:1409.3834 [hep-th].
[50] Johannes Noller, Scott Melville, The coupling to matter in massive, bi- and multi-gravity, J. Cosmol. Astropart. Phys. 1501 (2014) 003, arXiv:1408.5131 [hep-th].
[51] Adam R. Solomon, Jonas Enander, Yashar Akrami, Tomi S. Koivisto, Frank Könnig, et al., Cosmological viability of massive gravity with generalized matter coupling, J. Cosmol. Astropart. Phys. 1504 (2015) 027, arXiv:1409.8300 [astro-ph.CO].
[52] Angnis Schmidt-May, Mass eigenstates in bimetric theory with matter coupling, J. Cosmol. Astropart. Phys. 1501 (2015) 039, arXiv:1409.3146 [gr-qc].
[53] Lavinia Heisenberg, Quantum corrections in massive bigravity and new effective composite metrics, Class. Quantum Gravity 32 (2015) 105011, arXiv:1410.4239 [hep-th].
[54] A. Emir Gümrükçüoğlu, Lavinia Heisenberg, Shinji Mukohyama, Cosmological perturbations in massive gravity with doubly coupled matter, J. Cosmol. Astropart. Phys. 1502 (2015) 022, arXiv:1409.7260 [hep-th].
[55] Johannes Noller, On consistent kinetic and derivative interactions for gravitons, J. Cosmol. Astropart. Phys. 1504 (2015) 025, arXiv:1409.7692 [hep-th].
[56] A. Emir Gümrükçüoğlu, Lavinia Heisenberg, Shinji Mukohyama, Norihiro Tanahashi, Cosmology in bimetric theory with an effective composite coupling to matter, J. Cosmol. Astropart. Phys. 1504 (2015) 008, arXiv:1501.02790 [hep-th].
[57] Denis Comelli, Marco Crisostomi, Kazuya Koyama, Luigi Pilo, Gianmassimo Tasinato, Cosmology of bigravity with doubly coupled matter, J. Cosmol. Astropart. Phys. 1504 (2015) 026, arXiv:1501.00864 [hep-th].
[58] Kurt Hinterbichler, Rachel A. Rosen, A note on ghost-free matter couplings in massive gravity and multi-gravity, arXiv:1503.06796 [hep-th], 2015.
[59] Atsushi Higuchi, Forbidden mass range for spin-2 field theory in de Sitter space–time, Nucl. Phys. B 282 (1987) 397.
[60] A. Higuchi, Massive symmetric tensor field in space–times with a positive cosmological constant, Nucl. Phys. B 325 (1989) 745–765.
[61] Garrett Goon, Kurt Hinterbichler, Austin Joyce, Mark Trodden, Einstein gravity, massive gravity, multi-gravity and nonlinear realizations, arXiv:1412.6098 [hep-th], 2014.
[62] S.F. Hassan, Angnis Schmidt-May, Mikael von Strauss, On consistent theories of massive spin-2 fields coupled to gravity, J. High Energy Phys. 1305 (2013) 086, arXiv:1208.1515 [hep-th].
[63] Z. Berezhiani, D. Comelli, F. Nesti, L. Pilo, Spontaneous Lorentz breaking and massive gravity, Phys. Rev. Lett. 99 (2007) 131101, arXiv:hep-th/0703264.
[64] Valentina Baccetti, Prado Martin-Moruno, Matt Visser, Massive gravity from bimetric gravity, Class. Quantum Gravity 30 (2013) 015004, arXiv:1205.2158 [gr-qc].
[65] S.F. Hassan, Angnis Schmidt-May, Mikael von Strauss, Particular solutions in bimetric theory and their implications, Int. J. Mod. Phys. D 23 (2014) 1443002, arXiv:1407.2772 [hep-th].
[66] H. van Dam, M.J.G. Veltman, Massive and massless Yang–Mills and gravitational fields, Nucl. Phys. B 22 (1970) 397–411.
[67] V.I. Zakharov, Linearized gravitation theory and the graviton mass, JETP Lett. 12 (1970) 312.
[68] Marc Henneaux, Martin Pilati, Claudio Teitelboim, Explicit solution for the zero signature (strong coupling) limit of the propagation amplitude in quantum gravity, Phys. Lett. B 110 (1982) 123.
[69] G. Francisco, M. Pilati, Strong coupling quantum gravity. 3. Quasiclassical approximation, Phys. Rev. D 31 (1985) 241.
[70] D.S. Salopek, Hamilton–Jacobi solutions for strongly coupled gravity and matter, Class. Quantum Gravity 15 (1998) 1185–1206, arXiv:gr-qc/9802025.
[71] Max Niedermaier, The dynamics of strong coupling gravity, Class. Quantum Gravity 32 (2015) 015008.
[72] Nima Arkani-Hamed, Howard Georgi, Matthew D. Schwartz, Effective field theory for massive gravitons and gravity in theory space, Ann. Phys. 305 (2003) 96–118, arXiv:hep-th/0210184.
[73] Matthew D. Schwartz, Constructing gravitational dimensions, Phys. Rev. D 68 (2003) 024029, arXiv:hep-th/0303114.
[74] Clare Burrage, Nemanja Kaloper, Antonio Padilla, Strong coupling and bounds on the spin-2 mass in massive gravity, Phys. Rev. Lett. 111 (2013) 021802, arXiv:1211.6001 [hep-th].
[75] Henrik Nersisyan, Yashar Akrami, Luca Amendola, Consistent metric combinations in cosmology of massive bigravity, arXiv:1502.03988 [gr-qc], 2015.
[76] Ufuk Aydemir, Mohamed M. Anber, John F. Donoghue, Self-healing of unitarity in effective field theories and the onset of new physics, Phys. Rev. D 86 (2012) 014025, arXiv:1203.5153 [hep-ph].
[77] Claudia de Rham, Massive gravity, Living Rev. Relativ. 17 (2014) 7, arXiv:1401.4173 [hep-th].
[78] Kurt Hinterbichler, Mark Trodden, Daniel Wesley, Multi-field galileons and higher co-dimension branes, Phys. Rev. D 82 (2010) 124018, arXiv:1008.1305 [hep-th].
[79] Claudia de Rham, Gregory Gabadadze, Lavinia Heisenberg, David Pirtskhalava, Nonrenormalization and naturalness in a class of scalar-tensor theories, Phys. Rev. D 87 (2013) 085017, arXiv:1212.4128.
[80] A.I. Vainshtein, To the problem of nonvanishing gravitation mass, Phys. Lett. B 39 (1972) 393–394.
[81] Eugeny Babichev, Cédric Deffayet, An introduction to the Vainshtein mechanism, Class. Quantum Gravity 30 (2013) 184001, arXiv:1304.7240 [gr-qc].