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CREDIT MARKETS AND INVESTMENT IN RENEWABLE ENERGY UNDER UNCERTAINTY

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The present study examines the problem facing a resource-importing economy seeking to achieve energy independence by developing a renewable substitute. The invention of the substitute is assumed to follow a stochastic process that can be influenced by investment in energy research and development. I analyze the optimal investment strategy under alternative assumptions with respect to the economy's access to international financial markets, the terms on which credit is available, and the country's degree of dependence on resource imports. It is found that, in general, having access to capital markets does not necessarily lead to a higher investment rate. However, in the empirically relevant range of elasticity of intertemporal consumption substitution, the economy with access to credit invests more than under financial autarky. A higher degree of dependence on resource imports implies a lower optimal investment. Arrival of the substitute does not necessarily cause an immediate improvement in the net foreign asset position but may in fact cause its further deterioration.

Keywords: Nonrenewable Resource, Capital Market, Uncertainty, Backstop

1. INTRODUCTION

Private and public investment spending on projects devoted to research and development (R&D) of renewable energy sources ("backstops") is often motivated by concerns about exhaustion of nonrenewable energy resources, their volatile and ever-increasing market price, pollution caused by the use of fossil fuels, and climate change. If we look at the leading investors in energy R&D in per capita terms, we find Japan occupying the first place [IEA (2006a)]. Not surprisingly, this country is also well known for its heavy dependence on energy imports.[1] Among European economies leading the way in terms of the share of their national income devoted to renewable energy sources are Switzerland, Denmark, Finland, the Netherlands, and Sweden [IEA (2006b)]. These are again countries that do not possess large stocks of fossil fuels, making them heavily dependent on imports (except for the Netherlands, which do possess large reserves of natural gas, and Denmark, which is expected to continue its North Sea production of oil and gas in excess of its own demand until 2018.)

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The purpose of the present paper is to study the problem facing a resource-importing country, hereafter RIC, which seeks to achieve energy independence by developing a substitute for the nonrenewable importable input. This is assumed to require sustained investment in an R&D program. Arrival of the substitute follows a stochastic process, with the probability of a successful outcome per unit of time being a nondecreasing function of the rate of investment in R&D. Apart from trade in the resource market, RIC can also participate in the global financial market. This latter dimension is most often overlooked in the literature on backstop-technology adoption and resource management in general. As we shall see, however, access to international lending and borrowing is important in several dimensions, especially if a country is heavily dependent on imports of an essential factor of production.

The literature on backstop-technology adoption has its origins in the wake of the oil price shock of 1973. The early contributions focus on a closed economy, endowed with a known stock of an exhaustible resource, seeking to sustain its consumption in the long run by appropriately substituting a renewable backstop for the nonrenewable essential input. The arrival date of the substitute is assumed to be either known with certainty or uncertain but governed by an exogenous stochastic process [see, e.g., Dasgupta and Heal (1974), Dasgupta and Stiglitz (1981)]. The seminal contribution of Kamien and Schwartz (1978) extends this analysis by endogenizing the uncertain arrival date through investment in R&D. Hung and Quyen (1993) go further to determine the optimal time to initiate the R&D project, although their R&D investment policy is simplified to a single-date expenditure, after which a backstop may arrive with a constant Poisson rate.[2] The work of Dasgupta, Gilbert, and Stiglitz (1983) shows (in the context of a deterministic model) that the intention to develop a substitute and its eventual arrival can trigger a strategic response from resource owners. Harris and Vickers (1995) extend this analysis to a stochastic setting. More recently, Gerlagh and Liski (2011) and Jaakkola (2012) analyze dynamic interactions between resource importers and resource exporters.[3] Jaakkola (2012) and van der Ploeg (2012) add the climate change dimension to the analysis of substitute arrival and strategic behavior. Although these contributions are concerned with open economies, their analysis is limited to exchange of the resource for the consumption good, whereas the possibility of international lending and borrowing is ruled out.

The trade-theoretic literature, on the other hand, deals with problems related to exhaustible resource management and, in some cases, for countries that have access to foreign credit, but it does not address the problem of optimal investment in the development of a backstop technology.[4] Moreover, these contributions consider purely deterministic models and therefore exclude the possibility of uncertainty affecting behavior.[5] The purpose of the present study is to bridge the existing gap between the closed-economy analysis of investment in a backstop and open-economy models of trade in goods and financial assets within a fully dynamic stochastic optimization framework. This will make it possible to examine the role of international financial markets in influencing optimal investment strategies in a stochastic environment, an issue of increasing importance in a world where energy prices and international indebtedness are becoming dominant concerns.

To highlight the role of access to international credit, I first present in Section 2 a model of a resource-importing economy that initiates development of an energy substitute relying exclusively on domestic financing. I call this scenario "financial autarky." Section 3 extends the model to allow for international lending and borrowing. Section 4 solves the two models numerically and compares the optimal R&D investment rates. I also analyze the time profile of the net foreign asset position before and after the invention of a substitute (if this happens to occur). Access to international lending and borrowing allows more efficient intertemporal allocation of resources and higher lifetime welfare than in the case of financial autarky. Although this is generally to be expected, a comparison of the optimal investment rates under financial autarky and access to foreign credit enables us to address a number of entirely new issues. First, there is the question of how the degree of dependency on imported energy resources affects the economy's optimal investment in the development of a backstop. On one hand, greater dependency makes it more urgent to discover a substitute. On the other hand, it also implies a higher import bill prior to invention, which tightens the economy's budget constraint and makes any given investment program relatively more burdensome. My analysis shows that for empirically plausible values of the elasticity of intertemporal consumption substitution, a higher price of nonrenewables entails a lower investment rate in renewables R&D, wherease having access to credit is equivalent to a smaller share of nonrenewables in final output production. The second set of issues concerns the role of the cost of credit, which influences not only the time path of the country's net foreign asset position but also the optimal investment decision. The paper concludes in Section 5 with a summary of the main results.

2. FINANCIAL AUTARKY

Let me introduce the assumptions and the notation by starting with the simplest case of financial autarky. The term "financial autarky" refers to an economy that can transact in the domestic financial market but not in the international capital market, although it is open to trade in the global energy market. The domestic financial market offers an asset, denominated in current real consumption, that yields a domestic rate of return, endogenously determined. This economy—call it a resource-importing country (RIC for short)—produces a composite consumption good according to the production function

$$Y_t = F(R_t, L), \tag{1}$$

where $F(.,.)$ is a strictly increasing, concave, and twice differentiable function of both arguments, L is the constant labor input, and R_t is the resource input, which must be entirely imported from abroad. The price of the resource, measured in terms of the consumption good, satisfies $P_t = P_0 e^{rt}$, P_0 known, and r a constant

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growth rate.[6] RIC wishes to develop a backstop, i.e., to invent and produce a substitute for the resource, but this requires setting up and maintaining an R&D lab. Once the substitute is invented, RIC becomes its unique owner.[7] RIC may invest $m \geqslant 0$ units of the consumption good each period to keep the lab operational; that is, RIC must commit itself to a certain constant expenditure per unit of time to maintain the R&D activities. This implies that the sacrifice of current consumption becomes more and more difficult to support as time goes by without the substitute being invented.[8] The discovery of a substitute follows a stochastic process that can be influenced by the investment decision. Let τ be a random variable, which I call the arrival date of the substitute, and assume that the per-unit-time probability of discovering a backstop is given by $\lambda(m)$, with $\lambda'(m) > 0$ and $\lambda(0) \geqslant 0$.

If the backstop arrives, a known quantity B of the substitute becomes available every period at zero cost. The quantity B may represent a limit, imposed by Nature, on how much solar energy or hydro or wind power, for example, may be collected in any given year. Further, one may think of m as an investment that also includes the cost of building the necessary infrastructure, say solar panels. Once these are installed, the cost of production is negligible.[9] The input B simply substitutes for the resource in the production function. The flow of output is then constant and given by $\bar{Y} = F(B, L)$ and the resource is no longer imported. If B is not large enough, however, it may be optimal to continue importing energy from abroad until its price rises sufficiently to reduce the demand to the available per-period supply of the substitute. In the present paper I do not analyze the optimal timing of the switch from the nonrenewables to the backstop, which has already been studied elsewhere [see, e.g., Gerlagh and Liski (2011)] but wish to focus on the optimal investment strategy under uncertainty. In the rest of the analysis I therefore assume that B is sufficiently large, i.e., $B \geqslant g(P_0 e^{r\tau})\ \forall\tau$, where the function $g(.)$ is the inverse of the marginal productivity of the resource. In particular, it is sufficient to assume that $\partial F(B, L)/\partial B \leqslant P_0$, so it is not optimal to import the exhaustible resource if the substitute becomes available from the start.[10]

The social planner's objective is to maximize the expected discounted lifetime welfare by choosing the optimal consumption rate, c_t, the investment rate, m, and imports of the resource, R_t, given the constant rate of time preference, ρ, and the resource price path. The utility, $u(c_t)$, is a strictly increasing, concave, and twice differentiable function of consumption with $\lim_{c\to 0} u'(c) = \infty$. The objective is then

$$\max_{c_t, m, R_t} \int_0^\infty \left\{ \int_0^\tau u(c_t) e^{-\rho t} dt + \int_\tau^\infty u(\tilde{c}) e^{-\rho t} dt \right\} f_\tau d\tau, \tag{2}$$

subject to the constraints

$$c_t = F(R_t, L) - P_t R_t - m, \tag{3}$$

$$\tilde{c} = \bar{Y}, \tag{4}$$

$$f_\tau = \lambda(m) e^{-\lambda(m)\tau}. \tag{5}$$

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The consumption rate in Phase II, i.e., after the discovery has taken place, is denoted by $\tilde{c}$. Note that once the substitute has arrived, there is no more need to maintain the R&D investment.

This stochastic control problem can be analyzed with the aid of the Hamiltonian [see Boukas et al. (1990) or the Appendix]:

$$H = \left\{ u[F(R_t, L) - P_t R_t - m] + \lambda(m) \frac{u(\bar{Y})}{\rho} \right\} e^{-\rho t - z_t} + \nu_t \lambda(m), \qquad \textbf{(6)}$$

where z_t is an auxiliary state variable such that $\dot{z}_t \equiv \frac{dz_t}{dt} = \lambda(m)$, $z_0 = 0$, and ν_t is the associated co-state variable. The necessary conditions for optimality are

$$R_t : \quad u'(c_t) \left(\frac{\partial F_t}{\partial R_t} - P_t \right) e^{-\rho t - z_t} = 0, \qquad \textbf{(7)}$$

$$m : \quad \left\{ -u'(c_t) + \lambda'(m) \frac{u(\bar{Y})}{\rho} \right\} e^{-\rho t - z_t} + \nu_t \lambda'(m) = 0, \qquad \textbf{(8)}$$

$$z_t : \quad \left\{ u(c_t) + \lambda(m) \frac{u(\bar{Y})}{\rho} \right\} e^{-\rho t - z_t} = \dot{\nu}_t, \qquad \textbf{(9)}$$

and the budget constraint (3). Equation (7) is the efficiency-in-production condition, which requires that the marginal productivity of the resource input equal its price. Equation (8) guarantees the optimality of investment by equating the present value of the marginal investment cost, $u'(c_t)e^{-\rho t - z_t}$, to the present value of the marginal expected benefit, $\lambda'(m)\left[\frac{u(\bar{Y})}{\rho} e^{-\rho t - z_t} + \nu_t\right]$. Equation (9) describes the dynamics of the co-state variable.

Given the structure of the production technology (1), equation (7) relates the quantity of imports to the resource price as $R_t = g(P_t)$, where $g(.)$ is the inverse function of the marginal productivity of the resource with $g'(.) < 0$. Define the net output as $Y_t^n \equiv F(R_t, L) - P_t R_t$. Then the value of Y_t^n at each point in time is determined by the resource price:

$$Y_t^n = F(g(P_t), L) - g(P_t) P_t, \qquad \textbf{(10)}$$

with $\frac{\partial Y_t^n}{\partial P_t} = -g(P_t) < 0$, $\frac{\partial^2 Y_t^n}{\partial P_t^2} = -g'(P_t) > 0$. The budget constraint (3) may then be rewritten as

$$c_t = Y_t^n(P_t) - m. \qquad \textbf{(11)}$$

Solving for ν_t from (8), differentiating with respect to time, and inserting the result into (9) yields

$$-\frac{u''(c_t)c_t}{u'(c_t)} \hat{c}_t = \lambda'(m) \left[\frac{u(\bar{Y}) - u(c_t)}{u'(c_t)} \right] - \rho - \lambda(m), \qquad \textbf{(12)}$$

which, in combination with (10) and (11), provides an implicit solution for m. Note that the first term on the right-hand side corresponds to the economy's implicit rate of interest. It is endogenously determined and time-varying. Moreover, it depends on the possibility of the substitute being discovered at some future date, as represented by the future per-period utility, $u(\bar{Y})$, as well as by the marginal impact of R&D investment on the invention probability, $\lambda'(m)$.

The dynamic behavior of the key endogenous variables can be deduced from the system (10)–(12). First of all, equation (10) shows that if the resource price increases over time, the net output (i.e., total output minus the import bill) falls. Then equation (11) shows that if the net output declines over time while the investment rate remains constant, the consumption rate must fall over time until an eventual discovery of the backstop occurs. Given (11), equation (12) pins down the investment rate as a function of preferences, degree of impatience, and the characteristics of the backstop. When the invention occurs, the output jumps to the level $\bar{Y}$ and remains constant thereafter. The same happens to the consumption rate, whereas R&D investment rate and imports drop to zero.

Transactions with the rest of the world have been limited so far to the exchange of the consumption good for the resource. I examine next how the optimal investment strategy is affected if RIC has the possibility of lending and borrowing in the international financial markets. It is clear that access to a riskless saving technology makes it possible to implement a smoother optimal consumption path. However, the following questions remain: To what extent does access to foreign credit alleviate the burden of investment, facilitating development of a more ambitious project? What role does foreign credit play when RIC's dependency on energy imports is increased? What is the role of the cost of credit? What is the optimal time profile of the net foreign asset position and how is it affected by the arrival of the backstop? Sections 3 and 4 address these questions.

3. ACCESS TO WORLD FINANCIAL MARKETS

In this section I allow RIC to have access to international financial markets, where a single riskless asset, denominated in units of the consumption good, is costlessly traded. The asset yields a constant world rate of return, r.[11] By arbitrage, the growth rate of the resource price must also be equal to r, assuming that extraction is costless [Hotelling (1931)].[12]

Let a_t denote RIC's net foreign asset position at time t. Assuming that the time horizon is infinite, the budget constraints in the first and the second phases, respectively, are

$$\dot{a}_t = F(R_t, L) - c_t - P_t R_t - m + r a_t, \quad \forall t \in [0, \tau), \quad a_0 \text{ given,} \qquad (13)$$

$$\dot{a}_t = F(B, L) - \tilde{c}_t + r a_t, \quad \forall t \geq \tau, \qquad (14)$$

$$\lim_{t \to \infty} a_t e^{-rt} = 0. \qquad (15)$$

Equation (13) states that during the first phase, while the substitute is not yet available, the rate of accumulation of foreign assets is equal to the total output

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minus expenditure on consumption, resource imports and investment, plus interest earned (paid) on the accumulated assets (outstanding debts).[13] Equation (14) states that during the second phase, the change in the asset position is just equal to the constant flow of output minus consumption plus interest, and the resource is no longer imported. RIC's objective is to maximize (2) subject to (13)–(15).

The solution method consists of two steps. First the maximized value of discounted (time-τ) welfare in Phase II is obtained, given the net foreign asset position at $t = \tau$. I call this function $\Phi(a_\tau)$. Then the total lifetime welfare is maximized, given the relationship between a_τ and the welfare in Phase II (the detailed derivation is relegated to the Appendix).

Consider the optimization problem pertaining to Phase II and assume a standard CRRA utility $u(x) = \frac{x^{1-\theta}}{1-\theta}$, where $1/\theta$ is the elasticity of intertemporal consumption substitution (EICS). RIC seeks to maximize

$$\int_\tau^\infty u(\tilde{c}_t)e^{-\rho(t-\tau)}dt, \tag{16}$$

subject to (14)–(15) and with a_τ given. The solution for the optimal $\tilde{c}_t$ is obtained in a straightforward manner using the standard dynamic optimization technique, which yields a constant growth rate of consumption equal to the difference between the rate of interest and the rate of time preference, adjusted by EICS:

$$\tilde{c}_t = \tilde{c}_\tau e^{\frac{r-\rho}{\theta}(t-\tau)}, \ \ \forall t \geq \tau, \ \ \tilde{c}_\tau = \left(r - \frac{r-\rho}{\theta}\right)\left(a_\tau + \frac{\bar{Y}}{r}\right). \tag{17}$$

Then the maximized value of (16) is

$$\Phi(a_\tau) = u(\tilde{c}_\tau)\left(r - \frac{r-\rho}{\theta}\right)^{-1}. \tag{18}$$

The Hamiltonian associated with RIC's original optimization problem may then be written as

$$H = \Big\{u(c_t) + \lambda(m)\Phi(a_t)\Big\}e^{-\rho t - z_t} + \eta_t\big[ra_t + F(R_t, L) - c_t - P_tR_t - m\big] + \nu_t\lambda(m), \tag{19}$$

where η_t is the co-state variable associated with the constraint (13) and z_t is the auxiliary state variable, as in Section 2. The solution is implicitly given by the system

$$\theta\hat{c}_t = r - \rho - \lambda(m)\left[1 - \frac{u'(\tilde{c}_t)}{u'(c_t)}\right], \tag{20}$$

$$\tilde{c}_t = \left(r - \frac{r-\rho}{\theta}\right)\left(a_t + \frac{\bar{Y}}{r}\right), \tag{21}$$

$$\lambda'(m)\left[\rho\Phi(a_t) - u(c_t) - u'(\tilde{c}_t)\dot{a}_t\right] = u'(c_t)r + \lambda(m)u'(\tilde{c}_t), \tag{22}$$

$$\dot{a}_t = F(R_t, L) - c_t - P_tR_t - m + ra_t, \quad a_0 \text{ given.} \tag{23}$$

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Equation (20) describes the growth rate of consumption in Phase I. Note that if there is no uncertainty, the last term vanishes and the standard Keynes–Ramsey rule applies. When $\lambda(m) > 0$, the standard rule is modified to account for the effect of uncertainty. The term in square brackets is unambiguously positive because $\tilde{c}_t > c_t$ and therefore consumption grows at a lower rate than in the certainty case. The lower optimal growth rate (or more rapid decline) of consumption results in a higher dissaving rate at the beginning of the planning horizon in anticipation of the possible technological breakthrough. Moreover, the higher the flow of the substitute, B, in the event of a discovery, the lower the consumption growth rate and the higher the dissaving rate at the beginning of the planning horizon.

Equation (21) determines the time-τ consumption rate, i.e., the consumption rate to which the economy jumps at the moment when the backstop arrives. It depends negatively (positively) on the stock of debt (assets) accumulated up to the time of the invention.[14] From time τ onward the consumption rate during Phase II is no longer constant, as it was under financial autarky, but grows or contracts depending on the difference between the world rate of interest and RIC's rate of time preference, satisfying the standard Keynes–Ramsey rule. Without access to credit, Phases I and II were disconnected, in the sense that the optimal consumption rate in Phase II was independent of the variables pertaining to Phase I.[15] In the present setting, the two phases are connected through the net foreign asset position held at the time of invention. Equation (22) is the optimality condition for the choice of m, which states that the marginal expected benefit from undertaking the investment must be equal to the marginal cost, which also includes the opportunity cost of not investing in the capital markets. The system (20)–(23) is solved numerically and analyzed in the next section.

4. NUMERICAL ILLUSTRATION AND DISCUSSION

This section compares the solution to RIC's problem with access to credit (AC, for short) with the solution under financial autarky (FA, for short). The objective is to analyze how the economy's dependence on energy resources translates into the choice of m and what role access to international capital markets plays in this respect. I also examine the optimal borrowing/lending strategy in an uncertain environment.

Let the production function be of Cobb–Douglas type: $Y_t = AR_t^{\alpha}L^{1-\alpha}, \quad 0 < \alpha < 1, \quad A > 0$. I assume that the invention of the substitute follows a Poisson process with the arrival rate $\lambda(m)$. The arrival rate is positively related to the R&D investment rate, i.e., $\lambda'(m) > 0$. It is assumed that $\lambda''(m) \geqslant 0$ for $m \leqslant \bar{m}$ and $\lambda''(m) < 0$ for $m > \bar{m}$. That is, when the investment rate is relatively small, commitment to an additional unit of sustained investment has an increasing marginal impact on the probability of making a discovery. Alternatively, when the investment rate is already high, the impact of an extra unit on the arrival rate is diminishing.[16] A natural candidate for the λ-function is a sigmoid-type function, because it possesses the property that I have just outlined: convexity up to a certain

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TABLE 1. Benchmark parameter values

Labor	L	1
Technological parameter	A	1
Resource share	α	0.1
Substitute flow	B	0.5
Elasticity of marginal utility	θ	0.75
Rate of time preference	ρ	0.02
Resource price growth rate	r	0.02
Initial resource price	P_0	1
Initial asset holdings	a_0	0
Planning horizon	T	200
Minimum time to discover	$T_{\min}$	20

(inflection) point and concavity thereafter. I specify the exact functional form for $\lambda(m)$ to be

$$\lambda(m) = \left[T_{\min} + e^{(\mu-\gamma m)/\sigma}\right]^{-1}, \tag{24}$$

where $T_{\min} \geqslant 0$ is the shortest possible time needed for the development of a backstop,[17] and μ, γ, and σ are positive parameters calibrated as $\mu = \ln(T - T_{\min})$, $\gamma = 15$, $\sigma = 1$. The inflection point is given by $\bar{m} = \frac{\mu-\sigma \ln T_{\min}}{\gamma}$. A higher (lower) γ makes the slope of the λ-function steeper (flatter). The chosen values of μ and σ ensure that $\lambda(0) = 1/T$, where T is the length of the economy's planning horizon. This latter condition states that if the economy chooses a zero investment rate, there is still a chance of discovering a backstop once in T units of time.[18]

The parameter values for the benchmark simulation are presented in Table 1. Labor input, the level of technology, and the initial resource price are normalized to unity. The share of exhaustible resources in the production function is assumed to be 10%. A relatively high value of the resource share is chosen in order to highlight the economy's dependency on energy input. Simulation results for alternative values of α are discussed in Section 4.2. The value of θ is calibrated to guarantee that the elasticity of intertemporal consumption substitution lies in the empirically relevant range [see Hansen and Singleton (1982), Epstein and Zin (1991), and Vissing-Jørgensen (2002)]. Multiple calibrations of θ are examined, especially in the analysis of the relationship between energy dependence and investment choice. The rate of growth of the resource price in the world market and the rate of time preference, ρ, are set at 2% per annum.[19] The length of the planning horizon, T, is assumed to be 200 years, whereas the minimum average time needed to discover a substitute, $T_{\min}$, is 20 years. The value of B is calibrated in such a way that it no longer pays to import the resource when B becomes available: $\partial \bar{Y}/\partial B = A\alpha B^{\alpha-1}L^{1-\alpha} \leqslant P_0$.

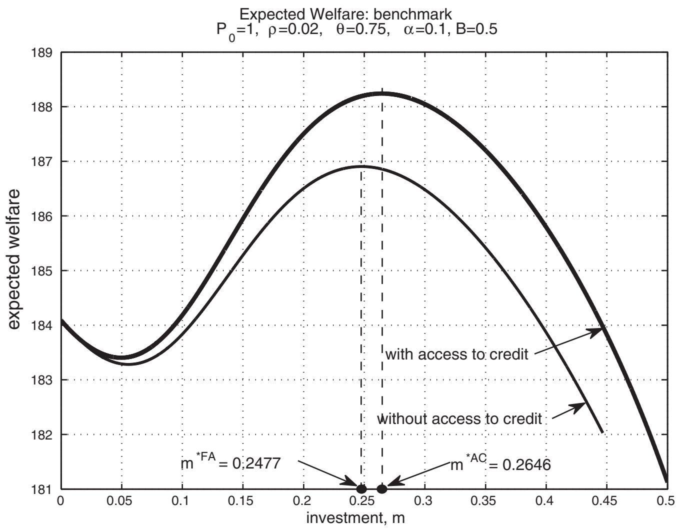

FIGURE 1. Expected welfare.

4.1. Solution for Optimal R&D Investment

The optimal investment rate is the one that maximizes expected lifetime welfare, given the planning horizon. Figure 1 plots RIC's expected welfare as a function of investment under financial autarky (thin line) and with access to credit (thick line). The optimal investment rate and the associated expected welfare level are higher and the average time to discover the backstop is shorter under the second scenario (AC) than under the first scenario (FA).

The following subsections provide a more detailed analysis of the optimal investment choice under the two scenarios. In particular, I will show that having access to credit markets does not necessarily imply that the economy will choose a higher investment rate than under financial autarky. Additionally, I will show how the optimal investment is affected by a change in the cost of credit or the economy's dependence on nonrenewables. I also discuss the optimal path of the net foreign asset position and how it is affected by the arrival of the renewable energy resource.

4.2. R&D Investment and Energy Dependence

Two interesting questions emerge at this point: First, does greater dependence on resource imports raise or lower the optimal investment rate, and second, what is the role of access to foreign credit in this respect? On one hand, greater dependence makes it more urgent to develop an alternative source of energy. On the other hand, a country that is more dependent spends a larger share of its GDP on resource imports. Its budget constraint is then tighter, making the burden of any investment

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project relatively heavier. In terms of the present model, either a higher growth rate of the resource price, r, a higher initial price, P_0, or a larger distributive share of energy in the production function, α, manifests itself in greater dependence on resource imports.

Equation (12) allows us to derive analytical results with respect to the reaction of m to a change in P_0 and r:

$$\frac{dm^{FA}}{dP_0} = \frac{\Delta_P}{\Delta_m} e^{rt} \gtrless 0, \tag{25}$$

$$\frac{dm^{FA}}{dr} = \frac{\Delta_P}{\Delta_m} t P_t \gtrless 0, \tag{26}$$

where

$$\Delta_m = g(P_t)\dot{P}_t \left[\frac{u'''(c)u'(c) - (u''(c))^2}{(u'(c))^2} \right] - \frac{u(\bar{Y}) - u(c)}{u'(c)} \left[\lambda''(m) + \frac{\lambda'(m)u''(c)}{u'(c)} \right], \tag{27}$$

$$\Delta_P = \left\{ \lambda'(m) \frac{\left[(u'(c))^2 + (u(\bar{Y}) - u(c))u''(c) \right]}{(u'(c))^2} - g(P_t)\dot{P}_t \left[\frac{u'''(c)u'(c) - (u''(c))^2}{(u'(c))^2} \right] \right\} g(P_t) - r^2 e^{rt} \frac{u''(c)}{u'(c)} \left[g'(P_t)P_t + g(P_t) \right]. \tag{28}$$

The term Δ_m is, in general, of ambiguous sign. However, for standard utility functions employed in the literature, such as CRRA and negative exponential, the term $u'''(c)u'(c) - [u''(c)]^2$ is nonnegative.[20] Because an infinitesimal additional unit of m produces an increasing marginal impact on the probability of a breakthrough on the convex segment of the λ-function, the optimal investment rate always lies on the concave segment; hence the term $\lambda''(m)$ is negative. This is sufficient to ensure that $\Delta_m > 0$. The term Δ_P is of ambiguous sign and therefore the effects of P_0 and r on the optimal investment rate are ambiguous (although they always work in the same direction). This is hardly surprising. An increase in the resource price generates two conflicting effects: On one hand, it tightens the economy's budget constraint as the import bill expands. On the other, it makes the development of the backstop more urgent as the economy's dependence on energy resources, whose market price rises exponentially, is increased. If the social planner of this economy is risk-neutral (or has an infinitely high intertemporal substitution elasticity), we

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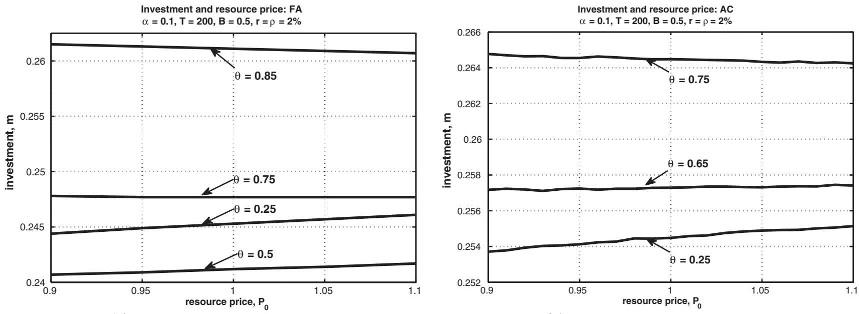


(a) Energy dependence under FA

(b) Energy dependence under AC

FIGURE 2. Price of nonrenewables and optimal investment.

obtain

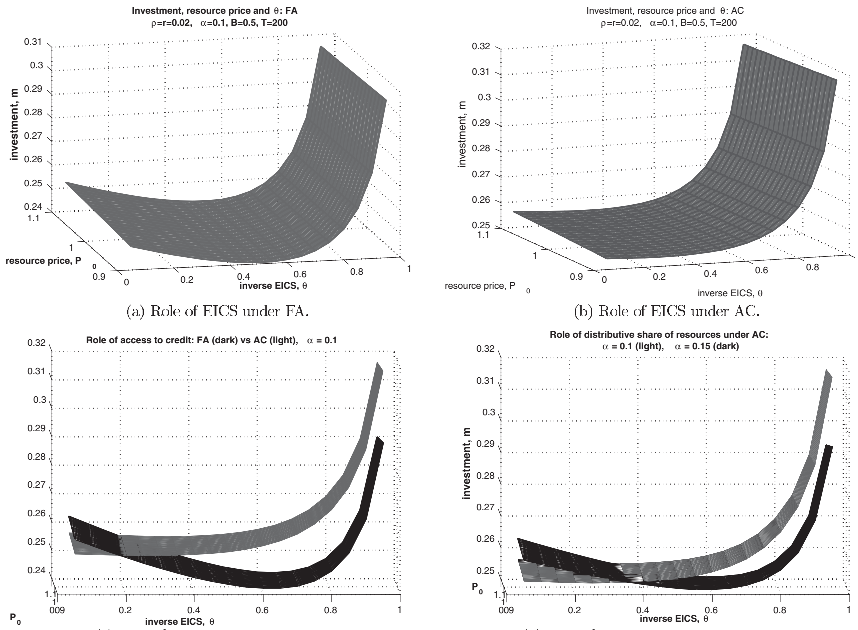
$$\frac{dm}{dP_0} = -\frac{\lambda'(m)g(P_t)e^{rt}}{\lambda''(m)\frac{u(\bar{Y})-u(c)}{u'(c)}} > 0, \quad \frac{dm}{dr} = -\frac{\lambda'(m)g(P_t)tP_t}{\lambda''(m)\frac{u(\bar{Y})-u(c)}{u'(c)}} > 0 \ \forall t > 0, \qquad \textbf{(29)}$$

where the numerators are unambiguously nonnegative and the denominators are negative if $\lambda(m)$ is concave or m lies in the concave region of $\lambda(.)$. A risk-neutral planner will therefore react to an increase in the resource price or its growth rate by increasing investment in R&D.

The response of m may be different, however, when the elasticity of intertemporal consumption substitution (EICS for short) is reduced to the empirically relevant range. It matters as well whether the country has access to international capital markets or not. Figures 2a and 2b plot m^* as a function of P_0 for several values of θ (the inverse of EICS) under "financial autarky" and "access to credit," respectively.

The slope of the relationship between m^* and P_0 changes from positive to negative as θ increases (i.e., EICS falls). To understand the intuition here, it is useful to think of the investment program as a plan to purchase, at every point in time, options that offer the prospect of stabilizing future consumption at a particular high level starting from an uncertain future date. Buying a larger number of options at each instant in the first phase advances the expected date of the payoff and lowers the chances of experiencing very low consumption rates at the end of Phase I. With a lower θ (i.e., higher EICS), the willingness to purchase such options diminishes as consumers care relatively less about the time profile of consumption as opposed to the total discounted consumption over the entire planning horizon.

An increase in P_0 lowers the economy's expected net income over the planning horizon. This raises the utility cost of investing at any given rate in the development of the backstop. At the same time, with a higher P_0, the benefit of investing is also greater, in the sense that the expected future jump in income that the investment program helps bring forward in time is larger. The impact of an increase in P_0 on the cost relative to the benefit of marginal investment is larger in the case of a



(a) Role of EICS under FA. (b) Role of EICS under AC.
(c) Role of access to capital markets. (d) Role of resource share in production, α.

FIGURE 3. Energy dependence under FA and AC.

higher degree of concavity of the utility function. Thus, when θ is relatively higher, the optimal investment rate declines with P_0, whereas with a lower θ, investment is positively related to P_0. Empirical studies of EICS conclude that the relevant range is below 2, which corresponds to $\theta > 0.5$.[21] Our numerical results show that in the benchmark calibration m^* is decreasing in P_0 for $\theta > 0.75$ under FA and for $\theta > 0.65$ under AC. Therefore, the optimal response of the R&D investment rate is more likely to be negative as the nonrenewable resource price rises.

In order to obtain a more comprehensive picture of the relationship among m^*, P_0, and θ, I plot in Figures 3a (FA case) and 3b (AC case) a 3-dimensional surface with m^* on the vertical axes and P_0 and θ on the two horizontal axes. Figure 3c superimposes both surfaces in one graph and demonstrates that the economy that has access to foreign capital markets does not necessarily invest more in renewables R&D than an economy under FA: for high EICS (low θ) $m^{*\text{FA}}$ (dark surface) is above $m^{*\text{AC}}$ (gray surface). In the empirically relevant range of EICS, however, $m^{*\text{AC}} > m^{*\text{FA}}$, so that having access to credit does help sustain a higher investment rate.

Finally, an increase in energy dependence may also be interpreted as an increase in the resource-use share in output production, i.e., a higher α. The optimal response of m to an increase in α (for a given P_0) also depends on θ, as shown in Figure 3d. The gray surface is the same as in Figure 3b, whereas the dark surface corresponds to α raised from 10 to 15%. Comparison of Figure 3c with 3d reveals

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that having access to credit is equivalent to having a lower distributive share of energy resources in production of final goods.

To summarize the results so far, (a) when the nonrenewable-resource price rises, the optimal response of renewables R&D investment is to fall because of the contractionary effect on the budget constraint; (b) the optimal investment rate in an economy with access to capital markets is higher than that in an economy without such access, provided the elasticity of intertemporal substitution is not too high; (c) having access to credit is equivalent to being less dependent on nonrenewable energy resources for production of final goods.

4.3. Role of the Cost of Credit

Evolution of net foreign asset position. So far, the analysis has proceeded under the simplifying assumption $\rho = r$, i.e., the economy's rate of time preference equals the world rate of interest. Variations in the cost of borrowing clearly affect RIC's optimal R&D investment rate, as well as its borrowing/lending decision. Interestingly, under specific conditions discussed in the following, RIC may find it optimal to have a positive net asset position (to be a lender) and at the same time to maintain a relatively high R&D investment rate (above the rate under financial autarky).

Let us examine the role of the world interest rate in more detail. Figure 4 shows the time path of asset holdings under two alternative calibrations: (a) the thin lines correspond to $r = 2.5\%$ (50 basis points above the benchmark) and (b) the thick lines are drawn for $r = 3\%$ per year. The solid lines illustrate the evolution of assets under the assumption that the substitute is never discovered, whereas the dashed schedules are drawn assuming that the discovery takes place at $t = 60$ for case (a) and at $t = 100$ for case (b).

Note that in spite of the fact that $r > \rho$, the economy is initially a net borrower under calibration (a). This is because of the effect of uncertainty, which, as we have seen in equation (20), tilts clockwise the time path of consumption in Phase I and thus contributes to dissaving (the optimal time profile of consumption is presented and discussed in Appendix A.3). Only if the substitute is eventually invented may RIC become a lender (see the shaded area), with the length of the lending span depending negatively on the invention date and positively on EICS and on the difference between r and ρ. The later the substitute arrives, the longer the period of borrowing and the shorter the subsequent period of lending (if it exists at all). Note, in addition, that the larger r is relative to ρ, the weaker is the incentive to borrow during Phase I. Thus for higher interest rates, the borrowing phase becomes shorter or even disappears, whereas the lending phase widens. Interestingly, for high enough r, the borrowing phase may not necessarily occur at the beginning of the planning horizon. As illustrated by the thick solid line, with $r = 3\%$, RIC is initially a net lender in spite of maintaining a relatively high investment rate. The net asset position in this case exhibits a wave-shaped time profile with the borrowing phase occurring at the end of the planning horizon. If r is relatively

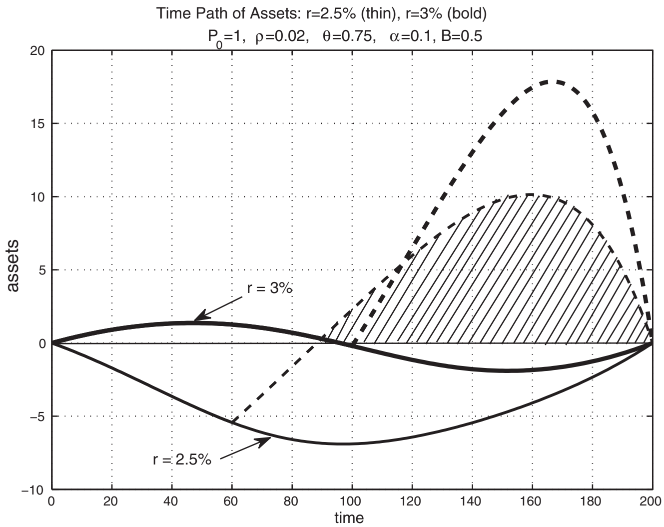

FIGURE 4. Asset position and intertemporal terms of trade: $r = 0.025$ (thin lines), $r = 0.03$ (thick lines).

high and the invention occurs relatively late, the time profile of the net asset position peaks twice, as in the case of calibration (b), where the invention occurs at $t = 100$.

Invention date and debt repayment. So far we have seen that the arrival of the substitute initiates repayment of the debt or further improves the asset position if it is positive: immediately after the invention the dashed lines are positively sloped and lie above the solid schedules (see Figure 4 and Figure A.1b in the Appendix). This, however, may not always be true. The optimal time path of the net foreign asset position after the invention depends on the relationship between r and ρ. It is clear that when $r < \rho$, the economy will consume at a declining rate during Phase II, i.e., $\hat{\tilde{c}}_t = \frac{r-\rho}{\theta} < 0$. Moreover, the difference between the market rate of interest and RIC's rate of time preference also affects the *initial* consumption rate in Phase II: The smaller (i.e., the more negative) is $r - \rho$, the larger is $\tilde{c}_\tau$ (see equation (21)). When r is sufficiently below ρ, the economy will in fact find it optimal to start Phase II with a consumption rate in excess of its income (net of interest and imports) which entails a further deterioration of the net asset position. This is illustrated in Figure 5, where r is set at 0.5%.

As before, the solid line is drawn under the assumption that the substitute never arrives, whereas the dashed lines are drawn assuming that the invention occurs at $t = 30$ (thin dashed line) and $t = 100$ (thick dashed line). Given that credit is relatively cheap, the net asset position continues to deteriorate even after the invention and reaches a minimum several years later than if the

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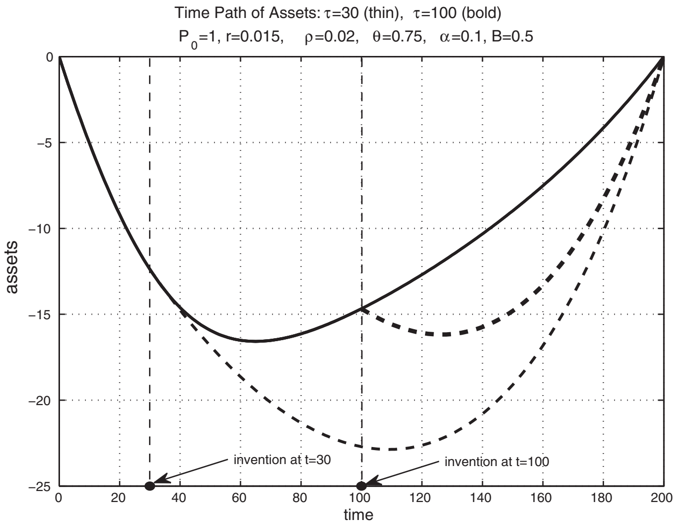
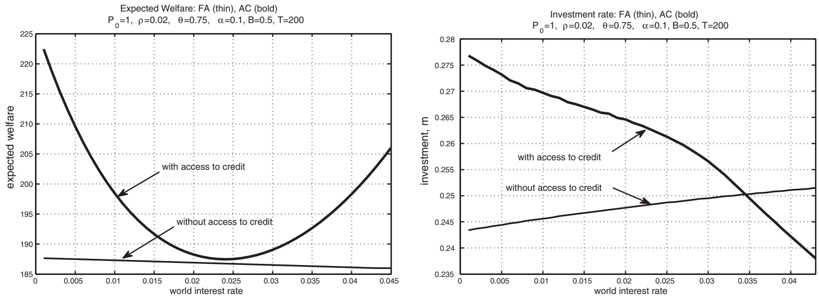


FIGURE 5. Deterioration of the asset position after the invention.

substitute had never arrived. Throughout the remainder of the planning horizon, the economy is more indebted than it would have been without the invention. Interestingly, the time profile of a_t may exhibit a double-trough pattern when invention occurs relatively late as, for example, at $t = 100$. The high levels of indebtedness, equal to a multiple of the economy's GDP, are nonetheless perfectly sustainable. This is true even if the backstop never arrives. In this case, the debt is repaid at the expense of current consumption, which falls over time.

Cost of credit and lifetime welfare. Further examination of the role of the market rate of interest leads us to consider its effect on the economy's expected lifetime welfare. When access to credit is available, r affects expected welfare through two channels. The first is the resource price: The higher the rate, the greater the rate of increase in P_t and the heavier the burden of future payments for resource imports. An additional channel emerges with the possibility of lending and borrowing. If RIC is a net borrower, an increase in r implies a heavier debt burden, so that both effects contribute to a lower expected welfare. On the other hand, if RIC is a net lender, a higher r represents an improvement in its intertemporal terms of trade, contributing to higher expected welfare. Whether RIC is a borrower or a lender is determined endogenously and depends on the structure of its preferences and its production technology, on the amount of the substitute it expects to obtain in the case of a technological breakthrough, and finally on the relationship between r and ρ. Thus, in general, the net effect of the world interest rate on the economy's expected welfare is nonmonotonic. It depends, in essence,



(a) welfare (b) investment

FIGURE 6. The effect of intertemporal terms of trade on welfare and investment rate.

on the volume of its trade in the resource market in relation to the volume of its net lending over the entire planning horizon. It is generally to be expected, however, that an economy's welfare is higher with free trade—in this case trade in the financial asset—than it is under autarky. This is illustrated in Figure 6a, where I show RIC's expected lifetime welfare, under the optimal investment strategy, as a function of the market interest rate, holding other parameters at their benchmark levels.

Under financial autarky, the expected welfare declines with r, as shown by the thin line. In this case, only the effect of r on the price path of the resource impinges on welfare. With access to credit, the schedule is U-shaped, reflecting the conflicting forces discussed earlier. Note that regardless of the value of r, the expected lifetime welfare with access to credit is always higher than that without such access. The possibility of improving the efficiency of the intertemporal allocation of resources by transacting in the international financial market has therefore an important welfare-enhancing role.

The effect of the cost of credit on the optimal R&D investment rate can be visualized in Figure 6b. First, observe that $m^{*\mathrm{FA}}$ is increasing in r, whereas $m^{*\mathrm{AC}}$ is decreasing in r. This difference in the optimal response hinges on the dual role of the interest rate in the latter scenario. Second, for a high enough interest rate in relation to ρ, $m^{*\mathrm{FA}}$ exceeds $m^{*\mathrm{AC}}$. In other words, economies that have access to capital markets but face a relatively high cost of credit tend to choose less ambitious investment projects than they would have chosen without access to credit. This is because holding foreign assets yielding a relatively high rate of return effectively substitutes for investment in R&D. The economy then chooses a lower investment rate and becomes a net lender in the capital market.

5. CONCLUSION

The paper attempts to answer three main questions: (i) What is the optimal investment rate in an R&D project thath may secure a given flow of income in the future, with the probability of success being dependent on the investment rate? (ii) How is the optimal investment choice affected by a higher degree of dependence on imports of an essential nonrenewable factor of production? (iii) To what extent does access to capital markets matter for the investment decision? The answers to these questions are analyzed in the context of a model of a resource-importing country (RIC), which seeks to achieve energy independence by developing a renewable substitute for a nonrenewable essential input. I assume that the invention of a substitute follows a stochastic process that can be influenced by appropriate investment in R&D. The focus of this paper is on the role of access to international lending and borrowing for the optimal choice of the investment rate in renewables R&D and of the net asset position under uncertainty. This role is highlighted by comparing the outcomes under two extreme assumptions about the economy's access to global capital markets: financial autarky vs. full access.

With access to foreign credit, the economy chooses a very different time path of consumption from the one obtained under financial autarky. Because of the presence of uncertainty, i.e., the possibility of a successful R&D outcome, the economy dissaves during an initial phase of its planning horizon and runs a negative foreign asset position, even when the rate of interest is above the rate of time preference. This type of behavior is exactly the opposite of precautionary saving in an environment with negative income shocks.

When it comes to the optimal choice of R&D investment rate, having access to capital markets does not necessarily imply that the economy systematically invests more than it does without such access. The outcome depends crucially on the value of the elasticity of intertemporal consumption substitution (EICS) and on the cost of credit. Numerical simulations show that in the empirically relevant range of EICS, R&D investment rate with unrestricted access to international credit markets always exceeds the investment rate under financial autarky.

Another key element influencing the optimal choice of R&D investment rate is the economy's dependence on foreign energy sources, as measured by the share of GDP absorbed by expenditure on resource imports. In the context of the present model, energy dependence is determined by the market price of the resource and the distributive share of energy in the production of final goods. An increase in the resource price may either boost or decrease the investment rate depending on EICS. The numerical results show that in the empirically relevant range of values for EICS, an increase in the resource price leads to a lower optimal investment rate. This result holds regardless of whether or not the economy has access to borrowing and lending. Having access to global capital markets, however, is shown to be equivalent to a smaller share of energy resources being needed for production of final goods.

Several interesting results emerge when we look at what role the cost of credit, r, plays in determining the optimal investment choice and the economy's net

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foreign asset position (NFA). First, it is shown that, depending on the relationship between r and the rate of time preference ρ, the lending phase may precede the phase of borrowing even when the economy maintains a relatively high investment rate (i.e., higher than under FA). Second, a successful R&D outcome causes an improvement in the NFA when r is not too low in relation to ρ but a *deterioration* in the NFA for low enough interest rates. Third, the economy's expected lifetime welfare with access to credit is U-shaped in r because of the dual role of the latter (resource price and intertemporal terms of trade) and always exceeds the welfare under financial autarky, regardless of the value of r. Finally, the optimal investment rate responds differently to variations in r depending on whether access to credit is available or not: it is an increasing function of r under financial autarky but a *decreasing* function of r under openness.

The present analysis motivates the desire to invent a substitute for a nonrenewable resource by its increasing market price and thus increasing dependence on energy imports. Introducing other motivations for a switch from nonrenewable to renewable sources of energy, such as an objective to meet a specific climate-policy target, would enrich the analysis even further. Assuming that the social planner dislikes pollution and the backstop is a clean energy source, there would be an additional incentive to invest in R&D.

NOTES

1. Although Japan is only the second largest oil importer after the United States, it meets a larger share of its energy needs through imports of oil than the United States does (U.S. Energy Information Administration, http://www.eia.doe.gov/country/index.cfm).

2. Tsur and Zemel (2003) propose an alternative (deterministic) framework of analysis, where the cost of the backstop falls continuously as the knowledge base accumulates through R&D. This ensures a continuous transition from the nonrenewable to the backstop. Their model advocates an R&D policy characterized by the most rapid approach path to the target-knowledge process, which should then be followed forever.

3. See also Gerlagh and Liski (2012).

4. Kemp and Long (1984) do consider resource replacement, but in a deterministic setting, where the resource price is exogenous and constant and there is no possibility of participating in the international financial markets. Djajić (1988) considers a two-country world, where both countries are endowed with some stock of the resource and can lend or borrow from each other at an endogenously determined rate of interest. The dynamics of his model is, however, limited to only two time periods, and neither country intends to develop a backstop.

5. An exception is Dasgupta et al. (1978), who do consider uncertainty related to future energy demand. They also introduce the possibility of accumulating a foreign asset yielding a constant rate of return, but focus on a resource-exporting economy that is not engaged in any R&D activity.

6. When resource extraction is costless, such a price path follows from the Hotelling (1931) model.

7. This occurs, for instance, if the substitute (or its production process) is specific to RIC's geographic location or if RIC can patent the invention. I do not, however, analyze issues related to patent races.

8. It is fairly easy to find arguments in favor of optimality of time-varying R&D investment rates. Ideally, one would prefer to have the possibility of adjusting the investment rate as frequently as possible in order to respond to any unanticipated changes in market conditions. However, in the

present setting, there are no unanticipated price changes, and moreover, government funding of R&D projects tends to be rigid once the project is formulated and is in progress.

9. Allowing a cost of production that is positive and constant or varying over time but exogenous will merely affect the relevant budget constraint in a straightforward manner. The qualitative results will remain intact.

10. See Amigues et al. (1998) for treatment of a capacity constraint on the flow of the substitute.

11. Treating r as exogenous is based on the assumption that RIC's borrowing to finance (in part) its R&D efforts does not have a perceptible impact on the world rate of interest. Given the size of the global financial markets in relation to that of a major investment project in any one country, this assumption is fairly realistic.

12. Assuming that extraction is costly will not fundamentally affect the mechanics of the model. If the marginal extraction cost, MC, is constant over time, the growth rate of the resource price is given by $\frac{\dot{P}_t}{P_t} = r - r\frac{\mathrm{MC}}{P_t} \equiv \tilde{r} < r$. If the marginal extraction cost is not constant but convex in the extraction rate, we have $\frac{\dot{P}_t}{P_t} = r - r\frac{\mathrm{MC}(R_t)}{P_t} + \frac{\mathrm{MC}'(R_t)\dot{R}_t}{P_t} \equiv \bar{r} < r$, where the last inequality holds because the extraction rate falls over time as the stock is being depleted. Thus the resource price increases over time at a rate $\tilde{r}$ or $\bar{r}$ that is lower than the rate of interest r.

13. An alternative approach to analyzing the role of access to foreign capital is to introduce a parameter $\beta \in [0, 1]$ that denotes the share of wealth invested in the international asset. When β is close to unity, the economy has full/unrestricted access to international lending and borrowing. In contrast, when β approaches zero, access to foreign credit is shut down.

14. Convergence of the integral in (16) requires that $\frac{r-\rho}{\theta} - r < 0$, so that $\partial\tilde{c}_\tau/\partial a_t = r - \frac{r-\rho}{\theta} > 0$.

15. This is why Kamien and Schwartz (1978) are able to summarize the value function pertaining to Phase II with the variable W, which is taken to be exogenous and, more importantly, independent of the arrival date of the backstop.

16. In the model of Kamien and Schwartz (1978) it is assumed that the probability of discovering a substitute depends on the *cumulative* R&D effort. The rate of growth of R&D effort is, in turn, a concave function of investment. In their suggestions for possible extensions K&S write that "successful development of a new technology may require a sustained commitment of resources above a minimal level." Here I follow this route in assuming that the probability of inventing a substitute depends on the *level* of the sustained investment rate as opposed to cumulative investment.

17. $T_{\min}$ imposes, essentially, an upper limit on the probability; that is, $\lim_{m\to\infty} \lambda(m) = 1/T_{\min}$. One may think of $T_{\min}$ as the time necessary to build the appropriate infrastructure for producing the substitute. Even if the breakthrough occurs before $T_{\min}$, the new technology cannot yet be marketed until the infrastructure is built (charging stations for electric cars are an example of such infrastructure).

18. Although the model is written as an infinite-horizon problem, the simulations require a finite horizon. The optimal paths in a finite-horizon model are, of course, the same as described for the infinite-horizon problems in Sections 2 and 3; only the initial and terminal points are affected. Also, the finiteness of the horizon requires the truncated PDF to be used: $f_\tau = \frac{\lambda(m)e^{-\lambda(m)\tau}}{1-e^{-\lambda(m)T}}$.

19. I ignore the possibility that RIC's investment project might alter the time path of the resource price on the global markets. Even the recent nuclear incident in Japan did not seem to have an impact on the price of nonrenewable energy resources in spite of having triggered a large drop in planned investment in nuclear power plants across a number of major economies, including Germany, Switzerland, and Japan.

20. This term is equal to zero for the class of negative exponential functions of the type $u(c) = -e^{-\theta c}$ and for linear utility functions. It is strictly positive for negative exponential utility of the type $u(c) = -e^{1/c}$ and for CRRA utility, which is widely used in the literature.

21. Vissing-Jørgensen (2002) estimates EICS for stock- and bondholders, distinguishing among three wealth groups, as well as for nonstockholders. Her estimates range from 0.29 for stockholders to 1.38 for bondholders, with higher estimates for top-wealth-layer households and close to zero estimates for nonstockholders. See also Epstein and Zin (1991) and Hansen and Singleton (1982).

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22. For the benchmark set of parameters, the probability of the discovery occurring by $t = 30$ is equal to 72.25%.

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APPENDIX

A.1. TRANSFORMING A STOCHASTIC CONTROL PROBLEM INTO A DETERMINISTIC CONTROL PROBLEM

In the case of financial autarky the optimization problem is to maximize

$$E_\tau\left\{\int_0^\tau u(c_t)e^{-\rho t}dt + \int_\tau^\infty u(\tilde{c}_t)e^{-\rho t}dt\right\}, \quad \textbf{(A.1)}$$

subject to $c_t = Y_t^n - m_t$ and $\tilde{c}_t = \bar{Y}$, where E_τ denotes the expectation operator with respect to the distribution of the arrival date. Given that

$$\mathcal{P}[\tau \in (t, t+dt)|\tau \geqslant t] = q(m_t)dt + o(dt),$$

the elementary probability on the interval $(t, t+dt)$ is given by $q(m_t)e^{-\int_0^t q(m_s)ds}dt$. Then (A.1) can be rewritten as

$$\int_0^\infty\left\{\int_0^t u(c_s)e^{-\rho s}ds + \int_t^\infty u(\tilde{c}_s)e^{-\rho s}ds\right\}q(m_t)e^{-\int_0^t q(m_s)ds}dt. \quad \textbf{(A.2)}$$

Because the consumption rate after the arrival of the backstop is constant at $\bar{Y}$, the last term in the curly braces equals $u(\bar{Y})\frac{e^{-\rho t}}{\rho}$, and (A.2) can be written as

$$\int_0^\infty\left\{\int_0^t u(c_s)e^{-\rho s}ds\right\}q(m_t)e^{-\int_0^t q(m_s)ds}dt + \frac{u(\bar{Y})}{\rho}\int_0^\infty q(m_t)e^{-\rho t-\int_0^t q(m_s)ds}dt. \quad \textbf{(A.3)}$$

Defining $\mathcal{U}(t) = \int_0^t u(c_s)e^{-\rho s}ds$ and $\mathcal{V}(t) = -e^{-\int_0^t q(m_s)ds}$, we can apply integration by parts to the first term to obtain

$$\int_0^\infty\left\{\int_0^t u(c_s)e^{-\rho s}ds\right\}q(m_t)e^{-\int_0^t q(m_s)ds}dt \quad \textbf{(A.4)}$$

$$= \int_0^\infty \mathcal{U}(t)d\mathcal{V}(t) \quad \textbf{(A.5)}$$

$$= \mathcal{U}(t)\mathcal{V}(t) - \int_0^\infty \mathcal{V}(t)d\mathcal{U}(t) \quad \textbf{(A.6)}$$

$$= -\int_0^t u(c_s)e^{-\rho s}ds\left[e^{-\int_0^t q(m_s)ds}\right] + \int_0^\infty e^{-\int_0^t q(m_s)ds}u(c_t)e^{-\rho t}dt. \quad \textbf{(A.7)}$$

The term $\mathcal{U}(t)\mathcal{V}(t)$ is zero in the limit as t goes to infinity because $\int_0^t u(c_s)e^{-\rho s}ds < \infty$ and $\int_0^\infty q(m_s)ds = \infty$. Thus the original objective in (A.3) becomes

$$\int_0^\infty e^{-\int_0^t q(m_s)ds}u(c_t)e^{-\rho t}dt + \frac{u(\bar{Y})}{\rho}\int_0^\infty q(m_t)e^{-\rho t-\int_0^t q(m_s)ds}dt$$

$$= \int_0^\infty\left\{u(c_t) + q(m_t)\frac{u(\bar{Y})}{\rho}\right\}e^{-\rho t-\int_0^t q(m_s)ds}dt. \quad \textbf{(A.8)}$$

Defining an auxiliary state variable $z_t \equiv \int_0^t q(m_s)ds$ with $\dot{z}_t \equiv \frac{dz}{dt} = q(m_t)$ and $z_0 = 0$, the objective function (A.8) becomes

$$\int_0^\infty \left\{ u(c_t) + q(m_t)\frac{u(\bar{Y})}{\rho} \right\} e^{-\rho t - z_t} dt, \quad \textbf{(A.9)}$$

which is used to construct the Hamiltonian (6) in the text.

A.2. OPTIMAL INVESTMENT WITH OPEN ACCESS TO INTERNATIONAL LENDING AND BORROWING

The optimal control problem pertaining to phase II is

$$\max_{\tilde{c}_t} \int_\tau^\infty u(\tilde{c}_t)e^{-\rho(t-\tau)}dt,$$

subject to

$$\dot{a}_t = B^\alpha L^{1-\alpha} - \tilde{c}_t + ra_t, \quad \forall t > \tau. \quad \textbf{(A.10)}$$

The current-value Hamiltonian may be written as

$$H = u(\tilde{c}_t) + \mu_t \left[B^\alpha L^{1-\alpha} - \tilde{c}_t + ra_t \right],$$

and the first-order conditions as

$$\tilde{c}_t : \quad u'(\tilde{c}_t) - \mu_t = 0, \quad \textbf{(A.11)}$$

$$a_t : \quad \mu_t r = \rho - \dot{\mu}. \quad \textbf{(A.12)}$$

Differentiating equation (A.11) with respect to time and inserting the result into (A.12) yields the standard Keynes–Ramsey rule

$$\hat{\tilde{c}}_t = \frac{r-\rho}{\theta}, \quad \forall t > \tau,$$

and therefore the consumption path

$$\tilde{c}_t = \tilde{c}_\tau e^{\frac{r-\rho}{\theta}(t-\tau)}.$$

Combining this with the budget constraint (A.10) makes it possible to solve for the consumption rate right after the discovery takes place, $\tilde{c}_\tau$, and for the time path of asset holdings:

$$\tilde{c}_\tau = \left(r - \frac{r-\rho}{\theta} \right) \left(a_\tau + \frac{B^\alpha L^{1-\alpha}}{r} \right), \quad \textbf{(A.13)}$$

$$a_t = a_\tau e^{\frac{r-\rho}{\theta}(t-\tau)} + \frac{B^\alpha L^{1-\alpha}}{r} \left(e^{\frac{r-\rho}{\theta}(t-\tau)} - 1 \right). \quad \textbf{(A.14)}$$

The maximized discounted (time-τ) welfare in Phase II is

$$\Phi(a_\tau) = \int_\tau^\infty \frac{\tilde{c}_t^{1-\theta}}{1-\theta} e^{-\rho(t-\tau)}dt = u(\tilde{c}_\tau) \left(r - \frac{r-\rho}{\theta} \right)^{-1}.$$

The Hamiltonian associated with the RIC's original optimization problem may be written as

$$H = \{u(c_t) + \lambda(m)\Phi(a_t)\} e^{-\rho t - z_t} + \eta_t(ra_t + R_t^{\alpha} L^{1-\alpha} - c_t - P_t R_t - m) + \nu_t \lambda(m),$$

where η_t is the co-state variable associated with the constraint (13) and z_t is the auxiliary state variable, such that $\dot{z}_t = \lambda(m)$. The optimality conditions are

$$R_t : \ \eta_t \left(\frac{\partial F_t}{\partial R_t} - P_t \right) = 0, \tag{A.15}$$

$$c_t : \ u(c_t) e^{-\rho t - z_t} - \eta_t = 0, \tag{A.16}$$

$$m : \ \lambda'(m)\Phi(a_t) e^{-\rho t - z_t} - \eta_t + \nu_t \lambda'(m) = 0, \tag{A.17}$$

$$a_t : \ \lambda(m) \frac{\partial \Phi_t}{\partial a_t} e^{-\rho t - z_t} + r\eta_t = -\dot{\eta}_t, \tag{A.18}$$

$$z_t : \ -\Big[u(c_t) + \lambda(m)\Phi(a_t)\Big] e^{-\rho t - z_t} = -\dot{\nu}_t. \tag{A.19}$$

Combining (A.16) with (A.18) yields the Keynes–Ramsey rule under uncertainty:

$$\theta \hat{c}_t = r - \rho - \lambda(m) \left[1 - \frac{u'(\tilde{c}_t)}{u'(c_t)} \right],$$

where I used $u'(\tilde{c}_t) = \frac{\partial \Phi_t}{\partial a_t}$. Isolating ν_t from (A.17), differentiating with respect to time, and inserting the result into (A.19) yields

$$u(c_t) = \frac{u''(c_t)\dot{c}_t - u'(c_t)[\rho + \lambda(m)]}{\lambda'(m)} + \rho\Phi(a_t) - u'(\tilde{c}_t)\dot{a}_t.$$

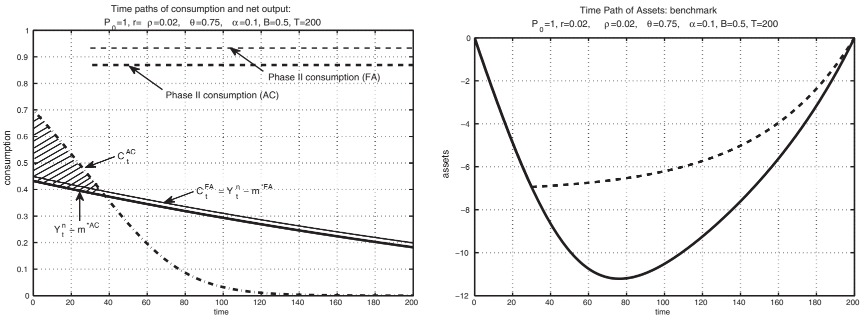
The expression in the square brackets can be rewritten in terms of consumption growth rate and then combined with the Keynes–Ramsey rule, so that we get equation (22) in the text:

$$\lambda'(m) \left[\rho\Phi(a_t) - u(c_t) - u'(\tilde{c}_t)\dot{a}_t \right] = u'(c_t) r + \lambda u'(\tilde{c}_t).$$

A.3. OPTIMAL PATHS OF CONSUMPTION AND ASSETS

The possibility of international lending and borrowing has important implications for the intertemporal allocation of consumption in an economy striving to achieve energy independence. Under the benchmark set of parameters, the economy is a net debtor. Borrowing from abroad (net of interest payments) can be visualized in Figure A.1a by the gap between the "c_t^{AC}" locus and the "$Y_t^n - m^{*AC}$" locus (the shaded area). The figure demonstrates that foreign credit has a dual purpose. It serves not only to finance the increase in the optimal investment rate but also to raise current consumption during the initial phase of the economy's planning horizon. The initial "overconsumption" in the present model is exactly the opposite of the precautionary saving phenomenon in models where an economy is subject to a random *drop* in income [see, e.g., Toche (2005) for the case of job loss].

Note that in the present calibration the rate of time preference, ρ, is identical to the rate of interest, r. In a deterministic environment, the economy's time path of consumption would have been flat. In a stochastic environment, however, the prospect of an upward jump in



(a) consumption (b) assets

FIGURE A.1. Optimal paths of consumption and net foreign asset position.

income results in a clockwise rotation of the consumption path. During the initial phase of the planning horizon $c_t^{*\mathrm{AC}}$ exceeds $Y_t^n - m^{*\mathrm{AC}}$, so that $\dot{a}_t - ra_t < 0$. Thus, in spite of ρ being equal to r, RIC's asset position initially deteriorates. This is shown in Figure A.1b.

Suppose that the invention happens to occur at $t = 30$.[22] Under both scenarios, consumption in Figure A.1a jumps upward (see dashed lines) and the economy switches from borrowing to repaying its debt (the dashed line in Figure A.1b). A higher value of B obviously causes a larger upward jump and a faster loan repayment (not shown in the figures). Note that the initial consumption rate in Phase II is *higher* under FA than it is with AC. The reason is that in both scenarios the arrival of the backstop ensures a constant flow of output, but in the second scenario the economy starts Phase II with a negative foreign asset position, which must be liquidated by the end of the planning horizon. In general, the initial consumption rate in Phase II and the subsequent time path of consumption under AC depend on the parameters of the model and in particular on the difference between r and ρ [see equation (21)]. If $r > \rho$ ($r < \rho$), consumption in Phase II exhibits a rising (falling) time path, whereas $\tilde{c}_\tau$ is below (above) the value obtained under the assumption $r = \rho$.

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