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A speed limit scheme to enhance carpooling in the presence of HOV lanes in degradable networks

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Abstract

Triggered by the principle that speed limits are able to adjust flow patterns in a network, this paper proposes a scheme imposing speed limits on the low occupancy vehicle (LOV) lanes in the presence of high occupancy vehicle (HOV) lanes to encourage travelers to carpool. Degradable traffic networks are taken into consideration. A bi-level programming model is built to detect the effects of this scheme. The lower level describes the travelers' choice of travel mode and route by a variational inequality while the upper level targets the maximum net economic benefit and minimal number of accidents. A multi-objective genetic algorithm for the computation of optimal values of the speed limits is applied to compute solutions to the Sioux Falls network example. The optimal solutions, the rough bound of dominate solution and the system performances with respect to various demands are described and discussed.

Keywords

traffic network modelling – speed limit scheme – net economic benefit and number of accidents – HOV lanes

1. Introduction

HOV lanes, which facilitate travelers to carpool and thereby reduce traffic volumes to alleviate traffic congestion, have been built in many places around the world, such as North America, Europe. However, the performance of HOV lanes is controversial. The favorable effects of HOV lanes have been well-documented (Li, 2001; Menendez and Daganzo, 2007) while some researchers indicates that the impacts of HOV lanes on congestion are not always positive. For example, (Dahlgren, 1998) showed that sometimes HOV lanes are less effective than general purpose lanes. (Kwon and Varaiya, 2008) found that HOV lanes do not always attract travelers so that overall congestion does not decrease. Also they demonstrated that HOV lanes neither increase the share of carpooling nor reduce the overall congestion in the case of HOV lanes implemented in California.

Compared to HOV lanes, HOT (high occupancy/toll vehicle) lanes, which are applied to attract more travelers who are willing to pay to use it, show better a performance than HOV lanes, especially in North America. Also there is a large body of literature with respect to the effects of HOT lanes (Cao, 2011; Gordon et al., 2004). Nevertheless, the effects of HOT lanes are not always desirable. Cho demonstrated that HOT lanes have limited effects on mitigating traffic congestion in terms of the performance of I-394 HOT lane on account of travelers inclining to jump a queue. Furthermore, imposing tolls probably needs toll stations, which would causes delays on a link. In addition, imposing tolls will lead to discontent among the public, especially in developing country like China. Therefore, more effective measures to encourage travelers to carpool need to be explored.

Recently some researchers demonstrated that speed limits are able to mitigate traffic congestion in a network. Yang et al. made the first attempt to research the effects of a speed limit scheme in a general network (Yang et al., 2012). The reason why speed limits are able to alleviate traffic congestion in a network is that they adjust flow patterns by increasing time costs on links. The relevant research is available, including optimal speed limits design aiming at various objectives (Wang, 2013; Yang et al., 2013; Liu et al., 2015), travelers' behavior under speed limits and empirical study (Nitzsche and Tscharaktschiew, 2013).

Triggered by the research above, we propose a scheme imposing speed limits on the LOV lanes to increase the cost on LOV lanes and thereby to attract more travelers to utilize HOV lanes. A speed limit scheme is easy to implemente without advanced technologies and reasonable speed limits are advocated by the public. Nevertheless, the results of research with regard to speed limits has shown that a speed limit scheme is a double-edge sword. Namely, it has either positive or negative effects on alleviating congestion (Wang, 2013). According to this conclusion we are not capable of guaranteeing the positive effects of a speed limit scheme on LOV lanes in

all circumstances. Therefore it is worth exploring the effects of speed limit scheme in a general network in the presence of HOV lanes.

Three factors are deemed to be essential to affect the route choices of travelers: travel time, travel time reliability and monetary cost (Abdel-Aty et al., 1995). Thus abundant research paid attention to travel time reliability. The causes of travel time reliability are divided into two aspects: supply and demand. From the perspective of supply, the degradable network is common due to causes such as weather, traffic accident and others. The definition of capacity reliability in a degradable network is the probability of the network for a given demand at a specific service level (Chen et al, 1999; Chen et al, 2002). Many studies have emerged for the traffic equilibrium in a degradable network. Lo and Tung (2003) proposed a probabilistic user equilibrium (PUE) model. Lo et al. (2006) extended the PUE model and used travel time budget (TTB) in a degradable network to demonstrate the system optimal. For speed limits, Yan et al. (2015) investigated the performances of a degradable network under a speed limit scheme and designed the optimal speed limits. In accordance with the results above we are capable of exploring the impacts of speed limit scheme on LOV lanes in a degradable network.

This paper detects the effects of the speed limit scheme in the presence of HOV lanes based on travel time reliability in a degradable network. The organizations is as follows: Section 2 presents the notation and formulation of this paper. Section 3 investigates the performances of this scheme in a small network. Section 4 builds the optimal speed limit scheme model. Section 5 demonstrates the algorithm adopted in this paper. Section 6 applies this model to the Sioux Falls network and analyzes the impacts. The concluding remarks are presented in section 7.

2. Model formulation

2.1 Notations and assumptions

We utilize the following notation in the model:

N	Set of nodes in the network
A	Set of links in the network
$\bar{A}$	Set of LOV lanes
W	Set of OD pairs in the network
L	Set of routes in the network
I	Set of types of travelers, $i \epsilon I, i = H, M$
M	Set of travel modes, $m \epsilon M, m = S, C$
d_w	Travel demand between OD pair w
$d_w^{i,m}$	Travel demand of type i for mode m between OD pair w
$f_{w,l}^{i,m}$	Traffic flow of type i for mode m on path l between OD pair w
v_a	Traffic flow on link a
v_a^{car}	Vehicle flow on link a
t_a	Travel time and mean travel time on link a
t_w^l	The travel time and mean travel time on path l between OD pair w
t_a^{sl}	The travel time under a given effective speed limit scheme on link a
s_a	The speed limit on link a
v_a^c	The critical vehicle flow on link a
C_a	The maximum capacity on link a
θ_a	The coefficient of capacity decrement
C_a^{sl}	The critical capacity of link a under a speed limit scheme

The following assumptions are made for the generality and feasibility in realistic circumstances:

(1) The degradable random link capacity is independent from each other.
(2) The degradable random link capacity follows a uniform distribution.
(3) The speed of a vehicles on a link is constant.
(4) All the travelers follow the speed limit regulations.
(5) Each HOV lane or LOV lane is a link.

2.2 Travel time distribution

In this paper, we use BPR function to capture link travel time:

$$t_a = t_a^0(1 + \rho(v_a^{car}/C_a)^n) \tag{1}$$

Under a speed limit scheme, the link travel time can be expressed as:

$$t_a = \begin{cases} t_a^{sl}, & v_c \geq v_a \\ t_a^0[1 + \rho(v_a^{car}/C_a)^n], & v_c \leq v_a \end{cases} \tag{2}$$

Assuming the lower bound of capacity on link a is $\theta_a C_a$, and the upper bound of capacity is C_a (Lo et al., 2006). According to (Yan et al., 2015), the critical capacity C_a^{sl} is

$$C_a^{sl} = v_a \sqrt[n]{\beta t_a^0/(t_a^{sl} - t_a^0)}\, t_a^0 \tag{3}$$

(Yan et al., 2015) indicated that link a is supposed to be in one of the three states under speed limits:

State 1: $C_a^{sl} \leq \theta_a C_a$

$$E(T_a) = t_a^{sl} \tag{4}$$

$$(\sigma_a^t)^2 = 0 \tag{5}$$

State 2: $\theta_a C_a \leq C_a^{sl} \leq C_a$

$$E(T_a) = \int_{\theta_a C_a}^{C_a^{sl}} t_a^0 \left(1 + \beta \left(\frac{v_a^{car}}{C_a}\right)^n\right) \frac{1}{C_a - \theta_a C_a} dC_a + \int_{C_a^{sl}}^{C_a} t_a^{sl} \frac{1}{C_a - \theta_a C_a} dC_a \tag{6}$$

$$(\sigma_a^t)^2 = \int_{\theta_a C_a}^{C_a^{sl}} \left[t_a^0 \left(1 + \beta \left(\frac{v_a^{car}}{C_a}\right)^n\right)\right]^2 \frac{1}{C_a - \theta_a C_a} dC_a + \int_{C_a^{sl}}^{C_a} (t_a^{sl})^2 \frac{1}{C_a - \theta_a C_a} dC_a - [E(T_a)]^2 \tag{7}$$

State 3:

$$E(T_a) = \int_{\theta_a C_a}^{C_a} t_a^0 \left(1 + \beta \left(\frac{v_a^{car}}{C_a}\right)^n\right) \frac{1}{C_a - \theta_a C_a} dC_a \tag{8}$$

$$(\sigma_a^t)^2 = \int_{\theta_a C_a}^{C_a} \left[t_a^0 \left(1 + \beta \left(\frac{v_a^{car}}{C_a}\right)^n\right)\right]^2 \frac{1}{C_a - \theta_a C_a} dC_a - [E(T_a)]^2 \tag{9}$$

$E(T_a)$ is the mean travel time and $(\sigma_a^t)^2$is the variance of travel time on link a. State 1 demonstrates that the speed limits result in a link capacity even lower than the lower bound, which implies that the mean travel time on link a is always t_a^{sl} and the variance is zero regardless of the capacity. State 2 indicates that when the critical capacity C_a^{sl} is in the range between maximum and minimum capacity, the speed limits is effective when travelers' speeds is faster than the speed limits. State 3 accounts for the situation when speed limits are in vain, as the speed limits are always higher than travelers' speed. More detail explanations could be found in (Yan et al., 2015).

Then the mean travel time and variance on path l could be expressed as:

$$E(T_l) = \sum_a [\delta_a^l \cdot E(T_a)] \tag{10}$$

$$\sigma(T_l) = \sqrt{\sum_a [\delta_a^l \cdot (\sigma_a^t)^2]} \tag{11}$$

2.3 Cost function

The travel budget time is defined as (Lo et al., 2006):

$$b(T_l) = E(T_l) + \lambda \cdot \sigma(T_l) \tag{12}$$

λ is the parameter to estimate the probability that the travelers could arrive at the destinations within travel budget time. In this paper, travelers are divided into two categories: travelers who have 50% probability to arrive at destinations in time and travelers who have 95% probability to arrive in time. Thus λ has two values: $\lambda = 0$ and $\lambda = 1.64$ (Lo et al., 2006). It implies that travelers of type one (mean time travelers) just value their travel time budget by mean time and travelers of type two (high reliability travelers) by mean time and reserve time. Each type of travelers have two travel mode choices: solo driving and carpooling. If they choose carpooling, they have carpooling costs on a trip (Yang and Huang, 1999). Then the cost functions can be denoted as follows:

$$c_{w,l}^{H,s} = \lambda_t \cdot [E(T_l) + \lambda \cdot \sigma(T_l)] \tag{13}$$

$$c_{w,l}^{H,c} = \lambda_t \cdot [E(T_l) + \lambda \cdot \sigma(T_l)] + \Delta \tag{14}$$

$$c_{w,l}^{M,s} = \lambda_t \cdot E(T_l) \tag{15}$$

$$c_{w,l}^{M,c} = \lambda_t \cdot E(T_l) + \Delta \tag{16}$$

H and M denote the high reliability travelers and mean time travelers respectively while S and C denote solo driving travelers and carpooling travelers. Δ is carpooling cost; λ_t is the coefficient to translate time into cost. For example, $c_{w,l}^{H,s}$ is the generalized cost of mean time travelers who choose solo driving on path l between OD pair w.

2.4 Model formulation

We assume that the travel demands are fixed. For the sake of the definition of travelers, it is known that the mean time travelers are risk-neutral and the high reliability travelers are risk-adverse. Thus we assume the portions of each type of travelers are constant. It follows relationships:

$$d_w^i = \eta_i \cdot d_w \tag{17}$$

$$\sum_i \eta_i = 1 \tag{18}$$

Then we use Logit mode to allocate the shares:

$$d_w^{i,m} = d_w^i \cdot \frac{exp\left(-\theta_1 l_w^{i,m}\right)}{\sum_m exp\left(-\theta_1 l_w^{i,m}\right)} \quad \forall\, i = H, M \;\; m = S, C \tag{19}$$

We utilize SUE principle, which is that no traveler believes that his/her travel time can be improved by unilaterally changing routes, to describe travelers' route choices.

$$f_{w,l}^{i,m} = d_w^{i,m} \cdot \frac{\exp(-\theta_2 c_{w,l}^{i,m})}{\sum_l \exp(-\theta_2 c_{w,l}^{i,m})} \tag{20}$$

A varitional inequality program is formulated for this traffic system. The feasible region Ω is stated as:

$$\sum_l f_{w,l}^{i,m} = d_w^{i,m} \qquad \left(l_w^{i,m}\right) \tag{21}$$

$$\sum_m d_w^{i,m} = d_w^i \qquad \left(l_w^i\right) \tag{22}$$

$$f_{w,l}^{i,m} \geq 0 \qquad \left(\mu_w^{i,m}\right) \tag{23}$$

$$v_a^{i,m} = \sum_l \delta_a^l f_{w,l}^{i,m} \tag{24}$$

$$v_a = \sum_i \sum_m v_a^{i,m} \tag{25}$$

$$v_a^{car} = \sum_i v_a^{i,s} + \sum_i \frac{v_a^{i,c}}{p_c} \tag{26}$$

where v_a^{car} is the vehicle flow on link a. p_c is the rough estimation number of carpooling travelers in one vehicle.

The complementarity is

$$f_{w,l}^{i,m} \cdot \mu_w^{i,m} = 0 \tag{27}$$

$$\mu_w^{i,m} \geq 0 \tag{28}$$

The VI program is to find $(f_{w,l}^{i,m}, d_w^{i,m}, d_w^i) \in \Omega$ which satisfies:

$$\sum_w \sum_i \sum_m \sum_l \left[c_{w,l}^{i,m} + \frac{1}{\theta_2}\left(f_{w,l}^{i,m}/d_w^{i,m}\right)\right]\left(f_{w,l}^{i,m} - f_{w,l}^{i,m^*}\right) + \sum_w \sum_i \sum_m \frac{1}{\theta_1} ln\left(d_w^{i,m^*}/d_w^i\right)\left(d_w^{i,m} - d_w^{i,m^*}\right) \geq 0$$

The VI above is equivalent to the equations (19), (20). See also (Wu and Lam, 2003)

Some approaches such as the projection and diagonalization algorithm have been developed for the solution of such problems. For simplicity and effectiveness, the block Gauss-Seidel decomposition approach together with the method of successive averages is applied to this VI program. However, solving the equilibrium problem is a non-additive question. There exists some literature for non-additive questions (Bernstein and Gabriel, 1997; Lo and Chen, 2006). The framework of solution algorithm proposed by (Lam et al., 2008) for SUE equilibrium is adopted in this paper.

3. The degradable network performances under speed limits

(Yang, 1999) demonstrated that the net economic benefit can be used to measure the network. Thus the net economic benefit in a degradable network with multi-mode travelers is defined as follows:

$$Z = -\sum_w \sum_i \sum_m \sum_l \theta_2 f_{w,l}^{i,m} ln f_{w,l}^{i,m} - \sum_a \sum_i \sum_m c_a^{i,m} \cdot v_a^{i,m} \tag{29}$$

where $v_a^{i,m}$ is the link flow of i type travelers selecting travel mode m on link a. If path l is on link a, $\delta_a^l = 1$; otherwise $\delta_a^l = 0$.

It is well known that speed limits have impacts on the safety of network. Here we adopt the formulation in (Yang et al., 2013) to estimate the safety.

$$Q = \sum_a \alpha_a \cdot {v_a}^{\kappa_a} \tag{30}$$

where $\alpha_a$and κ_a are the parameters on link a. For simplicity, we assume these parameters on each link are the same. Q is the number of accidents in a network, which represents the safety of whole network.

A simple example network is shown in Figure 1 in terms of four nodes and seven links. Link (1,3) and link (3,4) are HOV lanes. The free flow travel time and the capacities are shown in table 1. The OD pair is (1,4), and the travel demand is 300. The free flow travel speed is assumed to be 130km/h.

Figure 1 Simple network example

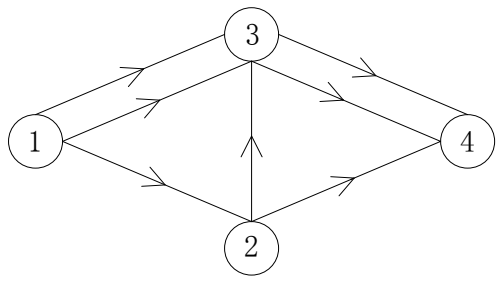
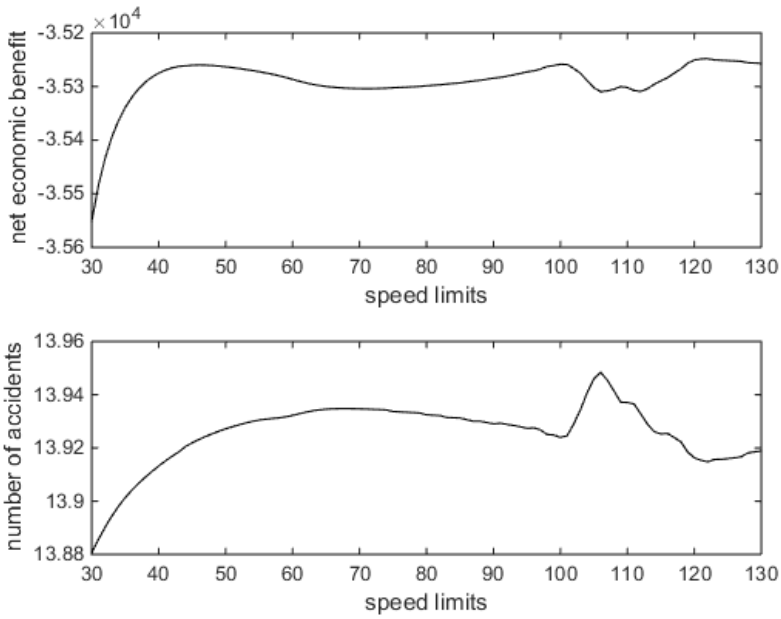


Table 1. The attributes of links

link	Free flow travel time t_a^0(min)	Capacity C_a(veh/h)	Degradable parameter θ_a
(1,2)	6	60	0.6
(1,3)	5	60	0.6
(2,3)	2	40	0.6
(2,4)	6	80	0.6
(3,4)	7	60	0.6
(1,3) (HOV lane)	5	15	0.6
(3,4) (HOV lane)	7	15	0.6

The speed limit is not binding if it is higher than free flow travel speed. Thus we vary the speed limit on link (3,4) between 30km/h~130km/h. The portions of high reliability travelers and mean time travelers are assumed to be 0.8 and 0.2 respectively. Figure 2 depicts how the net economic benefits and the numbers of accidents change in this network. In terms of the speed limit on link (3,4), both the net economic benefits and numbers of accidents are not monotonic. In addition, the speed limits which minimize the net economic benefits are approximately 100km/h and 120km/h. However, the speed limit which minimizes numbers of accidents is 30km/h, while the net economic benefit is maximum at this point. There is a trade-off between net economic benefit and number of accidents in a network. Thus it is necessary to consider the two objectives simultaneously in the speed limit design.

Figure 2. Network performance when imposing various speed limits on link (3,4)



4. Speed limit scheme design

A bi-level program is applied to design the optimal speed limit scheme. On the basis of the results above, the upper level aims to maximize the net economic benefit and minimize the number of accidents in a network. In order to ensure the speed limits are binding, we set the upper bound and lower bound of speed limits in advance. Thus we have the following program at the upper level:

$$\min(-Z, Q)$$

$$s_a^{min} \leq s_a \leq s_a^{max}$$

The path flow patterns and costs on paths are obtained from the low level program, namely, the VI program.

5. Solution algorithm

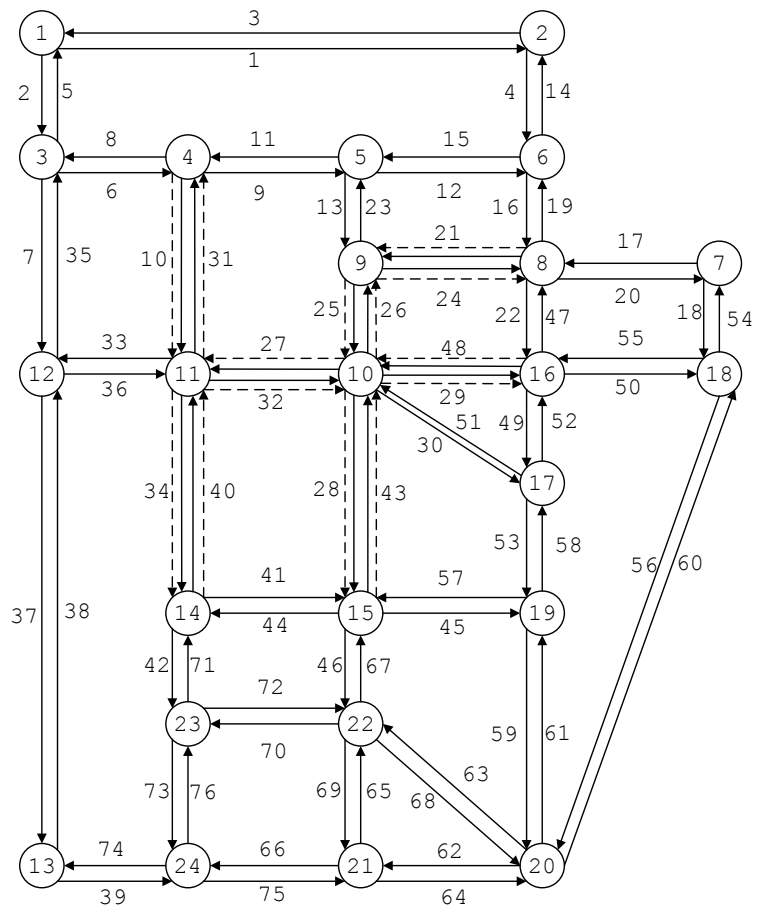
Some algorithms have been developed for bi-level programming. (Yang and Bell, 1998) developed sensitivity analysis-based method to solve this problem efficiently when first-order derivatives could be obtained easily. Nevertheless, the lower program is complex and the cost function is not differentiable everywhere. (Wang, 2013) provided an algorithm to design the optimal speed limits for a single objective. On account of the two objectives in the upper level, a genetic algorithm would be a straightforward and effective method to capture Pareto-improving solutions. Thus we adopted multi-objective genetic algorithm to solve this program.

6. Numerical example

We add some HOV lanes into Sioux Falls network and take it as an example. 14 HOV lanes are added to the network as shown in Figure 3. The dashed lines represent HOV lanes, whose nodes are (4,11) (11,14) (11,10) (10,15) (10,16) (10,9) (9,8) respectively. The OD pairs, link travel time and capacity are the same as in (Tam and Lam, 1999). The travel demand is set to be 300 pcu/h for each OD pair. The coefficient of capacity decrement θ_a is 0.6 on each link and the free flow travel speed is 130km/h. We also assume $\eta_H = 0.8$ and $\eta_M = 0.2$.

(Cascetta et al., 1996) proved that in an Italian traffic network there are limited paths with nonzero flows. (Shao et al., 2008) indicated that the path sets could be given and fixed for each OD pair. (Yan et al., 2015) indicated that there are fewer paths with nonzero flows in their Sioux Falls network example. Here we assume a given path set in advance. In a realistic network, the path sets could be obtained by observation and/or interview surveys (Cascetta et al., 1996; Shao et al., 2008).

Figure 3. Sioux fall network with HOV lanes



10 solutions generated as shown in Table 2. There are four dominant solutions: s_4,s_5,s_6,s_7. From the results we find that under speed limits the solo driving demands are decrease and the carpooling driving demands are increase. It shows that the speed limit is capable of shifting travelers to carpool. However, the net economic benefits decrease and the numbers of accidents decrease since speed limits may cause decrement of link capacities and have positive impacts on safety. Table 3 lists speed limit on every link of scheme s_4 for an example. 8 links have non-speed limit.

Table 2. The performances of Sioux Falls network under various speed limit schemes

Scheme	Net benefit economic	Decrement (%)	Number of accidents	Decrement (%)	Number of solo driving travelers	Decrement (%)	Number of carpooling travelers	Increment (%)
s_0	-2360670	—	2.0654	—	14169	—	14631	—
s_1	-2433193	0.03	2.0206	0.02	14096	0.51	14704	0.5
s_2	-2433193	0.03	2.0206	0.02	14096	0.52	14704	0.5
s_3	-2449206	0.04	2.0342	0.02	13993	1.24	14807	1.2
s_4	-2428112	0.03	2.0191	0.02	14098	0.5	14702	0.48
s_5	-2433869	0.03	2.0168	0.02	12805	9.63	15995	9.32
s_6	-2612535	0.11	1.8662	0.09	13900	1.9	14900	1.84
s_7	-2620395	0.11	1.8590	0.1	12556	11.38	16244	11.02
s_8	-2620817	0.11	1.8588	0.1	12556	11.38	16244	11.02
s_9	-2617827	0.11	1.8606	0.1	12582	11.2	16218	10.85
s_{10}	-2599616	0.1	1.8848	0.09	12568	11.3	16232	10.94

Table 3. The speed limit scheme s_4 (km/h)

Link	s_a	Link	s_a	Link	s_a	Link	s_a
1	95	20	120	39	50	58	—
2	110	21	120	40	100	59	105
3	120	22	—	41	70	60	120
4	130	23	120	42	—	61	115
5	125	24	130	43	125	62	130
6	110	25	65	44	95	63	—
7	125	26	—	45	90	64	130
8	120	27	110	46	110	65	105
9	100	28	105	47	110	66	130
10	110	29	105	48	110	67	105
11	70	30	—	49	100	68	95
12	65	31	120	50	70	69	55
13	115	32	130	51	120	70	105
14	110	33	130	52	130	71	—
15	90	34	120	53	110	72	125
16	105	35	110	54	110	73	45
17	100	36	100	55	75	74	130
18	105	37	130	56	130	75	125
19	—	38	95	57	110	76	30

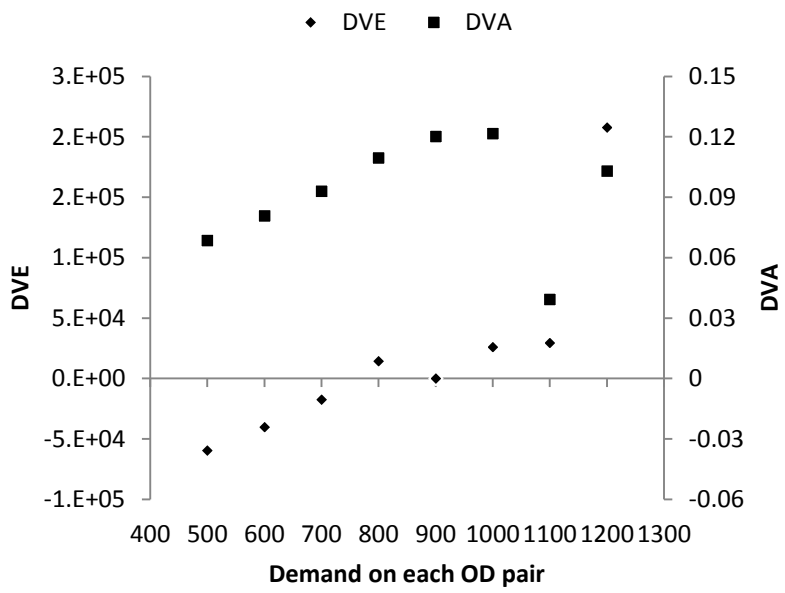
Note that the performances of net economic benefits under various speed limit schemes are always worse than that without speed limit scheme. Figure 4 displays the different values of net economic benefits and numbers of accidents under speed limit scheme s_4 and no speed limit scheme, with demand varying from 500 to 1200 on each OD pair.

DVE = net economic benefit with speed limit scheme s_4 – net economic benefit without speed limit scheme

DVA = number of accidents without speed limit scheme – number of accidents with speed limit scheme s_4

From demand 800, the net economic benefits increase and numbers of accidents decrease, which implies that the two objectives are optimized simultaneously. This indicates that the speed limit scheme is more appropriate for the traffic system with high demand.

Figure 4. Net economic benefits and numbers of accidents by demand level under speed limit scheme s_4



7. Conclusion

In a degradable network in presence of HOV lanes, we propose a scheme imposing speed limits on LOV lanes to urge travelers to carpool. A bi-level program is adopted to design the optimal speed limit scheme for optimal net economic benefit and number of accidents, and multi-objective genetic algorithm is applied to solve this problem. The results displays that this scheme is capable of prompting travelers to carpool and reduce uncertainty of the system. Nevertheless, due to the decrement of capacity caused by speed limits, the traffic system is not always optimized. Namely, the net economic benefit will decrease although the number of accidents will also decrease for low demands. For high demands, the net economic benefit and number of accidents can be optimized simultaneously.

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