# Silicon Retina and Cochlea with Asynchronous Delta Modulator for Spike Encoding 

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#### Abstract

Spike encoding in bio-inspired sensory chips such as silicon retina and cochlea has been more of an empirical practice drawing inspirations from neuroscience since the dawn of neuromorphic engineering. From a system design and optimization point of view, the two natural questions to ask are how faithfully the encoded spike train represents the original analog input, and what minimum level of encoding fidelity is required according to some output performance specifications like the classification accuracy of a spiking neural network classifier. This thesis develops from the first question in the context of spiking sensors with massively parallel spike encoders, and covers both computational spike coding analysis and silicon chip design of spiking sensors.

The difficulty in rigorous mathematical analysis of spike encoding integrity especially when nonidealities in practical silicon implementations are considered particularly lies in its irregular sampling and nonlinear nature. Although simple linear spike decoding is cost-efficient in terms of computational and hardware expenses, it does not fully appreciate the timing information, i.e. the precise timestamps contained in the output spike train. Therefore it appears that to achieve the same signal-to-distortion ratio (SDR) which is used as the evaluation metric of encoding quality, linear decoding requires much more encoding quantization levels compared to nonlinear decoding based on the so-called frame theory. Nonetheless linear decoding reveals that, when feedback delay in the encoders is taken into account, the encoding mechanism of self-timed reset (STR) used in prior silicon retina results in lower SDR than the asynchronous delta modulation (ADM) commonly adopted for level-crossing ADCs. To further study the impact of jitter in spike timestamps caused by signal-dependent comparison delay during spike generation and spike transmission queueing due to limited communication bandwidth, nonlinear decoding algorithms are developed for STR and ADM. Circuit analysis and queueing theory are used to emulate the jitter generation with the examples of a specific comparator topology and two queueing models. A quantitative link is established between the system and circuit parameters and the decoding SDR metric, which is useful for future specifications-guided design of spiking sensory systems.

One type of silicon retina called dynamic vision sensor (DVS) encodes temporal contrast (TC) change of light intensity into output spike trains. Prior DVSs normally have a TC sensitivity of about $10 \%$ and all use STR spike encoders. Although DVSs have been successfully used in object tracking, for other potential applications like optical neuroimaging, the TC sensitivity and spike encoding quality need to be improved. The major advantage of adopting DVSs over prevailing APS imagers is much reduced output data rate given the sparsity of neuronal activity in neuroimaging, which could largely save power in wireless data transmission and thus facilitate continuous monitoring of free-moving animals. To improve the TC sensitivity, a low-noise transimpedance photoreceptor with pFET common-gate feedback and a programmable gain amplifier (PGA) employing a compact two-stage Opamp with pseudo-cascode compensation are used. To improve the spike encoding, a compact asynchronous switched-capacitor circuit is proposed for in-pixel ADM. The measured results show that a $1 \%$ TC sensitivity is achieved with $35 \%$ relative standard deviation across the pixel array, and using an exemplary visual stimulus input, up to $3.5 \times$ more spikes are preserved compared to a prior DVS with STR encoding. A simulated optical neuroimaging experiment is demonstrated.


To aim for ubiquitous smart audio sensing in the context of internet of everything, a $0.5-\mathrm{V}$ ultra-lowpower binaural silicon cochlea with ADM for spike encoding is designed. The spike output of a silicon cochlea is the natural input to event-driven DSPs and spiking neural networks for cognitive audio infor-
mation processing. The cochlea has a parallel architecture with $64 \times 2$ channels. Besides bias distribution network, each binaural channel consists of a shared translinear loop for BPF $Q$-tuning, a pair of identical programmable attenuators, asymmetrical 1-zero 4-pole bandpass filters (BPFs), ADMs with adaptive selfoscillating comparators, and asynchronous logic blocks. The proposed asymmetrical BPF is composed of a low-power $4^{\text {th }}$-order source-follower-based lowpass filter (LPF) and a summing PGA. The proposed ADM employs latched comparators instead of commonly-adopted continuous-time comparators based on multistage amplifiers in clockless systems for improved power efficiency. The reset signal needed to initialize the regenerative latch for each comparison is generated through a self-oscillation loop, and the oscillation frequency is adaptive to the output spike rate of a local channel. The measured power consumption of the $0.5-\mathrm{V}$ core is about $55 \mu \mathrm{~W}$ at a 100 k spike/s output rate and the system has a $>70 \mathrm{~dB}$ dynamic range. Using the normalized power metric, this design is about $18 \times$ more power efficient compared to the best prior art. Moreover, the transfer functions between the corresponding binaural channels exhibit good matching and each channel has a wide $Q$-tuning range from 1 to about 40 . This cochlea targets integration with ultra-low-power spike processors to form a smart audio sensing SoC.

## Zusammenfassung

Die Impulskodierung (IK), welche in biologisch inspirierten Sensoren wie Silizium-Retinae und Silizium-Cochleae verwendet wird, ist seit den Anfängen der neuromorphen Ingenieurswissenschaften basiert auf empirischen Beobachtungen der Neurowissenschaften. Ausgehend von optimalem Systemdesign stellen sich zwei Fragen: Wie genau repräsentiert die Pulsefolge am Ausgang des Sensors das analoge Eingangssignal, und welche minimale Genauigkeit ist nötig um eine gewisse Leistung zum Beispiel eines Klassierers zu erreichen. Diese Doktorarbeit behandelt die erste Frage im Kontext von Sensoren mit vielen, parallel arbeitenden Impulskodierern.

IK im Detail mathematisch zu analysieren ist sehr schwierig, insbesondere wenn Nicht-Idealitäten von praktischen Schaltungen einbezogen werden sollen, da die Signalabtastung unregelmässig ist, und sich die Schaltungen in der Realität nicht-linear verhalten können. Eine einfache lineare IK ist effizient bezüglich Rechenaufwand und Hardwareanforderungen, jedoch wird damit die zeitliche Information der Impulsfolge nicht vollständig genutzt. Um debselben Signal-zu-Verzerrungs-Abstand (SDR) wie mit einer nicht-linearen, auf der sogenannten „Frame-Theorie" basierten IK zu erreichen, müssen bei einer linearen IK deshalb mehr Quantisierungsstufen benutzt werden. Trotzdem zeigt schon lineare IK, dass der in bisherigen Silizium-Retinae verwendete Kodierungsmechanismus des „selbst-rücksetzens" (STR) einen schlechteren SDR als der häufig in Schranken-Analog-zu-Digital-Konvertern verwendete asynchrone Delta-Modulator (ADM) zur Folge hat. Um auch den Einfluss von Ungenauigkeiten der Zeitstempel der Impulse genauer zu untersuchen, wurden nicht-lineare Dekodieralgorithmen für STR und ADM entwickelt. Die Ungenauigkeiten derZeitstempel werden verursacht durch signalabhängige Komparatorverzögerung und/oder Verzögerungen in der Kommunikation der Impulse durch die limitierte Bandbreite der Kommunikations-Schnittstelle. Eine Schaltungsanalyse der benutzen Komparatoren und zwei verschiedene Warteschlangenmodelle werden benutzt um die Zeitungenauigkeiten zu emulieren, und damit einen quantitativen Zusammenhang zwischen System- und Schaltungsparametern und dem SDR des Dekodierers herzustellen. Dieser Zusammenhang kann nützlich sein für die Entwicklung zukünftiger impulsbasierter Sensorsysteme.

Der am Institut für Neuroinformatik entwickelte dynamische Sehsensor (DVS) enkodiert die zeitliche Änderung (TC) des einfallenden Lichts in eine Impulsfolge am Ausgang. Bisherige DVS-Sensoren hatten eine TC-Empfindlichkeit von etwa $10 \%$ und benutzen ausschliesslich STR Kodierer. DVS werden benutzt für Standortverfolgung von Objekten, aber für Anwendungen zum Beispiel in neurologischen Bildgebungsverfahren müssen der TC und die Kodierqualität verbessert werden. Der Vorteil von DVS gegenüber konventionellen APS-Bildsensoren ist die reduzierte Datenmenge durch die geringe Dichte neuronaler Aktivität. Die verminderte Datenmenge würde einen tieferen Energieverbrauch von drahtlosen Sensoranbindungen und damit eine kontinuierliche Überwachung freilaufender Tiere ermöglichen.

Diese Doktorarbeit präsentiert einen neuen DVS-Sensor mit verbesserter TC-Empfindlichkeit. Der neue Sensor verwendet einen rauscharmen Transimpedanz-Fotorezeptor mit einem common-gate pFET im Signalrückführungspfad und einen programmierbarer Verstärker. Der benutzte zweistufige Operationsverstärker ist kompakt und verwendet eine Pseudo-Kaskoden-Kompensation. Um die Kodierqualität zu verbessern wird ein kompakter, asynchroner Schaltkreis mit geschalteten Kondensatoren verwendet, welcher einen Intrapixel ADM implementiert. Die Messungen zeigen eine TCEmpfindlichkeit von $1 \%$ und $35 \%$ relative Standardabweichung auf dem Pixelfeld. Mit einem spezifischen Stimulus werden im Vergleich zu bisherigen DVS-Sensoren mit STR-Kodierung bis zu 3.5-
mal mehr Impulse generiert. Die Fähigkeiten des Sensors werden auch an Hand eines simulierten Neuro-Bildgebungs-Experiments demonstriert.

In einem zweiten Teil dieser Doktorarbeit wurde im Kontext von intelligenten Audiosensoren für das „Internet of everything" eine verbrauchsarme 0.5 V Silizium-Cochlea mit ADM Impulskodierer entwickelt. Die Impulsfolge am Ausgang einer Silizium-Cochlea eignet sich sehr gut als Eingangssignal von impulsgesteuerten digitalen Signalprozessoren und neuronalen Netzwerken für die AudioSignalverarbeitung. Die Cochlea hat eine parallele Architektur mit zwei Mal 64 Kanälen. Jeder binaurale Kanal besteht aus einem asymmetrischen Bandpassfilter (BPF) mit verteilter translinearer Schlaufe für die BPF Qualitätsabstimmung, einem Paar programmierbarer Attenuatoren, ADMs mit adaptiven, eigenschwingenden Komparatoren und asynchroner Logik. Der BPF hat eine Nullstelle und 4 Pole und besteht aus verbrauchsarmen Source-Folger-basierten Tiefpassfiltern vierter Ordnung und einem programmierbaren Summierverstärker. Um den Stromverbrauch zu senken benutzt der ADM einen verriegelten Komparator anstelle des oft in taktlosen Systemen verwendeten zeitkontinuierlichen mehrstufigen Komparators. Das Signal zur Rückstellung der Verriegelung wird generiert durch eine eigenschwingende Schaltung, die Schwingungsfrequenz adaptiert sich an die Impulsrate des lokalen Kanals. Der gemessene Stromverbrauch des 0.5V-Kerns der Cochlea ist 55 Mikrowatt bei einer Ausgangsrate von hunderttausend Impulsen pro Sekunde, und das System hat einen Dynamikumfang von mehr als 70 dB . Wenn man den normalisierten Leistungsverbrauch vergleicht, ist das hier präsentierte System etwa 18 Mal effizienter als der Stand der Technik. Die Übertragungsfunktionen der binauralen Kanäle zeigen eine gute Abstimmung, und jeder Kanal hat einen grossen Qualitäts-Abstimmung-Bereich von 1 bis 40 . Diese Cochlea könnte zusammen mit sehr verbrauchsarmen Impulsprozessoren integriert werden um ein intelligentes Audiosensor-System zu bilden.

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## Chapter 1: Introduction

(2)hile neuroscientists are delving deep into the working mechanisms of the ultra-intricate neural networks in our central nervous system that has the number of synaptic connections a thousand times larger than the number of the stars in our Milky Way galaxy, engineers have been going to great lengths to emulate the extraordinary capabilities of our information sensing and processing organs, particularly the retina, the cochlea and the neocortex. This thesis is about another engineering endeavor at evolving artificial bio-inspired sensors, including the spiking silicon retina and cochlea.

### 1.1 Brief History of Spiking Sensors

### 1.1.1 Spiking Silicon Retina

In the late 1980s, Carver Mead brought in the IC community the concept of neuromorphic engineering trying to replicate the morphologies and functions of biological nervous systems with the fast-evolving silicon technologies, and a number of his protégées devoted great efforts in bridging the knowledge gap between biology and electronics. One such representative figure is Misha Mahowald who built the first ever silicon retina [1]. In a most simplified way, it mimicked the four layers of cells in a mammalian retina, namely the photoreceptor, the horizontal cell, the bipolar cell and the ganglion cell. Back then the design philosophy was dominated by mimicking biological details and did not emphasize possible practical applications and performance specifications. The results were demonstrations of interesting engineering feat with biological flavors other than something practically useful. Consequently, despite the pioneering creation of many circuit and system rudiments that are still academically in use today such as the adaptive logarithmic photoreceptor and the address event representation (AER) protocol for spike transmission, the industrial attention was mostly drawn to the famous active pixel sensor (APS) [2] which has a more straightforward working principle to comprehend and offers more reliable performances for commercial production with wide applications in consumer digital cameras, high-end telescopes for astrophysics, optical bioimaging, computer/machine vision etc.

Proliferated in 1990s, APS imagers capture images or record videos based on the notion of frame, the 2-D equivalence of the classic 1-D sampling with equidistance time steps. This uniform sampling has a well-established mathematical foundation [3], and is ubiquitously used in analog-to-digital converters (ADCs) to transform real-world continuous analog signals into discrete digital representation, both in amplitude and time. An APS pixel normally has a very simple circuitry composed of three or four transistors to transform photons to sampled voltages, which has substantial advantages in spatial resolution and uniformity. Because of these advantages, APS imagers have become the de facto standard in image/video acquisition. On the other hand, the original silicon retina and the subsequent improvements to morph all the five retinal layers (including the amacrine cells) [4] suffered from significant pixel mismatch due to the pixel complexity, making them barely useable. In addition, asynchronous non-uniform sampling associated with the spike generation of each individual pixel and the asynchronous (arbitrated) spike transmission in a sensor array is not as well understood by electronic engineers as its counterpart of synchronous uniform sampling.

The so-called octopus imager published in early 2000s [5] indicates a design style shift of spiking retinae. The grand idea of faithfully morphing biological retinae was displaced by keeping the functional essence for performance gain. Each pixel performs a nearly linear conversion of light intensity to spike rate, and the spike transmission follows the AER protocol. Although the $80 \times 60$ pixel array achieved large dy-


Figure 1.1. Simplified illustration of a dynamic vision sensor (DVS) pixel [6].
namic range, high effective frame rate, relatively low power, and reasonably small fixed pattern noise (FPN), the light-intensity-to-frequency encoding, i.e. the pulse frequency modulation (PFM), is not an efficient way of conveying information considering its temporal redundancy and in turn the heavy load on data transmission and post-processing.

Reduction of temporal redundancy could be done via frame-based methods by subtracting the sampled voltages in each pixel of two consecutive frames [7], [8], but the repetitive frame scan still wastes unnecessary energy especially when there is no temporal change at all, and the temporal resolution is limited to frame rate. It has been discovered that human eyes rely on constant microsaccades to prevent visual fading during fixation [9], which indicates that the ganglion cells in human retina produce almost no spike if no temporal contrast of light intensity $\Delta I$ is detected. The dynamic vision sensor (DVS) invented in 2006 by Lichtsteiner et al. [6], [10] exploited this mechanism of spike generation. As illustrated in Figure 1.1, the pixel functions as follows: light intensity temporal contrast $\Delta I$ is converted to voltage change $\Delta V ; \Delta V$ is amplified by $A$ times; the amplified AC signal is compared with two comparators with upper and lower thresholds; the asynchronous control block generates a spike if an ON or OFF threshold crossing is detected; the spikes are communicated off-chip via the on-chip peripheral AER circuits; the capacitivelycoupled amplifier is reset once the acknowledge in response to the request of this pixel comes back. Because of the AC coupling, a DVS pixel only generates a spike if the $\Delta I$ within some frequency passband is sufficiently large, which means no output spike is generated for a static scene or a scene with merely very high frequency content. This is similar to the experimental observations from human eyes [11], except the eye bandwidth is restricted within tens of Hz whereas the DVS can response to stimuli above kHz depending on the ambient illuminance and the circuit bias settings. The DVS is a big step for spiking retinae towards practical applications where dynamic content recording and analysis is of particular interest. The pixel-autonomous and frameless operation with in-pixel self-timed reset spike encoding largely reduces the temporal redundancy of output data and breaks the tight trade-off between data rate and power consumption in conventional APS imagers [12], [13]. The pixel uniformity is also greatly improved compared to prior spiking retinae thanks to the signal amplification before threshold crossing and the wellmatched gain among pixels using capacitance ratio. Further works around DVS include high-speed application in optical line sensors [14], improving temporal contrast sensitivity [15], [16] and adding the function of absolute light intensity acquisition [17], [18].

Extracting spatial contrast on the focal plane is also of interest because it reduces spatial redundancy in


Figure 1.2. Cascaded second-order sections (SOSs) and the detailed circuits of one SOS [23].
output data while keeping relevant contour information for shape and object recognition, which is dual in functionality to temporal redundancy reduction. Frame-based solutions [19], [20] give good output uniformity and low power consumption. Spiking sensors that use diffusive grid connecting neighboring pixels [21] for spatial contrast computation still suffer from large component mismatch which needs in-pixel calibration and consequently results in large pixel size with very small fill factor (e.g. 2\% in [22]).

### 1.1.2 Spiking Silicon Cochlea

Early in-silico cochlea modeling in Mead's lab focused on the frequency division of the basilar membrane as a function of place [23], [24]. Multiple (tens to hundreds of) second-order sections (SOSs) with each composed of two feedforward $\mathrm{g}_{\mathrm{m}}$ - C integrators and a feedback OTA are connected in series as shown in Figure 1.2. The bandpass filtering (BPF) outputs are obtained by subtracting $V_{x}$ and $V_{\text {out }}$ in each SOS. If $g_{m 1}=g_{m 2}=g_{m t}, g_{m 2}=g_{m Q}$, and $C_{1}=C_{2}=C$, the BPF transfer function of one single SOS can be then written as:

$$
H(s)_{\text {sos }}=\frac{V_{\text {out }}-V_{x}}{V_{i}}=-\frac{\frac{g_{m t}}{C} s}{s^{2}+\frac{2 g_{m t}-g_{m Q}}{C} s+\frac{g_{m r}^{2}}{C^{2}}}=-\frac{\omega_{0} s}{s^{2}+\frac{\omega_{0}}{Q} s+\omega_{0}^{2}}
$$

where $\omega_{0}$ is the angular central frequency and $Q$ is the quality factor, which can be written as:

$$
f_{0}=\frac{\omega_{0}}{2 \pi}=\frac{g_{m r}}{2 \pi C}, Q=\frac{1}{2-g_{m Q} / g_{m \tau}}
$$

where $f_{0}$ is the central frequency. All $f_{0}$ 's of the cascaded SOSs are proportional to the bias current $I_{\tau}$ of each individual SOS, and are geometrically scaled. $Q^{\prime}$ s are controlled by the bias current $I_{Q}$. The first SOS in the serial chain with its input normally connected to a preamplifier is a $2^{\text {nd }}$-order symmetrical BPF, and due to the accumulation of poles, the following stages have increasingly larger lowpass roll-off, theoretically $40 \mathrm{~dB} /$ decade more after each stage. The highpass roll-off is however always $20 \mathrm{~dB} /$ decade because the single zero is created by the subtraction of $V_{x}$ and $V_{\text {out }}$ in each SOS.

There are several problems with the cascaded structure: 1. $I_{\tau}$ was generated by using linearly-graded voltages to bias nFETs in subthreshold and thus had large mismatch; 2. Although the BPF transfer functions have progressively steeper roll-off as $f_{0}$ decreases, the frequency selectivity is poor. Partly this is because the $Q$ of each SOS has to remain low (usually $<2$ ) to prevent the prohibitively large passband


Figure 1.3. (a) Zero-crossing spike encoding; (b) Integrate-and-fire spike encoding.
gain due to gain accumulation; 3. Noise is aggregated along the cascaded chain, so the SNR of the lowfrequency stages could be much lower than the input stage. The random mismatch was addressed by using the compatible lateral bipolar transistors (CLBTs) in standard CMOS instead of subthreshold nFETs [25], which is basically the parasitic bipolar transistors often used in bandgap reference as the substitute of diodes [26]. The results showed significant improvement in linearity of logarithmic scaling of $f_{0}$ over SOS stages. The low $Q$ and noise aggregation problem can be circumvented by using a parallel architecture even though it does not quite resemble the structure of the basilar membrane that allows cascaded propagation of mechanical waves. In the analog bionic ear processor presented by Sarpeshkar et al. [27], the 16 parallel channels employing input-attenuated $4^{\text {th }}$-order symmetrical $\mathrm{g}_{\mathrm{m}}$ - C BPFs [28] is said to be capable of having $Q$ up to 10 . Parallel filter banks nevertheless fall short in providing steep roll-off unless multiple filter biquads are used. For example, $8^{\text {th }}$-order one-zero gammatone filter transfer function with 8 poles and 1 zero was implemented using 3 lowpass and 1 bandpass pseudo-differential class-AB logarithmic current-mode biquads [29]. The area is $2.25 \mathrm{~mm}^{2}$ for one channel in $0.35 \mu \mathrm{~m}$ CMOS and the power consumption is $3.4 \mu \mathrm{~W}$ without AGC at $f_{0}=3.3 \mathrm{kHz}$, whereas in [27] the area for one BPF is about 2.8 $\mathrm{mm}^{2}$ in $1.5 \mu \mathrm{~m}$ BiCMOS and the power consumption is $5.4 \mu \mathrm{~W}$ at $f_{0}=7.1 \mathrm{kHz}$. Considering the difference in the technology nodes, area and power are clearly the penalty for higher roll-off with more biquads.

Besides the cascaded and parallel architectures, new models have been proposed to better emulate the human cochlea. One example is to use the resistive passive coupling among parallel resonators to model the cochlea fluid [30]-[32], and another example takes inspiration from the function of the outer hair cells and uses active bidirectional coupling to counteract the destructive interference among channels in passive coupling [33]. The efforts towards more bio-realistic modeling on analog VLSI were unfortunately buried in unsatisfactory performances mainly due to the obstinate mismatch problem. In [31], the $f_{0}$ variation is as large as $15 \%$ while the ideal $f_{0}$ ratio between neighboring BPFs is less than 1.05 , and in [33], the expected $Q_{10}$ ( $f_{0}$ divided by the bandwidth taken at 10 dB below the peak) enhancement in software simulation is hindered by abrupt changes in basilar membrane properties in silicon.

Integration of spike encoders together with basilar membrane has rarely been implemented in silicon. About six such chips sparsely scattered spanning twenty years from 1990s to 2010s, and their encoding mechanisms mainly include zero-crossing [27], [34] and integrate-and-fire [33], [35]-[37], as illustrated in Figure 1.3. Zero-crossing rate is useful for relatively primitive sound processing, e.g. zero-crossingbased ultra-low-power voice activity detection [38]. However, zero-crossing alone usually does not constitute a complete representation of the original signal [34], [39] and thus certain information like amplitude could be lost or corrupted during encoding. On the contrary, integrate-and-fire can theoretically have perfect reconstruction even with absolute refractory period if certain conditions are satisfied [40]. For re-al-world-scenario applications, besides digit recognition that used the cochlea with zero-crossing encod-
ing [34], the spike trains of the binaural cochlea with integrate-and-fire encoding called AER-EAR was readily used for sound source localization by exploiting the interaural time difference even though the implemented encoders suffered from substantial mismatch among channels in one monaural cochlea and between binaural channels [36], [37].

Conventional sound acquisition uses high-precision synchronous ADCs, and the frequency analysis is done in digital domain by FFT which can still be power-consuming even with the state-of-the-art development in 32 nm CMOS [41]. For always-on smart sensing like voice activity detection, a spike-driven processor built on spiking silicon cochlea has the potential advantage of reducing the system power consumption due to the low-power analog filtering and adaptive digital power in response to the incoming spike trains. A recently developed voice activity detector with cochlea-like 16-channel analog BPFs and a mixed-signal decision-tree machine leaning kernel has shown a $\times 10$ reduction in power compared to prior arts [42], even though the microcontroller for configuring the decision tree is clocked and off-chip.

### 1.2 Other Related Works

### 1.2.1 Revival of Neuro-Inspired Computing

14 nm processors are in mass production from Intel and Samsung. Moore's law is predicted to have only 5 to 10 more years before it literally hits the end according to ITRS. The technology scaling that has been successfully lasting for half a century has undoubtedly aided the seamless penetration of electronics into our daily life, and yet nothing that is sufficiently intelligent to make a decision on its own in complex situations has come to life. Of course we should count out supercomputers sitting in a cozy lab and consuming immense electricity. The ever-increasing motivation of maintaining the prospects of the commercially profitable semiconductor industry at the end of Moore's law has spurred the diverse quests for alternative means of building smarter and more energy-efficient systems such as carbon-nanotube-based computing [43], inexact computing [44], [45], etc. Revived to be the neuro-inspired computing with more emphasis on taking functional inspiration other than trying to be bio-realistic, neuromorphic engineering has also reemerged to the surface of common technological interests after years of relatively quiet progress. In August 2014, IBM's announcement of the TrueNorth chip with a million digital spiking neurons has attracted a lot of public attention [46]. A visual recognition task was demonstrated with 72 mW power consumption at 775 mV power supply which is $\times 176,000$ and $\times 769$ more energy-efficient compared to running on a modern general-purpose microprocessor and the state-of-the-art multiprocessor neuromorphic platform SpiNNaker [47], respectively. TrueNorth largely deviates from the stereotyped Von Neumann architecture of centralized memory and processing units, and instead is composed of massively parallel neurons and adopts the distributed slow asynchronous spike communication among synaptic cores other than high-speed digital communication which is inevitable in conventional digital processors. As pointed out in [46], spiking sensors like the DVS and AER-EAR could be the perfectly suitable candidates for TrueNorth in live settings for real-time applications because their spike train output is exactly what TrueNorth needs as input. It is interesting to see that the campaign of neuro-inspired computing is expanding beyond the traditional neuromorphic engineering community, and has drawn the attention of engineers who are more on the industry-oriented IC design side: Jan Rabaey at UC Berkeley is travelling around the world to give the presentation 'The Return of Neuro-Inspired Computing - Why Now', Zhengya Zhang at University of Michigan is developing digital neural network IC with on-chip learning for image feature extraction [48], Mingoo Seok at Columbia University is also developing digital spiking neural network chip for brain-machine interface applications like spike sorting, just to name a few.


Figure 1.4. A TEM/TDP/TDM system, where information is represented and processed in time domain in the form of continuous-time and discrete-amplitude spikes [49].

### 1.2.2 Computational Spike Encoding/Processing/Decoding

The intense debate about the nature of neural coding within the neuroscience community - whether it is rate coding or temporal coding - has not yet settled. At least, from an engineering point of view, rate coding is far less data- and power-efficient than temporal coding, e.g. comparing the Octopus imager with the DVS. Although the Octopus is capable of acquiring absolute intensity information, temporal coding can do the same by using the information of inter-spike interval of a single spike pair [17]. Aurel Lazar shares the same belief in temporal coding. He pondered the system TEM/TDP/TDM as illustrated in Figure 1.4 [49], where TEM, TDP and TDM stand for time encoding machine, time domain processing and time decoding machine, respectively, in analogy to the conventional clocked ADC/DSP/DAC system. Signals are represented and processed all in time domain in the form of spike trains. The precise timestamps of the encoded spikes are deemed to contain all the information about the original input given that the parameters of the spike encoders are known. The importance of precise timestamps is substantiated by the observation of the cat lateral geniculate nucleus (LGN) where the temporal structure of the spiking response of an LGN neuron was found to have a finer timescale than that of the input stimuli filtered by the neuron's temporal receptive field (RF), and large timing jitter of spikes with its reciprocal comparable to the average firing rate results in much degraded signal reconstruction [50]. Lazar approached the analysis of the TEM/TDP/TDM system dealing with precise timestamps in a rigorous signal-processing
way, and studied many forms of TEM and their corresponding TDM, from single-input and single-output neuron to multi-input and multi-output neuron ensemble [51].

Mathematically TEM maps input analog waveforms to output spike timestamps following the orthodox $t$-transformation. Taking the simplest integrate-and-fire encoding depicted in Figure 1.4 as an example with immediate spike-triggered feedback reset and no refractory period, its $t$-transformation can be written as:

$$
\int_{t_{k}}^{t_{k+1}} x(t) d t=\delta
$$

where $\delta$ is the encoder threshold, $x(t)$ is the analog input, and $t_{k}$ is the output $k^{\text {th }}$ timestamp with $k \in Z$. It has been shown that perfect reconstruction of $x(t)$ from the output spike train $z_{k}$ is possible if the average spike rate is at least as large as the Nyquist rate and the dimensionality of $z_{k}$ is infinite [52]. A TDM implements the reconstruction process by giving appropriate weight to each spike in $z_{k}$ and then passing the weighted $z^{\prime}{ }_{k}$ through a brick-wall lowpass filter (LPF) with the same bandwidth of $x(t)$. Each spike is treated as an ideal Dirac-delta function so that its impulse response of an ideal LPF is a sinc function, and therefore the reconstructed signal $x^{\prime}(t)$ can be seen as superposition of differently-weighted sinc functions that are time-shifted to $t_{k}$. The method of calculating the weights is adapted from irregular sampling based on frame theory [51]. Simply put, in contrast to a basis with linearly independent elements in a vector space $V$ equipped with an inner product, the elements in a frame may be linearly dependent and thus could have more than one set of coefficients to represent an arbitrary element in $\boldsymbol{V}$. To form a frame $\left\{f_{k}\right\}_{k \in Z^{*}}$, it needs to satisfy the inequality below:

$$
A\|v\|^{2} \leq \sum_{k \in Z^{*}}\left|<v, f_{k}>\right|^{2} \leq B\|v\|^{2} \quad \forall v \in \boldsymbol{V}
$$

where $A$ and $B$ are the frame bounds, and have $0<A, B<\infty$. In TDM, a frame can be formed by the timeshifted sinc functions. TDP was first demonstrated in a video coding setup as shown in Figure 1.4 [53]. A video passes through multiple RFs with spatiotemporal filtering before being fed into identical spike encoders. The processed spike trains by the TDP are recovered back to analog waveforms which are then summed together after passing through the same RFs to obtain the recovered video. The TDP block is merely composed of switches that control the routing of spike trains among different RF channels. Identity preserving transformations such as video recovery, rotation and zooming can be achieved by this simple TDP.

The mathematical treatment of the TEM/TDP/TDM system provides a first rigorous insight into how spike timestamps play the role in real-world analog signal representation and processing. The TEM/TDM modeling is especially useful for evaluating the quality of spike encoding in spiking sensors. The change of information currency from binary digits $0 / 1$ to spikes however increases the analysis difficulties. This whole system including the TDP could be a primitive model for future neuro-inspired silicon SoC with integrated sensing and processing modules working in time domain without any clock, but it is yet to be shown that the functional mechanisms of timestamps in advanced processing tasks like classification through learning using more complex neural networks.

### 1.2.3 Clockless Continuous-Time ADC/DSP/DAC Systems

A conceptually related system to the TEM/TDP/TDM that has been implemented in silicon is the clockless continuous-time (CT) ADC/DSP/DAC [54] (Figure 1.5) proposed by Yannis Tsividis. Like


Figure 1.5. A clockless CT-ADC/DSP/DAC system, where usually the ADC uses level-crossing sampling, the DSP implements CT digital filtering, and the DAC performs zero-order-hold [54].

TEM, the CT-ADC only quantizes the input by its amplitude whereas in conventional ADC both time and amplitude are quantized. The conversion is based on the level-crossing sampling scheme [55] which is a mutation of the asynchronous delta modulation (ADM) proposed back in 1960s [56]. Every time the input passes a predefined amplitude level (normally the levels are equally spaced), an output spike (also called event) is generated and its polarity depends on the sign of the signal slope at the moment of level-crossing. Using the signal-to-quantization-noise ratio (SQNR) metric as in clocked ADCs, the performance can be quantified the same as the ADM with white Gaussian noise input [57]:

$$
\mathrm{SQNR}=18.8+30 \log _{10} \frac{\sigma}{\delta} \mathrm{~dB}
$$

where $\sigma$ is the rms amplitude and $\delta$ is the level-crossing threshold. Note that the SQNR increases 9 dB for every increased 1-bit resolution, i.e. halved $\delta$, in contrast to 6 dB in clocked ADCs. This is also true for sinusoidal input [58]. The encoded signal can be in either digital or spike (also called delta) format, and is sent to the CT-DSP for processing, where only digital filtering functions including FIR and IIR have been demonstrated so far. Taking the digital CT-FIR as an example, it is readily derived from the analog FIR with the output $y(t)$ written as:

$$
y(t)=\sum_{n=0}^{N} w_{n} x(t-n \tau)
$$

where $x(t)$ is the input, $\tau$ is the tap delay, $w_{n}$ is the weight for tap $n$, and $(\mathrm{N}+1)$ taps are used. Note that 'CT-DSP' is particularly distinguished from 'asynchronous DSP' like asynchronous microprocessors pioneered by Alain Martin et al. [59] - the former emphasize the exact timing of spikes while the later only cares to preserve the spike order. The CT-DAC usually performs zero-order-hold ( ZOH ) to produce the staircase-like output waveform. Recently the derivative level-crossing sampling is proposed for CT-ADC so that a much improved signal-to-distortion ratio (SDR) at the CT-DAC output can be obtained by ZOH and integration [60]. Thanks to the lack of clock and time quantization, the whole system is free of aliasing and has adaptive power consumption - if no spike is generated by the CT-ADC, the CT-DSP only dissipates leakage power.

Several circuit implementations of CT-ADC and CT-DSP have been presented in accord with different system requirements. For voice band applications, the circuit (a) at the bottom left of Figure 1.5 was used as CT-ADC [61]. The change of the crossing levels is achieved by using a feedback DAC in response to the outputs of the ON and OFF comparators. If $V_{\text {ON }}$ gets high, $V_{\text {refon }}$ and $V_{\text {refofF }}$ increase one LSB; if $V_{\text {OFF }}$ gets high, $V_{\text {refon }}$ and $V_{\text {refoff }}$ decrease one LSB. The encoded digital signal in spike/delta form is sent to the CT-DSP via asynchronous handshake. The tap delay circuit in the CT-DSP based on capacitor discharging is customized for delaying burst signals like spikes [62]. The weight multiplication and tap summation are implemented in asynchronous digital circuits. For GHz-range applications, the delay in the feedback loop of circuit (a) is too long for $V_{\text {refon }}$ and $V_{\text {refoff }}$ to track the input, and instead the circuit (b) was employed which is basically a clockless flash ADC with 7 comparators for 3-bit encoding [63]. The output of each comparator is encoded into rising and falling edges which are sent to the CT-DSP composed of charge-pump-based weight-multiplier and current-mode adder.

The CT-ADC/DSP/DAC system may be regarded as the first rational physical embodiment of the computational TEM/TDP/TDM in silicon which closely resembles the prevalent clocked ADC/DSP/DAC system. It involves development of novel clockless circuits and systems for encoding and processing signals in time domain. Recent advancements include adaptive-resolution CT-ADCs [64], [65], flexible CTDSPs that allow different digital input formats with variable data rate [66], etc. Spiking sensors like silicon cochlea and retina can in fact be regarded as 1-D and 2-D arrays respectively of CT-ADCs with analog preprocessing before spike encoding. Compact implementation is usually required considering the large number of array elements.

### 1.3 Thesis Contribution \& Organization

This thesis analytically and numerically studies the quality of sensory information representation in spike domain with non-idealities of silicon implementation considered. ADM is found in general to deliver better encoding quality over previously adopted self-timed reset (STR) or integrate-and-fire, and is chosen for spike encoder implementations in spiking silicon retina and cochlea. In additional to compact ADM designs, the fabricated silicon retina focuses on improvement of temporal contrast sensitivity for potential fine-texture recognition and optical neuroimaging applications, and the silicon cochlea emphasizes low supply voltage and low power consumption for wireless sensor networks in the context of internet of things or ambient intelligence where harvested energy is scarcely available. The rest of the thesis is organized as follows.

Chapter 2 first compares the encoding quality between the ADM and STR encoders using the SDR metric with linear decoding of the output spike train. With comparison delay $T_{D C}$ and queueing delay $T_{D Q}$ considered, the ADM encoder shows an increasingly higher SDR improvement over the STR as the input
signal frequency and the encoder's quantization bit number increase as long as no slope overload occurs in ADM. Second, the effect of jitter of spike timestamps caused by $T_{D C}$ and $T_{D Q}$ variations is quantitatively measured by the reconstruction SDR using nonlinear decoding algorithms based on frame theory. Because $T_{D C}$ and $T_{D Q}$ variations are related to several circuit and system parameters, this analysis can provide guidance for specifications-oriented design of spiking sensors.

Chapter 3 presents the spiking silicon retina with enhanced temporal contrast sensitivity and spike encoding quality with in-pixel ADM. One pixel is composed of a low-noise common-gate photoreceptor that logarithmically converts light intensity change $\Delta I$ to output voltage $\Delta V_{p r}$, a capacitively-coupled programmable gain amplifier that amplifies $\Delta V_{p r}$ with four levels of adjustable gains, a switched-capacitor ADM that encodes the amplified $\Delta V_{p r}$ into spike trains, and an in-pixel asynchronous logic block that generates the switching signals for the ADM and communicates spikes to the peripheral 2-D burst-mode word-serial AER. The chip named as ADMDVS was fabricated in UMC $0.18 \mu \mathrm{~m} 1 \mathrm{P} 6 \mathrm{M}$ RF/MM CMOS. The measurement results include noise, TC sensitivity, and the comparison of ADM spike encoding with a prior DVS using STR encoding. A simulated optical neuroimaging experiment is also performed to demonstrate a potential practical application.

Chapter 4 elaborates on the design of the $0.5-\mathrm{V} 55-\mu \mathrm{W} 64 \times 2$-channel binaural silicon cochlea. A pow-er-efficient source-follower-based 1-zero 4-pole asymmetrical BPF with tunable $Q$ is proposed which comprises a $4^{\text {th }}$-order source-follower-based LPF and a summing PGA. A self-oscillating comparison scheme is proposed to replace the CT-comparators in ADM with more energy-efficient dynamic latched comparators. The oscillation frequency is designed to be adaptive in response to the output spike activity to further save power. The biases of all the channels are geometrically scaled, covering a frequency range from 8 to 20 k Hz . The chip named as CochLP was fabricated in TowerJazz $0.18 \mu \mathrm{~m}$ 1P6M IS CMOS. The measurement results mainly include the transfer functions, noise and distortion of the BPFs, and the spike output using chirp and natural sound input to the chip. The $0.5-\mathrm{V}$ CochLP core is about $\times 18$ times power-efficient compared to the best prior art, exhibits good matching within a monaural ear and between both ears, and has a wide $Q$ tuning range.

Chapter 5 describes a wide dynamic range current reference array that provides all the biases for spiking sensors. To cover the current range from about 30 fA to $26 \mu \mathrm{~A}$, a hierarchical coarse-fine selection architecture is employed. A subthreshold PTAT master bias sends a 400 nA current to a coarse current divider and multiplier to generate eight coarse currents with a scaling ratio of 8 . Each bias branch selects one of the eight coarse currents and uses an 8-bit R-2R DAC based on MOSFET-only current-splitting technique to generate a fine-tuned current which is sent to a configurable buffer. A bias voltage is generated by the buffer via a diode-connected transistor. The measurement results are obtained from chips fabricated in UMC $0.18 \mu \mathrm{~m}$ CMOS.

Chapter 6 gives a brief summary of the thesis and some outlooks of possible future directions.

# Chapter 2: Analysis of Spike Encoding Mechanisms in Spiking Sensors 

$\mathscr{T}$his chapter presents the analysis of the spike encoding mechanisms that can be feasibly implemented in array sensors like silicon retina and cochlea, particularly the self-timed reset (STR) and asynchronous delta modulation (ADM). STR and ADM both belong to the direct-threshold-crossing encoding category without integration in the feedforward path in contrast to other encoding schemes like asynchronous sigma-delta modulation (ASDM) [52] and integrate-and-fire (IAF) [40] where an integrator is used before threshold-crossing. STR and ADM mainly differ in the feedback path as will be discussed later. STR has been used in all previous dynamic vision sensors (DVSs) [67] because of its simple circuit implementation. ADM has so far only been realized in the form of level-crossing sampling in clockless continuous-time ADCs [54]. Different encoding schemes have been shown to have largely different encoding quality [53]. With a nonlinear decoding algorithm derived based on frame theory, regardless of the encoding mechanisms, perfect reconstruction is possible provided the average spike rate is at least as large as the Nyquist rate and the time support is infinite [52]. In practice however, finite time support results in degradation of signal representation integrity, and with the same time length and similar spike number some encoding is superior compared to others measured by the signal-to-distortion ratio (SDR) metric that is used in the rest of this chapter and is defined as below:

$$
\begin{equation*}
\mathrm{SDR}=\frac{P_{\text {signal }}}{P_{\text {distortion }}}=\frac{\int_{t_{1}}^{t_{2}}(x(t))^{2} d t}{\int_{t_{1}}^{t_{2}}\left[x(t)-x_{r c}(t)\right]^{2} d t} \tag{2.1}
\end{equation*}
$$

where $P_{\text {signal }}$ is the power of the original input signal $x(t)$ and $P_{\text {distortion }}$ is the power of the reconstruction error which is the difference between $x(t)$ and the recovered signal $x_{r c}(t)$.

The calculated SDR depends not only on the encoding methods but also on the decoding algorithms. As pointed out in [52], linear decoding that uses simple integration and/or filtering [68] gives much lower SDR compared to nonlinear decoding, because the former does not consider the nonlinear nature of the encoding process where the threshold-crossing takes place. To obtain the same SDR, linear decoding requires much higher quantization resolution, i.e. lower threshold, and in turn leads to high average spike rate and high power consumption in silicon implementations. As shown in the simulation of levelcrossing sampling [69], with 4-bit amplitude quantization, nonlinear decoding gives a larger than 100 dB SNR in frequency domain for a sinusoidal input while linear decoding could only achieve less than 40 dB [70]. This infers that the information of the input waveform is entirely contained in the encoded timestamps, and therefore to evaluate the encoding integrity, nonlinear decoding should be used. However, nonlinear decoding is much more computational intensive, and hence if hardware efficiency is paramount, linear decoding should be employed. But for spike processing, it is unclear yet how to evaluate the tradeoff between encoding resolution and algorithm complexity with some targeted performance like classification accuracy of a spike-based classifier. This is something to be explored in the future.

According to the data-processing inequality in information theory [71], any information loss caused by the encoder cannot be retrieved by further processing, and therefore in a rational design, the SDR lower bound of a spike train that is encoded by a spike encoder (asynchronous ADCs) or a population of spike encoders (spiking sensors) should be determined by the performance requirement of a spike processing algorithm (e.g. classification accuracy of a classifier). With specified encoding quality, the encoder type


Figure 2.1. Ideal spike encoding model of STR named as silicon ON-OFF TEM in [53].
and resolution can be accordingly chosen. This chapter mainly compares two types of encoders, i.e. the STR and ADM.

The content of this chapter is organized as follows: Section 2.1 compares the STR and ADM with fixed feedback delay, including: Section 2.1.1 describes the abstract models of STR and ADM considering fixed feedback delay; Section 2.1.2 shows the SDR metric comparison of STR and ADM using linear decoding; Section 2.2 discuss the SDR degradation problem due to imprecise timestamps caused by communication delay variation, including: Section 2.2 . 1 provides the nonlinear spike decoding algorithms of STR and ADM; Section 2.2.2 quantifies two sources of delay variation, namely the comparison delay variation and the queueing delay variation; Section 2.2 .3 shows the encoding SDR degradation due to the two types of delay variation; Section 2.3 gives conclusion and remarks.

### 2.1 STR and ADM with Fixed Feedback Delay $\boldsymbol{T}_{\boldsymbol{D}}$

### 2.1.1 Modeling of STR and ADM

The STR was ideally modeled as the silicon ON-OFF time encoding machine (TEM) [53] as illustrated in Figure 2.1. Starting from the reset level, say 0 , the error signal $\boldsymbol{e}(t)$ tracks the input signal $\boldsymbol{x}(t)$. When $\boldsymbol{e}(t)$ crosses the threshold $\delta /-\delta$ at time $t_{k}{ }^{\mathrm{ON}} / t_{k}{ }^{\mathrm{OFF}}(\mathrm{k} \in \mathrm{Z})$, an ON/OFF spike is generated. The generated spike immediately reset $\boldsymbol{e}(t)$ back to 0 with no delay and $\boldsymbol{e}(t)$ instantaneously starts to track $\boldsymbol{x}(t)$ again. This process repeats to produce spikes over time which forms the spike train $\boldsymbol{z}(t)$. The assumptions of no feedback delay $T_{D}$ and zero reset time $T_{H}$ are invalid in practical silicon implementation. Two sources contribute to $T_{D}$. One is the comparison delay. The detection of threshold-crossing is normally done by a comparator that has a finite bandwidth set by its bias current and capacitive load. At nA-bias, the delay of a 2 T cur-rent-mode comparator is in the range of tens of $\mu \mathrm{s}$. The other source is the spike transmission delay. The generated spikes from each retina pixel or cochlea channel are transmitted via the address-eventrepresentation (AER) communication protocol. Besides the intrinsic delay of the customized on-chip asynchronous circuits and the time needed for an off-chip CPLD or FPGA to register the addresses and timestamps of the spikes, queueing caused by arbitration in a congested AER channel also contributes to transmission delay. The time for a full four-phase handshake in DVS128 is about $1.2 \mu \mathrm{~s}$ [72], and it can increase drastically without bounds when the average spike rate of a sensor array approaches the throughput capacity of an AER channel. The reset in STR is usually done by a MOS switch [6]. For $\boldsymbol{e}(t)$ to have a certain settling accuracy within e.g. a few percent of $\delta, T_{H}$ has to be larger than 0 and is determined by the slew rate and bandwidth of the STR amplifier. With a 10 fF capacitance and a 0.5 V threshold voltage, a 1 nA slewing current give a $5 \mu \mathrm{~s} T_{H}$. In addition, the refractory period deliberately introduced to limit the spike rate of a pixel may further increase $T_{H}$.

For the reasons mentioned above, the silicon ON-OFF TEM model needs to be modified to account for the practical non-idealities. We first assume that $T_{D}$ and $T_{H}$ have fixed values. Figure 2.2(a) shows the


Figure 2.2. Spike encoding model of (a) STR and (b) ADM with delay time $T_{D}$ and switch-hold period $T_{H}$.
STR model with $T_{D}$ and $T_{H}$ considered. The two sources of $T_{D}$ are lumped together and the timestamp of each spike is the moment of threshold-crossing plus $T_{D}$. The reset occurs after the acknowledge to each spike transmission comes back, and the $T_{H}$ represents the reset time and refractory period during which $\boldsymbol{e}(t)$ is held at $0 . \boldsymbol{e}(t)$ can be expressed as:

$$
\boldsymbol{e}(t)=\left\{\begin{array}{lc}
0 & \left(t_{k}^{i}<t \leq t_{k}^{i}+T_{H}\right) \\
\boldsymbol{x}(t) & \left(t_{k}^{i}+T_{H}<t<t_{k+1}^{j}\right)
\end{array} \quad(i, j=\mathrm{ON} / \mathrm{OFF})\right.
$$

The ADM model is similar to STR, as shown in Figure 2.2(b). The reset is replaced by an ideal integrator with the impulse response given below:

$$
\boldsymbol{y}_{k+1}(t)= \begin{cases}\boldsymbol{y}_{k}(t)+\delta & \left(t_{k}^{i}<t \leq t_{k+1}^{j}, i, j=\mathrm{ON}\right) \\ \boldsymbol{y}_{k}(t)-\delta & \left(t_{k}^{i}<t \leq t_{k+1}^{j}, i, j=\mathrm{OFF}\right) \\ 0 & (t \leq 0)\end{cases}
$$

And $\boldsymbol{e}(t)$ is now written as:

$$
e(t)=x(t)-y(t)
$$

The ADM model used here is adapted from the model given in [57], where the incremental value in $\boldsymbol{y}(t)$ at each $t_{k}$ is $2 \delta /-2 \delta$ instead of $\delta /-\delta$. Using a less than $2 \delta$ incremental step can help prevent false crossing of the complementary threshold ( $\delta$ and $-\delta$ are complementary thresholds for each other) due to noise or digital coupling. Here $\delta$ is chosen to be consistent with the ideal STR case.

### 2.1.2 Linear Decoding Comparison

Figure 2.3(a) shows the simulated waveform $\boldsymbol{e}(t)$ of both STR and ADM using MATLAB within a time window of $[0.78 \mathrm{~s}, 2.28 \mathrm{~s}]$. The input $\boldsymbol{x}(t)$ is a 1 Hz sinusoidal signal:

$$
\boldsymbol{x}(t)=0.316 \times \sin (2 \pi \times 1 \times t)
$$

$T_{D}$ and $T_{H}$ are both about 7.8 ms , and $\delta$ is set to $0.316 / 2^{3}=0.0395$ for a 4 -bit quantization ( 4 is called the quantization bit number ( QBN ) ). The signal loss due to complete reset and switch hold is labeled in red circle in STR , and so is the $\delta$ subtraction in ADM. The parts of the signal that are beyond $\delta$ and during $T_{H}$ are discarded in STR because in linear reconstruction, every spike is assumed to contribute a $\delta$ incremental which is not true. In ADM, the amount in each feedback subtraction is exactly $\delta$, and therefore ideally no signal loss occurs.

To compare the encoding quality using the SDR metric, a linear decoder composed of an ideal integrator and a brick-wall LPF is used as depicted in Figure 2.4. The lowpass corner frequency of the LPF is set


Figure 2.3. (a) Waveforms of $\boldsymbol{x}(t)$ and $\boldsymbol{e}(t)$ of the STR and ADM, and (b) the linear reconstruction errors $\Delta \boldsymbol{x}(t)$ and $\Delta \boldsymbol{x}_{f}(t)$ within a time window of $[0.78 \mathrm{~s}, 2.28 \mathrm{~s}]$.


Figure 2.4. Linear spike decoder comprised of an ideal integrator and a brick-wall LPF.
to 2 Hz . Let $\boldsymbol{x}_{r c}(t)$ and $\boldsymbol{x}_{r c f}(t)$ represent the reconstructed signal before and after LPF. The reconstruction errors are written as:

$$
\Delta x(t)=x(t)-x_{r c}\left(t+T_{D}\right), \Delta x_{f}(t)=x(t)-x_{r c f}\left(t+T_{D}\right)
$$

Note that $\boldsymbol{x}_{\boldsymbol{r c}}(t)$ and $\boldsymbol{x}_{r c f}(t)$ are shifted by $T_{D}$ to account for the spike delay. Figure 2.3(b) shows both $\Delta \boldsymbol{x}(t)$ and $\Delta \boldsymbol{x}_{f}(t)$ of STR and ADM within a time window $[0.78 \mathrm{~s}, 2.28 \mathrm{~s}]$. The maximum $\Delta \boldsymbol{x}_{f}(t)$ of STR is about $3 \mathrm{LSB}(\sim 0.12)$, and of ADM is less than 1LSB ( $\sim 0.04$ ). The calculated SDR using $\Delta \boldsymbol{x}_{f}(t)$ is 7.95 dB for STR and 21.0 dB for ADM, respectively. The SDR improvement $\triangle \mathrm{SDR}$ is 13.1 dB . It may seem that the SDR improvement comes from the more frequent sampling in ADM, 28 versus 20 spikes of STR during one $\boldsymbol{x}(t)$ period. However, if the threshold of ADM $\delta$ is increased so that ADM also takes only 20 samples per period, its SDR only slightly decreases to 19.0 dB , and if the STR threshold $\delta$ is decreased so that 28 samples are taken per period, its SDR is further decreased to 4.91 dB . This is because the higher the sampling frequency of STR, the larger portion of signal is discarded due to complete reset and switch hold.

The SDR improvement $\triangle \mathrm{SDR}$ is a function of both QBN and input frequency $f_{\text {in }}$ for a given signal amplitude. Figure 2.5 shows the dependence of $\triangle \mathrm{SDR}$ on QBN and $f_{\text {in }}$. QBN is swept from 2 to 11 in simulation, and $f_{\text {in }}$ from 1 to 20 Hz . The 20 Hz bandwidth is reported to contain most information in natural human vision [50]. Smaller and more practical $T_{D}$ and $T_{H}$ are used, $61 \mu \mathrm{~s}$ and $7.6 \mu \mathrm{~s}$ respectively. When QBN is less than $4, \triangle$ SDR is negligible within the frequency range, but it becomes increasingly large as QBN goes up above 5, and reaches the peak value of about 57 dB at $f_{i n}=2.5 \mathrm{~Hz}$ and $\mathrm{QBN}=11$. $\Delta$ SDR also generally increases with $f_{i n}$. The abrupt $\Delta$ SDR drop at large $f_{i n}$ and QBN is caused by slope overload in ADM where the feedback $\boldsymbol{y}(t)$ is not able to tract the input $\boldsymbol{x}(t)$ due to the feedback delay. In this region, $\boldsymbol{e}(t)$ can even exceed $\pm 2 \delta$. The critical parameter that determines the occurrence of slope overload is $T_{D}$. In the case of a sinusoidal input, the bounds on $f_{\text {in }}$ and QBN can be linked to $T_{D}$ as:


Figure 2.5. $\Delta \mathrm{SDR}$ of ADM over STR as a function of $f_{\text {in }}$ and QBN .

$$
\begin{equation*}
T_{D} \leq \frac{1}{\pi f_{i n} \cdot 2^{Q B N+1}} \tag{2.2}
\end{equation*}
$$

If an ADM is designed for a large $f_{i n}$, e.g. 1 k Hz , to have an $8 \mathrm{QBN}, T_{D}$ must be significantly reduced to about $0.62 \mu$ s to avoid slope overload. The input amplitude of a sinusoidal signal $V_{\sin }$ has been assumed to be fixed so far, but no additional analysis is needed for a varying $V_{\text {sin }}$ because with a given threshold $\delta$, it is directly linked to QBN by $V_{\text {sin }} / \delta=2^{\mathrm{QBN}}$.

Other spike encoding mechanisms like IAF with reset in the feedback also have the same problems of STR in practical implementations, i.e. complete reset and switch hold, which may also lead to severe signal loss using linear reconstruction.

### 2.2 SDR Degradation due to $\boldsymbol{T}_{\boldsymbol{D}}$ Variation

With the assumption of fixed $T_{D}$ and $T_{H}$, the numerical analysis in Section 2.1 reveals that ADM generally favors a higher SDR than STR with linear decoding, especially at large $f_{i n}$ and QBN, which is attributed to the uncertainty of signal representation in STR due to the complete reset and switch hold. It has been shown in [40] that the uncertainty caused by absolute refractory period in IAF neurons can be accounted for and a perfect reconstruction algorithm was thereof obtained based on the methods from irregular sampling. For both STR and ADM, similar algorithms can be developed and give much higher SDR. This section develops the nonlinear decoding algorithms to study the encoding quality of ADM and STR considering the variation of $T_{D}$ which is caused by several factors in spike transmission of spiking sensors.

### 2.2.1 Nonlinear Decoding

To develop the nonlinear decoding algorithms, the first step is to map the input signal to timestamps following the so-called $t$-transform [52]. Figure 2.6 illustrates the principle for STR and ADM. The waveforms are the modulated $\boldsymbol{x}(t)$, i.e. the $\boldsymbol{e}(t)$ in Figure 2.2. For simplicity, $\boldsymbol{x}(0)$ is assumed to be 0 ; otherwise an offset $\boldsymbol{x}_{0}=\boldsymbol{x}(0)$ can be added to $\boldsymbol{x}(t)$. The numbers 1 and 2 in the superscript of the timestamps represent ON and OFF spikes respectively. For STR, the timestamps have the following relationships:

$$
x\left(t_{2}^{1}-T_{D 2}\right)-x\left(t_{1}^{1}+T_{H}\right)=(-1)^{1-1} \delta, x\left(t_{3}^{2}-T_{D 3}\right)-x\left(t_{2}^{1}+T_{H}\right)=(-1)^{2-1} \delta
$$



Figure 2.6. Illustration of the $t$-transformation principle for STR and ADM.
And for ADM, the equations below hold:

$$
x\left(t_{1}^{1}-T_{D 1}\right)=\delta \cdot(1-2 \cdot 0), x\left(t_{2}^{1}-T_{D 2}\right)=\delta \cdot(2-2 \cdot 0), x\left(t_{3}^{2}-T_{D 3}\right)=-\delta \cdot(3-2 \cdot 2)
$$

$T_{D 1}, T_{D 2}$ and $T_{D 3}$ represent the different delays associated with each spike. From the specific examples given above, more general formulization can be derived. For STR, assume $T_{H}$ is fixed, and given $\forall k \in N^{+}$:

$$
\begin{equation*}
x\left(t_{k}^{i}-T_{D k}\right)-x\left(t_{k-1}^{j}+T_{H}\right)=(-1)^{i-1} \delta \quad(k \geq 2), x\left(t_{k}^{i}-T_{D k}\right)=(-1)^{i-1} \delta \quad(k=1) \tag{2.3}
\end{equation*}
$$

where $i, j=1,2, i$ and $j$ are independent of each other, and $T_{D k}$ is the delay associated with the $k^{\text {th }}$ spike. For ADM , given $\forall k, l \in N^{+}$:

$$
\begin{equation*}
x\left(t_{k}^{1}-T_{D k}\right)=\delta \cdot\left(k-2 \sum_{l \in N^{+}} 1_{\left[t_{l}^{2}<t_{k}^{1}\right]}\right), x\left(t_{k}^{2}-T_{D k}\right)=-\delta \cdot\left(k-2 \sum_{l \in N^{+}} 1_{\left[t_{l}^{1}<t_{k}^{2}\right]}\right) \tag{2.4}
\end{equation*}
$$

The inner product form of Eq. (2.3) can be written as:

$$
<x, \chi_{D, k}^{i}>-<x, \chi_{H, k-1}^{j}>=q_{k S}^{i}
$$

where $q_{k S}^{i}$ is the right side of Eq. (2.3), and $\chi$ is the sampling function and has the form of:

$$
\chi_{D, k}^{i}=g\left(t-t_{k}^{i}+T_{D k}\right), \chi_{H, k-1}^{j}=g\left(t-t_{k-1}^{j}-T_{H}\right) \quad(k \geq 2), \chi_{H, 0}^{j}=0
$$

In this chapter, $g(t)$ represents the sinc function $g(t)=\sin (\Omega t) / \pi t$. It is the impulse response of a brick-wall LPF with a cutoff frequency of $\Omega$.

The inner product form of Eq. (2.4) can be written as:

$$
\left\langle x, \chi_{k}^{i}\right\rangle=q_{k A}^{i}
$$

where $q_{k A}^{i}$ is the right side of Eq. (2.4), $i=1,2$, and the sampling function $\chi$ has the form of:

$$
\chi_{k}^{i}=g\left(t-t_{k}^{i}+T_{D k}\right)
$$

In light of the Proposition 1 and its proof in [53], in STR the recovered signal $\boldsymbol{x}_{r c}(t)$ from spike trains can be obtained by weighted summation of the representation functions $\eta_{k}(t)=g\left(t-t_{k}^{i}\right)$, which form a frame for the space of bandlimited functions:

$$
\begin{equation*}
x_{r c}(t)=\sum_{k \in N^{+}} c_{k} \eta_{k}(t) \tag{2.5}
\end{equation*}
$$

The weights $c_{k}$, i.e. the coefficient vector $\boldsymbol{c}$ can be computed as:

$$
\mathbf{c}=\mathbf{G}^{+} \mathbf{q}
$$

where $[\mathbf{q}]_{k}=q_{k s}^{i}, i=1,2 . \mathbf{G}^{+}$is the pseudo-inverse of the matrix $\mathbf{G}$. The elements in $\mathbf{G}$ are:


Figure 2.7. Reconstruction error of STR and ADM using nonlinear decoding.

$$
[\mathbf{G}]_{l l}=\left\langle\chi_{D, k}^{i}, \eta_{l}\right\rangle-\left\langle\chi_{H, k-1}^{j}, \eta_{l}\right\rangle
$$

for all $i, j=1,2$, and $k, l \in N^{+}, k \geq 2$. When $k=1$,

$$
\left[\mathbf{G}_{k l}=\left\langle\chi_{D, k}^{i}, \eta_{l}\right\rangle\right.
$$

Using Parseval's formula $[73],[\mathbf{G}]_{k l}$ can be computed as:

$$
\begin{gathered}
{[\mathbf{G}]_{k l}=\int_{-\infty}^{\infty} g\left(t-t_{k}^{i}+T_{D k}\right) g\left(t-t_{l}\right) d t-\int_{-\infty}^{\infty} g\left(t-t_{k}^{i}-T_{H}\right) g\left(t-t_{l}\right) d} \\
=\frac{1}{2 \pi} \int_{-\Omega}^{\Omega} e^{-i \omega\left(t_{k}-T_{D}-t_{l}\right)} d \omega+\frac{1}{2 \pi} \int_{-\Omega}^{\Omega} e^{-i \omega\left(t_{k}+T_{H}-t_{l}\right)} d \omega \\
=\mathrm{g}\left(t_{k}^{i}-T_{D k}-t_{l}\right)-\mathrm{g}\left(t_{k-1}^{j}+T_{H}-t_{l}\right) \quad(k \geq 2) \\
\quad[\mathbf{G}]_{k l}=\mathrm{g}\left(t_{k}^{i}-T_{D k}-t_{l}\right) \quad(k=1)
\end{gathered}
$$

Similarly, in ADM the recovered signal $\boldsymbol{x}_{r c}(t)$ from spike trains can be computed as:

$$
\begin{equation*}
x_{r c}(t)=\sum_{k \in N^{+}} c_{k}^{1} \eta_{k}^{1}(t)+\sum_{k \in N^{+}} c_{k}^{2} \eta_{k}^{2}(t) \tag{2.6}
\end{equation*}
$$

where $\eta_{k}^{i}(t)=g\left(t-t_{k}^{i}\right), i=1,2$. With $\mathbf{c}=\left[\mathbf{c}^{1} ; \mathbf{c}^{2}\right]$ and $\left[\mathbf{c}^{\mathbf{i}}\right]_{k}=c_{k}^{i}$, the vector of coefficients $\mathbf{c}$ can be computed as:

$$
\mathbf{c}=\mathbf{G}^{+} \mathbf{q}
$$

where $\mathbf{q}=\left[\mathbf{q}^{1} ; \mathbf{q}^{2}\right]$ with $\left[\mathbf{q}^{\mathbf{i}}\right]_{k}=q_{k A}^{i}, i=1,2$, and the matrix $\mathbf{G}$ can be written as:

$$
\mathbf{G}=\left[\begin{array}{ll}
\mathbf{G}^{11} & \mathbf{G}^{12} \\
\mathbf{G}^{21} & \mathbf{G}^{22}
\end{array}\right],\left[\mathbf{G}^{i j}\right]_{k l}=\left\langle\chi_{k}^{i}, \eta_{l}^{j}\right\rangle
$$

where $k, l \in N^{+} .\left[\mathbf{G}^{\mathrm{ij}}\right]_{k l}$ can be computed as:

$$
\begin{aligned}
{\left[\mathbf{G}^{\mathrm{ij}}\right]_{k l} } & =\int_{-\infty}^{\infty} g\left(t-t_{k}^{i}+T_{D k}\right) g\left(t-t_{l}^{j}\right) d t \\
& =\frac{1}{2 \pi} \int_{-\Omega}^{\Omega} e^{-i \omega\left(t_{k}-T_{D-}-t_{l}^{\prime}\right)} d \omega=\mathrm{g}\left(t_{k}^{i}-T_{D k}-t_{l}^{j}\right)
\end{aligned}
$$

With the same input signal $\boldsymbol{x}(t)$ and the encoding threshold $\delta$ that are used to obtain the plots in Figure 2.3, the reconstruction errors $\Delta \boldsymbol{x}(t)=\boldsymbol{x}(t)-\boldsymbol{x}_{r c}(t)$ of STR and ADM using the nonlinear decoding algorithms described above are shown in Figure 2.7. The time support is $8 \mathrm{~s}, \Omega=2 \pi \mathrm{rad} / \mathrm{s}$, and time step is about $3.8 \mu \mathrm{~s}$.

Obviously the reconstruction errors $\Delta \boldsymbol{x}(t)$ are reduced by orders of magnitude in both cases compared to linear decoding. The ADM has an SDR of 87 dB and even the STR has 80 dB , only 7 dB lower. The residual $\Delta \boldsymbol{x}(t)$ is caused by two sources, the finite time support and finite simulation accuracy in terms of non-infinitesimal time step. To study a more general case, a white Gaussian noise will be used as the input $\boldsymbol{x}(t)$ instead of a sinusoid.

### 2.2.2 Two Sources of $\boldsymbol{T}_{\boldsymbol{D}}$ Variation

### 2.2.2.1 Comparison Delay $\boldsymbol{T}_{D C}$ Variation

Comparators are used for the detection of threshold-crossing, and comparison delay $T_{D C}$ variation is the first source of $T_{D}$ variation because $T_{D C}$ is dependent on the slope $s_{x}$ of the input signal $\boldsymbol{x}(t)$ at the moment of threshold-crossing. For continuous-time comparators commonly used in clockless systems, the relevant parameters are the bias current and the DC gain with a given circuit topology. The following analysis will take the simplest 2 T common-source amplifier (CSA) comparator as an example which has been used in many DVSs [6]. Assuming one dominant pole and no slewing [74], the transfer function of the CSA can be written as:

$$
H(s)=\frac{A_{D C}}{s \tau_{c}+1}
$$

where $A_{D C}$ is the DC gain of the CSA, and $\tau_{c}$ is the time constant that is associated with the dominant pole and determined by the CSA bias and output load capacitance. The input signal change at the moment of threshold-crossing is approximated by $s_{x} \Delta t$, where $\Delta t$ is time needed for the CSA output to have a change of $\delta$, and the Laplace transform of $s_{x} \Delta t$ is $s_{x} / s^{2}$. The CSA output can be then written as:

$$
V_{\text {out }}(s)=\frac{s_{x}}{s^{2}} \cdot H(s)=\frac{A_{D C} s_{x}}{s^{2}\left(s \tau_{c}+1\right)}
$$

The inverse-Laplace transform of $V_{\text {out }}(s)$ is:

$$
\begin{aligned}
V_{\text {out }}(t) & =\mathcal{L}^{-1}\left\{V_{\text {out }}(s)\right\}=\mathcal{L}^{-1}\left\{\frac{A_{D C} S_{x}}{s^{2}\left(s \tau_{c}+1\right)}\right\} \\
& =A_{D C} s_{x} \mathcal{L}^{-1}\left\{\frac{1}{s^{2}}-\frac{\tau_{c}}{s}+\frac{\tau_{c}^{2}}{s \tau_{c}+1}\right\}=s_{x} A_{D C}\left(t-\tau_{c}+\tau_{c} e^{-t / \tau_{c}}\right)
\end{aligned}
$$

Using $T_{D C}$ to replace $t$ and making Taylor expansion on the exponential term, $V_{o u t}(t)$ is rewritten as:

$$
\begin{aligned}
V_{\text {out }}(t) & =s_{x} A_{D C}\left[T_{D C}-\tau_{c}+\tau_{c}\left(1-\frac{T_{D C}}{\tau_{c}}+\frac{T_{D C}^{2}}{2 \tau_{c}^{2}}-O\left(T_{D C}^{3}\right)\right]\right. \\
& \approx s_{x} A_{D C} \frac{T_{D C}^{2}}{2 \tau_{c}} \quad\left(T_{D C} \gg 3 \tau_{c}\right)
\end{aligned}
$$

$T_{D C}$ can be written as the function of $s_{x}$ in logarithmic:

$$
\begin{equation*}
\log _{10} T_{D C}=\frac{1}{2} \log _{10} \frac{2\left|V_{\text {out }}\right| \tau_{c}}{A_{D C}}-\frac{1}{2} \log _{10}\left|s_{x}\right|=y_{0}-\frac{1}{2} \log _{10}\left|s_{x}\right| \tag{2.7}
\end{equation*}
$$

where $y_{0}$ is the lumped indicator of comparison speed as the function of $A_{D C}$ and $\tau_{c}$ with a given $V_{\text {out }}$ range, usually from $10 \%$ to $90 \%$ rail-to-rail. The simulated $T_{D C}$ versus $s_{x}$ of a CAS comparator biased at 0.2 nA ,


Figure 2.8. Simulated comparison delay $T_{D C}$ versus input signal slope $s_{x}$ at different biases of a CSA comparator by Spectre in $0.18 \mu \mathrm{~m}$ CMOS, and the linear fittings using Eq. (2.7).

2 nA and 20 nA using Spectre in $0.18 \mu \mathrm{~m}$ CMOS is shown in Figure 2.8 as the scattered dots. The red lines are the linear fits using Eq. (2.7) with $y_{0}=-2.91,-3.30$, and -3.78 , respectively. It can be seen that Eq. (2.7) is a relatively good approximation of practical $T_{D C}$, and therefore in spike encoding simulation, it is used to generate the delays $T_{D k}$ in Eq. (2.3) and (2.4) that correspond to the signal slope $s_{x}$ at the moment of each threshold-crossing. However in signal recovery, $T_{D k}$ is not available due to the lack of the knowledge of $s_{x}$. One simple possible way of approximate $\boldsymbol{x}(t)$ recovery is to assume a fixed average delay at all the threshold-crossing moments according to some general characteristics of $\boldsymbol{x}(t)$, such as its bandwidth and power. In this case, all $T_{D k}$ 's are replaced by $T_{D a v g}$ and are independent of $k . T_{\text {Davg }}$ will be derived as follows with characteristic parameters of a given input signal.

For a more general analysis, a random noise $\boldsymbol{x}(t)$ with Gaussian amplitude distribution is used instead of a sinusoid as in Section 2.1. To obtain the $T_{D a v g}$, the average slope at threshold-crossing moments is approximated by the mean slope of $\boldsymbol{x}(t)$. The joint probability density function of $\boldsymbol{x}(t)$ 's amplitude $a_{x}$ and slope $s_{x}$ has been derived by S. Rice as [75]:

$$
\begin{equation*}
p\left(a_{x}, s_{x}\right)=\frac{\left(-\psi_{0} \psi_{0}{ }^{\prime \prime}\right)^{-1 / 2}}{2 \pi} \exp \left(-\frac{a_{x}^{2}}{2 \psi_{0}}+\frac{s_{x}^{2}}{2 \psi_{0}{ }^{\prime \prime}}\right) \tag{2.8}
\end{equation*}
$$

where $\psi_{0}$ and $\psi_{0}{ }^{\prime \prime}$ are the correlation function $\psi(\tau)$ of $\boldsymbol{x}(t)$ and the second derivative of $\psi(\tau)$ at $\tau=0$, respectively. The probability density function of $s_{x}$ can be computed as [57]:

$$
p\left(s_{x}\right)=\int_{-\infty}^{\infty} p\left(a_{x}, s_{x}\right) d a_{x}=\frac{\left(-\psi_{0}{ }^{\prime \prime}\right)^{-1 / 2}}{\sqrt{2 \pi}} \exp \left(\frac{s_{x}^{2}}{2 \psi_{0}{ }^{\prime \prime}}\right)
$$

The average absolute slope $s_{\text {xavg }}$ of $\boldsymbol{x}(t)$ can be calculated as:

$$
s_{x a v g}=2 \int_{0}^{\infty}\left|s_{x}\right| p\left(\left|s_{x}\right|\right) d\left|s_{x}\right|=\sqrt{\frac{2}{\pi}}\left(-\psi_{0}^{\prime \prime}\right)^{1 / 2}
$$

$\psi^{\prime \prime}(\tau)$ is related to $\boldsymbol{x}(t)^{\prime}$ 's power spectrum $\omega(f)$ by [76]:

$$
\psi^{\prime \prime}(\tau)=-(2 \pi f)^{2} \int_{0}^{\infty} \omega(f) \cos (2 \pi f \tau) d f
$$


(a)
row request batches,

each contains 1 to $n$

(b)

Figure 2.9. The queueing models of (a) 1-D sensors like a 1-D spiking silicon cochlea and (b) 2-D sensors like a 2-D spiking silicon retina.

For bandlimited white noise with zero mean, $\sigma^{2}$ power and $f_{n}$ bandwidth, $\psi_{0}{ }^{\prime \prime}$ is:

$$
\psi_{0}^{\prime \prime}=-\frac{4 \pi^{2} \sigma^{2}}{f_{n}} \int_{0}^{f_{n}} f^{2} d f=-\frac{4}{3}\left(\pi \sigma f_{n}\right)^{2}
$$

$s_{x a v g}$ can hence be written as:

$$
\begin{equation*}
s_{x a v g}=2 \sqrt{\frac{2 \pi}{3}} \sigma f_{n} \tag{2.9}
\end{equation*}
$$

Using Eq. (2.7), $T_{\text {Davg }}$ can be written as:

$$
\begin{equation*}
T_{\text {Davg }}=10^{y_{0}} s_{x a v g}^{-1 / 2} \tag{2.10}
\end{equation*}
$$

From Eq. (2.9) and (2.10), one can see that the required parameters for computing $T_{\text {Davg }}$ are $\boldsymbol{x}(t)$ 's bandwidth $f_{n}$, power $\sigma^{2}$, encoder's threshold $\delta$, and the comparator speed indicator $y_{0}$, which is related to comparator's DC gain $A_{D C}$ and time constant $\tau_{c}$ that is determined by its bias current and output load capacitance. The estimation of $s_{x a v g}$ using Eq. (2.9) may deviate from the actual value of an arbitrary $\boldsymbol{x}(t)$ as the threshold $\delta$ approaches $\sigma$. This is because in practice, $s_{x}$ only takes values at quantized amplitudes in ADM that are integer-multiple of $\delta$. In STR, the $s_{x}$ sampling occurs at more randomized $a_{x}$ due to complete reset and refractory period.

### 2.2.2.2 Queueing Delay $T_{D Q}$ Variation

Arbitration is essential for sparse spike encoding schemes like STR and ADM. Unlike some early spiking sensors that use pulse-frequency modulation (PFM) for spike encoding [5], [77] where several missing spikes due to collision in unfettered communication channels do not have significant impact on the average spike frequency within a time window, the reconstruction SDR from spike trains encoded by STR and ADM, i.e. the representation integrity of input signals in spike domain, relies on the presence and precise timestamp of each spike. Arbitration guarantees the transmission of every spike, but it compromises the timestamp accuracy. Because the spikes are only recorded or processed on the receiving end, if the generated spikes pass through the transmission channel freely without any waiting, the timestamps are intact and otherwise skewed if they need to wait for the transmission of earlier generated spikes. This kind of system can be studied using queueing models, and the variation of queueing delay $T_{D Q}$ can be characterized by the waiting time distribution $P(W)$.

Integrated CMOS sensors can be categorized as either 1-D sensors like the silicon cochlea and optical line sensors [14], or 2-D sensors like the silicon retina, and neural recording [78] and biochemical sensing [79] arrays. For 1-D spiking sensors as shown in Figure 2.9(a), each sensing element generates a stream of spike trains. The spike trains from all sensing elements are arbitrated according to the first-come-firstserve (FCFS) rule, which can be implemented as the 1-D version of the fair address-event-representation (AER) circuits [80]. Assuming an ideally fair arbiter which does not distinguish the identity among different sensing elements and assign priorities, all spikes enter one queueing line abiding by the time order of their generation. Each spike is served by a single server that is either a CPLD or an FPGA for off-chip recording, or a TDC for on-chip recording. The service time is normally fixed in the recording case. For a practical sensor under some input stimuli, the pattern of spike arrival forms a particular stochastic process. In general, the queueing model $\mathrm{G} / \mathrm{D} / 1$ can be used, where G indicates that the inter-spike interval of the arrival spike train has an arbitrary distribution, D means the deterministic service time, and 1 means a single server. Here we consider the Poisson arrival to demonstrate the effect of spike queueing on the integrity of signal representation using the SDR metric, and hence the $M / D / 1$ model is used where $M$ stands for the Poisson arrival.

To obtain the $P(W)$ of an $\mathrm{M} / \mathrm{D} / 1$ queue, two parameters the average rate of spike arrival $\lambda_{q 1 D}$ and the service rate $\mu_{q}$ need to be determined. $\mu_{q}$ is the reciprocal of the service time which is the sum of the intrinsic delay of the AER circuit and the fixed time assigned for registration of one spike address and timestamp. $\lambda_{q 1 D}$ depends on a specific sensor model. Taking silicon cochlea with $m$ total sensing elements as the example, each sensing element or channel contains a BPF with a central frequency of $f_{i}\left(i \in N^{+} \cap i \in[1\right.$, $\mathrm{m}]$ ) cascaded by a spike encoder with a threshold $\delta_{i}$. The $f_{i}^{\prime}$ s of all the BPFs are geometrically scaled with a ratio of $r_{f i}$. With the predefined frequency range from $f_{1}=f_{L}$ to $f_{m}=f_{H}, r_{f i}$ can be calculated as:

$$
r_{f i}=\left(\frac{f_{H}}{f_{L}}\right)^{\frac{1}{m-1}}
$$

If the spike rate of the $\mathrm{i}^{\text {th }} \mathrm{ADM}$ spike encoder is $\lambda_{i}, \lambda_{q 1 D}$ is the sum of all $\lambda_{i}$ :

$$
\lambda_{q 1 D}=\sum_{i=1}^{m} \lambda_{i}
$$

To calculate $\lambda_{i}$ of a bandpass-filtered white Gaussian noise input $\boldsymbol{x}(t)$, its $\psi_{0}{ }^{\prime \prime}$ is first computed as:

$$
\psi_{0}{ }^{\prime \prime}=-\frac{4 \pi^{2} \sigma_{i}^{2}}{f_{H i}-f_{L i}} \int_{f_{L i}}^{f_{H i}} f^{2} d f=-\frac{4}{3}\left(\pi \sigma_{i}\right)^{2}\left(f_{H i}^{2}+f_{H i} f_{L i}+f_{L i}^{2}\right)
$$

Table 2.1. Parameter values for computing $W_{\text {mean }}$ and $P(W)$ of queueing models.

| Symbol | Description | Value |
| :---: | :---: | :---: |
| $\sigma$ | Signal rms amplitude | 0.382 |
| $\delta$ | Encoder threshold | 0.125 |
| $f_{H}$ | Highest frequency for cochlea | 20 k Hz |
| $f_{L}$ | Lowest frequency for cochlea | 20 Hz |
| $f_{v i s}$ | Pixel bandwidth for retina | 20 Hz |
| $\mu_{q}$ | Service rate or spike departure rate | $10^{7} / \mathrm{s}$ |
| $p$ | Probability of a retina pixel being active | 0.16 |



(a)
(b)

Figure 2.10. (a) Mean waiting time $W_{\text {mean }}$ and (b) waiting time distribution $P(W)$ of an $\mathrm{M} / \mathrm{D} / 1$ queue with the specified parameters in Table 2.1.
where $\sigma_{i}$ is the rms amplitude, and $f_{H i}$ and $f_{L i}$ are the lowpass and highpass corner frequencies of the $i^{\text {th }}$ BPF. $f_{H i}$ and $f_{L i}$ are related to $f_{i}$ by the equations below:

$$
f_{H i}-f_{L i}=f_{i} / Q_{i}, f_{H i} f_{L i}=f_{i}^{2}
$$

where $Q_{i}$ is the quality factor of the $i^{\text {th }}$ BPF. Using Eq. (2.8), $\lambda_{i}$ can be calculated as:

$$
\lambda_{i}=\frac{2 \int_{0}^{\infty} s_{x} \int_{-\infty}^{\infty} p\left(a_{x}, s_{x}\right) d a_{x} d s_{x}}{\delta_{i}}=\sqrt{\frac{2}{\pi}} \frac{\sqrt{-\psi_{0}^{\prime \prime}}}{\delta_{i}}=\frac{2 \pi \sigma_{i}}{\delta_{i}} \sqrt{\frac{2\left(f_{H i}^{2}+f_{H i} f_{L i}+f_{L i}^{2}\right)}{3 \pi}}=2 \pi \frac{\sigma_{i} f_{i}}{\delta_{i}}\left[\frac{2}{3 \pi}\left(3+\frac{1}{Q_{i}^{2}}\right)\right]^{1 / 2}
$$

Note that an STR encoder gives a spike rate lower than the $\lambda_{i}$ computed above, depending on the values of $T_{D k}$ and $T_{H}$. For simplicity, it is assumed for an STR encoder to have the same $\lambda_{i}$ as an ADM with the consequence of underestimated reconstruction SDR. More realistic $\lambda_{i}$ estimation of both ADM and STR should take the actual input signal statistics into account. Further assumptions of identical $\sigma_{i}, \delta_{i}$ and $Q_{i}$ for all channels denoted as $\sigma, \delta$ and $Q$ give the following simplified explicit expression of $\lambda_{q 1 D}$ :

$$
\begin{equation*}
\lambda_{q 1 D}=\frac{2 \sigma}{\delta}\left[\frac{2 \pi}{3}\left(3+\frac{1}{Q^{2}}\right)\right]^{1 / 2} \frac{f_{m}\left(1-r^{-m}\right)}{1-r^{-1}} \tag{2.11}
\end{equation*}
$$

Now that we have both $\lambda_{q 1 D}$ and $\mu_{q}$, the mean waiting time $W_{\text {mean }}$ [81] and the waiting time distribution $P(W)$ [82] of an $\mathrm{M} / \mathrm{D} / 1$ queue can be computed by:

$$
\begin{gather*}
W_{\text {mean }}=\frac{\rho}{2 \mu_{q}(1-\rho)}  \tag{2.12}\\
P(W \leq w)=(1-\rho) \sum_{i=0}^{N} \frac{\left[-\lambda_{q 1 D}\left(w-\frac{i}{\mu_{q}}\right)\right]^{i}}{i!} e^{\lambda_{q 1 D}\left(w-\frac{i}{\mu_{q}}\right)} \quad\left(\frac{N}{\mu_{q}} \leq w<\frac{N+1}{\mu_{q}}\right) \tag{2.13}
\end{gather*}
$$

where $\rho=\lambda_{q 1 D} / \mu_{q}$ is the traffic intensity. With the parameter values given in Table 2.1, $W_{\text {mean }}$ and $P(W)$ as a function of the number of channels $m$ are plotted in Figure 2.10. The resolution of the waiting time $w$ in computation is 1 ns . As $m$ increases, $W_{\text {mean }}$ increases exponentially, the sigmoid $P(W)$ shifts towards larger waiting time with increasing variance, and the probability of immediate service with no delay decreases. Both $W_{\text {mean }}$ and $P(W)$ are slightly dependent on $Q$, and they will be later used for the computation of reconstruction SDR of the 1-D cochlea.

The first generation of 2-D AER implemented handshakes of the row and column requests of a sensing element $E(i, j)$ in series [80], and then the row and column addresses are transmitted in parallel. The average service rate of one pixel $\mu_{\text {qpixel }}$ in this transmission scheme is:

$$
\mu_{\text {qpixel }}=\frac{1}{T_{\text {row }}+T_{\text {col }}}
$$

where $T_{\text {row }}$ and $T_{\text {col }}$ are the durations needed for a row and a column handshake, respectively. A G/D/1 model can still apply to this case, and the 1-D sensor analysis above is also valid if the spike arrival is Poisson. To enhance the service rate, the second generation of 2-D AER employs the burst-mode wordserial transmission scheme [83]. As illustrated in Figure 2.9(b), for a pixel array of size $m \cdot n$, whenever one or several pixels in row $R_{i}\left(i \in N^{+} \cap i \in[1, \mathrm{~m}]\right)$ initiate the spike transmission, $R_{i}$ enters the queue of row requests waiting for its row acknowledge from the single server. The fair arbitration also follows the FCFS rule. By the time when the $R_{i}$ request gets served, i.e. the row address and the row request timestamp are recorded, all the 'active' pixels in $R_{i}$ ('active' means pixels having spike generated before $R_{i}$ gets served) starts to transmit their column addresses in burst from $C_{1}$ to $C_{n}$ and they all have the same timestamp as the $R_{i}$ request. After the column address of the last active pixel in $R_{i}$ is registered, the next row request waiting in the queue will be served, and the process repeats. Because one active pixel is only allowed to generate another spike after the current spike is served and acknowledged, the number of active columns in one row request cannot be larger than $n$. In this case, $\mu_{\text {qpixel }}$ is:

$$
\mu_{\text {qpixel }}=\frac{1}{T_{\text {row }} / n_{\text {avg }}+T_{\text {col }}} \approx \frac{1}{T_{\text {col }}} \quad\left(\text { if } 1 \ll n_{\text {avg }} \leq n\right)
$$



Figure 2.11. (a) Mean waiting time $W_{\text {mean }}$ and (b) waiting time distribution $P(W)$ of an $\mathrm{M}^{\mathrm{x}} / \mathrm{D} / 1$ queue with the specified parameters in Table 2.1.
where $n_{\text {avg }}$ is the average number of active columns in one row request. In silicon retina, $n_{\text {avg }}$ is mostly determined by stimulus patterns which are often spatiotemporally correlated. If $n_{\text {avg }}$ is large, $\mu_{q}$ is improved by a ratio of $1+T_{\text {row }} / T_{\text {col }}$. $T_{\text {row }}$ is usually not smaller than $T_{\text {col }}$, and could be even as twice as large because of the fair arbitration [84]. Therefore, $\mu_{q}$ can be improved by $\times 2 \sim \times 3$ at the cost of precise timestamps of the spikes in the same row request.

With no assumption on the arrival pattern of the row requests, the second generation of 2-D AER that is currently in use for DVSs [18] can be described by a $G^{X} / D / 1$ queueing model, where $X$ denotes the bulk arrival of multiple active columns in one row request and the number of active columns $n_{a}$ is drawn from another independent distribution. The model implies that the delay of a row request is the delay for all the active columns in this row, which is only true if all the active columns generate spikes at the same time. In practice, one pixel may become active at the moment just before the burst column transmission, which may be much later than its row request due to queue waiting, and consequently its delay is overestimated. In this sense, the delay distribution from the $\mathrm{G}^{\mathrm{X}} / \mathrm{D} / 1$ model gives an upper bound of $T_{D Q}$ variation. If a Poisson process is assumed for the row requests in the following analysis, the combination of multiple active columns and a Poisson row requests lead to the compound Poisson process [85], and the corresponding queueing model is $\mathrm{M}^{\mathrm{X}} / \mathrm{D} / 1$.

Let $\lambda_{q 2 D}$ be the mean arrival rate of row requests in a 2-D silicon retina. In light of the principle of spike number conservation of a whole 2-D array, the equality below holds:

$$
\lambda_{q 2 D} \cdot n_{\text {avg }}=\lambda_{p i x} \cdot m \cdot n \cdot p
$$

where $\lambda_{p i x}$ is the mean spike rate of one active/stimulated pixel, and $p$ is the probability of one pixel being active. $n_{\text {avg }}$ was shown to be positively dependent on the traffic intensity because the spikes contained in a row request can be accumulated during the queue waiting [86]. However, it will be shown later that this dependence is negligible in a particular case of silicon retina where $\lambda_{p i x}$ and $p$ are small and in turn the probability of spike accumulation is very low. With the assumption of binomial distribution of $n_{a}$, the probability mass function $p_{a}$ of spike number in a row request is written as:

$$
p_{a}=\binom{n-1}{a-3} p^{a-1}(1-p)^{n-a+2}
$$



Figure 2.12. (a) Waveform of the white Gaussian noise input $\boldsymbol{x}(t)$ used for simulation of SDR degradation; (b) exemplary encoded spike train $z(t)$ with both ADM and STR within the time window [ $2.5 \mathrm{~s}, 3 \mathrm{~s}$ ].
where $a$ has the support of $a \in N^{+} \cap a \epsilon[3, n+2]$. Note that $a$ is not from 1 to $n$ because $T_{\text {row }} \approx 2 T_{\text {col }}$ is considered. $n_{\text {avg }}$ therefore can be written as:

$$
n_{\text {avg }}=\sum_{a=3}^{n+2}(a-2) p_{a}
$$

For a bandlimited white Gaussian noise input, $\lambda_{p i x}$ of an ADM is [57]:

$$
\lambda_{p i x}=2 \sqrt{\frac{2 \pi}{3}} \frac{\sigma f_{v i s}}{\delta}
$$

where $f_{\text {vis }}$ is the visual signal bandwidth. With $\lambda_{\text {pix }}$ and $n_{\text {avg }}$ obtained from the equations above, $\lambda_{q 2 D}$ can be computed as:

$$
\begin{equation*}
\lambda_{q 2 D}=\frac{\lambda_{p i x} \cdot m \cdot n \cdot p}{n_{\text {avg }}} \tag{2.14}
\end{equation*}
$$

The mean waiting time $W_{\text {mean }}$ [87] and waiting time distribution $P(W)$ [88] of row requests in an $\mathrm{M}^{\mathrm{X}} / \mathrm{D} / 1$ queue can be computed as:

$$
\begin{gather*}
W_{\text {mean }}=\frac{\rho}{2 \mu_{q}(1-\rho)}\left(\frac{\sum_{a=3}^{n+2} a(a-1) p_{a}}{\sum_{a=3}^{n+2} a p_{a}}+1\right)  \tag{2.15}\\
P(W \leq w)=\sum_{k=0}^{N} \sum_{i=0}^{N+1-k} f_{i} g_{k}\left(\frac{N+1}{\mu_{q}}-w\right) \frac{N}{\mu_{q}} \leq w<\frac{N+1}{\mu_{q}} \tag{2.16}
\end{gather*}
$$

where $\rho$ is the traffic intensity that is defined as:

$$
\rho=\frac{\lambda_{q 2 D}}{\mu_{q}} \sum_{a=3}^{n+2} a p_{a}
$$

$f_{i}$ denotes the probability of the stationary distribution with $i$ row requests being held in the system, and $g_{k}(t)$ denotes the probability that exact $k$ new row requests enter the queue during an arbitrary time interval of length $t$ (the computation of $f_{i}$ and $g_{k}(t)$ sees Appendix). With the parameter values given in Table 2.1, $W_{\text {mean }}$ and $P(W)$ as a function of the total pixel number $m \cdot n$ are plotted in Figure 2.11. An aspect ratio


Figure 2.13. (a) Reconstruction SDR of ADM and STR as the function of the comparison speed indicator $y_{0}$. Hollow and solid symbols represent reconstruction using exact delay and average delay, respectively; (b) Exemplary recovered signal $\boldsymbol{x}_{r c}(t)$ at $y_{0}=-3$ within the time window of $[2.5 \mathrm{~s}, 3 \mathrm{~s}]$.


Figure 2.14. SDR versus the ratio $T_{D} / T_{\text {Davg }}$ for STR and ADM. $T_{D}$ is the actual average delay used for spike train decoding, and $T_{\text {Davg }}$ is the estimated average delay calculated by Eq. (2.10).
of $m: n=3: 4$ is used. The popular video graphics array (VGA) resolution and its scaled versions are labeled in Figure 2.11(a). Similar to the 1-D case, $W_{\text {mean }}$ increases exponentially as $m \cdot n$ increases, and so does the variance of $P(W)$. However, the $W_{\text {mean }}$ values are about $\times 100$ larger than the 1-D case with comparable traffic intensities (e.g. both around 0.88 for the largest $W_{\text {mean }}$ ), which is attributed to bulk arrival. Note that a decreased $m / n$ results in increased $W_{\text {mean }}$ even though $m \cdot n$ is kept constant.

We now verify an earlier statement that spike accumulation during waiting can be neglected in the 2-D silicon retina. In the worst delay case that we have considered when the array has a VGA size, the average accumulated spike number $N_{\text {accu }}$ can be calculated as:

$$
N_{\text {accu }}=\lambda_{p i x} n p W_{D}
$$

where $W_{D}$ is the waiting delay before the requesting row gets served. We take a large $W_{D}$ of $200 \mu \mathrm{~s}$ ac-


Figure 2.15. (a) SDR degradation in (a) 1-D silicon cochlea as the function of the channel number $m$ with two different central frequencies 3.4 k and 20 k Hz , and two different $Q^{\prime} \mathrm{s} 2$ and 10 evaluated, and (b) in 2D silicon retina as the function of the array size $m \cdot n$ for both ADM and STR.
cording to Figure 2.11 (b) which already has a very small chance to happen, and $N_{\text {accu }}$ is calculated to be 3.6. In the VGA case, $n_{\text {avg }}$ is calculated to be 103 , much larger than 3.6.

### 2.2.3 SDR Degradation

The white Gaussian noise shown in Figure 2.12(a) is used as the input $\boldsymbol{x}(t)$. It has zero mean, variance $\sigma$ of 0.382 and bandwidth $f_{n}$ of 20 Hz . Figure $2.12(\mathrm{~b})$ shows the example spike train $z(t)$ encoded by ADM and STR encoders within the time window [ $2.5 \mathrm{~s}, 3 \mathrm{~s}$ ] respectively, with threshold $\delta$ of 0.125 , fixed delay $T_{D}$ and refractory period $T_{H}$ both of 1 ms . It is clear that STR gives a $z(t)$ with a smaller sampling rate compared to ADM. With this input $\boldsymbol{x}(t)$ and using the nonlinear decoding algorithms described in Section 2.2.1 for signal reconstruction from $z(t)$, SDR degradation due to both $T_{D C}$ and $T_{D Q}$ variation will be discussed as follows.

The SDR degradation due to $T_{D C}$ variation is plotted as the function of the comparator speed indicator $y_{0}$ in Figure 2.13(a) for both ADM and STR. The time resolution in simulation is $1 \mu \mathrm{~s}$, and $T_{H}$ for STR is fixed at 1 ms . The exact delay $T_{D k}$ at each threshold-crossing is generated according to Eq. (2.7) with computed slope values. The average delay $T_{\text {Davg }}$ is calculated using Eq. (2.10). The hollow symbols and the solid symbols represent the SDR computed with exact delay and average delay, respectively. The SDR only accounts for the time interval [ $0.2 \mathrm{~s}, 4.8 \mathrm{~s}$ ] to exclude the relatively large distortion in the vicinity of time boundaries. Both ADM and STR maintain high SDR ( 87.6 dB and 68.8 dB in average) regardless of $y_{0}$ if exact delay is used for reconstruction. In practice however, $T_{D k}$ is not available due to the lack of instantaneous slope at threshold-crossings, therefore $T_{\text {Davg }}$ is used to approximately recover $\boldsymbol{x}(t)$, and SDR decreases by about 20 dB for the increase of $y_{0}$ by 1 . Intuitively this is because the variance of the delay error $T_{D k}-T_{\text {Davg }}$ increases with $y_{0}$. Figure 2.13(b) gives an example of recovered signal $\boldsymbol{x}_{r c}(t)$ at $y_{0}=-3$. With a given SDR specification, the $y_{0}$ value can be determined, and in turn the comparator parameters $A_{D C}$ and $\tau_{c}$. Larger SDR requires either larger $A_{D C}$ that can be achieved by using long channel-length transistors, or smaller $\tau_{c}$ by increasing comparator biases. Increased signal bandwidth with the same $A_{D C}$ and $\tau_{c}$ is expected to result in decreased SDR because of comparatively larger jitter in timestamps. Figure 2.14 examines the effectiveness of $T_{\text {Davg }}$ estimation using Eq. (2.10) with replacing the $T_{\text {Davg }}$ by a sweeping $T_{D}$
for decoding, which clearly shows that the calculated $T_{\text {Davg }}$ is not optimal in terms of the highest achievable reconstruction SDR. For example, at $y_{0}=-4.5$, the SDR obtained with $T_{\text {Davg }}$ is 8 dB lower than the best SDR at $T_{D}=0.5 T_{\text {Davg }}$ in STR, and 6 dB lower at $T_{D}=0.7 T_{\text {Davg. }} . T_{\text {Davg }}$ becomes an increasingly better estimation as the threshold $\delta$ decreases.

The SDR degradation in the 1-D silicon cochlea case is plotted as the function of the channel number $m$ in Figure 2.15(a). The error bar represents the SDR variance obtained in 20 runs. The waveform in Figure 2.12(a) is scaled to 5 ms with a bandwidth of 20 k Hz to be used as the input to the BPF bank. As examples, the input signal is filtered by two BPFs with different central frequencies at 3.4 k and 20 k Hz and different $Q^{\prime} \mathrm{s}$ of 2 and 10 before it is encoded into spikes. The 3.4 k and 20 k Hz are chosen for they are the telephony voice frequency band and the upper limit of human hearing frequency range. The time step for simulation is 1 ns , and $T_{H}$ is $5.9 \mu \mathrm{~s}$ for 3.4 k Hz and $1 \mu \mathrm{~s}$ for 20 k Hz . The exact delay $T_{D k}$ at each thresh-old-crossing is drawn from the distribution $P(W)$ in Figure $2.10(\mathrm{~b})$, and the average delay $T_{\text {Davg }}$ is from Figure 2.10 (a). The decoding bandwidth is set to the 3-dB lowpass corner frequency. With $m$ increasing from 8 to 194, the SDR approximately decreases by 30 dB for both ADM and STR in all combinations of central frequencies and $Q^{\prime}$ s. The $20 \mathrm{k}-\mathrm{Hz}$ case always gives a worse reconstruction SDR than the $3.4 \mathrm{k}-\mathrm{Hz}$ case because signals with higher frequency component result in finer timescale of the encoded spike train which is more prone to the negative impact of spike timestamp uncertainty due to time quantization. Although theoretically the decoding algorithm for STR given in Section 2.2.1 can perform perfect reconstruction, the lower SDR compared to ADM in Figure 2.15(a) can be attributed to the finite time support, i.e. less samples in a given period. $Q=10$ gives a higher $\operatorname{SDR}$ in ADM than $Q=2$, which is probably due to the fact of slightly smaller $T_{D Q}$ variation as can be seen in Figure 2.10(b). The less obvious benefit of a higher $Q$ in STR could be the result of its much lower reconstruction SDR, masking the effect of $Q$ dependence.

The SDR degradation in the 2-D silicon retina case is plotted as the function of the array size $m \cdot n$ in Figure 2.15(a). The waveform in Figure 2.12(a) is used as the input to spike encoders. The time resolution in simulation is $0.1 \mu \mathrm{~s}$, and $T_{H}$ is 1 ms . The exact delay $T_{D k}$ at each threshold-crossing is drawn from the distribution $P(W)$ in Figure 2.11(b), and the average delay $T_{\text {Davg }}$ is from Figure 2.11(a). The decoding bandwidth is set to 20 Hz . With $m \cdot n$ increasing from 19.2k (QQVGA) to 307.2k (VGA), the SDR approximately degrades by 25 dB for both ADM and STR.

### 2.3 Conclusion and Remarks

This chapter presents the performance analysis of and comparison between two spike encoding mechanisms, namely the self-timed reset (STR) and the asynchronous delta modulation (AMD), both of which have been implemented in silicon spiking sensors. Linear decoding of the spike trains produced by these two types of encoders shows that, with non-idealities considered like feedback delay due to comparison and spike queueing during spike transmission, and switch-holding during complete reset, STR usually results in less reconstruction SDR than ADM. Further analysis using nonlinear decoding algorithms quantitatively reveals the effects of jitters in spike timestamps caused by comparison and queueing on SDR: larger jitters results in more SDR degradation

For practical system designs of spiking sensors, the type of spike encoder can be chosen according to the specified SDR requirements. An STR encoder or an integrate-and-fire encoder should be adopted if sufficient SDR can be obtained because of the simpler and more area-efficient circuit implementations compared to an ADM encoder even though the later usually have a higher-fidelity spike-domain represen-
tation. Circuit parameters that are related to timestamp jitters caused by comparison delay $T_{D C}$ variation and queueing delay $T_{D Q}$ variation can also be determined by the targeted SDR. $T_{D C}$ variation is associated with the comparator DC gain and pole frequency, and $T_{D Q}$ variation is related to the encoder threshold, number of encoders, spike train service rate, and system architecture that can be modeled by a corresponding queueing model. The queueing models $M / D / 1$ and $M^{X} / D / 1$ used in the analysis above can be replaced by others if necessary which can more accurately describe the patterns of spike arrival/departure and server number of the system under consideration.

In the context of internet of things, the analysis in this chapter has implications in future low-power spiking sensor design. The comparator speed and the AER communication channel bandwidth can be reduced to the minimal level where the required encoding quality specifications (e.g. measured by SDR) can still be satisfied, and therefore the bias current of the comparators and the supply voltage of the AER circuits and be lowered to a certain degree to save system power. The SDR metric used here is directly relevant to faithful recording applications like optical neuroimaging [89] and electrical neural signal acquisition [78] where a small encoding error is of paramount importance. Compared to traditional clocked Nyquist sampling, asynchronous spike encoding schemes like STR and ADM have the advantage of reduced data redundancy, which is essential in minimizing RF transmission power for wireless sensors. For emerging smart sensing systems with low-power embedded processing for within-sensor classification and recognition [42], SDR may not be the best measure and further study is needed to establish the link between signal encoding quality and system performance in terms of classification/recognition accuracy.

### 2.4 Appendix

This appendix gives the method of computing $f_{i}$ and $g_{k}(t)$ in Eq. (2.16) based on the fully probabilistic analysis described in [88]. Recall that $g_{k}(t)$ denotes the probability that exact $k$ new row requests enter the queue during an arbitrary time interval of length $t$. If $j$ row requests arrive within $t$, the probability $\psi_{k}(j)$ of these $j$ row requests containing $k$ spikes can be derived by recursion (note that one row request is equivalent to two real spikes because of $T_{\text {row }} \approx 2 T_{\text {col }}$ ):

$$
\psi_{k}(1)=p_{k}(k \in N \cap k \in[3, n+2]), \psi_{k}(j)=\sum_{a=3}^{k-3(j-1)} p_{a} \psi_{k-a}(j-1)(k \geq 3 j), \psi_{k}(j)=0(k<3 j)
$$

where $p_{k}$ is the probability mass function of spike number in a row request. Given the assumption that the number of row request arrival within time $t$ is Poisson-distributed, $g_{k}(t)$ can be expressed as:

$$
\begin{equation*}
g_{k}(t)=\sum_{j=1}^{\infty} \psi_{k}(j) \frac{\left(\lambda_{q 2 D} t\right)^{j}}{j!} e^{-\lambda_{q 2 D} t} \tag{2.al}
\end{equation*}
$$

Let $f_{i}\left(t_{0}\right)$ denote the probability of the system holding $i$ row requests at time $t_{0}$. By conditioning on the number of spikes present at $t_{0}$, the equation below holds:

$$
f_{i}\left(t_{0}+D\right)=\sum_{l=0}^{1} f_{l}\left(t_{0}\right) g_{i}(D)+\sum_{l=2}^{i+1} f_{l}\left(t_{0}\right) g_{i+1-l}(D)\left(i \in N_{0}\right)
$$

where $D=1 / \mu_{q}$ is the deterministic service time. The stationary distribution $f_{i}$ is found by letting $t_{0} \rightarrow \infty$ :

$$
\begin{equation*}
f_{i}=\sum_{l=0}^{1} f_{l} g_{i}(D)+\sum_{l=2}^{i+1} f_{l} g_{i+1-l}(D)\left(i \in N_{0}\right) \tag{2.a2}
\end{equation*}
$$

To solve $f_{i}$ from Eq. (2.a2), the geometric tail approach described in [90] is used. For $i \geq M$ where M is a sufficiently large positive integer, $f_{i}$ is approximated as:

$$
f_{i} \approx f_{M} \varepsilon^{M-i}(i \geq M)
$$

The scaling factor $\varepsilon$ can be solved by setting the denominator of the probability generating function of the $\mathrm{M}^{\mathrm{X}} / \mathrm{D} / 1$ queue [91] to 0 :

$$
1-\varepsilon \cdot \exp \left[-\frac{\lambda_{q 2 D}}{\mu_{q}}\left(\sum_{a=3}^{n+2} p_{a} \varepsilon^{a}-1\right)\right]=0
$$

Eq. (2.a2) can be then written as:

$$
f_{i}=\sum_{l=0}^{1} f_{l} g_{i}(D)+\sum_{l=2}^{i+1} f_{l} g_{i+1-l}(D)(i<M)
$$

which is a linear equation system with $M$ dimension. The normalization equation can be written as:

$$
\sum_{i=0}^{M-1} f_{i}+f_{M} \sum_{i=M}^{\infty} \varepsilon^{M-i}=1 \Rightarrow \sum_{i=0}^{M-1} f_{i}+\frac{f_{M}}{1-\varepsilon^{-1}}=1
$$

Now an $(M+1)$-dimensional linear equation system is complete for $M+1$ variables $f_{i}, i \in[0, M]$. In the numerical simulation to obtain Figure 2.11(b), $M=200$ is used.

# Chapter 3: Silicon Retina with Enhanced Temporal Contrast Sensitivity and Spike Encoding 

$\mathscr{T}$his chapter elaborates on a new silicon retina design [89] based on the original dynamic vision sensor (DVS) [6] that is briefly introduced in Chapter 1. The new features of this design are down to $1 \%$ temporal contrast (TC) sensitivity with less than $35 \% 1 \sigma$ variation and improved spike encoding thanks to the employment of in-pixel asynchronous delta modulator (ADM).

Enhanced TC sensitivity can be useful for applications such as fine texture recognition, fluorometric calcium imaging and voltage-sensitive dye imaging (VSDI). For example, in VSDI, the transient fluorescence signal change is typically below $1 \%$ within tens of milliseconds [92], while in calcium imaging the signal is about $10 \%$ for a single action potential [93]. Several reported minimum TC sensitivity values of DVSs lie around $10 \%$. Setting the thresholds of spike encoders smaller than minimum TC sensitivity results in excessive noise spikes, which makes it difficult to reliably detect any transient visual change. A recent design improved the TC sensitivity to $1.5 \%$ by incorporating a subthreshold transimpedance preamplifier stage [16]. Despite the measured $2.1 \%-2.5 \% \mathrm{rms}$ equivalent input TC noise, the $1.5 \% \mathrm{TC}$ sensitivity in [16] was obtained only by averaging the output spikes over the entire sensor array. Hence a low noise design of the pixel front-end is essential for a reasonable signal-to-noise ratio (SNR) at the sensor output enabling an even smaller TC sensitivity setting.

As already discussed in Chapter 2, with linear decoding, ADM normally results in a better encoding quality than self-timed reset (STR) that is commonly used in prior DVSs. This is also true for nonlinear decoding with finite time support. Improved spike encoding can profit applications where video reconstruction from spikes is required. For example, the continuous-time signal dynamics is of interests in optical bioimaging. The input waveform needs to be recovered from the output spike train using the linear or nonlinear decoding methods presented in Chapter 2. Another example is the video decompression from vision sensors that can acquire both intensity images and asynchronous spikes in response to temporal intensity changes [94], [95]. It was pointed out in [94] that the signal loss due to STR encoding greatly degrades the decompression quality.

The content of this chapter is organized as follows: Section 3.1 describes the system architecture and design considerations of pixel gain division; Section 3.2 gives the detailed pixel design, including: Section 3.2.1, analysis of the photoreceptor; Section 3.2.2, the capacitively-coupled programmable gain amplifier (CC-PGA) for sensitivity enhancement including a compact two-stage Opamp with pseudocascode compensation; Section 3.2.3, the in-pixel ADM design; Section 3.2.4, the in-pixel asynchronous logic (IPAsyncL) to generate ADM control signals and for communication of generated spikes with peripheral address-event-representation (AER) circuitry; Section 3.3 presents the experimental results; Section 3.4 concludes the chapter with several remarks.

### 3.1 System Architecture

The system architecture of the new DVS (called ADMDVS) is shown in Figure 3.1. It consists of a $60 \times 30$ pixel array, the $\mathrm{X} / \mathrm{Y}$ address encoder that encodes the column and row addresses into 6 - and 5 -bits output respectively, the X/Y AER logic and arbiter for asynchronous transmission of pixel spike output, and the asynchronous state machine for the communication with off-chip complex programmable logic device (CPLD) or field programmable gate array (FPGA). The AER is a modified version [84] of the


Figure 3.1. System architecture of the ADMDVS chip, including the pixel array, address encoder, ad-dress-event representation (AER) logic and arbiter, and asynchronous state machine. The bias generator and shift-register configuration chain are not shown.
word-serial burst-mode protocol as proposed in [83]. The working mechanism of this AER scheme can be understood from the queueing model described in Chapter 2.

The building blocks of one pixel are shown in Figure 3.2. The photoreceptor that is composed of a photodiode and a transimpedance amplifier (TIA) logarithmically converts a small-signal photocurrent $i_{p h}$ into a voltage output $\Delta \mathrm{V}$, which is then buffered by a source follower (SF) before it is amplified by a CCPGA. The amplified analog signal is encoded into spikes by the ADM and the spikes are communicated off-chip by in-pixel asynchronous logic (IPAsyncL) and the peripheral AER circuitry. The bias currents of the TIA, SF and CC-PGA are adjustable to control the front-end bandwidth. The gain of the CC-PGA is programmable with 2 bits, and the threshold voltages of the ADM can be continuously adjusted for different TC sensitivity settings. The illustration of the pixel communication with the periphery AER is simplified; the complete sensor array has the 2-dimensional AER to communicate X and Y addresses of active pixels in a burst-mode word-serial fashion as depicted in Figure 3.1.

The aim of the ADMDVS design is to achieve below $1 \%$ TC sensitivity using ADM for spike encoding instead of STR. Sufficient front-end gain is necessary to amplifier small TC signals. For example, $1 \%$ TC corresponds to about 0.25 mV photoreceptor output. With a 250 mV ADM threshold, a gain of 60 dB is needed. If the gain is only provided by the CC-PGA, the total capacitance would be about 5 pF given a 5 fF feedback capacitor. In a $0.18 \mu \mathrm{~m}$ CMOS process, the unit capacitance of MIM is normally $1 \mathrm{fF} / \mu \mathrm{m}^{2}$, which leads to a $71 \times 71 \mu \mathrm{~m}^{2}$ capacitor size. Although large closed-loop gain reduces the CC-PGA's noise contribution (explained in Section 3.3), in order to have a pixel size of about $30 \times 30 \mu \mathrm{~m}^{2}$ that is comparable to previous DVS pixels, the ADM needs to take part of the gain. If the CC-PGA has a gain of 36 dB and the ADM 24 dB , the total capacitance is about 400 fF if both CC-PGA and ADM have a 5 fF feedback capacitor, and the area is about $20 \times 20 \mu \mathrm{~m}^{2}$ which is affordable by a $30 \times 30 \mu \mathrm{~m}^{2}$ pixel.


Figure 3.2. Building blocks of one pixel, including the photoreceptor, source follower (SF), capacitivelycoupled programmable gain amplifier (CC-PGA), asynchronous delta modulator (ADM) and in-pixel asynchronous logic (IPAsyncL).

### 3.2 Pixel Design

### 3.2.1 Photoreceptor

Two types of photoreceptors have been used in DVSs, and they differ in the TIA feedback element as shown in Figure 3.3: the one in Figure 3.3(a) uses an nFET, and is called a source-follower photoreceptor (SFPR) [6]; the one in Figure 3.3(b) uses a pFET, and is called a common-gate photoreceptor (CGPR) [16]. The detailed analysis of SFPR and CGPR is given below.

### 3.2.1.1 SFPR

The output DC $\mathrm{V}_{\text {outDC }}$ of SFPR is at $\mathrm{V}_{\mathrm{gs} 1}+\mathrm{V}_{\mathrm{gs} 3}$. $\mathrm{M}_{3 \mathrm{~s}}$ is usually in deep subthreshold because of small DC photocurrent $I_{P H}$, which can result in negative $\mathrm{V}_{\mathrm{g} 33}$. The headroom problem may arise when $\mathrm{V}_{\text {outDC }}$ is too low for $\mathrm{M}_{2 \mathrm{~s}}$ or even $\mathrm{M}_{1 \mathrm{~s}}$ to stay in saturation. $\mathrm{V}_{\text {outDC }}$ could always stay at a reasonably high level to guarantee the saturation of $\mathrm{M}_{2 \mathrm{~s}}$ in $0.35 \mu \mathrm{~m}$ CMOS [6], but not anymore as process scales and transistor threshold voltage decreases. Recent vision sensors with SFPR in $0.18 \mu \mathrm{~m}$ CMOS used 3.3-V nFETs for $\mathrm{M}_{1 \mathrm{~s}}$ and $\mathrm{M}_{3 \mathrm{~s}}$ to circumvent this problem [18].

The small signal equivalent circuit of SFPR is shown in Figure 3.4(a). The transfer function is accordingly calculated as in Eq. (3.1):

$$
\begin{equation*}
\frac{V_{\text {out }}}{V_{T} \frac{i_{p h}}{I_{P H}}}=\frac{V_{\text {out }}}{i_{\text {ph }}} g_{s 3} \quad \frac{A_{D C}\left(1-\frac{s}{\omega_{z}}\right)}{\frac{s^{2}}{\omega_{n}^{2}}+\frac{s}{\omega_{n} Q}+1}=\frac{\frac{A}{1+\kappa A}\left(1-s \beta \tau_{\text {out }}\right)}{s^{2} \frac{A}{1+\kappa A}(1+\alpha+\beta) \tau_{\text {in }} \tau_{\text {out }}+s \frac{A}{1+\kappa A}\left[\tau_{\text {in }}\left(\alpha\left(1+\frac{1}{A}\right)+\frac{1}{A}\right)+\tau_{\text {out }}(\beta(1-\kappa)+1)\right]+} \tag{3.1}
\end{equation*}
$$

where $V_{T}$ is the thermal voltage, $A_{D C}$ is the DC gain of the TIA, $\omega_{n}$ is the natural frequency, $\omega_{z}$ is the zero frequency, $Q$ is the quality factor, $\kappa$ is the subthreshold slope factor (assume the same for both nFET and pFET ), and the other parameters are defined as below:

$$
\begin{gathered}
A=\frac{g_{m \mathrm{l}}}{g_{\text {dsout }}}, \tau_{\text {in }}=\frac{\mathrm{C}_{\text {in }}}{g_{s 3}}, \tau_{\text {out }}=\frac{\mathrm{C}_{\text {out }}}{g_{m 1}}, \alpha=\frac{\mathrm{C}_{\mathrm{g} 3} 3}{\mathrm{C}_{\text {in }}}, \beta=\frac{\mathrm{C}_{\mathrm{g} 3} 3}{\mathrm{C}_{\text {out }}}, R_{c}=\frac{\mathrm{C}_{\text {in }}}{\mathrm{C}_{\text {out }}}, R_{i}=\frac{I_{\text {amp }}}{I_{P H}}, R_{\tau}=\frac{\tau_{\text {in }}}{\tau_{\text {out }}}=\kappa R_{c} R_{i}, \\
m=\kappa(1+\alpha+\beta), n=1+\beta(1-\kappa)
\end{gathered}
$$


(a)

(b)

Figure 3.3. Circuit diagrams of (a) the source-follower photoreceptor (SFPR) and (b) the common-gate photoreceptor (CGPR).


Figure 3.4. Small signal equivalent circuits of (a) the SFPR and (b) the CGPR.
where $g_{d s o u t}$ and $I_{\text {amp }}$ are the output conductance and the bias current of the TIA, respectively. If $A \gg 1$, we have:

$$
\mathrm{A}_{\mathrm{DC}}=\frac{1}{\kappa}, \omega_{n}=\sqrt{\frac{\kappa^{2}}{m \tau_{\text {in }} \tau_{\text {out }}}}, \omega_{z}=\frac{1}{\beta \tau_{\text {out }}}, Q=\frac{\sqrt{m R_{\tau}}}{R_{\tau}\left(\alpha+\frac{1}{A}\right)+n}
$$

The maximum $Q$ over all illuminance conditions is derived as:

$$
Q_{\max }=\frac{\sqrt{\frac{m A}{n(1+\alpha A)}}}{2} \text {, when } R_{\tau}=\frac{n A}{1+\alpha A}
$$

If $\alpha A \gg 1$, the $Q_{\max }$ is simplified as:

$$
\begin{equation*}
Q_{\max }=\frac{\sqrt{m / n \alpha}}{2}, \text { when } R_{\tau}=\frac{n}{\alpha} \tag{3.2}
\end{equation*}
$$

To have real poles in this $2^{\text {nd }}$-order system, $R_{i}$ has to satisfy:

$$
R_{i}<\frac{\left.-n\left(\alpha+\frac{1}{A}\right)+2 m-2 \sqrt{m\left(m-n\left(\alpha+\frac{1}{A}\right)\right.}\right)}{\kappa R_{c}\left(\alpha+\frac{1}{A}\right)^{2}} \text { or } R_{i}<\frac{-n\left(\alpha+\frac{1}{A}\right)+2 m-2 \sqrt{m\left(m-n\left(\alpha+\frac{1}{A}\right)\right.}}{\kappa R_{c}\left(\alpha+\frac{1}{A}\right)^{2}}
$$

With practical design parameters such as $A=100, \kappa=0.8, \alpha=0.05, \beta=0.25, Q_{\max }$ is about 2.03 at $R_{i}=4.38$. To have real poles, the range of $R_{i}$ is calculated to be: $R_{i}<6.83 \times 10^{-2}$ or $R_{i}>2.80 \times 10^{2}$. Under low illumination, e.g. $R_{i}=10^{4}$, the $3-\mathrm{dB}$ bandwidth is about:

$$
\omega_{3 d B}=13.4 / \tau_{i n}
$$

Several comments on the calculations above are given below.

1. If the $\mathrm{C}_{\mathrm{gs}}$ of $\mathrm{M}_{3 \mathrm{~s}}$ is not considered, $Q_{\max }$ is directly related to the gain of the amplifier $A$ :

$$
Q_{\max }=\frac{\sqrt{\kappa A}}{2}, \text { when } R_{\tau}=A
$$

which is about 4.47. With the presence of $\mathrm{C}_{\mathrm{gs}}$, if $\alpha \mathrm{A} \gg 1, Q_{\max }$ is not dependent on $A$ as shown in Eq. (3.2), and can be designed to be relatively small by choosing the size of $\mathrm{M}_{3 \mathrm{~s}}$. However, the parasitic $\mathrm{C}_{\mathrm{gs}}$ limits the speed: the $3-\mathrm{dB}$ bandwidth is only 13.4 times larger than that of the simple source-follower photoreceptor. For a relatively large range of photocurrent, the system has two real poles, and the speed is either determined by $\tau_{\text {in }}$ or $\tau_{\text {out. }}$. Another benefit of this configuration compared to CGPR might be that the photocurrent $I_{P H}$ could be larger than $I_{a m p}$, and $I_{P H}$ can be readily reused for APS intensity readout [18].
2. The DC gain is dependent on illumination level, which implies contrast inconsistency over a wide dynamic range. (1) Under low illumination, dark current is prominent, and $g_{s 3}$ is $\left(I_{P H}+I_{\text {dark }}\right) / V_{T}$ instead of $I_{P H} / V_{T}$. Therefore, the effective DC gain decreases. (2) The DC point of $V_{o u t}$ changes with $I_{P H}$, and so does the $\kappa$ of $\mathrm{M}_{3 \mathrm{~s}}$ [96]. Short channel devices have less $\kappa$ dependence due to the effect of the capacitance between the channel and the drain [97]. This is still valid in UMC $0.18 \mu \mathrm{~m}$ CMOS in Spectre simulation. (3) Under high illumination, $\mathrm{M}_{3 \mathrm{~s}}$ may enter moderate inversion, and the effective DC gain may become larger. So overall, $\mathrm{M}_{3 \mathrm{~s}}$ with a small L and large W is good for less $\kappa$ variation over a large illumination range, but it aggravates the headroom problem under low illumination due to more negative $\mathrm{V}_{\mathrm{g} 53}$.
3. The $C_{g s}$ of $\mathrm{M}_{3 \mathrm{~s}}$ is not constant but dependent on illumination level. It increases with the $V_{g s}$ of $\mathrm{M}_{3 \mathrm{~s}}$ [98], which is logarithmically dependent on the photocurrent. This may indicate larger photoreceptor bandwidth under low illumination than expected and smaller $Q$ under high illumination. It is shown that short-channel transistors have larger normalized $C_{g s}$ in deep sub-threshold region than long-channel transistors [99]. Therefore, smaller L does not necessarily guarantee smaller $C_{g s}$ or higher bandwidth, and simulation is needed for optimized $Q$ and speed.

The output noise power of SFPR can be analytically calculated using simple noise models [26]. The transfer function of the noise current from the photodiode and $\mathrm{M}_{3 \mathrm{~s}}$ are the same as that of the input small signal $i_{p h}$. Their contribution to output noise power spectral density (psd) can thus be written as:

$$
\bar{V}_{\text {nout }, p h}^{2}(\omega)=\bar{i}_{n, p h}^{2} \frac{V_{T}^{2}}{I_{P H}^{2}}|H(\omega)|^{2}=\bar{i}_{n, p h}^{2} \frac{V_{T}^{2}}{I_{P H}^{2}} \frac{\frac{1}{\kappa^{2}}\left(1+\frac{\omega^{2}}{\omega_{z}^{2}}\right)}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{n}^{2}}+1}, \bar{V}_{\text {nout }, M_{3}}^{2}(\omega)=\bar{q}_{n, M_{3}}^{2} \frac{V_{T}^{2}}{I_{P H}^{2}} \frac{\frac{1}{\kappa^{2}}\left(1+\frac{\omega^{2}}{\omega_{z}^{2}}\right)}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{n}^{2}}+1}
$$

The output noise psd from the forward amplifier is calculated as:

$$
\bar{V}_{\text {nout }, \text { anp }}^{2}(\omega)=\bar{i}_{n, a m p}^{2} \frac{V_{T}^{2}}{I_{P H}^{2}} \frac{\frac{1}{\kappa^{2}}\left(\frac{1}{\left(\kappa R_{i}\right)^{2}}+\frac{\omega^{2}}{\omega_{z^{*}}^{2}}\right)}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{n}^{2}}+1}, \text { where } \omega_{z^{*}}=\frac{1}{\beta \tau_{\text {out }}+\frac{\tau_{\text {in }}}{\kappa R_{i}}}
$$

It is often stated that the main noise sources in photoreceptors are shot noise from the photodiode and thermal noise from the feedback transistor $\mathrm{M}_{3 \mathrm{~s}}$, and the noise of the amplifier only becomes nonnegligible under high illumination [100], [101]. For a complete analysis, flicker noise ( $1 / f$ ) is also included. For small-sized transistors, $1 / f$ noise is actually believed to become random telegraph signal (RTS) noise with a Lorentzian-shaped psd [98]. Nevertheless, the simple $1 / f$ noise model is used here for approximate calculations to gain some insights. The noise current psd from the photodiode, $\mathrm{M}_{3 \mathrm{~s}}$, and the amplifier are written as:

$$
\bar{i}_{n, p h}^{2}=2 q I_{P H}, \bar{i}_{n, M_{3}}^{2}=2 q I_{P H}+\frac{K_{n}}{(W L)_{3} C_{o x}} \frac{1}{f} g_{m 3}^{2}, \bar{i}_{n, a m p}^{2}=4 q I_{a m p}+\frac{K_{p}}{(W L)_{4} C_{o x}} \frac{1}{f} g_{m 4}^{2}+\frac{K_{n}}{(W L)_{1} C_{o x}} \frac{1}{f} g_{m 1}^{2}
$$

Noise from the cascode $n$ FET $\mathrm{M}_{2 \mathrm{~s}}$ can be normally neglected, if the output resistance of $\mathrm{M}_{1 \mathrm{~s}}$ is large enough and signal frequency is sufficiently low [26]. We define the following transistor size ratios:

$$
\frac{(W L)_{3}}{(W L)_{4}}=R_{34}, \frac{(W L)_{3}}{(W L)_{1}}=R_{31}
$$

The total output noise can be calculated as in Eq. (3.3):

$$
\begin{align*}
\bar{V}_{\text {nout,tot }}^{2}(\omega) & \approx \frac{1}{\kappa^{2}} \frac{4 q V_{T}^{2}}{I_{P H}} \int_{0}^{B W} \frac{T_{1}+T_{2} \frac{\omega^{2}}{\omega_{n}^{2}}}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{n}^{2}}+1} d \omega+\frac{2 \pi K_{n}}{(W L)_{3} C_{o x}} \int_{0}^{B W} \frac{1}{\omega} \frac{F_{1}+F_{2} \frac{\omega^{2}}{\omega_{n}^{2}}}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{n}^{2}}+1} d \omega  \tag{3.3}\\
& x=\frac{\omega}{\omega_{n}} \\
& \Rightarrow \frac{4 k T}{C_{i n}} \sqrt{\frac{R_{i} R_{c}}{\kappa m}} \int_{0}^{\frac{B W}{\omega_{n}}} \frac{T_{1}+T_{2} x^{2}}{x^{4}+\left(\frac{1}{Q^{2}}-2\right) x^{2}+1} d x+\frac{2 \pi K_{n}}{(W L)_{3} C_{o x}} \int_{0}^{\frac{B W}{\omega_{n}}} \frac{1}{x} \frac{F_{1}+F_{2} x^{2}}{x^{4}+\left(\frac{1}{Q^{2}}-2\right) x^{2}+1} d x
\end{align*}
$$

where BW is the pixel bandwidth, and the coefficients $T_{1}, T_{2}, F_{l}$ and $F_{2}$ are defined as:

$$
T_{1}=1+\frac{1}{\kappa^{2} R_{i}}, T_{2}=\frac{\kappa R_{c}}{m}, F_{1}=1+\frac{1}{\kappa^{2}} \frac{K_{p}}{K_{n}} R_{34}+\frac{1}{\kappa^{2}} R_{31}, F_{2}=\left(1+R_{i}^{2} \frac{K_{p}}{K_{n}} R_{34}+R_{i}^{2} \frac{R_{c}^{2}}{\beta^{2}} R_{31}\right) \frac{\kappa \beta^{2}}{m R_{c} R_{i}}
$$

### 3.2.1.2 CGPR

The output DC $\mathrm{V}_{\text {outDC }}$ of CGPR is $\mathrm{V}_{\mathrm{bf}}+\left|\mathrm{V}_{\mathrm{gs} 3}\right| . \mathrm{V}_{\text {bf }}$ can be set at a reasonably high voltage so that unlike SFPR both $\mathrm{M}_{1 \mathrm{c}}$ and $\mathrm{M}_{2 \mathrm{c}}$ can stay in saturation even under low illumination. The TIA bias current $I_{\text {amp }}$ set by pFET $\mathrm{M}_{4 \mathrm{c}}$ should always be larger than the photocurrent $I_{p h}$ with some margin to supply sufficient current to $M_{1 c}$ and $M_{2 c}$. The $C_{d s}$ of $M_{3 c}$ in CGPR is much smaller than that of $M_{3 s}$ in SFPR, and hence no Miller effect needs to be considered which lends to speed advantage albeit with more stability concern. The body of $\mathrm{M}_{3 \mathrm{c}}$ is tied to VDD, sharing the same n-well with $\mathrm{M}_{4 \mathrm{c}}$ to save area.

The small signal equivalent circuit of the CGPR is shown in Figure 3.4(b). The transfer function is calculated as in Eq. (3.4):

$$
\begin{equation*}
\frac{V_{\text {out }}}{V_{T} \frac{i_{\text {ph }}}{I_{P H}}}=\frac{V_{\text {out }}}{\frac{i_{\text {ph }}}{g_{s 3}}}=\frac{A\left(1-\frac{1}{A^{\prime} R_{i}}\right) / A+\frac{\kappa}{A^{\prime}}(1+A)}{\frac{s^{2}}{\omega_{n}^{2}}+\frac{s}{\omega_{n} Q}+1}=\frac{\tau^{2} \frac{A}{A+\frac{\kappa}{A^{\prime}}(1+A)} \tau_{\text {in }} \tau_{\text {out }}+s \frac{A}{A+\frac{\kappa}{A^{\prime}}(1+A)}\left(\frac{A}{A}\left(1+\frac{A}{\kappa R_{i}}+\frac{A}{A^{\prime} R_{i}}\right)+\tau_{\text {out }} \frac{\kappa}{A^{\prime}}\right)+1}{} \tag{3.4}
\end{equation*}
$$

where $A^{\prime}$ is defined as:

$$
A^{\prime}=\frac{\kappa g_{s 3}}{g_{d s 3}}
$$

If $A$ and $A^{\prime}$ are much larger than $1, A_{D C}, \omega_{n}$ and $Q$ can be simplified as:

$$
A_{D C}=1, \omega_{n}=\sqrt{\frac{1}{\tau_{\text {in }} \tau_{\text {out }}}}, Q=\frac{\sqrt{R_{\tau}}}{R_{\tau}\left(\frac{1}{A}+\frac{1}{\kappa R_{i}}\right)+\frac{\kappa}{A^{\prime}}}=\frac{\sqrt{R_{\tau}}}{\frac{R_{\tau}}{A}+R_{c}+\frac{\kappa}{A^{\prime}}}
$$

The maximum $Q$ over all illumination conditions is derived as:

$$
Q_{\max }=\frac{\sqrt{\frac{A A^{\prime}}{\kappa+A^{\prime} R_{c}}}}{2}, \text { when } R_{\tau}=\frac{A}{A^{\prime}}\left(\kappa+A_{c}^{\prime} R_{c}\right)
$$

If $A^{\prime} R_{c} \gg \kappa$, the $Q_{\max }$ is simplified as Eq. (3.5):

$$
\begin{equation*}
Q_{\max }=\frac{\sqrt{A / R_{c}}}{2}, \text { when } R_{\tau}=A R_{c} \tag{3.5}
\end{equation*}
$$

To have real poles in this $2^{\text {nd }}$-order system, $R_{i}$ has to satisfy:

$$
R_{i}<\frac{2 A^{2}}{\kappa R_{c}}\left(1-\sqrt{1-\frac{R_{c}}{A}}\right)-\frac{A}{\kappa} \text { or } R_{i}<\frac{2 A^{2}}{\kappa R_{c}}\left(1+\sqrt{1-\frac{R_{c}}{A}}\right)-\frac{A}{\kappa}
$$

With practical design parameters such as $A=100, \kappa=0.8, R_{c}=5, Q_{\max }$ is about 2.24 at $R_{i}=125$. To have real poles, the range of $R_{i}$ is calculated as: $R_{i}<1.60$ or $R_{i}>9.75 \times 10^{3}$. Under low illumination, e.g. $R_{i}=10^{4}$, the 3dB bandwidth is about:

$$
\omega_{3 d B}=126 / \tau_{i n}
$$

Comments on the calculations above are given below.

1. Note that $Q_{\max }$ is not only dependent on $A$ but also $R_{c}$, as shown in Eq. (3.5). To keep a small $Q_{\max }$, large $R_{c}$ and small $A$ are needed. Therefore, if a CC-PGA is used, as depicted in Figure 3.2, an SF connected to the output of the photoreceptor is indispensable to avoid large capacitive loading. By tying the gate of $\mathrm{M}_{2 \mathrm{c}}$ to VDD, smaller $A$ could be obtained to keep $Q_{\text {max }}$ low, especially under high illumination. Without the Miller effect in contrast to the case of SFPR where $\mathrm{C}_{\mathrm{g} s 3}$ plays the role of Miller capacitance, the $3-\mathrm{dB}$ bandwidth of CGPR is over 9 times larger than that of SFPR at $R_{i}=10^{4}$. Under high illumination, the bandwidth is limited by $\tau_{\text {out }}$ in both cases, and the bandwidth difference becomes negligible.

The noise analysis follows the method used in the SFPR case. The output noise psd contributed by the photodiode is written as:

$$
\bar{V}_{\text {nout }, p h}^{2}(\omega)=\bar{i}_{n, p h}^{2} \frac{V_{T}^{2}}{I_{P H}^{2}}|H(\omega)|^{2}=\bar{i}_{n, p h}^{2} \frac{V_{T}^{2}}{I_{p h}^{2}} \frac{1}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{n}^{2}}+1}
$$

The output noise psd contributed by $\mathrm{M}_{3 \mathrm{c}}$ and the amplifier can be respectively calculated as:

$$
\begin{gathered}
\bar{V}_{\text {nout }, M_{3}}^{2}(\omega)=\bar{q}_{n, M_{3}}^{2} \frac{V_{T}^{2}}{I_{P H}^{2}} \frac{1+\frac{\omega^{2}}{\omega_{z}^{2}}}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{n}^{2}}+1}, \text { where } \omega_{z}=A \frac{g_{\text {dsout }}}{C_{\text {in }}} \\
\bar{V}_{\text {nout }, \text { amp }}^{2}(\omega)=\bar{i}_{n, \text { anp }}^{2} \frac{V_{T}^{2}}{I_{p h}^{2}} \frac{\frac{1}{\left(\frac{\left.A^{\prime} R_{i}\right)^{2}}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{z}^{2}}} \frac{\omega_{n}^{2}}{\omega_{n}^{2}}+1\right.}}{} .
\end{gathered}
$$

The noise current psd from the photodiode, $\mathrm{M}_{3 \mathrm{c}}$, and the amplifier can be written as:

$$
\bar{i}_{n, p h}^{2}=2 q I_{P H}, \bar{i}_{n, M_{3}}^{2}=2 q I_{P H}+\frac{K_{p}}{(W L)_{3} C_{o x}} \frac{1}{f} g_{m 3}^{2}, \bar{i}_{n, a m p}^{2}=2 q\left(2 I_{a m p}-I_{p h}\right)+\frac{K_{p}}{(W L)_{4} C_{o x}} \frac{1}{f} g_{m 4}^{2}+\frac{K_{n}}{(W L)_{1} C_{o x}} \frac{1}{f} g_{m 1}^{2}
$$

The total output noise can be calculated as in Eq. (3.6):

$$
\begin{align*}
\bar{V}_{\text {nout }, \text { tot }}^{2}(\omega) & \approx \frac{4 q V_{T}^{2}}{I_{p h}} \int_{0}^{B W} \frac{T_{1}+T_{2} \frac{\omega^{2}}{\omega_{n}^{2}}}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{n}^{2}}+1} d \omega+\frac{2 \pi \kappa^{2} K_{p}}{(W L)_{3} C_{o x}} \int_{0}^{B W} \frac{1}{\omega} \frac{F_{1}+F_{2} \frac{\omega^{2}}{\omega_{n}^{2}}}{\left(\frac{\omega^{2}}{\omega_{n}^{2}}\right)^{2}+\left(\frac{1}{Q^{2}}-2\right) \frac{\omega^{2}}{\omega_{n}^{2}}+1} d \omega  \tag{3.6}\\
& x=\frac{\omega}{\omega_{n}} \\
& \Rightarrow \frac{4 k T}{C_{i n}} \sqrt{\kappa R_{i} R_{c}} \int_{0}^{\frac{B W}{\omega_{n}}} \frac{T_{1}+T_{2} x^{2}}{x^{4}+\left(\frac{1}{Q^{2}}-2\right) x^{2}+1} d x+\frac{2 \pi \kappa^{2} K_{p}}{(W L)_{3} C_{o x}} \int_{0}^{\frac{B W}{\omega_{n}}} \frac{1}{x} \frac{F_{1}+F_{2} x^{2}}{x^{4}+\left(\frac{1}{Q^{2}}-2\right) x^{2}+1} d x
\end{align*}
$$

where BW is the bandwidth of the pixel analog front end, and the coefficients $T_{1}, T_{2}, F_{1}$ and $F_{2}$ are defined as:

$$
T_{1}=1, T_{2}=\frac{R_{c}}{\kappa}, F_{1}=1, F_{2}=\left(1+R_{i}^{2} R_{34}+\left(R_{i}-1\right)^{2} \frac{K_{n}}{K_{p}} R_{31}\right) \frac{R_{c}}{\kappa R_{i}}
$$

### 3.2.1.3 Noise Comparison

The bandwidth improvement of CGPR due to the lack of $\mathrm{C}_{\mathrm{gs}}$ Miller capacitance has been exploited to improve the minimum latency of DVSs [15], even though this is at the cost of increased $Q_{\max }$. In the context of designing a DVS with enhanced TC sensitivity, noise is what matters more because high noise level at the photoreceptor output can mask any transient visual signal with amplitude below the noise rms value. This is true for a single pixel. For an array of pixels, however, the visual detection of an object also depends on the percentage of its 2D geometrical size in the whole visual field. As pointed out in [102], statistically the minimum number of photons required to detect a spatial contrast in an image is inversely proportional to the object size, which also applies to the DVS case. In this regard, even though the TC signal may be below the noise rms, an object whose size is sufficiently large could still be detected. Because object size is unpredictable, in the following analysis, we still apply the most stringent criterion that the photoreceptor output noise should be below the aimed TC sensitivity.

The only difference between the two photoreceptors is the feedback transistor, an nFET in SFPR and a pFET in CGPR respectively. CGPR is expected to have a lower output noise because pFETs are often observed to have a lower $1 / f$ noise compared to nFETs. As shown by the theoretically calculated results

Table 3.1. Comparison of the output voltages of SFPR and CGPR with different $i_{p h}$ change.

|  | SFPR | CGPR | Notes |
| :---: | :---: | :---: | :---: |
| normalized DC gain $\boldsymbol{V}_{\text {out }} /\left(\boldsymbol{i}_{\boldsymbol{p} \boldsymbol{h}} / \boldsymbol{I}_{\boldsymbol{P H}}\right)$ | $V_{T} / \kappa$ | $V_{T}$ | $V_{T}:$ thermal voltage, $25.9 \mathrm{mV} ; i_{p h}$ : small signal |
| photocurrent; $I_{P H}$ : DC background photocur- |  |  |  |
| rent; $\kappa$ : subthreshold slope factor, 0.8. |  |  |  |
| $\boldsymbol{\Delta} \boldsymbol{V}_{\text {out }}(\mathrm{mV})$ at $10 \% \boldsymbol{i}_{\text {ph }}$ change | 3.05 | 2.59 |  |
| $\boldsymbol{\Delta} \boldsymbol{V}_{\text {out }}(\mu \mathrm{V})$ at $1 \% \boldsymbol{i}_{\text {ph }}$ change | 305 | 259 |  |

Table 3.2. Sizes of the transistors in Fig. 2.

| Device | Width $(\mu \mathrm{m}) /$ Length $(\mu \mathrm{m})$ |
| :---: | :---: |
| $\mathrm{M}_{1 \mathrm{~s}} / \mathrm{M}_{1 \mathrm{c}}$ | $0.24 / 2.00$ |
| $\mathrm{M}_{2 \mathrm{~s}} / \mathrm{M}_{2 \mathrm{c}}$ | $0.24 / 0.18$ |
| $\mathrm{M}_{3 \mathrm{~s}} / \mathrm{M}_{3 \mathrm{c}}$ | $0.60 / 0.20$ |
| $\mathrm{M}_{4 \mathrm{~s}} / \mathrm{M}_{4 \mathrm{c}}$ | $0.24 / 1.00$ |

listed in Table 3.1, for $1 \%$ TC detection by a single pixel, the rms integrated output noise should be less than $305 \mu \mathrm{~V}$ and $259 \mu \mathrm{~V}$ for SFPR and CGPR, respectively. For Spectre simulation, resistors are used to emulate the shot noise of a photodiode [103]. With the transistor sizes given in Table 3.2, the simulated output noise for both SFPR $V_{n, \text { outs }}$ and CGPR $V_{n, \text { outc }}$ within 100 Hz bandwidth are plotted in Figure 3.5(a) along with the numerically calculated results according to Eq. (3.3) and Eq. (3.6) with the parameters provided by UMC $0.18 \mu \mathrm{mF} / \mathrm{MM}$ CMOS. $\mathrm{M}_{1 \mathrm{~s}} / \mathrm{M}_{1 \mathrm{c}}$ is a $3.3-\mathrm{V}$ transistor in simulation to avoid the situation where $V_{\text {out }}$ is too low for $\mathrm{M}_{2 \mathrm{~s}}$ in SFPR to stay in saturation at low $I_{P H}$. For small $I_{P H}<100 \mathrm{fA}, V_{n, \text { outs }}$ and $V_{n, \text { outc }}$ are approximately the same because the photocurrent shot noise dominates. As $I_{P H}$ increases, the shot noise contribution decreases, and transistor $1 / f$ noise becomes prominent. $V_{n, \text { outs }}$ saturates to 350 $\mu \mathrm{V}$ at $I_{P H} \approx 1 \mathrm{pA}$, still above its $1 \%$ TC level. In comparison, $V_{n, \text { outc }}$ continues to decreases until $I_{P H} \approx 50 \mathrm{pA}$, and saturates at about $50 \mu \mathrm{~V}$, 7 times lower compared to the SFPR. Figure 3.5 (b) shows the simulated $V_{n, \text { outc }}$ as a function of both $I_{P H}$ and pixel front-end bandwidth using a $1.8-\mathrm{V} \mathrm{M}_{1 \mathrm{c}}$. Larger bandwidth results in increased $V_{n, \text { outc }}$ under the same illumination. The plateau of the $10^{4}-\mathrm{Hz}$ curve is due to the fact that the CGPR bandwidth under low illumination is already less than $10^{4} \mathrm{~Hz}$, and thus the integrated noise remains relatively constant. The pixel front-end bandwidth can be controlled via the CC-PGA as will be discussed in the next section.

From the comparison of the output noise between SFPR and CGPR, the latter is chosen in the design for its better noise performance under moderate to high illumination which ensures the goal of $1 \% \mathrm{TC}$ sensitivity. Note that a $1.8-\mathrm{V} \mathrm{M}_{1 \mathrm{c}}$ is used in fabrication for CGPR.

### 3.2.2 Capacitively-Coupled Programmable Gain Amplifier (CC-PGA)

CC-PGA is chosen for the additional in-pixel gain to amplify small TC signals. The previous solutions include using above-threshold common-source amplifiers (Figure 3.6(a) [15]) and subthreshold transimpedance amplifiers (Figure 3.6(b) [16]). The common drawback in these two approaches is that they both need a global feedback mechanism to control the bias of the preamplifiers $I_{\text {preamp }}$ in order to keep the transistors in saturation because $I_{\text {preamp }}$ is dependent on the local photocurrent. In Figure 3.6(a), it is done by tuning the source voltage of the nFET, and in Figure $3.6(\mathrm{~b})$, it is done by tuning the gate voltage of the pFET. This global feedback limits the intrascene dynamic range (to about 60 dB with a 3.3-V


Figure 3.5. Spectre simulation and theoretical calculation plots of (a) the output noise of both SFPR and CGPR bandlimited within 100 Hz , and (b) the output noise of CGPR at different bandwidths.
supply in $0.35 \mu \mathrm{~m}$ CMOS) because of possible large $I_{\text {preamp }}$ span across the whole pixel array. In addition, the obvious penalty of using the circuit in Figure 3.6(a) is large power consumption for above-threshold operation to obtain proper gain via transistor sizing, and in Figure 3.6(b) is the compromised output swing because of the stacking of diode-connected nFETs. To overcome the drawbacks in the previous designs, a compact in-pixel CC-PGA is used, and the design requirements are low power, large output swing and independence of bias current on local photocurrent. The proposed circuit is shown in Figure 3.7.

The closed-loop gain is defined by the capacitance ratio $A_{C C-P G A}=-\mathrm{C}_{\mathrm{in}} / \mathrm{C}_{\text {fbtot }}$, where $\mathrm{C}_{\text {fbtot }}$ is the total feedback capacitance. 2 digits $\mathrm{G}_{1} \mathrm{G}_{0}$ are used for 4-level programmable $A_{C C-P G A}$ control via combinational logic, from $18 \mathrm{~dB}\left(\mathrm{G}_{1} \mathrm{G}_{0}=00\right)$ to $36 \mathrm{~dB}\left(\mathrm{G}_{1} \mathrm{G}_{0}=11\right)$ with a 6 dB step. The capacitance values are given in Figure 3.7. Note that when the capacitors $\mathrm{C}_{\mathrm{fbi}}$ are not connected in the feedback loop, their right plates are connected to $V_{\text {ref }}$. This is adapted from [104] to prevent frequency response distortion at low frequencies where the off-state switch resistance can no longer be assumed to be infinitely large, especially in deep submicron technology. A simplified circuit shown in Figure 3.8(a) is used to explain the effect. Let $\mathrm{R}_{\mathrm{x}}$ denote the finite off-state switch resistance, and assume infinite open-loop gain of the Opamp. The


Figure 3.6. Previous circuit solutions to high in-pixel gain (a) above-threshold common-source amplifier [15] and (b) subthreshold transimpedance amplifier [16].


Figure 3.7. CC-PGA circuit diagram with the pseudo-resistor transistor implementation. The digital control bits and capacitor values are also listed.


Figure 3.8. (a) Simplified circuit for explaining the effect of finite off-state switch resistance; (b) Simulated results using Spectre at $G_{1} G_{0}=11$ with the right plates of $\mathrm{C}_{\mathrm{fbi}}$ connected (green line) or not connected (red line) to $V_{\text {ref }}$ when not used in the feedback.
closed-loop gain is calculated to be:

$$
\frac{V_{\text {out }}}{V_{\text {in }}}=-\frac{C_{1}\left(s+\frac{1}{R_{x} C_{x}}\right)}{C_{2}\left(s+\frac{1}{R_{x}}\left(\frac{1}{C_{x}}+\frac{1}{C_{2}}\right)\right)}
$$

The finite $\mathrm{R}_{\mathrm{x}}$ creates an additional pair of pole and zero. If the pole frequency is larger than the highpass corner frequency $f_{h c}$ determined by $\mathrm{C}_{\mathrm{fbtot}}$ and $\mathrm{R}_{\mathrm{fb}}$ in Figure $3.7, A_{C C-P G A}$ starts to degrade at a frequency higher than $f_{h c}$. By connecting the right plates of $\mathrm{C}_{\mathrm{fbi}}$ to $V_{r e f}, \mathrm{C}_{\mathrm{fbi}}$ is completely avoided in the feedback loop, and therefore the parasitic pole/zero pair is avoided. Figure 3.8 shows the simulated frequency response in both cases: the red curve represents $\mathrm{C}_{\mathrm{fbi}}$ not connected to $V_{\text {ref }}$, and the green curve represents $\mathrm{C}_{\mathrm{fbi}}$
connected to $V_{\text {ref. }}$. The highpass corner frequency is shifted from 942 mHz to 165 mHz , about 6 times improved. The sufficiently low $f_{h c}=165 \mathrm{mHz}$ for not filtering out low-frequency visual signals is ensured by using the pseudo-resistor $\mathrm{R}_{\mathrm{fb}}$ as depicted in Figure 3.7. It is composed of two off-state pFETs with the bulk connected to source [105]. The only conducting channel is formed by the back-to-back drain-bulk PN junctions whose leakage current provides the DC feedback.

The main challenge lies in the Opamp design. For the CC-PGA, the output swing needs to be maximized for wide input dynamic range, because unlike ADM it does not have any reset mechanism to pull its output towards the resetting level VDD/2 and therefore can be saturated by large input transient. This precludes the use of simple 5T one-stage amplifiers or the telescopic topology. Folded cascode with moderate output swing is not considered either for its four bias voltages needed which complicates the pixel array wiring. Another reason for not using one-stage amplifiers is the insufficient open-loop gain. Assume the open-loop gain of the Opamp in Figure 3.8(a) is $A_{\text {open }}$, and ignore $\mathrm{C}_{\mathrm{x}}$ and $\mathrm{R}_{\mathrm{x}}$. The actual closed-loop gain $A_{C C-P G A}^{\prime}$ is calculated as:

$$
A_{C C-P G A}^{\prime}=\frac{V_{\text {out }}}{V_{\text {in }}}=-\frac{C_{1}}{C_{2}} \frac{1}{\frac{\frac{C_{1}}{C_{2}}+1}{A_{\text {open }}}+1}=-A_{C C-P G A} \frac{1}{\frac{A_{C C-P G A}+1}{A_{\text {open }}}+1}
$$

At $\mathrm{G}_{1} \mathrm{G}_{0}=36 \mathrm{~dB}$, i.e. the desired $A_{C C-P G A}=64$, with $A_{\text {open }}=10^{2}, A_{C C-P G A}^{\prime}$ is only 38.8 ; with $A_{\text {open }}=10^{3}, A_{C C-P G A}^{\prime}$ is about 60.1; with $A_{\text {open }}=10^{4}, A_{C C-P G A}^{\prime}$ is about 63.6. Large $A_{\text {open }}$ also helps desensitize the variation of $A_{C C-P G A}^{\prime}$ to the variation of $A_{\text {open }}$. For example, given a $\pm 10 \%$ variation at $A_{\text {open }}=10^{3}, A_{C C-P G A}^{\prime}$ varies about $1.2 \%$; give a $\pm 40 \%$ variation at $A_{\text {open }}=10^{4}, A_{C C-P G A}^{\prime}$ varies about $0.6 \%$. Even though the variation at $A_{\text {open }}=10^{4}$ is about 4 times larger than that at $A_{\text {open }}=10^{3}$, the variation of $A_{C C-P G A}^{\prime}$ is only half. It is difficult to achieve a design goal of $A_{\text {open }}=10^{4}$ in a single-stage amplifier in $0.18 \mu \mathrm{~m}$ CMOS. The next reasonable option is the symmetrical amplifier which is virtually a single-stage amplifier but with additional gain that can be obtained from its inherent current-mirror structure. Although it offers a large output swing, this topology has twice more noise than a single stage amplifier [106], not mention that it is area- and powerconsuming if large gain is to be obtained from the transistor ratio of the current mirrors. Now the best candidate seems to be the Miller-compensated two-stage amplifier. It has a large output swing, almost the same noise as a single-stage amplifier thanks to the large gain of the first stage, and needs only 2 more transistors. However, as suggested by its name, it needs a compensation capacitor connected between the outputs of the first and second stages, which is usually fine for an Opamp with a given bandwidth. In DVS, on the other hand, the Opamp bandwidth is required to be widely tunable by changing the bias to define the pixel front-end bandwidth, which can result in insufficient phase margin of the Opamp because the right-half plane zero caused by Miller compensation can only be well cancelled at a particular bias current using a nulling resistor. The nulling resistor would have unacceptably large size given small bias current of the Opamp. It is possible to use a pFET operating in linear region as the nulling resistor whose resistance tracks the bias of the Opamp [26]; the cost is obviously the extra area for the resistancetracking circuits with at least 4 additional transistors.

The schematic of the Opamp used in the design is shown in Figure 3.9(a). The "Miller-like" cascode compensation is based on the technique proposed in [107] back in 1984. The compensation capacitor is connected between the sources of the cascode transistors $\left(\mathrm{M}_{1 \mathrm{~b} 2}\right.$ and $\left.\mathrm{M}_{3 \mathrm{~b} 4}\right)$ in the first stage and the output of the second stage. This configuration helps push the right-half plane zero to a much higher frequency compared to the original Miller compensation, and thus no nulling resistor is needed. Nevertheless, the


Figure 3.9. (a) Schematic of the Opamp used in the CC-PGA with pseudo-cascode compensation; (b) transistor channel length splitting, and the sweep of the $V_{g s}$ of an nFET as the function of its $L$ and $W$.
value of the compensation capacitance $\mathrm{C}_{\mathrm{c}}$ should be carefully chosen to avoid insufficient gain margin caused by gain peaking beyond the GBW as pointed out in [107]. Note that the cascode transistors $\mathrm{M}_{\mathrm{xby}}(\mathrm{x}$ is the transistor index, y is the number of unit transistors connected in parallel) is not an ordinary cascode transistor, because its gate is not connected to an external fixed bias voltage but the gate of the commonsource transistors $\mathrm{M}_{\mathrm{xaz}}$ (z is the number of unit transistors connected in series). The compensation in this Opamp topology is called pseudo-cascode compensation [108]. Compared to the traditional cascode compensation, besides the benefits such as lower VDD, and higher unit-gain frequency [108], pseudo-cascode compensation can save two extra biases and thus simplify the wiring in a pixel array. Usually $\mathrm{M}_{\mathrm{xaz}}$ is in deep triode region due to its small $\mathrm{V}_{\mathrm{ds}}$, and yet still relax the value of $\mathrm{C}_{\mathrm{c}}$ to a large extent as demonstrated in [109]. To increase the open-loop gain, body-biasing of $\mathrm{M}_{\mathrm{xby}}$ was used [108] to lower its $\mathrm{V}_{\text {th }}$ so that $\mathrm{M}_{\mathrm{xaz}}$ can move towards the edge of saturation. Not only is this impossible for using area-consuming deep n well in pixels, but also two extra wires for biasing voltages are needed. To circumvent the aforementioned problems, two small-size effects of MOSFETs are exploited in a split-transistor technique in subthreshold [110].

The schematic in Figure 3.9(b) shows the concept of the split-transistor technique. $n$ FET $\mathrm{M}_{\mathrm{ns}}$ is divided into two transistors $\mathrm{M}_{\mathrm{sa}}$ and $\mathrm{M}_{\mathrm{sb}}$ with equivalent channel length $L=L_{a}+L_{b} . \mathrm{M}_{\mathrm{sa}}$ and $\mathrm{M}_{\mathrm{sb}}$ form the pseu-do-cascode structure. In subthreshold, as long as the $V_{d s}$ of $\mathrm{M}_{\mathrm{sa}}$ is larger than 100 mV , it is considered to be in saturation and the output impedance is large. In contrast, to stay in saturation in above threshold, $V_{d s}$ usually needs to be larger than the overdrive voltage $\mathrm{V}_{\mathrm{ov}}>200 \mathrm{mV}$. Sizing $\mathrm{M}_{\mathrm{sa}}$ with a very small $W / L$ ratio and $\mathrm{M}_{\mathrm{sb}}$ with a very large $W / L$ ratio can also push $\mathrm{M}_{\mathrm{sa}}$ towards saturation, but it is as impractical as the body-biasing means with a restricted pixel area. Note that the transistor threshold voltage $V_{t h}$ varies with $L$ and $W$. The well-known short-channel effect (SCE) [111] indicates that in sub-micrometer processes $V_{t h}$ decreases with $L$ due to effects like the drain-induced barrier lowering (DIBL). To reduce the resulting increased leakage current at small $L$, the so-called halo implantation was devised to mitigate channel de-
pletion, and hence $V_{t h}$ may decrease slower or even increases within some range of $L$. This phenomenon is termed as reverse SCE (RSCE) [111]. Modern processes normally use shallow-trench-isolation (STI) instead of local oxidation of silicon (LOCOS) for device isolation to allow a denser integration. In STI, $V_{t h}$ decreases with decreasing $W$ due to an increased electrostatic potential near the channel edge. The phenomenon is termed as inverse narrow-width effect (INWE), in contrast to NWE observed in LOCOS [112]. To verify these effects by simulation in UMC $0.18 \mu \mathrm{~m}$ CMOS, the $V_{g s}$ of an nFET is plotted over its $L$ and $W$ at 1 nA bias current as shown in Figure $3.9(\mathrm{~b}) . V_{d s}$ is fixed at 0.9 V because $V_{t h}$ is also dependent on $V_{d s}$ [111]. It can be seen that $V_{g s}$ does not monotonically decrease as the $W / L$ ratio increases. At small $L$ and $W$, the small size effects play an important role: With fixed $W(L), V_{g s}$ first increases with decreasing $L(W)$ and then drops down rapidly after a certain point, indicating RSCE (INWE). The $V_{g s}$ sweep of PMOS shows the RSCE and the INWE as well, but they are less evident. If the sizes of $\mathrm{M}_{\mathrm{sa}}$ and $\mathrm{M}_{\mathrm{sb}}$ are chosen according to Figure 3.9(b) such that the $V_{g s}$ of $\mathrm{M}_{\mathrm{sa}}$ is at least 100 mV larger than that of $\mathrm{M}_{\mathrm{sb}}$, then $\mathrm{M}_{\mathrm{sa}}$ has its $V_{d s}>100 \mathrm{mV}$ and is considered to be in saturation. The size of $\mathrm{M}_{\mathrm{sa}}$ and $\mathrm{M}_{\mathrm{sb}}$ in this design are $(W / L)_{\mathrm{sa}}=0.48 / 0.36$ and $(W / L)_{\mathrm{sb}}=0.24 / 0.18$ in $\mu \mathrm{m}$ corresponding to the maximum $(116 \mathrm{mV})$ and minimum ( 235 mV ) $V_{g s}$ in Figure $3.9(\mathrm{~b})$. The size of $\mathrm{M}_{\mathrm{ns}}$ is $(W / L)_{\mathrm{ns}}=0.40 / 0.54$ in $\mu \mathrm{m}$, which has the same area as the sum of $\mathrm{M}_{\mathrm{sa}}$ and $\mathrm{M}_{\mathrm{sb}}$. Theoretical analysis of the gain ratio of the split transistor $A_{s}$ over the non-split transistor $A_{n s}$ is given as follows.

The ratio $A_{s} / A_{n s}$ can be written as:

$$
\frac{A_{s}}{A_{n s}}=\frac{g_{m s a} r_{\text {osa }} g_{m s b} r_{\text {osb }}}{g_{\text {mns }} r_{\text {ons }}}
$$

where $g_{m i}{ }^{\prime} \mathrm{s}$ and $r_{o i}{ }^{\prime} \mathrm{s}$ are the transconductance and output resistance of transistor $\mathrm{M}_{\mathrm{i}}(\mathrm{i}=\mathrm{sa}, \mathrm{sb}, \mathrm{ns})$ respectively. In subthreshold, the transconductance is $\kappa I / V_{T}$, but the output conductance can have different formulizations depending on the physical mechanisms considered. Let us first derive the output resistance with DIBL considered $r_{o D I B L}$. The $I_{d s}$ in subthreshold can be written as:

$$
\begin{equation*}
I_{d s}=\frac{W}{L} I_{0} e^{k\left(V_{s s}-V_{\text {hop }}+\Delta V_{\text {hbDBLL }}\right) / V_{T}}\left(1-e^{-V_{d s} / V_{T}}\right) \tag{3.7}
\end{equation*}
$$

where $I_{0}$ is a lumped parameter determined by unit gate capacitance, carrier mobility and subthreshold slope factor, $V_{t h 0}$ is the intrinsic transistor threshold voltage, and $\Delta V_{t h D I B L}$ is the threshold voltage shift due to DIBL. If in saturation, $I_{d s}$ can be simplified as:

$$
\begin{equation*}
I_{d s}=\frac{W}{L} I_{0} e^{\kappa\left(V_{g s}-V_{\langle h 0}+\Delta V_{l D D B L L}\right) / V_{T}} \tag{3.8}
\end{equation*}
$$

A typical expression of $\Delta V_{t h D I B L}$ is [98]:

$$
\Delta V_{t h D I B L}=\left[3\left(\phi_{b i}-\phi_{0}\right)+V_{d s}\right] e^{-L / /_{0}}
$$

where $\phi_{b i}$ is the drain/source-bulk pn junction built-in potential, $\phi_{0}$ is the surface potential of two-terminal MOS structure in strong inversion, and $l_{0}$ is the characteristic length or scale length [98], [113]. $r_{o D I B L}$ is calculated as:

$$
\begin{equation*}
r_{o D I B L}=1 / \frac{\partial I_{d s}}{\partial V_{d s}}=\frac{V_{T}}{\kappa I_{d s}} e^{\frac{L}{l_{0}}} \tag{3.9}
\end{equation*}
$$

The scale length $l_{0}$ can be written as:

$$
l_{0}=2 \sqrt{\frac{\varepsilon_{\mathrm{si}} t_{\mathrm{ox}}}{\varepsilon_{\mathrm{ox}}} W_{d m}}
$$

where $W_{d m}$ is the maximum depletion depth under gate, $\varepsilon_{s i}$ and $\varepsilon_{o x}$ are the permittivity of silicon and silicon dioxide respectively, and $t_{o x}$ is the thickness of the gate dioxide. For a uniformly doped channel, $W_{d m}$ can be calculated as:

$$
W_{d m}=\sqrt{\frac{2 \varepsilon_{s i}\left(2 V_{T} \ln \frac{N_{a}}{n_{i}}-V_{b s}\right)}{q N_{a}}}
$$

where $N_{a}$ is the channel doping concentration and $n_{i}$ is the intrinsic carrier concentration of silicon. For $V_{b s}=-0.1 \mathrm{~V}, W_{d m}$ is about 40.63 nm , and $l_{0}$ is about 45.22 nm . If the $r_{o i}$ of all the nFETs follow the exponential dependence on $L$ as in Eq. (3.9), $A_{s} / A_{n s}$ is calculated as:

$$
\frac{A_{s}}{A_{n s}}=\frac{\kappa I_{d s}}{V_{T}} \frac{\frac{V_{T}}{I_{d s}} e^{\frac{L_{s a}}{l_{0}}} \frac{V_{T}}{\kappa I_{d s}} e^{\frac{L_{s b}}{l_{0}}}}{\frac{V_{T}}{I_{d s}} e^{\frac{L_{n s}}{l_{0}}}}=1=0 \mathrm{~dB}
$$

As we will learn later, the resulting ratio 0 dB is not consistent with the simulation and testing results where $A_{s} / A_{n s}$ often shows to be much larger than 1 . This is probably due to the fact that the halo implantation suppresses the DIBL and modifies the dependence of $r_{o}$ on $L$ in light of the drain-induced-thresholdshift (DITS) effect [114]. Although the DITS is lately attributed to the difference between the source and drain barriers [115], we still use the widely accepted phenomenological long-channel DIBL model provided in BSIM where the $V_{t h}$ shift is written as [116]:

$$
\Delta V_{t h D I T S}=\frac{V_{T}}{\kappa} \ln \left[\frac{L}{L+D V T P 0 \cdot\left(1+e^{-D V T P 1 \cdot V_{d c s}}\right)}\right]
$$

where $D V T P 0$ and $D V T P 1$ are the DITS coefficients. $D V T P 0$ is only related to process parameters and DVTP1 depends on $L$ via the equation below [114]:

$$
D V T P 1=\frac{e^{-L_{p} / l_{0}}+2 e^{-2 L_{p} / l_{0}}}{V_{T}}
$$

where $L_{p}$ is the pocket length of halo doping that can be estimated with $L$ and the channel and halo doping concentration $N_{c h}$ and $N_{p}$ by [116]:

$$
L_{p}=\frac{N_{c h}}{N_{p}-N_{c h}} \frac{N l x+l N l x / L}{2}
$$

where $N l x$ is the lateral non-uniform doping parameter, and $l N l x$ is its corresponding binning parameter.
Similar to Eq. (3.9), the output resistance considering DITS can be derived as:

$$
\begin{equation*}
r_{o D I T S}=1 / \frac{\partial I_{d s}}{\partial V_{d s}}=\frac{\kappa L}{I_{d s}} \frac{e^{D V T P 1 \cdot V_{d s}}}{D V T P 0 \cdot D V T P 1}=\frac{V_{E}}{I_{d s}} \tag{3.10}
\end{equation*}
$$

where $V_{E}$ can be seen as the Early voltage and is proportional to $L$. Because $\mathrm{M}_{\mathrm{sb}}$ is clearly affected by the DIBL with its $V_{g s}$ at the nadir of the sweep in Figure 3.9(b), its $r_{o}$ still uses Eq. (3.9), and Eq. (3.10) ap-
plies to $\mathrm{M}_{\mathrm{sa}}$ and $\mathrm{M}_{\mathrm{ns}}$. Note that the $L$ in the equations above should be in fact the effective length $L_{\text {eff }}$ which can be calculated from the nominal drawn length $L_{\text {drawn }}$ as:

$$
L_{e f f}=L_{\text {drawn }}-2 d L
$$

where $d L$ accounts for the depletion length in channel due to heavily-doped drain/source. Herein, $A_{s} / A_{n s}$ can be again calculated as:

$$
\begin{equation*}
\frac{A_{s}}{A_{n s}}=\frac{\kappa I_{d s}}{V_{T}} \frac{\frac{\kappa L_{e f f s a}}{I_{d s}} \frac{e^{D V T P_{s s c} \cdot V_{d s s a}}}{D V T P 0_{s a} \cdot D V T P 1_{s a}} \frac{V_{T}}{\kappa I_{d s}} e^{\frac{L_{e f f b b}}{l_{0}}}}{\frac{\kappa L_{e f f i s}}{I_{d s}} \frac{e^{D V T P P_{n s} \cdot V_{d s s s s}}}{D V T P 0_{n s} \cdot D V T P 1_{n s}}}=\frac{L_{e f f s a} D V T P 1_{n s}}{L_{e f f n s} D V T P 1_{s a}} e^{D V T P P_{s a} \cdot V_{d s a s}-D V T P P_{n s} \cdot V_{d s s s}} e^{\frac{L_{e f f f b}}{l_{0}}}=3.96=12 \mathrm{~dB} \tag{3.11}
\end{equation*}
$$

$\mathrm{M}_{\mathrm{sa}}$ could be in linear region with its $V_{d s}<100 \mathrm{mV}$ due to variation of $V_{t h}$ in both $\mathrm{M}_{\mathrm{sa}}$ and $\mathrm{M}_{\mathrm{sb}}$. In this case, using Eq. (3.7), the output resistance of $\mathrm{M}_{\mathrm{sa}}$ is derived as:

$$
r_{o s a}=\frac{V_{T}}{I_{d s}} e^{\frac{V_{\text {las }}-V_{\text {lbb }}}{V_{T}}}
$$

With $r_{o s b}$ using Eq. (3.9) and $r_{o n s}$ using Eq. (3.10), $A_{s} / A_{n s}$ is calculated as:

$$
\begin{equation*}
\frac{A_{s}}{A_{n s}}=\frac{\kappa I_{d s}}{V_{T}} \frac{\frac{V_{T}}{I_{d s}} e^{\frac{V_{l a s}-V_{l u b}}{V_{T}}} \frac{V_{T}}{\kappa I_{d s}} e^{\frac{L_{\text {effib }}}{l_{0}}}}{\frac{V_{E}}{I_{d s}}}=\frac{V_{T}}{V_{E}} e^{\frac{V_{\text {las }}-V_{l u b}}{V_{T}}} e^{\frac{L_{e f f i b}}{l_{0}}} \tag{3.12}
\end{equation*}
$$

$V_{E}$ is simulated to be about 1.76 V for $\mathrm{M}_{\mathrm{ns}}$ biased at 10 nA . From the equation above, it is possible to have a gain ratio less than 1 if $V_{\text {tha }}-V_{t h b}<31.9 \mathrm{mV}$, which can happen due to process variation. For more robust gain increase, more $\mathrm{M}_{\mathrm{sa}}$ 's are connected in series and more $\mathrm{M}_{\mathrm{sb}}$ 's are connected in parallel, as in the case of the Opamp design in Figure 3.9(a).

The noise of the CC-PGA has a direct tradeoff with its closed-loop gain $A_{c}$. Considering only thermal noise, the input-referred noise can be approximated by [117]:

$$
V_{n, i r}=\sqrt{\frac{8 k T}{3 C_{c} A_{c}}}
$$

With $\mathrm{C}_{\mathrm{c}}=32.8 \mathrm{fF}, V_{n, i r}=73 \mu \mathrm{~V}$ and $202 \mu \mathrm{~V}$ when $A_{c}=36 \mathrm{~dB}$ and 18 dB respectively. If a $50 \mu \mathrm{~V}$ noise is contributed by the CGPR, a 6 dB SNR can still be obtained for a single pixel at $A_{c}=36 \mathrm{~dB}$ with a $1 \% \mathrm{TC}$ signal. Limiting the bandwidth by either lowering the bias current of the CC-PGA or using larger $\mathrm{C}_{\mathrm{c}}$ could further improve SNR, but they are limited by voltage headroom and pixel area, respectively. To minimize the $1 / f$ noise contribution from the Opamp, the nFET in the first stage are designed to occupy the most area of the CC-PGA.

Instantaneous signal dynamic range is limited by the output range of the CC-PGA. Let us assume that the output range is $0.2 \mathrm{~V}-1.6 \mathrm{~V}$ without severe output signal distortion or clipping. At $A_{c}=18 \mathrm{~dB}$, the input range of the CC-PGA is about 170 mV which allows 57 dB instantaneous $i_{p h}$ change. At $A_{c}=36 \mathrm{~dB}$, the input range is about 22 mV , which allows only 7.4 dB instantaneous $i_{p h}$ change. In terms of signal integrity, higher gain is at the cost of smaller allowable dynamic range of a dynamic visual scene. However, in


Figure 3.10. (a) Schematic of the ADM with illustration of the pseudo-resistor; (b) Timing diagram of the switch control signals $\varphi_{s}, \varphi_{l}$ and $\varphi_{h}$, and the $V_{o u t}$ waveform.
the long run, the CC-PGA output will eventually adapt to its DC level of 0.9 V through $\mathrm{R}_{\mathrm{fb}}$, and is independent of $I_{P H}$, therefore the intrascene dynamic range is only limited by the CGPR, in contrast to [16] where it is limited by the transimpedance preamplifier to about only 60 dB with a 3.3 V supply.

### 3.2.3 Asynchronous Delta Modulator (ADM)

Area constraint is the main challenge for the design of an in-pixel ADM. Previous implementations of ADMs as level-crossing ADCs often require a large area of resistive or capacitive feedback DACs [61], [65] which determines the resolution of the ADCs. The 1-bit capacitive DAC used in [118] can potentially reduce the area, but the switching of the control signals is complicated and because of the capacitive division, the input signal is attenuated after the DAC. With 36 dB gain from the CC-PGA, the $1 \% \mathrm{TC}$ signal is amplified to about 16 mV which is still too small for comparison considering the output DC variation of the ADM amplifier and the input offset of the comparators. Additional gain from the ADM is hence necessary.

The idea of realizing the $\delta$ subtraction feedback in ADM was first suggested in [70], but instead of using a full DAC for feedback, a novel asynchronous $\delta$-subtraction switched-capacitor circuit is proposed here for a more area-efficient implementation. The circuit diagram is shown in Figure 3.10(a). The
closed-loop gain of the ADM is $\mathrm{C}_{\mathrm{in}} / \mathrm{C}_{\mathrm{fb}}=24 \mathrm{~dB}$, and the Opamp that has a similar topology as in Figure 3.9(a) is optimized for slew rate instead of noise. The amplified $1 \%$ TC after the ADM is about 260 mV , and is detectable by comparators as long as $V_{\text {refh }}<1.16 \mathrm{~V}$ and $V_{\text {refl }}>0.64 \mathrm{~V}$ with $V_{\text {ref }}=0.9 \mathrm{~V}$. The control signals $\varphi_{x}(\mathrm{x}=\mathrm{s}, 1, \mathrm{~h})$ of switches $\mathrm{S}_{\mathrm{i}}(\mathrm{i}=0,1,2)$ are generated by the in-pixel asynchronous logic (IPAsyncL). The switching sequence for ON spikes is described as follows: (1) When $V_{o u t}$ exceeds $V_{\text {refh }}, \varphi_{h}$ becomes high, and $\mathrm{S}_{2}$ is connected. The top plate of $\mathrm{C}_{\text {rst }}$ is charged to $V_{\text {refh }}$. The row request $n R_{\text {req }}$ (active low) is communicated to the periphery AER; (2) after a four-phase AER handshake to transmit the row and column addresses of this pixel, the column acknowledge $n C_{a c k}$ (active low) is activated leading to $\varphi_{h}$ switching low and $\varphi_{s}$ switching high, and therefore disconnecting $\mathrm{S}_{2}$ and connecting $\mathrm{S}_{0} . V_{\text {out }}$ is reset towards $V_{\text {ref }}$, (3) after a certain reset time controlled by the IPAsyncL, $\varphi_{s}$ switches low and $\mathrm{S}_{0}$ is disconnected. The ADM is ready for the next communication cycle. For OFF spikes, the sequence is similar except that the top plate of $\mathrm{C}_{\text {rst }}$ is charged to $V_{\text {refl }}$ first by connecting $\mathrm{S}_{1}$ via a high $\varphi_{l}$. The switching sequence along with the $V_{\text {out }}$ waveform is illustrated in Figure 3.10(b). Comparing with the ADM model given in Chapter 2, the integration is done by ON/OFF charging on $\mathrm{C}_{\text {rst }}$ via $\mathrm{S}_{2} / \mathrm{S}_{1}$ connected to $V_{\text {refh }} / V_{\text {reff }}$, and the subtraction is done by connecting $\mathrm{S}_{0}$ so that the charge on $\mathrm{C}_{\text {rst }}$ is redistributed. Mathematically, the subtraction process can be explained as follows. For an ON spike, at time $t_{2}$, the voltage at the top plate of $\mathrm{C}_{\text {rst }} V_{x}$ is charged to $V_{\text {refh }}$, and before the end of $t_{3}, V_{x}$ is shorted to $V_{f b}$ which is approximately the same as $V_{\text {ref }}$ due to virtual ground. In light of charge conservation, the total charge on $\mathrm{C}_{\mathrm{in}}, \mathrm{C}_{\mathrm{fb}}$ and $\mathrm{C}_{\mathrm{rst}}$ at $t_{2}$ equals to that at $t_{3}$ :

$$
\begin{aligned}
& \left(V_{\text {in }}\left(t_{2}\right)-V_{f b}\left(t_{2}\right)\right) \times C_{\text {in }}+\left(V_{\text {out }}\left(t_{2}\right)-V_{f b}\left(t_{2}\right)\right) \times C_{f b}+\left(V_{\text {ref }}\left(t_{2}\right)-V_{x}\left(t_{2}\right)\right) \times C_{r s t} \\
& =\left(V_{\text {in }}\left(t_{3}\right)-V_{f b}\left(t_{3}\right)\right) \times C_{\text {in }}+\left(V_{\text {out }}\left(t_{3}\right)-V_{f b}\left(t_{3}\right)\right) \times C_{f b}+\left(V_{r e f}\left(t_{3}\right)-V_{x}\left(t_{3}\right)\right) \times C_{r s t}
\end{aligned}
$$

The equalities below hold:

$$
V_{f b}\left(t_{2}\right)=V_{f b}\left(t_{3}\right), \quad V_{r e f}\left(t_{2}\right)=V_{r e f}\left(t_{3}\right), \quad V_{x}\left(t_{2}\right)=V_{x}\left(t_{3}\right)+\delta
$$

where $\delta=V_{\text {refh }}-V_{\text {ref }}=V_{\text {ref }}-V_{\text {reff }}$. The charge conservation equation can then be simplified as:

$$
V_{\text {in }}\left(t_{2}\right) C_{\text {in }}+V_{\text {out }}\left(t_{2}\right) C_{f b}-\delta C_{r s t}=V_{\text {in }}\left(t_{3}\right) C_{\text {in }}+V_{\text {out }}\left(t_{3}\right) C_{f b}
$$

The output change $\Delta V_{\text {out }}=V_{\text {out }}\left(t_{3}\right)-V_{\text {out }}\left(t_{2}\right)$ can be written as:

$$
\begin{equation*}
\Delta V_{\text {out }}=-\Delta V_{i n} \frac{C_{i n}}{C_{f b}}-\delta \frac{C_{r s t}}{C_{f b}} \tag{3.13}
\end{equation*}
$$

where $\Delta V_{\text {in }}=V_{i n}\left(t_{3}\right)-V_{\text {in }}\left(t_{2}\right)$. It is clear from Eq. (3.13) that the subtraction does not interfere with the input change amplification, in contrast to STR where the input is blocked from the output during reset.
$\mathrm{S}_{1}$ and $\mathrm{S}_{2}$ are connected to low impedance voltage sources $V_{\text {reft }}$ and $V_{\text {reft }}$ respectively, and hence clock feedthrough usually has a more detrimental effect than charge injection on the precision of the sampled voltage at $V_{x}$. If both $\mathrm{S}_{1}$ and $\mathrm{S}_{2}$ use nFETs or pFETs , after being disconnected, $V_{x}$ would be $V_{\text {reft }}+\Delta V_{x}$ for OFF spikes and $V_{\text {refh }}+\Delta V_{x}$ for ON spikes. $\Delta V_{x}$ has a negative/positive sign if $n F E T s / p F E T s$ are used. The nominal subtracted voltages for OFF and ON spikes now become $-\left|\delta-\Delta V_{x}\right|$ and $\delta+\Delta V_{x}$, respectively. Because the reset capacitance $C_{r s t}$ is quite small compared to the parasitic $C_{g s} / C_{g d}, \Delta V_{x}$ could be as large as tens of mV . To reduce the subtraction imbalance, a pFET is used for $\mathrm{S}_{1}$, and an nFET for $\mathrm{S}_{2}$. The nFET and pFET are sized so that the $\Delta V_{x}^{\prime} \mathrm{s}$ are approximately the same. A complementary transmission gate is used for $\mathrm{S}_{0}$ to minimize the switching effects. Note that the reset capacitance is in effect $C_{r s t}$ plus the parasitic $C_{d b} / C_{s b}$ of all the switches connected to $C_{r s t}$. Therefore even though the clock feedthrough makes the


Figure 3.11. Representative transistor-level implementation of STR [6]. It comprises a differencing amplifier and two 2T current-mode comparators.
nominal subtracted voltage smaller than $\delta$ and $C_{r s t}$ is 1 fF smaller than the feedback capacitance $C_{f b}$, the overall actual subtracted voltage at the amplifier output $V_{\text {out }}$ is about $\delta$ in post-layout simulation. In retrospect, for signal reconstruction from spike trains, the subtraction imbalance does not necessarily have to be dealt with in encoding as long as it is taken into account in decoding by using different incremental values for ON and OFF spikes. For spike processing, the imbalance could also be absorbed in hardware design, especially if machine learning algorithms are adopted for classification tasks.

The pseudo-resistor $\mathrm{R}_{\mathrm{fb}}$ is used to establish the DC operating point of the Opamp, however it cannot employ the off-state pFETs in series as used in the CC-PGA because the switch $\mathrm{S}_{0}$ implemented by a complementary transmission gate has considerable leakage compared to the drain-bulk PN junction leakage of off-state pFETs, which can cause DC feedback to fail. On the other hand, the traditional pseudoresistor of two diode-connected pFETs in series [119] cannot provide sufficiently large resistance for a sub-Hz highpass corner frequency in $0.18 \mu \mathrm{~m}$ CMOS with $\mathrm{C}_{\mathrm{fb}}$ only about 7 fF . The proposed pseudoresistor shown in Figure 3.10(a) is composed of two diode-connected pFETs with bulk connected to VDD. This topology supplies both PN junction leakage and channel leakage currents. The connection of the bulk to VDD instead of the source not only prevents forward conduction of PN junction at large output swing and in turn distortion, but also increases the effective resistance. The simulated highpass corner frequency is $<0.25 \mathrm{~Hz}$ under room temperature. One benefit of continuous-time feedback via $\mathrm{R}_{\mathrm{fb}}$ instead of using reset switch to refresh the output DC level as is done in STR is the avoidance of background ON spikes that are not correlated to input stimulus. A representative transistor implementation of STR is shown in Figure 3.11. The reset switch $\mathrm{M}_{\mathrm{S}}$ connects the gate and drain of the input transistor $\mathrm{M}_{\mathrm{IN}}$ of the differencing amplifier every time after either the ON or OFF threshold set by the ON or OFF comparators is crossed. When $\mathrm{M}_{\mathrm{s}}$ is off, even with no input signal present, ON spikes can still occur due to the charging of $V_{x}$ towards VDD via the bulk-source junction leakage.

The equally spaced thresholds of the two 2T current-mode ON and OFF comparators in Figure 3.11 are derived as:

(a)

(b)

Figure 3.12. (a) Circuit diagram of the IPAsyncL and (b) timing diagram of the important signals involved in an ON spike transmission.

$$
V_{\theta O N}=\frac{V_{T}}{\kappa} \ln \frac{I_{O N}}{I_{A m p}}, \quad V_{\theta O F F}=\frac{V_{T}}{\kappa} \ln \frac{I_{A m p}}{I_{O F F}}
$$

To have an empirically minimum 50 mV threshold, the current ratio $I_{O N} / I_{O F F}$ is about 22 . The largely different biases of the ON and OFF comparators results in the speed gap, i.e. the OFF comparator has a much larger delay than the ON comparator, and hence severely limits the SDR of the encoded spike train, as analyzed in Chapter 2. To circumvent this problem, a two-stage uncompensated amplifier is used as the comparator whose threshold is directly set by a reference voltage and is independent of its bias current.

### 3.2.4 In-Pixel Asynchronous Logic (IPAsyncL)

The IPAsyncL communicates with the peripheral AER and generates the switch control signals $\varphi_{h}, \varphi_{l}$, and $\varphi_{s}$ for ADM. The circuit diagram is shown in Figure 3.12(a), and the timing diagram of the main signals for an ON spike is depicted in Figure 3.12(b). The IPAsyncL can be divided into three parts:


Figure 3.13. $2 \times 2$ pixel layout arranged in common centroid and the chip microphotograph.
(1). The comparison latch part is used to latch the threshold-crossing events detected by the two comparators in the ADM. In previous DVS designs, the active output of the comparators was directly used to trigger the row request $n R_{\text {req }}$. This is problematic because the comparator output can become inactive due to noise or digital coupling before the column request is sent for off-chip registration of the pixel address. Row-only spikes were often observed in prior DVSs which indicates failure of spike transmission due to missing column addresses. In the present design, two SR latches lock the positive state of thresholdcrossing for ON and OFF spikes respectively. The latch output ONo or OFFo stays active until $\varphi_{s}$ gets high which means the charge redistribution starts in the ADM; ONo and OFFo cannot get high until $\varphi_{s}$ gets low which means the charge redistribution is finished. As depicted in Figure 3.12(b), $V_{O N}$ goes low before $n C_{\text {reqon }}$ is sent, but ONo still stays high to finish the communication cycle.
(2). The communication logic is designed to be compatible with the peripheral AER circuits that implement the burst-mode word-serial spike transmission [83], [84]. Once either ONo or OFFo is active, after a small delay controlled by the rising edge delay element Delay1, the row request $n R_{\text {req }}$ gets low. After the row acknowledge $R_{\text {ack }}$ becomes high in response to $n R_{\text {req }}$, the column request either $n C_{\text {reqon }}$ or $n C_{\text {reqoff }}$ gets low. The column acknowledge $n C_{a c k}$ becomes low in response to active $n C_{\text {reqon }} / n C_{\text {reqoff }}$, and it becomes high after a certain time window (about $\sim 30 \mathrm{~ns}$ minimum in current design in $0.18 \mu \mathrm{~m}$ CMOS at 1.8 V ) controlled by the off-chip CPLD/FPGA. $R_{\text {ack }}$ only becomes low again after all the active pixels in this row have transmitted their column addresses. Delay1 is to ensure sufficient pulse width of $\varphi_{h}$ or $\varphi_{l}$ for charging the $\mathrm{C}_{\mathrm{rst}}$ in ADM.
(3). The three switch control signals $\varphi_{h}, \varphi_{l}$, and $\varphi_{s}$ for ADM are generated according to the timing of the handshake communication described above. $\varphi_{h}$ or $\varphi_{l}$ gets high in response to high ONo or OFFo, and gets low once $n C_{a c k}$ becomes active low. $\varphi_{s}$ only goes high if both $R_{a c k}$ and $n C_{a c k}$ are active. Its pulse width is controlled by the rising edge delay element Delay2. The SR latch associated with $\varphi_{s}$ is to prevent $\varphi_{s}$ goes low prematurely in response to $R_{\text {ack }}$ going low. However, the delay from Delay 2 cannot be too long,


Figure 3.14. (a) Measured DC-gain ratio $A_{s} / A_{n s}$ in dB of split to non-split nFETs over 10 samples at different bias currents, along with Monte Carlo simulation results; (b) Measured open-loop DC-gain distribution of the Opamp in Figure 3.9(a) over 10 samples, along with Monte Carlo simulation results.
otherwise $n C_{a c k}$ from the same column but another row could block $\varphi_{s}$ from going low and in turn block this pixel from requesting again until $\varphi_{s}$ could eventually get low. A better design is to disassociate the pulse width of $\varphi_{s}$ from the rising edge of $n C_{a c k}$ by only using simultaneously active $R_{a c k}$ and $n C_{a c k}$ as the trigger to a simple timer which independently determines the $\varphi_{s}$ pulse width.

### 3.3 Experimental Results

### 3.3.1 Pixel Layout and Chip Microphotograph

A $60 \times 30$ vision sensor prototype called ADMDVS was designed and fabricated in UMC $0.18 \mu \mathrm{~m}$ RF/MM CMOS. The pixel layout and the chip microphotograph are shown in Figure 3.13. The $2 \times 2$ pixel layout shows a common centroid arrangement. Each pixel has a pitch size of $31.2 \mu \mathrm{~m}$ and a fill factor of about $10.3 \%$. The $10 \times 10 \mu \mathrm{~m}^{2}$ photodiode is formed by the n -well/p-substrate junction. A metal ring made of M1 to M6 that is connected to ground shields the photodiode from any signal coupling from the other building blocks. To minimize the digital coupling, all request and acknowledge signals are wired using mostly M1 at the outer edge of block 4 and 5 . The analog biases using M3 and M4 are carefully shielded from digital signals using ground or VDD intermediate metal layers. The whole chip including the pads occupies $3.2 \times 1.6 \mathrm{~mm}^{2}$. Besides the X/Y AER and the asynchronous state machine (SM) for spike transmission, the chip includes a digitally programmable proportional-to-absolute-temperature (PTAT) bias generator array (see Chapter 5) [120] to provide all the current biases, a serial-to-parallel interface (SPI) to configure digital bits for pixel gain control, chip power down and debug, a test pixel and test structures for split transistors described in Section 3.2.2. The USB interface, firmware logic, and host side codes in jAER [121] are based on existing designs.

### 3.3.2 Split-Transistor Gain

To validate the split-transistor technique exploiting small-size effects, a compound split nFET composed of two $\mathrm{M}_{\mathrm{sa}}$ in series and two $\mathrm{M}_{\mathrm{sb}}$ in parallel $\left(\mathrm{M}_{\mathrm{sa}} / \mathrm{M}_{\mathrm{sb}}\right.$ has the same size as the one in Figure 3.9(b)) was fabricated along with a normal nFET with $L_{n s}=2 * L_{s a}+L_{s b}=0.9 \mu \mathrm{~m}$ and $W_{n s}=2\left(W_{s a} L_{s a}+W_{s b} L_{s b}\right) / L_{n s}=0.48$


Figure 3.15. (a) Experiment setup for measuring the noise of the ADMDVS array; (b) Measured average noise spike rate $R_{n}$ of the ADMDVS array and calculated average equivalent noise voltage $V_{r m s, n}$ at the output of one CGPR with two different bias current settings of the CC-PGA, 50 pA and 1 nA .
$\mu \mathrm{m}$. The $I_{d s}-V_{d s}$ characteristic curves of the split and non-split nFETs were measured using Keithley 236 over 10 samples from different dies and the DC gains were extrapolated from the slope of the $I_{d s}-V_{d s}$ curves in the saturation region. The calculated gain ratio $\Delta \mathrm{A}=20 \log _{10}\left(A_{s} / A_{n s}\right)$ in dB at different bias currents are plotted in Figure 3.14(a), along with the 500 Monte Carlo simulation trial results using Cadence Spectre. The error bar shows the simulated variance. The measured results give a maximum 16 dB DCgain increases of the split transistor within the $8-64 \mathrm{nA}$ range. The measured samples exhibit a large variation that can be attributed to the small device area. Some samples even have decreased DC gain which was implied by Eq. (3.12) if the $\mathrm{M}_{\mathrm{sa}}$ or compound $\mathrm{M}_{\mathrm{sa}}$ is not in saturation. The simulation generally overestimates the DC gain enhancement, especially at very low biases. Such large discrepancy could be the consequence of unreliable device modeling and inaccurate device parameter extraction. Particularly it is known that accurate $g_{d s}$ modeling of MOSFETs is difficult in deep subthreshold region.

With a 4 nA total current consumption sufficient for 500 Hz bandwidth in-pixel gain amplification at $\mathrm{G}_{1} \mathrm{G}_{0}=11$, the open-loop DC gain of the Opamp in Figure 3.9 (a) was measured over 10 samples by an SR780 network signal analyzer. The distribution of the measured and simulated DC gain is plotted in Figure 3.14(b). The 500 Monte Carlo simulation trials are scaled to 10 so that it is evident to be compared with 10 tested samples. Statistically the measured DC gain of the Opamp is less than the simulated one using split transistor while mostly larger than the simulated one using non-split transistors. In all, 9 out of the 10 samples have DC gains larger than 85.4 dB with a maximum up to 94.0 dB . Only one sample has a gain of about 77.1 , slightly less than the simulated mean 81.7 dB of the Opamp using non-split transistors.

### 3.3.3 Sensor Array Noise

Noise performance of the ADMDVS is important because it determines its minimum TC sensitivity. To estimate the average equivalent noise $V_{r m s, n}$ at the output of a CGPR, the average noise spike rate $R_{n}$ of the sensor array under DC illumination was measured with the setup shown in Figure 3.15(a). The chip is covered with an infrared blocking filter (IRBF) so that the illuminance value shown on the Tektronix J17 photometer approximately reflects the actual illuminance on the pixel array. The tunable light source was


Figure 3.16. Experiment setup for measuring the average TC sensitivity $\theta$ of the ADMDVS array

Table 3.3. Stimulus contrast $C_{s t i}$ used for measuring $\theta$ at different gain codes $\mathrm{G}_{1} \mathrm{G}_{0}$ of the CC-PGA.

| $\mathrm{G}_{1} \mathrm{G}_{0}$ | $C_{s t i}$ |
| :---: | :---: |
| 00 | 1.17 |
| 01 | 0.628 |
| 10 | 0.340 |
| 11 | 0.174 |

a QT-DE12R7s floodlight lamp with 500-W maximum power. A white Gaussian noise input is assumed for the ADMs in all pixels. Using Eq. (3.a1) in the Appendix, $V_{r m s, n}$ can be calculated as:

$$
\begin{equation*}
V_{r m s, n}=\frac{R_{n} \cdot \delta}{M \cdot N \cdot 2 \sqrt{\frac{2 \pi}{3}} f_{B W} A_{F E}} \tag{3.14}
\end{equation*}
$$

where $M \cdot N=1.8 \times 10^{3}$ is the total pixel number, $\delta$ is the ADM threshold, $f_{B W}$ is the CC-PGA bandwidth, and $A_{F E}$ is the front-end gain including the gain of CC-PGA and ADM.

Figure 3.15(b) shows the plot of the measured $R_{n}$ and calculated $V_{r m s, n}$ versus background illuminance $E_{v, B G}$ with the TC threshold set to about $2.4 \%$ under two different nominal biases of the CC-PGA ( 50 pA and 1 nA ). At 1 nA bias, the maximum $R_{n}$ of about 220 k spike/s occupies a considerable portion of the array's total bandwidth of 10M spike/s. $R_{n}$ would increase linearly as the array size increases. Limiting $f_{B W}$ by setting the bias to 50 pA lowers $R_{n}$, and also lowers $V_{r m s, n}$, because the noise contributed by CGPRs is further filtered. The $V_{r m s, n}$ curves resemble the simulated CGPR output noise in Figure 3.5(b). To keep $V_{r m s, n}$ less than the $1 \% \mathrm{TC}$ limit, $E_{v, B G}>2.5 \mathrm{k}$ lux is needed at 50 pA bias, and $E_{v, B G}>10 \mathrm{k}$ lux at 1 nA bias. Therefore, to achieve a $<1 \% \mathrm{TC}$ sensitivity in a single pixel, low pixel bandwidth and sufficiently high illumination are the two essential factors. In fact, it is observed that removal of IRBF from the chip coverage induces significant $R_{n}$ drop (up to $50 \times$ at about 8 k lux) which is believed to be the result of significantly increased photocurrent thanks to the exposure to a large amount of infrared. The flattened tail of the photoreceptor output noise at high illuminance limited by $1 / f$ noise in Figure 3.5 also becomes visible in measurement when IRBF is removed even though it is not shown in Figure 3.15(b).

### 3.3.4 Temporal Contrast Sensitivity

The experimental setup for the sensitivity test is shown in Figure 3.16. An SST-90 white LED from Luminus Devices with a maximum luminous flux of 2500 lm modulated by a sinusoidal signal $V_{\sin }(t)$ is used to provide the input stimulus for the chip through an integrating sphere. The peak-to-peak voltage of $V_{s i n}(t)$ and DC bias current of the LED determines the stimulus contrast $C_{s t i}$. The different $C_{s t i}$ values used for sensitivity testing (listed in TABLE 3.3) at different CC-PGA gain codes $\mathrm{G}_{1} \mathrm{G}_{0}$ are used to keep a relatively constant output spike rate. $C_{\text {sti }}$ is calculated as

$$
C_{s t i}=\ln \frac{I_{\max }}{I_{\min }}
$$

where $I_{\max }$ and $I_{\min }$ are the measured maximum and minimum illuminance from the LED. Using the signal


Figure 3.17. (a) Measured average TC sensitivity $\theta$ of the ADMDVS and (b) measured relative standard deviation of the TC sensitivity $\sigma_{\theta} / \theta$ at different threshold voltages $V_{\theta}$ of the ADM and different gain codes $\mathrm{G}_{1} \mathrm{G}_{0}$ of the CC-PGA.


Figure 3.18. Effect of different TC sensitivity settings on detecting fine palm lines. The vertically moving hand is divided into three different regions marked as I, II, and III. The ADMDVS is used to detect the lines of each region separately.
spike rate $R_{\text {sin }}$ of the array in response to the stimulus $V_{\sin }(t)$, the average detectable TC threshold, i.e. the TC sensitivity $\theta$ can be calculated as

$$
\begin{equation*}
\theta=\frac{2 M \cdot N \cdot f_{\text {sin }} \cdot C_{s t i}}{R_{\text {sin }}} \tag{3.15}
\end{equation*}
$$

where $f_{\sin }$ is the frequency of $V_{\sin }(t)$. To evaluate the standard deviation of the sensitivity $\sigma_{\theta}$ among all pixels, the signal spike rate of each pixel $R_{\text {sin,pixel }}$ is used to calculate the sensitivity for each pixel $\theta_{\text {pixel }}$, and $\sigma_{\theta}$ is obtained by applying a Gaussian fit to all $\theta_{\text {pixel }}$.

As pointed out in Section 3.3.3, the noise spike rate $R_{n}$ is quite considerable when the ADM threshold is set small even with a high illuminance, so the actual measured spike rate $R_{s i n+n}$ under stimulus is due to both the sinusoidal signal stimulus and the noise. Therefore, directly using $R_{\sin +n}$ to calculate $\theta$ results in overestimated sensitivity. However, with measured $R_{s i n+n}$ and $R_{n}$, the $R_{\text {sin }}$ can be calculated using Eq. (3.16) below (see Appendix) assuming the noise is white and Gaussian

$$
\begin{equation*}
R_{s i n+n}=\sqrt{\frac{2}{\pi}} R_{n} e^{-c^{2} / 4}\left[\left(1.25+0.63 c^{2}\right) I_{0}\left(\frac{c^{2}}{4}\right)+0.63 b^{2} I_{1}\left(\frac{c^{2}}{4}\right)\right] \tag{3.16}
\end{equation*}
$$

where the parameter $c$ is given as:

$$
c=\sqrt{\frac{\pi}{2}} \frac{R_{s i n}}{R_{n}}
$$

and $I_{v}(x)(v=0,1)$ is the modified Bessel function of the first kind. Note that this noise de-embedding is only necessary for low gain settings at $\mathrm{G}_{1} \mathrm{G}_{0}=00$ and 01 where $R_{n}$ is substantial when $V_{\theta}$ is small, and even though the sinusoidal input stimuli is logarithmically transformed to voltage signals by the photoreceptor, the ADM input can still be approximately regarded as a sinusoid because $\ln (1+\Delta) \approx \Delta$ when $\Delta$ is much less than 1 .

The TC sensitivity $\theta$ versus $V_{\theta}$ at different gain codes $\mathrm{G}_{1} \mathrm{G}_{0}$ of the CC-PGA and the corresponding relative standard deviation $\sigma_{\theta} / \theta$ versus $V_{\theta}$ are plotted in Figure 3.17. Figure 3.17(a) shows that $\theta$ increases by less than a factor of two at the same $V_{\theta}$ when $\mathrm{G}_{1} \mathrm{G}_{0}$ decreases one step, especially from 11 to 10 where $\theta$ increases by only $\times 1.25$ on average. This non-ideal gain step is because the underestimated parasitic capacitance in the fabricated chips is comparable to the feedback capacitance $\mathrm{C}_{\mathrm{fb}}$ in Figure 3.7. $\theta$ increases relatively linearly with $V_{\theta}$ for all $\mathrm{G}_{1} \mathrm{G}_{0}$. The minimum measured $\theta$ is about $0.54 \%$ at $\mathrm{G}_{1} \mathrm{G}_{0}=11$ and $V_{\theta}=100 \mathrm{mV}$; below this $V_{\theta}, R_{n}$ is too large for the array to detect any visual signal. The obvious penalty of small $\theta$ at high gain and low $V_{\theta}$ is the large $\theta$ variation as can be seen in Figure 3.17(b). For a reasonable $\sigma_{\theta} / \theta \approx 35 \%, \theta$ is about $1 \%$. At high gain, the feedback capacitance in CC-PGA is small, which causes significant capacitance mismatch and in turn large $\sigma_{\theta} / \theta$. The mismatch decreases as the capacitance increases, i.e. the gain decreases. At low $V_{\theta}$, the variation of the ADM amplifier's DC output and the input offset of the two comparators contribute substantially to $\sigma_{\theta} / \theta$, which is mitigated as $V_{\theta}$ increases because the DC variation and offset become a smaller portion of $V_{\theta}$.

The effect of different TC sensitivity settings is demonstrated by detecting the fine palm lines of a moving hand under office lighting ( $\sim 500 \mathrm{lux}$ ) as shown in Figure 3.18. The hand moves at a speed of about $15 \mathrm{~cm} / \mathrm{s}$ and it is about 6 cm away from the lens. The experiment was repeated for three different sensitivity settings. The palm lines in different parts of the hand are marked in the original hand image, and also in the accumulated-spike histogram images (over a time window of 30 ms for $\theta=0.5 \%, 4 \%$ and $30 \%$ ) wherever they are visually detectable. It is clear that a small $\theta$ setting helps detection of low contrast objects, although with a larger fixed-pattern noise. On the other hand, a large $\theta$ setting can be used to detect the contour of high contrast objects with minimal noise.

### 3.3.5 Spike Encoding Comparison

To verify the improved spike encoding of the proposed in-pixel ADM over STR, the ADMDVS chip is compared with a DAVIS chip with STR that is previously developed in our group [18], using a moving visual pattern as the input stimulus and a simple histogram reconstruction as output. For a fair comparison between the two chips, the following three factors were paid attention to in the designed experiment: 1 . To ensure that the front-end bandwidth is not limited by the photoreceptor, the 500 W floodlight lamp is used to provide additional lighting so that the illuminance at the position of the rotating image is about 4 k lux; 2. The front-end bandwidth is set to about 300 Hz by the source follower in STR pixel and the CCPGA in ADM pixel respectively so that the signals fed into the two encoders have approximately the same bandwidth; 3. The cutoff frequency of the amplifiers for spike encoding is set to about 500 Hz , and


Figure 3.19. Testing setup and the original image for the spike encoding comparison experiment.


Figure 3.20. Comparison of the spike-accumulated histogram images acquired by the chip with ADM encoding (ADMDVS) and the chip with STR encoding (DAVIS) [18] at different rotational speeds within time windows which are inversely proportional to the rotational speeds.
the sensitivity to about $15 \%$. As illustrated in Figure 3.19 , the sensors are mounted with a $1 / 3^{\prime \prime} 2.6 \mathrm{~mm}$ $\mathrm{f} / 1.6$ lens, and a pear image is attached to a disk driven by a motor with an adjustable rotational speed. Because the DAVIS has a $240 \times 180$ resolution, a $60 \times 30$ region was selected to match with the ADMDVS. The spike-accumulated histogram images in Figure 3.20 are acquired by the two sensors at different rotational speeds from 0.10 to 6.1 rps within time windows inversely proportional to the rotational speeds. The dark edges in the ADM images are more clearly defined and the bright edges become obscure more slowly as rotational speed increases compared to those in the STR images. Note that in Figure 3.20(a), the STR image shows clearly visible scattered white dots, which probably indicates background ON spikes caused by leakage charging of the reset switch $\mathrm{M}_{\mathrm{S}}$ in Figure 3.11.

To quantify the spike encoding improvement, the following method is proposed for comparing the number of produced spikes from the two sensors. Figure 3.21 shows the comparison of the average number of ON/OFF spikes ( $\mathrm{SN}=$ =spike number) from the 108 most active pixels of the images in Figure 3.19. The number of pixels chosen for SN averaging depends on the percentage of active pixels in the array. Ideally the average ON/OFF SN should stay constant with stimulus speed given infinite pixel bandwidth and instantaneous spike feedback. However because of the limited 300 Hz analog front-end bandwidth used in this experiment, the average SN in both STR and ADM decreases. Although the average ON SN in STR is approximately the same as that in ADM at 0.1 rps , implying the nearly identical sensitivity set-


Figure 3.21. Comparison of the average number of ON/OFF spikes (SN=spike number) over the 108 brightest/darkest pixels and the SN ratio of ADMDVS over DAVIS.
ting, it decreases faster with increasing rotational speeds as indicated by the increasing ON SN ratio up to 3 because of signal loss in STR during reset and refractory period. This ON SN ratio increase supports the SDR improvement of ADM against STR in model simulation as shown in Chapter 2. The much lower average OFF SN in STR even at low rotational speeds is due to severe signal loss caused by large feedback delay with the threshold-determined low bias of the OFF comparator, and the maximum SN ratio is up to $>3.5$. The irregular increase of average OFF SN in STR from 0.1 to 0.57 rps is due to the junction leakage of the reset switch $\mathrm{M}_{\mathrm{S}}$ in Fig. 11 in accordance with the comparison of the STR images between Figure 3.20(a) and 3.20(b). This is eliminated in ADM thanks to the continuous-time DC feedback using the pseudo-resistor.

### 3.3.6 Simulated Optical Neuroimaging using ADMDVS

The prototype ADMDVS is applied to a simulated optical neuroimaging experiment. A fluorescence imaging video recorded from a region in mouse cortex is displayed on a screen and the ADMDVS mounted with the same lens as in Section 3.3.5 is placed in front of the screen. The measured grayscaleluminance relationship using photometer is given in Figure 3.22(a). Assuming a lens transmissibility of 0.9 , the average illuminance on the chip is calculated to be about 6.9 lux with the measured screen luminance of about $25 \mathrm{~cd} / \mathrm{m}^{2}$ (using Eq. (3.b1) in the Appendix). The TC sensitivity of the ADMDVS is set to $2.7 \%$. Figure 3.22(b) shows one frame of the optical neuroimaging recording. The target neuron circled in yellow is used to demonstrate the effectiveness of temporal signal reconstruction from the ADMDVS output spikes.

The upper waveform in Figure 3.23 is the grayscale value over time averaged from $5 \times 5$ pixels within


Figure 3.22. (a) Measured grayscale-luminance relationship of the screen; (b) one frame of the optical neuroimaging video with the target neuron circled in yellow.


Figure 3.23. The temporal waveform averaged over a $5 \times 5$ pixel window within the target neuron of the video in (a) (upper plot); The linear signal reconstruction using the output ON and OFF spikes from one ADMDVS pixel whose visual field covers the region around the target neuron (lower plot).
the target neuron in the video. The lower waveform in Figure 3.23 is the simple linear reconstruction from the ON and OFF spikes recorded by one ADMDVS pixel that has the visual field around the target neuron. The corresponding peaks in the two waveforms are evident. The missing peak pointed by the green arrow is likely due to the fact that the ADMDVS pixel may not have the exact visual field of the $5 \times 5$ pixels. On the other hand, the rising edge circled in red in the upper waveform has a contrast of $61 \%$ calculated according to the measured grayscale-luminance relationship in Figure 3.22(a), and the one in the lower waveform is composed of 26 ON events corresponding to a contrast of $\ln \left[(1+2.7 \%)^{26}\right]=69 \%$, well close to $61 \%$. The long-term DC level fluctuation of the peaks in the reconstructed waveform is caused by the unbalanced ON and OFF thresholds of the fabricated ADM circuit.

### 3.4 Conclusion and Remarks

This chapter described a specific type of silicon retina, i.e. the dynamic vision sensor (DVS) with enhanced TC sensitivity and spike encoding. The performance is summarized and compared with prior DVSs in Table 3.4. Although the circuit area of this proposed pixel is approximately 3 times larger than

Table 3.4. Comparison with previous DVSs.

|  | This work | $2014[18]$ | $2013[16]$ | $2011[15]$ | $2011[17]$ | $2008[6]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Technology | $0.18 \mu \mathrm{~m}$ <br> MM/RF | $0.18 \mu \mathrm{~m}$ <br> IS | $0.35 \mu \mathrm{~m}$ <br> IS | $0.35 \mu \mathrm{~m}$ <br> $\mathrm{MM} / \mathrm{RF}$ | $0.18 \mu \mathrm{~m}$ <br> $\mathrm{MM} / \mathrm{RF}$ | $0.35 \mu \mathrm{~m}$ <br> $\mathrm{MM} / \mathrm{RF}$ |
| Resolution | $60 \times 30$ | $240 \times 180$ | $128 \times 128$ | $128 \times 128$ | $304 \times 240$ | $128 \times 128$ |
| Chip Area $\left(\mathrm{mm}^{2}\right)$ | $3.2 \times 1.6$ | $5 \times 5$ | $4.9 \times 4.9$ | $5.6 \times 5.5$ | $9.9 \times 8.2$ | $6.3 \times 6$ |
| Pixel Area $\left(\mu \mathrm{m}^{2}\right)$ | $31.2 \times 31.2$ | $18.5 \times 18.5^{\mathrm{a}}$ | $31 \times 30$ | $35 \times 35$ | $30 \times 30^{\mathrm{b}}$ | $40 \times 40$ |
| Fill Factor (\%) | 10.3 | 22 | 10.5 | 8.7 | $10^{\mathrm{c}}$ | 8.1 |
| Supply Voltage (V) | 1.8 | $1.8 / 3.3$ | 3.3 | 3.3 | $1.8 / 3.3$ | 3.3 |
| Power (mW) | $0.72^{\mathrm{d}}$ | $14^{\mathrm{e}}$ | $4^{\mathrm{d}}$ | $145^{\mathrm{d}}$ | $175^{\mathrm{e}}$ | $24^{\mathrm{f}}$ |
| Power/Pixel ( $\mu \mathrm{W}$ ) | 0.40 | 0.32 | 0.24 | 8.8 | 2.4 | 1.5 |
| Min. TC Sensitivity (\%) | $\mathbf{1}$ | 11 | 1.5 | 10 | 13 | 17 |
| DR (dB) | $130^{\mathrm{g}}$ | 130 | 120 | 100 | N.A. ${ }^{\mathrm{h}}$ | 120 |
| Intra-Scene DR (dB) | $\mathbf{1 3 0}$ | 130 | 60 | 56 | N.A. | 120 |
| Event Encoding | ADM | STR | STR | STR | STR | STR |

a. Including 4T APS; b. Including PWM imaging circuits; c. Only DVS photodiode; d. At $10^{5}$ event/s; e. High activity, including the imaging functionality; f. Non-optimized power-consuming biasgen; g. About 0.03 lux to $>100 \mathrm{k}$ lux; h. Only DR of PWM imaging given.
[18], it achieves comparable power consumption per pixel, and a $1 \% \mathrm{TC}$ sensitivity with a $35 \%$ relative standard deviation without sacrificing the intra-scene DR by using an in-pixel CC-PGA. The TC sensitivity record is $0.3 \%$ [122], but it was only demonstrated in a single pixel with a very limited pixel bandwidth. The prototype ADMDVS also employs an in-pixel ADM for spike encoding which has been in MATLAB simulation (Chapter 2) [123] and here experimentally verified to have a better encoding quality compared to STR (Note that a similar switched-cap circuit used in relaxation oscillator for constant charge subtraction like the one used for the in-pixel ADM here is almost simultaneously published on JSSC by a group at Michigan University [124]). These improvements together with the intrinsic low-latency sparse-output features of DVSs pave the way for applications like wireless in-vivo optical neuroimaging on free-moving animals, where the energy spent on RF data transmission can be reduced.

For VSDI with signal temporal contrast often less than $1 \%$ [92], further improved SNR at a high sensitivity setting is still necessary. One obvious means is to increase the photodiode size. Optimized photodiodes in a dedicated image sensor process with higher quantum efficiency and micro-lenses can be used to obtain high photocurrent so that shot noise is reduced. Transistors with large $L$ and $W$ help lower the contribution of $1 / f$ noise. Large compensation capacitance and large closed-loop gain can be used to reduce the CC-PGA noise at the cost of pixel area and power consumption.

Although an ADM improves the spike encoding integrity compared to STR, for real-time high-fidelity signal reconstruction [94], [95], the problems of sensitivity variation among pixels and unbalanced ON and OFF threshold remain. They could be addressed at the circuit level by using larger transistors and capacitors with increased pixel size. A novel encoding mechanism might give an area-efficient solution. For example, a threshold-variation-insensitive decoding algorithm was developed for the asynchronous sigma-delta modulation (ASDM) [52], although the ASDM generates idle output without any input signal
change and thus results in a much more limited pixel spike-output bandwidth.

### 3.5 Appendix

## 3.5.a Spike Rate of ADM

Let us assume a white Gaussian noise input $x_{n}(t)$ that has zero mean, rms amplitude $V_{r m s, n}$, and bandwidth $f_{b w, n}$, then the output spike rate $R_{n}$ of an ADM can be derived as [57]:

$$
\begin{equation*}
R_{n}=2 \sqrt{\frac{2 \pi}{3}} \frac{f_{b w, n} V_{r m s, n}}{\delta} \tag{3.a1}
\end{equation*}
$$

where $\delta$ is the ADM threshold. For a sinusoidal input $x_{\sin }(t)$, the output spike rate $R_{\text {sin }}$ is [57]:

$$
\begin{equation*}
R_{s i n}=4 \sqrt{2} \frac{f_{\text {sin }} V_{r m s, s i n}}{\delta} \tag{3.a2}
\end{equation*}
$$

where $f_{\sin }$ is the signal frequency, and $V_{r m s, s i n}$ is the rms amplitude of $x_{s i n}(t)$.
Let $x_{\sin +n}(t)=x_{\sin }(t)+x_{n}(t)$ represent the sum of a sinusoidal signal and a white Gaussian noise. To derive its corresponding spike rate $R_{s i n+n}$, the mean of the absolute slope needs to be obtained. The joint probability density function $p\left(a_{x}, s_{x}, t\right)$ of the amplitude $a_{x}$ and the slope $s_{x}$ of $x_{\sin +n}$ at time $t$ is given as [125]:

$$
p\left(a_{x}, s_{x}, t\right)=\frac{\pi N_{0}}{-\psi_{0}^{\prime \prime}} \varphi\left(y-c_{1} \cos \left(2 \pi f_{\text {sin }} t\right)\right) \varphi\left(x+c_{2} \sin \left(2 \pi f_{\text {sin }} t\right)\right)
$$

where the parameters $N_{0}, x, y, c_{1}, c_{2}$, and the function $\varphi(z)$ are given below:

$$
N_{0}=\frac{1}{\pi} \sqrt{\frac{-\psi_{0}^{\prime \prime}}{\psi_{0}}}, x=\frac{s_{x}}{\sqrt{-\psi_{0}^{\prime \prime}}}, y=\frac{a_{x}}{\sqrt{\psi_{0}}}, c_{1}=\sqrt{\frac{2}{\psi_{0}}} V_{r m s, s i n}, c_{2}=\sqrt{\frac{2}{-\psi_{0}^{\prime \prime}}} 2 \pi V_{r m s, s i n} f_{\text {sin }}, \varphi(z)=\frac{1}{\sqrt{2 \pi}} e^{-\frac{z^{2}}{2}}
$$

$\psi_{0}$ is the correlation function $\psi(\tau)$ of $x_{n}(t)$ at $\tau=0$, and $\psi_{0}{ }^{\prime \prime}$ is the second derivative of $\psi(\tau)$ at $\tau=0$. For bandlimited white noise that has a power of $V^{2}{ }_{r m s, n}$ within $f_{b w, n}, \psi_{0}$ and $\psi_{0}{ }^{\prime \prime}$ can be calculated as:

$$
\psi_{0}=\int_{0}^{f_{b w, n}} \frac{V_{r m s, n}^{2}}{f_{b w, n}} \mathrm{~d} f=V_{r m s, n}^{2}, \psi_{0}^{\prime \prime}=-4 \pi^{2} \int_{0}^{f_{b w, n}} \frac{V_{r m s, n}^{2}}{f_{b w, n}} f^{2} \mathrm{~d} f=-\frac{4}{3} \pi^{2} V_{r m s, n}^{2} f_{b w, n}^{2}
$$

The probability density function $p\left(s_{x}, t\right)$ of the slope $s_{x}$ can be obtained by integrating the amplitude over its range, namely from $-\infty$ to $+\infty$ :

$$
p\left(s_{x}, t\right)=\frac{\pi N_{0}}{-\psi_{0}{ }^{\prime \prime}} \varphi\left(x+c_{2} \sin \left(2 \pi f_{\text {sin }} t\right)\right) \int_{-\infty}^{+\infty} \varphi\left(y-c_{1} \cos \left(2 \pi f_{\text {sin }} t\right)\right) \mathrm{d} a_{x}=\frac{\pi N_{0}}{-\psi_{0}{ }^{\prime \prime}} V_{r m s, n} \varphi\left(x+c_{2} \sin \left(2 \pi f_{\text {sin }} t\right)\right)
$$

The mean of $\left|s_{x}(t)\right|$ can be calculated as:

$$
\begin{aligned}
& <\left|s_{x}(t)\right|>=2 \int_{0}^{\infty} s_{x} p\left(s_{x} t\right) \mathrm{d} s_{x}=\frac{2 \pi N_{0}}{-\psi_{0}{ }^{\prime}} V_{r m s, n} \int_{0}^{\infty} s_{x} \varphi\left(x+c_{2} \sin \left(2 \pi f_{s i n} t\right)\right) \mathrm{d} s_{x} \\
& \xrightarrow{v=c_{2} \sin \left(2 \pi f_{s i n} t\right)} 2 \pi N_{0} V_{r m s, n} \int_{0}^{\infty} x \varphi(x+v) \mathrm{d} x=2 \pi N_{0} V_{r m s, n}\left[-\frac{v}{2}+\varphi(v)+v \int_{0}^{v} \varphi(z) \mathrm{d} z\right]
\end{aligned}
$$

The mean of $\left|s_{x}\right|$ can be calculated by taking the limit below:

$$
\begin{aligned}
& <\left|s_{x}\right|>=\lim _{T \rightarrow \infty} \frac{1}{T} \int_{0}^{T}<\left|s_{x}(t)\right|>\mathrm{d} t=2 \pi N_{0} V_{r m s, n} \lim _{T \rightarrow \infty} \frac{1}{T} \int_{0}^{T}\left[-\frac{v}{2}+\varphi(v)+v \int_{0}^{v} \varphi(z) \mathrm{d} z\right] \mathrm{d} t \\
& =2 \pi N_{0} V_{r m s, n} \lim _{T \rightarrow \infty} \frac{1}{T} \int_{0}^{T}\left[-\frac{c_{2} \sin \left(2 \pi f_{s i n} t\right)}{2}+\varphi\left(c_{2} \sin \left(2 \pi f_{s i n} t\right)\right)+c_{2} \sin \left(2 \pi f_{s i n} t\right) \int_{0}^{c_{2} \sin \left(2 \pi f_{s i n} t\right)} \varphi(z) \mathrm{d} z\right] \mathrm{d} t \\
& \xrightarrow{\theta=2 \pi f_{s i n} t} 2 \pi N_{0} V_{r m s, n} \lim _{T \rightarrow \infty} \frac{1}{2 \pi f_{\sin } T} \int_{0}^{2 \pi f_{s i n} T}\left[-\frac{c_{2} \sin \theta}{2}+\varphi\left(c_{2} \sin \theta\right)+c_{2} \sin \theta \int_{0}^{c_{2} \sin \theta} \varphi(z) \mathrm{d} z\right] \mathrm{d} t \\
& =2 N_{0} V_{r m s, n} \int_{0}^{\pi}\left[\varphi\left(c_{2} \sin \theta\right)+c_{2} \sin \theta \int_{0}^{c_{2} \sin \theta} \varphi(z) \mathrm{d} z\right] \mathrm{d} \theta
\end{aligned}
$$

The last step is simply obtained based on the parity of each term in the square brackets. Therefore, together with Eq. (3.a1) and Eq. (3.a2), the output spike rate $R_{s i n+n}$ of an ADM with $x_{s i n+n}(t)$ as the input can be written as:

$$
\begin{align*}
R_{s i n+n} & =\frac{\left\langle s_{x}\right|>}{\delta}=\frac{4 f_{b w, n} V_{r m s, n}}{\sqrt{3} \delta} \int_{0}^{\pi}\left[\varphi\left(c_{2} \sin \theta\right)+c_{2} \sin \theta \int_{0}^{c_{2} \sin \theta} \varphi(z) \mathrm{d} z\right] \mathrm{d} \theta \\
& =\sqrt{\frac{2}{\pi}} R_{n} \int_{0}^{\pi}\left[\varphi\left(c_{2} \sin \theta\right)+c_{2} \sin \theta \int_{0}^{c_{2} \sin \theta} \varphi(z) \mathrm{d} z\right] \mathrm{d} \theta \tag{3.a3}
\end{align*}
$$

Note that the equation below holds:

$$
c_{2}=\sqrt{\frac{2}{-\psi_{0}^{\prime \prime}}} 2 \pi V_{r m s, s i n} f_{s i n}=\sqrt{\frac{3}{2}} \frac{2 V_{r m s, s i n} f_{s i n}}{V_{r m s, n} f_{b w, n}}=\sqrt{\frac{\pi}{2}} \frac{R_{s i n}}{R_{n}}
$$

Eq. (3.16) is the numerically integrated version of Eq. (3.a3) for faster computation.

## 3.5.b From Screen Luminance to Chip Illuminance

With the assumption of lumen conservation, the equation below holds:

$$
L_{\text {screen }} \cdot A_{\text {screen }} \cdot \frac{\pi \frac{d_{\text {lens }}^{2}}{4}}{d_{\text {scr-lens }}^{2}} \cdot T=I_{\text {sensor }} \cdot A_{\text {sensor }}
$$

where $L_{\text {screen }}\left(\mathrm{cd} / \mathrm{m}^{2}\right)$ is the screen luminance, $I_{\text {sensor }}$ (lux) is the illuminance at the focal plane of the ADMDVS chip, $A_{\text {screen }}$ and $A_{\text {sensor }}$ are the areas of the screen and the sensor respectively, $d_{\text {lens }}$ is the lens diameter, $d_{\text {scr-lens }}$ is the distance between the screen and the lens, and $T$ is the lens transmissibility. $A_{\text {screen }}$ and $A_{\text {sensor }}$ have the geometrical relationship below:

$$
\frac{A_{\text {screen }}}{A_{\text {sensor }}}=\frac{d_{\text {scr-lens }}^{2}}{d_{\text {sen-lens }}^{2}}
$$

where $d_{\text {sen-lens }}$ is the distance between the sensor and the lens. Therefore, $I_{\text {sensor }}$ can be expressed as:

$$
\begin{equation*}
I_{\text {sensor }}=L_{\text {screen }} \cdot \frac{d_{\text {scr-lens }}^{2}}{d_{\text {sen-lens }}^{2}} \cdot \frac{d_{\text {lens }}^{2}}{d_{\text {scr-lens }}^{2}} \cdot \frac{\pi T}{4}=L_{\text {screen }} \frac{\pi T}{4 f^{2}} \tag{3.b1}
\end{equation*}
$$

where $f$ is the lens aperture ratio.

## Chapter 4: Ultra-Low-Power Binaural Silicon Cochlea

9
his chapter presents an ultra-low-power binaural spiking silicon cochlea with 0.5 V power supply in $0.18 \mu \mathrm{~m}$ CMOS aiming for energy-scarce applications like voice activity detection and speaker identification in scenarios of wireless sensor networks (WSNs), internet of things (IoTs), etc. As projected by major semiconductor companies including Intel and Qualcomm, deployment of trillions of sensors is envisioned to interweave the physical and cyber worlds and facilitate human-environment, human-human interactions [126]. One of the key enabling technologies is smart ultra-low-power sensor nodes that are able to make simple local decisions for information transmission instead of power-consuming raw data transmission. Hence processing units should be tightly integrated with sensing front end. Conventional audio signal acquisition and processing are dominated by clocked ADCs and DSPs. The obvious drawback of this approach is the waste of energy, because sound signals are bursty and constant sampling in clocked systems produces redundant data for processing. As one essential functional block in audio DSPs, FFT has been optimized for very high energy efficiency [41], but it still falls short compared to analog frequency division at least in some classification tasks like voice activity detection where signal SNR requirement is low-to-medium [42].

The newly-developed silicon cochlea [127] features $64 \times 2$ channels with biomimetic asymmetrical analog BPFs covering frequency range from 8 to 20 k Hz and an asynchronous delta modulator (ADM) for spike encoding in each channel to facilitate event-driven clockless processing. To improve power efficiency, source-follower-based BPFs and ADMs with adaptive self-oscillating comparison are proposed. The SF-based BPF is composed of a $4^{\text {th }}$-order source-follower-based LPF and a summing PGA, and the self-oscillation is realized by employing dynamic latched comparators, simple logic gates and a delay element. The circuit details will be described in the subsections. The content of this chapter is organized as follows: Section 4.1 describes the system architecture and the composition of one cochlea channel; Section 4.2 gives the detailed $0.5-\mathrm{V}$ cochlea core design, including: Section 4.2.1, the geometrically-scaled channel bias generation; Section 4.2.2, the translinear loop for $Q$-tuning and the in-channel bias distribution circuits; Section 4.2.3, the programmable capacitive attenuator; Section 4.2.4, the source-followerbased asymmetrical BPF; Section 4.2.5, the ADM with adaptive self-oscillating comparison; Section 4.3 describes the system design considerations; Section 4.4 presents the measurement results.

### 4.1 Architecture

The whole system of the silicon cochlea is illustrated in Figure 4.1. Binaural audio signals are acquired and amplified by off-chip microphones and preamplifiers, respectively. The on-chip part consists of the $0.5-\mathrm{V}$ core and the $1.8-\mathrm{V}$ AER. The AER could be designed with a $0.5-\mathrm{V}$ supply as well, but we did not pursue this because of limited time budget during tape-out. The chip request $\mathrm{C}_{\text {req }}$, chip acknowledge $\mathrm{C}_{\mathrm{ack}}$, as well as the channel addresses are communicated between the AER and the off-chip FPGA. The 1-D AER contains an address encoder, an arbiter logic block and a binary-tree arbiter [84]. The address bus has 8 bits: the LSB $\mathbf{b}_{\text {AER } 0}$ for the spike polarity (whether ON or OFF), $\mathbf{b}_{\text {AER } 1}$ for the distinction of the left or right cochlea, and $\mathbf{b}_{\text {AER } 2}-\mathbf{b}_{\text {AER } 7}$ for the 64 channels. The biases of the 64 channels $\mathrm{I}_{\mathrm{ch} 0} \sim \mathrm{I}_{\mathrm{ch} 63}$ are geometrically scaled with a designed ratio of 1.108 covering the frequency range from about 32 to 20 k Hz . In each channel, the input signal passes through a programmable capacitive attenuator before being filtered by a $4^{\text {th }}$-order source-follower-based LPF. The subsequent programmable-gain amplifier (PGA) sums the output of the LPF with its internal nodes to create the zero for a BPF transfer function (TF). The central frequency of the $\operatorname{BPF} f_{c \mathrm{c}}$ is proportional to the channel bias $\mathrm{I}_{\text {chi }}\left(\mathrm{i} \in N_{0} \cup \mathrm{Ui} \in[0,63]\right)$. The quality factor $Q$ of


Figure 4.1. The complete binaural silicon cochlea system. The microphones, preamplifiers and FPGA are off-chip, and the on-chip part consists of the $0.5-\mathrm{V}$ cochlea core with $64 \times 2$ binaural channels and the $1.8-$ V AER. Each cochlea channel contains the left and right monaural branches, and the building blocks are the programmable capacitive attenuator, $4^{\text {th }}$-order LPF, summing programmable gain amplifier (PGA), asynchronous delta modulator (ADM), asynchronous logic, shared translinear loop (TLL) for $Q$-tuning, and in-channel bias distribution circuitry.
the BPF is tunable via a translinear loop. The filtered signal is modulated by an ADM with self-oscillating comparison whose oscillating frequency is adaptive in accordance with the spike output activity. The asynchronous logic generates the control signals for the ADM and communicates with the $1.8-\mathrm{V}$ AER abiding by the fourth-phase handshake protocol.

### 4.2 0.5-V Cochlea Core Design

### 4.2.1 64 Geometrically-Scaled Channel Bias Currents

The original cochlea design used subthreshold MOS transistors to produce the biases for the secondorder section (SOS) BPFs [23], [24]. The transistor gate voltages are linearly scaled by a resistive divider so that the drain currents are geometrically scaled because of the exponential $V_{g s}-I_{d s}$ relationship. With specified frequency range and scaling ratio, the voltages on the two terminals of the resistive divider can be determined. Besides the problem of large $I_{d s}$ deviation resulted from the mismatch of MOS threshold


Figure 4.2. Schematic of the circuit that generate4 64 geometrically-scaled bias currents.

Table 4.1. Mean $\mu$ and $3 \sigma$ variation of some of the output currents in Figure 4.2 in a 250 -run Monte Carlo simulation.

| Current | $\mathrm{I}_{\text {ch } 0}$ | $\mathrm{I}_{\text {ch } 8}$ | $\mathrm{I}_{\text {ch16 }}$ | $\mathrm{I}_{\text {ch24 }}$ | $\mathrm{I}_{\text {ch32 }}$ | $\mathrm{I}_{\text {ch40 }}$ | $\mathrm{I}_{\text {ch48 }}$ | $\mathrm{I}_{\text {ch56 }}$ | $\mathrm{I}_{\text {ch63 }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mu(\mathrm{A})$ | 50.1 n | 22.0 n | 9.63 n | 4.21 n | 1.83 n | 798 p | 346 p | 152 p | 76.6 p |
| $3 \sigma(\mathrm{~A})$ | 273 p | 122 p | 53.6 p | 23.2 p | 11.5 p | 4.54 p | 2.27 p | 966 f | 684 f |
| $3 \sigma / \mu(\%)$ | 0.54 | 0.55 | 0.56 | 0.55 | 0.63 | 0.57 | 0.66 | 0.64 | 0.89 |

voltages, a large area of resistors is needed to minimize the power consumption. The required two additional reference voltages can be defined by two diode-connected transistors with two defined currents flowing through each of them to counteract the global corner die-to-die threshold voltage variation; however, two buffers are needed to supply the current flowing on the resistive divider. To mitigate the problem of large $I_{d s}$ deviation using subthreshold MOS transistors, CMOS compatible lateral BJTs were used as the alternative owing to the well-matched device parameters of BJTs [25], and consequently the monotonicity of the central frequency scaling was greatly improved. Nonetheless, the base-emitter junction voltage $\mathrm{V}_{\mathrm{BE}}$ of a BJT is usually around $0.6 \sim 0.7 \mathrm{~V}$ that is already larger than the targeted supply voltage of 0.5 V , not to mention that the base current causes deviation of linear $\mathrm{V}_{\mathrm{BE}}$ scaling with resistive divider providing the base voltage.

Another approach that uses only MOS transistors is to employ the MOS-based current-splitting technique as described in [128], [129] for geometrical current scaling. The complete circuit is illustrated in Figure 4.2. All the unit pFETs have the same width and length. The numbers of vertical pFETs in series (lumped as $\mathrm{M}_{\mathrm{S}}$ ) and horizontal pFETs in parallel (lumped as $\mathrm{M}_{\mathrm{P}}$ ) are designed to be 9 and 10, respectively, and the chain is terminated with a single unit pFET $\mathrm{M}_{\mathrm{U}}$. The choice of 9 and 10 is justified as follows. As explained in [129], for a scaling ratio of $r$, the size ratio $R_{S P}$ of $\mathrm{M}_{\mathrm{S}}$ to $\mathrm{M}_{\mathrm{P}}$ should be:

$$
R_{S P}=\frac{(r-1)^{2}}{r}
$$

With a frequency range of 32 to 20 k Hz for 64 channels, $r \approx 1.108$, and in turn $R_{S P} \approx 0.011$. The arrangement of 9 vertical in series and 10 horizontal in parallel gives a size ratio of $1 / 90 \approx 0.011$, the same as the calculated $R_{S P}$. For correct operation of the current divider, the $V_{d s}$ 's of $\mathrm{M}_{\mathrm{S}}$ and $\mathrm{M}_{\mathrm{P}}$ need to be larger than 100 mV so that they can stay in saturation. The body voltages of the pFETs are all biased at half VDD, i.e.


Figure 4.3. (a) Translinear loop (TLL) that generates the BPF biases $\mathrm{I}_{\mathrm{BPF} 1}$ and $\mathrm{I}_{\mathrm{BPF} 2}$ whose multiplication equals to the square of $\mathrm{I}_{\text {chi }}$; (b) R-2R current DAC that adjusts $\mathrm{I}_{\mathrm{BPF} 1}$ with 8 programming digital bits.


Figure 4.4. Complete in-channel bias distribution network.

250 mV to relax the $W / L$ ratio with a gate voltage of larger than 200 mV . Given the theoretical $r$ of 1.108 , the ideal incremental of neighboring central frequencies is $10.8 \%$, which limits the variation of $\mathrm{I}_{\text {chi }}$ to be at least less than $\pm 5.4 \%$ to avoid crossover of central frequencies. Considering the variation accumulatively contributed by later stages in current copying, the budget at this stage should be much less. With a $W / L$ of $200 / 2$ in $\mu \mathrm{m}$ for a unit pFET and $\mathrm{I}_{\mathrm{in}}=50 \mathrm{nA}$, the mean $\mu$ and $3 \sigma$ variation of some of the output currents obtained from a 250 -run Monte Carlo simulation are listed in Table 4.1. As $\mathrm{I}_{\text {chi }}$ decreases, $3 \sigma / \mu$ generally increases, and the maximum is within $0.89 \%$, about $1 / 6$ of $5.4 \%$.

### 4.2.2 Translinear Loop for $\boldsymbol{Q}$-Tuning and In-Channel Bias Distribution

As will be shown in Section 4.2.4, the $Q$-tuning of the BPFs needs two bias currents $\mathrm{I}_{\mathrm{BPF} 1}$ and $\mathrm{I}_{\mathrm{BPF} 2}$ whose ratio is tunable and multiplication stays constant. A translinear loop (TLL) as shown in Figure 4.3(a) can fulfill the requirement. $\mathrm{M}_{1} \sim \mathrm{M}_{4}$ are in subthreshold and saturation, and their bulks are connected to their own sources. The currents should ideally satisfy the following TLL equation:

$$
\mathrm{I}_{\mathrm{chi}}^{2}=\mathrm{I}_{\mathrm{BPF} 1} \cdot \mathrm{I}_{\mathrm{BPF} 2}
$$

$\mathrm{I}_{\mathrm{BPF} 1}$ is generated by the circuit in Figure $4.3(\mathrm{~b})$, a 9-bit R-2R current DAC. $\mathrm{I}_{\mathrm{BPF} 1}$ can be expressed as:

$$
\mathrm{I}_{\mathrm{BPFI}}=\mathrm{I}_{\mathrm{chi}}\left(2^{-1}+\sum_{\mathrm{i}=0}^{7} \mathrm{~b}_{\mathrm{Q}(7-\mathrm{i})} 2^{-(\mathrm{i}+2)}\right)
$$

where $\mathbf{b}_{\mathbf{Q i}}\left(\mathrm{i} \in N_{0} \cup i \in[0,7]\right)$ are the 8 digital bits for programming $\mathrm{I}_{\mathrm{BPF} 1} . \mathrm{I}_{\mathrm{BPF} 2}$ can be accordingly generated from the TLL circuit. Note that even though the DAC has a 9-bit resolution, the number of digital pro-

Table 4.2. Mean $\mu$ and $3 \sigma$ variation of the multiplication and division of $\mathrm{I}_{\mathrm{BPF} 1}$ and $\mathrm{I}_{\mathrm{BPF} 2}$ in some of the channels in a 250 -run Monte Carlo simulation.

|  | channel | 0 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 63 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \sqrt{I_{\mathrm{BPF} 1} \times \mathrm{I}_{\mathrm{BPF} 2}} \\ \mathbf{B}_{\mathbf{Q}}=11111111 \end{gathered}$ | $\mu$ (A) | 50.2n | 22.0n | 9.65n | 4.22n | 1.84n | 800p | 347p | 152p | 77.0p |
|  | $3 \sigma$ (A) | 560p | 236p | 103p | 46.6p | 22.0p | 9.18p | 3.95p | 1.80p | 904f |
|  | $3 \sigma / \mu(\%)$ | 1.1 | 1.1 | 1.1 | 1.1 | 1.2 | 1.1 | 1.1 | 1.2 | 1.2 |
| $\begin{gathered} \mathrm{I}_{\text {BPF2 } 2} / \mathrm{I}_{\text {BPF1 }} \\ \mathbf{B}_{\mathrm{Q}}=11111111 \end{gathered}$ | $\mu$ | 0.982 | 0.986 | 0.989 | 0.991 | 1.01 | 1.01 | 1.01 | 1.00 | 1.00 |
|  | $3 \sigma$ | 26.6 m | 27.7 m | 27.1 m | 28.6 m | 24.7 m | 23.4 m | 23.6 m | 23.7 m | 25.4 m |
|  | $3 \sigma / \mu(\%)$ | 2.7 | 2.8 | 2.7 | 2.9 | 2.4 | 2.3 | 2.3 | 2.4 | 2.5 |
| $\begin{gathered} \sqrt{\mathrm{I}_{\text {BPF } 1} \times \mathrm{I}_{\mathrm{BPF} 2}} \\ \mathbf{B}_{\mathbf{Q}}=00101111 \end{gathered}$ | $\mu$ (A) | 50.1n | 22.0n | 9.63n | 4.21n | 1.83n | 798p | 346p | 152p | 76.8p |
|  | $3 \sigma$ (A) | 556p | 234p | 102p | 46.1p | 21.9p | 9.13p | 3.92p | 1.79p | 899f |
|  | $3 \sigma / \mu(\%)$ | 1.1 | 1.1 | 1.1 | 1.1 | 1.2 | 1.1 | 1.1 | 1.2 | 1.2 |
| $\begin{gathered} \mathrm{I}_{\text {BPF2 }} / \mathrm{I}_{\text {BPF1 }} \\ \mathbf{B}_{\mathbf{Q}}=00101111 \end{gathered}$ | $\mu$ | 2.79 | 2.80 | 2.81 | 2.81 | 2.85 | 2.85 | 2.84 | 2.84 | 2.83 |
|  | $3 \sigma$ | 78.8 m | 85.3 m | 84.2 m | 88.1 m | 73.0 m | 70.7 m | 70.9 m | 71.9 m | 75.2 m |
|  | $3 \sigma / \mu(\%)$ | 2.8 | 3.0 | 3.0 | 3.1 | 2.6 | 2.5 | 2.5 | 2.5 | 2.7 |



Figure 4.5. 3-bit programmable capacitive attenuator.
gramming bits is 8 , and in turn $\mathrm{I}_{\mathrm{BPF1}}$ is always larger than $0.5 \mathrm{I}_{\text {chi. }}$. The design rationale is that $\mathrm{I}_{\mathrm{BPF1}}$ less than $0.5 \mathrm{I}_{\text {chi }}$ is not needed for $Q$ larger than 1 as will be clear in Section 4.2.4. The simplified complete inchannel bias distribution network including the TLL and the R-2R DAC is shown in Figure 4.4. The $\mathrm{I}_{\text {chi }}$ is directly the current generated by the current-splitting circuit in Figure $4.2 . \mathrm{I}_{\text {BPF3 }}$ and $\mathrm{I}_{\text {BPF4 }}$ are for the second filter biquad, and $\mathrm{I}_{\text {others }}$ are distributed to the other building blocks including the PGA and ADM.

Monte Carlo simulation is performed to evaluate the variation of the multiplication of $\mathrm{I}_{\mathrm{BPF} 1}$ and $\mathrm{I}_{\mathrm{BPF} 2}$, and the division of $\mathrm{I}_{\mathrm{BPF} 2}$ over $\mathrm{I}_{\mathrm{BPF} 1}$. The results are listed in Table 4.2. Comparing with the channel biases in Table 4.1, the largest deviation of the square root of $\mathrm{I}_{\mathrm{BPF1} 1} \times \mathrm{I}_{\mathrm{BPF} 2}$ is about $0.55 \%$ at channel 32 with $\mathbf{B}_{\mathbf{Q}}=11111111$. The $3 \sigma$ variation is within $1.2 \%$, and together with the variation of the channel bias $\mathrm{I}_{\mathrm{chi}}$ itself, the total $3 \sigma$ variation is within $1.5 \%$ at the worst case channel 63 , which is still below $1 / 3$ of $5.4 \%$. The ratio $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\mathrm{BPF} 1}$ is related to the $Q$ of the BPFs, and its variation is more important for high $Q$ settings because the central frequency gain and $Q$ value are more sensitive to $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\mathrm{BPF} 1}$ variation as will be discussed in Section 4.2.4. $\mathbf{B}_{\mathbf{Q}}=00101111$ usually sets a high $Q$, and the worst $3 \sigma$ variation is about $3.1 \%$.

### 4.2.3 Programmable Capacitive Attenuator

A fixed capacitive attenuator has been used in a prior bionic ear chip to increase the maximum allowable input amplitude with a $5 \%$ output THD [28], and in wireless transceivers programmable resistive at-


Figure 4.6. (a) The complete RC-equivalent circuit of the attenuator including all important parasitics; (b) consider only the pseudo-resistor $\mathrm{R}_{\mathrm{DC}}$; (c) consider only the ON-resistance and parasitic capacitance when switches are connected to $\mathrm{V}_{\mathrm{in}}$; (d) consider only the ON -resistance when switches are connected to ground.
tenuators are used to limit signal power in both the receiving and transmitting paths [130]. In this work, a programmable capacitive attenuator is used before the main BPF to allow flexible linearity control in accordance with the input audio power. The circuit schematic is shown in Figure 4.5. 8 attenuation levels from 0 dB to -18 dB are selected by 3 digital bits $\mathbf{b}_{\text {atti }}(\mathrm{i}=[1,2,3])$. Because the output $V_{\text {out }}$ is directly connected to the gate of the BPF input transistor, a pseudo-resistor $\mathrm{R}_{\mathrm{DC}}$ is used to set the output DC level to $\mathrm{V}_{\text {ref }}$ whose generation will be discussed in Section 4.2.4. $\mathrm{R}_{\mathrm{DC}}$ is composed of two pFETs $\mathrm{M}_{3}$ and $\mathrm{M}_{4}$ with the gates and bulks connected to their own sources. The switches are connected to the left plates of the capacitors because their leakage currents can pull $V_{\text {out }}$ to largely deviate from $\mathrm{V}_{\text {ref }}$ if they are at the same side of $\mathrm{R}_{\mathrm{DC}}$.

The RC-equivalent circuit considering the parasitic resistance $\left(R_{i n}\right.$ and $\left.R_{o n}\right)$ and capacitance $\left(C_{p}\right)$ of the switches is shown in Figure 4.6(a). The capacitors and $R_{D C}$ form a highpass filter whose corner frequency needs to be at least lower than the lowest central frequency in cochlea channels, i.e. 32 Hz . The resistance of $\mathrm{R}_{\mathrm{DC}}$ is positively correlated with the $W / L$ of the pFETs. A conservative design with $W / L=0.5 / 5$ in $\mu \mathrm{m}$ gives a resistance that is larger than 56 G ohm at room temperature. With the simplified circuit in Figure 4.6(b), the highpass corner frequency $f_{c 1}$ is computed as:

$$
f_{c 1}=\frac{1}{2 \pi \times 8 C_{u} R_{D C}}
$$

With a relaxed $f_{c 1}=0.4 \mathrm{~Hz}<32 \mathrm{~Hz}, \mathrm{C}_{\mathrm{u}}$ is calculated to have the capacitance of about 910 fF which occupies an area of $30 \times 30 \mu \mathrm{~m}^{2}$. Larger $L$ of the pFET in $\mathrm{R}_{\mathrm{DC}}$ results in larger resistance, and consequently the capacitance and area of $\mathrm{C}_{\mathrm{u}}$ can be reduced.

When switches are connected to $V_{i n}$, the pole/zero pair caused by the parasitic resistance $\mathrm{R}_{\text {in }}$ and capacitance $\mathrm{C}_{\mathrm{p}}$ limits the attenuator bandwidth. The simplified circuit in Figure 4.6(c) assumes that the individual switch resistances and the capacitors between $V_{i n}$ and $V_{\text {out }}$ are lumped together. Let $C_{x}=a \cdot C_{u}$, the pole and zero are located at:

$$
\omega_{p}=\frac{1}{\left(C_{p}+a C_{u}-a^{2} C_{u} / 8\right) R_{i n}}, \omega_{z}=\frac{a+1}{\left(C_{p}+a C_{u}\right) R_{i n}}
$$

With $C_{p} « C_{u}$, the lowest $\omega_{p}$ is obtained when $a=4$, and the value of $\omega_{p}$ and $\omega_{z}$ are:

$$
\omega_{p}=\frac{1}{2 C_{u} R_{i n}}, \omega_{z}=\frac{1}{0.8 C_{u} R_{i n}}
$$



Figure 4.7. (a) AC characteristics of the attenuator. Eight gain levels have flat response within the cochlea passband; (b) the attenuator's output IM3 at $V_{\text {in }}=1 \mathrm{~V}_{\mathrm{pp}}$ and $\mathbf{B}_{\mathrm{att}}=111$.
$\omega_{p}$ should be larger than the highest cochlea channel frequency 20 k Hz , which gives the lower limit of the $W / L$ ratio of the complimentary $\mathrm{pFET} \mathrm{M}_{1}$ and $\mathrm{nFET} \mathrm{M}_{2}$ whose bulk voltages are biased at $\mathrm{V}_{\text {mid }}=0.25 \mathrm{~V}$ to further reduce the equivalent resistance. Another factor that constrains the ON-resistance of the switch is distortion. The odd-order harmonics mainly the $3{ }^{\text {rd }}$-order are of concern because the even-orders are largely suppressed thanks to the differential input. The theoretical analysis in [130] shows that with a given input amplitude, the $3^{\text {rd }}$-order intermodulation resulted from a single transistor switch is proportional to its cubic ON-resistance. Usually a $5 \%$ output THD is targeted in applications like cochlea implant [28], [29], and it corresponds to only a -16 dBc IM3 if HD3 is the main distortion component. In biological basilar membrane, the IM3 can decrease from about -17 dBc at 30 SPL fundamental tones spaced with a 1.1 ratio to less than -50 dBc at 90 SPL [131]. In this design, an $\mathrm{IM} 3<-60 \mathrm{dBc}$ at a 20 k Hz differential input of $V_{i n}=1 \mathrm{~V}_{\mathrm{pp}}$ and $\mathbf{B}_{\mathrm{att}}=111$ (no attenuation) is specified not to limit the system performance, and with the sizes of $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ shown in Figure 4.5, an IM3=-66 dBc is obtained in simulation at room temperature and slow process corner. IM3 decreases as input frequency decreases because it is inversely proportional to the cubic capacitor impedance [130].

When switches are connected to ground via $n F E T M_{0}$, the ON-resistance $R_{\text {on }}$ in series with the capacitor $\mathrm{C}_{\mathrm{y}}$ as shown in Figure 4.6(d) also creates a pole/zero pair which are located at:

$$
\omega_{p}=\frac{1}{\left(\frac{1}{8}+\frac{C_{x}}{8 C_{u}}\right) C_{y} R_{o n}}, \omega_{z}=\frac{1}{C_{y} R_{o n}}
$$

It is obvious that $\omega_{z} \leq \omega_{p}$, and hence $\mathrm{R}_{\mathrm{on}}$ should be sufficiently low so that $\omega_{z} / 2 \pi$ is at least larger than 20 $\mathrm{kHz} . \mathrm{M}_{0}$ is chosen to have the same size of $\mathrm{M}_{2}$ which can guarantee a sufficiently small $\mathrm{R}_{\mathrm{on}}$. To summarize the attenuator design, the simulation results of the transfer function and IM3 as the function of input signal frequency with a $V_{i n}=1 \mathrm{~V}_{\mathrm{pp}}$ and $\mathbf{B}_{\mathrm{att}}=111$ are plotted in Figure 4.7.

### 4.2.4 Source-Follower-Based Asymmetrical Bandpass Filter

Many types of active filters have been proposed for various applications ranging from wireless and wireline communication to biomedical signal acquisition. Opamp-RC, $\mathrm{g}_{\mathrm{m}}-\mathrm{C}$ and switched-cap are among the most widely used types, and more recently developed ones including the ring-oscillator-based [132]


Figure 4.8. (a) The original SF-based LPF biquad with local positive feedback via $n F E T s \mathrm{M}_{2}$ and $\mathrm{M}_{4}$, the biases for the $n F E T$ pairs $M_{1} / M_{3}$ and $\mathrm{M}_{2} / \mathrm{M}_{4}$ are the same, i.e. $\mathrm{I}_{0}$ [134], [135]; (b) the alternative SF-based LPF biquad with separate biases $I_{1}$ and $I_{2}$ for $n F E T s M_{1} / M_{3}$ and pFETs $M_{2} / M_{4}$ respectively [136], [137].

(a)

(b)

Figure 4.9. The RC-equivalent circuits for (a) the SF-based LPF biquad in Figure 4.8(a) and (b) the SFbased LPF biquad in Figure 4.8(b).
and switched-Opamp-based [133] specifically aim for systems with low supply voltages down to 0.55 V . If power efficiency is the most paramount requirement in system design, the source-follower-based (SFbased) filter is the best choice because of its simple topological configuration using minimal number of transistors. The patented SF-based LPF biquads are depicted in Figure 4.8. The first version of a $2^{\text {nd }}$-order biquad proposed in 2006 is shown in Figure 4.8(a) [134], [135]. The nFET pairs $M_{1} / M_{3}$ and $M_{2} / M_{4}$ form the composite source follower structure. To avoid gain loss, the bulks of the nFETs should be connected to their sources, which requires deep-Nwell in a p-substrate process. The bias current $\mathrm{I}_{0}$ and the capacitors $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ determine the LPF cutoff frequency. The nFETs can be equivalently seen as resistors which have the same small-signal resistance as in Eq. (4.1) if the nFETs are in weak inversion:

$$
\begin{equation*}
R=1 / g_{m}=\frac{V_{T}}{\kappa I_{0}} \tag{4.1}
\end{equation*}
$$

where $g_{m}$ is the transconductance, $V_{T}$ is the thermal voltage and $\kappa$ is the subthreshold slope factor. Note that all the transconductance symbols below like $g_{m}$ or $g_{m i}\left(i \in N_{0}\right)$ represent the reciprocal of the corresponding resistance symbols like R or $\mathrm{R}_{\mathrm{i}}$. The RC-equivalent circuit is illustrated in Figure 4.9(a). The equations related to the small-signal currents $I_{1}$ and $I_{2}$ are:

$$
I_{1}=\frac{V_{\text {in }}-V_{x}}{R}=I_{2}+V_{x} \cdot s C_{1}, I_{2}=-\frac{V_{x}+V_{\text {out }}}{R^{*}}=V_{\text {out }} \cdot s C_{2}
$$

Note that $\mathrm{R}^{*}$ has the same value as R , and the unusual expression of $I_{2}$ is due to the local positive feedback in the cross-coupled transistors. The lowpass transfer function can be then derived as:

(a)

(b)

(c)

Figure 4.10. The SF-based BPF biquads in (a) [141]; (b) [142]; (c) [143].


Figure 4.11. The RC-equivalent circuits of the BPF biquads in Figure 4.10.

$$
\begin{equation*}
H_{\text {out }}(s)=\frac{V_{\text {out }}(s)}{V_{\text {in }}(s)}=-\frac{1}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m}^{2}}+s \cdot \frac{C_{1}}{g_{m}}+1} \tag{4.2}
\end{equation*}
$$

Thanks to the local positive feedback, synthesis of complex poles becomes possible, and the quality factor can exceed 0.5 which allows steeper roll-off. This composite SF-based biquad was used to build a $4^{\text {th }}-$ order Bessel LPF which was fabricated in $0.18 \mu \mathrm{~m}$ CMOS and had a 10 MHz cutoff frequency and 79-dB dynamic range with a 4.1 mW power consumption.

An alternative way of constructing the biquad is shown in Figure 4.8(b), which was published in 2008 [136], [137]. Instead of stacking the normal nFET pair on top of the cross-coupled nFET pair, the two pairs are separated in two branches, and hence their bias currents do not have to be the same, although in [136] the same bias currents were used. Even though this topology does not have the current-reuse advantage as the one in Figure 4.8(a), the flexibility of choosing different biasing currents for $M_{1} / M_{3}$ and $\mathrm{M}_{2} / \mathrm{M}_{4}$ makes $Q$ tuning easy as will be evident later. With the aid of the RC-equivalent circuit in Figure 4.9 (b), the lowpass transfer function can be derived as:

$$
\begin{equation*}
H_{\text {out }}(s)=-\frac{1}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 1} g_{m 2}}+s \cdot\left(\frac{C_{1}}{g_{m 1}}+\frac{C_{2}}{g_{m 2}}-\frac{C_{2}}{g_{m 1}}\right)+1} \tag{4.3}
\end{equation*}
$$

Three of this biquad were cascaded to build a $6^{\text {th }}$-order LPF for UWB applications, which was fabricated in $0.13 \mu \mathrm{~m}$ CMOS and had a 280 MHz cutoff frequency with a $120 \mu \mathrm{~W}$ power consumption. Compared to the state-of-the-art $\mathrm{g}_{\mathrm{m}}$-C LPFs with large bandwidth [138]-[140], the SF-based solution retains the lowest power/pole/Hz with only slightly inferior linearity.


Figure 4.12. Proposed SF-based BPF biquad.
Bandpass filtering is needed in silicon cochlea for channel frequency division, and the previous works on building SF-based BPFs are first briefly summarized here [141]-[143]. The transistor implementations of the biquads and the corresponding RC-equivalent circuits are depicted in Figure 4.10 and Figure 4.11, respectively. The biquads (a) and (b) were for low-frequency biomedical applications, and (c) was for MHz-range IF baseband in heterodyne receivers. Their transfer functions can be derived as follows:

$$
\begin{gathered}
H_{a}(s)=-\frac{s \cdot \frac{C_{1}}{g_{m}}}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m}^{2}}+s \cdot \frac{C_{1}}{g_{m}}+1}, H_{b}(s)=-\frac{s \cdot \frac{C_{1}}{g_{m}}}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m}^{2}}+s \cdot\left(\frac{C_{1}}{g_{m}}+\frac{C_{2}}{g_{m}}\right)+1} \\
H_{c}(s)=\frac{\frac{g_{m 1}\left(s+\frac{g_{m 2}-g_{m 3}}{C_{1}}\right)}{s_{2}+s \cdot\left(\frac{g_{m 1}-g_{m 2}}{C_{1}}+\frac{g_{m 2}-g_{m 3}}{C_{2}}\right)+\frac{g_{m 1} g_{m 2}-g_{m 1} g_{m 3}+g_{m 2} g_{m 3}}{C_{1} C_{2}}}}{},
\end{gathered}
$$

The BPF biquad in Figure 4.10(a) has a straightforward RC-prototype as shown in Figure 4.11(a): a $1^{\text {st }}$-order HPF cascaded by a $1^{\text {st }}$-ordre LPF. The cross-coupled pFETs $\mathrm{M}_{2}$ and $\mathrm{M}_{4}$ allow the synthesis of complex poles, and in turn a $Q$ larger than 0.5 . $Q$ is equal to the square root of the capacitance ratio $C_{2} / C_{1}$, and for a $Q=10$, the ratio needs to be as large as 100 , which is not area-efficient. Also as will be discussed later, large $Q$ by large capacitance ratio results in its impractically high relative sensitivity to the variation of the $g_{m}$ ratio of $M_{2} / M_{4}$ over $M_{1} / M_{3}$. Higher-order BPFs can be built by repeatedly cascading the same biquads; however each cascading causes about $6-\mathrm{dB}$ gain loss in the passband due to the finite input impedance which is frequency dependent. The problem could be solved by inserting extra 2 T source followers between biquads. The BPF biquad in Figure 4.10 (b) does not have the gain loss problem in cascading owing to the gate input, but the lack of negative $\mathrm{g}_{\mathrm{m}}$ prevents the possibility of a $Q$ larger than 0.5 . The biquad in Figure 4.10(c) can both have $Q>0.5$ and avoid gain loss in cascading, nevertheless besides the complicated form of the transfer function, the major problem is that the values of $g_{m 1}, g_{m 2}$ and $g_{m 3}$ are not well-defined by bias currents and are dependent on the common-mode input voltage. This makes its transfer function very sensitive to any common mode fluctuation from input and power supply.

### 4.2.4.1 Proposed Source-Follower-Based BPF Biquad

The proposed SF-based BPF biquad used in cochlea channels are based on the LPF biquad in Figure 4.8(b). As depicted in Figure 4.12, this biquad has the same small-signal RC-equivalent circuit as in Figure 4.9 (b), except that the final output is not $V_{\text {out }}$ but the summation of $V_{x}$ and $V_{\text {out }}$. The transistor pairs $\mathrm{M}_{1} / \mathrm{M}_{3}$ and $\mathrm{M}_{2} / \mathrm{M}_{4}$ all have the bodies connected to their own sources to prevent gain loss, which requires


Figure 4.13. The impact of current source mismatch at (a) $Q=1$ and (b) $Q=10$.
deep-nWell for nFETs. The transfer function of $V_{x}$ can be derived as:

$$
\begin{equation*}
H_{x}(s)=\frac{s \cdot \frac{C_{2}}{g_{m 2}}+1}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 1} g_{m 2}}+s \cdot\left(\frac{C_{1}}{g_{m 1}}+\frac{C_{2}}{g_{m 2}}-\frac{C_{2}}{g_{m 1}}\right)+1} \tag{4.4}
\end{equation*}
$$

It is easy to obtain the bandpass transfer function by summing Eq. (4.3) and Eq. (4.4):

$$
\begin{equation*}
H_{B P F}(s)=H_{\text {out }}(s)+H_{x}(s)=\frac{s \cdot \frac{C_{2}}{g_{m 2}}}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 1} g_{m 2}}+s \cdot\left(\frac{C_{1}}{g_{m 1}}+\frac{C_{2}}{g_{m 2}}-\frac{C_{2}}{g_{m 1}}\right)+1} \tag{4.5}
\end{equation*}
$$

The central frequency $f_{0}$, quality factor $Q$ and passband gain $K$ are written as:

$$
\begin{equation*}
f_{0}=\frac{1}{2 \pi} \sqrt{\frac{g_{m 1} g_{m 2}}{C_{1} C_{2}}}, Q=\frac{\sqrt{\frac{g_{m 1} g_{m 2}}{C_{1} C_{2}}}}{\frac{g_{m 1}}{C_{1}}+\frac{g_{m 2}}{C_{2}}-\frac{g_{m 2}}{C_{1}}}, K=\frac{\frac{g_{m 1}}{C_{1}}}{\frac{g_{m 1}}{C_{1}}+\frac{g_{m 2}}{C_{2}}-\frac{g_{m 2}}{C_{1}}} \tag{4.6}
\end{equation*}
$$

In the simplest case, the same capacitance $C_{0}$ is used for both $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$, as in the case of the SOS-BPF in the original cochlea design [23]. The transfer function and the characteristic parameters are simplified as:

$$
H_{B P F}(s)=\frac{s \cdot \frac{C_{0}}{g_{m 2}}}{s^{2} \cdot \frac{C_{0}^{2}}{g_{m 1} g_{m 2}}+s \cdot \frac{C_{0}}{g_{m 2}}+1}, f_{0}=\frac{1}{2 \pi} \frac{\sqrt{g_{m 1} g_{m 2}}}{C_{0}}, Q=\sqrt{\frac{g_{m 2}}{g_{m 1}}}, K=1
$$

Or the same bias current $I_{0}$ is used for both $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$, as in the case of the original SF-based LPF in Figure 4.8(a). The transfer function and the characteristic parameters are simplified as:

$$
H_{B P F}(s)=\frac{s \cdot \frac{C_{2}}{g_{m 0}}}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 0}^{2}}+s \cdot \frac{C_{1}}{g_{m 0}}+1}, f_{0}=\frac{1}{2 \pi} \frac{g_{m 0}}{\sqrt{C_{1} C_{2}}}, Q=\sqrt{\frac{C_{2}}{C_{1}}}, K=\frac{C_{2}}{C_{1}}
$$

Although these two cases provide simple expressions of parameters which can be easily calculated, the following analysis will show that they cannot be practically employed in designs where relatively high $Q$, e.g. $Q=10$, is required. A design methodology is proposed to circumvent the problems.

First, the case of equal capacitance is considered. The main problem is identified as the mismatch of the upper and lower current sources ( $\mathrm{I}_{2 \mathrm{U}}$ and $\mathrm{I}_{2 \mathrm{D}}$ ) in the cross-coupled pFET branch, as illustrated in Figure 4.13. For simplicity, the capacitors and summation symbols are neglected. Assume the $g_{m}$ values of nFETs and pFETs are only functions of the bias currents. When $Q=1$, ideally $I_{2 U}=I_{2 D}=I_{1}=I_{0}$. If a $1 \%$ mismatch exists between $I_{2 U}$ and $I_{2 D}$, e.g. $I_{2 D}=1.01 I_{2 U}$ which is already not easy to achieve in MOS process, the currents flowing through the nFET and pFET pairs are $1.01 I_{0}$ and $I_{0}$, respectively. The central frequency considering the mismatch $f_{0 m i s}$ is calculated as:

$$
f_{0 m i s}=\frac{1}{2 \pi} \frac{\sqrt{1.01 g_{m 0} \cdot g_{m 0}}}{C_{0}}=1.005 f_{0}
$$

which means the actual central frequency is only $0.5 \%$ off the targeted value without mismatch. When $Q=10$, ideally $I_{2 U}=I_{2 D}=10 I_{0}$ and $I_{1}=0.1 I_{0}$. Still assume $I_{2 D}=1.01 I_{2 U}$. The currents flowing through the nFET and pFET pairs are $0.2 I_{0}$ and $10 I_{0}$, respectively. The central frequency considering the mismatch $f_{0 \text { mis }}$ is calculated as:

$$
f_{0 m i s}=\frac{1}{2 \pi} \frac{\sqrt{0.2 g_{m 0} \cdot 10 g_{m 0}}}{C_{0}}=1.414 f_{0}
$$

which means the actual central frequency is $41.4 \%$ off the targeted value without mismatch. This is unacceptable because as mentioned in Section 4.2.1, the variation of central frequency should be limited within $\pm 5.4 \%$.

To circumvent the $I_{2 U} / I_{2 D}$ mismatch problem, let us consider a more general case and write the capacitance and transconductance in the form below:

$$
\begin{equation*}
C_{1}=\frac{C_{0}}{k}, C_{2}=k C_{0}, g_{m 1}=\frac{g_{m 0}}{n}, g_{m 2}=n g_{m 0},(k, n \in R \bigcap \geq 1) \tag{4.7}
\end{equation*}
$$

The characteristic parameters in Eq. (4.6) can be then rewritten as:

$$
\begin{equation*}
f_{0}=\frac{g_{m 0}}{2 \pi C_{0}}, Q=\frac{1}{\frac{k}{n}+\frac{n}{k}-n k}, K=\frac{1}{1+\frac{n^{2}}{k^{2}}-n^{2}} \tag{4.8}
\end{equation*}
$$

Let $m_{i s}$ be the mismatch of the current sources and $t_{o l}$ the limit of tolerable relative central frequency variation. Assume $I_{2 D}=\left(1+m_{i s}\right) \cdot I_{2 U}=\left(1+m_{i s}\right) \cdot n \cdot I_{0}$, and $I_{1}=I_{0} / n$. The current flowing through $\mathrm{M}_{2} / \mathrm{M}_{4}$ is $n \cdot I_{0}$, and the current flowing through $\mathrm{M}_{1} / \mathrm{M}_{3}$ can be calculated as:

$$
I_{2 D}+I_{1}-n \cdot I_{0}=\frac{I_{0}}{n}+m_{i s} \cdot n \cdot I_{0}
$$

We have the following inequality:

$$
\sqrt{\left(\frac{1}{n}+m_{i s} \cdot n\right) \cdot n}-1 \leq t_{o l}
$$



Figure 4.14. Half-circuit of the proposed SF-based BPF biquad with noise sources shown in gray.
The scale parameter $n$ can be calculated to have the upper bound as:

$$
\begin{equation*}
n \leq \sqrt{\frac{t_{o l}^{2}+2 t_{o l}}{m_{i s}}} \tag{4.9}
\end{equation*}
$$

If $n$ is determined, the scale parameter $k$ can be accordingly calculated using the $Q$ expression in Eq. (4.8):

$$
\begin{equation*}
k=\frac{n-\sqrt{n^{2}-4 Q^{2} n^{2}\left(1-n^{2}\right)}}{2\left(1-n^{2}\right) Q} \tag{4.10}
\end{equation*}
$$

However, Eq. (4.9) only gives the upper limit of $n$; the determination of its lower limit is related to the impractical case of equal $g_{m}$. Define $r_{c}$ and $r_{g m}$ as follows:

$$
r_{c}=\frac{C_{2}}{C_{1}}=k^{2}, r_{g m}=\frac{g_{m 2}}{g_{m 1}}=n^{2}
$$

Using the $Q$ expression in Eq. (4.6), the relative sensitivity of $Q$ over $r_{g m}$ can be calculated as:

$$
S=\frac{\partial Q}{\partial r_{g m}} / \frac{Q}{r_{g m}}=\frac{1}{2} \frac{r_{c}-r_{g m}+r_{c} r_{g m}}{r_{c}+r_{g m}-r_{c} r_{g m}}
$$

If $r_{c}=1$, i.e. the equal capacitance case, $S$ is calculated to be 0.5 and independent of $r_{g m}$. With $r_{g m}=100$ for $Q=10$ and a $1 \%$ variation of $r_{g m}$, the variation of $Q$ is $0.5 \%$. If $r_{g m}=1$, i.e. the equal transconductance case, with $r_{c}=100$ for $Q=10, S$ is calculated to be 99.5 . This means a $1 \%$ variation of $r_{g m}$ results in $99.5 \%$ variation of $Q$. The $r_{g m}$ variation comes from both the variation of $I_{1}$ and $I_{2}$, and the parameter variation of the nFET and pFET pairs like the subthreshold slope factor. Now we can see why the equal transconductance case is impractical as well in terms of a well-controlled $Q$ value that should be relatively insensitive to $r_{g m}$ variation. If the upper limit of $S$ is specified as $S_{0}$, the inequality below can be obtained:

$$
\left(2 S_{0}+1\right) \cdot n^{2}-2 Q \cdot n+1-2 S_{0} \geq 0
$$

With $S_{0} \geq 0.5$, the lower limit of $n$ is calculated as:

$$
\begin{equation*}
n \geq \frac{Q+\sqrt{Q^{2}+4 S_{0}^{2}-1}}{2 S_{0}+1} \tag{4.11}
\end{equation*}
$$

Therefore the value of the scale parameter $n$ can be found between the upper and lower bounds calculated


Figure 4.15. Simulated noise power spectrum density (PSD) of the proposed biquad at (a) $Q=1$ and (b) $Q=10$.
by Eq. (4.9) and Eq. (4.11), respectively. With $m_{i s}=0.02, t_{o l}=0.025$ and $S=20$, the range of $n$ at $Q=10$ is calculated to be $1.249 \leq n \leq 1.591$. In the fabricated design, $r_{c}$ is set to be $26 / 15$, and using Eq. (4.10) $n$ is calculated to be 1.450 at $Q=10$, which is within the targeted range. This design methodology is verified by the results of Monte Carlo simulation of the complete asymmetrical BPF in Section 4.2.4.3.

### 4.2.4.2 Noise and Linearity

The half-circuit of the proposed SF-based BPF biquad with noise sources is shown in Figure 4.14. $V_{n 1}$ and $V_{n 2}$ are the equivalent gate noise voltages of transistors $\mathrm{M}_{1} / \mathrm{M}_{3}$ and $\mathrm{M}_{2} / \mathrm{M}_{4}$, respectively, and $V_{n l 1}$, $V_{n 12 D}$ and $V_{n I 2 U}$ are of the current sources $\mathrm{I}_{1}, \mathrm{I}_{2 \mathrm{D}}$ and $\mathrm{I}_{2 \mathrm{U}}$, respectively. The noise transfer function of $V_{n 1}$, $H_{n 1}(s)$, is the same as the BPF transfer function $H_{B P F}(s)$ given in Eq. (4.5). The noise transfer functions of $V_{n 2}, V_{n I 1}, V_{n I 2 D}$ and $V_{n I 2 U}$ are denoted as $H_{n 2}(s), H_{n 11}(s), H_{n I 2 D}(s)$ and $\left|H_{n I 2 U}(s)\right|$, respectively, and have the expressions shown below:

$$
\begin{gathered}
\left|H_{n 2}(s)\right|=\left|\frac{s \cdot \frac{C_{2}-C_{1}}{g_{m 1}}-1}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 1} g_{m 2}}+s \cdot\left(\frac{C_{1}}{g_{m 1}}+\frac{C_{2}}{g_{m 2}}-\frac{C_{2}}{g_{m 1}}\right)+1}\right|,\left|H_{n I 1}(s)\right|=\left|\frac{s \cdot \frac{g_{m I 1} C_{2}}{g_{m 1} g_{m 2}}}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 1} g_{m 2}}+s \cdot\left(\frac{C_{1}}{g_{m 1}}+\frac{C_{2}}{g_{m 2}}-\frac{C_{2}}{g_{m 1}}\right)+1}\right| \\
\left|H_{n I 2 D}(s)\right|=\left|\frac{s \cdot \frac{g_{m I 2 D} C_{2}}{g_{m 1} g_{m 2}}}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 1} g_{m 2}}+s \cdot\left(\frac{C_{1}}{g_{m 1}}+\frac{C_{2}}{g_{m 2}}-\frac{C_{2}}{g_{m 1}}\right)+1}\right|,\left|H_{n I 2 U}(s)\right|=\left|\frac{s \cdot \frac{g_{m I 2 U} C_{1}}{g_{m 1} g_{m 2}}+\frac{g_{m I 2 U}}{g_{m 2}}}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 1} g_{m 2}}+s \cdot\left(\frac{C_{1}}{g_{m 1}}+\frac{C_{2}}{g_{m 2}}-\frac{C_{2}}{g_{m 1}}\right)+1}\right|
\end{gathered}
$$

Note that in subthreshold, $g_{m I 1} \approx g_{m 1}$ and $g_{m I 2 D} \approx g_{m I 2 U} \approx g_{m 2}$. The zeros of $\left|H_{n 2}(s)\right|$ and $\left|H_{n I 2 U}(s)\right|$ are located at:

$$
\omega_{z H n 2}=\frac{g_{m 1}}{C_{2}-C_{1}}, \omega_{z H n 3}=\frac{g_{m 1}}{C_{1}}
$$

These zeros are either larger or comparable to the angular central frequency $\omega_{0}=2 \pi f_{0}$ of $H_{B P F}(s)$, and therefore $\left|H_{n 2}(s)\right|$ and $\left|H_{n I 2 U}(s)\right|$ are almost flat below $\omega_{0}$, and behave similarly to a lowpass transfer function. On the other hand, $\left|H_{n I 1}(s)\right|$ and $\left|H_{n I 2 D}(s)\right|$ has a bandpass shape that is the same as $H_{B P F}(s)$. Consequently
at low $Q$, the major noise contributions to the output $V_{B P F}$ are from $V_{n 2}$ and $V_{n I 2 U}$, i.e. the pFET pair $\mathrm{M}_{2} / \mathrm{M}_{4}$ and the current source pair $\mathrm{I}_{2 \mathrm{U}}$. This is validated by the power spectrum density (PSD) simulation of the output referred noise ( ORN ) at $Q=1$ and $f_{0}=20 \mathrm{k} \mathrm{Hz}$ and the individual transistor noise contribution as shown in Figure 4.15(a). Note that the noise curves of $\mathrm{I}_{1}$ and $\mathrm{I}_{2 \mathrm{D}}$ almost overlap. Because of the lowpassalike behavior of $H_{n 2}(s)$, the use of pFETs for $\mathrm{M}_{2} / \mathrm{M}_{4}$ is beneficial in terms of lower flick noise compared to nFETs with the same area. Be aware that all the noise simulation results in this chapter are obtained by using the process parameters from UMC even though the chip is fabricated in TowerJazz who admitted that their noise parameter extraction in subthreshold is extremely unreliable. As $Q$ increases, the low frequency noise PSD decreases because $g_{m 2}$ increases, but the in-band noise PSD increases significantly because of gain peaking as shown in Figure 4.15 (b) where $Q=10$. The most dominant in-band noise contributions are from the current sources $\mathrm{I}_{2 \mathrm{U}}$ and $\mathrm{I}_{2 \mathrm{D}}$. This can be attributed to their large in-band gain. For comparison, the ratios of the central frequency gain of the noise transfer functions to the BPF's central frequency gain $K$ given in Eq. (4.6) are listed below:

$$
\frac{K_{n 1}}{K}=1, \frac{K_{n 2}}{K}=\sqrt{\frac{\left(C_{2}-C_{1}\right)^{2} g_{m 2}^{2}}{C_{2}^{2} g_{m 1}^{2}}+\frac{C_{1} g_{m 2}}{C_{2} g_{m 1}}}, \frac{K_{n I 1}}{K} \approx 1, \frac{K_{n I 2 D}}{K} \approx \frac{g_{m 2}}{g_{m 1}}, \frac{K_{n I 2 U}}{K} \approx \sqrt{\frac{C_{1}^{2} g_{m 2}^{2}}{C_{2}^{2} g_{m 1}^{2}}+\frac{C_{1} g_{m 2}}{C_{2} g_{m 1}}}
$$

Using the capacitance and transconductance ratios calculated in Section 4.2.4.1, the gain ratios are:

$$
\frac{K_{n 1}}{K}=1, \frac{K_{n 2}}{K}=1.44, \frac{K_{n I 1}}{K} \approx 1, \frac{K_{n 12 D}}{K} \approx 2.16, \frac{K_{n \mid 2 U}}{K} \approx 1.68
$$

These calculated values are approximately consistent with the simulation results. In both low and high $Q$ cases, the dominant output noise contributions are related to the noise sources of the cross-coupled branch. The noise PSD can only be reduced by larger bias currents and in turn larger capacitor area.

It is well-known that MOSFET in above-threshold is superior compared to BJT in terms of smaller distortion because the latter have an exponential I-V characteristics while the former generates no $3^{\text {rd }}-$ order distortion if the ideal square law is assumed [106]. However, with a $0.5-\mathrm{V}$ power supply, the transistor pairs $M_{1} / M_{3}$ and $M_{2} / M_{4}$ work in deep-subthreshold, and the I-V curve becomes exponential, resembling BTJ's. While the SF-based LPFs with above-MHz cutoff frequencies for transceiver applications can easily have less than -40 dBc IM3 with hundreds of mV input amplitude especially when the signal frequency is far below the cutoff frequency [134], [136], [144], the kHz-range SF-based BPFs usually exhibit merely around -30 dBc IM3 with tens of mV input signals at low $Q$ [141], [142], and the input is more limited at high $Q$ because of increased passband gain. The inferior linearity performance in those low-frequency BPFs is attributed to both subthreshold operation of MOSFETs which is the same reason for poor linearity of low-frequency biomedical $\mathrm{g}_{\mathrm{m}}-\mathrm{C}$ filters [145], and the restricted effective operating frequency range where the nonlinear V-I conversion occurs more than the case of SFB-LPF at frequencies well below its cutoff frequency. In other words, at low frequencies, the intrinsic negative feedback due to the current source in a source follower allows the input transistor to have negligible voltage-to-current conversion which is the source of nonlinearity, and the input and output signals are in phase; as input frequency increases, the loading capacitance at the output reduces the effective impedance of the current source, so the voltage-to-current conversion of the input transistor becomes significant, and the phase difference between input and output signals increases. Nevertheless, our biological basilar membrane in cochlea also has only around -30 dBc IM3 at 60 SPL input which is the loudness of our normal speech [131], and we still perform recognition task well. This of course relies on the functions of the intricate neural networks in our auditory cortex.


Figure 4.16. Complete asymmetrical SF-based BPF with 4 poles and 1 zero. The building blocks, including the SF-biquad, the input DC reference generator, and the bias are respectively illustrated.

The $0.5-\mathrm{V}$ supply enforces boundaries on signal swing and transistor voltage overhead. The branch with cross-coupled pFETs is determinant because three transistors are stacked between ground and VDD. The expression below holds:

$$
V_{s i g, s w i n g}+2 V_{d s, s a t}+V_{g s}=0.5
$$

A minimum $100-\mathrm{mV} V_{d s, s a t}$ needs to be ensured for the saturation of the upper and lower current sources because the overall linearity can benefit from their larger output resistance [144]. If a $50-\mathrm{mV}$ single-ended signal swing $V_{\text {sig,swing }}$ is assumed, the pFET pair $\mathrm{M}_{2} / \mathrm{M}_{4}$ can have a maximum $V_{g s}$ of 250 mV , and the $W / L$ ratio can be determined accordingly with a specific bias current. The bias current, as mentioned in the noise part, is related to the required noise PSD and the BPF central frequency.

### 4.2.4.3 4-Pole, 1 -Zero 4 $^{\text {th }}$-Order Asymmetrical BPF

The BPFs in silicon cochlea are usually designed to be asymmetrical to mimic the biological cochlea transfer function. The original SOSs were cascaded to obtain the accumulatively steeper lowpass roll-off while the highpass roll-off is kept to be $1^{\text {st }}$-order [23], [24]. For parallel architectures, one example is the one-zero gammatone filter transfer function which was implemented by cascading four log-domain cur-rent-mode biquads where three are LPFs and one is BPF, and therefore it has 8 poles and 1 zero [29]. Another recent example is constructed by cascading one $2^{\text {nd }}$-order BPF, one $2^{\text {nd }}$-order tunable LPF and one $5^{\text {th }}$-order elliptic LPF, which makes it a 9-pole 1-zero filter [146]. Symmetrical $4^{\text {th }}$-order BPFs have also


Figure 4.17. Simulated transfer functions of the $4^{\text {th }}$-order asymmetrical BPF at different $Q^{\prime}$ s. The right graph is the zoomed-in around the central frequency of the left graph.

Table 4.3. $Q$ value as the function of the bias ratio $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\mathrm{BPF} 1}$.

| $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\text {BPF1 }}$ | Q |
| :---: | :---: |
| 1 | 1.43 |
| 1.5876 | 2.00 |
| 2.1492 | 3.01 |
| 2.4649 | 4.04 |
| 2.6504 | 5.02 |
| 2.7889 | 6.03 |
| 2.8866 | 7.02 |
| 2.9653 | 8.03 |
| 3.0276 | 9.04 |
| 3.0800 | 10.09 |
| 3.1684 | 12.46 |
| 3.2364 | 15.23 |
| 3.2797 | 17.65 |
| 3.3197 | 20.63 |

been used [28], and it is not clear yet how the BPF order can have impact on some post-processing tasks. For example, simple $2^{\text {nd }}$-order BPFs that are used in a voice activity detector with a 16 -channel cochlealike analog frontend and a subsequent mixed-signal processing stage implementing a decision-tree machine learning algorithm seems to provide reasonable classification accuracy [42]. However, good frequency selectivity that is associated with relatively high $Q$ and steep roll-off is speculated to be of importance in more advanced tasks like speech recognition considering the ability of fine-pitch discrimination in human auditory systems. Steeper roll-off normally comes from higher filter orders at the cost of more power consumption in parallel architectures. One exception could be a previous work from CSEM Neuchâtel, i.e. the bank of a hundred $2^{\text {nd }}$-order resonators coupled via resistive network which creates several hundreds of $\mathrm{dB} /$ decade lowpass roll-off because of destructive interferences of properly weighted frequency components in adjacent channels [31]. Besides the poor monotonicity of BPF central frequency scaling, the penalty is largely reduced dynamic range from 50 dB to 25 dB when the coupling is turned on.

A reasonable tradeoff between power consumption and cutoff steepness is to choose a moderate filter order, like the symmetrical $4^{\text {th }}$-order BPF in [28]. In this design, an asymmetrical $4^{\text {th }}$-order BPF is implemented with 4 poles and 1 zero, as illustrated in Figure 4.16. Two cascaded SF-based LPF biquads give rise to 4 poles, and the summation of $V_{x}$ and $V_{\text {out }}$ to obtain the final output $V_{B P F}$ at the second biquad produces 1 zero. Several design considerations of the proposed asymmetrical BPF are listed below:

1) As will be discussed later in Section 4.2.4.4, the summation at the $2^{\text {nd }}$ biquad is implemented via a capacitively-coupled PGA (CC-PGA). The input capacitors $\mathrm{C}_{0}$ of the PGA are in effect the capacitive loading of the $2^{\text {nd }} \mathrm{SF}$-biquad and determine the pole positions. The four $\mathrm{C}_{0}$ 's need to have the same capacitance in order to make the same gain for both $V_{x}$ and $V_{o u t}$ during summation, as required by the principle of the BPF transfer function generation shown in Eq. (4.5). We already know that with the same capacitance at $V_{x}$ and $V_{\text {out }}$, obtaining high $Q$ by tuning the ratio of $\mathrm{I}_{\text {BPF4 }}$ over $\mathrm{I}_{\text {BPF3 }}$ is unacceptable because of significant central frequency variation. Therefore, the task of $Q$-tuning is borne by the $1^{\text {st }}$ biquad with $C_{2} / C_{1}=26 / 15$. To superimpose the resonance frequencies of the two biquads, the capacitances should satisfy $C_{1} \times C_{2}=C_{0} \times C_{0}$, and the bias currents should satisfy $\mathrm{I}_{\mathrm{BPF1}} \times \mathrm{I}_{\mathrm{BPF} 2}=\mathrm{I}_{\mathrm{BPF} 3} \times \mathrm{I}_{\mathrm{BPF} 4}$, where


Figure 4.18. $3 \sigma$ variation of (a) $Q$, (b) central frequency and (c) peak gain at different mean $Q$ obtained from 250 -run Monte Carlo simulations of the $20 \mathrm{k}-\mathrm{Hz}$ channel.
$\mathrm{I}_{\mathrm{BPF} 1}$ and $\mathrm{I}_{\mathrm{BPF} 2}$ are tunable via the DAC and TTL described in Section 4.2.2, and $\mathrm{I}_{\mathrm{BPF} 3}$ and $\mathrm{I}_{\mathrm{BPF} 4}$ are the same as the channel bias $\mathrm{I}_{\text {chi }}$ generated by the circuit in Figure 4.2.
2) The $Q$-tuning DAC illustrated in Figure 4.3 (b) has a 9 -bit resolution even though the number of digital controlling bits is 8 . The DAC's output is the BPF bias current $\mathrm{I}_{\mathrm{BPF} 1}$, and $\mathrm{I}_{\mathrm{BPF} 2}$ is generated via the TLL. The resolution of the DAC is determined by the granularity of the $Q$-tuning steps. The simulated transfer functions of the asymmetrical BPF at different $Q$ s for the $20 \mathrm{k}-\mathrm{Hz}$ channel are shown in Figure 4.17, and the corresponding values of $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\mathrm{BPF} 1}$ are given in Table 4.3. It is easy to see that as $Q$ increases, the interval between the bias ratios becomes smaller. If a $Q$-tuning step of 1 is aimed when $Q<10$, the necessary number of tuning bits is calculated to be:

$$
-\log _{2}\left(\frac{1}{\sqrt{3.0276}}-\frac{1}{\sqrt{3.0800}}\right)=7.7
$$

At least 8 bits are needed for the aimed $Q$-tuning resolution. One extra bit, i.e. 9-bit resolution in total, is assigned to the DAC for some design margin. The lowest $Q$ value 1.43 is obtained when $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\mathrm{BPFl}}=1$. Lower $Q$ is not considered to be of much use because of its poor frequency selectivity, and for $Q$ within 20, the bias ratio is always less than 4 . These two factors help limit the tuning range of $\mathrm{I}_{\mathrm{BPF} 1}$ within [ $0.5 \mathrm{I}_{\text {chi }}, \mathrm{I}_{\text {chi }}$ ], and hence 8 controlling bits are sufficient for the DAC.
3) To verify the design methodology of controlling the variations of BPF's central frequency and $Q$ described in Section 4.2.4.1, 250 -run Monte Carlo simulations are performed on the complete $4^{\text {th }}$-order BPF of the $20 \mathrm{k}-\mathrm{Hz}$ channel and the results are shown in Figure 4.18. The x-axis is the mean $Q$ value, and the $y$-axes are the $3 \sigma$ variation of $Q$, central frequency and peak gain. Different $C_{2} / C_{1}$ ratios are used. $C_{2} / C_{1}=26 / 15$ is the one used in the fabricated chip. For the same $Q, C_{2} / C_{1}=23 / 17$ has more balanced capacitance $C$ and requires larger $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\mathrm{BPF} 1}$, whereas larger $C_{2} / C_{1}=39 / 10$ results in smaller $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\mathrm{BPF} 1}$, i.e. more balanced transconductance $g_{m}$. More balanced $g_{m}$ favors smaller central frequency variation, especially at high $Q$. This is validated by the curves in Figure 4.18(b). More balanced $C$, on the other hand, favors less $Q$ sensitivity on $g_{m}$ ratio. Figure 4.18 (a) indeed indicates that the $Q$ varia-


Figure 4.19. Simulated results at different $Q$ values of (a) the maximum BPF rms output with in-band IM3 $=-26 \mathrm{dBc}$ and $\mathrm{THD}=-40 \mathrm{~dB}$, respectively, (b) the output-referred noise (ORN) and (c) the calculated DR. All the results are from the $4^{\text {th }}$-order BPF of the $20 \mathrm{k}-\mathrm{Hz}$ channel.
tion of $C_{2} / C_{1}=26 / 15$ is smaller than that of $C_{2} / C_{1}=39 / 10$. The reason why the case of $C_{2} / C_{1}=23 / 17$ has a larger $Q$ variation than that of $C_{2} / C_{1}=26 / 15$ might be increased $g_{m}$ ratio variation due to increased $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\mathrm{BPF1}}$. The peak gain variation in Figure 4.18 (c) resembles the $Q$ variation. Except the tradeoff between the variations of central frequency and $Q$, practical circuit considerations also put constraints on the allowable $C_{2} / C_{1}$ ratio. If $C_{2} / C_{1}$ is too small, $\mathrm{I}_{\mathrm{BPF} 2} / \mathrm{I}_{\mathrm{BPF1}}$ needs to be very large for high $Q$ values, which significantly limits signal swing in the cross-coupled pFET branch especially under low supply voltage because of the increase $\mathrm{V}_{\mathrm{gs}}$ of $\mathrm{M}_{2} / \mathrm{M}_{4}$. If $C_{2} / C_{1}$ is too large, not only is it not area-efficient, but also a DAC with an impractically high resolution much higher than 9 bits is needed for fine $Q$-tuning. Therefore $C_{2} / C_{1}=26 / 15$ is an optimal choice, in terms of both BPF parameter variations and feasibility of circuit implementations.
4) The input DC level of each SF-biquad should be properly set so that the signal swing headroom at $V_{x}$ and $V_{\text {out }}$ can be maximized. $\mathrm{V}_{\text {ref1 }}$ and $\mathrm{V}_{\text {ref2 }}$ set the input DC level of the $1^{\text {st }}$ and the $2^{\text {nd }}$ SF-biquads separately via the pseudo-resistors, one in the attenuator as described in Section 4.2.3 and one as the $\mathrm{R}_{\mathrm{DC}}$ in Figure 4.16. The transistor implementation of $\mathrm{R}_{\mathrm{DC}}$ is the same as the one in the attenuator. With the very low $0.5-\mathrm{V}$ power supply, the two biquads are AC-coupled to avoid the impact of the output DC level variation of the $1^{\text {st }}$ biquad due to $Q$-tuning and process variation on the $2^{\text {nd }}$ biquad, in contrast to the three SF-based LPF biquads directly DC-cascaded in [136] with a $1.2-\mathrm{V}$ supply. The DC reference generator is used to stabilize the DC level of $V_{x}$ and set it to the same voltage as $\mathrm{V}_{\text {DCin }}$, if the nFET $M_{D C}$ size is proportionally scaled with respect to that of $M_{1} / M_{3}$ as their bias currents are. The downscaling of the channel bias current from high to low frequency channels reduces the $\mathrm{V}_{\mathrm{gs}}$ of $\mathrm{M}_{2} / \mathrm{M}_{4}$ to a value where the input signal swing can cause the pFETs to operate at the brink of triode region. To avoid this, the $W / L$ of $\mathrm{M}_{2} / \mathrm{M}_{4}$ as well as $\mathrm{M}_{1} / \mathrm{M}_{3}$ is scaled down every four other channels.
5) The linearity of the $4^{\text {th }}$-order BPF of the $20 \mathrm{k}-\mathrm{Hz}$ channel is evaluated by the in-band IM3 and THD. In the simulation of IM3, one of the two test tones is at the central frequency $f_{c}$ and the other is at $f_{c}+1 \mathrm{k}$. The maximum output rms amplitudes $V_{B P F o u t}$ with $\mathrm{IM} 3=-26 \mathrm{dBc}(5 \%)$ and $\mathrm{THD}=-40 \mathrm{~dB}(1 \%)$ are plotted in Figure 4.19(a) at different $Q$ values. Even though the THD requirement is more stringent, its


Figure 4.20. Circuit diagram of the fully differential capacitively-coupled PGA. The input signals $V_{\text {in } 1+} / V_{\text {in } 1-}$ and $V_{\text {in2 } 2} / V_{\text {in } 2}$ are from $V_{\text {out }} / V_{\text {out }}$ - and $V_{x+} / V_{x-}$ of the $2^{\text {nd }}$ SF-biquad in Figure 4.16.
maximum $V_{B P F o u t}$ values are still always higher than the ones bounded by the relaxed IM3 requirement, which is attributed to the filtering effect of out-of-band harmonics. To target for a $60-\mathrm{dB}$ DR at the lowest $Q$ using the THD metric, an output-referred noise (ORN) of less than $26.5 \mu \mathrm{~V}$ should be achieved. As discussed in Section 4.2.4.2, the output noise is dominated by the cross-coupled pFETs $\mathrm{M}_{2} / \mathrm{M}_{4}$ and the current sources $\mathrm{I}_{2 \mathrm{U}}$ at low $Q$. Because the output noise of the $1^{\text {st }} \mathrm{SF}$-biquad is filtered by the $2^{\text {nd }}$ one, the total ORN of the $4^{\text {th }}$-order BPF can be approximated by considering the noise contribution only from the $2^{\text {nd }}$ SF-biquad. If only thermal noise is considered, the ORN is analytical written as:

$$
\mathrm{ORN}=\sqrt{\frac{2 k T}{\kappa C_{0}}}
$$

A $C_{0} \geq 14.7 \mathrm{pF}$ is calculated with $\kappa=0.8$ and $T=300$. In simulation, a $C_{0}=10.13 \mathrm{pF}$ is found to be sufficient, and according to the ratio of $C_{2} / C_{1}=26 / 15, C_{1}=3.75 \mathrm{pF}$ and $C_{2}=6.51 \mathrm{pF}$ are used. The simulated ORN is shown in Figure 4.19(b). It is clear that the ORN increases as $Q$ increases due to gain peaking at the central frequency. The calculated DR is given in Figure 4.19(c). With the THD metric, the BPF DR is above 50 dB at all $Q$ values, and has a peak value of 60.3 dB ; with the IM3 metric, the highest DR is 58.1 dB and lowest 37.7 dB . The programmable attenuator described in Section 4.2 .3 can add another $18-\mathrm{dB}$ DR and makes the total maximum DR 78.3 dB at $Q=1.43$ with $\mathrm{THD}=-40 \mathrm{~dB}$, although it has no effect on improving SNR. Note that the maximum channel bias current 50 nA given in Section 4.2.1 is determined by the calculated capacitance values and the central frequency.

### 4.2.4.4 Summing Programmable Gain Amplifier

The fully-differential CC-PGA for summing the $V_{x}$ and $V_{\text {out }}$ of the second SF-biquad in Figure 4.16 to ultimately obtain the $4^{\text {th }}$-order asymmetrical BPF transfer function while simultaneously provides programmable gain is illustrated in Figure 4.20. The input signals $V_{i n 1+} / V_{\text {in1- }}$ and $V_{\text {in2 } 2} / V_{\text {in2- }}$ are connected to $V_{\text {out }} / V_{\text {out }}$ and $V_{x+} / V_{x}$, respectively. Owing to the virtual ground at the input of the Opamp with large


Figure 4.21. Circuit diagram of the fully-differential two-stage Opamp used in the CC-PGA in Figure 4.20 including the biasing and the common-mode feedback (CMFB) circuits.

Table 4.4. Body biasing of the transistors in the Opamp circuit in Figure 4.21.

| Transistors | First 32-channel | Last 32-channel |
| :--- | :--- | :--- |
| $\mathrm{M}_{1 \mathrm{a}}, \mathrm{M}_{\mathrm{b}}, \mathrm{M}_{5 \mathrm{a}}, \mathrm{M}_{5 \mathrm{~b}}$ | $\mathrm{~V}_{2}$ | 0.5 V |
| $\mathrm{M}_{22}, \mathrm{M}_{2 \mathrm{~b}}, \mathrm{M}_{6 \mathrm{a}}, \mathrm{M}_{6 \mathrm{~b}}, \mathrm{M}_{3}, \mathrm{M}_{7}$ | $\mathrm{~V}_{3}$ | 0 V |
| $\mathrm{M}_{4}, \mathrm{M}_{8}, \mathrm{M}_{9}, \mathrm{M}_{\mathrm{bl}}, \mathrm{M}_{\mathrm{b} 2}, \mathrm{M}_{\mathrm{b} 5 \mathrm{a}}, \mathrm{M}_{\mathrm{b} 5 \mathrm{~b}}, \mathrm{M}_{\mathrm{c2a}}, \mathrm{M}_{\mathrm{c} 2 \mathrm{~b}}, \mathrm{M}_{\mathrm{c4a}}, \mathrm{M}_{\mathrm{c4b}}$ | $\mathrm{~V}_{1}$ | 0.5 V |
| $\mathrm{M}_{\mathrm{c} 1}, \mathrm{M}_{\mathrm{c} 3}$ | 0.4 V | 0 V |

open-loop gain, the four input capacitors are actually the same capacitors $\mathrm{C}_{0}$ 's in Figure 4.16, and provide the loading capacitance for the second SF-biquad. The PGA is designed to have four gain levels with thermometer code controlling bits $\mathbf{b}_{\text {PGAi }}(\mathrm{i}=0,1,2)$. The unit feedback capacitance $\mathrm{C}_{\mathrm{fb0}}$ is 50.6 fF , and $C_{0}$ is 10.13 pF as calculated earlier according to noise requirement. Hence the four gain levels are $18 \mathrm{~dB}, 26 \mathrm{~dB}$, 32 dB and 40 dB . The same as the CC-PGA in Chapter 3, the DC feedback is to establish the input DC level and simultaneously needs to give a sufficiently low highpass corner frequency. Because of the low supply voltage, the input DC level is set to 100 mV for pFET input transistors. With the output commonmode DC level set to 250 mV for maximum output swing, the DC feedback needs to provide $2 / 5$ voltage division. Given all the requirements, unit diode-connected pFET pseudo-resistor $\mathrm{R}_{\mathrm{fb}}$ is used and the connection is shown in the insert $\mathbf{b}$. Small $W / L$ is used for a highpass corner frequency at least smaller than the lowest channel frequency, and the unit pFET area is kept reasonably large to minimize its contribution to output DC offset. The switch for feedback capacitance selection is depicted in the insert $\mathbf{c}$, a complimentary transmission gate composed of $\mathrm{pFET} \mathrm{M}_{0}$ and $\mathrm{nFET} \mathrm{M}_{1}$. To reduce the ON -resistance, the bulks of $\mathrm{M}_{0}$ and $\mathrm{M}_{1}$ are connected to 0.1 V and 0.4 V respectively controlled by $\mathrm{S}_{\text {bulkn }}$ and $\mathrm{S}_{\text {bulkp }}$ when $\mathbf{b}_{\text {PGAi }}=1$. The voltage divider to provide the 0.1 V and 0.4 V voltages is composed of five diode-connected pFETs in series between ground and VDD. When $\mathbf{b}_{\text {PGAi }}=0$, the bulks of $\mathrm{M}_{0}$ and $\mathrm{M}_{1}$ are connected to VDD and ground respectively for large OFF-resistance.

A two-stage Opamp is chosen for the PGA core, considering sufficient open-loop gain, low noise and wide output range. The circuit diagram is shown in Figure 4.21. The biasing current $\mathrm{I}_{\text {bias }}$ of the Opamp in each channel is proportional to the corresponding channel bias currents $\mathrm{I}_{\text {chi }}$ as described in Section 4.2.1.


Figure 4.22. Detailed circuit diagrams of the compensation capacitors in Figure 4.12: (a) $\mathrm{C}_{1}$ and $\mathrm{C}_{4}$; (b) $\mathrm{C}_{2}$ and $\mathrm{C}_{5}$; (c) $\mathrm{C}_{3}$ and $\mathrm{C}_{6}$.

Because of the large current span, some of the transistor's body-biasing voltages are different in the Opamps of the first 32 and last 32 channels, which are listed in Table 4.4. The transistors not mentioned in the list have bulk connected to VDD in pFETs and ground in nFETs. Even though complimentary $\mathrm{nFET} / \mathrm{pFET}$ input stage can improve power efficiency, and has been used in many neuron-activity recording biomedical ICs [147]-[150], the input transistors here are simple pFETs $M_{1 a}, M_{1 b}, M_{5 a}$ and $M_{5 b}$ because it is much easier to design considering current scalability. In some low-supply-voltage designs, the tail current of the first stage is avoided to save voltage headroom, and a feedforward cancellation technique is employed to suppress the common-mode gain [151]. Here however, the tail current transistor $\mathrm{M}_{9}$ is still kept for current scaling. The voltage headroom is of less concern because of the relatively low GBW requirement for audio applications. The same pseudo-cascode compensation technique as in the CC-PGA Opamp of the retina pixel is again used here for sufficient phase margin over a wide biasing range and avoiding the use of area-consuming zero-nulling resistors. The compensation capacitors $\mathrm{C}_{1}, \mathrm{C}_{2}$, $C_{4}$ and $C_{5}$ are connected from the middle nodes of the split-transistors $M_{1}, M_{5}, M_{2}$ and $M_{6}$ instead of the output of the first stage to the output of the second stage. The problem of potentially insufficient gain margin of cascode Miller-compensation caused by gain peaking beyond GBW [107] is circumvented by the feedforward capacitors $\mathrm{C}_{3}$ and $\mathrm{C}_{6}$ [152].

For a one-stage Opamp, the input-referred noise (IRN) of a CC-PGA composed of this Opamp is only dependent on its closed-loop gain and loading capacitance [117]; for a two-stage Opamp like the one in Figure 4.21, the IRN is dependent on the compensation capacitance:

$$
\begin{equation*}
\mathrm{IRN}=\sqrt{\frac{2 k T}{\kappa C_{\text {ctot }} A_{c}}} \tag{4.12}
\end{equation*}
$$

where $C_{\text {ctot }}$ is the total compensation capacitance and $A_{c}$ is the CC-PGA closed-loop gain. To keep a relatively constant IRN while programming $A_{c}, C_{c t o t}$ should be accordingly changed with ideally $C_{\text {ctot }} \times A_{c}$ being constant. Meanwhile, the CC-PGA bandwidth should be constant and not less than the BPF central frequency with the presence of variations of bias current and capacitance. The bandwidth can be written as (see Appendix 4.6.a):

$$
\begin{equation*}
\mathrm{BW}_{\mathrm{CC}-\mathrm{PGA}}=\frac{g_{m 1}}{4 \pi C_{c t o t} A_{c}} \tag{4.13}
\end{equation*}
$$

where $g_{m 1}$ is the transconductance of the input transistors that is determined by the bias current of the first


Figure 4.23. (a) Conventional CMFB loop that is composed of $-g_{m 1},-g_{m 2}$ and $-g_{m 3}$ blocks. The 5T amplifier is a representative implementation of $-g_{m 3}$; (b) CMFB loop in the proposed Opamp in Figure 4.21.
stage of the Opamp. The requirement of constant $C_{\text {ctot }} \times A_{c}$ for BW is consistent with that for noise. For $A_{c}=40 \mathrm{~dB}$, if an $\operatorname{IRN} \leq 12 \mu \mathrm{~V}$ noise is aimed, $C_{\text {ctot }}$ is calculated to be larger than 720 fF . About 700 fF is used in the design, and is split between $\mathrm{C}_{1}$ and $\mathrm{C}_{2}\left(\right.$ or $\mathrm{C}_{4}$ and $\mathrm{C}_{5}$ ), i.e. $C_{1}=C_{2}=350.4 \mathrm{fF}$ at $A_{c}=40 \mathrm{~dB}$. With $C_{c m u}=350.4 \mathrm{fF}$ and $C_{c f u}=987.4 \mathrm{fF}$, the complete circuits of $\mathrm{C}_{1}, \mathrm{C}_{2}$ and $\mathrm{C}_{3}$ (the same for $\mathrm{C}_{4}, \mathrm{C}_{5}$ and $\mathrm{C}_{6}$ ) are shown in Figure 4.22. Simple nFET switches are used in $\mathrm{C}_{1}$ and $\mathrm{C}_{3}$. Even though the bulks are connected to ground, the nFETs can have sufficiently low ON -resistance because their sources are connected to the middle node of the split-transistor $\mathrm{M}_{2}\left(\right.$ or $\left.\mathrm{M}_{6}\right)$ which has a nominal voltage of less than 50 mV . pFET switches are used in $\mathrm{C}_{2}$ because the middle node of the split-transistor $\mathrm{M}_{1}$ (or $\mathrm{M}_{5}$ ) where the pFETs' sources are connected to has a nominal voltage larger than 350 mV . To further reduce the high ON resistance of pFET switches due to its lower mobility and higher threshold compared to nFETs , their bulks are connected to the switch control signals. Simulated results show worse-case less than $10-\mathrm{pA}$ source-to-bulk leakage current when the switch is turned on.

A common-mode feedback (CMFB) circuit is an indispensable integral part for a fully differential Opamp with balanced output. Its main function is to set the Opamp output DC level at half VDD for maximum swing, i.e. 250 mV with a $0.5-\mathrm{V}$ supply. A secondary function is to suppress the common-mode output swing in response to input signals and supply noise. One representative CMFB circuit for a twostage Opamp [153]-[156] is illustrated within the red box in Figure 4.23(a) together with its symbolic small-signal abstraction $-g_{m 3}$ as part of a two-stage Opamp which consists of two transconductance blocks $-g_{m 1}$ and $-g_{m 2}$ and a compensation capacitor $\mathrm{C}_{\mathrm{cpDM}}$. The complete CMFB loop can be seen as a three-stage amplifier in unity-gain feedback configuration. This conventional design follows the rule of thumb described in [157], particularly concerning the sharing of the main differential amplifier part with the CMFB circuit so that high common-mode open-loop gain and bandwidth can be naturally obtained. For stability consideration, the CMFB circuit is only allowed to produce a non-dominant pole whereas the dominant pole is still at the output of the $-g_{m 1}$ block. This imposes a relatively high lower boundry on the bias current of the CMFB $\mathrm{I}_{\mathrm{cmfb}}$, which is usually $20 \%$ of the main amplifier's according to previously reported results [154], [158] and our own simulations. The $20 \%$ power overhead may not seem to be significant but it is already comparable to the total power consumption of the two SF-biquads in the $4^{\text {th }}$-order BPF. We argue that, in this specific system, the CMFB loop does not have to have the same high GBW as the main differential amplifier for the two reasons below:

1) The SF-biquads behave like LPFs with $Q$ always less than 0.5 for common-mode signals, and therefore any common-mode input with frequencies higher than the BPF central frequency is filtered.
2) When the output of the summing CC-PGA is processed by the capacitively-coupled amplifier in the asynchronous delta modulator (ADM) as will be discussed in Section 4.2.5, common-mode signals will be suppressed to some extent.

Therefore, the GBW of the CMFB loop is designed to be only several times higher than the CC-PGA bandwidth to save power, and the symbolic diagram is shown in Figure 4.23(b). Only two $g_{m}$ blocks $-g_{m 2}$ and $g_{m 3}$ are in the loop. The first stage $g_{m 1}$ achieves its output DC balance via the diode-connected-pFET pseudo-resistors $\mathrm{M}_{10}$ and $\mathrm{M}_{11}$ in Figure 4.21. The dominant pole in the CMFB loop now is at the output of the $g_{m 3}$ block. Additional compensation by the capacitor $\mathrm{C}_{\mathrm{cpCM}}$ is needed to ensure stability. $\mathrm{C}_{\mathrm{cpCM}}$ is implemented as $\mathrm{C}_{\mathrm{c} 3}$ and $\mathrm{C}_{\mathrm{c} 4}$ in Figure 4.21. The power consumption of the CMFB circuit is now less than $1 \%$ of the main differential amplifier, and the CMRR of the summing CC-PGA together with the ADM amplifier as will be described in the next section is found to be larger than 50 dB around the central frequency in the worst case in Monte Carlo simulations.

One more criterion of the CMFB circuit design is that the common-mode signal detector should have a linear characteristic [157]. A commonly used detector comprises two resistors ( $\mathrm{R}_{\mathrm{c} 1}$ and $\mathrm{R}_{\mathrm{c} 2}$ ) and two capacitors $\left(\mathrm{C}_{\mathrm{c} 1}\right.$ and $\left.\mathrm{C}_{\mathrm{c} 2}\right)$ with the connections shown in Figure 4.21 . The resistors and capacitors sense the low- and high-frequency common-mode signals, respectively. For good linearity, ideally the resistor and capacitors can be implemented by poly-silicon and MIM structure. However, to avoid resistive loading at the Opamp output, very large resistance is needed which is impractical in terms of area. Pseudo-resistor has to be used instead. For example in [149], diode-connected pFETs are used as $\mathrm{R}_{\mathrm{c} 1}$ and $\mathrm{R}_{\mathrm{c} 2}$, like $\mathrm{M}_{10}$ and $\mathrm{M}_{11}$ at the Opamp's first stage. If the pseudo-resistance is sufficiently large, mainly the capacitors $\mathrm{C}_{\mathrm{c} 1}$ and $\mathrm{C}_{\mathrm{c} 2}$ play the role in detecting common-mode signals, which should be reasonably linear. The problem of this solution is that for large output swing, the asymmetrical characteristic of diode-connected pFETs causes DC drift of the generated common-mode voltage $V_{c m}$, and in turn drift of the DC level of the Opamp's output, which can quickly lead to signal clipping especially under a very low supply voltage. The DC drift is solved by using a symmetrical pseudo-resistor that is composed of two pFETs $\mathrm{M}_{\mathrm{c} 6}$ and $\mathrm{M}_{\mathrm{c} 7}$ as shown in the red box insert in Figure 4.21. It is adapted from the one in [104] where the working principle is detailed. The difference is that no tunable level-shifting is needed here between $\mathrm{M}_{\mathrm{c} 6}{ }^{\prime} \mathrm{s}\left(\mathrm{M}_{\mathrm{c} 7}{ }^{\prime} \mathrm{s}\right)$ bulk and $\mathrm{Mc}_{7}{ }^{\prime} \mathrm{s}\left(\mathrm{M}_{\mathrm{c} 6}\right.$ 's) gate for adjustable resistance.

### 4.2.5 Asynchronous Delta Modulator with Self-Oscillating Comparison

The block diagram of the asynchronous delta modulator (ADM) is illustrated in Figure 4.24. The input signals $V_{\text {inadm+ }}$ and $V_{\text {inadm- }}$ are from $V_{o P G A^{+}}$and $V_{o P G A-}$ of the summing CC-PGA in Figure 4.20, respectively. $V_{\text {inadm }}{ }^{+}$and $V_{\text {inadm- }}$ are not differentially connected to the input capacitors of the ADM Opamp unlike the VGA block in [149], because in that case the Opamp's input voltages $V_{r e f}$ and $V_{r s t}$ would have large com-mon-mode fluctuation due to the lack of feedback between the Opamp's output and its positive input, which is not allowed according to the charge conservation requirement of an ADM as explained in Section 3.2.3 in Chapter 3. To convert the differential input to single-ended output, either $V_{\text {inadm+ }}$ or $V_{\text {inadm }}$ needs to be inverted before summed together. Here $V_{\text {inadm }}$ is inverted by the unity-gain inverting amplifier, i.e. the '-1' block, and then combined with $V_{\text {inadm- }}$ through the capacitively-coupled amplifier employing a differential-input single-ended-output two-stage Opamp. The Opamp's input DC level $V_{r e f}$ is as well set to 100 mV through the diode-connected-pFET pseudo-resistor $\mathrm{R}_{\mathrm{fb}}$ to give sufficient headroom for the design of the Opamp's input stage. The equilibrium voltage of $V_{r s t}$ is thus also at 100 mV . The passband gain of the capacitively-coupled amplifier is 2 with $C_{a d m}=101.3 \mathrm{fF}$, and the working principle of the asynchro-


Figure 4.24. Block diagram of the ADM with adaptive self-oscillating comparison.
nous switched-capacitor circuit for delta modulation is the same as the ADM circuit in the retina pixel except that here $\mathrm{C}_{\text {rt }}$ is programmable. Adaptive self-oscillating comparison is proposed to improve energy efficiency compared to using continuous-time comparators. Dynamic latched comparators are used, and their psudo-clock Reset signal is generated by detecting the completion of each comparison. The flag signals $O N$ and $O F F$ indicating spike generation only become valid when the amplifier output $V_{\text {adm }}$ is above or below the comparison thresholds $\mathrm{V}_{\text {thH }}$ and $\mathrm{V}_{\text {thL }}$, respectively. Once the asynchronous logic is triggered by the $O N$ or $O F F$ signal, it communicates with the peripheral AER to transmit the generated spike and generates the switching signals $\varphi_{r s t} \varphi_{H}$ and $\varphi_{L}$ for $\operatorname{ADM}$. $\operatorname{High} \varphi_{H}$ and $\varphi_{L}$ are also responsible for adaptively increasing the frequency of self-oscillation which is kept at a lower value when there is no spike generation and communication. The following subsections will detail the designs of the ADM building blocks in Figure 4.24.

### 4.2.5.1 Unity-Gain Inverting Amplifier

Ideally the unity-gain inverting amplifier should have a gain of 1 and cause no additional phase shift at the output with respect to its input signal. Any mismatch of gain and phase between the input and output can degrade the CMRR, which is of course inevitable in practical circuits and can only be minimized. Power overhead is another main concern, and for this reason, a simple common-source amplifier is used as the core of the inverting amplifier. The complete circuit and its symbolic abstraction are illustrated in Figure 4.25. The body biasing voltages of the relevant transistors are listed in Table 4.5, and the other nFETs and pFETs by default have their bulks connected to ground and VDD, respectively. As shown in Figure $4.25(\mathrm{a})$, the 2 T common-source amplifier $\left(\mathrm{M}_{1}\right.$ and $\left.\mathrm{M}_{2}\right)$ is configured in a unity-gain closed-loop feedback by capacitors $C_{1}$ and $C_{2}$ with equal capacitance. The biasing current is copied via $\mathrm{M}_{\mathrm{b} 3}$. A pseu-do-resistor $\mathrm{R}_{\mathrm{DC}}$ that has the same topology as the one used in the attenuator in Figure 4.5 blocks the low impedance path of the diode-connected $M_{b 3}$ for the input signal. The associated highpass corner frequency is sufficiently low due to $\mathrm{R}_{\mathrm{DC}}$ 's large resistance. To set the DC level of $V_{\text {out }}$ at 250 mV for maximum

(a)

(b)

Figure 4.25. (a) Circuit diagram and (b) symbolic abstraction of the unity-gain inverting amplifier.
Table 4.5. Body biasing of the transistors in the amplifier in Figure 4.25.

| Transistors | First 32-channel | Last 32-channel |
| :--- | :---: | :---: |
| $\mathrm{M}_{2}, \mathrm{M}_{\mathrm{fb} 2}, \mathrm{M}_{\mathrm{fb} 4}$ | $\mathrm{~V}_{1}$ | 0.5 V |
| $\mathrm{M}_{\mathrm{fb} 1}, \mathrm{M}_{\mathrm{fb} 3}$ | 0.4 V | 0 V |

swing, a feedback loop is formed by the 5 T amplifier $\left(\mathrm{M}_{\mathrm{fb} 1}-\mathrm{M}_{\mathrm{fb} 5}\right)$ and $\mathrm{M}_{2}$, which however creates a zero that should be placed at a much lower frequency than the channel central frequency.

To analyze the frequency response in detail and determine the design parameters, the complete transfer function is derived according to the small-signal symbolic abstraction in Figure $4.25(\mathrm{~b})$ where $C_{\text {in }}$ is the capacitance of $\mathrm{C}_{1}$ and $\mathrm{C}_{2}, C_{L}$ is the capacitance of $\mathrm{C}_{3}, C_{g s}$ and $C_{g d}$ are the gate-source and gate-drain capacitances of $\mathrm{M}_{1}, g_{m 1}$ is the transconductance of $\mathrm{M}_{1}$ and $\mathrm{M}_{2}, g_{m f b}$ is the transconductance of the 5 T feedback amplifier, and $g_{d s 0}$ is the output conductance of the common-source amplifier:

$$
H_{U G I A}(s)=\frac{C_{i n} \cdot\left[s^{2} \cdot\left(C_{i n}+C_{g d}\right)-s \cdot g_{m 1}\right]}{s^{2} \cdot\left[C_{i n} C_{L}+\left(C_{i n}+C_{L}\right)\left(C_{i n}+C_{g d}+C_{g s}\right)+C_{g d} C_{g s}\right]+s \cdot\left[\left(C_{i n}+C_{g d}\right) g_{m 1}+\left(2 C_{i n}+C_{g s}+C_{g d}\right) g_{d s 0}\right]+\frac{\left(2 C_{i n}+C_{g d}+C_{g s}\right) g_{m 1} g_{m g b}}{C_{g b}}}
$$

With approximation, the locations of zeros and poles can be obtained as:

$$
z_{1}=0, z_{2}=\frac{g_{m 1}}{C_{i n}}, p_{1}=\frac{2 g_{m f b}}{C_{f b}}, p_{2}=\frac{g_{m 1}}{C_{i n}+2 C_{L}}
$$

The highpass corner frequency is at $p_{1}$. If $2 C_{L} « C_{i n}, z_{2}$ overlaps with $p_{2}$ and $H_{U G I A}(\mathrm{~s})$ has a highpass characteristic. Otherwise, the lowpass corner is at $p_{2}$. To minimize the phase shift around the channel central frequency $f_{c i}, p_{1}$ and $p_{2}$ should be at least $10 \times$ away from $2 \pi f_{c i}$. The passband gain $A_{0 U G I A}$ can be derived as:

$$
A_{0 U G I A}=\frac{1}{1+\frac{2 g_{d s 0}}{g_{m 1}}+\frac{C_{g d}}{C_{i n}}+\frac{g_{d s 0}\left(C_{g s}+C_{g d}\right)}{g_{m 1} C_{i n}}}
$$

Increasing the open-loop gain of the common-source amplifier and reducing the $C_{g s}$ and $C_{g d}$ by proper sizing can help $A_{0 U G I A}$ approach 1 . Connecting $\mathrm{C}_{\mathrm{fb}}$ between $V_{f b}$ and VDD instead of ground significantly


Figure 4.26. Circuit diagram of the differential-input single-ended-output two-stage Opamp used in ADM.
Table 4.6. Body biasing of the transistors in the amplifier in Figure 4.26.

| Transistors | First 32-channel | Last 32-channel |
| :--- | :---: | :---: |
| $\mathrm{M}_{1 \mathrm{a}}, \mathrm{M}_{1 \mathrm{~b}}, \mathrm{M}_{3 \mathrm{a}}, \mathrm{M}_{3 \mathrm{~b}}$ | $\mathrm{~V}_{1}$ | $\mathrm{~V}_{4}$ |
| $\mathrm{M}_{\mathrm{b} 5}, \mathrm{M}_{7}$ | $\mathrm{~V}_{2}$ | 0.5 V |
| $\mathrm{M}_{2 \mathrm{a}}, \mathrm{M}_{2 \mathrm{~b}}, \mathrm{M}_{4 \mathrm{a}}, \mathrm{M}_{4 \mathrm{~b}}, \mathrm{M}_{6 \mathrm{a}}, \mathrm{M}_{6 \mathrm{~b}}$ | $\mathrm{~V}_{3}$ | 0 V |

improves PSRR because the AC coupling stabilizes the $V_{g s}$ of $M_{2}$; in fact, if $\mathrm{C}_{\mathrm{fb}}$ is connected to ground, the PSRR has a gain of 2 at the square root of $p_{1} \times p_{2}$, which is in vicinity of $f_{c i}$. The PSRR can be further optimized by reducing the $C_{g d}$ of $\mathrm{M}_{2}$. Another constraint on $\mathrm{C}_{\mathrm{fb}}$ is the stability of the DC feedback loop. If the non-dominant pole at $V_{\text {out }}$ is to be $3 \times$ the GBW of the loop for sufficient phase margin, the value of $\mathrm{C}_{\mathrm{fb}}$ has to satisfy:

$$
C_{f b} \geq \frac{3 g_{m 1} g_{m b b} C_{L}}{g_{d s 0}^{2}}
$$

### 4.2.5.2 Differential-Input Single-Ended-Output Two-Stage Opamp

The schematic of the differential-input single-ended-output Opamp is given in Figure 4.26. It has the same topology as the Opamp used in the retina pixel described in Chapter 3. Its body-biasing voltages are listed in Table 4.6. The noise of this Opamp is of less concern thanks to the gain of the previous circuit stages. The speed is more important because of the fast settling requirement of the output during the charge redistribution phase in ADM. The GBW of the Opamp can be estimated as follows. According to Eq. (4.a1) in Appendix 4.6.b, to achieve a $\eta \%$ settling accuracy within time $t$, the equation below should be satisfied:

$$
p_{1} \cdot \frac{A_{0} C_{2}}{C_{1}+C_{2}+C_{r s t}} t=\ln \frac{100}{\eta}
$$

where $p_{1}$ and $A_{0}$ are the dominant pole frequency and DC gain of the Opamp. Note that $p_{1} \times A_{0}=2 \pi \times \mathrm{GBW}$. Assuming a sinusoidal input for a channel with a central frequency of $f_{c i}$, if each period produces $N$ spikes,


Figure 4.27. Detailed switched-capacitor circuits used in ADM.
Table 4.7. Programmable $\delta$-subtraction of the ADM amplifier in one charge redistribution operation.

| $\mathbf{b}_{\text {rst1 }} \mathbf{b}_{\text {rst0 }}$ | 00 | 01 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: |
| $\Delta V_{\text {adm }}$ | $\delta$ | $1.5 \delta$ | $2 \delta$ | $2.5 \delta$ |

then the settling has to be at least within the time of $1 /\left(f_{c i} \times N\right)$. The GBW can be finally expressed as:

$$
\mathrm{GBW} \geq \frac{1}{2 \pi} \ln \frac{100}{\eta} \cdot f_{c \mathrm{c}} \cdot N \cdot \frac{C_{1}+C_{2}+C_{r s t}}{C_{2}}
$$

For example, for the $20 \mathrm{k}-\mathrm{Hz}$ channel, if $N=16, \eta=2$ and $C_{1}=2 C_{r s t}=4 C_{2}$, GBW needs to be at least 1.4 M $\mathrm{rad} / \mathrm{s}$. This is a tradeoff between the encoding quality and power consumption, because smaller $\eta$ and larger $N$ results in more faithful encoding at the cost of large GBW As discussed in Chapter 2, small $N$ (3to 4 -bit encoding, i.e. $N$ takes value from 8 to 16) can already give high encoding quality in light of the decoding SDR using nonlinear reconstruction algorithms based on frame theory. Therefore the power consumption of the sensing frontend can be reduced by using a small $N$ at the cost of possibly more postprocessing power, which might be beneficial in wireless sensor applications due to scarcely available energy at sensor nodes.

### 4.2.5.3 ADM Switched-Capacitor Circuits

The detailed circuit of the ADM switched-capacitor network is shown in Figure 4.27. The reset capacitor $\mathrm{C}_{\text {rst }}$ in Figure 4.24 is programmable by two digital bits $\mathbf{b}_{\text {rst0 }}$ and $\mathbf{b}_{\text {rst1 }}$. It consists of three capacitors with $C_{r s t 0}=C_{r s t 2}=C_{a d m}$ and $C_{r s t 1}=0.5 C_{a d m}$. Hence the ultimate change at the output of the ADM amplifier $\Delta V_{a d m}$ can have four different values, and are listed in Table 4.7 where $\delta=\mathrm{V}_{\text {refH }}-\mathrm{V}_{\text {refM }}=\mathrm{V}_{\text {refM }}-\mathrm{V}_{\text {refL }}$. Because in idle state, $V_{r s t}$ is 100 mV set by the negative feedback in the ADM amplifier, the middle level reference voltage $\mathrm{V}_{\text {refM }}$ is set to 100 mV as well. $\mathrm{V}_{\text {refL }}$ can be adjusted between ground and any voltage less than 100 mV , and in practice usually set to ground. For symmetry $\mathrm{V}_{\text {refH }}$ is set to 200 mV , which gives a $\delta$ of 100 mV . When $\varphi_{L}$ is high, the nFET switches $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ have a $V_{g s}$ of 500 mV that is enough to give a sufficiently small ON-resistance determined by the requirement of charging speed. When $\varphi_{H}$ is high, however, the pFET switches $\mathrm{M}_{3}$ and $\mathrm{M}_{4}$ only have a $\left|V_{g s}\right|$ of 200 mV , leading to unacceptably large ON-resistance. To circumvent this problem, the circuit in the red box in Figure 4.27 is used to generate an $n \varphi_{H}$ swinging between -300 and 200 mV , and thus a $500-\mathrm{mV}\left|V_{g s}\right|$ can also be obtained for $\mathrm{M}_{3}$ and $\mathrm{M}_{4}$ when $\mathrm{C}_{\text {rst }}$ needs to be charged to $\mathrm{V}_{\text {refH }}$. The same circuit is used to generate $n \varphi_{r s t}$ which controls the pFET switch $\mathrm{M}_{5} . \mathrm{R}_{\mathrm{DC}}$ is the same pseudo-resistor used in the attenuator as depicted in Figure 4.5. The coupling capacitor $\mathrm{C}_{\mathrm{AC}}$


Figure 4.28. Circuit diagrams of the ON- and OFF-comparators and the asynchronous logic that generates the $C L K$ and $n C L K$ for the comparators. The circuit that generates the enabling signal $E N_{\text {busy }}$ for fast selfoscillation is in the red box.
should be chosen such that $n \varphi_{r s t}$ and $n \varphi_{H}$ stay approximately at -300 mV during a large pulse-width of $\varphi_{r s t}$, particularly in low frequency channels where the ADM amplifier takes a long time to settle in each charge redistribution phase because of the scaled bias current. The pole associated with the ON-resistance $R_{o n}$ of the transmission gate switch composed of $\mathrm{M}_{5}$ and $\mathrm{M}_{6}$ can be approximated as $1 /\left(R_{o n} C_{r s t}\right)$ according to Eq. (4.a2) given high open-loop gain of the Opamp. This pole should be located at a higher frequency than the pole $p_{2}$ in Eq. (4.a1) determined by the GBW of the Opamp, which sets the upper limit on $R_{o n}$ as:

$$
R_{o n} \leq \frac{1}{\ln \frac{100}{\eta} \cdot f_{c \mathrm{c}} \cdot N \cdot C_{r s t}}
$$

### 4.2.5.4 Latched Comparators and Self-Oscillation Logic Circuits

Discrete-time (DT) comparators that employ latch with positive feedback generally have higher comparison speed and power efficiency compared to continuous-time (CT) comparators based on multi-stage amplifiers, especially at low supply voltages (see comparison analysis in Appendix 4.6.c). However, DTcomparators usually need clock to reset the regenerative latch to its initial state after each comparison is completed. In asynchronous systems, clock is normally not available. In order to internally generate the clock, i.e. the Reset signal in Figure 4.24, a self-oscillation loop can be formed in which Reset is derived from the completion of one comparison. This method has actually been used in SAR ADCs to automati-


Figure 4.29. Impact of the threshold setting on maximum allowable comparison delay $t_{d m a x}$.
cally determine the optimal time needed for each comparison [159], [160]; nonetheless, to the author's best knowledge, no such scheme has been employed in any asynchronous systems for signal modulation.

The two-stage DT-comparators used in the design are depicted in Figure 4.28. They are based on the topology proposed in [161] which has less stacking between ground and VDD compared to the conventional StrongARM latch [162]. The ON- and OFF-comparators use nFET and pFET input transistors respectively, because the nominal upper and lower threshold voltages $\mathrm{V}_{\text {thH }}$ and $\mathrm{V}_{\text {thL }}$ are 400 and 100 mV respectively. The working principle is briefly described here taking the ON-comparator as the example. When $C L K=0$ and $n C L K=1$, the comparator is reset. The tail $n F E T \mathrm{M}_{5 \mathrm{n}}$ is off and the $\mathrm{pFETs} \mathrm{M}_{2 \mathrm{n}}$ and $\mathrm{M}_{4 \mathrm{n}}$ are on charging the output of the first stage $V_{1 n} / V_{2 n}$ to VDD. Consequently, the input pFETs of the second stage $\mathrm{M}_{7 \mathrm{n}}$ and $\mathrm{M}_{11 \mathrm{n}}$ are off, cutting off the current path from VDD to ground while the output of the second stage $V_{3 n} / V_{4 n}$ are set to ground by $\mathrm{M}_{9 \mathrm{n}}$ and $\mathrm{M}_{13 \mathrm{n}}$. The final output $O N$ and $n O N$ are both low. When $C L K=1$ and $n C L K=0$, the comparison starts. If $V_{i n}>V_{t h H}, V_{1 n}$ drops faster than $V_{2 n}$, and the difference between $V_{1 n}$ and $V_{2 n}$ increases over time as long as the input $n F E T s \mathrm{M}_{1 n}$ and $\mathrm{M}_{3 n}$ are still in saturation. As the common-mode voltage of $V_{1 n}$ and $V_{2 n}$ decreases, $\mathrm{M}_{7 \mathrm{n}}$ and $\mathrm{M}_{11 \mathrm{n}}$ start to turn on, and $V_{3 n}$ is charged faster than $V_{4 n}$. Finally the positive feedback loop that consists of $\mathrm{M}_{6 \mathrm{n}}, \mathrm{M}_{8 \mathrm{n}}, \mathrm{M}_{10 \mathrm{n}}$, and $\mathrm{M}_{12 \mathrm{n}}$ takes over, and $V_{3 n}$ goes up to VDD and $V_{4 n}$ goes down to ground. The comparison is completed with $O N=1$ and $n O N=0$. On the other hand, if $V_{\text {in }}<V_{\text {th }}$, the final output is $O N=0$ and $n O N=1$. The comparator is ready for the next comparison cycle. The OFF-comparator is summarized as follows: when $C L K=0$ and $n C L K=1$, the comparator is reset, and $O F F=0$ and $n O F F=0$; when $C L K=1$ and $n C L K=0$, the comparison starts, and the result is either $O F F=1$ and $n O F F=0$ if $V_{\text {in }}<V_{t h L}$ or $O F F=0$ and $n O F F=1$ if $V_{\text {in }}>V_{\text {thL }}$.

A self-oscillating loop can be formed based on the fact that when the comparator is in reset, both outputs are low, and after the completion of a comparison, one output is low and the other is high. Therefore, the reset and comparison completion can be detected by an OR gate. The simplified self-oscillating logic is shown on the right side of Figure 4.28. After reset in response to $C L K=0$ and $n C L K=1$, all four inputs to the two OR gates are low, which leads to $C L K=1$ and $n C L K=0$. After comparison completion in response to $C L K=1$ and $n C L K=0$, each OR gate receives a high input which leads to $C L K=1$ and $n C L K=0$ after some delay. The delay is controlled by the current-starved inverter INV*, and it determines the frequency of the self-oscillation. For further power saving, the oscillating frequency is set to a low value by connecting the switch $\mathrm{S}_{\text {INV* }}$ to a smaller starving current $\mathrm{I}_{\text {idle }}$ when no threshold crossing occurs, i.e. no spike output is detected. Once $\varphi_{H}$ or $\varphi_{L}$ is triggered to high by output spikes, $E N_{b u s y}$ is set to high and a larger starving current $\mathrm{I}_{\text {busy }}$ is connected to $\mathrm{INV}^{*}$, resulting in a higher oscillation frequency. Both $\mathrm{I}_{\text {idle }}$ and $\mathrm{I}_{\text {busy }}$ are derived proportionally from the corresponding channel bias $I_{\text {chi. }}$. The circuit in the red box in Figure 4.28 acts like an analog timer. It keeps $E N_{\text {busy }}$ high as long as the time interval between two consecutive $\varphi_{H}$ or $\varphi_{L}$ pulses is less than a certain time threshold $t_{t h}$. The $t_{t h}$ is determined by the leakage current of the pFET $\mathrm{M}_{14}$ and the capacitance of $\mathrm{C}_{0} . E N_{b u s y}$ becomes low when $V_{E N}$ is discharged low. $t_{t h}$ can be designed to be


Figure 4.30. (a) In-channel asynchronous logic circuits that generate the control signals $\varphi_{H}, \varphi_{L}$ and $\varphi_{r s t}$ for ADM, and communicate with the $1.8-\mathrm{V}$ peripheral AER circuit via request signals $R_{\text {eq }}^{\text {ON }}, R_{\text {Req }}^{\text {OFF }}$ and acknowledge signals $A c k_{O N}$ and $A c k_{O F F}$. The inverters with star marks have long channel-length pFETs; (b) Exemplary timing diagram of an ON spike.
several seconds in accordance with the time interval of normal human speech utterance, or other values depending on the application scenarios.

The frequency of the self-oscillation sets the worst-case comparison delay. If a threshold-crossing occurs right after the comparator enters the regenerative phase, the comparator output will not become valid until this oscillation cycle is completed. Hence we approximately have:

$$
t_{d \max } \approx \frac{1}{f_{o s c}}
$$

where $t_{\text {dmax }}$ is the maximum delay and $f_{\text {osc }}$ is the oscillation frequency. Note that $f_{\text {osc }}$ is not constant because the actual comparison time $t_{\text {comp }}$ between the moment when $C L K$ becomes valid and the end of the latch regeneration is dependent on the slope of the input signal. However, $f_{\text {osc }}$ is mostly determined by the time constant of INV* when $t_{\text {comp }}$ is comparatively small. The low supply voltage put hard limit on $t_{d m a x}$, which is explained by the illustration in Figure 4.29. In Figure 4.29 (a), the upper threshold $\mathrm{V}_{\text {th }}$ is set to 400 mV . If the extra delays caused by in-channel asynchronous logic and AER communication are neglected, within $t_{d \max }$, the increase of the ADM amplifier output $\Delta V_{a d m}$ has to be less than 100 mV to avoid clipping before $V_{a d m}$ is pulled back towards 250 mV in response to delta subtraction. In Figure 4.29(b), the $\Delta V_{\text {adm }}$ is relaxed to 150 mV with $\mathrm{V}_{\mathrm{thH}}$ set to 350 mV . It is clear that a lower threshold allows a larger $t_{d m a x}$ given the same $V_{a d m}$ slope, at the cost of more severe threshold mismatch [89]. The following calculation considers the case of $\mathrm{V}_{\mathrm{thH}}=400 \mathrm{mV}$. In the worst case, the two inputs of the ADM amplifier $V_{\text {inadm }+}$ and $V_{\text {inadm- }}$ in Figure 4.24 have maximum amplitude of 250 mV assuming sinusoidal signals, and $V_{\text {adm }}$ in turn has amplitude of 1 V . The maximum slope is therefore $2 \pi \times 20 \times 10^{3}$ for the $20 \mathrm{k}-\mathrm{Hz}$ channel, and $t_{\text {dmax }}$ is calculated to be $0.1 /\left(2 \pi \times 20 \times 10^{3}\right)=796 \mathrm{~ns}$, which means $f_{\text {osc }}$ has to be at least 1.26 M Hz . Corner simulation results show that, with $V_{\text {adm }}$ set to 250 mV , $f_{\text {osc }}$ has minimum value of 1.92 M Hz in busy mode and 0.81 M Hz in idle mode. When $V_{\text {adm }}$ is set to $401 \mathrm{mV}, 1 \mathrm{mV}$ above $\mathrm{V}_{\mathrm{thH}}$, the minimum $f_{\text {osc }}$ in busy mode degrades


Figure 4.31. Chip microphotograph.
to 1.48 M Hz . The idle-mode $f_{\text {osc }}$ is $36 \%$ less than the minimum required 1.26 M Hz , which suggests that any $V_{\text {inadm }+}$ and $V_{\text {inadm- }}$ with amplitude larger than 160 mV might get $V_{\text {adm }}$ clipped at the first spike. However, keep in mind that the $1.26 \mathrm{M}-\mathrm{Hz}$ requirement is overestimated by using the maximum slope of a sinusoid. The simulated worst-case power consumption of the complete circuits in Figure 4.28 in busy mode is 290 nW , and 164 nW in idle mode which is $43 \%$ less.

### 4.2.5.5 In-Channel Asynchronous Logic

The in-channel asynchronous logic is shown in Figure 4.30(a) and the exemplary ON-spike timing diagram is shown in Figure 4.30(b). The main functions of the logic are to generate the control signals $\varphi_{H}$, $\varphi_{L}$ and $\varphi_{r s t}$ for ADM, and serve as the communication interface between the $0.5-\mathrm{V}$ cochlea channels and the $1.8-\mathrm{V}$ peripheral AER. Because the output of a DT-comparator is valid only at the end of the comparison phase and become invalid during the reset phase, an SR latch is indispensable to store the generated spike. Taking ON-spike as the example, an ON spike immediately sets $\varphi_{H}$ high for charging the $\mathrm{C}_{\text {rst }}$ to $\mathrm{V}_{\text {refH }}$ in the ADM. The inverters in the red box 1 in Figure 4.30(a) have pFETs with long $L$, and thus the request $n R e q_{O N}$ becomes valid after a certain delay which sets the pulse width of $\varphi_{H}$. Once $n R e q_{O N}$ is pulled low, a valid-high $A c k_{O N}$ is sent back from the $1.8-\mathrm{V}$ peripheral AER, and brings $n R e q_{O N}$ back high as long as either nFET $\mathrm{M}_{1}$ or $\mathrm{M}_{2}$ stays off after $A c k_{O N}$ goes back low; otherwise $n R e q_{O N}$ will be pulled low again, generating false ON spikes. The circuit in the red box 3 is used to guarantee the overlapped offstate of $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ after $A c k_{O N}$ goes low. The weak pull-up current source charges the gate of $\mathrm{M}_{1}$ slowly so that the time needed for $\mathrm{M}_{2}$ to be turned off by valid $\varphi_{r s t}$ setting the SR latch output low is sufficient. Once the gate of $\mathrm{M}_{2}$ goes low the NOR gate in the red box 3 turn on $\mathrm{M}_{1}$ immediately, ready for the next spike transmission. The weak pull-up is necessary because otherwise the initial state of the whole logic could be in deadlock and unable to send off any spike after power up if $\mathrm{M}_{1}$ is initially off. The circuit in the red box 2 is to generate the $\varphi_{\text {rst }}$ pulse after the completion of charging $\mathrm{C}_{\text {rst }}$ with $\varphi_{H}$ or $\varphi_{L}$ turning low. The current-starved inverter controls the pulse width of $\varphi_{r s t}$ for sufficient settling time of the ADM amplifier, and the current source is derived proportionally from the channel bias $\mathrm{I}_{\text {chi }}$. The signal flow of an OFF spike is similar, so the details will not be repeated here.

### 4.3 System Design and Chip Microphotograph



Figure 4.32. Measured transfer functions of the right ear at (a) $Q \approx 1.3$ and (b) $Q \approx 9.2$.

The chip was designed and fabricated in TowerJazz $0.18 \mu \mathrm{~m}$ IS CMOS with triple well, except the noise was simulated using UMC models. Figure 4.31 shows the chip microphotograph. It occupies an area of $10.5 \times 4.8 \mathrm{~mm}^{2}$ including the pads. The 128 binaural channels are placed in an interleaved manner, i.e. the left and right branches of the same channel are mirrored and put in neighbor. The 1-D AER has 256 channels (ON and OFF spike communications use separate AER channels) with fair arbitration. To obtain each individual transfer function of all the channels, the differential output of the summing PGAs are buffered by on-chip pFET source followers with a $1.8-\mathrm{V}$ supply before connected to a common bus, and a shift register chain is used to select the channel connected to pads for measurements. The source follower buffer is designed to have sufficiently high bandwidth and low noise. The latches in each channel for storing configuration bits have the $0.5-\mathrm{V}$ supply, and the timing diagram for channel selection and bit writing is similar to the one in Chapter 5 for programming the bias generator array. The digital nFETs in the $0.5-$ V core are built within deep N -well to mitigate noise injection into substrate. The digital and analog pads are arranged such that their power supplies and grounds can be easily separated. A programmable bias generator that will be described in Chapter 5 [120] is integrated but only for biasing AER, source follower buffer and test structures. A diagnostic shift register is used to configure the digital bits for test structure, which contains a testing summing PGA and a testing ADM. The USB interface, firmware logic, and host side codes in jAER [121] are based on existing designs.

### 4.4 Measurement Results



Figure 4.33. Central frequency scaling as the function of channel number and the ratios of neighboring central frequencies of the transfer functions in (a) Figure 4.32(a) and (b) Figure 4.32(b).

 (b) $Q \approx 9.2$.

### 4.4.1 BPF Transfer Functions

All the frequency-related measurements are done with the SR780 network signal analyzer. The transfer functions of the right cochlea at low $Q(Q \approx 1.3)$ and high $Q(Q \approx 9.2)$ are plotted in Figure 4.32. The 8 bits for $Q$-tuning are set to 255 for all 64 channels at low $Q$, and are individually adjusted for each channel to have a $50-\mathrm{dB}$ peak gain at high $Q$. Because peak gain and $Q$ are positively correlated, the channels have approximately the same $Q$ as well. The high $Q$ has a $17-\mathrm{dB}$ more peak gain compared to the low $Q$. Both of them have monotonically scaled central frequencies $f_{c}$ 's, as can be seen in Figure 4.33 where the ratios of neighboring $f_{c}^{\prime}$ s are also plotted. The mean scaling ratio is about 1.13 in both cases, $2 \%$ larger than the designed value 1.108 . However, accumulated over 64 stages, this seemingly small discrepancy which might be attributed to underestimated pFET leakage in the current divider in Figure 4.2 pushes the intended 32 Hz lowest frequency to around 8 Hz . The mismatches of $f_{c}^{\prime}$ s and $Q^{\prime}$ s between two ears are plotted in Figure 4.34. The higher $f_{c}$ mismatch at low $Q$ can be mostly ascribed to the difficulty in determining the peak gain frequency due to the flatter passband and the higher $Q$ mismatch at high $Q$ is the


Figure 4.35. $Q$-tuning examples of 4 different channels.


Figure 4.36. Noise PSD of channel 00 and channel 54 at the output of the summing PGA.
Table 4.8. Measured integrated rms noise at the PGA output.

| Channel | 00 |  |  |  |  |  |  |  | 18 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Q$ | 1 |  |  |  | 10 |  |  |  | 1 |  |  |  | 10 |  |  |  |
| PGA gain (dB) | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 |
| $\mathrm{V}_{\text {noise,rms }}(\mathrm{mV})$ | 0.30 | 0.77 | 1.5 | 3.1 | 0.76 | 1.9 | 3.7 | 7.6 | 0.33 | 0.82 | 1.6 | 3.5 | 0.78 | 2.0 | 3.8 | 8.1 |
| Channel | 36 |  |  |  |  |  |  |  | 54 |  |  |  |  |  |  |  |
| $Q$ | 1 |  |  |  | 10 |  |  |  | 1 |  |  |  | 10 |  |  |  |
| PGA gain (dB) | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 |
| $\mathrm{V}_{\text {noise,rms }}(\mathrm{mV})$ | 0.32 | 0.74 | 1.6 | 3.3 | 0.73 | 1.9 | 3.7 | 7.4 | 0.32 | 0.82 | 1.6 | 3.4 | 0.79 | 2.0 | 3.9 | 8.2 |

direct tradeoff for well-controlled $f_{c}$ matching, as analyzed in Section 4.2.4.1. $Q$-tunings of BPFs in four channels are demonstrated in Figure 4.35. The lowest $Q^{\prime}$ s are around 1.3 and the highest can get close to 40. Note that the larger $f_{c}$ variation during $Q$-tuning in low-frequency channels especially in channel 54 might be the indication of larger mismatch between $\mathrm{I}_{2 \mathrm{U}}$ and $\mathrm{I}_{2 \mathrm{D}}$ in Figure 4.13 due to the much smaller currents compared to high-frequency channels or nonidealities in the TLL for $Q$ tuning.

### 4.4.2 BPF Noise, Distortion and Dynamic Range

The noise PSD at the output of the PGA with different PGA gains is measured for the same four channels and Figure 4.36 shows the plot of channel 00 and channel 54 . The noise floor increases as the PGA gain increases and has gain peaking at the central frequency when $Q=10$. The integrated output rms noise


Figure 4.37. Measured distortion of $\mathrm{THD}=-40 \mathrm{~dB}$ and $\mathrm{IM} 3=-26 \mathrm{~dB}$ of channel 00 and channel 54 with a PGA gain of 18 dB .
values for all the four channels are listed in Table 4.8. At the same PGA gain and $Q$, the four channels have approximately the same output noise, which is expected because the filter noise is determined by the same loading capacitance used in all channels and the PGA noise is designed to be constant over four different gains.

The distortion of one-tone and two-tone tests at $Q=1$ and $Q=10$ are plotted in Figure 4.37 for channel 00 and channel 54 at an $18-\mathrm{dB}$ PGA gain. In the THD plots, the $3^{\text {rd }}$-order harmonics generally dominates over the $2^{\text {nd }}$-order thanks to the differential signal path, and it is worth noting that when $Q=10$, the dominant nonlinearity is the non-harmonic components rising beside the fundamental tone. The maximum rms

Table 4.9. Measured PGA rms output amplitude at $\mathrm{THD}=-40 \mathrm{~dB}$ and $\mathrm{IM} 3=-26 \mathrm{~dB}$, and the calculated SNR .

| Channel | $Q$ | $\begin{aligned} & \hline \text { PGA } \\ & \text { gain } \\ & \text { (dB) } \\ & \hline \end{aligned}$ | $\mathrm{V}_{\text {BPFout }}\left(\mathrm{mV}_{\text {rms }}\right)$ | SNR (dB) | Channel | $Q$ | $\begin{aligned} & \hline \text { PGA } \\ & \text { gain } \\ & \text { (dB) } \\ & \hline \end{aligned}$ | $\mathrm{V}_{\text {BPFout }}\left(\mathrm{mV}_{\text {rms }}\right)$ | SNR (dB) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | THD/IM3 | THD/IM3 |  |  |  | THD/IM3 | THD/IM3 |
| 00 | 1 | 18 | 165/91 | 55/50 | 18 | 1 | 18 | 181/98 | 55/49 |
|  |  | 26 | 272/232 | 51/50 |  |  | 26 | 312/250 | 52/50 |
|  |  | 32 | 261/324 | 45/47 |  |  | 32 | 303/370 | 46/47 |
|  |  | 40 | 246/303 | 38/40 |  |  | 40 | 285/347 | 38/40 |
|  | 10 | 18 | 92/37 | 42/34 |  | 10 | 18 | 98/43 | 42/35 |
|  |  | 26 | 233/94 | 42/34 |  |  | 26 | 248/109 | 42/35 |
|  |  | 32 | 251/181 | 37/34 |  |  | 32 | 298/199 | 38/34 |
|  |  | 40 | 236/297 | 30/32 |  |  | 40 | 284/349 | 31/33 |
| 36 | 1 | 18 | 157/79 | 54/48 | 54 | 1 | 18 | 170/89 | 55/49 |
|  |  | 26 | 299/203 | 52/49 |  |  | 26 | 257/233 | 50/49 |
|  |  | 32 | 268/338 | 44/46 |  |  | 32 | 224/331 | 43/46 |
|  |  | 40 | 232/301 | 37/39 |  |  | 40 | 190/293 | 35/39 |
|  | 10 | 18 | 70/34 | 40/33 |  | 10 | 18 | 76/29 | 40/31 |
|  |  | 26 | 181/87 | 40/33 |  |  | 26 | 194/82 | 40/32 |
|  |  | 32 | 263/191 | 37/34 |  |  | 32 | 252/162 | 36/32 |
|  |  | 40 | 216/297 | 29/32 |  |  | 40 | 220/305 | 29/31 |

Table 4.10. CMRR and PSRR at central frequencies measured at the output of the ADM amplifier.

| Channel | 00 |  |  |  |  |  |  |  | 18 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Q$ | 1 |  |  |  | 10 |  |  |  | 1 |  |  |  | 10 |  |  |  |
| PGA gain (dB) | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 |
| CMRR (dB) | 51 | 49 | 50 | 49 | 40 | 40 | 40 | 40 | 53 | 50 | 51 | 50 | 37 | 37 | 37 | 36 |
| PSRR (dB) | 46 | 51 | 48 | 47 | 49 | 49 | 48 | 48 | 46 | 52 | 47 | 45 | 42 | 42 | 41 | 40 |
| Channel | 36 |  |  |  |  |  |  |  | 54 |  |  |  |  |  |  |  |
| $Q$ | 1 |  |  |  | 10 |  |  |  | 1 |  |  |  | 10 |  |  |  |
| PGA gain (dB) | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 | 18 | 26 | 32 | 40 |
| CMRR (dB) | 52 | 52 | 52 | 51 | 43 | 43 | 44 | 44 | 51 | 49 | 49 | 51 | 47 | 48 | 47 | 49 |
| PSRR (dB) | 36 | 44 | 45 | 44 | 49 | 48 | 48 | 47 | 28 | 37 | 44 | 46 | 49 | 46 | 44 | 44 |

output values at $\mathrm{THD}=-40 \mathrm{~dB}$ and $\mathrm{IM} 3=-26 \mathrm{~dB}$ for all the four channels are summarized in Table 4.9, as well as the corresponding SNR. The maximum SNR for both $Q=1$ and $Q=10$ occurs at the lowest PGA gain of 18 dB . For small input signals, high PGA gains need to be used to obtain sufficiently large output swing for subsequent ADM. Ideally the gain adjustment should be achieved by automatic gain control which can be integrated in the system in the future. The system dynamic range can be extended by another 18 dB from the maximum SNR owing to the input attenuator.

### 4.4.3 CMRR and PSRR

The CMRR and PSRR are measured at the output of the ADM amplifier, i.e. the voltage $V_{\text {adm }}$ in Figure 4.24. The example CMRR curves for channel 00 and channel 54 are shown in Figure 4.38, and the example PSRR curves for channel 00 are shown in Figure 4.39. The CMRR and PSRR values at central frequencies for channel $00,18,36$ and 54 are given in Table 4.10. The significant PSRR degradation below the channel central frequencies is deemed to be aggravated by the ' $\mathbf{- 1}$ ' analog inverter block used in the ADM. Around central frequencies, the $\mathrm{C}_{\mathrm{fb}}$ in Fig. 4.25 can keep the gate voltage of $\mathrm{M}_{2}$ tracking VDD,


Figure 4.38. Measured CMRR at the output of the ADM amplifier.


Figure 4.39. Measured PSRR at the output of the ADM amplifier.
namely the source voltage of $\mathrm{M}_{2}$, and hence the output voltage is relatively insensitive to VDD. At low frequencies, the $I_{d s}$ of $\mathrm{M}_{2}$ is modulated by VDD, which directly affects the output voltage by the openloop gain of the common-source amplifier.


Figure 4.40. Spike cochleagram in response to an exponential chirp input with BPF transfer functions set as in Figure 4.32(a) for $Q \approx 1$ and Figure 4.32(b) for $Q \approx 10$.


Figure 4.41. Output spike train samples of channel 18 in response to a $2.8 \mathrm{kHz} \operatorname{sinusoid}$ at (a) $\mathbf{b}_{\text {rst1 }} \mathbf{b}_{\text {rst0 }}=01$ and (b) $\mathbf{b}_{\text {rst1 }} \mathbf{b}_{\text {rst0 }}=11$; (c), (d) the corresponding amplitude spectra of the unfiltered reconstructed waveform with compensated and uncompensated ON and OFF thresholds; (e), (f) the uncompensated reconstruction waveform filtered by a $6^{\text {th }}$-order Butterworth HPF with a 300 Hz corner frequency and an ideal LPF with a 2.94 k Hz corner frequency.

### 4.4.4 Spike Output



Figure 4.42 . (a) Total spike train output of the cochlea chip with a two-tone input at 2.8 k Hz and 1.4 k Hz ; (b) spike train of channel 18; reconstructed waveforms from the spike trains of (c) channel 18 and (d) channel 24, and their corresponding amplitude spectra (e) and (f).

The spike cochleagram of the chip with BPF transfer functions set as in Figure 4.32 in response to an exponential chirp input sweeping from 6 Hz to 21 k Hz are shown in Figure 4.40. To plot the cochleagram, for each channel the number of spikes is binned every 0.1 s , and the maximum spike number is normalized to 1 . The spike response over channels is linear with time because of the geometrically scaled central frequencies of the BPFs. It is obvious that the high $Q$ cochleagram has a smaller channel response overlap in time than the low $Q$ cochleagram because of the better frequency selectivity at high $Q$.

To quantify the encoding performance of a single channel, the output spike train is used to reconstruct the analog input signal employing the linear decoding methods described in Chapter 2. Spikes are recorded by the off-chip FPGA with $1-\mu$ s time resolution. Taking channel 18 as the example, the input signal is a single-tone $2.8-\mathrm{kHz} 5-\mathrm{mVpk}$ sinusoid, and the channel is configured to have a $Q=10$. The spike train samples with the configuration bits $\mathbf{b}_{\text {rst1 }} \mathbf{b}_{\text {rst0 }}$ of $C_{\text {rst }}$ set to 01 and 11 are shown in Figure 4.41 (a) and 4.41(b) respectively within a $4-\mathrm{ms}$ time window. Using linear decoding without filtering, the $10^{6}$-pt FFT amplitude spectra of the reconstructed signals are shown in Figure 4.41(c) and 4.41(d). The black curve is


Figure 4.43. Illustration of the reconstruction method for human speech synthesis from spike trains produced by the cochlea chip.
obtained by using equal ON and OFF thresholds in reconstruction, and the dark red curve using a slightly higher ON threshold to compensate the unbalanced ON and OFF thresholds in silicon and keep a relatively stable DC level in reconstruction. The uncompensated unbalanced threshold obviously causes significantly raised noise floor at low frequencies, even though in the compensated case the ON threshold is only about $3 \%$ larger than the OFF threshold. The known channel identity can be exploited to improve the quality of reconstruction by bandpass filtering. The DC drift caused by unbalance thresholds is circumvented by passing the integrated spike train through a $6^{\text {th }}$-order Butterworth HPF with a $300-\mathrm{Hz}$ corner frequency which is the lower end of the frequency range used in telephony. The high frequency harmonics are filtered by an ideal LPF with a corner frequency that equals to the mean of the central frequencies of channel 18 and 19 , i.e. 2.94 k Hz . After filtering, the example reconstructed time-domain waveforms with uncompensated thresholds are shown in Figure 4.41(e) and 4.41(f). Assuming the signal energy is concentrated within 20 Hz around the central frequency and using the truncated waveform for FFT ( 0.1 s after and before the start and end of the 1.05 -s full-length waveform), the SNDR values of (e) and (f) are calculated to be 26.5 dB and 26.4 dB , respectively. As for the case of compensated thresholds, the SNDR values of the reconstructed waveform filtered by the same filters as described above at $\mathbf{b}_{\text {rst1 }} \mathbf{b}_{\text {rst } 0}=01$ and 11 are 26.8 dB and 26.2 dB , respectively.

A two-tone test is performed on channel 18 and 24 with $Q$ set to about 10 . The input signal is the combination of a 2.8 k Hz and a 1.4 k Hz sinusoid, both with $5-\mathrm{mVpk}$ amplitude. The configuration bits $\mathbf{b}_{\text {rst1 }} \mathbf{b}_{\text {rst0 }}$ are set to 11 . The total output spike train without distinguishing the spike addresses is shown in Figure 4.42(a), and the spike train from channel 18 alone is shown in Figure 4.42(b). The reconstructed waveform of channel 18 and 24 obtained by passing the integrated spike trains with uncompensated thresholds through a $6^{\text {th }}$-order Butterworth HPF with a 300 Hz corner frequency and an ideal LPF with corner frequencies at 2.94 k Hz and 1.48 k Hz respectively are shown in Figure 4.42 (c) and 4.42(d). The corresponding amplitude spectra are shown in Figure 4.42(e) and 4.42(f).

For real-world scenario test, speech is used as the chip input. Signals are directly generated from a computer's soundcard by playing the .WAV files. The reconstruction method is depicted in Figure 4.43. The output spike train of channel $i$ is integrated and filtered by an ideal LPF with the corner frequency at the mean of the central frequencies of channel $i$ and $i+1$.The filtered signals from all 64 channels are summed and then further filtered by a $6^{\text {th }}$-order Butterworth HPF to obtain the final reconstructed waveform. Figure 4.44 (a) and 4.44 (b) are the spectrogram and time-domain waveform of a male speech saying '18174', respectively. Figure 4.44 (c), 4.44 (e) and $4.44(\mathrm{~g})$ are the spike cochleagram, the spectrogram and time-domain waveform of the reconstructed signal at $Q \approx 1$. Figure 4.44(d), 4.44(f) and 4.44(h) are at


Figure 4.44. (a) Spectrogram and (b) time-domain waveform of the input male speech '18174'; (c) output spike cochleagram, and (e) spectrogram and (g) time-domain waveform of the reconstructed speech at $Q \approx 1$; (d), (f) and (h) at $Q \approx 10$.
$Q \approx 10$. The .WAV files of the original and the reconstructed audio can be downloaded from the links below:
'https://dl.dropboxusercontent.com/u/26642184/15CochLP/18174mOrigin.WAV', original '18174';
'https://dl.dropboxusercontent.com/u/26642184/15CochLP/18174mQ1.WAV' recovered at $Q \approx 1$;


Figure 4.45. (a) Spectrogram and (b) time-domain waveform of the input female speech "she had your dark suit in greasy wash water all year '; (c) output spike cochleagram, and (e) spectrogram and (g) timedomain waveform of the reconstructed speech at $Q \approx 1$; (d), (f) and (h) at $Q \approx 10$.
'https://dl.dropboxusercontent.com/u/26642184/15CochLP/18174mQ10.WAV' recovered at $Q \approx 10$.
Figure 4.45 (a) and 4.45 (b) are the spectrogram and time-domain waveform of a female speech 'she had your dark suit in greasy wash water all year', respectively. Figure $4.45(\mathrm{c}), 4.45(\mathrm{e})$ and $4.45(\mathrm{~g})$ are the spike cochleagram, the spectrogram and time-domain waveform of the reconstructed signal at $Q \approx 1$. Fig-


Figure 4.46. Spectrogram of (a) 'heed' and (d) 'had'; normalized histogram of spike number (SN) of 'heed' at (b) $Q=1$ and (c) $Q=10$; normalized histogram of SN of 'had' at (e) $Q=1$ and (f) $Q=10$.
ure $4.45(\mathrm{~d}), 4.45(\mathrm{f})$ and $4.45(\mathrm{~h})$ are at $Q \approx 10$. The .WAV files of the original and the reconstructed audio can be downloaded from the links below:
'https://dl.dropboxusercontent.com/u/26642184/15CochLP/shefOrigin.WAV', original sentence;
'https://dl.dropboxusercontent.com/u/26642184/15CochLP/shefQ1.WAV' recovered at $Q \approx 1$;
'https://dl.dropboxusercontent.com/u/26642184/15CochLP/shefQ10.WAV' recovered at $Q \approx 10$.
Subjective evaluation of the reconstruction quality indicates that the decoding methods as depicted in Figure 4.43 can synthesize well-recognizable speech from the output spike trains produced by the cochlea chip. This implies that essential speech information is preserved during the transformation from analog signals to parallel multi-frequency-channel spike trains. The high $Q$ case is perceived to have larger distortion, which could be caused partly by the prolonged ringing of high $Q$ BPFs, especially at large input amplitudes, or the larger distortion of the BPFs. However a high $Q$ BPF bank may still better facilitate subsequent event-driven classification processing like voice activity detection in terms of more distinct acoustic feature extraction [163]. Figure 4.46 shows a simple example. The most two energy-significant F1 and F4 formants of the vowel 'i:' are marked in the spectrogram of the word 'heed', and the most two energy-significant F1 and F2 formants of the vowel 'æ' are marked in the spectrogram of the word 'had'. The histogram plots of normalized spike number ( SN ) of the two words are obtained by accumulating the spikes produced from each cochlea channel when the sound signals are sent to the chip input. As shown in Figure 4.46(b), (c) for 'heed', and 4.46(e), (f) for 'had', the formant peaks are better separated at $Q=10$ than at $Q=1$, and therefore it is easier to classify words with different vowels with a higher $Q$.

### 4.4.5 Comparison with Prior Arts

Table 4.11. Cochlea chip performance summary and comparison with prior works.

|  | Liu, et al. TBCAS 2014 [37] | $\begin{gathered} \text { Wen, et al. } \\ \text { TBCAS } \\ 2009 \text { [33] } \\ \hline \end{gathered}$ | $\begin{gathered} \text { Fragnière } \\ \text { ISSCC } \\ 2005 \text { [31] } \\ \hline \end{gathered}$ | Sarpeshkar, et al. ISSCC 2005 [164] | This work 2015 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Architecture | Cascade | Active coupling | Passive coupling | Parallel | Parallel |
| Technology ( $\mu \mathrm{m}$ ) | 0.35 CMOS | 0.25 CMOS | 0.5 CMOS | 1.5 BiCMOS | 0.18 CMOS |
| Power supply (V) | 3.3 | 2.5 | 3.3 | 2.8 | 0.5 |
| Power ( $\mu \mathrm{W}$ ) | $14000^{\text {a }}$ | $35900^{\text {b }}$ | $1700^{\text {b }}$ | $60^{\text {c }}$ | 55 |
| Channel number | $64 \times 2$ | 360 | 100 | 16 | $64 \times 2$ |
| Frequency range (Hz) | 50-50k | 210-14k | 200-20k | 100-5k | 8-20k |
| Q range | $1.5 \pm 0.4$ | $1.16 \pm 0.92^{\text {d }}$ | 0.25-12 | <10 | $1.3-39^{\text {e }}$ |
| Normalized power ${ }^{\text {f }}(\mu \mathrm{W})$ | 291 | 605 | 78 | 56 | 3.2 |
| Area per channel (mm ${ }^{2}$ ) | 0.107 | 0.0304 | 0.174 | 5.53 | 0.262 |
| Dynamic range (dB) | 52 | 52 | 50 | 75 | $\begin{aligned} & 73^{\mathrm{e}, \mathrm{~g}} @ \mathrm{Q}=1.3 \\ & 60^{\mathrm{e}, \mathrm{~g}} @ \mathrm{Q}=10 \end{aligned}$ |
| Spike encoding | PFM | PFM | Thresholdcrossing | Zerocrossing | ADM |
| Spike readout | AER | AER | Scanning | Scanning | AER |
| Binaural | Yes | No | No | No | Yes |

a. preamplifier and AER readout included;
b. analog part only;
c. consist of BPFs, envelope detector and bias;
d. -10 dB bandwidth Q ;
e. data from channel 18 , other channels are slightly different;
f. assuming geometrical scaling of currents along channels, the power consumption of the highestfrequency channel normalized to 20 kHz is:

$$
P_{\text {norm }}=\frac{P_{\text {total }}(1-r)}{1-r^{n}} \cdot \frac{20 \mathrm{k}}{f_{H}}, r=\left(\frac{f_{L}}{f_{H}}\right)^{1 /(n-1)}
$$

where $P_{\text {total }}$ is the total power consumption, $f_{H}$ and $f_{L}$ are the highest and lowest channel frequencies, and $n$ is the channel number;
g. at $1 \% \mathrm{THD}$, including 18 dB of the attenuator.

The performance of this cochlea chip is summarized and compared with prior works in Table 4.11. The power consumption of the $0.5-\mathrm{V}$ core is $55 \mu \mathrm{~W}$ at 100 k spike/s output rate. Using the normalized power metric as defined below Table 4.11 , this cochlea is about $17.5 \times$ more power efficient than the best prior art, namely the bionic ear developed by Sarpeshkar et al. at MIT [164]. Besides, this cochlea has presented one of the best matching among channels not only at the BPF output but also at the output of the spike encoders. The only cochlea chip that has been used to demonstrate the information integrity from the output spike trains is the AEREAR2 [37], and the reconstruction methods described in a recent paper [165] is adapted from a linear mapping method that was used to reconstruct the auditory stimulus of a ferret from its spike responses in auditory cortex. The results show recognizable reconstructed digit speech as well, but the method requires learning procedure of new mapping if the input audio is changed from digits to some other arbitrary speech sequence. The reason that no straightforward decoding method like the one depicted in Figure 4.43 could be developed is mostly likely because of the not well-controlled circuit and system parameters in AEREAR2.


Figure 4.47. (a) Alternative summation scheme, (b) super-source-follower-based LPF [144], and (c) and (d) alternative topologies to obtain one zero.

### 4.5 Conclusion and Discussion

An ultra-low-voltage and ultra-low-power $64 \times 2$-channel binaural silicon cochlea is presented in this chapter. Two main techniques are proposed to achieve the $17.5 \times$ improvement in power efficiency compared to the best prior art [164]. One is the source-follower-based BPF, and the other is the ADM with adaptive self-oscillating comparison. The source-follower-based BPF is composed of a source-followerbased LPF and a summation block which in this design is particularly implemented as a summing PGA. In general, the summation block can have any form. One much more power-efficient solution is illustrated in Figure 4.47 (a), where the summation is achieved simply by two AC-coupling capacitors. An alternative way of obtaining one zero is illustrated in Figure 4.47(c). The circuit in Figure 4.47(b) is the so-called super-source-follower-based LPF [144], where the input and output are the gate and source of $\mathrm{M}_{1}$, respectively. By simply changing the output from the source to the drain of $M_{1}$, a BPF transfer function can be obtained:

$$
H(s)=-\frac{s \cdot \frac{C_{2}}{g_{m 2}}}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 1} g_{m 2}}+s \cdot \frac{C_{1}}{g_{m 1}}+1}, f_{0}=\frac{1}{2 \pi} \sqrt{\frac{g_{m 1} g_{m 2}}{C_{1} C_{2}}}, Q=\sqrt{\frac{g_{m 1} C_{2}}{g_{m 2} C_{1}}}, K=\frac{g_{m 1} C_{2}}{g_{m 2} C_{1}}
$$

where $g_{m 1}$ and $g_{m 2}$ are the transconductance of $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$, respectively. One obvious shortcoming of this topology is that its peak gain is the square of its $Q$, and hence its input dynamic range is largely limited with a high $Q$ setting. However, this topology can be used as a low- $Q$ BPF cascaded by the SF-based LPF so that a high $Q$ BPF can be achieved by tuning the $Q$ of the LPF using the same method presented in Section 4.2.4.1. A topological mutation similar to the one in [166] for RF applications is shown in Figure 4.47 (d). The only difference is that the capacitor $\mathrm{C}_{1}$ is now connected between the gate and source of $\mathrm{M}_{2}$ instead of its gate and drain. The transfer function and the parameters are given below:

$$
H(s)=-\frac{s \cdot \frac{C_{1}}{g_{m 2}}}{s^{2} \cdot \frac{C_{1} C_{2}}{g_{m 1} g_{m 2}}+s \cdot \frac{C_{2}}{g_{m 2}}+1}, f_{0}=\frac{1}{2 \pi} \sqrt{\frac{g_{m 1} g_{m 2}}{C_{1} C_{2}}}, Q=\sqrt{\frac{g_{m 2} C_{2}}{g_{m 1} C_{1}}}, K=\frac{C_{2}}{C_{1}}
$$

This topology does not have the drawback of quadratic dependence of $K$ on $Q$, but large transconductance ratio and capacitance ratio are still needed to obtain a high $Q$.

Several open questions remain to be answered to further advance silicon cochlea and its related systems. How an automatic gain control block can be integrated to dynamically control the gain and $Q$ of the analog front-end so that a larger audio input dynamic range can be self-adaptively accommodated? Existing examples can be found in cochlea papers [29] or even in wireless receivers [167]. From an SoC system point of view, if this cochlea is to be integrated with a spike-processing stage for functions like voice activity detection, how the linearity of the analog front-end and the spike encoding quality can affect the ultimate performance, e.g. classification accuracy? What kind of sensing and processing co-optimization can be done to achieve the best possible performance with constraints like available power budget? A recent interesting overview article [163] written by Marian Verhelst et al. shared some useful points for future research in the era of internet of everything (IoE). She specifically stated that unlike in traditional multimedia applications where faithful signal reconstruction is often necessary, many IoE applications do not have such requirement and therefore during the analog-to-digital conversion or analog-to-spike in our case, the sampling rate could be much lower than the Nyquist rate. Only essential information or feature should be preserved in the conversion, and thus saving both conversion and processing energy. This statement in my opinion coincides with the biological reality. The spiral ganglion cells in the cochlea can hardly achieve a firing rate larger than 1 k spike/s, but we are still able to effortlessly understand human speech which undoubtedly contains frequency components above 500 Hz . Collaboration with information theorists is indispensable to better understand the working principle of smart spiking sensory systems and implement them in silicon with ever-decreasing power and increasing performance.

### 4.6 Appendix

## 4.6.a Bandwidth of the Summing CC-PGA

For a summing CC-PGA, if the Opamp has a finite DC gain of $A_{0}$ and a dominant pole at $p_{1}$, the closed-loop transfer function and the cutoff frequency $f_{-3 d B}$ can be written as:

$$
H_{C C-P G A}(s)=-\frac{A_{0} p_{1} C_{i n}}{s \cdot\left(2 C_{i n}+C_{f b}\right)+p_{1}\left(2 C_{i n}+C_{f b}+A_{0} C_{f b}\right)} \approx \frac{C_{i n} / C_{f b}}{s \cdot 2 C_{i n} /\left(A_{0} p_{1} C_{f b}\right)+1}, f_{-3 d B}=\frac{1}{2 \pi} \frac{A_{0} p_{1} C_{f b}}{2 C_{i n}}=\frac{1}{2 \pi} \frac{\mathrm{GBW}}{2 A_{c}}
$$

where $C_{i n}$ and $C_{f b}$ are the input and feedback capacitances, respectively, and GBW and $A_{c}$ are the Opamp gain-bandwidth product and CC-PGA closed-loop gain, respectively. Therefore, given the requirement of $f_{-3 d B}$ larger than the BPF central frequency for minimum peak gain loss, the GBW should be $2 \pi \cdot 2 A_{c}$ times larger instead of intuitively $2 \pi \cdot A_{c}$ times larger. Unfortunately, this was not realized during the design until approaching the tape-out deadline. The penalty of GBW $<2 \pi \cdot 2 A_{c} f_{-3 d B}$ is gain loss at the BPF central frequency and increased peak gain variation. In the fabricated design, $\mathrm{GBW} \approx 2 \pi \cdot 1.3 A_{c} f_{-3 d B}$, and the peak gain loss is around 6 dB .

## 4.6.b Transient Analysis of the ADM Amplifier


(a)

(b)

(c)

Figure 4.a1. ADM amplifier models considering (a) the limited Opamp bandwidth, (b) the switch resistance and (c) the source resistance.


Figure 4.a2. (a) StrongARM latch DT-comparator [162] and (b) a commonly-used CT-comparator based on a two-stage amplifier without compensation.

The abstract model of the ADM amplifier is illustrated in Figure 4.a1. First let us consider the effect of limited Opamp bandwidth. We use the model in Figure 4.a1(a) where $S_{0}$ is an ideal switch with zero resistance. The initial voltages across $\mathrm{C}_{1}, \mathrm{C}_{2}$ and $\mathrm{C}_{\text {rst }}$ are assumed to be 0,0 and $\delta$, respectively. According to KCL and the Opamp's transfer function, we have the following equations:

$$
\left(s \cdot V_{1}-\delta\right) \cdot C_{r s t}+s \cdot V_{1} \cdot C_{1}+s \cdot\left(V_{1}-V_{2}\right) \cdot C_{2}=0, V_{1} \cdot(-A(s))=V_{2}, A(s)=\frac{A_{0}}{s / p_{1}+1}
$$

where $A_{0}$ is the DC gain of the Opamp, and $p_{1}$ is the dominant pole. The output $V_{2}$ is solved as:

$$
V_{2}(s)=-\frac{\frac{A_{0} C_{r s t} \delta}{C_{1}+C_{2}+C_{r s t}}}{s\left(\frac{s}{p_{1}}+1+\frac{A_{0} C_{2}}{C_{1}+C_{2}+C_{r s t}}\right)}
$$

With inverse Laplace transform, the $V_{2}$ in time domain can be found as:

$$
\begin{equation*}
V_{2}(t)=-\frac{A_{0} C_{r s t} \delta}{C_{1}+C_{2}+C_{r s t}+A_{0} C_{2}}\left(1-e^{-p_{2} t}\right), p_{2}=p_{1} \cdot\left(1+\frac{A_{0} C_{2}}{C_{1}+C_{2}+C_{r s t}}\right) \tag{4.a1}
\end{equation*}
$$

Figure 4.a1(b) and Figure 4.a1(c) are the abstract models considering the switch resistance $R_{\text {on }}$ and the source resistance $\mathrm{R}_{\text {in }}$ (the output resistance of the last circuit stage), respectively. Infinite Opamp bandwidth is assumed in both cases. Using the same methods above in derivation, the amplifier output $V_{2}$ in Figure 4.al(b) can be obtained as:

$$
\begin{equation*}
V_{2}(t)=-\frac{A_{0} C_{r s t} \delta}{C_{1}+C_{2}+C_{r s t}+A_{0} C_{2}}\left(1-e^{-p_{3} t}\right), p_{3}=\frac{C_{1}+C_{2}+C_{r s t}+A_{0} C_{2}}{\left(C_{1}+C_{2}+A_{0} C_{2}\right) C_{r s t} R_{o n}} \tag{4.a2}
\end{equation*}
$$

And the $V_{2}$ in Figure 4.a1(c) can be obtained as:

$$
\begin{equation*}
V_{2}(t)=-\frac{A_{0} C_{r s t} \delta}{C_{1}+C_{2}+C_{r s t}+A_{0} C_{2}}\left(1+\frac{C_{1}}{C_{2}+C_{r s t}+A_{0} C_{2}} e^{-p_{4} t}\right), p_{4}=\frac{C_{1}+C_{2}+C_{r s t}+A_{0} C_{2}}{\left(C_{2}+C_{r s t}+A_{0} C_{2}\right) C_{1} R_{i n}} \tag{4.a3}
\end{equation*}
$$

## 4.6.c CT- and DT-Comparator Power Consumption Comparison

The average power consumption of a representative DT-comparator, the StrongARM latch as shown in Figure 4.a2(a) can be estimated as [162]:

$$
P_{\text {avg }}=\left(2 C_{P, Q}+C_{X, Y}\right) f_{C L K} V D D^{2}
$$

where $C_{P}$ and $C_{Q}$ are the parasitic capacitances at the nodes $P$ and $Q$, and $C_{X}$ and $C_{Y}$ are at the nodes $X$ and $Y$. This power estimation holds also for the two-stage DT-comparator used in the ADM design. For the $20 \mathrm{k}-\mathrm{Hz}$ cochlea channel, an oscillation frequency that is $114 \times$ the central frequency is obtained in simulation under typical corner when the input voltage difference is 1 mV , i.e. $f_{C L K}=2.28 \mathrm{M} \mathrm{Hz}$. For simplicity, let us assume that $C_{P, Q}=0.5 C_{X, Y}=C_{p a r 0} . C_{X, Y}$ is made larger considering capacitive loading at the output. With $\mathrm{VDD}=0.5 \mathrm{~V}$, the average power becomes:

$$
P_{D T}=2.28 \times 10^{6} C_{p a r 0}
$$

The largest comparison delay in the oscillation loop has the upper limit of $1 / f_{C L K}$. Now let us compute the power consumption of a CT-comparator with a step response delay of $1 / f_{C L K}$. The CT-comparator is a two-stage open-loop amplifier without compensation as shown in Figure 4.a2(b). The open-loop DC gain is usually above 5000 , and thus the sensitivity is at least 0.1 mV given a $500-\mathrm{mV}$ output swing. Two dominant poles are located at nodes $M$ and $N$, and they are assumed to have the same capacitance as $C_{P, Q}$ and $C_{X, Y}$, respectively. Setting the two pole frequencies the same is a good tradeoff between comparison speed and power consumption, and if the input step is 1 mV , the pole locations can be calculated as [74]:

$$
p_{M}=p_{N}=\frac{f_{C L K}}{\sqrt{1 m V / 0.1 \mathrm{mV}}}=7.21 \times 10^{5} \mathrm{rad} / \mathrm{s}
$$

where $p_{M}$ and $p_{N}$ are the pole frequencies of nodes $M$ and $N$ respectively. If transistors with $L=0.5 \mu \mathrm{~m}$ is used, the same as those in the DT-comparator, the Early voltages of the nFET and pFET are simulated to be $V_{E n}=7.91$ and $V_{E p}=7.66 \mathrm{~V}$, respectively. The power consumption of the CT-comparator can be then calculated as:

$$
P_{C T}=\frac{p_{M} \cdot C_{p a r 0} \cdot 2+p_{N} \cdot 2 C_{p a r 0}}{\frac{1}{V_{E n}}+\frac{1}{V_{E p}}} \cdot V D D=5.61 \times 10^{6} C_{p a r 0}
$$

This is more than twice the $P_{D T}$. In fact, much more power consumption is needed for CT-comparators in high-frequency channels under a low supply voltage because large $W / L$ ratios are necessary to keep tran-
sistors in saturation which results in large extra parasitic capacitances. In DT-comparators, only the initial saturation of the input pair transistors are required to provide certain voltage gain before the regenerative latch takes effect, and this is guaranteed by charging the drain nodes to VDD in the reset phase (discharging to ground in the case of pFET input); therefore the transistor sizes in DT-comparators can be much smaller than those in CT-comparators. The two-stage DT-comparator [161] is used in the fabricated design instead of the conventional StrongARM latch because of its speed advantage with less stacking between ground and VDD [168].

## Chapter 5: Addressable Current Reference Array for Spiking Sensors

$\mathscr{T}$his chapter describes the addressable current reference array with wide dynamic range [120] that has been used in many of the spiking sensors developed in Sensors Group at Institute of Neuroinformatics, including the silicon retina and the silicon cochlea presented in Chapter 3 and in Chapter 4 , respectively. Coarse-fine architecture is employed to cover a 174 dB DR (from 50 fA to $25 \mu \mathrm{~A}$ ) of digitally programmable bias currents with extendible number of bias branches.

Configurable bias currents are desirable in many analog/mixed-mode circuits and systems, especially in array sensors [37], [89] and neuron array [169], where the circuit parameters require wide tuning. The tuning range can span from strong inversion of transistors to below leakage currents. A previous design implemented a digitally programmable current array with a nominal 22-bit precision [170]. It used shift-ed-source current mirror (SSCM) to enable sub-leakage current copying [129], and in turn increased the effective current DR. The capability will become increasingly important as the channel leakage continues increasing with technology scaling. However, the current array was not addressable because all the shift registers (SRs) for digital configuration were connected in series, i.e. to reprogram one bias, the whole SR chain needs to be rewritten even though other biases are in no need of changing. Each bias achieved 110 dB DR with the constant 22-bit precision; however, for large current values, such high precision adjustment is not necessary. A more reasonable design would be to adjust a nominal bias with certain precision and the nominal bias can be roughly tuned over a wide DR. This coarse-fine tuning strategy is exploited in the design of the new addressable current reference array.

The content of this chapter is organized as follows: Section 5.1 describes the coarse-fine system architecture and the timing of the digital signals for bias programming; Section 5.2 gives the detailed circuit implementation, including: Section 5.2.1, the proportional-to-absolute-temperature (PTAT) master current; Section 5.2.2, the eight coarse currents with a scaling ratio of 8 generated by using compact current divider and multiplier; Section 5.2.3, the coarse-fine current interface; Section 5.2.4, the 8-bit compact current DAC; Section 5.2.5, the configurable current buffer to generate the output bias voltage; Section 5.3 presents the experimental results; Section 5.4 concludes the chapter with several remarks.

### 5.1 Coarse-Fine System Architecture

The overall architecture of the addressable current reference array and the timing of the main digital configuration signals are given in Figure 5.1. To select one bias branch in the array for programming its current, the address SRs receive the 6-bit address which is then decoded by the one-hot 6-to-64 decoder. After the bias branch selected by the decoder, the data SRs receive the 15 -bit configuration data which are loaded into the selected branch, and the final output bias voltage $\mathrm{V}_{\text {outi }}(\mathrm{i} \in[1, \mathrm{~N}], \mathrm{N} \leq 61)$ generated from the diode-connected half of a current mirror (CM) in the output buffer settles accordingly after some time. New configuration bits can be loaded into the data SRs without changing the address. At a clock frequency of 100 k Hz , new data can be written to program one branch in $<200 \mu$ s which is much shorter than the several ms in the previous long SR chain design [170]. Because both the address and data SRs share the same clock, latch and bit input to save pads, the one bit ADSEL is used to steer the flow of these inputs to either address (ADSEL=1) or data (ADSEL=0) SRs. The 15 configuration bits are composed of: 3 bits for coarse current selection, 8 bits for fine current selection, and 4 bits for output buffer configuration. Note that the number of bias branches is 61 instead of 64 , because 3 addresses are reserved for 2 branches that


Figure 5.1. (a) Coarse-fine architecture of the addressable current reference array; (b) timing diagram of the main digital signals for programming bias currents.
generate regulated voltages $\mathrm{V}_{\text {SSN }}$ and $\mathrm{V}_{\text {SSP }}$ to support SSCM configuration, and 1 branch that generates the buffer biases $\mathrm{V}_{\text {BBN }}$ and $\mathrm{V}_{\text {BBP }}$ needed in the configurable buffer in each regular bias branch.

The 8 coarse currents are derived from a classic PTAT master bias current, ranging from about 13 pA to $25 \mu \mathrm{~A}$, and are broadcasted in voltages to all the bias branches through a half CM. A selected copied coarse current in each bias branch is fed into a compact DAC with 8-bit resolution to generate a finetuned current which is then buffered to generate the output bias voltage $\mathrm{V}_{\text {outi }}$. For debugging and measuring the generated bias current, each one of the $\mathrm{V}_{\text {outi }}$ 's can be selected by an analog mux to $\mathrm{V}_{\text {test }}$ as the gate voltage to an nFET or pFET with a large $\mathrm{W} / \mathrm{L}$ ratio whose drain current can be directly measured by a Keithley 236.


Figure 5.2. Circuit diagram of the PTAT master bias.

### 5.2 Circuit Implementation

### 5.2.1 PTAT Master Bias

The core of the PTAT master bias uses the classic positive-feedback CM topology in subthreshold [171] as circled in 'PTAT core' in Figure 5.2. The sizes of $M_{3}$ and $M_{5}$ are identical with that of $M_{4}$ and $M_{6}$, respectively. $M_{1}$ and $M_{2}$ are composed of multiple unit transistors $M_{u}$ and the number of $M_{u}$ in $M_{1}$ is 4 times that in $\mathrm{M}_{2}$. The current in the off-chip resistor R, i.e. the master bias current, can be derived as:

$$
\begin{equation*}
\mathrm{I}_{\mathrm{R}}=\frac{V_{T}}{\kappa R} \ln \frac{(W / L)_{1}}{(W / L)_{2}}=\frac{V_{T}}{\kappa R} \ln 4 \tag{5.1}
\end{equation*}
$$

where $V_{T}$ is the thermal voltage, $\kappa$ is the subthreshold slope factor, and $R$ is the resistance of the off-chip resistor. $V_{T}$ is proportional to absolute temperature, and so is $\mathrm{I}_{\mathrm{R}}$. $\mathrm{C}_{1}$ is added to prevent the limit-cycle oscillation induced by the parasitic capacitance $\mathrm{C}_{\mathrm{par}}$. With $R=100 \mathrm{k} \Omega$, the simulated $\mathrm{I}_{\mathrm{R}}$ under room temperature is about 389 nA . The start-up circuit is to force the PTAT core not to enter the zero-current mode when the circuit is powered up. The digital control signal $\mathrm{V}_{\mathrm{PD}}$ at the gate voltage of $\mathrm{M}_{9}$ is to power down the master bias and in turn the whole current reference array when it is high. Two copies of $\mathrm{I}_{\mathrm{R}}, \mathrm{I}_{\text {coarsed }}$ and $\mathrm{I}_{\text {coarsem }}$ are sent to the coarse divider and multiplier respectively, which will be described in the next section. Cascode CMs are used to mitigate the copying inaccuracy caused by channel length modulation.

### 5.2.2 Eight Coarse Currents

The generated eight coarse currents are listed in Table 5.1 with the PTAT master bias current denoted as $\mathrm{I}_{\text {coarse3 }}$. The scaling ratio is approximately 8 . Hence, to obtain $\mathrm{I}_{\text {carse2 }}$ and $\mathrm{I}_{\text {coarsel }}, \mathrm{I}_{\text {coarse3 }}$ needs to be multiplied by $8^{1}$ and $8^{2}$, respectively, and to obtain $\mathrm{I}_{\text {coarse } \sim} \sim \mathrm{I}_{\text {coarses }}, \mathrm{I}_{\text {coarses }}$ needs to be divided by $8^{1} \sim 8^{5}$, respectively. Traditional CMs can be used for the circuit implementation which, however, is not area-efficient. The current division thus utilizes the same compact current-splitting technique [128] that has been employed to generate the geometrically scaling currents in silicon cochlea in Chapter 4. The proposed current multiplication as illustrated in Figure 5.3 can be seen as a reverse application of the technique. The

Table 5.1. Simulated nominal coarse current under room temperature.

| Coarse Current | $\mathrm{I}_{\text {coarse1 }}$ | $\mathrm{I}_{\text {coarse2 }}$ | $\mathrm{I}_{\text {coarse3 }}$ | $\mathrm{I}_{\text {coarse4 }}$ | $\mathrm{I}_{\text {coarse5 }}$ | $\mathrm{I}_{\text {coarse6 }}$ | $\mathrm{I}_{\text {coarse7 }}$ | $\mathrm{I}_{\text {coarse8 }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value (A) | $24.8 \mu$ | $3.16 \mu$ | 388 n | 49.7 n | 6.39 n | 821 p | 105 p | 13.4 p |
| LSB (A) | 97.01 n | 13.34 n | 1.647 n | 202.5 p | 25.21 p | 3.198 p | 409.9 f | 55.72 f |
| Current Ratio | $\frac{\mathrm{I}_{\text {coarsel }}}{\mathrm{I}_{\text {coarse2 }}}$ | $\frac{\mathrm{I}_{\text {coarse2 }}}{\mathrm{I}_{\text {coarse3 }}}$ | $\frac{\mathrm{I}_{\text {coarse3 }}}{\mathrm{I}_{\text {coarse4 }}}$ | $\frac{\mathrm{I}_{\text {coarse4 }}}{\mathrm{I}_{\text {caarse5 }}}$ | $\frac{\mathrm{I}_{\text {coarse5 }}}{\mathrm{I}_{\text {caarse6 }}}$ | $\frac{\mathrm{I}_{\text {coarse6 }}}{\mathrm{I}_{\text {caarse7 }}}$ | $\frac{\mathrm{I}_{\text {coarse7 }}}{\mathrm{I}_{\text {coarse8 }}}$ |  |
| Value | 8.05 | 8.02 | 7.81 | 7.78 | 7.81 | 7.83 | 7.91 |  |



Figure 5.3. Circuit diagram of a compact current multiplier.
validity of its function can be checked by the following calculation. All the transistors are composed of multiple unit transistors $M_{u}$ with the red number labeled close to their sources as the $M_{u}$ number. Assume all the transistors are in above-threshold for the relatively large currents from $\mathrm{I}_{\text {coarsel }}$ to $\mathrm{I}_{\text {coarse } 3}$ in Table 5.1. With the simplest square law [26], $\mathrm{I}_{\text {in }}$ and $\mathrm{I}_{\text {out }}$ in Figure 5.3 can be written as:

$$
\begin{gathered}
\mathrm{I}_{\text {in }}=\frac{1}{2} \mu_{p} C_{o x}\left(\frac{W}{L}\right)_{u}(N-1)\left(V_{2}-V_{1}-V_{t h}\right)^{2} \\
\mathrm{I}_{\text {out }}=\frac{1}{2} \mu_{p} C_{o x}\left(\frac{W}{L}\right)_{u}(N-1)\left(V_{2}-V_{D D}-V_{t h}\right)^{2}
\end{gathered}
$$

$M_{3}$ is in linear region, and its $I_{d s}$ is the sum of $I_{i n}$ and the $I_{d s}$ of $M_{1}$ :

$$
\frac{N}{N-1} \mathrm{I}_{\mathrm{in}}=\mathrm{I}_{\mathrm{d} s \mathrm{M} 3}=\mu_{p} C_{o x}\left(\frac{W}{L}\right)_{u} \frac{N}{N-1}\left[\left(V_{2}-V_{D D}-V_{t h}\right)\left(V_{1}-V_{D D}\right)-\frac{1}{2}\left(V_{1}-V_{D D}\right)^{2}\right]
$$

The equation above gives rise to the equality below:

$$
\frac{1}{2}\left(V_{2}-V_{D D}-V_{t h}\right)^{2}=\frac{N}{N-1}\left[\left(V_{2}-V_{D D}-V_{t h}\right)\left(V_{1}-V_{D D}\right)-\frac{1}{2}\left(V_{1}-V_{D D}\right)^{2}\right]
$$

Therefore, it is easy to draw the conclusion:

$$
\begin{equation*}
\mathrm{I}_{\mathrm{out}}=(N-1) \mathrm{I}_{\mathrm{dsM} 3}=N \mathrm{I}_{\mathrm{in}} \tag{5.2}
\end{equation*}
$$

Eq. (5.2) still holds if subthreshold current equation is applied. If $N=8, M_{2}$ is composed of $N-1=7 M_{u}$, and $M_{3}$ is composed of $N /(N-1)=8 / 7 M_{u}$. In practical implementation, $M_{2}$ should have $49 M_{u}$, and $M_{3} 8 M_{u}$. In order to keep the number of $M_{u}$ in $M_{2}$ low for the sake of reduced area, $N /(N-1)$ could be replaced by 1 [129]. Eq. (5.2) needs to be modified as:


Figure 5.4. Complete circuit diagram of the current multiplier and divider for generation of eight coarse currents, and their diode-connected nFET loads.

$$
\begin{equation*}
\mathrm{I}_{\text {out }}=(N+1-\Delta) \mathrm{I}_{\text {in }} \quad(0 \leq \Delta<1) \tag{5.3}
\end{equation*}
$$

Approximate octal weight among $\mathrm{I}_{\text {coarse } 1} \sim \mathrm{I}_{\text {coarse8 }}$ can be achieved by making $\mathrm{N}=7$ or 7.5 . $\mathrm{N}=7.5$ is chosen for larger scaling and is implemented as a $13: 2 \mathrm{M}_{\mathrm{u}}$ number ratio (e.g. the ratio of $\mathrm{M}_{2}$ to $\mathrm{M}_{3}$ ). The complete circuit of generating the eight coarse currents together with the diode-connected nFET loads is illustrated in Figure 5.4. Compared to conventional CM implementations [172], the adopted current multiplier and divider save considerable chip areas, especially with such wide current span. The sizes of the diodeconnecteed nFET loads are largely different to accommodate the large span of the coarse currents. For example, the size of the nFET for $\mathrm{I}_{\text {coarsel }}$ is eight $(W / L)=2 \mu \mathrm{~m} / 5 \mu \mathrm{~m}$ in parallel with its nominal output gate voltage $\mathrm{V}_{\text {coarsel }}=583 \mathrm{mV}$, and the size of the nFET for $\mathrm{I}_{\text {coarse8 }}$ is eight $(W / L)=0.5 \mu \mathrm{~m} / 20 \mu \mathrm{~m}$ in series with nominal $\mathrm{V}_{\text {coarse } 8}=179 \mathrm{mV}$. Note that the nFET sizes for $\mathrm{I}_{\text {coarse } 5} \sim \mathrm{I}_{\text {coarse8 }}$ are identical to save areas in individual bias branches as will be explained in the next section. The generated voltages $\mathrm{V}_{\text {coarsel }} \sim \mathrm{V}_{\text {coarse8 }}$ are broadcasted to all the bias branches, and one of the coarse currents is copied in each bias branch via the other half of the CM.

### 5.2.3 Coarse-Fine Current Interface

One of the eight coarse currents is selected in each bias branch by a one-hot 3-to-8 decoder which has the output of $\mathrm{S}_{\mathrm{i}}(\mathrm{i}=1 \sim 8)$ to control the selection switch. Because the smallest coarse current $\mathrm{I}_{\text {coarse }}$ is about 13 pA , close to the channel leakage of a normal nFET in off state in $0.18 \mu \mathrm{~m} \mathrm{CMOS}$, the selection circuits need careful design to minimize the impact of the leakage from the selection switches on the accuracy of current copying, especially on $\mathrm{I}_{\text {coarses. }}$. Figure 5.5 (a) shows the conventional current selection scheme. If $I_{\text {coarsei }}$ is not selected, $S_{i}$ is disconnected; if $I_{\text {coarsei }}$ is selected, $S_{i}$ is connected. The drain terminals of all the nFET switches are connected together. Ideally, $\mathrm{S}_{\mathrm{i}}$ would contribute no current to the common line when it is disconnected; however, with its $\mathrm{V}_{\mathrm{gs}}=0$ and $\mathrm{V}_{\mathrm{ds}}=\mathrm{V}_{\text {common }}$ that is close to VDD, the channel leakage from the switch nFET is in the range of pA , and has large variation depending on process and temperature. In the case of selecting $\mathrm{I}_{\text {coarse }}$, it is completely overwhelmed by all the summed leakage currents, and in turn $\mathrm{I}_{\text {coarse8 }}$ cannot be accurately copied for the subsequent fine current selection. The leakage may be reduced to an acceptable level by using very long channel length, but the consequently large ON-resistance results


Figure 5.5. (a) Conventional current selection scheme using a single nFET as the switch; (b) current selection with switch leakage current suppression using two $n F E T s\left(S_{i}\right.$ and $\left.S_{i b}\right)$ and one $\mathrm{pFET}\left(\mathrm{S}_{\mathrm{ia}}\right)$.


Figure 5.6. Complete circuit of the coarse-current selection.
in too large voltage drop for large coarse currents like $\mathrm{I}_{\text {coarsel }}$. To circumvent the switch leakage problem, a leakage suppression scheme is proposed as illustrated in Figure 5.5(b). Two auxiliary switches $\mathrm{S}_{\mathrm{ia}}$ and $\mathrm{S}_{\mathrm{ib}}$ are added $\mathrm{S}_{\mathrm{i}}$. When $\mathrm{S}_{\mathrm{i}}$ is disconnected, $\mathrm{S}_{\mathrm{i}}$ is connected and $\mathrm{S}_{\mathrm{ib}}$ is disconnected so that the source voltage of $S_{i}$ is elevated to VDD. This way the $S_{i} n F E T$ has $V_{g s}=-V D D$ and $V_{d s}=V_{\text {common }}-V D D$ that is close to 0 . Therefore the leakage current from $\mathrm{S}_{\mathrm{i}}$ is largely suppressed under various process and temperature corners to the range of fA.

The complete circuit of the coarse current selection is shown in Figure 5.6. A 3-to-8 decoder generates the 8 control signals for switches $\mathrm{S}_{1}-\mathrm{S}_{8} . \mathrm{I}_{\text {coarse1 }}$ to $\mathrm{I}_{\text {coarse }}$ are copied via separate nFETs , and the selection is done by using the leakage suppression scheme in Figure $5.5(\mathrm{~b})$. $\mathrm{I}_{\text {coarse }}$ to $\mathrm{I}_{\text {coarse8 }}$ share the same $n$ FET by respectively connecting its gate to $\mathrm{V}_{\text {coarse5 }}$ to $\mathrm{V}_{\text {coarse8. }}$. Compared to using separate $n$ FETs for $\mathrm{I}_{\text {coarse }}$ to $\mathrm{I}_{\text {coarse8, }}$ $n$ FET sharing saves about $38 \%$ area for each bias branch. The current selection switch $\mathrm{S}_{5-8}$ that is controlled by a four-input NOR gate is a simple $n$ FET as in Figure 5.5(a). No leakage suppression is needed for two reasons: the leakage from $S_{5-8}$ alone is negligible to coarse currents $I_{\text {coarsel }}$ to $I_{\text {coarse4 }}$; the leakage from $S_{i a}$ in its off state may affect the accuracy of small currents like $I_{\text {couarse }}$.

The selected $\mathrm{I}_{\text {coarsei }}$ is copied via an auto-configured pFET CM before being fed into the 8-bit current


Figure 5.7. Auto-configured CM to convey the selected coarse current to an 8 -bit current DAC for finecurrent selection.


Figure 5.8. 8-bit R-2R current-splitting DAC for fine-current selection.

DAC. The auto-configured CM as shown in Figure 5.7 is composed of two different CMs. The CM1 has large $W / L$ for coarse currents $\mathrm{I}_{\text {coarse1 }}$ and $\mathrm{I}_{\text {coarse2 }}$, and the CM 2 has small $W / L$ for coarse currents from $\mathrm{I}_{\text {coarse }}$ to $\mathrm{I}_{\text {coarses. }}$. The CM selection logic which is a simple OR gate uses the Bit2 and Bit1 in Figure 5.6. Diodeconnected $\mathrm{pFET} \mathrm{M}_{5}$ provides the gate voltage to the 8 -bit current DAC as will be explained in the next section.

### 5.2.4 8-bit Current DAC

Illustrated in Figure 5.8, the compact 8-bit current DAC has the R-2R topology using pFETs. It is based on the same current-splitting principle used in Section 5.2.2 except here with binary scaling instead of octave. The number of unit transistors is marked at the source of each pFET. To the first-order approximation, the compound $\mathrm{pFET} \mathrm{M}_{\mathrm{com}}$ comprising all the DAC pFETs in Figure 5.8 is equivalent to a pFET comprising two unit pFETs in parallel with the gate voltage $\mathrm{V}_{\mathrm{gDAC}}$ from $\mathrm{M}_{5}$ in Figure 5.7. $\mathrm{M}_{5}$ is also composed of two unit pFETs in parallel, and hence $\mathrm{M}_{\text {com }}$ and $\mathrm{M}_{5}$ form the cascode stage of the CM in Figure 5.7, enabling a more accurate current copying. In previous designs [170], the gate voltage of $\mathrm{M}_{\mathrm{com}}$ is connected to $\mathrm{V}_{\mathrm{MBN}}$ in Figure 5.2, which not only loses the benefit of cascading but also compromises the saturation of the vertical pFETs. The eight fine currents $\mathrm{I}_{\text {finel }} \sim \mathrm{I}_{\text {fine8 }}$ are either collected at the output node to


Figure 5.9. Fine-current buffer to generator the output bias voltage.
the next buffer stage as $\mathrm{I}_{\text {fineout }}$, or at the drain of the diode-connected $\mathrm{nFET} \mathrm{M}_{\text {dump. }} . \mathrm{I}_{\text {fineout }}$ can be written as:

$$
\begin{equation*}
\mathrm{I}_{\text {fineout }}=\mathrm{I}_{\text {finein }} \sum_{i=1}^{8} 2^{-i} \mathrm{~B}_{i} \tag{5.4}
\end{equation*}
$$

where $\mathrm{B}_{i}$ is the value of the i -th bit. The LSB of each coarse current is given in Table 5.1.

### 5.2.5 Configurable Output Buffer

The reconfigurable output buffer receives the output current of the 8 -bit DAC $\mathrm{I}_{\text {fineout }}$ described in the previous section and generates a bias voltage $\mathrm{V}_{\text {out }}$ to bias other circuit modules, such as retina pixels and cochlea channels, in the form of CM even though the pair transistors are distant from each other. The simplified buffer circuit is illustrated in Figure 5.9. The features of this buffer are described as follows.
(1). If $\mathrm{I}_{\text {coursel }}$ is selected in a bias branch, the cascode mode of the buffer will be automatically invalid, i.e. the switches $S_{2 n}$ and $S_{2 p}$ are always connected so that $M_{3}$ and $M_{8}$ are disabled. This is to prevent voltage headroom problems, i.e. $\mathrm{V}_{\text {inn }} / \mathrm{V}_{\text {inp }}$ from being too high/low for the transistors in the previous circuit stage, namely the 8 -bit DAC, to stay in saturation. If one of $\mathrm{I}_{\text {coarse } 2} \sim \mathrm{I}_{\text {coarse }}$ is selected, the cascode mode can be enabled by disconnecting $S_{2 n}$ and $S_{2 p}$.
(2). The source-follower structures formed by $\mathrm{M}_{5} / \mathrm{M}_{6}$ and $\mathrm{I}_{\mathrm{BN}}$, and $\mathrm{M}_{9} / \mathrm{M}_{10}$ and $\mathrm{I}_{\mathrm{BP}}$ are used to stabilize $\mathrm{V}_{\text {outn }}$ and $\mathrm{V}_{\text {outp }}$ respectively in case of digital coupling disturbance. For $\mathrm{I}_{\text {corasel }} \sim \mathrm{I}_{\text {coarse }}$, the transistor with low threshold $\mathrm{M}_{5} / \mathrm{M}_{9}$ is used to keep $\mathrm{V}_{\text {inn }} / \mathrm{V}_{\text {inp }}$ not too high/low. For $\mathrm{I}_{\text {coarse }} \sim \mathrm{I}_{\text {coarses }}$, the transistor with normal threshold $M_{6} / M_{10}$ is used to maintain the saturation of $M_{1} / M_{7}$. The control logic is derived from the


Figure 5.10. Layout of the current reference array.
Bit2 in Figure 5.6. The nominal values of $\mathrm{I}_{\mathrm{BN}}$ and $\mathrm{I}_{\mathrm{BP}}$ are both 50 nA , which are tunable by changing the $\mathrm{V}_{\text {BBN }}$ and $\mathrm{V}_{\text {BBP }}$ in Figure 5.1. However, they cannot be too high to avoid voltage headroom problems.
(3). Normally the sources of $M_{1}$ and $M_{7}$ are connected to ground and VDD respectively. In the noncascode mode, to enable sub-off current generation, the source of $M_{1}$ needs to be lifted up above ground by connecting to $V_{S S N}$ via $S_{1 n}$, and $M_{7}$ be shifted down from VDD by connecting to $V_{\text {SSP }}$ via $S_{1 p}$. Together with the source-follower structure mentioned in (2), the shifted-source mode is enabled [129]. $\mathrm{V}_{\text {SSN }}$ and VDD $-V_{\text {SSP }}$ are usually around $200 \sim 300 \mathrm{mV}$.
(4). If the bias branch is configured to send $V_{\text {outn }}$ to $V_{\text {out }}$ by connecting $S_{5 n}$ and $S_{6}, S_{2 n}$ can be connected or disconnected for the selection of the cascode mode. If instead $V_{\text {outp }}$ is sent to $V_{\text {out }}$ by connecting $S_{5 p}$ and $\mathrm{S}_{6}, \mathrm{~S}_{2 \mathrm{n}}$ is always disconnected to enable more accurate current copying via the cascode CM formed by $M_{1} \sim M_{4}$ except for the case of $I_{\text {corasel }}$.
(5). The output voltage $\mathrm{V}_{\text {out }}$ can be selected as one of $\mathrm{V}_{\text {outn }}, \mathrm{V}_{\text {outp }}$, and $\mathrm{V}_{\text {rail }}$. $\mathrm{V}_{\text {rail }}$ is hard-wired to either ground or VDD in layout, and is supplied in reset to ensure minimum current consumption during start-up before the configuration data is loaded.

### 5.3 Experimental Results

The addressable current reference array has been fabricated and verified to be functional in both UMC $0.18 \mu \mathrm{~m}$ RF/MM CMOS and TowerJazz $0.18 \mu \mathrm{~m}$ CIS processes. The quantitative measurements are from the UMC run along with the first version of DAVIS array [18]. Figure 5.10 shows the layout of the array with 19 bias branches. Each branch occupies an area of $360 \times 22 \mu \mathrm{~m}^{2}$ that is only $25 \%$ of the previous design [170] thanks to the removal of the long SR chain in each bias branch.

The measured master bias $\mathrm{I}_{\mathrm{R}}$ is about 390 nA under room temperature, which is consistent with the simulated $\mathrm{I}_{\text {couarse }}$ given in Table 5.1. Figure 5.11 gives the measured current tuning curves. The x -axis is the 8 -bit fine current selection codes with the value from 1 to 256 in logarithm. The 3 -bit coarse current selection codes are label along each curve. The lowest 3 curves are measured with shifted-source enabled. The specific and leakage current levels extracted from a $2 \times 2 \mu \mathrm{~m}^{2}$ transistor are also marked. The largest


Figure 5.11. Measured currents as the function of the 8 -bit fine code and 3-bit coarse code.
and smallest currents are about $25.83 \mu \mathrm{~A}$ and 31.13 fA , respectively. This gives a current dynamic range of 178 dB , close to the simulated 173 dB . The current range overlap between neighboring coarse codes allows reasonably fine resolution at all current scales, which is not the case in a non-hierarchical coarsefine architecture. All 8 tuning curves are approximately linear with the fine code value. Note that the subleakage current capability is compromised at higher temperatures. According to simulation, the junction leakage currents contributed by the switches in the 8 -bit R-2R DAC can reach to several pA at $80^{\circ} \mathrm{C}$, which degrade the current dynamic range to about 130 dB . Within one die, the worst-case $1 \sigma$ mismatch among all bias branches over the whole current range is within $\pm 10 \%$. The worst-case DNL is found to be about 5LSB at $\mathrm{I}_{\text {coarsel }}$, which happens at the transition of the MSB of the 8-bit fine codes.

### 5.4 Conclusion and Remarks

This chapter detailed the design and measurement of an addressable current reference array with coarse-fine architecture. Compared to previous designs [170], the main benefits are: area-saving in each bias branch and faster current programming thanks to the avoidance of long SR chain; extended current dynamic range with less resolution bits owing to the hierarchical coarse-fine structure; adaptive power consumption in each bias branch in accordance to the selected coarse current, instead of a fixed upscaling of the master bias.

To improve the linearity of the 8 -bit current-splitting DAC, besides using larger unit transistors, transistors working in linear region with regulated identical voltages at the outputs of the selected current and the dumped current could be adopted at the cost of increased area. This method has been proven valid in an untrimmed 10-bit SAR ADC with -79 dB THD using MOS-only DAC [173].

## Chapter 6: Summary and Future Work

ЯThis thesis starts with addressing the impact of the non-idealities of the massively-parallel asynchronous spike encoding in artificial bio-inspired sensors on encoding performances by mathematical modeling and numerical simulation. Specifically, two spike encoding mechanisms are studied, namely the self-timed reset (STR) that was implemented in prior dynamic vision sensors (DVSs), i.e. the spiking silicon retina, and the asynchronous delta modulation (ADM) that was used to implement contin-uous-time ADCs in the form of level-crossing sampling. The effects of spike transmission delay and switch holding (refractory period) of STR on the encoding quality in terms of signal-to-distortion ratio (SDR) of reconstructed input from spike trains are compared between STR and ADM using linear decoding. The STR is susceptible to signal loss due to the aforementioned non-idealities, whereas the ADM usually gives higher decoding SDR especially at high input frequencies and large quantization bit number because unlike STR its feedback does not disruptively interfere with the input. Nonlinear decoding based on frame theory results in much higher SDR than linear decoding, which facilitates the study of the impact of delay variation during spike transmission on encoding quality degradation. Delay variation comes from the input-slope dependent comparison delay and spike queueing owing to the AER arbitration. Using circuit analysis and queueing theory, both of the delay variation sources are quantitatively related to circuit and system design parameters like comparator biasing and AER bandwidth, and in turn the encoding quality measured by the SDR of reconstructed signals using nonlinear decoding. The results can be useful for future specifications-guided design of spiking sensors.

Two spiking sensors using ADM are developed. The new silicon retina focuses on improving previous designs in two aspects: the temporal contrast sensitivity and the encoding quality. The former is achieved by employing a low-noise photoreceptor with pFET common-gate feedback and a PGA for sufficient frontend gain, and the latter is achieved by using a proposed compact asynchronous switched-capacitor circuit for in-pixel ADM. The new $0.5-\mathrm{V} 55-\mu \mathrm{W}$ silicon cochlea features a proposed low-power pro-grammable-gain source-follower-based BPF composed of a $4^{\text {th }}$-order LPF and a summing PGA, and an ADM using latched comparators with adaptive self-oscillation loop to provide the pseudo-clock for improved energy efficiency. Both chips have been fabricated in $0.18 \mu \mathrm{~m}$ CMOS and experimentally validated.

The improved temporal contrast sensitivity and encoding quality of silicon retina can facilitate its application in areas like in-vivo optical neuroimaging where the fluorescence dynamics is of particular interest. The intrinsic sparse output characteristic of event encoding can help significantly reduce the output data redundancy and thus save power in RF transmission which is beneficial to long-term monitoring of free-moving animals. As already discussed in Chapter 3, several technical hurdles including the ADM threshold mismatch still need to be overcome before any practical use. Alternative encoding mechanisms like asynchronous sigma-delta modulation (ASDM) may solve the threshold mismatch problem at the cost of much limited output bandwidth because ASDM produces idle spikes without any transient input change. However, ASDM could be used in the design of ultra-low-voltage spiking image or vision sensors by directly encoding photocurrent into spike timing instead of A/D conversion in voltage domain. One example is the Octopus imager [5] where the integrate-and-fire encoding can be seen as a special case of ASDM [174]; another example is the free-running-oscillator-pixel imager with synchronous frame sampling [175]. Neither of them exploited time-domain interpolation of the sensor output nor explored ultra-low-voltage design like in some $0.5-\mathrm{V}$ imagers with pulse-width-modulation readout [176], [177],


Figure 6.1. Four possible architectures for sound acquisition and processing: (a) The traditional architecture implementing an energy-based voice activity detection (VAD) algorithm with clocked ADC and DSP including FFT [41]; (b) A clockless system with CT-ADC/DSP. So far the CT-ADCs mainly implement level-crossing sampling and existing CT-DSPs only have FIR/IIR functions. The insert is the testing results from [61] showing adaptive system power consumption in response to a segment of sound signal; (c) Feature extraction of the energy in each frequency band is moved to the analog frontend. By exploiting the decision tree machine learning algorithm for VAD, unused analog filters are dynamically disabled to save power [42]; (d) Proposed architecture with cochlea-like signal acquisition and event-driven hybrid analog/digital processing.
and in both of the prior cases the problem of threshold mismatch among pixels still needs to be circumvented if no calibration or post-processing is desired. One interesting encoding scheme published recently [178] resembles ASDM except it does not produce any idle event output when the input DC is at 0 . Preliminary modeling and simulation using a nonlinear decoding algorithm have shown its big advantage over ADM, i.e. the reconstructed signal does not suffer from DC drift with unbalanced ON and OFF thresholds, although the reconstruction error is still sensitive to threshold variations. One possible means of workaround is to adopt the so-called time-domain correlated double sampling [179] so that the voltage crossing represented by two spike duplets has minimized variation across pixels.

Ultra-low-voltage and ultra-low-power designs are deemed as one of the key enabling technology for ubiquitous embodiment of the concept internet of everything, which is the theme of ISSCC 2016. The cochlea design in this thesis echoes with this trend. Not only does the sensor core itself operate under a $0.5-\mathrm{V}$ supply and dissipate merely tens of $\mu \mathrm{W}$ with $64 \times 2$ channels, but also the spike output is the natural input to event-driven signal processing systems with activity-dependent power consumption like the event-driven DSPs [61] and spiking neural networks [46]. Further improvement on the power efficiency of the sensor core can be achieved on both circuits and sub-system levels. For example, the summing method of creating one zero proposed in this thesis requires the bandwidth of the summing PGA to be as at least twice the BPF central frequency for non-degraded passband gain. One method of reducing the PGA bandwidth requirement is to create the zero by a source-follower-based BPF [141], [142], [166] so
that the summation can be avoided. A more radical method is to replace the closed-loop PGA with a more power-efficient open-loop OTA using the spike encoding scheme proposed in [178]. Functionally it is beneficial to add automatic gain control to the silicon cochlea in order to fully harness the system dynamic range which is especially important under a low voltage supply.

In the future, the cochlea is expected to be directly integrated with spike-processing ASICs to form ul-tra-low-power smart sensor nodes. The first step is to implement some simple functions like voice activity detection, sound source localization and emergency word detection, which is my proposed postdoc project to work with Prof. Mingoo Seok and Prof. Yannis Tsividis at Columbia University. Although IBM's pure digital neural network approach represented by the TrueNorth chip is becoming popular among academic communities, analog computing may still have a place in further improving computational energy efficiency [180]-[182]. The complete SoC is envisioned to contain analog preprocessing (bandpass filtering in the case of silicon cochlea), spike encoding, and digital or analog/digital hybrid spike processing as depicted in Figure 6.1(d). In terms of system optimization, one fundamental question to ask is that how the requirement of spike processing performance measured by metrics like classification or recognition accuracy enforces constraints on the analog frontend design. In principle, this type of SoC architecture can be extended to vision systems. Despite the advantage of reduced number of spike encoders and thus spike transmission bandwidth [183], an analog feature extraction pre-processing stage added before spike encoding is usually considered too area-consuming to be accomplished in focal plane of vision sensors. The area difficulty might be overcome by the maturing 3D integration technology [184]-[186]; a more challenging problem is how to minimize the performance variation in feature extraction and how much the post-processing can compensate for the front-end imperfections with a targeted system performance goal. It is also worth mentioning that the research in biomedical IC systems also seems to be following this trend [187].

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