Doctoral Thesis

Time Reversal of Dispersive Waves in Nondestructive Testing Applications

Author(s):
Ernst, Robert

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Time Reversal of Dispersive Waves in Nondestructive Testing Applications

A dissertation submitted to

ETH ZURICH

for the degree of
Doctor of Sciences

presented by

ROBERT ERNST

MSc. Mech.-Eng. ETH
born June 5, 1984
citizen of Aarwangen (BE)

accepted on the recommendation of

Prof. Dr. J. Dual, examiner
Prof. Dr. M. Ruzzene, co-examiner

2015
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Finally, I would like to express my deepest gratitude to my wife Ellen for her support, time management and motivation throughout my thesis.

Robert Ernst
Zurich, Juni 2015
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Abstract

Guided wave testing has become an important branch of nondestructive testing (NDT) because of its inspection efficiency in large 1D and 2D structures. Similarly to ultrasonic testing, guided waves are used in time of arrival (ToA) mode, by which the time difference between excited pulse and reflection from defect is evaluated to locate scatterers. While this principle is straightforward in ultrasonic testing, its application to guided wave testing is more challenging. In general, the propagation velocity of guided waves is frequency dependent. If the frequency components of a wave pulse travel with different velocities, the shape of the wave pulse changes with distance. This effect is known as dispersion. Obviously it is difficult to assign an accurate arrival time to a wave pulse that is permanently changing its shape. Moreover, as the pulse disperses, its duration typically increase and the pulse is likely to interfere with reflections and other wave signals. This complicates the extraction of an individual pulse from the measured signal. The extraction however is necessary to obtain information on the characteristics of the defect. Therefore, notable effort has been spent to find ways to decrease or compensate the effect of dispersion in guided wave signals.

The method presented tackles the problem from another side. Instead of avoiding dispersion, dispersion constitutes the basis of a novel method for detecting, localizing and characterizing defects. Dispersion causes the pulse shape to change with distance, hence the pulse shape of a dispersed wave contains information about the wave’s propagated distance. The key to access this information is found in a time reversal (TR) simulation. Hereby, the measured signal is reversed in time and set as the boundary condition of a numerical simulation. The boundary condition causes the wave to virtually propagate back in a numerical model of the structure, while also reversing the effect of dispersion. At the virtual origin of the backward propagating dispersive wave, the original pulse is retrieved. Given that a crack is the source of the generated dispersive wave, the crack position is easily obtained from the calculated displacement field by determining the maximal amplitudes.

The combination of dispersion and a TR process has the following advantages:

- The time difference between excitation and reflection is irrelevant. This simplifies the experimental setup.
- A single dispersive wave mode in the signal suffices to detect and localize defects.
- Additional signal content such as noise, reflections or other wave modes hardly affect the localization performance.
- Identification of specific signal features such as arrival time, pulse duration etc. is not required. Instead, the measured waveform is processed as a whole.
Abstract

This substantially increases the method’s robustness in low signal-to-noise environments.

The TR process proposed is analytically investigated for a Timoshenko beam and a Mindlin plate. The presence of non-propagating wave modes, known as evanescent modes, was shown to affect the TR process in guided wave applications. However, this effect could be compensated in all applications presented. Other influences such as measurement position, boundary condition or signal manipulation are analyzed and discussed. On the basis of these investigations, applications are proposed for acoustic emission (AE) and guided wave testing.

Acoustic emissions are elastic waves originating from discontinuities in materials that are subject to mechanical loading. Damage mechanisms may lead to a sudden redistribution of internal forces resulting in elastic waves outgoing from the damage zones. Detection and localization of these AE sources is an important tool in NDT and allows monitoring of the structural health of critical mechanical parts. Acoustic emissions are typically broadband and the resulting elastic waves propagate dispersively in beam and plate-like structures. Up till today, localization of AE in a 1D structure requires at least two sensors. The minimum number of sensors to localize a source in a 2D structure is three. By applying the TR numerical simulation concept as described above to the AE localization problem, the minimum number of sensors for both 1D and 2D structures is reduced to one. In addition, the difficulties arising with dispersion are elegantly bypassed and used to the method’s advantage. Quantitative comparison of different AE events at different positions becomes possible because the wave signal is restored at the virtual position of the AE source. In this dissertation, localization results are shown for artificially created AE’s (Hsu Nielsen Source) on various aluminum beams, unidirectional carbon fiber reinforced beams and aluminum plates. Multiple AE’s could be localized from a single one point measurement.

The TR numerical simulation concept has also been implemented in a guided wave inspection technique. The method is based on the effect of mode conversion at asymmetric defects in beams. Up to a frequency-thickness range of about 1 MHz mm for steel, a longitudinal wave impinging on a crack is reflected and transmitted, while part of its energy is converted into propagating flexural waves and local oscillations. A Gaussian shaped longitudinal wave propagating along the structure causes similarly shaped flexural waves at every crack. The origins of these flexural waves indicate the positions of the cracks which are determined from the TR simulation. Moreover, while the extraction of an individual flexural wave in the signal is not possible directly from the measurement, the flexural wave can be extracted from the TR numerical simulation. This allows the calculation of a mode conversion ratio from longitudinal to flexural wave which in turn allows the quantitative characterization of the defect. The method is validated on a range of aluminum beams and pipes. Three out of three machined notches were localized from a single one point measurement in a 2 m long beam. The depth of electrical discharge machined notches, ranging between 2 mm and 0.5 mm was reliably predicted over a broad frequency band.
Zusammenfassung


In dieser Arbeit wird ein alternatives Vorgehen vorgeschlagen. Anstatt den Effekt der Dispersion zu verhindern, bildet neu die Dispersion einen zentralen Bestandteil für die hier vorgestellte Methode zur Detektion und Lokalisierung von Defekten. Da die dispersive Welle ihre Form während der Ausbreitung ändert, ist die Information über den zurückgelegten Weg schon in der Form der Welle enthalten. Um auf diese Information zugreifen zu können, wird eine sogenannte Zeitumkehrsimulation, in Englisch time reversal (TR) simulation, vorgeschlagen. Dabei wird das im Experiment gemessene Signal zeitlich invertiert als Randbedingung für eine Simulation benutzt. Dabei wird auch der Effekt der Dispersion invertiert, sodass die Amplitude des zurück laufenden Pulses am virtuellen Ursprung die grössste Amplitude erreicht. Falls die Welle an einem Defekt entstanden ist kann dessen Position einfach aus der Simulation gelesen werden.
Zusammenfassung

Ein solches Vorgehen hat die folgenden Vorzüge:

- Die Zeitdifferenz zwischen Puls und Echo ist irrelevant. Das ermöglicht einen vereinfachten experimentellen Aufbau.
- Ein dispersiver Wellenmode im Signal genügt zur Detektion und Lokalisation von Defekten.
- Zusätzliche Komponenten im Signal wie beispielsweise Rauschen, Reflexionen und andere Wellen Moden beeinträchtigen die Lokalisation nur minimal.
- Das Erkennen bestimmter Signal Merkmale wie Ankunftszeit, Pulsdauer etc. entfällt. Das gemessene Signal wird als ganzes in der Simulation verwendet. Das erhöht deutlich die Robustheit der Methode in Applikationen mit schlechtem Signal-Rausch-Verhältnis.


Eine weitere vorgestellte Anwendung ist die aktive Prüfung von Strukturen mittels geführten Wellen. Diese Methode basiert auf dem Effekt der Mode-Umwandlung.
an asymmetrischen Defekten im Balken. Für Frequenz-Strukturdicken Produkte bis 1 MHz mm für Stahl erzeugt eine einfallende Längswelle neben eine reflektierten und transmittierten Längswelle auch stehende und laufende Biegewellen. Die entstehenden Biegewellen zeigen starke Dispersion. Indem eine gaussförmige Längswelle in der Struktur angeregt wird, entstehen an allen Rissen neue Biegewellen welche am Defekt noch der erzeugenden Welle gleichen. Der Ursprung einer solchen Biegewelle, und damit auch der Riss, lässt sich wie eingangs beschrieben über die TR Simulation ermitteln. Darüber hinaus kann mithilfe der TR Simulation ein individueller Biegepuls vom Signal isoliert werden, was direkt vom gemessenen Signal aus nicht möglich ist. Damit kann das Verhältnis zwischen erzeugender Längswelle und erzeugter Biegewelle berechnet werden, was wiederum die Berechnung der Risstiefe ermöglicht. Diese Anwendung wurde für verschiedene Aluminium Balken sowie ein Aluminium Rohr experimentell validiert. Von drei gefrästen Kerben unterschiedlicher Größte in einem 2 m Balken konnten alle mittels einer einzigen Messung lokalisiert werden. Die Tiefe von Draht erodierte Kerben zwischen 2 mm und 0.5 mm konnte über ein breites Frequenzspektrum zuverlässig bestimmt werden.
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AE</td>
<td>Acoustic emission</td>
</tr>
<tr>
<td>FDTD</td>
<td>Finite difference time domain method</td>
</tr>
<tr>
<td>FE</td>
<td>Finite element</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier transform</td>
</tr>
<tr>
<td>NDT</td>
<td>Nondestructive testing</td>
</tr>
<tr>
<td>PC</td>
<td>Personal computer</td>
</tr>
<tr>
<td>SD</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>SFEM</td>
<td>Spectral finite element method</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise ratio</td>
</tr>
<tr>
<td>SSMAL</td>
<td>Single sensor modal acoustic localization</td>
</tr>
<tr>
<td>TF</td>
<td>Transfer function</td>
</tr>
<tr>
<td>ToA</td>
<td>Time of Arrival</td>
</tr>
<tr>
<td>TR</td>
<td>Time reversal</td>
</tr>
</tbody>
</table>
# List of Symbols

## Roman alphabet

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>[m]</td>
<td>Side length/ radius</td>
</tr>
<tr>
<td>$A$</td>
<td>[m$^2$]</td>
<td>Cross-sectional area</td>
</tr>
<tr>
<td>$A_i$</td>
<td>[varying]</td>
<td>Amplitude coefficients associated with $k_1$</td>
</tr>
<tr>
<td>$b$</td>
<td>[m]</td>
<td>Side length</td>
</tr>
<tr>
<td>$B$</td>
<td>[m]</td>
<td>Beam width</td>
</tr>
<tr>
<td>$B_i$</td>
<td>[varying]</td>
<td>Amplitude coefficients associated with $k_2$</td>
</tr>
<tr>
<td>$B_{TF}$</td>
<td>[ ]</td>
<td>Amplitude of arbitrary plate edge transfer function</td>
</tr>
<tr>
<td>$c$</td>
<td>[m s$^{-1}$]</td>
<td>Phase speed</td>
</tr>
<tr>
<td>$C_A$</td>
<td>[m$^2$]</td>
<td>Group velocity fundamental antisymmetric Lamb mode</td>
</tr>
<tr>
<td>$C_{crack}$</td>
<td>[m$^2$]</td>
<td>Crack surface</td>
</tr>
<tr>
<td>$c_g$</td>
<td>[m s$^{-1}$]</td>
<td>Group velocity</td>
</tr>
<tr>
<td>$C_i$</td>
<td>[m]</td>
<td>Amplitude coefficients associated with $k_3$</td>
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<tr>
<td>$c_p$</td>
<td>[m s$^{-1}$]</td>
<td>Primary phase velocity</td>
</tr>
<tr>
<td>$c_{pred}$</td>
<td>[m s$^{-1}$]</td>
<td>Flexural phase speed used in simulations</td>
</tr>
<tr>
<td>$C_S$</td>
<td>[m s$^{-1}$]</td>
<td>Group velocity fundamental symmetric Lamb mode</td>
</tr>
<tr>
<td>$c_s$</td>
<td>[m s$^{-1}$]</td>
<td>Secondary wave velocity</td>
</tr>
<tr>
<td>$c_{texttrue}$</td>
<td>[m s$^{-1}$]</td>
<td>Flexural phase speed observed in experiment</td>
</tr>
<tr>
<td>$D$</td>
<td>[m]</td>
<td>Diameter</td>
</tr>
<tr>
<td>$d_p$</td>
<td>[m V$^{-1}$]</td>
<td>Piezoelectric charge coefficient</td>
</tr>
<tr>
<td>$D_P$</td>
<td>[N m]</td>
<td>Plate stiffness $= E h^3/(12(1 - \nu^2))$</td>
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<tr>
<td>$D_{ss}$</td>
<td>[m]</td>
<td>Source-Sensor distance</td>
</tr>
<tr>
<td>$E$</td>
<td>[kg m$^{-1}$ s$^{-2}$]</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>$f$</td>
<td>[s$^{-1}$]</td>
<td>Frequency</td>
</tr>
<tr>
<td>$F(i,j)$</td>
<td>[ ]</td>
<td>non axially symmetric modes</td>
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<td>$f_0$</td>
<td>[s$^{-1}$]</td>
<td>Central frequency of a narrow band pulse</td>
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<td>$F_0$</td>
<td>[N]</td>
<td>External force</td>
</tr>
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<td>$f_2$</td>
<td>[s$^{-1}$]</td>
<td>Cut-off frequency of $F(2,1)$</td>
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<td>[varying]</td>
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<td>$f_{spat}$</td>
<td>[m$^{-1}$]</td>
<td>Spatial frequency</td>
</tr>
<tr>
<td>$g$</td>
<td>[V]</td>
<td>Piezoelectric voltage constant</td>
</tr>
<tr>
<td>$G$</td>
<td>[kg m$^{-1}$ s$^{-2}$]</td>
<td>Shear modulus</td>
</tr>
<tr>
<td>$G_{ij}$</td>
<td>[ ]</td>
<td>Element shape function</td>
</tr>
<tr>
<td>$h$</td>
<td>[m]</td>
<td>Half plate thickness</td>
</tr>
<tr>
<td>$I$</td>
<td>[m$^4$]</td>
<td>Second moment of area</td>
</tr>
<tr>
<td>$i$</td>
<td>[ ]</td>
<td>Complex number or index number when used as subscript</td>
</tr>
<tr>
<td>$k$</td>
<td>[rad m$^{-1}$]</td>
<td>Wavenumber</td>
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<tr>
<td>$K_0$</td>
<td>[kg m$^{-0.5}$ s$^{-2}$]</td>
<td>Impact compliance</td>
</tr>
<tr>
<td>$k_1$</td>
<td>[rad m$^{-1}$]</td>
<td>Flexural mode wavenumber</td>
</tr>
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<td>Symbol</td>
<td>Description</td>
<td></td>
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<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>$k_2$</td>
<td>Shear-thickness mode wavenumber</td>
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<tr>
<td>$k_3$</td>
<td>Twisting wave</td>
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<tr>
<td>$K_I, K_{II}$</td>
<td>Stress intensity factors</td>
<td></td>
</tr>
<tr>
<td>$K_{ij}$</td>
<td>Element stiffness coefficients</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
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</tr>
<tr>
<td>$l_{acc}$</td>
<td>Acceleration distance</td>
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<tr>
<td>$M$</td>
<td>Bending Moment</td>
<td></td>
</tr>
<tr>
<td>$m$ $m_M$</td>
<td>Mass of piezo and backing plate</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>Mass</td>
<td></td>
</tr>
<tr>
<td>$M_0$</td>
<td>Nodal bending moment</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>Integer</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>Axial force</td>
<td></td>
</tr>
<tr>
<td>$n_d$</td>
<td>Depth of crack/ notch</td>
<td></td>
</tr>
<tr>
<td>$n_b$</td>
<td>Notch/ crack breadth</td>
<td></td>
</tr>
<tr>
<td>$n_d$</td>
<td>Notch/ crack depth</td>
<td></td>
</tr>
<tr>
<td>$n_n$</td>
<td>Notch orientation</td>
<td></td>
</tr>
<tr>
<td>$n_w$</td>
<td>Notch/ crack width</td>
<td></td>
</tr>
<tr>
<td>$p_i$</td>
<td>$i^{th}$ flexural wave front</td>
<td></td>
</tr>
<tr>
<td>$P(t)$</td>
<td>Transverse point force history</td>
<td></td>
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<tr>
<td>$Q_0$</td>
<td>Nodal shear force</td>
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<tr>
<td>$r$</td>
<td>Radial distance</td>
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<td>$R_1$</td>
<td>Amplitude ratio associated with $k_1$</td>
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</tr>
<tr>
<td>$R_2$</td>
<td>Amplitude ratio associated with $k_2$</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>Vector of kinematic variables</td>
<td></td>
</tr>
<tr>
<td>$s_d$</td>
<td>Compliance at constant electric displacement</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>Standard deviation</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>Arbitrary time constant</td>
<td></td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Arrival time difference</td>
<td></td>
</tr>
<tr>
<td>$t_0$</td>
<td>Impact duration</td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>Axial displacement</td>
<td></td>
</tr>
<tr>
<td>$u_{\text{eff}}$</td>
<td>RMS voltage</td>
<td></td>
</tr>
<tr>
<td>$u_i$</td>
<td>Incident longitudinal wave</td>
<td></td>
</tr>
<tr>
<td>$u_r$</td>
<td>Reflected longitudinal wave</td>
<td></td>
</tr>
<tr>
<td>$u_t$</td>
<td>Transmitted longitudinal wave</td>
<td></td>
</tr>
<tr>
<td>$V$</td>
<td>Voltage amplitude</td>
<td></td>
</tr>
<tr>
<td>$V_0$</td>
<td>Impact velocity</td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>Transversal displacement</td>
<td></td>
</tr>
<tr>
<td>$W$</td>
<td>Total work</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>Axial distance or plate coordinate</td>
<td></td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>Distance increment</td>
<td></td>
</tr>
<tr>
<td>$\Delta x_{\text{err}}$</td>
<td>Error in AE/ notch distance prediction</td>
<td></td>
</tr>
<tr>
<td>$x_0$</td>
<td>Excitation location</td>
<td></td>
</tr>
<tr>
<td>$x_{AE}$</td>
<td>Location of acoustic emission</td>
<td></td>
</tr>
<tr>
<td>$x_E$</td>
<td>Offset distance from boundary</td>
<td></td>
</tr>
</tbody>
</table>
$x_m$ [m] Measurement location

$x_n$ [m] Position of notch n

$x_{pred}$ [m] Predicted position of notch

$x_{prop}$ [m] Propagation distance AE-sensor/ notch-sensor

$y$ [m] Beam deflection or plate coordinate

$y_0$ [m] Deflection at excitation location

$\bar{y}$ [m] Deflection after manipulation

$y_{er}$ [m] Reflected evanescent wave

$y_{et}$ [m] Transmitted evanescent wave

$y_{Extr}$ [m] Extracted part of deflection history

$y_{FS}$ [m] Forward simulated deflection history

$y_m$ [m] Deflection at measurement location

$y_r$ [m] Reflected flexural wave

$y_t$ [m] Transmitted flexural wave

$\bar{y}$ [m] Deflection measured at free end

$y^{TR}$ [m] Time reversed deflection history

$\hat{}()$ Frequency domain transformed quantity

**Greek alphabet**

$\alpha$ [V] Amplitude scale of excitation in time domain

$\Lambda$ [varying] Matrix relating kinematic variables with acting loads at Notch

$\tilde{\alpha}$ [V] Amplitude scale of excitation in frequency domain

$\epsilon$ [ ] Permittivity

$\kappa$ [ ] Timoshenko coefficient

$\lambda$ [m] wavelength

$\nu$ [ ] Poisson’s ratio

$\Phi$ [ ] Phase value

$\psi_{TR}$ [ ] Time reversed beam slope

$\psi$ [ ] Beam slope

$\Delta\Psi$ [°] Phase change

$\rho$ [kg m$^{-3}$] Mass density

$\sigma$ [s$^{-1}$] Time scale of excitation function

$\bar{\sigma}_c$ [m s$^{-1}$] Averaged SD of measured phase speeds

$\tilde{\sigma}$ [f$^{-1}$] Frequency scale of excitation function

$\tau_g$ [s] Group delay

$\Theta$ [J] Potential energy

$\omega$ [rad s$^{-1}$] Circular frequency


1 Introduction

1.1 Overriding Research Question

The extensive use of technology is characteristic for our society. Technology has increased comfort, safety, and prosperity. However, as technology advanced, consequences of its failure became more and more severe. An early example of such a technological tragedy occurred on steamship Sultana on April 27th, 1865. Sultana headed along the Mississippi river when suddenly three of four steam boilers exploded. Between 1200 and 1600 people lost their lives in this catastrophe.

Figure 1.1: Explosion of the steamer Sultana, April 28th, 1865. Harpers Weekly, May 20, 1865. Library of Congress

This was not the first boiler accident in that time and as a result, a year after the Sultana disaster, the company Hartford Steam Boiler Inspection and Insurance Company was founded in an effort to address this problem. It became clear, that safe operation of machines and tools required periodic inspection. This was a major step in the evolution and awareness of nondestructive testing (NDT), a term used to describe methods and techniques which provide information on the condition of an examined object. Up until now, technical evolution continues to increase the need for NDT.

Current important applications of NDT include evaluation of materials during production, testing of aerospace components, monitoring of civil structures and medical imaging. NDT today employs all kinds of physical principles to evaluate the integrity of structures and systems. A comprehensive overview over the variety of NDT methods can be found in.

An important branch of NDT is based on mechanical wave propagation, in NDT terms best known as ultrasonic testing. A local transient disturbance in the elastodynamic equilibrium does not remain local. Instead the local response to the disturbance affects the surrounding media. Energy therefore spreads out and this is
what we then conceive as a propagating wave. Discontinuities in the media cause the wave to reflect or diffract, depending on the wavelength-to-discontinuity ratio and the incident angle of the wave. The wave signal can therefore be altered by the media through which it is propagating. As a result, the wave signal contains information on the media through which it passed. This information may be encoded as a time delay, amplitude, phase, or frequency change of the wave. Wave propagation is therefore very attractive to NDT because it enables examination of the inner bulk of material from outside.

In 3D solid isotropic media two fundamental wave types exist. These are known as primary waves (P-wave) and secondary waves (S-wave). For plane waves, the difference between these two wave types is that particle motion in P-waves is in line with the wave’s propagation direction whereas particle motion in S-waves is perpendicular to the propagation direction. These wave types are also known as compressional and shear waves respectively. The terms primary and secondary refer to the observation that the P-wave arrives before the S-wave. The term ultrasonic in ultrasonic testing implies that the frequencies are above the audible frequency range which generally means above 20 kHz. The choice of such high frequencies is motivated by the equation:

\[ \lambda = \frac{c}{f} \]  

which relates the wave’s wavelength \( \lambda \) to the phase speed \( c \) and the frequency \( f \) of the wave. In order for a wave to be reflected by a discontinuity, the wavelength-to-discontinuity ratio ideally must be smaller than one. In consequence, detecting small defects requires high frequencies which is why ultrasound waves are so prominent in NDT. Increasing the frequency of the interrogating waves comes at the price of higher attenuation and hence lower inspection range. Ultrasonic waves illuminate only the local field below the transducer which is often limited to a range between 1 cm–10 cm. Due to the limited range, ultrasonic testing is typically performed by scanning the area of interest with a hand-held transducer. This is practical when the location of potential defects is known a priori and accessible by a technician or the inspected part is small such that its surface is readily scanned. Testing of large areas or complete structures such as pipes, cables or rails is less convenient using ultrasonic methods. In these cases, guided waves may offer a better solution.

Guided waves, also known as structural waves, are essentially again composed of P- and S-waves. However, the wavelength of these fundamental waves is now in the order of the thickness of the structure. This has the effect that P- and S-waves are repeatedly reflected by the boundaries of the structure. Visually spoken, in the ultrasonic case the wave sees itself surrounded by an infinite bulk of material whereas in the guided wave case, the wave is permanently interacting with the structure’s boundaries and is therefore guided along. The beauty of this phenomenon is that the reflecting P- and S-waves interfere with each other in a way that they form a variety of different wave modes. The character of these wave modes is frequency dependent and one can imagine these wave modes as propagating cross-sectional vibration modes. Although these wave modes are formed by P- and S-waves their behavior is fundamentally different. In the case of plate structures, we speak of symmetrical (s) and anti-symmetrical (a) Lamb modes. In the lower frequency range,
these modes can be approximated by using strength of materials theories like Timoshenko beam- or Mindlin-Reissner plate theories. Then, the a0 mode is referred to as flexural wave and the s0 mode as the longitudinal wave mode. One important characteristic is that in general, the propagation speed of these wave modes is a function of frequency. This has the effect that a wave pulse, which is composed of a band of frequencies, will disperse while propagating. Knowing the dispersive behavior of the structure is key to guided wave testing applications. Another difference between ultrasonic and guided waves is the interaction with a discontinuity. The effect of a discontinuity on the incident wave is governed by the change in the structure’s stiffness and therefore also a function of the defect’s position, size and orientation and not just by the ratio wavelength-to-discontinuity, as in the ultrasonic case. The response in the guided wave case is quite complex since the interaction can cause mode conversion from one mode into several other modes, of which some are not even propagating and remain local oscillations. To summarize, using guided waves for NDT is more complex due to dispersion and complicated response at defects. But it offers some distinct advantages compared to ultrasonic testing. Guided waves can be used to investigate the entire cross-section of large structures outgoing from one point. This increases efficiency and can reduce inspection cost. Moreover, guided waves can investigate inaccessible parts such as pipes buried in soil, covered by insulation or if personal access is too dangerous due to extreme pressure and temperature in the specimen.

Both, ultrasonic and guided wave testing are referred to as active techniques, because the interrogating waves are actively excited by a suitable transducer. The idea of sending a wave into the media and measuring its response is both old and successful and has found numerous applications. But it is not the only way to take advantage of mechanical wave propagation in NDT. Acoustic emission (AE) testing is based on the fact that the structure itself generates elastic waves at damage zones when loaded high enough. Therefore it is not necessarily required to actively excite waves into the structure to examine its condition. Acoustic emission testing is therefore categorized as a passive technique. The use of sound as an indicator for imminent danger or failure e.g. the sound of breaking ice under one’s feet or the sound of a breaking branch, is deeply ingrained in humans. Early scientific use of AE analysis is related to seismology. The release of energy by active faults is observable as earthquakes which in our terms can be understood as AE waves in earth’s crust. It is therefore of no wonder that in NDT fields other names such as microseismic emission were in use for this technique. In 1980s the method became increasingly popular through ASTM and ASME standardization codes and the term acoustic emission became the standard term. Sources of AE are divided into three categories: primary, secondary, and noise. Primary sources are of impulsive nature and typically associated with material damage such as crack initiation and crack growth at stress concentrations. In composites, AE primary sources are delaminations, matrix cracking, and fiber fracture. The sudden release of energy results in broadband AE signals ranging from DC to MHz or even higher. Secondary sources are less impulsive and therefore less broad band and may be caused by friction, fluid leaks etc. It is worth noting that the elastic waves associated with AE events propagate as a variety of guided wave modes in beam and plate-like structures. However,
because higher order modes attenuate faster, the main contribution away from the
AE source are the fundamental wave modes such as flexural and longitudinal wave
modes.

Both, active and passive techniques are typically based on evaluating the time dif-
ference between excited wave and its reflection at a defect, or the time difference of
a AE wave at different sensors. In the following these techniques are classified as
time of arrival (ToA) or time of flight (ToF) methods. Knowing the arrival times
together with the knowledge of the propagation speed of the wave suffices to cal-
culate the distance between sensor and reflector, or between sensor and AE source.
This is a very successful concept which is applied to many other wave propagation
techniques far beyond NDT. In ultrasonic testing, ToA is straight forward since the
wave velocity is usually constant within the investigated region and the wave prop-
agates without dispersion. In guided wave application, ToA is complicated by the
dispersive behavior of many wave modes. Obviously, it is difficult to attribute an
exact time of arrival to a wave, that is constantly changing. It can even be difficult
to identify which part of the signal belongs to a particular wave pulse. It is there-
fore common practice to use narrow band signals in order to reduce dispersion or,
if possible, use wave modes which are non-dispersive. This is not an option in AE
applications because the AE waves can not be influenced. Even worse, the signal
content is typically broadband and strongly dispersive. Thus, although conventional
ToA methods are well established, its applications to guided wave testing and AE
testing remains a difficult task. Recently, notable efforts were taken to overcome
these difficulties. Typical strategies include the use of wavelet transformation to
generate time-frequency diagrams, from which arrival times can be computed for in-
dividual frequency components e.g. in [27]. Other approaches compensate the effect
of dispersion with a priori known dispersion information for the respective wave or
transform the time-space information of the wavefield into frequency-wavenumber
domain [61].

In this thesis we propose an alternative to the ToA method. Instead of avoiding
or compensating dispersion, the method presented treats dispersion as an informa-
tion for how far the wave has traveled. This is achieved by combining an experiment
with a time reversal (TR) numerical simulation. In a TR simulation a measured
time series is reversed and set as the boundary condition. This has the effect that
the fastest component of a wave is excited last in the simulation. At the position
of the wave’s origin, maximal amplitude is achieved because all components are
superimposed. This idea is inspired by the TR experiments reported by Fink et. al. [23, 22, 57]. Performing the time reversal step as a numerical simulation was
first presented by Leutenegger et al. [15], who applied a similar technique to defect
localization in pipes. Instead of using the propagation speed difference of different
frequency components as it is presented here, Leutenegger used the propagation
speed difference of different wave modes. Another conceptually similar approach to
dispersion is reported by Wilcox [83]. The intention in his work is to remove the
effect of dispersion and subsequently apply ToA to the signal.

The overriding question in this thesis is how to maximally exploit a one point
measurement in a guided wave test. In particular, dispersion, mode conversion, and
reflection of waves shall be used for AE and guided wave defect localization and quantification. As will be shown in the following chapters, dispersion in conjunction with a numerical model can be utilized to achieve the following advantages over ToA in dispersive media:

- The time difference between excitation and reflection is irrelevant. This simplifies the experimental setup.
- A single dispersive wave mode in the signal suffices to detect and localize defects.
- Additional signal content such as noise, reflections or other wave modes hardly affect the localization performance.
- Identification of specific signal features such as arrival time, pulse duration etc. is not required. Instead, the measured waveform is processed as a whole. This substantially increases the method’s robustness in low signal-to-noise environments.

The contribution of this work to science is twofold. In a strict sense, a new approach is demonstrated for the quantitative inspection of 1D and 2D structures. Although ToA is a proven method for pulse-echo and AE testing, the use of a time reversal simulation can offer some distinct advantages, especially when confronted with dispersive wave propagation. The methodology is validated for isotropic and anisotropic beam structures, aluminum pipes, and aluminum plates. Other NDT applications are imaginable, such as rail testing or cable testing. Recent improvements in numerical tools for wave propagation simulations together with ever increasing computing hardware capabilities make the direct use of numerical simulation tools much more obvious for NDT inspection. Beside application examples, the physical background of the time reversal process is investigated and conclusions are drawn for the limitations and pitfalls of the method for guided waves.

In a broader sense the findings of this thesis are not restricted to NDT. Similar to ToA, the method proposed may be applied to all kind of wave propagation phenomena. Dispersion is also known to occur in optics, e.g. optical fibers. Knowing the dispersive behavior of the fiber, dispersion could be compensated with a TR simulation approach, allowing e.g. higher bit rates in long-haul systems. In geophysics, Rayleigh waves show dispersion due to the density gradient in earth’s crust. As a result, lower frequencies which penetrate deeper into the crust propagate faster. The source-sensor distance between epicenter and station should therefore be determinable from the waveform of a single measurement and a subsequent TR simulation. Another group of surface waves showing dispersion are water waves, where especially in deep water, the propagation speed is a function of wavelength. The sudden emergence of an abnormally high water wave, known as Rogue wave, is partially attributed to the effect of dispersion. Dispersion, which in general is known to reduce the amplitude of waves, can as well be used to focus energy at a point in time and space, when its effect is reversed in time. Although it is not believed that
Chapter 1. Introduction

time is running backwards in deep sea, a particular phase constellation in the water surface oscillation may have a similar effect in that the waves accumulate and form an abnormally high wave for a short time instant.
These examples show, that the combination of dispersion and a time reversal processes may find applications beyond NDT.

1.2 Structure of the Thesis

This thesis is structured as a cumulative dissertation, a collection of papers which are all first authored by the Ph.D. candidate. Three papers are included which either are already published in a peer reviewed journal or which are currently being reviewed. Due to the structure of this thesis, some arguments may occur repeatedly. This however has the advantage, that the particular sections can be read independently of each other.

This chapter is concerned with a global introduction to the field of NDT and introduces the overriding research question. It is followed by an outline of the thesis’ structure and concludes with an introduction of the applied methodologies, which were not discussed in detail in the particular papers.

Chapter 2 presents two implementations for AE testing. The first paper is presented in section 2.1 and is entitled Acoustic Emission Localization in Beams based on Phase Dispersion and is published in Ultrasonics in 2014. The paper investigates the use of TR simulations for flexural waves in beams. The TR process for a Timoshenko beam is analytically modelled. Results are presented for both isotropic and anisotropic structures as well as structures with nonuniform cross-sections. The effect of noise in the signal is discussed.

The second paper is presented in section 2.2 and is entitled One Sensor Acoustic Emission Localization in Plates which at the time of this writing is reviewed in Ultrasonics. In this article, the TR process for a Mindlin plate is analytically modelled. Finite element simulations are used to localize AE events in plates. It is shown, how the location of an AE source can be determined from only a single unidirectional velocity measurement of the plate’s surface. To the knowledge of the authors, this is still a novelty since all other methods require at least 2 to 3 sensors for the same task.

Chapter 3 presents an implementation of the method for guided wave testing. In section 3.1 a paper entitled Quantitative Guided Wave Testing by Applying the Time Reversal Principle on Dispersive Waves in Beams is presented which will be published in Wave Motion in 2015. The analytical model of the Timoshenko TR process presented in section 2.1 is extended. The method employed different excitation methods to localize notches in beams and pipes. Using an analytical model based on stress intensity factors, crack depths are evaluated from measured amplitude ratio’s of longitudinal and flexural waves.
1.3 Methodologies

This section introduces the reader to some experimental and numerical methodologies extensively used during the project. In most academic journal articles, experimental details are often omitted or presented in a very condensed manner due to size limitations of these periodicals. This comes at the cost of confirmability and reproducibility of the reported experiments. This section’s objective is therefore to facilitate reproduction of the results presented in the papers.

1.3.1 Excitation of Waves

In the presented studies, the excitation method was chosen with regard to the characteristics of the excitation signal. The excitation signal in turn depended on the dimension of the structure or the phenomena which the signal should emulate, e.g. acoustic emissions. Important aspects of the excitation method were frequency range, linearity, and efficiency. For the guided wave tests of the small test specimens, a Gaussian shaped longitudinal wave pulse with frequencies up to 200 kilohertz was specified. In this frequency band flexural waves show pronounced dispersion. Doubling the specimen’s diameter halves the required bandwidth in order to cover the same range in the dispersion relation. Thus in general, the smaller the test structure, the higher the required excitation frequencies. The smallest defect still possible to detect is limited by the signal to noise ratio (SNR). Therefore, the minimal detectable size of a defect is proportional to the excited wave amplitude, which is why transduction efficiency plays an important role. For this reason, piezoelectric excitation, electromagnetic acoustic transduction (EMAT) and impact excitation were considered valid transduction options. EMATS have many advantages for wave applications due to contact-less operation and wide frequency response for excitation and detection (20 kHz -10 MHz). However, applications are restricted to metallic materials and limited by the rather low transduction efficiency [72, 24]. Long-range inspection and applications in reinforced plastics are therefore not possible. Therefore piezoelectric transduction was used for the small cross-section specimens. For the larger pipe specimen impacting steel balls were used. Although the impact type excitation is not as repeatable as piezoelectric excitation and multiple modes are triggered at the impact location, its frequency response was found to be ideal for the present purpose and the produced amplitudes were high enough such that good SNR’s were obtained without averaging over multiple ensembles, as done with piezoelectric actuation.

1.3.1.1 Piezoelectric Wave Excitation

The following subsection covers the modeling and measuring of the transfer function (TF) for piezoelectric excitation and practical aspects of specimen preparation. Most of the guided wave experiments were performed with piezoelectric transducers. The reason is that actuation is relatively linear in a wide frequency range and analytical models are available to specifically design and optimize the piezoelectric transducer assembly. In [15], the mechanics of the piezoelectric transducer system is modelled as a 1D system of elastic bodies. In order to design and optimize actuation, we
implemented the equations in [15] to derive the analytical TF of the system beam-piezo-backing mass. The TF relates the applied voltage with the displacement at the rod-piezo interface.

Figure 1.2: Illustration of the piezoelectric transducer assembly. The adhesive layers were neglected in the model of the analytical transfer function.

In order to validate the analytical model, a similar TF was experimentally determined. Both TF’s are shown in Fig. 1.3 (d). An approximate design TF for the long wavelength limit is given in section 3.1. The experimental setting for the measurement of the TF is illustrated in Fig. 1.3, details about the instrumentation is listed in table 1.1. In contrast to the analytical model, the experimentally derived TF includes, beside the backing mass-piezo-beam system also the adhesive layers, vibrometer system, and voltage divider. Therefore only a qualitative comparison between analytical and experimental TF is given. The piezoelectric plate was driven with a linear sweep voltage signal with frequencies between 10 kHz and 200 kHz, see Fig. 1.3 (a). The axial displacement response was measured, see Fig. 1.3 (b), the first pulse was extracted and shifted back in time thus restoring the axial displacement at the excitation location. The analytical model is in good agreement with the experiment up to 180 kHz, see Fig. 1.3 (c). The experimentally determined TF breaks down above 200 kHz due to lack of energy in the linear sweep signal. The experimentally determined TF was used to design the input voltage for the generation of a Gaussian shaped longitudinal wave. A signal as shown in Fig. 1.3 (d), results in an axial displacement response at the end face of the beam as shown in Fig. 1.3 (e) by the solid line. This signal matches well with the ideal analytical signal, plotted as dotted line. The guided wave tests of the aluminium beams were performed with this excitation signal. Properties of the beam, piezoelectric plate and backing mass are listed in Tables 1.2, 1.3 and 1.4.
1.3. Methodologies

<table>
<thead>
<tr>
<th>Component</th>
<th>Type/ Program</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC</td>
<td>GenericSignalGenerator.vi</td>
<td>connected via GPIB</td>
</tr>
<tr>
<td>Function Generator</td>
<td>DS 345 Stanford Research Systems</td>
<td>Triggered via PC</td>
</tr>
<tr>
<td>Power Amplifier</td>
<td>Krohn Hite 7500</td>
<td>Variable gain</td>
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<tr>
<td>Voltage Divider</td>
<td>1:100</td>
<td></td>
</tr>
<tr>
<td>Laser Head</td>
<td>PSV 400</td>
<td></td>
</tr>
<tr>
<td>Demodulater</td>
<td>OFV 5000 DD 900</td>
<td></td>
</tr>
<tr>
<td>Oscilloscope</td>
<td>LeCroy WaveSurfer 424</td>
<td>connected to USB drive</td>
</tr>
</tbody>
</table>

Table 1.1: Instrumentation details to Fig.1.3

The assembly of the piezoelectrically actuated specimens is illustrated in Fig. 1.2. The piezoelectric plate was bonded to the beam with a conductive epoxy adhesive (EPO-TEK® H20e). This allowed contacting one wire to the side face of the aluminum beam. The other wire was bonded to the side of the piezoelectric plate, the electrode contacted with a film of conductive silver paint. The wires were fixed by two small droplets of a two component adhesive (X60) and electrically contacted with conductive silver paint. After installation of the piezoelectric plate and the wires, the backing mass was bonded to the piezoelectric plate with a layer of suppository hard fat (Witepsol®). To do so, the beam was positioned such that the free face of the piezoelectric plate pointed upwards, the backing mass was heated and placed upon the piezoelectric plate, in between a layer of Witepsol®. The warmth of the backing mass melted the layer of Witepsol® which then became fluid. Gravitation firmly closed the gap between backing plate and piezoelectric plate. This procedure allowed a very small gap between backing mass and piezoelectric plate and facilitated the experimentation with different backing masses.

<table>
<thead>
<tr>
<th>Material</th>
<th>Aluminum EN6082</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>$E \quad 6.82 \times 10^1$ GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu \quad 0.34$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho \quad 2.7 \times 10^3$ kg m$^{-3}$</td>
</tr>
<tr>
<td>Timoshenko coefficient</td>
<td>$\kappa \quad 0.985$</td>
</tr>
</tbody>
</table>

Table 1.2: Material parameters used in the numerical simulations.

1.3.1.2 Impact Wave Excitation

A major disadvantage of piezoelectric wave excitation is the transducer system’s inherent high-pass behavior. In applications requiring most energy below 10 kHz we found steel balls impacting the test specimen a valuable alternative. The impact excited high amplitude waves into the structure, with velocity pulse shapes resembling a Gaussian distribution curve. Compared to piezoelectric excitation, this approach has lower reproducibility and the trigger time remains unknown. While the latter is problematic to ToA methods, it was found that the TR method did not require knowledge about the time instant of excitation.
Table 1.3: Properties of the piezoelectric plate.

<table>
<thead>
<tr>
<th>Material</th>
<th>Ferroperm Pz26</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge coeff.</td>
<td>$d_p = 3.28 \times 10^{-10}$ m V$^{-1}$</td>
</tr>
<tr>
<td>Compliance</td>
<td>$s_D = 1.05 \times 10^{-11}$ m$^2$ N$^{-1}$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho = 7.7 \times 10^3$ kg m$^{-3}$</td>
</tr>
<tr>
<td>Permittivity</td>
<td>$\epsilon = 7.00 \times 10^2$</td>
</tr>
<tr>
<td>Length</td>
<td>$L = 1.00$ mm</td>
</tr>
<tr>
<td>Width</td>
<td>$a = 6.35$ mm</td>
</tr>
<tr>
<td>Height</td>
<td>$b = 6.35$ mm</td>
</tr>
</tbody>
</table>

Table 1.4: Properties of the backing mass.

<table>
<thead>
<tr>
<th>Material</th>
<th>Densimet 185</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>$E = 3.85 \times 10^2$ GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu = 0.203$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho = 1.85 \times 10^4$ kg m$^{-3}$</td>
</tr>
<tr>
<td>Length</td>
<td>$L = 4.00$ mm</td>
</tr>
<tr>
<td>Width</td>
<td>$a = 6.35$ mm</td>
</tr>
<tr>
<td>Height</td>
<td>$b = 6.35$ mm</td>
</tr>
</tbody>
</table>

The pulse duration is controlled by the speed, mass, and elastic properties of the impacting sphere and plate. Graff [29] reviewed the problem of an elastic sphere impacting a rod. By considering Hertzian contact laws, a nonlinear differential equation is obtained, characterizing the mechanics of the impact. Approximate solutions for the contact duration $t_0$ are presented in the book *Formulas for mechanical and structural shock and impact* by Szuladzinski [70]. In the case of a sphere impacting an elastic plate, impact duration is approximately:

$$t_0 = \frac{3.214}{V_0^{0.2}} \left( \frac{M_2}{K_0} \right)^{0.4}$$

$$\frac{1}{K_0} = \frac{3}{4} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right) \frac{1}{\sqrt{r_2^2}}$$

with $K_0$ being the calculated coefficient of compliance. The sphere’s velocity before impact is $V_0$ [m s$^{-1}$], the parameters $E_i$ [Pa], $\nu_i$, $M_2$ [kg] and $r_2$ [m] are Young’s modulus, Poisson’s ratio, mass and radius of plate and sphere, where indices i differentiate between plate (1) and sphere (2). Equation 1.3 produced plausible results when compared to the measurements. Note that increasing the speed of impact reduces the time duration of the contact. This effect was used to further optimize the frequency content of the impact excitation.

The actual implementation of the impact excitation used a neodymium magnet and four steel balls with mass $M_2 = 4.07 \times 10^{-3}$ kg and radius $r_2 = 5 \times 10^{-3}$ m. A graphical illustration is given in Fig. 1.4. The balls roll on a rail fixed on a positioning table. An aluminium plate is attached concentrically to the end-face of
1.3. Methodologies

the pipe providing the surface for the ball to impact on. The initial setup is such, that three balls are on the side facing towards the pipe and the trigger ball is held off the magnet by a distance \( l_{\text{acc}} \). When the trigger ball is released, the magnetic force accelerates the trigger ball until it impacts on the magnet. Momentum is carried through the four bodies and the last ball shoots towards the plate. When the ball rebounds from the plate, it is again caught by the magnetic force, assuring only a single impact is produced. By measuring the lateral displacement of the pipe, the positioning table could be adjusted such that only minimal bending is introduced into the pipe. This setup allowed good reproducibility, control over the impact location, and, by changing the accelerating distance \( l_{\text{acc}} \), control over the impact velocity. The resulting axial velocity pulses are measured at the end of an aluminium rod for two different accelerating distances and are plotted in Fig: 1.5. In Fig: 1.5 (a), three repetitions of impacts with \( l_{\text{acc}} = 25 \text{ mm} \) are shown. In Fig: 1.5 (c), three repetitions of impacts with \( l_{\text{acc}} = 31 \text{ mm} \) are shown. Note the good reproducibility. Amplitudes were normalized for better comparability, however the high velocity impact has about 3 times the amplitude of the low velocity impact. The effect of impact speed on pulse duration is clearly visible and further illustrated in 1.5 (b) by the impacts’ amplitude spectra.

1.3.1.3 Pencil Lead Break (Hsu-Nielsen Source)

Acoustic emissions (AE) are typically triggered by sudden changes in the structure’s stress distribution. The underlying cause can be e.g. growing cracks, breaking fibres or matrix cracks and occur typically when the structure is subjected to high loads. A substitute for real AE was required in order to test AE localization techniques at various positions on a variety of structures. A common way to test the sensitivity of acoustic emission equipment is by breaking a pencil lead on a test structure. This method is described by the ASTM Standard (ASTM E976 - 10) and was originally described in a technical review at Bruel og Kjaer 1981 by Arved Nielsen [59]. The original implementation suggested a guide ring to ensure that the lead always breaks at the same angle. Preliminary tests of different lead sizes (0.35 mm - 0.5 mm - 0.7 mm) and different grades (2H - HB - B) showed that reproducibility was sufficient even without guide ring and the different leads mainly affected only the amplitude of the resulting AE signal. A 0.35 mm 2H pencil was used for breaking the leads on the test specimen. The lead was protruded 3 mm before breaking. Splitting the vibrometer signal into a signal used for the source localization algorithm and a signal for the trigger channel proved to be very helpful. The signal for the trigger channel is first fed to a high-pass filter such that low frequency movements introduced by the test operator did not inadvertently trigger the AE recording system.

1.3.2 Measurement of Dispersion Relation

Knowledge of the dispersion relation is key to understand and solve wave propagation problems. The dispersion relation establishes the relationship between wavenumber and frequency and describes how a pulse propagates through a structure. Most methods presented are a combination of experiment and numerical simulation. The
simulation requires material input data which can be obtained by measuring the dispersion relation for certain modes and fitting an analytical model to the measured points. The objective of the simulation used in this study is to accurately model flexural wave propagation. Therefore, deriving the necessary material parameters by measuring the dispersion relation of the flexural wave mode seems to be most adequate. However, this only ensures that the analytical model with the derived material data is in optimal agreement with the measurements. If the numerical simulation deviates from the analytical model, the material parameters may require further tuning. By using spectral elements for simulation, accuracy is independent of the discretization and is exact with regard to the implemented theory. However, by using standard finite element models, and material properties derived by fitting e.g. Timoshenko theory, deviations between simulation and experiment may still occur. In this case, either the numerical simulation must be improved or separate material values are iteratively determined for the simulations. As an example, part of the plate simulations were done with finite element (FE) method. The FE model did deviate slightly from the Mindlin theory and in consequence the material parameters for the FE calculations were adjusted to better fit the dispersion relation measured in the experiments.

Many different methods exist for the experimental determination of the dispersion relation. Ideally, all wave modes are excited in an infinitely long structure over the frequency band of interest. The surface motion is captured by an infinite number of points in space and time. Transforming the measured data into frequency domain, both from time to frequency and from space to wavenumber, reveals the frequency relation for the different wavenumbers. In practical applications, data is often corrupted with noise and range and resolution especially in the spatial dimension is often insufficient. This is why the Fourier transformation approach by itself does not always yield satisfactory results. Alternatively, spectrum estimation algorithms exist which approximate the measurement signal with harmonic models, the method being known as Prony’s method. In the Institute of mechanical systems at ETH, Vollmann [79] used a related method, known as linear prediction method, to extract the wavenumber spectrum from spatially sampled vibration data of cylindrical shells. This approach reduced the required number of samples in the dataset at least by a factor six compared to FFT in the spatial domain, and allowed the determination of the complex wavenumber-frequency dependence from broad band multimode signals.

In the studies presented, essentially only the flexural mode’s dispersion relation was of interest. In the chosen frequency range, a single wave pulse could easily be isolated from the displacement signal because the experiments are set up such that only one wave mode is present in the signal. Hence, the dispersion relation is obtained by measuring the phase velocity \( c(f) \) of a single propagating wave mode. This allows a very accurate and precise measurement of the flexural wave dispersion relation. The experimental methodology used for measuring phase velocities is well described in [13, 68]. Narrow band pulses for different central frequencies \( f_0 \) were excited by piezoelectric actuation. The deflection \( y(x,t) \) is measured at \( n \) equally spaced locations, \( \Delta x \) being the distance between two measurement points. Assuming a plane wavefront, the phase value changes linearly with the distance and hence
phase velocity is given by:

\[ c(f) = \frac{2\pi f}{k(f)} = 2\pi \frac{\Delta x f}{\Delta \Psi} \quad (1.4) \]

where \( \Delta \Psi \) is the change in phase angle. When measuring phase velocities of circular wavefronts, initially the phase does not change linearly with distance. However, measuring at two wavelengths away from the excitation source, equation (1.4) is again valid. Because all phase values are considered to be in the interval \( \pm \pi \), the phase difference \( \Delta \Psi \) must be corrected by \( \pm n\pi \). Although this can be done automatically in current mathematical software, the result is often incorrect. The approach used here was to create an array of \( \pm n\pi \) modified \( \Delta \Psi \) values. This leads to an array of different phase velocity values from which the ones are picked that have equal phase velocity values and were apparently modified with the correct \( \pm n\pi \).

### 1.3.3 Numerical Simulations

A large number of numerical simulation methods exist to analyze dynamic problems. Traditionally, finite difference time domain (FDTD) methods were used to obtain numerical solutions to wave problems which were not easily accessible by an analytical approach. Using the FDTD approach, the governing equations of the problem are directly discretized with finite differences. Central differences are used for both the spatial and temporal partial derivatives. In contrast to FE method, FDTD requires constant grid spacing and the meshing is therefore less convenient compared to FEM, although FDTD is numerically more efficient. Good reference articles for FDTD methods are [63] and [30]. Using FE, the governing equations are reformulated using the variational principle to obtain the weak form of the problem. Instead of finding an exact solution valid over the whole domain, as required in the strong form, parametrized shape functions are introduced which are set up such that they provide an averaged solution to the problem. The best approximate solution to the problem, i.e. the best parameters in the shape functions, are found by applying Hamilton’s principle. This allows modeling of geometrically complex structures compared to FDTD. Both methods however have stability criteria for the temporal and spatial discretization increment. The discretization increments used in the FE analysis presented are discussed in [2.1.2.5] and [2.2.5.1]. The stability criterion demands sufficient sampling of the waves in time and space. A too large time step in the explicit formulation of the FE model leads to a breakdown of the simulation process and is therefore immediately noticeable. On the other hand, a too coarse mesh is more critical since it leads to increased numerical dispersion which affects the simulation result and is hard to recognize. To avoid errors due to numerical dispersion, the shortest wavelength must be sampled at least 8 times \[67\]. In addition, convergence studies are advisable where one decreases iteratively element length until no change in the simulation result is noticeable anymore. This however requires apriori knowledge of the frequency content of the signals used in the simulations. Sample source files controlling the FEM simulations for beams, cylindrical hollow shells and plates are attached in the appendix.
In both FDTD and FEM methods, modeling fidelity depends on the level of discretization, which in turn governs the computational cost. Higher frequencies lead to smaller wavelengths which require a finer mesh. A valuable alternative is found in the spectral finite element method (SFEM). Contrary to classical FEM, SFEM uses interpolation functions that are functions of frequency and wave type. For uniform waveguides, this means that the modeling fidelity is independent of the spatial discretization and exact according to the implemented theory. Essentially, the time dimension is transformed into the frequency domain using the fast Fourier transform (FFT) algorithm. The boundary value problem is solved as a pseudo-static problem at discrete frequencies. At the institute of mechanical systems, Mario Weder established a Matlab toolbox based on Doyle’s book *Wave Propagation in Structures* [12]. Many of the beam simulations were done using this toolbox which is accessible in the FSI group’s intern folder\(^1\). The theory regarding the spectral element approach is discussed in [12, 81], an overview of the implementation is given here. The program flow of a typical SFEM simulation is illustrated in Fig. 1.6. The problem is set as a boundary value problem. Different structural elements can be combined to accommodate different loading types. The theories incorporated in the elements were Timoshenko beam theory and Love rod theory. The loading must be decomposed into axial and flexural force, and bending moment. Torsion was not included. In the initialization step, an appropriate time vector \( t \), interpolation grid \( x \) and an external force vector \( F_0 \) are chosen. Time and force vectors are transformed into frequency domain via FFT. Since the FFT assumes periodic signals, ‘start’ and ‘end’ of the signal must be equal. Therefore, the chosen time window must be large enough such that the simulated wave pulses have enough time to fade out. Otherwise, wraparound effects may affect the simulation result [12]. Over a large For loop, the problem is solved at each frequency up to the Nyquist frequency. Hereby, dynamic element stiffness matrices \( \hat{K}_{ij} \) are calculated. Similar to conventional FEM, these element stiffness matrices are pieced together to form the global stiffness matrix \( \hat{K}_{Gkl} \) which relate the nodal forces to the nodal displacements. For the Timoshenko element nodal forces are shear force \( Q_i \) and bending moment \( M_i \), nodal displacements are deflection \( y_i \) and rotation \( \psi_i \). In Fig. 1.6, this is exemplary shown for a 2-noded and a throw off element. A throw-off or 1-noded element is a peculiarity of spectral elements representing a semi-infinite structure. This is easily implemented by dropping the reflection terms in the formulation of the stiffness matrix and shape functions. Next, the system of equations is solved by inverting the global stiffness matrix to obtain the nodal displacements. If the boundary value problem is stated for given displacements instead of forces, the respective forces can be calculated using the throw-off element stiffness:

\[
\begin{pmatrix}
\hat{Q}_0 \\
\hat{M}_0 \\
\end{pmatrix} =
\begin{pmatrix}
\hat{K}_{11} & \hat{K}_{12} \\
\hat{K}_{21} & \hat{K}_{22} \\
\end{pmatrix}
\begin{pmatrix}
\hat{y}_0 \\
\hat{\psi}_0 \\
\end{pmatrix}
\]  

(1.5)

where \( K_{ij} \) are given in [12]. After nodal displacements are obtained for a particular frequency, element shape functions \( \hat{G}(\omega_n, x) \) are determined and the displacement field is obtained by multiplication of the element shape functions with the nodal dis-

\(^1\text{\textbackslash ortrud\textbackslash groups\textbackslash FSI-Group\textbackslash SpectralElementToolbox}
placements. In the case of a throw-off element, this has the form:

\[
\begin{pmatrix}
\hat{y}(x) \\
\hat{\psi}(x)
\end{pmatrix}
= \begin{pmatrix}
\hat{G}_{11}(x) & \hat{G}_{12}(x) \\
\hat{G}_{21}(x) & \hat{G}_{22}(x)
\end{pmatrix}
\begin{pmatrix}
\hat{y}_0 \\
\hat{\psi}_0
\end{pmatrix}
\] (1.6)

where $\hat{G}_{ij}$ are given in [12]. After accomplishing this process for each frequency, the displacement field is transformed back into the time domain by the inverse FFT algorithm.

Advantages of SFEM over FEM are its analytical accuracy, much lower computational cost, simulation of semi-infinite and infinite structures, and the inherent decomposition of the signal into different wave modes. The method is less convenient for modeling geometrically complex structures, although this is possible by combining spectral elements with spectral superelements. Superelements are a sub-structure modelled as FEM from which a dynamic stiffness matrix is extracted, see [12]. However, since we are concerned with guided waves, the structure to investigate is typically rather uniform. In the studies presented, SFEM was used for isotropic and anisotropic materials, beams, and cylindrical hollow shells. Details for modelling more complex anisotropic structures and shells with SFEM are found in [28].
Figure 1.3: (a) Experimental setup used to experimentally determine the transfer function of the excitation system. Details about the components used are listed in Tab. 1.1. Plot (b) shows the input voltage, plot (c) the output voltage from the vibrometer system. The propagated pulse is cut out and used to derive the TF of input and output voltage. The magnitude of the measured TF is given in plot (d) together with the analytically derived TF using the model in [15]. The TF is then used to calculate the necessary input voltage as shown in plot (d) to produce an axial displacement at the end of the beam as plotted in (e). The measured axial displacement is in good agreement with the analytical model.
1.3. Methodologies

Figure 1.4: Illustration of the impact excitation assembly. A steel ball, accelerated by a neodymium magnet over a distance $l_{acc}$, supplies the momentum with which another ball impacts the pipe upon an additional plate.

Figure 1.5: Plot (a) and (c) show velocity curves measured by the laser vibrometer in axial direction at the opposite side of the plate. Two different pulse lengths were achieved by changing the acceleration distance of the impacting ball. Plot (b) shows the amplitude spectra of the six velocity curves.
Chapter 1. Introduction

Figure 1.6: Spectral element simulation program flow. The dynamic problem is solved in a series of pseudo-static problems at discrete frequencies and then transformed back into the time domain.
2 Acoustic Emission Localization
2.1 Acoustic Emission Localization in Beams

Ernst, R., & Dual, J.
Institute of Mechanical Systems, Swiss Federal Institute of Technology, ETH Zurich, Switzerland

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**Abstract**

The common approach for the localization of acoustic emission sources in beams requires at least two measurements at different positions on the structure. The acoustic emission event can be located by evaluating the difference of the arrival times of the elastic waves. Here, a new method is introduced, which allows the detection and localization of multiple acoustic emission sources with only a single, one point, unidirectional measurement. The method makes use of the time reversal principle and the dispersive behavior of the flexural wave mode. Whereas time of arrival methods struggle with the distortion of elastic waves due to phase dispersion, the method presented uses the dispersive behavior of guided waves to locate the origin of the acoustic emission event. Therefore, the localization algorithm depends solely on the measured wave form and not on arrival time estimation. The method combines an acoustic emission experiment with a numerical simulation, in which the measured and time reversed displacement history is set as the boundary condition.

In this paper, the method is described in detail and the feasibility is experimentally demonstrated by breaking pencil leads on aluminum beams and pultruded carbon fiber reinforced plastic beams according to ASTM E976 (Hsu-Nielsen source). It will be shown, that acoustic emissions are successfully localized even on anisotropic structures and in the case of geometrical complexities such as notches, which lead to reflections, and cross sectional changes, which affect the wave speed. The overall relative error in localizing the acoustic emission sources was found to be below 5%.
2.1.1 Introduction

Acoustic emission (AE) in the field of nondestructive testing according to ASTM E1316 - 11b standard terminology for nondestructive examinations refers to transient elastic waves that are generated by the rapid release of energy from localized sources within a material. These AE signals may arise due to growing cracks in monolithic materials or breaking fibers or fiber-matrix cracking in composite materials, see e.g. Salinas et al. (2010) [64] or Kundu et al. [44], and are used as an indicator in nondestructive testing applications to monitor the integrity of critical structures. An important aspect of AE testing is the ability to determine the location of AE sources. The standard procedure for AE source localization relies on the identification of precise arrival times and the knowledge of an appropriate propagation velocity. With these parameters, a triangulation method can be established where the source is identified as the intersection of three circles, whose centers are the sensors’ location [23], also known as time of arrival (ToA) methods. This approach might sound straightforward, however in reality, several difficulties may arise because of anisotropic materials, reflections and mode conversion due to inhomogeneities and distortion of the waveform due to geometrically induced phase dispersion [34]. However, ToA methods were successfully extended to adapt to these difficulties. Ciampa et al [11] extended the method with additional transducers to account for anisotropy of composite plates with arbitrary layup. The effect of dispersion on the recorded waveform was studied by Aggelis et al [2]. He accounted for dispersive effects on AE parameters such as duration or maximal amplitude of an AE event that are typically affected by dispersion. This allowed him to compare AE parameters from sources at different locations.

The detection and/or localization of AE sources in geometrically complex structures was often approached by means of neural network-type of methods, that rely on previously obtained training data [8, 34]. However, the gathering of proper training data can be quite cumbersome.

Beside ToA methods and neural network methods, there is a group of methods, that explicitly uses Lamb wave modes for the localization of AE sources. These methods are known as single sensor modal analysis location (SSMAL) methods, since they allow the determination of the transducer-source distance with only one sensor [8, 58]. The sudden release of energy results in a broadband spectrum and depending on the type of source, the emitted AE cover frequencies from kHz to MHz range. Therefore, in plates and beams, these AE events lead to a range of Lamb wave modes. Studies have shown [35], that most of the energy of an AE event propagates in the first symmetric S0 and first antisymmetric A0 Lamb mode and higher modes tend to have lower amplitudes and attenuate faster. If one can determine the arrival time difference of these two wave modes, the distance $D_{ss}$ to the source is:

\[
D_{ss} = \Delta t \left( \frac{C_s C_A}{C_s - C_A} \right)
\]  

where $\Delta t$ is the time difference of the two arriving modes, $C_S$ and $C_A$ the group velocity of the symmetric and antisymmetric mode for a particular frequency [35]. Maji et al. [48] used the arrival time difference of several frequencies of only one dispersive mode and then performed the same calculation as Holford [35]. These
methods have the advantage of using fewer transducers but are prone to misinterpre-
tation due to mode conversion, reflections or noise in the measurement, since
they assume, that the structure is infinite and uniform.

The method presented here is related to the above mentioned SSMAL methods in
the sense, that it operates with fewer transducers and that it exploits the dispersive
wave nature of AE in 1D structures. A comparison of the SSMAL and the presented
time reversal (TR) method is given in table 2.3 in the conclusion section of this
paper together with a discussion of the advantages and limitations of the method
presented.

The main difference is however, instead of using a single equation such as equation
2.1, the method presented uses a time reversal simulation to reverse the process of
dispersion and therefore retrieve the origin and shape of the AE event. We will
refer to this method as a TR dispersion method, for brevity also just TR method.
This approach takes advantage of the ease of using fewer transducers as found in
SSMAL techniques, is more robust because no arrival times must be picked, has
wider applicability because the test specimen must not be uniform, and reconstructs
the original shape of the AE. The application of a TR numerical simulation for
guided wave testing was previously studied by Leutenegger and Dual [45] and Ernst,
Weder and Dual [19] for beams. These studies have shown promising results for the
localization of notches and cracks in tubes and beams. Other TR applications in
guided wave settings for composite plates were studied e.g. by Park et al. [55, 54]
and Veidt and Normandin [78], who studied the sensitivity of the method to sense
nonlinearities due to delaminations, bonded masses and through holes. The concept
of using TR mirrors in wave propagation problems was first investigated by Fink et
al. [23].

2.1.2 Method

Acoustic emissions are typically broadband and therefore give rise to many propagat-
ing modes in a beam. Holford et al. [35] found, that most of the energy is carried by
the first symmetric and antisymmetric Lamb modes. The first antisymmetric Lamb
mode, here referred to as flexural wave, shows strongly dispersive behavior at low
frequency - thickness numbers. Without a priori knowledge of the actual shape of
an AE event, the source is assumed to be locally focused in time and space. At the
location of the AE origin, the frequency components of the waveform are assumed
to be in phase. However, the part of the disturbance that propagates as a flexural
wave distorts and diverges as it travels away from the source location.

The method presented for locating AE sources consists of two steps. In a first step,
the transverse deflection, caused by the incident flexural wave, is measured at \( x_m \).
In a second step, the measured displacement data is reversed in time and set as the
boundary condition at \( x_m^{TR} \) in a numerical model of the beam. In the simulation,
the distorted flexural wave form recompresses and reaches maximal amplitude at
the location of its previous origin in the numerical model \( x_0^{TR} \). This allows the de-
tection of the AE source by finding the local maximum in a time-space diagram.
This two-step procedure is illustrated in Fig. 2.1.
2.1. Acoustic Emission Localization in Beams

Experiment
Acoustic Emission

Numerical Simulation

Figure 2.1: Illustration of the TR method. On the left hand side, an AE event at $x_0$ on a beam structure leads to dispersing flexural waves. The resulting displacement is recorded at $x_m$ by means of a laser vibrometer. On the right hand side, a numerical model of the beam is used to simulate the TR experiment. The dispersed waveform converges at the origin of the source of the AE.

After having converged at the location of the AE source, the different frequency components of the flexural wave diverge again, leading to a decrease in the amplitude of the flexural wave. This allows the detection of multiple AE events with a single measurement, even if the different AE waves overlap each other. The method described requires only a single, one point, unidirectional measurement, because only the dispersion of the flexural wave mode is used. Multiple AE can be detected because the wave amplitudes decrease again after having converged. Multiple AE sources at an identical position are temporally separated in the time-space diagram. Acoustic emission parameters extracted from the simulation of several AE sources at different positions can easily be compared because dispersion effects are compensated as a natural side effect of this method. However, the effect of damping on the wave form is not yet compensated.

2.1.2.1 Time Reversal Process in a Timoshenko Beam

Here, a beam with the properties given in table 2.1 has been considered. The free motion of a Timoshenko beam is governed by the following coupled set of differential equations, where $Q$ and $M$ are the shear force and the bending moment, $G$, $A$ and $I$ are the shear modulus, cross sectional area and area moment of inertia, respectively.

$$ GAK \left( \frac{\partial \psi}{\partial x} - \frac{\partial^2 y}{\partial x^2} \right) + \rho A \frac{\partial^2 y}{\partial t^2} = 0 \quad (2.2) $$

$$ GAK \left( \frac{\partial y}{\partial x} - \psi \right) + EI \frac{\partial^2 \psi}{\partial x^2} - \rho I \frac{\partial^2 \psi}{\partial t^2} = 0 \quad (2.3) $$

The transverse deflection of the center line of the beam is described by $y(x)$ and $\frac{\partial y}{\partial x} - \psi$ is the shear angle.
By eliminating $\psi$, we may reduce the two governing equations to a single equation

$$EI \frac{\partial^4 y}{\partial x^4} - \frac{I}{A} \left(1 + \frac{E}{G\kappa}\right) \frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{\partial^2 y}{\partial t^2} + \frac{\rho I}{GA\kappa} \frac{\partial^4 y}{\partial t^4} = 0 \quad (2.4)$$

Time invariance holds for equation 2.4 because all time derivatives are of even order. Assuming $y(x, t)$ is a solution to equation 2.4 then $y(x, -t)$ must also be a valid solution. However, we must restrict the method to systems without or with only minimal damping, since damping typically involves time derivatives of first order.

A further requirement for a TR experiment is spatial reciprocity. Illustratively spoken, spatial reciprocity states, that the ray paths traversed by a pulse from point A to point B will also be traversed if the same pulse is sent from point B to point A [6]. That reciprocity holds in the 1D case of flexural waves is shown by Achenbach [1].

Using Timoshenko theory, a transverse disturbance gives rise to two modes, described by the wave numbers $k_1$ and $k_2$. The wave mode associated with wave number $k_1$ will be referred to as flexural wave. The wave mode associated with wave number $k_2$ will be referred to as thickness-shear mode, according to [49]. These wave numbers are plotted in Fig. 3.1 for frequencies up to 500 kHz. The second mode $k_2$ shows a particularly interesting feature in that it exhibits a cut-off frequency, below which the wave number is purely imaginary. While wave modes with real wave numbers correspond to propagating waves, wave modes with purely imaginary wave numbers are called ‘evanescent’ waves and correspond to exponentially decaying vibrations. This behavior will be of importance in the following sections.

Figure 2.2: Dispersion diagram showing the wavenumbers $k_1$ and $k_2$ as a function of frequency according to Timoshenko theory for the investigated aluminium beam. The second mode has a cut-off frequency at 280 kHz.
2.1. Acoustic Emission Localization in Beams

<table>
<thead>
<tr>
<th>Material</th>
<th>AlMgSi1</th>
<th>CFRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>$E$</td>
<td>$E_{11}$</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>$G$</td>
<td>$G_{12}$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td>$\nu_{12}$ and $\nu_{23}$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>$\rho_{23}$</td>
</tr>
<tr>
<td>Timoshenko coefficient</td>
<td>$\kappa$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Material parameters used for the numerical simulations. For the CFRP beam, $E$ corresponds to $E_{11}$, $G$ corresponds to $G_{12}$, $\nu$ is the mean of $\nu_{12}$ and $\nu_{23}$, and $\kappa$ corresponds to $\kappa_{12}$.

2.1.2.2 Time Reversal Process for an Infinite Beam

Consider a beam long enough, such that we do not encounter reflections from the free end. We use $x$ to describe the axial location and $y$ to describe the transverse deflection in the beam. We restrict the method to harmonic waves and suppress the time dependence $e^{i\omega t}$. The general transverse displacement due to waves propagating in the positive $x$-direction for a Timoshenko beam can be written as

\[
y(x) = R_1 A_0 e^{-ik_1 x} + R_2 B_0 e^{-ik_2 x} \tag{2.5}
\]

\[
\psi(x) = A_0 e^{-ik_1 x} + B_0 e^{-ik_2 x} \tag{2.6}
\]

where $A_0$ and $B_0$ are the amplitude coefficients and $R_i$ are the amplitude ratios in the form given by Doyle [12]

\[
R_i = \frac{ik_i GA\kappa}{GA\kappa k_i^2 - \rho A\omega^2} \tag{2.7}
\]

Suppose, a transverse, point like disturbance in the form of $y_0(t)$ excites flexural waves at $x = 0$ in the beam. The displacement due to waves propagating in the positive $x$-direction for this Timoshenko beam is

\[
y_0 = R_1 A_0 + R_2 B_0 \tag{2.8}
\]

\[
\psi_0 = A_0 + B_0 = 0 \tag{2.9}
\]

The rotational degree of freedom, $\psi_0$, is assumed to be zero at $x = 0$ because of symmetry considerations and the assumption of a point like disturbance. Assuming further, that the disturbance, $2y_0(t)$, consists of frequencies below the cut-off frequency of the second mode exclusively, i.e. below frequencies of about 280 kHz in our case, and that the measurement location, $x_m$, is several wavelengths away from $x = 0$, we can write the measured displacement as

\[
y_m(x = x_m) = R_1 A_0 e^{-ik_1 x_m} \tag{2.10}
\]

Note that, under the assumption made above, we have lost the displacement information contained in the evanescent mode. In a second step, we excite the
beam at \( x_m \) with the time reversed time series \( y_m(-t) \). However, due to multimode dispersion, this has again the effect of splitting the excitation displacement into two modes.

\[
\begin{align*}
y^\text{TR}_m(x = x_m) &= -R_1 A_1 e^{-ik_1(x_m-x)} - R_2 B_1 e^{-ik_2(x_m-x)} = (R_1 A_0 e^{-ik_1 x_m}) (2.11) \\
\psi^\text{TR}_m(x = x_m) &= A_1 e^{-ik_1(x_m-x)} + B_1 e^{-ik_2(x_m-x)} = 0 (2.12)
\end{align*}
\]

The \( \text{TR} \) step in the frequency domain is similar to taking the complex conjugate \( (\cdot)^* \) of \( y_m \). Note, that \( R_1^* = -R_1 \) and \( (R_2^2)^* = R_2^2 \). The rotational degree can be arbitrarily chosen in the simulation and is set to zero. This facilitates the relation between \( A_1 \) and \( B_1 \) and its ratio becomes \(-1\). Solving for \( A_1 \) in terms of \( A_0 \) gives the following deflection amplitude at \( x = 0 \)

\[
y^\text{TR}_0(x = 0) = -R_1 \left\{-R_1 A_0 e^{-ik_1 x_m + ik_1 x_m} \right\}_{=1} + R_2 \left\{-R_1 A_0 e^{-ik_2 x_m + ik_1 x_m} \right\}_{=0} (2.13)
\]

In the first term of Eq. \( 2.13 \) the effect of the \( \text{TR} \) process is visible in that the phase is reset to zero. The second term however vanishes because of the evanescent character of the second mode. Apparently, \( y^\text{TR}_0 \) is not equal to \( y_0 \) and the \( \text{TR} \) process failed. Multimode dispersion and the evanescent character of the second mode are the reasons for this behavior. However, if the assumption made so far holds, we can compensate for these effects. Reconsidering, that the original signal was composed of \( A_0 \) and \( B_0 \), we can extract \( A_0 \) from the measured amplitude:

\[
\begin{align*}
\tilde{A}_0 &= (y^\text{TR}_0)^* \frac{R_1 + R_2^*}{R_1^2} (2.14) \\
\tilde{B}_0 &= -\tilde{A}_0 (2.15) \\
\tilde{y}_0 &= R_1 \tilde{A}_0 + R_2 \tilde{B}_0 (2.16)
\end{align*}
\]

The \( ^\text{”}^\text{-”} \) stands for the corrected values and equation \( 2.15 \) reflects the assumption, that the original disturbance had no rotational component.

### 2.1.2.3 Time Reversal Process for a Semi-Infinite Beam

By measuring deflections at only one point between the ends of a beam, the propagation direction of the traveling waves remains unknown. Therefore, we measure the deflection at the free end of the beam. However, at the free end, an incident wave gives rise to a reflected propagating wave and a decaying wave. Having the free end now at \( x = 0 \), we measure the sum of all the amplitudes

\[
\tilde{y}_m(x) = R_1 A_0 e^{-ik_1 x} - R_1 A_1 e^{+ik_1 x} - R_2 B_1 e^{+ik_2 x} (2.17)
\]
2.1. Acoustic Emission Localization in Beams

The "tilde" symbol indicates "measured at the free end". No forces and moments at the free end add two more equations:

\[ Q(x = 0) = -EI \frac{\partial^2 \psi}{\partial x^2} - \rho I \omega^2 \psi = 0 \quad (2.18) \]
\[ M(x = 0) = EI \frac{\partial \psi}{\partial x} = 0 \quad (2.19) \]

Solving for \( A_0 \) in terms of the measured amplitude \( \tilde{y}_m \) gives

\[ A_0 = \frac{\tilde{y}_m (q + k_1 k_2 m)(k_1 - k_2)}{(2(q - k_1^2 m)(R_1 k_2 - R_2 k_1))} \quad (2.20) \]
\[ q = \rho I \omega^2 \quad (2.21) \]
\[ m = EI; \quad (2.22) \]

Having \( A_0 \) isolated from the total displacement at the free end, we proceed with the TR step as outlined in section 2.1.2.2 by substituting \( y_m \) with

\[ y_m = \tilde{y}_m R_1 \frac{(q + k_1 k_2 m)(k_1 - k_2)}{(2(q - k_1^2 m)(R_1 k_2 - R_2 k_1))} \quad (2.23) \]

Fig. 2.3 shows a numerical experiment of a TR process using Timoshenko theory and spectral elements according to Doyle [12]. The simulated semi-infinite beam has section properties according to table 2.1. Fig. 2.3 (a) shows the excitation amplitude \( y_0 \) at \( x = 0 \) within the beam. Fig. 2.3 (b) shows the free end deflection. This deflection amplitude is recorded, reversed in time, and set as the boundary condition in a subsequent simulation, as seen in Fig. 2.3 (c). The response at \( x = 0 \) is plotted in Fig. 2.3 (d). Apparently, the waveform differs from the original signal \( y_0 \). The amplitude is too high, because free end effects were not accounted for, and the shape does not resemble the original pulse because the evanescent mode was lost both in the forward and in the TR simulation. Equations 2.16 and 2.23 were used to get a different excitation amplitude \( \tilde{y}_m^{TR} \). Although only slightly different to \( y_m^{TR} \), the resulting amplitude after TR at \( x = 0 \) matches well with the original signal \( y_0 \), as shown in 2.3 (f).
Figure 2.3: Numerical TR process for a semi-infinite Timoshenko beam for a broadband input signal simulated with spectral elements. The x-axis displays a time scale in ms. The y-axis displays the deflection amplitude on a normalized scale. (a) Excitation signal $y_0$ at $x = 0$. (b) Deflection $y_m$ at the free end of a beam. (c) Time reversed excitation $y_{TR}^m$ at free end. (d) Reconstructed amplitude $y_{TR}^0$ at $x = 0$. (e) Time reversed and processed excitation signal $\bar{y}_{TR}^m$ that accounts for free end effects and the evanescent mode $k_2$. (f) Reconstructed amplitude $\bar{y}_{TR}^0$ at $x = 0$ that fits perfectly the original signal $y_0$.

2.1.2.4 Experiments

The major requirement for the method presented is that the numerical simulation accurately reflects the dispersion of the flexural mode in the experiment. Therefore, the material parameters determining the wave propagation speed for the flexural mode are evaluated by fitting material parameters using Timoshenko theory to experimentally determined phase velocities.

All experiments are done on an aluminum beam with the length $L = 2$ m and square cross section with the side length $b = 0.006$ m and a pultruded carbon fiber reinforced plastic (CFRP) beam with circular cross section, diameter $D = 0.008$ m and length $L = 2$ m.
The elastic properties of a transversely isotropic material are described by 5 independent material parameters. However, we achieved a reasonable fit of the flexural mode by using Timoshenko theory and fitting only three material parameters: Young’s modulus $E$ corresponding to $E_{11}$, shear modulus $G$ corresponding to $G_{12}$, and Poisson’s ratio $\nu$ corresponding to the mean of $\nu_{12}$ and $\nu_{23}$.

The measured and fitted dispersion diagrams for the flexural mode for the aluminium and composite beams are given in Fig. 2.4. Note, that for frequencies below 60 kHz, phase velocities of the flexural mode are dominated by axial stiffness and are higher in the composite beam while for frequencies above 60 kHz, phase velocities are dominated by shear stiffness and are higher in the aluminium beams. This behavior is explained by the high $E/G$ ratio of the CFRP beam. The fitted Young’s modulus of the CFRP beam is substantially lower than typical literature values. The reason for this is that the transversely isotropic composite beam was modeled as an isotropic material with an independent shear modulus parameter $G$. From a physical point of view, due to the fact that $G_{12}$ is much smaller than $E_{11}$, the shear correction in the Timoshenko theory is much more pronounced. The effect of rotary inertia is weaker and the assumption of linear variation of normal stresses does not hold in the anisotropic case [65]. In consequence, if one is interested in the exact evaluation of material parameters, more accurate theories are necessary for transversely isotropic beams, see e.g. Sayir [65] or King [40]. In the present case however, the flexural mode is sufficiently well captured by Timoshenko theory as evident from Fig. 2.4 (b).

Figure 2.4: Experimentally determined dispersion diagrams of the flexural mode for the aluminium beam (a), and the CFRP beam (b). Young’s modulus is used as a fitting parameter for the aluminium beam, Young’s modulus and Shear modulus are fitting parameters for the CFRP beam.

After having obtained the necessary material parameters, AE experiments were performed. In order to validate the method, we artificially created AE events by breaking pencil leads on the surface of a beam, according to ASTM E976. The
experimental setup is depicted in Fig. 2.5. We performed AE measurements on four different beams, all being 2 m long. The three aluminium beams have rectangular cross sections, the CFRP beam has a circular cross section see Fig. 2.6. The origin of the axial coordinate is now set to the left end of the beams. Beam 1 is uniform along its length and has a quadratic cross section with a side length of 6 mm. Beam 2 is a similar beam, but between $x = 1.6$ m and $x = 1.7$ m, the cross section is reduced to $4 \text{mm} \times 6 \text{mm}$. Beam 3, again similar to beam 1, has at $x = 0.7$ m, $x = 1.0$ m and $x = 1.9$ m notches machined into the beam, with a depth of 2 mm and a width of 1 mm. Beam 2 and beam 3 were designed to investigate the performance of the method for nonuniform structures. Beam 4, the CFRP beam, has a diameter of 8 mm and was used to investigate the performance of the method for anisotropic structures.

Figure 2.5: Experimental setup for AE experiments. Pencil leads were broken at locations $x_{AE} = [1.3 \text{ m}, 1.5 \text{ m}, 1.7 \text{ m}]$ for the uniform beams and at $x_{AE} = [0.8 \text{ m}, 1.3 \text{ m}, 1.5 \text{ m}]$ for the two nonuniform beams.
2.1. Acoustic Emission Localization in Beams

2.1.2.5 Time Reversal Simulations

The overall process for localizing AE in the beam structure is illustrated in Fig. 2.7. Before the measured data can be used in the TR-simulation, the following data preprocessing is done:
• Find time window of the measured displacement history.

• Remove high frequency noise using Savitzky-Golay filters.

• Compensate for evanescent mode $k_2$ and free end effects according to section 2.1.2.3 in the frequency domain.

• Reverse the processed displacement history in time domain.

The time window was chosen manually so that the amplitude vector starts and ends at a zero crossing to avoid leakage. The time window length was approximately 2 ms so as to capture flexural waves with frequencies as low as 5 kHz.

In order to reduce the high frequency noise content in the measured signals, we applied a Savitzky-Golay smoothing filter. Savitzky-Golay filters are known to preserve the high frequency content of a signal better than standard moving average filters. Most of the energy of the flexural wave is found below 30 kHz.

The numerical simulation is done with Abaqus Explicit, a commercial finite element (FE) analysis product. This has the advantage, that even complex geometries
can easily be modeled. For the validation of the analytical derivations in section 2.1.2.3, a spectral element approach, as outlined by Doyle [12] was used. This significantly reduces computational cost, has Timoshenko-accuracy and elements with a semi-infinite extension are a natural side benefit of the spectral element method. These features are very convenient for the use in the method proposed. The aluminium beams were modeled as a 2D plane stress structure, using quadratic linear quadrilateral elements of type CPS4R. The spatial discretization was chosen to sufficiently sample the smallest wavelength $\lambda_{\text{min}}$. Schubert [67] proposed, that the shortest wave length should be sampled at least eight times in order to keep the numerical dispersion at a reasonably low level. In the present frequency band we estimate the smallest wavelength that we want to capture to be approximately 0.04 m. The element length was set to $\Delta x = 0.001$ m. This gives a dimensionless spatial sampling frequency of about

$$f_{\text{spat}} = \frac{\lambda_{\text{min}}}{\sqrt{2}\Delta x} = 28$$  \hspace{1cm} (2.24)

The criterion for stable temporal sampling [21] gives a time increment $\Delta t$

$$\Delta t = \frac{\Delta x}{\sqrt{2}c_p} = 1.13 \cdot 10^{-7} \text{ s}$$  \hspace{1cm} (2.25)

where $c_p$ is the primary wave speed in aluminium. The CFRP beam was simulated with spectral elements.

The structure's length receives particular attention. While figure 2.8 shows a TR simulation of the exact length of the beam, the results shown in table 2.2 were obtained from simulations, where the length of the beam was modeled longer than in reality. The reasons to model the structure longer than in reality are twofold. On one hand, we want to investigate the methods capability to detect and locate an AE event on the effect of dispersion alone. It was clear beforehand, that by letting several reflected waves propagate back along their previous paths, they will superpose at the AE's origin, regardless on the waves dispersive properties. However, this effect is not the subject of the presented study and there are many applications possible where no reflections at a free end occur, e.g. pipelines or rails. On the other hand, the performance under the presence of noise is even improved when no reflections occur during the simulated time span. This effect is discussed in section 2.1.4.2.

### 2.1.2.6 Localization of Acoustic Emission Events

The localization of an AE event was done on the basis of the transverse displacement data along the beam. The origin of the AE source is assumed to be at the maximum of the transverse displacement amplitude, as resulting from the TR process. Since the problem is one dimensional in space, we can draw time-space diagrams, also known as Lagrange diagrams, with the transverse displacement amplitude being the third dimension, displayed by the gray tone. Such a diagram is shown in Fig. 2.8.
2.1.3 Results

Table 2.2 lists the results from the AE measurements. At every location $x_{AE}$, three AE’s were simulated by breaking pencil leads on the surface of a beam. The AE excites flexural waves, that propagate a length $x_{prop}$ to the sensor, where the displacement is recorded. The AE location was then predicted by the method described in the previous sections. Columns $\Delta x_{err}$ list the absolute difference between the predicted location $x_{AE \text{ predicted}}$ and the true location $x_{AE}$ for all three measurements. For the nonuniform beam 2 at $x_{AE} = 0.8$ m, in one experiment, we identified the first maximum as an outlier and chose the second maximum as the ”right” location. In section 2.1.4.4 we discuss, how this outlier has been identified.
2.1. Acoustic Emission Localization in Beams

<table>
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<th>Structure</th>
<th>$x_{AE}$</th>
<th>$x_{prop}$</th>
<th>$\Delta x_{err1}$</th>
<th>$\Delta x_{err2}$</th>
<th>$\Delta x_{err3}$</th>
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<tr>
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<td>0.7</td>
<td>0.002</td>
<td>0.001</td>
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<tr>
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<td>0.003</td>
<td>0.001</td>
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<tr>
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<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
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<td>0.7</td>
<td>0.004</td>
<td>0.001</td>
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</tr>
</tbody>
</table>

$x_{AE}$: AE location  
$x_{prop}$: distance AE - sensor  
$\Delta x_{erri}$: $|x_{AE} - x_{AE \, predicted}|$  
(*)*: outlier  

Table 2.2: Localization results of the AE tests

2.1.4 Discussion

Every simulated AE was easily identifiable as such and the averaged localization error over all 36 measurements was 3.05 mm with a standard deviation (SD) of 3.5 mm. The averaged relative error with respect to the source-receiver distance was below 5%. Reproducibility and accuracy was good considering an uncertainty of ± 1 mm in the triggering of each AE. Overall, there is a tendency, that the localization error increases with smaller source-sensor distances. However, the data set is too small to see statistically significant effects. The effect of uncertainties in the material parameters on the localization error is discussed in section: 3.1.3.2 for a Timoshenko beam.

2.1.4.1 Multiple Acoustic Emission Detection

Multiple AE detection is especially difficult in dispersive systems, because the individual waveforms of the AE’s are superimposed in the measurement signal. One AE event may lead to several propagating modes and reflections and it becomes difficult to assign certain features in the signal to a particular AE. The TR method acts as a filter which converges information propagated as flexural mode and disperses all other information. Flexural wave mode information will converge at its source location leading to a maximum in amplitude and a minimum in temporal and spatial extent. No matter whether the individual AE’s were triggered at different times and / or different spatial locations, the TR method makes them separable in the Lagrange diagram. As an example, the Lagrange diagram in Fig. 2.8 shows a TR simulation of a signal measured at $x = 2 \text{ m}$ due to a pencil lead break at $x = 0.8 \text{ m}$. Note, how the waves focus on $x = 0.793 \text{ m}$, $t = 0.845 \text{ ms}$, where
the pencil lead originally was broken and the first event was produced. However, a second focal spot can be identified at the same spatial location, but at $t = 0.67$ ms, when apparently a second event was produced. This peak originates from a second disturbance produced by the pencil tip hitting the beam after the lead was broken. The breaking of a pencil lead gives a sharper peak than the blunt impact of the tip on the beam surface. Also the sign of the two peaks change from negative to positive, from a sudden pressure decrease due to the breaking pencil lead, to a pressure increase due to the impact of the pencil tip, respectively. This is clearly visible in the time trace in Fig. 2.8. To summarize, individual AE’s are separated with the TR method and therefore multiple AE events can be identified and localized from one measurement with one sensor even if the waveform of several AE’s cannot be separated at the measurement location.
2.1.4.2 Uniform Beam, Robustness under Noise

Since beam 1 is completely uniform, no reflections along the beam occur. The robustness of the method was investigated by superposing synthetic white noise upon the smoothed TR data for an AE at \( x_{AE} = 1.5 \text{ m} \), see Fig. 2.9. The white noise had the same effective amplitude as the TR data and the signal to noise ratio was therefore calculated to be 3 dB according to

\[
SNR = 20 \log_{10} \frac{u_{\text{eff, signal}}}{u_{\text{eff, noise}}} \tag{2.26}
\]

Regardless of the noise, the method determined the location of the AE successfully to \( x_{TR}^* = 1.502 \text{ m} \), without any filtering of the TR signal. The displacement data at the location of the AE origin is still noisy and the reproducibility might be slightly worse than in the case without noise.

![Amplitude spectra](image)

Figure 2.9: Plot (a) shows the processed data \( \bar{y}_{TR} \). A pencil lead was broken 0.5 m away from the end of the uniform beam, where the displacement was recorded. Plot (b) shows the same data with superposed synthetic white noise of the same root mean square (RMS) amplitude as the original data. The plot on the right hand side (c) shows the amplitude spectra of both time series.

In the presence of noise, the TR process seems to have a filter function on the noise. Since we simulate the beam twice as long as it is in reality, waves can travel outside of the time window, that is considered in the localization algorithm. Since the noise consists of different frequencies, the signal will immediately start to disperse while it propagates through the structure. The further away from the excitation, the more time is covered with the noise and the faster frequency components move to the front of the wave while the slower frequency components lag behind. Therefore, in the case of an infinite long beam, a decomposition of the frequencies takes place. The behavior is different for the AE wave data, that, as opposed to the uniform noise, has a specific phase relation, that allows the wave to concentrate itself on the origin of the AE event, thereafter it rapidly disperses. This effect is useful for the
localization of the AE as all the phases belonging to the AE event are compressed during the TR simulation, and noise stemming from other sources is scattered. To verify this hypothesis, we plotted the SD with respect to zero displacement of the time series along the axial coordinate for a TR simulation and for pure white noise, see Fig. 2.10.

![Diagram](image)

Figure 2.10: Plot (a) and (b) show the TR simulation due to an input signal as shown in Fig. 2.9 (a). Plot (b) displays the SD along the spatial coordinate for the data shown in plot (a). The same procedure was applied to a TR-simulation with pure noise as the input signal. In the case of pure noise, the SD in the considered window decreases almost continuously whereas in the case for the TR data, it remains high until the waves reached their origin and then decreases rapidly.
The SD of the time series indicates the average amplitude along the beam for the considered time span. The frequency band of the noise covered the high and low dispersion regime. The SD decreases along the beam, because more and more wave energy travels out of the considered time-space window. This decrease is also affected by the dispersion characteristic of the beam and the frequency composition and distribution of the signal. The very fast decrease of the SD close to $x = 2\, \text{m}$ for both data sets is a result of the very low phase speed at low frequencies. These frequency components are somewhat bounded to the location, where the waves were excited. This effect is visible for both data sets, however much more pronounced for the noise data because this data has it’s low frequency content spread over the whole time span as opposed to the TR data.

The important difference in the behavior of the SD of the TR signal and the noise signal is, that the SD almost monotonically decreases along the beam for the noise signal, whereas the SD of the TR signal remains on a constant high level till the wave achieves the origin of the AE event, thereafter the SD rapidly decreases. This is the result of the wave being concentrated at the source of the AE, before the energy of the wave bleeds out of the window. This result suggests, that the proposed method performs better under substantial noise than ToA methods, especially if the frequency spectrum of the noise overlaps the frequency spectrum of the AE and the location of the AE event is away from the end of the structure, such that the fastest noise components do not interfere with the AE wave after being reflected at the end of the structure.

2.1.4.3 Nonuniform Beam 2

The geometry of beam 2 is similar to beam 1 but has a symmetric reduction of the cross section between 1.6 m and 1.7 m from 6 mm $\times$ 6 mm to 4 mm $\times$ 6 mm. Waves, that were excited by the pencil lead, pass this section before they are recorded by the laser vibrometer at the end of the beam. Despite this non-uniformity, all AE events were again localized with a averaged error of 1.45 mm. These TR simulations accounted for the reduced cross section of beam 2. By using a model of a straight beam without reduction of the cross section in the TR simulation, the AE was still detected but with a localization error of 25 mm. A simple hand calculation using Bernoulli theory makes this result plausible. Using 10 kHz as the center frequency of the AE, we can calculate the group velocity for the two beam sections.

$$c_{gi} = 2 \sqrt{\frac{EI_i}{\rho A_i \omega}}$$ \hspace{1cm} (2.27)

where $I_i$ is the respective area moment of inertia and $A_i$ the respective cross section area. The time $t_0$ during which the wave propagates from the source of the AE to the sensor is:

$$t_0 = \frac{x_1}{c_{g1}} + \frac{x_2}{c_{g2}}$$ \hspace{1cm} (2.28)

with $x_1$ being 1.2 m and $x_2$ being 0.1 m for $x_{AE} = 1.3\, \text{m}$. The localization error due to an inaccurate geometric model without the reduced cross section is in that
case:

\[ \Delta x = c_{g1} t_0 - (x_1 + x_2) = 0.0225 \text{ m} \]  \hspace{1cm} (2.29)

This is very close to the observed difference of 25 mm. For precise localization of the AE source, we need an accurate numerical model of the structure. However, if the model has some inaccuracies, the AE can still be identified and detected. As opposed to traditional ToA methods, the geometry can be rather complex without affecting the localization of the AE.

![Figure 2.11: Lagrange diagram of a TR simulation of nonuniform beam 2, containing a reduced cross section between \( x = 1.6 \text{ m} \) and \( x = 1.7 \text{ m} \). The AE was triggered at \( x = 1.5 \text{ m} \). The waves are partially reflected at the geometrical discontinuity.](image)

If taken into account, the geometrical discontinuity scatters the flexural wave also in the TR simulation, resulting in a reflected and transmitted wave. In Fig. 2.11, the Lagrange diagram of the TR simulation is plotted. The two horizontal lines indicate the border of the reduced cross section. Note, that the part of the wave that is reflected at the discontinuity shows also a focusing behavior, but its focus does not point to the original location of the AE event. While the reflected wave does not focus on the spatial origin, it does focus on the same temporal location, indicated by the white dotted line in Fig. 2.11. By recording the displacement at only one point, in one direction and for a finite time window, such reflections can not be fully compensated in the TR simulation. However, with the help of a Lagrange diagram, we can identify such artificial maxima. This shows again, how convenient the analysis in a time-space diagram is.

In table 2.2, we identified a local maximum as an outlier. The particular simulation is plotted in Fig. 2.12. All time traces are superimposed, such that the largest amplitudes can be identified along the length of the beam. The max search algorithm found the maximum displacement to be at \( x = 0.75 \text{ m} \). This is a relatively large
error of 50 mm from the true position. By looking closer at the respective simulation, one sees that the amplitude of that maximum is positive. However, all other 26 TR simulations had a negative amplitude at the predicted location of the AE. When a broad band flexural wave propagates dispersively through the structure, the difference in phase velocities leads to positive and negative phase interference. This can easily be seen in Fig. 2.12 in that positive and negative amplitudes alternate while the wave propagates through the structure. The outlier can therefore be corrected by choosing the next negative local maximum in the Lagrange diagram. Unfortunately, the sign of the AE amplitude is probably not always known a priori. Due to symmetry considerations, i.e. no rotational displacement at the AE origin and symmetry of the left and right going waves, we expect the TR peak to be symmetric. Therefore, since the peak at \( x = 0.8 \) m is more symmetric than the peak at \( x = 0.75 \) m, this is assumed to be a further indication for the maximum at \( x = 0.8 \) m is an outlier.

Figure 2.12: TR simulation of beam 2, where all time traces are superimposed. The AE event was triggered at \( x = 0.8 \) m. The max search algorithm found the maximum displacement at \( x = 0.75 \) m. The true AE event is expected to be at \( x = 0.8 \) m.

2.1.4.4 Nonuniform Beam 3

The nonuniform beam 3 contains several sharp notches, as illustrated in Fig. 5. Waves, originating from pencil lead breaks, hit these notches and are partially reflected at these discontinuities. Regardless of that, the largest part of the energy is transmitted without any noticeable change in the waveform. Staudenmann discussed reflection and transmission coefficients for beams with transverse notches and reported, that although frequency dependent, the transmission coefficient is always much larger than the reflection coefficient for an incident flexural wave, see e.g. Staudenmann [68] for a discussion of the transmission and reflection coefficients for flexural waves at transverse notches in beams. Therefore, all AE’s were successfully localized with a mean error of 0.002 m. The localization error was about the same also for simulations not incorporating the notches in the modeled geometry. This result suggests for the simulations, that it is more important, to accurately capture the overall phase speed characteristic than to incorporate all tiny details, to achieve good AE localization results. On the other hand, a one point measurement of only a short period of time, as it was used in these experiments, might not suffice to recover the exact waveform of the AE if substantial reflections within the structure are encountered.
2.1.4.5 Anisotropic Beam 4

The good results for the CFRP beam suggest that anisotropy is of no problem for the method, as long as the numerical model is able to simulate the a0 mode in the particular frequency range given by the input signal. Relatively large localization errors were found for the AE being triggered closest to the sensor. An assumption in the model of the TR process is that the evanescent mode $k_2$ is decayed at the measurement location. Since the wavelengths in the composite beam being notable longer than in the aluminium beam, this assumption may be incorrect for short source-sensor distances and therefore explain the larger errors in the predictions of the AE closest to the measurement location.

2.1.5 Conclusion

In the paper presented, AE sources in both uniform and nonuniform, isotropic and anisotropic beams were successfully localized using a single one point, unidirectional measurement with a laser vibrometer, a subsequent numerical processing using Timoshenko theory and a TR simulation. AE signals were produced by breaking pencil leads on different test specimens. No ToA information was used. The method requires, that the structure is of one dimensional shape, that the AE triggers flexural waves at frequencies for which dispersion occurs, and that the sensor is sensitive in the particular frequency range. The method performed very robustly even under the presence of white noise with a SNR of 1:1. Noise even in the frequency range of the signal and reflections due to geometric discontinuities did not affect the localization results. Depending on the amount of reflections occurring within the structure, the waveform of the AE event is recovered during the TR simulation. This allows to compare different AE events within the structure.

It was experimentally validated that multiple AE could be identified from one measurement, see Fig. 2.8.

Both the TR method presented and the SSMAL method exploit the modal nature of guided waves in 1D structures. A comparison of the two types of methods is given in table 2.3.

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<th>TR</th>
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<td>1</td>
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<td>Based on wave form</td>
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<tr>
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<tr>
<td>Specimen assumptions</td>
<td>uniform structure</td>
<td>also nonuniform structure</td>
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</table>

Table 2.3: Comparison of SSMAL with the TR method presented. \(^{1}\) The method of Maji et al. \([48]\) uses arrival times of multiple frequencies.

The FE simulations of the TR problems for the beams done with Abaqus Explicit took about 45 minutes each on a 2.6 GHz computer using a standard hard disk drive. That said, the spectral element simulations performed in Matlab took about
While the SSMAL-type methods perform much faster, the TR method has some considerable advantages:

- **Robustness** The SSMAL technique requires the identification of the s0 and a0 mode. With only one sensor, this may be a difficult task, depending on the AE source. The TR method does not require such a separation and the sensor can be optimized for out of plane sensitivity. Moreover, even if considerable amounts of energy is carried by other modes, the TR method does only converge waves containing a0 phase information, all other modes are dispersed i.e. filtered out in the TR simulation. For the same reason, TR is much less affected by noise than ToA methods, for which phase picking can be very cumbersome in a noisy environment.

- **Accuracy** Since the whole waveform is used for the localization of the AE, the method is expected to perform better than methods, that require the extraction of a certain feature in the waveform, e.g. ToA information. Especially in the case of dispersive wave propagation, by which ToA information depends on frequency, one has to determine the arrival time of a single frequency component by means of e.g. wavelet analysis. This inherently involves the uncertainty principle or Gabor-limit, which states, that it is impossible to exactly localize a signal in both frequency and time domain [26].

- **Versatility** As shown with the nonuniform beam 3, the test specimen may have changes in the cross section that alter the local wave velocity without affecting the localization result, as long as a sufficiently accurate model of the test specimen is available. Similarly, anisotropic and complex structures can be tested if the a0 mode for the structure can be simulated with a numerical model.

The method’s limitations are strongly related to the limitations of the chosen numerical simulation method. Commercially available software was used for the aluminum beam simulations. Larger structures or higher frequencies may require other simulation approaches such as spectral finite element simulations, in order to limit the computational effort. Further, the dispersion relation of the a0 mode of the test specimen must be known or measured prior to the AE measurements. However, this is true for most of the localization approaches and especially for the SSMAL type of methods.
2.2 One Sensor Acoustic Emission Localization in Finite Plates

Ernst, R., & Zwimpfer, F., & Dual, J.
Institute of Mechanical Systems, Swiss Federal Institute of Technology, ETH Zurich, Switzerland


Abstract  Acoustic emissions are elastic waves accompanying damage processes and are therefore used for monitoring the health state of structures. Most of the traditional acoustic emission techniques use a trilateration approach requiring at least three sensors on a 2D domain in order to localize sources of acoustic emission events. In this paper, we present a new approach which requires only a single sensor to identify and localize the source of acoustic emissions in a finite plate. The method proposed makes use of the time reversal principle and the dispersive nature of the flexural wave mode in a suitable frequency band. The signal shape of the transverse velocity response contains information about the propagated paths of the incoming elastic waves. This information is made accessible by a numerical time reversal simulation. The effect of dispersion is reversed and the original shape of the flexural wave is restored at the origin of the acoustic emission. The time reversal process is analyzed first for an infinite Mindlin plate, then by a 3D FEM simulation which in combination results in a novel acoustic emission localization process. The process is experimentally verified for different aluminum plates for artificially generated acoustic emissions (Hsu-Nielsen source). Good and reliable localization was achieved for a homogeneous quadratic aluminum plate with only one measurement.
2.2. One Sensor Acoustic Emission Localization in Finite Plates

2.2.1 Introduction

Many damage mechanisms involve some sort of sudden stress release or stress redistribution in the loaded structure. The transient nature of these mechanisms leads to elastic waves propagating away from the damage zone. These waves traditionally are called acoustic emissions (AE) and are used for monitoring the structural health of a mechanical part. In contrast to ultrasonic inspection, AE monitoring is a passive inspection technique which does not require an interrogating wave in order to scan the structure. AE waves are generated by the damage mechanism itself and the method is therefore ideal for monitoring the structural health of mechanical parts in operation. However, only active damage processes are detectable and the detection of such damage zones depends on the type of loading.

Locating an AE event is traditionally done by trilateration, which requires arrival time identification of a wave at different sensors, very similar to the localization of the epicenter of an earthquake from arrival times. More recently, researchers developed alternative techniques for localization of AE in mechanical structures. A thorough review of the latest AE localization techniques is given by Kundu [43]. He found two main research directions, one striving for minimal a priori knowledge requirements, the other for using as few sensors as possible. Beam-forming and optimization algorithms allow robust localization results using at least four sensors. Often, AE are to be detected in structural components and hence the AE waves are multimode and propagate dispersively. Therefore, considering the modal nature of these AE wave problems, fewer sensors suffice to localize AE events and these techniques are known as modal acoustic emission (MAE), see e.g [69, 74, 37]. The benefit of using fewer transducers are of either economical nature or offer monitoring capabilities in situations, where only limited access is possible. In the case of plate structures, MAE typically requires the identification of arrival times of extensional and flexural wave modes. Since the two modes propagate with different group velocities, the propagated distance of the waves can be calculated with the knowledge of the respective velocities. Three main concerns come with the application of MAE. (1) First of all, the measurement signal is often composed of many wave reflections and of multiple AE’s. As a result, identification and separation of flexural and extensional wave modes from the same event is not straightforward, especially when using only one transducer [37]. Methods to decouple incident and reflected waves in plates are reported in [61]. (2) Errors in arrival time determination are a main source of the localization uncertainty. Accurate arrival time picking is further complicated by noise in the signal. (3) The MAE method implies that the measured waves propagated along direct paths from AE source to sensor. Non uniformities or interruptions in the wave paths are not compensated in the calculation [43]. This paper provides an alternative method closely related to MAE, however, avoids many of the previously mentioned issues while requiring even fewer sensors. The method is based on the dispersion of the flexural wave mode and the time reversal (TR) principle in linear wave propagation. The AE’s are analyzed in a TR numerical simulation for a specific frequency band, similarly to what is proposed by Leutenegger and Dual [45]. Constraints for the chosen frequency band come from two sides. On the one hand, the signal must contain flexural waves in a frequency range where dispersion is present. On the other hand, the implemented finite element model is
reasonably accurate only up to 50 kHz. We therefore band-limited the AE signal to frequencies between 5 kHz and 50 kHz. The method was previously applied to AE localization in beams [17] and is extended here to AE localization in plates.

2.2.2 Axisymmetric Time Reversal Process for Flexural Waves in Plates

In this paper, we define the TR process as follows: Starting from a transverse displacement disturbance \(w_0\) at location \(r_0\), the disturbance propagates through the material as a number of wave modes, one of them being the flexural wave mode. At another location \(r_m\) the resulting transverse displacement \(w_m\) is recorded, reversed in time and used to re-excite waves in the plate such that the transverse displacement at \(r_m\) has the form \(w_m^{TR}(t) = w_m(T - t)\), where \(T\) is an arbitrary time constant. Part of this second disturbance propagates to the original location \(r_0\), resulting in a displacement response \(w_0^{TR}\). If the TR process succeeds, \(w_0^{TR}\) is similar to \(w_0\).

This process is now investigated for flexural waves, using Mindlin plate theory [50]. While other modes also contribute to the transverse response, we assume that the signal has been band-limited such that only flexural and shear-thickness wave modes contribute to the transverse response, higher order modes and longitudinal modes are ignored and later shown to be not relevant.

The three wavenumbers for Mindlin plate theory are given by:

\[
k_1 = \pm \frac{1}{c_p \kappa} \sqrt{\frac{\omega}{2h(1 - \nu)}} \sqrt{h \omega(2 + \kappa^2(1 - \nu)) + \sqrt{12\epsilon_p^2\kappa^4(1 - \nu)^2 + h^2\omega^2(2 - \kappa^2(1 - \nu))^2}}
\]

\[
k_2 = \mp \frac{i}{c_p \kappa} \sqrt{\frac{\omega}{2h(1 - \nu)}} \sqrt{-h \omega(2 + \kappa^2(1 - \nu)) + \sqrt{12\epsilon_p^2\kappa^4(1 - \nu)^2 + h^2\omega^2(2 - \kappa^2(1 - \nu))^2}}
\]

\[
k_3 = \mp \frac{\sqrt{\pm 1}}{c_p h} \sqrt{\frac{2}{1 - \nu}} \sqrt{\frac{3}{2} \epsilon_p^2\kappa^2(1 - \nu) - h^2\omega^2}
\]

with \(c_p = \sqrt{\frac{E}{\rho(1 - \nu^2)}}\) being the longitudinal plate velocity. The other parameters are: non dimensional factor \(\kappa\), here \(\kappa = \sqrt{\frac{\pi}{12}},\) the plate thickness \(2h\), the circular frequency \(\omega\), Poisson’s ratio \(\nu\), Young’s modulus \(E\), density \(\rho\). The test specimen used in theory and experiment is a 2 mm thick aluminium plate and Mindlin plate theory predicts a frequency spectrum according to Fig. 2.13.
2.2. One Sensor Acoustic Emission Localization in Finite Plates

Figure 2.13: Frequency spectrum showing the wavenumbers $k_1$, $k_2$ and $k_3$ as a function of frequency according to Mindlin plate theory. The plate thickness is 2 mm and is made of T6 AW-6082 aluminum. The first mode ($k_1$) will be referred to as flexural wave, the second mode ($k_2$) as thickness-shear mode with cut-off frequency 0.79 MHz. The third mode ($k_3$) is referred to as twisting wave and has a cut-off frequency of 1.58 MHz.

2.2.2.1 Infinite Plate

The transverse axisymmetric response for an infinite plate subjected to a point force can be written as a flexural wave and a shear boundary layer, see Fromme [25]:

$$w(r,t) = \sum_{n=0}^{\infty} \left[ A_{0n} H_2^0(k_1nr) + B_{0n} H_1^0(-k_2nr) \right] e^{i\omega_n t}$$ (2.33)

$$\text{(2.34)}$$

Below the cut-off frequency of the $k_2$ mode, the term $H_1^0(-k_2r)$ is associated with an evanescent wave. The Hankel functions of the first and second kind are denoted as $H_1^0$ and $H_2^0$ respectively.

A forced transverse displacement $w_0(t)$ of a circular area with radius $a < \lambda_{\text{min}} \approx 0.01$ m centered at $r = 0$ is assumed. For the frequencies of interest, we can use the property $\lim_{z \to 0} H_2^0(z) + H_1^0(iz) = 1$, to obtain the amplitude coefficients $A_{0n} = B_{0n} = \hat{w}_{0n}$. This approximation is valid at least up to 100 kHz for the considered plate in Fig. 2.13. Note that $\hat{w}_0(\omega_n)$ is the discrete spectral representation of $w_0(t)$. The resulting flexural wave can be written as:

$$w(r,t) = \sum_{n=0}^{\infty} \hat{w}_0(\omega_n)(H_2^0(k_1nr) + H_1^0(-k_2nr))e^{i\omega_n t}$$ (2.35)

At point $M$ which is a distance $r_{0M} > \lambda_{\text{max}} \approx 0.1$ m away from the initial disturbance, the transverse motion has the following spectral representation:

$$\hat{w}(r_{0M}, \omega) = \hat{w}_0 H_2^0(k_1r_{0M})$$ (2.36)
Chapter 2. Acoustic Emission Localization

Note that the evanescent wave is assumed to have vanished at the measurement location. The TR operation in frequency domain is achieved by taking the complex conjugate of the measured signal

$$\hat{w}_{TR}(r_{0M}, \omega) = \hat{w}_0^* H_0^2(k_1 r_{0M})^*$$  \hspace{1cm} (2.37)

where $^*$ denotes complex conjugation. Enforcing a displacement history according to Eq. 2.37 at the measurement position results in a transverse motion at the initial position $r_0$

$$\hat{w}_{TR}(\omega) = [\hat{w}_0(H_0^2(k_1 r_{0M}))]^* H_0^2(k_1 r_{0M})$$  \hspace{1cm} (2.38)

Using the approximation for Hankel functions for large arguments $H_0^2(z) = \sqrt{\frac{2}{\pi z}} e^{-i(z-\pi/4)}$, we obtain

$$\hat{w}_{TR}(\omega) = \left[ \frac{2}{\pi k_1 r_{0M}} \right]^* \left[ \frac{2}{\pi k_1 r_{0M}} \right] e^{-i(k_1 r_{0M} - \pi/4)}$$  \hspace{1cm} (2.39)

In the spectral representation this equation simplifies to

$$\hat{w}_{TR}(\omega) = \frac{2}{\pi k_1 r_{0M}}$$  \hspace{1cm} (2.40)

and can be again converted into the time domain

$$w_{TR}(t) = \sum_{n=0}^{\infty} \frac{2}{\pi k_1 r_{0M}} e^{i\omega_n t}$$  \hspace{1cm} (2.41)

Note, that the evanescent term associated with the Hankel function of the first kind is lost in equation (2.40) due to $r_{0M}$ being large enough such that the $k_2$ mode vanished. However the phase of the TR signal is still restored due to the point force assumption. This is not always the case, as for example a study of a TR process for Timoshenko beams has shown [17]. Here, the amplitude has decreased due to the circular spread of energy which occurred in the forward and in the TR step. The simple structure of (2.40) lends itself to compensation of the TR process, meaning, that the original shape of the signal can be recovered, if the wavenumber $k_1(\omega)$ and the distance $r_{0M}$ is known a priori. This is demonstrated in Fig. 2.14, in which a TR process is performed by exciting waves with an instrumented hammer, making the excitation force history accessible. In Fig. 2.14 (a) this force history is displayed as a blue line, however converted to a velocity-time signal according to equation:

$$\hat{w}(t) = \frac{1}{8\sqrt{\rho h D_P}} P(t)$$  \hspace{1cm} (2.42)

derived by Doyle [12] using Kirchhoff plate theory. $D_P = E h^3 / (12 (1 - \nu^2))$ is the plate flexural stiffness, $P(t)$ a transverse point force.

According to equation (2.35), the signal $\hat{w}_0(t)$ can be decomposed into a propagating and an evanescent wave mode. This is displayed in Fig. 2.14 (a) as a dashed and a dot-dashed line. The propagating mode is measured at $r_{0M} = 0.2$ m away from the excitation location, resulting in the gray line of Fig. 2.14 (b). In addition, the
reference signal $\dot{w}_0(t)$ is used in a finite element (FE) simulation to reproduce the experiment, resulting in a signal $\dot{w}_{FE}(t)$. Experiment and simulation show good agreement except for very low frequencies. The simulation signal at $r_{0M} = 0.2\, m$ is reversed in time, resulting in a signal $\dot{w}_{TR}(0.2, t)$ given in Fig. 2.14 (c) and subsequently used as excitation signal for a TR simulation. However, since we assume an infinite plate, the simulation is replaced by the analytical expression:

$$w_{TR}(r_{M0}, t) = \sum_{n=0}^{\infty} \hat{w}_{TR}(H_0^2(k_1nr_{M0}) + H_0^1(-k_2nr_{M0}))e^{i\omega_n t}$$

(2.43)

and leads to the velocity signal given in Fig. 2.14 (d). Compared to the original signal, displayed as the gray line $\dot{w}_0(T - t)$, a peak is achieved at the correct time instant but with a 20 times smaller amplitude and the frequency content lacking high frequency content. By transforming $\dot{w}_{TR}$ into frequency domain, multiplying it with the inverse of the bracket term in equation (2.40), and again transforming back to time domain results in the signal shown in (c) and the subsequent TR simulation recovers the original signal, as apparent in subplot (f). When measuring the direct flexural wave in a finite plate, one can recover the original disturbance signal by using a TR process modified with equation (2.40). We call this process infinite MATR, for infinite modified analytical time reversal process in what follows. This process is of relevance for quantitative AE testing, as it allows a quantitative comparison of the amplitude and phase of different wave modes measured at arbitrary locations and moreover the extraction of reflection from the signal.

### 2.2.2.2 General Boundary

The above analyzed TR process assumed the absence of reflections from edges and other boundaries. However, in real AE problems, waves may interact with boundaries before they are measured by a sensor. Reflection of flexural waves at edges have been studied by many authors, one of the first being T. R. Kane [38]. In general, the character of the reflected wave depends on incident angle and type of boundary. Using classical plate theory, the only case at which waves are simply mirrored is the case of a pinned edge. All other cases may involve a phase change of the reflected wave as well as local vibrations and generation of edge waves. Local vibrations and edge waves will not propagate to the sensor and are not captured in the TR process. We assume that it is sufficient to model the TR wave-boundary interaction by assigning an arbitrary phase and amplitude change to the reflected wave. In other words, the reflected wave may be written as a multiplication of the incident wave with a complex transfer function $B_{TF}e^{i\phi_B}$.

Considering a point disturbance at point 0 on a semi infinite plate, flexural waves are generated similar to the case of the infinite plate. At point E, which is a distance $r_{0E} > \lambda_{max}$ away from the disturbance, the wave has reached an edge. The spectral representation of the transverse displacement due to the incident wave at the general edge can be written as

$$\hat{w}_l(r_{0E}, \omega) = \hat{w}_0 \left( \sqrt{\frac{2}{\pi k_1 r_{0E}}} e^{i(k_1 r_{0E} - \pi/4)} \right)$$

(2.44)
again making use of the approximation for Hankel functions for large arguments. The resulting reflected wave at the same location is:

\[ \hat{w}_r(r_{0E}, \omega) = \hat{w}_0 B_{TF} \sqrt{\frac{2}{\pi k_1 r_{0E}}} e^{i(k_1 r_{0E} - \pi/4 + \phi_B)} \] (2.45)

At location M, which is a distance \( r_{EM} \) away of point E, the reflected wave is measured, time reversed, and used to excite waves in a simulation. As a result, in the simulation at the edge location E, the incident time reversed wave is

\[ \hat{w}_{iTR}(r_{ME}, \omega) = \left[ \hat{w}_0^* B_{TF} \sqrt{\frac{2}{\pi k_1 (r_{0E} + r_{EM})}} e^{-i(k_1 (r_{0E} + r_{EM}) - \pi/4 + \phi_B)} \right] \]

\[ \sqrt{\frac{2}{\pi k_1 r_{ME}}} e^{i(k_1 r_{ME} - \pi/4)} \] (2.46)

Again the boundary leads to a reflected TR wave

\[ \hat{w}_{rTR}(r_{ME}, \omega) = \left[ \hat{w}_0^* B_{TF} \sqrt{\frac{2}{\pi k_1 (r_{0E} + r_{EM})}} e^{-i(k_1 (r_{0E} + r_{EM}) - \pi/4 + \phi_B)} \right] \]

\[ B_{TF} \sqrt{\frac{2}{\pi k_1 r_{ME}}} e^{i(k_1 r_{ME} - \pi/4 - \phi_B)} \] (2.47)

At the initial position 0 of the disturbance, the reflected wave has the following form:

\[ \hat{w}_{rTR}(r_{E0}, \omega) = \left[ \hat{w}_0^* B_{TF} \sqrt{\frac{2}{\pi k_1 (r_{0E} + r_{EM})}} e^{-i(k_1 (r_{0E} + r_{EM}) - \pi/4 + \phi_B)} \right] \]

\[ B_{TF} \sqrt{\frac{2}{\pi k_1 r_{ME} + r_{E0}}} e^{i(k_1 (r_{ME} + r_{E0}) - \pi/4 - \phi_B)} \] (2.48)

Apparently, the phase change due to the boundary is recovered after the TR process, but the amplitude change is squared and the signal can be simplified to:

\[ \hat{w}_{rTR}(r_{E0}, \omega) = B^2_{TF} \left[ \frac{\hat{w}_0^*}{\pi k_1 r_{m}} \right] \] (2.49)

This has two consequences. On the one hand, it is not sufficient to only use an infinite axisymmetric TR simulation to detect the origin of the AE source, because waves reflected at boundaries do not have the correct phase at the origin and may interfere destructively with other waves which propagated on a direct path. On the other hand, if the numerical model can reproduce the effect of the boundary, than the phase is recovered in the numerical TR simulation although the effect of the boundary is still recognizable as a change in amplitude.
2.2.3 Acoustic Emission Source Modeling

A number of assumptions are made regarding the mechanical character of an AE source in order to optimize the AE localization method. However, the only necessary requirement is that the AE event is of short duration and triggers flexural waves. Assumptions regarding the actual shape of the AE allow an improved localization accuracy but are not necessary otherwise. The short duration implies a broad frequency spectrum which should contain frequencies in which the flexural wave mode shows dispersive behavior. Holford et al. [35] found that in slender structures, most energy in AE waves is carried by the first symmetric and antisymmetric lamb modes. The breaking of a pencil lead is a well established method for mimicking AE events and testing AE equipment, see ASTM E976 standard. Although different types of AE lead to different mechanical responses, in this study we optimize the technique for pencil lead break induced AE. The loading of the pencil lead is much slower compared to the unloading due to the lead fracturing. Thus the load history of a pencil lead break (PLB) resembles a step function. This assumption is validated by producing a PLB in the center of a quadratic aluminum plate with 2 m side length. The lateral velocity is measured 0.12 m away from the source with a laser interferometer, see Fig. 2.15 (a). Next, the measured signal is used for MATR simulation with compensation of the frequency dependent amplitude loss. At the lead break position, a step-like velocity signal is observed in the simulation, see Fig. 2.15 (b) and confirms the above mentioned assumption. However, the AE localization process requires additional filtering of the measured signal because the finite element simulation has constraints on the frequency bandwidth of the signal. In addition, due to the low propagation velocity of the low frequency components, cutting the tail of the signal reduces the low frequency content of the signal and thus the step like shape is lost. As a result, the modified time reversed signal exhibits a maximum and minimum shape at the signal’s origin, see the gray line in Fig. 2.15 (b). However neither of these maxima indicate the correct time instant of the lead break. A solution to this problem is found by taking the time derivative of the filtered signal, i.e. analyzing the TR problem in the acceleration domain. By doing so, the acceleration signal exhibits a maximum indicating the correct time instant of the AE event. Modeling the AE velocity step signal as half Gaussian, with standard deviation of 5 µs and using such a signal in a numerical simulation gives a very good match with the measured response, see Fig. 2.15 (a) and (d).
Figure 2.14: Numerical validation of a TR process. The excitation signal $\dot{w}_0(t)$, obtained by exciting a plate with an instrumented hammer, is plotted in (a). This signal can be decomposed into an evanescent and a propagating mode. The out of plane velocity is measured 0.2 m away from the excitation and is plotted as a gray line in (b). This signal is also generated synthetically using $\dot{w}_0(t)$ as excitation in a FE model. The synthetic response is displayed as the blue line. The FE calculated response is reversed in time and plotted in (c). Using this signal as excitation condition for the TR-simulation results in the signal $\dot{w}_{TR}(0, t)$, given in subplot (d) as the blue line. A symmetric peak at the correct time instant is achieved, however with an altered frequency content and about 20 times smaller peak amplitude compared to the reference amplitude, plotted as a gray line. By transforming $\dot{w}_{TR}$ into frequency domain, multiplying it with the inverse of the bracket term in equation (2.40) and transforming the result back into time domain results in the signal shown in (e) and therefore the MATR simulation restores the original signal, see Fig. (f).
2.2. One Sensor Acoustic Emission Localization in Finite Plates

Figure 2.15: AE source modeling. Plot (a) shows the velocity response of a PLB 0.12 m away from the source. The signal is applied in an axisymmetric MATR simulation to recover the original source velocity signal of the PLB. Apparently, the PLB produced a step like velocity signal, see the blue line in subplot (b). In the AE localization process, the measured signal is bandpass filtered and the TR simulated velocity source signal does not resemble a step like form anymore but rather looks like the gray line plotted in (b). We modeled the PLB excitation signal as half Gaussian step function, according to plot (c). Using this signal for a FE TR simulation results in a signal given in plot (d), which is quite similar to the signal in (a) apart from the lack of high frequency content.
2.2.4 Procedure

The results from the previous sections suggest the following procedure for the detection and localization of AE using only one sensor.

1. Record out of plane velocity signal $\dot{w}_m(t)$.
2. Bandpass filtering of $\dot{w}_m(t)$ with zero-phase or linear-phase filter.
3. Extract the first wavefront and two additional reflections from the signal.
4. Investigate the acceleration field of the axisymmetric infinite TR simulation to obtain the time instant and the radial distance of the recovered direct wave.
5. Repeat the TR simulation on a numerical 3D model of the test specimen.
6. Analyze the acceleration field of the simulation at the time instant and along a circle determined from the previous infinite plate simulation.

The monitoring system ideally has two channels, a trigger channel and a recording channel. If the trigger channel detects a signal whose amplitude exceeds a predefined level, the recording channel is used to capture a finite section of the out of plane velocity signal. This signal is then filtered with a bandpass with cut-off frequencies 5 kHz and 50 kHz in order to avoid numerical issues in the TR simulation. Care must be taken to ensure that the phase information is not changed by the bandpass filter because the phase relation of the signal carries the information about the wave’s traveled path. To do so, a zero-phase filter [52] is used which ensures that the filtered signal is not phase shifted. A linear-phase filter is also valid since this corresponds to a constant time shift for all frequencies. After the filtering, the signal still contains many reflections and other content which, when applied on a numerical model, lead to a very disturbed wave field in the TR simulation that is rather difficult to interpret. However, the analysis of the TR process for an infinite plate showed, how a signal can be propagated back and forth in time in a numerical simulation. It is therefore possible, to synthetically reverse a flexural wave signal in time, locate the focused wavefront in a time-space diagram, extract the focused wave pulse, again propagate the signal forward to the measurement location and, by iterating these steps, build up a new signal, which is similar to the original signal but contains only a choice of reflections. This technique is illustrated in section 2.2.6.

In addition, the axisymmetric infinite plate simulations indicate the time instant, when the wavefronts converge due to the reversed effect of dispersion and the radial distance between sensor and convergence point. This information is very helpful in analyzing the TR simulation of the exact structure, because it reduces the search domain to a circle and to a small time window. Because simulation and experiment do not match exactly, it is not enough to only analyze the TR simulation at a single time instant.

The origin of the AE event is determined by the highest acceleration magnitude for uniform plates or at the position where most wavefronts intersect for more complex geometries involving internal boundaries. The reason for this is, that internal boundaries influence the energy distribution in the plate. With regard to a specific
sensor position, shadow zones exist behind geometric features, where less wave energy propagates from the sensor position in the TR simulation. If an AE is triggered in the shadow zone, the TR simulation still produces a peak at the correct position, however, other peaks may have larger amplitudes. A more reliable localization is achieved by considering the number of wavefronts that should intersect at the correct position since the number of reflections in the signal is manipulated by the operator.
2.2.5 Implementation

In order to validate the method proposed, a range of laboratory experiments were performed. Pencil lead breaks (Hsu-Nielsen sources) were used to mimic AE events on aluminum plates. The chosen pencil used 0.3 mm 2H leads and the lead was protruded 3 mm before breaking it on the plate. The elastic properties were measured prior to the AE test in order to match the numerical simulation with the experiment. This was achieved by exciting narrow band flexural waves of different central frequencies on the plate and measuring the deflection at several positions. From the Fourier transformed measurements, the dispersion curve of the flexural wave was calculated in both x- and y-direction in order to ascertain isotropy of the plate. The deviation between x- and y-direction phase speed was small and an average Young’s modulus was fitted to the experimentally determined dispersion curve. A comparison of velocity traces between experiment and simulation is given in Fig. 2.14 (b). The measured and fitted dispersion curves together with the material parameters are given in Fig. 2.16.

![Figure 2.16: Flexural wave dispersion curve displayed as phase velocity vs. frequency. Phase velocities were measured in two in-plane directions on the plate, here plotted as red and green crosses together with ± standard deviation. These points were fitted using Mindlin plate theory, resulting in an analytical dispersion curve. Using the resulting material parameters in the FE simulation gives calculated phase velocities presented as black circles.](image)

The experimental setup is illustrated in Fig. 2.17. Elastic waves were excited by breaking a pencil lead on the test specimen. A laser vibrometer was used to measure the out of plane velocity of the plate. The recording system is triggered by the same
2.2. One Sensor Acoustic Emission Localization in Finite Plates

laser vibrometer, but after high pass filtering the signal. This was necessary to avoid preliminary triggering due to the test operator contacting the plate.

![Diagram of experimental setup](image)

Figure 2.17: Illustration of experimental setup. AE were simulated by breaking pencil leads on an aluminum plate. A laser vibrometer was used to measure the out of plane velocity signal. The signal is split to feed the trigger and the data recorder.

For the experimental evaluation of the method presented, two quadratic aluminum plates were used as test specimen, with a thickness of 2 mm and side length 0.75 m. One plate was uniform, the other nonuniform plate included a rectangular cut-out at the center of the plate, with dimensions \( L_x \times L_y = 20 \text{ mm} \times 40 \text{ mm} \). The coordinates of the sensor positions and the AE locations are given in Table 2.4, the origin of the coordinate system is at the center of the plate.

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<thead>
<tr>
<th>Uniform Plate</th>
<th>x</th>
<th>y</th>
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<td>Sensor</td>
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<td>-155</td>
</tr>
<tr>
<td>AE 1</td>
<td>-200</td>
<td>150</td>
</tr>
<tr>
<td>AE 2</td>
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<td>200</td>
</tr>
<tr>
<td>AE 3</td>
<td>300</td>
<td>-250</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>Nonuniform Plate</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>AE 3</td>
<td>-10</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.4: Table of sensor and AE location coordinates (mm) with respect to the center of the plates
2.2.5.1 Numerical Setup

Two types of numerical simulations were used. As described in section 2.2.2.1, the axisymmetric infinite plate simulations were done using the closed form solution of Mindlin plate theory. The FE simulations were done with ABAQUS explicit solver. The model was meshed with $2\,\text{mm} \times 2\,\text{mm} \times 1\,\text{mm}$ C3D8I elements and a fixed time increment of $1\times10^{-7}\,\text{s}$. Prior to the FE TR simulation, the measurement signal was bandpass filtered with cut-off frequencies of $5\,\text{kHz}$ and $50\,\text{kHz}$. The model was excited by prescribing the processed velocity signal for the out-of-plane direction of the node at the measurement position. Out-of-plane acceleration for the top surface nodes was used for the analysis of the AE location.

2.2.6 Results and Discussion

The different steps of the method presented are listed in section 2.2.4. The method is demonstrated by measuring AE’s at position AE 3 for the case of on an uniform plate. The AE is produced by breaking a pencil lead on the surface of the plate. The recorded signal is low-pass filtered and used in an axisymmetric infinite plate TR simulation to identify the various wavefronts in the measured signal. This is done in a time-space diagram similar to Fig. 2.19, where one axis is given by the radial distance outgoing from the measurement location, the other axis displays time. Due to the reversed dispersion effect, these wavefronts converge to a maximum at the distance corresponding to the traveled source-sensor ray distance in the experiment. This ray path may include an arbitrary number of reflections. However, these converged wavefronts can all be ascribed to the same AE event, if they converge at equal time instants. At the convergence position, the wave has minimal temporal and spatial extent, which allows convenient extraction of an individual wave pulse from the signal. The extracted wave is sent back to the measurement position in an additional forward simulation. This process can be repeated for an arbitrary number of reflections to build up a new signal which only contains the choice of reflections. Fig. 2.18 shows the first three wavefronts, after they were extracted at their convergence location and sent back to the measurement position. In Fig. 2.18 (d), the three dispersed wavefronts are added together and compared to the original signal measured. The absence of later reflections and the particular character of the flexural dispersion relation, by which higher frequencies propagate faster than lower frequencies, explain why the synthetic signal lacks more and more detail at later time points.

The new signal, $p_1 + p_2 + p_3$ is again applied in an infinite axisymmetric TR simulation and the three wavefronts are clearly visible in the time-space diagram, see Fig. 2.19 (a). The time instants and the radial distances of the converged waves can be extracted from this diagram. In Fig. 2.19 (b), the peak at $x = 0.136\,\text{m}$ indicates the convergence point of the direct wave, which traveled a distance of $0.136\,\text{m}$ from the AE location to the measurement location. The radial distance between measurement point and AE source can therefore already be identified, assuming a direct path between source and sensor exist. The other reflections converge at time instants quite close to the direct wave. The reflections undergo phase changes at the edge, which obviously are not modeled in the infinite model and therefore the
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Figure 2.18: Plot (a), (b) and (c) shows the extraction of the first three wavefronts \( p_1 \), \( p_2 \), and \( p_3 \) by iteratively forward and backward simulation of the measurement signal. In plot (d), the measurement signal is compared to the summation of \( p_1 \), \( p_2 \), and \( p_3 \).

particular convergence time differs slightly. Otherwise, the source location could already be determined geometrically by drawing and folding circles into the specimens contour, see Fig. 2.20. The radii of these circles are defined by the distance between sensor and converged wavefronts. In the TR process, reflections act as additional sensors, mirrored at the boundaries of the structure. This effect together with the effect of reversed dispersion allows the 2D localization of the AE source. This effect is reported in detail by Park et al. [55]. The interpretation of the full 3D TR-simulation is greatly facilitated by incorporating the axisymmetric simulation into the 3D simulation results. In the 3D simulation, the individual wavefronts become apparent only for a short moment, when the different frequency components are in phase, which again is helpful in determining the correct time instant.
Figure 2.19: (a) TR, infinite axisymmetric plate simulation, displayed in a time-space diagram. The color tone is a measure for the acceleration amplitude. (b) Extracted vertical trace from the time-space diagram, showing the acceleration amplitude at $t = 2.366$ ms along the radial coordinate. (c) Extracted horizontal trace, showing the acceleration amplitude at $r = 0.136$ m for the whole time window.
Figure 2.20: Circular wavefronts of the direct $p_1$ and the reflected wavefronts $p_2$, $p_3$ at the time instant by which the waves have converged. Note the mirroring effect of the boundaries, and how well the drawing resembles the 3D simulation field shown in Fig. 2.21c.
2.2.6.1 Uniform Plate

The results of the TR simulations are given by means of screen-shots of the acceleration amplitude field of the plate. The time instant at which the screen-shot is taken is chosen from the axisymmetric simulations, but slightly adapted to capture the maximum amplitude at the convergence point. In the screen-shots, the measurement location, which is the excitation location in the TR simulation, is indicated by a white circle. The true position, where the AE was triggered in the experiment, is indicated by a white X.

Figure 2.21: Screenshots of the acceleration amplitude for the uniform plate AE tests. Measurement position is indicated by a white circle, true position of AE is indicated by a dashed X.

All AE locations are unambiguously identifiable by finding the maximal amplitude along the circular wavefront of the direct source-sensor wave. The radius of this wavefront is obtained by the preceding axisymmetric TR simulation. With no information about where to look in the 3D TR simulation, the AE location is not always unambiguously determined by the highest amplitude in the simulated acceleration field. The interference of different wavefronts lead to various other peaks and therefore the largest amplitude does not necessarily indicate the correct location. However, since we included only a specified number of reflections in the manipulated measurement signal, namely $p_1 + p_2 + p_3$, see subsection 2.2.4 and Fig. 2.18, the AE position is further indicated by the intersection of 3 wavefronts. The various circular wavefronts are clearly recognizable. Illustration 2.20 demonstrates how the individual circular wavefronts can be folded into the structure. As discussed before, the radii of these fronts are determinable from the axisymmetric TR simulation, up to a phase error due to missing boundary reflections and a small discrepancy between experiment and FE simulation. Unlike the axisymmetric simulation, the phase change at the boundary is compensated in the 3D simulation as discussed in subsection 2.2.2.2. With the information of the radii of the wavefronts, a geometrical drawing of the wavefronts can be obtained, which facilitates interpretation of Fig. 2.21 (c). In fact, for simple geometries like the presented quadratic plates, the origin of the AE can often be constructed from the axisymmetric simulation alone.
2.2.6.2 Nonuniform Plate

In the above section, it was found that the direct wave path is important to identify, with the help of the axisymmetric simulation, the radius of the circle on which the AE location must be expected. What if the direct source-sensor path is disturbed by a geometric feature? This question motivated further tests of nonuniform plate, where no direct path exists between sensor and AE event. The line between AE and measurement location is disrupted by a rectangular cut-out, that refracts the incident wave before it is captured by the measurement system. The results of these tests are again displayed as screen-shots in Fig. 2.22.

Figure 2.22: Screen-shots of the acceleration amplitude for the nonuniform plate AE tests. Measurement position is indicated by a white circle, true position of AE is indicated by a dashed X.

Due to the large wavelength-to-cut-out ratio, the wave approaching the cut-out, is bent around and captured by the measurement system as an almost direct wave. As a result, source AE 1 can correctly be identified by finding the highest magnitude along the direct wave radius, see Fig. 2.22 (a).

In Fig. 2.22 (b), an AE was triggered closer to the cut-out and, while still exhibiting a maximum close to the true position, the TR acceleration field displays additional peaks along the circular direct wavefront, which in cases are of even higher amplitude. It is not possible anymore to unambiguously locate the AE by only considering the amplitude. However, by considering also the intersections of the circular wavefronts, knowing that three wavefronts were extracted from the measurement signal, the true AE position is again identifiable.

An additional test was performed by exciting an AE at the edge of the cut-out, see Fig. 2.22 (c). While the correct radial distance again is implied by the circular wavefront, no dominant peak is observable at the true location. But by considering the most prominent wavefronts, the true AE position may still be discovered. One reason for the absence of a clear peak at the cut-out is that in the vicinity of the feature, local oscillations interfere with the incoming waves. This effect is not compensated in the presented one sensor TR process. An additional effect is related to the measurement position with regard to the boundaries of the structure. The measurement signal is the result of oscillations due to several propagating waves. By
using the signal in a subsequent simulation, the information about the propagation
direction is lost, and leads to additional wave peaks in the wave field. These peaks
are artifacts and may affect the interpretation of the TR wave field. Additional
scatterers, as for example the cut-out in the present plate, can increase the number
of artifacts. This effect is illustrated in Fig. 2.23 for a 1D semi-infinite structure
with an AE event and a scatterer.

Figure 2.23: Schematic of a TR process of a semi-infinite 1D structure with a scat-
terer. In this configuration, one AE event is measured as 4 individual
waves due to reflection at the boundary and reflection at a scatterer.
However, in the TR-simulation, 8 waves propagate in the structure,
and another 2 are introduced by the scatterer. Due to dispersion, all
waves show a convergence at the same time, but only those 4 which
travel along the original path converge at the right position. The other
6 waves produce artificial peaks.

The AE event triggers left and right-going waves which propagate in two di-
rections. Due to the boundary close to the measurement position, a total of four
waves are measured. Applying the time reversed signal in a simulation, four waves
propagate in the original direction, another four in the reverse direction. Due to
dispersion, the waves recover their original shape after a certain distance accord-
ing to the distance traveled in the experiment. As a result, all eight waves recover
their shape at the same time but only four at the original position. The other four
produce artifacts symmetrically before and after the true position. However the
waves interacting with the scatterer produce again two artifacts and interpretation
of the wavefield becomes challenging. Depending on the location of the AE and the
sensor-to-boundary distance, these artifacts may even interfere destructively with
the waves at the correct position. The mechanisms in a 2D structure are the same
but interpretation can be even more confusing. Additional scatterers in the sys-

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2.2. One Sensor Acoustic Emission Localization in Finite Plates

and then transforming the data to the frequency-wavenumber domain [62]. Here a simulated experiment with only one measurement point is presented. The stiffener is a 750 mm × 1.6 mm × 0.4 mm stripe made from the same material and welded to the plate. Two measurement locations were simulated, one within the plate, one at the edge. In Fig. 2.24 (a) and (b) screenshots of the TR velocity field are displayed. The measurement positions were within the plate and at the edge of the plate for plot (a) and (b), respectively. As illustrated in Fig. 2.23, more artifacts were produced when measuring within the plate as compared to measuring at the edge of the plate. However, in contrast to the 1D case, measuring only at one point at the edge does not completely prevent the formation of additional wavefronts. Though interpretation of Fig. 2.24 (b) is easier than Fig. 2.24 (a), the true AE position is found in both cases. When comparing this result to results reported using the frequency-wavenumber approach [62], one must be aware that here, full knowledge of the properties of the test specimen is required, while the results in [62] do not require a model of the structure. This is characteristic for most NDT problems which either need an exact model and few measurements or many measurements and no model.

As seen in Fig. 2.24, the positioning of the sensor is important. It has been observed that symmetries in the wave field emerge, if the sensor is positioned at symmetry lines with respect to the structure’s edges. For example the square plate used in the validation tests exhibit four symmetry lines. If the measurement location is set to the midpoint of a square plate, four focal points will occur in the TR wave field and it is not clear, which of them is the correct position.

Figure 2.24: Screenshots of the velocity amplitude for several numerical plate AE simulations. Measurement position is indicated by a white circle, true position of AE is indicated by a dashed X. Both, initial AE event and TR process were simulated numerically, the AE event was modeled as a gaussian velocity peak. Therefore the TR process is analyzed in the velocity domain.

In Fig. 2.24 (c), the measurement position is at a horizontal symmetry line, and the TR simulation indicates four locations for the AE. While the left two peaks are obvious, the edge on the right hand side mirrors the wave field and as a result an
additional two peaks develop. By choosing a measurement location away from the symmetry lines, unambiguous localization of the AE is possible.

In terms of geometry of plates, so far only square plates were investigated in the present validation study. Localizing AE’s in other plate geometries is assumed to work as well and in general, the fewer symmetries, the better the localization performance. Internal boundaries or features may be more problematic and the localization performance depends on several factors such as wavelength-to-feature-size, energy distribution in the plate and quality of the numerical model. As seen in Fig. 2.23 scatterers in the plate disturb the overall wavefield, leading to artificial maxima. In order to recover the original displacement field in a plate, a closed line of transducers around the plates’ edge would be required. Referring to Huygens principle, the recovered TR displacement field would be established by the superposition of all circular waves emanating from the individual transducers. By exciting only at one point however, the overall displacement field lacks the contribution of the other superposing wave components. The best possible result achievable in the one sensor configuration is that the waves which were propagating from AE to sensor again reach the AE origin with recovered phase and along the same wave paths. However, at every other position, the wave field in the TR simulation and in the experiment will be different.

Energy transfer between AE source and sensor may become an issue for certain plate geometries. While a local maximal amplitude is expected even in complex plate shapes, other maxima may be more pronounced if the energy flux to the true AE position is low. In such situations, more sensors are required or the measuring position must be optimized.

### 2.2.6.3 Noise

When AE systems are used as monitoring systems for structures in service, noise in the measurement signal is often a concern. As pointed out by Kundu [43], arrival time determination is problematic in noisy signals and false arrival time picks are a main source of errors in AE localization.

The performance of the method proposed is therefore also validated for signals with increased noise content. The reference signal was produced by a numerical simulation and is assumed to be noise free. White noise with half the root mean square amplitude of the reference signal was used to corrupt the reference signal. Both signals are plotted in Fig. 2.25 (c). Performing the same steps as outlined in section 2.2.4, the TR FE simulations lead to the results shown in Fig. 2.25.

Apparently, the AE detection quality is hardly impaired by the noise. While the prediction of an arrival time in the 7 dB corrupted signal in Fig. 2.25 (c) seems to be difficult, the localization accuracy and uniqueness in the TR FE simulation is almost identical to the reference simulation. This has two main reasons. Prior to the 3D FE simulation, the measurement signal is modified by a number of axisymmetric infinite TR simulations in order to extract individual wave pulses and to identify the radial distance and the time instant of the AE origin. In these simulations, the signal phase relation is modified according to the flexural dispersion relation of the plate. Therefore, signal content, with a phase relation according to the flexural mode, is
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Figure 2.25: Screenshots of the acceleration amplitude for the reference and the noise simulation. Measurement position is indicated by a white circle, true position of AE is indicated by a X. The source signals are presented on the right hand side.

accumulated at the original location of the AE, whereas signal content with arbitrary phase relation is further dispersed. The TR process has therefore a filtering behavior which conserves the simulated wave mode and filters out other signal content, see [17] for discussion of the filter effect in a 1D TR process. In addition, by building up a new signal from back propagated extracts of the converged wave signal, a substantial amount of noise is further eliminated, because the converged wave pulse has minimal temporal and spatial width. However, if the noise level is too high, i.e. above approximately 0 dB, the extraction process may fail completely and the subsequent FE simulation gives no meaningful result anymore.

2.2.7 Conclusion

A novel technique for AE localization in a 2D structure is presented and validated for a number of AE experiments. In case of an uniform isotropic plate, it was shown that high accuracy localization was possible using only one sensor. Since the method presented does not use arrival time estimates, many problems related to arrival time determination are completely avoided. Traditional modal AE (MAE) methods struggle with the accurate time of arrival determination of dispersive wave modes and time-frequency transformations have typically insufficient time resolution for low frequency signals. The method presented was shown to perform very robustly under significant noise because, amongst other effects, the complete waveform is used in the analysis instead of only a single feature in the signal. Since the method only requires the measurement of one mode, the sensor system can be optimized for this particular mode and mode separation is not necessary. Other single sensor MAE methods [69, 74, 87], which can only be used to determine the direct radial distance between source and sensor, require measurement and separation of two modes which is difficult using only one sensor. In the TR process, the original waveform is recovered at the AE location. This allows better interpretation and comparison of the damage process as compared to traditional analysis of parameters.
such as rise time, ring down counts or maximal amplitude. Since the geometry is incorporated in the TR model, also complex geometries can be tested. However, using only a single sensor, structural features and non-uniformities in the specimen may compromise accuracy and uniqueness of the predicted AE source location. In that case, it is suggested to use additional sensors, or optimize the sensor location. It was shown, that care must be taken when choosing the sensor position in order to avoid symmetries arising in the wave field due to the boundaries of the plate. This prevents an unambiguous solution to the AE location problem. A main limitation of the method is the availability of an accurate and efficient numerical model of the specimen. Computation time to solve 3 ms of a $10^7$ degrees of freedom model using commercial FE software was about 6 hours on a conventional PC system. However, more efficient simulation approaches exist, i.e. spectral element approaches \cite{12, 28} which could reduce computation time to fractions of a minute, since only flexural waves must be modeled. Another approach worth testing is to first evaluate the individual propagation distances of the incident waves and then geometrically construct the origin of the AE wave. This would allow the localization of AE’s without expensive FE simulation and has been shown in Fig. 2.20. However, the effect of boundaries on the phase of the reflected waves must then be incorporated separately. Finally, the measurement system should capture the phase of the wave signal without influencing it, because the information of the propagation distance is contained in the phase of the signal. 

If these requirements are fulfilled, the method presented provides a number of advantages as compared to other currently available AE localization methods. To summarize, these features are:

- One point measurement suffices.
- Capability to test complex geometries with indirect wave paths.
- Improved robustness in low SNR situations.
3 Guided Wave Testing in Beams
Chapter 3. Guided Wave Testing in Beams

3.1 Quantitative Guided Wave Testing by Applying the Time Reversal Principle on Dispersive Waves in Beams

Ernst, R., & Dual, J.
Institute of Mechanical Systems, Swiss Federal Institute of Technology, ETH
Zurich, Switzerland

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Abstract A method for the localization and characterization of defects in waveguide-like structures is presented in this paper. In contrast to traditional ultrasound and guided wave techniques, a broadband signal is used to enforce strong dispersion of the flexural wave mode. Since dispersion is well compensated in a time reversal experiment we use a time reversal numerical simulation to identify the origin and the original shape of the flexural wave excited at a local non-uniformity due to mode conversion. Limitations of the time reversal process for broadband signals due to multimode and evanescent behavior of guided waves are discussed and eliminated using a Timoshenko beam model. The resulting novel process which uses both flexural and longitudinal wave information allows detection, localization and size estimation of several defects in a beam with only a single measurement. The method proposed is experimentally validated on rectangular solid beams and cylindrical hollow beams with notches of different sizes and positions. Up to three notches could be localized from one measurement, with a maximum error of 3% with respect to the propagation distance. The size was accurately predicted for notches as small as 0.5 mm depth or 8.3% of the cross section, using a generalized spring model of a crack.
3.1. Quantitative Guided Wave Testing by Applying the Time Reversal Principle on Dispersive Waves in Beams

3.1.1 Introduction

For almost three decades, researchers have investigated the use of guided waves for nondestructive testing and structural health monitoring. Many applications have found their way to industry. The reason for this demand is that guided waves are more convenient for large beam, plate and cylindrical structures due to their ability to interrogate larger distances compared to ultrasonic testing. A recent review on the use and advantages of guided wave techniques has been given by Rose [60].

Advantages of using guided waves come at the price of a more complex wave propagation behavior, i.e. dispersion and multimode propagation [4]. The former refers to the fact that the propagation speed of guided waves in general is a function of frequency. As a consequence, a wave pulse consisting of many frequencies distorts during propagation. It is clear that the determination of exact arrival times is more difficult for dispersed waves. Multimode propagation means, depending on the excitation frequency, that more than one wave mode is generated. This is also observed for waves interacting with boundaries and non-homogeneities, e.g. a wave hits a crack and is split into different modes. This complicates the interpretation of the measured signal. Finally, the combination of multimode propagation and dispersion makes it difficult to extract a single wave mode from the signal, because the signal consists of different dispersed modes and reflections which overlap each other. This poses a problem for quantitative nondestructive testing, where one tries to determine the size of a crack from amplitude ratios of wave modes.

All these challenges have long been known and researchers have developed ways to overcome them. Dispersion is generally minimized by working with narrowband signals. The determination of arrival times of dispersed modes is achieved in the time-frequency plane, the transformation done with fast Fourier transformation (FFT) or wavelet analysis, e.g. Kishimoto et al. [41]. However, this has the drawback that frequency precision comes at the cost of time precision due to the uncertainty principle or Gabor-limit, which states, that it is impossible to exactly localize a signal in both frequency and time domain [26]. The extraction of a single mode from a multimode signal is achieved by measuring at multiple locations and then transferring the measured data from time-space domain into frequency-wavenumber domain, see i.e. Alleyne et al. [3] or Hayashi et al. [32] or by using linear prediction algorithms, see Vollmann et al. [79]. Good damage characterization results in beams were also reported in [7] using a damage index formulation. The use of dispersion as an indicator for the distance a wave has traveled has already been reported by Wilcox et al. [82], however not in combination with a time reversal (TR) process. The use of a broadband signal in combination with a TR experiment on plates was reported by Gangadharan [27]. It was reported, that, while the method cannot be used as a baseline free technique, an increase in signal to noise ratio and an improvement in spatial resolution was observed. The combination of an experiment with a subsequent numerical TR simulation for cylindrical structures was reported by Leutenegger et al. [45]. The method requires the measurement of the full 3D wave field at an end-face of the structure and allowed localization of single defects at arbitrary location and orientation.

Here, a method originally developed for acoustic emission testing in beams [17] is implemented in a guided wave setting. The method bypasses the typical difficulties
found in guided wave testing without the use of frequency transformations or linear prediction. In the cited acoustic emission implementation, only active damage zones are detectable, the method is based on only one wave mode and the character of the waves cannot be influenced. In the present application, an actively excited wave is used to quantitatively inspect the structure for cracks and determine their size. In contrast to acoustic emission localization, this is done by evaluating mode conversion between longitudinal and flexural wave modes. The method makes use of the TR principle that has been extensively studied by Fink et al. [23]. The main difference of the present method with regard to most other guided wave testing methods is that dispersion is not avoided, but builds a main cornerstone in the process. Dispersion is used as a measure for the distance the wave has traveled. The key to resolve this information is a TR numerical simulation. As will be shown, using flexural and longitudinal waves as working modes and a Gaussian shaped excitation signal, the TR simulation not only facilitates the localization of multiple defects from only a single measurement but also allows the separation of a wave mode from a single dispersive signal with multiple reflections overlapping in the measured signal. So far, the separation of multiple wave modes which overlap in the time domain required several measurements at different positions. Transforming the measured data into frequency-wavenumber domain then separates e.g. incident and reflected waves [61]. Having isolated longitudinal and flexural waves from the signal, the depth of cracks can be calculated by comparing the mode conversion ratio of longitudinal and flexural waves with an analytical model of the crack based on stress intensity factors. Effects of evanescent modes, choice of measurement location, noise, and processing of the measured signal on the TR process are discussed. Using the results of the TR process analysis in Timoshenko beams in [17], two equations are derived to compensate for evanescent waves and multimode propagation effects that otherwise break the TR process. With the help of these findings, a method is proposed for the localization and characterization of several cracks in a beam with only a single, one-point measurement of the lateral displacement component. The method is validated with experiments on aluminum beams with different notch locations and sizes.

The motivation for investigating TR concepts on guided waves in one dimensional structures are twofold. On one hand, there are many applications of beam-like structures such as rails, rotor blades, and general truss structures. On the other hand, by restricting the study to a simple one dimensional structure, it is hoped that a better and more intuitive understanding of the physics behind a guided wave TR experiment is achieved. Non-propagating modes, also known as evanescent modes and multimode propagation strongly influence the TR process. These effects occur not only in beam-like structures but also in cylindrical or plate-like structures. However, due to additional complexities such as 2D and 3D effects and higher order modes, the TR process may be more difficult to study in these structures.

In terms of applicability, the method presented may as well be considered for more complex structures since the first symmetric and antisymmetric mode of motion behave very similar at low frequency-thickness numbers.
3.1.2 Presentation of Method

In order to test the integrity of a structure, a two-step procedure is suggested. In a first step, a guided wave test is performed on the structure of interest. A broadband longitudinal wave pulse in the form of a Gaussian function is excited into the structure. As the wave hits one or several defects, partial mode conversion from the nearly dispersion-free longitudinal mode to a dispersive flexural mode occurs. The transverse displacement history is recorded by means of a laser interferometer. After compensating for certain TR effects, presented in section 3.1.2.1 we proceed with the second step. A numerical simulation is set up in which the modified and time reversed transverse displacement history is set as the boundary condition of the structure modeled. Note, that the structure is modeled in its pristine state. Since dispersion is well compensated in a TR process [54], the flexural wave pulse converges while propagating toward it’s origin and reaches maximal amplitude and minimal duration at the location of the defect. Therefore, by considering only dispersion effects, the method presented is capable of detecting and localizing defects without time-of-flight measurements or prior obtained baseline data and accounts for geometric and material variations in the structure [17]. Subsequently, the size of a crack can be determined by evaluating the mode conversion ratio of the excited flexural wave and the transmitted longitudinal wave. The non-dispersive longitudinal wave can be extracted right from the measured signal because the pulse remains compact and propagates with the highest group velocity. The flexural wave can then be extracted with the help of the TR simulation because the time reversed flexural wave has minimal duration at it’s origin.

The present numerical TR process intentionally simulates flexural waves exclusively. This is a major difference to the experimental TR process discussed in [54], by which the measurement and the TR excitation is done on the same structure using the same transducers. The TR process presented also differs from the numerical TR process presented in [45], by which the TR simulation mimics the dynamics present in the experiment. The approach in [45] requires the measurement of the complete displacement field and is therefore experimentally more expensive. By simulating only flexural waves in the TR simulation, two advantages emerge. First of all, a one point unidirectional measurement suffices because the relevant information for the crack localization is contained in the phase of the dispersed flexural wave. The other advantage is that only information which in the experiment propagated as flexural wave is recovered in the simulation. Other information, although recorded by the measurement system, immediately disperses in the TR simulation. As will be seen in section 3.1.2.1 this results in a filtering effect because noise and other wave modes have decreased amplitudes in the TR simulation result.

3.1.2.1 Time Reversal Process for a Timoshenko Beam

The method presented uses results derived from a TR process in Timoshenko beams [17]. In the present paper only the problem setting and the results are presented. Further details are given in [17]. A Timoshenko beam with properties given in table 3.1 is considered. A transverse disturbance gives rise to two modes, described by the wave numbers $k_1$ and $k_2$. The wave mode associated with wave number $k_1$ will
be referred to as flexural wave. The wave mode associated with wave number \( k_1 \) will be referred to as thickness-shear mode, according to [49]. These wave numbers are plotted in Fig. 3.1 for frequencies up to 500 kHz. The second mode \( k_2 \) shows an interesting feature in that it exhibits a cut-off frequency, below which the wave number is purely imaginary. While wave modes with real wave numbers correspond to propagating waves. Wave modes with purely imaginary wave numbers are called 'evanescent' waves and correspond to exponentially decaying vibrations. This behavior will be of importance in the following sections.

Figure 3.1: Dispersion diagram showing the wavenumbers \( k_1 \) and \( k_2 \) as a function of frequency according to Timoshenko theory for the investigated aluminium beam with a square cross section of 6 mm side length. The second mode has a cut-off frequency of 280 kHz. The first mode, \( k_1 \), will be referred to as flexural wave. The second wave mode, \( k_2 \), will be referred to as thickness-shear mode.

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<th>Specimen</th>
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<th>Pipe</th>
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</tr>
<tr>
<td>Timoshenko coefficient</td>
<td>( \kappa ) 0.985</td>
<td>0.5337</td>
</tr>
</tbody>
</table>

Table 3.1: Material parameters used in the numerical simulations.

An infinite beam for which \( x \) describes the axial position, \( y \) the transverse deflection, and \( \psi \) the rotational degree of freedom is considered. We restrict the method to harmonic waves and suppress the time dependence \( e^{i\omega t} \), where \( i \) and \( \omega \) are the imaginary unit and the angular frequency, respectively.

The general transverse displacement due to waves propagating in the positive \( x \)-direction for a Timoshenko beam can be written as

\[
y(x) = R_1 A_0 e^{-ik_1 x} + R_2 B_0 e^{-ik_2 x} \\
\psi(x) = A_0 e^{-ik_1 x} + B_0 e^{-ik_2 x}
\]  

(3.1)  

(3.2)
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where \( A_0 \) and \( B_0 \) are the amplitude coefficients and \( R_i \) are the amplitude ratios in the form given by Doyle [12]

\[
R_i = \frac{ik_i G A_k}{G A_k k_i^2 - \rho A \omega^2} \quad (3.3)
\]

where \( G \) is the shear modulus, \( A \) the cross sectional area, \( \kappa \) the Timoshenko coefficient and \( \rho \) the mass density of the beam. Suppose, a transverse, point like disturbance in the form of \( 2y_0(t) \) excites flexural waves at \( x = 0 \).

The rotational degree of freedom \( \psi_0 \) is assumed to be zero at \( x = 0 \) because of symmetry considerations and the assumption of a point like disturbance. The frequency content of \( y_0(t) \) shall be limited to 280 kHz, i.e. below the cut-off frequency of the shear thickness mode. Assuming the measured deflection signal \( y_m(t) \) is time reversed and subsequently used to define the transverse motion at \( x_L \), while the rotational degree of freedom is fixed, what would be the response \( y_0^{TR} \) at the original position \( x_0 \)? The answer is that the TR response at \( x = 0 \) is not anymore equal to the original signal. Multimode propagation and the evanescent character of the second mode causes this behavior. Because the second mode is lost in both the forward and the TR step, we refer to this effect as evanescent mode leakage. The splitting of the excitation signal into different modes depends on the frequency content and the boundary conditions during excitation. If these conditions are known, the following equation compensate for these effects in the case the response is measured and again excited within the ends of the structure:

\[
H_{within} = \frac{y_0}{y_0^{TR}} = \frac{R_1 - R_2}{R_1^2} (R_1 + R_2^*) \quad (3.4)
\]

where \( * \) denotes complex conjugate. This manipulation will be referred to as within processing.

By measuring deflections at only one point on a beam, the propagation direction of the wave remains unknown. One may measure at a free end position to circumvent this problem. This however complicates the measured signal in that it consists not only of the incident wave but also of a reflected propagating wave and an evanescent wave. By considering the boundary condition of the free end, the incident flexural wave can be extracted from the measured wave with the following equations [17]:

\[
A_0 = \frac{\tilde{y}_m (q + k_1 k_2 m)(k_1 - k_2)}{2(q - k_1^2 m)(R_1 k_2 - R_2 k_1)} \quad (3.5)
\]

\[
q = \rho I \omega^2 \quad (3.6)
\]

\[
m = EI; \quad (3.7)
\]

where \( I \) is the area moment of inertia. Having the amplitude coefficient \( A_0 \) isolated from the total displacement at the free end, the same procedure is applied as for the measurement within the beam. Obviously, other boundary conditions than free end can be implemented. The TR signal at the original position is recovered if
the measured signal is manipulated with the following equation:

\[
H_{\text{free}} = \frac{y_0}{y_{0TR}} = R_1 \frac{(q + k_1 k_2 m)(k_1 - k_2)}{(2(q - k_1^2 m)(R_1 k_2 - R_2 k_1))} \frac{R_1 - R_2}{R_1^2} (R_1 + R_2^*) \tag{3.8}
\]

where the first part extracts the incident wave at the free end and the second part is similar to the transfer function for within processing. This manipulation will be referred to as \textit{free end processing}.

In the following section, we want to investigate the physical meaning of Eq. (3.4) and Eq. (3.8). This will be done by discussing numerical simulations of a TR process on an isotropic beam. Spectral elements are used because they allow the simulation of semi-infinite structures which is very convenient for the analysis of the flexural wave mode. Further, the simulation of the flexural wave dynamics is based on Timoshenko theory with the advantage that accuracy and limitations of this approach are well known and not dependent on choice of discretization and element type. As an example, Fig. 3.2 illustrates the notation for a typical TR process. Beginning with Fig. 3.2 (a), a transverse disturbance \(y_0\) with zero rotation \(\psi\) is assumed to excite flexural waves at \(x = 0\) within a semi-infinite beam. In Fig. 3.2 (b), the transverse deflection \(y_m\) is measured at the free end of the beam. The rotational component \(\psi\) is not recorded, however it could be calculated with the assumption of vanishing forces and moments at the free end. In Fig. 3.2 (c) the TR step is performed by assigning the time reversed deflection history \(y_{mTR}\) without further processing to the end of the beam at \(x = x_m\). In the case shown in Fig. 3.2 (c), the rotational component \(\psi\) is arbitrarily set to zero, as denoted with \(\psi = 0\). This is equivalent to loading the end of the beam with a certain bending moment. In Fig. 3.2 (d), the deflection \(y_{0TR}\) at \(x = x_0\) is measured and compared with the original deflection \(y_0\). In an ideal TR process, \(y_{0TR}\) and \(y_0\) match perfectly.

![Figure 3.2: Illustration of a TR process on a semi-infinite beam. In (a), \(y_0\)excites waves at \(x = 0\) in the beam. In (b), the transverse deflection at the free end is recorded. In (c), \(y_m\)is reversed in time and set as the boundary condition for a subsequent simulation. In (d), the deflection is measured at \(x = 0\). In an ideal TR process, \(y_0 = y_{0TR}\).](image-url)
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Multimode and Free End Effects  By comparing the original signal \( y_0 \) (blue) with the time reversed signal \( y^{TR}_0 \) (green) in Fig. 3.3, different configurations of the TR process can be compared and evaluated. All traces are computed from spectral element simulations using Timoshenko theory. The shape of the excitation signal \( y_0(t) \) is chosen to be a scaled Gaussian function:

\[
y_0(t) = \alpha e^{-\frac{1}{2} \left( \frac{t-t_0}{\sigma} \right)^2} \tag{3.9}
\]

where \( \alpha \) scales the signal amplitude and \( \sigma = T_1^4 = 2.5 \mu s \) the duration of the pulse. This leads to a pulse duration \( T_1 \) of 10 \( \mu s \). The choice of a Gaussian function is motivated by the property, that the Fourier transform of a Gaussian function again is a Gaussian function. Therefore, the amplitude spectrum of \( y_0(t) \) is given by:

\[
\tilde{y}_0(f) = \tilde{\alpha} e^{-\frac{1}{2} \left( \frac{f}{\tilde{\sigma}} \right)^2} \tag{3.10}
\]

\[
\tilde{\sigma} = \frac{1}{2\pi\sigma} \tag{3.11}
\]

In other words, the amplitude spectrum is like a scaled normal distribution with a mean \( \mu = 0 \) and a standard deviation of \( \tilde{\sigma} \simeq 64 \text{kHz} \).

Fig. 3.3 (a) and Fig. 3.3 (b) show \( y^{TR}_0 \) after manipulating the measured signal with the free end processing. The "-" denotes "manipulated value". Fig. 3.3 (c) and Fig. 3.3 (d) show \( y^{TR}_0 \) without free end processing. In Fig. 3.3 (a), the rotational component \( \psi \) during TR excitation was set to zero. This reflects the assumption made in the derivations for Eq. 3.8. As a result, the time reversed signal fits perfectly the original excitation. In Fig. 3.3 (b), the boundary condition in the TR step is changed from \( \psi = 0 \) to \( M = 0 \). This is different from the assumptions in Eq. 3.8 and as a result, the time reversed deflection at \( x = x_0 \) is not congruent with the original excitation. In Fig. 3.3 (c), the rotational component in the boundary condition is set to zero in the TR step. This results in a strong deviation of the time reversed trace \( y^{TR}_0 \) compared to \( y_0 \). Finally, in Fig. 3.3 (d), the moment is set to \( M = 0 \) and compared to the trace in the case of unmanipulated signals. The resulting time reversed trace \( y^{TR}_0 \) is very close to the original deflection signal. However, a slight phase and amplitude mismatch is still visible.

To summarize, complete recovering of the time reversed signal is achieved by accounting for free end effects, multimode propagation and evanescent mode leakage with Eq. (3.8). In addition, the dispersion characteristic of the A0 mode is such that low frequencies travel with very low velocities. As a consequence, there is always a certain low frequency leakage for a finite time measurement. This effect is further discussed in section 3.1.2.1. Considering boundary conditions of the TR simulation, if the recorded data can not be processed to account for these effects, the best match is achieved by simply reversing the measured transverse deflection \( y_m \) and setting the bending moment at the excitation point to zero. While in the experimental step, no external forces and moments act at the measurement location, in the TR step, some sort of forcing is needed to excite the structure. Nonetheless, omitting bending actuation seems to be closer to the experimental situation than using both transverse forces and bending moments.
Figure 3.3: TR process for a semi-infinite Timoshenko beam for a broadband input signal. The x-axis displays a time scale in 10µs, the y-axis the deflection amplitude on a normalized scale. The TR signal in (a) and (b) is manipulated with Eq. (3.8) to compensate for evanescent mode and free end effects. The TR signal in (c) and (d) is not manipulated. The boundary condition for the TR simulation in (a) and (c) allow no rotation ψ whereas the boundary condition in (b) and (d) allows no bending moment M at the excitation location.

In Fig. 3.3 the effect of free end processing is shown for one particular signal. To provide a more general analysis, the amplitude and phase spectrum of the data processing transfer function is plotted in Fig. 3.4. Shown in Fig. 3.4 (a) and (c) is the transfer function of the within processing according to Eq. (3.4). The transfer function of the free end processing according to Eq. (3.8) is shown in Fig. 3.4 (b) and (d). In addition, the amplitude ratio $R_2/R_1$ is given in Fig. 3.5. We start with the discussion of the within processing, in which no reflections occur during the TR process. At low frequencies, the transfer function has an amplification of factor 2 and no phase shift. The amplitude ratio at zero frequency is one, with a phase shift of $-\pi/2$. We assume rotation free transverse excitation at $x = x_0$ that gives rise to left- and right-going flexural waves. Considering only the left-going waves and suppressing the spatial dependence, we can write for the local oscillation at $x = x_0$:

$$y_0 e^{i\omega t} = A_0 e^{i(\omega t + \phi_0)} - A_0 e^{i(\omega t + \phi_0 - \pi/2)}$$  \hspace{1cm} (3.12)

where the first and the second term on the right hand side of the equation are the $k_1$-mode and $k_2$-mode respectively. The amplitudes are of similar magnitude but with opposite signs due to the specified boundary condition $\psi(x = 0) = 0$. The negative phase shift of $-\pi/2$ of the second term stems from the amplitude ratio $R_2/R_1$, see Fig. 3.5. The amplitude $A_0$ and the phase $\phi_0$ can easily be identified in the complex plane:
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\[ A_0 = \frac{1}{\sqrt{2}} y_0 \] (3.13)

\[ \phi_0 = -\frac{\pi}{4} \] (3.14)

In other words, the transverse disturbance \( y_0 \) gives rise to two modes with equal magnitudes of \( \frac{1}{\sqrt{2}} y_0 \) and a phase shift of \(-\frac{\pi}{4}\) and \(-\frac{3\pi}{4}\) for the propagating and evanescent mode respectively. At higher frequencies, the magnitude of the amplitude ratio \( R_2/R_1 \) decreases and the split of the initial disturbance becomes more involved. Since the evanescent mode does not propagate in the chosen frequency range, it is not detectable at the measurement-location. Therefore, the TR excitation at the measurement-location will be the following:

\[ \frac{1}{\sqrt{2}} y_0 e^{-i(\omega t - \frac{\pi}{2})} = A_1 e^{i(\omega t + \phi_1)} - A_1 e^{i(\omega t + \phi_1 - \frac{\pi}{2})} \] (3.15)

Note the switch of sign in the exponent of the term on the left hand side of the equation resulting from the TR operation. Again, the excitation signal is split into two modes. The resulting amplitude and phase shift are:

\[ A_1 = \frac{1}{2} y_0 \] (3.16)

\[ \phi_1 = 0 \] (3.17)

Interestingly, the phase shift is compensated during the TR process. This is valid over the whole frequency range, as shown in Fig. 3.4 (c). The amplitude of the propagating mode is half the original excitation for zero frequency. Therefore, to compensate for the evanescent mode leakage, we have to amplify the measured signal by a frequency dependent factor, starting at two for zero frequency, see Fig. 3.4 (a).

Considering the free end processing, the only difference is that at the measurement location, being the free end of the beam, additional wave amplitudes are measured. These additional waves are the incident wave, a reflected \( k_1 \) mode and a reflected \( k_2 \) mode. From the condition of zero shear force and zero moment, the incident wave can be isolated from the sum of all three wave amplitudes. This is done in Eq. (3.5). Obviously, amplitudes are higher at the free end than within the structure. To compensate for this, the data processing transfer function magnitude is less than 1. The phase shift starts at \( \frac{\pi}{4} \). This is in accordance with Bernoulli theory, that predicts a constant frequency shift of \( \frac{\pi}{4} \) between incident wave and total deflection at the free end over all frequencies. Using Timoshenko theory, we find that this phase shift is frequency-dependent and is influencing the TR process at the free end.

In summary, a broadband signal is not completely restored in a TR process for a Timoshenko beam. This corresponds with the observations of Wang et al. [80] and Park et al. [55] who report a 'frequency dependence of the TR operator' in Mindlin plate theory. The here explained effects of evanescent mode leakage and low frequency leakage may help in understanding part of what is previously identified as a 'frequency dependence of the TR operator' in guided wave experiments. In the
Figure 3.4: Transfer function spectra of the data processing Eq. (3.8) and Eq. (3.4) for the cases of free end (b) and (d), and within processing (a) and (c), respectively. Plot (a) and (b) show the magnitudes of the transfer functions while plot (c) and (d) show the phase spectra.

These findings have the following consequences for the method presented:

In order to localize a defect:

- And when the TR process involves no reflections from the free end, no manipulation of the measured signal is necessary, because the phase shift is fully compensated during the TR process. The maximal amplitude is achieved at the original location of the defect, albeit with a different and smaller amplitude.

- And when the TR process involves reflections from the free end, manipulation of the measured signal may become necessary, especially for higher frequencies, because a frequency dependent phase shift occurs at the free end.

In order to characterize the size of a defect:

- No matter whether the TR process involves reflections from the free end or not, manipulation of the measured signal becomes necessary for a meaningful calculation of mode conversion coefficients.
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Effect of Measurement Location

As seen in the previous section, the measurement location may have an influence on the TR process. Measuring at a free end of the structure has two advantages. Higher signal-to-noise ratio is achieved by measuring at the free end. In addition, there is no ambiguity about the propagation direction of the incident wave. In general, the information about the wave’s propagation direction is lost with a one point measurement. As a result, the TR simulation will result in a point symmetric wave field with, for example two identical focal spots, although only one scatterer was present. This ambiguity can be resolved if the longitudinal wave, with which the flexural waves originally were excited, is also recorded. The propagation direction of the longitudinal wave is certainly known and its characteristic can be drawn into a time-space diagram, as illustrated in Fig. 3.6. The intersection of the longitudinal and flexural wave indicates the true scatterer location.

Yet another experimental configuration measures close to the free end, but with an offset $x_E$. Similar to the free end configuration, three wave amplitudes are measured, however, now the waves have different phase angles. This configuration was again investigated with spectral element simulations and its results are discussed on the basis of Fig. 3.7.

In Fig. 3.7(a), the rotation free excitation $y_0$ is applied on a semi-infinite beam several wavelengths away from the free end. In Fig. 3.7(b), the deflection response caused by this disturbance is captured at $x = x_m$. The offset from the free end is denoted as $x_E$.

The measured deflection $y_m$, see Fig. 3.7(c), is reversed in time and without further manipulation set as the excitation condition at the same location $y_m$ in the TR simulation. The resulting deflection at $x = x_0$ is plotted in Fig. 3.7(d). Instead of only one single pulse, we observe three individual pulses. Two propagating wave amplitudes were measured at the measurement location, being the incident wave

![Figure 3.5: Amplitude spectrum (blue) and phase spectrum (green) of the amplitude ratio $R_2/R_1$ according to Eq. 3.3](image)
Figure 3.6: Illustration of a guided wave TR experiment in form of two time-space diagrams. In the experimental step, a non-dispersive longitudinal wave pulse excites flexural waves at the scatterer. The displacements due to longitudinal and flexural waves are recorded at the measurement location. In the TR simulation step, the recorded displacement history is reversed in time and applied to a numerical model of the structure. The resulting waves travel symmetrically to the left and right side from the TR excitation, indicating two symmetrical scatterers. This ambiguity can be resolved by drawing the characteristic of the longitudinal wave into the time-space diagram. The intersection point of the longitudinal and flexural waves indicate the true scatterer position, while the other focal point is an artifact.

and the reflected wave. The incident wave’s phase is $k_1x_1$ with $x_1 = x_m - x_0$, the reflected wave’s phase is $k_1x_2$ with $x_2 = x_m - x_0 + 2x_E$. Similar to the ambiguity problem discussed before, waves cannot be given a particular propagation direction and therefore, four propagating waves are excited in a TR experiment. However, only two of them propagate along their original paths and form the sharp peak in the middle. This pulse still differs from the original signal $y_0$ in amplitude and shape due to the effects described in section [3.1.2.1] but nevertheless reaches the highest amplitude at the true location $x_0$. Two side-pulses are found symmetrically to the left and right side of the middle peak resulting from the two waves which did not travel along their original paths. This symmetry phenomenon is very typical for any TR process close to a border and also typical for multimode propagation effects, as demonstrated by Park et al. [54]. If the distance $x_E$, the material and the section properties of the beam are known, we can isolate the incident wave amplitude from the superposition of the three wave amplitudes.

This procedure was applied to $y_m$ and the effect of evanescent mode leakage was compensated. The resulting signal $\tilde{y}_m^{TR}$ is shown in Fig. 3.7 (e). As expected, the time reversal of signal $\tilde{y}_m^{TR}$ reproduces exactly the original excitation signal $y_0$, as shown in Fig. 3.7 (f). The numerical model is changed to an infinite beam in order to avoid reflections from the free end. The isolation of the incident wave is a delicate procedure because the manipulations are done on the phase of the three wave terms
and even a small deviation from the true $x_E$ has a high impact on the result.
In Fig. 3.7 (g) this is demonstrated by again isolating the incident wave, but with
a deviation of 0.5 mm from the true value $x_E$ in the calculation. This results in a
strong distortion of the signal $\bar{y}_m^{TR}$ as shown in Fig. 3.7 (g). Despite this strong
distortion, the TR process is still able to produce a meaningful reproduction $\bar{y}_0^{TR}$
of the original signal $y_0$ as presented in Fig. 3.7 (h). With regard to practical
implementations, this sensitivity of the parameter $x_E$ in the isolation of the incident
wave is quite problematic. It is therefore advisable to measure either far away from
the end of a structure so reflections can be eliminated in the time domain or to
measure directly upon the end.
Figure 3.7: Spectral element simulation of a TR process close to the free end. X-axis displays time on a ms scale, y-axis displays deflection on a normalized scale. Excitation deflection is given in (a), the resulting response close to a free end is given in (b). The time reversed signal is plotted in (c) and the response at the original excitation position is plotted in (d). Note the side-pulses in (d). Extracting the incident wave from the total response and compensating for evanescent mode leakage results in signal (e) which then is able to recover the excitation signal, see plot (f). This extraction process is sensitive to changes in the input parameters. Using a 0.5 mm deviation from the true free-end measurement distance in the extraction process results in a strongly disturbed signal, (g). The resulting TR deflection at \( x_0 \) is plotted in (h).
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Effect of Manipulating Signal With regard to practical implementation of the method, the measured deflection signal is likely to undergo additional manipulation as e.g. truncation, windowing, noise, and filtering. The effect of such changes to the signal on the TR process is investigated with a range of spectral element simulations, shown in Fig. 3.8. The plotted traces were truncated for a better overview.

Figure 3.8: Spectral element simulations showing the effect of various manipulations to the TR excitation signal $y_m$ on the converged signal at $x = x_0$. Beside the manipulations to compensate for the evanescent mode leakage, the manipulations are: (a) none; (c) windowing with a rectangular window; (e) windowing with a Hanning window; (g) adding noise; (i) filtering signal with 2nd order Butterworth bandpass. The x-axis displays time on a 100 $\mu$s scale, the y-axis displays deflection on a normalized scale.

In Fig. 3.8 (a), by compensating for evanescent mode leakage, perfect reconstruction of the original excitation is achieved, see Fig. 3.8 (b).

In plot (c), the signal $\tilde{y}_m^{TR}$ is multiplied with a rectangular window to mute the signal after approximately 290$\mu$s. This imitates the effect of recording with a limited number of samples. Due to the dispersion characteristics of the flexural wave, truncating the signal reduces its low-frequency content. As a result, also the TR simulation lacks the low-frequency content, which is apparent by the non-flat basis
of the peak. However, the signal still resembles its original and maximal amplitude is achieved at the correct time instant, see Fig. 3.8 (d).

In Fig. 3.8 (e), the signal is multiplied with a Hanning window of the same length as the rectangular window. The effect on the time reversed signal is very similar to the multiplication with a rectangular window. Again, the overall reconstruction of the signal is quite good and especially the phase of the signal is similar to the original signal despite the loss of some frequency content, see Fig. 3.8 (f).

Adding noise to the signal, as illustrated in Fig. 3.8 (g), has very little impact on the TR process. Phase and amplitude of the reconstructed signal is similar to the original signal, see Fig. 3.8 (h). Since the presented model assumes linear wave propagation, superposition holds and noise and wave information propagates independently. The boundary condition in the TR simulation is set up such that only flexural waves propagate in the structure. This automatically implies, that only information originally propagated as flexural waves will refocus in the TR simulation. This explains the insensitivity of the present TR process to noise and other disturbing sources like reflections and other wave modes. This robust behavior is a major advantage of the method and may be important for industrial implementations.

Finally, the effect of filters to the TR process is investigated in Fig. 3.8 (i) and (j). A second order Butterworth bandpass with cut-off frequencies of 10kHz and 150kHz was simulated and applied to the signal $\bar{y}_{TR}$. Signal $\bar{y}_{TR}$ has a frequency distribution of a half-sided Gaussian probability density function with frequency band width up to 200kHz. While the effect of filtering is not easily visible in plot (i), the effect on the time reversed signal is clearly visible. The reconstructed signal is not symmetric and the maximum amplitude is not at $x = x_0$. Modifying the signal with a filter seems to be detrimental for the TR process, since a filter not only changes the amplitude spectrum but, depending on the filter type and settings, also changes the phase spectrum. A signal passing such a filter is delayed in a frequency dependent manner. This delay is known as group delay $\tau_g$, and defined as:

$$\tau_g = -\frac{d\phi(\omega)}{d\omega} \quad (3.18)$$

where $\phi$ is the phase of the filter. In an experimental setup, components with a frequency dependent group delay may alter the overall dispersion characteristic of the system and impair the TR process. Therefore, if filtering of the recorded signal $y_m$ is necessary, the filter should have a constant group delay over the frequency range of interest. This is not the case for the simulated Butterworth bandpass filter as shown in figure 3.9.
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Figure 3.9: Characteristics of the simulated 2nd order Butterworth bandpass filter with cut-off frequencies of 10 kHz and 150 kHz. Red line: amplitude spectrum. Blue line: phase spectrum. Green line: Group delay. Group delay is most pronounced in the vicinity of the cut-off frequencies.

Application to Circular Cylindrical Shells  Circular cylindrical shells are a very popular form of structural element and the applicability of the presented method to tube- and pipe-like structures is investigated in this section. Depending on the wavelength and shell thickness to diameter ratio, a cylindric shell may behave more like a beam for long wavelengths or like a plate for very short wavelengths. In between these extremes, a cylinder has a peculiar complex behavior, with a variety of circumferential modes. The dispersion relation for the three first non-axially symmetric circumferential modes F(1,1) - F(3,1) according to Naghdi and Cooper [51] is illustrated in Fig. 3.10 together with an approximation of the Timoshenko flexural wave. While for the lowest frequencies, the agreement between Timoshenko theory and shell theory is very good, it breaks down with the emergence of higher order circumferential modes in the cylindrical shell. In the case of our test specimen, this is predicted at $f_2 = 5.748$ kHz. The cut off frequency for the F(2,1) mode can be estimated by the following equation:

$$f_2 \approx \sqrt{\frac{3E}{5\rho(1-\nu^2)}} \frac{h}{2\pi r^2} \quad (3.19)$$

where $h$ is the shell thickness, $r$ the radius of the middle surface of the circular cylindrical shell [51].

In order to localize defects with the help of a TR simulation, one could either work in the cylindrical wave regime, using an appropriate numerical model of the specimen or work in the beam regime, using a simpler approximation of the structure. The first approach was reported by Leutenegger et al. [45], who achieved good localization results by measuring the complete 3D displacement field at 53 points around the circumference of the pipe. Dispersion is, however, not as pronounced for non-axially symmetric modes at higher frequencies and the localization of the defect is mainly achieved by the concurrent arrival of the different modes at the defect in contrast to the recovering of an individual dispersive mode. In addition, compensating for the
many evanescent modes in the cylindrical wave regime is probably more complex and would require measuring at multiple points in order to achieve highest localization accuracy.

As will be shown later by the validation tests, equally good localization results are achieved by the much simpler approach of exciting frequencies below the cylindrical wave regime. Hereby, the structure can be treated as a beam which has fewer demands on the numerical model of the structure, and requires only a one point measurement of the out-of-plane component of the displacement field. This comes at the cost of crack detection sensitivity, because the mode conversion ratio is low at low frequencies and the resolution for the detection of multiple defects is worsened due to the long wavelengths of the flexural wave.

### 3.1.2.2 Crack Characterization

Consider a longitudinal wave, with amplitude $u_i$, incident on an off-axis defect in a beam. This situation is illustrated in Fig. 3.11 for the analytical model. The incident wave in the considered frequency range triggers the following wave modes:

- $u_r$, reflected longitudinal wave
- $u_t$, transmitted longitudinal wave
- $y_{fr}$, reflected flexural wave
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- \( y_t \), transmitted flexural wave
- \( ye_r \), reflected evanescent mode
- \( ye_t \), transmitted evanescent mode

As apparent from the result of both the numerical and the analytical model in Fig. 3.11, the transverse motion is quite similar but not identical to the axial motion at the notch. The transverse motion is a superposition of the evanescent modes \( ye_r + ye_t \) and the propagating wave modes \( y_r + y_t \). Some wavelengths beyond the defect, the evanescent mode has decayed. The transverse motion on the surface of the beam consists only of the lateral contraction due to \( u_t \) and the deflection due to \( y_t \). Ideally, these two wave modes can be measured and the depth of the crack can be evaluated from the amplitude ratio, \( y_t/ u_t \), using an appropriate model for the crack. Different approaches exist for modeling cracks and to derive mode conversion- reflection-, and transmission-coefficients for defects. Alleyne and Cawley [4] investigated the interaction of Lamb waves with notches by means of plane strain FE simulations and supporting experiments. More recently, Castaings et al. [10] analyzed the crack-defect interaction via a modal decomposition approach which allows an accurate displacement field at the crack interface while still being 100 times faster than a comparable FE model. A good physical interpretation of the wave-crack interaction is presented by Lowe et al. [46], again by utilizing FE simulations, experiments, and an analytical model employing the concept of crack-opening displacement.

Another way of modeling the crack in guided wave settings is known as a generalized spring approach, by which the defect is modeled as a discrete spring element, introducing discontinuities in the kinematic variables of the beam. Such models were first presented by Gudmundson [31] to predict the change of eigenfrequencies of slender structures due to cracks and later extended by many other authors, see for example references [53, 16, 68, 42]. Here, we adopt the procedure of Dual [14] to calculate amplitude ratios for our test specimen.

Figure 3.11: Analytical and numerical models of a cracked beam.
An additional discontinuity $\delta s$ in the kinematic variables is assumed, having a linear relation with the acting loads $f$ at the crack location.

$$\delta s = \Lambda f$$  \hspace{1cm} (3.20)

where $\Lambda$ is a $3 \times 3$-matrix, $\delta s$ having components of longitudinal displacement $\delta u$, angle of rotation of the cross section $\delta \psi$ and transverse displacement $\delta y$ and $f$ having components of normal force $N$, bending moment $M$ and shear force $Q$. The forces can be written as functions of the wave amplitudes:

$$N = EA \frac{du}{dx}$$  \hspace{1cm} (3.21)

$$M = EI \frac{d\psi}{dx}$$  \hspace{1cm} (3.22)

$$Q = -EI \frac{d\psi^2}{dx^2} - \rho I \omega^2 \psi$$  \hspace{1cm} (3.23)

Note that $\Lambda$ has three diagonal entries and two off diagonal entries $\Lambda_{12} = \Lambda_{21}$, which couple longitudinal and bending waves. This linear behavior is assumed to be valid for large wavelength-to-beam-thickness ratios as used here. The unknown coefficients of $\Lambda$ are obtained from energy considerations of linear-elastic fracture mechanics. Following Irwin, [36], the potential energy $\Theta$ stored in the vicinity of a crack is reduced by the growing crack to generate new surface. The total amount of potential energy which is released as the crack grows from zero up to a given crack surface $C_{crack}$ is obtained by:

$$-\Theta = \int_0^{C_{crack}} \frac{1}{E} \left( K_I^2 + K_{II}^2 \right) Bdn_d$$  \hspace{1cm} (3.24)

where $K_I$ and $K_{II}$ are the stress intensity factors which are a function of the crack depth and are obtained from the Stress Analysis of Cracks Handbook [71]. Further, $B$ is the width of a beam with rectangular cross section and $n_d$ the depth of a breathing crack. To close the crack by external forces, the total work $W$ needed is:

$$W = \frac{1}{2} (f^T \delta s)$$  \hspace{1cm} (3.25)

Combining Eq. (3.20) with Eq. (3.25), one can derive the components of $\Lambda$ by comparing the coefficients in Eq. (3.24) with the coefficients in Eq. (3.25). The individual wave amplitudes are then obtained by the interface conditions:

$$s^+ = s^- + \delta s$$  \hspace{1cm} (3.26)

$$F^+ = F^-$$  \hspace{1cm} (3.27)

where $-$ and $+$ indicate the left and right hand side of the crack, in the case of an incident wave from the left hand side, see Fig. 3.11. This procedure allows the calculation of amplitude ratios with respect to an incident wave. As an example, Fig. 3.12 shows amplitude ratios resulting from an incident wave.
longitudinal wave in the case of a 2 mm deep crack. Note, that the generated left and right-going flexural waves have the same amplitude. This is a reasonable behavior for an infinitely narrow crack but might not be what one observes at a machined notch of non-zero width. Also the mode conversion ratio for the running flexural mode $y_t$ monotonically increases with frequency. This ratio will be used for the crack characterization. The pulse shape of the transmitted flexural pulse will be different from the incident wave as each frequency component is reflected differently, see Figs. 3.11 and 3.12. This effect is most noticeable for broadband pulses.

Although such mode conversion ratios have been available for some time, their applicability is often limited due to the difficulty of isolating an individual wave mode from a multimode dispersive signal \[4\]. In most applications, narrowband pulses are used to minimize dispersion effects and therefore maximize the compactness of the pulse. However, this comes at the expense of pulse length and reflections are likely to overlap with the incident wave. However, methods are reported, for example Alleyne and Cawley \[4\] and more recently, Hayashi et al. \[32\], which extract a single mode out of a multimode dispersive signal via 2D FFT processing, by which in the latter the frequency-wavenumber image is filtered for the mode of interest. This of course requires measuring at many points along the structure. Here, the combination of a broadband signal and a TR simulation offers a considerable advantage. Using a broadband excitation in the form of a narrow Gaussian peak, the time and spatial extent of the wave pulse initially is short. The TR simulation restores the shape of every generated flexural wave at its individual origin. The advantage
of using broadband pulses in TR processes has also been reported by Gangadharan [27]. Resolving scatterers that are located close to each other is therefore facilitated. Using a broadband signal has the additional advantage, that amplitude ratios can be compared over a broad frequency range, making the prediction of the crack size more robust.

In the time-space diagram of the TR simulation, all individual flexural waves and their reflections are separated at their origin. Moreover, the boundary condition of the TR simulation excites only flexural waves and as a consequence, only signals produced by flexural waves will get restored. Signal content stemming from other sources such as noise or other modes will get dispersed. In this setting, the TR process acts as a filter for the flexural wave mode. The extraction of the initial flexural wave mode is a natural effect of the TR simulation as used in this study. To characterize the extent of the defect, the mode conversion ratio is evaluated from the measured lateral contraction of the non-dispersive longitudinal wave and the restored flexural wave from the TR simulation. This procedure is feasible from a one point, multimode, multireflection dispersive measured signal.

### 3.1.3 Experimental Validation

The method presented was validated with a range of experiments. Five aluminium beams and one aluminum pipe with machined notches were used as test specimens. All beams have a quadratic cross section of 6 mm × 6 mm. Beam 00 was used for the localization test, is 1900 mm in length and incorporates 3 different notches. The other four beams were used for the crack characterization tests, have lengths of 845 mm and incorporate only one notch per beam. To achieve the smallest possible notch width, electrical discharge machining was used for the crack characterization beams. The aluminum pipe has a length of 2000 mm, shell thickness of 2 mm and a radius of the outer surface of 16 mm. The position and geometry details of the notches are listed in Table 3.2.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Notch</th>
<th>$x_n$</th>
<th>$n_n$</th>
<th>$n_d$</th>
<th>$n_w$</th>
<th>$n_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 00 (L = 1900 mm)</td>
<td>1</td>
<td>700</td>
<td>-</td>
<td>2</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1000</td>
<td>+</td>
<td>2</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1150</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Beam 01 (L = 845 mm)</td>
<td>1</td>
<td>250</td>
<td>-</td>
<td>0.5</td>
<td>0.3</td>
<td>6</td>
</tr>
<tr>
<td>Beam 02 (L = 845 mm)</td>
<td>1</td>
<td>250</td>
<td>-</td>
<td>1</td>
<td>0.3</td>
<td>6</td>
</tr>
<tr>
<td>Beam 03 (L = 845 mm)</td>
<td>1</td>
<td>250</td>
<td>-</td>
<td>1.5</td>
<td>0.3</td>
<td>6</td>
</tr>
<tr>
<td>Beam 04 (L = 845 mm)</td>
<td>1</td>
<td>250</td>
<td>-</td>
<td>2</td>
<td>0.3</td>
<td>6</td>
</tr>
<tr>
<td>Pipe (L = 2000 mm)</td>
<td>1</td>
<td>800</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 3.2: Notch locations and geometries of the specimens tested. Abbreviations: $x_n$: Notch location [mm], $n_n$: Notch at top (+) or bottom (−) edge, $n_d$: Notch depth [mm], $n_w$: Notch width [mm], $n_b$: Notch breadth [mm].
3.1. Quantitative Guided Wave Testing by Applying the Time Reversal Principle on Dispersive Waves in Beams

3.1.3.1 Experimental Setup

The experimental setup is shown in Fig. 3.13. The test specimens were supported by several foam pieces. For the aluminum beams, waves were excited by piezoelectric crystals bonded to the beams. The choice of excitation method depends on the dispersion characteristic of the longitudinal and flexural wave. At low frequency-thickness numbers, flexural waves are highly dispersive while longitudinal waves are almost non-dispersive. However, for high spatial resolution, high frequencies are required. At high frequency-thickness numbers, dispersion of flexural wave decreases while dispersion of longitudinal wave becomes more pronounced. This needs to be considered when choosing the frequency range of the excitation. A long wavelength approximation for the voltage-displacement transfer function for longitudinal piezoelectric excitation with a backing plate is presented in [15]:

\[ H_{\text{piez}} = \frac{u}{d_p V} = -\frac{2m_M + m}{m_M + m} \frac{\omega}{\omega - i \sqrt{\frac{\rho A}{m_M + m}}} \]  

where \( u \) is the axial displacement in the rod, \( d_p \) the piezoelectric charge constant, \( V \) excitation voltage, \( m_M \) and \( m \) the mass of the backing plate and piezoelectric element, respectively. Apparently, the system has a high-pass behavior, making it hard to excite a peak-shaped wave signal. Increasing the backing mass increases the low frequency gain, but there are physical limitations to this tuning parameter. Note that the cross section area \( A \) decreases the low frequency gain which means that piezoelectric excitation in beams with large cross sections is not ideal for the excitation of peak-shaped wave signals. Impact-type excitation may be an alternative since this delivers more low frequency energy into the structure.

The used piezoelectric plates have the dimensions 6.35 mm × 6.35 mm × 1 mm and vibrate in thickness extension mode. The piezoelectric plates were tuned with copper-tungsten alloy backing-plates in order to enhance the power transfer into the beams in the frequency range between 10 kHz and 200 kHz. The backing plate was designed with the help of an analytical model [15], and the transfer function between excitation voltage and displacement amplitude was experimentally determined. The accurate measurement of the transfer function is necessary to excite the specifically designed broadband pulses into the beams. For the actual guided wave tests, a pulse in the form of a Gaussian distribution with standard deviation \( \sigma = \frac{T}{4} = 2.5 \mu s \) was used.

For the aluminum pipes, waves were excited by impacting a steel ball, diameter = 10 mm, weight = 4 g, on the center of the surface of an aluminum plate bonded to the pipe, see Fig. 3.13 for a schematic illustration. The duration of such an impact was approximately 0.045 ms. Beside the longitudinal waves, the impacting ball also triggered flexural waves and other modes due to the ball impacting slightly off the pipe center-line. The resulting longitudinal wave will excite non-axially symmetric waves at the defect, of which most energy will be carried by the flexural wave, see Fig. 3.10.

The displacement amplitude was measured with a one point laser interferometer system. The demodulated voltage signal was sent through a band pass filter with 0.1 kHz and 200 kHz cut-off frequencies.
Young’s modulus and Poisson’s ratio were determined to best fit the experimentally determined dispersion curve for flexural waves in a range between 10 kHz to 200 kHz for the beams. Therefore, a good match of the phase velocities in experiment and simulation is guaranteed which is important for the accuracy of the method presented. Fig. 3.14 shows the measured phase velocities together with the fitted curve. The material parameters for the pipe were taken from [45] since the same test specimen was used in the present paper.

The TR simulations were performed with Timoshenko spectral elements, on the basis of Doyle [12]. The beam was modeled as a semi-infinite structure to avoid reflections and facilitate interpretation in the time-space diagram. This is by no means an essential requirement. The measured transverse displacement history was modified according to section 3.1.2.1 to account for evanescent mode leakage and free end effects. The modified signal was then reversed in time and set as the transverse displacement end condition. The axial and rotational degree of freedom was constrained to zero to be in accordance with the modeling assumptions in section 3.1.2.1. Therefore, only flexural waves are excited in the simulation.

Figure 3.13: Illustration of the experimental setup.
3.1. Quantitative Guided Wave Testing by Applying the Time Reversal Principle on Dispersive Waves in Beams

Figure 3.14: Dispersion measurement and fit using Timoshenko theory, for an aluminum beam with quadratic cross section of 6 mm × 6 mm.

3.1.3.2 Crack Localization Results and Discussion

The localization results are given as time-space diagrams in Fig. 3.15, Fig. 3.16, Fig. 3.17, and Fig. 3.18. In these figures, the blue tone is a measure for the lateral displacement amplitude along the beam’s axial coordinate. From there, defects are identified as local maxima in the displacement field. The time and space coordinate of a detection is illustrated by the intersection of a horizontal and vertical line. The horizontal line cuts the diagram along the time coordinate at the detected notch position. The associated time trace is plotted below the figure. The vertical line cuts the diagram along the space coordinate for the detected time instant. The associated space trace is plotted on the right hand side of the time-space diagram. The time reversed displacement data prescribes the boundary condition at x = 1.9 m for the beam and at x = 2 m for the pipe. All notches were identified from one single point unidirectional measurement at x = 1.9 m normal to the beam surface and at x = 2 m normal to the cylinder surface. The localization values are listed in Table 3.3.

In Fig. 3.15 the time-space diagram indicates a sharp local maximum at x = 0.681 m and t = 1.0174 ms corresponding to the notch at x = 0.7 m. The amplitude of the peak correctly indicates the orientation of the notch. The shape of the peak resembles a Gaussian function, however the basis of the peak is not flat and apparently lacks some low frequency content. This has several reasons. The excited longitudinal wave itself will deviate slightly from a perfect Gaussian, due to limitations in the transducer system. Further, as documented by a FE simulation of a notched beam in Fig. 3.11 the arising flexural wave is not completely identical to the incident longitudinal wave. Moreover, the finite recording time also results in a reduction of low frequency content in the TR flexural wave, according to the discussion in section 3.1.2.1. Eventually, other wave oscillations are visible in the signal and may superpose the basis of the peak. In the time-space diagram, the amplitude of the wave oscillates between positive and negative which is a result of dispersive propagation behavior.
In Fig. 3.16 the same time-space diagram is plotted however now investigating the time trace at $x = 0.988\,\text{m}$ and the spatial trace at $t = 0.9562\,\text{ms}$. On that time trace, apparently two sharp peaks exist, one at $t = 0.5951\,\text{ms}$ and one at $t = 0.9562\,\text{ms}$. They both were excited at the same notch, once by the direct longitudinal wave and once by the reflected longitudinal wave. The sign change of the amplitude is due to the free end reflection of the longitudinal wave. The amplitude of the first peak in the TR simulation is significantly smaller mainly because the corresponding wave was truncated earlier.

The localization error for the pipe resulted in $5\,\text{mm}$. This is considered a good result with regard to the relatively long wavelengths used. When comparing the present result to the result obtained by Leutenegger et al. [45], who reported a localization error of $15\,\text{mm}$ for the same specimen, one may be surprised, that the more elaborated measurement approach involving 53 measurement points in all 3 directions does not increase the location prediction. However, these measurement data were not adjusted for free end and evanescent mode effects. The approach of Leutenegger also allowed the determination of the breadth and the circumferential position of the notch, which is not achieved by the present 2D approach.

The wave field for the pipe as shown in Fig. 3.17 is more disturbed than the wave field shown in Fig. 3.15 for the beam. This might be due to the steel ball impacting the pipe slightly off axis resulting in higher order waves and flexural waves. However, since the simulation treated all displacement information as flexural waves, only the part of the signal that originally propagated as a flexural wave signal is recovered in the TR simulation. This again emphasizes the method’s robustness in that a simple impact of the structure, which produces different waves at the excitation location, suffices to accurately detect a notch in the structure. However, the limitation in excitation frequency, which was chosen such that the flexural behavior can be described by Timoshenko theory, may impair the ability to detect closely-located defects. Adapting the spectral element method with a suitable shell theory would remove this frequency limitation. Details for implementing shell theories into a spectral element setting are found in [28].

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Notch nr.</th>
<th>$x_n$</th>
<th>$x_{\text{pred}}$</th>
<th>$\Delta x_{\text{err}}$</th>
<th>$x_{\text{prop}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 00 ($L = 1900,\text{mm}$)</td>
<td>1</td>
<td>700</td>
<td>681</td>
<td>19</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1000</td>
<td>988</td>
<td>12</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1150</td>
<td>1129</td>
<td>21</td>
<td>750</td>
</tr>
<tr>
<td>Pipe ($L = 2000,\text{mm}$)</td>
<td>1</td>
<td>800</td>
<td>795</td>
<td>5</td>
<td>1200</td>
</tr>
</tbody>
</table>

Table 3.3: Localization results. Abbreviations: $x_n$: Notch location [mm], $x_{\text{pred}}$: Predicted location [mm], $\Delta x_{\text{err}}$: Error [mm], $x_{\text{prop}}$: Source - sensor distance [mm].

**Unveiling small scatterers** Notch 1 and 2 were identified from one TR simulation, notch 3 required some additional processing. Notch 3 is much smaller and located close to the larger notch 2, increasing the difficulty of it’s detection in the time-space diagram.
3.1. Quantitative Guided Wave Testing by Applying the Time Reversal Principle on Dispersive Waves in Beams

The TR process offers several ways to iteratively unveil small scatterers. One concept was presented by Prada et al. [57], known as D.O.R.T - method. Selective focusing is achieved by the decomposition of the TR operator and requires an array of transmitting-receiving transducers. Another procedure is suggested by Anderson et al. [5] and is adopted in the method presented.

Fig. 3.19 (a) shows the measured transverse displacement $y_m$, which has been cropped and modified to account for free end and evanescent mode leakage effects. The signal is assumed to have the following form:

$$y_m = A_1 e^{i(k_1(L-x_1)+\omega t)} + A_2 e^{i(k_1(L-x_2)+\omega t)} + A_3 e^{i(k_1(L-x_3)+\omega t)} + \sum A_r e^{i(k_1(L-x_i)+\omega t)} + \sum A_l e^{i(k_1(L-x_i)+\omega t)}$$  \hspace{1cm} (3.29)

This equation is helpful in explaining the physical idea behind the reduction of large scatterers from the measured signal and is not involved in the actual process. It illustrates, that $y_m$ is a superposition of direct and reflected waves excited at the individual notches. In particular, $A_1$, $A_2$ and $A_3$ are the amplitudes from the direct flexural waves excited at notch 1, 2, and 3. $A_r$ are amplitudes from the reflected waves and $A_l$ are amplitudes from longitudinal waves. Due to dispersion, the different waves interfere at the measured position. By time reversing $y_m$ in
a numerical simulation, we get a displacement field as plotted in the time-space diagram in Fig. 3.15 and Fig. 3.16. From there, we can identify strong scatterers and extract them from the total displacement. As an example, notch 2 is identified at \( x = 0.988 \text{ m} \) due to the sharp peak in the time trace \( y_{\text{TR1}} \) at \( t = 0.9562 \text{ s} \). At this location, the wave \( A_2 \) is assumed to have restored its original phase and therefore its original shape. At this position we can extract \( A_2 \) and its reflection from the time trace. Assuming, we have identified notch 1 and its reflections, we can extract \( y_{\text{Extr}} \) from \( y_{\text{TR1}} \), see Fig. 3.19 (b). Now, \( y_{\text{Extr}} \) can again be reversed in a TR simulation, calling it a forward simulation, which leads to the deflection \( y_{\text{FS}} \) at the free end of the numerical beam. Note the similarity of \( y_{\text{FS}} \) and \( y_{\text{m}} \) in Fig. 3.19 (c) and Fig. 3.19 (a). Subtracting \( y_{\text{FS}} \) from \( y_{\text{m}} \) leads to the response of the beam at the free end due to the incoming wave \( A_3 \), see Fig. 3.19 (d):

\[
y_{\text{m}} - y_{\text{FS}} = A_3 e^{i(k_1(L-x_3)+\omega t)}
\]

An additional TR simulation finally reveals notch 3 based on the restored \( A_3 \) wave which results in a peak as shown in Fig. 3.19 (e).

\[
y_{\text{TR2}} = A_3 e^{i\omega t}
\]
To summarize, the combination of one experiment with multiple TR simulations allows one to iteratively exclude the highest scatterer from the data, revealing the next stronger scatterer. This works best for broadband pulses which have a small duration compared to narrowband pulses of the same frequency. Assuming a specimen with features known to act as scatterers, e.g. holes or stringers on a blade, this procedure may enable the detection of small defects close to such features without a reference measurement. We would therefore claim the method as baseline free.
Figure 3.18: Localization result beam 00, notch 3, piezoelectric excitation, after subtracting the larger scatterers from the signal.
3.1. Quantitative Guided Wave Testing by Applying the Time Reversal Principle on Dispersive Waves in Beams

Figure 3.19: Unveiling a small scatterer localized close to a strong scatterer. The y-axis displays transverse displacement with 2 nm per division, the x-axis displays time with 0.5 ms per division. In (a), \( y_m \) is a segment of the measured lateral displacement at the end of the beam. In (b), \( y_{TR1} \) is the time reversed time trace at \( x = 0.988 \) m, as shown in Fig. 3.16. \( y_{extr} \) is the part of \( y_{TR1} \), which presumably belongs to notch 1 and 2, not notch 3. In (c), the simulated deflection \( y_{FS} \) at the free end due to the forward simulation of \( y_{extr} \) is plotted. In (d) \( y_{FS} \) is subtracted from \( y_m \). In (e), \( y_{TR2} \) is the converged time trace indicating the location of notch 3, see Fig. 3.18.
Uncertainty analysis The localization accuracy strongly depends on how well the numerical model predicts the flexural wave speed in the test specimen. Ideally, one initially measures the dispersion relation of the specimen to bring the numerical simulation in accordance with the measurements. This might not always be practical and one may employ material values known from literature for the numerical simulation.

How sensitive is the phase speed to a mismatch in the input parameters of the model? A sensitivity analysis showing the percentage change in phase speed for a 1% change in the input parameters for Timoshenko theory is given in Fig. 3.20. The formula was linearized around the parameters for the aluminum beam listed in Table 3.1. As apparent from Fig. 3.20, phase speed is relatively robust to small changes in the input parameters. This means that the method performs robust even if the provided material parameters are not very accurate.

Mass density and Young’s modulus have the strongest effect, with about $\Delta c = 0.3\%$ for a 1% change. The effect of all parameters except cross section $A_0$ increases with frequency.

Note, all parameters show a frequency dependent behavior. As a consequence, for a strong error in the input parameters of the simulation, both the position and the shape of the maximum displacement deviates from the true situation. Overall, for the investigated case of aluminum beams, the results presented are not severely affected by the material uncertainty. However, in the general case, depending on the geometry and material of the test specimen, some parameters, such as Timoshenko shear coefficient, may have higher effects on the wave speed than what was found here.

In the present case, the phase speed was measured directly for 20 frequency steps at several points on the beam, see Fig. 3.14. Therefore, multiple phase speed values are determined for every frequency step. In Fig. 3.14 mean values and standard
deviations are plotted. The average standard deviation versus phase speed $\bar{c}$ for all measured values was 1.2%. This measurement uncertainty in the evaluation of the dispersion curve leads to an uncertainty in the simulation and affects the localization predictions. The localization error $\Delta x_{err}$ due to an error in the predicted phase speed $c_{pred}$ is approximated by:

$$\Delta x_{err} \approx x_{prop} \left(1 - \frac{c_{pred}}{c_{true}}\right) \approx x_{prop} \bar{c}$$  (3.32)

Notch 1 and 2 were predicted with a deviation of $\Delta x_1 = 19$ mm and $\Delta x_2 = 12$ mm respectively, see Table 3.3. This is close to the calculated error $\Delta x_{err1} = \pm 15$ mm and $\Delta x_{err2} = \pm 11$ mm respectively. This suggests that the deviation is within measurement uncertainty. Notch 3 however has a higher deviation $\Delta x_3 = 21$ mm compared to the calculated error of $\Delta x_{err3} = \pm 9$ mm. This may be attributed either to a lower signal-to-noise ratio or to the manipulations described in section 3.1.3.2.

### 3.1.3.3 Crack Characterization Results and Discussion

Four beams, B01 - B04, with varying notch depths, 0.5 mm to 2.0 mm, were tested by measuring the transverse displacement 100 mm off the free end. At the free end, lateral contraction due to the longitudinal wave is zero and the longitudinal wave cannot be measured with the laser beam being normal to the beam’s axis. The complete measured signal, as it was obtained from the vibrometer system, down sampled by a factor 100, is presented in Fig. 3.21. Note the first two peaks belonging to the lateral displacement of the incoming and reflected longitudinal wave. These two peaks are present in all four measurements. The subsequent oscillation tail is the result of the flexural wave. The magnitude of the flexural wave is very sensitive to the depth of the notch. The reflected waves which interfere with the incoming waves make it difficult to calculate the mode conversion ratios from these signals. However, the modified TR simulation allows the extraction of the individual wave pulses and therefore allows the calculation of the mode conversion ratios over a wide frequency band.

The corresponding notch depth evaluation results are presented in Fig. 3.22. The result for every test (a-d) is illustrated with two plots per row, for example (a.1) and (a.2) for beam B01. The plots on the left-hand side, show the converged flexural wave extracted from the TR simulation (green line), and the longitudinal wave calculated from the measured lateral displacement (blue line). Note the deviation from an ideal Gaussian of the longitudinal wave signal due to small dispersion. The shown flexural and longitudinal amplitudes were divided in the frequency domain leading to frequency dependent amplitude ratios, displayed as red crosses in the plots on the right-hand side. In addition, analytical amplitude ratios for 4 different crack depths ranging from 0.5 mm to 2.0 mm are plotted. The notch depth is estimated by comparing the measured and analytical amplitude ratios. So far, dissipation is not considered in the comparison of flexural and longitudinal wave modes. Since the flexural wave mode travels twice the distance of the longitudinal mode (source to sensor and sensor to source) before its amplitude is evaluated, this effect would need to be considered for
As can be seen from Fig. 3.22, all notch depths are accurately predicted at frequencies between 10 kHz to 40 kHz. With increasing notch depth, the flexural wave amplitude increases while the longitudinal wave amplitude decreases. At frequencies below 10 kHz and above 40 kHz, the notch depth is generally overestimated. The prediction error below 10 kHz is ascribed to the lack of energy in the signals at that frequency range. For the increasing prediction error above 40 kHz, no clear physical explanation has yet been found. The frequency content of both longitudinal and flexural wave signals is higher than 80 kHz. Possible reasons could either be frequency dependent dissipative effects or a discrepancy between the machined rectangular, finite width notch in the test specimen and the infinitely narrow crack in the analytical model. Indicators for the latter are found e.g. in Lowe et al. [46], who reported a strong dependence of the fundamental anti-symmetric Lamb mode A0 reflection coefficient on the notch-width-to-wavelength ratio.
3.1. Quantitative Guided Wave Testing by Applying the Time Reversal Principle on Dispersive Waves in Beams

Figure 3.22: Results of the crack-sizing tests. Four beams, B01 - B04, with varying notch depths, 0.5 mm to 2.0 mm, were tested. The result for every test (a-d) is illustrated with two plots, e.g. (a.1) and (a.2) for beam B01. The plots on the left-hand side show the converged flexural wave extracted from the TR simulation (green line) and the longitudinal wave calculated from the measured lateral displacement (blue line). The plots on the right-hand side show analytical amplitude ratios (lines) and the measured amplitude ratio (red crosses).
3.1.4 Conclusion

This paper has two main parts. In the first part an analysis of a time reversal (TR) process for dispersive signals in a Timoshenko beam is presented. The effect of truncation, filtering and noise in the TR process is discussed. While noise in and truncation of the recorded signal is shown not to be critical for the correct prediction of crack location, filtering may be detrimental, since it alters the dispersion characteristic of the overall system, especially around the filter’s cut-off frequencies. As a rule of thumb, the cut-off frequencies for a high or low-pass should be chosen one order of magnitude lower or higher than the signal content of the wave. It is shown that the TR process, without modification, fails to recover the original shape of a flexural wave below the cut-off frequency of the shear-thickness mode. While the shear-thickness mode does contribute to the response of a beam due to a local disturbance, it does not propagate and hence can not be measured at the receiver location. The TR excitation of the measured signal at the receiver location can therefore not establish the original signal at the original disturbance location, due to what we referred to as evanescent mode leakage. This effect occurs not only for beams and flexural waves, but for all multimode propagation phenomena at frequencies, where certain modes have imaginary wave numbers. In the present case however, the effect of evanescent mode leakage is compensated because the loading condition at the defect is known a priori.

In the second part, the insights gained on the TR process are incorporated in a baseline free, guided wave testing setup for the localization and quantification of open cracks in beam structures. Most of the guided wave testing methods struggle with distortion of wave pulses due to dispersion. Therefore, the excitation signal has usually a narrow frequency band to minimize dispersion effects. In contrast, the method described here uses broadband signals to achieve strong dispersion of the flexural wave mode. The method consists of two steps. The first step is a guided wave experiment, in which a longitudinal wave in the form of a Gaussian broadband signal is excited in the test specimen. The transverse displacement is measured at one point and mainly consists of the lateral contraction due to the incident longitudinal wave and the deflection signal of the flexural wave. In the second step, the measured signal is modified to compensate for evanescent mode leakage and free end effects, reversed in time and applied in a numerical model of the test specimen. The TR simulation reverses the effect of dispersion, such that the flexural wave recovers its initial shape at the location of the crack. On one hand, this allows a relatively easy localization of multiple scatterers from a single one point measurement, on the other hand, the converged flexural wave amplitude can easily be isolated in the TR simulation which is not possible directly from the measured signal because reflections and other modes interfere with the dispersed flexural wave signal. Finally, dividing the recovered flexural wave signal from the simulation by the measured longitudinal wave signal allows the characterization of the depth of the crack.

It is shown that the same procedure can successfully be applied to circular cylindrical shells, if the excitation frequency is chosen low enough so that the structure’s response is mainly that of a beam. Using higher frequencies, dispersion is not as pronounced as in the case of beams, higher order non-axially symmetric modes appear and the method may not perform as well as in the examples presented.
3.1. Quantitative Guided Wave Testing by Applying the Time Reversal Principle on Dispersive Waves in Beams

The above outlined method is validated with a number of experiments on aluminum beams and an aluminum pipe, all incorporating machined notches of different depths and locations. Up to 3 notches were localized from one measurement, with a maximal error of 3% with respect to the propagation distance. The size was accurately predicted for notches as small as 0.5 mm depth or 8.3% of the cross section. It is found that the localization accuracy is mainly determined by how accurate the numerical model matches the wave propagation speed observed in the experiment. The accuracy of the crack characterization is found to be frequency dependent. Between 10 kHz and 40 kHz, the prediction of the notch depth is very accurate, above this frequency margin it is believed that the discrepancy between the ideal analytical model of a crack and the realization of the machined notch becomes substantial.

As a general rule, the better the model the less information is needed for a reliable prediction of the structure’s integrity. This explains why the approach presented is capable of extracting information about location and size of multiple surface breaking cracks out of only one measurement. On the one hand, one benefits from maximal a priori knowledge of the test specimen by using a numerical model of the structure and on the other hand, one achieves maximal exploitation of the measurement since the TR process makes use of the complete waveform instead of interpreting only certain features such as time of arrival.

In the opinion of the authors, the method’s accuracy, robustness and ease of implementation make it attractive for further development and industrial application.
4 Conclusions and Outlook

4.1 Global Objectives and Achievements

The overall objective in NDT is to determine the structural integrity of a component through measurements. The driving question at the beginning of the thesis was:

| Given a numerical model of the test specimen, what is the minimum experimental effort required to examine the structure’s condition? |

In general, the better the model, the fewer information is required to solve an inverse problem. It seemed to be obvious to use a numerical model in combination with an experiment to examine the structure’s condition with minimal experimental effort. Inspired from the work of Fink [23] and Leutenegger et al. [15], a TR approach was chosen to maximally exploit the information contained in the measured data. It was shown in a previous thesis [15] that the multimode character of guided waves can be used in a TR simulation, where the different modes propagate back and superimpose at the location of the defect. This, however, still requires a notable measurement effort because the complete displacement field must be recorded and reapplied in the simulation to correctly excite the different wave modes. The approach taken here is based on the observation that pulse distortion is proportional to the traveled distance of the wave. If the cause of this distortion is dispersion, this distortion can be reversed in a TR process. Thus the hypothesis for the present study was that defects and AE sources can be located by only considering one dispersive mode. Testing of this hypothesis and discussing possible applications and limitations was the objective of this work.

While individual conclusions were given in chapters 2.1.5, 2.2.7 and 3.1.4 for the different applications of AE testing in beams, plates, and guided wave testing in beams respectively, here a global conclusion is drawn.

- The effect of evanescent mode leakage was identified to cause the TR process in general guided waves settings to fail. Evanescent mode leakage refers to the fact that the transient disturbance not only generates propagating waves, but also evanescent waves which do not propagate and hence can not be recorded at the measurement point. Additionally in the TR simulation step, the TR signal is again split into a propagating and an evanescent wave. In the TR simulation at the original position of the disturbance, the arriving waves lack the information contained in the evanescent waves. These evanescent waves however are required in order to restore the original shape of the disturbance.
• **Compensation of evanescent mode leakage** has been achieved in all the applications presented based on numerical manipulations of the measurement signal. This required a priori assumptions on the boundary conditions at the origins of the waves, namely that no rotation occurs at the crack tip and that the AE can be modeled as a point force. These assumptions are justified for open surface cracks and AE sources simulated by breaking pencil leads.

• **The robustness of the method** is a major advantage when compared to other approaches such as e.g. ToA where noise and reflections may affect arrival time picking or mode separation. Reflections and multiple modes in the signal hardly affected the localization performance of the method. Neither did truncation, time delaying, or windowing of the signal cause errors in the localization results. The reasons for this robust behavior is that the complete wave form is used in the process, the process is in a linear elastic range such that the superposition principle holds, and the location information is contained in the phase of the signal. However, care must be taken to not distort the phase relation of the signal. Phase distortion may happen when filtering the measured signal with other than linear- or zero-phase filters.

• **Separation of pulses** in the measured signal is greatly facilitated, because the dispersed broadband pulses again converge in the TR simulation to a short, high amplitude peak. This allows wave mode separation and subsequently evaluation of mode conversion ratios in situations, where mode separation was not possible for conventional methods. This increases the range of applications for quantitative NDT. So far, separation of e.g. incident and reflected wave pulses which partially overlap in the time domain require at least a two point measurement in order to separate the individual pulses [68, 61].

• **Broadband excitation signals** were used instead of narrow band signals. This can reduce the time duration of the wave pulse at its origin for a given frequency range which improves temporal and spatial resolution. Moreover, the mode conversion ratio can be evaluated over a broad frequency band which improves accuracy and robustness of the crack size estimation.

• **Small scatterers can be unveiled**, even when they were covered by larger scatterers in the measured signal. This was achieved by iteratively removing the highest peaks in the signal after forwarding and time reversing the measured signal in simulations. This is of great value to the method presented since the identification of closely located defects or defects close to other geometrical features still poses a major challenge. Note, that no baseline data is required to unveil small scatterers from the signal.

• **Multiple cracks** and multiple AE’s could be detected from a single one point measurement on 1D structures. Again, separation of broadband pulses in the TR simulation was key to resolve multiple events. To what extent multiple sources or events can be identified in a 2D domain is not clear yet. The presented approach discussed in 2.2 which is based on retaining a fixed number of reflections in the measured signal would probably work against the detection
of multiple AE events. In addition, the interactions of multiple AE waves with geometrical features in a plate would create a wave field which is difficult to interpret. This is also the case for other AE techniques and it would be very interesting to make further investigations in how the TR method can be used to the localization of multiple AE’s in 2D structures. A possible starting point could be to look at the time instant when the reversed wave forms converge. While the individual reflections travel at different lengths until they converge, the waves belonging to a specific AE event should all converge at the same time.

- Localization of AE sources from only a single measurement in a 2D structure is arguably the most intriguing result presented in the thesis. The TR process recovers both the effect of dispersion and the effect of reflections which in combination allows the 2D localization of the AE source. According to the author’s knowledge, no other AE localization method has achieved this before.

- Spectral FE method proved to be advantageous over conventional FE methods. Numerical efficiency and accuracy are important factors for the method and these properties are improved in spectral FE compared to conventional elements. The possibility to work with e.g. solely Timoshenko elements was an advantage because localization required only flexural waves. If the signal contains e.g. flexural and longitudinal waves, only flexural waves are recovered and longitudinal waves immediately disperse in the TR simulation. This is also the case for noise and other sources of disturbance and contributes to the robust performance of the method. Moreover, the possibility to model the structure as infinite or semifinite structure, i.e. avoiding reflections facilitates the interpretation of the TR simulation.

### 4.2 Promising Applications and Open Questions

The present thesis reports on TR approaches which exploits the dispersive behavior of guided waves. Exemplary applications were shown for guided waves and AE testing which, on the one hand proved to be very promising and capable, but on the other hand were rather academic. It is therefore suggested that the method proposed is now applied to relevant problems in industry. On the one hand, this would reveal the methods practicality and on the other hand, a numerical simulation tool could be chosen and optimized for a particular problem. Since an accurate numerical model is very important for the success of the presented method, external know how may be required for more complex structures/ materials. Ideally, this would be done in collaboration with external academic or industrial partners. Thorough experience in modeling of composite structures for NDT applications is e.g. found in the group of Prof. Veidt, see [76, 77].

Pipe inspection might be the most prominent application of guided wave testing. In section 3.1 it was shown how defects can be localized in pipes. However, the
working frequencies were limited such that the pipe could be modeled as a beam. This is clearly a limitation which could be overcome by substituting Timoshenko theory with an appropriate shell theory. A good starting point is the implementation of a spectral FE for cylindrical shells, according to [47].

**Cable inspection** for e.g. bridges and cable cars represent another application field for guided wave testing. Treyssede et al. [75] investigated wave propagation problems in multi-wire strands with and without prestress using the semi-analytical FE approach. While the dispersive behavior at low frequency-thickness numbers is similar to beams it becomes very complex at higher frequencies due to inter-wire coupling and helical winding. This strong dispersive behavior could be an advantage for the method presented if an efficient numerical model is found for the cable.

**AE localization in nonuniform plate geometries** is a major concern in current AE implementations according to a recent comprehensive review of AE techniques [43]. Most methods, including the one presented in section 2.2, assume at least one direct wave path from source to sensor. However, using more than one sensor in combination with a numerical simulation, it is believed that the TR process would overcome such difficulties, for example encountered in complex structures such as e.g. wing ribs, shear webs, stringers, and spars, while still requiring fewer sensors as compared to ToA based methods.

**Characterization of real acoustic emission sources.** In Fig. 2.15 we have seen that the transverse time source function of a pencil lead break could be reconstructed from a single measurement on a plate. Solving the inverse source problem, i.e. recovering the moment tensor components of the source event has long been and still is an important research question in both engineering and seismology [39]. The concept of a TR simulation, which has been shown to be capable of reconstructing the initial flexural wave form at the AE event, may be a promising approach for solving the inverse problem. By taking into account the individual mode shapes of the AE generated waves and reconstructing the waves with the help of TR simulations, one could possibly reconstruct the moment tensor by retrieving the original wave form of the various modes and then applying superposition.

**The generation of sharp transient pulses** in another adaption of the method. Instead of detecting the origin of a broad band flexural wave pulse, the method could be adapted to produce a sharp pulse at a specific location. By using a waveguide for dispersive flexural waves and an excitation system, capable of exciting an arbitrary wave signal, locally concentrated high intensity pulses can be generated. Opposite to the presented NDT applications, the time reversal simulation is used first to derive the necessary excitation signal. The signal is then reversed in time and used in an experiment such that the dispersed flexural wave signal converges in the structure. By knowing the dispersive behavior of a particular structure, the transfer function of the electro-mechanical transducer system and the geometry of the beam, energy can be pumped into the waveguide over an extended time period which is then focused to a specific location and time. This effect is further complemented by constructive
interference of reflections at the waveguide’s boundaries, see Fig. 4.1. This would allow e.g. the study of crack propagation due to highly dynamic loads without the load introduction interfering with the crack.

![Figure 4.1: On the left hand side, a time-space diagram displays two flexural wave pulses traveling dispersively in a slender beam. The waves are excited at x = 1.5 m, the first packet is reflected at x = 0 and both waves interfere constructively at x = 0.75 m, resulting in a sharp pulse as shown in the figure on the right hand side, which immediately disperses again.](image)

A number of questions survived the investigations during this study. Throughout this study AE were mimicked by the breaking of pencil leads. In consequence, AE’s were modeled as step forces. An obvious next step is the validation of the method for material induced AE. This could be approached by cyclic loading of a pre-notched beam, as reported e.g. by Schlums et al. [66]. The growth of a fatigue crack at the notch is known to be accompanied by AE’s which may be recorded with a laser vibrometer. The AE localization technique is then easily validated since the AE source-sensor distance is determined by the position of the notch.

In case of a beam, multiple AE’s could be identified from one measurement. This is an important A further inaccuracy in the AE tests presented is the absence of external loads. Typically, AE’s are excited while the structure is under load. However, wave propagation speed is load dependent. Consequently, the loading condition in the experiment should be implemented in the numerical model used for the TR simulation.

An open question in guided wave testing is as follows: Given an accurate model of the cracked beam together with the complete information about the reflected, transmitted and mode converted waves; can the original wave that was incident on the crack be recovered in a TR simulation? Do the reflected and transmitted waves vanish after they have interacted with the defect in the simulation? While this thought experiment has never found an application in the examples presented, one could think of a new method for damage characterization that makes use of this idea. Given that in the ideal case, all reflected and transmitted waves vanish in the TR simulation after having interacted with the crack, an optimization algorithm could be established, which tries to minimize the reflected and transmitted waves
past their origin by varying the properties of the crack.
Related to this question is the following: In the TR simulations, the structure has always been modeled damage free. However, the TR process was designed with the assumption that the structure is similar in experiment and in TR simulation. Obviously, there must be some deviation from the TR simulated response to the response at the crack in the experiment. For the methods presented this effect was found to be negligible. Nevertheless, a closer look at this inconsistency might be rewarding and may reveal further insights to the TR process.

The author sees great potential in the combination of time reversal processes and dispersive waves. This thesis provides a first impression on the various application options and the strengths and limitations of this methodology. It is hoped that research continues on that topic and eventually leads to a valuable complement to time of arrival techniques.
5 Appendix

5.1 Impact Localization Demonstration

In order to demonstrate the working principle of the presented time reversal localization method, a laboratory experiment has been set up which allows almost real time impact localization on an aluminum beam. This demonstration experiment is based on the AE localization implementation presented in section 2.1 and was shown several times to visiting researchers as part of a lab show or to students during class in the wave propagation course. The experiment can be performed interactively with students or visitors due to its robustness and may be used to discuss longitudinal and flexural waves, dispersion, time reversal principle and effect of signal filtering and group delay.

The experimental setup and the instrument settings are illustrated in Fig. 5.1. The necessary Matlab scripts and LabView file can be found in the FSI group’s intern folder. The test specimen is a 2 m aluminum beam with quadratic cross section of 6 mm side length. Material properties for this beam are stored in the Matlab script GWP2013MatProp.m. The beam may be supported by several foam pieces and a tape rule is placed beside the beam for reference. An impact is then generated by gently hitting the beam with a small hammer or a similar object while memorizing the impact location. The resulting transverse velocity signal at the end of the beam is measured with a laser interferometer system and the data is acquired via the PC’s DAQ card. Triggering and data acquisition is controlled over the LabView file DAQ Laser only-0.0.vi with settings chosen according to Fig. 5.2. The measured data is then processed by the Matlab script Locate_V4.m which runs a semi-infinite time reversal simulation of the recorded data. The simulation result is then plotted as a time-space diagram and the impact position is indicated.

Since the simulation takes only a few seconds and the results are then showed on screen, this setup lends itself to experimenting and exploring with different kinds of impacts. The original velocity time function at the impact location as well as the deflection line at the impact time instant is restored and plotted on the screen. This can help building intuition for impact dynamics due to the immediate response on the screen after hitting the beam. By changing the impact location, dispersion can be illustrated by the transverse velocity response which is also plotted on the LabView GUI. By measuring the transverse response away from the free end, both flexural and longitudinal waves are visible in the signal. Measuring at the edge of the free end, the lack of a reaction force results in vanishing lateral contraction and hence the longitudinal wave is not in the transverse signal anymore. Finally, the effect of

\[\text{\textit{\textcopyright ortrud\groups\FSI-Group\Impact Localization Demo Experiment}}\]
Figure 5.1: Experimental setup for impact localization experiments. The beam’s oscillatory response due to an impact is captured by the laser interferometer which measures the transverse velocity at the free end of the beam. The signal is split and then fed to channel AI0 and AI1 of the DAQ card of the Lab. PC. The signal at channel AI0 is band limited in order to be used as a robust trigger signal. Data acquisition is controlled by the LabView file DAQ Laser only-0.0.vi and the subsequent processing is controlled by the Matlab script Locate_V4.m

filtering the signal can be studied as well. The present setup splits the decoder signal in a trigger signal connected to channel AI0 and a raw signal connected to channel AI1. Filtering the signal does alter the signal’s phase relation and therefore change the overall dispersion characteristic of the system. By switching the cables at AI0 and AI1, This becomes apparent when switching the cables at AI0 and AI1. The computed time reversal simulation will now show incorrect localization results, because the filter has affected the phase relationship originally determined by the dispersion characteristics of the flexural wave.
5.1. Impact Localization Demonstration

Figure 5.2: Settings used in LabView GUI.
Appendix

6.1 Abaqus sample scripts

Numerical simulations of wave propagation problems were done either in Matlab using spectral elements or in Abaqus 6.9 as FE calculations. The simulations were defined using Abaqus’ scripting functionality which uses Python scripting language to control the generation of the input file and post-processing. To facilitate reproduction of the presented simulation results, three sample python scripts are listed in the following sections. Furthermore, the scripts together with the amplitude input files can be found in the FSI group’s intern folder.

6.1.1 Abaqus Beam Python Script

```
# Sample 2D Beam simulation
# Conversion study confirmed up to 100 kHz

from part import *
from material import *
from section import *
from assembly import *
from step import *
from interaction import *
from load import *
from mesh import *
from job import *
from sketch import *
from visualization import *
from connectorBehavior import *

# Intended to start a number of subsequent simulation.
# However here, only Sim. 'C1'B01_D00007' is generated.
jobName = ['C1'B01_B00000',
'C1'B01_B00001',
'C1'B01_B00002',
'C1'B01_C00003',
'C1'B01_C00004',
'C1'B01_C00005',
'C1'B01_D00006',
'C1'B01_D00007',
'C1'B01_D00008']

for Jname in jobName:
    Excitname = 'TRv04%a.txt' %(Jname)
    JnameODB = '%a_straight' %(Jname)

    # geometry
    L=5000
t=6
    L_c=2600 #Position of Crack (L_c<L-1)
d=1 #depth of the crack
w=100 #width of the crack
E=6.82e4 #from dispersion relation
nu=0.34
mi=0.34

    # Parameters for Analyse
    time=3e-3 #time for which analyzing should be done
    dt=1e-8 #Step time

    # Variables
    mySketch=mdb.models['Model-1'].sketches
```
myModel=mdb.models['Model-1']

# Geometry definition
mdb.models['Model-1'].ConstrainedSketch(name='__profile__', sheetSize=1300.0)

myCoord=[
    (0.0, 0.5*t), #1
    (L_c, 0.5*t), #2
    (L_c, (0.5*t)+d), #3
    (L_cw, (0.5*t)-d), #4
    (L_cw, 0.5*t), #5
    (L, 0.5*t), #6
    (L, 0.0), #7
    (L, 0.0, 0.5*t), #8
    (L, (0.5*t)+d), #9
    (L, (0.5*t)-d), #10
    (L_c, (0.0, 0.5)t), #11
    (0.0, 0.0), #12
    (0.0, 0.5*t) #13
]

for i in range(len(myCoord)-1):
    mySketch["__profile__"].Line(point1=myCoord[i], point2=myCoord[i+1])

myModel.Part(dimensionality=THREE_D, type=PLANAR, name="Part-1").BaseShell(sketch=mySketch["__profile__")

# material definition
myModel.Material(name="Material-1")

myModel.m XIerials["Material-1"] = Elastic(table=((E, mu),))

# section definition
myModel.HomogeneousSolidSection(name="Section-1", thickness=None)

myModel.parts["Part-1"].SectionAssignment(offset=0.0, offsetType=MIDDLE_SURFACE, offsetField="厚度", regionName="Section-1")

# Creating Instances
myModel.rootAssembly.DatumCsysByDefault(CARTESIAN)

# Creating step
myModel.ExPLICITDynamicStep(linearBulkViscosity=0.0, name="Step-1", nlgeom=OFF, timeIncrementationMethod=FINITE_INCREMENTAL, timePeriod=0.0, userDefinedInc=dt)

# open datafile
Exci tationpaths=K:\/01_Diss Projekt\AB_TRNS//03_transformation/\%A' %Excitname # ! Adjust Path! data = open(Excitationpath)

# build x and y list
x=[]; y=[]
for line in data:
    xval, yval = line.split()
    x.append(float(xval))
    y.append(float(yval))

# build data list (list of tuples)
datavec=[]
for i in range(len(x)):
    datavec.append((x[i],y[i]))

myModel.TabularAmplitude(data=datavec, name="Amp-1", smooth=SOLVER_DEFAULT, timeSpan=STEP)

# creating meshing and features
P1=myModel.rootAssembly.DatumPointByCoordinate(coords=(L_c, 0.5*t, 0.0))
P2=myModel.rootAssembly.DatumPointByCoordinate(coords=(L_c, 0.5*t, 0.0))
P3=myModel.rootAssembly.DatumPointByCoordinate(coords=(L_cw, 0.5*t, 0.0))
P4=myModel.rootAssembly.DatumPointByCoordinate(coords=(L_cw, 0.5*t, 0.0))

# two variants; with or without notch
if w != 0.0 or d != 0.0:
    with notch
        a = mdb.models['Model-1'].rootAssembly
        fl = a.instances['Bar-1'].faces
        pickedFaces = fl.findAt(((1.0, 0.0),))
        dl = a.datums
        a.PartitionFaceByShortestPath(point1=dl[4], point2=dl[5], faces=pickedFaces)
        fl = a.instances['Bar-1'].faces
        pickedFaces = fl.findAt(((1.0, 0.0),))
        dl = a.datums
        a.PartitionFaceByShortestPath(point1=dl[6], point2=dl[7], faces=pickedFaces)
        a = mdb.models['Model-1'].rootAssembly
        fl = a.instances['Bar-1'].faces
        pickedRegions = fl.findAt(((1.0, 0.0),))
        mdb.models['Model-1'].rootAssembly.setMeshControls(regions=pickedRegions, elemShapes=QUAD, technique=STRUCTURED)
        pickedRegions = fl.findAt(((1.0, 0.0),))
        mdb.models['Model-1'].rootAssembly.setMeshControls(regions=pickedRegions, elemShapes=QUAD, technique=STRUCTURED)
6.1. Abaqus sample scripts

```
pickedRegions = f1.findAt(((Lc+1.0,0.),))
mb.addModels([Model-1], rootAssembly.setMeshControls(regions=pickedRegions, 
  elemShapes=QUAD, techniques=STRUCTURED)
myModel.rootAssembly.generateMesh(regions=myModel.rootAssembly.instances['Bar-1'],)
myModel.rootAssembly.seedPartInstance(deviationFactor=0.1, regions= 
  myModel.rootAssembly.instances['Bar-1'], size=1.0)
myModel.rootAssembly.generateMesh(regions=myModel.rootAssembly.instances['Bar-1'],)

# Creating Sets
myModel.rootAssembly.Set(edges=myModel.rootAssembly.instances['Bar-1'], edges.getSequenceFromMask( 
  ([#40] , ) , name='A')
myModel.rootAssembly.Set(edges=myModel.rootAssembly.instances['Bar-1'], edges.getSequenceFromMask( 
  ([#200] , ) , name='B')

# Creating boundary condition
myModel.DisplacementBC(amplitude='Amp-1', createStepName='Step-1', distributionType=UNIFORM, 
  fieldName='', fixed=OFF, localCsys=None, name='BC-1', 
  regions=Region(edges=myModel.rootAssembly.instances['Bar-1'], edges.findAt(((L,0.0,0.0),)), 
  ub=UNSET, u2=1.0, ur3=UNSET)

else:
  myModel.rootAssembly.seedPartInstance(deviationFactor=0.1, 
  regions=(myModel.rootAssembly.instances['Bar-1'],), size=1.0)
a = mdb.models['Model-1'].rootAssembly
f1 = a.instances['Bar-1'].faces
pickedRegions = f1.at(((L,0.0,0.0),))
mb.addModels([Model-1], rootAssembly.setMeshControls(regions=pickedRegions, elemShapes=QUAD, 
  techniques=STRUCTURED)
myModel.rootAssembly.generateMesh(regions=(myModel.rootAssembly.instances['Bar-1'],))

# Creating boundary condition
myModel.DisplacementBC(amplitude='Amp-1', createStepName='Step-1', distributionType=UNIFORM, 
  fieldName='', fixed=OFF, localCsys=None, name='BC-1', 
  regions=Region(edges=myModel.rootAssembly.instances['Bar-1'], edges.findAt(((L,0.0,0.0),)), 
  ub=UNSET, u2=1.0, ur3=UNSET)

# Creating Node sets
# Center Nodes Label List
N0=[7421]
N1=range(22015,35006,5)
N2600=[4718]
N3000=[2315]
N2701=[11]
N2700=[10025,21516,5]
N3000=[2315]
Ncent=[]
Ncent+=N0+N1+N2600+N2601+N2700+N2701+N3000

# Creating Sets
# n2 = a.instances['Bar-1'], nodes
n2 = a.instances['Bar-1'].nodes
nodes2 = n2.getSequenceFromMask((Ncent))
a.Set(nodes=nodes2, name='Set-cent')

# Deleting default output
def del mdb.models['Model-1'].fieldOutputRequests['F-Output-1']

# Creating field output, Global
regionDefcnt=myModel.rootAssembly.sets['Set-cent']
myModel.FieldOutputRequest(name='F-Output-cent', 
  createStepName='Step-1', variables=('U', 'UT', 'UR'), timeInterval=1e-6, 
  timeMarkers=ON, region=regionDefcnt, sectionPoints=DEFAULT, rebar=EXCLUDE)

# Create Job
newJob=mdb.Job(name=JnameODB, models=['Model-1'], description=Jname, type=ANALYSIS, 
  atTime=None, waitMinutes=0, waitHours=0, queue=None, memory=90, 
  memoryUnits=PERCENTAGE, getMemoryFromAnalysis=True, 
  explicit=PRECISIoN, nodalOutputPrecisions=PRECISIoN, echoPrints=OFF, 
  modelPrint=OFF, contactPrint=OFF, historyPrints=OFF, userSubroutine='', 
  scratch='', parallelizationMethod=explicit=DOMAIN, 
  multiprocessingMode=DEFAULT, numDomains=1, numCpus=1)

# Wait for the job to complete.
# newJob.submit()
# newJob.waitForCompletion()
```

6.1.2 Abaqus Pipe Python Script

# Sample 3D Pipe simulation
# Conversion study confirmed up to 30 kHz
# Discussion of optimal parameters for pipe
# Simulation found in Semester Thesis Tobias
# Renz, Guided wave testing by time-reversal-simulation
# of dispersive waves in pipes, 2013

# coding: mbcs
from material import *
from section import *
from assembly import *
from step import *
from interaction import *
from load import *
from mesh import *
from job import *
from sketch import *
from visualization import *
from connectorBehavior import *

# Define jobs
jobName = ['TRsig_compens_C1notchalupipe_00008 ']
for Jname in jobName:
    Excitname = ' TRsig_compens_C1notchalupipe_00008 '
    JnameODB = ' TRsig_compens_C1notchalupipe_00008 '

# Geometry
R=0.015
# radius to middle surface
L=6
# length
t=0.002
# shell thickness
Lc=2
# position of crack (Lc<L−1)
d=1
# width of the crack
E=6.58e10
# from Diss. D. Gsell (2002)
rho=2700
# from Diss. D. Gsell (2002)
nu=0.338
# from Diss. D. Gsell (2002)

# Mesh
nel =40
# number of circumferential elements

# Geometry definition
myModel. ConstrainedSketch(name='profile', sheetSize=0.1)
mySketch['profile'].CircleByCenterPerimeter(center=(0.0, 0.0), point1=(0.0, R))

# Mesh properties
acc=ON
# 2nd order accuracy
hgc=ENHANCED
# hourglass control
ey=S4R
# element type

# Parameters for Analysis
time=5e−3
# time for which analyzing should be done
dt=2.7e−7
# step time

# Variables
mySketch=mdb.models['Model-1'].sketches
myModel=mdb.models['Model-1']

# Creating instances
myModel.rootAssembly.Instance(dependent=OFF, name='Part-1'),
part=myModel.parts['Part-1'])

# Creating step
myModel.ExplicitDynamicsStep(name='Step-1', previous='Initial',
timeIncrementationMethod=FIXEDUSER_DEFINED, timePeriod=time,
userDefinedInc=dt, linearBulkViscosity=0.0, quadBulkViscosity=0.0)

# Define amplitude
Excitationpath = 'K:/01_DissProjekt/PipeTesting/Abaqus/Files_Tobias/amplitude/%s.txt ' %Excitname
data = open(Excitationpath)

# Build x and y list
x= []; y=[]
for line in data:
xval, yval=line.split()
x.append(float(xval))
y.append(float(yval))

# Build data list (list of tuples)
datavec=[]
for i in range(len(x)):
    xy=tuple([x[i], y[i]])
    datavec.append(xy)

# Create sets
6.1 Abaqus sample scripts

```python
# create mesh and define properties
myModel.rootAssembly.setMeshControls(elemShape=QUAD, regions=[myModel.rootAssembly.instances['Part-1-1'].faces.findAt(((0.003893, 0.014486, 2.666667),)), technique=SWEEP)
myModel.rootAssembly.seedEdgeByNumber(edges=myModel.rootAssembly.instances['Part-1-1'].edges.findAt(((R, 0.0, L),)), number=nel)
myModel.rootAssembly.generateMesh(regions=[myModel.rootAssembly.instances['Part-1-1'].faces.findAt(((0.003893, 0.014486, 2.666667),))])
myModel.rootAssembly.setElementType(elemTypes=[ElemType(elemCode=ety, elemLibrary=EXPLICIT, secondOrderAccuracy=acc, hourglassControl=hgc), ElemType(elemCode=S3R, elemLibrary=EXPLICIT)], regions=[myModel.rootAssembly.instances['Part-1-1'].faces.findAt(((0.003893, 0.014486, 2.666667),))])

# Creating boundary condition
myModel.DisplacementBC(amplitude='Amp-1', createStepName='Step-1', distributionType=UNIFORM, fieldName='', fixed=OFF, localCSys=None, name='BC-1', region=Region(edges=myModel.rootAssembly.instances['Part-1-1'].edges.findAt(((R, 0.0, 0.0),)), ui=UNSET, u2=1e-6, u3=0.0, url=UNSET, ur2=UNSET, ur3=UNSET)

# Creating Node sets
# Upper Nodes Label List
N1=range(1,1700,1)

# Creating Sets
myModel.rootAssembly.Set(nodes=myModel.rootAssembly.instances['Part-1-1'].nodes.sequenceFromLabels((N1)), name='SetUpperNodes')

# Deleting default output
del myModel.historyOutputRequests['H-Output-1']
del myModel.fieldOutputRequests['F-Output-1']

# Creating field output, Global
myModel.FieldOutputRequest(name='F-Output-Global', createStepName='Step-1', variables=('U',), timeInterval=1e-06, timeMarks=ON, directions=ON, regions=MODEL, sectionPoints=DEFAULT, rebar=EXCLUDE)

# Create Job
newJob = mdb.Job(name=JnameODB, model='Model-1', description=Jname, type=ANALYSIS, atTime=None, waitMinutes=0, waitHours=0, queue=None, memory=90, memoryUnits=PERCENTAGE, getMemoryFromAnalysis=True, explicit=PRECISION, nodalOutputPrecision=SINGLE, echoPrint=OFF, modelPrint=OFF, contactPrint=OFF, historyPrint=OFF, userSubroutine=' ', scratch='', parallelizationMethod=EXPLICIT, multiprocessingMode=DEFAULT, numDomains=1, numCpus=1)

# Wait for the job to complete.
newJob.submit()  # newJob.waitForCompletion()
```

6.1.3 Abaqus Plate Python Script

```python
# Sample 3D Plate simulation
# Conversion study confirmed up to 50 kHz

# --- coding: mbcs ---
from part import *
from material import *
from section import *
from assembly import *
from step import *
from interaction import *
from load import *
from mesh import *
from job import *
from sketch import *
from visualization import *
from connectorBehavior import *
import regionTools

# Material data: Aluminum
E=68.2e9  #Pa
rho=2700  #kg/m^3
nu=0.31  #-

#Timing
stepTime1=2.7e-3  #s
stepTime2=3.0e-3-stepTime1  #_tot=step
dt=1e-7  #s

# Geometry of rectangular plate
Lx=75  #m
Ly=75  #m
t=0.002  #m

# Element size
```
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E1 = 0.002;

# Geometry definition
myModel = mdb.models['Model-1'].sketches
s = myModel.ConstrainedSketch(name='profile', sheetSize=2.0)

# Corner
e21 = (-0.375, 0.375, 0.002)
e22 = (0.375, 0.375, 0.002)
e23 = (0.375, -0.375, 0.002)
e24 = (-0.375, -0.375, 0.002)
e26 = (-0.01, 0.02, 0.002)
e27 = (0.01, 0.02, 0.002)
e28 = (0.01, -0.02, 0.002)
e29 = (-0.01, -0.02, 0.002)
e11 = (-0.375, 0.375, 0.001)
e12 = (0.375, 0.375, 0.001)
e13 = (0.375, -0.375, 0.001)
e14 = (-0.375, -0.375, 0.001)
e16 = (-0.01, 0.02, 0.001)
e17 = (0.01, 0.02, 0.001)
e18 = (0.01, -0.02, 0.001)
e19 = (-0.01, -0.02, 0.001)
e01 = (-0.375, 0.375, 0)
e02 = (0.375, 0.375, 0)
e03 = (0.375, -0.375, 0)
e04 = (-0.375, -0.375, 0)
e06 = (-0.01, 0.02, 0)
e07 = (0.01, 0.02, 0)
e08 = (0.01, -0.02, 0)
e09 = (-0.01, -0.02, 0)
p1 = (-0.375, 0.375)
p2 = (0.375, 0.375)
p3 = (0.375, -0.375)
p4 = (-0.375, -0.375)
p6 = (-0.01, 0.02)
p7 = (0.01, 0.02)
p8 = (0.01, -0.02)
p9 = (-0.01, -0.02)
pA = (-0.2, 0.15, 0.002)
pB = (0.25, 0.2, 0.002)
pC = (0.3, -0.25, 0.002)
pD = (-0.01, 0.0, 0.002)
pE = (-0.01, 0.0, 0.002)
ps = (0.2, -0.155, 0.002)

xyCoord=[p1, p2, p3, p4, p1,]
for i in range(len(xyCoord)-1):
    mySketch['profile'].Line(point1=xyCoord[i], point2=xyCoord[i+1])

xyCoord=[p6, p7, p8, p9, p6,]
for i in range(len(xyCoord)-1):
    mySketch['profile'].Line(point1=xyCoord[i], point2=xyCoord[i+1])

# Part: extrude
p = myModel.Part(name='AluPlate', dimensionality=THREE_D, type=DEFORMABLE_BODY)
p = myModel.parts['AluPlate']
p.BaseSolidExtrude(sketches=sheet, depth=)

# View
session.viewports['Viewport:1'].setValues(displayedObject=p)
session.viewports['Viewport:1'].view.setProjection(projection=PARALLEL)
session.viewports['Viewport:1'].view.rotate(aAngle=0, yAngle=0, zAngle=0, mode=TOTAL)

# Material definition
myModel.Material(name='Aluminum')
myModel.materials['Aluminum'].Density(table=(rho,))
myModel.materials['Aluminum'].Elastic(table=((E, nu),))

# Section definition
myModel.HomogeneousSolidSection(name='AluSection',

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```plaintext
material='Aluminum', thickness=None)
c = p.cells
cells = c.getSequenceFromMask(mask=['#1'],
region = regionToolset.Region(cells=cells)

# Section assignment
p.SectionAssignment(region=region, sectionName='AluSection', offset=0.0,
offsetType=MIDDLE_SURFACE, offsetField='')

# Instance definition
a = myModel.rootAssembly
a.DatumCsysByDefault(CARTESIAN)
a.Instance(name='AluPlate-1', part=myModel.parts['AluPlate'],
dependent=OFF)

# Element Type
f1 = a.instances['AluPlate-1'].faces
pickedRegions = f1.findAt(((0.0, 0.0),))

myModel.rootAssembly.setElementType(elemTypes=(
ElemType(elemCode=C3D8I, elemLibrary=EXPLICIT,
secondOrderAccuracy=ON, distortionControl=DEFAULT, linearBulkViscosity=0.0,
quadraticBulkViscosity=0.0),
ElemType(elemCode=C3D6, elemLibrary=EXPLICIT),
ElemType(elemCode=C3D4, elemLibrary=EXPLICIT)),
regions=[
myModel.rootAssembly.instances['AluPlate-1'].cells.findAt(((0.1, 0.2e-3),))])

# Partitioning via three points
p = myModel.parts['AluPlate']
# first Partition
p.DatumPointByCoordinate(coords=e26)
dp1 = p.datums.keys()[-1] # aktuell letzter der datum points

p.DatumPointByCoordinate(coords=e27)
dp2 = p.datums.keys()[-1] # aktuell letzter der datum points

p.DatumPointByCoordinate(coords=e07)
dp3 = p.datums.keys()[-1] # aktuell letzter der datum points

volumen = p.cells[0]
p.PartitionCellByPlaneThreePoints(point1=p.datums[dp1],
point2=p.datums[dp2], point3=p.datums[dp3],
cells=volumen)

# second Partition
p.DatumPointByCoordinate(coords=e28)
dp1 = p.datums.keys()[-1] # aktuell letzter der datum points

p.DatumPointByCoordinate(coords=e29)
dp2 = p.datums.keys()[-1] # aktuell letzter der datum points

p.DatumPointByCoordinate(coords=e09)
dp3 = p.datums.keys()[-1] # aktuell letzter der datum points

volumen = p.cells[-1]
p.PartitionCellByPlaneThreePoints(point1=p.datums[dp1],
point2=p.datums[dp2], point3=p.datums[dp3],
cells=volumen)

# Seed thickness (for 50 kHz, 1x2x2 mm necessary!)
e1 = a.instances['AluPlate-1'].edges
pickedEdges = e1.findAt(((e11,), (e12,), (e13,), (e14,), (e15,), (e16,), (e17,), (e18,), (e19,)))
a.seedEdgeBySize(edges=pickedEdges, size=0.001)

# Seed faces
epx = (0.001, 0.0)
epy = (0.0, 0.01)
pickedEdges = e1.findAt(((e01,), (e02,), (e03,), (e04,), (e05,), (e06,), (e07,), (e08,), (e09,)))
a.seedEdgeBySize(edges=pickedEdges, size=EI)
pickedEdges = e1.findAt(((e21,), (e22,), (e23,), (e24,), (e25,), (e26,), (e27,), (e28,), (e29,)))
a.seedEdgeBySize(edges=pickedEdges, size=EI)
a.regenerate()

# # Generate mesh
a = mdb.models['Model-1'].rootAssembly
session.viewports['Viewport:1'].setValues(displayedObject=a)
a = mdb.models['Model-1'].rootAssembly
partInstances = a.instances['AluPlate-1'].parts

# # Datum points for AE Sources

# Impact
a.DatumPointByCoordinate(coords=pA)
a.features.changeKey(fromName='Datum_pt-1', toName='DP_A')
a.DatumPointByCoordinate(coords=pB)
a.features.changeKey(fromName='Datum_pt-1', toName='DP_B')
a.DatumPointByCoordinate(coords=pC)
a.features.changeKey(fromName='Datum_pt-1', toName='DP_C')
```

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Datum Points for Node Sets for Measurements

#NS1
# a. DatumPointByCoordinate (coords=pD)
a. features.changeKey(fromName='Datum pt-1', toName='DP_D')
a. DatumPointByCoordinate (coords=pE)
a. features.changeKey(fromName='Datum pt-1', toName='DP_E')

# NS2
# a. DatumPointByCoordinate (coords=ps)
a. features.changeKey(fromName='Datum pt-1', toName='DP_s')

# Node Sets

# Excitation
a. SetFromNodeLabels(name='NSA', nodeLabels=(( 'AluPlate-1', (227777),))).
a. SetFromNodeLabels(name='NSB', nodeLabels=(( 'AluPlate-1', (267627),))).
a. SetFromNodeLabels(name='NSC', nodeLabels=(( 'AluPlate-1', (72032),))).
a. SetFromNodeLabels(name='NSD', nodeLabels=(( 'AluPlate-1', (236590),))).
a. SetFromNodeLabels(name='NSE', nodeLabels=(( 'AluPlate-1', (34),))).

# NS1
a. SetFromNodeLabels(name='NSs', nodeLabels=(( 'AluPlate-1', (63135),))).

# Selected nodes
# importNodes = open ('K:\Masterarbeit\Simulationen\AE_TR_Sim075\NodeList075.txt')
# selNodes =[]
# for nLabel in importNodes:
#   selNodes.append(int(nLabel))
# selNodes=tuple(selNodes)
a. SetFromNodeLabels(name='Selected Nodes', nodeLabels=(( 'AluPlate-1', (selNodes),))).

# Top layer
f1 = a.instances[ 'AluPlate-1' ].faces
topFace = f1.findAt(((.3,.3,.002),),(-.3,0,.002),(.3,0,.002),(-.3,-.3,.002),)
a. Set(name='topFaceNodes', faces = topFace)

# Creating steps
myModel.ExplicitDynamicsStep(name='excitation', previous='Initial', timePeriod=steptime1, timeIncrementationMethod=FIXED_USER_DEFINED, userDefinedInc=dt, linearBulkViscosity=0.0, quadBulkViscosity=0.0)

mdb.models[ 'Model-1' ].ExplicitDynamicsStep(name='Propagation', previous='excitation', description='Propagation', timePeriod=steptime2, timeIncrementationMethod=FIXED_USER_DEFINED, userDefinedInc=dt, linearBulkViscosity=0.0, quadBulkViscosity=0.0)

# Creating Amplitude
# Amplitude Definition; get data from external file
# open data file; must be in workdirectory
data = open ('K:\01_DissProjekt\AE-Plate\PLB_Measurements\TR_Mod_C2_not_200007.txt')
# build x and y list
x =[]; y=[]
for line in data:
xval , yval=line.split()
x.append(float(xval))
y.append(float(yval))

# build data list (list of tuples)
datavec=[]
for i in range(len(x)):
xy=tuple([x[i],y[i]])
datavec.append(xy)

myModel.TabularAmplitude(name='Amp', timeSpan=STEP, smooth=SOLVER_DEFAULT, data=datavec)

# Boundary Condition - Excitation
region = a.sets[ 'NSA' ]
myModel.VelocityBC(name='BC_TR', createStepName='excitation', region=region, v1=UNSET, v2=UNSET, v3=1, vr1=UNSET, vr2=UNSET, vr3=UNSET, amplitude='Amp', distributionType=UNIFORM, fieldName='', localCsys=None)

# region = a.sets[ 'NSExc' ]
# mdb.models[ 'Model-1' ].ConcentratedForce (name='Load-1',
# createStepName='excitation', region=region, cF3=1.0, amplitude='Amp',
# distributionType=UNIFORM, field=' ', localCsys=None)
6.1. Abaqus sample scripts

```python
# Field Output Request

del mdb.models['Model-1'].fieldOutputRequests['F-Output-1']
del mdb.models['Model-1'].historyOutputRequests['H-Output-1']
regionDef= sets['topFaceNodes']
myModel.FieldOutputRequest(name='topFaceNodes1', createStepName='excitation', variables=('AT', ), timeInterval=2e-06, region=regionDef, sectionPoints=DEFAULT, rebar=EXCLUDE)

myModel.FieldOutputRequest(name='topFaceNodes2', createStepName='propagation', variables=('AT', ), timeInterval=2e-06, region=regionDef, sectionPoints=DEFAULT, rebar=EXCLUDE)
```
7 Appendix

7.1 Analytical Crack Model

In order to characterize the depth of cracks, an analytical crack model was used. The model establishes a 6x6 system of equations for the unknown wave amplitudes, namely

$$A \mathbf{s} = \mathbf{b}$$  \hspace{1cm} (7.1)

Vectors s and b have entries $s = [u_t, u_r, y_t, y_{st}, y_{r}, y_{sr}]^T$ and $b = [u_i, \psi_t, y_i, N_i, M_i, Q_i]^T$. For more detail on the modeling approach, the reader is referred to section 3.1.2.2.

In the provided scripts, variables in the above equations are denoted $\mathbf{A} = \mathbf{A}A$, $\mathbf{s} = \mathbf{s}V$ and $\mathbf{b}V = \mathbf{b}$. Matrix $\mathbf{A}$ is derived in Matlab’s symbolic math. environment MuPAD. The file is named CrackModel.mn. Matrix $\mathbf{A}$ cannot be inverted algebraically and therefore required a Matlab script CrackModelAnalysis.m to invert $\mathbf{A}$ numerically in the frequency domain. Both scripts are listed here for reference and can be found in the FSI group’s intern folder.

7.1.1 Matlab Script CrackModelAnalysis.m

```matlab
function [] = CrackModelAnalysis()
% Inverting numerically Matrix A, defined in a MuPAD script.
% Simulating the time response due to an incident longitudinal wave

% ! make sure 'CrackModel.mn' is evaluated from beginning to end! after
% calling it at the following line!
CrackModel_Long_inc = mupad('CrackModel.mn')
AASub = getVar(CrackModel_Long_inc, 'AASub');
bVSub = getVar(CrackModel_Long_inc, 'bVSub');
omegac = double(getVar(CrackModel_Long_inc, 'omegac'));
cracksize = double(getVar(CrackModel_Long_inc, 'a'));
thick = double(getVar(CrackModel_Long_inc, 'b'));

% Define Signal
T1 = 10e-6;
T = 200*T1;
dT = T/20;
N = round(T/dT);
n = (0:N-1)';
t = n*dT;
u0 = normpdf(t,50*T1,T1/4)*T1*1e-6;
% Fourier transform
[even omega u0] = time2freq(t, u0);
Q = floor(N/2)+1;
% Number of omega increments
N = 50
omega = linspace(1000,T1/4,omega, N);
freq = omega/(2*pi);

sV = zeros(Q,6);
bVSub_new = zeros(Q,6);
bVSub_new(:,1) = -u0(1);
bVSub_new(:,4) = u0(3);
for j=1:Q
   AASub_new = subs(AASub,omega(j));
end
```

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% VSub_neu = sub(s(VSub, omega(j))); % sV(j,:) = vpa(AA(VSub,nu,6)/VSub_neu(j,:));
end
[t ut ur wt wst u0] = freq2time(even, omega(sV(:,1), sV(:,2), sV(:,3), sV(:,4), u0);

function [even omega varargout] = time2freq(t, varargin)

figure(1)

subplot(3,1,1)
plot(t,u0)
ylim([a,b])
legend('u_{incident}''

subplot(3,1,2)
plot(t,(ut))
ylim([a,b])
xlim([c,d])
legend('u_{transmitted}''

subplot(3,1,3)
plot(t,(ur))
ylim([a,b])
xlim([c,d])
legend('u_{reflected}''

figure(2)

subplot(3,1,1)
plot(t,(wt))
ylim([a,b])
xlim([c,d])
legend('y_{k1}''

subplot(3,1,2)
plot(t,(wst))
ylim([a,b])
xlim([c,d])
legend('y_{k2}''

subplot(3,1,3)
plot(t,(wst+wt))
ylim([a,b])
xlim([c,d])
legend('y_{k1}+y_{k2}''

figure(3)

[x, h1, h2] = plotyy(t, u0/max(u0), t, -2*(wt+wst)/max(u0))

hold(ax(2), 'on')
plot(ax(2), t, -1*wt/max(u0), 'r', t, -1*wst/max(u0), 'k')

xlim(ax(2), [-0.5*1.2 0.5*1.2])
ylim(ax(1), [-1.2 1.2])

end

function [even omega varargout] = time2freq(t, varargin)

figure(3)

[x, h1, h2] = plotyy(t, u0/max(u0), t, -2*(wt+wst)/max(u0))

hold(ax(2), 'on')
plot(ax(2), t, -1*wt/max(u0), 'r', t, -1*wst/max(u0), 'k')

xlim(ax(2), [-0.5*1.2 0.5*1.2])
ylim(ax(1), [-1.2 1.2])

end
function [t varargout] = freq2time( even, omega, varargin)
    odd = ~even;
    \% Timing properties
    f = omega/(2*pi);
    df = mean(diff(f)); \% time increment
    Q = length(f); \% number of frequencies
    \% Frequency sampling for single-sided spectrum
    T = 1/df; \% length of time signal
    t = linspace(0,T,N+1); t(end) = []; \% time vector
    \% Loop all input variables
    for i = 1:nargin-2,
        X = varargin{i};
        \% Expand to two-sided spectrum
        if even,
            X = [X(1:end-1,:) abs(X(end,:))]
        else
            X = [X conj(X(end-1:1,-1:2,:))]
        end
        \% Inverse Fourier transform
        X(~isfinite(X)) = 0;
        x = ifft(X,[],1,'symmetric');
        varargout{i} = x;
    end

7.1.2 MuPAD Script Crack_Model.mn
Generalized Spring Model of a Crack with simplified Stress functions

Stress Functions

\[ F_{11} = \begin{pmatrix} 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \end{pmatrix} \]

\[ F_{22} = \begin{pmatrix} 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \\ 3.22 \times 10^{-14} & 1.06 \times 10^{-14} & 3.38 \times 10^{-15} & 1.23 \times 10^{-15} & 2.95 \times 10^{-15} & 1.39 \times 10^{-15} \end{pmatrix} \]

Strain Energy Functions

\[ E = \frac{G}{2} \int \left( \sum_{i=1}^{6} 
\sum_{j=1}^{6} \left( \frac{\partial \varepsilon_{ij}}{\partial x} \right) \right) \]

\[ \text{where} \]

\[ \epsilon_{ij} = \left( \frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}} \right) \]

Coupling of lateral displacement and rotation: \( k \cdot \varepsilon_{ij} \cdot \theta \)

Equations:

Axial displacement incendent longitudinal wave \( u \) is chosen as 1. incident normal wave as \( D \)

\[ u = c_{L} \cdot \sin \left( \frac{\pi x}{L} \right) \cdot \cos \left( \frac{\pi t}{T} \right) \cdot (1 \cdot \sin \left( \frac{\pi \omega t}{T} \right)) \]

\[ \text{wave:} \quad c_{L} = \frac{E}{\rho} \cdot \sqrt{\frac{A+1}{A+1/2}} \]

\[ \text{wave:} \quad \omega = \sqrt{k \cdot m} \]
\[
\text{eq4 := } -(-u_t) - u_i + u_r | [u_i = 1, u_w = 0]\\
\text{b} = 0\\
\text{w} = 0\\
\text{omega} = 1000\\
\]
Substituting everything in:

\[
\begin{align*}
&= 0.000000603 \omega(\omega)^2 + 0.0000000015625 \pi^4 \\
&= 1.292343056 \\
&= 4.451461566 \\
&= 0.0000000243 \\
&= 3.288990432 \\
&= 4.84628646 \\
&= 0.7558272 \\
&= 15 \\
&= 21
\end{align*}
\]
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Curriculum vitae

Robert Ernst

Born on June 5, 1984 in Schwyz, Switzerland
Citizen of Aarwangen BE, Switzerland

Education

1999 - 2003 High School Kantonsschule Kollegium Schwyz, Switzerland
Graduation with matura, Physics and Mathematics

2003 - 2009 Studies in mechanical engineering at the Swiss Federal Institute
of Technology (ETH) Zurich, Switzerland
Graduation as MSc. Mech.-Ing. ETH

2010 - 2015 Doctoral student at the Institute of Mechanical Systems (IMES,
Center of Mechanics), Swiss Federal Institute of Technology
(ETH) Zurich, Switzerland

Professional experience / practical trainings

Sept. - Apr. 2008 Practical internship at HILTI AG, Schaan,
Liechtenstein

Sept. - Dec. 2009 Structural Engineer, RUAG Aerospace, Emmen,
Switzerland

2010 - 2015 Research and teaching assistant at the Institute of Mechanical
Systems (IMES, Center of Mechanics), Swiss Federal Institute
of Technology (ETH) Zurich, Switzerland

2015 - Development Engineer, VESTAS Wind Systems A/S, Aarhus,
Denmark

Extracurricular activities

2003 J&S Ski Mountaineering guide, Switzerland

2013 - 2014 Danish (I-IV) University Zurich, Switzerland
List of publications


Ernst, R., Dual, J., Acoustic Emission Localization in Beams based on Time Reversed Dispersion, Ultrasonics 54, 6, 1522-1533, 2014. DOI: 10.1016/j.ultras.2014.04.012

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