Doctoral Thesis

Effects of parking on urban traffic performance

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EFFECTS OF PARKING ON URBAN TRAFFIC PERFORMANCE

A thesis submitted to attain the degree of DOCTOR OF SCIENCES of ETH ZURICH (Dr. sc. ETH Zurich)

presented by
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2016
Foreword

The short-term interactions between parking and traffic performance, although rather important, have been drastically overlooked in the past. In her doctoral dissertation, Ms. Jin Cao combines a number of mathematical tools in a rather innovative manner to model these interactions both locally and at the network level. Ms. Cao first investigates the parking search phenomenon, and the impact of different parking policies on the traffic network. She does so with a groundbreaking macroscopic model that uses operations research concepts and probability theory to replicate the interactions between the parking and the traffic system. Ms. Cao then investigates the effects of parking maneuvers on the traffic stream. She uses a combination of queuing theory and kinematic wave theory to develop some general guidelines regarding the locations of on-street parking spots with respect to signalized intersections. Lastly, Ms. Cao investigates different parking data issues, in particular the collection and use of parking information. To this end, she uses dimensional analysis combined with probability theory to first determine the accuracy of parking patrol surveys, and then to evaluate the benefits of parking guidance systems.

Ms. Cao’s dissertation is rather interesting from both a practical and a scientific perspective, with many methodological developments and theoretical findings, as well as clear applications. Additionally, it sets the groundwork for further studies to better understand and model different traffic phenomena in relation to parking, as well as the implementation of different parking policies aiming to improve the performance of the traffic system.

On behalf of the Traffic Engineering research group at the Swiss Federal Institute of Technology, Zurich, I thank Ms. Cao for using her never-ending energy and enthusiasm to explore such an important topic from so many different perspectives. I hope that her vision of a tangible product that helps users, parking providers, and regulators, becomes a reality, and drives parking into a new era.

Dr. Monica Menendez
Director of research group Traffic Engineering
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Abstract

The interactions between the urban parking and traffic systems, can have both, long-term effects (i.e., parking policies can affect travel demand, and vice versa), and short-term effects (i.e., parking policies can affect traffic operations, and vice versa). While the long-term effects have attracted lots of research attention, the short-term effects have not been well researched yet. This is unfortunate, as the short-term interactions between parking and traffic can be highly significant and influential to the performance of both systems. In this dissertation, we propose methodologies to analyze such interactions, and evaluate their effects on urban congestion.

The parking system can affect the traffic system through two processes: parking search, which can cause higher traffic density; and on-street parking maneuvers, which can directly cause additional delay. Both can lead to more congested traffic conditions, and lower travel speeds on the network. On the other side, the traffic system can also affect the parking system. Average travel speeds highly influence the arrival rate to the parking facilities, and ultimately, the level of parking usage. To represent all these interactions, the following work has been carried out. The three main parts of the dissertation are briefly introduced below.

I. In the first part of the dissertation, we propose a macroscopic model to analyze the immediate interactions between traffic performance and parking usage. For any given time interval, the transitions of vehicles between different parking related states (i.e., non-searching, searching, and parking) can be shown. Similarly, both the parking and traffic indicators can be updated (e.g., traffic density, average travel speed, average delay, parking occupancy, parking supply, and share of parking searchers within traffic). When the time intervals are small, the macroscopic model reflects
the dynamics of both, the parking and the traffic systems. This provides a short-term prediction tool for future parking and traffic conditions, indicating the cruising-for-parking traffic and the ensued delays. Numerical examples and a case study based on the city of Zurich, Switzerland are used to illustrate the proposed methodology.

The macroscopic model proposed in this study has low data requirements, but it still provides reasonable aggregate results to make informed decisions about network management and control, as well as to assess the impact of different parking related strategies. With this approach, a better optimization scheme for the system can be developed in terms of both traffic performance and parking usage.

II. In the second part of the dissertation, we propose a methodology to estimate the impact of on-street parking maneuvers on traffic delay and the service rate of the nearby intersections. We focus on the analysis of the maneuvers that might cause service rate reductions, as they can trigger lingering delays over multiple signal cycles. Such cases are specially important because their effects may spread over to the whole network.

Using the hydrodynamic theory of traffic flow, we are able to formulate the relation between the service rate reduction of the nearby intersections, the time duration and the location of the maneuver (i.e., distance between the maneuver and the intersection). Based on that, the model can provide not only estimations on the reduction, but also suggestions on the parking locations (on road links) so that large service rate reductions can be avoided.

III. In the third part of the dissertation, we look into different parking data issues, including the accuracy of parking patrol surveys; and the potential benefits of parking guidance and information systems (PGIs).

In the first study, an evaluation and correction method is proposed to help surveyors to better understand patrol surveys and obtain high-quality results, while keeping costs to a minimum. In the second study, a method is given to assess the effectiveness of PGI systems. The results show that the cruising time saved through PGIs can be significant (only) during the time that the parking
system is approaching or leaving the saturation state (i.e., close to 100% occupancy).

In general, in this dissertation we discuss various topics related to urban parking and traffic performance, including both practical and theoretical issues. The models proposed aim to provide clear, easy-to-use, generalized and applicable tools for solving parking-related problems with the minimum amount of data requirements and costs. The conclusions can be used to guide parking planning, management, and investments under different sets of conditions, as well as to assist the development of parking policy (regulations). Potential beneficiaries from this dissertation include, traffic managers, local authorities, practitioners, consultants on the parking industry, owners and developers of private parking houses.
Zusammenfassung


I. Im ersten Teil der Arbeit wird ein makroskopisches Modell zur Analyse der unmittelbaren Wechselwirkung zwischen Verkehrstätigkeit und Parkplatznutzung vorgestellt. Für jegliche Zeitspannen kann der Wechsel von Fahrzeugen von einem in einen anderen Parkzustand (d.h. nicht auf der Suche, auf der Suche und parkierend) aufgezeigt werden. Weiterhin können Indikatoren bezüglich des Parkierens und des Verkehrs (d. h. Verkehrsdichte,


III. Im dritten Teil der Dissertation werden verschiedene Probleme im
Zusammenfassung

Zusammenhang mit Parkdaten analysiert. Diese beinhalten die Verlässlichkeit von Parkkontrollen und die potenziellen Vorteile von Parküberwachungs- und Informationssystemen (PÜIs).

In der ersten Untersuchung wird eine Beurteilungs- und Korrekturmethode vorgestellt, um verkehrsüberwachenden Personen bei Parkkontrollen zu unterstützen, damit hochqualitative Ergebnisse erzielt und die Kosten tief gehalten werden.

In der zweiten Untersuchung wird eine Methode zum Testen der Effektivität eines PÜI-Systems vorgestellt. Die Resultate zeigen auf, dass die für die Parkplatzsuche benötigte Zeit durch PÜIs beträchtlich vermindert werden kann. Dies jedoch (nur) während der Zeit, in der das Parksysteem fast ausgelastet (d.h. annähernd 100% der Parkplätze sind besetzt) ist.

Zusammenfassung
Acknowledgments

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Obviously, my advisor Monica Menendez is the person I would like to thank the most. Being an outstanding researcher herself, she passed me her knowledge, spent her time and patience generously on supporting and providing valuable feedback. More important is the fact that she respected and always had faith in me and my work. I am also very thankful for her strong sense of responsibility, not only in our daily work, but also in other things such as keeping high standards within the academic field and in assisting her students to grow and develop. She has been a good example and has influenced me in many different ways. In return, I have, as a matter of fact, copied her way of thinking, way of reasoning, even a bit her way of talking and hand-writing. Hopefully, my “Monication” will make me a tremendous mentor one day, just like her.

I would also like to thank my committee members, professor Kay W. Axhausen, professor Yafeng Yin and professor Yanfeng Ouyang for serving as my committee. I want to thank you for your brilliant comments and suggestions. I would especially like to thank my dear colleagues in my group at SVT, ETH Zurich. All of you have been there when I needed support, encouragement, and comfort. Your generosity and thoughtfulness made SVT a reliable and close team; and our office, an interesting, dynamic, and enjoyable place to spend each day.

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Chapter 1

Introduction

1.1 Motivation

The urban parking and the urban traffic systems are essential components of the overall urban transportation structure. The interactions between these two systems, can have both, long-term effects (i.e., parking policies can affect travel demand, and vice versa), and short-term effects (i.e., parking policies can affect traffic operations, and vice versa). While the long-term effects have attracted lots of research attention, the short-term effects have not been well researched yet. This is unfortunate, as the short-term interactions between parking and traffic can be highly significant and influential to the performance of both systems. In this dissertation, we look at different aspects of such short-term interactions, propose methodologies to analyze them, and evaluate their effects on urban congestion.

It is well known that urban parking affects travel demand, and hence, affects traffic, mostly in the medium- to long-term. Both the parking experience and the parking policies can influence travellers’ future trip decisions. In other words, future traffic can be partially determined by the sensitivity of travellers to parking attributes such as parking fee, walking distance, and search time (see Axhausen et al., 1988, 1990, 1991 and Weis et al., 2011 for models on parking choice behavior). Based on that, many efforts have been devoted to modify parking policies with the aim to improve urban planning and development (see Polak et al., 1989, 1990, 1990b, 1990a; Marshall and Garrick, 2006, 2008 and McCahill, 2010). Summaries of other research topics,
Chapter 1. Introduction

all related to the parking system, including parking relocation, distribution, route choice, utility optimization, and searching strategies, are included in Young et al. (1991) and Feeney (1989). There are also many research works focused on parking economics and its effects on traffic, e.g., Glazer and Niskanen (1991) and Arnott et al. (1991), (2006), (2009). More recently, mechanism designs for parking pricing and parking slot assignment attracted lots of attention, such studies can be seen in Zo et al. (2015), He et al. (2015), Mackowski et al. (2015), Liu et al. (2014b), and Liu et al. (2014a). In this dissertation, however, we will not cover these studies in detail. Although evidently parking impacts on travel demand will ultimately influence traffic in urban areas, here we will focus on the short-term interactions between the urban parking and traffic systems, as well as their immediate effects on the overall system performance. In other words, we will focus on how, within the day, parking and traffic changes, hence, we will not consider change in travel demand.

As of today, even if the travel demand is known and/or accurately forecasted, without the aid of costly traffic simulators it is not yet possible to predict or measure how well (or how bad) urban traffic will perform as a function of parking. Within the frame of this dissertation, we plan to develop a methodology to analyze the impact of urban parking on traffic in the short-term, mostly for operational purposes (e.g., traffic management and control). We will focus on two specific processes that can potentially affect traffic performance in a network: (i) parking search, and (ii) on-street parking maneuvers.

i Under certain conditions, vehicles searching for parking might not be able to find it right away, so they are forced to drive longer (i.e., stay longer in the road network). This can lead to higher vehicle accumulations, and lower average speeds on the network.

ii Vehicles parking on-street can sometimes influence the cars behind them while making the parking maneuver (i.e., they can block (partially or totally) traffic for a few seconds while accessing or departing from the on-street parking spot). Depending on the traffic conditions, these blockages can lead to delays that could spread through the network, and linger for periods of time much longer than the duration of the maneuvers themselves.

The impact of the two processes described above has not yet been very well documented in the literature, as for a while most travellers
1.1. Motivation

and even traffic experts believed their effects to be marginal, and the cost associated with their analysis too large to be worth it. However, recent studies have demonstrated that the effects of both processes can actually be significant (although a general model to estimate them are still lacked), potentially influencing traffic performance considerably. Main reasons for previous misperceptions regarding the two processes, and recent studies that dispute such misperceptions are described below.

i Vehicles that are searching for parking are hard to observe/measure as often times they do not present any clear distinctions in comparison with other vehicles in the network (i.e., they are typically hidden among other vehicles in the traffic stream). Moreover, they are not easily predictable through conventional methods, as their number highly depends on the constantly changing supply of available parking. Geroliminis (2009) used a traffic simulation of the city of San Francisco, U.S., to perform a quick evaluation of the impact that parking searchers can have on network traffic conditions, and concluded that these could be significant. In addition, during the last decades, many studies, often based on surveys, have been able to quantify some of the characteristics associated with the parking search phenomenon. For example, through the review of 13 studies of mostly American and European cities, Shoup (2006) found that, on average, 30% of traffic is cruising for parking during rush hours and the average cruising time is 7.8 minutes. More recently, the global parking report conducted by IBM (2011) also revealed similar results.

ii The effects of on-street parking maneuvers are hard to observe/measure, given their dispersed spatial and temporal distributions. Moreover, on-street parking maneuvers typically last a very short period of time, and the scope of their influence (i.e., number of cars affected) is often limited. However, evidence now suggests (Cao and Menendez, 2015 and 2015a) that both, the quantity of on-street parking maneuvers, and the delay that on average they generate, can be high. This is especially true for city centers with (normally) high traffic levels, high density of intersections, and high number of on-street parking spots.

Such misperceptions have motivated us, in this dissertation, to look for scientific methodologies that could evaluate the impact of the two
processes described above. Within the dissertation, for each process mentioned, analytical models are built, mostly followed by data collections validating them, or field tests showing the use of them. Additionally, another 2 chapters focusing on parking data related issues are also included. The dissertation outline will be explained in section 1.3.

1.2 Literature Review and Research Goals

In the previous section, we have given a short but general literature review covering all different aspects of parking related research topics, below, we provide a review of studies that are highly related to our topic. The literature is divided into two categories following the same structure from the previous part, i.e., impact from parking searching vehicles, and impact from on-street parking maneuvers.

The research goals of this dissertation will then be stated, specially in comparison to the existing literature.

1. The parking search process and its impact on traffic congestion

   - Modelling methods
     - Searching strategies and routing
       Thompson (1998) described a model of parking search behaviour. Relationships for estimating the utility of a car park including access, waiting, direct and egress cost components were developed. A function was then proposed to estimate the perception/utility of the attributes of parking. A “stopping rule formulation” and “direction of search” were provided. However, as a microscopic model, the paper did not clarify how the parking demand was assigned onto the network; and how real time information about parking and searching vehicles are embedded into travellers’s decision.

     Gallo and D’Acierno (2011) proposed an assignment model on urban networks to simulate parking choice, and the impact of parking search on traffic congestion. Users’ cost function mainly included driving, parking, and walking. The driving cost depended on the flow on each link and the parking cost depended on the parking occupancy of each facility. When a parking facility was almost full, a very high fee could be imposed, so the user would prefer
to cruise to another parking facility. The parking location and the route choice were then determined through a stochastic user equilibrium (SUE) model. Based on that, the traffic condition induced by the parking search process could be found. However, to do so, a relatively long time interval was needed (e.g., one hour) for traffic assignment. Within each interval, the traffic and parking conditions remained steady, so that the equilibrium could be reached. Therefore, the system was not really dynamic, which limits its application as both traffic and parking conditions can change rapidly. Moreover, parking fees were assumed to be dependent on the parking occupancy. This, although reasonable, is only adopted in very few cities (to the author’s knowledge). Therefore, the results obtained upon such assumptions may lack generality.

Bodenbender et al. [2013] developed a computable general equilibrium model for parking. The model distinguished between people driving through the network, and those looking for on-street parking and off-street parking. By considering the probability of not finding a parking spot, the model accounted for the parking search process. However, the model was built based on certain assumptions including that, drivers are fully informed about all the alternatives and their features, have consistent and stable preferences, and optimize their utility over time.

– Effects on the network level

Based on newly developed methodology and analytical tools, the understanding of traffic performance at the network level has been largely enhanced in the last dozen years (see Yang, Meng and Bell, 2000; Daganzo and Geroliminis, 2007 and 2008; Leclercq and Geroliminis, 2013). Such developments provide scholars a new approach to model and analyze the network level disturbances. For example, Geroliminis (2009) modelled the extra delay caused by vehicles searching for parking in a network using a macroscopic view of the urban traffic system. He modelled the additional travel distance needed for parking searchers to find parking spots. Based on that, the delay in the system caused by parking searchers was measured. However, in that study, the transition of travellers from searching
to finding the parking was not defined in detail. Instead, it was substituted with Bernoulli trials with a given probability.

Using a different perspective (i.e., an economic one), Arnott et al. (1991, 2006) defined some parking related states of vehicles on the local network, including moving, cruising, and parked. Then, they suggested different pricing schemes where part of the objective was to reduce searching time. The model provided a macroscopic view of urban traffic in relation to parking, although the connection between parking search and traffic performance was not analyzed.

- Agent based simulations

Agent based approaches are widely used to simulate travel behavior for a large number of users. A spatially explicit environment (e.g., a non-homogeneous road network) can be set up as an input in the simulation, as well as travelers’ personal references. The output of the simulation can also contain very detailed results. Using this approach, parking search time and its relation to parking inventory can be obtained (Martens and Benenson, 2008; Waraich and Axhausen, 2012; Horni and Montini, 2013); while the impact of parking search on urban traffic congestion can also be found (Geroliminis, 2009). This method, though comprehensive and powerful, relies on many preliminary models such as car following models, route and parking choice models, etc. Therefore, the accuracy of the final output can be affected through many aspects. In addition, agent based simulations require an excessive amount of detailed data for the specified conditions, such as the detailed network of the city, and the travel behavior and searching habits of the citizens. Hence, the transferability of the approach (and results) across cities and/or population types is rather difficult.

- Empirical methods

Besides conventional methods like questionnaires (Shoup, 2006 and IBM’s global parking survey, 2011), GPS data is now used more and more often for extracting data on vehicles searching for parking (Montini and Rieser-Schüssler, 2012; Van Der Waerden et al. 2015). Some data collected shows the existence
of a relation between urban parking and traffic performance, e.g., lower travel speeds are typically adopted near parking facilities (Montini et al., 2012). However, as the data extraction methods are still under development, the precision and generality of such conclusions are not known, and therefore results cannot yet be used.

ii On-street parking maneuvers and its impact on traffic performance

Several studies have been carried out, analyzing the influence of on-street parking maneuvers on traffic performance. In the early stages, scholars focused on the road space required by the parking lane, and the ensuing road capacity reduction (e.g., Chick, 1996; Vallely, 1997). With time, scholars tried to gain better understanding of how on-street parking maneuvers disrupt traffic flow. For example, the ensued delay for the following vehicles was analyzed based on both the physical process of parking maneuvers such as friction and blockage delay (Ye and Chen, 2011) and traditional traffic flow theory tools (Cao and Menendez, 2015). However, these models measure the traffic delay microscopically, thus cannot macroscopically present the aggregate impact of parking maneuvers on urban traffic conditions.

Above, we described the main methods currently used in addressing the relation between urban parking and traffic systems as proposed here. As shown, most of the existing analysis focuses on the microscopic effects of parking searchers or parking maneuvers. Moreover, most research has also overlooked the fact that parking processes, in turn, can also be influenced by the changes in traffic conditions. Therefore, we believe that our methods can fill the gap in the existing literature. Our research goals encompass at least four aspects:

1. to include the effects of both parking searchers and parking maneuvers;
2. to dynamically evaluate the impacts of the parking system on urban traffic performance, and vice versa;
3. to evaluate such impacts without excessive data requirements, and to provide general relations between the final outputs and the model inputs (such as travel demand and parking durations); and
Chapter 1. Introduction

4. to provide a prediction tool that could be easily adjusted for usage in different areas or across different sets of conditions.

The chapters presented in this dissertation will focus on the conditions where parking search and parking maneuvers affect traffic on the network level, analyze the process of such effects, and calculate/evaluate the level of the influence on network performance. That being said, individual delays can be measured as well, but are not the main focus here.

1.3 Dissertation Outline

Corresponding to the aforementioned topics, this dissertation consists of three main parts with two or three chapters within each part.

Part I: Effects of on-street parking search
- Chapter 2. A macroscopic model of urban parking system
- Chapter 3. Parking search model: analysis and application

In this part, the interactions between the urban parking system and the urban traffic system are presented with a focus on the estimation of parking search traffic. Chapter 2 presents the macroscopic model where the time-varying conditions of parking usage and the traffic performance are modelled to estimate the dynamics of both systems. Chapter 3 shows a numerical example on the dynamics of both parking and traffic systems and, it further extends it to a new parking policy test. Additionally, a case study assessing the cruising conditions in the city of Zurich is given.

Part II: Effects of on-street parking maneuvers
- Chapter 4. Effects of downstream on-street parking maneuvers on the intersection
- Chapter 5. Effects of upstream on-street parking maneuvers on the intersection
- Chapter 6. Preferred on-street parking locations on urban links with respect to the intersections
In this part, a generalized methodology is provided to analyze the traffic delay and the service rate reduction of nearby intersections caused by on-street parking maneuvers. Chapter 4 analyzes the situation where downstream parking maneuvers occur, and generate car queues which block the intersection and cause lingering delay at the intersection. It estimates and compares the service rate of nearby intersections with and without the disturbance of the maneuver. Chapter 5 analyzes the situation where upstream parking maneuvers occur and starve the intersection as they cause traffic blockage. Chapter 6 generalizes these two chapters and shows guidelines on the location to place parking spaces in road links to avoid problematic scenarios where traffic discharge rate can be largely damaged for the nearby intersections. In all cases, the presented models are validated with real data from the city of Zurich.

**Part III: Parking data related issues**

- Chapter 7. Cost and accuracy of parking patrol surveys
- Chapter 8. Parking guidance and information systems (PGIs)

In this part, evaluations improvements on data collection of parking usage are proposed. Chapter 7 shows a method to improve parking patrol surveys. This traditional kind of parking survey is often used to assess parking duration, occupancy and turnover for dispersed small parking areas. Using parking usage data, Chapter 8, provides a methodology to assess the effectiveness of parking guidance and information systems on reducing cruising based on the model proposed in Chapter 2.

Chapter 9 concludes the overall dissertation.
Part I

Effects of cruising-for-parking
Abstract

The urban parking and the urban traffic systems are essential components of the overall urban transportation structure. The short-term interactions between these two systems can be highly significant and influential to their individual performance. The urban parking system, for example, can affect the searching-for-parking traffic, influencing not only overall travel speeds in the network (traffic performance), but also total driven distance (environmental conditions). In turn, the traffic performance can also affect the time drivers spend searching for parking, and ultimately, parking usage. In this part, we propose a methodology to model macroscopically such interactions and evaluate their effects on urban congestion. The proposed model can be used to estimate both, how parking availability can affect traffic performance (e.g., average time searching for parking, number of cars searching for parking); and how different traffic conditions (e.g., travel speed, density in the system) can affect drivers ability to find parking. Moreover, the proposed model can be used to study multiple strategies or scenarios for traffic operations and control, transportation planning, land use planning, or parking management and operations.

In chapter 2, the model is built on a matrix describing how, over time, vehicles in an urban area transition from one parking-related state to another. With this model it is possible to estimate, based on the traffic and parking demand as well as the parking supply, the amount of vehicles searching for parking, the amount of vehicles driving on the network but not searching for parking, and the amount of vehicles parked at any given time. More important, it is also possible to estimate the total (or average) time spent and distance driven within each of these states. Based on that, the model can be used to design and evaluate different parking policies, to improve (or optimize) the performance of both systems.

In chapter 3, a numerical example and a case study based on real data are provided to show how the model (built in chapter 2) can be used to efficiently evaluate the urban traffic and parking systems. Parking policies such as increasing parking supply or shortening the maximum parking duration allowed (i.e., time controls) are tested, and their effects on traffic are estimated. The preliminary results show that time control policies can alleviate the parking-caused traffic issues without the need for providing additional parking facilities.
This part is based on the following research papers:

  


- Cao, J., M. Menendez and Waraich R.A. (2016) Cruising-for-parking: macroscopic modelling and insights on parking policies based on a case study from the city of Zurich, submitted for Transportation Research Board (Presentation only), 2016.
Chapter 2

A macroscopic model of urban parking system

2.1 Introduction

The urban parking and the urban traffic systems, as two essential components of the overall urban transportation structure can interact with each other and affect the overall system performance. As a matter of fact, searching-for-parking or cruising-for-parking can have significant influence to traffic. Shoup (2005), based on the review of 16 studies of mostly American and European cities, concluded that cars searching for free parking spaces contribute to over 8% of the total traffic in a city, reaching 30% in business areas during rush hour. Although this part of traffic is caused by inefficient parking provision, its corresponding externalities are endured by the traffic system as a whole. Such externalities have been studied from an economic point of view (e.g., Arnott, 2006) and could have a significant influence on traffic performance, causing congested or hyper-congested traffic conditions (Geroliminis, 2015).

Studies like these, all provide some insights on how the urban parking system (both supply and policies) can influence traffic performance. Nevertheless, although different parking policies including pricing schemes have been analyzed, proposed and implemented; to the authors’ knowledge, no study has provided yet a generalized methodology to macroscopically model the relation between parking demand, parking avail-
ability, and traffic conditions.

In this Chapter, we develop a parking-state-based matrix showing the system dynamics of urban traffic. It aims to model macroscopically a dynamic urban parking system. Basic assumptions for the matrix include a traffic demand over a period of time (e.g., a day), the distribution of parking durations, the length and the traffic properties of the network. Within the matrix, the likelihood of a parking searcher to access/find parking spots in an urban network is estimated, as well as other transition events such as starting to search for parking and departing from it. The model then provides an approximation of the proportion of cars searching for parking, as well as the time cars spent searching for parking, or traveling through the system. Moreover, traffic density and travel speed are also estimated over time based on different background conditions. These results are useful to evaluate both, how traffic performance (e.g., speed, density, flow) affects drivers’ ability to find parking; and how parking availability affects traffic performance.

The main contributions of the model are twofold.

1. This study looks at the relation between parking and traffic performance macroscopically. Most of the existing research looks at the problem microscopically, modeling the parking behavior of individual agents. The agent-based studies can require huge amounts of data, and high levels of detail both on the demand and the supply side. Here, we look at the problem macroscopically, and focus only on average values and probability distributions across the whole population. This is valuable, as all data requirements correspond to aggregated values at the network level and there is no data requirement for individual drivers or parking spots. This macroscopic approach, compared to microscopic methods, saves not only on data collection efforts (e.g., drivers’ preferences, individual driving routes, individual parking spots turnovers), but also reduces the computation costs significantly. Such efficiencies are especially useful when the network of interest is large and/or data is scarce.

2. This study allows us to model two dynamic systems interacting with each other. For the traffic system, the model is able to analyze overcrowded situations, where time-varying traffic conditions are provided as traffic performance indicators; they are also taken into consideration for the evolution of the matrix. For
2.2 Literature Review

Currently, there are three general approaches that are used for understanding and estimating traffic-parking interactions: empirical, analytical, and multi-agent (MA) simulations tools.

Empirical studies rely on driver surveys (e.g., Axhausen, 1994; IBM’s global parking survey, 2011), videotaping (e.g., King, 2010), driving test cars and searching for parking (Shoup, 2006), GPS data (e.g., Van Der Waerden et al., 2013; Montini et al., 2012), and parking occupancy data (e.g., Millard-Ball et al., 2014).

Empirical data is often collected for local projects as it is mostly specific to an area or a city. Thus, since the data observed is based on localized conditions, it is difficult to draw generalized conclusions from it. For example, driver surveys generally stop people at intersections to ask if they are seeking parking, or ask people emerging from their cars about their experience finding a parking place. As one would expect, the results are then very much based on local drivers’ preference for parking, and their value-of-time, as well as the time of day. The same is true for studies that rely on video or other visual techniques. Methods using GPS and parking occupancy data can be used for a wider range of cities as they can provide more generalized conclusions. However, GPS data extraction tools are still under development, so the precision and generality of conclusions drawn with them are not yet known. As for the use of parking occupancy data, this one typically does not include
any traffic information and thus, the parking-caused traffic still needs to be derived through other methods. Therefore, a macroscopic model that does not require any physical devices and yet can provide both more generalized conditions and results is desirable.

Notable theoretical contributions on the interaction between parking and traffic on the network level include literature on economics, macroscopic traffic models and traffic assignment models. The literature on economics includes Arnott et al. (1999, 2006, 2010). Based on two traffic assignment methods, user equilibrium and social optimal, the externalities of parking system on traffic congestion are presented (1999). However, the model does not represent traffic performance, e.g., a fixed value travel speed is assumed for all conditions. More connected to our study, Arnott et al. (2006) defined different types of vehicles (moving, cruising, and parked), then provided very useful relations between these types. However, the model is based on stationary-state conditions, and cannot describe the dynamics of the system (i.e., time-varying conditions).

Daganzo (2007) described an adaptive control approach to improve urban mobility and relieve congestion; the basic idea is to monitor and control aggregate vehicular accumulations in an area. This idea is further expanded with the theory of the Macroscopic Fundamental Diagram (MFD) in Daganzo and Geroliminis (2008) and verified empirically in Yokohama, Japan. Data collected in the latter study shows that the trip length for vehicles travelling a specific region of a city is time-invariant. This assumption is relaxed when vehicles have to travel longer while cruising for parking (Geroliminis, 2015). In these two papers, it is assumed that cruising-for-parking starts after the travelers arrive at their destinations (after they drove a fixed trip length) and the percentage of cruising traffic is inversely proportional to the percentage of available parking spots.

Gallo and D’Acierno (2011) proposed an assignment model on urban networks to simulate parking choice, and the impact of parking search on traffic congestion. The cost function of users included driving, parking, and walking. Then the traffic conditions induced by the parking search process could be found. However, within each interval of the traffic assignment (1 hour), the traffic and parking conditions remained steady, limiting the application of the model, as in reality both traffic and parking conditions can change rapidly. Bodenbender et al. (2013) developed a model which considered the probability of not finding a parking. The model was used to test different parking
policies based on the urban network of Zurich, Switzerland. Same as the previous study, the static traffic assignment neglected all the time-varying conditions such as the parking usage and traffic performance. In addition, the study assumed that travelers were fully informed about parking including the probability to find one in each link. Boyles et al. (2015) also described an equilibrium formulation for incorporating parking search into traffic network assignment models. The equilibrium problem was formulated as a variational inequality. Numerical results showed that neglecting parking search could substantially underestimate network flows, and quantitatively demonstrated the relationship between parking duration effects and the cost of time spent walking relative to driving.

MA simulation tools are widely used to simulate travel behavior for a large number of users. These tools allow inputs such as a non-homogeneous network and personal preferences. The output of the simulations can contain very detailed results, e.g., parking search traffic and impact on traffic performance (Martens and Benenson 2008; Waraich and Axhausen 2012; Horni et al. 2013). This method, though comprehensive and powerful, relies on many preliminary models such as car following models, route and parking choice models, etc. Therefore, the accuracy of the final output can be affected through many aspects. In addition, agent-based simulations require a large amount of very detailed data for the specified conditions, including the detailed network of the city and its parking system, as well as the travel behavior and searching habits of its citizens. Hence, the transferability of the approach (and results) across cities and/or population types is rather low.

2.3 Overall Methodology and Matrix Framework

2.3.1 System dynamics of urban traffic and parking-related-states

Consider a round-trip going into an urban area as a tour instead of 2 single trips. The vehicle may experience three parking-related states separated by five parking-related transition events. The three parking-related states are:

- Non-searching state: Vehicles in this state are not searching for
Figure 2.1: Parking-related states and the parking-related transition events of vehicles in an area.

- Searching state: Vehicles in this state are searching for parking.
- Parking state: Vehicles in this state are parked (i.e., staying in parking spots).

The states are shown in Figure 2.1 linked by the transition events. Figure 2.1(a) describes the transition events based on one single vehicle trip. Figure 2.1(b) describes the transition events for all vehicles (the overall traffic movements) in an urban area; the shaded part represents the parking system in this area, and the rest represents the traffic system. Notice that we assume the parking maneuvers (access/depart) are instantaneous although in reality they are not. More details on the specific effects of these maneuvers can be found in Cao and Menendez (2015a); Cao et al. (2015).

The five parking-related transition events are also shown in Figure 2.1. They are:

- Enter the area: These vehicles enter the “non-searching” state.
- Start to search: These vehicles transition from the “non-searching” to the “searching” state.
- Access parking: These vehicles transition from the “searching” to the “parking” state.
Table 2.1: Key variables in a time slice.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{ns}^i$</td>
<td>Number of vehicles in the state “non-searching” at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$N_s^i$</td>
<td>Number of vehicles in the state “searching” at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$N_p^i$</td>
<td>Number of vehicles in the state “parking” at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$n_{ns}^i$</td>
<td>Number of vehicles that enter the area and transition to “non-searching” during time slice $i$ (enter the area).</td>
</tr>
<tr>
<td>$n_{ns/s}^i$</td>
<td>Number of vehicles that transition from “non-searching” to “searching” during time slice $i$ (start to search).</td>
</tr>
<tr>
<td>$n_{s/p}^i$</td>
<td>Number of vehicles that transition from “searching” to “parking” during time slice $i$ (access parking).</td>
</tr>
<tr>
<td>$n_{p/ns}^i$</td>
<td>Number of vehicles that transition from “parking” to “non-searching” during time slice $i$ (depart parking).</td>
</tr>
<tr>
<td>$n_{ns/}^i$</td>
<td>Number of vehicles that leave the area and transition from “non-searching” during time slice $i$ (leave the area).</td>
</tr>
</tbody>
</table>

- Depart parking: These vehicles transition from the “parking” to the “non-searching” state.
- Leave the area: These vehicles finish the “non-searching” state as they exit the area of interest.

Consider a very small time slice, $i$, (e.g., 1 minute), Table 2.1 shows the notation for the number of vehicles in each state at the beginning of the time slice and the number of vehicles experiencing each transition event during the time slice.

2.3.2 Changes to the number of vehicles in each state

The transition of the whole system between consecutive time slices is shown in Figure 2.2 and formulated as Eq. 2.1, 2.2 and 2.3.

In Figure 2.2(a), the traffic composition (i.e., the number of vehicles in each parking-related state) at the beginning of time slice $i$ is
Chapter 2. A macroscopic model of urban parking system

Figure 2.2: Illustrative plot of queuing diagram of vehicles in the area.

shown, as well as the transition events during time slice $i$. Based on
them, the traffic composition at the beginning of the next time slice
can then be obtained. When repeating the process described above, a
queuing diagram for a longer period that consists of many time slices
can be found (Figure 2.2(b)). In Figure 2.2(b), at a given time, the
vertical distance between two neighboring curves indicates the number
of vehicles in a state, e.g., at the beginning of time slice $i$, the vertical
distance between the curves “start to search” and “access parking” is the
number of vehicles in the “searching” state, $N^i_s$. Notice that the number
of vehicles in the “non-searching” state, $N^i_{ns}$, includes two families
of vehicles: new vehicles that just entered the area (on the top of the
figure) and vehicles that are about to leave the area (on the bottom
of the figure). For a given period of time, the average horizontal distance
between two neighboring curves is the average time that vehicles spend
in that state.

Eq. 2.1 updates the number of “non-searching” vehicles. During
time slice $i$, vehicles that enter the area (i.e., $n^i_{ns}$) and vehicles that
depart parking (i.e., $n^i_{p/ns}$) join this state; vehicles that start to search
(i.e., $n^i_{ns/s}$) and vehicles that leave the area (i.e., $n^i_{ns/}$) quit this state.

$$N^{i+1}_{ns} = N^i_{ns} + n^i_{ns} + n^i_{p/ns} - n^i_{ns/s} - n^i_{ns/} \quad (2.1)$$
2.3. Overall Methodology and Matrix Framework

Figure 2.3: Interactions between the urban parking and traffic systems.

Eq. 2.2 updates the number of “searching” vehicles. During time slice $i$, vehicles that start to search (i.e., $n_{ns/s}^i$) join this state; vehicles that access parking (i.e., $n_{s/p}^i$) quit this state.

$$N_{s}^{i+1} = N_{s}^{i} + n_{ns/s}^{i} - n_{s/p}^{i}$$ (2.2)

Eq. 2.3 updates the number of “parking” vehicles. During time slice $i$, vehicles that access parking (i.e., $n_{s/p}^i$) join this state; vehicles that depart parking (i.e., $n_{p/ns}^i$) quit this state.

$$N_{p}^{i+1} = N_{p}^{i} + n_{s/p}^{i} - n_{p/ns}^{i}$$ (2.3)

2.3.3 Changes to the number of vehicles going through each transition event

Time slices are sufficiently small. Hence, the trips can be generated simultaneously in each time slice. Vehicles can have only one parking-related state during one time slice. They may or may not transition to another state at the end of a time slice, such decision is endogenously generated within the model.

A framework showing all the interactions within urban traffic/parking systems is given in Figure 2.3. More important, it also indicates, conceptually, how the number of vehicles in each parking-related state affects the transition events.
As shown in Figure 2.3, in general, there are three ways that vehicles within each parking-related state may affect the transition events:

- The number of parked vehicles (i.e., within “parking” state) and the time they accessed parking, affects the number of vehicles “accessing parking” and “departing parking”.

- The number of parking searchers (i.e., within “searching” state) and the time they started to search affects the number of vehicles “accessing parking”.

- The number of vehicles driving (i.e., within the “non-searching” and “searching” states) and the time these vehicles joined the state affects the travel speed and further influences all the transition events except “enter the area”.

In other words, based on the number of vehicles in each parking-related state and the time they joined that state, each transition event is modeled.

- For the “enter the area” transition, it is assumed to be the same as the travel demand (known or assumed). Details are explained in section 2.4.1.

- For the “start to search” transition, it is assumed to happen after a vehicle drives a given distance. The time needed for this transition depends on the corresponding traffic conditions (travel speeds) during that period. Details are explained in section 2.4.2.

- For the “access parking” transition, it is modeled based on the likelihood of finding an available parking spot based on the conditions during that time slice. Details are explained in section 2.4.3.

- For the “depart parking” transition, it is obtained based on the arrival time of vehicles to the parking facilities and the distribution of parking durations. Details are explained in section 2.4.4.

- For the “leave the area” transition, it is obtained also after a vehicle drives a given distance (two distances are assumed respectively for through traffic to leave the area and for parked cars to leave the area). Details are explained in section 2.4.5.
2.3. Overall Methodology and Matrix Framework

The development of the matrix can be programmed based on the definitions/equations/models provided here. Therefore, the system can either end when all traffic have left the area (this is the case for our numerical example), or any other time as needed.

2.3.4 Basic information for analytical model

Basic model assumptions, inputs, and expected outputs are briefly described below.

2.3.4.1 Assumptions

The network is relatively small, compact, and homogenous. On average, all existing parking spaces (not only the available ones) are uniformly distributed on the network. Moreover, they are all identical. We thus do not address the role of parking fees or walking distance when allocating more or less desirable parking spots to users.

Having no consideration of fee, walking distance, allows us to avoid drivers’ preference on parking location and price, etc. This makes the model simple enough that one can focus on the searching process of travellers in the network. The basis for this assumption is that the network should not be too large so that the drivers can be more or less indifferent to parking spaces at different locations and take the first one they find.

The arrival rate of traffic into the area, the size of the network, and the distribution of parking durations and the traffic properties of the network are known. Trips are uniformly distributed along the network and the parking demand (vehicles that are searching for parking) is homogenously distributed within the overall driving traffic.

As vehicles that use parking garages do not typically search for parking (they treat the parking garage as the target destination), it is not realistic to model them as searching traffic. Considering that, we define a portion of travel demand as through traffic which represents trips that do not search for parking. They include users of off-street, dedicated/private parking facilities or vehicles that are simply driving through the network. In this way, the parking garages do not need to be modeled explicitly, but the vehicles using them are still taken into account.
2.3.4.2 Inputs

Corresponding to the assumptions described above. Table 2.2 shows all the model’s independent variables.

Table 2.2: Independent variables (inputs).

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_i^{ns}$</td>
<td>New arrivals to the network during time slice $i$ (i.e., travel demand).</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Proportion of new arrivals during time slice $i$ that corresponds to through traffic.</td>
</tr>
<tr>
<td>$L$</td>
<td>Size (length) of the network.</td>
</tr>
<tr>
<td>$A$</td>
<td>Total number of existing parking spots (for public use) in the area.</td>
</tr>
<tr>
<td>$t$</td>
<td>Length of a time slice.</td>
</tr>
<tr>
<td>$t_d$</td>
<td>Parking duration.</td>
</tr>
<tr>
<td>$f(t_d)$</td>
<td>The probability density function of the parking duration.</td>
</tr>
<tr>
<td>$v$</td>
<td>Free flow speed, i.e., maximum speed on the network.</td>
</tr>
<tr>
<td>$Q_{max}$</td>
<td>Maximum traffic flow rate that can be adopted on the network.</td>
</tr>
<tr>
<td>$k_c$</td>
<td>Optimal/critical traffic density on the network. If the traffic density is higher than this value, then congestion occurs.</td>
</tr>
<tr>
<td>$k_j$</td>
<td>Jam density.</td>
</tr>
<tr>
<td>$l_{ns/s}$</td>
<td>Distance that must be driven by a vehicle before it starts to search for parking.</td>
</tr>
<tr>
<td>$l_i$</td>
<td>Distance that must be driven by a vehicle before it leaves the area without parking.</td>
</tr>
<tr>
<td>$l_{p/s}$</td>
<td>Distance that must be driven by a vehicle before it leaves the area after it has parked.</td>
</tr>
<tr>
<td>$N_{ns}^0$</td>
<td>The initial condition of non-searching state.</td>
</tr>
<tr>
<td>$N_s^0$</td>
<td>The initial condition of searching state.</td>
</tr>
<tr>
<td>$N_p^0$</td>
<td>The initial condition of parking state.</td>
</tr>
</tbody>
</table>

One can see four distinct sets of input variables.

The first set corresponds to the travel demand and supply, including the traffic demand, the proportion of through traffic, the distribution of the parking durations, and the parking supply in the area. These data can be assumed based on some historical data, e.g., traffic data.
on main roads to enter the network; parking data from one day’s data collection, etc.

The second set corresponds to the traffic network, including the total length, the traffic flow properties such as the saturation flow, free flow speed and jam density. These data can be estimated based on real measurements, the kinematic wave theory of traffic flow, the macroscopic fundamental diagram, and/or simulation results.

The third set corresponds to the distances one needs to drive before transitioning into the next state. These values can be assumed based on the length of the network, and other data collected from travellers.

The four set corresponds to the initial conditions of the parking-related states. These values can be measured, assumed or simulated.

2.3.4.3 Outputs

The model is able to provide, among others, the following outputs:

- Indicators for traffic conditions: vehicle accumulation on the network (traffic density), it includes vehicles in both non-searching and searching states; average travel speed, obtained based on the traffic density; total and average distance driven.

- Indicators for parking conditions: arrival to parking facilities (transition event “access parking”); departure of parking facilities; parking occupancy; available parking supply; parking demand (i.e., parking searchers).

Besides these, indicators specific to parking searchers can also be obtained, such as the average search time and distance; share of searching traffic and non-searching traffic, etc.

2.4 Analytical Formulations for Transition Events

For the modeling of the number of vehicles that go through a transition event in a time slice, we establish some more detailed assumptions. The urban network is abstracted as one ring road with cars driving in a single direction. Notice that, the model and the formulations are valid and representative for other network topologies such as a grid network or a ring-radial network. The ring road is used here only as a representation of a homogenous network, the physical shape, structure and
### Chapter 2. A macroscopic model of urban parking system

#### Table 2.3: Intermediate variables.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^i$</td>
<td>Number of available parking spots at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$k^i$</td>
<td>Average traffic density in time slice $i$.</td>
</tr>
<tr>
<td>$v^i$</td>
<td>Average travel speed in time slice $i$.</td>
</tr>
<tr>
<td>$d^i$</td>
<td>Maximum driven distance of a vehicle in time slice $i$.</td>
</tr>
<tr>
<td>$s^i$</td>
<td>Spacing between vehicles that are searching for parking at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$m^i$</td>
<td>Maximum number of vehicles that can pass by the same place on the network during time slice $i$.</td>
</tr>
<tr>
<td>$d^i_r$</td>
<td>Remainder of the division $\frac{d^i}{s^i}$ when $d^i &gt; s^i$.</td>
</tr>
</tbody>
</table>

The topology of the real network does not affect the validity of the model results as soon as the network is homogeneous. The assumption of a single travel direction simplifies the model without affecting the model results: the traffic demand can be seen as homogenously distributed on the network, whether vehicles travel in a single direction or two. We assume no overtaking takes place. This, although seems unrealistic, does not affect the model results: for any given number of available parking spaces and searchers, the average number of vehicles finding parking spaces in a time slice should not change even if cars can overtake each other.

During a given time slice, vehicles drive at the same speed. The available parking spots might be visited by several vehicles, but they only accommodate the first one that passes by. The vehicles that pass afterwards, see it full and continue searching for the next available parking spot.

Additionally, we define some new variables, they are listed in Table 2.3. They are used as the foundation to quantify the number of vehicles that experience each transition event in a time slice.

Their functions are written as Eq. 2.4 - 2.10

$A^i$ is the parking capacity minus the number of parking spaces that are occupied at the beginning of a given time slice, $A^i \leq A$.

$$A^i = A - N^i_p$$  \hspace{1cm} (2.4)
2.4. Analytical Formulations for Transition Events

\( k^i \) is the total number of vehicles on the road network at the beginning of a given time slice divided by the length of the network.

\[
    k^i = \frac{N^i_s + N^i_{ns}}{L}
\]  

\( v^i \) is formulated based on a triangular Fundamental Diagram. Hence, it assumes that vehicles travel at speed \( v \) when traffic is not congested, and at a lower speed once the traffic density exceeds \( k_c \) (i.e., traffic starts to be congested). The form of \( v^i \) can be assumed differently in other cases if necessary. For a larger network, a macroscopic fundamental diagram (MFD) theory which takes into account aggregate conditions across the network might be more suitable. In case another network is used instead of a simple ring road, exchanging the triangular FD with an MFD, should not affect, however, any of the presented methodology. As a matter of fact, any model relating network densities to speed could be used.

\[
v^i = \begin{cases} 
    v, & \text{if } 0 \leq k^i \leq k_c \\
    \frac{Q_{\text{max}}}{k_c - k_j} \cdot \left(1 - \frac{k_j}{k^i}\right), & \text{if } k_c < k^i \leq k_j 
\end{cases}
\]  

\( d^i \) and \( s^i \) are formulated as explained by their definitions.

\[
d^i = v^i \cdot t
\]

\[
s^i = \frac{L}{N^i_s}
\]

Notice that, if \( k^i \) exceeds or equals \( k_j \) at a given time, a gridlock situation will be immediately generated. Since this moment, \( v^i = 0 \) and \( d^i = 0 \), i.e., no vehicle on the network is able to travel any further and no transition events can be reached (except for “depart parking”). Such a situation may occur when a set of unfavorable conditions are met. For example, a combination of small parking supply, long parking durations and large parking demand, etc.

As the maximum number of vehicles that can pass by the same place on the network, \( m^i \) is formulated based on the maximum distance a vehicle can drive and the spacing between two consecutive vehicles. Note that, all locations on the network could be potentially visited by \( m^i - 1 \) cars.
\begin{equation}
m^i = \left\lfloor \frac{d^i}{s^i} \right\rfloor (2.9)
\end{equation}

\(d^i_r\) is formulated based on its definition.

\begin{equation}
d^i_r = d^i - \left\lfloor \frac{d^i}{s^i} \right\rfloor \cdot s^i \text{ for } d^i > s^i (2.10)
\end{equation}

Assume that at the beginning of time slice \(i\), the starting point of a car is location \(x_c\). Then the starting points of the vehicles behind this original car are locations \(x_c - s^i, x_c - 2 \cdot s^i, x_c - 3 \cdot s^i\), etc. At the end of time slice \(i\), depending on the ratio between \(d^i\) and \(s^i\) (i.e., the amount of overlap between vehicles’ trajectories), one of those vehicles behind the original car will not be able to drive beyond \(x_c + d^i_r\) (the vehicle with starting point \(x_c - \left\lfloor \frac{d^i}{s^i} \right\rfloor \cdot s^i\)). All the other vehicles with starting points between \(x_c - \left\lfloor \frac{d^i}{s^i} \right\rfloor \cdot s^i\) and \(x_c\) will be able to drive further. In other words, within the area of \([x_c, x_c + d^i_r]\), a maximum of \(m^i\) cars can pass by (i.e., this is the area of maximum overlap between vehicles’ trajectories); and within the area of \([x_c + d^i_r, x_c + s^i]\), a maximum of \(m^i - 1\) cars can pass by (i.e., this is the area of minimum overlap between vehicles’ trajectories). This will later be used to obtain \(n^i_{s/p}\).

Based on these values, we can now find the number of vehicles that go through each transition event during time slice \(i\).

### 2.4.1 Enter the area, \(n^i_{/ns}\)

\(n^i_{/ns}\) is an input to the model. Within \(n^i_{/ns}\), a percentage \(\beta^i\) is through traffic that will directly leave the area after driving a distance \(l_\text{/}\), the rest will go through all transition events.

### 2.4.2 Start to search, \(n^i_{ns/s}\)

Vehicles start to search after driving a distance of \(l_{ns/s}\) (since they enter the area). \(l_{ns/s}\) can be fixed or drawn out of a given probability distribution function. Here, for simplicity, we assume it is fixed. The expression then for the number of vehicles starting to search for parking during time slice \(i\) is written as Eq. \(\text{[2.11]}\) and explained below.

\begin{equation}
n^i_{ns/s} = \sum_{i' = 1}^{i-1} (1 - \beta^{i'}) \cdot n^i_{/ns} \cdot \gamma^i_{ns/s} \text{ term 1} \cdot \gamma^i_{ns/s} \text{ term 2} (2.11)
\end{equation}
2.4. Analytical Formulations for Transition Events

\[ \gamma_{ns/s}^{i'} = \begin{cases} 
1, & \text{if } l_{ns/s} \leq \sum_{j=i'}^{i-1} d^j \text{ and } \sum_{j=i'}^{i-1} d^j \leq l_{ns/s} + d^{i-1} . \\
0, & \text{if otherwise.} 
\end{cases} \]

\( n_{ns/s}^{i'} \) may consist of vehicles that entered the area in any time slice between 1 and \( i - 1 \). Use \( i' \) to denote such a time slice, \( i' \in [1, i - 1] \) (notice that vehicles that enter the area in time slice \( i \) are not included as they already experience one transition event during this time slice). In time slice \( i' \), \( n_{ns/s}^{i'} \) vehicles entered the area. Term 1 in Eq. 2.11 represents the portion of those which need to park (i.e., all vehicles except through traffic). Term 2 is a binary variable (0 or 1) indicating whether these vehicles start to search for parking in time slice \( i \). For \( \gamma_{ns/s}^{i'} \) to be equal to 1, two conditions must be satisfied: the vehicles have driven enough distance to start searching, and they have not started the search before.

2.4.3 Access parking, \( n_{s/p}^{i} \)

After drivers start searching, their driving time/distance is not identical anymore (during the searching state). It depends on the current conditions (i.e., the density of available parking spaces, the density of searchers and traffic performance) and their luck finding an available parking spot (their own location, that of the available parking spots and the competitors). To find each vehicles’ driving time/distance, one needs to record the location of all the cars and parking spots throughout the different time slices, this requires lots of additional details/efforts. However, we do not address who takes which parking space, but only the average number of travellers that access parking. Therefore, these efforts are saved. In other words, the model does not provide information about which vehicle parked, or which parking space was taken, or how far each vehicle drove before finding parking. We do know, however, the average number of vehicles that found parking spaces during this time slice, and the total/average searching distance driven during this time slice.

At the beginning of each time slice, the number of available parking spots and the number of parking searchers are found based on the matrix. These numbers are recorded over time. However, their locations are not tracked. The following two assumptions are used in the model: First, at the beginning of each time slice, the locations of the available parking spots are random. Second, at the beginning of each time slice, the locations of parking searchers are uniformly distributed.
on the network. The first assumption represents the stochasticity of the parking availability. The second assumption guarantees that the demand is homogeneously generated.

The second assumption is necessary, as during each time slice, the model provides an average amount of parking spots being taken. This average value only stands for a condition where, more or less, all searchers are uniformly distributed on the network. This, evidently, limits the model. For example, if in reality all searchers focus in one street where parking spots are scarce while parking spots are available somewhere else, then the model would most likely overestimate the real value of the amount of parking spots being taken. However, it does provide us an idea how the traffic might behave under general conditions where the parking demand (vehicles that are searching for parking) is homogeneously distributed within the overall driving traffic. We are interested on whether there is on average at least one car that is able to take an available parking spot. This, does not require us to know exactly the actual location of each car.

Notice that, at the beginning of each time slice, the locations of parking searchers and available parking spots are reset (independently from the previous time slice). This guarantees that we obtain the average number of vehicles that find parking, without being influenced by the randomness of vehicles’ location at that specific time. A simulation could be further developed to compare the assumed situation and the real situation. To represent the real situation, the locations of cars and parking spaces need to be recorded over different time slices. Meanwhile, the locations of vehicles that newly started searching, or parking spaces that newly became available need to be assumed. The simulation could start with two sets of inputs which indicate: first, the number and the locations of the vehicles that started searching in each time slice; and second, the number and the locations of the parking spaces that became available in each time slice. Using a simple software such as matlab, this simulation can be easily carried out. However, due to time limitations, it is not presented in this dissertation, but it can be included in future extensions.

To find then the value of \( n_{s/p}^i \), we define three different scenarios based on the relation between \( d^i \), \( s^i \) and \( L \).

**Scenario 1:** if \( d^i \in [0, s^i] \).

Under this scenario, the maximum driven distance of a vehicle is shorter than the spacing between two consecutive vehicles. Therefore,
2.4. Analytical Formulations for Transition Events

no two vehicles’ trajectories will ever overlap during a single time slice. As a result, a parking spot can be visited at most by one car (recall Eq. 2.9).

Assume a parking spot is located at \(x\) and the rest \(A^i - 1\) parking spots are located at \(x_r\), for \(r \in \{1, 2, ..., A^i - 1\}\). The searching vehicles’ initial positions are \(x_c\), for \(c \in \{1, 2, ..., N_s^i\}\). Then, there are two conditions to guarantee that this parking spot at location \(x\) becomes occupied during time slice \(i\).

- First, the parking spot must be within the reach of a car, i.e., \(x \in [x_c, x_c + d^i]\) for any \(c \in \{1, N_s^i\}\). The probability of that is \(\sum_{c=1}^{c=N_s^i} \int_{x_c}^{x_c+d^i} \frac{1}{L} \, dx\).

- Second, there is no other parking spots between the car at location \(x_c\) and this parking spot at location \(x\), i.e., \(x_r \notin [x_c, x]\) for \(r \in \{1, A^i - 1\}\). The probability of that is \(\prod_{r=1}^{r=A^i-1} \left(1 - \int_{x_c}^{x} \frac{1}{L} \, dx_r\right)\).

Therefore, the probability of a random parking spot been taken during time slice \(i\) is the product of these two probabilities. As this is the same for all parking spots, the average number of parking spots been taken during time slice \(i\) equals to \(A^i\) times the product of the two probabilities detailed above; it is written as Eq. 2.12. A simplified equation for this scenario is written in Eq. 2.19.

\[
\text{if } d^i \in [0, s^i], \quad n_{s/p}^i = A^i \cdot \sum_{c=1}^{c=N_s^i} \int_{x_c}^{x_c+d^i} \frac{1}{L} \, dx \cdot \prod_{r=1}^{r=A^i-1} \left(1 - \int_{x_c}^{x} \frac{1}{L} \, dx_r\right) \tag{2.12}
\]

**Scenario 2:** if \(d^i \in (s^i, L)\).

Under this scenario, vehicles’ trajectories can overlap and a parking spot can be visited by more than one car (although it only accommodates the first one).

Assume a parking spot is located at \(x\) and the rest \(A^i - 1\) parking spots are located at \(x_r\), for \(r \in \{1, 2, ..., A^i - 1\}\). The searching vehicles’ initial positions are \(x_c\), for \(c \in \{1, 2, ..., N_s^i\}\).

To formulate the probability of this parking spot at location \(x\) been taken during time slice \(i\), we define three sub-scenarios. They are based on the relation between \(A^i\) (i.e., the number of available parking spots) and \(m^i\) (i.e., the maximum number of searching vehicles that can pass
- Sub-scenario 2.1: if $m^i > A^i$

Since in this scenario $d^i < L$, then according to Eq. 2.8 and 2.9, $N^i_s \geq m^i$. Therefore, there is more parking demand than supply ($N^i_s > A^i$). Recall also that any parking spot on the network could be potentially visited by $m^i - 1$ cars ($m^i - 1 \geq A^i$). Therefore, any available parking spot will be taken by one of these cars, as there are simply too many cars searching and they drive a distance that is long enough to reach all available parking spots. Hence, all the parking spots will be taken, and still some vehicles will remain searching at the end of the time slice. $n^i_{s/p}$ is written as Eq. 2.13.

$$\text{if } d^i \in (s^i, L) \text{ and } m^i > A^i, \quad n^i_{s/p} = A^i$$

(2.13)

- Sub-scenario 2.2: if $m^i = A^i$

Still, we use $x$ as the location of the considered parking spot. As defined before,

- If $x \in [x_c, x_c + d^i_r]$, a number of $m^i$ cars ($A^i$ in this sub-scenario) could drive by that parking spot at $x$. If a parking spot is located within this area, it will be taken (see theory described in scenario 1). Thus, the probability of a parking spot located within this range and been taken is $\sum_{c=1}^{c=N^i_s} \int_{x_c}^{x_c+d^i_r} \frac{1}{L} \, dx$.

- If $x \in [x_c + d^i_r, x_c + s^i]$, a number of $m^i - 1$ cars ($A^i - 1$ in this sub-scenario) could drive by that parking spot at $x$.

Denote $p_f(n=m^i-1)$ as the probability of this parking spot not being taken, i.e., the probability that all the cars that could reach location $x$ park before arriving at $x$. Thus, the probability of a parking located within this range and been taken is $\sum_{c=1}^{c=N^i_s} \int_{x_c}^{x_c+s^i} \frac{1}{L} \cdot \left(1 - p_f(n=m^i-1)\right) \, dx$.

Combining these two ranges of $x$, for sub-scenario 2.2, the probability of a parking spot been taken can be written as $\sum_{c=1}^{c=N^i_s} \left\{ \int_{x_c}^{x_c+d^i_r} \frac{1}{L} \, dx + \int_{x_c+d^i_r}^{x_c+s^i} \frac{1}{L} \cdot \left(1 - p_f(n=m^i-1)\right) \, dx \right\}$. Since there are a number of $A^i$ parking spots, $n^i_{s/p}$ can be written as Eq. 2.14.
if \( d^i \in (s^i, L) \) and \( m^i = A^i \),
\[
n^i_{s/p} = A^i \cdot \sum_{c=1}^{c=N^i_s} \left\{ \int_{x_c}^{x_c+d^i_r} \frac{1}{L} dx + \int_{x_c+d^i_r}^{x_c+s^i} \frac{1}{L} \cdot (1 - p_f(n=m^i-1)) \cdot dx \right\}
\]
(2.14)

where

\[
p_f(n) = \frac{A^i-1}{\sum_{z_n=n}^{A^i-1} A^i-1-z_n} \cdot \prod_{j=1}^{n-1} p_{f_j} \quad (2.15)
\]

\[
p_{f_j} = \sum_{z_j=j}^{z_{j+1}} A^i \cdot \left( \frac{\int_{-(j-1)s^i}^{x} \frac{1}{L} dx}{\int_{-j \cdot s^i}^{x} \frac{1}{L} dx} \right)^{z_j} \cdot \left( \frac{\int_{-(j-1)s^i}^{x} \frac{1}{L} dx}{\int_{-j \cdot s^i}^{x} \frac{1}{L} dx} \right)^{z_{j+1}-z_j} \quad (2.16)
\]

In Eq. 2.15, \( n \) stands for the number of vehicles that can potentially reach \( x \). Within these \( n \) cars, the probability that the furthest vehicle (to \( x \)) parks before it arrives at \( x \) is shown in term 1; the probability that the rest \( n-1 \) vehicles all park before they arrive at \( x \) is shown in term 2. A simplified equation of \( n^i_{s/p} \) for this sub-scenario is written in Eq. 2.19.

- Sub-scenario 2.3: if \( m^i < A^i \)

Similar to sub-scenario 2.2, we define two ranges of \( x \).

- If \( x \in [x_c, x_c+d^i_r] \), a number of \( m^i \) cars could drive by that parking spot at \( x \). Thus, the probability of a parking located within this range and been taken is \( \sum_{c=1}^{c=N^i_s} \int_{x_c}^{x_c+d^i_r} \frac{1}{L} \cdot (1 - p_f(n=m^i)) \cdot dx \).
Chapter 2. A macroscopic model of urban parking system

- If \( x \in [x_c + d^i_r, x_c + s^i] \), a number of \( m^i - 1 \) cars could drive by that parking spot at \( x \). Thus, the probability of a parking spot located within this range and been taken is the same as that defined in sub-scenario 2.2, i.e.,

\[
\sum_{c=1}^{c=N^i_s} \int_{x_c + s^i_r}^{x_c + d^i_r} \frac{1}{L} \cdot (1 - p_f(n=m^i-1)) \, dx.
\]

Combining these two ranges of \( x \), for sub-scenario 2.3, the probability of a parking spot been taken is written as

\[
\sum_{c=1}^{c=N^i_s} \left\{ \int_{x_c + d^i_r}^{x_c + s^i} \frac{1}{L} \cdot (1 - p_f(n=m^i)) \, dx + \int_{x_c}^{x_c + d^i_r} \frac{1}{L} \cdot (1 - p_f(n=m^i-1)) \, dx \right\}.
\]

Since there are a number of \( A^i \) parking spots, \( n^i_{s/p} \) can be written as Eq. 2.17. A simplified equation of \( n^i_{s/p} \) for this sub-scenario is written in Eq. 2.19.

\[
n^i_{s/p} = A^i \cdot \sum_{c=1}^{c=N^i_s} \left\{ \int_{x_c + d^i_r}^{x_c + s^i} \frac{1}{L} \, (1 - p_f(n=m^i)) \, dx + \int_{x_c}^{x_c + d^i_r} \frac{1}{L} \, (1 - p_f(n=m^i-1)) \, dx \right\} (2.17)
\]

**Scenario 3:** if \( d^i \in (s^i, L) \) and \( m^i < A^i \),

\[
n^i_{s/p} = A^i \cdot \sum_{c=1}^{c=N^i_s} \left\{ \int_{x_c + d^i_r}^{x_c + s^i} \frac{1}{L} \, (1 - p_f(n=m^i)) \, dx + \int_{x_c}^{x_c + d^i_r} \frac{1}{L} \, (1 - p_f(n=m^i-1)) \, dx \right\} (2.17)
\]

Under this scenario, each car can drive around the whole network at least once, so all cars will park if there are enough parking spots. Otherwise, all spots will be taken. The result is written as Eq. 2.18.

if \( d^i \in [L, \infty) \),

\[
n^i_{s/p} = \min\{A^i, N^i_s\} (2.18)
\]

The expression of \( n^i_{s/p} \) for all scenarios described above is written as Eq. 2.19. For the convenience of the reader, here some of the equations have been further simplified with respect to what has been shown before for the description of each scenario.
2.4. Analytical Formulations for Transition Events

\[ n_{s/p}^i = \begin{cases} 
N_s^i \left[ 1 - (1 - \frac{d^i}{L})^A_i \right], & \text{if } d^i \in [0, s^i] \\
A_i^i & , \text{if } m^i > A_i \\
A_i^i(1 - \frac{N_s^i}{L A_i} \cdot p_2) & , \text{if } m^i = A_i \\
A_i^i(1 - \frac{N_s^i}{L A_i} \cdot p_1 - \frac{N_s^i}{L A_i} \cdot p_2) & , \text{if } m^i < A_i \\
\min\{A_i, N_s^i\} & , \text{if } d^i \in [L, \infty) 
\end{cases} 
\] 

(2.19)

where

\[ p_1 = \int_0^{d^i-(m^i-1)s^i} \left\{ \sum_{i_{m^i} = m^i}^{A_i-1} C_{A_{i-1}}^{i_{m^i}} \cdot [(N_s^i - m^i + 1)s^i - x]^{A_{i-1} - i_{m^i}} \right\} dx \]

\[ \sum_{i_{m^i-1} = m^i-1}^{i_{m^i}} C_{i_{m^i}}^{i_{m^i-1}} \cdot \sum_{i_{m^i-2} = m^i-2}^{i_{m^i-1}} C_{i_{m^i-2}}^{i_{m^i-1}} \cdot \left( \frac{x}{s^i} \right)^{i_{m^i-1}} \]

\[ p_1 = \int_{d^i-(m^i-1)s^i}^{s^i} \left\{ \sum_{i_{m^i-1} = m^i-1}^{A_i-1} C_{A_{i-1}}^{i_{m^i-1}} \cdot [(N_s^i - m^i + 2)s^i - x]^{A_{i-1} - i_{m^i-1}} \right\} dx \]

2.4.3.1 Approximation of \( n_{s/p}^i \) and the probability of finding parking

This subsection simplifies and proposes an approximation of the function of \( n_{s/p}^i \). The final approximation can be found in Eq. 2.28. Compared to the original function Eq. 2.19, Eq. 2.28 saves computational costs significantly and will be used in the following numerical examples and applications.

Simplification

Let us use \( N_p \) to denote the number of available parking space instead of \( A_i^i \). Since when \( m = N_p \), \( p_1 = \ldots \sum_{i_{m^i} = m^i}^{N_p-1} \ldots = 0 \). Therefore, \( n_{s/p}^i \) can be written as below (based on Eq 2.19).
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\[ n^i_{s/p} = \begin{cases} 
N_s \cdot \left[ 1 - \left( 1 - \frac{d}{L} \right)^{N_p} \right], & \text{if } d \in \left[ 0, \frac{L}{N_s} \right] \\
N_p \cdot \left( 1 - \frac{N_s}{L N_p} \cdot p_1 - \frac{N_s}{L N_p} \cdot p_2 \right), & \text{if } m > N_p \\
\min\{N_p, N_s\}, & \text{if } d \in \left( \frac{L}{N_s}, \frac{L}{N_p} \cdot N_s \right) \\
N_p \cdot \left( 1 - \left( 1 - \frac{d}{L} \right)^{N_p} \right), & \text{if } d \in \left[ \frac{L}{N_s}, \frac{L}{N_p} \cdot N_s \right] \cup \left( \frac{L}{N_p} \cdot N_s, \frac{L}{N_p} \cdot L \right) \\
N_s, & \text{if } d \in \left[ \frac{L}{N_s}, \frac{L}{N_p} \cdot L \right] \cup \left( \frac{L}{N_p} \cdot L, \infty \right) 
\end{cases} \tag{2.20} \]

As \( m = \left\lfloor \frac{d}{s} \right\rfloor \) and \( N_p \) is integer, \( m > N_p \) is equivalent to \( \frac{d}{s} > N_p \); \( m \leq A \) is equivalent to \( \frac{d}{s} \leq N_p \). Additionally, reverse the two sub-equations in the middle where \( d \in \left( \frac{L}{N_s}, L \right) \).

\[ n^i_{s/p} = \begin{cases} 
N_s \cdot \left[ 1 - \left( 1 - \frac{d}{L} \right)^{N_p} \right], & \text{if } d \in \left[ 0, \frac{L}{N_s} \right] \\
N_p \cdot \left( 1 - \frac{N_s}{L N_p} \cdot p_1 - \frac{N_s}{L N_p} \cdot p_2 \right), & \text{if } d \in \left( \frac{L}{N_s}, \frac{L}{N_p} \cdot N_s \right) \\
\min\{N_p, N_s\}, & \text{if } d \in \left( \frac{L}{N_p} \cdot N_s, \frac{L}{N_s} \right) \\
N_p, & \text{if } d \in \left[ \frac{L}{N_p} \cdot N_s, \frac{L}{N_s} \right] \cup \left( \frac{L}{N_s}, \infty \right) 
\end{cases} \tag{2.21} \]

Notice that, \( N_p \) and \( N_s \) are assumed to be integers and larger than 0.

Use \( v \cdot t \) to represent \( d \). Then, \( n^i_{s/p} \) can be written as below that two cases where \( N_p \leq N_s \) and \( N_p \geq N_s \) are explicitly shown.

\[ n^i_{s/p} = \begin{cases} 
N_s \cdot \left[ 1 - \left( 1 - \frac{\frac{v \cdot t}{L}}{N_p} \right)^{N_p} \right], & \text{if } t \in \left[ 0, \frac{L}{v \cdot N_s} \right] \\
N_p \cdot \left( 1 - \frac{N_s}{L N_p} \cdot p_1 - \frac{N_s}{L N_p} \cdot p_2 \right), & \text{if } t \in \left[ \frac{L}{v \cdot N_s}, \frac{L}{v \cdot \frac{N_p}{N_s}} \right], \text{if } N_p \leq N_s \\
N_s, & \text{if } t \in \left[ \frac{L}{v \cdot \frac{N_p}{N_s}}, \infty \right) \tag{2.22a} \\
N_p \cdot \left[ 1 - \left( 1 - \frac{\frac{v \cdot t}{L}}{N_p} \right)^{N_p} \right], & \text{if } t \in \left[ \frac{L}{v \cdot \frac{N_p}{N_s}}, \infty \right] \\
N_p \cdot \left( 1 - \frac{N_s}{L N_p} \cdot p_1 - \frac{N_s}{L N_p} \cdot p_2 \right), & \text{if } t \in \left[ \frac{L}{v \cdot \frac{N_p}{N_s}}, \frac{L}{v} \right), \text{if } N_p \geq N_s \tag{2.22b} \\
N_s, & \text{if } t \in \left[ \frac{L}{v}, \infty \right) \tag{2.22c} 
\end{cases} \]

Notice, in Eq. 2.22 the functions have NOT been changed from Eq. 2.19

**Approximation**

The computational cost of the original equations is very large due to the summations and combinations in \( p_1 \) and \( p_2 \). Therefore, the sub-equations where \( p_1 \) and \( p_2 \) exist are replaced.

With the replacements of the two sub-equations, we construct an approximation of \( n^i_{s/p} \) in Eq. 2.23
2.4. Analytical Formulations for Transition Events

\[ n_{s/p}^i = \begin{cases} 
N_s \cdot [1 - (1 - \frac{v \cdot t}{L})^N_p] & \text{if } t \in [0, \frac{L}{v \cdot N_s}] \\
N_p + N_s \left[ \frac{N_p}{N_s} - 1 + \left( 1 - \frac{1}{N_s} \right)^N_p \right] \cdot \log_{N_p} \frac{N_s}{N_p} \cdot \frac{v \cdot t}{L} & \text{if } t \in \left[ \frac{L}{v \cdot N_s}, \frac{L}{v} \cdot \frac{N_p}{N_s} \right], \text{if } N_p \leq N_s \\
N_s \cdot \left[ 1 + \left( 1 - \frac{1}{N_s} \right)^N_p \cdot \log_{N_s} \frac{v \cdot t}{L} \right] & \text{if } t \in \left[ \frac{L}{v} \cdot \frac{N_p}{N_s}, \frac{L}{v} \right], \text{if } N_p \geq N_s \\
N_s & \text{if } t \in \left[ \frac{L}{v}, \infty \right) 
\end{cases} \] (2.23a) (2.23b)

Notice that, using this equation, the values of \( N_p \) and \( N_s \) do not have to be integers anymore. However, the values of \( N_p \) and \( N_s \) have to be larger than 1.

To show more information about the approximated part in both sub-equations, Eq. [2.23] is converted to Eq. [2.28], i.e., the probability of finding parking \( p = \frac{n_{s/p}^i}{N_s} \).

The approximations are explained below.

We construct two functions based on the logarithmic function,

- The 2nd sub-equation in Eq. [2.22a] is replaced by the equation below.

\[ n_{s/p}^i = N_p + N_s \cdot \left[ \frac{N_p}{N_s} - 1 + \left( 1 - \frac{1}{N_s} \right)^N_p \right] \cdot \log_{N_p} \frac{N_s}{N_p} \cdot \frac{v \cdot t}{L} \] (2.24)

\[ p = \frac{N_s}{N_p} + \left[ \frac{N_p}{N_s} - 1 + \left( 1 - \frac{1}{N_s} \right)^N_p \right] \cdot \log_{N_p} \frac{N_s}{N_p} \cdot \frac{v \cdot t}{L} \] (2.25)

This function guarantees that the value of \( p \) is continuous:

- when \( t = \frac{L}{v \cdot N_s} \), \( p = 1 - (1 - \frac{1}{N_s})^N_p \), and

- when \( t = \frac{L}{v} \cdot \frac{N_p}{N_s} \), \( p = \frac{N_p}{N_s} \).

With this function, as \( t \) increases, the value of \( p \) is increasing at a decreasing rate. This is the same pattern as the original equation.

- The 2nd sub-equation in Eq. [2.22b] is replaced by the equation below.

\[ n_{s/p}^i = N_s \cdot \left[ 1 + \left( 1 - \frac{1}{N_s} \right)^N_p \cdot \log_{N_s} \frac{v \cdot t}{L} \right] \] (2.26)
$p = 1 + \left(1 - \frac{1}{N_s}\right)^{N_p} \cdot \log_{N_s} \frac{v \cdot t}{L}$  \hspace{1cm} (2.27)

This function guarantees that the value of $p$ is continuous:

- when $t = \frac{L}{v \cdot N_s}$, $p = 1 - (1 - \frac{1}{N_s})^{N_p}$, and
- when $t = \frac{L}{v \cdot N_s} \cdot \frac{N_p}{N_s}$, $p = 1$.

With this function, as $t$ increases, the value of $p$ is increasing at a decreasing rate. This is the same pattern as the original equation.

**Approximation on the probability of finding parking**

Denote $p$ as the probability of searching vehicles to find parking,

\[ p = \frac{n_i}{N_s}. \]

\[
p = \begin{cases} 
1 - (1 - \frac{v \cdot t}{L})^{N_p} & \text{if } t \in [0, \frac{L}{v \cdot N_s}] \\
\frac{N_p}{N_s} + \left[\frac{N_p}{N_s} - 1 + (1 - \frac{1}{N_s})^{N_p}\right] \log_{N_s} \frac{N_p}{N_s} \cdot \frac{v \cdot t}{L} & \text{if } t \in \left[\frac{L}{v \cdot N_s}, \frac{L}{v} \cdot \frac{N_p}{N_s}\right], \text{if } N_p \leq N_s \\
\frac{N_p}{N_s} & \text{if } t \in \left[\frac{L}{v} \cdot \frac{N_p}{N_s}, \infty\right) \\
1 - (1 - \frac{v \cdot t}{L})^{N_p} & \text{if } t \in [0, \frac{L}{v \cdot N_s}] \\
1 + \left(1 - \frac{1}{N_s}\right)^{N_p} \cdot \log_{N_s} \frac{v \cdot t}{L} & \text{if } t \in \left[\frac{L}{v \cdot N_s}, \frac{L}{v}\right) \\
1 & \text{if } t \in \left[\frac{L}{v}, \infty\right) 
\end{cases} \quad \text{(2.28a)}

\[
p = \begin{cases} 
\frac{N_p}{N_s} & \text{if } t \in \left[\frac{L}{v \cdot N_s}, \frac{L}{v}\right) \\
1 & \text{if } t \in \left[\frac{L}{v}, \infty\right) 
\end{cases} \quad \text{(2.28b)}
\]

Notice that the approximation satisfies the following two conditions:

- The value of $p$ is continuous for $t \in [0, \infty)$, it is also derivable in each of the three intervals.

- The upper bound of the error (between the original and the approximation) is limited. It is shown below.

Figure 2.4 illustrates Eq.2.28

In Figure 2.4, the left part of the curves represents Eq.2.28(a) where $N_p \leq N_s$; the right part of the curves represents Eq.2.28(b) where $N_p \geq N_s$. In each part, there are three lines representing the three ranges of $t$ as described above. Based on detailed values of $L$, $N_p$, $N_s$, $v$, the exact positions of the three lines might be shifted up or down, but the relative position between them remains the same. Notice that in reality $N_p \in [0, A]$. If $N_s \geq A$, then only the part where $N_p \leq N_s$, i.e., Eq.2.28(a) can exist.
2.4. Analytical Formulations for Transition Events

Figure 2.4: Probability of finding parking \( p \) in relation to different values or ranges of \( N_p \) and \( t \).

Upper bound of the error

Corresponding to different combinations of \( N_s, N_p \) and \( t \), the error differs. Here, a contour plot is given to show the upper bound of the error corresponding to various sets of combinations of \( N_s \) and \( N_p \). In each set, the upper bound means the largest error within all the errors corresponding to different \( t \).

Notice that, Eq. 2.19 was not used in plotting Figure 2.5 due to the high computational cost. Instead, Eq. 2.19 was used. For each combination of \( N_s, N_p \) and \( v \cdot t/L \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\} \), 1000 experiments were run to find the error; the average value of the 1000 experiments is taken to represent the result. The value shown in Figure 2.5 represents the largest value across \( v \cdot t/L \), for each combination of \( N_s \) and \( N_p \).

In the test, \( N_s \) ranges from 0 to 105, \( N_p \) ranges from 0 to 1005 (only the range between 0 and 205 is shown in the figure though). Since the number of searchers is normally limited while the size of the parking supply can be large, we choose a smaller range for \( N_s \) compared to \( N_p \). However, when considering a small area, 105 parking searchers can already represent a really large value.

As shown in the figure above, for to a given value of \( N_s \), the upper-
bound of the error first rises and then drops as $N_p$ increases. For the ranges of $N_s$ that we investigate, the upper bound of the error remains below 12%. On the other hand, for a larger $N_s$, the largest error for any $N_p$ is larger. In other words, if $N_s$ is extremely large, there is chance that $p$ has a much larger error. However, in reality, such situation can seldom occur. Therefore, it is reasonable to assume that the very largest error of the approximation is around 12%.

**Numerical example**

A numerical example is shown here. $N_s = 50$, $N_p = 50$. Figure below shows a comparison between the real value of $p$ and the approximation (obtained from Eq.2.28).

Figure 2.5: Contour plot of the upper bound of the error

Figure 2.6: Real probability and the approximated probability to find parking
2.4. Analytical Formulations for Transition Events

In Figure 2.6, the x axis is $\frac{v \cdot t}{L}$. Since the value of $v$ and $L$ are fixed, the x axis depends on the value of $t$. In other words, the curves in Figure 2.6(a) represent an increasing probability of finding parking as the searching time grows. The curves in Figure 2.6(b) represent the absolute value of the error as the searching time grows. As shown, the highest error is about 6% and it happens when $\frac{v \cdot t}{L} = 0.1$.

2.4.4 Depart parking, $n_{p/ns}^i$

As we know the number of vehicles accessing parking in all former time slices, we can find $n_{p/ns}^i$ based on the distribution of parking durations (an input to the model). Eq. 2.29 shows the number of vehicles that depart parking in time slice $i$.

$$n_{p/ns}^i = \sum_{i' = 1}^{i-1} n_{s/p}^{i'} \cdot \int_{(i-i') \cdot t}^{(i+1-i') \cdot t} f(t_d) \, dt_d$$ (2.29)

$n_{p/ns}^i$ may consist of vehicles that accessed parking in any time slice between 1 and $i - 1$. Use $i'$ to denote such time slice, $i' \in [1, i - 1]$. Notice that the vehicles that access parking during time slice $i$ are not included, as they already experience one transition event during this time slice. The number of vehicles that accessed parking in time slice $i'$ is $n_{s/p}^{i'}$. The probability that these vehicles depart parking in time slice $i$ equals to the probability of the parking duration being between $(i - i') \cdot t$ and $(i + 1 - i') \cdot t$, i.e., $\int_{(i-i') \cdot t}^{(i+1-i') \cdot t} f(t_d) \, dt_d$.

Some distributions are more suitable to describe parking duration than others, see Richardson (1974), Lautso (1981) and Cao and Menendez (2013); although theoretically, any distribution can be used, e.g., negative binomial, poisson. Eq. 2.29 therefore, remains general enough to fit any distribution for describing parking duration in this model.

2.4.5 Leave the area, $n_{ns}^i$

Vehicles leave the area after they drive for a given distance. The starting point for counting that distance corresponds to the moment they enter the area (if they are through traffic and do not park in the area), or the moment they depart the parking facilities. For these two cases, the required distances are $l_/$ and $l_{p/}$, respectively. They can be fixed
values, or values drawn out of any given probability distribution function. Here, for simplicity, we assume they are fixed. Eq. 2.30 shows the number of vehicles that leave the area during time slice $i$.

$$n_{ns}^i = \sum_{i' = 1}^{i - 1} \left( \beta^{i'} \cdot n_{ns}^{i'} \cdot \gamma_{/}^{i'} + n_{p/ns}^{i'} \cdot \gamma_{p/}^{i'} \right)$$  \hspace{1cm} \text{(2.30)}$$

where

$$\gamma_{/}^{i'} = \begin{cases} 1, & \text{if } l_{/} \leq \sum_{j=i'-1}^{j=i-1} d^j \text{ and } \sum_{j=i'-1}^{j=i-1} d^j \leq l_{/} + d_{i-1} \\ 0, & \text{if otherwise.} \end{cases}$$

$$\gamma_{p/}^{i'} = \begin{cases} 1, & \text{if } l_{p/} \leq \sum_{j=i'-1}^{j=i-1} d^j \text{ and } \sum_{j=i'-1}^{j=i-1} d^j \leq l_{p/} + d_{i-1} \\ 0, & \text{if otherwise.} \end{cases}$$

As shown in Eq. 2.30, $n_{ns}^i$ consists of two parts, the vehicles that leave the area without parking, i.e., $\beta^{i'} \cdot n_{ns}^{i'}$ for all time slices $i' \in [1, i - 1]$ (through traffic); and the vehicles that leave the area after they have parked, i.e., $n_{p/ns}^{i'}$ for all time slices $i' \in [1, i - 1]$. $\gamma_{/}^{i'}$ and $\gamma_{p/}^{i'}$ are binary variables (0 or 1) indicating whether these two groups of vehicles leave the area in time slice $i$. $\gamma_{/}^{i'}$ and $\gamma_{p/}^{i'}$ are equal to 1 if the vehicles that transitioned into the “non-searching” state in time slice $i'$ have reached (during time slice $i - 1$) the given distance required to leave the area.

### 2.5 Conclusions

In this study, we develop a macroscopic model to analyze the interactions between the urban parking and traffic systems. Using the system dynamics of urban traffic in an urban area based on its parking-related states, a queuing diagram can be provided. This can show the cumulative number of vehicles that go through each parking-related transition event as a function of time, as well as the number of vehicles within each state at any given time. The model can also furnish other traffic related metrics, such as total time vehicles spend in each state, total distance driven, and total delay.

The whole framework/model provides a new perspective for looking at parking systems and their interactions with traffic. Below we highlight some advantages of the model:

1. In comparison to microscopic models or MA simulation tools which are typically used when analyzing parking-caused traffic
2.5. Conclusions

issues, the macroscopic model proposed here has several advantages.

- The model has very little data requirements, while most of the tools used nowadays to analyze parking-related traffic require a lot of detailed data that is really hard to get. Our model, on the other hand, is macroscopically built and only needs some general inputs, distributions and probability theory.

- This macroscopic model allows us to compute the results without the use of complex simulations, as it can be easily solved with a simple numerical solver such as excel or matlab. This is in part possible because we only have a few parameters, and all of them have a physical interpretation. Moreover, they can all be obtained from field data. In addition, there is no need to run the model many times in order to account for its stochasticity, as it is based on probability functions (i.e., the stochasticity is already implicit within the model formulations).

- The simpler form of the macroscopic model might provide additional insights that cannot be delivered by microscopic models (e.g., insights into the mathematical relation between parking availability and traffic speeds).

2. The proposed model represents a dynamic system, where the time-varying conditions can be considered, and the time-based results or average values across time can be found. This gives it clear planning and operational applications (e.g., provide short term forecasting of traffic conditions based on the parking system, or parking usage based on traffic conditions; evaluate the total/average effect of different parking policies onto the traffic system over time and vice versa).

- The model provides the proportion of vehicles looking for parking on an urban network. In reality, vehicles looking for parking are hard to distinguish from normal driving vehicles, hence significant investments must be made to collect empirical data through the use of GPS and other devices. This model, however, provides some analytical results, which could be very helpful for cities to estimate their parking search condition with very limited investment.
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- The model provides a method to find the influence of parking searchers (or the parking system) on the non-searching vehicles (e.g., through traffic). This is interesting, as city governments and individual travelers often do not realize that parking can be a source of traffic jams (general congestion instead of a distinctive bottleneck). To this end, the model helps to detect the portion of traffic congestion which is caused by parking issues (i.e., detect parking-caused problems) as well as the magnitude of such negative effects.

- The model provides the total distance driven on the network, including the extra distance driven due to the search for parking. Even if there is no traffic jam, considering the same amount of trips, the longer the distance travelled, the worse it is for the environment (i.e., more air pollution). Based on this model, this part can be estimated as well, and further taken into account for policies such as pricing, etc.

- The model provides new insights and tools to evaluate the performance of parking systems over time (i.e., considering dynamic conditions). In other words, it also provides new aspects for parking systems to consider and new goals for them to reach when they are being planned and designed. Eventually, it can assess and assist parking provision such as dynamic pricing schemes and time control policies, to avoid the traffic deterioration caused by parking systems.

Overall, the usage/application of the proposed model is far beyond what we have illustrated in the numerical example. The model can provide the relation between the proportion of through traffic, the traffic conditions, and the likelihood of vehicles to access parking, for example. This is not included in the paper as the through traffic was assumed to be zero for simplification purposes. Also, the values of the driven distances needed for certain transition events (i.e., \( l_{ns/s} \), \( l_s \), and \( l_{p/} \)) can be generated by distributions to better duplicate reality, instead of using fixed values. An in-depth sensitivity analysis (Ge et al., 2014b, 2015, 2014a, Ge and Menendez, 2016) could further shed some light on the importance of these parameters and their distributions.

In addition, the off-street parking facilities can be modeled more explicitly, such as multiple parking spots at the same location; so the network can be easily expanded to include different kinds of parking supplies. Then, the decision of users between on-street and off-street
parking (especially when on-street parking spaces are not available) can also be modelled. All of these extensions, although not directly presented here, can be achieved easily based on the current model. The future research work could incorporate, however, a non-homogeneous environment (e.g., where both, the parking demand and supply are inhomogeneously distributed temporally and spatially); and incorporate different adjacent networks, where parking decisions can be made based on the conditions of more than one network. Also, in future studies, the bottleneck and delays caused by on-street parking maneuvers to traffic flow will be taken into account as well based on some other studies by the authors (Cao et al., 2015; Cao and Menendez, 2015a).

In summary, the proposed model, despite its simplicity, can be used to efficiently evaluate the urban traffic system macroscopically. The system dynamics of urban traffic based on its parking-related-states can be used to estimate both, how parking availability can affect traffic performance (e.g., average time searching for parking, number of cars searching for parking); and how different traffic conditions (e.g., travel speed, density in the system) can affect drivers’ ability to find parking. Moreover, the proposed model can be further exploited to study multiple strategies or scenarios for traffic operations and control, transportation planning, land use planning, or parking management and operations (e.g., evaluation of parking time/pricing controls, location and number of parking stalls).
Chapter 3

Parking search model: analysis and application

3.1 Introduction

In the previous Chapter, a macroscopic parking model is proposed linking urban traffic and urban parking state. In this Chapter, we explore the use of the concept and methodology through a numerical example and a case study. The Chapter is structured as follows.

Section 3.2 provides a sensitivity analysis showing the influence of each parameter to the final model output.

Section 3.3 provides a numerical example illustrating how to develop the system dynamics of urban traffic based on its parking-related-states and exploit the useful information it provides. Additionally, we use the numerical example to test different parking policies and evaluate their influence on the traffic system.

Section 3.4 provides a case study based on a central shopping area, namely “Jemoli” in the city of Zurich. The parking conditions there were investigated as well as the traffic demand. In the end, the traffic conditions in the area during a typical working day are estimated using our methodology.

Section 3.5 concludes the chapter.
3.2 Sensitivity Analysis

In this part, specifically, a 500 meter radius downtown area is considered. This represents a small but typical downtown center, e.g., around 10 standard blocks in Manhattan network. It also produces a walking distance less than 1 kilometer between any two points, which guarantees users’ indifference to the specific parking locations. For simplicity, the area is assumed to obey a triangular macroscopic fundamental diagram (MFD) where the jam density is 120 veh/km-lane while the backward shockwave speed is 20 km/h. In this way, the optimal density and the capacity can be estimated once the free flow speed is known. Notice that although 120 veh/km-lane might be considered too low of a jam density for a single link, given the typical inhomogeneities even in small networks, this value is not unreasonable as the average jam density for the MFD. To imitate a typical rush hour, we assume a fixed traffic arrival rate to the network for 2 hours.

The basic network conditions are detailed in Table 3.1. We test different combinations of their values to better understand and estimate their influence on the system performance.

More details on the values (ranges) from Table 3.1 are given below.

- The traffic demand (arrival flow to the network) is a fixed rate per time slice, including a number of $N_p$ vehicles which will park, and a number of $N_{tt}$ vehicles which are through traffic. The multiple combinations between these 2 variables represent not only different levels of demand, but also different compositions for those demands.

- The network length $L$ is chosen within 5, 10, 15. These values represent approximately, 3x3, 6x6 and 9x9 networks (i.e., 9, 36, 81 intersections), respectively (i.e., average block sizes of 350, 175, and 100 meters).

- The parking supply in the area ranges between 20 and 500. The lower and higher bound might be quite unrealistic for some of the networks considered, e.g., 500 parking spaces might be too much for a 5-kilometer network. However, these extreme combinations will help illustrate the general trends for the relation between parking supply and overall network conditions.

- The parking durations obey a gamma distribution where the shape parameter ranges between 1 and 5, and mean values be-
Table 3.1: Variables used in the model.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Unit</th>
<th>Value</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic demand entering the network and headed to parking.</td>
<td>$N_p$</td>
<td>veh/hr</td>
<td>{120,300,600}</td>
</tr>
<tr>
<td>Traffic demand entering the network and not headed to parking (through traffic).</td>
<td>$N_{tt}$</td>
<td>veh/hr</td>
<td>{600,1200,1800}</td>
</tr>
<tr>
<td>Network length.</td>
<td>$L$</td>
<td>km</td>
<td>{5,10,15}</td>
</tr>
<tr>
<td>Number of existing parking spaces in the area (total parking supply).</td>
<td>$A$</td>
<td>veh</td>
<td>[20,500]</td>
</tr>
<tr>
<td>Shape parameter of distribution of parking durations $t_d$ (for Gamma distribution).</td>
<td>$t_1$</td>
<td>-</td>
<td>[1,5]</td>
</tr>
<tr>
<td>Mean value of parking durations $t_d$ (for Gamma distribution).</td>
<td>$\bar{t}$</td>
<td>min</td>
<td>[10,60]</td>
</tr>
<tr>
<td>Distance to drive to transition between non-cruising and cruising.</td>
<td>$l_s$</td>
<td>km</td>
<td>[0,0.5]</td>
</tr>
<tr>
<td>Distance to drive to leave the area after the car finished parking.</td>
<td>$l_p$</td>
<td>km</td>
<td>[0,0.5]</td>
</tr>
<tr>
<td>Distance to drive for through traffic inside the area.</td>
<td>$l_{tt}$</td>
<td>km</td>
<td>[0.5,1]</td>
</tr>
<tr>
<td>Free flow speed of network (with intersections, etc.)</td>
<td>$v$</td>
<td>km/h</td>
<td>[10,40]</td>
</tr>
</tbody>
</table>

between 10 and 60 minutes. The gamma distribution is a two-parameter family of continuous probability distributions. The shape parameter indicates the position of the peak in the probability density function graph, whereas the mean is the product of the shape parameter and the scale parameter. Studies showed such assumption for distribution of parking durations is reason-
able (Cao and Menendez 2013; Richardson 1974). Notice that the parking durations mentioned here refer to only on-street parking spaces. Therefore, the upper bound is kept relatively low, i.e., 60 minutes.

- Recall that the diameter of the network area is 1 kilometer, the values of $l_s$ and $l_p$ range between 0 and the radius of 0.5 kilometers, while $l_{tt}$ ranges between the radius and the diameter of 1 kilometer.

- The free flow speed ranges between 10 and 40 km/h. The upper bound assumes almost no interruptions in the network with a resulting average free flow speed close to the free flow speed on links. The lower bound emulates networks with many interruptions (e.g., many intersections, on-street parking maneuvers affecting the traffic stream, pedestrian crossings).

A total number of 108000 scenarios are generated to conduct the sensitivity analysis. The idea is to have enough scenarios to cover the input space as much as possible, while keeping the computational requirements within reasonable limits (Ge and Menendez 2014b). The results from the simulations are shown below.

### 3.2.1 Factors affecting cruising-for-parking

Denote ACT as the average cruising time, and ACD as the average cruising distance.

Based on the results from all the simulations, a sensitivity analysis is carried out. Figure 3.1 shows the results. Figure 3.1(a) shows the influential factors of ACT, Figure 3.1(b) shows the influential factors of ACD. In both diagrams, each labelled point represents a variable. If the variable locates on the right side of the V curve, it has positive influence on the result. If it locates on the left side of the V curve, it has negative influence on the result. In (Ge et al. 2014b), the $\mu - \sigma$ plot is used to analyze the impacts of multiple parameters. The wedge in this plot is formed by two red lines corresponding to $\mu = \pm 2SEM$ (SEM = Standard Error of the Mean). The variable lying outside the wedge with surely be influential to the result. Moreover, if the variable locates on the right side of the wedge, it has positive influence on the result (i.e., any increase of the variable will make the model output increase). If it locates on the left side of the wedge, it has negative influence on the result. More details can be found in (Ge et al. 2014b).
3.2. Sensitivity Analysis

The results show that for ACT, the most influential factors are the parking supply ($A$), the mean parking duration ($\bar{t}$) and the parking demand ($N_p$). Not surprisingly, ACT is inversely proportional to $A$, and directly proportional to $\bar{t}$ and $N_p$. Notice that, $N_p$ is not shown in Figure 3.1 as its values are fixed instead of been generated through a distribution, but its effect is shown by the results. In other words, to reduce ACT, a smaller mean parking duration, a smaller demand, and a larger parking supply are preferred. For ACD, the most influential factors also include $N_p$, $\bar{t}$, and $A$, as well as the free flow speed $v$. ACD is directly proportional to $v$, i.e., the higher the free flow speed, the larger the driven distance. Recall that our analysis focuses on rush hour traffic, where parking saturation often occurs. In this case, parking occupancy can be 100%. Hence, a higher speed does not improve

Figure 3.1: Sensitivity analysis of average cruising time and average cruising distance.
the searchers’ ability to find parking, but it does increase their driven
distance. The network length, \( L \), has some small effect. Both \( ACT \)
and \( ACD \) are directly proportional to \( L \).

The other parameters are not influential to the results. This also
means that to run this model, perfect information is not needed for
those parameters. The shape parameter of the distribution of parking
durations, \( t_1 \), demand of through traffic, \( N_{tt} \) and distance thresholds
\( l_s, l_p, l_{tt} \) are not very influential. They can thus be assumed as a fixed
value. Nonetheless, the smaller these values are, the better the system
becomes (i.e., less cruising on average), thus, an early start-to-search
should be advocated for the benefit of parking users themselves.

### 3.2.2 Cruising analysis: time and distance

Define \( \mu \) as the level of parking supply (Equation 3.1).

\[
\mu = \frac{A}{N_p \cdot t} \quad (3.1)
\]

\( \mu \) is a dimensionless variable obtained as a combination of the three
most influential variables identified above. It represents the average
portion of parkers that the parking supply could accommodate. By
plotting many values of \( \mu \) (from the 108000 scenarios) and the corre-
sponding \( ACT \), Figure 3.2 is obtained.

In Figure 3.2, the scatter plot between \( ACT \) and \( \mu \) follows a clear
pattern (Equation 3.2), revealing that the results are quite general con-
sidering a small downtown area despite the specific values of the other
factors.

\[
ACT = \alpha \cdot \mu^\theta \quad (3.2)
\]

where \( \alpha > 0 \) and \( \theta < 0 \). In our study, \( \alpha = 3.13 \) and \( \theta = -3.88 \). Not
surprisingly, \( ACT \) decreases as \( \mu \) grows, but it does so at a decreasing
rate. This means that the incremental benefits of increasing the parking
supply, or decreasing the demand or parking duration are larger for a
small \( \mu \) than for a large \( \mu \).

Based on \( ACT \), one can estimate the upper bound of \( ACD \) (\( ACD =
ACT \cdot v \)). When there is no congestion (i.e., the system always free
flows at speed \( v \)), \( ACD = \overline{ACD} \). When there is congestion (i.e., the
average travel speed drops below \( v \)), \( ACD < \overline{ACD} \). More information
on congestion detection is provided in the next section.
3.2. Sensitivity Analysis

3.2.3 Congestion control

Cruising vehicles occupy road space during the additional cruising time; and hence, increase the accumulation of vehicles on the network. This may raise the probability of congestion and even gridlock (Daganzo, 2007). In this section, we look at the congested and gridlocked scenarios throughout all the simulations. The idea is to find out the scenarios which generate a high probability of congestion or gridlock. Understanding these scenarios can help us avoid or at least minimize the occurrence of congestion/gridlock.

Within the 108000 tested scenarios, 15194 cases (14%) experienced a speed drop (congestion), and within those, 10358 cases reached gridlock (68%).

Denote \( \rho \) as the flow intensity for the network, expressed as Equation 3.3. It is the ratio between the hourly parking demand and the maximum number of vehicles that can exist on the road network (not including vehicles in parking spaces).

\[
\rho = \frac{N_p}{K_j \cdot L} \tag{3.3}
\]

Recall that \( \mu \) is the level of parking supply. Based on \( \mu \) and \( \rho \), the

![Figure 3.2: Relation between ACT and \( \mu \).](image-url)
Chapter 3. Parking search model: analysis and application

Figure 3.3: Scatter plot between the flow intensity $\rho$ and level of parking supply $\mu$ for the free flow, the congested (non-gridlock), and the gridlocked scenarios.

Causes for traffic congestion can be analyzed as twofold: high arrival flow to the network, i.e., large $\rho$, and small departure flow from the network (to parking spaces), i.e., small $\mu$. Therefore, intuitively, the larger $\rho - \mu$ is, the more likely the system is to experience congestion. From the simulation results, a scatter plot between $\rho$ and $\mu$ is shown in Figure 3.3.

In Figure 3.3, each simulation is shown as a single mark. The free flow scenarios are shown as the red circles, the congested scenarios (non-gridlock) are shown as black triangles, and the gridlock scenarios are show as the blue crosses. The graph shows 3 clear zones. The boundaries between them are written as Equation 3.4 and Equation 3.5.

\[ \rho - \mu = 0 \]  
\[ \rho - \mu = 0.33 \]

Equation 3.4 defines the boundary between the free flowing and the congested scenarios. Equation 3.5 defines the boundary between the congested and the gridlock scenarios. As shown, the main factor is
3.2. Sensitivity Analysis

$\rho - \mu$, a larger $\rho - \mu$ represents a high flow demand compared to both the network and the parking system. Using its range, we can make a general prediction of the traffic performance.

- $\rho - \mu \in (-\infty, 0)$, right bottom triangle in Figure 3.3. There is a high probability of a free flowing (not congested) traffic state. In our study, all of the cases contained in this zone are free flow scenarios, which covers 93% of all the free flow cases. In this case, the level of parking supply is larger than the flow intensity. In other words, the transition rate of traffic into parking is sufficient for the flow intensity arriving to the network.

- $\rho - \mu \in (0, 0.33]$, middle area in Figure 3.3. There are different possible outcomes for this set of conditions. In our study, 58% of the cases contained in this zone have free-flowing conditions, 39% have congested conditions (non-gridlock), and 4% are gridlocked. In this case, the level of parking supply is smaller than the flow intensity but not too much. In other words, the parking system is not always sufficient to meet the flow intensity thus causing some delay. Notice that the value of 0.33 is a variable that may change under different conditions, e.g., larger area, different jam density, etc.

- $\rho - \mu \in [0.33, \infty)$, left upper triangle in Figure 3.3. There is a high probability the system will be gridlocked. In our study, 99% of the cases contained in this zone are gridlocked and the rest 1% scenarios are highly congested. In this case, the level of parking supply is much smaller than the flow intensity. In other words, the parking system is not able at all to accommodate the flow intensity thus causing gridlock.

Evidently, the best scenario is $\rho - \mu \in (-\infty, 0)$. Under these conditions, the system will mostly be free flowing. If these conditions cannot be met, by controlling at least $\rho - \mu \in (0, 0.33]$ the gridlocks or largely congested states can be avoided. Notice that these conclusions are approximations. However they can be used as a rule of thumb to assess the parking provisions in a small urban area like the one assumed here.
3.3 Numerical Example

3.3.1 General context

The total travel demand contains 200 trips. Each time slice lasts for 1 minute, i.e., \( t = 1 \) min.

The entry time of the vehicles to the area obeys a gamma distribution, where the average arrival time is 20 minutes after the observation period starts (more precisely, the shape parameter is 4 and the scale parameter is 5).

The parking durations also obey a gamma distribution, where the average duration is 10 minutes (more precisely, the shape parameter is 2 and the scale parameter is 5). Notice that, the application of the analytical model is not limited to a specific distribution, other distributions besides gamma could also be assumed.

Although the model is suitable for cases with through traffic, for simplicity and due to space constraints, we assume there is no through traffic (i.e., \( \beta^i = 0, \forall i \)). Other inputs include: \( L = 1 \) km; \( A = 21 \) spaces; \( v = 30 \) km/h; \( k_c = 60 \) veh/km/lane; \( k_j = 150 \) veh/km/lane; \( Q_{max} = 1800 \) veh/h/lane; \( l_{ns/s} = 0.5 \) km and \( l_{p/s} = 0.5 \) km. Note that for this specific case, \( A = 21 \) is the smallest integer where gridlock conditions are avoided, larger values are tested in section 3.2.2.

Section 3.2.1.1 provides some important outputs of the model/matrix. Section 3.2.1.2 explains these outputs and how to use them to better understand and evaluate the interactions between the urban parking and traffic systems.

3.3.1.1 The matrix and resulting queuing diagram

A queuing diagram is shown in Figure 3.4 with five curves, indicating the cumulative number of vehicles going through each parking-related transition event.

As mentioned before, the vertical distance between each pair of consecutive curves is the number of vehicles in a state (notice that the non-searching state consists of two parts). Also, the area between two consecutive curves is the total time vehicles spend within that state.

The area between the curves “enter the area” and “start to search” is the total time vehicles spend within the network before they start to search for parking; the area between the curves “depart parking” and “leave the area” is the total time vehicles spend within the network after they depart their parking spots. The sum of these two areas constitutes
the total time vehicles spend in the “non-searching” state. If there is no congestion and the average travel speed remains $v$, this area would equal to 400 vehicle-minutes (as each vehicle would drive for a period of $\frac{l_{ns\rightarrow s} + l_{p_1}}{v} = 2$ min). If there is congestion, then this area could be larger due to a lower travel speed. The obtained values are listed in Table 3.2.

Table 3.2: Average time vehicles spend in non-searching and searching states (numerical example).

<table>
<thead>
<tr>
<th>State</th>
<th>Total time (vehicle-minutes)</th>
<th>Average time per vehicle (minutes)</th>
<th>Average delay per vehicle (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-searching state</td>
<td>2100</td>
<td>10.5</td>
<td>8.5</td>
</tr>
<tr>
<td>Searching state</td>
<td>6180</td>
<td>30.9</td>
<td>30.9</td>
</tr>
<tr>
<td>Total</td>
<td>8280</td>
<td>41.4</td>
<td>39.4</td>
</tr>
</tbody>
</table>

As shown in Table 3.2, the average time spent by a vehicle in the non-searching state is 10.5 minutes, i.e., a total time of 2100 vehicle-minutes. This contains an average delay (during non-searching state) of 8.5 minutes, i.e., a total delay of 1700 minutes (i.e., 28.3 hours).

Moreover, while searching, each vehicle spends on average 30.9 minutes. This is, evidently, an extreme case and not very realistic, as drivers spend three times longer searching for parking than actually...
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Figure 3.5: Proportion of traffic “searching for parking” and parking occupancy over time (numerical example).

parked. However, it illustrates the potential negative effects that a limited parking supply can have on the traffic system if the demand is not altered. In total, each vehicle is delayed for 39.4 minutes during driving (within the non-searching and the searching states).

Figure 3.5 shows both the proportion of traffic searching for parking, and the parking occupancy over time. It can be seen that the values of both of them are high for a large portion of the observation period. The peak of the parking occupancy starts earlier than that of the share of traffic searching for parking, indicating the causal relationship. Notice that once the parking occupancy reaches 100%, it stays there for most of the observation period, indicating that vacated parking spots get filled with new searchers right away within the same time slice. This is not surprising, as in this example the demand for parking is much larger than the supply.

3.3.1.2 Interactions between parking and traffic systems

In this section we illustrate these interactions with the results of the numerical example, e.g., the correlation between traffic speed, total
3.3. Numerical Example

Figure 3.6: Model output (numerical example): traffic density, average travel speed, total distance driven.

As seen in Figures 3.6 (a) and (b), traffic congestion occurs between minutes 18 and 80 (i.e., the traffic density is higher than $k_c$ and the average speed is lower than $v$). Additionally, near-gridlock conditions are reached between minutes 37 and 55 as the speed drops below 2
km/h.

Figure 3.6 (a) shows a continuous growth of traffic density before minute 40, there are three reasons contributing to this. First, the parking system starts to saturate, thus vehicles take longer to find a parking space (i.e., they spend more time in the “searching” state). Second, as the traffic becomes more congested, the vehicles can drive a smaller distance within a time slice, and this also influences their ability to find parking. Third, the distance vehicles can drive within a time slice becomes smaller, vehicles in the “non-searching” state (after parking) need a longer time to leave the area. Notice that the congestion also reduces the number of vehicles transitioning between the states of “non-searching” and “searching”; this influences the traffic composition, but it does not affect the overall traffic density.

Figure 3.6 (c) shows the total distance driven within each time slice. There are two peaks on this curve, the first one occurs approximately at minute 18 and the second one occurs approximately at minute 80. Notice that these two times correspond to the moments when the average travel speed starts to drop from 30 km/h, and when it reaches back 30 km/h, respectively. Before the first peak and after the second peak, the average travel speed remains at the maximum level, and the curve of the total driven distance follows the same pattern as that of the traffic density. Between the two peaks, the traffic density reaches the critical density, leading to a rather low speed across the system. Then the speed changes become more significant than the density changes, and the curve of the total driven distance follows the same pattern as that of the travel speed (Figure 3.6(b)).

Table 3.3 shows the average and total driven distance within the non-searching and the searching states.

<table>
<thead>
<tr>
<th>State</th>
<th>Total driven distance</th>
<th>Average driven distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-searching</td>
<td>219 km</td>
<td>1.1 km/veh</td>
</tr>
<tr>
<td>Searching</td>
<td>1175 km</td>
<td>5.9 km/veh</td>
</tr>
<tr>
<td>Total</td>
<td>1394 km</td>
<td>7.0 km/veh</td>
</tr>
</tbody>
</table>

Recall that vehicles can only transition into the next state at the end of each time slice (not immediately); therefore, the average driven distance for non-searching vehicles is 1.1 km/veh, slightly higher than
that assumed (i.e., $l_{ns/s} + l_{p/}=1$ km). Nevertheless, the distance driven by non-searching vehicles constitutes a small portion of the total driven distance. The average driven distance of searching vehicles is 5.9 km/veh (over four times more than that of non-searching vehicles).

Overall, vehicles drive 1394 km (recall that the size of the network is 1km and there are only 200 trips). This distance can be used to measure energy consumption, air pollution, and other externalities caused by parking issues.

### 3.3.2 Assessment of alternative parking policies

In the numerical example, the traffic problems observed are highly related to the parking supply. Hence, in this section, we test two sets of parking policies and compare them by quantifying their effects on the total system delay and driven distance. The two sets of policies are (1) increasing the parking supply, and (2) limiting the maximum parking duration. They are independent of each other. Notice that the specifics of these policies as tested here are very simplistic (e.g., a 10 minute maximum parking duration is not realistic for most networks). However, they are only used to illustrate the effects that such policy types may have on traffic, and not to draw specific conclusions about their optimal values.

The results indicate that a proper time control scheme can highly improve the system without enlarging parking supply, which is typically harder to implement as it is a more expensive and controversial policy.

**Increasing the parking supply**


A2: provide 23 parking spaces instead of 21.

**Limiting the maximum parking duration**

B1: the longest parking duration is 20 minutes. Vehicles who wish to park shorter than 20 minutes are not affected. Vehicles who wish to park longer than 20 minutes have to leave at the end of the 20 minutes maximum parking duration.

B2: the longest parking duration is 10 minutes. Vehicles who wish to park shorter than 10 minutes are not affected. Vehicles who wish to park longer than 10 minutes have to leave at the end of the 10 minutes maximum parking duration.

Table 3.4 shows the comparison between the original conditions and policies A1, A2, B1 and B2. Values within parenthesis indicate the
percentage change driven by the different policies with respect to the original conditions.

Table 3.4: Traffic effects of different parking policies (numerical example).

<table>
<thead>
<tr>
<th>Policy</th>
<th>Non-searching time (min/veh)</th>
<th>Searching time (min/veh)</th>
<th>Delay (min/veh)</th>
<th>Total driven distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>10.5</td>
<td>30.9</td>
<td>39.4</td>
<td>1394</td>
</tr>
<tr>
<td>A1 (22 parking spaces)</td>
<td>7.9 (-24.8%)</td>
<td>29.5 (-4.5%)</td>
<td>35.4 (-10.2%)</td>
<td>1449 (+3.9%)</td>
</tr>
<tr>
<td>A2 (23 parking spaces)</td>
<td>6.7 (-36.2%)</td>
<td>27.9 (-9.7%)</td>
<td>32.6 (-17.3%)</td>
<td>1447 (+3.8%)</td>
</tr>
<tr>
<td>B1 (20 minutes maximum)</td>
<td>9.7 (-7.6%)</td>
<td>29.3 (-5.2%)</td>
<td>37.0 (-6.1%)</td>
<td>1365 (-2.1%)</td>
</tr>
<tr>
<td>B2 (10 minutes maximum)</td>
<td>5.4 (-48.6%)</td>
<td>22.2 (-28.2%)</td>
<td>25.6 (-35.0%)</td>
<td>1295 (-7.1%)</td>
</tr>
</tbody>
</table>

Not surprisingly, it can be seen that the non-searching time, searching time and delay per vehicle can be reduced both by increasing the parking supply and by limiting the maximum parking duration. Also, stronger policies are more effective at reducing delays, i.e., A2 reduces delay by an additional 7% compared to A1; B2 reduces delay by an additional 29% compared to B1.

Interestingly, not all the policies reduce the total driven distance. As a matter of fact, compared to the original conditions, policies A1 and A2 result in longer distances (3.9% and 3.8% longer respectively) despite reducing the delay. This is important to notice as it highlights the need for estimating/optimizing multiple traffic metrics (besides delay) when evaluating different parking policies from the traffic prospective.

To understand these seemingly controversial results, Figure 3.7 is provided to show the traffic density, the average travel speed and the total driven distance over time based on the tested policies. Figure 3.7 (a) compares policies A1 and A2 to the original conditions; Figure 3.7 (b) compares policies B1 and B2 to the original conditions.

As shown in Figure 3.7 (a3) and (b3), the values of total driven distance in some time slices are larger than for the original conditions whereas for other time slices they are smaller. Increases in driven dis-
3.3. Numerical Example

Figure 3.7: Traffic conditions obtained from the matrix based on different parking policies, including the traffic density, the travel speed and the total driven distance in each time slice.

tance are caused by the higher average speeds on the network (Figure 3.7 (a2) and (b2)), and the still relatively high traffic density (Figure 3.7 (a1) and (b1)). Reductions in driven distance happen when congestion disappears earlier.

For policies A1 and A2, the increased driven distance during the congested period is larger than the saved driven distance after congestion (notice that the congested period finishes only a couple of minutes earlier compared to the original conditions). Even though the speed is higher than with the original conditions, the ability of vehicles to find parking spots (during congestion) is kept low as the parking occupancy is high. In other words, these drivers take the same time to find parking, but since the speed is faster than with the original conditions, they
drive longer distances.

On the contrary, for policies B1 and B2, the increased driven distance during the congested period is smaller than the saved driven distance after congestion. Therefore, these two policies are more effective in reducing both the delay and the total driven distance in this case.

These findings are relevant, as they highlight the importance of estimating multiple metrics when designing or evaluating new parking policies. A good parking policy should be aiming to (at least) not only enhance the traffic performance but also the total driven distance, and this is not automatically achieved, as these two metrics might react to the policy in very opposite directions.

3.4 Case Study: City of Zurich, Switzerland

3.4.1 The parking supply and the road network

In this section, a case study is given based on the central shopping area Jelmoli, within the city of Zurich, Switzerland. This area is located in the old town with many shopping centers but also a lot of offices from the financial sector. The radius of the area is 300 meters. The area contains a total of 539 parking spaces for public usage, including 207 on-street parking spaces and 332 off-street parking spaces. As the area is rather small, we consider the sum of 539 parking spaces as the capacity of the parking supply. Figure 3.8 shows the parking conditions and network in the area.

Figure 3.8: layout of the area (case study).
3.4. Case Study: City of Zurich, Switzerland

- Off-street parking: 332 spaces inside the area.
  Parkhaus Jelmoli: Steinmühlplein 1, 8001 Zürich. 222 Plätze.
  Parkhaus Talgarten: Nüsslerstrasse 31, 8001 Zürich. 110 Plätze.

- On-street parking: 207 public parking spaces, and another 78 parking spaces only for delivery and private usage. The maximum parking duration allowed is 120 minutes for most of them.

- Streets/links: there are in total 106 streets/links in this area, with a total length of 7.7 link kilometers. Assuming the streets have two directions, and one lane per direction on average, then the network is $2 \times 7.7 = 15.4$ lane kilometers. There are two streets with only a small portion inside the area, their lengths are 263 meters, and 175 meters. Since their lengths are quite small compared to the network length, they are still included.

Additionally, we assume that the average free flow travel time is $v=15$ km/hr (this includes time spent at intersections), the critical traffic density is $k_c=25$ veh/km, and the jam density is $k_j=55$ veh/km (Ortigosa et al., 2014; Ge and Menendez, 2012).

3.4.2 Traffic and parking demand

The daily traffic data arriving to this network has been simulated based on previous measurements in an agent-based model Matsim (Waraich and Axhausen, 2012). There are 2534 agents entering the area during a typical working day. Figure 3.9(a) shows the cumulative number of vehicles that enter the area and leave the area. Figure 3.9(b) shows the histogram of the durations of the activities of the travellers. The average parking duration is 227 minutes. The shape of the histogram is similar to a gamma distribution with the shape parameter being 1.6 and the scale parameter being 142.

After the calibration of the parking demand with the real parking occupancy in public parking spaces in the area, the results show that approximately 77% of the daily traffic uses public parking space, the other 23% uses private parking. If we assume vehicles start to search since they enter this small area, the searching demand can be seen as 77% (for each time slice) of the trips. 23% of the trips are assigned to dedicated parking spaces in the center (e.g., private parking houses, parking provided by their company or employer) and thus do not search.
for parking. Figure 3.9(a) shows the total demand (including both, the one for public and the one for private parking spaces).

Also, at the beginning of the day, i.e., at 00:00, there is a total of 183 vehicles inside the area, they mostly entered the area during the previous day. This pattern repeats itself, as at the very end of the day, i.e., 24:00, there is also some vehicles in the area.

![Figure 3.9: Model input (case study).](image)

### 3.4.3 Parking usage estimation and cruising analysis

We assume all parking spaces (whether they are on-street or off-street) are uniformly distributed in the area, travellers take the first they see (i.e., users are indifferent to different parking spaces). In this case, the trips heading to garage and on-street parking are not distinguished. The activity start times are not used, instead, the macroscopic model is used to find the time they start-to-search for parking and access the parking space.

As we discussed above, assuming that 77% of all travellers need to park in public parking spaces (for each time slice), and using the model we proposed in chapter 2, we find that the total searching time is 6310 minutes, i.e., 105 hours in one day. In other words, each traveler spends on average 3.0 minutes searching for parking. However, the main searching delay occurs between 11:00 and 15:00, because that is the period when the parking system approaches saturation (i.e, 100% occupancy). Figure 3.11(a) shows the parking occupancy inside the
area. Figure 3.11(b) shows the model results with the cumulative number of vehicles that enter the area, access parking, depart parking, and leave the area. Figure 3.12 shows the number of searchers along the number of available parking spaces over time.

Figure 3.10: Average search time before parking over a typical working day.

Figure 3.11: Model output (case study): parking occupancy, cumulative number of vehicles that enter the area, find parking, depart parking, and leave the area.
Figure 3.12: Model output (case study): number of searchers and number of available parking spaces.

In Figure 3.12 it is shown that:

- between 06:00 and approximately 11:00:
  - The number of searching vehicles increases at a very low rate.
  - The number of available parking spaces drops continuously until all parking spaces are occupied.

- between 11:00 and 16:00:
  - The number of searching vehicles increases significantly and reaches a peak of approximately 30 vehicles at around 12:00, then it starts to fluctuates, at around 14:00, it reaches another peak of 27 searching vehicles, then it starts to drop and reaches nearly zero at the end of this period.
  - All parking spaces are occupied during this period, i.e., when a parking space becomes available, it is immediately taken.

- between 16:00 and 22:00:
  - The number of searching vehicles stays at a very low level, i.e., nearly zero.
  - The parking system starts to become undersaturated, more and more parking spaces become available.
3.4.4 Validation

Figure 3.13 shows a comparison between the estimated parking occupancy and the real data. Parking occupancy were collected between 1st and 22nd of April, 2016. Data from a total of 12 working days are included and averaged to obtain the dotted line (real data) shown in the graph. To represent a working day demand, only Tuesdays-Fridays are included.

As shown in Figure 3.13, the two curves show rather similar patterns, although there is a small time shift between the real data and the estimated one. The real data shows that the traffic arrive at the area about an hour earlier. This can be explained by the dates of the data collection (i.e., 1st -22nd April). Zurich is a city with clear seasonal change on light duration. For example, on January 1st 2016, sun rises at 08:13; whereas on April 1st 2016 the sun rises already at 07:04 (source: http://www.timeanddate.com/). Thus, the travel demand shifts seasonally, earlier in summer and later in winter. However, besides this shift in time, the curves in Figure 3.13 are very similar; for example, both curves show parking saturation for a similar period of time, the recovering of the parking system (the pattern when it gets more and more vacancy) is also very alike.

Besides that, a survey was carried out during May 2016, and parking users in this area were randomly chosen and asked to fill a questionnaire. In total, there were 88 parking users surveyed. According to their
statements, 27% of the users had spent 0-5 minutes looking for parking and 13% users had spent 5-10 minutes looking of parking. The rest 0 minute. Figure 3.14 shows the distribution of these users based on the hour of the day.

![Histogram of the cruising time in relation to the time of the day](image)

Figure 3.14: Histogram of the cruising time in relation to the time of the day

In Figure 3.14, two graphs are shown representing two areas where the parking users are surveyed. Both of these areas are either inside or by the border of the Jelmoli area for which we did the analysis in the previous sections. In each graph, there are two kinds of columns (noted by different colour), representing the cruising of 0-5 minutes and 5-10 minutes. In area 1, cruising for parking mostly happens between 12 PM and 4 PM, but there is no clear pattern for the time they spent on searching. In area 2, cruising for parking mostly happens between 11 PM and 2 PM, with the vehicles that arrive between 12 and 2 PM spending more time searching than the vehicles that arrive at other hours. Both the hour of the day where searching occurs and the time spent on searching show similar pattern to our estimations as shown in Figure 3.10. The longer cruising time between 12 PM and 2 PM (in area 2) matches also our results.

The model have produced a large number of useful indicators, most of them can even represent time-varying conditions. These results can be used to understand the urban parking dynamics, better plan the urban parking system, design better parking policies, and develop real time control schemes. The comparison between the estimated parking occupancy and real data shows that the model results are reasonable
and can quite well represent the realistic conditions. The validity of the model is further reinforced with the statistics from anonymous user questionnaires. The time distribution (over the day) of cruising obtained by our model and the questionnaires show similarities to each other, the values of the average duration spent on cruising are also close. Besides the rigorous model process and the trustworthy model results, in the next section, we will show some interesting and important applications of the model in testing and proposing parking policies.

3.5 Conclusions

This chapter is a continuation of the previous one where a macroscopic parking model was introduced.

In this chapter, we carried out an in-depth sensitivity analysis which shed some light on the importance of the parameters used in the model. The results showed that the average searching time is inversely proportional to the total parking supply $A$, and directly proportional to average parking duration $\bar{t}$ and the parking demand $N_p$. For the average cruising distance, the most influential factors also include $N_p$, $\bar{t}$, and $A$, as well as the free flow speed $v$ (directly related). This is quite meaningful as it shows that actually only three input variables can significantly affect the model results. Based on this finding, the model is more applicable and easier to use in more practical and realistic cases. Based on that, we also proposed several mathematical methods to directly estimate the average searching time and detect traffic performance in the system.

Some of these methods are illustrated with a numerical example and a case study.

Concerning the numerical example, in spite of being rather simple, it shows very optimistic results for the use of the proposed model. It provides an idea on how different parking policies can affect traffic in the short-term. It is also evident from the presented results that multiple traffic metrics should be considered when studying the potential impacts of parking policies, e.g., a given policy can reduce traffic delay (i.e., by increasing the travel speed), but simultaneously increase total distance travelled. This is a very interesting fact, and relevant as well, as it shows the importance of considering the total driven distance as an indicator of the policy/system, in addition to the direct traffic indicators such as speed. This could guide policy makers into a more
sustainable direction, rather than short-sighted decisions which could be detrimental to the environment.

Concerning the case study of the Jelmoli area in the city of Zurich, the model is ran using only four inputs: the parking supply, the daily traffic, the parking durations and additional information of the road network. Based on that, the model predicts the parking occupancy and parking search state inside the area. Part of the results (i.e., parking occupancy) are compared to the real data for validation. The case study shows the applicability of the model, and its easiness to use due to the limited data requirements.
Part II

Effects of on-street parking maneuvers
Abstract

An on-street parking maneuver can often start a temporary bottleneck, leading to additional delay endured by the following vehicles. If the maneuver occurs near a signalized intersection, the service rate of the intersection might be reduced. When this happens, the corresponding traffic delay can linger over several signal cycles afterwards, and the delays may spread over to the whole network. It is therefore important to avoid such situations. In this part, models are built to analyze the effects of parking maneuvers on the intersection service rate. Based on the hydrodynamic theory of traffic flow, the perturbation caused by the parking maneuver is analyzed. Using dimensional analysis, we illustrate the relation between the traffic conditions, the distance from the parking area to the intersection, and the intersection service rate. With this relation, one can then compute the service rate reduction caused by existing on-street parking areas. A minimum distance between the parking area and the intersection to avoid such reduction can accordingly be found. Numerical examples based on empirical data from the city of Zurich, Switzerland, are provided to illustrate the practical applications. Although the analysis is based on streets with a single lane per direction, the findings can provide some insights regarding more general situations.

In chapter 4, downstream parking maneuvers (with respect to the intersections) are analyzed. In chapter 5, upstream parking maneuvers are analyzed. In chapter 6, the parking locations to avoid a service rate drop are suggested, based on the minimum distances from the potential parking maneuver to the intersections. The results show that, generally, for an undersaturated intersection, parking downstream of the intersection causes less negative effects on the service rate than parking upstream. However, for an oversaturated intersection, parking at a close distance downstream of the intersection might reduce the service rate significantly. We hope the conclusions can be used to guide parking planning/management under different sets of conditions, and to assist the development of parking regulations (such as dynamic parking supply according to real-time traffic conditions).
This part is based on the following research papers:


Chapter 4

Effects of downstream on-street parking maneuvers on the intersection

4.1 Introduction and Background

On-street parking maneuvers can often start temporary bottlenecks, potentially affecting some following vehicles which might have to endure an extra delay. When close to signalized intersections, such delay can sometimes linger over multiple cycles, affecting vehicles which arrive much later. In this paper, we provide a generalized methodology to identify and further prevent such problematic cases of traffic delay caused by parking maneuvers.

Several studies have already been carried out analyzing the influence of on-street parking on traffic performance. In the early stages, scholars focused on the road space required by the parking lane, and the ensuing road capacity reduction (Chick, 1996; Valleley, 1997). With time, scholars changed the focus to better understand how on-street parking maneuvers disrupt traffic flow. For example, Yousif and Purnawan (1999, 2004) analyzed the differences in delay caused by parallel and angle parking maneuvers. Portilla et al. (2009) analyzed the average
link journey times under the influence of on-street parking maneuvers and badly parked vehicles using queuing theory. Ye and Chen (2011) analyzed traffic delays caused by parking maneuvers depending on the cycle length of the nearby intersection; the results showed that the delay increases with a longer cycle. In that study, individual vehicle delay was found based on the physical process of a parking maneuver (e.g., friction delay, blockage delay), and later aggregated to obtain the total delay. Our study will analyze the delay using a different methodology (based on traditional traffic flow theory tools), to provide more general conclusions with the aid of dimensional analysis. More recently, using variables such as effective lane width and number of parking maneuvers, Guo and Gao (2012) proposed a model to estimate travel time, taking into account the effects of on-street parking. Additionally, some studies have also focused on other externalities of on-street parking such as the impacts of cruising-for-parking (Shoup, 2005). Similar studies on bus stop locations can be found in Gu et al. (2013, 2014); Gayah et al. (2015); Luthy et al. (2016); Nikias et al. (2016) or on space removal in Ortigosa and Menendez, 2014, 2016.

Overall, it is generally accepted that parking maneuvers can cause traffic delay. However, not much attention has been paid to it in the literature, since in most cases, the effects (e.g., delay) of on-street parking maneuvers are not problematic, as the scope of the influence (i.e., number of cars affected) is limited. That being said, if the parking area is near a signalized intersection, the delay caused by the parking maneuver can either affect vehicles during a single cycle, or linger over multiple cycles. Evidently, the second case (which happens when the parking maneuver reduces the service rate at the intersection) is the most unfavorable, as the perturbation might spread across the network. Hence, instead of looking at random on-street parking locations within an urban network, in this paper we focus on the parking locations near the network nodes (i.e., intersections) and their impact on the intersection service rate.

When a queue from a downstream parking location spills back into an intersection, the discharging rate of the intersection could be easily reduced. Therefore, the parking location can be considered as an essential parameter for calculating the negative effects of parking maneuvers on traffic.

In this study, we analyze the effects of the on-street parking maneuvers on traffic performance and its relation to, among others, the distance between the parking area and the intersection. Notice that
the analysis is based on streets with a single lane per direction, where the parking areas do not occupy any driving lane. As no conclusion on this have been drawn yet, our study will fill the gap in the literature by providing general guidelines. Since the resulting delay is highly related to the service rate at the intersection, we use the latter as the indicator to measure the effects of parking on traffic performance. Through dimensional analysis, the relation between the reduction on intersection service rate and the distance from the parking area to the intersection is illustrated. Also, a generalized function to compute the minimum distance to avoid severe delays is provided.

Notice that the relation mentioned above also takes into account the duration of the parking maneuvers (i.e., the time they actually block traffic), the signal control settings, and the demand volume of arriving traffic. That means, our model can also guide the development of other traffic management schemes (e.g., signal control) to avoid lingering delays caused by parking maneuvers.

The remaining sections of this chapter are organized as follows. Section 4.2 shows the analytical model illustrating the relations described above. Section 4.3 shows two validation experiments with real data, and demonstrates the applications of the proposed methodology with detailed examples. Section 4.4 summarizes the findings of this study.

4.2 Analytical Model

This section is divided into two parts. First, all the assumptions and variables used in the analysis are defined and/or derived. Second, the two output variables from our model (service rate reduction at the intersection, and minimum distance required to avoid that reduction) are introduced.

4.2.1 Definitions and formulations

4.2.1.1 Basic definitions

Assume the parking maneuver is conducted by the car passing the intersection $\Delta t$ time after the start of the green light. The maneuver happens at a distance $l_d$ downstream of the intersection, and blocks traffic (i.e., vehicles must stop) for a period of time $p$ ($p \leq g$). Notice that, in this study, since we are looking at parking maneuver that normally block traffic for a short time, the delay $p$ is assumed to be smaller
than the green time $g$. If there are two parking maneuvers happening at approximately the same time, or a single parking maneuver which lasts longer than a green light; the methodology might yield some errors.

Figure 4.1 depicts the situation. Note that, we are not considering any other interruptions from downstream except the parking maneuvers. The parking areas do not occupy any driving lane, thus parked vehicles do not affect the road capacity nor the intersection service rate. However, the parking or unparking maneuvers, while taking place, do interrupt the proper usage of the driving lane and generate delay.

Figure 4.1 depicts a one lane street (for the travel direction of interest) where there is no lane change or other influence from the opposite travel direction. To generalize the situation where multiple lanes exist, a lane change rate could be assumed, where vehicles following the parking vehicle on the blocked lane can divert to other lanes. Such lane change rate can be estimated or assumed based on local driving behavior. For example, in the city of Zurich, based on our survey results, lane change maneuvers seldomly happen under this condition. More details are provided in the model validation part.

The length of the signal cycle is $c$ and the green time is $g$. Denote $\alpha$ as the ratio between $p$ and $g$, $\alpha \in [0,1]$.

$$\alpha = \frac{p}{g} \quad (4.1)$$

To model traffic conditions, the hydrodynamic theory of traffic flow is used (Lighthill and Whitham, 1955; Richards, 1956), with a triangular relationship between flow and density - triangular fundamental diagram (FD). Empirical evidence suggests that this is reasonable (Banks, 1989; Cassidy, 1998; Hall et al., 1986; Windover and Cassidy, 2001). Figure 4.2(a) depicts an illustrative FD with the different traffic states observed in the scenarios described above. State 1 represents
4.2. Analytical Model

Figure 4.2: Illustrative triangular fundamental diagram for the link and illustrative time space diagram.

the traffic flow arriving from upstream; state 2 represents the stopping traffic; and state 3 represents the traffic discharging from the queue. The shockwaves (i.e., interfaces) between those traffic states are indicated by the lines connecting each pair of points. Figure 4.2(b) shows an example of a possible time-space diagram; the numbers indicate the traffic states shown in the FD; the dark lines correspond to the shockwaves, and the dashed line shows a possible vehicle trajectory. The two diagrams assume that the geometry of the road is the same both upstream and downstream of the intersection, so the parking area does not occupy any space from the driving lanes. They also assume that the portion of right turning vehicles is small, so their effect is not considered.

As indicated in Figure 4.2(a), $S$ is the saturation flow rate (i.e., maximum flow) during the effective green signal, $q$ is the arrival flow rate (i.e., demand volume), and $K_J$ is the jam density. The maximum number of vehicles that can pass within the green signal is $S \cdot g$. Denote $\beta$ as the minimum space required to store those vehicles.

$$\beta = \frac{S \cdot g}{K_J} \quad (4.2)$$

The capacity of the intersection is $S \cdot g$ and the volume-to-capacity ratio, $x$, can be expressed as Eq. 4.3 ($x \in [0,c/g]$).

$$x = \frac{q \cdot c}{S \cdot g} \quad (4.3)$$
In the absence of parking maneuvers, \(x \in [0,1]\) and \(x \in (1,c/g]\) represent the undersaturated and oversaturated conditions of the intersection, respectively.

Note that \(x \beta = \frac{q \cdot c}{K_J}\). It represents the minimum space required to store all the vehicles that have arrived in this cycle (i.e., \(q \cdot c\)).

As indicated in Figure 4.2(b), the parking maneuver occurs at time \(c - g + \Delta t + \frac{l_d}{v}\). assume \(v\) is the free flow speed, and at the location of \(l_d\) downstream of the intersection. \(t\) is the theoretical time at which the last car affected (i.e., delayed) by this parking maneuver would pass the intersection.

\[
t = \frac{c - (1 - \alpha)g}{c - xg} \cdot c \quad (4.4)
\]

However, if the red light starts before this theoretical moment (i.e., \(c < t\)), then due to the blockage caused by the parking maneuver, some vehicles would have to wait until the next green signal to cross the intersection. In other words, when \(c < t\), the intersection becomes oversaturated. This condition is equivalent to \(x \in (1 - \alpha, c/g]\).

Hence, when a parking maneuver occurs, the intersection is oversaturated not only when \(x \in (1,c/g]\), but also when \(x \in (1-\alpha,1]\). Consider both situations with and without the parking maneuver, three possible scenarios arise depending on \(x\):

**Scenario 1**: \(x \in [0,1-\alpha]\): The intersection is undersaturated with or without a parking maneuver.

**Scenario 2**: \(x \in (1-\alpha,1]\): The intersection becomes oversaturated with the parking maneuver (it is undersaturated in the absence of a parking maneuver).

**Scenario 3**: \(x \in (1,c/g]\): The intersection is oversaturated with or without the parking maneuver.

The throughput of the intersection is the number of vehicles that are discharged in a cycle. Use \(\mu\) to define the average throughput over the cycle, i.e., the intersection service rate in the presence of a parking maneuver. The upper bound of it, \(\bar{\mu}\), corresponds to the condition where no delay from downstream occurs, e.g. no parking maneuver takes place. It is equal to the demand volume when the intersection is undersaturated, and the capacity otherwise.
4.2. Analytical Model

\[
\mu = \begin{cases} 
q & \text{if } x \in [0,1] \\
S \cdot \frac{g}{c} & \text{if } x \in (1,\frac{c}{g}] 
\end{cases} \tag{4.5a}
\]

\[
\mu = \begin{cases} 
q & \text{if } x \in [0,1] \\
S \cdot \frac{g}{c} & \text{if } x \in (1,\frac{c}{g}] 
\end{cases} \tag{4.5b}
\]

Thus, \(\mu - \mu\) represents the reduction of the intersection service rate caused by the parking maneuvers, it further indicates the impact on the traffic performance. Hence, we can conclude that:

- when \(\mu - \mu = 0\), the performance of the intersection is not affected by the parking maneuver at all, and the delay (if it exists) is concentrated into a single signal cycle; we define it as local delay; and

- when \(\mu - \mu > 0\), the parking maneuver does affect the intersection, and the delay persists over multiple signal cycles; we define it as lingering delay. In the special case when \(\mu = 0\), the parking maneuver affects every vehicle going through the intersection in that cycle, making them wait at least until the next green signal.

4.2.1.2 Time variables

The intersection service rate is only affected (i.e., \(\mu - \mu > 0\)) if the queue created by a parking maneuver spills back into the intersection and causes a blockage. In this part, we introduce two time variables demonstrating how the duration of the intersection blockage decreases with \(l_d\).

- \(t_b\) is the time when the back of the queue reaches the intersection.

- \(t_f\) is the time when the front of the queue reaches the intersection.

Thus, \(t_f - t_b\) is the duration of the intersection blockage caused by the parking maneuver. The equations for \(t_b\), and \(t_f\) are given below. They are obtained through the time-space diagrams. The slopes of the shockwaves can be found based on an assumed FD diagram (see Figure 4.2).

\[
t_b = \begin{cases} 
c - g + \Delta t + g \cdot \frac{l_d}{\beta} & \text{if } l_d \leq \frac{c - g}{c - xg} \cdot x\beta - \frac{\beta}{g} \cdot \Delta t \\
\frac{c}{x} \cdot \left( \frac{l_d}{\beta} + \frac{\Delta t}{g} \right) & \text{if } l_d > \frac{c - g}{c - xg} \cdot x\beta - \frac{\beta}{g} \cdot \Delta t 
\end{cases} \tag{4.6a}
\]

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Figure 4.3: Time-space diagrams depicting time variables for different values of $l_d$.

$$t_f = c - g + \frac{\Delta t}{\beta} + p$$  \hspace{1cm} (4.7)

In Figure 4.3, three time-space diagrams are given to illustrate $t_b$ and $t_f$. Time zero corresponds to the start of the red signal for the current cycle, and time $c-g$ corresponds to the start of the green signal. $l_d$ indicates the location of the parking maneuver. The solid lines represent the shockwaves between different traffic states, the dotted lines are auxiliary lines to indicate the values.

In Figure 4.3, the following variables are illustrated: $l_d$ represents the distance between the location of the parking maneuver and the stopping line; $p$ represents the time the parking vehicle blocks traffic; $x$ represents the volume-to-capacity ratio; $c$ and $g$ represent the cycle and green light time; the maximum number of vehicles that can pass within the green signal is $S \cdot g$; $\beta$ represents the minimum space required to store those vehicles; $\Delta t$ represents the virtual time at which the parking vehicle would pass the intersection (after the start of the green light).

As seen in Figure 4.3 and Eq. 4.6, a distance of $l_d = \frac{c - g}{c - xg} \cdot x\beta - \frac{\beta \cdot \Delta t}{g}$ is critical, because it is the furthest distance that causes the intersection to be blocked for time $p$.

- When $l_d$ is smaller than this distance (Figure 4.3(a)), the back of the queue reaches the intersection before the upstream queue
(due to the red signal) has fully dissipated, so the intersection is blocked for time $p$.

- When $l_d$ is larger than this distance (Figure 4.3(c)), the back of the queue reaches the intersection after the upstream queue has dissipated, so the intersection is blocked for a period shorter than $p$.

### 4.2.1.3 Traffic throughput

Two new distance variables, $l_1$ and $l_2$, are defined based on $t_f$ and $t_b$. The two variables are important in finding the throughput of the intersection (which leads to the service rate) when there is a parking maneuver. $l_1$ is the distance for which $t_f = c$. In other words, when $l_d = l_1$, the front of the queue reaches the intersection exactly at time $c$. Similarly, $l_2$ is the distance for which $t_b = c$. The formulas for $l_1$ and $l_2$ are given in Eq. 4.8 and 4.9 respectively.

$$l_1 = \left(1 - \alpha - \frac{\Delta t}{g}\right) \cdot \beta \quad (4.8)$$

$$l_2 = \begin{cases} 
(x - \frac{\Delta t}{g}) \cdot \beta & \text{if } x \in (1 - \alpha, 1] \\
(1 - \frac{\Delta t}{g}) \cdot \beta & \text{if } x \in (1, \frac{c}{g}] 
\end{cases} \quad (4.9a)$$

Recall that $\beta$ is the minimum space required to store the maximum number of vehicles that can be discharged in a cycle; and $x\beta$ is the minimum space required to store all the vehicles arriving in the cycle.

Figure 4.4 shows three $t$-$x$ diagrams for different ranges of $l_d$; all correspond to situations where $x \in (1-\alpha, c/g]$.

In the figure above, the following variables are illustrated: $l_d$ represents the distance between the location of the parking maneuver and the stopping line; $p$ represents the time the parking vehicle blocks traffic; $c$ and $g$ represent the cycle and green light time; $t_f$ represents the time when the front of the queue reaches the intersection; $l_1$ represents the distance between the parking maneuver and the intersection for which the front of the queue reaches the intersection at time $c$; $l_2$ represents the distance between the parking maneuver and the intersection for which the back of the queue reaches the intersection at time $c$.

Three ranges of $l_d$ are categorized and briefly explained below.

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Figure 4.4: Intersection throughput based on different values of \( l_d \) when \( x \in (1-\alpha, c/g] \).

- \( l_d \in [0, l_1] \) and \( x \in (1-\alpha, c/g] \), Figure 4.4(a): The front of the queue caused by the parking maneuver reaches the intersection before the end of the current cycle \( (t_f \leq c) \). The intersection is blocked for time \( p \), throughput equals \( S \cdot (g-p) \).

- \( l_d \in [l_1, l_2] \) and \( x \in (1-\alpha, c/g] \), Figure 4.4(b): The front of the queue caused by the parking maneuver reaches the intersection after the end of the current cycle \( (t_f > c) \). The intersection is blocked for a period shorter than \( p \), and does not always discharge at \( S \), but also at \( q \) for some time. The throughput equals \( S \cdot [t_f - p - (c-g)] \), equivalent to \( S \cdot (\Delta t + g \cdot \frac{L_d}{\beta}) \) (based on Eq. 4.7).

- \( l_d \in [l_2, \infty) \) and \( x \in (1-\alpha, c/g] \), Figure 4.4(c): The queue does not reach the intersection, therefore no throughput reduction occurs.

As defined, \( \mu \), is the average throughput over the cycle. It can be easily found based on our analysis shown until here. Results are given in the next section.

4.2.2 Results

4.2.2.1 Formulations of the service rate reduction

Based on the analysis presented in the previous part, the service rate with a parking maneuver, \( \mu \), can be written as below.
Scenario 1: if $x \in [0, 1 - \alpha]$, $\mu = q$ \hfill (4.10a)

Scenario 2: if $x \in (1 - \alpha, 1]$,

\[
\mu = \begin{cases} 
  S \cdot \frac{g}{c} \cdot (1 - \alpha) & \text{if } l_d \in [0, l_1] \\
  S \cdot \left(\frac{\Delta t}{c} + \frac{g}{c} \cdot \frac{l_d}{\beta}\right) & \text{if } l_d \in [l_1, l_2] \\
  q & \text{if } l_d \in [l_2, \infty)
\end{cases}
\] \hfill (4.10b)

Scenario 3: if $x \in (1, \frac{c}{g}]$,

\[
\mu = \begin{cases} 
  S \cdot \frac{g}{c} \cdot (1 - \alpha) & \text{if } l_d \in [0, l_1] \\
  S \cdot \left(\frac{\Delta t}{c} + \frac{g}{c} \cdot \frac{l_d}{\beta}\right) & \text{if } l_d \in [l_1, l_2] \\
  S \cdot \frac{g}{c} & \text{if } l_d \in [l_2, \infty)
\end{cases}
\] \hfill (4.10c)

Therefore, based on Eq. 4.5 and Eq. 4.10, we can obtain the formulation of $\bar{\mu} - \mu$, i.e., the reduction of the service rate.

Scenario 1: if $x \in [0, 1 - \alpha]$, $\bar{\mu} - \mu = 0$ \hfill (4.11a)

Scenario 2: if $x \in (1 - \alpha, 1]$,

\[
\bar{\mu} - \mu = \begin{cases} 
  q - S \cdot \frac{g}{c} \cdot (1 - \alpha) & \text{if } l_d \in [0, l_1] \\
  q - S \cdot \left(\frac{\Delta t}{c} + \frac{g}{c} \cdot \frac{l_d}{\beta}\right) & \text{if } l_d \in [l_1, l_2] \\
  0 & \text{if } l_d \in [l_2, \infty)
\end{cases}
\] \hfill (4.11b)

Scenario 3: if $x \in (1, \frac{c}{g}]$,

\[
\bar{\mu} - \mu = \begin{cases} 
  S \cdot \frac{g}{c} \cdot \alpha & \text{if } l_d \in [0, l_1] \\
  S \cdot \left(\frac{g - \Delta t}{c} - \frac{g}{c} \cdot \frac{l_d}{\beta}\right) & \text{if } l_d \in [l_1, l_2] \\
  0 & \text{if } l_d \in [l_2, \infty)
\end{cases}
\] \hfill (4.11c)

Notice in scenario 2, $l_2 = (x - \frac{\Delta t}{g}) \cdot \beta$, and in scenario 3, $l_2 = (1 - \frac{\Delta t}{g}) \cdot \beta$. 

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Figure 4.5 illustrates $\bar{\mu} - \mu$, based on three scenarios of $x \in [0, 1 - \alpha]$, $x \in (1 - \alpha, 1]$ and $x \in (1, \frac{c}{g}]$. In each scenario, corresponding to the value of $l_d$, the reduction of the intersection service rate caused by the parking maneuver is shown.

We can see that no reduction is caused on the intersection service rate for scenario 1. For scenarios 2 and 3 where $x \in (1 - \alpha, \frac{c}{g}]$:

- The service rate can be reduced. Such reduction is larger for scenario 3. As expected, for the same parking location, a parking maneuver is more problematic (i.e., has more negative effects) when the intersection is already oversaturated.

- The worst conditions arise when $l_d < l_1$. For any value of $l_d$ smaller than $l_1$, the front of the queue always reaches the intersection before the end of the current cycle, affecting it for the maximum possible time period, $p$. Therefore, parking supply should be minimized (or avoided) within a distance $[0, l_1]$ downstream of the intersection (i.e., stopping line).

- For $l_d \in (l_1, l_2]$, $\bar{\mu} - \mu$ decreases with the value of $l_d$ by the same rate/slope for the two scenarios. No service rate reduction is caused after $l_d$ reaches $l_2$, since $\bar{\mu} - \mu = 0$. However, $l_2$ has different values for scenarios 2 and 3. It means that, to totally avoid affecting the intersection with the parking maneuvers, a different distance limitation for parking should be applied based on the volume-to-capacity ratio (i.e., $x$). We define this distance limitation as the minimum distance. It is addressed in the next part.
4.2.2.2 Minimum distance formulations

As analyzed earlier, to completely avoid the throughput reduction, a minimum distance, $L$, can be applied.

\[ L = \begin{cases} 
0 & \text{if } x \in [0, 1-\alpha] \\
(x - \frac{\Delta t}{g}) \cdot \beta & \text{if } x \in (1-\alpha, 1] \\
(1 - \frac{\Delta t}{g}) \cdot \beta & \text{if } x \in (1, \frac{c}{g}] 
\end{cases} \quad (4.12) \]

Figure 4.6 illustrates these conditions for the worst possible situation, $\Delta t=0$ (i.e., the parking maneuver is performed by the first vehicle that passes the intersection on a given cycle).

The line in Figure 4.6 shows three distinct parts, corresponding to the three scenarios previously described:

**Scenario 1**: $x \in [0, 1-\alpha]$. No lingering delay or throughput reduction will happen no matter how near the parking location is to the intersection ($L=0$).

**Scenario 2**: $x \in (1-\alpha, 1]$. The minimum $l_d$ required to avoid a throughput reduction increases with the volume-to-capacity ratio ($L = x \beta$).

**Scenario 3**: $x \in (1, c/g]$. The minimum $l_d$ required to avoid a throughput reduction corresponds to the space needed to store all
vehicles that discharge from the intersection, and is independent of the demand \((L = \beta)\).

Note that there are two cases not considered in the explanation of Figure 4.6. 

\(\Delta t > 0\): The larger \(\Delta t\) is, the later the parking vehicle passes the intersection. Hence, less cars are affected by the parking maneuver. In such case, the minimum distance is actually smaller than that shown in Figure 4.6. The value of \(\Delta t\), although not included in Figure 4.6, is considered in the model (see Eq. 7.1) and the validation part.

Multiple maneuvers happen in the same cycle: This might lead to longer delays or multiple temporary bottlenecks on the segment of the road. Therefore, in such case, the minimum distance is actually larger than the result obtained from Eq. 7.1.

4.3 Model Validation and Application

4.3.1 Validation

In this section, we use real data to validate the model proposed above, specifically the value of \(\mu\) (i.e., service rate). Two sets of data were collected with the aid of video camera during July and October, 2013. We define the real service rate with a parking maneuver as \(\mu_{\text{real}}\). The validation is based on the comparison between \(\mu\) (from the model) and \(\mu_{\text{real}}\) (from the data collected). The survey locations are shown in Figure 4.7.

4.3.1.1 Data set 1 (Zurich, Switzerland)

The observed section of Dreikonigstrasse has 2 lanes (only one direction). Along the street, downstream of the intersection between Beethovenstrasse and Dreikonigstrasse, there are 22 parking stalls: 10 on the left side and 12 on the right. We assume both lanes have the same traffic demand, and a parking maneuver occurring on one side does not influence traffic on the other side. Hence, variables such as \(q\) and \(\mu_{\text{real}}\) are estimated as half of the observed values to be suitable for the model (one lane street). More than 70 maneuvers were recorded, but only 37 maneuvers caused delay \((p > 0 \text{ and } q > 0)\) and were used for validation: 26 of them caused a throughput reduction and hence a lingering delay \((\bar{\mu} - \mu > 0)\), and 11 only caused a local delay \((\bar{\mu} - \mu = 0)\). The
4.3. Model Validation and Application

Figure 4.7: Survey locations for the two data sets.

analysis of the other 33 maneuvers \((p=0\) or \(q=0\)) is trivial and does not require a model.

From the video, fixed-value variables were obtained: \(g=20\) sec, \(c=50\) sec, \(K_J=125\) veh/km, \(S=1620\) veh/h. \(K_J\) and \(S\) are averaged values across multiple cycles, \(K_J\) was measured with the queue at the red signal and \(S\) was the saturation flow rate discharging from the queue (both are considered for one lane). Other values such as \(\Delta t\), \(p\), \(l_d\), and \(q\) were found based on the specific parking maneuver.

Plotting the observed values into the model, \(\alpha\), \(\beta\), \(x\), \(\mu\) were found, as well as \(\mu\). The histogram of accuracy (of \(\mu\) to \(\mu_{real}\)) levels obtained is shown in Figure 4.8(a). The X axis shows different ranges of the accuracy (i.e., values below 100% indicate we underestimated the service rate of the intersection), the Y axis stands for the frequency of observations with such accuracy.

The accuracy ranged between 63% and 133%. There were 17 cases (46% of the cases) with an error smaller than 10% (accuracy between 90% and 110%). The average error was 103%. The largest error was 37%, and it was observed only once. There are two possible reasons for the inaccuracy. First: turning vehicles at the intersection (they are counted as served in \(\mu_{real}\) but not in \(\mu\)). Even though the number of turning vehicles is small, it leads to an underestimation of the service
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Figure 4.8: Histogram of accuracy levels (surveyed in Zurich and Munich).

rate, as the queue length after the parking vehicle is, in reality, shorter than what the model predicts, due to turning cars not joining the queue. Second: some disturbances from pedestrian sidewalk and bicycles, these disturbances lead to an overestimation of the throughput. Despite such inaccuracies, overall the model seems to perform quite reasonably. It predicts the throughput reduction with an error smaller than 20% in at least 73% of the cases (27 cases).

4.3.1.2 Data set 2 (Munich, Germany)

The observed section of Frauenstraße has one lane per direction. There are 25 parking stalls along the street, downstream of the intersection between Frauenstraße and Reichenbachstrasse (towards Thomas-Wimmer-Ring direction). Here we used 42 maneuvers for validation, 15 of them caused a throughput reduction and hence a lingering delay ($\mu>0$), and 27 only caused a local delay ($\mu=0$). Data was extracted and processed as in Zurich. The fixed-value variables were $g=60$ sec, $c=80$ sec, $K_J=125$ veh/km, $S=1548$ veh/h.

The histogram of accuracy levels obtained is shown in Figure 4.8(b). The accuracy ranged between 59% and 157%. There were 35 cases (80% of the cases) with an error smaller than 20% (accuracy between 80% and 120%). The average error was 109%. Overall, the results overestimated the throughput with a parking maneuver. This was mostly driven by multiple parking maneuvers happening during one cycle due to the long green light and abundant supply of parking stalls.

For both histograms shown in Figure 4.8, the patterns seem similar
4.3. Model Validation and Application

to normal distribution, and for both histograms, more than 70% of the cases are covered within the accuracy range of [80%, 120%]. However, they do show slight differences from one another. Figure 4.8(a) shows more underestimations using the data collected in Zurich while Figure 4.8(b) shows more overestimations using the data collected in Munich. This difference, as explained earlier, is due to the specific conditions of the surveyed street including the street and parking layout (e.g., number of lanes and parking spaces), the turning vehicles, and other disturbances such as pedestrian and cyclists.

Another interesting difference between the two datasets includes the values of $p$ and $l_d$. They are 7.7 seconds and 35.9 meters for the dataset collected in Zurich, and 10.6 seconds and 82.3 meters for the dataset collected in Munich. Notice that the ratio between maneuvers which generated a reduction on the service rate and the ones which did not generate reduction in this data set, cannot be generalized to other situations/locations. Recall that all maneuvers for which either $p=0$ or $q=0$ were excluded from the analysis. Hence, the data presented here cannot be used to justify a high frequency of parking maneuvers influencing traffic.

4.3.2 Application

In this section we illustrate the use of the model through three examples. Some of them employ empirical data (shown in Figure 4.9) from the city of Zurich, Switzerland. We first introduce the data.

$p$ represents only the time during which the parking maneuver blocked traffic, and not the total duration of the parking maneuver. Data for $p$ were collected at 6 intersections (Stampfenbachstrasse, Nueschelerstrasse, Staufacherquai, Talstrasse, Dreikoenigstrasse, and Museumstrasse) during April and May 2013. Data includes 66 parking maneuvers that were videotaped. $l_d$ represents the distance between the intersection and the nearest on-street parking stall downstream of the intersection. Data for $l_d$ were extracted from 161 intersections in the central area of the city (Zone 10), and only covers signalized intersections with parking downstream.

As shown in Figure 4.9, the value of $p$ ranges from 2 seconds to 19 seconds, with an average of 8.94 seconds. The value of $l_d$ ranges from 10 meters to 190 meters, with an average of 70.6 meters.

We now present three examples showing possible applications of the model proposed earlier.
4.3.2.1 Example 1

Determine $L$ to avoid the reduction on the intersection service rate (considering $\Delta t=0$) based on $q=500$ veh/h, $S=1500$ veh/h, $K_J=200$ veh/km, $c=60$s, $g=24$s, $p=8.94$s.

Step 1: Find basic values, $\alpha=0.3725$ (Eq. 4.1); $\beta=50$ meters (Eq. 4.2), and $x=0.83 \in (1-\alpha,1]$ (Eq. 4.3).

Step 2: Find $L=41.5$ meters (Eq. 7.1(b)).

4.3.2.2 Example 2

Determine the service rate reduction based on a known value of $l_d=25$ meters, the average observed value of $p$ (i.e., 8.94 seconds), and the same values of $\Delta t$, $g$, $c$, $S$, $K_J$, $q$ as above.

Step 1: As found above, $\alpha=0.3725$; $\beta=50$ meters; $x=0.83$ and $x \in (1-\alpha,1]$.

Step 2: Find $l_1=31.375$ meters (Eq. 4.8) and therefore, $l_d \in [0, l_1]$.

Step 3: Find $\bar{\mu}=500$ veh/h (Eq. 4.5(a)) and $\mu=376.5$ veh/h (Eq. 4.10(b)).

Step 4: Find $\bar{\mu}-\mu=123.5$ veh/h (Eq. 7.1(b)).

That means, the service rate of the intersection is reduced by 123.5 veh/h, or in other words, 24.7% (i.e., ($\bar{\mu}-\mu$)/$\bar{\mu}$).
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4.3.2.3 Example 3

Estimate the overall conditions considering the throughput reduction for a set of intersections. Assume all intersections are oversaturated, \( x \in (1, c/g] \). Use Figure 4.9(b) as the distribution of \( l_d \) for the intersections, the values of \( p, g \) and \( c \) remain the same as in Example 2.

Step 1: As found above, \( \alpha = 0.37 \), \( l_1 = 31.375 \) meters.

Step 2: Find \( l_2 = 50 \) meters (Eq. 4.9(b)) and \( \bar{\mu} = 600 \) veh/h (Eq. 4.5(b)). We can draw Figure 10 according to Figure 4.5(c) and Eq. 4.11(b).

Step 3: The reduction occurs when \( l_d \leq 50 \) meters. The maximum reduction, 37.25% (i.e., \((600-376.5)/600\)), occurs when \( l_d \leq 31.375 \) meters. Based on the distribution of \( l_d \) of surveyed intersections in the city of Zurich shown in Figure 4.9(b), the percentage of intersections that might be disrupted by parking maneuvers downstream, and the extent of such disruption is shown in Figure 4.11.

We can see:

- Intersections with \( l_d \leq 31.375 \) are shown on the left side, and represent 10% of the 161 intersections surveyed. For this 10% intersections, they will have a high reduction (37.25%) of the service rate.

- The middle part represents intersections with \( l_d \in (31.375, 50] \). These intersections (22%) will endure some kind of reduction of the service rate (although smaller than 37.25%).

- The rest of the intersections, 68%, should not be affected.
Figure 4.11: Overall conditions considering the throughput reduction for a set of intersections. Example 3, $x \in (1, c/g] = (1, 2.5]$.

Notice that for a more accurate macroscopic analysis of the impact of parking maneuvers at a network level, one must also consider the actual frequency of parking maneuvers (i.e., proportion of cycles in which parking maneuvers occur), the actual distribution of $l_d$ and $\Delta t$, and the temporal/spatial variations of traffic demand.

### 4.4 Conclusions

In this paper, we study the negative effects that a parking maneuver can have on traffic, especially on the discharging flow of a signalized intersection. We provide a methodology to calculate the reduction in the service rate as a function of the parking location (with respect to the intersection), and a minimum distance to avoid such reduction. Notice that the service rate reduction could also be found based on other methodologies such as queuing theory (Portilla et al., 2009).

We consider multiple factors such as signal control, arriving traffic flow rate, and the duration of the parking maneuver. As a result, the
4.4. Conclusions

The model is general enough to fit different background conditions for single lane streets.

For streets with multiple lanes, lane changes (or overtaking maneuvers) may lead to a higher traffic throughput, i.e., our model could underestimate the resulting service rate at the intersection. To account for that, one can either assume an overtaking rate, or discount the time length for which a parking vehicle blocks traffic.

The volume-to-capacity ratio indicates the saturation condition of the intersection, and it was found essential to the result. Notice, however, that exact traffic volumes might be difficult to use for policy or design purposes. Instead, we can estimate different saturation levels based on time of day. We now summarize the findings based on 3 different levels of saturation.

- The intersection is very undersaturated.

  When volume-to-capacity ratio is lower than $1 - p/g$, the intersection is undersaturated with or without a parking maneuver. In other words, the parking maneuver has no effect on the throughput as the arriving flow rate is so low that the duration of the blockage, $p$, does not reduce the service rate.

  Therefore, under this condition, no reduction can be caused regardless of the location of the parking area. Based on this, one can justify a large parking supply in certain streets (e.g., local streets) where the intersections are normally uncongested.

- The intersection is near saturation level.

  When volume-to-capacity ratio is higher than $1 - p/g$, but smaller than 1, the intersection can become oversaturated when the parking maneuver occurs (it remains undersaturated in the absence of a parking maneuver). In this case, when the traffic demand (arriving flow rate) rises, a longer distance is required to avoid the reduction on the service rate. If $l_d < L$, then the magnitude of the reduction grows with a larger traffic demand.

  Consider a fixed demand, then, the larger the distance between the parking area and the intersection is, the less the service rate is reduced. As a matter of fact, it is possible to completely avoid the reduction by ensuring that $l_d > x\beta$ (recall that $x\beta$ is the minimum space required to store all the vehicles arriving in the cycle, and $\beta$ is the minimum space required to store the maximum number
of vehicles that can pass with the green signal). To be completely safe, one could always make \( l_d > \beta \) (in this case, \( x\beta < \beta \)).

- The intersection is oversaturated.

When volume-to-capacity ratio is higher than 1, the intersection is oversaturated with or without the parking maneuver. This is the most problematic situation out of the three.

In this case the minimum distance, \( L \), stays at a constant value of \( \beta \), corresponding to the amount of space required for storing the maximum number of vehicles that can pass with the green signal. Compared to the previous situation (same background data), for the same parking location with \( l_d \leq \beta \), the service rate reduction is larger in this situation (recall Figure 4.5).

Under this condition, the parking area should not be located closer than \( \beta \). By the same token, it is not necessary to make \( l_d \gg \beta \), as any \( l_d > \beta \) should avoid the reduction.

As shown with the validation experiments, this model, despite its simplicity, can provide reasonably accurate results. Note that the model does not consider parking maneuvers that last longer than the green signal, nor the lower speeds that vehicles often adopt when approaching the parking stall. In addition, the model is, by design, rather conservative, in order to minimize the effects of all possible parking maneuvers. Nonetheless, it is still able to predict more than 73% of the cases with at least 80% accuracy.

Current efforts are underway to extend the model proposed here for finding the effects of parking maneuvers upstream of intersections. The ultimate goal is to further generalize this methodology in order to cover larger areas (i.e., networks with multiple intersections). This could allow city agencies to better plan the location of on-street parking in a city, and/or implement traffic management strategies for diminishing the negative effects in situations where they cannot be completely avoided. A similar methodology could also be used to evaluate the effects of other types of fixed disruptions on nearby signalized intersections. Additionally, there might be a chance to combine these findings with signal control algorithms in the presence of connected and autonomous vehicles \( \text{Guler et al., 2014} \) \( \text{Yang et al., 2016} \) to further improve the performance of the system.

The general guidelines proposed on this paper only consider how the location of the parking area affects the service rate of nearby intersec-
tions, but it does provide a generalized relation between the intersection service rate and signal design, or duration of parking maneuvers. Nevertheless, for the proper design of parking areas, other factors (out of the scope of this paper) must also be considered (e.g., benefits to parking drivers, pedestrians, businesses). The final decision about parking supply (including location), cannot be based only on traffic operations considerations, but also on a more holistic analysis of the transportation system and local urban planning policies.
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Chapter 5

Effects of upstream on-street parking maneuvers on the intersection

5.1 Introduction

The previous chapter looked at the reduction of intersection service rate caused by on-street parking maneuvers, but only when the parking maneuvers happen downstream of the intersection. Results showed that the distance between the parking location and the intersection is highly influential to the performance of the intersection. As a result, a minimum distance to avoid the service rate reduction based on different background conditions can be found.

In this chapter, we look at the situation where parking maneuvers occur upstream of the intersection. We use the service rate at the intersection as the indicator to measure the effects of parking on traffic performance. An analytical model is built, to find the relation between the service rate reduction and the distance of the parking maneuver to the intersection. Notice that the relation mentioned above also takes into account the duration of the parking maneuvers (i.e., the time they actually block traffic), the signal control settings, and the demand vol-
ume of arriving traffic. That means, our model can also guide the development of other traffic management schemes (e.g., signal control) to avoid significant delays caused by parking maneuvers. Based on the model, one can, not only suggest the parking arrangement with the least influence to the intersection, but also compare the impact on traffic between upstream and downstream parking maneuvers.

The remaining sections of this chapter are organized as follows. Section 5.2 shows the analytical model illustrating the relations described above. Section 5.3 shows a validation experiment with real data. Section 5.4 summarizes the findings of this study.

5.2 Analytical Model

5.2.1 Assumptions and variables

5.2.1.1 Basic assumptions

Denote the distance between the parking maneuver and the intersection as $l_u$. Notice that $l_u$ is the absolute value; the larger it is, the further upstream the parking maneuver occurs. Figure 5.1 depicts the situation where the parking vehicle blocks the following vehicles. Note that we are not considering any other traffic interruption. Figure 5.1 depicts a one lane street (for the travel direction of interest) where there is no lane change or other influence from the opposite travel direction. To generalize the situation where multiple lanes exist, a lane change rate could be assumed, where vehicles following the parking vehicle on the blocked lane can divert to other lanes. Such lane change rate can be estimated or assumed based on local driving behavior. For example, in the city of Zurich, based on our survey results, lane change maneuvers seldomly happen under this condition. More details are provided in the model validation part.

Same as the previous chapter, $S$ is the saturation flow rate (i.e., maximum flow), $q$ is the arrival flow rate (i.e., demand volume), $v$ is the free flow speed, and $K_J$ is the jam density; $c$ is the cycle length, $g$ is the green time for the direction of interest; $p$ is the time that the traffic is blocked by the parking maneuver; $\alpha$ is the ratio between $p$ and $g$. Notice that, in this study, since we are looking at parking maneuver that normally block traffic for a short time, the delay $p$ is assumed to be smaller than the green time $g$. If there are two parking maneuvers happening at approximately the same time, or a single parking maneu-
Figure 5.1: Illustration of $l_u$ and the general situation.

Figure 5.2: Illustrative time-space diagram.

ver which lasts longer than a green light; the methodology might yield some errors. Assume the parking maneuver is conducted by the car passing the intersection $\Delta t$ time after the start of the green light. The capacity of the intersection is $S \cdot g/c$ and the volume-to-capacity ratio is $x$, $x \in [0,c/g]$. Use $\mu$ to denote the intersection service rate in the presence of a parking maneuver, i.e., the average throughput over the cycle. The upper bound of it, $\overline{\mu}$, corresponds to the service rate when there is no parking maneuver.

Assume all the vehicles that arrived in the previous cycle were successfully discharged. The parking maneuver is conducted at time $T$ (this is counted from the start of the red signal of the current cycle). $T = c - g + \Delta t - \frac{l_u}{v}$ assume $v$ is the free flow speed. Figure 5.2 illustrates the situation.

When there is no traffic interruption (i.e., no parking maneuver), the last vehicle that is delayed by the red signal should pass the intersection at time $t_r$.

$$t_r = \frac{S}{S - q} \cdot (c - g) \quad (5.1)$$
5.2.1.2 Parking areas

As assumed, the vehicle parks at time $T$ and location $l_u$. Thus, $l_u/v$ indicates the time this vehicle would require to reach the intersection, and $l_u/v + T$ represents the earliest time it would cross the intersection if it was neither parking nor facing any other delay. Let $\beta$ represent this time period as a percentage of the whole cycle length.

$$\beta = \frac{l_u/v + T}{c}$$  \hspace{1cm} (5.2)

If $\beta \leq t_r/c$ (i.e., $l_u/v + T \leq t_r$), the parker would have experienced the queue from the red signal under normal circumstances (i.e., no parking maneuver). When making the maneuver, however, the parking vehicle can either park after it discharges from the queue (leaves the front of the queue) or before it joins the queue (reaches the back of the queue). Define $l_1$ and $l_2$ as the front and the back of the queue at time $T$, then these two ranges for the parking location can be written as $l_u \in [0, l_1]$ and $l_u \in [l_2, \infty)$. Time space diagrams are shown in Figure 5.3(a) and (b) as examples to illustrate these two conditions.

$$l_1 = \frac{S \cdot v}{K_J \cdot v - S} \cdot (T + g - c) = \frac{S}{K_J} \cdot (\beta \cdot c + g - c)$$  \hspace{1cm} (5.3)

$$l_2 = \frac{q \cdot v}{K_J \cdot v - q} \cdot T = \frac{S}{K_J} \cdot x \cdot g \cdot \beta$$  \hspace{1cm} (5.4)

We define two areas where the vehicle can park:

- **Area A**: the parking vehicle does meet the queue due to the red signal (see the shaded area in Figure 5.3(a)). It happens when $\beta \leq t_r/c$ and $l_u \in [0, l_1]$. The vehicle parks after it discharges from the queue.

- **Area B**: the parking vehicle does not meet the queue (see the shaded area in Figure 5.3(b)). It contains two situations, they are listed below.
  
  - When $\beta \leq t_r/c$ and $l_u \in [l_2, \infty)$, the vehicle parks before reaching the queue (Figure 5.3(b1)).
  
  - When $\beta > t_r/c$, the vehicle parks after the queue has fully dissipated and therefore never meets the queue (Figure 5.3(b2)).
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Figure 5.3: Time-space diagrams illustrating the two parking areas.

5.2.1.3 Additional time variables

Shown in Figure 5.3, \( t_a, t_b \) and \( t \) are newly defined time variables based on a single cycle (i.e., without any consideration for the traffic perturbation caused by the next red signal). \( t_a \) is the time at which the last vehicle before the parker crosses the intersection, and \( t_b \) is the time at which the first vehicle behind the parker crosses the intersection. Thus, the intersection is starved of traffic for a period of \( t_b - t_a \). \( t \) is the theoretical time at which the last vehicle affected (i.e., delayed) by a parking maneuver would pass the intersection. Their expressions are written as Eq. 5.5-5.7

\[
t_a = \begin{cases} 
\beta \cdot c & \text{if } \beta \leq \frac{t_r}{c} \text{ and } l_u \in [0, l_1] \\
\frac{q}{S} \cdot \beta \cdot c + c - g & \text{if } \beta \leq \frac{t_r}{c} \text{ and } l_u \in [l_2, \infty) \\
\beta \cdot c & \text{if } \beta > \frac{t_r}{c}
\end{cases}
\] (5.5)

\[
t_b = \beta \cdot c + p
\] (5.6)
$$t = \begin{cases} \frac{S}{S - q} \cdot (c - g + p) & \text{if } \beta \leq \frac{t_r}{c} \text{ and } l_u \in [0, l_1] \\ \beta \cdot c + \frac{S}{S - q} \cdot p & \text{if } \beta \leq \frac{t_r}{c} \text{ and } l_u \in [l_2, \infty) \\ \beta \cdot c + \frac{S}{S - q} \cdot p & \text{if } \beta > \frac{t_r}{c} \end{cases} \quad (5.7a)$$

The service rate reduction caused by the parking maneuver will be analyzed based on these variables in the next part.

### 5.2.2 Service rate reduction

#### 5.2.2.1 Parking area A

If the car parks in area A, based on the saturation condition, we define three cases.

- $x \in [0, 1 - \alpha]$. Shown in Figure 5.4(a1), the intersection is very undersaturated, so that the traffic perturbation caused by the parking maneuver dissipates before the start of the next cycle (i.e., $t \leq c$). No service rate reduction occurs (Eq. 5.8(a)).

- $x \in [1 - \alpha, 1]$. The intersection is undersaturated without the presence of the parking maneuver, but becomes oversaturated with a parking maneuver (i.e., $t_r \leq c \leq t$). A service rate reduction occurs (Eq. 5.8(b)).

- In Figure 5.4(b1), $\beta \in [1 - g/c, 1 - p/c]$, the starvation period ends before the start of the next cycle (i.e., $t_b \leq c$). The traffic discharges not only before $t_a$, but also after $t_b$.

- In Figure 5.4(b2), $\beta \in [1 - p/c, t_r/c]$, the starvation period ends after the start of the next cycle (i.e., $t_b \geq c$). The traffic discharges only before $t_a$.

- $x \in [1, c/g]$. The intersection is oversaturated with or without a parking maneuver (i.e., $c \leq t_r$). A service rate reduction occurs (Eq. 5.8(c)).

- In Figure 5.4(c1), $\beta \in [1 - g/c, 1 - p/c]$ and $t_b \leq c$. The traffic discharges not only before $t_a$, but also after $t_b$. 

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- In Figure 5.4(c2), $\beta \in [1 - p/c, 1]$ and $t_b \geq c$. The traffic discharges only before $t_a$.

- In Figure 5.4(c3), $\beta \in [1, t_r/c]$, the starvation period only starts in the next cycle (i.e., $t_a \geq c$). Thus, no service rate reduction occurs in the current cycle, and this part is excluded from Eq. 11.

Figure 5.4: Time-space diagrams illustrating the situations where the vehicle parks in area A.

When $\beta \leq t_r/c$ and $l_u \in [0, l_1]$, $\bar{\mu} - \mu =$
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Scenario 1: if \( x \in [0, 1 - \alpha] \), \( \bar{\mu} - \mu = 0 \)  \hfill (5.8a)

Scenario 2: if \( x \in (1 - \alpha, 1] \),
\[
\bar{\mu} - \mu = \begin{cases} 
q - S \cdot \left( \frac{g}{c} \right) \cdot (1 - \alpha) & \text{if } \beta \in \left[ \frac{1 - g}{c}, \frac{1-p}{c} \right] \\
q - S \cdot \left( \frac{g}{c} \right) + S \cdot (1 - \beta) & \text{if } \beta \in \left[ \frac{1 - p}{c}, \frac{c - g}{c - g \cdot x} \right] 
\end{cases}
\]  \hfill (5.8b)

Scenario 3: if \( x \in (1, \frac{c}{g}] \),
\[
\bar{\mu} - \mu = \begin{cases} 
S \cdot \left( \frac{g}{c} \right) \cdot \alpha & \text{if } \beta \in \left[ \frac{1 - g}{c}, \frac{1-p}{c} \right] \\
S \cdot (1 - \beta) & \text{if } \beta \in \left[ \frac{1 - p}{c}, 1 \right] 
\end{cases}
\]  \hfill (5.8c)

Notice that no car can discharge before the green signal starts (i.e., time \( c - g \)). Therefore, \( \beta \in [0, 1 - g/c] \) is excluded in Eq. 5.8. Also, \( t_r/c = \frac{c-g}{c-g \cdot x} \), the value of it depends on \( x \). Figure 5.5 illustrates Eq. 5.8 In Figures 5.5(a) and (b), \( t_r/c \leq 1 \); and in Figure 5.5(c), \( t_r/c \geq 1 \).

Figure 5.5: Relation between the service rate reduction and the value of \( \beta \) based on different saturation conditions.

In Figure 5.5, it can be seen that the parking maneuver only reduces the intersection service rate if \( x \geq 1 - \alpha \). For a given value of \( x \) within this range, as \( \beta \) increases, the service rate reduction caused by the parking maneuver either stays the same or decreases. Since \( \beta = \frac{l_u/v+T}{c} \), if \( v, T \) and \( c \) are constant, then the further upstream the car
parks, the less negative effect it has on the intersection. Additionally, consider the same parking maneuver, a larger service rate reduction is caused when the intersection is oversaturated (i.e., $x \in [1, c/g]$) than when it is undersaturated (i.e., $x \in [1 - \alpha, 1]$).

### 5.2.2.2 Parking area B

If the car parks in area B, three cases can be defined based on the saturation condition.

Notice that the case when $\beta \in \left[0, \frac{c - p - g}{c - g \cdot x}\right]$ is not included in Figure 5.6. Under this condition, the traffic perturbation caused by the parking maneuver does not reach the front of the queue due to the red signal and the traffic discharge rate at the intersection is not affected. Thus, no service rate reduction can occur.

- $x \in [0, 1]$. The intersection is undersaturated, the service rate reduction is written as Eq. 5.9(a).
  - In Figure 5.6(a1), $\beta \in \left[\frac{c - p - g}{c - g \cdot x}, 1 - \frac{p}{c - g \cdot x}\right]$, the queue due to the parking maneuver dissipates before the next cycle (i.e., $t \leq c$). No service rate reduction occurs.
  - In Figure 5.6(a2), $\beta \in \left[\frac{c - p - g}{c - g \cdot x}, 1 - \frac{p}{c - g \cdot x}\right]$, the starvation period ends before the start of the next cycle (i.e., $t_b \leq c$). Some traffic discharges before $t_a$, and some after $t_b$. The queues created by the parking vehicle and the red signal might or might not interact.
  - In Figure 5.6(a3), $\beta \in \left[1 - \frac{p}{c}, 1\right]$, the starvation period ends after the start of the next cycle (i.e., $t_b \geq c$). Hence, only vehicles in front of the parking vehicle can discharge (i.e., a number of $q \cdot \beta \cdot c$ vehicles). The queues created by the parking vehicle and the red signal might or might not interact.

- $x \in [1, \frac{c}{c - p}]$. The service rate reduction is written as Eq. 5.9(b).
  - In Figure 5.6(b1), $\beta \in \left[\frac{c - p - g}{c - g \cdot x}, 1 - \frac{p}{c}\right]$, the starvation
period ends before the start of the next cycle (i.e., \( t_b \leq c \)). Some traffic discharges before \( t_a \), and some after \( t_b \).

- In Figure 5.6(b2), \( \beta \in \left[1 - \frac{p}{c} \frac{1}{x}\right] \), the starvation period starts before the next cycle (i.e., \( t_a \leq c \)) and ends after it (i.e., \( t_b \geq c \)). Only vehicles in front of the parking vehicle can discharge (i.e., a number of \( q \cdot \beta \cdot c \) vehicles).

- In Figure 5.6(b3), \( \beta \in \left[\frac{1}{x}, 1\right] \), the starvation period would start after the end of the current cycle (i.e., \( t_a \geq c \)). Hence, no service rate reduction occurs in the current cycle.

- \( x \in \left[\frac{c}{c-p} \frac{c}{g}\right] \). No service rate reduction occurs (Eq. 5.9(c)).

In Figure 5.6(c1), \( \beta \in \left[\frac{c-p-g}{c-g \cdot x}, 1\right] \), the starvation period would start after the end of the current cycle (i.e., \( t_a \geq c \)).

---

**Figure 5.6:** Time-space diagrams illustrating the situations where the vehicle parks in area B.
When $\beta \leq \frac{t_r}{c}$ and $l_u \in [l_2, \infty)$ or when $\beta > \frac{t_r}{c}$, $\bar{\mu} - \mu =$

Scenario 1' : if $x \in [0, 1], \bar{\mu} - \mu =$

$$\begin{cases} 0 & \text{if } \beta \in \left[0, 1 - \frac{p}{c - g \cdot x}\right] \\ q - S \cdot \frac{c - p}{c} + (S - q) \cdot \beta & \text{if } \beta \in \left[1 - \frac{p}{c - g \cdot x}, 1 - \frac{p}{c}\right] \\ q - q \cdot \beta & \text{if } \beta \in \left[1 - \frac{p}{c}, 1\right] \end{cases}$$

(5.9a)

Scenario 2' : if $x \in \left(1, \frac{c}{c - p}\right], \bar{\mu} - \mu =$

$$\begin{cases} 0 & \text{if } \beta \in \left[0, \frac{c - p - g}{c - g \cdot x}\right] \\ S \cdot \frac{g + p - c}{c} + (S - q) \cdot \beta & \text{if } \beta \in \left[\frac{c - p - g}{c - g \cdot x}, 1 - \frac{p}{c}\right] \\ S \cdot \frac{g - q}{c} \cdot \beta & \text{if } \beta \in \left[1 - \frac{p}{c}, 1\right] \\ 0 & \text{if } \beta \in \left[1, 1\right] \end{cases}$$

(5.9b)

Scenario 3' : if $x \in \left(\frac{c - p}{c - g \cdot x}, \frac{c}{c - p}\right], \bar{\mu} - \mu = 0$

(5.9c)

Figure 5.7 illustrates Eq. 5.9

![Figure 5.7](image)

Figure 5.7: Relation between the service rate reduction and the value of $\beta$ based on different saturation conditions.

In Figure 5.7 it can be seen that the service rate reduction can only be generated when $x \in [0, c/(c - p)]$. For a given value of $x$ within this range, the largest reduction occurs when $\beta = 1 - p/c$. For $x \in [0, 1]$ (in comparison to $x \in [1, c/(c - p)]$, a wider range of $\beta$ can cause a service rate reduction. Also, for a given value of $\beta$, the reduction could be larger if $x \in [0, 1]$. 

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This part provided a methodology to find the service rate when a parking maneuver happens upstream of the intersection. Depending on the area where the vehicle parks, $l_u$ may have different effects on the service rate.

- When the vehicle parks in area A, as $\beta$ increases, the service rate reduction of the intersection decreases until it becomes zero. If $l_u$ is the only independent variable, then the further the car parks, the less reduction it causes.

- When the vehicle parks in area B, the parking location does not always influence the service rate in the same way and magnitude. In other words, it is possible for a further upstream parking to generate a higher reduction. No clear suggestion can be made from this part, it depends on the specific situation. Even so, the model can be used to quantify the effect that the parking maneuver has on the intersection.

Table 5.1 shows a complete set of background conditions, and the corresponding Equation for the service rate reduction.

Table 5.1: Roadmap to equations for different conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>$\beta \leq \frac{t_r}{c}$</th>
<th>$\beta &gt; \frac{t_r}{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_u \in [0, l_1]$</td>
<td>Eq. 5.8(a)</td>
<td>If $l_u \in [l_2, \infty)$ Eq. 5.9(a)</td>
</tr>
<tr>
<td>$x \in [0, 1-\alpha]$</td>
<td>Eq. 5.8(b)</td>
<td>Eq. 5.9(b)</td>
</tr>
<tr>
<td>$x \in [1-\alpha, 1]$</td>
<td>Eq. 5.8(c)</td>
<td>Eq. 5.8(c)</td>
</tr>
<tr>
<td>$x \in [1, \frac{c}{c-p}]$</td>
<td>Eq. 5.8(c)</td>
<td>Eq. 5.9(c)</td>
</tr>
<tr>
<td>$x \in \left(\frac{c}{c-p}, \frac{c}{c-p} g\right]$</td>
<td>Eq. 5.9(c)</td>
<td></td>
</tr>
</tbody>
</table>

### 5.3 Validation

In this section, we use real data to validate the model proposed above, specifically the value of $\mu$ (i.e., service rate). Data were collected with the aid of video cameras for nine hours during June, 2014.

We define the real service rate with a parking maneuver as $\mu_{\text{real}}$. It corresponds to the traffic throughput divided by the cycle length.
5.3. Validation

The traffic throughput is counted directly from the survey data. The accuracy here (Figure 5.8) refers to the service rate (instead of the reduction), i.e., \( \frac{\mu}{\mu_{\text{real}}} \).

The survey was conducted on a section of Dreikonigstrasse, located upstream of the intersection between Claridenstrasse and Dreikonigstrasse, in the city of Zurich, Switzerland. It has 2 lanes (only one direction) and 22 parking stalls: 10 on the left side and 12 on the right. We assume both lanes have \( \mu \) similar traffic demand, and a parking maneuver occurring on one side does not influence traffic on the other side. Hence, variables such as \( q \) and \( \mu_{\text{real}} \) are estimated as half of the observed values to be suitable for the model (one lane street). During the survey, more than 90 maneuvers were recorded, but only 32 of them affected traffic and were used for validation.

From the video, fixed-value variables were obtained: \( g=20 \) sec, \( c=50 \) sec, \( K_J=125 \) veh/km, \( S=1620 \) veh/h. \( K_J \) and \( S \) are averaged values across multiple cycles, \( K_J \) was measured with the queue at the red signal, and \( S \) was the saturation flow rate discharging from the queue (both are considered for one lane). Other values such as \( T, p, l_u, \) and \( q \) were found based on the specific parking maneuver. For example, among the 32 parking maneuvers used for validation, the value of \( T \) ranged from 6 to 39 seconds; the value of \( p \) ranged from 2 to 32 seconds; the value of \( l_u \) ranged from 15.5 to 87.5 meters; and the value of \( q \) ranged from 288 to 648 veh/h. Also, note that at the observed location, the lane changing maneuvers triggered by the parking maneuver were possible. Since they were not taken into account by the proposed methodology, they might have partly reduced the accuracy of the presented results.

Plotting the observed values into the model, \( x, t_r, \beta, l_1 \) and \( l_2 \) were found, as well as \( \mu \). The histogram of accuracy levels obtained is shown in Figure 5.8. The X axis shows different ranges of the accuracy (i.e., values below 100% indicate we underestimated the service rate of the intersection). The Y axis stands for the frequency of observations with such accuracy.

The accuracy ranged between 89% and 136% with an average accuracy of 111%. There were 15 cases (48%) with an error smaller than 10% (accuracy between 90% and 110%). Overall the model seems to provide reasonable results. It predicts the service rate with an error smaller than 30% in at least 93% of the cases (30 cases).
Chapter 5. Effects of upstream on-street parking maneuvers on the intersection

5.4 Conclusions

In this chapter, we described the phenomenon when a vehicle parks upstream of a signalized intersection and analyzed its influence on traffic. As the parking maneuver can occur at different locations and times, it might or might not cause a reduction of the service rate at the intersection. When no reduction occurs, the delay caused by the parking maneuver affects only a very limited number of cars arriving in the current cycle and thus, it is not problematic. However, under certain conditions, a service rate reduction does occur. Then, the negative effects caused by the parking maneuver are magnified. This chapter provides a methodology to define those cases and find the corresponding reduction caused to the service rate.

According to our analysis, the amount of the service rate reduction depends very much on the saturation conditions (i.e., volume-to-capacity ratio, $x$) and the virtual arrival time of the parking vehicle to the intersection (i.e., $\beta \cdot c$). Also, the reduction differs when the parking vehicle parks before joining the queue due to the red signal or after departing from the queue (also upstream or downstream of the intersection).

Applications on parking planning and design based on this research cover at least two aspects. First, one can easily estimate the service rate of a nearby intersection where parking is supplied, and assess the parking design according to its location and its impact on traffic delay. Second, a comparison of the impact on the intersection and traffic between an upstream parking (maneuver) and a downstream one can be useful to guide on-street parking planning policies.

![Figure 5.8: Histogram of accuracy levels based on data collected.](image)
Chapter 6

Preferred on-street parking locations on urban links with respect to the intersections

6.1 Introduction

In the previous two Chapters, the influence of downstream and upstream on-street parking maneuvers with respect to the intersection service rate were modeled, respectively. In this Chapter, the service rate reduction caused by an upstream parking maneuver at a distance \( l_u \) (from Chapter 5) is compared to that caused by a downstream parking maneuver at a distance \( l_d \) (from chapter 4).

6.2 Comparison Between Different Parking Areas

For a car which would cross the intersection at \( \beta \cdot c \), there are 5 areas it may choose to park, areas A and B are upstream of the intersection and defined in section 2; areas 1, 2 and 3 represent three areas downstream of the intersection. They are shown in the time-space diagram in Figure
Chapter 6. Preferred on-street parking locations on urban links with respect to the intersections

Figure 6.1: Comparison of the service rate reduction between an upstream parking maneuver and a downstream parking maneuver.

6.1(a). Within these 3 areas, area 1 is the nearest to the intersection, and area 3 is the farthest. The boundaries between areas 1, 2 and 3 can be found in chapter 1.

The service rate reduction caused by a parking maneuver based on parking areas A, B and 1 are shown in Figure 6.1(b). The service rate reduction is indicated by the lines, and the horizontal axis indicates the value of $\beta$. The graphs are drawn based on the conclusions from chapter 4 and 5 after modification (the derivation and equations are available directly from the author).

Notice that the service rate reduction caused by a vehicle parking in area 2 is between zero and the reduction caused by a vehicle parking in area 1. As it is a range (instead of a single value), it is not drawn in Figure 6.1(b). Vehicles that park in area 3 do not generate any service rate reduction, they are also not included in Figure 6.1(b). Evidently, area 3 is the optimum location to provide parking spaces as parking maneuvers in this area do not affect the intersection. However, Figure 6.1 is useful when considering providing additional parking supply (e.g., near the intersection).
It can be seen in Figure 6.1 that,

- when \( x \in [0, 1 - \alpha] \), the cars who park in area B (i.e., before joining the queue due to the red signal) may reduce the service rate. The largest reduction happens when \( \beta = 1 - p/c \), any other value may cause either a smaller reduction or no reduction at all. In other words, for an often very undersaturated intersection, a parking maneuver in area B causes larger service rate reduction than a parking maneuver in other areas. From this perspective, it might be better to provide parking spaces in other areas instead of area B.

- when \( x \in [1 - \alpha, 1] \), parking upstream in areas A, B and parking downstream in areas 1, 2 may all cause a service rate reduction. It can be seen in Figure 6.1(b2) that parking in area 1 (the nearest area downstream of the intersection) always causes a higher reduction than parking in area A (the nearest area upstream of the intersection). On the other hand, parking further upstream (in area B) has a wider range of \( \beta \) to cause reduction. As a matter of fact, the largest service rate reduction occurs when the car parks in area B with the value of \( \beta = 1 - p/c \).

- when \( x \in [1, c/(c - p)] \), parking near the intersection (areas A and 1) can cause a very high service rate reduction, especially parking downstream in area 1 because the high reduction occurs with a wider range of \( \beta \). When the intersection is oversaturated, to avoid high reduction on the service rate, the cars should park either before they reach the queue due to the red signal or further downstream to enhance the traffic performance.

- when \( x \in [c/(c - p), c/g] \), the service rate reduction caused by cars that park in area A and area 1 is very high (same as in the previous scenario). However, a car that parks in area B (before joining the queue due to the red signal) does not generate any reduction. Therefore, under this scenario, parking further upstream and further downstream causes less traffic problems than parking very near the intersection.

## 6.3 Conclusions

Generally, the results showed that if the intersection is undersaturated (Figures 6.1(b1) and (b2)), the service rate reduction might be higher
when the vehicle parks before it joins the queue due to the red signal. If the intersection is oversaturated (Figures 6.1(b3) and (b4)), the results are the opposite, the reduction is always higher when the cars park after they discharge from the queue. Using this information, it might be suitable to place parking areas downstream of the intersection, or upstream but near to it if the intersection is often undersaturated, and place the parking area further upstream if the intersection is often oversaturated. If the traffic conditions vary much during the day, a dynamic parking supply based on real-time traffic conditions might be very helpful.

Based on the comparison between upstream and downstream parking maneuvers and their influence on the intersection, we do not suggest to provide downstream parking spaces close to the intersection unless the intersection is often very undersaturated, as parking at those locations always cause a relatively high service rate reduction.

Actually, if the intersection is seldom undersaturated (often oversaturated), the best choice is to place the parking area far downstream (areas 2 and 3), the second best choice is to place the parking area far upstream (area B), the third best choice is to place the parking area upstream and close to the intersection (area A), and the last choice is to place the parking area downstream and close to the intersection (area 1).

The findings can be used to guide parking provision, especially on location selection.

- First, the findings can assist decisions on choosing the proper links on the network to provide on-street parking spaces. In the simplest case, as shown, the streets which have lower traffic demand are preferred.

- Second, for a given link on the network, the proper location of parking provision can be suggested based on the background conditions such as traffic demand. As a matter of fact, dynamic parking spaces can be suggested for streets where traffic demand varies significantly over time. For example, in New York, some delivery parking is forbidden during rush hours. Concepts such as this should be advocated with the support of our findings.

- Third, for any link and parking location, one can use the model to evaluate the effects of parking maneuvers supporting the decisions such as parking removal or new supply. Notice that, suggested
parking locations on a link and the corresponding effects of the maneuver highly depends on the length of the link. In other words, for some situations where the link is short and the traffic demand is high, full removal of parking spaces may be suggested based on our study. These parking spaces could be move to other streets where, for example, traffic demand is lower or the link itself is longer.
Chapter 6. Preferred on-street parking locations on urban links with respect to the intersections
Part III

Parking data related issues
Abstract

Although technologies used in the parking industry advanced greatly in the past decade, the concepts and the methodologies behind their usage are often not yet fully understood by practitioners or researchers. In this part, we discuss two parking data related tools in order to improve our understanding, and thus our effectiveness when using them.

In chapter 7, parking patrol surveys are investigated. Patrol surveys are frequently used to estimate average parking duration. However, error in estimating is unavoidable and yet unpredictable. Therefore, a trade off is often conducted between the survey accuracy and the survey budget. In this chapter, the relationship between the survey budget, the estimated average parking duration (i.e., the survey result), and the survey error is illustrated through dimensional analysis. On the basis of this analysis, objective criteria are found to evaluate patrol surveys, a budget test is provided to find the optimal survey budget, and a correction method is proposed to improve survey accuracy. An example application is provided to illustrate how to use the proposed approaches to choose a survey budget and to improve survey accuracy.

In chapter 8, parking guidance and information systems (PGIs) are investigated. PGIs are typically promoted in urban areas to reduce parking related negative effects such as the cruising-for-parking phenomenon. However, their effectiveness often remains unknown. This chapter provides a generalized methodology to evaluate the potential reduction of cruising time through PGIs. Based on probability theory, the relation between the system cruising time reduction, the parking occupancy, and the number of searchers is found. Using this relation, the most effective time periods and conditions for the implementation of PGIs can be estimated, as well as the cruising time reduction during these periods. The results show that, the cruising time saved through PGIs can be significant when the parking system is approaching or leaving the saturation state (i.e., 100% occupancy). However, except for these periods, the PGIs are not very effective. Therefore, the cost and benefit of installing and operating PGIs aiming to reduce system cruising should be carefully assessed for each individual scenario. Examples are given to illustrate how stakeholders (e.g., owners and developers of private parking houses, traffic managers, local authorities and consultants on the parking industry) can use this model.
This part is based on the following research papers:


- Cao, J., M. Menendez (2016) The effectiveness of parking guidance and information systems (PGIs) in reducing cruising-for-parking: a macroscopic probabilistic analysis, working paper, IVT, ETH.
Chapter 7

Cost and accuracy of parking patrol surveys

7.1 Introduction

Despite being one of the most widespread methods for parking data collection, patrol surveys and the theory behind them are not fully understood yet by practitioners. As the relation between survey accuracy and survey cost is unclear, surveyors nowadays still find themselves in the dilemma of balancing the two sides. The goal of this research is to provide basic tools for a better understanding of the typical biases in the collected data, and to quantify the trade-offs between cost and accuracy.

In patrol surveys, patrolling observers check the parking area at fixed time intervals and record the plate number of the car occupying each stall. Among the results given by the data, surveyors are often interested in the average parking duration. This value can be estimated by averaging the parking duration of each car observed. The parking duration of a parked vehicle can be obtained by multiplying the number of times it is observed by the interval of observation (Richardson, 1974).

Lately, many new technologies, especially for data collection, have made it into the parking world. Nevertheless, patrol surveys are still very helpful and widely used in multiple situations and a large number of regions; e.g., on-street parking or off-street parking areas with no advanced facilities, temporary parking areas, and developing coun-
tries with comparatively lower labor cost. Besides, the irreplaceable advantage of a patrol survey is the simplicity of preparation, and its suitability for any kind of parking area.

An intrinsic problem of patrol surveys, however, is their tendency to under sample short stay parkers as their arrival and departure times might occur within the same observation interval. To account for this bias, correction factors were developed by Cleveland (1963), but the results were not completely reliable according to Bonsall (1991). Richardson (1974) claimed that it is possible to get more accurate results at a lower survey cost by using correction coefficients, but the evaluation is only based on data from one single survey. A linear correction model was built upon the parking duration derived from negative exponential distribution by Lautso (1981), but only part of the error was analysed and the improvement in terms of accuracy was not well defined. In general, existing literature provides ideas to improve accuracy, but to the authors’ knowledge, none has provided a generalized method to illustrate how the survey input affects the accuracy.

Intuitively, shorter observation intervals are preferred as more vehicles can be covered in the survey, but they also generate higher data collection costs driven by higher labor requirements. In developing countries, the intervals are chosen between 5 and 15 minutes (Tong et al., 2004) as the labor costs are relatively low compared to the U.S. or Europe, where they are normally between 30 and 60 minutes. As one would expect, there is a trade-off between data collection costs and study accuracy in most patrol surveys. On one hand, a survey with long time intervals may have relatively low costs, but the data can be quite inaccurate, and could potentially generate unusable results. On the other hand, 100% accuracy might demand extremely high and unreasonable costs. Then the question is: at what point does the incremental change in accuracy becomes too expensive? Or similarly: at what point does an increment in a unit of cost yields almost no improvements in terms of accuracy.

Although the error between estimated and real average parking duration is inherent to the survey, it can be analyzed and the data can be manipulated afterwards to improve the accuracy. In this article, an analytical model is built to analyze this error and its relation to survey budget. Through dimensional analysis, the relation between survey budget and survey error is illustrated both mathematically and graphically. Using probability theory, the whole process can be simulated based on certain assumptions regarding arrival time and parking dura-
tion. Simulation-based examples are presented to extend the model to more realistic and generalized situations. Lastly, real life data is used to validate the proposed methodology and to show how to apply it. The final results show that with this approach, a comprehensive method to evaluate the survey can be provided, and the balance between accuracy and cost can be found.

The remaining sections of this chapter are organized as follows. Section 7.2 shows the analytical model illustrating the relation between the survey budget, survey results, and accuracy. Section 7.3 uses simulation-based examples to extend the basic model to a more general framework with more realistic distributions. Section 7.4 shows the validation and application of the proposed methodology using real data. Section 7.5 summarizes the findings of this study.

7.2 Analytical Model

7.2.1 Assumptions and basic definitions

For a given parked vehicle, denote \( t_a \) as the arrival time, \( t \) as the parking duration. The shortest parking duration across all vehicles is \( t^{min} \), and the longest is \( t^{max} \). A basic assumption of the analytical model is that the parking duration is uniformly distributed between these two unknown values, hence the real average parking duration is \( T = (t^{min} + t^{max})/2 \); and the PDF (i.e., probability density function) of parking duration is \( f(t) = 1/(t^{max} - t^{min}) \). This assumption, although restrictive, allows us to formulate all the results analytically; it will be relaxed in the Extended Models section.

Similarly, we assume that the arrival time of vehicles to the parking area is uniformly distributed in the interval \([0, C]\), where \( C \) is the length of the cycle (e.g., 1 day). Hence, the PDF of arrival times is \( f(t_a) = 1/C \). This distribution is only used to describe the arrival time of vehicles which successfully park. Notice that the arrival time is independent of the individual parking duration in the model, and in reality it does not need to follow the same distribution whatsoever.

Denote \( \delta \) as the survey interval, so the patrolling observer checks the parking area every \( \delta \) time units. Use \( M \) to identify the total number of checking intervals, \( M = C/\delta \). The checking time for each interval is \( m \cdot \delta, m \in [1, M] \). The whole survey duration is \((M - 1)\delta\) as no check happens at time 0.

Denote \( L \) as the survey cost. It grows by \( q \) units each time the
surveyor checks the parking area (assume for now that the fixed cost is negligible).

\[ L = M \cdot q = C \cdot q/\delta \]  

(7.1)

Denote \( p_i \) as the probability of a car being observed \( i \) times, \( i \in \lfloor a \rfloor, \lceil b \rceil \); where \( a = t_{\text{min}}/\delta \), and \( b = t_{\text{max}}/\delta \). Hence, \( T \) and \( f(t) \) can also be presented as \( T = (a + b) \cdot \delta/2 \) and \( f(t) = 1/(b - a)/\delta \).

Finally, let \( \beta = t_{\text{max}}/t_{\text{min}} = b/a \).

7.2.2 Model

Considering all parked vehicles, \( p_{\text{obs}} \) denotes the percentage of vehicles that are observed.

\[
p_{\text{obs}} = \sum_{m=1}^{M} \left\{ \int_{(m-1)\delta}^{m\delta-t_{\text{min}}} f(t_a) \left[ \int_{m\delta-t_a}^{t_{\text{max}}} f(t) \, dt \right] \, dt_a + \int_{m\delta-t_{\text{min}}}^{m\delta} f(t_a) \left[ \int_{t_{\text{min}}}^{t_{\text{max}}} f(t) \, dt \cdot \right] \, dt_a \right\}
\]

(7.2)

The combination of terms 1 and 2 represents the probability of vehicles to arrive within a given interval and stay long enough to be observed. Term 1 covers vehicles that are only observed if they stay longer than \( m \cdot \delta - t_a \). Term 2 covers vehicles that will always be observed no matter how long they stay (this is possible given that the difference between their arrival time and the next checking time is no smaller than \( t_{\text{min}} \)).

In real surveys, the number of times a given vehicle is observed in the same place, multiplied by \( \delta \), gives the individual parking duration; so the duration of a vehicle that has been observed \( i \) times is counted as \( i\delta \). Using \( \bar{n}_{\text{obs}} \) to represent the average number of times a car is observed, the estimated average parking duration \( \tilde{T} \) can be interpreted as:

\[
\tilde{T} = \delta \cdot \bar{n}_{\text{obs}} = \delta \sum_{i=\lfloor a \rfloor}^{\lceil b \rceil} i \cdot \frac{p_i}{p_{\text{obs}}}
\]

(7.3)
From this equation, we can tell that $\hat{T}$ is never smaller than $\delta$. Moreover, two reasons which cause the difference between $\hat{T}$ and $T$ (i.e., error) can be found: 1) A portion of the parked vehicles can not be observed (i.e., $p_{obs} < 100\%$) and the parking duration of those vehicles is not reflected in $\hat{T}$; 2) The individual parking durations are only approximate, they are counted as $i\delta$ but their real value could be different.

Use $Y$ to measure the (relative) survey error, the closer $Y$ is to 0, the more accurate $\hat{T}$ is. $\hat{T}$ is an overestimate of $T$ if $Y$ is positive, an underestimate if $Y$ is negative.

$$Y = \frac{\hat{T}}{T} - 1$$  \hspace{1cm} (7.4)

We further employ two dimensionless variables: survey intensity $X$ and relative survey cost $Z$.

$$X = 1 - \frac{\delta}{\hat{T}} = 1 - \frac{1}{n_{obs}}$$ \hspace{1cm} (7.5)

$$Z = \frac{L}{C \cdot q/\hat{T}} = \frac{T}{\delta} = \frac{1}{(1 - X) \cdot (Y + 1)}$$ \hspace{1cm} (7.6)

$X$ is known from the survey ($X \in [0, 1]$) and it defines the level of checking frequency based on individual parking areas. $Z$ reflects the survey cost $L$ in a unit-less way. The relations among $X$, $Y$ and $Z$ can be found based on our analysis.

Given a parking area, a higher checking frequency (i.e., larger $X$) corresponds to a smaller error (i.e., smaller $Y$), but it requires a higher budget (i.e., larger $Z$). $X = 1$ is equivalent to continuous observation (e.g., video camera), then $Y = 0$. In practice, surveyors often evaluate the quality of the survey by checking the value of $X$, although there was no theoretical basis for it. General experience suggested that the results would show a high error for $X$ values below 0.5. The findings from this work will prove that this is a very reasonable rule of thumb.

If $\delta > t_{max}$, the survey accuracy is very low because only a very small percentage of vehicles can be recorded; if $\delta < t_{min}$, the survey can be unnecessarily expensive. Therefore, we only consider $\delta \in [t_{min}, t_{max}]$ in the model (i.e., $a \leq 1 \leq b$). The relation between $Y$ and $X$ can be calculated by eq. 7.7 (derivation of equations is available from the author). Based on eq. 7.6 and 7.7, the relation between $Y$ and $Z$ can be further written as eq. 8. Figure 1 illustrates both relations.
\[ Y = \frac{\delta}{T} \cdot \frac{1}{1 - X} - 1 = \frac{1}{1 + \beta} \cdot \frac{1}{1 - \frac{\beta}{2} \cdot \sqrt{(\beta^2 - 1) \cdot X}} \cdot \frac{1}{1 - X} - 1 \quad (7.7) \]

\[ Y = \frac{4Z\beta^2 - 4Z}{4Z\beta^2 - 2\beta - 1 - (\beta - 2Z)^2} - 1 \quad (7.8) \]

Note that the requirement \( \delta \in [t_{\min}, t_{\max}] \) can be easily verified using the same survey data:

- \( \delta \leq t_{\min} \) when all the observed cars are observed at least twice, then the observation interval is simply shorter than needed, survey cost could be reduced by extending it.
- \( \delta \geq t_{\max} \) when \( X = 0 \), then the observation interval is too long and clearly the error is large. In reality, this would mean that surveyors can only observe cars at most once, so \( \tilde{T} = \delta \).

Additionally:

- When the percentage of cars being observed only once (i.e., \( p_1/p_{\text{obs}} \)) is much lower than the percentage of cars being observed twice or more times, then \( \delta \approx t_{\min} \) and it is possible to increase the value of \( \delta \) without losing much accuracy.
- When the percentage of cars being observed only once (i.e., \( p_1/p_{\text{obs}} \)) is much higher than the percentage of cars being observed twice or more times, then \( \delta \approx t_{\max} \) and the surveyor should reduce the value of \( \delta \). As a matter of fact, when \( \bar{p}_{\text{obs}} \) is smaller than 1.5 (i.e., \( X < 1/3 \)), one can safely assume that a comparatively large portion of the cars are observed only once. The results, hence, might not be representative enough, so the data should not be considered reliable. This is highly important, as we can now easily determine if a patrol survey is valid or not based on the value of \( X \). The survey could be considered unusable if \( X < 1/3 \), and valid otherwise. Note that here, unusable does not necessarily means a high survey error, but an unpredictable error. Based on this finding, figure 1 only includes values of \( X \) larger than or equal to 1/3.
The relation between $X$ and $Y$ is on the left side of figure 7.1. The relation between $Z$ and $Y$ is on the right side. With those graphs, one can find the survey error by either knowing the survey intensity or the relative cost of the survey. Note that the curves on the right graph show scenarios with fixed cost equal to zero. They could be shifted to the right by the ratio between fixed cost and $q$ when including the fixed cost of the survey.

Not surprisingly, for a given $\beta$, $Y$ decreases as $X$ or $Z$ increases. In other words, the survey error gets smaller for more intensive surveys (i.e., more investment considering a specific parking area). Other conclusions include:

1. The error is always positive; in other words, $\tilde{T}$ is always an overestimate of $T$. When $\delta = t^{max}$, $X = 0$ and the error reaches its maximum value $1 - 2/(1 + \beta)$ (not included in the graph). When $\delta = t^{min}$, $X$ reaches its maximum value $1 - 2/(1 + \beta)$ and the error is 0. Notice that the curves are only symmetric at the boundaries.

2. For a given value of $X$ or $Z$, a smaller $\beta$ drives a smaller survey error. Since a basic range of $\beta$ could be obtained from a survey by approximating the longest and shortest parking duration, one can find the possible range of errors for the $X$ obtained from the survey.

3. The relation between $Z$ and $Y$ shows diminishing returns. In
other words, the marginal improvement in accuracy (i.e., reduction in error) decreases as the relative cost increases.

If Figure 7.1 is generalized (it will be later tested with different arrivals) for any parking durations which follow a uniform distribution, then one can actually calculate how much accuracy could be gained by modifying the survey budget. For example, assume $\beta = 10$, the survey error $Y$ can decrease from 26% to 7% when $Z$ is doubled from 1, from 12% to 2% when $Z$ is doubled from 1.5, and from 7% to 1% when $Z$ is doubled from 2.

Also, if the value of $Z$ for the current survey is known, we can calculate the range of the accuracy and what happens if the budget is increased by certain amount. For example, let's assume $\beta \in [10, 20]$. If the survey cost is 500 USD and $Z$ comes out as 1.7, we know the survey error is between 9% and 13%. If we were to add 100 USD into the survey, $Z$ would be $1.7 \times 600 / 500 = 2.04$, then the error would be between 7% and 10%.

In this section, we used uniform distribution to represent both arrival time and duration distributions. We can see by the analysis, that the survey results always overestimate real values, but the survey error can be controlled within 30% when $X \geq 1/3$. The curves showed a clear discipline based on the value of $\beta$, the reason will be discussed in the next section. Based on the generalized relation between $Y$ and $Z$, the expenses required to obtain certain level of accuracy can be found; and the accuracy obtained by certain amount of survey budget can also be calculated. Moreover, the analysis provided an evaluation method for patrol surveys considering both accuracy and survey cost. The next section introduces more realistic distributions. However, the methodology proposed here is still useful as it allows us to explore many different assumptions and distributions when combined with simulation tools.

### 7.3 Extended Models

In this section we will expand the use of the analytical model to cover more realistic situations and we will test if some of the results from the analytical model can be extended to other distributions. The first part looks at the influence of different distributions for the parking durations. The second part looks at different distributions for the arrival time of vehicles.
7.3.1 Distribution of parking duration

A negative exponential distribution has been suggested (and used) the most to represent the parking duration (see Richardson [1974], Cleveland [1963], Blunden and Black [1984]). Possibly, like many other service functions, the parking duration distribution can be expressed by an Erlang function. Besides, Richardson (1974) recommended two additional distributions in his research: a gamma distribution when the standard deviation is less than the mean duration, and a hyper-exponential distribution otherwise. As gamma distribution is the generalized form of both negative exponential and Erlang, we will use it to simulate parking duration. In addition, we will use a hyper-exponential distribution to cover cases where two or more groups of parkers with different purposes park in the same area.

7.3.1.1 Gamma distribution

The gamma distribution can be expressed in the form of $G(k, T)$ and its PDF can be written as $f(t; k, T) = \frac{1}{[t \cdot \Gamma(k)]} \cdot \frac{(kt/T)^k \cdot e^{-kt/T}}{\Gamma(k)}$. Note that when $k = 1$, it turns into a negative exponential distribution. When $k > 10$, the gamma distribution approaches a normal distribution, and the skewness of the PDF approaches zero. This does not represent well the distribution of parking duration, which typically shows a skewness to shorter durations. Therefore, we consider situations only with $k \leq 10$ in our analysis.

7.3.1.2 Hyper-exponential distribution

The hyper-exponential distribution is an example of a mixture density. Its name comes from the fact that its coefficient of variation is greater than that of the exponential distribution, whose coefficient of variation is 1. Its PDF can be written as $f(t; r, T) = \sum r_i \cdot \frac{2r_i}{T} \cdot e^{-2r_i/T}$ where $r_i \in (0, 1), \sum r_i = 1$, (Morse, 2004).

In Richardson (1974), the hyper-exponential distribution fits real parking distribution better than gamma distribution when the standard deviation is greater than the mean duration. One example could be when the parking cars consists of two or more separate sub-populations, obeying negative exponential distributions but with different values of variables, then $r_i$ can be seen as the percentage of all users that belong
to sub-population $i$. In the simulation, we assume that there are two sub-populations, let $w = r_1/r_2$.

### 7.3.1.3 Simulation results

Assume 2000 vehicles arrive in the parking area during a day following a gamma or hyper-exponential distribution. 1000 simulations have been ran for each value of $k$ among 1, 2, 3, 4, 5, 10 when the duration obeys Gamma distributions, and also for each value of $w$ among 1, 3, 5, 7, 9 when the duration obeys hyper-exponential distribution. The obtained relation between $Y$, $X$ and $Z$ is shown in Figure 7.2 where solid lines are average parking durations obeys gamma distribution ($k$ is the shape parameter) and dotted lines are average parking durations obey hyper-exponential distribution ($w$ is the shape parameter).

Notice that the lines in Figure 7.2 represent the average across 1000 simulations. Evidently the results of a single simulation will show more fluctuations. Below are some conclusions that can be drawn from these graphs when the duration obeys gamma distribution.

1. As before, for a given $k$, $Y$ decreases as $X$ or $Z$ increases. In other words, the survey error gets smaller for more intensive (costly) surveys.

2. The survey result is typically an overestimate of real parking duration (not always as in the analytical model). Here in some (very few) cases the results are slight underestimates.
3. For a given value of $X$, a smaller $k$ drives a larger survey error (notice that this is exactly the opposite as with $\beta$ in the analytical model presented before).

4. The most influential factor is $k$. Same as $\beta$ in the analytical model with a uniform distribution, $k$ is the shape parameter of the PDF. The shape parameter is more influential than scale parameters because all the variables in the analysis are dimensionless making the scale of the distribution less critical.

Although the specific values differ, the overall results from simulation based on Gamma distribution are very similar to the results from the analytical model. This shows that the conclusions from the model can also be applied to realistic situations with gamma distribution representing parking durations. Notice that the curve for $w = 1$ and the curve for $k = 1$ fully overlap. When $w = 1$, the two sub-populations have the same amount of parkers; when $k = 1$, it means parking durations obey a negative exponential distribution.

When the durations obey a hyper-exponential distribution (compared to gamma distribution), not surprisingly, the survey produces larger errors for the same $X$. Moreover, the larger the difference between the size of the 2 sub-populations (i.e., larger $w$), the larger the error is. Given the magnitude of these errors, it would be reasonable to use highly intensity surveys where a hyper-exponential distribution is suspected. Considering the economic aspect, the shape of the curves is again similar to the results from the analytical model, only the error is comparatively larger for a given value of $Z$. With relative cost growing, the marginal utility (marginal improvement of accuracy) diminishes.

Through the simulations, we expanded the results and conclusions obtained from the analytical model to a wider range of situations where parking durations obey other distributions. It proved that the relation between variables $X$, $Y$ and $Z$ is very similar (just follows a different scale) for different distributions of parking durations. Hence, the overall conclusions obtained with the analytical model can be generalized.

### 7.3.2 Distribution and number of arrivals

Given the lack of data about arrival distributions, having that as one of the inputs to our methodology could limit its applicability. Hence, here we will test how different distributions and total number of vehicles parked can affect the results shown before. We use both uniform $U(1,9)$
and Gamma $G(2,4.5)$ distributions to represent the background data of parking duration (note that gamma distribution is represented as $G(k,T)$, so the average is 4.5 hours for both); and assume a total number of 5000 parked cars. We then generate multiple scenarios with arrival times following different distributions (uniform and gamma). The curves shown in Figure 7.3 represent the average of 100 simulations for each scenario. Figure 7.3(a) shows the results for different average arrival times compared to the beginning of the survey, e.g., $U(0,2)$ and $U(0,4)$. Figure 7.3(b) shows the results for the same average arrival time but different arrivals’ start times compared to the beginning of the survey, e.g., $U(2,8)$ and $U(3,7)$. Figure 7.3(c) shows the results for different number of vehicles parked: 2000, 500, and 200. Background data in this case includes gamma distributions for both arrivals and durations.

Below are some findings:

1. The value of $Y$ typically decreases (i.e., accuracy increases) as the average arrival time becomes smaller (i.e., average arrival time approaches beginning of the survey)- Figure 7.3(a).

2. Fluctuations increase as the start time for arrivals increases (i.e., arrivals start later compared to the beginning of the survey). However, this is only noticeable when the parking arrivals obey a uniform distribution- Figure 7.3(b).

3. For different numbers of parked vehicles, the relative difference between graphs is always below 2 percentage points (except when $k = 1$)- Figure 7.3(c).

Based on the findings, we suggest surveyors to pay especial attention to situations where the start time of arrivals is much later than the starting time of the survey. That being said, in most situations the influence from arrivals and total number of vehicles parked is quite weak and the total results are still encouraging, especially for arrivals and durations obeying gamma distributions. Hence, the graphs from Figures 7.1 and 7.2 can still be used.

### 7.4 Numerical Examples and Applications

#### 7.4.1 Numerical examples for model validation

Two sets of real parking duration data are used here: (1) Max-Bill-Platz in Zurich (Switzerland), with 60 parking stalls; (2) Ballston Garage in Arlington (U.S.), with nearly 2800 parking stalls. The recorded data
Figure 7.3: Relation between survey error ($Y$) and survey intensity ($X$) with different arrivals.

includes the arrival and departure time of each car ($\delta=0, X=1, Y=0$). This way we can test the results for different $\delta$.

In Max-Bill-Platz, a total number of 148 cars arrived and departed
between 8am and 6pm (data from November 2nd, 2012), $\overline{T}=63$ minutes. In Ballston Garage, 879 cars arrived and departed between 5am and 8pm, $\overline{T}=478$ minutes (data from June 13th, 2012). The results from the two parking areas are shown in Figures 7.4(a) and 7.4(b), respectively. Graphs on the left show the distributions of parking durations. As seen, the shape is similar to gamma distribution. As a matter of fact, $k=1.5$ and $k=5.0$ can be drawn respectively using the data fitting function in matlab. Graphs on the right show the $Y-X$ relation derived from the real data and the theoretical curves from Figure 7.2. The theoretical curves are extended for $X < 1/3$ for comparison purposes; the real curves have been divided into two parts accordingly.
As shown in Figure 7.4, the real and the theoretical curves match each other quite well when $X \geq 1/3$ for both parking areas. For $X < 1/3$, the curves show no clear discipline. However, this fact coincides with the statement before, the error is unpredictable when $X < 1/3$. In such cases, data obtained is very incomplete as cars are mostly observed only once, and the magnitude of the error is highly dependent on the fortuitous combination of the arrival times and the start time of the survey.

### 7.4.2 Applications

Based on the results and conclusions obtained so far, one can reach a better understanding of survey accuracy and its connection to survey cost. Moreover, a $Z$ test and a correction method can be provided to find a comparatively low budget while achieving high accuracy. In this section, we illustrate with the real data from Max-Bill-Platz, how to use these two methods to conduct patrol surveys with a higher capital-utility.

For preparation, we first need to define the basic PDF shape of parking durations and a lower bound of $\bar{T}$ based on typical parking purposes, time controls, and any other available data. Here we assume the PDF follows a gamma distribution (the corresponding results in Table 7.1 will be used) and $\bar{T}>40$ minutes. Although $C$ and $q$ are known to surveyors in practice, we need to assume values here, let $C = 240$ minutes and $q = 20$ USD.

As mentioned before, the main issues with patrol surveys are how to decide the budget and how to enhance the accuracy. Below, a $Z$ test and a correction method will be provided to solve these two problems.

#### 7.4.2.1 $Z$ test to recommend survey budget

The $Z$ test will be used to detect if a budget $L$ is enough to guarantee certain accuracy. As explained before, $X = 1/3(0.33)$ can be used to distinguish usable and unusable results. Corresponding to this $X$ and a given $k$, a value for $Z$ can be found in Table 1. To guarantee this test is suitable for any value of $k \leq 10$, we use the highest value of $Z = 1.33$ ($k = 10$) as the minimum required $Z$ to produce usable result. In other words, misleading survey results with unpredictable error may be generated when $Z < 1.33$.

Through eq. 7.6 and our assumptions, $Z$ can be linked to the survey budget $L$:
Table 7.1: Results of Y and Z as a function of X and k assuming parking durations obey a gamma distribution

<table>
<thead>
<tr>
<th>k=</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>X=0.33</td>
<td>0.77</td>
<td>0.36</td>
<td>0.23</td>
<td>0.16</td>
<td>0.12</td>
<td>0.05</td>
</tr>
<tr>
<td>X=0.44</td>
<td>0.55</td>
<td>0.23</td>
<td>0.13</td>
<td>0.09</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>X=0.58</td>
<td>0.35</td>
<td>0.13</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>X=0.65</td>
<td>0.28</td>
<td>0.09</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>k=</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>X=0.33</td>
<td>0.79</td>
<td>1.02</td>
<td>1.13</td>
<td>1.2</td>
<td>1.24</td>
<td>1.33</td>
</tr>
<tr>
<td>X=0.44</td>
<td>1.06</td>
<td>1.33</td>
<td>1.45</td>
<td>1.51</td>
<td>1.55</td>
<td>1.61</td>
</tr>
<tr>
<td>X=0.58</td>
<td>1.57</td>
<td>1.89</td>
<td>2</td>
<td>2.05</td>
<td>2.08</td>
<td>2.12</td>
</tr>
<tr>
<td>X=0.65</td>
<td>1.96</td>
<td>2.29</td>
<td>2.4</td>
<td>2.45</td>
<td>2.46</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 7.2: Result comparison based on Z test and correction method under different survey budgets (unit of L, δ, T are USD, minutes, and minutes)

<table>
<thead>
<tr>
<th>Input</th>
<th>Survey results</th>
<th>Z Test</th>
<th>Survey error Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>δ</td>
<td>T</td>
<td>X</td>
</tr>
<tr>
<td>100</td>
<td>48</td>
<td>85</td>
<td>0.44</td>
</tr>
<tr>
<td>160</td>
<td>30</td>
<td>71</td>
<td>0.58</td>
</tr>
<tr>
<td>200</td>
<td>24</td>
<td>68</td>
<td>0.65</td>
</tr>
</tbody>
</table>

\[ Z = \frac{T}{C \cdot q} \cdot L > \frac{40}{240 \cdot 20} \cdot L = \frac{L}{120} \]

Hence, for example, L=100 USD generates Z > 0.83 and this budget fails the Z test. Therefore, 100 USD is not good as Z still has a large chance to be smaller than 1.33. On the contrary, a budget of 200 USD generates Z > 1.66, so this budget is acceptable.

Moreover, one can find that L ≥ 160 USD gives Z > 1.33. Since 160 USD is the lowest budget satisfying this standard, it will be the recommended survey budget. The results of the Z test under different budgets can be seen in Table 7.2 including the recommended budget.
7.4.2.2 Correction method to enhance survey accuracy

The method provides a new estimate for the average parking duration, $\tilde{T}'$. Compared to $\tilde{T}$, the possible maximum error will be reduced.

Using Table 7.1, $Y^{\text{min}}$ (when $k=10$) and $Y^{\text{max}}$ (when $k=1$) are known for a given $X$, then we can find the lower and higher bound $[\bar{T}^{\text{min}}, \bar{T}^{\text{max}}]$ for $\bar{T}$ using survey result $\tilde{T}$ (eq. 7.4). $\tilde{T}'$ can then be found with eq. 7.9

$$\tilde{T}' = \frac{2}{\frac{1}{\bar{T}^{\text{min}}} + \frac{1}{\bar{T}^{\text{max}}}} = \frac{2\tilde{T}}{Y^{\text{max}} + Y^{\text{min}} + 2} \quad (7.9)$$

With this correction, the upper bound of the error is reduced and the absolute value of $Y$ is now located in a smaller range. The maximum error happens when $\bar{T}$ equals to $\bar{T}^{\text{min}}$ or $\bar{T}^{\text{max}}$ (eq. 7.9). Although this method does not always give a smaller value of the exact error, the range of the error is always narrowed, so the quality of the survey is guaranteed in general.

$$\text{Maximum survey error} = \left|\frac{\tilde{T}'}{\bar{T}^{\text{min}}} - 1\right| = \left|\frac{\tilde{T}'}{\bar{T}^{\text{max}}} - 1\right| \quad (7.10)$$

Take $L=160$ as an example, survey results are $\tilde{T}=71$ minutes and $X=0.58$. From table 1 we can find $Y^{\text{min}}=0.01$ ($k=10$) and $Y^{\text{max}}=0.35$ ($k=1$). Accordingly $\bar{T}^{\text{min}}=53$, $\bar{T}^{\text{max}}=70$ (eq. 7.4), and $\tilde{T}'=60$ minutes (eq. 7.4). Therefore, using the correction, the estimation of average parking duration is reduced from 71 to 60 minutes. More important, the possible maximum error is reduced by 21 percentage points from 35% to 14%, the range of error becomes [-14%, 14%] (eq. 7.10). If we compare the new estimate to the real data $\bar{T}=63$, the proposed correction reduces the error from 13% to -4%. In Table 7.2 results under different budgets are shown.

Below are some advantages of the proposed methods:

- From the $Z$ test, we detect that $L=100$ generates unusable results, thus the capital waste can be avoided.

- From the correction, we can see the reduction of survey error from 13% to -4% for $L=160$ and from 8% to -3% for $L=200$. 

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• Compared to a higher budget of $L=200$, the recommended budget of 160 USD not only generates a high accuracy (with correction) but also saves 40 USD.

Although the results above come from a single numerical example, they show the applicability and benefits of the $Z$ test and the correction method. We hope the ideas can guide practitioners to conduct patrol surveys with higher capital-utility.

7.5 Conclusions

Although patrol surveys are widely used and also supported by new technology nowadays (Bonsall and O’Flaherty [1997]), the idea has not changed much and the bias of the data still exists. In this paper, we established a scientific method to find a relation between survey cost and survey accuracy in a generalized way. By defining dimensionless variables $X$ (survey intensity), $Y$ (survey error), and $Z$ (relative cost), the conclusions drawn hold for any unit or scale concerned in the survey. With an analytical model and simulations covering the most suggested (and used) distributions of parking durations, we obtained the relation among the three variables analytically and graphically. The results are quite generalized as they are drawn based on comparisons of different arrival distributions and parking duration distributions. With this approach, one is able to evaluate the accuracy of a patrol survey and the effects of additional investments. It is also possible to detect if a survey budget is high enough to produce usable results and then to guarantee the accuracy with our correction method. Both the resulting graphs, and the proposed methodology have been validated with real data from two different parking areas.

Below is a summary of the findings and how they could be useful in real surveys:

1. The distribution of arrival times is not highly relevant to the relation between $X$, $Y$ and $Z$ when the quantity of parked vehicles is reasonable (i.e., enough to be representative of the distribution). Hence, surveyors do not need any information on arrival rates. They can directly use the graphs shown in this paper (Figures 7.1 and 7.2) as reference for their surveys after making an educated guess regarding the distribution of parking durations.
2. The shape parameter of parking durations distribution is the decisive factor to the curves obtained in the relations among X, Y and Z (e.g., $\beta$, $k$, $w$ for uniform, gamma, hyper-exponential distribution, respectively). The curves showing those relations follow a clear discipline according to the value of the shape parameter. By defining or conjecturing the range of the parameter, surveyors can obtain the range of possible error.

3. The survey intensity X and relative cost Z can be chosen as the quality criteria of any patrol survey; the survey error Y directly depends on them. For $X < 1/3$ the error is possibly unpredictable, hence data should be discarded. In the case that parking durations obey a gamma distribution ($k \leq 10$), $Z < 1.33$ can be used as a standard to obtain an acceptable budget.

4. The survey result is typically an overestimate for average parking durations obeying uniform, gamma or hyper-exponential distributions. Accordingly, we developed a method to reduce the survey error Y without the need for additional labor.

5. The range of possible errors for $X \geq 1/3$ (before the use of correction) is within 30% when parking durations obey a uniform distribution; 40% when they obey a gamma distribution (except negative exponential distribution when $k=1$); and 70% when they obey a hyper-exponential distribution.

6. Given the value of X or Z, the average survey error is larger when parking durations obey a hyper-exponential distribution compared to gamma or uniform (see point 5). This means that a higher value of X may be required to guarantee the accuracy when the parking area includes parkers with different parking purposes (especially if the sizes of the different sub-populations of parkers are very different).

Note that the results shown here have not been verified for all possible distributions of parking durations. Before adopting this method and data, surveyors should make sure the parking durations for the specific parking area obey one of the three distributions discussed above. The parking duration assumptions can be made based on the parking purpose and area served by the parking (e.g., shopping versus working).

Even though we have tried to use the three most recommended parking durations distributions, there are still some limitations. For
example, in the model we have counted all the vehicles that arrived within the day, while in practice, the vehicles with no information on arrival interval or departure interval are typically eliminated or modified. The errors caused by this data manipulation were not considered. Further research is necessary to extend and generalize the findings to even more distributions and other parking scenarios.
Chapter 8

Parking guidance and information systems (PGIs)

8.1 Introduction

Some researchers have estimated that upwards of 30%, and maybe as much as 50%, of traffic on a given downtown street is comprised of people cruising-for-parking (Shoup, 2006). These cruising vehicles endure longer travel distances to look for parking opportunities, causing additional delays to the whole traffic stream by significantly impacting the travel time in downtown areas (Geroliminis, 2015; Cao and Menendez, 2015b). Also, the additional distance travelled while cruising can increase emissions, and hence, damage the local environment. An estimation from Shoup (2005) shows that cruising in one small area of Los Angeles produces 3600 miles (around 5794 kilometers) of excess travel each day - equivalent to two round trips to the moon each year.

To reduce such negative effects of cruising, parking guidance and information systems (PGIs) are often promoted. In general, PGIs help searchers to find parking spaces by providing information on available parking locations. In other words, PGIs can help to better allocate users to the available resources, and consequently reduce cruising.

Empirical and analytical studies have been carried out in the past to evaluate the effects of PGIs on travellers’ choice and cruising time.
For example, in Axhausen et al. (1993), surveys were conducted to evaluate the effectiveness of PGIs in two cities, Nottingham, UK, and Frankfurt, Germany. The results showed that, in Nottingham, the PGIs have a clear benefit in terms of reducing parking search time for those who adjust their behaviour before they begin their journey; while in Frankfurt, the benefits for the individual drivers remain unknown. In Asakura and Kashiwadani (1994), the effects of different types of parking availability information (i.e., full or not, number of available parking spaces, and waiting time) on system performance were assessed using a simulation model. Results found that the difference of effects among information types depends on the congestion level of the system.

More recently, Waterson et al. (2001), based on survey data from Southampton, UK, and the network traffic simulation model RGCONTRAM, evaluated the potential savings in travel time offered by PGIs. Interestingly, that study argued that PGIs do impact the average travel time of individual vehicles, but the network-wide level effects are rather limited. This will be further discussed in our study. In Spencer and West (2004), an on-going project incorporating PGIs is presented. They showed some survey results of PGIs implementation. For example, Yokohama, Japan, found that 82% of motorists appreciated the PGIs system; and the use of on-street parking decreased by 18% while off-street parking usage increased by 21%. In Aachen, Germany, within 3 years of PGIs implementation, the number of motorists searching for parking was reduced from 30% to 20%, while traffic volume increased by 16%. Caicedo (2010) developed a demand assignment model to evaluate the benefits of manipulating information with the objective of reducing the time and distances involved in finding a parking space; including the walking distances involved. Using the full search procedure it was found that improvements of approximately 10% in efficiency could be achieved, but only at high computational costs. Jonkers et al. (2011) compared the effects of Intelligent Parking Service, IPS (parking reservation, navigation and automated payment) and traditional PGIs (guidance on parking availability). In the micro simulation model, the parking conditions were simulated for the city of Eindhoven, Netherlands. Compared to PGIs, IPS provides a higher reduction on the travel time and the travel distance in inner city centres (45%), but less reduction in outer city centres. Overall, the cruising time was reduced by 22% using either system. In Mei et al. (2012), a guiding parking reliability model was constructed by analyzing the parking choice under guiding and optional parking lots. Results show that, the parking guid-
ing variable message signs achieved the best benefit when the parking supply was close to or was less than the demand. [Moini et al. (2013)] assessed the impact of on-street PGIs on network mobility and vehicular greenhouse gas (GHG) emissions. The findings showed that the most significant reductions in vehicular emissions and delays are realized under conditions where the availability of parking spaces is less than the demand. [Chen et al. (2016)] showed that, parking reservation, compared to conventional parking information provision, is more efficient in improving parking usage.

Despite these studies, to the authors’ knowledge, a generalized methodology to quantify the cruising reduction due to PGIs has not been proposed yet. In this chapter, the average cruising time with and without PGIs is estimated. First, we provide an analytical methodology to evaluate the potential reduction of average cruising time through PGIs under static conditions, i.e., a given parking occupancy and a given number of searchers. Afterwards, we extend the methodology to take into account the time-varying conditions of parking occupancy and searching vehicles.

Based on the proposed methodology, it is possible to assess the effectiveness of PGIs by simply estimating the parking demand and recording the parking occupancy. The methodology is generalized enough for areas with different network layouts and different parking supplies, and can be easily applied. Notice that, the results can also be used to study and evaluate the implementation of PGIs inside parking houses. Using the model, one is able to find the most effective time periods for the implementation of PGIs, and predict the cruising reduction during those periods.

The content of this chapter is organized as follows. Section 8.2 summarizes the methodology to formulate the probability of finding parking without PGIs. Section 8.3 describes the probability of the best (i.e., optimal state) that the system could reach with the assistance of PGIs. Section 8.4 models and quantifies the performance (i.e., effectiveness) of the PGIs. Section 8.5 provides a case study based on real data from the city of Zurich to illustrate and validate the proposed methodology. Section 8.6 summarizes this study.
8.2 Probability of finding parking without PGIs

The model upon which this part is built is shown in Chapter 2, where a homogenous network is considered such as a small network with on-street parking supply, or inside a large parking house.

Although the methodology is the same as shown in Chapter 2 some of the notations are changed for simplification purposes. Denote $A$ as the capacity of the parking supply, i.e., the total number of parking spaces, including occupied and available parking spaces; $N_p$ as the number of currently available parking spaces; $N_s$ as the number of searching vehicles; $L$ as the length of the network, i.e., the total street length (km) in the network (or the driveways inside the parking house).

Denote $p$ as the average probability of finding parking (without the aid of PGIs) within a period of time $t$. Assume that searching vehicles travel at speed $v$. In general, $p$ is a function of $N_s$, $N_p$ and $\frac{v}{L}$ (the ratio between the travel distance and the total network length).

Eq. 8.1 provides a mathematical approximation of the probability of $p$, i.e., finding parking without the assistance of PGIs.

\[
p = \begin{cases} 
1 - (1 - \frac{v}{L})N_p & \text{if } t \in [0, \frac{L}{vN_s}] \\
\frac{N_p}{N_s} + \left[ \frac{N_p}{N_s} - 1 + \left(1 - \frac{1}{N_s}\right)^N_p \right] \log_{\frac{N_p}{N_s}} \frac{N_p}{N_s} \frac{v}{L} & \text{if } t \in \left[ \frac{L}{vN_s}, \frac{L}{vN_s}, \frac{N_p}{N_s} \right] \text{ if } N_p \leq N_s \\
1 - (1 - \frac{v}{L})N_p & \text{if } t \in \left[ \frac{L}{v}, \frac{N_p}{N_s}, \infty \right] \\
1 + \left(1 - \frac{1}{N_s}\right)^N_p \cdot \log_{N_s} \frac{v}{L} & \text{if } t \in \left[ \frac{L}{vN_s}, \frac{L}{v} \right] \text{ if } N_p \geq N_s
\end{cases}
\] (8.1a)

Eq. 8.1(a) represents the situation where the searching vehicles outnumber the available parking spaces, i.e., not all searchers can find parking. Eq. 8.1(b) represents the opposite situation, where the available parking spaces are enough to accommodate all the searching vehicles. Notice that in both cases it is possible that some (or all) searchers are unable to find parking in a given time period (e.g., if $t$ is very small, or $v$ is very low).
8.3 Optimal probability of finding parking with PGIs

The optimal probability of finding a parking corresponds to the situation where either all parking spaces are immediately taken, or all vehicles find parking immediately. Denote $p_{opt}$ as the optimal probability, it is formulated as Eq. 8.2

$$p_{opt} = \begin{cases} \frac{N_p}{N_s}, & \text{if } N_p \leq N_s \\ 1, & \text{if } N_p \geq N_s \end{cases} \quad (8.2a)$$

Eq. 8.2(a) represents the situation where the searching vehicles outnumber the available parking spaces, thus only a maximum number of $N_p$ vehicles can find parking. Eq. 8.2(b) represents the situation where all searching vehicles are able to find parking.

As mentioned before, by publishing location information of available parking spaces, PGIs improve the searching routes of vehicles and allocate vehicles to parking spaces in a shorter time. In other words, PGIs might improve the system by increasing the probability of finding parking. Although $p_{opt}$ is not directly the probability of finding parking with the aid of PGIs, it does represent the upper bound for the performance that could be achieved when PGIs are deployed. In reality, the longer $t$ is, the more representative $p_{opt}$ is for the situation with PGIs. Hence, here we use $p_{opt}$ as an indicator for the probability of finding parking with the aid of PGIs.

Notice that, the values shown in Eq. 8.2 represent the largest potential probability of Eq. 8.1, however, the formulation of Eq. 8.2 has no connection to the formulation of Eq. 8.1. Eq. 8.2 is built on thoroughly optimistic conditions, despite the form of the network, the traffic distribution, the searching behavior, and the parking preference. It is simply the maximum number of parking spaces that can be found according to the number of searching vehicles. In other words, Eq. 8.2 is the optimal probability of finding parking in any kind of situation in real life. On the other hand, Eq. 8.1 is drawn based on some assumptions, for example, the network needs to be homogeneous and compact; since the area we consider is small, the preference of drivers on parking spaces are neglected. Nevertheless, as mentioned in part I, Eq. 8.1 can be applied to any physical form of the network, e.g., grid, ring, ring-radial.
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8.4 Evaluation of PGIs effectiveness

Below, section 8.4.1 proposes a PGIs’ effectiveness indicator under static conditions (i.e., fixed value of $N_s$ and $N_p$) and it indicates the general conditions where PGIs can be useful. Section 8.4.2 analyzes the real searching time reduction considering time-varying conditions of $N_p$ and $N_s$.

### 8.4.1 PGIs effectiveness under static conditions

Denote $\beta$ as the total potential reduction of cruising time in the system due to PGIs under static conditions. The larger it is, the higher potential PGIs have. $\beta$ is defined in Eq. 8.3.

\[
\beta = N_s \cdot \int_0^\infty (p_{opt} - p) dt = \begin{cases} 
N_s \cdot \int_0^{L_v} \frac{N_p}{N_s} (p_{opt} - p) dt, & \text{if } N_p \leq N_s \\
N_s \cdot \int_0^{L_v} (p_{opt} - p) dt, & \text{if } N_p \geq N_s 
\end{cases} \tag{8.3a}
\]

Evidently, based on Eq. 8.1-8.3, $\beta$ is a function of $N_p$, $N_s$, and $\frac{v}{L}$. The contour plot of $\beta$ is shown in Figure 8.1 using two dimensionless variables, $\frac{N_p}{A}$ and $\frac{N_s}{A}$, as the axes. Notice that the value of $\beta$ is not related to $A$, but using $\frac{N_p}{A}$ and $\frac{N_s}{A}$ (instead of $N_p$ and $N_s$) is advantageous because it provides a dimensionless scale for the axes, thus makes the graph generalized. As a matter of fact, $\frac{N_p}{A} \in [0, 1]$ has a physical meaning: the percentage of parking spaces that are available. Meanwhile, $\frac{N_s}{A} \in [0, \infty)$ is only a ratio without a practical meaning behind it. In Figure 8.1, the range of $\frac{N_s}{A}$ is $[0, 2]$ as the plot can already cover the general pattern of $\beta$ using this range.

Based on Figure 8.1, the following conclusions can be drawn.

- The largest $\beta$ occurs when $\frac{N_p}{A}$ and $\frac{N_s}{A}$ are both around 100%. This situation corresponds to large values of $N_s$ and $N_p$ (in relation to $A$) at the same time (the middle point on the right side of the graph). However, in reality, when $N_p$ is large, $N_s$ is often relatively small as it is rather easy for searchers to find parking. Hence, this situation can hardly occur.

- The smallest $\beta$ occurs on the left top corner and the right bottom corner of the graph.
8.4. Evaluation of PGIs effectiveness

Figure 8.1: Contour plot of $\beta$ in relation to $N_p/A$ and $N_s/A$. The pattern of the graph is general whereas the scale depends on $A$ and $v/L$. ($A=50$, $v=10$ and $L=100$ are used in this example).

- The left top corner represents a highly oversaturated situation with many searchers but low parking availability. In this case, PGIs are not advantageous as there is simply no parking information to provide. In an extreme case, when $N_p=0$, i.e., parking occupancy is 100%, PGIs cannot provide any benefit.

- The right bottom corner represents a highly undersaturated situation with few searchers but many parking spaces free. In this case, PGIs are not advantageous either as searchers can find a parking space easily even without the assistance of PGIs, due to the abundant number of available parking spaces in the network.

- For any given value of $N_p/A$, the largest $\beta$ occurs when $N_s$ equals to $N_p$, i.e., $\beta$ drops as $N_s$’s value drifts away from $N_p$’s. Notice that the value of $N_p/A$ is equivalent to 1 minus the parking occupancy. Hence, for any given parking occupancy, PGIs would be the most effective if there is a similar amount of searchers as the number of available parking spaces.

The pattern of the plot shown in Figure 8.1 stays the same despite
the specific values of $A$, $L$, $N_s$ or $N_p$. This is valuable, as it means that the pattern of $\beta$ in relation to $N_p/A$ and $N_s/A$ is very general, although the specific magnitude (i.e., value of $\beta$) might differ according to the input values.

Therefore, by simply comparing the values of $N_s$ and $N_p$, or $N_s/A$ and the value of one minus the current parking occupancy, the overall effectiveness of PGIs under static conditions can be directly estimated. As mentioned earlier, large values of $N_s$ and $N_p$ at the same time can seldom happen. Similar levels of $N_s$ and $N_p$ may occur when they are both relatively small (that is when the parking occupancy is relatively high). In other words, when the parking occupancy is relatively high but the parking is not totally saturated, PGIs have a lot of potential in reducing cruising. In the next sub-section, we focus on these high potential periods, to further model the reduction of cruising time due to PGIs.

### 8.4.2 PGIs effectiveness under dynamic conditions

The previous section developed an indicator for the effectiveness of PGIs under static conditions, i.e., under a fixed parking occupancy and a fixed number of searchers. In this section, the model is further developed to quantify the cruising time that can be potentially reduced through PGIs over a period of time, taking into account the time-varying conditions of parking occupancy and number of searchers.

Within the time period of interest, time is cut into very thin slices of length $t$. The conditions are assumed steady (non-changing) within each time slice. Assume the time period contains a number of $I$ time slices, i.e., total time period of $t \cdot I$. Notice that, to choose a proper value of $t$, two aspects should be taken into account. On one hand, $t$ cannot be too small so that $p_{opt}$ can represent the real probability of finding parking with the assistance of PGIs. On the other hand, $t$ cannot be too large so that the dynamics of parking usage and traffic conditions can be incorporated.

#### Model inputs

To consider the general situation, we need, besides $t$, another two data inputs.

- $N^i$ is the number of vehicles that start to search for parking at the beginning of time slice $i \in [1, I]$ (i.e., parking demand); and
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- \( \text{occ}^i \) is the parking occupancy at the beginning of time slice \( i \in [1, I] \).

The values for \( N^i \) and \( \text{occ}^i \) are known over time. \( N^i \) is the same with or without PGIs and it can be estimated based on traffic measurements or demand modeling. \( \text{occ}^i \) represents the parking occupancy without PGIs and it can be directly recorded from the system, or estimated using models such as the one in \cite{Cao and Menendez, 2015} and presented in Chapter 2.

**Searching time without PGIs**

Eq.8.4 defines \( N^i_p \). It is the number of available parking spaces at the beginning of time slice \( i \in [1, I] \).

\[
N^i_p = (1 - \text{occ}^i) \cdot A \tag{8.4}
\]

Considering a situation where PGIs are not yet adopted, we define two additional variables.

- \( N^i_s \) is the number of searchers at the beginning of time slice \( i \). It is calculated according to Eq.8.5.

In Eq.8.5, \( p^{i-1} \) can be found through Eq.8.1. \( N^{i-1}_s \cdot (1 - p^{i-1}) \) represents the number of searching vehicles who did not find parking in the previous time slice; \( N^i \) represents the number of vehicles who started to search at the beginning of the current time slice; and \( N^i_s \) is the summation of these two terms.

\[
N^i_s = N^{i-1}_s \cdot (1 - p^{i-1}) + N^i \tag{8.5}
\]

- \( N^i_d \) is the number of vehicles that departed parking during time slice \( i \). The value is assumed to be same with or without PGIs. It is calculated according to Eq.8.6.

\[
N^i_d = N^i_p - N^{i-1}_p + p^{i-1} \cdot N^{i-1}_s \tag{8.6}
\]

Within Eq.8.6, \( N^i_p - N^{i-1}_p \) stands for the net change in the number of available parking spaces with respect to the previous time slice; \( p^{i-1} \cdot N^{i-1}_s \) stands for the number of searchers that found parking in the previous time slice. This equation will be used to estimate the parking occupancy in the presence of PGIs. Notice that the
number of vehicles that departed is independent of whether we implement PGIs or not.

- \( T \) is the total cruising time over the whole period \( i \in [1, I] \) without PGIs. It is calculated according to Eq.8.7

In Eq.8.7, \( N^i_s \cdot t \) represents the total searching time spent during time slice \( i \). This assumes that \( t \) is short enough so even the vehicles that find parking in time slice \( i \) do drive for time \( t \).

\[
T = \sum_{i=1}^{I} N^i_s \cdot t
\]  

Figure 8.2 shows the transition between time slice \( i - 1 \) and time slice \( i \) without any assistance of PGIs.

As shown in Figure 8.2, \( N^i_s \) can be found based on Eq.8.4 and \( N^i_p \) can be found based on Eq.8.5 which also uses the values of previous time slice (i.e., \( p^{i-1} \) and \( N^{i-1}_s \)). Then, \( p^i \) can be found using Eq.8.1

**Optimal searching time with PGIs**

Using the same dataset, the model below estimates the optimal cruising time assuming PGIs are installed. To do so, we define several variables for a system with PGIs: \( p^{i}_{opt} \) is the probability to find parking during time slice \( i \), \( \text{occ}_{opt}^i \) is the parking occupancy at the beginning
of time slice \( i \), \( N_{p_{opt}}^i \) is the number of available parking spaces at the beginning of time slice \( i \), \( N_{s_{opt}}^i \) is the number of searching vehicles at the beginning of times slice \( i \), and \( T_{opt} \) is the total searching time during \( i \in [1, I] \).

The initial conditions can be considered the same as the situation without PGIs, i.e., \( \text{occ}_{i=1}^i = \text{occ}_{i=1}^1 \) and \( N_{s_{opt}}^i = N_{s_{opt}}^1 \). For \( i \in [2, I] \), the formulations of \( \text{occ}_{i_{opt}}^i \), \( N_{s_{opt}}^i \) and \( T_{opt} \) are shown in Eq. 8.9, Eq. 8.11 and Eq. 8.12, respectively.

- Recall that, \( p_{opt} \) can be found through Eq. 8.2. Since the time slice is introduced in this section, \( p_{opt}^i \) is written as Eq. 8.8 to avoid confusion.

\[
p_{opt}^i = \begin{cases} 
\frac{N_{p_{opt}}^i}{N_{s_{opt}}^i}, & \text{if } N_{p_{opt}}^i \leq N_{s_{opt}}^i \\
1, & \text{if } N_{p_{opt}}^i \geq N_{s_{opt}}^i 
\end{cases} 
\tag{8.8a}
\]

\[
p_{opt}^{i-1} \cdot N_{s_{opt}}^{i-1} \tag{8.8b}
\]

- \( \text{occ}_{i_{opt}}^i \) is the predicted parking occupancy at the beginning of time slice \( i \) with PGIs. With the aid of PGIs, more searchers can find parking and this can increase the parking occupancy. \( \text{occ}_{i_{opt}}^i \) is calculated according to Eq. 8.9.

\[
\text{occ}_{i_{opt}}^i = \text{occ}_{i_{opt}}^{i-1} + \frac{1}{A} \cdot (p_{opt}^{i-1} \cdot N_{s_{opt}}^{i-1} - N_d^i) 
\tag{8.9}
\]

In Eq. 8.9, \( p_{opt}^{i-1} \) can be found through Eq. 8.8. \( p_{opt}^{i-1} \cdot N_{s_{opt}}^{i-1} \) represents the number of searchers that found parking with the aid of PGIs in the previous time slice, i.e., number of parking spaces that are newly occupied; \( N_d^i \) represents the number of vehicles that departed parking in the previous time slice from Eq. 8.6. Notice that \( N_d^i \) was not needed for the system without PGIs as we directly recorded the occupancy from the real data. In this case, however, we need to estimate the occupancy. As stated before, \( N_d^i \) is independent of whether we implement PGIs or not. Therefore, \( p_{opt}^{i-1} \cdot N_{s_{opt}}^{i-1} + N_d^i \) represents the net change of occupied parking spaces in the time slice \( i - 1 \).

- The number of available parking spaces with PGIs, at the beginning of time slice \( i \), is written in Eq. 8.10.
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\[ N_{p_{opt}}^i = (1 - occ_{opt}^i) \cdot A \]  \hspace{1cm} (8.10)

- \( N_{s_{opt}}^i \) is the predicted number of searchers at the beginning of time slice \( i \) with PGIs. It is calculated according to Eq.8.11. In Eq.8.11 \( p_{opt}^{i-1} \) can be found through Eq.8.8. The rest is similar to Eq.8.5:

\[ N_{s_{opt}}^i = N_{s_{opt}}^{i-1} \cdot (1 - p_{opt}^{i-1}) + N^i \]  \hspace{1cm} (8.11)

- \( T_{opt} \) is the total cruising time over the whole period \( i \in [1, I] \) with PGIs. It is calculated according to Eq.8.12. This equation is similar to Eq.8.7:

\[ T_{opt} = \sum_{i=1}^{I} N_{s_{opt}}^i \cdot t \]  \hspace{1cm} (8.12)

Figure 8.3 shows the transition between time slice \( i - 1 \) and time slice \( i \) where PGIs are deployed.

![Diagram of transition steps between time slice \( i - 1 \) and time slice \( i \) with PGIs.](image)

As shown in Figure 8.3, first, \( occ_{opt}^i \) can be found based on Eq.8.9 using results obtained in the previous time slice and \( N_d^i \) which can be obtained by Eq.8.6; second, \( N_p^i \) and \( N_{s_{opt}}^i \) can be found based on Eq.8.10.
and third, $p_{opt}^i$ can be found using Eq.8.8.

**Potential searching time reduction due to PGIs**

As $T$ and $T_{opt}$ are found, the total reduction in cruising time during this period can also be formulated.

$$T - T_{opt} = \sum_{i=1}^{l} (N_{si}^i - N_{s_{opt}}^i) \cdot t$$

(8.13)

In the next section, a numerical example is given to illustrate the use of the proposed methodology.

### 8.5 Application

In this section, a case study is provided based on the central shopping area Jelmoli in the city of Zurich, Switzerland. This area is located in the old town with many shopping centers but also a lot of offices from the financial sector (see Figure 8.4). The radius of the area is 300 meters. The length of the network within this area is approximately 7.7 km-links, i.e., $L=7.7\text{km}$.

The area contains a total of 539 parking spaces for public usage, including 207 on-street parking spaces and 332 off-street parking spaces (parking garage Jelmoli with 222 spaces, and Parking garage Talgarten with 110 spaces). As the area is rather small, we consider the 539 parking spaces as the capacity of the parking supply, i.e., $A=539$. Additionally, we assume the travel speed of vehicles is $v=15\text{ km/h}$, this is considered as the average speed in the network taking into account the stopping time at intersections. This value is consistent with observations in the city of Zurich (Ortigosa et al., 2014). In the numerical example, the average travel speed is assumed steady (i.e., no traffic congestion). However, in other cases, this value can be modified over time as needed.

Figure 8.5(a) shows the number of vehicles that enter the area (the data is aggregated into 1 minute intervals), i.e., $t = 1\text{ min}$. The daily traffic data arriving to this network has been simulated with the agent based model MatSim (Waraich and Axhausen, 2012) based on previous data measurements. The parking occupancy in the area (Figure 8.5(b)) has been simulated using the model from Cao and Menendez (2015b) and presented in Chapter 2. The results have been validated with real
parking data from the city of Zurich (http://www.pls-zh.ch/), and represent realistic conditions. In other words, \( \text{occ}^i \) values for each minute within the analysis period (i.e., one day), \( i \in [1, 1440] \) are known. Details on the validation can be found in section 3.4.

As shown in Figure 8.5, the peak of traffic demand entering the area occurs between the hours of 07:00 and 14:00. The highest value is 9 vehicles per minute and it happens around 09:30. Additionally, the parking occupancy grows rapidly between 04:00 and 11:00. The system gets saturated around 11:00, and becomes undersaturated again around 15:00.

Notice that Figure 8.5(a) does not represent \( N^i \), since it also includes trips heading towards dedicated/private parking spaces. To find out the portion which uses public parking spaces, the daily traffic data was calibrated with the public parking usage in the area. The results show that the public parking spaces only accommodated approximately 77% of the trips. In other words, the other 23% uses private parking.

If we assume vehicles start to search since they enter this small area, then \( N^i, i \in [1, 1440] \) is 77% of the traffic demand shown in Figure
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Figure 8.5: Model input (case study): number of vehicles entering the area and parking occupancy.

8.5(a). Here for simplicity, we assume this proportion remains invariable throughout the day.

The following subsections describe the parking search conditions without and with PGIs, and show the potential reduction of searching time due to PGIs.

8.5.1 Parking conditions without PGIs ($N_s^i$ and $N_p^i$)

For $i \in [1, 1440]$, the number of searching vehicles $N_s^i$ is found through Eq. 8.1 and 8.5 based on the traffic entering the area and the parking data; the number of available parking spaces $N_p^i$ is obtained through occupancy data (Eq. 8.4). The curve is basically the vertical reflection of Figure 8.5(b). They are plotted in Figure 8.6(a) and (b) respectively.

The combination of these figures show that:

- Between 00:00 and approximately 11:00:
  - The number of searching vehicles increases at a relatively low rate.
  - The number of available parking spaces increases first until around 04:00, and then drops continuously until all parking spaces are occupied. The first increase is given by vehicles that started parking the day before and leave after midnight. The drop afterwards happens because of new vehicles coming
between 11:00 and 15:00:

- The number of searching vehicles increases significantly and reaches a peak of approximately 30 vehicles at around 12:00. Then it starts to fluctuates. At around 14:00 it reaches another peak of 28 searching vehicles, then it starts to drop until the end of this period.

- All parking spaces are occupied during this period, i.e., when a parking space becomes available, it gets immediately taken.

between 15:00 and 24:00:

- The number of searching vehicles stays at a very low level, i.e., between 0 and 5 vehicles.

- The parking system becomes undersaturated, more and more parking spaces become available. By midnight, around 350 parking spaces are available.
8.5.2 PGIs effectiveness analysis

8.5.2.1 Static analysis ($\beta$)

Based on $N^i_s$ and $N^i_p$, $i \in [1, 1440]$, the value of $\beta$ can be found over time using Eq. 8.3. Figure 8.7 illustrates the results.

![Figure 8.7: Value of $\beta$ over time.](image)

As shown in Figure 8.7, the high values of $\beta$ occur in two periods: 10:30-11:30 and 15:00-16:00. As a matter of fact, these periods correspond to an approximate range of parking occupancy within (95%, 99%). The first period exhibits an increasing parking occupancy, and the second period exhibits a decreasing parking occupancy. In other words, in this example, the most effective periods of PGIs happen when the parking system is either approaching the saturation state (loading), i.e., 10:30-11:30 or leaving it (unloading), i.e., 15:00-16:00. Notice that, $\beta$ is only an indicator, it does not convey any practical meaning.

8.5.2.2 Dynamic analysis of time periods with high PGIs effectiveness

In this section we analyze the potential reduction of cruising time due to PGIs during the 10:30-11:30, and 15:00-16:00 periods, using Eq. 8.5-8.13. Figure 8.8 shows the cumulative number of vehicles that find
parking, with and without PGIs. Figure 8.8(a) corresponds to the time period between 10:30 and 11:30. Figure 8.8(b) corresponds to the time period between 15:00 and 16:00. In both graphs, three curves are shown. The curve at the top represents the cumulative number of vehicles that start searching. The curve in the middle represents the cumulative number of vehicles that find parking with PGIs. The curve at the bottom represents the cumulative number of vehicles that find parking without PGIs. In both figures, a background flow rate of 2.2 veh/min (i.e., 132 veh/h) is deducted so that the curves become distinguishable.

![Figure 8.8: Cumulative counts of vehicles that start searching and find parking over time (with and without PGIs).](image)

Not surprisingly, the cumulative number of vehicles that find parking with the aid of PGIs is always larger than the number of vehicles that find parking without PGIs. The area between the curve at the top and the curve at the bottom represents the cruising time without PGIs. It is equivalent to 435 minutes (7.25 hours) of cruising-for-parking in Figure 8.8(a) and 391 minutes (4.5 hours) in Figure 8.8(b). The area between the curve at the top and the curve in the middle represents the cruising time with PGIs. It is equivalent to 180 minutes (3 hours) in Figure 8.8(a) and 191 minutes (3.2 hours) in Figure 8.8(b). The area between the curve in the middle and the curve at the bottom represents the cruising reduction due to PGIs. In Figure 8.8(a), PGIs save 255 minutes (4.25 hours) of cruising time (59%). In Figure 8.8(b), they save 200 minutes (3.3 hours) of cruising time (51%).

In other words, during these two periods, the total time spent on
searching for parking is 13.75 hours. By implementing PGIs, 55% (7.55 hours) of this cruising time can be saved.

However, as mentioned before, except for these two periods, PGIs are not very effective. As a matter of fact, the potential savings in cruising time for the other periods of the day is only 411 minutes total (savings of 7%). Table 8.1 lists all the values of cruising time with and without PGIs for the whole day (00:00-24:00), the high PGIs effectiveness time periods, and the low PGIs effectiveness time periods, respectively.

Table 8.1: Model output (case study): cruising times and reductions due to PGIs within different periods of the day.

<table>
<thead>
<tr>
<th>Period</th>
<th>whole day</th>
<th>high PGIs effectiveness</th>
<th>low PGIs effectiveness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>00:00-24:00</td>
<td>10:30-11:30</td>
<td>15:00-16:00</td>
</tr>
<tr>
<td>$T$</td>
<td>6310 min</td>
<td>435 min</td>
<td>391 min</td>
</tr>
<tr>
<td>$T_{opt}$</td>
<td>5444 min</td>
<td>180 min</td>
<td>191 min</td>
</tr>
<tr>
<td>Reduction</td>
<td>866</td>
<td>255 min</td>
<td>200 min</td>
</tr>
<tr>
<td>Percentage</td>
<td>14%</td>
<td>59%</td>
<td>51%</td>
</tr>
</tbody>
</table>

The case study, despite its simplicity, shows that, during rush hours or under specific conditions where the number of searching vehicles is similar to the number of available parking spaces, PGIs can assist in the reduction of cruising time significantly. However, PGIs yield limited benefits outside of those periods.

8.6 Conclusions

In this Chapter, we propose a new methodology to analyze the effectiveness of parking guidance and information systems (PGIs) in reducing parking search time. Most of the existing literature has estimated this through empirical methods where surveys were conducted to evaluate the changes on parking choice and travel behavior. However, until now, to the best of the authors’ knowledge, a generalized method to evaluate parking search reduction due to PGIs did not exist.

Here, a model based on probability theory and traffic flow theory is proposed to, macroscopically, estimate the likelihood of finding parking
with and without the assistance of PGIs. Based on that, the model is further developed to estimate the cruising time reduction within specific periods of time. Results show that, for any given parking occupancy, PGIs are most effective when the number of searching vehicles is similar to the number of available parking spaces. As the ratio between the number of searching vehicles and the total parking supply is normally kept to a relatively low level, this implies that the most effective periods of PGIs happen when the parking system is approaching or leaving a saturation state (where the ratio between the number of available parking spaces and the total parking supply is also low). When the parking system is approaching saturation, the number of searching vehicles accumulates to a similar level as the number of available parking spaces. When the system is leaving saturation, the number of available parking spaces increases to a similar level as the number of searching vehicles. During these time periods, the cruising time saved through PGIs can be significant. However, except for these time periods, the PGIs are not very effective. When parking occupancy is low, vehicles do not need to search. When the parking is full and the searching demand is high, a parking space will be immediately taken when it becomes vacant. Hence, city governors or traffic/facility managers of an area need to balance the cost of installing and operating PGIs with the benefits they provide in reducing cruising time.

Besides its novelty, the model proposed here has additional advantages. First, the model is macroscopic which means that the requirements for data inputs are kept to a minimum. As a matter of fact, only five inputs are needed to run the model: the average travel speed of the searching vehicles \( v \); the network length \( L \) (km), the total size of the parking supply \( A \), the parking occupancy, and an estimation of the parking demand. Additionally, the computational cost is rather low, and the outputs (e.g., equations and results) are produced in a simple manner.

Second, the model provides not only generalized results, but also detailed formulations to more accurately estimate cruising time savings under dynamic conditions. The generalized results, including the \( \beta \) (i.e., PGIs' effectiveness indicator) pattern shown in Figure 8.1, hold for all conditions. The combinations of \( N_s \) and \( N_p \) that correspond to high (or low) PGIs effectiveness can be directly found without any knowledge of model inputs. The detailed formulations, on the other hand, are advantageous because they incorporate time-varying conditions such as the number of searchers and the parking occupancy. More
important, they provide quantitative results such as the reduction of cruising vehicles over different time periods of the day.

Third, given its flexibility, the model can be used not only with on-street parking systems but also private parking garages where cruising typically occurs inside the building. Additionally, results can be used for different purposes such as promoting PGIs, evaluating existing PGIs, or suggesting prices for private services on parking information.

That being said, as of now, the model only considers a small area where the network, the overall parking supply (not only the available ones), and the parking demand are homogeneous. Although city centers often contain small zones where our model can be directly applied, an extension towards heterogeneous networks with various parking patterns (e.g., different densities or prices) can be incorporated in future studies. Also notice that the results shown here only represent the maximum benefit that can be potentially achieved by using PGIs. In other words, the effectiveness level of PGIs could in reality be lower than that modeled.

Despite these limitations, the proposed model can be used efficiently to evaluate the effectiveness of parking guidance and information systems. It can predict the parking search conditions assuming PGIs are implemented, based on little knowledge of the system. It provides both generalized conclusions on cruising reduction and specific estimations under dynamic conditions. From the pragmatic perspective, the model can be used to design standards for the cost of private services on parking information, guide local authorities or developers on facility investment.
Chapter 9

General Conclusions

9.1 Summary

This dissertation looked at various aspects revolving around one concept: urban parking provision and urban traffic performance are inextricably linked with each other. Analyzing the two systems as a whole leads to a more comprehensive and efficient solution in the long term.

The urban parking system has always been a controversial point in the process of urban planning and development, because decisions on it are influential to many stakeholders who have thoroughly different perspectives and interests. There are in general four different kinds of stakeholders, their perspectives are described below.

- Car owners (individual travelers): for historical reasons, parking has long been considered as a public good and its proper provision (affordable, easy to access and near to destination, etc.) seems to be a compulsive job for the government to fulfill.

- Urban residents and local authorities: parking supply motivates private transportation and thus have a significant influence on car ownership and traffic demand. This can further affect the local environment and the area livability.

- Government. Parking relates to three important roles for the government:
  - first, government is responsible for social services including parking;
– second, government benefits from parking. Parking services might generate a considerable amount of revenue for the government, this includes both the regular parking payment and the violation fines;

– third, government cares about long term outcomes such as future use and urban development, especially because parking might consume a large amount of urban space.

- Developers (estate or logistic companies, etc.) and local business (restaurants and shops, etc.): parking policy influences developers’ investment decisions on location, scale and pricing. Also, for local business, parking provisions might affect the number of customers and the economic development of the region.

Taking then into account the perspectives from the different stakeholders, we find ourselves facing a series of questions: Should we provide parking? If yes, how many and how should we distribute them? What policy should we adopt on the pricing and the time limit? Also, more importantly, how would the traffic respond to the decisions we have made (especially in the short term)? For example, how much traffic delay can be caused by parking under certain circumstances? How congested (or uncongested) would the system become if certain parking policies are introduced?

In this dissertation, we endeavor to build methodologies to model and evaluate the interactions between the parking and traffic systems and to provide answers to some of these questions.

- In chapters 2 and 3, a model is built to estimate the conditions of both traffic and parking systems in the urban area based on certain assumptions of the network, the demand and the parking system. The model is time dependent, this means the time varying conditions of the system are immediately taken into account during the next time slice, for example the number of available parking spaces or the travel speed of vehicles on the streets. This makes the model more realistic.

In the end, the model provides not only the parking usage and traffic conditions (e.g., speed) at any given time, but also the number of vehicles searching for parking and the percentage of such vehicles on the whole network. Based on that, we can also evaluate the parking-caused traffic delays.
The model can be used to evaluate parking search conditions in a given city or area, or to test different parking policies and their effects on traffic performance. While chapter 2 focused on describing and building the methodology, chapter 3 deepened the understanding of the model and showed several examples of how to apply it in real situations. As it is a macroscopic model, the data requirements and computational cost are both kept to a minimum. It is shown in chapter 3 that the model is not only generalized (can be applied in very different situations), but also easy to use (only very basic data inputs are required.)

- In chapters 4, 5 and 6, the influences of different on-street parking maneuvers on the service rate of nearby intersections are discussed. Models are built based on the hydrodynamic theory of traffic flow, in order to analyze the perturbation caused by the parking maneuver.

  In chapter 4, the downstream parking maneuvers (with respect to the intersection) are investigated. It describes the situation where the queue caused by downstream maneuvers spills over to the intersection and blocks vehicles from discharging.

  In chapter 5, the upstream parking maneuvers (with respect to the intersection) are investigated. It describes the situation where the maneuver blocks vehicles from arriving to the intersection, and starving the intersection of flow.

  Both situations may generate a service rate reduction not only at the current signal cycle, but also lingering over several future cycles. Such cases are specially analyzed since their effect may spread over to the whole network. Based on this analysis, in chapter 6, the parking locations on a road link which do not generate service rate reduction to the nearby intersections are proposed.

- Chapters 7 and 8 describe some parking data related topics.

  In chapter 7, parking patrol surveys are investigated, providing a method to evaluate and improve survey accuracy without increasing (and possibly even lowering) cost. The relationship between the survey budget, the estimated average parking duration (i.e., the survey result), and the survey error is illustrated through dimensional analysis and well validated with real parking data. The results from this chapter can help surveyors to better understand
patrol surveys and obtain high-quality results, while keeping costs to a minimum.

In chapter 8, parking guidance and information systems (PGIs) are investigated. It provides a generalized methodology to evaluate the potential reduction of cruising time through PGIs. Based on probability theory, the relation between the system cruising time reduction, the parking occupancy, and the number of searchers is found. The results showed that, cruising time saved through PGIs can be significant when the parking system is approaching or leaving the saturation state (i.e., 100% occupancy). This is valuable as the peak periods for parking may occur within congestion periods, when removing cruising may highly improve traffic performance. However, except for these periods, the PGIs are not very effective. Some examples are shown, aiming to illustrate how stakeholders (e.g., owners and developers of private parking houses, traffic managers, local authorities and consultants on the parking industry) can use this model.

9.2 Outlook

This dissertation brings some new ideas to look at the relation between urban parking and traffic performance. The methodology used is solid and mostly validated with real data. However, given the complexity of both parking and traffic systems, there are plenty of extensions and improvements that can be made. The following extensions are suggested.

- The integration of demand in response to parking policy:

  Since the focus of this dissertation is on traffic operations and performance, we have paid little attention to the changes in travel demand, but more to the changes in traffic conditions in response to a given demand.

  Taken as a background condition, in our model, the travel demands are given as time-dependent rates. Although this is already more realistic than fixed rate assumptions, it does not reflect the choice of travelers as a result of the parking attributes such as parking search time.

  Given the traffic demand itself is a huge topic to research on, and possibly highly depend on the local conditions (travel behavior, preference, culture, etc.), it might be a good idea to start the
integration of parking demand to our model with a case study in a given city.

- The applicability to larger networks:

In our models, we focused on urban areas where the traffic and the parking spaces are more or less homogeneously distributed. This, however, requires the area to be comparatively small and uniformly designed. For example, the width of the streets and the traffic that different roads can carry should be similar, each parking space on the network should be equally attractive (as trip destinations), drivers should be indifferent to the location where they park.

These assumptions made it possible to build a macroscopic model which has many merits from the modeling and computational point of view. However, it restricts the representation of a large network.

Therefore, it is a very meaningful extension to have multiple networks where the spatial distribution of traffic demand and parking supply can be described. One can start building the model by combining multiple small but homogeneous networks.

- The integration of parking pricing:

Parking pricing, as a main tool to mitigate parking-driven traffic congestion, is a very effective measure to adjust travelers’ mode choices. In other words, it works on demand of private transportation. However, parking pricing, is also part of transport economics and relates to revenue management, social equity and local development. Therefore, it is more connected to parking than simply through the traffic demand.

By creating the macroscopic model, we now have a methodology to look at the parking search time as a function of demand and time. This offers a new perspective for parking pricing, as parking search can be seen as a tradable element of the overall trip cost.

- The understanding on the influence of urban parking on traffic properties in urban areas:

Urban traffic is a self-organized system, it behaves in a certain way. For example, the Macroscopic Fundamental Diagram has been introduced to described such behavior or property. However, as far as the authors’ see it, this property has significant
connection to parking settings because parking is the final destination of traffic (vehicles). In other words, parking can be a main constraint for it. Therefore, it could be a real interesting and promising direction to work on, especially for urban areas with oversaturate parking conditions, for example, the capacity of the system without any parking constraint.

- The extension to creative concepts such as dynamic parking lane and parking reservation systems:
  Dynamic parking lane means a road/lane which can have two functions according to the current time and traffic state: a parking lane or a driving lane (normal lane). From chapter 4 and 5, it is already shown that parking maneuvers may have significant impact on traffic delay (or not at all), depending on the traffic flow. Therefore, dynamic parking lanes could be a potential idea for cities that have very different traffic demands throughout a day.

  Based on the model developed in chapter 1, parking reservation systems, could be potentially used for rush hour parking. This will reduce the parking search time in the system, correspondingly mitigate congestion and increase the system efficiency.


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PUBLICATIONS


4. Cao, J., M. Menendez (2013) Methodology to evaluate cost and accuracy of parking patrol surveys. Transportation Research


Under review:

1. Cao, J., M. Menendez and R.A. Waraich (2016) Impacts of urban parking system on cruising traffic and policy development: The case of Zurich downtown area, Switzerland. Submitted to Transportation.


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