The influence of soil moisture and vegetation changes in differential interferometry

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presented by

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Abstract

Differential interferometry is a radar remote sensing technique with which movements of the earth’s surface can be estimated with centimetric precision, for instance from satellites. These estimates have become central to a wide range of scientific disciplines, including geophysics and glaciology. They are formed by combining two radar acquisitions taken at different instants of time, between which the soil moisture content and the vegetation can change. The influence of these changes on the deformation estimates has long been suspected, but the prevalence, magnitude and origin of these effects remain uncertain.

The thesis contributes to addressing the influence of soil moisture changes and vegetation growth using both data-driven and theoretical modelling approaches. To this end, simple mechanisms that can potentially give rise to these effects in radar data are presented and analysed. These hypotheses are subsequently assessed by confronting their predictions with observations. The analyses suggest the transmission of the microwaves through many agricultural crops to be the dominant vegetation-related mechanism that influences the observations. The impact of soil moisture changes is similarly inferred to be related to the propagation of the electromagnetic waves through the soil: an increase in soil moisture lengthens the optical path between the antenna and subsurface heterogeneities within the soil, corresponding to a movement away from the antenna.

The insights gained from the data analyses and previous studies form the basis for the development of a scattering model of bare soil based on Maxwell’s equations, which allows us to address two important questions. First, can soil moisture changes be estimated using differential interferometry in the absence of displacements? And second, can they be separated from deformations, e.g. to provide corrections for the estimation of displacements? Both the data and the theoretical analyses indicate that soil moisture estimation at the field scale is indeed feasible when displacements can be ruled out. However, they also suggest that, in practice, the second question cannot be answered in the affirmative without additional prior assumptions, chiefly due to the similar influence of displacements and soil moisture changes on the measurements.

Even though it is difficult to separate soil moisture from displacements, the subtle differences may provide a way to detect the impact of soil moisture changes and also that of other systematic processes. To this end, a statistical test is introduced. When applied to measurements over high-latitude regions, it indicates the impact of snow metamorphism and melt in differential interferometry. Moreover, it detects significant influences on the measurements in ice-rich permafrost regions, especially in late summer. While the origin of these influences is not clear – possible mechanisms include soil moisture changes, the influence of open water surfaces, and heterogeneous displacements –, their non-random nature suggests that they may have a systematic and deleterious impacts on the estimated movements.

As the measurements indicate the importance of subsurface scattering, we conduct a radar experiment in which the subsurface contributions can be resolved. The observations are consistent with the central tenet of the scattering model – the soil moisture effects in differential interferometry are chiefly due to the changing propagation characteristics of electromagnetic waves through the soil – but they also indicate shortcomings of the way it is
parameterized. As the depth-resolved measurements are closely related to soil moisture changes, we conclude that such depth-resolved observations have considerable potential for estimating soil moisture profiles.
Zusammenfassung


Zur näheren Analyse von Streumechanismen, welche von unterhalb der Bodenoberfläche stammen, führen wir ein Radarexperiment durch, mit welchem diese Mechanismen als Funktion der Tiefe abgebildet werden können. Die Beobachtungen stützen die zentrale Idee des Streumodells, nämlich dass die von der Bodenfeucht abhängigen Ausbreitungseigenschaften innerhalb des Bodens für die differenzielle Interferometrie von essenziel-
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Chapter A

Introduction

A.1 Motivation

Differential radar interferometry is quite unique in that it can provide maps of even subtle movements of the Earth’s surface at regional scales [17, 33]. And such movements, and indeed soil moisture and vegetation changes, are ubiquitous. Consequently, differential interferometry has found widespread use in geophysics, its synoptic information providing vital input to studies of earthquakes, aseismic tectonic activity, volcanoes, ground subsidence, and many more phenomena [11, 27, 28]. It is equally important for monitoring the behaviour of glaciers and ice sheets, and how they respond to climate change [22].

It is also useful for understanding surface movements associated with erosion and mass wasting. These processes are commonly associated with changes in moisture: heavy precipitation or rapid snow melt can trigger sudden landslides [4]; they can also speed up slow downslope creep [34]. Moisture is also related to deformations on flat areas. For instance, ice-rich permafrost soils shrink and lose cohesion when they thaw, inducing surface displacements [13]. The topographic changes can lead to the impoundment of water, which in turn alters the thermal state of the soils and can induce further thawing [21]. Spurious soil moisture signals may hence systematically bias the spatial and temporal patterns in the displacement estimates produced by differential interferometry, thus potentially limiting the applicability of differential interferometry when studying such processes, and more generally impacting its reliability and robustness.

A.2 Background

Radar interferometry In what way could differential radar interferometry, or rather its measurements, be influenced by soil moisture and vegetation changes? It may be expedient to start to address this question by first looking at the general concept of interferometry. In interferometry, waves (e.g. sound waves or electromagnetic waves) are combined, i.e. they are made to interfere [18]. In radar interferometry, which is the topic of this thesis, the waves are electromagnetic in nature, and they are typically microwaves [1, 33]. The most common wavelengths that are used in satellite-borne and air-borne applications range between 1 cm and half a metre.

Interferometry’s hallmark is the great sensitivity with which information about the waves, and hence about the environment that the waves have propagated in and interacted with, can be extracted [12]. The key piece of information about the waves that are made to interfere is their phase difference. In many setups, this phase difference can be related to an optical path difference, which in turn depends on the geometric path lengths and the propagation velocities [17]. In arguably one of the most famous interferometric experiments of all time, Michelson and Morley found that the observed phase difference was at odds with prevailing ideas regarding the propagation of light [19], i.e. the geometric path lengths (given by Galilean geometry) and the propagation velocities (defined with respect to the aether medium). Those ideas have since been replaced, and also the practice of interferometry has continued to develop. Its many applications now include seismology (probing the Earth’s structure with elastic waves), solid mechanics (e.g. for inferring stress distributions), radar remote sensing, and many more [12, 42].

Radar remote sensing of the earth’s surface can be conducted from diverse platforms, ranging from systems that are located at the surface, to systems that are mounted on aircraft or satellites [33]. Antennas are crucial to radar remote sensing because they transmit the electromagnetic waves, which subsequently interact with the scene of interest, e.g. glaciers, water surfaces, forests, or soils. This interaction modifies the waves – they are absorbed, refracted, scattered, etc. The measurement of the modified waves with a receiving antenna allows one to make inferences about the scene [6]. For instance, changes in soil moisture induce changes in the scattering behaviour. The intensity of the received scattered
Figure A.1: Satellite soil moisture retrieval using the intensity of the scattered electromagnetic waves. The satellite antenna emits microwaves, which are bounced back from the earth’s surface and recorded using the same antenna. The intensity of the scattering is related to the moisture content of the soils.

Figure A.2: A displacement $d$ changes the optical path length of the electromagnetic waves emitted and received by the radar antenna. Using differential interferometry, the corresponding phase difference can be converted into an estimate of the displacement.

waves can hence be related to the soil moisture content [6, 39, 41]. The process of emitting microwaves that are bounced back from the earth’s surface and recorded using, in this case, the same antenna is illustrated in Fig. A.1. By using a synthetic aperture radar (SAR) system, two-dimensional images of the surface reflectivity can be generated; even from satellite platforms, spatial resolutions of several metres can be achieved [24].

In radar remote sensing, interferometric measurements can be effected in a number of ways. Depending on the observational setup, a range of parameters or properties of the earth’s surface can be inferred [1, 33]. The defining characteristic of differential interferometry is that there is a time separation between the measurements that are subsequently combined coherently [17], i.e. made to interfere. The presence of a time gap implies that the scene may have changed. For instance, there may have been movements due to an earthquake. These movements correspond to differences in the optical path lengths, and this is where interferometry becomes useful. The measured phase difference between the two radar acquisitions can be used to infer the displacement (Fig. A.2). Changes in the optical path length and hence non-zero phase differences can, however, also be due to other phenomena. For instance, changes in the atmosphere, e.g. the humidity, affect the propagation velocities (and also the travel paths predicted by geometric optics, but that is a second-order effect) and hence the optical paths [17]. Optical path differences can also be due to different positions of the antennas that are used to measure the electromagnetic waves; such optical path differences can be used to infer the position of objects [33]. The first applications in remote sensing focussed on such geometric information, for instance, the estimation of the surface topography [45]. In these cases, it is typically advantageous to keep the time separation as small as possible, so as to reduce the influence of temporal changes such as movements. To this end, the two antennas may be mounted on the same platform (for instance, a ground-based system, an airplane, or even a space shuttle), or they may be on separated platforms that are in close proximity to each other [8, 24].
Subsurface scattering on any soil specific parameters, to numerical solutions to Maxwell’s equations, which are sensitive to the way the soil is analyzed. An analytical expression, as the one developed by De Zan et al. [7] describing subsurface scattering, which does not rely than what has been found in other studies [16, 35].

Observations consistent with dominant surface scattering [43]. However, these effects are an order of magnitude smaller when wetting. Pure surface scattering has also been suggested to play a role [10, 35], and a laboratory experiment has yielded estimates linking the interferometric phase to the increased attenuation of the waves upon wetting [31] (Fig. A.3c). As wave penetration decreases with soil moisture, the predicted phase corresponds to a movement towards the antenna upon wetting (Fig. A.3a), they concluded that the observed phases were related to moisture-induced surface movements (i.e. changes in the travel path), rather than due to the changing propagation velocity within the soil. In this latter case (Fig. A.3b), assuming the surface to remain in place, the apparent movement would have been in the opposite direction, as the optical path between the antenna and subsurface scatterers increases upon wetting. Several years later, Rudant et al. [35] conducted laboratory experiments where wetting of the soils was generally associated with such an apparent movement away from the antenna. They observed the same general trend when plants were sprinkled with water. The accumulation of dry snow is also known to induce such a phase trend [15].

A further mechanism that has been explored in order to explain the impact of soil moisture changes on the displacement estimates links the interferometric phase to the increased attenuation of the waves upon wetting [31] (Fig. A.3c). As wave penetration decreases with soil moisture, the predicted phase corresponds to a movement towards the antenna upon wetting. Pure surface scattering has also been suggested to play a role [10, 35], and a laboratory experiment has yielded observations consistent with dominant surface scattering [43]. However, these effects are an order of magnitude smaller than what has been found in other studies [16, 35]. A number of these ideas have been incorporated into mathematical models. Their complexity ranges from simple analytical expression, as the one developed by De Zan et al. [7] describing subsurface scattering, which does not rely on any soil specific parameters, to numerical solutions to Maxwell’s equations, which are sensitive to the way the soil is...
A.3 Significance

Is it any useful?

**Mapping of surface deformations, soil moisture, and vegetation properties**  An improved understanding of the influence of soil moisture in differential interferometry may be of considerable use, as it would permit researchers to reliably quantify the effects and the associated uncertainties, and possibly to correct for the soil moisture signal. This would be particularly useful when observing displacements, which are often related to soil moisture changes, but may also improve the estimation of the surface topography or the vegetation height.

The correction for soil moisture changes requires a quantitative model that describes these effects. Ideally, it should be both simple (so as to be useful in practice) but powerful enough to account for the different magnitudes and signs of the effects. It should also be able to characterize their polarization dependence. Similarly, the influence of vegetation changes, e.g. the growth of agricultural plants, has to be characterized if it is to be detected and compensated when estimating displacements.

Rather than only being a nuisance, the influence of soil moisture and vegetation changes on interferometric observations may also present opportunities. The great sensitivity of interferometry in general suggests that such observations may provide novel ways of estimating such changes from satellite or airborne data [2, 7].

**Microwave remote sensing**  The sensitivity of interferometric observations may also provide additional insight into the interaction of microwaves with natural objects. Such observations may hence inform standard non-interferometric studies, e.g. by improving the parameterization of backscatter models. However, such standard models typically have to be extended and made more complex if they are to make predictions of interferometric observables [44]; indeed, this is one of the key topics of this thesis. Once this has been achieved, however, these models with interferometric capabilities may be applied to backscatter observations as well, and interferometric observations may provide a more powerful test of their assumptions (that are also relevant to the backscatter predictions) than what would be possible with non-interferometric observations alone.

Interferometric observations thus potentially allow us to assess hypotheses that are relevant to all, also non-interferometric, radar observations. The associated questions regarding model structure and complexity are, I find, remarkably similar to those that arise in hydrological modelling, which traditionally used to focus on the predictions of runoff in response to rainfall [3]. The propagation of the runoff ‘pulses’ is distinct from that of the water molecules themselves. For instance, headwater catchments commonly respond rapidly to rainfall, but the water supplied to the stream is old, i.e. to a large extent it does not originate from the precipitation event [23]. In other words, the pulse propagation celerity exceeds the surface or subsurface flow velocity [29]. While the propagation celerities are more directly observable – and have hence provided the empirical basis for the majority of studies –, the flow velocities are more difficult to measure routinely, but they can be probed using tracer experiments. It has been argued that traditional celerity-based modelling approaches should more often incorporate explicit flow velocity information, as this may enable powerful assessments of hypotheses about relevant hydrologic processes and hence lead to improved parameterizations and hydrological models [29]. Such models that represent the actual processes accurately rather than only reproducing the observations owing to extensive previous calibration should, or are at least generally expected to, be more widely applicable to new or changing conditions [23]. In analogy to hydrological modelling, microwave backscattering modelling of snow, ice, vegetation, or soil, may also profit from new types of information and observations. This thesis identifies differential interferometry as particularly promising: in the case of soil, it is more directly sensitive to subsurface scattering than backscatter measurements [36] – like tracer observations are to subsurface flow paths in hydrology. By extending scattering models to differential interferometry, one may be able to conduct more stringent tests of assumptions regarding certain scattering phenomena like subsurface soil scattering. Conversely, backscatter measurements (e.g. with a sufficiently high bandwidth to resolve subsurface scattering) may lead to a better understanding of the interferometric observations, thus reinforcing the potential of jointly studying interferometric and non-interferometric measurements. Within a modelling framework, such a combination of a range of distinct observations may hence give rise to models that apply to a wider range of conditions (e.g. radar frequencies, soil types) because they – as J. Kirchner put it – give the ‘right answers for the right reasons’ [23].
A.4 Research objectives and questions

What’s in store?

Influence of soil moisture changes and vegetation growth A number of possible, not necessarily mutually exclusive, mechanisms that may give rise to soil moisture effects have been proposed in the literature. Conversely, only Rudant et al. [35] have investigated the influences of vegetation changes, more precisely, the role of leaf wetness. In chapter B, I study airborne L-band radar data acquired during two campaigns; the observed agricultural fields display a wide range of plant densities (including bare soil) and crop types. Figure A.4 shows the spatial variability of the interferometric phase formed from two acquisitions taken within one of the campaigns. The phase shows pronounced jumps at the borders between certain agricultural fields, which differ with respect to the residue cover and soil properties. Are these phase patterns associated with soil moisture? If so, what underlying mechanism gives rise to them? In order to address these questions, I statistically quantify the effects by comparing the interferometric results to ground measurements of soil moisture and vegetation parameters. The inferred influence of soil moisture and vegetation as a function of polarization can then be contrasted with what the proposed mechanisms predict. The soil moisture effects in both campaigns are consistent with dominant subsurface scattering, in particular because the soil appears to move away from the antenna as it gets wetter, which is what one would expect if subsurface scattering were dominant. While the sign of this observed movement is consistent across the fields, its magnitude varies by about one order of magnitude. The impact of vegetation growth displays the same sign, suggesting a similar mechanism that originates in the propagation through the canopy. In contrast to the soil moisture effects, the magnitude of this influence depends strongly on the polarization.

Polarization diversity of the interferometric phase over agricultural fields The polarization dependence of the vegetation effects is addressed in chapter C. How does it relate to scattering in the canopy, to plant properties and changes therein? Are these effects relevant for displacement estimation and if so, can they be removed? May such observations be useful to characterize vegetation changes? I address these questions using analytical models and the observed radar and ground data. To this end, I frame a number of hypotheses about the relevant scattering processes, quantify their effect on the observable polarimetric response, and compare these predictions to the observed polarimetric phase diversity and
Combining bare soil surface and subsurface scattering in an electromagnetic model  Returning to soil, the most striking question that arises from chapter B is this: is there a way to account for the different sizes and signs of the soil moisture effects observed in the agricultural fields, as well as in different studies? In order to address this question, I propose to combine subsurface and surface contributions in chapter D. The model is based on Maxwell’s equations, whereby the soil is abstracted as a heterogeneous half-space with a rough surface. The roughness of the interface causes surface scattering, the heterogeneities give rise to subsurface volume scattering. The parameterization of the heterogeneities rests on simple and pragmatic but admittedly ad hoc assumptions. As the resulting predictions are based on a first-order approximation to Maxwell’s equations, they are intrinsically polarimetric. They comprise not only the phase, but also the coherence magnitude (a measure of interferometric consistency) and the closure phases, also called phase triplets [9]. These latter describe the degree of temporal consistency of three phases: in other words, for three subsequent radar measurements, they quantify the deviation of the measured displacement between times one and three from the sum of the intermediate displacement estimates. These closure phases have been observed to be closely associated with soil moisture changes [7].

Suitability of differential interferometry for estimating soil moisture changes  The closure phases and the coherence magnitude are both promising observables from which soil moisture changes could be estimated. De Zan et al. [7] were the first to use them for this purpose, showing their potential over bare soil in L-band. What makes them particularly interesting is the fact that neither is affected by deformations, as opposed to the phase. In chapter E, I explore the suitability of all three of these observables for estimating soil moisture in both the presence and the absence of deformations. The reliable estimation of soil moisture at the field scale is of considerable practical importance, as such information can inform a wide spectrum of applications and disciplines, ranging from ecology, to agriculture and water resources management [40]. In what circumstances is differential interferometry a viable tool for field-scale monitoring of soil moisture? Which observables and polarizations are most suitable? Owing to the equally wide applicability of differential interferometry for estimating deformations [17], I also address the separability of these two influences. Can the soil moisture impact be removed in order to improve the estimation of displacements?

Subsurface volume scattering in soils: How does it vary with depth?  The observed soil moisture influence on the interferometric measurements suggests the importance of subsurface volume scattering, also for non-interferometric observations [7, 36]. Soils are intrinsically variable in depth; in fact, their depth variability is central to their functioning as e.g. substrates for plant growth or agents of water retention. Microwave radar imaging techniques can be applied in order to create depth-resolved profiles of soils. In order for such observations of subsurface volume scattering to become useful in practice, we need to understand the subsurface scattering of soils, i.e. to have appropriate models. In turn, such observations could be used to inform and assess such models, including those that predict the standard backscatter and interferometric signals that do not resolve the depth component. To this end, I analyse depth-resolved imaging data in chapter F in order to test the central tenets and the ad-hoc assumptions of the model developed in chapter D. Can these observations be used to improve its parameterization? Can such observations be combined with subsurface scattering models to give information about the soil moisture profile?

Detection of soil moisture and other non-random error sources in differential interferometry using closure phases  However, the scope of these modelling attempts remains desperately narrow, focussing on bare mineral soils and, to a lesser extent, agricultural crops. Such simplified scenarios are rarely encountered in practice. Rather, a range of surface processes, including displacements, soil moisture variations, vegetation changes, and snow processes, occur simultaneously, at a range of spatial and temporal scales. One such place are high latitude permafrost regions, many of which have been observed to be responding rapidly to climate change. Displacement estimates are excellent indicators for understanding and monitoring permafrost thaw in ice-rich soils, and differential interferometry is hence a promising tool for studying permafrost landscapes [26, 37]. However, all these other processes are present as well, and they are often closely connected to such surface movements. It would hence be good to have a tool that can detect whether the observed interferometric signals can be explained by deformations and noise alone, or whether there is evidence for the presence of additional influences like soil moisture changes. In chapter G, I propose such a tool, which analyses the closure phases that are not influenced by homogeneous deformations, but instead by noise and other sources. With this tool, one may get a clue as to whether there are statistically significant phase errors that affect the estimation of surface movements.

Bibliography


Chapter B

Assessment of soil moisture effects on L-band radar interferometry

S. Zwieback, S. Hensley and I. Hajnsek

Key findings:
- soil moisture has a measurable impact on differential interferometry
- can correspond to spurious displacements exceeding 2 cm in L-band, i.e. much larger than the precision
- nature of the soil moisture impact is consistent with subsurface dielectric scattering
- effect of agricultural vegetation growth comparable in magnitude but more strongly dependent on polarization

The author’s contributions:
- framed the hypotheses based on previous research
- devised and conducted the statistical analyses
- interpreted the results and wrote the manuscript

The co-authors’ contributions:
- both co-authors provided radar data and helped with the interpretation of the imagery and ground measurements
- both co-authors assisted in the statistical analysis and interpretation of the results
- both co-authors were instrumental in presenting the results and writing the manuscript
Assessment of soil moisture effects on L-band radar interferometry

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Abstract

Differential SAR interferometry, a popular technique for measuring displacements of the Earth’s surface, is potentially influenced by changes in soil moisture. Different mechanisms for this impact have been proposed, but its magnitude, sign and even presence remain poorly understood. In this study the dependence of the phase, the coherence magnitude as well as the phase triplets on soil moisture was inferred empirically with regression techniques: this was done for two airborne data sets at L-band. The phase dependence was significant (at a significance level of 0.05) for more than 70% of the fields at HH polarization, its sign corresponding to an increase in optical path upon wetting, and the magnitude of the associated deformation commonly exceeding 2 cm for a change in soil moisture of 20%. This trend was similar in both campaigns, whereas the prevalence of soil moisture-related decorrelation differs. These results are only consistent with a dielectric origin of the soil moisture effects, and not with soil swelling or the penetration depth hypothesis. Changes in vegetation impact the phase depending on the crop and polarization, with the vegetation influence at VV being more pronounced for the agricultural crops present in the study area.

1 Introduction

Radar interferometry is an established technique for the observation of a broad range of phenomena. These include volcanology (Massonnet et al., 1995), tectonics (Massonnet et al., 1993), permafrost studies (Liu et al., 2010), or the analysis of groundwater-related subsidence (Galloway and Hoffmann, 2007). It works by coherently combining two radar images. When these images are acquired at different times, the technique is sensitive to displacements on the scale of the radar wavelength, i.e. typically 1 - 10 cm (Gabriel et al., 1989; Rosen et al., 2000). These two images can also be taken from different positions, in which case height information can be derived from the data (Bamler and Hartl, 1998).

When there is a time gap between the two acquisitions, not only can there be deformations, but also the vegetation and soil moisture can change. If this is the case, such soil moisture \( m_v \) changes can lead to systematic errors in the estimated deformations. However, the prevalence and magnitude of these influences are not well understood. A possible influence of variations in soil moisture on the interferometric signal was initially postulated by Gabriel et al. (1989) due to an observed correspondence of the phase \( \phi \) and thus the estimated deformations with hydrological units such as agricultural fields. However, dedicated observational studies have been scarce and limited to a handful of laboratory experiments (Rudant et al., 1996; Nesti et al., 1998; Morrison et al., 2011; Yin et al., 2014), as well as a few air- or satellite-borne campaigns (Nolan, 2003a; Hajnsek and Prats, 2008; Hensley et al., 2011; Barrett et al., 2012, 2013). Simultaneously, different mechanisms and models that could describe some of these effects have been proposed, alongside electromagnetic simulations based on Maxwell’s equations (Rabus et al., 2010). These explanations attribute the change in \( \phi \) to deformations (Gabriel et al., 1989), changes in the optical path due to soil moisture variations \( \Delta m_v \) (Rudant et al., 1996; De Zan et al., 2014), or differences in the penetration depth of electromagnetic waves (Nolan, 2003b).

Despite these analyses, there is no consensus on the magnitude, sign and even presence of these effects (Rudant et al., 1996; Rabus et al., 2010; Morrison et al., 2011). This is partly due to the lack of suitable data. The speckle patterns tend to decorrelate over time, which implies that the phase cannot be estimated reliably (Zebker and Villasenor, 1992; Barrett et al., 2013). The lack of temporal stability of many areas (especially those covered by vegetation) has led to the development of algorithms that estimate deformations using only stable, point-like scatterers (Ferretti et al., 2011). When the data over the less stable areas are to be analysed with respect to the influence of soil moisture on the phase, a small time gap and preferably bare soil are required. In addition, the radar signals are also influenced by other parameters such as the elevation (for non-zero spatial baselines), deformations, and vegetation properties. Furthermore, there are sizeable differences between the different studies with regards to the wavelength, incidence angle, soil type, vegetation cover, etc., and these render comparisons and model assessments difficult (Barrett et al., 2013). The proposed explanations have not yet been assessed with extended data sets or compared with each other.

In view of these open questions, we want to study these soil moisture effects in two L-band airborne campaigns. The low frequency, short revisit times, small spatial baselines, and (in one campaign) absence of vegetation cover are expected
to reduce the impact of these additional influences such as the topography and vegetation-related processes. The soil moisture effects, by contrast, are expected to be more dominant and thus detectable. In particular, this allows us to address the question of the sign, magnitude and statistical significance of these effects. We do so by using regression techniques whereby we describe the interferometric observables as a function of the change in soil moisture $\Delta m_w$. Furthermore, we want to assess the plausibility of the different conjectured mechanisms that could describe these effects. This assessment is made by comparing their predictions with the empirically found impact of soil moisture on the interferometric data. As the applicability and relevance of these explanations are not well understood, we focus on the differences between these explanations rather than particular models and parameterizations. This analysis is conducted for different polarizations, as the sensitivity to soil moisture is not necessarily identical. In most previous studies (both observational and models), the polarimetric aspect was not addressed explicitly, often due to lack of suitable data or because the proposed physical explanations did not involve any polarimetric differences (Nolan, 2003a; De Zan et al., 2014).

The interferometric observables along with the notation and sign conventions of this paper are introduced in Sec. 2. Subsequently, the study sites and data sets are outlined, followed by an overview of the SAR processing and the statistical methods. The results of these analyses are presented in Sec. 6; in Sec. 7 they are scrutinized and compared to the predictions of the different hypotheses.

2 Radar interferometry

In a polarimetric framework (Cloude, 2009), from which the standard single channel scenario arises as a special case, each single look complex (SLC) pixel is described by a scattering vector $q$: in the lexicographic basis (reciprocal backscatter situation), $q = [S_{HH}, \sqrt{2}S_{HV}, S_{VV}]$. From two SLC images $q_1, q_2$ – they usually differ in their acquisition time and/or position – one derives the scalar quantity called complex coherence (Cloude and Papanastassiou, 1998)

$$
\gamma_{12}(\omega) = \frac{\omega^{\dag}(q_1q_2^{\dag})\omega}{\sqrt{\omega^{\dag}(q_1q_1^{\dag})\omega^{\dag}(q_2q_2^{\dag})\omega}}
$$

where $\omega$ is a polarimetric measurement functional (e.g. $[1, 0, 0]^T$ for HH). The $\langle \rangle$ denotes an ensemble average, which can be estimated by spatial multilooking (Gabriel et al., 1989; Bamler and Hartl, 1998). This averaging applies if the target is treated as a distributed one, i.e. as realization of a random process. The coherence can be factored as $\gamma = |\gamma| \exp(i\phi)$. From this factorization, the three observables (phase $\phi$, coherence magnitude $|\gamma|$, and phase triplets $\Xi$) used in this study can be derived.

The phase $\phi$ (the $\exp(i\omega t)$ convention is employed throughout) is sensitive to the geometry and displacements. After flat earth phase removal, spectral filtering, and neglecting noise and propagation effects in e.g. the atmosphere, $\phi$ of a point target can be approximated as $\phi = \kappa_z z + 2k_0d$, where the first part determines the impact of the elevation above a reference surface $z$, and the second one to displacements $d$ along the RADAR look direction. The first coefficient of proportionality is given by $\kappa_z \equiv \frac{\phi_0}{\delta z} \propto k_0 B_{\perp} R^{-1}$, where $B_{\perp}$ is the antenna offset perpendicular to the look direction, $R$ the distance to the target, and $k_0$ the wavenumber in free space. The sensitivity to displacements is given by twice the wavenumber in free space.

The coherence magnitude $|\gamma|$ can be interpreted as a measure of the correlation of the speckle patterns in $q_1$ and $q_2$ (Rosen et al., 2000). A value less than one can e.g. be caused by volume scattering for $B_{\perp} \neq 0$, or by changes in the arrangement and physical properties of the target for non-simultaneous acquisitions, as well as noise (Tsang et al., 2000). The phase triplets (Ferretti et al., 2011; De Zan et al., 2014) are a combination of the phases of the three interferograms formed from three SLC images $\Xi_{123} = \phi_{12} + \phi_{23} + \phi_{13}$; they are only different from zero if $|\gamma_{ij}| \neq 1$. In astronomy they are usually referred to as closure phases (Monnier, 2007) and have proven useful due to their insensitivity to a phase offset (e.g. due to the atmosphere) in any of the acquisitions.

3 Hypotheses

The four hypotheses about the origins of the soil moisture effects that have been framed in the literature will each be briefly presented. The focus will be less on the implementation of these mechanisms in particular parameterized models, but rather on the physical basis and the predictions that can be formulated based on them. The sign of the dependence of the interferometric observables on soil moisture changes $\Delta m_w$ for each of the mechanisms is summarized in Tab. 1. These explanations, although distinct, are not necessarily mutually exclusive.

3.1 Null hypothesis (Null)

The null hypothesis states that there is no relationship between the moisture content and the interferometric observables, including the phase $\phi$; this is schematically depicted in Fig. 1a. This hypothesis is implicitly assumed in virtually
Figure 1: Schematic depictions of the postulated mechanisms giving rise to the interferometric phase $\phi$; left image: dry soil, right one: moist soil. A positive $\frac{\Delta m_v}{m_v}$ corresponds to an increase in the optical path upon wetting (e.g. due to subsidence).

Table 1: Model predictions for the sign of the sensitivity of an observable on $m_v$: + positive, - negative, 0 no influence, ? not explicable. The volume hypothesis is used for the Diel mechanism.

<table>
<thead>
<tr>
<th></th>
<th>Null</th>
<th>Defo</th>
<th>Pene</th>
<th>Diel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\Delta m_v}{m_v}$</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$\Xi(m_{v:0.2})$</td>
<td>0</td>
<td>0</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\neq$ 0</td>
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</tbody>
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all interferometric studies (Ferretti et al., 2011), where soil moisture effects are either not considered, minimized by excluding soil, or deemed negligible.

3.2 Deformation (Defo)

$\phi$ variations whose patterns match those of hydrological units such as field boundaries have previously been interpreted as deformations (Gabriel et al., 1989; Nolan, 2003a). Certain types of soils (e.g. montmorillonite clay) are known to swell upon wetting (Norrish, 1954; Mitchell, 1991), and such deformations have indeed been studied (and compared with in-situ measurements) with differential interferometry (te Brake et al., 2013). The influence of an expanding soil on the phase of the coherence is illustrated in Fig. 1b. The impact on the magnitude of the coherence depends intricately on the detailed mechanism: a piston-like shift would in general not lead to decorrelation, whereas non-uniform deformations easily could. The generality of these effects is, however, doubtful, as these swelling and shrinking behaviours are restricted to certain types of soil (Mitchell, 1991). Furthermore, the sensitivity of $\phi$ to these effects diminishes with decreasing radar frequency. For example, at L-band $\phi = \pi$ corresponds to a deformation of about 5 cm, at X-band to 0.5 cm. Especially at such longer wavelengths, observed phase values that are of this magnitude have been deemed too large to be plausibly caused by deformations (Hensley et al., 2011).

3.3 Penetration depth (Pene)

Nolan (2003b) suggested that the penetration into the lossy soil governs the phase signal. When the soil is treated as a homogeneous uniform dielectric medium, the former can be parameterized in terms of the wavelength and the dielectric constant $\varepsilon$ (the dependence on soil moisture is described in terms of a mixing model, e.g. (Mironov et al., 2013)). The soil moisture content governs the characteristic length scale $\delta$ at which the wave attenuates (Tsan et al., 2000). According to the penetration depth hypothesis, the phase is then related to a difference in $\delta$, cf. Fig. 1c. There are two inherent problems in this approach. Firstly, it is not clearly stated how the change in penetration depth is related to the observable $\phi$: $\delta$ is a characteristic length, the scaling of which is somewhat arbitrary. Furthermore, in free space the propagation phase is related to the distance $R$ by $\varphi = -2k_0R$ in a dielectric medium the free-space wavenumber $k_0$ should be replaced by the one specific to this medium and frequency. It is not clear which conversion (Nolan, 2003b) apply. Secondly, the model does not predict the coherence $|\gamma|$. Indeed, it does not explain how a non-zero correlation can be achieved, as there is no physical mechanism postulated that could give rise to such correlations. One possibility in a non-uniform soil consisting of two layers with highly correlated rough surfaces was studied by Rabus et al. (2010) and shown to give rise to such a phase signal but also to be exceedingly sensitive to the geometrical parameters.

Let the penetration depth $\sigma$ be defined to be the depth at which the two-way propagation at an incidence angle of 34° leads to reduction in power by a factor of $\frac{1}{e}$. At L-band the commonly used Hallikainen model predicts a value for $\sigma$ of 2 cm for wet conditions of $m_v = 0.4 \text{ m}^3 \text{ m}^{-3}$, and 4 cm for dry conditions of $m_v = 0.1 \text{ m}^3 \text{ m}^{-3}$. The Peplinski model, for which the absorption in the soil is smaller (Hallikainen et al., 1985; Peplinski et al., 1995), predicts in these conditions values of $\delta$ of 5 and 13 cm, respectively.

3.4 Dielectric mechanism (Diel)

The dielectric properties of a medium govern the complex wave vector $k$: an example applicable to DInSAR are changes in atmospheric properties that influence the measured $\phi$. This idea was extended by Rudant et al. (1996) to the vegetation
overlying the soil as well as the soil itself, as the dielectric properties of both are known to depend on the moisture content (Tsang et al., 2000). This effect has been observed (as well as modelled) for a buried reflector in a laboratory experiment (Morrison et al., 2013); it can also be interpreted as the mechanism giving rise to the phase signal observed in a numerical electromagnetic model of an inhomogeneous soil (Rabus et al., 2010). De Zan et al. (2014) described the soil as an aggregate of such inhomogeneities and derived the first-order scattering solution of the coherence $\gamma$: the radar signal is thus modelled as the superposition of contributions from a large number of scatterers within the soil (surface scattering is not considered). Note that non-zero phase triplets $\Xi$ are predicted by this model.

Within the context of this first-order scattering model (FOSM), a change in the dielectric constant corresponds to a change in the wavenumber $k$ in the medium: the real part of the latter encodes how rapidly the phase changes with position (the spacing of the wavefronts in Fig. 1d), the absorption with depth is governed by the imaginary part (the darkness of the wavefronts in Fig. 1d). Under the independent scattering assumption and if the positions of the inhomogeneities are uncorrelated, it is the real part that give rise to a non-zero $\phi$, conversely to the penetration depth approach (De Zan et al., 2014).

Even in the absence of inhomogeneities within the soil, a change in the dielectric properties is predicted to lead to phase changes by rough surface scattering models (assuming identical geometric properties for the two acquisitions). For sufficiently small wavelengths (compared to the radius of curvature and root-mean-square (RMS) height of the rough surface, the Geometric Optics (GO) Kirchhoff approximation applies, whereas the Small Perturbation Model (SPM) is adequate when the wavelength is large compared to the RMS height (Ishimaru, 1997). A small example in Tab. 2 shows that the sign of the phase dependence of these models is opposite for the dielectric model by Hallikainen et al. (1985) and indeterminate for the one by Peplinski et al. (1995); the magnitude, however, is much smaller for the surface models. This small dependence of $\phi$ has often been considered negligible compared to the noise level or other influences (Rudant et al., 1996; De Zan et al., 2014).

## 4 Data sets

The airborne L-band data acquired during two campaigns are considered in this study. The first one is the Agrisar 2006 campaign in north-eastern Germany, the second one is the Canadian Experiment for Soil Moisture 2010. In both cases in-situ soil moisture measurements are available. The campaigns differ with respect to the temporal intervals of the acquisitions and the vegetation cover. Neither was designed for the analysis of soil moisture effects on interferometry, and this is particularly evident with respect to vegetation dynamics and additional sources of decorrelation such as ploughing.

### 4.1 Agrisar 2006

The objective of this campaign was to provide data for an improved understanding of remote sensing measurements over agricultural vegetation. The study site is located around G örmín, Mecklenburg-Western Pomerania, Germany ($53^\circ 58'N$ $13^\circ 16'E$). The temporal extent effectively covers one growing season for the crops present (German Aerospace Center, 2008): winter wheat, maize, rape, sugar beet, and barley; several small towns and patches of mixed forest are also included (see Fig. 3. The topography is flat, with a slight slope towards a nearby river and several drainage features perpendicular to the latter (Zwieback and Hajnsek, 2014). The soil is dominated by sandy loam and similar textures, with clay contents of less than 10% (German Aerospace Center, 2008).

The airborne L-band ($\lambda = 0.23\,\text{m}$) SAR data were acquired by the E-SAR system in intervals of one to two weeks at a range (azimuth) resolution of about 2 (1) meters (German Aerospace Center, 2008). The subset of images used in this
study were recorded from the same track at a nominal baseline of 0 m. They were acquired around noon local time on the dates shown in Fig. 4. The interferometric phase and coherence with respect to the image taken at DOY 163 of a wheat field are shown in the first panel.

Two corner reflectors provide a reliable phase reference in their vicinity. As shown in Fig. 3, there are considerable large-scale phase trends present in the data. The interferometric phase has clear correspondences with certain field boundaries, which correspond to differential deformation estimates between the fields. The impact of the additional phase patterns is minimized by using stable scatterers in addition to the corner reflectors, cf. Sec 5.1.3.

Several soil- and plant-related parameters were measured contemporaneously with the airborne acquisitions: the ones used in this investigation are soil moisture $m_v$ and biomass $b$. The volumetric soil moisture $m_v \text{[m}^3\text{m}^{-3}]$ was measured manually at three locations (by Time Domain Reflectometers (TDR); 0-5 cm depth) within each field within one to two hours of the radar acquisitions. These measurements were subsequently averaged. The sampling of the measurements of the wet biomass $b \text{[kg m}^{-2}]$ and vegetation height $h \text{[cm]}$ was conducted analogously, the former being determined by clearing 1 m$^2$ and weighing. The fields for which such data are available are compiled in Tab. S1. The temporal evolution of these parameters is shown for one of the fields in Fig. 4. The soil moisture tends to decrease over time until June 13, when it reaches $m_v=0.12 \text{ m}^3\text{m}^{-3}$. Subsequently, there is a rain event followed by a dry-down period. The biomass $b$ of the wheat field increases monotonically until June 21, when it reaches a value of 5.6 kg m$^{-2}$, before it declines as the plants reach the period of senescence.

4.2 CanEx2010

The aim of the Canadian Experiment for Soil Moisture in 2010 (Magagi et al., 2013) was to support calibration, validation and algorithm development activities for the SMOS and SMAP satellite missions; to this end, in-situ measurements of soil and vegetation properties were taken from June 2-14 2010. Among the remote sensing data gathered are six UAVSAR acquisitions over the Kenaston, Saskatchewan, Canada, test site (51° 30’ N, 106° 18’ W). This flat area is characterized
The quadpol UAVSAR data (L-band: $\lambda = 0.24$ m) have a resolution of 1.7 m (0.8 m) in range (azimuth) (Jones and Davis, 2011). They were acquired in irregular intervals between one and three days with a nominal baseline of 0 m at 3 pm local time. The lack of corner reflectors implies that the unknown phase offsets in the data have to be eliminated by e.g. forming differences with stable scatterers. The phase patterns are more stable than in the Agrisar campaign, which can be seen in the interferogram of Fig. 5.

Volumetric soil moisture $m_v$ was measured hourly at permanent stations by Environment Canada (EC) using Stevens Hydraphobe probes in several depths, of which the 0-5 cm vertical sensor (using an improved factory calibration) will be considered. In addition, visual assessments of the tillage, vegetation cover and crop type are available for most fields (Magagi et al., 2013). The fields for which such measurements are available at all acquisitions are compiled in Tab. S2. The locations of these measurements are shown in Fig. 5 and annotated by the number of the field. Note that the dynamic range in the measured soil moisture, i.e. the difference between the maximum and minimum value, varies by about a factor of 10 between the different fields. This might be related to e.g. variations in soil texture and other soil properties, as well as the microtopography.

The temporal pattern of the soil moisture evolution is, by contrast, very stable across the different probes. One example is given in Fig. 6, where the changes with respect to the master acquisition at DOY 165 ($m_v = 0.23$ m$^3$ m$^{-3}$) are given. They show a close correspondence to the precipitation measured at field 136, which led to a wetting in the middle of the campaign.
5 Data analysis

5.1 SAR Processing

5.1.1 Interferogram formation

The complex interferograms were formed for all possible pairs, which results in a total of 28 interferograms in the Agrisar campaign, and 15 in the CanEx2010 experiment. The number can depend on the field (due to harvesting or ploughing) and can be found in tables S1 and S2. A flat earth and topographic phase correction, along with range spectral filtering, was applied (Bamler and Hartl, 1998). The Agrisar data showed larger deviations from the intended zero $B_\perp$ condition, with the height sensitivity $|\kappa_z|$ generally bounded by 0.05 m$^{-1}$; a difference in the elevation model error at two points of 2 m would thus correspond to a phase error of 0.1 rad, which is considerably smaller than the dynamic range observed in the data. Larger height errors are only expected for phase references on top of isolated man-made structures such as buildings.

5.1.2 ROI definition

Within each field a region of interest (ROI) in the shape of a rectangle (50 m in range, 100 m in azimuth) was chosen in such a way that the target was as homogeneous as possible. For the Agrisar campaign, the choice of location of the ROI was dictated by the presence of the spurious phase patterns. The ROI was taken to be as close to a persistent scatterer as described in Sec. 5.1.3 as possible. In the CanEx2010 data set, the ROI was taken to be as close to the soil moisture probe as possible. Difficulties and heterogeneities of the observed phases are mainly related to the presence of water surfaces that are partially covered with vegetation. In either campaign and at each of the four corners of this rectangle, 172 looks $L$ were averaged to obtain the coherence information, corresponding to a rectangular box car filter. These subROIs are coded by their field number (three digits) followed by a letter: v-y for Agrisar, a-d for CanEx.

5.1.3 Phase reference

The unknown phase offset in each interferogram and for each ROI can be removed by forming differences with respect to stable scatterers. In the absence of corner reflectors, one commonly resorts to a data-driven approach (Ferretti et al., 2011). We do so by finding persistent scatterers (PS) using the amplitude-based signal-to-clutter ratio method (Kampes, 2006). As the amplitude criterion is not sufficient for a pixel to be stable, the three closest ones are taken for each ROI, and among these two are chosen. The first, the nearest one, is based on the rationale that phase errors due to e.g. inaccuracies in the flight track increase with distance. The second – the one among the three with the smallest phase variance – will potentially rule out unstable targets, as these are characterized by larger fluctuations. The impact of this decision rule is studied empirically in Sec. 6.4.

For a given ROI and its persistent scatterer, the difference between the phase over the field and over the PS is formed. This so-called double difference (Kampes, 2006) will be referred to as simply the phase $\phi$ of the ROI. This implicitly assumes that the phase of the PS is stable. The deformation associated with this phase $\phi$ thus corresponds to an apparent movement of the ROI with respect to the persistent scatterer. This phase did not have to be unwrapped as no phase wrapping was observed in the data. The double-difference phase is still affected by phase patterns associated with orbital errors and tropospheric influences. Their impact is expected to be mainly determined by the spatial separation of the PS and the ROI (Hanssen, 2001). These distances are given for each field in Tab. S1 and S2.

In the Agrisar campaign, the spurious phase patterns have a typical wavelength of more than 2.5 km (see e.g. Fig. 3). For an average spatial distance to the PS of 100 m, this corresponds to a phase error of 14°, which is not expected to be correlated with soil moisture changes. In the CanEx2010 data, the typical separation of 300 m corresponds to a phase error 5° given a spatial scale of the phase patterns of 20 km. The tropospheric influence is partially included in these numbers, and is expected to be smaller ($<2^\circ$) (Goldstein, 1995; Emardson et al., 2003).

5.2 Regressions

The connection between the observables and explanatory parameters (such as $m_v$) is studied by regressions, whereby it is assumed that the observable can be described by simple functions of said parameters. In the Agrisar campaign, during which sizeable vegetation growth and senescence occur, the vegetation is described by the wet biomass $b$. This parameter is expected to be closely related to the backscattered power and the optical path through the canopy (Ulaby et al., 1987). As the vegetation height was also measured, it will be included in separate regression to study the robustness with respect to the characterization of the vegetation. In general, these vegetation parameters are denoted by $v$. For the CanEx campaign, the fields were bare or covered by harvest residues, so that no vegetation terms are included in the model.
5.2.1 Regression model

For the coherence $|\gamma| \in [0, 1]$, which is invariant to the choice of master/slave, the following structure is postulated:

$$\log(|\gamma_{ij}|) = \beta_0 + \beta_{m_v} |\Delta m_v| + \beta_{|\Delta t|} + \beta_v |\Delta v| + \epsilon_{ij}$$

(2)

i.e. the terms are assumed to affect $|\gamma|$ in a multiplicative fashion, thus rendering negative coherences impossible. The coefficient $\beta_{m_v}$ represents the decorrelation due to $|\Delta m_v|$, whereas $\beta_v$ denotes the impact changes in the vegetation parameter $\Delta v$ have on the coherence magnitude. The temporal decorrelation is assumed to be related to the time separation between the two acquisitions $|\Delta t|$; it is quantified by the coefficient $\beta_t$. The error term of the $i,j$ interferogram is denoted by $\epsilon_{ij}$ (with expected value of zero), and $t$ is the temporal separation.

The phase $\phi$ is assumed to be governed by

$$\phi_{ij} = \beta_{m_v} |\Delta m_v| + \beta_v |\Delta v| + \epsilon_{ij}$$

(3)

where the absence of an intercept term is due to the expectation that no change in the exogenous variables corresponds to zero $\phi$.

The structure of the phase triplets is complicated, as they i) depend on three acquisitions; and ii) deviate from zero only in the presence of decorrelation. They are thus analysed separately in Sec. 5.3.

5.2.2 Stochastic model

The phase noise $\epsilon_{ij}$ is often considered to be due to two kinds of components (Kampes, 2006): the first $\varepsilon_{ij}$, due to the correlation properties of the particular interferogram, can e.g. be described within the Gaussian speckle model (Bamler and Hartl, 1998); the second $\xi$, due to a phase offset of each SLC image $i$ from e.g. the atmosphere, is here assumed to be stationary in time. Thus $\epsilon_{ij} = \varepsilon_{ij} + \xi_i - \xi_j$, with $\varepsilon$ and $\xi$ assumed uncorrelated. The second moments of $\epsilon_{ij}$ then evaluate to

$$\langle \epsilon_{ij} \epsilon_{kl} \rangle = \langle \varepsilon_{ij} \varepsilon_{kl} \rangle + \langle \xi_i \xi_k \rangle - \langle \xi_j \xi_l \rangle - \langle \xi_i \xi_l \rangle$$

$$= \sigma_{ij}^2 \delta(i,j,k,l) + \Sigma^2 [\delta(i,k) + \delta(j,l) - \delta(i,l) - \delta(j,k)]$$

(4)

where $\delta(u,v)$ is a Kronecker delta and the correspondence between the two equations is term by term. The value of $\sigma_{ij}^2$ is estimated from the observed $\gamma_{ij}$ based on the Gaussian speckle model Cramer-Rao bound, relative to $\sigma_i^2$, which is the value for the given number of looks $L$ that would be obtained for $|\gamma| = 0.5$. The ratio $\kappa = \Sigma^2/\sigma_i^2$ is estimated in the regression model from the data. A simplified analysis without autocorrelations is also conducted to study the robustness with respect to the stochastic model.

5.2.3 Implementation

The regressions for $|\gamma|$ are obtained by ordinary least squares, the ones for $\phi$ by Maximum Likelihood estimation assuming Gaussian noise using the nlme package in R (Pinheiro and Bates, 2000; Pinheiro et al., 2013). Standard regression diagnostics (Judge et al., 1983) are used to remove outliers (Bonferroni-corrected t-test of studentized residuals with $\alpha = 0.05$). In addition to the point estimates of $\beta$, the decisions of the Shapiro-Wilk test for normality at $\alpha = 0.05$, and the 95% confidence intervals for $\beta$ are reported. Note that the latter are i) based on the normality hypothesis, and ii) are only approximate if $\kappa$ is estimated.

5.3 Quantile regressions of $\Xi$

In contrast to all other hypotheses, the dielectric volume model predicts non-zero phase triplets. As the sign of these phase triplets depends on the ordering of the acquisitions, we propose to focus on their magnitude $|\Xi_{ijk}|$ and parameterize it as a function of max$|\Delta m_v| > 0$, i.e. the maximum of all three $|\Delta m_v|$:

$$|\Xi_{ijk}| = \beta_0 + \beta_{m_v} \max |\Delta m_v| + \beta_v \max |\Delta v|$$

(5)

where the vegetation change term $v$ is only included for the Agrisar campaign. The regressand $|\Xi_{ijk}|$ is non-negative: its spread and thus its median is expected to increase with max$|\Delta m_v|$ in the dielectric framework, but in a complicated fashion as it depends on all three values of $m_v$ and the particular parameterization and model. In order to study this behaviour in a robust way, median regression is employed to estimate the coefficients $\beta$. The standard errors are computed according to the rank-inversion method (Koenker, 2005, 2013).
6 Results

6.1 Exploratory data analysis

The impact of $m_v$ on the different observables is exemplified in several scatter plots in Fig. 7; these were chosen to be representative of the different kinds of relations found in the data. The phase values (Fig. 7a-h) predominantly show a positive, approximately linear trend with $\Delta m_v$. The correlation, however, differs: in field 230x, for instance, the scatter around the fitted curve is smaller in HH than in VV, where it seems to increase with the temporal separation $\Delta t$. Differences are also conspicuous with regards to the magnitude of the effect, e.g. field 307a.

The magnitudes of the coherence $|\gamma|$ in subfigures i) and k) both decrease with $|\Delta m_v|$, with the apparent influence of $\Delta t$ being more pronounced for 230x. The influence of this temporal decorrelation is also evident in Fig. 4, where both biomass and soil moisture return on DOY 192 to the values observed during the master scene. However, the coherence magnitude does not return to values close to one. By contrast, such a return can be observed in Fig. 6, where both soil moisture and coherence show similar temporal behaviour.

Also the phase triplets $|\Xi|$ exhibit such diversity: the ones in field 230x (subfigure j)) are an order of magnitude larger on average than the ones of 136a (l)), with neither displaying a conspicuous dependence on the maximum soil moisture.

6.2 Regression results

6.2.1 Agrisar campaign

The estimated soil moisture regression coefficients $\beta$ of both phase and correlation are plotted in Fig. 8 for the Agrisar campaign. For $\log|\gamma|$ in HH, all but a single one are negative. The deviation, however, is not necessarily significantly different from zero ($\alpha = 0.05$): such significant effects are found for 50% of the samples in HH, 21% in HV, and 36% in VV. However, between these polarizations, the differences of the size of the effect for any particular field are generally
The impact of soil moisture variations on the phase $\phi$ is found to be significant for 71% of the samples in HH, 39% in HV, and 43% in VV. All the estimated effects of these significant samples are positive. The size of the effect $\beta_{\Delta m_v}$ exceeds 2 rad m$^{-3}$ m$^3$ (this corresponds to a phase change of 23° for $\Delta m_v = 0.2$ m$^3$ m$^{-3}$) in 82% of the samples in HH, 53% in HV and 46% in VV.

Not only the soil moisture $m_v$, but also the biomass $b$ is seen to impact the observables: the estimated regression coefficients are shown in Fig. 9. The effects on $|\gamma|$ are predominantly negative (a notable exception being 101v), and significantly different from zero for about 35%-55% of the samples. An increase in biomass is seen to affect $\phi$ in the same direction as an increase in $m_v$: all significant effects $\beta_{\Delta b}$ but two are positive. This significance is found for 32% of the samples in HH, 61% in HV, and 82% in VV. The impact over all the crops except sugar beet is less than 1 rad kg$^{-1}$ m$^2$, and more pronounced in VV than in HH over wheat (230, 250) and barley (440), cf. Tab. S4.

6.2.2 CanEx campaign

The analysis of the CanEx data, where the vegetation is not subject to significant changes, also reveals a positive dependence of $\phi$ on $m_v$, albeit with different magnitudes. These are also exceedingly variable, with e.g. field 201 showing a 10 times larger impact than field 109; the magnitude of the latter (2-4 rad m$^{-3}$ m$^3$) being comparable to the one found in the Agrisar data set. The fields with the highest coefficients $\beta_{\Delta m_v} > 10$ rad m$^{-3}$ m$^3$ are those in which the soil moisture measurements exhibit small variations < 0.1 m$^3$ m$^{-3}$, see Tab. A2.

These effects $\beta_{\Delta m_v}$ are significant at $\alpha = 0.05$ for 86% in HH and HV, and 82% in VV; between these polarizations, the difference in magnitude is generally smaller than the uncertainty, see Tab. S4. Exceptions include field 307, where also the variation of the effect observed in different parts of the field exceeds the uncertainty. The estimated soil moisture impact on the coherence magnitude is found to be significant for around 57% of the samples in all polarizations. Negative

Figure 8: The soil moisture coefficients of the regression models for the magnitude (top row) and phase (bottom row) of the complex coherence grouped according to the fields in the AGRISAR campaign: for each field there are four subROIs v-y. The error bars indicate the 95% confidence intervals, diamonds a significant deviation from 0.
6.3 Quantile regression

The median of $\left| \Xi \right|$ turns out not to depend significantly on $\max |\Delta m_v|$ in the Agrisar campaign (Fig. 11). The significant deviations from zero are in line with what would be expected based on random variations under the null hypothesis. This also applies to most fields in the CanEx campaign (Fig. 12): ROIs 307 and 326 form an exception, the former in particular in HV, the latter in HH.

6.4 Sensitivity analysis

The previous analysis is based on several algorithmic choices introduced in Sec. 5. As the main trends and patterns turn out not to be severely affected, the relevant figures and tables are attached as supplementary materials. The impact of each of the identified choices will be briefly sketched in the following.

6.4.1 Reference phase

The phase observable $\phi$ is sensitive to the reference phase with respect to which it is determined. The second implementation of its retrieval, which is based on the identified persistent scatterer for which the smallest phase variance is obtained, yields the phase sensitivities shown in Fig. S1 for the Agrisar campaign and for the CanEx data set in Fig. S2. For the former the sensitivities decrease for fields 230 and 440 (partially ceasing to be significant), whereas the opposite trend is observed over field 460. A similar decrease in the effect occurs for fields 109 and 201 in the CanEx campaign, whereas fields 326 and 331 cease to display any sensitivity to $m_v$. 

Figure 9: The biomass coefficients of the regression models for the magnitude (top row) and phase (bottom row) of the complex coherence, cf. Fig. 8 for a description.
6.4.2 Vegetation parameterization

The amount of vegetation as well as changes therein can also be parameterized by the vegetation height \( h \) instead of the wet biomass \( b \) in the Agrisar campaign. The soil moisture sensitivities of \( \phi \) and \( \log(|\gamma|) \) are plotted in Fig. S3: neither those of the coherence nor those of the phase change by more than one standard error. In the vast majority of cases the differences are much smaller than that; exceptions are the phase sensitivities of the rape fields, for which the model based
on \( h \) yields significant \( \Delta m_v \) terms in HH and HV. The results of the median regression on \( |\Xi| \) are virtually unaffected, see Fig. S4.

### 6.4.3 Autocorrelation

The regressions on \( \phi \) rely on the stochastic model of Eq. 4: it represents correlations between the observations, and these are parameterized by \( \kappa \). The results of the simplified stochastic model for which this parameter is set to 0 are shown in Fig. S5 and S6. The former shows that for the Agrisar campaign the \( \beta_{\Delta m_v} \) coefficients remain predominantly positive and of similar magnitude; differences include the appearance of a significant negative instance (101 in HV and VV) or significant positive ones (e.g. 440 in VV) and the loss of significance for the field 250 in VV. In the CanEx data set, no similar changes occur.

### 6.4.4 Normality

The confidence intervals of the regressions of \( \phi \) and \( \log|\gamma| \) rely on the assumption of normality for the error term \( \epsilon_{ij} \) in e.g. Eq. 2. The results of the relevant hypothesis test (see Sec. 5.2) are summarized in Tab. A5. The null hypothesis is discarded for around 15% of the subROIs at \( \alpha = 0.05 \), which in the Agrisar campaign corresponds to about 5 instances. In most cases the numbers of rejections are more than what would be expected if the error terms were normally distributed in all subROIs and independent of each other. A closer inspection of the results, however, shows that the deviations from normality tend to be clustered according to the fields, thus rendering the assumption of independence inherent in the binomial test invalid. Irrespective of this violation, the total number of instances with significant deviations from normality is rather small, e.g. 1-7 for the \( \phi \) models in either campaign.

### 7 Discussion

#### 7.1 Impact on observables

##### 7.1.1 Phase

The positive phase coefficients \( \beta_{\Delta m_v} \) – they tend to prevail in both campaigns and all polarizations – are only consistent with the dielectric hypothesis of Tab. 1, but not with the penetration depth or deformation explanation. The magnitude, although highly variable, generally exceeds the sensitivities obtained from the surface-only models in Fig. 2 by about two orders of magnitude. Instead, it is more congruous with a volume contribution. The scatter in magnitude could be partially explained by the difficulty of calibrating \( m_v \) and soil permittivity measurements, their spatial scale and the depth dependence. The connection between the size of the phase coefficients \( \beta_{\Delta m_v} \) and the dynamic range of the measured soil moisture is indicative of such a problem affecting the soil moisture measurements. It could also reflect differences in physical properties that affect both the scattering and the hydrological behaviour of the soil. These might be related to soil texture or surface roughness. A non-linear dependence of the phase on soil moisture could also result in such an apparent relation between the soil moisture dynamic range and the interferometric phases.

The phase coefficients of the biomass term \( \beta_{\Delta b} \) in the Agrisar campaign tend to be positive as well, and all the significant ones (\( \alpha = 0.05 \)) are. This sign is also consistent with a dielectric mechanism: an increase in biomass (assumed...
closely related to total water content) above affects the optical path of the wave in a similar way that an increase in $m_v$ does in the soil dielectric model. This influence was observed previously for vegetation at X-band in laboratory experiments by (Rudant et al., 1996).

The positive sign of the soil moisture dependence inferred from the regression analysis was also found by Rudant et al. (1996) in the previously mentioned study: for a sandy soil the authors partitioned this phase change into a compaction (due to the particular kind of application of the water; measured independently) and a complementary effect, both of which had the same sign. They did likewise for a swelling soil and concluded that the complementary effect had the same sign as for the sandy soil, but could not provide a more quantitative analysis due to lack of near surface soil moisture measurements. Similar measurements between 1.5 and 10 GHz showed the same sign of the phase change. The interpretation of these results is hampered by i) lack of displacement measurements, and ii) the sizeable deviations from uniform soil water profiles (Rudant et al., 1996; Nesti et al., 1998). In a different experiment, Morrison et al. (2013) found only negligible phase changes upon wetting of a homogeneous sand sample (C-band laboratory experiment; deformations monitored by monoscopic photogrammetry). Application of the same measurement constellation to a different homogeneous sand sample found the opposite phase trend (Morrison et al., 2011), i.e. in line with the penetration depth hypothesis or surface scattering. The sensitivity of the phase to $\Delta m_v$ was observed to vary by about one order of magnitude over the homogeneous soil sample. These phases could not be explained by deformations. They are similar in size and sign to the ones observed in a laboratory experiment by Yin et al. (2014) in S-band, who concluded that the SPM surface scattering model could explain the observed phases.

Based on L- and C-band satellite data over three fields in Ireland, Barrett et al. (2013) found linear dependencies of $\phi$ on $\Delta m_v$ of both signs. In the majority of cases, these linear dependencies were, however, not significant. Over a swelling soil, te Brake et al. (2013) also found a negative dependence of $\phi$ on $\Delta m_v$ but concluded that it was consistent with the observed deformations.

7.1.2 Coherence

The majority of estimated $|\Delta m_v|$ coefficients for the coherence $|\gamma|$ are negative: this means that changes in soil moisture are associated with a loss of correlation. Such decorrelation is consistent with the volume dielectric hypothesis, but not with a pure surface contribution. The penetration depth explanation does not allow a prediction of this trend, whereas the latter depends on the detailed modelling assumptions for the deformation hypothesis. Note that this dependence was found to be significant at $\alpha = 0.05$ for less than one half of the samples. This low percentage is particularly pronounced for the Agrisar campaign at all polarizations considered, and could be due to

1. the larger time gap between acquisitions
2. $m_v$ and $b$ are correlated with each other and with time: this multicollinearity inflates the standard errors of $\beta_{|\Delta m_v|}$
3. pronounced vegetation growth dominating decorrelation
4. additional temporal decorrelation due to wind-induced movements and dielectric changes within the plant

The latter are difficult to quantify, especially given the large structural changes due to plant growth and the lack of suitable wind speed measurements. The impact of wind-induced decorrelation on vegetated areas is well known (Zebker and Villasenor, 1992), in particular for forests (Lavalle et al., 2012). In a tropical rain forest, Hamadi et al. (2014b,a) observed that wind-related movements, structural changes of the canopy and permittivity fluctuations of plant tissue led to decorrelation at different time scales. The influence of vegetation dynamics on decorrelation was also observed by Barrett et al. (2012) in agricultural fields in Ireland at both C and L-band: the effect of soil moisture changes was small in comparison. Srivastava and Jayaraman (2001); Weydahl (2001) deduced that decorrelation was related to changes in plant and soil moisture, but did not provide quantitative results due to a lack of soil moisture observations. The decorrelation observed in this study is also consistent with the results of Hensley et al. (2011). Using the CanEx data set but different ROI definitions and statistical techniques, they deduced that the time lag alone cannot explain the decorrelation but that soil moisture information is needed. Furthermore, the magnitude of these decorrelation effects is similar to the ones obtained by Nesti et al. (1998) in a laboratory experiment at higher frequencies.

7.1.3 Phase triplets

The results in Fig. 11 and 12 reveal that the median of $|\Xi|$ has an insignificance linear dependence on $\max|\Delta m_v|$ except for two fields in the CanEx campaign: this is consistent with the deformation, null, penetration depth and surface-based dielectric hypothesis, but not necessarily with the volume dielectric explanation. The origins of this insignificant dependence appear to be distinct for different fields. Firstly, there are fields for which the linear model for $\phi$ can describe the data exceedingly accurately: in this case the $\Xi$ are close to 0 and the fit for the median of $|\Xi|$ is good and close to 0 as well. Among these fields are CanEx 109, as well as Agrisar 230 and 250. The second category includes those fields whose fit to both $\phi$ and $|\Xi|$ is inaccurate, such as Agrisar 102 and 460, as well as CanEx 331. Note that this cannot be due to phase offsets alone (such as those arising from the phase referencing) as these cancel when the phase triplets are
formed. Thirdly, there are fields for which the linear model for $\phi$ applies reasonably well, but for which the $|\Xi|$ (which indicate deviations from linearity) are not clearly dependent on $\max|\Delta m_v|$, e.g. CanEx 136 (see Fig. 7) or Agrisar 140.

De Zan et al. (2014) compared the phase triplets with the predictions of the volume-only first-order scattering model for field 222 in the Agrisar data set, for which they found a clear correspondence. The model-independent results of Fig. 11 do not confirm a dependence on soil moisture, but neither do they rule out different functional forms of dependence, i.e. the assumed dependence of Eq. 5 might be incapable of capturing the soil moisture effects (see Sec. 5.3). This suggests the need for more detailed assessments of such physical models.

7.2 Assessment of explanations

The previous analyses of the observed relation between the DInSAR observables and soil moisture permit certain inferences about the plausibility of the four proposed hypotheses of Sec. 3.

7.2.1 Null hypothesis

The null hypothesis – there is no impact on any observable – cannot explain the significant and non-zero dependences observed for $\phi$ and $|\gamma|$. These inferred connections can, of course, be spurious, e.g. due to omitted variables or only partially considered phenomena that influence the observables. These include the phase influences due to DEM errors and orbit inaccuracies. However, these are not expected to be correlated with soil moisture changes, and estimated to be smaller than the inferred soil moisture contributions on the phase. Another prominent example of such a phenomenon is vegetation, which exerts a considerable influence in the Agrisar campaign. In the CanEx data set, however, the limited vegetation cover and growth, along with the short repeat periods, provide more support for the actual presence of soil moisture effects. Secondly, the soil temperature $T_s$ has been repeatedly shown to impact the soil dielectric constant (Mironov et al., 2013); however, $T_s$ only varies by about 4 K for the different acquisitions in the CanEx campaign, which translates to changes in permittivity that are much smaller than the ones due to soil moisture (in the model by Mironov et al. (2013), it is equivalent to $\Delta m_v \approx 0.001 \text{ m}^3\text{m}^{-3}$). The results of Fig. 8 and 10 thus provide evidence for the presence of soil moisture effects.

7.2.2 Deformation

The deformation hypothesis (soil swelling) predicts a different sign of $\beta_{\Delta m_v}$ for $\phi$ than the one observed in both data sets. It can thus be ruled out as the sole origin of an $m_v$ influence of the phase. The opposite deformation behaviour (swelling upon drying) would be consistent with the sign of $\beta_{\Delta m_v}$; we have, however, found no reference to such a soil in the literature, except for rain compaction (Moore and Singer, 1990). The latter is, however, too small a deformation to explain the measured $\phi$: it can reach almost $\pi$ (no apparent wrapping has been observed); this corresponds to a deformation of $\lambda/2$, i.e. 0.05 m, and this appears to be a peculiarly large displacement (Hensley et al., 2011).

7.2.3 Penetration depth

The penetration depth mechanism also predicts the opposite sign for the phase dependence than the one present in the data. In conjunction with the inherent inadequacies of this hypothesis (in particular the inability to explain $|\gamma|$), these empirical results contradict the penetration depth explanation.

7.2.4 Dielectric mechanism

The dielectric volume explanation can account for the observed dependencies of both $\phi$ and $|\gamma|$ (bearing in mind the difficulties with the latter identified in Sec. 7.1.2). The magnitudes of the phase effects are furthermore inconsistent with a pure surface dielectric effect as predicted by the surface scattering models considered in Tab. 2. The volume-only model of De Zan et al. (2014) predicts not only the sign but also the right order of magnitude for $\beta_{\Delta m_v}$ of $\phi$. This model also predicts non-zero phase triplets $\Xi$. Their dependence on soil moisture as expressed by Eq. 5 is found insignificant for the majority of the fields, but the functional relation predicted by the model by De Zan et al. (2014) is different as well as more complex than the one assumed in the regression. The absence of significance found in this study thus does not imply the absence of soil moisture effects in the phase triplets.

A change of the structure function, i.e. the variation of the scatterers with depth, in the volume model could render the model more flexible with regards to the soil moisture sensitivities to account for their observed variability. It would e.g. enable the prediction of larger sensitivities: these are consistent with a large concentration of inhomogeneities deeper in the soil and less scattering from the upper parts, cf. the buried target of Morrison et al. (2013). The depth of the buried target governs the sensitivity of $\phi$ with respect to $m_v$. In natural soils, a layered soil could have a similar impact on the interferometric observables (Rabus et al., 2010). Also the variation of soil moisture with depth and the choice of dielectric mixing model are expected to impact the interferometric signals (De Zan et al., 2014; Rabus et al., 2010). A model combining surface dielectric effects and volume scattering could similarly account for some of the variability of the soil moisture effects across different fields.
7.3 Differences between polarizations

Even though the soil moisture impact is similar for HH, HV and VV, the differences between the polarizations can potentially provide further insight into the scattering physics that give rise to the $m_v$ and vegetation effects. In particular the sizeable impact of $\Delta m_v$ on the HV phase might seem surprising given that the soil contribution at this channel is expected to be considerably smaller than at HH or VV. Such a dependence has also been found by Barrett et al. (2013) in C and L-band, but for the former with the opposite sign.

In the CanEx data set the $\beta_{\Delta m_v}$ for the phase $\phi$ at HH and VV are similar: their differences do not show a consistent pattern regarding their sign or size, and are significantly different from zero for only 14% of the samples, see Tab. S4. In the Agrisar campaign the phase coefficients tend to be larger for HH than for VV, but this difference is only significant at $\alpha = 0.05$ in 25% of the samples, cf. Fig. 8 and Tab. S3. These discrepancies appear to be related to the crop: the difference in both fit and magnitude is particularly pronounced for wheat (230, 250), the difference in location for barley (440) and partially also for rape (101,140). For maize HH and VV behave similarly, but there are marked differences between the subROIs. The linear model for $\phi$ is not accurate for sugar beet (102, 460) in any polarization. Note that all crops except the latter consist of dominantly vertically oriented scatterers, for which the interaction with electromagnetic waves is expected to be significantly stronger in the vertical than the horizontal polarization, as the horizontal extent of the scatterers is small compared to the wavelength except for mature maize. The increased interaction of the waves with vegetation in VV is consistent with smaller (i.e. larger magnitude) $\beta_{\Delta m_v}$ for the phase $\phi$ as well as the larger scatter observed in VV.

The reupted change in the effective propagation due to forward scattering in the vegetation affects the optical path (cf. Sec. 7.1.1), and this polarization dependence is evident in the slope term of $\phi$ with respect to $\Delta b$ in Tab. S3. Its magnitude is expected to be larger for VV than HH for vertically oriented crops: empirically, this difference is significant at $\alpha = 0.05$ for 230 and 250 (both wheat) as well as 440 (barley) and for two samples in 101 (rape).

The coefficients for $|\gamma|$ barely depend on the choice of polarization in the two campaigns; note, however, that the decorrelation appears not to be dominated by soil moisture effects in the Agrisar campaign, cf. Sec. 7.1.2. The vegetation-related decorrelation is, however, not strongly polarization-dependent (Tab. S3) either.

7.4 Robustness to implementation

The assumptions pointed out in Sec. 5 – the choice of reference phase, the parameterization of the vegetation, the estimation of the autocorrelation and the normality of the error term – are shown in Sec. 6.4 to have only minor impacts on the overall patterns found using regression analysis. Noticeable changes do occur for particular fields, e.g. due to representing the vegetation by its height rather than its biomass or by using a different reference phase. These are, however, limited in both number and extent. The lack of sensitivity with respect to the error model in the $\phi$ regressions (for example with respect to the consideration of the autocorrelation) tallies with the rationale of referencing the phase with scatterers in close proximity, thus limiting the influence of unmodelled phase patterns. The conclusions drawn from the statistical analyses thus appear to be reasonably robust to these assumptions.

7.5 Relevance of the soil moisture effects

As radar interferometry is commonly applied to estimate displacements and elevations, a question regarding the impact of soil moisture changes arises: how do they affect the estimation of displacements and elevations? The two data sets exhibit phase excursions of up to $\pi$, e.g. in Fig. 7, and we attributed these to soil moisture effects due to their conspicuous dependence on $\Delta m_v$. Phases of this magnitude were associated with coherence magnitudes of 0.5-0.8, which are commonly considered adequate for differential interferometry (Ketelaar and Hanssen, 2003; Crosetto et al., 2011).

Such a value of $\phi = \frac{\pi}{2}$ corresponds to a displacement of 2-3 cm at L-band. This spuriously inferred movement is of similar or larger magnitude than most geophysical deformation mechanisms commonly studied using DInSAR. Furthermore, it can occur on comparatively rapid time scales of minutes to days. Also the spatial scales at which these changes occur can in certain cases be comparable to those of the deformation processes. These soil moisture effects can thus potentially render such deformation analyses unreliable. However, many common processing systems avoid such areas by deriving the phase information from stable point-like scatterers, but there have been numerous approaches (e.g. Berardino et al., 2002; Ferretti et al., 2011)) to include areas such as soil, that do not act as point-like targets. These algorithms commonly map the phase information of as many interferograms as possible to one time series with a fixed master scene. This mapping could only reduce the soil moisture signal significantly if the non-linear terms of the soil moisture dependence of $\phi$ dominate. Otherwise, if the linear components dominated as in many of the fields observed in this study, the soil moisture variations will be preserved in the phase time series, from which the displacements are inferred. These estimated deformations are commonly assumed to exhibit a particular temporal behaviour, such as a movement with a constant velocity (Berardino et al., 2002). Under such an assumption, the deformation estimate would only be impacted if the soil moisture variations corresponded to that particular temporal model.

In the estimation of elevations using repeat-pass interferometry, the impact of a soil moisture phase term of $\frac{\pi}{2}$ (assumed independent of the baseline) on the inferred height will depend on the baseline. More specifically, it will scale with the
height of ambiguity \( h_a = \frac{2\pi}{|\kappa z|} \). This spurious elevation can be compared to the one due to phase noise: apart from areas where interferometry is hardly feasible due to decorrelation, the number of looks is generally chosen so that this phase noise is much smaller than \( \frac{\pi}{2} \) (Bamler and Hartl, 1998). Thus the soil moisture effect can dominate the noise; it will also generally exhibit spatial patterns that can be related to the topography. Soil moisture variations can thus induce errors in the estimated elevation models that are both significant and relevant.

8 Conclusions

We conducted an empirical study of the soil moisture effects of DInSAR observables at L-band in a model-independent way. These analyses reveal that there is a dependence of the phase \( \phi \) on changes in soil moisture \( \Delta m_v \) in both data sets examined. The decorrelation is also related to such changes; this relation is more pronounced in the CanEx data set, where the time difference between the acquisitions is smaller. The sign of the phase dependence and also its magnitude point towards volume scattering within the soil. This mechanism can also explain moisture-related decorrelation, and existing models could be adapted to capture the observed variabilities of the sensitivities respect to soil moisture. The observed \( \phi \) dependence on \( m_v \) is inconsistent with both the swelling soil and the penetration depth explanation.

These inferences only relate to the two L-band data sets analysed: their generality can only be established with additional studies. Future research might elucidate the impact of the soil characteristics (e.g., roughness or swelling behaviour), the properties of the vegetation cover, as well as the radar frequency. The wavelength governs the sensitivity to movements and roughness, as well as the penetration depth, which suggests a possible impact on the soil moisture effects. At higher frequencies such as X and Ku-band, temporal decorrelation is expected to be more pronounced. Frequent acquisitions, which can for example be made using ground-based radars, will thus be instrumental in providing the data that is necessary to improve the understanding and modelling of these effects.

Owing to the size of the observed phase values of up to \( \phi = 0.5\pi \), corresponding to a displacement of 2-3 cm at L-band, these soil moisture effects deserve to be considered in repeat-pass InSAR studies. They have the potential to induce errors in the estimated deformations in a wide range of temporal and spatial scales, thus indicating the importance of the study of these effects. Advances in the processing schemes and the physical modelling of these influences might lead to improvements in the estimates of the deformations and their uncertainties, thus contributing to the study of geophysical phenomena as diverse as tectonics, mass-movements and permafrost degradation.

9 Acknowledgements

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References


Assessment of soil moisture effects on L band radar interferometry
Supplementary Material

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Introduction The additional figures contain the regression results that are obtained with different assumptions/algorithmic choices than those in the manuscript.

The additional tables contain more detailed descriptions of the agricultural fields used in this study (1-2), and an exhaustive collection of the statistical results (3-5).
Table S1: Overview of the fields in the Agrisar campaign: besides the field number and the crop, the maximum vegetation height $h$ observed in-situ during the campaign is given, along with the difference between the maximum and minimum soil moisture value $m_v$. The mean radar incidence angle $\theta$ is the average angle at which the field is observed with the radar instrument. The distance of the ROI to the persistent scatterer is given in meters, along with the area of the field. The number of interferograms used in the analysis depends on the field: when only measurements up to a certain DOY were included in the analysis (due to harvest or lack of data), this DOY is given in the column restriction (– indicates the last day of the campaign).

<table>
<thead>
<tr>
<th>Field</th>
<th>Crop</th>
<th>Maximum $h$ [cm]</th>
<th>$m_v$ dynamic range [$m^3 m^{-3}$]</th>
<th>$\theta$ [°]</th>
<th>Distance to PS [m]</th>
<th>Area[ha]</th>
<th>Number of Interferograms</th>
<th>Restriction (DOY)</th>
</tr>
</thead>
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<tr>
<td>101</td>
<td>rape</td>
<td>172</td>
<td>0.206</td>
<td>50</td>
<td>55</td>
<td>80</td>
<td>28–</td>
<td>171</td>
</tr>
<tr>
<td>102</td>
<td>sugar beet</td>
<td>28</td>
<td>0.160</td>
<td>51</td>
<td>208</td>
<td>15</td>
<td>15</td>
<td>–</td>
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<td>15</td>
<td>22</td>
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<td>112</td>
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<td>Field</td>
<td>Code</td>
<td>Crop</td>
<td>Cover [%]</td>
<td>Soil texture</td>
<td>$m_v$, dynamic range $[m^3 m^{-3}]$</td>
<td>$\theta$ [$^\circ$]</td>
<td>Distance to PS [m]</td>
<td>Area [ha]</td>
</tr>
<tr>
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<tr>
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<td>?</td>
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<td>30</td>
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<tr>
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<td>55</td>
</tr>
<tr>
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<td>K4</td>
<td>peas</td>
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<td>silt loam</td>
<td>0.062</td>
<td>32</td>
<td>550</td>
<td>62</td>
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</tbody>
</table>
Table S3: Polarimetric differences in estimated sensitivities for each ROI in the Agrisar campaign. \(x:y:z\) gives the difference between the estimated \(\beta_x\) coefficient for observables \(y - z\). A star indicates significance at \(\alpha = 0.05\).

\[
\begin{array}{cccccccc}
\text{ROI} & m_x:|\gamma_{VV}|:|\gamma_{HH}| & m_x:|\gamma_{HV}|:|\gamma_{HH}| & m_x:|\gamma_{VV}:\gamma_{HH}| & m_x:|\gamma_{HV}:\phi_{HH}| & b_x:|\gamma_{VV}|:|\gamma_{HH}| & b_x:|\gamma_{HV}|:|\gamma_{HH}| & b_x:|\phi_{VV}:\phi_{HH}| & b_x:|\phi_{HV}:\phi_{HH}| \\
101v & -0.88 & 0.34 & -2.18 & -0.13 & 0.00 & 0.00 & -0.00 & 0.06 & 0.05 \\
101w & 0.59 & -1.11 & -4.91* & 0.75 & 0.05 & 0.06 & 0.41* & -0.06 & \\
101x & -0.31 & 1.01 & -1.62 & 0.56 & -0.02 & -0.10 & -0.01 & 0.35* & \\
101y & -0.39 & -0.57 & -3.19* & -0.90 & 0.09 & 0.03 & 0.38* & 0.16 & \\
102v & 1.27 & 4.98 & -10.57* & 0.76 & 0.60* & 1.53* & -1.30 & 0.07 & \\
102w & -4.64 & -6.00 & -6.54 & 4.74 & 0.08 & 0.99* & -0.93 & 1.56* & \\
102x & 1.11 & -4.37 & -1.33 & -4.17 & 0.26 & 1.26* & -0.45 & -1.35* & \\
102y & -0.99 & -3.23 & 1.61 & 2.28 & -1.16* & 0.21 & -0.17 & -0.61 & \\
104v & 0.13 & -0.17 & -1.53 & -0.38 & -0.09 & -0.12 & 0.11 & 0.00 & \\
104w & -1.00 & 1.21 & -0.07 & 1.19 & -0.03 & -0.13 & -0.03 & -0.22 & \\
104x & -1.02 & -1.87 & -1.31 & -0.41 & 0.01 & -0.01 & 0.12 & 0.04 & \\
104y & -1.18 & -0.46 & -2.40 & -0.23 & 0.01 & -0.15 & 0.14 & 0.06 & \\
222v & 1.13 & -0.04 & 0.16 & -4.75* & -0.19 & -0.24 & -0.27 & 0.11 & \\
222w & 1.43 & -1.09 & 0.61 & -1.95 & -0.08 & -0.46* & -0.22 & 0.50 & \\
222x & -0.41 & -0.75 & 0.37 & -3.19 & -0.05 & -0.16 & 0.07 & 0.13 & \\
222y & 0.66 & 0.44 & 1.04 & 0.34 & -0.18* & -0.15* & 0.23 & 0.07 & \\
230v & -0.45 & -0.56 & -2.23* & 1.55 & 0.03 & -0.05 & 0.33* & -0.09 & \\
230w & 1.07 & -0.22 & -0.93 & 0.00 & 0.06 & 0.03 & 0.40* & -0.16* & \\
230x & 0.45 & -2.54* & -2.89* & 3.12* & 0.11* & 0.00 & 0.38* & -0.20* & \\
230y & -0.40 & 0.25 & 0.24 & -0.40 & -0.05 & 0.01 & 0.37* & -0.10 & \\
250v & -0.14 & -1.46* & -2.36* & 0.99 & 0.01 & 0.04 & 0.47* & -0.03 & \\
250w & 0.08 & -0.65 & -0.76 & -3.65 & -0.03 & -0.10* & 0.54* & -0.24 & \\
250x & -0.75 & -1.02 & 1.04 & -0.80 & -0.01 & -0.02 & 0.21* & -0.10 & \\
250y & -0.16 & -0.56 & -0.07 & -2.14 & -0.05 & 0.02 & 0.54* & -0.28 & \\
440v & -0.09 & -0.55 & -4.92* & 3.64 & 0.07 & -0.01 & 0.95* & -0.60* & \\
440w & 0.90 & 0.82 & -2.17 & 3.13 & 0.06 & 0.05 & 0.48* & -0.29* & \\
440x & 0.87 & 1.25 & -2.64 & 2.63 & 0.17 & -0.04 & 0.54* & -0.39* & \\
440y & 0.79 & -0.05 & -3.37 & 1.61 & 0.13* & 0.10 & 0.71* & -0.30* & \\
460v & -1.02 & -3.55 & -0.72 & -0.60 & -0.01 & -0.24 & 0.11 & -0.52 & \\
460w & 1.96 & 6.05 & -5.60 & -0.49 & -0.03 & -0.15 & 0.17 & -1.39* & \\
460x & -0.03 & -0.35 & -1.62 & 10.64 & 0.22* & 0.21 & -0.55 & -0.23 & \\
460y & -2.95 & -7.19 & -9.24 & 14.87 & 0.12 & -0.11 & 0.24 & -0.23 & \\
\end{array}
\]
Table S4: Same as table A3 but for the CanEx campaign.

| ROI  | \( m_{\nu V} |\gamma_{VV}| |\gamma_{HH}| \) | \( m_{\nu V} |\gamma_{HV}| |\gamma_{HH}| \) | \( m_{\nu V} |\phi_{VV}| |\phi_{HH}| \) | \( m_{\nu V} |\phi_{HV}| |\phi_{HH}| \) |
|------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 109a | 0.18                            | -0.31                           | 0.28                            | 0.02                            |
| 109b | 0.64                            | -0.86                           | -0.56                           | 0.62                            |
| 109c | 0.20                            | -0.80                           | -0.02                           | 0.28                            |
| 109d | 0.03                            | -2.67*                          | 0.76                            | 1.48                            |
| 136a | -1.52                           | -4.01                           | -1.86                           | 0.69                            |
| 136b | 0.50                            | -6.72                           | 0.28                            | 1.95                            |
| 136c | 0.91                            | -12.43                          | 0.85                            | 0.04                            |
| 136d | 0.18                            | -3.31                           | 0.79                            | 5.37                            |
| 201a | -4.01                           | -1.10                           | 9.62                            | -4.50                           |
| 201b | -2.40                           | -2.16                           | 7.62                            | -7.29                           |
| 201c | -1.84                           | 2.41                            | 12.40                           | 2.98                            |
| 201d | -1.50                           | -0.74                           | 3.54                            | 0.19                            |
| 206a | 2.15                            | -24.72                          | 3.43                            | 2.71                            |
| 206b | 3.97                            | -9.11                           | 3.26                            | -0.45                           |
| 206c | 7.18                            | -16.06                          | 3.95                            | 2.06                            |
| 206d | 5.06                            | -0.01                           | 1.81                            | -3.99                           |
| 307a | 1.02*                           | -0.14                           | -1.25                           | 1.88                            |
| 307b | 1.08                            | -0.19                           | 1.06                            | 1.50                            |
| 307c | 0.54                            | -0.28                           | 0.87                            | 0.59                            |
| 307d | 0.44                            | -1.44*                          | 1.11*                           | 0.98                            |
| 326a | -4.08                           | -0.88                           | -6.09                           | 1.36                            |
| 326b | 2.52                            | -4.01                           | -4.00                           | 0.58                            |
| 326c | 6.37                            | 1.04                            | -3.29                           | 2.75                            |
| 326d | 1.61                            | -5.44                           | -4.74                           | 1.52                            |
| 331a | 8.31                            | 0.05                            | -4.28                           | 4.17                            |
| 331b | -1.71                           | -24.17                          | -2.89                           | 3.00                            |
| 331c | -1.46                           | -8.74                           | -1.36                           | 5.51                            |
| 331d | 2.15                            | 3.54                            | -1.31                           | -4.45                           |
Table S5: Results of the normality test of the residuals for each observable and campaign. $N_s$ gives the number of occurrences, for which the deviation from normality is significant at $\alpha = 0.05$. The relative number of these occurrences (normalized by the total number of ROIs) is given by $p_s$.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Campaign</th>
<th>$N_s$</th>
<th>$p_s$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>log($</td>
<td>\gamma_{HH}</td>
<td>$)</td>
<td>Agrisar</td>
</tr>
<tr>
<td>log($</td>
<td>\gamma_{HV}</td>
<td>$)</td>
<td>Agrisar</td>
</tr>
<tr>
<td>log($</td>
<td>\gamma_{VV}</td>
<td>$)</td>
<td>Agrisar</td>
</tr>
<tr>
<td>$\phi_{HH}$</td>
<td>Agrisar</td>
<td>7</td>
<td>21.9</td>
</tr>
<tr>
<td>$\phi_{HV}$</td>
<td>Agrisar</td>
<td>4</td>
<td>12.5</td>
</tr>
<tr>
<td>$\phi_{VV}$</td>
<td>Agrisar</td>
<td>1</td>
<td>3.1</td>
</tr>
<tr>
<td>log($</td>
<td>\gamma_{HH}</td>
<td>$)</td>
<td>CanEx</td>
</tr>
<tr>
<td>log($</td>
<td>\gamma_{HV}</td>
<td>$)</td>
<td>CanEx</td>
</tr>
<tr>
<td>log($</td>
<td>\gamma_{VV}</td>
<td>$)</td>
<td>CanEx</td>
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<td>CanEx</td>
<td>5</td>
<td>17.9</td>
</tr>
<tr>
<td>$\phi_{HV}$</td>
<td>CanEx</td>
<td>4</td>
<td>14.3</td>
</tr>
<tr>
<td>$\phi_{VV}$</td>
<td>CanEx</td>
<td>3</td>
<td>10.7</td>
</tr>
</tbody>
</table>
Figure S1: Same as the three phase panels of Fig. 4 but using the alternative phase reference.

Figure S2: Same as the three phase panels of Fig. 5 but using the alternative phase reference.
Figure S3: Same as the three phase panels of Fig. 4 but using the alternative description of the plant state (vegetation height $h$).

Figure S4: Same as the three phase panels of Fig. 7 but using the alternative description of the plant state (vegetation height $h$).
Figure S5: Same as the three phase panels of Fig. 4 but using the alternative stochastic model, i.e. assumption of no autocorrelation of the phase noise.

Figure S6: Same as the three phase panels of Fig. 5 but using the alternative stochastic model, i.e. assumption of no autocorrelation of the phase noise.
References
Chapter C

Influence of vegetation growth on the polarimetric zero-baseline DInSAR phase diversity – implications for deformation studies

S. Zwieback and I. Hajnsek

Key findings:
- the polarimetric DInSAR phase diversity is closely related to vegetation growth for certain agricultural crops
- the ambiguity of the displacement estimate can correspond to 3 cm at L-band
- the observations suggest birefringence within the canopy as dominant mechanism for wheat and barley

The author’s contributions:
- developed the modelling framework and suggested the hypotheses
- introduced the tests for confronting the hypotheses with the observations
- interpreted the results and wrote the manuscript

The co-author’s contributions:
- suggested analysing the polarimetric coherence regions using physical models
- provided radar data and helped with the interpretation of the imagery and ground measurements
- interpreted the results and wrote the manuscript
Influence of vegetation growth on the polarimetric zero-baseline DInSAR phase diversity – implications for deformation studies

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Abstract

The polarization diversity of the phase causes ambiguities in the estimation of displacements using differential interferometry. Over natural surfaces such as vegetated areas, the magnitude of these ambiguities is potentially related to complex dynamic processes such as vegetation growth. As the properties and possible origins of such diversity (besides noise-like influences) over changing vegetation canopies are virtually unknown, we propose to investigate them empirically using an L band zero-baseline data set covering one growing season over different agricultural crops. We frequently observe HH-VV phase differences exceeding $0.5\pi$, corresponding to a displacement discrepancy of 3 cm. The HH-VV phase difference and other properties of the polarimetric coherence regions (e.g. the shape and the relation to the in-situ observed biomass) vary with the crop type. The observations over wheat, barley and, to a lesser extent, rape suggest the presence of birefringence within the canopy. By contrast, those over maize and sugar beet – while also showing phase diversity – cannot be explained by birefringence or similarly simple models. Irrespective of the origin of this dependence, its presence and systematic nature indicate the potential importance of vegetation effects in differential interferometry, which may limit the accuracy of the estimated deformations over vegetated areas.

1 Introduction

Differential Synthetic Aperture Radar Interferometry (DInSAR) is a popular remote sensing technique with which movements of scatterers along the line of sight can be estimated [1]. It is routinely applied to monitor deformations that are due to tectonic and volcanic processes [2], or associated with mining activities and groundwater dynamics [3], as well as cryospheric phenomena [4]. The measurement is achieved by combining two SAR images acquired at different times, i.e. by forming an interferogram and computing the complex coherence $\gamma$. Its argument, the phase $\phi$, is proportional to the deformation once it has been referenced to account for e.g. the effects of the atmosphere and the topography. The magnitude of $\gamma$, the coherence $|\gamma|$, indicates the reliability, i.e. how well $\phi$ can be estimated [1]. Many SAR systems acquire several polarimetric channels: this polarimetric information has found a number of applications and has proved particularly suitable for characterizing vegetation properties [5]. When the SAR acquisitions contain more than one polarization, the interferogram will also consist of multiple polarimetric channels. From each of these a deformation estimate can be retrieved. This wealth of polarimetric information can be represented in terms of a coherence region [5] and is particularly suitable for characterizing vegetation properties [5]. When the SAR acquisitions contain more than one polarization, the interferogram will also consist of multiple polarimetric channels. From each of these a deformation estimate can be retrieved. This wealth of polarimetric information can be represented in terms of a coherence region [5] and is often exploited with the aim of producing the least noisy deformation estimate. As the noise level is indirectly related to the coherence magnitude $|\gamma|$, this is achieved by coherence maximization [6, 7]. This approach is based on the intrinsic assumption that there is no systematic impact on the polarimetric dependence of $\phi$ due to deformations or other processes, such as vegetation growth.

We propose to assess this assumption empirically by focussing on the L-band coherence regions over agricultural fields. For non-zero baseline data, the height of the scatterers, which include the ground surface and various vegetation elements, influences the interferometric phase. This effect can be modelled and exploited in order to estimate vegetation parameters such as the canopy height [8, 9]. As our data set was acquired with zero baseline, the height of the scatterers does not influence the interferometric phase [9]. For zero baseline acquisitions and a static scene, that is if there are no temporal changes, no polarimetric phase differences are expected [10, 11]. By contrast, the presence of vegetation growth may lead to such differences. We intend to study the impact of vegetation growth because agricultural fields change markedly at time scales of weeks to months, which are relevant for DInSAR deformation studies. The data set covers an entire growing season during which the vegetation canopies undergo complex changes. Nevertheless, the L-band SAR data are expected to contain reliable phase information as they retain considerable coherence over the entire study period [12]. Using this data set, we analyse how the polarimetric coherence region is related to the crop type and vegetation growth, focussing particularly on systematic differences in the phase $\phi$ between different polarimetric channels. We analyse possible origins of such phase diversity using a first-order scattering model [13]. These include polarimetrically diverse changes in the scattering phase and differential movements of scatterers within the canopy.

Another possible origin of zero-baseline phase diversity – birefringence, i.e. polarization-dependent propagation velocity – is not only relevant to zero-baseline DInSAR, but has been hypothesized to impact standard polarimetric backscatter, such as the co-polarimetric phase difference [14, 15, 16], as well as single-pass interferometric data [17, 18]
with non-zero baselines. In order to assess the plausibility of birefringence as origin of the observed phase diversity, we propose several quantitative features for which predictions can be formed and compared to observations. These features allow us to draw inferences regarding the importance of birefringence but also of other simple mechanisms such as the aforementioned differential movements. They also provide a quantitative summary of the prevalence and magnitude of the observed phase diversity and the associated ambiguities of the displacement estimates.

2 Theoretical considerations

2.1 DInSAR

The elementary observable in a polarimetric single-look complex (SLC) SAR image is the scattering matrix. Assuming reciprocity and backscattering geometry, it can be represented by a three dimensional scattering vector $\bar{k}^u$. We will represent the scattering vector in the lexicographic basis [5]. This vector characterizes the scattering properties of the target. Due to speckle, the targets are often considered to be a realization of a random process, in which case the covariance matrix $C^{u,u} - \langle \bar{k}^u \bar{k}^u \rangle$ gives a more useful description of the scattering behaviour [5]. When estimating these quantities from observations, the ensemble average $\langle \rangle$ is most commonly replaced by spatial averaging (multi-looking).

In interferometry, the scattering vectors of two acquisitions $u$ and $v$ are combined coherently by forming the interferometric covariance matrix $\bar{C}^{u,v}$ [5]. The relation between these two acquisitions can be described by a normalized quantity called coherence $\gamma^{u,v}$ [6]

$$\gamma^{u,v} = |\gamma^{u,v}| \exp i \phi^{u,v} = \frac{\bar{C}^{u,v} \bar{C}^{u,v}}{\sqrt{\bar{C}^{u,u} \bar{C}^{v,v} \bar{C}^{u,u} \bar{C}^{v,v}}}$$

whose magnitude $|\gamma^{u,v}| \in [0, 1]$ encodes the correlation and thus the similarity between $u$ and $v$. The coherence depends on the polarization functional $\bar{\omega}$, which describes the polarimetric properties of the transmit and receive antennas [6]. For instance, it can represent the HV or the HH channel. The set of all possible coherences as a function of $\bar{\omega}$ constitutes the coherence region [5].

The phase $\phi^{u,v}$ of a point scatterer in a zero-baseline scenario (which will be assumed from now on) corresponds to changes in the optical path between the scatterer and the antenna position. When the target moves away from the sensor by a distance $d$, $\phi^{u,v} = 2kd$, where $k$ is the wavenumber [1]. The phase $\phi^{u,v}$ is independent of polarization $\bar{\omega}$ in such an idealized scenario. In more complex situations, polarization dependence of both $|\gamma|$ and $\phi$ is, however, frequently observed [6]. This polarimetric diversity implies that there is no unique inferred displacement $d$, as it also depends on $\bar{\omega}$. A common way of enforcing uniqueness is by coherence optimization [6, 7]: one chooses the phase $\phi^{u,v}$ corresponding to the polarization $\omega^*$ at which $|\gamma^{u,v}|$ (or a related function) is maximized. The rationale of this optimization is that the standard error of the estimated phase increases as $|\gamma|$ decreases (for a fixed amount of averaging) [1]. The most precise phase estimate is thus obtained at $\omega^*$. The validity of this approach is based on the implicit assumption that the phenomena giving rise to polarimetric phase diversity are noise-like in nature, not systematic.

2.2 Phase diversity

2.2.1 First-order scattering model

Several possible origins of such phase diversity can be studied using the first-order scattering approximation [19]. According to this approach the target is assumed to be a random collection of elementary scatterers, i.e. a realization of a stochastic process, and only the contribution from each scatterer due to the coherent incident wave is included. The scattering vector is then a summation of contributions

$$\bar{k} = \sum_j I_j^{u} \bar{T}_{j}^{u} \bar{\pi}_{j}^{u}$$

where $I_j^{u}$ is an indicator function that is equal to one if scatterer $j$ is present at acquisition $u$ and zero otherwise. The scattering vector of particle $j$ is denoted by $\bar{T}_{j}^{u}$, and is not defined if $I_j^{u} = 0$. The contribution from the transmission between the particle and the antenna is contained in $\bar{T}_{j}^{u}$. Treating the propagation of the coherent wave within the vegetation canopy as equivalent to that in a deterministic channel, the propagation from the antenna to scatterer $j$ during acquisition $u$ can be described by a Jones matrix $\bar{M}_{j}^{u}$ [5]. Its transpose expresses the propagation in the reverse direction (i.e. from scatterer $j$ to the antenna) if the medium is reciprocal. For such a reciprocal medium, the two-way transmission
operator $\mathbf{T}_j$ that transforms the scattering vector $\mathbf{u}_j$ is given by

$$
\mathbf{T}_j = \begin{pmatrix}
M_{11} & \sqrt{2}M_{12} & M_{21} \\
\sqrt{2}M_{11}M_{12} & M_{11}M_{22} + M_{12}M_{21} & \sqrt{2}M_{11}M_{22} \\
M_{21} & M_{22} & M_{22}
\end{pmatrix}
$$

(3)

$$
\mathbf{M}_j = \begin{pmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{pmatrix}
$$

(4)

where the subscripts in the components of $\mathbf{M}_j$ have been dropped for simplicity.

It is often useful to conceive of the vegetation canopy as a dielectric medium. In a geometric optics (GO) approximation [20], the Jones matrix $\mathbf{M}_j$ is then obtained by integrating the local $\mathbf{N}(r)$ matrix along the propagation path (neglecting its polarization dependence as it is a second order effect)

$$
\mathbf{M}_j = \exp \left(-ik_0 \int_{r_0}^{r} \mathbf{N}(r) dr \right)
$$

(5)

This $\mathbf{N}(r)$ matrix is in turn closely connected to the refractivity tensor of the dielectric medium [21]: in a birefringent medium whose principal dielectric axes coincide with the coordinate directions and for propagation in the xz plane, it can be expressed in the associated linear polarization basis as

$$
\mathbf{N} = \begin{pmatrix}
\sqrt{n_x^2 \sin^2 \theta + n_y^2 \cos^2 \theta} & 0 \\
0 & n_y
\end{pmatrix}
$$

(6)

where $\theta$ is the angle between the z axis and the wave vector.

The second-order statistics follow from (2) upon ensemble averaging $\langle \cdot \rangle$

$$
\mathbb{E}^{\alpha, \beta}_{i, j} = \langle \mathbb{E}^{\alpha, \beta}_{i, j} \rangle - \sum_{j^f} \sum_{j^f} I_{j^f} I_{j^f}^* (\mathbf{T}_j^* \mathbf{q}_j \mathbf{q}_j^*) \mathbf{T}_j^* \\
- \sum_{j} I_{j} I_{j}^* (\mathbf{T}_j^* \mathbf{q}_j \mathbf{q}_j^*) \mathbf{T}_j^* 
$$

(7)

where the second line follows under the independent scattering assumption [19]. The interferometric phase $\phi^{\alpha, \beta}$ at a certain scattering polarization $\mathcal{P}$ is the argument of $\mathbb{E}^{\alpha, \beta}_{i, j} \mathcal{P}$

$$
\phi^{\alpha, \beta}_{i, j} = \sum_{j} I_{j} I_{j}^* \mathcal{P} \mathbf{T}_j^* \mathbf{q}_j \mathbf{q}_j^* \mathbf{T}_j^* \\
\mathcal{P} = \alpha^{\alpha, \beta}(\mathcal{P}) \exp(i \phi^{\alpha, \beta}(\mathcal{P}))
$$

(8)

Each contribution $j$ is represented by a real scattering amplitude $\alpha^{\alpha, \beta}_{i, j}$ and phase $\varphi^{\alpha, \beta}_{i, j}$, which can both depend on the polarization $\mathcal{P}$. When the phase $\phi^{\alpha, \beta}$ depends on $\mathcal{P}$, we will refer to this as $\mathbb{E}^{\alpha, \beta}$ exhibiting phase diversity.

### 2.2.2 Possible origins of phase diversity

Several of these will now be studied within the framework of the first-order scattering solution (8). When neither $\varphi_j$ nor $\alpha_j$ depend on $\mathcal{P}$, $\mathbb{E}^{\alpha, \beta}$ will not exhibit polarimetric phase diversity. There are thus three ways how such phase diversity might arise:

1. $\varphi_j$ depends on $\mathcal{P}$, $\alpha_j$ does not
2. $\alpha_j$ depends on $\mathcal{P}$, $\varphi_j$ does not
3. both do

The first case leads to phase diversity via $\varphi_j$, which in turn can depend on $\mathcal{P}$ via propagation effects ($\mathbf{T}_j$) or scattering effects ($\langle \mathbf{q}_j \mathbf{q}_j^* \rangle$). The former occur when the properties of the birefringent medium change with acquisition (due to e.g. vegetation growth, see Fig. 1), but not if the target moves in a medium whose polarimetric difference of optical paths in $\mathbf{M}_j$ is acquisition invariant. The latter could arise through changes in the properties of the scatterer, e.g. variations in the shape or dielectric properties.

In the second case, it is the polarization dependence of $\alpha_j$ that modulates the polarization-independent $\varphi_j$. The latter must be different for different $j$ and the modulation requires some systematic connection between the $\alpha_j$ and
Figure 1: A homogeneous, birefringent medium, for which the refractive index differs for polarizations A and B. This is illustrated by the different wavelengths (spacing of the wavefronts) of the two vertically incident waves. The propagation phase difference between A and B of a scatterer \( j \) at the bottom of the medium is different for the two acquisitions and thus leads to a polarization-dependent \( \varphi_j \).

Figure 2: Phase diversity caused by different values of \( \varphi_j \) (caused by movement, indicated by the red arrows). These are modulated by the scattering contribution (represented by the shape, with e.g. the horizontal particles preferentially scattering in HH and the vertical ones in VV).

\( \varphi_j \). The polarization dependence of \( \alpha_j \) can again arise via propagation effects (dichroism) or through scattering effects. The polarization-independent values of \( \varphi_j \) (which must differ between scatterers), can also be due to scattering effects \( \langle \tilde{q}^-_j \tilde{q}^+_j \rangle \) or transmission effects. The former can be caused by the nature of the scatterers (such as the shape) and changes therein, whereas the latter are due to variations in the optical path, i.e. movements or changes in the refractive index of the medium. The modulation of the \( \varphi_j \) (caused by movement) due to the scattering properties (shape-induced polarization dependence of \( \alpha_j \)) is illustrated in Fig. 2. The systematic connection is brought about by the dependence of the movement on the particle shape.

For the third way, all the previously mentioned effects can occur simultaneously.

The reasoning behind these explanations is based on the first-order scattering solution; however, the decomposition into contributions with different magnitudes \( \alpha_j \) and phases \( \varphi_j \) is also valid in a perturbative analysis of multiple scattering [22]: the double bounce scattering, for instance, is commonly represented as one of the terms in (2) [10].

Concerning possible origins of phase diversity we consider birefringence to be a plausible cause, as it has been frequently postulated (albeit rarely directly observed) to exist in canopies of vertically oriented crops [17, 10, 23]. It is, furthermore, a simple mechanism that can make crisp predictions without introducing many additional assumptions. This is in general not the case for the other (not necessarily mutually exclusive) explanations described above, as these include either complex scattering effects and/or changes in the optical path (e.g. due to differential movements). As none of these has – to our knowledge – been systematically analysed before, we will treat them not as comprehensively as birefringence (see Sec. 4.2).

2.2.3 Birefringence

In the case of birefringence, the phase diversity is due to the differential propagation path of scatterers within the canopy. We consider scatterers close to the ground to be the most relevant ones, as the upper parts of the vegetation canopy change...
Figure 3: The incident wave (in red) interferes with the one scattered in the forward direction (in purple) to give rise to one (in orange, identical polarization assumed) with changed amplitude and phase. This superposition is visualized by the addition of the phasors in the inset.

Birefringence in the microwave range within a medium that consists of dielectrically isotropic constituents is commonly described either in a static approximation or attributed to scattering. The former case applies when the heterogeneities are much smaller than the wavelength [24], e.g. snow in the microwave spectrum. As the size of the particles increases, scattering effects become more important. For the propagation of the wave, it is the forward scattering that matters. This is illustrated for the simplest case (1D scenario, one scatterer) in Fig. 3: in the forward direction, the scattered wave interferes with the incident one. This leads to a change of both amplitude (related to absorption, represented by the reduction of the magnitude of the phasor of the resulting wave compared to the incident one) and phase (represented by the angular difference). When this phase differs for different polarizations, this will correspond to birefringence.

Irrespective of its origin, birefringence manifests itself in the refractive index and thus (5) and (6): for vertically oriented symmetric crops whose constituents consist of isotropic materials, the $\mathbf{N}$ matrix is expected to be diagonal in a coordinate system aligned with the vertical direction ($z$ axis), and thus so will $\mathbf{M}_j$ and $\mathbf{T}_j$:

$$
\mathbf{T}_j^u = \begin{pmatrix}
\beta_{j,HH}^u e^{i(\phi_j^u + \epsilon_j^u)} & 0 & 0 \\
0 & \beta_{j,HV}^u e^{i\epsilon_j^u} & 0 \\
0 & 0 & \beta_{j,VV}^u e^{-i(\phi_j^u + \epsilon_j^u)}
\end{pmatrix}
$$

(9)

where $\beta_{j,\text{PQ}}^u$ encodes the absorption for scatterer $j$ at acquisition $u$ in channel PQ, and $\phi_j^u$ the phase relative to the HV propagation phase. The latter will thus be halfway between the HH and VV phases, if $|\psi_j^u| < \frac{\pi}{2}$. The latter two phases will be the extreme values of the propagation phase as a function of polarization.

This polarimetric dependence of the propagation phase directly affects the observable $\mathbf{C}_{uv}$. When all the stable scatterers ($I_j^u - I_j^v - 1$) are at ground level, (7) simplifies as the propagation matrices $\mathbf{T}_j$ become common to all scatterers:

$$
\mathbf{C}_{uv} = \mathbf{T}^u \sum_{I_j^u - I_j^v - 1} \langle \tilde{\eta}_j^u \tilde{\eta}_j^v \rangle \mathbf{T}^v
$$

(10)

In combination with (9), and assuming no scattering phase effects in $\mathbf{G}_{uv}$ as well as $|\psi^u - \psi^v| < \frac{\pi}{2}$, this implies that the HV phase is halfway between the HH and VV interferometric phase. Quantitatively, $l = 0.5$ for

$$
l = \frac{\phi_{HV} - \phi_{VV}}{\phi_{HH} - \phi_{VV}}
$$

(11)

where the differences of phases are assumed to be corrected for wrapping effects.

Under the same assumptions and if $\mathbf{C}_{uv}$ (and thus also $\mathbf{C}_{uv}$) is diagonal, the phases for all channels $\vec{\omega}$ will be between $\phi_{\text{HH}}$ and $\phi_{\text{VV}}$, i.e. $m - 1$ for

$$
m = \frac{\phi_{\text{HH}} - \phi_{\text{VV}}}{p} - \frac{\phi_{\text{CP}}}{p}
$$

(12)
where \( p \) is the maximum phase spread of the coherence region and \( \phi_{cp} \) defines the phase difference between HH and VV. In general, however, \( \overline{C}^{u,v} \) will not be diagonal, so that \( m < 1 \). The deviations from diagonality can be quantified by the sum of the absolute values of the correlation coefficients of \( \overline{C}^{u,v} \):

\[
\alpha = \frac{1}{2} \sum_{i \neq j} \left| \frac{C_{i,j}^{u,v}}{\sqrt{C_{i,i}^{u,v} C_{j,j}^{u,v}}} \right|
\]

(13)

where \( c_{i,j}^{u,v} \) is the \((i,j)\) element of \( \overline{C}^{u,v} \). In the presence of birefringence, one thus expects \( m \approx 1 \) if \( \alpha \ll 3 \).

The co-polarized phase \( \phi_{cp} \) is expected to be related to changes in the biomass \( \Delta b \), as models and observations of birefringence [14] suggest that the polarization difference of the propagation phase is proportional to the biomass. This is illustrated in Fig. 1, where the height of the medium represents the biomass, and the propagation phase for a fixed polarization is proportional to the number of wave fronts. For a given acquisition, the phase difference between polarizations A and B of a point at the bottom of the medium is proportional to the biomass; thus the interferometric phase between acquisitions 1 and 2 for this scatterer is proportional to \( \Delta b \). Under these assumptions, \( \phi_{cp} \) is expected to be related to changes in biomass between acquisition \( \Delta b \) and independent of the mean biomass \( \Gamma b \). Also the underlying soil return may contribute to \( \phi_{cp} \), e.g. due to soil moisture changes. Empirical analyses at L-band indicate that this effect is small compared to the influence of vegetation changes [25]. We thus do not consider them explicitly.

### 3 Observational analysis

#### 3.1 Data

The AGRISAR 2006 [26] campaign covers one growing season over a flat agricultural site around Görmin, Mecklenburg-Western Pomerania, Germany (53°58’N 13°16’E). The scene is dominated by fields of different crops (winter wheat, maize, rape, sugar beet, and barley), although several settlements and small forest patches are also present. The topography is flat, with a slight slope towards a nearby river and several drainage features perpendicular to the latter [27].

The airborne L band (\( \lambda = 0.23 \) m) SAR data, which were acquired by DLR’s E-SAR system [26] in intervals of one to two weeks, have a single look resolution of about 2 m in slant range and 1 m in azimuth [26]. The 11 images used in this study are denoted by the day of year (DOY) of their acquisitions. They were recorded from the same track at a nominal baseline of 0 m. Deviations from this zero-baseline scenario introduce a height dependence of the phase \( \phi \): the latter changes with height \( \delta z \), to first order, as \( \delta \phi = k_z \delta z \). The values of [\( k_z \)] in the data set are generally less than 0.05 rad m\(^{-1}\). For a canopy height of 1 m and \( k_z \approx 0.05 \) m\(^{-1}\), dominant scattering at the bottom in polarization \( \omega_b \) and dominant scattering at the top in polarization \( \omega_t \) would thus introduce a phase diversity between \( \omega_b \) and \( \omega_t \) of 0.05 (3°), which is considerably smaller than the observed differences. The impact of \( k_z \neq 0 \) will thus be neglected.

In addition to the SAR images, in-situ data of soil moisture, vegetation height and wet biomass were acquired in 8 fields, an overview of which can be found in Table 1. The properties of these in-situ data are summarized in Table 2. Within each field, the volumetric soil moisture \( \eta \) [m\(^3\)m\(^{-3}\)] was measured at three locations (by Time Domain Reflectometers (TDR); 0-5 cm depth) close to the time of the radar acquisitions and subsequently averaged. Such averaging was also performed for the wet biomass \( b \) [kg m\(^{-2}\)] and vegetation height \( h \) [m]; the former was estimated by clearing 1 m\(^2\) and weighing. This sampling of the in-situ measurements introduces scale differences with respect to the SAR images. These are expected to be more pronounced in fields that exhibit pronounced heterogeneity in the interferometric data, such as the maize field 222 analysed in Sec. 3.3. The temporal evolution of the measured biomass \( b \) for different kinds of crops is shown in Fig. 4. It shows a general increase in the biomass in the beginning of the growing season, which differs for the various crop types. Several crops, such as rape, wheat and barely, lose biomass towards the end of their growth cycle as they become increasingly dry.

#### 3.2 Methods

The interferograms are obtained by cross-multiplication of the co-registered SAR acquisitions and subsequent averaging by a rectangular filter (10 pixels in range, 50 in azimuth). This filter corresponds to a cell with approximately equal extension in ground range and azimuth due to unequal oversampling. Owing to the oversampling [28], each sample corresponds to 236 independent looks. For this number of looks, the standard error of the interferometric phase for \( \gamma \approx 0.5 \), which is a typically observed value, is less than 5° [29]. All the samples within any field (conspicuous heterogeneities such as houses were excluded; see Table 1 for an overview of the number of samples) are included in the analysis. For each sample the statistics (e.g. a correlation coefficient with biomass changes) are computed for the set of available interferograms, and subsequently the frequencies of these statistics over all the samples within one field. The separate computation of the statistics for different positions within a field is intended to account for the effect of the spatial variations observed in the data. By considering all available interferograms at once, the analysis focusses on the dominant temporal patterns observed over different crops.
Table 1: List of fields for which ground data are available along with additional information: the crop grown, the mean incidence angle $\theta$ and the number of samples used when computing the statistics.

<table>
<thead>
<tr>
<th>Number</th>
<th>Crop</th>
<th>$\theta$ [°]</th>
<th>Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>rape (r)</td>
<td>50</td>
<td>1462</td>
</tr>
<tr>
<td>102</td>
<td>sugar beet (sb)</td>
<td>51</td>
<td>251</td>
</tr>
<tr>
<td>140</td>
<td>rape (r)</td>
<td>28</td>
<td>640</td>
</tr>
<tr>
<td>222</td>
<td>maize (m)</td>
<td>36</td>
<td>914</td>
</tr>
<tr>
<td>230</td>
<td>wheat (w)</td>
<td>47</td>
<td>851</td>
</tr>
<tr>
<td>250</td>
<td>wheat (w)</td>
<td>46</td>
<td>767</td>
</tr>
<tr>
<td>440</td>
<td>barley (b)</td>
<td>38</td>
<td>276</td>
</tr>
<tr>
<td>460</td>
<td>sugar beet (sb)</td>
<td>47</td>
<td>541</td>
</tr>
</tbody>
</table>

Figure 4: Temporal evolution of the measured biomass $b$ for 5 different fields: 101 (rape), 102 (sugar beet), 222 (maize), 230 (wheat), and 440 (barley).

The statistics to be considered are summarized in Table 3. The median and median absolute deviation (MAD) of $l$ of (11) are taken over all the interferograms for each pixel: they are denoted by $l_m$ and $l_d$, respectively. The median and MAD are robust measures of the location and spread – the impact of unrepresentative acquisitions or interferograms is minimized by this choice of robust statistics. These are also applied to the $m$ and $o$ statistics of (12) and (13). Values of $l$ and $m$ are not considered in the computations of these statistics if the phase spread $p = 2\pi$ (i.e. the coherence region overlaps the origin); cf. the restrictions on the predictions of $l$ and $m$ in Sec. 2.2.2.

This phase spread $p$ as well as $\phi_{cp}$ are also of interest in their own right, as they are predicted to be related to changes in biomass if birefringence effects dominate. This dependence can be analysed using partial correlation coefficients $P_{z_1,\ldots,z_N}(x,y)$ [30], where $y$ is the radar observable ($p$ or $\phi_{cp}$) and $x$ is the vegetation parameter (the mean biomass $b$ or the difference $\Delta b$). These partial correlations control for other parameters $z_1,\ldots,z_N$; they express the correlation between those components of $x$ and $y$ that are not explainable by the controlling variables $z_1,\ldots,z_N$. For estimation purposes, these components are taken to be the residuals $r_x$ ($r_y$) of the least-squares regression of $x$ ($y$) versus $z_1,\ldots,z_N$.

In our case, these controlling variables are the other vegetation parameter (e.g. the mean biomass when the parameter of interest is $\Delta b$) and the mean and difference in soil moisture, $\Gamma_{m_v}$ and $\Delta m_v$, respectively. The inclusion of $m_v$ statistics is motivated by the observation that changes in soil moisture influence the DInSAR observables [31, 32]. Previous analyses indicate that the soil moisture dependence of $\phi_{cp}$ is approximately linear and that the size of the effect is smaller than the observed phase spreads [33, 34, 25].

These partial correlation coefficients express the degree of linear dependence; in order to get an estimate of the size of this dependence, we also compute least-square regression coefficients $\beta_{z_1,\ldots,z_N}(y;x)$. The underlying model has the

Table 2: Overview of the in-situ measurements, all of which are averages of three measurements within each field.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>volumetric soil moisture</td>
<td>$m_v$</td>
<td>m$^3$m$^{-3}$</td>
<td>Time Domain Reflectometers (TDR); 0-5 cm depth</td>
</tr>
<tr>
<td>vegetation height</td>
<td>$h$</td>
<td>m</td>
<td>manual measurement</td>
</tr>
<tr>
<td>wet biomass</td>
<td>$b$</td>
<td>kg m$^{-2}$</td>
<td>destructive sampling of 1 m$^2$ and weighing</td>
</tr>
</tbody>
</table>
Table 3: Overview of the statistics used in this study. These are computed for each field based on all samples and all the possible interferograms.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_m$</td>
<td>Median of $l$ statistic</td>
<td>see (11)</td>
</tr>
<tr>
<td>$l_d$</td>
<td>MAD of $l$ statistic</td>
<td>see (11)</td>
</tr>
<tr>
<td>$m_m$</td>
<td>Median of $m$ statistic</td>
<td>see (12)</td>
</tr>
<tr>
<td>$m_d$</td>
<td>MAD of $m$ statistic</td>
<td>see (12)</td>
</tr>
<tr>
<td>$o_m$</td>
<td>Median of $o$ statistic</td>
<td>see (13)</td>
</tr>
<tr>
<td>$o_d$</td>
<td>MAD of $o$ statistic</td>
<td>see (13)</td>
</tr>
<tr>
<td>$P_{\Gamma,h,\Delta m_v,\Gamma m_v} (\phi_{cp}, \Delta b)$</td>
<td>partial correlation of $\phi_{cp}$ with $\Delta b$</td>
<td>control for $\Gamma b$, $\Delta m_v$, $\Gamma m_v$</td>
</tr>
<tr>
<td>$P_{\Delta b,\Delta m_v,\Gamma m_v} (\phi_{cp}, \Gamma b)$</td>
<td>partial correlation of $\phi_{cp}$ with $\Gamma b$</td>
<td>control for $\Delta b$, $\Delta m_v$, $\Gamma m_v$</td>
</tr>
<tr>
<td>$P_{\Gamma h,\Delta m_v,\Gamma m_v} (\phi_{cp}, \Delta h)$</td>
<td>partial correlation of $\phi_{cp}$ with $\Delta h$</td>
<td>control for $\Gamma h$, $\Delta m_v$, $\Gamma m_v$</td>
</tr>
<tr>
<td>$\beta_{\Gamma h,\Delta m_v,\Gamma m_v} (\phi_{cp}, \Delta h)$</td>
<td>slope of $\phi_{cp}$ vs. $\Delta b$</td>
<td>control for $\Gamma h$, $\Delta m_v$, $\Gamma m_v$</td>
</tr>
</tbody>
</table>

Figure 5: Coherence regions for samples within four different fields based on images at DOY 130 and 171. The symbols denote $\gamma$ at HH (purple diamond), HV (blue triangle), and VV (orange square).

3.3 Results

3.3.1 Exploratory data analysis

The observed polarimetric diversity is exemplified by the coherence regions of samples in four different fields in Fig. 5. The first two panels (wheat fields) show HH and VV coherences falling on the border of the coherence set. The associated values of $m$ are close to 1, and the extreme values of the phase are associated with lower $|\gamma|$. The HV coherences are either similar or smaller in magnitude, with the phases being halfway between HH and VV ($l_{0.5}$). The maize field in Fig. 5c, whose coherences in all polarimetric channels are less than 0.5, does not exhibit such a clear pattern. Neither does the barley field in Fig. 5d.

The observed relation between $\phi_{cp}$ (fixing the first acquisition at DOY 130) and the biomass change is plotted for the two wheat and the maize fields in Fig. 6. For the wheat fields, there is a clear relation between these two quantities, but not for the maize field.

Polarimetric phase diversity is also evident in the spatial representations of Fig. 7. $\phi_{cp}$ in sub-figure a) reaches values close to $\pm \pi$. Jumps in $\phi_{cp}$ clearly correspond to field boundaries. Also the spatial heterogeneity varies between the fields: for instance, the sugar beet fields 102 and 460 are less homogeneous than the wheat fields 230 and 250. For these wheat fields the estimated displacement difference between HH and VV is on the order of 3 cm. The phase difference between HH and the optimized $\vec{\omega}$ (at which the coherence magnitude is maximized) behaves similarly in Fig. 7 b). Apart from the correspondence to the crop type, these two phases also appear to be related to the incidence angle $\theta$. They tend to be smallest in magnitude at near-range and to increase with incidence angle $\theta$.

3.3.2 Quantitative statistics

The homogeneity and the size of the quantities of Table 3 are reflected in the observed frequencies of these statistics. Those of the medians and MAD of the $l$ feature are plotted for each field in Fig. 8. The values of $l_m$ cluster around the predicted value 0.5. The variations are however only $< 1$ for fields 230, 250 and 440 (with accompanying small values
Figure 6: Time series of the HH-VV interferometric phase difference $\phi_{cp}$ and the change in biomass, both referenced to the image at DOY 130. The locations within each of the three fields correspond to those in Fig. 6.

(a) Field 250, wheat
(b) Field 230, wheat
(c) Field 222, maize

Figure 7: Spatial depiction of the extent of the coherence region of images at DOY 130 and 171, along with an overview of the fields.

<table>
<thead>
<tr>
<th>$l_d$</th>
<th>$m_d$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

of $l_d \ll 1$), whereas the other fields exhibit larger heterogeneity both within and between the samples. The rape field 101 constitutes an intermediate case. Note that a clustering of $l_m$ can also arise in the presence of a large spread of $l$ between the different interferograms: in this case $l_d$ is elevated, as in field 102. The medians of the $m$ statistic in Fig. 9, which is expected to be close to 1 in case of birefringence and $\alpha \ll 3$, attain high levels $\approx 1$ for field 250 in particular, but also in fields 230 and 440 (Fig. 10). These fields are characterized by $m_d < 0.3$ and by values of $\alpha \approx 1$. The remaining fields tend to be heterogeneous in $m_m$ with $\alpha \gg 0$ and subject to larger values of $m_d > 0.25$.

The partial correlations of $\phi_{cp}$ with changes in biomass $\Delta b$ in Fig. 11 are markedly different from zero and negative for fields 101, 140, 230, 250 and 440. They are predominantly positive in the sugar beet field 102, and mostly concentrated around 0 in fields 222 (maize) and 460 (sugar beet). The diverse behaviour of the sugar beet fields 102 and 460, which
are observed at similar incidence angles, is also evident in the correlations with mean biomass in Fig. 11: 102 is the only one whose values are not centered around 0.

The connection of \( \phi_{cp} \) with \( \Gamma b \) is weak for all fields except 102, for which negative partial correlation coefficients \( P_{\Delta b, \Delta m_p, \Gamma m_p}(\phi_{cp}, \Gamma b) \) prevail.

4 Discussion

4.1 Birefringence

4.1.1 Plausibility

The structure of the coherence region implied by the birefringence hypothesis, namely \( l \approx 0.5 \) and \( m \gg 0 \) for \( o \ll 3 \), matches the observations over the wheat (230, 250) and barley (440) fields closely. Its relation to the biomass, i.e. non-zero correlation between \( \Delta b \) and \( \phi_{cp} \), as well as vanishing partial correlation of these features with \( \Gamma b \), is also in line with the data for these same fields.

The expected impact of biomass (\( \Delta b \) and \( \Gamma b \)) on \( \phi_{cp} \) is also observed for the rape fields 101 and 140. The former is generally more heterogeneous (temporally: in terms of \( l_d \) and \( m_d \); spatially in terms of the spread of \( l_m \) and \( m_m \)) than the wheat and barley fields. Despite this variability, the values of \( l_m \approx 0.5 \) and \( m_m \approx 0.6 \) are consistent with birefringence as \( o_m \approx 1.5 \). The rape field 140 is characterized by \( l_m \approx 0.5 \), but the large values of \( l_d > 1 \) indicate the presence of additional influences on the phase diversity, such as those introduced in 2.2.2. However, the birefringence model underpinning this interpretation is based on a number of assumptions, e.g. that the dominant part of the coherent scattering is at ground level. The observations may also be affected by any violation of these additional assumptions, e.g. due to coherent volume scattering within the rape canopy.

The maize field 222 exhibits \( l \) values with considerable dispersion \( > 1 \) in MAD, albeit centred around 0.5. As \( o_m \approx 2 \), the birefringence hypothesis cannot make predictions about \( m \). The correlations with \( \Gamma b \) are weak, whereas that with \( \Delta b \) are positive for less than 70\% of the samples within this field. These findings do not point towards birefringence as a dominant mechanism. In addition the low coherence magnitudes for this field (e.g. \( \ll 0.5 \) for a temporal baseline of 6 weeks) render the evaluation of the birefringence hypothesis challenging.

The coherence regions of the remaining sugar beet fields (102, 460), where the vegetation is not predominantly vertically oriented, exhibit \( o \) values \( > 2 \) and are characterized by large variations in \( l \), albeit centered around 0.5. The copolar phase \( \phi_{cp} \) is related to \( \Delta b \) for the field 102, but not for the other sugar beet field 460. The mean biomass \( \Gamma b \) appears to be
related to $\phi_{cp}$ only for field 102. We will address possible explanations for this behaviour in Sec. 4.2, as the observations over these fields are inconsistent with the birefringence hypothesis.

4.1.2 Quantitative description

Birefringence and other propagation properties such as dichroism within a volume of discrete particles (assumed dielectrically isotropic) are governed by the interference of the incident wave with the scattered contributions from the particles [20].

Assuming Foldy’s approximation [22], the propagation properties of the medium arise from the interference of the forward scattering contribution from the particles with the incident wave, see Fig. 3. Ulaby et al. [14] suggested to model the associated $\overline{N}$ matrix as

$$\overline{N} = i 2 \eta k_0^{-2} \overline{S}_f$$

(15)

where $\overline{S}_f$ is the forward scattering matrix of a single scatterer as defined by [35]. The canopy is characterized by $\eta \left[ \text{m}^{-2} \right]$ scatterers per unit area. These are taken to be vertical dielectric cylinders in the model by [14]. They derived predictions for the two-way transmission of the wave, i.e. $\overline{P}$. The total propagation phase difference $\varphi_p = \varphi_{HH} - \varphi_{VV}$ of a scatterer at ground level is proportional to the vegetation height $h$, i.e. $\varphi_p = w h$. The modelled $w$ are shown to be sensitive to the cylinder radius $a$ for $a \ll \lambda$ in Fig. 12. Note that they are linear in the number density $\eta$, and that they also depend on the dielectric constant $\varepsilon_r$. The proportionality factor $w$ further depends on the incidence angle $\theta$, as the propagation path length increases with $\theta$. In interferometry (assuming that all the coherent scatterers are at ground level and constant $\varepsilon_r$) $\varphi_{p,\text{\scriptsize{\Delta h}}} = \varphi_{p}^{\text{\scriptsize{\Delta h}}} = \varphi_p - w (h^u - h^v) = -w \Delta h$, where the minus sign is due to the different conventions of forming differences/interferograms.

The model predicts that $w$ increases with incidence angle for fixed canopy properties (e.g. $\Delta h$). A corresponding increase in $\phi_p$ is evident in Fig. 6 within field 230. The differences between the two rape fields 101 (far range) and 140 (near range) are also consistent with this prediction. For fields where in-situ height information is available, $w$ can be estimated using regression techniques. It is given by $\dot{w} = \beta_p \Delta m_{\text{\scriptsize{\Delta h}}} (\phi_{cp}; \Delta h)$ of table 3. The results for each field are summarized in Table 4. For fields 230, 250, and 440 this table also contains model estimates $w_m$ using representative values for wheat/barley [36, 37, 38]: $\eta = 300 \text{m}^{-2}$, $\varepsilon_r = 6 - i 0.5$ and $a = 0.002 \text{m}$. These estimates are consistent in sign with the inferred ones. Also the flip in sign expected for the maize field due to the much larger radius of the stems based on Fig. 12 is consistent with the empirical results in Fig. 11. Returning to the maize and barely fields, we note that the simulated $w_m$ are too small by about a factor of 10 on average.

Figure 9: Observed frequencies of the median and MAD of the $m$ statistic (12), which equals one if the maximum phase spread equals $\phi_{cp}$; see Fig. 8 for an explanation of the symbols.
Figure 10: Observed frequencies of the median and MAD of the $o$ statistic (13); see Fig. 8 for an explanation of the symbols.

Table 4: Overview of the $w$ statistics estimated from the data $\hat{w}$ or modelled according to [14] $w_m$. $\hat{w}_{0.25}$, $\hat{w}_{0.5}$, and $\hat{w}_{0.75}$ are the first quartile, median, and third quartile of the estimates within a given field, respectively. See table 1 for a description of the fields and the crop types.

<table>
<thead>
<tr>
<th>Field</th>
<th>Crop</th>
<th>$\hat{w}_{0.25}$</th>
<th>$\hat{w}_{0.5}$</th>
<th>$\hat{w}_{0.75}$</th>
<th>$w_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>r</td>
<td>-0.728</td>
<td>0.298</td>
<td>1.347</td>
<td>-</td>
</tr>
<tr>
<td>102</td>
<td>sb</td>
<td>-3.975</td>
<td>-1.052</td>
<td>1.363</td>
<td>-</td>
</tr>
<tr>
<td>140</td>
<td>r</td>
<td>0.534</td>
<td>0.667</td>
<td>0.818</td>
<td>-</td>
</tr>
<tr>
<td>222</td>
<td>m</td>
<td>-0.316</td>
<td>-0.179</td>
<td>-0.041</td>
<td>-</td>
</tr>
<tr>
<td>230</td>
<td>w</td>
<td>2.240</td>
<td>2.803</td>
<td>3.134</td>
<td>0.293</td>
</tr>
<tr>
<td>250</td>
<td>w</td>
<td>0.244</td>
<td>0.823</td>
<td>1.734</td>
<td>0.277</td>
</tr>
<tr>
<td>440</td>
<td>b</td>
<td>1.977</td>
<td>2.291</td>
<td>2.563</td>
<td>0.179</td>
</tr>
<tr>
<td>460</td>
<td>sb</td>
<td>-2.966</td>
<td>-0.787</td>
<td>1.344</td>
<td>-</td>
</tr>
</tbody>
</table>

This discrepancy may be due to the inadequacy of the model or the analysis, or the choice of parameters. In the latter case, the most crucial parameter is the radius $a$, which varies in a non-linear fashion (with fluctuations in both sign and magnitude, cf. Fig. 12). The number density $\eta$ impacts on $w_m$ in a linear fashion (i.e. $\eta$ would have to increase by a factor of 10 to mimic the observations), whereas the dependence on $\varepsilon_p$ is more complex. Appropriate ground data or regional reference values are, however, not available. This absence also impacts the estimation of $\hat{w}$ and the simulation of $w_m$ indirectly, as it forces us to assume constant values of $\eta$, $a$, and $\varepsilon_p$. This simplification – which concerns the other possible source of mismatch, i.e. modelling assumptions – is not expected to be valid, as e.g. $\varepsilon_p$ is known to have varied for the observed fields due to increasing desiccation of the plants [12]. We also made the simplification that only scatterers at ground level contribute, i.e. we have neglected volume scattering from the canopy. Volume scattering has been shown by [39, 40] to influence the polarimetric (not interferometric) HH-VV phase difference and coherence. Their model for tropical forest canopies may in the future be extended to agricultural crops and differential interferometry. In particular, the dependence of the phase diversity on the incidence angle is expected to be a useful indicator of volume scattering. Ground-based experiments where the canopy is observed from a range of incidence angles [41] may provide insight into the importance of volume scattering.

Further assumptions that cannot easily be circumvented without additional information concern the representation of the plants (vertical homogeneous cylinders) and the computation of $\overline{N}$ (Foldy’s approximation). To our knowledge, the birefringence implied by these assumptions and thus the model by [14], have not been directly compared to observations.
before, apart from one study over a maize field [23]. In that study, the predictions of a slightly more general model matched the measured absorption at H and V polarizations reasonably well within a range of frequencies. They were also consistent with the observed birefringence at L band; however, the scatter of the latter was so large that the null hypothesis of no birefringence could not be rejected.

Despite the existence of models such as the one described above, and the commonly held belief that birefringence is present in vegetation canopies at microwave frequencies [17, 10], observations are very scarce. [23] inferred the presence of birefringence of a maize field in C band, whereas the results at L band were inconclusive. For a pine canopy at 1.6 GHz, [42] inferred that there was no significant anisotropy in the propagation properties, i.e. neither birefringence nor dichroism. The latter phenomenon has been studied empirically somewhat more intensively [43], using both direct measurements of transmission [44] and indirect ones based on scattering [45] or emission [46] measurements. Concerning agricultural crops at L band, [23] found significant differences in absorption between H and V in a maize field, and [44] observed dichroism in wheat and soybeans. Using passive observations, dichroism has been reported by [46] in, amongst

Figure 11: Observed frequencies of the partial correlation coefficients of \( \phi_{cp} \) with \( \Delta b \) and \( \Gamma b \); see Fig. 8 for an explanation of the symbols.

Figure 12: The dependence of \( w_m \) in the model by [14] on \( \frac{\omega}{\lambda} \) for \( \eta = 300m^{-2}, \varepsilon_{p} = 6-i0.5, \) and \( \theta = 40^\circ \).
others, wheat and grass. By contrast, [47] found no significant polarization difference in maize and soybeans, albeit at an incidence angle of 10 degrees.

4.2 Other explanations

Apart from birefringence, several other possible origins of phase diversity can be deduced from the first-order scattering model. Among those mentioned in Sec. 2.2, we will address i) polarimetric diversity in the scattering phase, ii) dichroism, iii) the differential movement of dominant scatterers, and iv) noise-like decorrelation.

If scattering phase effects i) dominate, one cannot obtain any predictions about $l$ or $m$ without additional assumptions about the origins of these effects. The associated changes in the polarimetric diversity of the scattering phase, however, are in general not expected to be directly related to the total biomass or changes therein, as they are local in origin, e.g. occurring at the leaf scale. Such a relation, inferred from non-zero $P_{b}(\phi_{cp})$, is, however, commonly observed in all fields except 460. On the other hand, the partial correlations may be influenced by confounding variables, such as the growth stage, which can provide a link between the scattering phase and changes in biomass.

Dichroism ii) can modulate (as a function of polarization) the sensitivity with respect to the depth of the scatterer. Depending on the position within the canopy, the scatterers must have different $\varphi_{j}$ due to differential movements – polarization-independent – or scattering effects – possibly polarization-dependent. However, similarly to the previous hypothesis, neither the movements nor the scattering effects are expected to be related to changes in biomass or the average biomass in a simple fashion. We thus expect only low correlations between these quantities and the $\phi_{cp}$. This is not consistent with the observations except for field 460.

The third scenario, the differential movement of dominant scatterers, implies that scatterers are displaced in a systematic way depending on their dominant scattering polarization (which governs the polarization dependence of $\alpha_{j}$ in (8)). These hypothetical movements are presumably not directly related to the mean biomass or changes therein, which is at odds with the $P_{b}(\phi_{cp})$ observed in all fields except 460.

When noise-like decorrelation iv) dominates, the phase diversity does not arise from polarimetric differences in the underlying expected value, but rather due to the imprecision of the estimates. As the precision of the phase measurements decreases with the coherence [29] – and changes in biomass have been reported to cause decorrelation [25]– one might expect a correlation between the phase spread of the coherence regions $\gamma$ and changes in biomass if the latter are always of the same sign. However, one also expects that both positive and negative signs of $\phi_{cp}$ will be equally likely (with the magnitude depending on $\Delta b$). $P_{b}(\phi_{cp})$ will thus be close to zero: empirically, this is only found in field 460. For this field, these results and the low coherences generally observed suggest that decorrelation is a relevant factor in the observed phase diversity.

The remaining fields, where birefringence or noise-effects are not consistent with the observations, suggest the presence of complex and possibly multiple origins for the phase diversity. None of the simple scenarios considered is able to explain the observed dependence of $\phi_{cp}$ on the mean biomass $\Gamma b$ for field 102, for instance. However, a single vegetation parameter such as the biomass cannot capture all the physical changes of the vegetation canopy. Rainfall interception and dew, for example, may influence the DInSAR observables but the extent and mechanisms have yet to be explored. By contrast, soil moisture changes have already been shown to influence the DInSAR observables [31, 25, 32]. The partial correlations $P$ account for soil moisture statistically based on in-situ soil moisture observations. There may, however, still be residual soil moisture effects, e.g. due to measurement errors or spatial variability. The parameters $l$ and $m$ may also be affected even though [25] found that the polarimetric DInSAR diversity was dominated by changes in vegetation biomass rather than soil moisture. Furthermore all scenarios are based on the assumption of first-order scattering. Different models (e.g. including multiple scattering) may have to be invoked to account for the diversity in patterns observed in the data.

4.3 Relevance for estimation of displacements

Irrespective of the origin of the observed polarimetric diversities of the phase, there arises the question whether these are relevant for the estimation of deformations by differential interferometry. The observed phase differences between the HH and VV channel – the most commonly employed ones for single-polarization studies – can exceed $0.5\pi$ ($0.25\pi$) for a temporal baseline of 6 (2) weeks (see Fig. 5 and 7). Similar magnitudes $\approx 0.5\pi$ are also found for the phase differences between the HH channel and the one with maximum coherence. A phase value $\phi - 0.5\pi$ corresponds to a deformation of $\approx 3$ cm at L band: such a value is an order of magnitude larger than the observed phase noise of the L band interferometric satellite ALOS [48]. The associated deformation rate of $26$ cm yr$^{-1}$ (corresponding to a temporal baseline of 6 weeks) also exceeds commonly studied deformation processes by a similar factor [49].

These phase values, as well as the corresponding deformations, only pertain to differences between two polarizations. Unless one of these two channels is not affected by the presence of vegetation, the deformations estimated using either one will be erroneous. Such a vegetation impact has been inferred at L band at both HH and VV channels [25]: the sign of these influences was the same for these channels for e.g. wheat and rape. This implies that the ‘actual’ deformation will not be between those corresponding to the two channels, but rather smaller or larger than either. Such a behaviour is also predicted by the birefringence model of [23] for thin stems ($\frac{\lambda}{b} < 0.1$).
These vegetation effects will only influence the final deformation product if the corresponding areas are included in the processing. The inclusion of distributed targets in typical processing chains can be determined by the value of the absolute coherence $|\gamma|$, with only those exceeding a certain threshold being considered [50]. The values of these thresholds depend on the processing algorithm employed; large spreads from 0.25 to 0.95 are reported in the literature [51, 52, 53]. The observed values for e.g. wheat fall in this range ($\approx 0.5$, see. Fig. 5) for most of the growing season. As the time separation increases, in particular after harvest or tilling, the coherence drops to increasingly lower values. Both the choice of threshold and the study period will correspondingly be important in determining whether any particular field will be included in the DInSAR analysis. On the other hand, the systematic impact of vegetation changes on the phase diversity suggests the possible benefit of additional screening tests besides the coherence: when multi-polarimetric data are available, the interferometric analysis could, for example, be restricted to regions where the magnitude of $\phi_{cp}$ is sufficiently small.

5 Conclusions

The polarimetric zero-baseline DInSAR phase diversity, which had hitherto not been systematically explored, has been found to depend on the crop. At L-band the phase diversity of wheat, barley and, to a lesser extent, rape fields appears to be governed by birefringence within the canopy. These inferences are drawn based on theoretical studies of first-order scattering. We proposed several features that can be derived from the DInSAR coherence regions and for which quantitative predictions can be made for a vegetation canopy whose DInSAR response is dominated by birefringence. This property, however, cannot well explain the observations over maize and sugar beet, and neither can similarly simple mechanisms, such as dichroism or differential movement of scatterers within the vegetation canopy. The DInSAR scattering properties of these crops, as well as more generally those at frequencies besides L-band, are thus an interesting question for future studies. Similarly, the effects of morphological and phenological changes deserve attention, as this study has only been able to address the relation of the bulk biomass to the phase diversity.

The observed phase diversity, irrespective of its presumed origin, may be considered in deformation studies over agricultural fields. At lower frequencies such as L-band, the coherences can remain comparatively high even when vegetation growth leads to significant phase differences between polarizations. As some of the studied features of the phase diversity can show a clear correspondence to vegetation changes, they might be used to exclude areas where these vegetation effects increase the uncertainty of the deformation estimates, or even possibly bias them systematically. The presence or size of such a bias has not been analysed in this study (only polarization differences) and thus remains an important open question. These polarization differences, however, already imply that there is a range of displacements that correspond to the observations. The lack of uniqueness puts emphasis on the estimation algorithm, the choice of which will impact the retrieved displacements. This choice may thus also influence the inferred spatial patterns, their magnitude and consequently also the way the deformation estimates can be related to geophysical phenomena such as post- or inter-seismic creep, groundwater-related subsidence or hill-slope mass movements.

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References


Chapter D

A polarimetric first-order model of soil moisture effects on the DInSAR coherence

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Key findings:
- surface and subsurface scattering combined in a model based on Maxwell’s equations
- model can explain variable sign/magnitude of soil moisture effects on three observables
- although fully polarimetric, it does not apply to the HV channel

The author’s contributions:
- developed the idea and derived the analytical expressions
- calibrated and validated the model using airborne measurements
- interpreted the results and wrote the manuscript

The co-authors’ contributions:
- S.H. provided radar data and helped with analysing the modelling assumptions/implications
- I.H. interpreted the results and helped with analysing the modelling assumptions/implications
- both co-authors contributed to writing the manuscript
A polarimetric first-order model of soil moisture effects on the DInSAR coherence

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Abstract

Changes in soil moisture between two radar acquisitions can impact the observed coherence in differential interferometry: both coherence magnitude $|\gamma|$ and phase $\phi$ are affected. The influence on the latter potentially biases the estimation of deformations. These effects have been found to be variable in magnitude and sign, as well as dependent on polarization, as opposed to predictions by existing models. Such diversity can be explained when the soil is modelled as a half-space with spatially varying dielectric properties and a rough interface. The first-order perturbative solution achieves – upon calibration with airborne L band data – median correlations $\rho$ at HH polarization of 0.77 for the phase $\phi$, of 0.50 for $|\gamma|$, and for the phase triplets $\Xi$ of 0.56. The predictions are sensitive to the choice of dielectric mixing model, in particular the absorptive properties; the differences between the mixing models are found to be partially compensatable by varying the relative importance of surface and volume scattering. However, for half of the agricultural fields the Hallikainen mixing model cannot reproduce the observed sensitivities of the phase to soil moisture. In addition, the first-order expansion does not predict any impact on the HV coherence, which is however empirically found to display similar sensitivities to soil moisture as the co-pol channels HH and VV. These results indicate that the first-order solution, while not able to reproduce all observed phenomena, can capture some of the more salient patterns of the effect of soil moisture changes on the HH and VV DInSAR signals. Hence it may prove useful in separating the deformations from the moisture signals, thus yielding improved displacement estimates or new ways for inferring soil moisture.

1 Introduction

The techniques repeat pass radar interferometry and differential interferometric SAR (DInSAR) are routinely applied to derive deformations of the Earth’s surface, digital elevation models or vegetation heights (Bamler and Hartl, 1998). A possible influence of soil moisture changes on the observables in DInSAR has been surmised for decades (Gabriel et al., 1989). Such an impact can impair the estimation of the aforementioned parameters but might also open the possibility for soil moisture retrieval by this technique. Since the first conjecture of the presence of such effects on the phase $\phi$ and the coherence $|\gamma|$ (Gabriel et al., 1989), several explanations about the origins have been advanced, and both observational studies (e.g. Nolan (2003), Barrett et al. (2013)) and dedicated experiments (e.g. by Rudant et al. (1996); Morrison et al. (2011)) have been conducted. The empirical evidence tended to be inconclusive and some results were inconsistent. This lack of agreement might be due to different study designs – e.g. radar wavelength and soil properties – and shows that the prevalence and size of these effects on the observables remains poorly understood.

This knowledge gap also encompasses the predictive power of the proposed explanations. Their applicability has recently been analysed empirically at L band by Zwieback et al. (2015): for the two airborne campaigns studied, the inferred dependence of $\phi$ on soil moisture changes $\Delta m$ was not consistent with the penetration depth hypothesis (Nolan, 2003) or soil swelling, but only with a dielectric volume scattering mechanism. More than 20 years ago the possibility of volume scattering within the soil was considered by Gabriel et al. (1989), but they preferred to attribute the observed phase patterns to deformations as the soils studied were prone to swelling; the observed sign of $\phi$ was mostly consistent with such swelling. The sign of the phase change was also studied by Rudant et al. (1996) in two laboratory experiments: they interpreted the phase signal not due to deformation as one of dielectric origin. A numerical model of a heterogeneous soil (which gives rise to scattering) based directly on Maxwell’s equations predicts this sign of the phase dependence (Rabus et al., 2010), as does the analytical approach by De Zan et al. (2014). They considered volume scatterers embedded in a soil bounded by a smooth surface: in the first-order scattering solution the scattered field could be expressed as the superposition of the return from all the particles, the phase and magnitude of each depending on its position and the bulk dielectric constant of the soil. The complex coherence $\gamma$ followed by ensemble averaging assuming no correlation between the returns from different particles; it did not depend on the polarization.

At higher frequencies the opposite sign of the phase dependence has been observed in laboratory experiments. The homogeneous samples of soil (S-band measurements by Yin et al. (2014)) and pure sand (C-band measurements by Morrison et al. (2011)) were put into containers and the surface was smoothed. The observed phase dependence on soil
moisture was found to be consistent with the Small Perturbation Model by Yin et al. (2014). That is, the signals could be explained by scattering from a slightly rough surface.

As both surface scattering and sub-surface volume scattering seem the most promising hypotheses for explaining the effects on $\gamma$ for different non-swelling soils and radar frequencies, we want to determine whether these can be combined in an analytical model based directly on Maxwell’s equations Jackson (1999). The relative importance of the two scattering contributions is intended to account for the variations in sign and magnitude of the observed soil moisture effects. To this end, we propose to extend De Zan et al. (2014) and model the soil in a first order Born approximation, where the zero-order scenario corresponds to a flat interface and a homogeneous soil. The first order expansion incorporates the roughness of the surface and spatial variations in permittivity. The resulting parametric solutions allow us to make predictions about the DInSAR observables – in particular the phase $\phi$ – for all polarizations, and compare them to observations. The evaluation of the proposed model focusses on the consistency of the predictions for different observables and polarizations, as well as the patterns in the data that the model cannot explain.

2 Proposed Model

The proposed model combines scattering from two origins, i.e. spatial variations of the soil permittivity and the rough surface. The first-order impact of the permittivity fluctuations on the scattered $E$ field will be studied first, followed by an analysis of the surface roughness. Subsequently we will combine these two and derive explicit expressions for the observables based on parameterizations of the permittivity fluctuations. We will represent $E$ as a phasor throughout, and employ the $\exp \omega t$ sign convention.

2.1 Permittivity fluctuations

The soil is represented by the half space $b$ in Fig. 1a. Its dielectric properties are characterized i) a mean scalar permittivity $\varepsilon$ and ii) a fluctuating part $\varepsilon_f(\tau)$ with mean $\langle \varepsilon_f(\tau) \rangle = 0$. The mean permittivity $\varepsilon$ is mainly governed by the amount of liquid water present in the soil but also e.g. the ice content and soil texture (Zwieback et al., 2012; Mironov and Savin, 2015). This dependence can be described by dielectric mixing models such as those by Hallikainen et al. (1985); Peplinski et al. (1995).

Let a plane wave of angular frequency $\omega$ impinge upon this half space. Following Tsang et al. (2000), let $E$ and $E_b$ be the electric field above (region $a$, $V_a$) and inside (region $b$, $V_b$) the medium, respectively. These electric field vectors are each split into two parts: a perturbation term and an unperturbed component $E^0$ or $E^0_b$. These latter correspond to the E-field in the absence of permittivity fluctuations. The following set of integral equations is readily obtained from Maxwell’s equations (see Tsang et al., 2000, chap. 6.2.3):

$$\bar{E}(\tau) = E^0_b(\tau) + \int_{V_b} \bar{G}^{0}_{ab}(\tau, \tau_b) \cdot \bar{E}_b(\tau_b) Q(\tau_b) d\tau_b \quad (1a)$$

$$\bar{E}_b(\tau) = E^0_b(\tau) + \int_{V_b} \bar{G}^{0}_{bb}(\tau, \tau_b) \cdot \bar{E}_b(\tau_b) Q(\tau_b) d\tau_b \quad (1b)$$

where $Q(\tau) = \omega^2 \varepsilon \varepsilon_f(\tau)$ and the fields with superscript 0 denote the solutions in the absence of fluctuations, whereas $\bar{G}^{0}_{ij}$ represents the dyadic undisturbed Green’s function for a source in region $j$ and an evaluation point in region $i$. Equation 1a relates the measurable field above the interface $\bar{E}(\tau)$ to the unperturbed field $E^0_b(\tau)$ and to the field $\bar{E}_b(\tau_b)$ at all points inside the half-space. The latter has to fulfill the consistency relation of Eq. 1b.

An approximate solution, which does not obey energy conservation, can be obtained using a perturbative approach. In the first-order Born approximation the unknown field $\bar{E}_b$ on the right-hand side is replaced by the unperturbed one $\bar{E}^0_b$. This field corresponds to a plane wave, whose wave vectors both above and below the interface are shown in Fig. 1a, and whose magnitude is determined by the Fresnel transmission coefficient for refraction into the lower half-space. For a smooth interface in the far-field, using the plane-wave approximation for the phase, i.e. $\frac{k_v R}{\pi} \ll 1$ where $R$ is the distance to the antenna

$$\bar{G}^{0}_{ab}(\tau, \tau_b) \cdot E^0_b(\tau_b) = \frac{1}{W} \exp \left[ 2i (\bar{k}_a \cdot \tau - \bar{k}_b \cdot \tau_b) \right] \left( T_h \bar{P}_h + T_v \bar{P}_v \right) \cdot \tau \quad (2)$$

where $W [m^{-1}]$ can be considered a constant (Tsang et al., 2000). The rank one dyad $\bar{P}_h$ projects any vector onto the horizontal polarization vector in medium $b$ and maps it to the horizontal polarization vector in medium $a$; $\bar{P}_v$ does likewise for the vertical polarization. The factors $T_h$ and $T_v$ are the two-way amplitude Fresnel coefficients (see Supplement), and $\tau$ is the unit polarization vector of the incident field in region $b$.

Let $q_m = [q^m_H, q^m_V, q^m_{HH}, q^m_{VV}]$ be the scattering vector in the lexicographic basis (Cloude, 2009) for acquisition $m$. It is a column vector as indicated by the transposition operation $T$ and encodes the components of the scattered E-field.
Figure 1: Description of the geometric setup: the circle at in the soil (halfspace $V_a$) is illuminated by the antenna at height $H$ in $V_a$. The coordinate vectors with respect to the origin (black circle at the surface) are shown in the left-hand panel, along with the wave vectors $\vec{k}_a$ in red. The panel on the right illustrates the coordinate definitions used in Sec. 2.3.

$E(\vec{r})$ for different transmit polarizations $\vec{r}$ (Cloude, 2009). In interferometry two such acquisitions, e.g. $m$ and $n$, are combined: the position of the antenna can change $\mathbf{r}_m \neq \mathbf{r}_n$, as can the dielectric properties of the half-space $b$, $\varepsilon_m \neq \varepsilon_n$. The interferogram is formed by cross-multiplying the scattering vectors of the two acquisitions to yield the interferometric covariance matrix $[C_{mn}]_{p} = \langle q_m q_n \rangle$ (Bamler and Hartl, 1998). Its predicted value according to the model under the Born approximation follows by rearranging (1) using (2) and grouping the results according to both incident and scattered polarization components. Denoting the acquisition with an additional index, we get

$$[C_{mn}]_{p} = \frac{1}{W^2} \int_{V_a, V_b} \left[ \langle Q_m(\mathbf{r}_{b.m})Q_n(\mathbf{r}_{b.n}) \rangle \right]$$

$$\cdot \exp \left[ 2i \left( \mathbf{r}_{a,m} \cdot \mathbf{r}_m - \mathbf{k}_{a,m}^* \cdot \mathbf{r}_n \right) - \left( \mathbf{k}_{b,m} \cdot \mathbf{r}_{b,m} - \mathbf{k}_{b,n}^* \cdot \mathbf{r}_{b,n} \right) \right] d\mathbf{r}_{b,m} d\mathbf{r}_{b,n}$$

(3)

$$[T_{mn}] = \begin{bmatrix} T_{h,m} T_{h,n}^* & 0 & T_{h,m} T_{v,n}^* \\ 0 & 0 & 0 \\ T_{v,m} T_{h,n}^* & 0 & T_{v,m} T_{v,n}^* \end{bmatrix}$$

(4)

where the dielectric fluctuations are encoded in $\langle Q_m(\mathbf{r}_{b,m})Q_n(\mathbf{r}_{b,n}) \rangle$. These variations represent small-scale variations in the dielectric constant, due to e.g. pores, clumps or stones. If the fluctuations are assumed to be uncorrelated in space, this reduces to $Q_{mn}(\mathbf{r}_{b,m})\delta(\mathbf{r}_{b,m} - \mathbf{r}_{b,n})$, which eliminates one of the integrals in (3).

The polarimetric behaviour is only governed by the transmissivity matrix $[T]$ in (3); a more general representation could be obtained using an effective covariance matrix $[T]^T$ encoding both transmission and scattering effects.

### 2.2 Surface term

When the interface between regions $a$ and $b$ is not smooth, backscattering occurs even in the absence of inhomogeneities. The perturbative analysis can be extended to the surface roughness (characterized by a non-zero characteristic height $\eta$): the first order scattered field consists of i) the volume contribution excited by the zero-order fields (3); ii) the first order contribution from the surface (Small Perturbation Model, SPM) (Rice, 1951). The latter approximation is valid for small roughness heights $\langle k \eta \cos \theta \rangle \ll 1$ and small slopes $\frac{\eta}{\epsilon_a} \ll 1$. The scattering vector $q_{s,m}$ at acquisition $m$ is (Cloude, 2009)

$$q_{s,m} = Z \begin{bmatrix} \cos \theta - \sqrt{\varepsilon_m - \sin^2 \theta} \\ \cos \theta + \sqrt{\varepsilon_m - \sin^2 \theta} \\ (\varepsilon_m - 1) \left( \sin^2 \theta - \varepsilon_m (1 + \sin^2 \theta) \right) \end{bmatrix}^T$$

(5)

where $Z$ is a factor that does not depend on the dielectric constant, but on the roughness. Assuming that $Z$ is constant and that the volume and surface contributions are uncorrelated, the total scattering covariance matrix $[C_{mn}] = [C_{mn}]_v +$
\[ [C_{mn}]_s - [C_{mn}]_v + \langle q_{sa,m} q_{sb,m}^\dagger \rangle. \] This assumption is commonly done in both polarimetric (Yamaguchi et al., 2005; Hajnsek et al., 2009) and interferometric modelling (Treuhaft and Cloude, 1999; Cloude, 2009) of surface and ‘volume’ terms. Note that the surface contribution \([C_{mn}]_s\) can in principle also be modelled differently, e.g. by the Kirchhoff Geometric Optics model, a constant or by semi-empirical formulae (Ishimaru, 1997).

### 2.3 Propagation phase

The integration kernel in the volume contribution of (1) modulates both the magnitude and the phase of the return from any point in the halfspace. The propagation phase \(\varphi_m\) in (2), i.e. its phase term, depends on the two-way propagation of the plane wave in half space \(a\) via the real part of its wavevector \(\Re k_{a,m}\) and similarly for half space \(b\) via \(\Re k_{b,m}\) (see Fig. 1a)

\[
\varphi_m = 2(\Re k_{a,m} \cdot \overline{r} - \Re k_{b,m} \cdot \overline{r}_b)
\]

(6)

\[
-2k_0 \sqrt{H_{a,m}^2 + (y_i - y_{a,m})^2} + 2k_0 w_m \sqrt{z^2 + (y_s - y_{i,m})^2}
\]

(7)

where the second line uses the coordinate definitions of Fig. 1b and the scenario is kept two dimensional for simplicity. \(y_i\) is an internal variable determined by Snell’s law, and \(w_m \rightarrow \left[ \sin^2 \theta^i + \Re \left( \sqrt{\varepsilon_m - \sin^2 \theta^i} \right) \right]^{1/2} \) – it corresponds to the index of refraction in the absence of absorption – such that

\[
\Re k_{b,m} = k_0 w_m (0, \sin \theta^i, \cos \theta^i)^T
\]

(8)

As described in more detail in the supplement, the interferometric phase of a point scatterer is given by the difference in the propagation phases and evaluates to

\[
\phi_{mn} = \varphi_m - \varphi_n - \tilde{\phi}_{mn} + \tilde{\phi}_{mn}
\]

(9)

where the second line is obtained by linearization with respect to the position of the antenna \((\mathbf{F}_B = k_0 B^2 R^{-1} \ll 1,\) where \(B_{\perp}\) is the projection of the baseline vector perpendicular to the line of sight), \(\phi_{mn}\) is the part due to different background dielectric constants, whereas \(\tilde{\phi}_{mn}\) is a correction due to an antenna offset in height \(\Delta H\) and in the horizontal direction \(\Delta y_s\).

For scatterers close to the interface it is expedient to simplify the expressions even further by linearization with respect to the position. Particularly relevant are the changes with respect to \(z\) at constant range \(R_m\) – the vertical interferometric wavenumber \((\frac{\partial \phi}{\partial z})^{R_m}\) – and with respect to \(R_m\) at constant height – the range wavenumber \((\frac{\partial \phi}{\partial R_m})^z\). The total differentials evaluated just below the surface – the antenna positions being held constant – of the two terms in (9) are derived in the supplement:

\[
d\tilde{\phi}_{mn} = \left[ -2k_0 \frac{B_{\perp} \cos \theta^i}{R} \right] dy_s + \left[ -2k_0 \frac{B_{\perp} \cos \theta^i \sin \theta^i}{\sqrt{w_m^2 - \sin^2 \theta^i}} \right] dz
\]

(10)

\[
d\tilde{\phi}_{mn} = \left[ -2k_0 \left( \sqrt{w_m^2 - \sin^2 \theta^i} - \sqrt{w_n^2 - \sin^2 \theta^i} \right) \right] dz
\]

(11)

The interferometric wavenumbers follow from (10) and (11), cf. also Dall (2007) for the vertical one:

\[
\left( \frac{\partial \phi}{\partial z} \right)_{R_m} = -2k_0 \left( \sqrt{w_m^2 - \sin^2 \theta^i} - \sqrt{w_n^2 - \sin^2 \theta^i} + \frac{B_{\perp}}{R \sin \theta^i} \frac{w_m^2 \cos \theta^i}{\sqrt{w_m^2 - \sin^2 \theta^i}} \right)
\]

(12)

\[
\left( \frac{\partial \phi}{\partial R_m} \right)_{z} = -2k_0 \frac{B_{\perp} \cos \theta^i}{R \sin \theta^i}
\]

(13)

Note that the range wavenumber does not depend on the permittivity and is identical to the one above the interface (Oveisgharan and Zebker, 2007), thus opening the possibility to do range spectral filtering (Gatelli et al., 1994). The linear approximation for a difference in \(\phi\) of two scatterers separated by \(\Delta z\) is appropriate if \(\mathbf{F}_z = k_0 (\Delta z)^2 R^{-1} \ll 1\).

### 2.4 Interferometric observables

In interferometric SAR studies it is commonplace to normalize the measured covariance of (3) to obtain the coherence \(\gamma_{mn}\)

\[
\gamma_{mn}(\overline{\varepsilon}) = \frac{\overline{\varepsilon} [C_{mn}]_{\overline{\varepsilon}}}{\overline{\varepsilon}^2 + [C_{mn}]_{\overline{\varepsilon}}^2}
\]

(14)
where \( \mathbf{\varpi} \) is a polarimetric projection vector (Cloude, 2006). The phase of \( \gamma_{mn} \) is denoted by \( \phi_{mn} \), its magnitude by \( |\gamma_{mn}| \).

In astronomic interferometry the bispectrum is analysed when no adequate phase reference can be established (Monnier, 2007). It is defined as a triple product of complex visibilities (Monnier, 1999), which correspond to the interferometric covariances \( C_{mn} \). We propose to adapt this bispectrum to the bicoherence \( \Gamma_{mn} \) in InSAR by forming the analogous triple product of the normalized covariance, i.e. the coherence

\[
\Gamma_{mno} = \gamma_{mn}\gamma_{no}\gamma_{mo}. \tag{15}
\]

Its argument is the phase triplet (De Zan et al., 2014) (or closure phase) \( \Xi_{mno} = \phi_{mn} + \phi_{no} - \phi_{mo} \). For \( N \) acquisitions there are only \( \binom{N-2}{3} \) independent triplets, as opposed to \( \binom{N}{3} \) phases (Monnier, 2007). The lost information consists of the phase offset of each slave acquisition (Lannes, 1990), which implies that these phase triplets are insensitive to deformations, atmospheric influences and general phase offsets.

### 2.5 Parametric solutions

In order to arrive at closed-form expressions, we propose to represent the dielectric fluctuations as simple functions of depth. The magnitude of these fluctuations consequently does not depend on the horizontal position but only on \( z \). Using these representations, along with the linearized phase terms of (12) and (13) in (1) will yield such analytic expressions. Let \( \delta_0 \) be a representative penetration depth and \( z' = z\delta_0^{-1}(k' - k\delta_0) \) the dimensionless depth (a dimensionless wavenumber).

The ensemble average in (3) will be assumed real and parameterized as

\[
\left< Q_m(z') \right> Q_n(z') - Q_{mn}(z') = Q_{mn}(z') - Q_{nn}(z') = a(z') \tag{16}
\]

where \( a(z') \) is a polynomial of degree \( A \), which we represent in the Laguerre basis. Note that this assumes i) invariance of the dielectric fluctuations between acquisitions, i.e. only the mean \( \varepsilon \) changes; and ii) a lack of spatial correlation. These assumptions mean that these fluctuations are like white noise added to the background dielectric constant. For simplicity only the cases \( A = 0 \), i.e. \( a(z') = a_0 \), and \( A = 1 \), i.e. \( a(z') = a_0 + a_1(1 - z') \), will be considered (Abramowitz and Stegun, 1964).

These two cases are illustrated in Fig. 2. The representation as a zero-order polynomial \( A = 0 \) implies constant heterogeneities with depth, which is shown in the left panel. The linear dependence with depth \( A = 1 \) is illustrated for \( a_1 > 0 \) in the right panel: the soil becomes increasingly homogeneous as \( z' \) increases.

The assumption of a real and acquisition-invariant profile of the spatially uncorrelated dielectric fluctuations renders (3) similar to the standard first-order scattering model employed in interferometry, with \( a(z') \) corresponding to the structure function, as employed – also as a series expansion – in coherence tomography (Cloude, 2006).
In the following it will be assumed that the $B_{\perp}$ term in (12) is negligible, i.e. that the phase contribution due to different elevations is much smaller than $\pi, k_0 B_{\perp} R^{-1} \delta_0 \ll 1$. We also assume that range spectral filtering (Gatelli et al., 1994) has been applied so that the range wavenumber vanishes. These assumptions are made because the data used in this study were acquired from the same track. In this case the exponential term in (3) reduces to $2i \left( b_{b,m} - b_{b,n}^{*} \right) \cdot \hat{z} z'$ where $\hat{z}$ is a unit vector in the vertical direction; this is the same functional form as in De Zan et al. (2014). The integration in the horizontal direction (which should include the antenna pattern) can be absorbed into the normalization constant $W$, as it cancels when the coherence (14) is formed. Owing to the lack of spatial correlation of $Q$ as parameterized by $a(z')$ in (16), one of the integrals in (3) drops out

$$[C_{mn}]_v = -\frac{1}{W^2}[T_{mn}] \int_{z'=0}^{z'} a(z') \exp \left[ -2i \left( b_{b,m} - b_{b,n}^{*} \right) \cdot \hat{z} z' \right] dz'$$

(17)

For $A - 0$ the integral in (17) yields

$$[C_{mn}]_v = -\frac{i a_0}{2W^2}[T_{mn}] \left( b_{b,m} \cdot \hat{z} - b_{b,n}^{*} \cdot \hat{z} \right)^{-1}$$

(18)

whereas $A - 1$ gives

$$[C_{mn}]_v = -\frac{1}{4W^2}[T_{mn}] \left( 2i(a_0 + a_1) \left( b_{b,m} \cdot \hat{z} - b_{b,n}^{*} \cdot \hat{z} \right) + a_1 \right) \left( b_{b,m} \cdot \hat{z} - b_{b,n}^{*} \cdot \hat{z} \right)^{-2}$$

(19)

The terms $a_0$ and $a_1$ capture not only the spatial variability of the dielectric fluctuations but also their absolute size. This size is usually not known in practice, and also presumably difficult to determine in-situ, which is discussed in more detail in Sec. 5.3. Similarly, the absolute value of the surface contribution depends on the roughness properties, which are generally not known a-priori. For practical reasons, we thus introduce a weighting parameter, the volume-to-surface ratio $f$. It expresses the relative importance of the two contributions (the absolute values are not necessary if only the coherence is looked at) at a reference moisture value:

$$[C_{mn}] = \frac{f}{f_0_v}[C_{mn}]_v + \frac{1}{f_0_s}[C_{mn}]_s$$

(20)

where $f_{0,v}$ and $f_{0,s}$ are normalization magnitudes, corresponding to the element of $[C_{mn}]_v$ or $[C_{mn}]_s$ at a reference moisture state in a specified polarization. The scaling of $[C_{mn}]$ is immaterial for the evaluation of $\gamma$, $a_0$ will be set to one. The case of $A - 0$ and $f \to \infty$ (pure volume) corresponds to the scenario studied by De Zan et al. (2014): its coherence is independent of frequency (and thus the free space wavelength) for frequency-independent $\varepsilon$ as the half space has no intrinsic length scale.

The simulated impact of a varying volume-to-surface contribution $f$ on the coherences is illustrated in Fig. 3a. For a fixed change in soil moisture of $\Delta m_v = 0.1 \ m^3m^{-3}$, an increase in $f$ makes the absolute values $|\gamma|$ decrease (i.e. the soil decorrelates more) and the magnitude of the phases grow. This increase in sensitivity is due to the fact that the sensitivity of the phase to soil moisture changes is much smaller (and of opposite sign) for the surface term than for the bulk volume. A similar difference in sensitivity is present for $|\gamma|$, which is predicted to be equal to one for the surface, which also implies vanishing phase triplets. The volume term, by contrast, leads to decorrelation: $|\gamma|$ is predicted in Fig. 3b to decrease monotonically as the absolute change in soil moisture $|\Delta m_s|$ increases, whereas the magnitude of the interferometric phase is expected to increase. This relation is non-linear: the phase $\phi$ saturates as $\Delta m_s$ increases, and it is not a function of $\Delta m_v$ alone but depends on both values of $m_v$. This is illustrated for two different dielectric mixing models in Fig. 3c and 3d, whereas Fig. 3e and 3f shows the impact of varying $a_1 (A - 1)$.

3 Calibration and model assessment

In order to assess the accuracy and validity of the model, we compare its predictions of the radar observables to airborne radar observations. These model predictions are based on the soil moisture that was measured in-situ at the time of the radar acquisitions. As the model contains unknown parameters, these will be estimated in a calibration step. The calibrated model can subsequently be directly compared to the radar measurements.

3.1 Study site and data

3.1.1 Study site

The available data were acquired within the Canadian Experiment for Soil Moisture in 2010 (Magagi et al., 2013) and cover the time period from June 2-14 2010. The test site near Kenaston, Saskatchewan, Canada ($51^\circ 30'\ N, 106^\circ 18'\ W$) covers an area of around 270 km$^2$. Rainfed agricultural fields, pastures and grassland predominate, but at least 1.5 % of the area is covered with open water surfaces. This percentage is likely to have been even larger due to the wet conditions before and during the campaign (CanEx-SM10, 2010). The majority of the agricultural fields had been tilled prior to the measurements and several were covered with crop residues to varying degrees (Magagi et al., 2013).
(a) Coherence values (Pep, A = 0) for m_{no}^{m} = 0.2 (0.3) m_{no}^{n} for the master (slave) and varying f.

(b) Coherence values (Pep, A = 0) for f = 100, a master m_{no}^{m} of 0.2 m^{3}m^{-3} and varying slave m_{no}^{n}.

(c) Interferometric phases \phi_{mn} (A = 0) for f = 25 and different m_{no}^{m} and slave m_{no}^{n} = m_{no}^{m} + \Delta m_{o}.

(d) Interferometric phase triplets (A = 0) \Xi_{mno} for f = 25 and m_{no}^{m} = 0.3 m^{3}m^{-3} and different m_{no}^{n} and m_{no}^{n}.

(e) Interferometric phases \phi_{mn} (Hal model) for (f) Interferometric phase triplets (Hal) \Xi_{mno} for f = volume-only and different m_{no}^{m} and slave m_{no}^{n}, using 25 and m_{no}^{m} = 0.3 m^{3}m^{-3} and different m_{no}^{n} and m_{no}^{n}, A = 0 (solid line) and A = 1 (a_{1} = -0.7, dashed using A = 0 (solid line) and A = 1 (a_{1} = -0.7, dashed line).

Figure 3: Model predictions for different observables (HH polarization) using the Peplinski mixing model Pep (Peplinski et al., 1995) or the Hallikainen (Hallikainen et al., 1985) model Hal. In panel a) the volume-to-ground ratio f varies, in the remaining ones the soil moisture varies.

3.1.2 Soil moisture data

Environment Canada operates a network of permanent stations where soil moisture is recorded by Stevens Hydra Probe II sensors at different depths (CanEx-SM10, 2010), from which we use the data measured by those installed vertically at 0-5 cm depth. Due to battery failure, the presence of standing water in some fields and limited image extent, only the data of ten fields are available, of which we use the same seven as in Zwieback et al. (2015); the remainder are put aside for future inversion studies. The sensor outputs are converted to values of volumetric soil moisture m_{o} [m^{3}m^{-3}] using a calibration procedure based on soil texture.

The m_{o} time series of these sensors show similar temporal patterns, but both the mean values and dynamic ranges differ, the latter by about a factor of 10. An illustration of this diversity is given in Fig. 4 for two different fields.
Figure 4: Time series of measured phases $\phi$ and absolute coherences $|\gamma|$ in two fields, where the master acquisition $m$ is June 5. The soil moisture values $m_v$, measured in the respective field are shown in the second panel; their dynamic ranges differ by about one order of magnitude.

### 3.1.3 Radar data and processing

The fully polarimetric (quadpol) UAVSAR data (L-band: $\lambda = 0.24$ m) have a resolution of 1.7 m (0.8 m) in range (azimuth) (Jones and Davis, 2011). They were acquired in irregular intervals between one and three days with a nominal baseline of 0 m and from a height of 13 km. The incidence angle ranges between 22° and 66° from the near to the far range of the image (CanEx-SM10, 2010).

Using all possible combinations of the six SLC images, 15 interferograms were formed. As in Zwieback et al. (2015), a rectangle (extent: 50 m in range, 100 m in azimuth) was manually delineated in the immediate surroundings of the location of each probe; it was chosen so as to maximize the homogeneity within this region of interest (ROI) and avoid pools of water. At each of the four corners of these rectangles, the interferograms were averaged (number of looks $L = 172$, area: 20 m$^2$) to estimate the coherence. These subROIs are denoted by the field number (three digits) and a letter a-d for each of the corners. For each of the fields, we reference the phase with respect to the closest stable scatterer as in Zwieback et al. (2015), where it was observed for the same data that the results are not sensitive (less than 10% change on average) to the choice of reference target.

As the flight tracks during the acquisition deviated from the nominal ones – the UAVSAR team aims at a track accuracy of 10 m and achieved one of 5 m in this campaign – the interferometric phase depends on the height, thus introducing an additional source of uncertainty. In the worst case scenario for an incidence angle of $\theta = 30^\circ$, the height of ambiguity $H_a \approx 190$ m (the height corresponding to a phase difference of one cycle). For a height difference between the field and the phase reference of 5 m, we thus expect a maximum phase error of $10^\circ$, which is about one order of magnitude smaller than the maximum phase differences expected from the model or observed in the data. An example of these observed phases is shown in Fig. 4: as the soil becomes more moist between June 6 and 9, the measured phases with respect to a drier acquisition on June 5 take on values of around 70° or $\frac{7\pi}{18}$ rad. They subsequently decline, and so does the soil moisture, whereas the absolute coherences increase again. The radar observations in field 206 (Fig. 4b) exhibit similar temporal behaviour, but the magnitude is smaller by a factor of 2. The soil moisture observations also display a smaller dynamic range (by about 75%).

### 3.2 Rationale

In order to assess the validity of the proposed model of (20), we compare its predictions with these L-band measurements. The predictions are governed by two parameters that are not known a priori: $f$ and $a_1$. Assuming the model can capture all the influences on the coherence – thus neglecting phenomena such as vegetation dynamics – we estimate these via minimization of misfit functions: these measure the difference between the predicted observables (based on prescribed soil moisture) and the measured ones. This estimation is done for each subROI separately. The overall aim of the calibration is to first estimate the parameters, and then assess the quality of fit.

### 3.3 Minimization of misfit

We focus on the two normalized observables of Sec. 2.4, i.e. $\gamma$ and $\Gamma$, and separate the magnitude and argument of these complex quantities. For the coherence we propose to minimize a generalized root mean square (RMS) deviation $\mu_\gamma(\zeta)$.
where $\zeta$ weights the relative importance of magnitude and argument:

$$
\mu_2^2(\zeta) = \sum_{n>m} \left[ (1 - \zeta) w_{mn}^0 (1 - \cos(\hat{\phi}_{mn} - \phi_{mn})) + \zeta w_{mn}^1 (|\hat{\gamma}_{mn}| - |\gamma_{mn}|)^2 \right]
$$

(21)

where the weights $w_{mn}^0$ are the reciprocals of the estimated variances (based on the Cramer-Rao bound of the Gaussian speckle model, see Bamler and Hartl (1998)) and are normalized such that $\sum w_{mn} = 1$. This normalization condition is also enforced for the uniform weights $w_{mn}^1$. The predicted values are denoted by hats and depend on the parameters $f$ and $a_1$, as well as on the prescribed soil moisture values. Note that the $\cos$ dependence of the phase misfit is not susceptible to phase wrapping and that the metaparameter $\zeta$ weights the influence of $\phi$ and $\gamma$, with $\zeta = 0.5$ corresponding to a roughly equal importance.

The bicoherence misfit function

$$
\mu_2^2(\zeta) = \sum_{a=n} (1 - \zeta) w_{m_a,n,o}^\infty (1 - \cos(\Xi_{m_a,n,o} - \Xi_{m_a,n,o})) + \zeta w_{m_a,n,o}^\Gamma (|\hat{\Gamma}_{m_a,n,o}| - |\Gamma_{m_a,n,o}|)^2
$$

(22)

is structured along the same lines; the weights $w_{m_a,n,o}^\infty$ are obtained assuming that the deviations of the phases $\phi_{mn}$ are uncorrelated for different acquisitions, and the $w_{m_a,n,o}^\Gamma$ are again uniform. The first acquisition $m_0$ in the bicoherence is held fixed as otherwise redundancy would be introduced (Monnier, 2007).

### 3.4 Evaluation

In order to assess the impact of different parameterizations, such as the dependence on the dielectric mixing models, we employ bootstrap resampling: Bias Corrected and accelerated (BCa) confidence intervals are derived using case resampling (DiCiccio and Efron, 1996). These quantities are estimated by taking random subsamples from the data, with which one can then approximate the distribution of the quantity of interest. These are the difference of the estimates of a parameter (e.g. the volume-to-ground ratio $f$) between two parameterizations.

For any parameterization of the model, the minimized value of the misfit function $\mu$ is an indicator of the discrepancy between model and observations. Apart from this value, the quality of fit will also be assessed by the correlation $\rho$ and the sensitivity ratio $s$. $\rho$ is the Pearson’s correlation coefficient between predictions and measurements, whereas the sensitivity ratio $s(\zeta)$ is given by the ratio of the standard deviation of the predictions to that of the data. A value $s > 1$ thus corresponds a larger spread of the predicted values than the observed ones, whereas $s < 1$ indicates a comparatively larger scatter in the data. This can be due to a larger $m_\rho$, sensitivity of the observations than predicted by the model, but also due to influences not represented by the model. In particular, it is expected that a poor model fit will lead to small values of $s$ upon calibration.

### 4 Results

#### 4.1 Examples

Such differences between the measured DInSAR observables and their predictions are found in the data, e.g. in the examples shown in Fig. 5. Panel a) shows the predicted $\phi_{VV}$ of both a volume-only ($f \rightarrow \infty$) and the calibrated version ($f$ estimated to yield $f$ using $\zeta = 0$). For both versions, there is an approximately linear relation between predictions and observations of the correct sign but the observed values are greater in magnitude than the predictions ($s = 0.56$). These simulated values of the two parameterizations cannot be distinguished as $f \gg 1$. Such a difference does occur for $\phi_{HH}$ in Fig. 5b, where the sensitivity ratio $s = 2.93$ is reduced, upon calibration, to 0.76. Similar variations in the sensitivity are also found for the phase triplets $\Xi$, e.g. in Fig. 5c and d).

#### 4.2 Coherence

##### 4.2.1 Calibration with $A = 0$

The calibration results of all the fields for the simple case of $A = 0$ – i.e. the fluctuation term $Q$ is independent of depth – are summarized for the HH polarization in Taylor plots in Fig. 6. The location of each point encodes the correlation $\rho$ via its angle and the relevant misfit $\mu$ via its distance from the origin; the colour depends on the field according to the topmost legend and the sensitivity ratio $s$ is represented by the symbol as indicated by the legend underneath. The three panels a) to c) show the evaluation results for the phase $\phi$, i.e. the correlation and sensitivity ratio between predicted and measured $\phi$ as well as the phase misfit $\mu(0)$. The evaluation measures of the same models but applied to the coherence magnitude $|\gamma|$ are displayed underneath.

In panel a) the Peplinski-based model was calibrated on phase data alone, i.e. $\zeta = 0$. The subROIs results tend to cluster with respect to the field, achieving a median correlation $\rho$ of 0.71. Typical misfits $\mu(0)$ vary between 0.1 and 0.25, with only field 201 (light blue) showing worse fits. For this field, the model cannot replicate the interferometric
observables based on the measured soil moisture data. This is even more evident for the coherence $|\gamma|$ in panel d), as negative correlations are obtained. This field shows considerably larger variations in the interferometric observables than the model can predict: even in the pure volume case, the sensitivity ratio $s < 1$. Such low values of $s$ for the volume-only case are not observed in the other fields. The exceptional properties of this field 201 are also reflected in the results obtained using $\zeta = 0.5$ (roughly equal weight on $\phi$ and $|\gamma|$). As seen in Fig. 6b, the overall fits and correlations are similar to the phase-only case using $\zeta = 0$.

The phase-only calibration $\zeta = 0$ using the Hallikainen mixing model contrasts with the Peplinski model in two important aspects. Firstly, according to Fig. 7d the model cannot reproduce the magnitude of the observed phases, i.e. $s < 1$. Secondly, the median correlation of the phase increases to 0.77, and the one of the coherence magnitude from 0.5 to 0.58. There is no clear trend in the misfits $\mu_s(0)$. They are significantly different between the two mixing models (at a significance level $\alpha = 0.05$) in 45% as shown in Fig. 8a. Non-significant differences occur e.g for fields 109 and 136, corresponding to confidence intervals that overlap with $\mu_s(0)_{\text{Pep}} - \mu_s(0)_{\text{Hal}} = 0$, i.e. equal misfit of the two mixing models.

The Taylor plots for the VV phase-only calibration (i.e. $\zeta = 0$) in Fig. 9 reveal similar patterns for the Pep and Hal mixing model: more than 90% of the correlations are between 0.3 and 0.95 (for $\phi$: median 0.78 and 0.70 with Hal and Pep, respectively). The two versions of the model cannot always reproduce the observed magnitudes of the interferometric observables. As seen in Fig. 7e and 7f, this occurs for the same fields as in the HH polarizations. The misfits $\mu_s(0) < 0.4$, except for field 201, with significant differences between Pep and Hal occurring in 36% of the cases (Fig. 8b). The volume-to-ground ratios $f$ estimated independently for HH and VV phases do not show a consistent offset in Fig. 8c.

The model of (20) predicts zero contribution in the cross-pol channel from both the surface and the volume term. As the observed signals are similar to the ones found in HH and VV (Zwieback et al., 2015), we study them using a volume-only model. In this approach the surface contribution is not considered ($f \to \infty$), and the volume model of (17) is adopted (using the appropriate Fresnel terms). Its correlations (median: 0.73) and misfits for the HV phase are plotted against those of the calibrated HH model applied to $\phi_{HH}$ in Fig. 7c and 7f. The correlations tend to be larger for the HH channel – however more than 85% exceed 0.5 in the HV channel –, and the misfits $\mu(0)$ smaller.

### 4.2.2 Calibration with $A = 1$

The inclusion of an additional term in the expansion of the dielectric fluctuations, i.e. letting $A = 1$, always reduces the misfit of the best-fitting model in both HH (Fig. 10a) and VV (Fig. 10b) channels. This reduction is less than 5% for all samples except two. Note that the misfit cannot increase as the $A = 1$ model includes the $A = 0$ model as a special case. The correlations $\rho$ increase for more than 90% of the samples (see Fig. 10c and 10d).

### 4.3 Bicoherence

The calibration procedure based on the bicoherence is achieved by minimizing the misfit $\mu_T$ of (22); the relevant Taylor plots for HH are shown in Fig. 11. The subfigures a) and b) reveal that for $A = 0$, the correlations $\rho$ are typically between 0.3 and 0.75, with median values of 0.56 and 0.58 for the Hal and Pep mixing model, respectively. The correlations and also the misfits display as much intra-field as inter-field variability. Panel c) reveals that when $\alpha_1$ is estimated as well ($A = 1$), the correlations tend to increase (median: 0.56) along with the sensitivity ratios. The great majority of these sensitivity ratios $s$ are less than one, even if the volume-to-surface ratio $f \to \infty$, in which case only 22% exceed 1 for the Hallikainen model.

The correlations obtained at VV (see Fig. 12) are comparable, with a median value of 0.59 for both mixing models. The misfits, by contrast, are smaller by around 30%.
5 Discussion

The comparison of the measured observables with the measured soil moisture changes and the model predictions has revealed certain agreements and inconsistencies. These will subsequently be discussed and assessed with respect to the uncertainties of the radar observations and the soil moisture measurements. The differences between the polarizations will be analysed first, followed by the impact of different parameterizations of the model and possible extensions or generalizations.

5.1 Polarimetry

In the proposed model, both the term due to the dielectric heterogeneities (3) and the one due to the surface scattering (5) predict zero backscattering in the cross-pol channel HV. This prediction, which is a general feature of such first-order expansions of dielectrically isotropic scattering scenarios (Ishimaru, 1997), is known to fail in practice, and has thus led to the introduction of both more sophisticated analytical and simpler semi-empirical models (Hajnsek et al., 2003; Oh et al., 2002). The discrepancy between predictions and observations is found to also apply to the interferometric coherence: soil moisture changes impact the DinSAR signals at HV (Zwieback et al., 2015). The measured phases and their observed dependence on soil moisture changes cannot be explained by noise or the uncertainty due to the phase referencing. Rather, they can also be described by the volume scattering of (3) provided that the HV channel is treated like the co-polar ones. The correlations \( \rho \) between predicted and observed phases obtained in this way (see Fig. 7) are of comparable size to the negative correlations are not shown but listed in the grey box. The top panels give the relevant quantities for the phase \( \phi \) (using different mixing models and \( \zeta \)), the bottom ones do the same for the coherence magnitude.

These values of the phase correlations \( \rho \) at HH and VV indicate that the model can capture the most salient features of the observed phases. These are thus considered to be larger than decorrelation noise and the uncertainties due to phase
Figure 7: The first two columns contain scatter plots of the phase correlation and misfit for HH and VV between volume-only and calibrated versions of the model. The third column shows a comparison between HV and HH fit using the Hallikainen mixing model. The colours of the symbols represent the agricultural fields as in Fig. 6.

Figure 8: Bootstrap confidence intervals of differences in $\mu_\gamma$ or $f$ between two parameterizations. For each quantity and each ROI, the estimated difference is shown by the symbol (a diamond if it is significantly different from zero at $\alpha = 0.05$, a circle otherwise), along with the 95% confidence intervals.

referencing (see 3.1) and their sign and general dependence on moisture changes can be described by the model. However, these $\rho$ values do not reveal the mismatch between the sensitivities to soil moisture changes of the model and the data. For four fields the observed ones tend to exceed the maximum sensitivities obtainable using the Hal mixing model: these are achieved with the volume-only version as $f \rightarrow \infty$, see Fig. 3a. Such a case is displayed in Fig. 5a: as the sensitivity cannot be further increased and $f$ is the only parameter that can be adapted, the optimized and the volume-only model coincide. The tendency of a reduction of sensitivity with decreasing $f$ becomes evident in one of the counterexamples, shown in Fig. 5b.

Such a discrepancy in the magnitude between the modelled and the observed impact of soil moisture can be caused by inconsistent definitions and calibrations of the input data (e.g. the soil moisture), or by deficiencies in the model itself, cf. Sec. 5.2. In particular in this data set, the dynamic ranges of the measured soil moisture time series differ by about one order of magnitude despite similar soil textures and the absence of vegetation (Magagi et al., 2013). Manual soil moisture sampling on some of the days indicated a smaller inter-field variability of the soil moisture dynamics than the
observations by the permanent probes used in this study. The interferometric observables show a smaller variability in magnitude than the in-situ measurements, cf. Fig. 4 and Zwieback et al. (2015). More generally, such discrepancies are not uncommon in backscatter-based estimation of soil moisture: this is why the correlation coefficient remains one of the most popular accuracy metrics (Entekhabi et al., 2010), and the differences in both mean and dynamic range have to be considered when comparing and merging these data, as well as for model integration (Liu et al., 2012; Zwieback et al., 2012).

For many practical purposes, the link between soil moisture and the phase will be more important than the one with the coherence $|\gamma|$, not least because there are many processes that lead to decorrelation over time (Rocca, 2007; Zebker and Villasenor, 1992). The data set used in this study is characterized by a repeat period of 1-3 days, which is smaller than that of most satellite missions. The impact of the soil moisture on the coherence is evident in the timeseries of Fig. 4, where the coherences increase again after a rain event has temporarily led to wet conditions and decorrelation. Except for field 201, for which negative correlations $\rho$ between predicted and modelled $|\gamma|$ are obtained, the $\rho$ are similar to those

![Taylor plots for the phase $\phi$ and the coherence $\gamma$ at VV polarization; cf. Fig. 6 for a description of these plots.](image1)

Figure 9

![Comparison of phase correlation and misfit for HH and VV between $A = 0$ and $A = 1$, using the Hal mixing model. The median values of the estimated $a_1$ are 0.12 (HH) and -0.06 (VV). Refer to Fig. 6 for the colour code.](image2)

Figure 10

![Taylor plots for bicoherence calibration based on HH phase triplets: $\zeta = 0$; cf. Fig. 6 for a description of these plots. The value of the estimated $a_1$ in c) is -0.65.](image3)

Figure 11
observed for the phase \( \phi \), see Fig. 6 and 9. The sensitivity of these predictions to soil moisture is, like for \( \phi \), smaller than expected \((s < 1)\) in certain cases.

Such a small sensitivity is also observed for the phase triplets. As for the comparison with \( \phi \), the correlations with the observations vary typically between 0.3 and 0.9. These larger sensitivities observed in the data correspond to negligible surface contributions (as in Fig. 5d), as the surface term leads to zero closure phases for all values of \( m_v \), resulting in identical predictions for the volume-only and the calibrated models.

5.2 Parameterization and uncertainties of soil moisture dependence

The dependence on the parameterization is particularly pronounced for the link between the dielectric constant and \( m_v \), i.e. the mixing model and the calibration of the soil moisture data. The difference between the Hal and the Pep mixing model is also evidenced by the internal sensitivity analysis of Fig. 3: compared to the Hal model, the Pep model predicts i) a higher sensitivity of \( \phi \) to \( \Delta m_v \), ii) increased decorrelation for fixed \( |\Delta m_v| \), and iii) in general larger magnitudes of the phase triplets \( \Xi \). The main difference between these two models is the loss tangent, and thus the absorption, both of which are smaller for the Pep model (Peplinski et al., 1995). For a fixed value of \( m_v \), the wave penetrates deeper into a soil whose dielectric properties are governed by the Pep model. There is consequently a proportionally larger amount of scattering from heterogeneities deeper in the soil; when \( m_v \) changes, these heterogeneities add contributions with a larger change in propagation phase than the ones closer to the surface. This leads to i) larger changes in the interferometric phase \( \phi \) (as the deeper parts have a larger contribution), ii) lower values of \( |\gamma| \) (due to the larger effective diversity in propagation phases), and iii) larger phase triplets (as they vanish for surface-scattering and increase in magnitude with decreasing loss tangent).

The overall similarity in the results despite the difference in the sensitivities can be partially explained by the compensating effect of a varying \( f \). A relative increase in volume scattering acts similarly to a decrease in absorption: it also leads to larger phase sensitivities, more decorrelation and larger phase triplets. Empirically, the calibrated \( f \) are consistently larger for the Hal model than for Pep. However, the compensation of the difference in the loss tangent is not perfect, as more than 40% of the samples show significant – at \( \alpha = 0.05 \) – differences in the misfit (Fig. 8a and 8b), i.e. the differences are larger than the uncertainties. There is no clear tendency towards any of the two, but the majority of fields can be modelled more accurately by the Hal model, which on average also achieves higher correlations. This points towards more complex disparities between the two mixing models and the predictions obtained with them. The deviations in the latter, such as the nonlinear components, are seen in Fig. 3c and 3d to increase with differences in soil moisture. The latter figure indicates the differences in the predictions for \( \Xi \), the impact of which on the misfit is seen to be similar to the one of \( \phi \) in Fig. 11 and 12.

The small impact of extending the expansion of the fluctuations to \( A = 1 \) on the quality of fit and the correlations is also consistent with such a balancing effect of \( f \). A negative \( a_1 \) corresponds to an increase in the fluctuations \( Q_{mn}(z') \) with depth. Deeper parts of the soil thus exert a relatively larger influence on the scattering – similar to the Pep model compared to the Hal mixing model, or to an increase in \( f \). The corresponding predictions of Fig. 3e and 3f for \( \phi \) and \( \Xi \), respectively, reflect this similarity: the sensitivity to changes in soil moisture tends to increase when \( a_1 < 1 \). Upon extension of the expansion of the fluctuations, the empirically determined misfits \( \mu_{(0)} \) always decrease. This decrease is always below 5%, and the size of the misfits remains stratified according the field.
5.3 Extensions and underdetermination

The parameterization of the scattering contribution as a function of depth – in the present model it is encoded in (16) – thus holds potential for improving the model fit. An analogous depth variation appears conceivable for the mean dielectric constant (De Zan et al., 2014), but without direct observation (e.g. using tomographic techniques), there is little physical basis for either. Rather, it would presumably lead to an overparameterization, given the typically available data and the compensating effects mentioned above.

Arguably more critical issues from the point of view of structure of the model – as they concern barely justified assumptions implicit in (16) – are i) the postulated invariance of the permittivity fluctuations with the acquisition, and ii) their lack of spatial correlation. Both were dictated mostly by practical reasoning: in particular i) could – if relaxed – easily explain any interferometric phase $\phi$ as $Q_{mn}(z)$ would be represented by an arbitrary complex number for $m \neq n$; similar observations apply to $|\gamma|$ if $m - n$ is also taken into account separately. The inclusion of spatial correlations ii) would, in the isotropic case, mainly impact the overall backscatter from these fluctuations rather than their polarimetric properties (see Tsang et al., 2000, chap. 6.2.3.). Anisotropic correlations, on the other hand, could possibly affect the latter, especially the HV component. As these fluctuations are somewhat germane to the model (they are not amenable to direct measurements except perhaps at the pore scale) and the relaxation of assumptions greatly increases the possible set of measurements explainable by the model, such an extension would suffer from underdetermination that could not easily be resolved.

Similar reasoning applies to the use of more general surface and volume scattering models. For instance, the use of discrete particles embedded in the soil as opposed to spatially varying dielectric properties, could yield similar results. Such discrete particles form the core of the model by De Zan et al. (2014), but their properties (and thus e.g. the polarimetric characteristics of the scattered fields) were not specified. More general models would also permit a relaxation of certain limitations such as the lack of energy conservation, the small RMS roughness height or the lack of correlation between surface and volume scattering. The latter might be particularly critical when the expansions are extended to include more terms. Such higher-order models could also introduce multiple scattering, e.g. within the subsurface or of the surface (Ivarez Prez, 2012). These higher-order contributions could also give rise to soil moisture effects in the interferometric returns from the soil. For instance, the double bounce between e.g. a vegetation stalk and a subsurface heterogeneity involves the same propagation phase within the soil as the direct return from the heterogeneity. It will thus introduce a similar dependence on soil moisture.

6 Conclusions

We introduced a fully polarimetric model that describes the impact of soil moisture changes on the measurements obtained using radar interferometry. It incorporates both volume scattering effects due to heterogeneities and a surface contribution in a first-order Born approximation of Maxwell’s equations. This combination is intended to make the model capable of accounting for varying contributions of these two sources, which have previously only been studied separately. The volume contributions are found to be dominant in an airborne L-band data set, when the model is calibrated using the Hal-lakainen mixing model to estimate their relative importance. The estimation of the relative size of the volume contributions was hampered by uncertainties due to the calibration of the soil moisture probes. Regarding the temporal dynamics, the uncertainties in the interferometric phase were considered to be an order of magnitude smaller than the observed soil moisture signals. The predominantly positive correlations between these observations and predictions of the calibrated model indicate that the model can capture some of the most salient features of the impact of soil moisture changes on the coherence $|\gamma|$ (median correlation at HH: 0.50), the phase $\phi$ (0.77), and the phase triplets $\Xi$ (0.56). However, the presence of these effects in the HV channel cannot be reconciled with the model. The description of these effects at HV is thus an open question for which the inclusion of anistropic heterogeneities or higher order expansions seem promising.

At all polarizations the model predictions depend on the parameterization of the depth-dependence of the volume scattering and the soil moisture-dependence of the dielectric constant of the soil, in particular the absorptive properties. The variation of these parameters impacts the predictions in a way similar to a change of the relative importance of volume and surface scattering. This parameter can thus partially compensate these differences, and similar qualities of fit can be achieved. These findings indicate that even though the model predictions are sensitive to these parameterizations, data-driven calibration of the model might render it robust enough to be used in future studies. Possible future accuracy assessments might elucidate the impact of the radar frequency and the soil properties on the validity of the model.

The qualitative agreement, as expressed by the correlations, points towards the possible use of such a model to estimate soil moisture. It might also be instrumental in accounting for changes in soil moisture in the estimation of deformations – a task for which the phase triplets are most promising, as they are insensitive to movements. They can thus potentially be used to remove the soil moisture effects from the estimated deformations due to, for instance, volcanic activity or groundwater-related subsidence.
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8 Author Contributions

All three authors devised the study. The model development and data analysis were done by S.Z. All three authors interpreted the results. S.Z and I.H. wrote the manuscript.

9 Conflicts of Interest

The authors declare no conflict of interest.

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Yin, Q.; Hong, W.; Li, Y.; Lin, Y. Analysis on Soil Moisture Estimation of SAR Data Based on Coherent Scattering Model. EUSAR, 2014.


A polarimetric first-order model of soil moisture effects on the DInSAR coherence
Supplementary Material

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1 Fresnel coefficients

The reflection and refraction of a plane electromagnetic wave at a plane surface which separates two homogeneous media are described by Fresnel’s equations. Their polarimetric properties are best described by decomposing the incident wave into two parts: the first one (horizontal h, or TE) has its E-field perpendicular to the normal vector of the plane, the second one (vertical v, or TM) has its H-field perpendicular to the normal vector (Jackson, 1999).

The $[T_{mn}]$ term in the model contains the two-way transmission coefficients $T_{v,m}$ and $T_{h,m}$, which pertain to the horizontal and the vertical part, respectively. In these expressions $m$ refers to the acquisition. These coefficients describe the change in amplitude that the E-field of the incident wave experiences when it i) penetrates into the soil, and ii) is transmitted back from the soil into the upper halfspace (with $\varepsilon = 1$). Assuming nonmagnetic media, these can be expressed as (Jackson, 1999)

$$T_{v,m} = \frac{2 \cos \theta^i}{\cos \theta^i + \sqrt{\varepsilon_m - \sin^2 \theta^i}} \frac{2 \cos \theta^t}{\cos \theta^t + \sqrt{\varepsilon_m - \sin^2 \theta^t}}$$
$$T_{h,m} = \frac{2 \sqrt{\varepsilon_m \cos \theta^i}}{\varepsilon_m \cos \theta^i + \sqrt{\varepsilon_m - \sin^2 \theta^i}} \frac{2 \sqrt{\varepsilon_m \cos \theta^t}}{\varepsilon_m \cos \theta^t + \sqrt{\varepsilon_m - \sin^2 \theta^t}}$$

In each of this terms, the first part represents the transmission into the soil, and the second one back into the air.

2 Phase terms

The interferometric phase $\phi_{mn}$ of a point scatterer embedded in the half-space consists of two terms (after linearization of (7) with respect to the antenna position). The first one $\tilde{\phi}_{mn}$ is due to the change in the background dielectric constant; if there is no such change, this term will vanish.

$$\tilde{\phi}_{mn} = 2k_0 \left[ w_m \sqrt{z^2 + (y_s - y_{i,m})^2} - w_n \sqrt{z^2 + (y_s - y_{i,n})^2} \right]$$

(1)

The second term $\bar{\phi}_{mn}$ accounts for the spatial baseline $B$, i.e. the variability of the antenna position between the acquisitions (Oveisgharan and Zebker, 2007). The spatial baseline can be decomposed into its horizontal $\Delta y_a = B \cos(\alpha)$ and vertical $\Delta H = B \sin(\alpha)$ components, where $\alpha$ is the angle between the baseline and the horizontal (Dall, 2007).

$$\bar{\phi}_{mn} = 2k_0 \left[ \sin \theta^i \Delta y_a - \cos \theta^i \Delta H \right]$$
$$= 2k_0 B \sin(\theta^i - \alpha)$$

(2)

Both these terms depend on the position of the point scatterer ($\phi_{mn}$ implicitly via the incidence angle). As volume scattering consists of contributions from different positions, it is expedient to study their dependence on the position. For small penetration depths (see the main document for details) only the first derivatives are important. We will evaluate these just below the surface, i.e. $z = 0$, by parameterizing the location of the point scatterer in terms of $y_s$ and $z$. The coordinates of the antenna and the surface are fixed parameters. All additional quantities, in particular the incidence and transmission angles as well as $y_i$, are thus determined too.
2.1 Total differentials

The total differentials of $\phi_{mn}$ with respect to $y_s$ and $z$ follow from Snell’s law and the geometric definitions given in the main document. Starting with $\tilde{\phi}_{mn}$, we find that the partial derivative with respect to $y_s$ vanishes. As the partial derivative with respect to the respective $y_i$ cancels in each of the terms (Snell’s law), we have

$$
\left( \frac{\partial \tilde{\phi}_{mn}}{\partial y_s} \right)_z = 2k_0 \left( w_m \frac{y_s - y_{i,m}}{\sqrt{z^2 + (y_s - y_{i,m})^2}} - w_n \frac{y_s - y_{i,n}}{\sqrt{z^2 + (y_s - y_{i,n})^2}} \right)
= 2k_0 (\sin \theta^i - \sin \theta^j)
= 0
$$

(3)

where the second line follows from Snell’s law as well.

The derivative with respect to $z$ is computed using the same identities:

$$
\left( \frac{\partial \tilde{\phi}_{mn}}{\partial z} \right)_y = 2k_0 \left( w_m \frac{z}{\sqrt{z^2 + (y_s - y_{i,m})^2}} - w_n \frac{z}{\sqrt{z^2 + (y_s - y_{i,n})^2}} \right)
= -2k_0 \left( \sqrt{w_m^2 - \sin^2 \theta^i} - \sqrt{w_n^2 - \sin^2 \theta^i} \right)
$$

(4)

We now turn to the part that depends on the baseline, i.e. $\bar{\phi}_{mn}$. Keeping $y_s$ fixed (implicit in the following equations), we have via the chain rule

$$
\left( \frac{\partial \bar{\phi}_{mn}}{\partial z} \right)_y = \frac{\partial \bar{\phi}_{mn}}{\partial \sin \theta^i} \frac{\partial \sin \theta^i}{\partial y_i} \frac{\partial y_i}{\partial z}
$$

(5)

In turn this term evaluates to:

$$
\frac{\partial \bar{\phi}_{mn}}{\partial \sin \theta^i} = 2k_0 B \frac{1}{\cos(\theta^i - \alpha)} \cos(\theta^i - \alpha)
$$

whereas

$$
\frac{\partial \sin \theta^i}{\partial y_i} = H \cos^{-3}(\theta^i)
$$

as $H$ is held constant. The last term follows from Fig. S1a

$$
\frac{\partial y_i}{\partial z} = \tan \theta^i
$$

Collecting these terms and expressing $\theta^i$ in terms of $\theta^t$, one finds

$$
\left( \frac{\partial \bar{\phi}_{mn}}{\partial z} \right) = \left[ -2k_0 B \frac{\cos \theta^i \sin \theta^i}{R \sqrt{w_m^2 - \sin^2 \theta^i}} \right]
$$

(6)

where $B_\perp = \cos(\theta^i - \alpha)$.

The partial derivative with respect to $y_s$, $\left( \frac{\partial \bar{\phi}_{mn}}{\partial y_s} \right)_z$, can be evaluated by similar means. As we evaluate $z$ just below the surface

$$
\left( \frac{\partial y_i}{\partial y_s} \right)_z \bigg| \frac{z}{\pi} \to 1
$$

i.e. the horizontal coordinate of the piercing point coincides with the one of the scatterer. Omitting the $z$ subscript for clarity, we get

$$
\frac{\partial \bar{\phi}_{mn}}{\partial y_s} = \frac{\partial \bar{\phi}_{mn}}{\partial \sin \theta^i} \frac{\partial \sin \theta^i}{\partial y_i} \frac{\partial y_i}{\partial y_s}
= \left( -2k_0 B \cos(\theta^i - \alpha) \frac{1}{\cos \theta^i} \right) \left( \frac{1}{R \cos^2 \theta^i} \right)
= -2k_0 B \frac{1}{R} \cos \theta^i
$$

(7)
2.2 Interferometric wavenumbers

These are the partial derivatives with respect to range \( R_m \) and \( z \), keeping the other one constant. They follow from the chain rule using the geometric relation shown in Fig. S1b. According to the highlighted triangle, a small change \( \delta z \) entails a change \( \delta y_s \) if \( R_m \) is to remain constant:

\[
\left( \frac{\partial y_s}{\partial z} \right)_{R_m} = \frac{\delta y_s}{\delta z} = \cot \theta_t
\]

where this is again evaluated just below surface.

The vertical interferometric wavenumber follows from the terms due to \( \bar{\phi}_{mn} \) and \( \hat{\phi}_{mn} \) and simplifying:

\[
\left( \frac{\partial \phi_{mn}}{\partial z} \right)_{R_m} = \left( \frac{\partial \bar{\phi}_{mn}}{\partial z} \right)_{y_s} \left( \frac{\partial z}{\partial z} \right)_{R_m} + \left( \frac{\partial \hat{\phi}_{mn}}{\partial z} \right)_{y_s} \left( \frac{\partial z}{\partial z} \right)_{R_m} + \left( \frac{\partial \bar{\phi}_{mn}}{\partial y_s} \right) \left( \frac{\partial y_s}{\partial z} \right)_{R_m} + \left( \frac{\partial \hat{\phi}_{mn}}{\partial y_s} \right) \left( \frac{\partial y_s}{\partial z} \right)_{R_m}
\]

\[
= -2k_0 \left[ \sqrt{\frac{w_m^2 - \sin^2 \theta_t}{w_n^2 - \sin^2 \theta_t}} - \frac{B_{\perp}}{R \sin \theta_t} \frac{w_m^2 \cos \theta_t}{\sqrt{w_m^2 - \sin^2 \theta_t}} \right]
\]

and similarly for the range interferometric wavenumber:

\[
\left( \frac{\partial \phi}{\partial R_m} \right)_z = -2k_0 \frac{B_{\perp} \cos \theta_t}{R \sin \theta_t}
\]

References


Chapter E

Suitability of differential SAR interferometry for soil moisture estimation: Can soil moisture changes be separated from displacements based on DInSAR data alone?

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Draft

Key findings:
- all observables are insensitive to certain temporal soil moisture variations
- all observables and polarizations are useful (to a different extent) for estimating soil moisture in L-band
- model cannot explain phase triplets and coherence at Ku-band, hence inversion only feasible for phase
- separation from displacements based on DInSAR data alone difficult
- separation difficult due to limited information content and sensitivity to noise/model misspecifications

The author’s contributions:
- developed and implemented the inversion procedure
- developed and implemented the calibration approach
- analysed and interpreted the results; wrote the manuscript

The co-authors’ contributions:
- I.H. suggested to develop an optimization-based inversion approach
- both co-authors provided radar data and helped with analysing the assumptions/limitations
- both co-authors contributed to writing the manuscript
Suitability of differential SAR interferometry for soil moisture estimation: Can soil moisture changes be separated from displacements based on DInSAR data alone?

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Abstract

Differential SAR interferometric (DInSAR) data are sensitive to displacements and to soil moisture ($m_v$) changes, but in different ways. Here, we present theoretical and observational analyses that show that when displacements can be ruled out or compensated for, the referenced DInSAR phase can be a suitable means to estimate $m_v$, time series up to an overall offset. In an L-band and a Ku-band campaign, temporal correlations with in-situ measurements between 0.75 and 0.9 are achieved. DInSAR data additionally comprise two observables that are also sensitive to soil moisture: the phase triplets and the coherence magnitude. As they are insensitive to displacements, they may be used to retrieve soil moisture so that the DInSAR phase and hence the displacement estimates can be corrected for the $m_v$ influence. However, our model results show that neither contains enough information for this purpose, i.e. it is not possible to estimate soil moisture uniquely without additional assumptions. The soil moisture correction of the phase/displacement estimates is hence ambiguous too. In practice, the applicability of these two observables is furthermore limited by their proneness to model misspecifications and decorrelation. Consequently, the separation of soil moisture changes and displacements using DInSAR data alone is difficult in practice, and additional data (e.g. external $m_v$ estimates) or assumptions (e.g. temporal model for displacements) may be required when the soil moisture effects on the displacement estimates (around 10% of the radar wavelength) are comparable to the magnitude of the movements.

1 Introduction

The remote sensing technique Differential SAR interferometry (DInSAR) is unique in that it can produce dense, synoptic estimates of movements of the earth’s surface with down to sub-centimetre precision [18, 40]. The key observable for estimating displacements is the interferometric phase, which is obtained by combining two time-lapse radar images [1]. In between these two radar acquisitions, the soil moisture content can change. Soil moisture changes have an impact on the measured phase, which can correspond to a displacement error on the order of 10% of the radar wavelength over bare soil [2, 42, 53]. For operational SAR satellites, this corresponds to errors of up to 3 mm to 3 cm (X-Band to L-band). These error magnitudes are larger than the precision of the estimates and than many movements of scientific interest. For instance, differential interferometry provides valuable information for studying frost heave and thaw subsidence in permafrost regions (typical magnitudes: cm per season) [29, 43] and for monitoring slow mass movements, the stability of dikes, and land surface subsidence (mm to cm per year) [7, 21, 41], Many such surface movement processes are closely coupled to soil moisture dynamics [41]. In the example of seasonal thaw subsidence, the active layer on the top of permafrost commonly becomes increasingly dry during the summer season [44]. As a decrease in soil moisture corresponds to spurious DInSAR displacement estimates towards the radar instrument [11, 42, 53], the apparent moisture-induced uplift may mask the actual surface subsidence.

The question hence arises how these two influences on the phase can be separated. Previous displacement estimation studies have focussed on detecting and masking areas where soil moisture and vegetation changes affect the deformation estimates [8, 14, 21], or on attributing the observed phases to geophysical displacements, atmospheric and soil moisture influences based on their expected spatio-temporal patterns [18, 19, 22, 36]. In particular, time series studies often implicitly assume that the temporal patterns and magnitudes of the soil moisture effects are such that they need not be accounted for explicitly when deriving long-term displacement estimates [28, 38], especially when filtering techniques are employed to account for the influence of orbit errors and atmospheric delays [20, 38]. Alternatively, soil moisture information may in the future be used to correct the DInSAR phase and thus account for moisture changes explicitly when estimating displacements. There are promising indications that soil moisture can be estimated from the DInSAR data directly [11], as three DInSAR observables are sensitive to soil moisture changes; apart from the phase, also the phase triplets and the coherence magnitudes are influenced by soil moisture changes [2, 12, 36, 53, 54]. Such DInSAR soil moisture estimation may furthermore complement more established soil moisture remote sensing approaches [47]. While the sensitivity of the DInSAR measurements to soil moisture changes has been documented in a range of experimental and observational
Figure 1: Soil moisture changes over time (a), which influences the DInSAR measurements, including the phase triplets. These are insensitive to displacements, but multiple soil moisture time series correspond to identical phase triplet observations. The non-uniqueness of the soil moisture estimation (b) implies that it is not possible to correct the interferometric phase unambiguously for soil moisture effects. When soil moisture effects are accounted for incorrectly (or not at all), this yields displacement errors that can be on the order of 10% of the radar wavelength (c; assuming no displacements actually occurred).

Studies [12, 35, 42, 53], De Zan et al. [11] were to our knowledge the only ones that tried to quantitatively estimate soil moisture changes, using the phase triplets and the coherence magnitude. The accuracy of the estimates and limitations of all three observables in the presence and absence of displacements have yet to be compared and quantified.

Displacements and moisture may potentially be separated based on DInSAR data, because they leave distinct imprints on the measured DInSAR observations, as De Zan et al. [11] have shown for interferometric stacks (three or more radar acquisitions). Specifically, displacements are temporally consistent. For three acquisitions at times $t_1$, $t_2$, $t_3$, the displacement between times $t_1$ and $t_3$ is equal to the sum of the two intermediate displacements; the same then also applies to that part of the phase due to displacements. The influence of soil moisture is different in that the phases are not consistent, i.e. they do not obey phase closure [12]. A measurement that quantifies the magnitude of the closure error is the phase triplet [14]. As the phase triplets are not affected by displacements, soil moisture may be derived from them even when displacements (and uncompensated atmospheric delays) occur [11]. Furthermore, the coherence magnitude is also insensitive to displacements but likewise influenced by soil moisture changes, and may hence contribute to this task. However, additional systematic influences on the coherence magnitude over soils are commonly observed [3, 19, 53], and these may bias or even preclude their reliable use in soil moisture estimation.

If it were possible to estimate soil moisture changes unambiguously from the coherence magnitude or the phase triplets, one could correct the DInSAR phases for the soil moisture influence without making any assumptions about the temporal evolution of the displacements or the soil moisture. However, the phase triplets contain less information than the phases themselves [25]. In particular, the model predictions by De Zan et al. [11] and Zwieback et al. [54] indicate that there are multiple soil moisture time series that correspond to identical phase triplets, even in the absence of additional unknown influences. One such ambiguity in phase triplet-based soil moisture estimation is shown in Fig. 1: in addition to the correct soil moisture time series, there is also an alternative, incorrect time series for which the model predicts identical phase triplets (in both cases, the phase triplets are exactly zero). These two soil moisture time series hence cannot be distinguished based on the phase triplets. Furthermore, when the estimated soil moisture time series are employed to correct the DInSAR displacement estimates (panel c in Fig. 1), one can obtain correct displacement estimates, but also spurious estimates when a wrong or no soil moisture correction is applied.

Here, we pursue two closely connected objectives in order to address the previously outlined knowledge gap. Our first objective is to analyse to what extent one can estimate soil moisture unambiguously from the DInSAR data. We concentrate on the two extreme cases: a) when displacements (and atmospheric effects) are known or can be ruled out, and b) without making any assumptions about the displacements (or phase offsets due to the atmosphere) or about the soil moisture evolution. To this end, in Sec. 2 we introduce the three DInSAR observables and how they are related to soil moisture changes. We show that in practice none of the three observables allows for the unique, unambiguous estimation of soil moisture when using the model by Zwieback et al. [54], and that these ambiguities are related to symmetry properties of the observables, i.e. they are expected to also apply to other models that link soil moisture changes to the DInSAR observations.

Our second objective is to quantify the real-world potential and the achievable accuracies of DInSAR soil moisture estimation over bare soil. In light of the theoretical limitations discovered in Sec. 2, we focus on the simplest case, when displacements can be ruled out and when the phase is referenced (no orbital and atmospheric influence). In this case, it may be possible to estimate soil moisture from all three observables, and we introduce an optimization framework for this purpose in Sec. 3. Our assessment of the real-world potential takes into account model inaccuracies and uncertainties, e.g. due to additional decorrelation sources or errors in the model parameterization. We do so by using a range of model parameterizations and by employing simulations when studying an L-band data set (CanEx-SM10 [30]), for which the DInSAR measurements can be described accurately as a function of soil moisture alone [54]. In order to characterize the frequency dependence of the achievable accuracies and limitations, we also estimate soil moisture from a large Ku-band data set (NoSREx, >200 acquisitions, [27]). In Sec. 4, we synthesize the theoretical and empirical findings pertaining to
2 Theoretical considerations

2.1 DInSAR and soil moisture changes

In radar interferometry, two radar acquisitions – sampled at different instances of space and/or time – are combined. The scattering of the microwaves, which is generally conceived of as a stochastic process [46], can be described by the second-order statistics: if the radar system is capable of receiving and transmitting different polarizations, they can be represented by the covariance matrix $\mathbf{C}_{m,n}$ of acquisitions $m$ and $n$ [10]. From the latter, the complex correlation coefficient $\gamma_{m,n}$ (the coherence) can be computed for fixed polarizations of the emitted and recorded wave, encoded by the $\omega$ projection vector [9]:

$$\gamma_{m,n} = \frac{\omega^H \mathbf{C}_{m,n} \omega}{(\omega^H \mathbf{C}_{m,m} \omega)(\omega^H \mathbf{C}_{n,n} \omega)^{1/2}}$$  \hspace{1cm} (1)

Its argument is the phase $\phi$, its magnitude the coherence magnitude $|\gamma|$.

The bicoherence $\Gamma_{mno}$ is the combination of three coherences (based on three acquisitions $m$, $n$, and $o$) [54]:

$$\Gamma_{m,n,o} = \gamma_{m,n} \gamma_{n,o} (\gamma_{m,o})^*$$  \hspace{1cm} (2)

whose argument is the phase triplet (or closure phase) $\Xi_{m,n,o}$

$$\Xi_{m,n,o} = \phi_{m,n} + \phi_{n,o} - \phi_{m,o}$$  \hspace{1cm} (3)

The closure phase does not require phase calibration, and it is invariant to atmospheric or topographic effects, as well as uniform displacements [12, 56].

All three observables $\phi$, $|\gamma|$, and $\Xi$ are sensitive to soil moisture changes. In the absence of actual moisture-induced surface elevation changes (e.g. clay swelling), an increase in soil moisture has been found to correspond to an interferometric phase $\phi$ that is associated with a lowering of the surface at L-band [11, 53] and at higher frequencies up to 12 GHz [42]. Both the sign and the magnitude of the soil moisture effect on the phase are consistent with subsurface volume scattering: the phase velocity of the electromagnetic waves decreases as the soil gets moister, corresponding to an increase in the optical path. Quantitatively, the effect can be described by analytic models like those by De Zan et al. [11] and Zwieback et al. [54], which can also successfully predict the phase triplets and coherence magnitudes. The latter model also includes surface scattering contributions, which have been directly observed in laboratory experiments [50]. The varying relative importance of these two contributions can partially account for the observed variability in the magnitude of the observed soil moisture effects [53, 54].

We use this model [54] that accounts for surface and subsurface scattering by introducing a free parameter, the volumeto-surface ratio $f$. The subsurface scattering is assumed to be due to dielectric heterogeneities, which are described as spatial variations in the permittivity. These heterogeneities are represented by point scatterers in Fig. 2, and the subsurface scattering contributions they give rise to are depicted as spherical wavelets. The surface contributions are also indicated in the figure, but the phase contribution of these interface reflections hardly changes as the soil becomes wetter [50]. Conversely, the scattering from within the soil is affected in two ways. Firstly, the real part of the refractive index of the soil increases upon wetting – or equivalently, the spacing of the phase fronts decreases, which is depicted in the figure. The subsurface heterogeneities thus appear to be further away from the sensor as the wave takes more time to travel along the path [11, 53]. Secondly, the absorption increases (indicated by the thickness of the wavefronts): heterogeneities deeper within the soil contribute less to the overall signal than for a dry soil. When applied to an interferometric pair of acquisitions, the first effect leads to a shift in the measured phase, and both of them combine to lead to decorrelation.
The first-order solution to Maxwell’s equations of this scenario consists of the incoherent addition of the rough surface scattering (small perturbation model, SPM) and a ‘volumetric’ term due to the heterogeneities. When the magnitude of the permittivity fluctuations is independent of depth, one obtains the following covariance matrix of the volumetric term

\[ C_{m,n}^v = \frac{1}{4W'} \left[ \langle 2i (k_{b,m} \cdot \hat{z} - k_{s,n}^* \cdot \hat{z}) \rangle \right] \]

where \( W' \) is a constant that depends on the magnitude of the dielectric fluctuations and the antenna patterns. It drops out upon forming \( \gamma \), as it appears in both the numerator and the denominator. \( T_{m,n} \) is a matrix that contains Fresnel transmission coefficients, and \( k_{b,m} \cdot \hat{z} - k_{s,n}^* \cdot \hat{z} \) is the vertical component of the dimensionless wavenumber in the soil at acquisition \( n \). The dimensionless wavenumber is obtained by using a reference penetration depth as length scale [54]. As the soil moisture (assumed constant with depth) changes between two acquisitions, so does the background dielectric constant (described by a mixing model) and with it the wavenumber.

The combination of this ‘volume’ term \( C_{m,n}^v \) with that due to the surface \( C_{m,n}^s \) yields the total scaled covariance matrix

\[ C_{mn} = \frac{f}{f_0,s} C_{m,n}^v + \frac{1}{f_0,v} C_{m,n}^s \]

where \( f_{0,s} \) and \( f_{0,v} \) are normalization magnitudes (at a fixed value of \( m_v \)), and \( f \) is the volume-to-surface ratio, which determines the relative importance of these two terms [54].

The volume-to-surface ratio \( f \) plays a crucial role in the soil moisture sensitivity of the phase \( \phi \). As illustrated in Fig. 3a, this sensitivity increases with increasing \( f \), as it is the volume term that shows pronounced \( m_v \) dependence. For a dominant volume contribution (\( f \rightarrow \infty \)), the magnitude of the interferometric phase is predicted to be on the order of 0.5\( \pi \), which is roughly consistent with observations of heterogeneous soils [2, 42, 53]. By contrast, strongly reflecting buried targets can give rise to much larger interferometric phase values: for a fixed soil moisture change, the phase is proportional to the depth of the target [34]. The observed values of \( \phi \approx 0.5\pi \) typical for heterogeneous soils correspond to an apparent depth of less than 10\% of the wavelength in free space. The assumption of uniform soil moisture profiles is hence expected to be accurate when the soil moisture does not change appreciably between the surface and depths of around 20-30\% of the wavelength, which also corresponds to typical penetration depths in moist soils [17]. However, the influence of non-uniform soil moisture profiles on the interferometric phase has previously been observed in laboratory experiments [42].

The interferometric phase for a heterogeneous soil is predicted to depend nonlinerly on the soil moisture values. This is evident in two ways: firstly, the predicted \( \phi \) does not only depend on the soil moisture difference \( \Delta m_v \), but also on that of the ‘master’ acquisitions (red and blue lines). Secondly, the phase response saturates as \( \Delta m_v \) increases. This saturation depends on the mixing model, as illustrated in Fig. 3b: it is more pronounced for the mixing model by [39] (Pep) than for the model proposed by [17] (Hal), both of which apply to L-band frequencies. The former also shows stronger decorrelation for fixed \( \Delta m_v \) (cf. Fig. 3c). The dissimilarities in the predictions are mainly related to the differences in the implied wave attenuation [54]. Owing to these differences in the predictions of the forward model, it is expected that they also carry over to the estimation of soil moisture from interferometry; this will be studied using both simulated and observed L-band data. At Ku-band, the mixing model by [32] leads to predictions that are similar to those by the Peplinski model. Irrespective of the mixing model, the model (5) predicts zero surface and volume backscatter at HV polarization [54]. It hence cannot reproduce the \( m_v \) dependence of the HV interferometric observables, which has been found to be similar to that at HH and VV [54].
Alternatively, the influence of soil moisture changes $\Delta m_v$ on the interferometric phase can also be described by a linear model, i.e.

$$\phi_{m,n} = \beta \Delta m_v - \beta(m_{v,n} - m_{v,m})$$  \hspace{1cm} (6)

Barrett et al. [2] observed significant linear correlations between these two quantities at C and L-band, and Zwiebesch et al. attributed the observed significant linear relations observed at L-band to subsurface volume scattering [53]. However, the linear model of (6) predicts zero phase triplets, in contrast to observations at L-band and nonlinear volume scattering models [11, 54].

The phase triplet information can thus be used to infer soil moisture changes [11], or more precisely, to constrain it. After all, the example in the introduction (Fig. 1) shows that even for the nonlinear model, there are situations when such unambiguous soil moisture estimation based on the phase triplets is not possible even in the absence of noise and model deviations. Furthermore, the phase triplets are often small compared to the interferometric phases, or in other words the linear model describes the soil moisture impact on the interferometric phases well [53]. The phase triplets are hence expected to be sensitive to noise or to model deviations that are e.g. associated with the mixing model in Fig. 3 [12, 54, 56].

We will now study how the limited information content of the phase triplets and the other two observables restricts our ability to estimate soil moisture time series from DInSAR data in both the presence and absence of displacements.

2.2 Lack of uniqueness in soil moisture estimation

2.2.1 Limited information content

We will first show that even in the absence of displacements and other phase offsets (e.g. atmosphere), the DInSAR phase observations $\phi$ are expected to be insensitive to certain temporal variations in soil moisture, implying that soil moisture cannot be retrieved uniquely from these observations. The lack of uniqueness is related to a permutational symmetry of the phase. Let the interferometric phase $\phi_{i,j}$ be described as a function of a state variable $s$ at times $i$ and $j$, i.e. $\phi_{i,j} = \phi(s_i, s_j)$. In our case the state is the surface soil moisture $m_v$, but the reasoning applies just as well to the surface location or the atmospheric delay. The phase observable is antisymmetric with respect to permutations [1], i.e. $\phi_{i,j} = -\phi_{j,i}$, so that it vanishes for any such model if $s_i = s_j$. In other words, the phase $\phi_{i,j}$ is expected to be exactly equal to zero for identical states, e.g. when the soil moisture is identical at times $i$ and $j$. When including more than two times, we see that all the phases $\phi_{i,j}$ will be zero if the time series of states is a constant, the value of which thus cannot be recovered [11, 18]. In the case of soil moisture, one can hence infer based on the phase observations $\phi_{i,j}$ that the soil moisture is constant, but not what value it has.

More generally, the ambiguity in $\phi$-based soil moisture estimation is not limited to time series of constant $m_v$. If the phase is modelled as a linear function of the difference of soil moisture as in (6), the soil moisture time series will only be retrievable up to an unknown offset. Put differently, any state vector representing a constant value of $m_v$ will be in the null space of the mapping $A$ from all moisture values $m_v$ to all phases $\phi$. This is in general not the case for a nonlinear model like (5). However, it suggests a diminished sensitivity to a time-invariant offset, and this is indeed found empirically in Fig. 4a. In order to generate the figure, the phases of all possible interferograms based on each pair of scenarios (red and blue soil moisture time series), which differ by an offset, are formed. The maximum mismatch between these two sets of phases is indicated in the top left corner. All of these mismatches are about one order of magnitude smaller than the typical error of the model [54], indicating that the two soil moisture time series are virtually indistinguishable given typical noise levels or uncertainties in the specification of the model.

In practice, it will hence typically only be possible to estimate the soil moisture time series up to an unknown offset, assuming the phase to be otherwise known (compensated for displacements, atmospheric effects, orbit errors). If we cannot adjust the phase for these effects, we may look at the phase triplets to estimate soil moisture [11]. As the phase triplets capture a subset of the entire phase information $\phi$ [25], we expect that such soil moisture estimation will only work up to an offset as well.

However, even when the irretrievable soil moisture offset is known, the phase triplets cannot always be inverted unambiguously. This is illustrated in the example in the introduction (Fig. 1). The reason is again closely related to a permutational symmetry. The phase triplets $\Xi_{i,j,k}$ are antisymmetric with respect to odd permutations and symmetric with respect to even ones [25], such that e.g. $\Xi_{i,j,k} = -\Xi_{j,i,k} = \Xi_{k,i,j}$. Any model whose predictions $\Xi_{i,j,k} = \Xi(s_i, s_j, s_k)$ are based exclusively on the states will thus predict zero phase triplets if among these three states there exists a pair of identical states, such as $s_j = s_k$. It will consequently also predict exclusively vanishing $\Xi$ for any time series whose samples only take on one or two distinct values. From the point of view of inversion, this implies that any such model cannot distinguish between a constant time series of states and one where the state takes on only two distinct values, e.g. a step or a sawtooth pattern, as in the example in the introduction. This also pertains to the nonlinear soil moisture model (5). Furthermore, we find that the phase triplets predicted by this model are relatively insensitive to the addition of such patterns, even when the soil moisture is not restricted to only two distinct values. In Fig. 3b, we find that the addition of an offset, a step or a sawtooth pattern to a soil moisture time series has a limited impact (smaller than the typical noise level or the accuracy of the model) on the predicted phase triplets. Conversely, the phase triplets predicted by the linear soil moisture model (6) are always zero and hence completely insensitive to soil moisture changes.

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Finally, we turn to the coherence magnitude, which also turns out to be insensitive to certain transformations of the soil moisture time series due to a permutational symmetry. The coherence $|\gamma_{ij}|$ is symmetric to the permutation of the two acquisitions [1], i.e. $|\gamma_{ij}| = |\gamma_{ji}|$. Let it be modelled as a function of states (under the same assumptions as above): $|\gamma_{ij}| = |\gamma(s_i, s_j)| = |\gamma(s_j, s_i)|$; the second equality is a consequence of the permutational symmetry. The permutational symmetry has two consequences; $|\gamma|$ is relatively insensitive to offsets, and wetting and drying are difficult to distinguish. The simplest empirical models will describe $|\gamma_{ij}|$ as a function of $|\Delta s| = |s_i - s_j|$; $|\gamma_{ij}| = f(|\Delta s|)$, where $f(x)$ might e.g. be an exponential function [19, 53]. If $f(x)$ is bijective (for appropriately defined domain and codomain), one can infer $|\Delta s|$ uniquely from $|\gamma|$. Even in the absence of such a one-to-one correspondence, it may still be possible to infer all $|\Delta s|$ in a time series as the problem is over-determined. The knowledge of all the $|\Delta s|$ does not necessarily recognize the reconstruction of the states: as it depends on the differences, it is invariant to shifts of all the states, i.e. as for the linear phase model, the offset is not recoverable. Furthermore, one cannot distinguish between two flipped versions of the time series of states $s$ and $s'$, where $s_i - s_j = -(s'_i - s'_j)$. This lack of retrievability does not necessarily occur for a more general model like (5), where $|\gamma|$ is not only a function of $|\Delta s|$. However, it suggests a reduced sensitivity, which is indeed observed for (5): Fig. 4c shows such cases, where the two scenarios differ by offsets and in one case also by mirroring, in which the role of drying and wetting is reversed. The differences in the predicted observables are much smaller than the accuracy of the model [54], and thus not likely to be distinguishable in practice.

### 2.2.2 Practical consequences

The limited information content of all three observables impacts the separation of soil moisture changes and displacements based on DInSAR data alone. If we wanted to estimate the soil moisture time series from the phase triplets in order to correct the displacement estimates, we would have to account for their lack of sensitivity to step changes or sawtooth patterns (or flipped time series when using $|\gamma|$). After all, the addition of such a soil moisture time series corresponds to a non-zero interferometric phases, which would hence correspond to different displacement time series (see the example in Fig. 1). In practice, these patterns could for instance be constrained by using a priori information or external measurements of the soil moisture times series.

The limited information content also impacts the estimation of soil moisture in the absence of displacements. In light of these theoretical limitations, we will impose a priori knowledge when inverting the DInSAR observations to estimate soil moisture. As all observables are insensitive to offsets of the soil moisture time series, we will fix the first soil moisture value [11]. We will restrict ourselves to referenced phases (pre-processed so as to account for the atmosphere, topography, etc.) and zero displacements when estimating soil moisture using the interferometric phase. For the phase triplets and the coherence magnitudes, which are insensitive to displacements and atmospheric influences, we will assume to have implicit a priori knowledge, which is given by the initial values of the inversion. These initial values may e.g. be obtained from a linear phase model assuming no displacements. We will show using simulations in Sec. 3.2 that such a regularization based on initial values can be sufficient when the observations fit the model predictions accurately. The insights gained from the inversions in this study may in the future be harnessed to account for the structural limitations of the DInSAR observables more explicitly in a data fusion framework that extends the inversion approach we introduce here. However,
we will briefly pick up on this point in the preliminary discussion in Sec. 4.3.

3 Soil moisture inversion in the absence of displacements

3.1 Inversion as an optimization problem

The proposed inversion is framed as an optimization problem, in which the unknown soil moisture values are adjusted so that the discrepancy between the predictions and the observations is minimized. Specifically, the inversion consists of two steps (Fig. 5). In the first step, all the available images are combined to compute all possible interferograms, from which the observables such as the phase \( \phi \) are derived. Based on this set of DInSAR observations, the second step proceeds by retrieving a soil moisture time series by minimizing the misfit between observations and predictions. The predictions are based on either the full model (5) or the linear model of (6), and the only unknowns are the soil moisture values; all other parameters are fixed (i.e. assumed to be known or determined before the inversion). The inversion is done for each observable separately using a dedicated misfit function.

The misfit between the simulated phase \( \hat{\phi} \), which depends on the soil moisture time series, and the observed one \( \phi \) is taken to be a least squares criterion

\[
\mu_\phi^2 = \sum_{n > m} (\hat{\phi}_{m,n} - \phi_{m,n})^2
\]

and similarly for the coherences \( |\gamma| \) and the phase triplets \( \Xi \)

\[
\mu_{|\gamma|}^2 = \sum_{n > m} (|\gamma_{m,n}| - |\gamma_{m,n}|)^2
\]

\[
\mu_\Xi^2 = \sum_{n > m} (\hat{\Xi}_{m,n,o} - \Xi_{m,n,o})^2
\]

where one acquisition \( m_0 \) is held fixed in \( \mu_\Xi^2 \) to avoid redundancy [33]. The combinations of acquisitions \( n, m, \) and \( o \) can be restricted further to exclude cases where the model is not expected to apply, e.g. where sufficiently long temporal separations lead to decorrelation that cannot be solely attributed to soil moisture changes.

The model of (5) relies on reference backscatter values to define the volume-to-surface ratio \( f \). These are obtained using a reference soil moisture \( m_{v,0} = 0.2 \text{m}^3\text{m}^{-3} \); the same value is also used to compute the penetration depth for obtaining the dimensionless wavenumber in (5). The soil moisture value at the first acquisition is held fixed owing to the small sensitivities to offsets found in Sec. 2.2.

3.2 CanEx-SM10: L-band

The proposed inversion scheme is applied to simulated and measured DInSAR observations at L-band. The simulated data allow us to study the achievable accuracies when the model is known perfectly and to quantify the impact of model deviations (e.g. noise level, model misspecification) on the estimates. We first introduce the data set and the study area, followed by a description of the simulation study. Subsequently, we present the inversion results of the simulated and the actually observed data.

Figure 5: Overview of the inversion processing chain. Starting from the top: the soil properties at each instance of time \( t_i \) give rise to the measured SAR image. From all the images, all possible interferograms are formed and the observables (in this case the phase) derived. The collection of observations is inverted to yield a soil moisture time series.
3.2.1 Data and study area

The radar and in-situ data were acquired within the Canadian Experiment for Soil Moisture in 2010 [30] over the Kenaston, Saskatchewan, Canada, study site (51°30'N, 106°18'W). The area is characterized by rainfed agricultural fields, grassland, and pastures. The relief is flat, and at least 1.5% of the area are covered by open water surface. This percentage was likely higher during the campaign (June 2 - 14, 2010), as it had been preceded by wet weather conditions. The soil is predominantly loamy [30], so that no widespread shrinking and swelling is expected. Neither are deformations due to tectonic or groundwater-related processes (owing to the location and the time scale of two weeks). If such displacements were present, their spatial patterns would differ from the observed ones and their impact on the phase is expected to be much smaller than the observed values [19, 53].

Volumetric soil moisture \( m_v \) was measured hourly at permanent stations in ten fields by Environment Canada (EC) using Stevens Hydraphobe probes in several depths, of which the 0-5 cm vertical sensor will be considered [30, 53]. The fields were all bare or partially covered with harvest residues [30]. Among these ten monitored fields, seven have already been used to assess the electromagnetic scattering model [54]. The remaining three ‘focus’ fields (numbers: 203, 219, 330; all characterized by loamy soils and partially covered with crop residues) will be studied in greater detail.

The UAVSAR system acquired 6 L-band radar images in irregular intervals over the entire study area at zero baseline [19]. These data comprise four polarizations (HH, HV, VH, and VV) and have a resolution of 1.7 m in range, and 0.8 m in azimuth [15, 22]. Here, we combine the radar data interferometrically (\( L = 172 \) looks) and thus estimate covariance matrices \( C_{l,m} \) between acquisitions \( l \) and \( m \) [10]. These raw interferograms contain an unknown phase offset and trends. When analysing the ten monitored fields separately, we reference the offset with respect to a persistent scatterer [23]; see [53] for an analysis of the sensitivity with respect to the referencing. When inverting soil moisture over the entire study area, we remove a third-order polynomial trend over the entire image. However, the images contain kilometre-scale residual phase contributions due to the atmosphere and orbital errors, which we do not remove. In the inversion for each of the ten fields, the first soil moisture value is taken to be the one measured by the corresponding in-situ probe; conversely, for the inversions over the entire study area it is set to 0.35 m\(^3\) m\(^{-3}\) (an average value of all in-situ measurements).

The quantitative assessment of the quality of the inversion is hampered by the brevity of the soil moisture time series. Not only does it prevent us from calibrating the measured and estimated values with respect to each other, but it also renders the computation of RMS differences problematic. Such a calibration is generally deemed necessary because of differences in the spatial scale and uncertainties in the calibration of the remote sensing and in-situ data [47, 57]. We thus restrict the assessment to the sample correlation coefficient \( \rho \), as it is insensitive to additive and multiplicative biases. Due to the limited number of samples, its standard error is expected to exceed 0.05 and it is also generally a biased estimator of the population correlation coefficient [37].

3.2.2 Simulation study: methods

As we cannot estimate the model parameters \( f \) or \( \beta \) in a calibration procedure – a typical situation in practice, e.g. when no soil moisture reference data are available –, we test the impact of a wrong model specification in a simulation study. The simulated data mimic the observations in length and temporal variability. Specifically, our objectives are to i) estimate the performance inversion as a function of the noise level, and ii) to study the impact of a wrong mixing model and misspecified model parameters.

The data are simulated according to distinct scenarios, which differ with respect to the mixing model, the values of the volume-to-surface ratio \( f \), and signal-to-noise (SNR) ratios, see Tab. 1. In order to achieve varying SNR levels, the \( C_{m,m} \) second-order statistics are contaminated with additive noise that is uncorrelated for distinct acquisitions \( m \). The noise can correspond to thermal measurement noise but also to decorrelation processes such as temporal decorrelation due to changing surface roughness [51]. In the simulations, the prescribed soil moisture time series is the same in all scenarios: it mimics observations and is shown in Fig. 7. Based on the covariance matrix simulated according to (5) and the Gaussian speckle model [16], \( L \) samples of the scattering vectors are drawn for each of the \( N \sim 500 \) independent ensemble members. The six inversion methods, which correspond to the forward models and parameterizations shown in Tab. 1, are applied to each of the \( N \) ensemble members, resulting in \( N \) estimated soil moisture time series. In order to assess the quality of these inversions, the average correlation coefficient with the reference, \( \bar{\rho} \), and also the RMSE are computed.

3.2.3 Simulation study: results

The inversions based on the phase \( \phi \) reproduce the temporal patterns accurately. The accuracy of the estimated soil moisture is summarized by the two metrics \( \bar{\rho} \) and RMSE, which are shown in Fig. 6. For the inversions based on the phase \( \phi \), the correlation coefficients \( \bar{\rho} \) are found to generally exceed 0.95 (exception: the Peplinski-based inversion method P when the phase is simulated using the Hallikainen mixing model). While the temporal patterns are reproduced accurately, a misspecification of the model (e.g. wrong mixing model or \( f \)) affects the overall scaling of the inverted time series, and hence the RMSE error. The assumed noise level has limited impact on the temporal correlations (differences in \( \bar{\rho} < 0.05 \)), which is also reflected in the respective estimated soil moisture time series of Fig. 7.
Table 1: Model configurations used in the L-band CanEx-SM10 inversions: in the simulations, the nonlinear model configurations are used to predict the observables (subsequently perturbed by noise). All model configurations can be employed in the inversions of the observed or simulated data.

<table>
<thead>
<tr>
<th>Name</th>
<th>Observables</th>
<th>Polarizations</th>
<th>Model (equation)</th>
<th>Mixing model</th>
<th>ln f [-]</th>
<th>β [rad m$^3$ m$^{-3}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>φ,</td>
<td>γ</td>
<td>, Ξ</td>
<td>HH, VV</td>
<td>Nonlinear (5)</td>
<td>Hallikainen</td>
</tr>
<tr>
<td>H'</td>
<td>φ,</td>
<td>γ</td>
<td>, Ξ</td>
<td>HH, VV</td>
<td>Nonlinear (5)</td>
<td>Hallikainen</td>
</tr>
<tr>
<td>P</td>
<td>φ,</td>
<td>γ</td>
<td>, Ξ</td>
<td>HH, VV</td>
<td>Nonlinear (5)</td>
<td>Peplinski</td>
</tr>
<tr>
<td>P'</td>
<td>φ,</td>
<td>γ</td>
<td>, Ξ</td>
<td>HH, VV</td>
<td>Nonlinear (5)</td>
<td>Peplinski</td>
</tr>
<tr>
<td>L</td>
<td>φ</td>
<td>HH, HV, VV</td>
<td>Linear (6); inversion only</td>
<td>–</td>
<td>–</td>
<td>5</td>
</tr>
</tbody>
</table>

The coherence $|γ|$ can also be used to retrieve the temporal patterns accurately when the SNR is high (10 dB; similar exceptions as for the phase). However, for lower SNR, the loss of accuracy is evident in the estimated soil moisture time series of Fig. 7. The inversion wrongly attributes the drop of coherence introduced by the random noise to changes in soil moisture.

Also the phase triplet inversions are prone to noise, and they are furthermore sensitive to model misspecification. Even when the noise level is low – and in contrast to the other two observables – the impact of the mixing model and the choice of $f$ are not only evident in the RMSE values but also in the correlations. When the noise level is higher (SNR of 3 dB), the proposed inversion strategy cannot reliably estimate $m_v$ even with a correctly specified model, as large spurious deviations with the true time series are frequent (Fig. 7). The deviations are reminiscent of the patterns identified in Sec. 2.2 to which the phase triplets are very insensitive, e.g. step changes.

3.2.4 Observed data: results

In contrast to the simulation study, the soil moisture is not known a priori in the observational data; rather, the corresponding in-situ measured $m_v$ will be taken as reference for each field. Owing to the limited impact of the choice of $f$ on the obtained correlations in the simulations, as well as the previously inferred values of $f$ in the data set [54], the following comparisons will be restricted to the inversion methods corresponding to larger $f$ (predominant volume scattering): H (Hallikainen), P (Peplinski), and L (Linear) from Tab. 1.

The inverted soil moisture (based on the HH and VV phase) in Fig. 8 shows a temporal behaviour that is similar to the in-situ measured $m_v$, with the wetting before the third acquisition and subsequent drying are present in all of them. The magnitudes of the temporal variations are around 70% smaller for the inversion based on the Peplinski model (P, see Tab. 1) than for the Hallikainen one (H) or the linear model (L). There is no clear pattern regarding the temporal correlations $ρ$: the two polarizations HH and VV and all three models achieve similar correlations for the three focus fields, and between 0.78 and 0.85 on average, i.e. for all 10 fields, cf. Fig. 9.

The linear model (L) was applied to the entire HH image, thus yielding a spatially continuous soil moisture time
series, of which two instances are shown in Fig. 10. They depict the wetting between the second (6 June) and the third acquisition (9 June), which is evident for almost all fields. The areas that do not follow this trend are mainly water bodies and fields for which coherences $|\gamma| < 1$ are observed in Fig. 10. The same linear model, when used to estimate $m_v$ from the HV phase, yields soil moisture time series for all fields that are comparable to both HH and VV, with similar magnitudes and correlations in Fig. 9.

Estimated soil moisture from the coherence $|\gamma|$ (HH: Fig. 11) shows average correlations with in-situ $m_v$ of around 0.85 for both the H and the P model. The spread of these $\rho$ values is slightly larger than for the phase, in particular in VV. However, for the three focus fields, lower correlations are not observed in Fig. 11. These time series also do not show the flipping effect, and neither does any of the remaining seven fields.

Slightly lower average correlations of 0.67 - 0.86 are obtained when comparing the in-situ measurements with the inversions based on the Hallikainen model and the phase triplets $\Xi$, see Fig. 9. The Peplinski model achieves lower correlations (average $< 0.1$ at HH, 0.75 at VV), as outliers are common. For both mixing models, such fields where the inversions do not correspond to the in-situ measurements occur, e.g. field 330 in Fig. 11. The Hallikainen-based inversion suggests a wetting on June 15, as opposed to the in-situ measurements. This step-like pattern that characterizes the offset between the inverted and the in-situ measured time series is comparable to the jump in one of the simulations in Fig. 4c, which cannot be resolved reliably owing to the structural limitations of the phase triplets.

### 3.3 NoSREx: Ku-band

We now apply the inversion framework to Ku-band data. At such high frequencies, the physical model of (5) has not been assessed previously. We hence focus on the applicability of DInSAR to estimate soil moisture at Ku-band, for which e.g. additional decorrelation effects may be expected to be more pronounced than at lower frequencies [3].

#### 3.3.1 Data and study area

Within the Nordic Snow Radar Experiment (NoSREx), a ground-based radar time series of a forest clearing in Sodankylä, Finland (67.36N, 26.63E), was acquired between 2009 and 2013 [27]. The forest clearing is characterized by mineral soil (sandy loam) and sparse lichen and moss cover. This Intensive Observation Area (IOA) also contained additional instruments, such as a soil moisture impedance probe (Delta-T ML2X) in 2 cm depth at a distance of around 20 metres.

Despite the campaign’s focus on snow cover, radar measurements were also made every four hours during the summer months of 2012; we use those between 1 July and 1 October. During this period, 182 mm of rain were recorded by a drop-counting rain gauge located at a distance of around 500 m from the forest clearing [27]. The daily maximum temperatures typically varied between 15 and 25°C during July and August and dropped to values of around 5°C at the end of September. According to a nearby soil temperature probe (2 cm depth), soil freezing did not occur during this period.

The fully polarimetric radar measurements were acquired in the frequency range between 9.2 and 17.8 GHz [27]. We use data at an incidence angle of 30°, a bandwidth of 5 GHz of bandwidth centered at 12 GHz and $L = 50$ looks. The phase of each radar acquisition is referenced with respect to a quasi-simultaneous observation of the calibration sphere [26]. The resulting phase-referenced SLC values are subsequently combined interferometrically with temporal baselines of up to 16 h, i.e. a maximum span of $\Delta a = 4$ acquisitions. This value was chosen as the radar data typically decorrelate

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1 Videos are available online at [52]

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Figure 7: L-band simulation study results, showing the estimated soil moisture $m_v$. The columns correspond to the three observables on which the inversion is based, the rows to different noise levels (SNR of 10 and 3 dB for the top and bottom row, respectively). Each panel shows 75 inverted $m_v$ time series (lines) and the underlying reference time series (black diamonds). The observables were simulated according to (5) using the H configuration (Hallikainen mixing model, $\ln(f) - 10$). The nonlinear inversion was based on the same correct model configuration (mixing model and value of $f$).
The observed interferometric phases $\phi$ only show a clear correspondence with the in-situ soil moisture differences once the latter have been calibrated. Without calibration, there is no clear relation ($R^2 = 0.03$ for $\phi_{HH}$ during TP-C, see Fig. 12). The radar data are, however, overdetermined as $\Delta a = 4$ interferograms are formed for each acquisition, and these interferograms are internally consistent with the linear model of (6). When the noise of the in-situ data is considered in the calibration (quantified by a parameter $\kappa$ as described in Sec. A.1), the calibrated soil moisture differences show a stronger correspondence to the interferometric phases ($R^2 = 0.99$ for $\phi_{HH}$ during TP-C, see Fig. 12). The sign and size of the linear component are consistent with dominant volume scattering. However, the correspondence with the full nonlinear model is weaker, as the larger phases (esp. $|\phi| > 0.25\pi$) cannot be predicted accurately: the observed nonlinear component is smaller than predicted by the model. The limitations of the nonlinear component of the phase implies that the phase triplets are greatly overestimated and they are barely correlated with the observations Fig. 12. Furthermore, the
Figure 10: Inverted soil moisture \([m^3 m^{-3}]\) based on the linearized model for the phase \(\phi_{HH}\) in the CanEx-SM10 campaign before (left, June 6) and after (middle, June 9) a rain event. The linearized model with the uniform parameterization is applied over the entire image, including to fields that decorrelate during the campaign (e.g. tilling) or to open water bodies. The right panel gives the coherence \(|\gamma_{HH}|\) between the first (June 5) and last (June 15) image. Note that the phase is not atmospherically corrected.

![Graph showing soil moisture and phase](image)

Figure 11: Soil moisture \(m_v\) \([m^3 m^{-3}]\) estimated using the DInSAR coherence magnitude (left) and phase triplets (right) at HH for the three focus fields (number in upper left corner) of the CanEx-SM10 campaign. The DInSAR inversion is based on the nonlinear model using the Hallikainen mixing model (H, red circle) and using the Peplinski mixing model (P, purple triangle). The in-situ measurements (M) are shown in yellow. The numbers to the right of each panel give the correlations \(\rho\) between the in-situ measurements and the inversions for each method.

![Graph showing soil moisture and phase](image)

observed coherence magnitudes cannot be reconciled with the model that assumes that soil moisture changes are the only source of decorrelation, see Fig. 12. Owing to these limitations, the analysis of the inversion results will be restricted to the phase and focussed on the linear model.

The inverted soil moisture time series based on the co-polarizations HH and VV in Fig. 13 capture the gradual drying

![Graph showing soil moisture and phase](image)

Figure 12: Scatter plots of HH observables obtained during the calibration phase TP-C of the NoSREx data set \((\Delta \alpha = 4)\). In a), uncalibrated soil moisture differences measured in-situ are plotted against the observed interferometric phases. In b) the observed phase \(\phi_{HH}\) is compared to one that is predicted by the calibrated linear model using the corrected soil moisture time series \(m_v\), \(\phi_{HH}\). Panels c-e) compare the measured to the predicted observables (c) phase, d) coherence magnitude, e) phase triplets) based on the calibrated nonlinear model.
in July and August, and the increasingly wet conditions in September. They disagree with the in-situ records regarding the timing of the latter as they indicate a sudden wetting on 1 September. So does the radar backscatter but not the in-situ sensor (2 cm depth), which suggests a more gradual increase in \( m_v \). Such disparate responses to precipitation events (e.g. also 31 July) are the most conspicuous differences apart from the diurnal variations. This similarity to the in-situ measurements is smaller for the HV polarization, with deviations exceeding 0.02 \( m^3 m^{-3} \) occurring for instance during the second half of September. The associated correlation coefficients \( \rho \) in Tab. 2 reflect these differences, with HH and VV achieving values between 0.85 and 0.9 during the total time period TP-T, as opposed to a value of 0.75 for HV. These quantities are substantially lower (\( < 0.4 \)) during the validation period TP-V, during which the temporal variability is less pronounced (\( < 0.03 m^3 m^{-3} \) according to the in-situ probe). However, the root mean square deviations are similar in both time periods, with typical values around 0.012 \( m^3 m^{-3} \) (difference of around 0.005 \( m^3 m^{-3} \)).

At HH and VV, the inversion results are not sensitive to changes in the maximum temporal span of the interferograms \( \Delta \alpha \) (see Fig. 13). Conversely, the deviations of the HV phase inversion to the in-situ reference are less pronounced when using fewer interferograms, i.e. \( \Delta \alpha = 2 \) instead of 4. Overall, the inverted time series do not show a conspicuous temporal drift, which may have been expected as the inversion is a differential technique that may be prone to the accumulation of errors when applied over longer time spans.

The nonlinear model of Eq. (5) achieves higher correlation coefficients and lower RMSD than the respective linear model for both polarizations and either evaluation period (see Tab. 2). The increased agreement with the in-situ data is not reflected in the internal consistency with respect to the overdetermined radar measurements: the misfits \( \mu_\phi^2 \) of Eq. (7) increase by about a factor of 100 compared to the linear model.

4 Discussion

The theoretical results, the two case studies and the simulations indicate the potential suitability but also limitations of the three DInSAR observables \( \phi \), \( \gamma \), and \( \Xi \) for estimating soil moisture. We will first synthesize these findings to characterize the extent to which each observable can be used for soil moisture estimation in the presence and absence of displacements, i.e. its information content (objective 1). Subsequently, we assess the achievable accuracies and limitations
in the presence of noise and additional decorrelation sources, when there are no displacements (objective 2). We will then discuss the implications of these findings for displacement studies, in which movements and soil moisture changes occur simultaneously.

### 4.1 Objective 1: Information content in the presence and absence of deformations

In the absence of deformations, the phase observable is suitable for soil moisture estimation up to an offset, assuming the phase can be referenced accurately (e.g. removal of the atmospheric phase screen). The entire soil moisture time series can hence be reconstructed when the first (or any other) value is fixed or assumed known (Fig. 4).

In the presence of arbitrary displacements, the phase cannot be used directly to estimate soil moisture changes as the displacements also affect the observed phases. For $N$ acquisitions, in the best case – i.e. when all acquisitions remain coherent – there are $\binom{N}{2}$ phases, but only $N - 1$ displacements and soil moisture changes each. The algebraic structure of the phases is such that they can be split into two parts [25]: one that is influenced by displacements and soil moisture (and the atmosphere and phase offsets) and which has $N - 1$ degrees of freedom, and a remainder. The latter contains the phase triplets and is influenced by nonlinear changes (soil moisture, but not displacements, atmosphere, or phase offsets). Without making any assumptions about the displacements, only the remainder, i.e. the phase triplets, can be used for soil moisture estimation. There are two problems with using this latter part for this purpose. Firstly, its signals are small, affected by noise, and difficult to model [12, 56]; these aspects will be addressed in the next subsection. More fundamentally, even when the model applies and is calibrated perfectly, there are structural limitations.

The permutational symmetries of the phase triplets imply that a constant soil moisture time series cannot be distinguished from step changes or sawtooth patterns (Sec. 2.2). The fact that this limitation is related to an intrinsic property of the observable indicates that it is not restricted to the particular model of (5). For this model, the simulations indicate that the phase triplets are insensitive to the addition of offsets [11], step changes or sawtooth patterns, see Fig. 4. Thus, even when the model applies perfectly, the phase triplets alone are not sufficient to reliably estimate soil moisture dynamics, but they contain valuable information in constraining the soil moisture evolution. In practice, one needs a priori information (e.g. in the form of initial values or constraints in an optimization framework) about the soil moisture. Conversely, a priori information about the displacements is not helpful as they do not influence the phase triplets.

The coherence magnitude $|\phi|$ is also sensitive to soil moisture changes but not to displacements. Measurements of $|\gamma|$ hence have potential to constrain the soil moisture time series in the presence of displacements. However, like the other DInSAR observables, $|\gamma|$ is of limited use in estimating the absolute value of soil moisture, as two constant time series cannot be distinguished from one another [11]. Furthermore, it has limited sensitivity when it comes to distinguishing drying from wetting, illustrated by almost identical coherence magnitudes when flipping the time series (see Fig. 4). In practice, however, the models often do not apply very well, as other effects on $|\gamma|$ besides soil moisture changes cannot be neglected [3, 36]. Their impact on the achievable accuracies is discussed in the following.

### 4.2 Objective 2: Real-world suitability in the absence of displacements

The retrieval of soil moisture based on the phase $\phi$ seems to be the most robust one in the absence of deformations, if the phase is referenced accurately. Apart from its structural richness – only an overall offset cannot be determined – it is least affected by noise and model misspecifications in the simulation study. Model misspecifications (e.g. wrong value of $f$) mainly affect the overall scaling. In practice they may induce spurious spatial patterns (e.g. Fig. 10) or unphysical soil moisture values (Fig. 6), but even an uncalibrated model can give reliable estimates of the relative temporal dynamics. In the CanEx-SM10 experiments, the retrieved soil moisture compares well, according to the temporal correlation $\rho \approx 0.8$, to in-situ measurements in all instrumented fields and at all polarizations (Fig. 9). The phase-based inversion results obtained in the NoSREx campaign also indicate a close correspondence with soil moisture (RMSE $\leq 0.02$ m$^3$ m$^{-3}$), despite certain temporal deviations compared to the in-situ record. These are partially related to precipitation events (e.g.
the last two days of the calibration period) that are only captured in either the estimates or the in-situ measurements. They also indicate the presence of additional processes that are possibly not directly related to soil moisture changes, e.g. the larger deviations at HV or the diurnal variations.

The similarity of the results obtained with the nonlinear and the linear $\phi$ inversions indicate that in the simulations and the case studies, soil moisture changes act predominantly like displacements on the entire phase information $\phi$. If there were larger discrepancies between the predictions by the nonlinear and linear models, this would imply that a larger part of the phase signal were in that part of the phase information space that cannot be due to displacements. Such larger discrepancies are expected when larger soil moisture changes $\Delta m_\phi \approx 0.1 \text{m}^3\text{m}^{-3}$ occur, according to the model simulations in Fig. 3; and when the subsurface scattering contributions are more spread out with depth [54]. These larger discrepancies would particularly affect that part of the signal not due to displacements, i.e. the phase triplets.

The soil moisture estimation based on the phase triplets is limited by their structural limitations and their small magnitude. The structural limitations discussed in the previous section imply that additional information (e.g. a priori constraints) is necessary for robust estimation results. The indirect regularization based on the initial values in the optimization quickly becomes inadequate as the noise level increases or when the model calibration becomes less accurate (Fig. 6 and 7). As opposed to the phase, the inversion based on the phase triplets using an uncalibrated model cannot reproduce the temporal dynamics reliably. These limitations are closely connected to the small size of the phase triplets, as both noise and errors due to model misspecifications can quickly dominate this observable. In the L-band case study, the inversion based on the Hallikainen model works well, but not that based on the Peplinski model (e.g. field 330 in Fig. 11). The discrepancy is likely related to the sensitivity of the nonlinear part of the phases, and hence the $\gamma$ observable, to the model parameterization (it may also be partially due to an inadequate value of $f$). The observation that the phase triplets can be difficult to model is also evident at Ku-band, where the forward model (5) does not apply. In addition to violations of the intrinsic model assumptions (the subsurface scattering models by De Zan et al. and of (5) have not been assessed at such high frequencies), and the parameterization of the surface scattering and the mixing model, the poor fit may also be related to the in-situ soil moisture data. These exhibit a lower dynamic range ($< 0.05 \text{m}^3\text{m}^{-3}$) than what may generally be expected, which in turn may result in biased calibration estimates. However, when the model applies and the noise level is sufficiently low, our simulations and inversions at L-band confirm that the phase triplets contain useful information with which the soil moisture can be constrained in combination with other information [11].

The coherence magnitude is also related to soil moisture changes and can be used for soil moisture estimation in certain circumstances. Like the phase triplets, $|\gamma|$ is also insensitive to displacements, and furthermore it has fewer structural limitations. It is only relatively insensitive to a flip in addition to an overall offset. However, the simulations show that additional sources of decorrelation such as random noise bias the estimates, as the inversion attributes all sources of decorrelation to soil moisture changes. In the relatively short CanEx-SM10 experiment, soil moisture changes dominate the coherence magnitude in the fields with in-situ measurements, and the inversion hence works well (correlation coefficients $\rho > 0.7$). By contrast, at Ku-band other influences cannot be neglected (Fig. 12), despite the short temporal baseline and the limited vegetation cover. For more typical temporal baselines of several days or weeks, previous studies indicate that soil moisture is rarely the only dominant decorrelation source [3, 49, 53]. These are furthermore difficult to model [3] and can only be corrected for to a limited extent, so that the coherence magnitude is likely not useful in most practical studies. When it is, its lack of sensitivity to uniform displacements may make it valuable in correcting the DInSAR displacement estimates for the influence of soil moisture.

### 4.3 Implications for correcting DInSAR displacement estimates

Soil moisture changes can typically change the phase by up to about $0.5\pi$, corresponding to a spurious displacement of $\approx \frac{1}{2} \pi$. For much larger movements (e.g. due to earthquakes), the soil moisture effect may hence be small by comparison. For sub-wavelength displacements, however, the DInSAR results may be substantially affected. In practice, the phase measurements are also influenced by the atmosphere and geometric uncertainties. The treatment of the atmospheric influence is generally considered to be the one of the most difficult aspects in differential radar interferometry [18, 38]. It is illustrative to compare its influence on the radar data with that of soil moisture: whereas soil moisture in addition to the DInSAR phase also influences the backscatter and the other DInSAR observables (coherence magnitude, phase triplets), the atmosphere only has a substantial impact on the phase. In contrast to the influence of the atmosphere on $\phi$, that of soil moisture may hence be separated from that of displacements based on DInSAR data alone.

Here, we showed that the structural limitations (objective 1) and the lack of robustness in practice (objective 2) render the phase triplets and the coherence magnitude by themselves unsuitable for correcting the DInSAR phase for displacements. While these two observables are not affected by displacements, their lack of sensitivity to step changes or sawtooth patterns (phase triplets) or flips (coherence magnitude) in the soil moisture time series implies that the corresponding temporal patterns in the displacement history cannot be constrained. In Fig. 1, the unresolved step change leads to multiple combinations of displacement and soil moisture time series that are consistent with the DInSAR phases and phase triplets. The additional information in the coherence magnitudes as implemented by De Zan et al. [11] helps to reduce the structural limitations of the phase observables, but the lack of robustness to other decorrelation sources limits the applicability of $|\gamma|$ for quantitative $m_\phi$ correction of the DInSAR phases. However, both the phase triplets and the coherence magnitudes are useful for detecting – rather than quantifying – influences like soil moisture and vegetation changes: the phase...
triplets (but also polarimetric interferometric phase differences or backscatter variations) can be beneficial in this context [12, 56, 55], and the coherence magnitude is commonly employed for masking areas for which displacements are not estimated [14, 24].

Additional soil moisture information is hence advantageous if one wants to correct the DInSAR phases for soil moisture effects using a scattering model like (5). Such information may come from the radar data themselves (e.g., backscatter, polarimetric information) or from ancillary sources such as hydrological models, other remote sensing instruments or in-situ measurements. In any case, the model that links these soil moisture data with the phase, for instance (5), would generally have to be calibrated for this purpose. This is in contrast to atmospheric corrections based on meteorological information, where the model that links the atmospheric state to the DInSAR phase does not have to be calibrated [18]. Surface soil moisture is furthermore notoriously difficult to model and measure on the relevant spatial scales, so that the availability of reliable soil moisture data may be a common limitation [48].

It thus seems expedient to additionally impose assumptions about the deformation signal itself, when the impact of soil moisture on the phase is comparable to that of the deformations. After all, such assumptions are commonly employed when mitigating the atmospheric contribution, which cannot be separated from displacements based on the phase triplets or the coherence magnitude. Atmospheric mitigation approaches in flat terrain are typically based on the assumption that the atmospheric contribution is strongly correlated in space, but not in time, whereas displacements are correlated in time [20, 24]. However, the spatial and temporal variability of the soil moisture effects can be even more complex as a large range of scales and processes are involved, ranging from the inter-field variability on sub-weekly time scales observed in Fig. 10 to seasonal variations on regional scales [13, 48]. Another complicating factor is the common interaction between soil moisture changes and surface displacements: for instance, an increase in moisture can speed up mass movements; subsidence-induced changes in water flow can lead to local wetting; both surface position and soil moisture may exhibit seasonal variability; and certain of soils swell when they become moist [5, 41, 44, 45]. Depending on the application and the data availability, a spectrum of processing approaches may be considered in the future. These approaches may deal with soil moisture implicitly or explicitly. Implicitly, the effect of soil moisture could be mitigated by stacking or temporal filtering in time-series models, with the filtering and uncertainty parameters being adapted to the temporal correlation properties of soil moisture [18, 20]. Explicit soil moisture information may also be incorporated into time-series models, so that the soil moisture calibration constants (e.g. \( f \) in our model) become part of the estimated parameters, like the linear displacement rate or the topographic error [20, 24].

5 Conclusions
The DInSAR phase is routinely used to estimate surface displacements, but it is is also influenced by soil moisture changes. Their effect corresponds to spurious displacements of around 10-20% of the radar wavelength. There are two more DInSAR observables, the phase triplets and the coherence magnitude, which are not influenced by uniform displacements, but by changes in soil moisture.

In the absence of displacements and when the phase can be referenced, we find that the phase can be useful for estimating soil moisture changes. However, the overall soil moisture time series can only be estimated up to a constant offset. Empirically, using acquisitions over bare soil that remain coherent, we find temporal correlations of the estimates with in-situ measurements in the range of 0.7-0.9 at L- and Ku-band.

When displacements and soil moisture changes occur simultaneously, the question arises whether these two influences can be separated based on the DInSAR data. Here, we studied the suitability of the phase triplets (which contain all phase information that cannot be due to displacements) and coherence magnitudes (not affected by displacements either) for estimating soil moisture in the presence of displacements. This soil moisture information could then be used to correct the phase before estimating the displacements. However, we identify two problems in this context. The first one is theoretical: the soil moisture estimation not unique, i.e. neither the phase triplets nor the coherence magnitude contain enough information to estimate soil moisture and correct the phase (see Fig. 1). The second one is practical: we find that soil moisture estimation using these two observables is hampered by their sensitivity to noise, model misspecifications (e.g. calibration errors), and additional unmodelled influences like changes in the surface roughness.

In practice, the feasibility of separating soil moisture changes from displacements based on DInSAR data alone is hence limited when no assumptions about the spatio-temporal variability of soil moisture or the deformations are made. In order to mitigate the influence of soil moisture changes on DInSAR displacement estimates, the masking of affected areas based on e.g. the phase triplets and the coherence magnitude may be an option, but in many applications one is interested in the deformation of these areas. Alternatively, one may introduce assumptions about the spatio-temporal characteristics of the deformations and the displacements, e.g. that the latter remain temporally correlated on longer time scales. For instance, this may be achieved by extending filtering and stacking approaches that try to minimize the influence of the atmosphere. However, the spatio-temporal variability of soil moisture is complex, and it is also often related to surface movements. In the long run, it may hence be worthwhile to explicitly integrate soil moisture information from external data and also from the radar data themselves (e.g. the phase triplets) in DInSAR time-series approaches, so that subtle displacements can be estimated in the presence of soil moisture changes.
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A Appendix

A.1 Calibration of SnowScat data

The aim of the calibration is the estimation of unknown model parameters [4], i.e. \( \beta \) for the linear model of Eq. 5 or \( f \) for the non-linear one of Eq. 6. The calibration relies on direct soil moisture observations \( m_v \), which are taken from the probe. The model parameters are estimated by solving an optimization problem, which accounts for the uncertainty in both the radar observations (i.e. the phase \( \phi \)) and the in-situ soil moisture records [4]. The motivation for including the latter kind of uncertainty is the small temporal separation (<1 day), at which reliable interferograms can be formed. At these time scales, the changes of \( m_v \) are likely to be frequently of comparable size to the noise of the in-situ probe. The associated least-squares misfit function of the calibration problem consists of two parts [31]

\[
\mu_C^2 = \sum_j (\hat{m}_{v_j} - m_{v_j})^2 + \kappa \sum_k (\hat{\phi}_k - \phi_k)^2
\]

where the first one (the sum over all in-situ measurements \( j \) during the calibration phase TP-C) accounts for the uncertainty in the in-situ soil moisture records. The second term measures the difference between the observed phase \( \phi_k \) and the predicted one \( \hat{\phi}_k \) in all interferograms \( k \), of which there are approximately \( \Delta a - 4 \) as many as acquisitions. This predicted phase depends on the estimated soil moisture \( \hat{m}_v \) and the model parameters: the two quantities are estimated by minimizing \( \mu_C^2 \) with respect to them. In order to solve this calibration optimization problem, the weighting parameter \( \kappa \) has to be set a priori. In error-in-variables least squares theory [6, 31], \( \kappa \) is the ratio of the variance of the in-situ data to that of the phases. It thus encapsulates the relative uncertainty of these two kinds of observations. Empirically, the impact of this choice of \( \kappa \) on the estimate of the linear model parameter \( \hat{\beta} \) (for \( \phi_{HH} \)) is limited. It is less than 30% when \( \kappa \) varies by about three orders of magnitude. A priori, we would expect the in-situ records to be ‘more’ reliable than the phases, i.e. \( \kappa \ll 1 \) m\(^6\) m\(^{-6}\) rad\(^{-2}\). However, when this value becomes smaller than \( 10^{-6} \) m\(^6\) m\(^{-6}\) rad\(^{-2}\), the parameter estimate \( \hat{\beta} \) changes sign, i.e. it would not be consistent with volume scattering according to the nonlinear model (5). This is in contrast to slightly larger values of \( \kappa \), such as \( \kappa_{\text{lin}} = 10^{-4} \) m\(^6\) m\(^{-6}\) rad\(^{-2}\), for which \( \hat{\beta} \) corresponds to volume scattering. The value of \( \kappa_{\text{lin}} \) is in line with a priori expectations regarding the uncertainty: if the RMSE of the in-situ probe were 0.01 m\(^3\) m\(^{-3}\), the associated phase error would be \( \approx 60^\circ \), respectively. For this value of \( \kappa \), which will be assumed for all calibrations, the residuals \( (\hat{m}_{v_j} - m_{v_j}) \) show a pronounced variability with a period of one day, as the diurnal frequency component alone accounts for \( 20\% \) of the total variation. These corrections (whose mean absolute value is 0.003 to 0.004 m\(^3\) m\(^{-3}\)) have a clear impact on the relation between \( \Delta m_v \) and the observed phases \( \phi \), as seen in Fig. 12. The associated estimates of the parameter \( \beta = -194.4 \) rad m\(^3\) m\(^{-3}\) of the linearized model \( \kappa_{\text{lin}} \) are well reconcilable with the full model (5). It corresponds to the linear term of the phase sensitivity predicted by the volume-only nonlinear model at \( m_v = 0.07 \) m\(^3\) m\(^{-3}\). At VV and HV, the values are comparable (-177.5 and -269.7 rad m\(^3\) m\(^{-3}\), respectively). When calibrating the nonlinear model, we find dominant subsurface volume scattering, as the volume-to-surface ratio \( \hat{f} \approx 1 \) (ln\( (\hat{f}) \) of 16 and 11 at HH and VV, respectively). Note that the calibration procedure based on the linear model can adjust to the input data more successfully than the nonlinear version, i.e. the misfits \( \mu_C \) are smaller by a factor of two.

References


Chapter F

Depth-resolved backscatter and differential interferometric radar imaging of soil moisture profiles: observations and models of subsurface volume scattering

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Draft

Key findings:
• depth-resolved tomographic profiling observations of sandy soils are sensitive to soil moisture changes
• backscatter and differential interferometry governed by soil moisture-dependent transmission properties
• evidence for increasing subsurface backscatter with soil moisture (in addition to changing transmission properties)
• potential of depth-resolved observations for monitoring dynamic soil moisture profiles

The author’s contributions:
• suggested studying soils using Tomographic Profiling
• developed a model for depth-resolved radar measurements
• assisted in the experiment and the processing of the data
• assessed the modelling assumptions and wrote the manuscript

The co-authors’ contributions:
• K.M. led the experiment and the analysis, A. E.-S. assisted
• K.M. and A. E.-S. were responsible for calibration and data processing
• all co-authors interpreted the results and co-wrote the manuscript
Depth-resolved backscatter and differential interferometric radar imaging of soil moisture profiles: observations and models of subsurface volume scattering

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Abstract

Depth-resolved radar imaging at L- to X-band has become a powerful tool for characterizing spatially extended targets such as forest canopies or snow packs. Soils have received less attention. Rather, they are commonly studied using ground penetrating radars, typically at lower frequencies, or radar systems that cannot resolve the depth component. These latter measurements are typically interpreted as being due to surface scattering alone, even though evidence (e.g. from differential interferometry) points towards the importance of subsurface volume scattering, which may itself contain valuable information about the subsurface water content. Here, we want to characterize the interferometric and backscatter characteristics of subsurface volume scattering at C-band. We image the wetting and drying of sandy soil using a ground-based radar with a depth resolution of 5-10 cm. The volumetric subsurface backscattered power is strongly affected by soil moisture (>15 dB): it appears to be governed by the local soil moisture content (local dielectric constant and its spatial variability) and the soil moisture content above (absorption). When the soil moisture changes, the observed interferometric response is consistent with the notion that the subsurface return is governed by the changing wave propagation within the soil. However, existing models of the depth-averaged interferometric coherence do not include variations in the volume scattering power induced by soil moisture changes, which the backscatter observations indicate exist. These findings indicate the potential of depth-resolved observations to provide direct, spatially extensive information about the vertical variability of soil moisture.

1 Introduction

Microwave radar imaging techniques based on tomographic principles (e.g. SAR tomography, Tomographic Profiling) in L- to X-band have shown their potential to characterize vertical variations of natural media including snow packs and forest canopies [9, 22, 24]. Their application to the near-surface parts of soils has received less attention, chiefly due to inherent limitations imposed by the penetration of the electromagnetic waves (cm to dm) and also the resolutions currently attainable from air- and space-borne platforms [20, 23]. By contrast, lower radar frequencies, such as those employed by ice sounders or most ground penetrating radar systems, permit larger sensing depths exceeding several metres, albeit at reduced resolutions [1, 8, 28]. At higher frequencies (L to X-band), air- or space-borne radar approaches often do not resolve the vertical coordinate. For bare soils, the measured depth-averaged backscattered power is commonly interpreted as being dominated by scattering from the rough soil surface: it is routinely analysed to estimate surface properties such as surface soil moisture, soil roughness, and freeze/thaw state [11, 38, 47].

There are, however, strong indications that subsurface scattering can frequently play a non-negligible role in radar measurements at L- to X-band [19, 17, 27]. For one, the models of the depth-averaged measurements that only account for surface contributions often do not work very well [37]. Their surface roughness parameters, which are purported to be well-defined physical quantities, are commonly considered as mere fudge factors in practice, as they are found to vary with incidence angle and radar frequency [15]. They have further been observed to depend on soil moisture, highlighting the difficulty of deriving soil moisture from backscatter measurements based on these models [15, 40]. More directly, an inverse relation between backscatter and soil moisture can be observed in certain circumstances: this can more easily be explained by a decrease in the subsurface scattering upon wetting (increase in absorption) rather than by surface scattering, for which the opposite would be expected [17, 25]. Such subsurface scattering is also commonly observed using depth-resolving ground penetrating radar systems. The so-called clutter is associated with quasi-random three-dimensional subsurface heterogeneities such as soil clods and stones [27, 34].

The contribution of sub-wavelength scale heterogeneities to the depth-averaged radar measurements has also been invoked to explain the observed soil moisture impact in differential radar interferometry (DInSAR) [6, 43]. This technique, which can be used to estimate movements of the soil surface [4], has found widespread application in the monitoring of subsidence and mass movements, and tectonic studies [4, 16]. Soil moisture changes correspond to spurious displacement
estimates. In non-swelling soils, previous observations indicate a dominant contribution by subsurface ‘volume’ scattering: as the soil becomes wetter, its refractive index and hence the apparent distance between the antenna and the subsurface scatterers increases, corresponding to a movement away from the antenna [6, 30, 43, 44]. The moisture-dependent wave propagation properties of the subsurface soil are hence crucial to the observed interferometric phase. Even though the sensitivity of differential interferometric phase is so large that the subsurface scatterers only have to be just below the surface (within less than one tenth of the free-space wavelength in order to account for the observations), the inferred dominance of the subsurface component is at odds with the previously described notion that attributes the observed backscatter to the return from the rough surface. While DInSAR is very sensitive to changes in the refractive index of the soil and hence to subsurface scattering, it is also sensitive to movements. It may hence be argued that the DInSAR observations only provide indirect evidence towards the importance of subsurface scattering. Similarly, our understanding of the role of subsurface scattering in backscatter studies of soils in the presence of surface scattering is limited by the paucity of direct observations of the subsurface component at L- to X-band [27, 31].

Here, we directly observe subsurface volume scattering using a depth-resolving radar system, that is, we image vertical backscatter and DInSAR profiles of a sandy soil. Our key objective is to characterize the impact of changes in the soil moisture profile on the depth-resolved radar backscatter and DInSAR profiles. The assessment of physical models of subsurface volume scattering is a vital part of this characterization. As previous models are either not interferometric in nature [27, 31, 34] or not applicable to depth-resolved observations [6, 44], we develop a depth-resolved interferometric model by extending those by [6, 44]. The comparison of its predictions with observations of the subsurface backscatter and interferometric response allows us to assess its structural assumptions and mechanisms more directly than what would be possible using depth-averaged observations or backscatter measurements alone. Our long-term aim is to contribute to improving our understanding of subsurface volume scattering [35], which in turn may lead to the application of depth-resolved interferometric data to infer subsurface soil properties and moisture profiles.

Specifically, we observe the wetting and drying of a sandy soil at C-band with a depth resolution of 5-10 cm during a period of 5 weeks. To this end, we adapt the Tomographic Profiling (TP) technique [24] so that we can observe both the backscatter and the differential interferometric response as a function of depth (Sec. 2). Subsequently (Sec. 3), we present and analyse our depth-resolved subsurface scattering model, which is an extension of a model that was originally developed to explain the impact of soil moisture changes on the depth-averaged DInSAR observations [44]. The actually observed radar measurements are presented in Sec. 4. Subsequently, we confront these observations with the model predictions in Sec. 5, with the aim of assessing the model and more generally of characterizing the impact of soil moisture changes on the depth-resolved radar observations. We also explore the implications of these findings for depth-averaged observations (in particular for modelling the effect of soil moisture changes on DInSAR observations in displacement studies), and provide a first tentative assessment of the potential suitability of depth-resolved observations of volumetric subsurface scattering for estimating soil moisture profiles.

2 Experimental setup

2.1 Tomographic profiling

Tomographic Profiling (TP) is a radar imaging technique which maps vertical reflectivity profiles of targets such as snow or soil [24]. As opposed to the more common side-looking imaging geometry, the radar antenna points downwards or is tilted along the scan direction, see Fig. 1, along which the raw radar data are acquired. In the image reconstruction, the beam in the along-track direction is sharpened by synthetic aperture processing. The characteristic property of TP is the location of this synthetic subaperture, which for each reconstructed point is chosen such that this point is imaged at the beam in the along-track direction is sharpened by synthetic aperture processing. The characteristic property of TP is

\[ h(\tilde{X}', Z'; x, k, \theta) = W_x(x - \tilde{X}')W_k(k - k_0)exp \left( -2ik\sqrt{(x - \tilde{X}')^2 + Z'^2} + 2ik_0\sqrt{(\tilde{X}' - X')^2 + Z'^2} \right) \]  

where a window function \( W_x(\Delta x) \) is introduced that centres the subaperture at \( \tilde{X}' \) and limits the extent of the kernel \( h \) to \( \Delta x \) samples. The range window function \( W_k(\Delta k) \) similarly weights the wave numbers \( k \) around the central wave number \( k_0 \). The tomographic profiling processor uses Hamming windows for both \( W_x \) and \( W_k \). The application of the filter \( h \) to
Figure 1: Illustration of the geometry of the data acquisition and the TP image reconstruction. The antenna is moved along a track; its phase centre defines the zero position of the vertical coordinate \( z \). The mid-point of the synthetic subaperture \( \tilde{X} \) is given by the antenna position at which a point scatterer at \((X, Z)\) is located at incidence angle \( \theta \) along the slant range direction. The cross-scan direction points out of the page.

A point target at \((X, Z)\) yields the impulse response \( I(\tilde{X}', Z'; X, Z) \)

\[
I(\tilde{X}', Z'; X, Z) = \sum_{\Delta x} \sum_{\Delta k} W_x(\Delta x) W_k(\Delta k) \cdot \exp\left( -2i(k_0 + \Delta k) \sqrt{[\Delta x + (\tilde{X}' - \tilde{X}) - Z \tan \theta]^2 + Z'^2} + 2i k_0 \sqrt{(\Delta x - Z' \tan \theta)^2 + Z'^2} - 2ik_0 \sqrt{(\tilde{X}' - X)^2 + Z'^2} \right)
\]

The impulse response is centred around the actual depth \( Z' = Z \) and along-track position \( \tilde{X}' = \tilde{X} = X - Z \tan \theta \), cf. [10]. The phase at its peak, \( \varphi = -2k_0 \frac{Z}{\sin \theta} \), corresponds to the distance to the point target viewed from an angle \( \theta \), i.e. this algorithm is phase preserving [2]. The phase of neighbouring pixels can thus be compared, which makes it possible to account for (apparent) displacements by coregistration. There is an along-track ramp across the impulse response, as the phase at \( \tilde{X}' + \delta \tilde{X} \) differs from that at the centre of the impulse response by \( -\sin \theta \cdot \delta \tilde{X} \). The along-track ramp term is typical for squinted acquisition geometries [2]: it corresponds to the change in range for a displacement along the horizontal direction, when the synthetic subaperture is held constant [10]. The presence of phase ramps poses a challenge in interferometry as the interferometric phase will be biased unless the two images are perfectly aligned [2]. The phase bias is proportional to the coregistration error, i.e. the uncompensated offset between the two images. The maximum horizontal offset found in this study is on the order of 2 cm. Assuming the accuracy of the coregistration to be at least a factor of two better, this would correspond to a worst case phase error of about 30°. As the accuracy of the coregistration increases with the similarity (as measured by the coherence magnitude) of the two acquisitions, such a worst case phase error is expected to occur when the coherence \( |\gamma| \ll 1 \), i.e. in situations when the phase cannot be measured accurately.

The matched filter in the image reconstruction does not take into account the refraction that occurs at the air-soil interface, so that points beneath the soil surface are not imaged at the right depth [8], which is accounted for by the coregistration. The changing refractive index also has an impact on the focussing quality itself [8], however these associated artefacts will be shown to be small compared to the observed changes for the experiment in this study.

2.2 Experimental setup

The experiment was conducted at C-band, at a central frequency of 5 GHz. The bandwidth of 2 GHz (stepped frequency continuous wave acquisition) corresponds to a vacuum slant range resolution of 10 cm, see Tab. 1. The cross slant range resolution depends on the incidence angle and the distance to the antenna; representative values of these resolutions for the processing squint angle \( \theta = 20° \) are given in Tab. 1. The three polarizations HH, VH and VV were acquired sequentially, with each scan taking about 6 minutes. A linear trough of dimensions \( 4 \times 1 \times 0.9 \) m (height) was positioned underneath the scanning installation, with the long side of the trough aligned with the along track direction, see Fig. 2. The trough was filled to the brim with sand and its surface smoothed. The trough contained pure kiln-dried sand, except for a central 1 m portion. This section contained a mixture of sand (90%) and gravel (10%) down to a depth of 20 cm. In its middle, a trihedral corner reflector (12 cm side length) was installed at a depth of 26.5 cm and aligned so as to give the maximum return for \( \theta = 20° \) in free space. In addition to this buried trihedral B, smaller trihedral reflectors were installed on the sides of the trough (S), see Fig. 2.

Starting on 17 February 2015, water was applied uniformly to the \( 1 \times 1 \) m treatment are above the gravel-sand mixture (Fig. 2). The first 1000 ml were applied in increments of 200 ml using a hand-held spray bottle. The addition of water and the radar scans were performed alternately in immediate succession. This alternation continued during the second part of the watering period, when water was added in increments of 2000 ml with a watering can up to a total of 10 litres. The following day, a first scan was made and subsequently the procedure was continued until a cumulative amount of 26 litres had been added. Subsequently, the scans were continued for more than 5 weeks until day 37, every 80 minutes. We refer to this period as the drying period. The data acquisition was not possible during the first two nights, i.e. between days 1
<table>
<thead>
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<th>Parameter</th>
<th>Value</th>
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</thead>
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<tr>
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<td>Frequency step</td>
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<td>Cross-slant-range resolution $r_c$</td>
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<td>14 cm</td>
</tr>
<tr>
<td>2.5 m distance</td>
<td>24 cm</td>
</tr>
</tbody>
</table>

Table 1: Parameters of the radar acquisitions and tomographic profiling. The cross-slant range direction is orthogonal to the slant range direction and the corresponding spatial resolution depends on the distance. The resolutions refer to targets in free space.

Figure 2: The sand trough imaged by the Tomographic Profiling system. a) Buried trihedral partially buried within the sand within the trough. It was subsequently covered with a gravel-sand mixture. b) The sand trough containing the treatment area (wetted, slightly darker patch; the buried trihedral is 26.5 cm below the surface) and the control area (continuously dry). The trihedrals on the side of the trough act as reference targets. c) A side view of the trough, indicating the treatment and the control area.

and three. Furthermore there were errors in the alignment of the scanner along the track during the first 13 scans on the day 1; the impact on the interferometric phase will be studied in Sec. 5.1.

2.3 Data analysis

The tomographic profiling images were derived as described in Sec. 2.1 and additionally phase referenced using the observed leakage signal. The leakage, or coupling, between the antennas provides a useful reference signal with which to measure the stability of the measurement system, unaffected by the scene. Thus, its fluctuations in phase (e.g. due to temperature variations) can be monitored continuously. We phase reference the imagery with respect to the coupling signal to account for temporal drifts of the radar system, which would otherwise produce spatially-invariant phase offsets in the imagery.

Hence, the signal in any image can be meaningfully compared with that in any other image, with an expected ra-
Figure 3: Backscatter images obtained with Tomographic Profiling at $\theta = 20^\circ$. The large image shows the trihedrals (annotated in orange), the standard and the alternative treatment profile to either side of the buried trihedral, and the control profile (no water applied). The notches indicate the layers 1 to 8, starting at just above the surface. The horizontal axis corresponds to $\tilde{X}$, the vertical axis to $Z$. The yellow L-shaped marks indicate the subset of the treatment area of which the three images at the bottom indicate the temporal changes during the experiment. The buried trihedral is clearly visible on day one before the application of water (left) and at the end of the drying period (right), but its backscatter is comparable to that of the surrounding soil at the end of the water application phase (middle).

The backscattered power $p_m$ for acquisition $m$ is estimated for each layer by averaging horizontally over $K$ samples corresponding to a total of around 14 independent looks

$$p = \sum_{k=1}^{K} k u_m \cdot k u_m^*$$

As we focus on changes with soil moisture we reference the power with respect to a master acquisition (completely dry) and report the dimensionless result as $\Delta p$, expressed in decibels (dB). Spatial variations of the referenced power can be measured by differences of the logarithmic $\Delta p$, expressed as a double difference $\Delta \Delta p$.

The complex coherence $|\gamma_{m,n}|$ is estimated for each soil layer from two acquisitions $m$ and $n$ according to

$$|\gamma_{m,n}| = \frac{\sum_{k=1}^{K} k u_m \cdot k u_n^*}{\sqrt{\sum_{k=1}^{K} k u_m \cdot k u_m^* \sum_{k=1}^{K} k u_n \cdot k u_n^*}}$$

As the optical path between the antenna and a subsurface point increases upon wetting – a fact which is not considered by the imaging algorithm –, the positions at which the subsurface targets are imaged vary over time. We determine the offset by coherent coregistration based on the HH polarization. The change in the optical path between the antenna and the target furthermore varies along the synthetic subaperture, which can induce defocussing. However, the synthetic subaperture is sufficiently small so that, for the buried corner reflector in the experiment, the neglected phase offset due to refraction
between the centre and the edge of the synthetic subaperture changes by only $< 0.15 \pi$ as the soil becomes saturated. As this value is considerably smaller than $2\pi$, the focussing quality is not expected to be change appreciably during the experiment [1], and the effect is further reduced by applying a window across the aperture. For the given experimental setup and the windowing in Sec. 2.1, it corresponds to a power difference of less than 0.5 dB.

The radar observables are computed for all acquired polarizations HH, VH, and VV. The presentation of the results will focus on the HH channel, as previous depth-averaged measurements did not suggest a strongly polarized interferometric return from subsurface scattering in soil [31, 44]. The electromagnetic models that we employ reflect such a limited polarization diversity.

3 Model predictions

3.1 Model formulation

We now adapt existing analytical models of subsurface scattering from depth-averaging to depth-resolving radar systems. We start by looking at the return from an ideal point target buried at depth $z_t$ in the soil as measured by a depth-averaging system: it is ideal in the sense that its return depends only on the propagation properties of the electromagnetic waves within the soil. These propagation properties are governed by the dielectric constant of the soil, which in turn is mainly affected by changes in soil moisture $m_v$ [37, 38]. The interferometric response from the point target (point A in Fig. 4), assuming a spatially homogeneous dielectric medium (spatially uniform soil moisture) and identical antenna positions during acquisitions $m$ and $n$, can be described by (cf. [6, 25])

$$u_m u^*_n = (s_m s^*_n) \exp \left( -2i \left( k_{z,m} - k^*_{z,n} \right) z_t \right)$$  \hspace{1cm} (5)

where $k_{z,m}$ is the vertical component of the wave number within the soil for acquisition $m$. This complex quantity describes both the phase velocity (via its real part) and the attenuation (via its imaginary part), both of which depend on soil moisture. The quantity $s_m$ is the scattering matrix of the target during acquisition $m$. For ideal targets, it is independent of the surrounding medium and thus the acquisition. The corresponding interferometric phase $\phi_{m,n}$, i.e. the argument of $u_m u^*_n$, is thus governed by the propagation term within the exponential factor. It is given by the difference of the propagation phases $\phi_{m,n} = \varphi_m - \varphi_n$ where the propagation phase is given by $\varphi_m = \arg (-2ik_{z,m}z_t)$ [44]. Note that this expression neglects the phase contribution due to propagation in the horizontal direction because it cancels in interferometry due to Snell’s law [6], see (5). An increase in soil moisture corresponds to an increase in the magnitude of the propagation phase, i.e. a movement away from the antenna [43]. Observations of buried corner reflectors are consistent with this assumption [26].

Even in the absence of such buried targets or reflecting layers, ground penetrating radar measurements are affected by so-called ‘clutter’, associated with volumetric subsurface scattering caused by soil heterogeneities [27, 34]. Depth-averaged interferometric measurements over natural soils also indicate the presence of subsurface scattering, which is evident due to the moisture-dependent propagation properties [30, 43, 45]. Changes in soil moisture have been shown to be related to decorrelation and to interferometric phases that correspond to a movement away from the antenna upon wetting. De Zan et al. [6] described the return from such a soil as the superposition of contributions from heterogeneities within the soil, such as small stones, clods, etc. They modelled the soil as a ‘volume’ of discrete scatterers: the return from each such scatterer was affected like that of the idealized point scatterer in (5). The ensemble average $\langle \cdot \rangle$ of the interferometric return was modelled as

$$\langle u_m u^*_n \rangle = \int_{z=0}^{\infty} f(z) \exp \left( -2i \left( k_{z,m} - k^*_{z,n} \right) z \right) \, dz$$  \hspace{1cm} (6)

where $f(z)$ is a backscattering density function that describes the backscatter per unit depth for a particular polarization. Similarly to $(s_m s^*_n)$ of the idealized point target, it is a real and positive quantity that was furthermore assumed...
independent of soil moisture. The associated complex coherence $\gamma_{m,n}$ is defined by

$$\gamma_{m,n} = \frac{\langle u_m^* u_n^* \rangle}{\sqrt{\langle u_m^* u_m^* \rangle \cdot \langle u_n^* u_n^* \rangle}} \equiv |\gamma_{m,n}| \cdot \exp i\phi_{mn}$$

(7)

This scattering model also makes predictions about the backscattered power from a heterogeneous soil. These predictions, which are given by $\langle u_m^* u_n^* \rangle$, have not been assessed yet. However, the model itself is similar in nature to discrete particle models which have been successfully employed to describe GPR clutter [34, 35] and subsurface volume scattering in depth-averaged radar observations [17]. In these models, the geometric (e.g., sphere radius) and dielectric (e.g., permittivity contrast with respect to the surrounding medium) properties of the discrete particles are commonly chosen to mimic the observed quasi-random heterogeneity of the soils.

Instead of representing the heterogeneities as discrete scatterers, the interferometric model by [44] conceives them as spatially continuous fluctuations of the dielectric constant. The mean or background dielectric constant was described by a mixing model as a function of soil moisture, whereas the fluctuations were described as an additive, zero-mean stochastic process. The response is essentially identical to (6) when the fluctuations are spatially uncorrelated and independent of soil moisture. If the latter assumption were to be relaxed, the backscattering density would become dependent on soil moisture and hence the acquisitions, i.e. $f_m, m(z) \neq f_n, n(z) \neq f_n, n(z)$. In the model, also the impact of the rough soil surface is considered, and its transmissivity affects the phase and magnitude of the subsurface return [6]. Furthermore, it itself also contributes to the backscatter and the interferometric phase; for the latter, such a surface contribution has indeed been found dominant in homogeneous soils in laboratory experiments [42]. The surface contribution and the transmissivity are predicted to affect the linear copolar polarizations HH and VV differently. However, the impact of the transmissivity on the interferometric phase and coherence is much smaller than the contribution from volume scattering [44], so that we will not consider it explicitly. At HV polarization the first-order model by [44] predicts zero return owing to the assumed isotropic nature of the fluctuations.

So far, the presented models focussed on soils with uniform dielectric properties and thus soil moisture. The validity of this assumption is questionable in the experiments outlined in Sec. 2.2. For instance, in the beginning small amounts of water are added to a perfectly dry soil, which is expected to lead to a nonuniform $m_z$ profile close to the surface (compared to the penetration depth). In order to relax this assumption, the differential optical path in the exponential term in (5) will be replaced by a more general expression [24, 26]

$$\left(k_{z,m} - k_{z,n}^*\right) z \to \int_{z'=0}^{z} \left(k_{z,m}(z') - k_{z,n}^*(z')\right) dz' \equiv o_{m,n}(z)$$

(8)

This geometric optics approximation neglects the scattering return induced by the spatially varying mean dielectric constant [29]. Its real part is closely related to the propagation phases, i.e. $\text{Re}(-2io_{m,n}(z)) = \varphi_{m,n}(z) - \varphi_{n}(z)$.

The interferometric tomographic profiling of Sec. 2.1 has the potential to resolve such nonuniform changes in soil moisture. As such, the single scattering model of (6) has to be limited to a layer that extends from depth $z_t$ to $z_b$ (see Fig. 4)

$$\langle u_m^* u_n^*\rangle(z_t, z_b) \equiv \exp \left(-2io_{m,n}(z_t)\right) \int_{z=z_t}^{z_b} \frac{f(z) \exp \left(-2i \int_{z'=z}^{z_t} \left(k_{z,m}(z') - k_{z,n}^*(z')\right) dz'\right) dz}{v_{m,n}(z_t, z_b)} \equiv \nu_{m,n}(z_t, z_b)$$

(9)

where we assume a rectangular antenna pattern for simplicity. The first term before the integral $\exp \left(-2io_{m,n}(z_t)\right)$ contributes an overall phase and attenuation: it is related to the soil moisture contents between the soil surface and the top of the layer. The integral term $\nu_{m,n}(z_t, z_b)$ accounts for the scattering contributions from within the soil layer.

3.2 Analysing the model predictions

According to this model, both the soil above and within the layer influence the observed power. For a layer extending from $z_t$ down to $z_b$, the backscattered power $p_m$ for acquisition $m$ is defined as $\langle u_m^* u_m^*\rangle(z_t, z_b)$. When the backscattering profile $f(z)$ is independent of soil moisture, only absorption (the imaginary part of the wavenumber $k_{z,m}$) governs the response. An increase in soil moisture between the surface and $z_t$ reduces the return from the layer as the absorption above $z_t$ increases, thus reducing the absolute value of the complex propagation term $\exp \left(-2io_{m,n}(z_t)\right)$. Similarly, a soil moisture increase within the layer, i.e. between $z_t$ and $z_b$, also causes the power $p_m$ to decrease. This time the decrease is due to a reduction in $\nu_{m,n}(z_t, z_b)$, which is also caused by the increased attenuation of the electromagnetic waves. When the power is referenced with respect to a predefined acquisition and moisture state, this logarithmic power difference $\Delta p$ can similarly be decomposed into two parts, one being due to soil between the surface and $z_t$, the other one depending on the soil between $z_t$ and $z_b$. The simulations in Fig. 5 illustrate the impact of soil moisture changes above and below the top of the layer $z_t$ on the measured $\Delta p$. 

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Figure 5: Simulated interferometric observables and power obtained from a subsurface layer \((z_t = 5 \text{ cm}, z_b = 13 \text{ cm})\) and varying soil moisture \(m_v\). The soil moisture value of the master acquisition is fixed at 0.1 \(m^3 m^{-3}\). The simulations are based on C-band observables predicted by (9) using the Hallikainen mixing model [12] and a depth invariant backscattering density \(f(z)\). The colours correspond to different scenarios: a capital T indicates that the soil moisture above \(z_t\) changes, a lowercase t that it remains the same, and similarly for the letter b indicating changes below \(z_b\). Panel a) shows the power difference \(\Delta p\) as a function of \(m_v\), b) the interferometric phase, and c) the coherence (the results for TB and tB are identical).

\[
\gamma_m,n(z_t, z_b) = \exp \left(i \left( \varphi_m(z) - \varphi_n(z) \right) \right) \frac{v_{m,n}(z_t, z_b)}{\sqrt{v_{m,m}(z_t, z_b) \cdot v_{n,n}(z_t, z_b)}}
\]

Figure 6: Simulated interferometric observables for a subsurface layer of varying depth in C-band as predicted by (9). The model settings and the definition of the scenarios is identical to Fig. 5. Panel a) shows the phase \(\phi\) for a layer of \(z_b - z_t = 0.08 \text{ m}\) depth when the soil moisture changes from 0.23 to 0.20 \(m^3 m^{-3}\). Panels b) and c) show the simulated phase \(\phi\) and coherence magnitude \(|\gamma|\), respectively, when the soil moisture changes from 0.26 to 0.20 \(m^3 m^{-3}\) for variable layer depth \(z_b - z_t\) and fixed \(z_t = 0.05 \text{ m}\).

The complex coherence (7) and hence the phase also depend on both the soil within and above this layer, but the coherence magnitude only depends on the changes within the layer. The complex coherence \(\gamma\) can be expressed as

\[
\gamma_m,n(z_t, z_b) = \exp \left(i \left( \varphi_m(z) - \varphi_n(z) \right) \right) \frac{v_{m,n}(z_t, z_b)}{\sqrt{v_{m,m}(z_t, z_b) \cdot v_{n,n}(z_t, z_b)}}
\]

which illustrates the influence of the soil above (via the first term containing the propagation phases \(\varphi(z_t)\) to the top of the layer) and the soil within the layer (via the second term). The soil above the layer only influences the phase (and not the coherence magnitude) via the propagation phases in the first term, as this term’s magnitude is equal to one. The second term leads to a phase change (via changes in the relative propagation path within the layer) and decorrelation (in addition to the phase diversity within the layer also influenced by depth dependent attenuation). The impact of soil moisture changes above and within the layer on the phase and coherence as predicted by this model is illustrated in the simulations of Fig. 5. Both the phase and coherence depend on the position and depth of the analysed layer. Several scenarios of inhomogeneous soil moisture changes are shown in Fig. 6.

In summary, the observables of a subsurface layer are predicted to be influenced by the soil moisture profiles in distinct ways, as compiled in Tab. 2. None of them is expected to be influenced by changes below the layer. The logarithmic
Observable Influence of \( mv \) changes above within below

<table>
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<th>Influence of ( mv ) changes</th>
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<td>( \phi )</td>
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</tr>
<tr>
<td>( \Delta p )</td>
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</tr>
<tr>
<td>(</td>
<td>\gamma</td>
</tr>
<tr>
<td>( \Delta\phi )</td>
<td>✗  ✓  ✓</td>
</tr>
<tr>
<td>( \Delta\Delta p )</td>
<td>✗  ✓  ✓</td>
</tr>
</tbody>
</table>

Table 2: Predictions by the model of (9) as to how the observables corresponding to a subsurface layer are influenced by changes in soil moisture above, within, or below this layer. For \( \Delta\phi \) and \( \Delta\Delta p \), these refer to above the upper layer, between the top of the upper and the bottom of the lower layer, and below the lower layer, respectively.

power difference \( \Delta p \) and the phase \( \phi \) depend on the soil moisture within and above the layer of interest. Conversely, the coherence magnitude \( |\gamma| \) is only influenced by the local value of soil moisture, not its distribution above the layer. For the phase and the power, the influence of the overlying soil can be removed by forming spatial differences, i.e. spatial phase differences \( \Delta\phi \) and logarithmic power double differences \( \Delta\Delta p \). Similarly to the coherence, these two quantities are then expected to be only influenced by soil moisture changes between the top of the upper layer and the bottom of the lower layer. As they are also exclusively influenced by local soil moisture changes, they are expected to show a pronounced covariability with each other and also with the coherence magnitude.

4 Results

4.1 Observed power

The observed response of the backscatter to soil moisture variations varies as a function of depth along the profile; both increases and decreases of the power are observed as the soil moisture increases during the watering period (Figs. 7 and 8). The power retrieved from the near-surface (approx. top 10 cm, layers 1-3) first decreases but subsequently increases by 3-5 dB compared to the dry soil as more water is added. The increase of the backscattering power is most pronounced in the VH channel.

The application of water also affects the deeper soil (layers 5-8): initially during the 200 ml phase, the backscatter \( \Delta p \) first decreases by 1-2 dB and subsequently increases to up to 4 dB compared to the dry reference scan (1.0 - 1.8 l of water added). As more water is applied (> 2 l) during the remainder of the watering period, \( \Delta p \) drops by up to -16 dB in HH. This contrasts with the buried corner reflector (B), whose backscatter \( \Delta p \) decreases continuously with the application of water (to less than -30 dB) and stays below its initial value for the entire experiment (Fig. 8). As the soil dries, the referenced power \( \Delta p \) increases more quickly for the buried trihedral B than it does for the soil profile.

The side trihedral S is characterized by backscatter fluctuations \( \Delta p \) that are generally less than 1-2 dB, see Figs.7–8. The backscatter variations observed along the control profile are smaller than those over the treatment profile (2 dB vs. up to 15 dB). The largest values of up to 5 dB in magnitude occur in the top 5-10 cm during the watering period (within the first 26 hours of the experiment).

4.2 Observed interferograms

The addition of water is also evident in the complex coherences, i.e. the interferometric phases and coherence magnitudes, based on interferograms of two subsequent acquisitions. Figure 9a) shows a vertical profile of those coherences at HH, along with those of the control profile, the buried corner reflector and the reference reflector on the side of the trough. The topmost 15 cm (layers 1-4) respond to the initial addition of water in 200 ml increments by decorrelation and a positive phase change. Underneath (down to a depth of 38 cm, in layers 5-8), the loss of correlation is smaller but they also exhibit an increase in the interferometric phase. Their magnitude of around 0.15\( \pi \) is comparable to that of the buried reflector. These patterns are also reflected in the VH and VV polarization of Fig. 10. During the second phase of the watering period (2 l increments), the lower layers exhibit low coherences and interferometric phases within the entire \( 2\pi \) interval. This response seemingly saturates on the second day, especially for layers 1-4. After the end of the watering period coherence magnitudes \( |\gamma| \ll 1 \) only occur over night, when the time gap exceeds 12 h. The associated phases \( \phi \) during drying have the opposite sign to those during the watering period, i.e. they are negative. So are also those corresponding to time gaps of less than one hour, but these are an order of magnitude smaller.

The soil moisture dependence of the interferograms is also evident when the latter are formed by always using the first acquisition as the master \( m \) (Fig. 9b). These interferograms initially indicate progressively positive phases (\( \frac{\partial \phi}{\partial t} > 0 \)) during the first watering period. Those of the buried reflector are similar to those of the soil at comparable depths (layers
Figure 7: The backscattered power in a polarimetric representation for the a) the watering period, i.e. the first three days of experiment; and b) the drying period (starting on the fourth day). The top four horizontal panels show the backscattered power $\Delta p$ at $\theta = 20^\circ$ (logarithmic) relative to the measurement obtained at the first scan during the first three days of the experiment. This normalization is done for each polarimetric channel; these are colour coded: HH red, VH green, VV blue. The horizontal axis corresponds to the scan number. The first panel shows the standard treatment profile (water added) with the first layer at the top, the second one the control profile (no water added), the third one the buried trihedral $B$, the fourth one the side trihedral $S$. The time between successive scans is given in the penultimate panel, the amount of water added since the previous scan is shown in the last panel.

Figure 8: Relative backscattered power $\Delta p$ (with respect to the first scan) at HH polarization during the wetting phase, with red colours indicating a decrease and blue colours and increase with respect to the first scan. The panels represent the same profiles and targets as in as Fig. 7.

During the addition of 2l increments, the phases of the latter change rapidly ($-\pi < \phi < \pi$). By contrast, the top layers 2-4 respond on much longer time scales.

The drying period during the remaining 32 days of the experiment (starting on the fourth day) is associated with less rapid moisture variations and phase changes. The HH interferograms in Fig. 11 share a common master scene $m$, which is the last acquisition of the experiment, making the phase $\phi_{m,n}$ appear 'inverted' compared to the previous plots. All phases approach the value 0 towards the end of the experiment. The magnitude of the phase change tends to increase with depth. The lower part of the treatment profile to a depth of 38 cm (layers 5-8) behaves similarly to the buried reflector at a comparable depth (especially in the last 3 weeks), whose phase changes by more than one cycle $2\pi$. The upper part of the profile exhibits a different temporal behaviour: for instance, the phase at 10-15 cm depth (layer 3) is generally below 0.1 rad during the last three weeks of the experiment. The transition between the upper and the lower part is associated with diminished coherence magnitudes compared to the rest of the profile. This layer 4 also exhibits the opposite sign behaviour, i.e. $\phi_{m,n}$ increases rather than decreases over time. This behaviour is even more pronounced in the alternative profile in Fig. 9b, the results of which otherwise resemble those of the standard profile.

The covariability between the observed power and the interferometric phase differs between the buried trihedral and the subsurface soil layers. The last scan serves as the reference and hence corresponds to zero power and phase differences in Fig. 12. Earlier acquisitions correspond to different soil moisture profiles and also non-zero power and phase
5 Discussion

5.1 Assumptions and uncertainties

The radar observations of the soil moisture profiles are subject to a number of possible sources of uncertainty. A key requirement for the reliable analysis of temporal changes is the stability of the radar system. Instabilities and drifts may affect the magnitude (resulting in a drift of the power) and the overall phase (impacting the interferometric phase). The fluctuations of the power of the reference trihedral on the side of the trough in Fig. 8 are generally less than 1-2 dB. Consequently, they are more than an order of magnitude smaller than the dynamic ranges of the soil and the buried trihedral that are on the order of 5-50 dB, thus indicating the suitability of the system for monitoring backscatter variations of this size. However, the surface layers of the control profile also respond significantly (up to 5 dB) during the watering period.

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Figure 11: Interferograms at HH polarization during the drying period of the experiment for a) the standard treatment profile and b) the alternative profile. The very last scan is fixed as the master acquisition. The scans (slave acquisitions) are aligned along the horizontal axis and the days along the horizontal axis refer to the time before the last scan. Otherwise same as Fig. 9.

Figure 12: Scatterplots of the power double difference $\Delta \Delta p$ versus the phase difference $\Delta \phi$ at HH polarization for four combinations of locations in panels a) to d). The differences are formed between the location in bold and that in normal font (e.g. trihedral B referenced with respect to S in the first panel). The power double difference $\Delta \Delta p$ (in dB) is the difference between the relative powers $\Delta p$ (normalized with respect to the last scan). The phase difference is the corresponding difference in the interferometric phases and is a ‘wrapped’ quantity, i.e. only known to modulo 2 $\pi$. The corresponding coherence of the bold location is encoded in the marker colour.

period (Fig. 8 and 7). The reasons are not clear but the spatial (only the top 5-10 cm are affected) and temporal (only during the watering period) patterns suggest that it is rather related to the addition of water than to the radar system. Isolated drops of water were observed adjacent to the treatment area, which may have affected the control profile as well. Another error source when measuring the subsurface power $\Delta p$ is induced by the changing dielectric properties, which are not considered in the image reconstruction. However, its magnitude (estimated to be less than 0.5 dB in 2.3) is also small compared to the observed dynamic changes.

The phase stability of the radar system also appears to be sufficient for tracking the interferometric phase over the soil profile. The dynamic changes in the latter commonly exceed 0.5 $\pi$, whereas the observations over the reference trihedral are typically less than 0.1 $\pi$ (e.g. Fig. 11). Larger values ($\approx 0.2\pi$) over the latter are observed on the first day (Fig. 9b),

Figure 13: Scatterplots of the coherence versus the spatial phase difference at HH polarization in three different depths (panels a to c). The interferograms are formed using the last scan as the master. The coherence $|\gamma|$ is evaluated at the layer corresponding to the depth given above each panel (layer 4, 5, and 6, respectively), the spatial phase difference $\Delta \phi = \phi_{\text{below}} - \phi_{\text{above}}$ is formed using the interferometric phases of the layer above and that below.
when positioning errors of the scanning device occurred (Sec. 2.2). The phase variations concomitant with the antenna positioning errors are also evident in the control profiles in the dry sand. The associated phase errors do not show a clear depth dependence (Fig. 9b). There is in general no such depth dependence in the control profile and the phase variations are comparable to those of the reference trihedral. Furthermore, the coherences are stable ($|\gamma| \gg 0.7$).

The radar observations of the soil layers may also be affected by random noise, in particular in situations when the signal is weak. A low return due to intrinsically low backscatter is expected for wet soils with pronounced absorption and for dry sand, e.g. the control profile. The observed coherences (typically $|\gamma| > 0.8$ unless water is added rapidly, see e.g. Fig. 11), however, indicate that the noise level is not a limiting factor. This is also in line with the smooth transition of the coherences towards 1 observed during the drying period (Fig. 11) when the last scan serves as the common master.

In Sec. 2.2 we further identified the influence of the buried trihedral on the signals from the soil as a possible limiting factor, which may be related to its sidelobes or its influence on the soil moisture distribution. The comparison of profiles on either side of the corner reflector (standard and alternative) reveals generally similar patterns (Fig. 11b). However, differences occur, e.g. in the alternative profile, layer 6 does not conform to the temporal patterns of the adjacent layers. Apart from the influence of the buried target due to its sidelobes and its obstruction of vertical drainage, differences between the two treatment profiles may also be related to horizontal heterogeneities of the soil moisture distribution. Even when water is added uniformly, the resulting soil moisture distribution can be horizontally heterogeneous [5]. Such unstable flow, i.e. the formation of flow fingers, is expected in sand when the flow rate is below a critical value (typically several metres per hour in sand) [5]. However, as the typical size of the fingers is comparable ($≈ 10$ cm) to the resolution of the system (see Tab. 1), it is an open question to what extent these can actually be resolved.

5.2 Sensitivity to soil moisture profiles and comparison with model predictions

Flow instabilities and the influence of the buried target contribute to the uncertainties in the spatial and temporal distribution of the soil moisture within the trough. As the spatial distribution is not directly known, the analysis of the results and their comparison with model predictions are not straightforward. However, it is possible to make assumptions about the soil moisture distribution and to assess the consistency of the predictions, e.g. those of the phase and the coherence.

5.2.1 Interferometry

The interferometric model of (9) is broadly consistent with the depth-resolved interferometric observations. The observed covariability of the spatial phase difference $\Delta \phi$ and the coherence $|\gamma|$ supports the key assumption of the family of models, i.e. that the interferometric response is governed by changing transmission properties. According to Sec. 3, these two quantities are predicted to be indicative of the local change in soil moisture content, and they are found to exhibit the expected covariation (Fig. 13); however, the relation is not uniform as a function of depth. This suggests the presence of additional effects, such as non-uniform soil moisture changes. Owing to the lack of direct observations of the spatial soil moisture distribution, it is difficult to attribute this variability to any particular cause.

Indirect reasoning provides further corroboration for the model, based on assumptions about the soil moisture profile. During the watering period, the phase $\phi$ and coherence $|\gamma|$ respond to the addition of water. During the first part of this period (200 ml increments) they do so in a way that is consistent with the slow propagation of a wetting front as predicted by (9): the layers below a depth of 20 cm respond almost uniformly regarding the phase, and the coherence (based on successive scans) remains large ($|\gamma| > 0.8$), see Fig. 9a. The top layers, on the other hand, do decorrelate ($|\gamma| < 0.7$) towards the end of this period. For the entire profile, the interferometric phase with respect to the first acquisition increases with the cumulative water content (see Fig. 9b), which is consistent with the expectations. Furthermore, the magnitude of $\phi$ increases with depth – after 1.5 litres had been added, the phase at a depth of 35 cm is about twice as large as that at a depth of 15 cm. In conjunction with the decorrelation $|\gamma| \approx 0.5$, this suggests gradual wetting of the soil profile according to the model (9). In the subsequent phase of the watering (2l increments), the pronounced decorrelation and the depth-dependence of the phase in 25-35 cm depth suggest that the infiltration front quickly reaches these layers. The phase changes $\phi$ of the lower layers and the buried trihedral wrap around, i.e. they exceed $2\pi$. Such large phase changes have previously only been observed (using depth-averaging systems) over buried, strongly reflecting targets [26] that can be modelled accurately considering only the transmission effects as in (5). The comparable response of the trihedral and the deeper soil layers, which is predicted by the model (see Fig. 5), thus points towards the dominant impact of the transmission between the soil surface and the subsurface soil layers.

During the drying period, the interpretation of the interferograms in terms of the model suggests the formation of a dry layer close to the surface. Specifically, the measurements (Fig. 11) indicate limited change in the top layers 2-3 and a gradual transition towards the final scan in layers 5-8. The sign of the interferometric phases is consistent with drying of the soil. For any given scan, e.g. 14 days before the last acquisition, the magnitude of $\phi$ tends to increase with depth (Fig. 11). As according to Sec. 3 the vertical phase difference $\Delta \phi$ is an indicator of the local soil moisture change, this suggests drying within the entire profile. $\Delta \phi$ is largest at a depth of around 10-15 cm, indicating that the soil moisture change is most pronounced at this depth. Such a vertical partitioning is not unreasonable according to the physical representation of the evaporation process by Lehman et al. [14]. They describe a so-called second stage drying of soil in which the water
transport is limited due to the existence of a dry near-surface layer. In this layer the water flux occurs in the vapour phase. Its depth depends on the grain size distribution of the soil, with typical values of 10-12 cm for sand [14].

However, all these interpretations are based exclusively on the radar signals rather than on direct soil moisture observations. Such direct measurements would permit a more comprehensive analysis of the radar signals and the model assumptions. They would for instance make it possible to analyse the volume backscattering density \( f(z) \) in (6) in more detail. In the existing models this volume return is assumed to be invariant: all the interferometric changes (non-zero \( \phi \)) are due to the changing propagation properties. Future dedicated experiments may thus try to analyse the validity of this assumption. For instance, a change in the local scattering phase would correspond to complex values of \( f_{m,n}(z) \), which would then depend on the acquisitions \( m \) and \( n \). A dependence of the magnitude of \( f_{m,m}(z) \) on the acquisition \( m \), e.g. owing to a relation to soil moisture, may furthermore influence the backscattered power.

5.2.2 Backscattered power

The observations indicate that the backscattered power from sub-surface layers depends not only on the absorption within the soil (between the surface and the particular layer as well as within the latter) but also directly on the soil moisture content. As \( f(z) \) is assumed independent of soil moisture in (9), only the contribution due to absorption is considered in the model. Its dominant role is confirmed by the overall tendency of the power from within the soil to decrease with the cumulative water content (e.g. Fig. 7). It thus supports the model of (9), where the absorption within and above a soil layer determines its backscatter. However, certain temporal patterns do not conform to this model. On the first day the power of e.g. layers 6 and 8 briefly increases by more than 2dB (Fig. 8). Furthermore, the power drop (compared to the first acquisition) observed over the buried corner reflector during the second half of the watering period consistently exceeds that of the soil at the same depth by 5-30 dB. This difference may also be related to the obstruction of vertical drainage by the corner reflector. Deviations from the expectations also occur during the drying period. These become evident as inconsistencies between the interferometric and the backscatter signals. During the last 10 days of the experiments (after day 25), the interferometric response of the top 15 cm indicates little change \((|\gamma| > 0.75, |\phi| < 0.1\pi \text{ with respect to the last acquisition, see Fig. 11})\). The phase of the layers below is still changing at that point (from 0.3 \( \pi \) to 0). If we take these interferometric observations to be an indication that the top 15 cm have essentially dried up and that the soil underneath is still drying (Sec. 5.2.1), we can make predictions according to (9) for the backscatter from the layer 5 just underneath. Firstly, the backscattered power should be lower than before the application of water on day 1, as i) the absorption from the soil above is the same (if completely dry) or larger (residual soil moisture present) and ii) the absorption within layer 5 is larger due to the increased soil moisture. In contrast to these expectations, the observed power is around 6dB larger than at the beginning of the experiment, when the soil was dry. Furthermore, the gradual desiccation during the last days (corresponding to a decreasing phase \( \phi_{m,n} \), when the last acquisition is fixed as the master \( m \)) should cause the backscatter to increase. However, the opposite behaviour is observed, whereas the corner reflector conforms to these expectations (Fig. 12).

Taken together, these deviations from the model predictions indicate insufficiencies of the assumptions on which the model is based. They are consistent with an increase of the backscattering density \( f_{m,m}(z) \) upon wetting, as could be brought about by an increase in the dielectric contrast, i.e. the magnitude of the dielectric fluctuations [44], or alternatively the scattering cross section of the subsurface particles [6]. Ground penetrating radar (GPR) models of the ‘clutter’ due to heterogeneities within natural soils typically do not directly account for the dependence of the variability of the permittivity on soil moisture [27, 35]. However, the clutter power has been observed to change during infiltration experiments [34, 35]. We conjecture that in our experiment the dielectric contrast increased upon wetting. In unsaturated soils, such an increase seems plausible as the magnitude of the dielectric constant of water is more than a factor of 10 greater than that of air or sand grains. The replacement of air within a pore by water may thus enhance the local dielectric fluctuations. This in turn is expected to increase the backscattering density \( f_{m,m}(z) \) [44], consistent with the observed changes in the subsurface scattering. However, the backscattering density \( f_{m,m}(z) \) is furthermore expected to depend on the spatial variability (autocorrelation length scale) of the dielectric fluctuations, or on the size of the particles for discrete particle models [6, 34]. Previous in-situ measurements of the centimetre-scale variability of the soil permittivity have revealed that the length scale depends on the dynamics of the soil water changes [34]. The observed increase of the magnitude of the local scale variability may, however, saturate or even decrease for very wet conditions, similar to what has been observed on larger scales [39].

5.2.3 Subsurface and surface scattering

The subsurface scattering envisioned by the model of Sec. 3 is volumetric in origin in that it is based on two distinct length scales. The soils are heterogeneous on the smaller, or ‘microscopic’, length scale, which is on the order of the wavelength within the soil. The heterogeneity may arise from local soil moisture variations, clods, pores filled by water and air, etc [27, 44]. The ‘macroscopic’ properties, such as the bulk soil moisture, are assumed to change on length scales that are large compared to the wavelength. They govern the propagation of the electromagnetic waves through the soil.

The impact of the vertical soil moisture distribution on the volumetric subsurface scattering profiles is hence twofold, according to this model. The subsurface soil volume scatters because it is dielectrically heterogeneous. The dielectric
contrast and hence the magnitude of the permittivity fluctuations and the backscattering density were inferred to increase with soil moisture: wet subsurface soil volumes scatter more than dry ones. However, the relation between the permittivity fluctuations and the soil moisture – e.g. in the presence of gradients or as soils approach saturation – remains uncertain. In addition to the effective dielectric contrast, the actually measured contribution from a subsurface soil layer is also dependent on the soil above this layer. Increasing soil moisture in the layer above induces increased absorption of the downgoing and the upgoing electromagnetic waves, thus reducing the observed backscatter. It also makes the optical path between the antenna and the subsurface scatterers longer: the interferometric phase of the subsurface scatterers thus corresponds to a movement away from the antenna. These two contributions – scattering and propagation – are depicted schematically in Fig. 14. The way soil moisture profiles impact the observables is broadly consistent with the predictions of the first-order model of Sec. 3 (see Tab. 2).

As the assumed separability of the two length scales may not be valid in practice, future modelling efforts should look into more general approaches. Actually, this assumption, which is implicit in the geometric optics approximation of (8), may frequently break down in practice: soils are layered; the capillary fringe above the water table can be small compared to the wavelength; or metallic objects may be buried in the soil [3]. The impact of the soil moisture variability at the wavelength (continuous profiles) or sub-wavelength (step changes, e.g. layers) scale on the observed radar signals can be exploited in depth-resolved full waveform GPR approaches for estimating soil moisture profiles [13, 21]. At the C-band frequency employed here, step-like soil moisture-related permittivity variations at the subsurface wavelength scale of 1-3 cm are likely to have occurred, especially during the initial watering period. Scattering from permittivity variations at a range of length scales can be described using computational electromagnetics approaches, but at the cost of increased model complexity [27, 29]. Within simple analytical models like that of Sec. 3, the influence of soil moisture profiles such as step-like changes may be accounted for by using an effective backscattering density \( f_{m,n}(z) \), especially in the interferometric case (e.g. by also adapting \( f_{m,n}(z) \)). Model extensions may also include multiple reflections, as strong subsurface reflectors may furthermore cause appreciable higher order scattering [28], which are neglected in the first-order model of Sec. 3. However, arguably the biggest obstacle is that this assumption is virtually always violated at the soil surface. It gives rise to the notion of surface scattering, which is described separately in the model by [44] (Sec. 3).

There appears to be a complex trade-off between the contributions by surface and sub-surface scattering in the experiment. The distinct surface backscattering contribution cannot be observed directly, only the combined surface and near-surface (≈ 6 cm) backscatter (cf. the simulation in Fig. 6a). The latter does not conform entirely to the model predictions either: the power of the topmost layers first decreases (during the 200 ml increment phase), and subsequently increases upon the addition of more water, reaching its maximum one day after the end of the watering period. Such an increase is generally consistent with surface scattering models such as the small perturbation model (SPM), which is used in the approach by Zwiebach et al. [44]. Conversely, the initial decrease is not expected if surface scattering dominates, unless surface roughness changes during the watering period also occurred. These cannot be ruled out; however, as the surface roughness remained much smaller than the wavelength throughout the experiment, one may also consider the increased absorption as a possible explanation. Previous depth-averaged observations over smooth sandy soil at L-band by Liu et al. [17] also exhibited a decrease of the backscatter with soil moisture when the soil was sufficiently dry (\( m_w < 0.07 \text{m}^3\text{m}^{-3} \)), followed by the typically expected increase as more water was added. The authors interpreted the initial dimming to be indicative of subsurface volume scattering, whose contribution was reduced by the increased absorption, and the subsequent brightening to be due to the surface scattering contribution becoming dominant. A trade-off between surface and subsurface scattering may also contribute to the non-monotonic soil moisture dependence of the backscattered power observed with C-band scatterometers over sandy deserts [38]. The presence of subsurface scattering

![Figure 14: The impact of soil moisture on subsurface scattering appears to be twofold: it increases subsurface scattering within the soil volume of interest, and it governs the propagation – the absorption and the optical path length – above and within.](image-url)
may furthermore contribute to limitations in the applicability of surface scattering models for the estimation of surface parameters [15, 37], as e.g. evidenced by the need to calibrate the surface roughness as a function of soil moisture. In summary, the depth-resolved observations highlight the complexity of the microwave scattering properties of bare soil due to surface and subsurface contributions, which may limit the applicability of surface scattering models to soil moisture retrieval based on depth-averaged radar observations [17].

Measurements in different polarizations may provide vital clues as to the relative contributions of surface and subsurface scattering [7, 33]. However, the model by [44] is known not to be adequate for the VH/HV cross polarization, as it predicts zero HV backscattering for both the surface (small perturbation model) and the subsurface contribution (first order backscatter from isotropic permittivity fluctuations). In this experiment, we observe the largest changes $\Delta \rho$ in near-surface scattering in the cross polarization. In deeper layers we find non-zero VH backscatter as well, and also an interferometric response that is consistent with subsurface volume scattering. Previous depth-averaged DInSAR observations at Ku and L-band [44, 46] also indicate a non-zero subsurface response in HV that is similar to that in HH and VV. The model by De Zan et al. [6] treats the subsurface scattering cross-section in any polarization as a free parameter so that it can account for the observed signal in HV. By contrast, the volumetric subsurface modelling by [27] is restricted to two dimensions and hence is not applicable to the HV polarization. The polarimetric response is expected to relate to the shape and orientation of subsurface particles, or – in a spatially continuous description – to the three dimensional variability of the dielectric fluctuations, including their anisotropy [36, 44]. Future studies may investigate this link between the subsurface structure and the polarimetric response, as such insight and improved models could help to better characterize the relative importance of surface and subsurface scattering, also in the depth-averaged case.

5.3 Volumetric subsurface scattering and soil moisture profile estimation

The observed sensitivity of the depth-resolved observations of volumetric subsurface scattering to soil moisture changes suggests significant potential for estimating dielectric profiles, e.g. soil moisture profiles or for determining the active layer depth [41]. In many situations, however, these approaches will also have to take into account the presence of non-volumetric subsurface scattering (see Sec. 5.2.3), associated with strong reflectors, subsurface layers or non-uniform soil moisture profiles. In fact, previous approaches using ground penetrating radars have focussed on the latter, treating the volumetric ‘clutter’ as more of a nuisance [13, 21, 32]. The presence of both effects indicates the potential of combined approaches. Moreover, such techniques for soil moisture estimation may in the future incorporate differential interferometry, which has been shown to be a sensitive indicator of moisture changes for both volumetric (Sec. 4) and non-volumetric scattering (e.g. the buried reflector B, or [25]). They may similarly be applied to snow, ice or vegetation canopies: depth-resolved differential interferometry has already shown its potential to image changes – for instance due to movements – within forests [18, 22].

6 Conclusions

Tomographic Profiling was used to image the vertical radar backscatter profile of a sandy soil mixed with gravel, whose heterogeneous nature gave rise to subsurface volume scattering. We observed the infiltration of water applied to the surface of the initially dry soil, followed by subsequent drying. The polarimetric C-band measurements (4-6 GHz, 5-10 cm resolution) show that the subsurface radar backscatter profile associated with small-scale heterogeneities responds to changes in soil moisture (exceeding 15 dB). Its observed dependence on the soil moisture profile suggests that several physical phenomena are relevant. The moisture-dependent absorption of the microwaves within the soil appears to be the dominant factor. However, there are several lines of evidence which suggest that also the subsurface ‘volume’ scattering power responds to changing soil moisture contents. Subsurface scattering also plays a key role in determining the interferometric radar response of soils, as it can give rise to signals that correspond to spurious displacement measurements. The depth-resolved observations provide novel and direct support for existing models of these effects. They are consistent with their central tenet, namely that the conventional (depth-averaged) radar signals are influenced by the moisture-induced changes of the microwave propagation within the soils. However, these models do not consider the observed dependence of the subsurface scattering power on the local soil moisture content (in addition to the changing absorption). Depth-resolved measurements may thus serve as input to future model improvements that incorporate this effect. As volumetric subsurface scattering is commonly neglected in conventional backscatter studies, depth-resolved observations can also potentially inform the analysis and modelling of the depth-averaged backscatter, e.g. in soil moisture studies.

Depth-resolved microwave imaging of subsurface volume scattering, or ‘clutter’, may in future provide a more comprehensive characterization of near-surface soils, which are known to commonly exhibit pronounced depth variability in their properties and moisture content. Such monitoring requires additional experiments that further elucidate the relation between the depth-resolved radar signals, the soil moisture profiles, and the soil properties. The experiment was restricted to only one type of soil, and the lack of reference soil moisture observations was an additional limiting factor. The observations did however indicate the importance of i) the surface scattering contribution, ii) the volume scattering due to sub-surface heterogeneities, and iii) the propagation (e.g. absorption) of the microwaves within the soil. An improved
understanding of the role of these factors, including their dependence on various soil properties, is required in order for Tomographic Profiling and related depth-resolving microwave techniques at X- to L-band to become useful in practical applications. The subsurface volume scattering may also be exploited in ground penetrating radar studies of soils, which so far have neglected such information when inferring soil moisture profiles, focussing e.g. on strong reflectors instead. The combined use of volume scattering and these existing techniques may extend the applicability of these approaches (e.g. in the absence of strong reflectors), and also their accuracy, especially when including interferometric information, which we show to be particularly sensitive to soil moisture changes. Hydrology, archaeology and irrigation management are such fields of application that could profit from an improved ability to map the depth variability of soils and of their moisture content.

References


Chapter G

A statistical test of phase closure to detect influences on DInSAR deformation estimates besides displacements and decorrelation noise: two case studies in high-latitude regions

S. Zwieback, X. Liu, S. Antonova, B. Heim, A. Bartsch, J. Boike and I. Hajnsek


Key findings:
- statistical significance testing can indicate whether the phase triplets can be explained by noise alone
- evidence for spurious influences on differential interferometry in high latitude regions
- closure errors associated with snow melt, soil moisture changes, and unknown processes

The author’s contributions:
- suggested and developed the significance test for analysing DInSAR data in high latitude regions
- performed the simulations for assessing its statistical properties
- participated in field campaign for second case study
- analysed the data and wrote the manuscript

The co-authors’ contributions:
- X.L. (BSc student) analysed and interpreted the data of the second case study
- S.A., B.H., A.B., and J.B. were responsible for field campaign and provided input (second case study)
- all co-authors interpreted the results and contributed to writing the manuscript
A statistical test of phase closure to detect influences on DInSAR deformation estimates besides displacements and decorrelation noise: two case studies in high-latitude regions

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Abstract
Displacements of the earth’s surface can be estimated using differential interferometric SAR (DInSAR). The estimates are derived from the phase difference between two radar acquisitions. When at least three such acquisitions are available, one can compute the displacement between the first and the third one and compare it to the sum of the two intermediate displacements. These two are expected to be equal for a piston-like, spatially uniform deformation. However, this is not necessarily the case in measured data. Such a lack of phase closure can be due to decorrelation noise alone. It has also been attributed to complex scattering processes such as soil moisture changes or multiple scattering sources. However, the nature of these non-random effects is only poorly understood in cold regions, as the role of snow and freeze/thaw processes has not been studied to date. In order to distinguish the noise-like and the systematic effects, an asymptotic Wald significance test is proposed. It detects situations when the observed closure error cannot solely be explained by noise. Such situations with $p < 0.05$ are observed at Ku-band during snow metamorphism and melt, and following a summer precipitation event in Sodankylä, Finland. They can also be prevalent (25%) in X-band observations of ice-rich permafrost regions in the Lena Delta, Russia, indicating the presence of processes that can have systematic and deleterious impacts on the estimation of surface movements. Satellite-based monitoring of these displacements is thus possibly subject to complex error sources in high-latitude regions.

1 Introduction
DInSAR radar interferometry is an active remote sensing technique for estimating displacements [1, 2], and has found applications in hydrology [3], geophysics [4], and the monitoring of infrastructure and permafrost-related subsidence [5, 6]. These displacement estimates are derived from the phase $\phi$, which is obtained by combining two radar acquisitions taken at different times. Apart from influences due to e.g. the elevation and the atmospheric conditions, the phase $\phi$ is proportional to the displacement that occurred in between [7]. When not only two but more radar acquisitions are available, an entire time series of deformations can be derived. The relevant algorithms have to take into consideration that the derivation of these deformations can cease to be unique as soon as three acquisitions are available [8]. From three acquisitions, three phases and thus three displacement estimates can be derived. However, the displacements themselves are constrained, as the displacement between times $t_1$ and $t_3$ equals the sum of the two intermediate ones (see Fig. 1). As the appropriately referenced phase $\phi_{ij}$ for a single target is proportional to the displacement between times $t_i$ and $t_j$, one expects – in the absence of noise and other influences – the phases to also fulfill this consistency criterion [9, 10]. We refer to this consistency as phase closure or phase triangularity [11].

By contraposition, the presence of an inconsistency (or non-zero closure phase) implies the presence of additional phenomena besides piston-like deformations. These include the impact of decorrelation noise, but also non-random effects different from noise have been reported. De Zan et al. [11] observed that changes in soil moisture and in the water content of vegetation could lead to the absence of phase closure. The observations and electromagnetic models indicate that the loss of phase closure is induced by dielectric changes associated with changes in the moisture content [12, 13]. Similar changes in the dielectric properties also occur when soil freezes and when snow melts, but their impact on DInSAR phase triangularity has not been studied before. A further factor that can lead to non-zero closure phases is related to the three dimensional structure of the target: when the radar acquisitions are not made from the same position, the scattering from multiple objects within a resolution cell can break the consistency of the phase [11]. Apart from noise, these phenomena have the potential to systematically bias the deformation estimates [12], as they render the straightforward conversion of referenced phases to displacements erroneous. Many common interferometric time series approaches do not account for systematic closure errors [11].
In order to study such non-random effects on the DInSAR signals, we introduce a significance test that determines whether the phase closure errors can be explained by noise alone, or whether they are significant instead. When the closure errors are large compared to the phase noise (and thus significant), they may be the limiting factor in the accuracy of the displacement estimates. The detection of such significant effects in the closure phase could therefore help to characterize the uncertainty of the deformation estimates, even though the impact of e.g. soil moisture changes can be considerably larger on the phase – and thus the deformation estimates – than on the closure phase [14]. Such an approach could also be used to exclude areas for which displacements are computed in the first place. Such flagging has previously been suggested by [9] within the SqueeSAR approach for the analysis of interferometric stacks. Their phase triangulation algorithm (PTA) estimates the DInSAR phase history using the assumption that the observed phase closure errors are due to decorrelation noise alone. The flagging of areas that are to be excluded is based on the test statistic $\gamma_{PTA}$, which measures the misfit between the observed phases and the estimated phase history that is free of closure errors. However, this flagging does not account for the magnitude of the phase noise, i.e. loss of correlation, which can vary in space and time.

The significance test that we propose takes account of the decorrelation, and thus the phase noise, to flag affected areas and acquisitions. This Wald test [15] is derived from the asymptotic properties of the Maximum Likelihood (ML) estimators under the Gaussian speckle model [16]. We employ this method in two case studies to analyse the prevalence and possible causes of such significant closure errors. We focus on high latitude areas characterized by cold region processes such as snow melt, soil freezing and thawing, and moisture changes in organic soils. Their impact on the phase closure has not been studied before, despite the increasing number of DInSAR applications dealing with snow, seasonally frozen ground and permafrost [17, 6, 18, 19]. The first case study concentrates on a tower-based zero-baseline Ku-band time series of a forest clearing in Finland: the data were acquired over a snow pack that underwent complex changes in terms of snow accumulation, metamorphism and melting [20, 21]. The data set also extends to the snow-free period, during which changes in soil moisture occurred. The second case study analyses the spatial and temporal patterns that occur at X-band over a heterogeneous permafrost area in the Lena Delta, Russia. There, surface displacements are closely connected to permafrost thaw [22]. Thawing is related to the release of old, previously frozen organic carbon, which in turn can lead to the emission of greenhouse gases and thus affect the global climate [23, 24]. The reliable monitoring of these displacements using DInSAR can provide insight into these thawing processes [25, 26]. However, it requires an adequate understanding of potential systematic error sources such as moisture changes [17, 6, 18, 19], which we propose to characterize with respect to the closure phase errors.

2 Interferometry

2.1 First-order statistics

These non-zero phase closures do not occur for idealized point scatterers, but they can be found over more complex targets, i.e. when the typical resolution cell of interest can be thought of as an accumulation of smaller entities (such as surface patches or parts of vegetation) [16]. A homogeneous patch of such a distributed target will give rise to a granular texture in the radar image, i.e. it is subject to the speckle effect [16]. The radar measurements of such targets are commonly conceived of as realizations of a random variable, and the target is similarly considered to be a stochastic quantity [27]. According to the first-order scattering model [28] of monostatic radar measurements, the complex SLC (single look complex) value $y_m$ of a given polarization arises from the superposition of the numerous smaller entities present in the resolution cell:

$$y_m = \sum_u u^s \exp(2i k_0 u \cdot l)$$

The contribution from each scatterer $u$ at $u \cdot P$ stems from its complex scattering amplitude $u^s$ and the propagation phase of the wave with free-space wavenumber $k_0$ between the antenna and the scatterer. This propagation is encoded in the
Figure 2: Multiple scatterers (dark dots) within a dielectric medium (in purple). In the geometric optics approximation, the propagation of the electromagnetic waves can be characterized by wavefronts of constant phase (e.g. points A and B), to which the rays are orthogonal. These undergo refraction where the refractive index $n$ changes ($P'$).

The optical path $u_l$, defined as [29]

$$u_l = \int_P^R n \, ds$$  \hspace{1cm} (2)

It is the integral of the refractive index along the ray path, which obeys Fermat’s principle. It is thus subject to refraction, which is illustrated for the scatterer at point P in Fig. 2, which is located within a dielectric medium (in purple, the refraction occurs at $P'$).

### 2.2 Second-order statistics

In radar interferometry, two SLC radar acquisitions $y_m$ and $y_n$ are combined by cross-multiplying and (for distributed targets) by averaging $\langle \cdot \rangle$

$$C_{m,n} = \langle y_m y_n^* \rangle$$ \hspace{1cm} (3)

The complex coherence is obtained by normalization

$$\gamma_{m,n} = |\gamma_{m,n}| e^{i \phi_{m,n}} = \frac{C_{m,n}}{\sqrt{C_{m,m} C_{n,n}}}$$  \hspace{1cm} (4)

and its argument is called the phase $\phi_{m,n}$. This phase is proportional to the change in the optical path length for a single target, assuming identical antenna positions at acquisitions $m$ and $n$, as well as discounting phase offsets, noise, and a change of scattering phase. A displacement $d$ along the line of sight in a medium with $n - 1$ leads to change of the optical path length (2) of $d$, and a phase difference $\phi_{m,n} = 2k_0 d$.

The expected phase and coherence of a distributed target can be modelled by extending (1); it is commonly assumed that the returns from different scatterers are uncorrelated [30], thus yielding

$$C_{m,n} = \sum_u \langle u_s_m u_s_n^* \rangle \exp(2i k_0 (u_m - u_n^*))$$ \hspace{1cm} (5)

or in a continuous description [31]

$$C_{m,n} = \int f_{m,n}(x) \exp(2i k_0 (l_m(x) - l_n^*(x))) \, dx.$$  \hspace{1cm} (6)

For such distributed targets the closure phase or phase triplet can be non-zero. It is defined by [32]

$$\Xi_{m,n,o} = \phi_{m,n} + \phi_{n,o} - \phi_{m,o}$$ \hspace{1cm} (7)

or alternatively (sidestepping problems due to phase wrapping) as the argument of the bicoherence [14]

$$\Xi_{m,n,o} = \arg(\gamma_{m,n} \gamma_{n,o}^* \gamma_{m,o}^-)$$ \hspace{1cm} (8)

The closure phase is a phase-like quantity, to which a corresponding displacement can be defined

$$D_{m,n,o} = \frac{\Xi_{m,n,o}}{2k_0}$$  \hspace{1cm} (9)
2.3 Origins of non-zero phase closure

This closure phase is not affected by any phase offset that affects all scatterers in the same way [11]. This includes piston deformations, when all scatterers are displaced by the same amount, see Fig. 1. It also pertains to unknown phase offsets due to e.g. the calibration or propagation within the atmosphere [11]. The latter is illustrated in Fig. 2, where it is assumed that the optical path length between the antenna and all points along the plane wavefront A-B is identical.

2.3.1 Non-zero baselines

When the radar antenna is not at the same position at all acquisitions, i.e. in the presence of non-zero spatial baselines, the difference in the optical path lengths in (5) of any scatterer \( u \) depends on its position. For an extended target at the same height (assuming range spectral filtering has been applied [33]), this phase contribution to the position will be identical for all such scatterers. It will thus not impact the phase triplet [11]. As soon as two targets at different heights are present, non-zero closure phases can be obtained. For two such scatterers in (5), the magnitude of the phase triplet can be estimated by specifying the relative amplitude \( \alpha \) (independent of acquisition) of the two scatterers [11]

\[
C_{i,j}(\alpha, \varphi_{i,j}) = \alpha \exp\left(-\frac{1}{2}k\varphi_{i,j}\right) + (1 - \alpha) \exp\left(\frac{1}{2}k\varphi_{i,j}\right)
\]

where \( \varphi_{i,j} = \kappa_{i,j}\Delta h \) is the phase contribution due to the difference in heights \( \Delta h \) with interferometric height wavenumber \( \kappa_{i,j} \). This phase, which is not directly observable, obeys the closure relation \( \varphi_{i,j} + \varphi_{j,k} - \varphi_{i,k} \) [2]. However, the interferometric phase \( \phi_{ij} = \arg(C_{i,j}) \) does not in the two scatterer case. The worst case phase triplet \( \Xi' \) is obtained by maximizing the magnitude of the closure phase with respect to \( \alpha \)

\[
\Xi'(\varphi_{1,2}, \varphi_{2,3}) = \max_\alpha \left| \arg\left(C_{1,2}C_{2,3}C_{1,3}^*\right) \right|
\]

This worst case scenario is plotted in Fig. 3, where the lines at which the closure phase changes abruptly correspond to vanishing bicoherences. In astronomical interferome-try, an analogous dependence of the observable phase triplets on the unobservable quantities \( \alpha \) and \( \varphi_{i,j} \) (corresponding to the spatial separation of the stars) is exploited to model unresolved binary stars [34] as well as more complex structures [32, 35, 11].

2.3.2 Zero baseline

When the antenna always remains at the same position, the phase variety causing non-zero closure phases [11] cannot be due to scatterers at different heights because the interferometric wavenumber \( \kappa_{ij} \) is zero. Instead, it may be induced by heterogeneous variations of the scattering phases \( \arg\left(\langle u^s_m v^s_n \rangle\right) \). It could also be due to differential, i.e. heterogeneous, movements: two scatterers \( i, j \) that move with respect to each other (by an amount \( \Delta d_{ij} \) along the line of sight) will yield worst-case phase triplets as illustrated in Fig. 3, where \( \varphi_{i,j} \) now corresponds to \( 2k_0\Delta d_{ij} \).

The propagation in a changing dielectric – such as soil with a varying amount of soil moisture – has been shown to have a similar effect [13]: an increase in the real part of the refractive index \( n \) in (2) for two embedded stationary targets has the same effect as an increased separation of the scatterers, i.e. it acts like an apparent differential movement. Such a changing refractive index, related to a change of moisture content in soils and vegetation [11], has been linked to observed non-zero closure phases [13].
3 Statistics of estimated phase triplets

3.1 Interferometric covariance matrix

The statistical properties of the observed closure phases are related to those of the radar acquisitions from which they are derived. For an idealized homogeneous distributed target, the Gaussian speckle model (fully developed speckle) is typically adequate to describe the statistics of the SLC acquisitions [27]. At least two such SLC images, say \( N \), are combined in interferometry. The stacking of the complex SLC values for one pixels gives an \( N \)-dimensional vector \( y \):

\[
y = [y_1 \cdots y_N]^T
\]

the distribution of which – assuming fully developed speckle – is given by the circularly-symmetric complex normal distribution [15] with probability density

\[
f(y) = \frac{1}{\pi^N|C|} \exp(-y^T C^{-1} y)
\]

The covariance matrix \( C \) encapsulates all the decorrelation and phase properties described in Sec. 2 [36]. It is not known in practice but has to be estimated from the data – the Maximum Likelihood (ML) estimate is given by [2]

\[
\hat{C} = \frac{1}{L} \sum_{l=1}^{L} y_l y_l^T
\]

where \( L \) independent samples \( y_l \) (also called looks) are averaged. This Maximum Likelihood estimate \( \hat{C} \) follows a complex Wishart distribution [37]: \( \mathcal{W}^C, \hat{C} \sim \mathcal{W}^C(K, \frac{1}{N} C) \).

3.2 Parameterization of the covariance matrix

The covariance matrix \( C \) is Hermitian [37], and as such it is completely specified by its elements along and above the diagonal. It is convenient to separate these two parts: the coherences are gathered in the correlation matrix \( P = \Delta^{-1} C \Delta^{-1} \) where \( \Delta = (\text{diag} \ C)^{1/2} \). This correlation matrix contains the complex coherences as off-diagonal elements

\[
P = \begin{pmatrix}
1 & e^{-i\phi_{1,2}} & \ldots & e^{-i\phi_{1,N}} \\
e^{i\phi_{1,2}} & 1 & \ldots & e^{-i\phi_{2,N}} \\
\vdots & \vdots & \ddots & \vdots \\
e^{i\phi_{1,N}} & e^{i\phi_{2,N}} & \ldots & 1
\end{pmatrix}
\]

It contains \( \binom{N}{2} \) phases \( \phi_{i,j} \) with \( i < j \). These phases can be grouped into two categories, based on the master acquisition \( i \) and the definition of the phase triplets in (7) [38, 35]:

- those with \( i = 1 \); there are \( N - 1 \) such terms
- those with \( i \neq 1 \): \( \phi_{i,j} = \Xi_{1,j} + \phi_{1,j} - \phi_{1,i} \); there are \( \binom{N-1}{2} \) such terms

By virtue of this separation, one can parameterize the correlation matrix \( P \) in (15) in terms of \( \binom{N}{2} \) coherences \( \gamma_{i,j} \), \( N - 1 \) phases \( \phi_{1,j} \) with a common master acquisition, and \( \binom{N-1}{2} \) phase triplets \( \Xi_{1,i,j} \) with a common master acquisition.

All \( \binom{N}{2} \) phases \( \phi_{i,j} \) will be grouped in a vector \( \phi \), the \( \binom{N-1}{2} \) independent phase triplets in another one \( \Xi \). This triplet vector \( \Xi \) is expected to vanish when phase closure holds, e.g., in the presence of piston deformations.

The splitting of the \( \binom{N}{2} \) phases into two groups is a valid parameterization of the covariance matrix \( C \), and as such it is equivalent to the one in (15). Having estimated \( \hat{C} \) via ML from (14) and represented it in terms of that parameterization (15), one can derive the \( \binom{N-1}{2} \)-dimensional estimated \( \hat{\Xi} \) via (7). As the ML solution is invariant to the parameterization of \( C \) [39], these estimates \( \hat{\Xi} \) are also the ML estimates of the phase triplet vector \( \Xi \).

3.3 Statistical properties of the triplet estimator

The statistical properties of these estimates \( \hat{\Xi} \) are determined by those of the \( \binom{N}{2} \) estimated phases \( \hat{\phi} \), which in turn are related to those of \( \hat{C} \). The latter – in particular the moments of the elements – have been compiled by [40]:

\[
\text{Cov}(\hat{C}_{ij}, \hat{C}_{kl}) = \frac{1}{L} C_{kj} C_{il}
\]
the covariance between two elements of the estimate \( \hat{C} \) (with \( i \neq j \) and \( k \neq l \)) depends on the number of looks \( L \) and two elements of the underlying \( C \) matrix. These covariances can also be expressed in terms of the real and imaginary parts of the components

\[
\text{Cov}((\Re(\hat{C}_{ij})), \Im(\hat{C}_{ik})), (\Re(\hat{C}_{kl})), \Im(\hat{C}_{kl})) = \frac{1}{2L} \left( \Re(C_{ik}C_{jl} + C_{ik}C_{jk}) \right) \left( \Re(C_{ij}C_{jk} + C_{ij}C_{kl}) \right) \left( \Re(C_{ik}C_{jl} - C_{ik}C_{jk}) \right)
\]

The second-order moments of the phase estimates \( \hat{\phi}_{ij} \) are obtained from (17) using the Jacobian matrix of the transformation to polar coordinates around the parameter value \( C_{ij} \). The linear approximation inherent in this uncertainty propagation becomes increasingly accurate as the uncertainty in the estimated phase \( \hat{\phi}_{ij} \) decreases:

\[
\text{Cov}(\hat{\phi}_{ij}, \hat{\phi}_{kl}) = \frac{1}{2L} |\gamma_{ik}| |\gamma_{jk}| \cos(\Xi_{i,j,k} - \Xi_{i,k,l}) + |\gamma_{ij}| |\gamma_{jk}| \cos(\Xi_{i,j,k} + \Xi_{i,k,l})
\]

These covariances make up the covariance matrix of the phase estimates, \( \Sigma_{\hat{\phi}} \). For its diagonal elements, i.e. the variance of \( \hat{\phi}_{ij} \), (18) reduces (by setting \( i = k \) and \( j = l \)) to the Cramer-Rao bound [41]

\[
\text{Var}(\hat{\phi}_{ij}) = \frac{1}{2L} \frac{1 - |\gamma_{ij}|^2}{|\gamma_{ij}|^2}
\]

The off-diagonal elements and thus also the correlations between phase estimates depend on both the coherences and the phase triplets (the case for vanishing phase triplets has been studied by [11]), which is illustrated in Fig. 4.

As the \( (N^2 - 1) \)-dimensional phase triplet vector \( \Xi \) is a linear function of all the \( \binom{N}{3} \) phase estimates \( \phi \) [38] – in matrix notation \( \Xi = A\phi \), the covariance matrix of the estimated phase triplets \( \Sigma_{\hat{\Xi}} \) follows via standard variance propagation from the covariance matrix of the estimated phases \( \Sigma_{\hat{\phi}} \)

\[
\Sigma_{\hat{\Xi}} = A \Sigma_{\hat{\phi}} A^T
\]

### 3.4 Significance test

The estimator of the phase triplets (7) is the ML solution and in general it will hence be consistent (converge to the actual value \( \Xi_0 \)) and asymptotically normal [15]:

\[
\Xi - \Xi_0 \xrightarrow{\Delta} \mathcal{N}(0, \Sigma_{\hat{\Xi}})
\]

Asymptotically, its covariance matrix \( \Sigma_{\hat{\Xi}} \) in (20) thus determines its distribution. Although it is not known, but rather has to be estimated from the data based on the inferred coherences and phase triplets, even an estimated version allows one to construct approximate confidence intervals and perform significance tests [15]. Such a significance test can be based on the chi-squared distributed test statistic \( s_{\chi^2} \), which measures the deviation of the closure phase estimates from \( \Xi_0 \) relative to their uncertainties

\[
S_{\hat{\Xi}} = (\hat{\Xi} - \Xi_0)^T \Sigma_{\hat{\Xi}}^{-1} (\hat{\Xi} - \Xi_0) \sim \chi^2_k
\]
Figure 5: Study of the impact of the linearization and normality assumptions on (a) the accuracy of the phase variance, and (b) the accuracy of the estimated $p$ values. The latter compare the observed type I error probability (simulated using $M = 50000$ samples and the covariance matrix $C$ of Fig. 4) with its expected value of $\alpha \sim 0.05$, indicated by a grey line.

where the number of degrees of freedom $k = (N-1)/2$. In order to test the assumption that $\Xi = \Xi_0$ (where $\Xi_0$ is in this paper the zero-vector, i.e. it is hypothesized that the phase triplets vanish) one can compute the $p$-value $p_w$ of the observed test statistic $s_\Xi$ based on the $\chi^2$ distribution [15]. This is the probability that the test statistic $S_\Xi$ is more extreme than the observed one assuming the null hypothesis distribution of (22)

$$p_w = P_{\chi^2}(S_\Xi > s_\Xi) \quad (23)$$

3.5 Underlying assumptions

This distribution and the associated test (generally known as a Wald test [15]) are accurate under the previously stated assumptions:

1. fully developed, homogeneous speckle
2. linearization of the phase covariance matrix ($\sqrt{\text{Var} \phi_{i,j}} \ll 2\pi$)
3. asymptotic normality of ML estimation ($L \to \infty$)

The first of these assumptions, the joint circular normality of the SLC image vector $\gamma$, can be examined by the Mardia test [42, 43]. The rationale of this approach is based on the comparison of the kurtosis (i.e. the third-order moments) with those expected under a normal distribution, applied to each SLC image separately [44]. The outcome of this procedure is a $p$-value $p_\text{M}$.

The second and third assumptions depend on the number of looks $L$ (i.e. the processing), as well as the coherences and phase triplets (i.e. the underlying data). We propose to explore their validity and limitations using Monte Carlo simulations. Based on $M = 50000$ samples drawn from (13) with $N = 2$, the accuracy of the linearized formula for the phase variance is found in Fig. 5a to increase with the number of looks: the ratio $r$ of the linearized to the correct (estimated from the $M$ samples) variance approaches 1, the two values thus becoming identical. For a fixed number of looks, the quality of this approximation depends on the coherence $|\gamma|$, with larger coherences corresponding to higher accuracies. The variances obtained by linearization are generally too large, such that large deviations are in reality not as common as they are predicted to be based on the linearized variance. This translates to a test that is conservative: the actual type I error probability $\hat{p}$ is smaller than the one specified a priori $\alpha$ (i.e. the theoretical false alarm rate or significance level). This is made evident in Fig. 5b, which illustrates how the combined effect of assumptions two and three – the distributional assumption in (21) using the estimated phase covariance matrix – leads to increasingly conservative tests as $|\gamma|$ and $L$ decrease, whereas the effect of $\Xi_{i,j,k}$ on $\hat{p}$ is much smaller than 0.01 (not shown).

4 Case study 1: NoSREx

4.1 Data & study area

We study the phase triplet magnitudes, the uncertainty of their estimates, and their significance over snow using the ground-based Ku-band system SnowScat [20]. A dense quadpol radar time series – its repeat period is 4 hours – was

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sensor type</th>
<th>Sensor name</th>
<th>Location</th>
</tr>
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<tr>
<td>Air temperature</td>
<td>resistance thermometer</td>
<td>Pentronic PT100</td>
<td>2 m height</td>
</tr>
<tr>
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<td>resistance thermometer</td>
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<td>2 cm depth</td>
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<td>acoustic sensor</td>
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<tr>
<td>Soil moisture</td>
<td>dielectric impedance probe</td>
<td>Delta-T ML2X</td>
<td>2 cm depth</td>
</tr>
</tbody>
</table>

Table 1: Relevant in-situ measurements acquired in the forest clearing during the NoSReX campaign [45].

acquired during the Nordic Snow Radar Experiment (NoSReX) in Sodankylä, Finland [45]. The main purpose of this campaign was the study of the backscatter properties of snow: apart from the radar acquisitions, also microwave brightness temperatures along with snow, soil and atmospheric parameters were regularly measured. The measurements analysed in this study are summarized in Tab. 1. The radar data have since also been used for differential interferometry [21].

During the time period from November 2011 to May 2013, a snow cover of up to 70 cm depth accumulated and subsequently melted twice. We study the data acquired in the clearing, shown in Fig. 6a, with an elevation angle of 40°. Each acquisition was processed in the frequency band around 12 GHz with a total bandwidth of 5 GHz [21]. From each such acquisition, $L = 100$ independent looks are extracted; by thinning we also obtain samples of $L = 33$ and $L = 50$.

The region is shown in both the photo and the radar image of Fig. 6.

When computing the interferometric covariance matrices and p-values, we vary the time windows $T$ and time gaps between the images $\Delta t$. For a fixed time window $T$, the number of acquisitions $N = T / \Delta t + 1$ that are interferometrically combined increases as the time gap is reduced. The associated covariance matrix is referred to by the time of acquisition of the first image.

4.2 Results

Significant closure errors occur during snow melt when the radar acquisitions are analysed on a time scale of $T \approx 3$ days. The results for $N = 9$ acquisitions and $L = 50$ looks are shown in Fig. 7. The significant closure errors during the snow season include most prominently the spring snow melt (mid-April to mid-May 2012 and 2013) for all polarizations, but also minor warming or melting events in mid-winter, such as on 2012-02-21 and the last week of December 2012. These melt events are also associated with pronounced drops of the coherence magnitude $|\gamma| < 0.7$, compared to values exceeding 0.9 during sub-zero temperatures.

The number of acquisitions $N$ has an impact on the results of the significance test. During the spring snow melt in 2012 (Fig. 8), the $p_w$ values are smaller when more acquisitions $N$ are analysed within the window of $T \approx 3$ days. In particular, no significant values with $p_w < 0.05$ are observed when $N = 6$ (Fig. 8) or $N = 3$ (Fig. 7 for all studied values of $L$) are analysed. This lack of significance occurs even when the magnitude of the phase triplets is large $|\Xi| > \frac{\pi}{2}$, which occurs not only during snow melt but also at the onset of freezing on 2012-10-18 (only at $L = 33$).

After the disappearance of the snow cover, significant closure errors coincide with precipitation in early summer, i.e. sudden changes in soil moisture (see Fig. 7 and 9). After these early summer rain events, the phase triplets are typically smaller than $\frac{\pi}{2}$ while at the same time the coherences $|\gamma| \approx 0.6$ are markedly smaller than during winter. No significant closure errors are observed in this time period.

The validity of the normality assumption inherent in the phase closure test varies seasonally according to the Mardia normality test. There is comparatively little evidence against this assumption when a snow cover is present ($p_m$ typically exceeding 0.05). Conversely, during summer (e.g. June 2012) lower p-values $p_m < 0.05$ occur more frequently (Fig. 7).
5 Case study 2: Lena Delta

5.1 Data & study area

The Lena River Delta is the largest Arctic river delta, covering an area of almost 30000 km$^2$. It is underlain by continuous permafrost of about 500 m depth [46] and characterized by a multitude of river channels and islands belonging to different geological terraces [47]. It is located within the Köppen ET tundra climate zone, with a mean annual temperature of -13 °C [48] and precipitation of around 250 mm [49], which is partitioned into approximately equal amounts of rain and snow. During the study period (Summer 2013 - Summer 2014), the autumnal freeze-back occurred in the third week of September and the subsequent spring melt in the third week of May 2014, with the annual flood reaching its peak on 31 May [50, 51]. The following autumn, the freeze-back occurred after the end of the study period (2014-09-12).

We focus on the islands Samoylov and Kurungnakh and their surroundings (approximately 15 km by 30 km) shown in Fig. 10. Samoylov island belongs to the first geological terrace and consists of an active floodplain and a late-Holocene river terrace characterized by polygonal tundra (see Fig. 11) [47]. These polygons, whose spatial extent typically varies between 5 and 10 m, are composed of elevated rims surrounding lower-lying, often inundated, centres. Neighbouring polygons are separated by ice wedges, which can also be inundated. Kurungnakh forms part of the third terrace, a
Yedoma or ice complex [52, 22]. These sediments of around 30 m elevation accumulated during the Pleistocene and are rich in ground ice and organic carbon. This terrace is marked by incisions caused by previous permafrost thawing, i.e. thermokarst lakes and basins as well as thermo-erosional valleys (see Fig. 11) [52].

During summer, the near-surface layer (active layer) thaws, with maximum thicknesses of typically 20 to 50 cm occurring in September [53]. The melting ground ice causes the surface to subside during the warm season [54]. Frost heave induces upwards movements during freeze-back but the surface position is not necessarily restored to the previous year’s height, leading to secular subsidence [55, 56]. Additionally, spatially localized, often abrupt, permafrost degradation processes such as thermokarst, thermo-erosion, and mass wasting on slopes and coasts continuously shape the landscape of the Lena Delta [52].

As TerraSAR-X X-band data might be a suitable means for studying these movements [57], we analyse a stack of 36 acquisitions made between 29-06-2013 and 12-09-2014, typically every eleven days (see Fig. 12). The HH polarized SLC data have a resolution of 0.9 and 2.4 m in slant range and azimuth, respectively. The multilooked ($L = 101$) interferograms are given in 28 m resolution. We combine three such interferograms (only those of subsequent acquisitions owing to low coherences) to estimate the phase triplets and their significance.

We study the intra-annual variations of these observables by focussing on four regions of interest (ROIs). These were chosen according to their representativeness of particular land surface types and the future availability of in-situ observations of surface displacements. Each ROI consists of 48 independent samples of $L = 101$ looks each within a rectangular region. We extract the coherences $|\hat{\gamma}|$, the phase triplets $|\hat{\Xi}|$ and the p-values of (23). Two of these ROIs are on Kurungnak: ROI A on the ice complex, and ROI B in the thermokarst basin (alas) shown in Fig. 10a. The remaining two ROIs cover the first terrace polygonal tundra (C) and the active floodplain (D) on Samoylov.

5.2 Results

The observed coherences are generally low; they typically vary between 0.1 and 0.5 (Fig. 12). The lowest values tend to be observed during the freeze-back in autumn, in mid-winter, during thaw and early summer. The loss of coherence during these latter two periods (June-July) is more pronounced in Samoylov, where the active floodplain is inundated during the spring flood in late May/early June, than in the two ROIs in Kurungnak. There, the coherence subsequently dips during summer, only to rebound in late August when values of $|\gamma| \approx 0.4$ are found. By contrast, the floodplain (D) in Samoylov is characterized by stable values of similar magnitudes throughout the summer, whereas the coherences on the first terrace polygonal tundra (C) remain between 0.1 and 0.3, with maximum values that exceed 0.4 only occurring in late summer. During this time the highest coherences $> 0.7$ in the entire study area are found on sandbanks (e.g. the one marked by a red square in Fig. 10) and on some but not all first terraces (e.g. the red circle).

The magnitude and statistical significance of the phase triplets vary seasonally and depend on the land cover type and the coherence. In late summer, the sand banks in Fig. 10c exhibit phase triplets $|\hat{\Xi}| \ll \pi$, of which around 25% are significant ($p = p_{sw} < 0.05$). In Samoylov the two geological formations differ with respect to the observed phase closure: while the frequency of significant phase triplets ($p < 0.05$) is usually below 10% on the floodplain, more than 40% are
significant on the first terrace polygonal tundra in late summer (Fig. 12). During the preceding early summer period and the subsequent freeze-back, the magnitude of the closure phases is about twice as large, while the frequency of significant phase triplets drops to less than 10% as the coherence magnitudes are also smaller. The magnitude of these closure phases generally tends to decrease as the coherence increases, a phenomenon which is also observed in the spatial representation of Fig. 10c (e.g. the first terrace on Samoylov and the one marked by a red circle). However, additional patterns are present, such as the slopes surrounding the drained lake basin B where, as opposed to the surrounding flat areas, the closure phases tend to be smaller, despite similar coherences on the surrounding ice complex and drained thermokarst basins.

6 Discussion

6.1 Scattering behaviour

Snow melt appears to induce significant closure phases, i.e. phase triplets whose magnitude cannot be explained by the pronounced decorrelation alone. By contrast, snow accumulation is not associated with systematic closure phase errors in the Ku-band case study in Finland (Fig. 7). This absence of significant phase triplets is consistent with the notion that dry snow mainly affects the return from the underlying ground rather than producing pronounced backscattering itself. In a Ku-band DInSAR study over dry snow, [21] found that the phase could be well described as being due to the return at the ground surface whose optical path length to the antenna increased with increasing snow water equivalent [58]. According to this description, the structure function of (6) would effectively reduce to a narrow spike, corresponding to only one scatterer in (5). Under these circumstances according to Sec. 2.3, one does not expect closure phase errors in excess of those due to noise. At lower frequencies such as X-band, the scattering contribution from the snow grains is expected
Figure 12: Time series of interferometric quantities for all four ROIs in the Lena Delta. The ticks at the top indicate the dates of the acquisitions. The closure phase magnitudes (marker shows the median of the 48 samples, the shaded area the interquartile range) in the top panel always refer to the first acquisition of the three subsequent ones. The red markers in the lower panel indicate the percentage of samples with $p < 0.05$ as determined by the Wald test. The orange markers refer to the median of the medians of the three coherence magnitudes, the shaded area to the medians of the maximum and minimum $|\gamma|$.  

A) ROI A: Ice complex, Kurungnakh  
B) ROI B: Alas, Kurungnakh  
C) ROI C: Polygonal tundra, Samoylov  
D) ROI D: Floodplain, Samoylov  

To be even smaller [30]. However, in the Lena Delta significant phase triplets are observed in winter. These and the low coherences $|\gamma| < 0.5$ could instead be related to the non-zero baselines. However, the ROIs where chosen such that their vertical extent is much smaller (<5 m) than the heights of ambiguity. The snow depth is also typically much smaller than the heights of ambiguity (0.3-0.6 m) [53], thus limiting the impact of the spatial baselines. Another possible reason for these observations in winter could be related to complex changes in the scattering behaviour during the time period of around three weeks. In this area, changes such as pronounced dry-snow metamorphism, or spatially heterogeneous changes in the snow cover (e.g. snow drift over ice-wedge polygons) and spatially variable frost heave [53] have been observed. The latter two could lead to phase diversity within the averaging area and thus non-zero phase triplets. The first one, dry-snow metamorphism, has been found to strengthen the scattering from the snow grains [59]. It is thus possible that the assumption of a dominant soil return ceases to be valid [60, 21]. This phenomenon has been reported to be even more pronounced when the snow becomes wet [61, 60]. Given the expected increase of the absorption in the snow pack and its enhanced dielectric contrast [62], a volume description similar to Fig. 2 might be more fitting [63]. In the presence of a multitude of scattering grains and pores, the changing background permittivity (itself related to variations in liquid water content [62]) can induce depth-dependent changes in optical path length $l$. This, along with possible changes of the structure of the snow pack, can lead to non-zero closure phases [11], like those actually observed in the NoSReX campaign.

Systematic closure errors also coincide with rain events, i.e. rapid changes in soil moisture. The mechanism that gives rise to these effects has been attributed to volume scattering from within the soil [13, 14]. Significant closure phase errors in the NoSReX data that appear to be linked to soil moisture changes are mainly observed during the wet period at the end of June and beginning of July 2012. A similar phenomenon occurs when the soil freezes in October 2012: Given the similar dielectric responses of a drying and a freezing soil [64, 65], this might account for the decorrelation and the large closure phase magnitudes. Such values are not present during the dry period in mid-summer and the slight wetting in September. During these periods, the coherences generally increase compared to early summer, while the magnitude of the phase triplets shrinks by more than 50%. Besides the decreased temporal dynamics of soil moisture, also the vegetation cover (sparse lichen and moss) and thus its scattering contribution may evolve more slowly.

Changes in soil moisture are only one of the conceivable origins for the observed significant closure phase errors in the Lena Delta during the snow free season. In late summer, particularly large numbers of significant phase triplets are observed in the polygonal tundra (e.g. polygonal tundra in Samoylov, ROI C), which is characterized by pronounced lateral variability on scales of metres [49]. Possibly relevant processes that exhibit distinct sub-resolution spatial variability include changes in inundated and saturated areas as well as vertical movements [53]. The heterogeneity corresponds to a multitude of scatterers with variable phase contributions in (5), which may induce non-zero closure phases and coherences $|\gamma| < 1$. During time periods when the coherence magnitudes are particularly low ($|\gamma| < 0.5$), a comparatively lower
percentage of significant phase triplets is observed.

6.2 Interpretation of the significance test

The observed relation between the coherence magnitude and the number of significant phase triplets suggests that the power of the test has to be considered when interpreting the results of the test. The loss of coherence leads to an increase in noise and consequently a decrease of the power of the test. This means that small but systematic phase closure errors are more difficult to detect. Additionally, the simulations of Sec. 3.5 indicate that the test becomes increasingly conservative as the coherence gets smaller, implying that even in the absence of systematic effects the number of significant phase triplets is expected to decrease. These two reasons might explain the commonly occurring low (≤ 5%) percentages of significant phase triplets in mid-summer or early winter in diverse geological terraces in the Lena Delta (Fig. 12).

The statistical properties of the test are also related to the way the radar data are processed, namely the number of looks \( L \). The two mathematical assumptions (2 and 3 of Sec. 3.5) are only valid for sufficiently large values of \( L \). The simulations indicate that for small \( L \) they lead to an increasingly conservative test, and furthermore a loss of power is expected. The observed dependence of the number of detected significant closure errors on \( L \) in Fig. 8 is thus consistent with these theoretical considerations. The absence of significant closure errors as detected by the test consequently does not imply the absence of processes that systematically induce a loss of phase closure. Such systematic effects may still be present and they may even be comparatively large when the coherences are low. However, in this latter case – when the phase cannot be estimated reliably –, the potential of using these data to estimate deformations is severely limited owing to the lack of coherence alone. By contrast, when the coherence is higher, systematic non-zero closure errors may limit the accuracy of displacement estimates and in this case, the test has better power to detect them.

6.3 Impact on deformation estimates

Systematic phase closure errors are a potential error source for deformation estimates, as they indicate that the simple proportionality between the (properly referenced) phase and a piston-like displacement is not valid any more [9]. The actual impact on the measured phase \( \phi \) may well be considerably larger than the magnitude of the closure error: this has indeed been observed for soil moisture [14], but it may also apply to growing vegetation [12, 66] or to dry ‘transparent’ snow [21], for which systematic closure errors are not expected. On the other hand, the presence of non-random closure errors is not necessarily relevant, which may for example be the case when the associated phase errors are much smaller than the movements of interest or the achieved accuracy. This accuracy depends on the coherence magnitude (see Eq. 18) and many studies thus introduce a threshold value for the coherence [67, 68, 69], but there are an increasing number of studies in which low coherence areas are analysed [70, 57].

One application where low coherences are common is the monitoring of vertical movements related to thawing permafrost [57, 26, 71]. Previous studies have been partially successful but also hampered by such low coherences [72, 57, 6, 73]. The particularly low (<0.3) values observed in certain ice-rich regions in the Lena Delta during most of the summer season indicate that the retrieval of deformation time series will be difficult. This is possibly compounded by the presence of persistent closure errors, especially in the ice-rich first and third terraces. One way to circumvent these problems may be the use of lower frequencies [57] or higher spatial resolutions, owing to the spatial heterogeneities.

The choice of algorithm will also be relevant when estimating displacements from interferometric stacks [11], as many popular techniques do not consider phase closure errors explicitly [67, 9]. The documented prevalence of systematic closure errors in this and previous studies [11, 14] points towards potential improvements in DInSAR deformation estimates and analyses of their uncertainty by incorporating phase closure estimates and their uncertainties into the estimation procedure.

7 Conclusions

Heterogeneous surface movements, soil moisture changes, and snow metamorphism are only three complex processes that can affect the phase measured in DInSAR radar interferometry: they then also impact the estimated displacements. The phases due to spatially homogeneous piston-like displacements obey phase closure (or triangularity), so that the observed absence of this closure can provide evidence for such additional systematic influences on the phase. However, also decorrelation noise can break phase closure. The uncertainty of the closure phases due to decorrelation noise can be described quantitatively, and based on this we developed a statistical test to check their significance, i.e. whether the observed inconsistencies can or cannot be explained by noise alone. This test permits the screening of an interferometric stack, and in the future it might contribute to the understanding of the scattering processes that give rise to non-zero closure errors, or to the modelling of the uncertainties of the estimated deformation time series.

An improved knowledge of the role of phase closure errors in the estimation of deformations is particularly relevant when non-zero closure phases occur regularly and when they are not due to noise. Systematic closure errors related to physical processes remain barely explored, especially in high-latitude regions characterized by permafrost, seasonally
frozen ground, and seasonal snow packs. We observed their persistence during snow metamorphism and melt in tower-based Ku-band data. They were also found to occur after rain events in summer, when the moisture content of the mineral soil changed rapidly. In TerraSAR-X acquisitions over the Lena Delta in the continuous permafrost zone, significant closure errors also occur both in winter and in summer. They are particularly common in late summer, especially over polygonal tundra, suggesting that persistent and systematic phase closure errors occur during the seasonal thaw period. Their non-random nature indicates that the impact of processes such as moisture changes can also systematically affect the robustness and accuracy of DInSAR monitoring of subsidence related to permafrost thaw.

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References


Chapter H

Conclusions

H.1 Summary

H.1.1 Influence of soil moisture in differential interferometry: description and attribution

Changes in soil moisture and vegetation growth influence interferometric radar measurements. In particular, they affect the phase, thus giving rise to errors in the estimation of displacements. The observations in chapter B show that soil moisture effects in differential interferometry can be significant at L-band, i.e. they are not negligible compared to the measurement precision. They are relevant in a wide range of applications, as they were observed to correspond to displacements exceeding 2–3 centimetres. A closer analysis of their properties indicates that they cannot be explained by soil swelling or by the penetration depth hypothesis, which links the measured displacements to changes in the moisture-dependent attenuation of the electromagnetic waves. Instead, they appear to be due to subsurface scattering: heterogeneities within the top few centimetres of the soil – such as stones or clods – scatter the waves [4, 13]. As the optical path between the antenna and these scatterers changes with soil moisture, a non-zero interferometric phase can be observed. The size of the sensitivity of the phase to soil moisture changes was found to vary between different fields. While non-uniform calibrations of the soil moisture measurements cannot be ruled out, the magnitude of its variability suggests that it may not be spurious, but instead related to the soil properties.

H.1.2 Growth of agricultural vegetation systematically influences the interferometric phase

Also growing agricultural vegetation systematically influences the interferometric phase. The effects are similar in magnitude, as they can induce spurious displacements of several centimetres within weeks to months at L-band (chapter B). As opposed to the soil moisture effects, the influence of vegetation growth strongly depends on the polarization. In chapter C, I characterized the polarimetric response for a range of different crops. I hypothesized that birefringence within the canopy could introduce systematic polarization differences. The observations over wheat and barley fields were largely consistent with this hypothesis, in contrast to sugar beet and maize fields, although the observations over these fields were also associated with vegetation growth and changes in biomass. This suggests the presence of other phenomena, such as the preferential movement of scatterers within the canopy, but this and other hypotheses are so general in their predictions that they are difficult to rule out. Nevertheless, the size of the polarimetric phase differences – corresponding to several centimetres at L-band – indicates that the vegetation influence on the interferometric phase may be relevant for a wide range of displacement studies. Moreover, its persistence and clear relation to biomass changes, especially for wheat and barley fields, suggest that differential interferometry has the potential provide quantitative estimates of vegetation properties.

H.1.3 Combination of surface and subsurface scattering helps to explain the variability of observed soil moisture effects in differential interferometry

As also the soil moisture effects are non-negligible in many applications, I investigated ways of modelling and describing them in chapter D. Existing models based on surface or subsurface scattering were not capable of replicating the differences in magnitude observed in chapter B, nor the varying signs found in previous studies [4, 11, 13]. By combining surface and subsurface scattering in a bottom-up approach based on Maxwell’s equations, the model can account for such differences. According to the model, they arise from the varying importance of surface and subsurface scattering: the former is governed by the surface roughness, the latter by the inherent heterogeneity of the soil. The model is polarimetric, but it cannot describe the scattering in the HV polarization as it is only a first-order approximation and the subsurface heterogeneities are assumed isotropic. For the HH and VV channels, on the other hand, it gives useful predictions for the phase, coherence, and phase triplets once the model is calibrated. However, the generality of the model remains an open question. Besides the limitation to L-band data in this chapter, other processes besides soil moisture changes usually...
occur simultaneously. For instance, tilling and ploughing can induce decorrelation, or displacements can affect the phase – these changes are not considered in the model.

**H.1.4 DInSAR can be useful for soil moisture estimation in the absence of displacements**

Owing to the sensitivity of differential interferometry to soil moisture changes and displacements, I investigated in chapter E to what extent they can be estimated from measurements. Soil moisture estimation in the absence of deformations is feasible at both L-band and Ku-band. The phase is the most suitable observable, as the entire soil moisture time series can be retrieved up to an unknown shift. The coherence and the phase triplets are in addition also insensitive to other kinds of transformations of the soil moisture time series, such as a flip in which wetting and drying are reversed. The coherence is furthermore affected by other sources of decorrelation, e.g. changes in the surface roughness [21]; these changes are wrongly attributed to soil moisture changes when the inversion is based on the coherences. In Ku-band, the model was not able to describe the observed coherences despite the short repeat interval of four hours. The phase triplets or closure phases are also difficult to describe reliably as the predictions are very sensitive to the parameterization of the model and because their magnitude is often comparable to the influence of other sources such as decorrelation noise. The phase triplets express the deviation of the interferograms from a linear model, in which the phase is proportional to a change in a state variable. This state variable can be soil moisture or the position of the surface. As this deviation from a linear model is small compared to the overall size of the effects, this implies that changes in soil moisture behave predominantly like a change in the surface position, i.e. displacements. Without a priori knowledge, these two are hence difficult to separate based on phase information alone. In combination with the susceptibility of the remaining observable, the coherence magnitude, to other sources of decorrelation, this implies that deformations and soil moisture changes are difficult to separate in practice.

**H.1.5 Depth-resolved radar measurements indicate the presence of subsurface scattering: opportunities for model improvements and soil moisture profile estimation**

Even though the soil moisture influence in differential interferometry can be due to moisture-induced movements [7, 16], in the settings studied in this thesis it is mainly associated with subsurface scattering. This conclusion suggests that subsurface scattering is also relevant to backscatter observations [14]. Furthermore, it points towards the possibility of observing it directly using depth-resolved radar imaging systems. Such depth-resolved observations may provide an opportunity to monitor variations in the soil moisture profiles and to test the models of subsurface scattering with which the depth-averaged observations can be explained. Using a technique called Tomographic Profiling [10], my co-authors and I found in chapter F that soil moisture changes have indeed a measurable impact on the depth-resolved backscatter and interferometric observations, hence suggesting the potential of such observations for inferring soil moisture profiles. For this purpose, reliable models would be useful. The experimental findings support the central tenet of the model presented in chapter D, namely the dominant impact of the moisture-dependent propagation properties – the absorption and the phase velocity. They also indicate that the dependence on soil moisture is more complex, as several lines of evidence suggest that the backscattering power of a subsurface layer depends, even when considering the changing propagation properties, on soil moisture. Such a dependence may be explained by the changing dielectric contrast within the soil: as the soil gets wetter, water fills the pore space, thus replacing air. As both the real and the imaginary part of the dielectric constant of water are larger than those of air and the soil grains, the dielectric constant increases, giving rise to more intense subsurface scattering.

**H.1.6 The statistical analysis of closure phases can help to detect influences on the DInSAR deformation estimates besides homogeneous displacements and decorrelation noise**

Subsurface scattering – in contrast to uniform displacements or atmospheric changes – can give rise to non-zero closure phases. Also changes in vegetation properties have previously be shown to do so [5]. The closure phases may hence contain useful information about systematic influences on the interferometric phase, but they are also affected by noise. In chapter G, I suggest a statistical test that determines whether the observed closure phases can be explained by noise alone. Two case studies in high latitude regions show that such situations occur regularly. In a tower-based study of bare soil in Ku-band, they are associated with snow melt and sudden changes in soil moisture due to precipitation. In the Lena Delta, both the magnitude and the significance of the closure phases varies in both space and time. While the origin is not clear – possible mechanisms also include vegetation changes or heterogeneous displacements – the non-random nature suggest that they can have a deleterious impact on the estimated displacements, even though the magnitude of this impact cannot be estimated reliably.
H.2 Implications & Outlook

The influence of soil moisture changes in differential interferometry, especially the impact on the interferometric phase and hence the displacement estimates, raises important questions for deformation studies. While the analyses in chapter E suggest that the removal of the soil moisture effects based exclusively on DInSAR data will be difficult in practice, future studies should address the use of additional information. Such data may be remotely sensed as well – for instance, the radar backscatter also provides information about the soil moisture content [8, 19] – or it may be derived from hydrological considerations; for instance, the soil moisture time series could be constrained using a hydrological model forced by precipitation and atmospheric data [2]. Irrespective of the origin of the additional information, the model that links soil moisture changes to the phase will have to be calibrated using appropriate assumptions. These approaches should also be designed in such a way that they are robust to decorrelation processes that are not explicitly assumed in the model. Such decorrelation is very common and occurs on a variety of time scales as it is associated with a large number of processes (e.g. tilling, weather-induced roughness changes) [1, 21, 15]. Any operational approach will hence have to be able to account for such changes automatically.

The closure phases remain a promising source of information for improving the estimation of displacements in the presence of soil moisture changes. However, any approach to reduce or estimate the moisture-induced uncertainty of the displacement estimates will have to be able to account for their sensitivity to decorrelation and to the limited information content outlined in chapter E. Attempts to reduce the influence of soil moisture changes may include the screening of the closure phases, similar to the SqueeSAR approach [6]. This may also reduce the impact of a range of additional processes (e.g. snow melt, changing vegetation moisture content) that have been shown to lead to non-zero phase triplets by [5] and in chapter G. However, as the precise nature of these effects remains uncertain, future studies should quantify their impact on the phase triplets.

The growth of agricultural vegetation also has a systematic impact on the differential interferometric phase. Our understanding of this impact is still very much incomplete, despite the enumeration of several not necessarily mutually exclusive mechanisms in chapter C. As some of these proposed mechanisms are very difficult to quantify or to rule out using single-frequency radar data alone, future experiments may include multi-frequency data or employ depth-resolved observations. As the polarimetric phase diversity appears to exhibit reasonable sensitivity and specificity for vegetation changes [9], it may be harnessed to detect and quantify vegetation growth or to improve deformation estimates, at least for those crops for which birefringence in the canopy seems to be important. The detection of vegetation processes in the DInSAR data based on the polarimetric phase diversity may also be used to screen such data in interferometric processing chains, especially given the observed stability of the coherence in certain agricultural fields [3].

There are several insufficiencies in the model of the soil moisture effects presented in chapter D. One aspect of the model that deserves clarification is its sensitivity to the dielectric mixing model. Especially the closure phases are affected: this may be one of the reasons why the Ku-band closure phase observations in chapter E cannot be predicted accurately. However, maybe the most important insufficiency is the lack of ability to predict the HV channel, which may be amended by extending the model to higher order expansions of the integral equation on which it is based, by using more general surface scattering models [8, 17], and by improving the parameterization of the dielectric fluctuations. Moreover, the assumed lack of soil moisture dependence of these fluctuations is at odds with the change in subsurface scattering power detected using depth-resolved observations in chapter F, thus indicating possible future model extensions. These will likely come at a price, namely that of increased model complexity and a larger number of parameters.

Depth-resolved observations may more generally facilitate the interpretation and exploitation of the data obtained with a range of microwave techniques [12, 20], for instance by improving the parameterization of backscatter models. Subsurface scattering has repeatedly been invoked as one of the reasons for the perceived lack of accuracy of depth-averaged backscatter models that are exclusively based on surface scattering [14, 18]. In practice, subsurface scattering is associated with volumetric clutter, but also with subsurface layers, buried targets, or non-uniform soil moisture profiles. There are a number of ground penetrating radar approaches to estimating soil moisture that make use of such strong reflectors below the surface. Can also observations of the volume ‘clutter’ be used to infer soil moisture profiles and changes therein? Can these approaches be combined? How do depth-resolved observations depend on the soil properties, e.g. clay content and soil organic matter concentration? Could also such soil parameters be inferred as a function of depth?

Bibliography


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