

# Who speaks mathematics: a semiotic case study

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**Publication Date:**
2010-03

**Permanent Link:**
https://doi.org/10.3929/ethz-b-000121576

# WHO SPEAKS MATHEMATICS?
# A SEMIOTIC CASE STUDY

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## 1. Introduction

This paper uses a case study to perform a semiotic analysis of the enunciative position in a mathematical text. My attempt is to demonstrate the (post)structural complexity of this enunciative position in order to link the research of mathematical textual practices to perspectives of critical theory.

Traditional philosophy of mathematics accounts for the mathematical subject from epistemological and ontological vistas. The subject position is hardly ever taken as a main concern, but it does come up, and requires accounting for. Links between various forms of intuitionism and platonism on the one hand and Kantian or Husserlian transcendental subjectivity on the other appear to provide such accounts (e.g. [van Atten], [van Atten & Kennedy] and [Tieszen]). Formalism, on the other hand, flirted with crossing the line between subjectivity and mechanical processing, especially around the notion of Turing' thinking machine.

The linguistic turn brought with it Wittgenstein's philosophy of mathematics ([Wittgenstein1] and [Wittgenstein2]), where the subject position is populated by an interactively learning, rule following, symbol manipulating form of life. More recent sociological accounts developed in the context of Science Studies led to the articulation of a social subject of mathematics (for a 'generic' version see [Ernest1], for some culture and gender specific discussions see [Ernest2]). The only theoretic structure, however, which focuses primarily on the role of the mathematical text's enunciative position as a semiotic agency, was presented by Rotman in [Rotman1] and [Rotman2].

What seems to be common to all the above discussions of mathematical subject and enunciative positions is an overwhelming tendency to avoid working with actual mathematical texts. The analyses are justified by imagined, generic and abstract mathematical texts, with an occasional allusion to some basic arithmetic or formal logic. While this might be somewhat excusable when discussing mathematical transcendental subjects, this is no longer legitimate when discussing social subjects and the semiotic function of a mathematical enunciative position (some recent research into mathematical semiotics does analyse concrete textual examples, e.g. [Lefebvre 2002] and [Netz], but such research is not centred around analyses of the enunciative position).

In order to rectify this situation, this paper will perform a hands-on analysis of the enunciative position in a specific case study. The texts I chose to investigate are Gödel's proofs of his first incompleteness theorem. Such concrete analysis, of course, has its price: it cannot be immediately generalised to *any* mathematical text. Indeed, Gödel's texts have many exceptional features. But the status of this analysis depends on the articulation of the enunciative position in Gödel's text, which, as the examples quoted in this paper demonstrate, is not all that exceptional. At any rate, regardless of how (a-)typical these texts are, even a single case study can demonstrate the potential complexity of mathematical enunciative positions, and embark on a journey to counter stereotypes that reduce them to simplistic, robust, or clear and distinct arrays. I hope that the restricted *vignette* which follows will motivate analyses of other case studies, which may eventually allow for a comparative semiotic study of the multiform variety of contemporary and historic mathematical enunciative positions.

I read Gödel's proofs in two texts: van Hijenoort's 1967 translation of the original paper from 1931, and the 1965 published notes of the 1934 Princeton lectures. Both versions were approved and revised by Gödel himself. References to the texts will be denoted by [1931] and [1934] respectively, and page numbers will refer to the [Gödel] edition.

I made an effort to keep the analysis below as untechnical as possible, and to choose examples which do not require going into mathematical detail. But some introduction to the argument is, of course, indispensable. The best introduction to Gödel's proof for an uninitiated reader is, as far as I can tell, still [Nagel & Newman]. For those who can settle for a very brief 'list of ingredients' survey of the theorem and proof, I present a very brief sketch in the appendix.

Theoretically, the analysis in this paper draws mainly on Barthes' *S/Z* [Barthes1] and Foucault's *Archeology of Knowledge* [Foucault1]. Earlier drafts of the work presented here had more explicit references to the concepts and techniques developed in these books. However, the streamlining of the presentation left little explicit evidence for the role of these authorities in the formation of this work, with the exception of a couple of quotations, some guiding questions, and a general mood.

The main conclusion of my analysis is that the mathematical text's successful functioning (making sense) depends on an enunciative position that forces an overlap between voices and modalities which are not quite clearly distinct. My claim is that the text makes sense by combining conflicting voices and modalities, and moreover, would fail to make sense if it were to clearly distinguish and separate these conflicting voices and modalities. This situation is not a failure of Gödel or of mathematics, and is not a hitch that needs mending by philosophers or semioticians. This situation, I claim, as in the case of many other kinds of texts, is a constitutive condition for the successful functioning of a mathematical text.

One of the referees of this paper made the following objection concerning my level of analysis: **In case my complaint about remaining at the rhetorical level is not clear, let me point out that the standard epsilon-delta way of writing about limits, which has to do with the succession of quantifiers for all epsilon there exists a delta ... . This is sometimes written as a mock dialogue. If you choose epsilon, then I can choose delta ... . That does not turn the argument into such a dialogue. If one were to analyse such a dialogue, one would have to reconstruct it in terms of quantifiers to see the structure of the argument**[1]. My stance in this paper is diametrically opposite. I am concerned here with mathematical practice, not with

[1] I employ the convention of using boldface for quotations.

formal reconstruction. mathematics can be practiced without formal reconstructions, but has never histrocially existed without surface dialogues. I do not mean to dismiss formal reconstructions. I do object to the dictum that one **would have to reconstruct it in terms of quantifiers** to analyse the successful functioning of mathematics. I am trying to study how mathematics works, not how it should work.

The analysis is not completely at odds with a Wittgensteinian analysis. However, where a Wittgensteinian analysis seeks to set apart and distinguish the different language games involved in mathematical practices (e.g. [Wittgenstein2, 118–122], especially §8), I point out that it is precisely the 'undecidable' meshing together of different language games that allows mathematics to make sense.

My approach employs techniques of structural discourse analysis in order to diagnose the different voices and modalities in the text. However, by problematising the articulation into voices and modalities my analysis undermines the distinction between elements of the analysis *found within the text* and analytic elements forced *on the text from without*. For these reasons the semiotic analysis presented in this paper can be labeled post structural.

The project presented here may be continued in two ways. The first is broadening the scope of the analysis towards an overall post-structural semiotic analysis of the text. This is done in [Wagner1] (for a different kind of post structural semiotic analysis of mathematical texts see [Wagner2]). The other direction is to extend the semiotic analysis of the enunciative position to other mathematical texts. I hope that this project too will get the attention it merits.

The paper is organised as follows. The first section brings up the structural complexity of the personal pronouns 'I' and 'we'. This analysis is carried out in a tradition that can be traced to many thinkers. Here I chose to refer to Benveniste due to his semiotic focus, and due to his concern with the quotability of the personal pronoun. The second section studies the enunciative position in the mathematical texts through the verbs it contracts. The analysis engages with Rotman's division of the mathematical enunciative position into three persons, critically examines the tenability of this division, and offers a more complex picture of the interaction between the various voices included in the enunciative position. The third section goes on to examine the enunciative position through the temporality, spatiality and modalities that it employs. Here too a modal web (or perhaps *rhizome*) is traced, which makes sense through its incongruent superpositions. The fourth section looks at the articulation of the enunciative position in the texts as polar to a constructed coalition of its contemporaries. The final section complements the above picture with a brief look at the few instances of placing the texts' enunciator in an object position. This last section also serves to conclude the argument and engage in some brief theoretic reflection. The appendix presents a brief summary of Gödel's argument.

## 2. We and I

The enunciative position in Gödel's texts is marked by a **we**. In order to study the enunciative position as articulated in Gödel's text, we shall analyse the grammar of this **we**. But before we do, I feel bound to acknowledge that we needn't read too much into this **we**. **We** is simply part of the code. Its use is as imposed upon Gödel as is using **I** when I speak to a friend. So we needn't read too much into **we**, no more, say, than Emile Benveniste reads into **I**.

**I**, Benveniste explains, is an interface between an individual and language. **Language is so organised that it permits each speaker to *appropriate to himself* an entire language by designating himself as *I*** [Benveniste, 226]. It is this precise interface, according to Benveniste, which turns the individual into a subject, and without which subjectivity could not be formed. ***I*** achieves that as it **refers to the act of individual discourse in which it is pronounced**. In fact, ***I* is "the individual who utters the present instance of discourse containing the linguistic instance *I*."** [Benveniste, 226, 218].

This unified functional view of appropriative **I** would be very convincing, if it weren't for the multitude of different **I**'s appearing in Benveniste's text. Despite the statement of the first quotation above the ***I***s in the previous paragraph do not function to appropriate language to individual, but are signifiers referring to this appropriative interface (as witnessed by the italics and the constructions ***I*** **refers** and ***I*** **is**, rather than 'I refer' and 'I am').

But this is not all. The non actual, non appropriative ***I*** breaks down into smaller constituents. The last quotation above, under pains of circularity, must generate two different grammatical positions (**referent** and **referee**, according to Benveniste). It begins with ***I*** ... **the individual**, and ends with **the linguistic instance** ***I***. Surely, even if **Man is a sign**, or as Benveniste puts it, **"subjectivity"** ... **is only the emergence in the being of a fundamental property of language** [Benveniste, 224], it is not the individual who is a linguistic instance (at least not according to any point of view which insists on articulating the individual and the subject as distinct).

We so far have an appropriative **I**, which a speaker designates himself as in order **to** ***appropriate to himself*** **an entire language**, the reported, non-actualised ***I***, who is the speaking individual who uses this interface, and the **linguistic instance** ***I***. If only things were so simple! consider that **the indicators** ***I*** **and** ***you*** **cannot exist as potentialities, they exist only insofar as they are actualised in the instance of discourse** [Benveniste, 224]. Here ***I*** is not that which a speaker designates himself as, but that which a speaker designates himself with (the speaker Emile does not designate himself as an indicator when he designates himself as ***I***). Nor is it the uttered linguistic instance, because an indicator is far more functional than a mere linguistic instance. But this ***I***, that is, the indicator, is obviously not the ***I*** who is **"the individual who utters** ... **"**, because indicators are not individual people. We therefore have a fourth, indicator ***I***. But then we're still not done. The ***I***, which appears in **the indicators** ***I*** **and** ***you*** **cannot exist as potentialities** can't be an indicator either, because **indicators** have, in that very sentence, been denied the possibility of existing un**actualised in the instance of discourse**, whereas the indicative function of the last ***I*** is still strictly a potentiality, the very potentiality which would indicate when and if it were eventually actualised in discourse. It is, so to speak, the signifier of the indicator.

It seems that whenever we try to articulate **I**, another structural dimension is forced upon it. Whatever instance of ***I*** we are given, science appears to objectify its function, and then signify that object. But there is yet another, deeper rooted reason for the plurality of **I**'s, which goes beyond the trap of infinite regress. For if ***I*** referred to a particular individual, **a permanent contradiction would be admitted into language, and anarchy into its use**. In fact, **Each *I* has a unique reference and corresponds each time to a unique being who is set up as such**. This is especially apparent when considering **I**'s quotability, that is the fact that **If I perceive two successive instances of discourse containing *I*, uttered in the same voice, nothing guarantees to me that one of them is not a reported discourse, a quotation in which *I* could be imputed to another**. On the other hand, and at the same time, **It is by identifying himself as a unique person pronouncing *I* that each speaker sets himself up in turn as the "subject."** [Benveniste, 218, 220, 226].

I am led to conjecture an extension to Benveniste's claim, which the following paragraphs will attempt to validate: *it is not only intersubjectivity, but substantial components of the very experience of making mathematical sense too, which arise, at least in part, from confounding all these* ***I****'s and all these functional positions.*

## 3. What we do

How does the enunciative position in Gödel's text, **we**, fare compared to Benveniste's **I**'s? Gödel's 1931 text contains only three **I**'s, and the 1934 text only one. These **I**'s are used in an acknowledgment, a reference to previous work, a reference to a lecture, and to comment on the relation between Gödel's work and Hilbert's programme. But the person narrating, articulating, and conducting the proof is **we**. And since analysing the grammatic structure of the personal pronoun has led us to a futile proliferation of structural positions, let us concentrate on what **we** do. Taking an inventory of Gödel's texts we find that **we** abbreviate, accomplish, add, adjoin, allow, apply, assign, associate, assume, attach, carry out, come, compare, consider, construct, deduce, define, denote, depend, describe, derive, eliminate, employ, establish, exclude, express, find, find convenient, generalise, give, have, include, insert, intend, let, list, make, make use, map, mean, note, observe, obtain, order, proceed, prove, put, replace, require, restrict, say, see, shift, show, sketch, substitute, take into account, turn to considerations, understand, use, wish, and write.

To make sense of what **we** do, let us follow Foucault, who suggests that a mathematical text articulates various subject positions. **Take the example of a mathematical treatise. In the sentence in the preface in which one explains why this treatise was written** ... **the position of the enunciative subject can be occupied only by the author** ... **only one possible subject** ... **On the other hand, if in the main body of the treatise, one meets a proposition like 'Two quantities equal to a third quantity are equal to each other', the subject of the statement is the absolutely neutral position, indifferent to time, space, and circumstances, identical in any linguistic system, and in any code of writing or symbolisation, that any individual may occupy when affirming such a proposition. Moreover, sentences like 'We have already shown that...' necessarily involve statements of precise contextual conditions that were not implied by the preceding formulation: the position is then fixed within a domain constituted by a finite group of statements** ... **The subject of such a statement** ... **will not be described as an individual who has really carried out certain operations, who lives in an unbroken, never forgotten time** [Foucault1, 94].

We may try to make systematic sense of these positions by relating them to the list of verbs above. Different verb types may articulate different functional positions taken by **we**. Fortunately, such typological analysis has already been conducted in the general context of mathematical texts by Brian Rotman in his [Rotman1] and in the first chapter of [Rotman2]. His account, which claims to derive from Peirce, is as follows.

Three characters take part in a mathematical text.

(1) The first is called the *Person*, who speaks in the meta-Code. He speaks about mathematics, but not in mathematics. He considers the idea or story behind the proof. He has access to natural language, the personal pronoun **I**, and indexicals such as **here** and **now**. His function is to be persuaded by the proof and to understand it.

(2) The second character is the *Subject*. The Subject is the entity which defines, derives, considers and proves. His pronouncements are, according to Rotman, voiced in a collective imperative: **let us consider, define, demonstrate...** [Rotman1, 71]. The Subject is abstracted of any temporal, spatial and cultural considerations. **The subject's psychology, in other words, is transcultural and disembodied** [Rotman2, 15]. The Subject makes mathematical statements, which are, in fact, predictions on the outcomes of sign manipulations. Thus, $x + y = y + x$ is the prediction that if he substitute any number-signs for $x$ and for $y$, and manipulate these signs according to the orders coded in the '+' sign, the results of the two sides of the equality will be the same.

(3) The last character is the *Agent*. The Agent is a reduced image of the Subject, who performs the sign manipulations, of which the Subject makes predictions. He is the one adding, counting, substituting in a sort of thought-experiment or a dream which the subject is experiencing. The **Agent, unlike the Subject, has no ability to imagine and can only respond to signs in their truncated, skeletonised form as signifiers devoid of intentioned meaning. In other words, the Agent is considered as an automaton, a wholly mechanical and formal proxy for the Subject** [Rotman1, 76]. The Agent's actions are expressed in an exclusive imperative mood[2] (add!, count!, integrate!).

The **Person constructs a narrative, the leading principle of an argument, in the meta-Code; this argument or proof takes the form of a**

[2]An exclusive imperative is an imperative which does not include the person giving the order. 'Eat!' is an exclusive imperative. 'Let's eat!' is an inclusive one.

**thought experiment in the Code; in following the proof, the Subject imagines his Agent to perform actions and observes the results; and in the light of the narrative, the Person is persuaded that the assertion being proved — which is a prediction about the Subject's sign activities — is to be believed** [Rotman2, 35]. **The hierarchy of semiotic agencies here, from imagined Agent to imagining Subject to indexically conscious Person structuring a thought experiment is isomorphic to that on which any dream rests: the Agent maps onto the figure dreamed about, the Subject the dreamer dreaming the dream, and the Person the dreamer awake, consciously interpreting and recognising the dream** ... **the dream-code** ... **is restricted in various ways, not least by the lack of the ability to recognise the dream *as* dream, which makes it impossible *for the dreamer* to articulate the dreamer's kinship to the imago he or she dreams into being. Likewise, the restrictive nature of the Code in which the Subject operates, in particular its lack of indexicality, prevents the mathematical Subject from articulating the status of *its* created fiction** [Rotman1, 78–79].

To explain his division (which is akin to — but does not refer to and is not identical with — Foucault's), Rotman appeals to the imagery of the dream. But his account fails to acknowledge a very unsettling, albeit not very common, occurrence: *lucid dreaming*. Lucid dreaming is the experience where a dreamer is aware that she is dreaming, and where control over the dream is negotiated between various subjective parts. We must not neglect the possibility that the dream-code may be lucid, and so, occasionally, we should consider waking from the lucid dream. Such experience, I recall, can be extremely unpleasant. Prominent physicist Richard Feynman, however, provides a somewhat more optimistic account. **During the time of making observations in my dreams, the process of waking up was a rather fearful one. As you're beginning to wake up there's a moment when you feel rigid and tied down, or underneath many layers of cotton batting. It's hard to explain, but there's a moment when you**

**get the feeling you can't get out; you're not sure you can wake up. So I would have to tell myself — after I was awake — that that's ridiculous. There's no disease I know of where a person falls asleep naturally and can't wake up. You can *always* wake up. And after talking to myself many times like that, I became less and less afraid, and in fact I found the process of waking up rather thrilling — something like a roller coaster: After a while you're not so scared, and you begin to enjoy it a little bit** [Feynman, 50]. Let us, then, muster the courage to elucidate Rotman's **dream**, and talk ourselves awake.

Rotman's divisions are based on both semantic and morphological markers. Morphologically, the Person is the one using the pronoun **I** as well as indexicals and cultural markers. The Subject is characterised by collective imperatives, and the Agent by exclusive imperatives. Unfortunately, such markers are absent from Gödel's text. Rotman claims that **Even a cursory examination of an arbitrary chosen item of mathematical communication will reveal two fundamental features of mathematical discourse: its organisation as an exhoratory, command-giving formalism and its complete lack of any indexical terms** [Rotman1, 71]. I do not dispute that if one tries hard enough, one could find such mathematical texts (some concise accounts of geometric constructions might be a good place to look). But our **arbitrary** texts, Gödel's 1931 and 1934 texts (and, as far as I can tell, most mathematical texts), do not conform to these characterisations.

First, there are very few imperatives in these texts. Perhaps the imperative mood was considered ill-suited for civilised written communication in the Vienna and Princeton circles. Perhaps it never occurred to writers in these circles to dominate a mathematical text with the imperative mood. Instead of imperatives, we have the frequent use of the indicative mood in both active and passive voices, often attributed to the character **we**. In addition, the texts contain 9 **here**s and 33 **now**s, some of which would be difficult to marginalise to the **meta-Code**, as they are used to order and demarcate the formal deductive process. These indexicals form part of a network of locative and temporal adverbial structures which operates inside the text.

This does not mean that Rotman's analysis has broken down. The lack of morphological markers does not mean we cannot establish a structural division of the enunciative position. We may regroup verbs into subsets, and identify the Subject with occurrences of **we** bundled up with some verbs, the Agent with occurrences of **we** bundled up with others, and the Person as related to yet another set of verbs and adverbials. But here we should take into account one of Barthes' observations concerning the threading of sequences of verbs. **Actions**, he explains, **can fall into various sequences which should be indicated merely by listing them, since the** sequence of actions **is never more than the result of an artifice of reading: whoever reads the text amasses certain data under some generic titles for actions** ... **and this title embodies the sequence; the sequence exists when and because it can be given a name, it unfolds as this process of naming takes place, as a title is sought or confirmed; its basis is therefore more empirical than rational, and it is useless to attempt to force it into a statutory order; its only logic is that of the "already-done" or "already-read"** [Barthes1, 19]. A formal grouping of verbs risks being an arbitrary empirical compulsion rather than a well grounded structural conclusion.

Bearing this cautionary statement in mind, let's try to allocate some verbs appearing in the texts to the Person, Subject and Agent. Consider three examples. I do not introduce the details which are required for a technical understanding of the quotations below. But, as will become evident, the question of distribution of actions between characters does not depend on such understanding.

Example I: **we can, for example, find a formula** $F(v)$ **of *PM* with one free variable** $v$ **(of the type of a number sequence) such that** $F(v)$**, interpreted according to the meaning of the terms of *PM*, says:** $v$ **is a provable formula**. Footnote: **It would be very easy (although somewhat cumbersome) to actually write down this formula** [1931, 147].

This text appears in the introduction, in a paragraph which opens with the statement: **Before going into details, we shall first sketch the main idea of the proof**, and must therefore be attributed to the Person. But it is obviously not the Person who **find**s formulas. The Person is the one who might try to **write down** the formula, since he is the only one who has enough embodiment to experience how **cumbersome** it is. But **find**ing the formula, which in the context of this example is a tedious formal procedure, is a task which the Subject should narrate and the Agent should perform. Now, which is the character who does the **interpret**ing? Is it the Person who has an overview of all the different layers and significations of the argument, or is it the Subject, for whom interpreting would stand for establishing a correspondence between the provability of a meta-statement and the provability of a formula, which the Agent can obtain by symbolically manipulating the number sequence substituted for $v$? And once the Person, Subject and Agent have thus collaborated, how come it is the formula, rather than any one of them, who, like Balaam's ass, is suddenly conferred with the power to **say**?[3]

Example II: **we can define relations to be classes of ordered pairs, and ordered pairs to be classes of classes; for example, the ordered pair** $a, b$ **can be defined to be** $((a), (a, b))$ [1931, 153].

The notions **relation**, **ordered pair** and **class** are all already determined from the point of view of the mathematical Person. The authority to represent one by another is therefore contingent on the Person's consent, and it is therefore he who **can define**. But the one to actually make the definition, and perhaps prove the formal adequacy of the definition, is the Subject. Finally, it is the Agent, who, whenever hearing the Subject speak of **the ordered pair** $a, b$ must replace such sign sequence by $((a), (a, b))$, and only then carry out his further manipulative tasks. In the Person's voice, the above statement reads: I agree to define. In the Subject's voice it reads: I define (and, perhaps, verify that the definition is adequate). In

[3]Some logicians object that formulas don't say, and shouldn't be described as saying. This normative judgment is irrelevant here. I am analysing how a specific mathematical text works. According to this text, Formulas do say. In [Wagner3] I take more of an issue with attempts to silcene formulas.

the Agent's voice it reads: I make the symbolic manipulations required by the definition.

Example III: **We have noted that $x\mathcal{B}y$ is a recursive relation; and we can also prove that $\sigma(x,y)$ is recursive, where $\sigma(x,y)$ is the number of the formula which results when we replace all free occurrences of $w$ by $z_y$ in the formula whose number is $x$** [1934, 359–360].

This statement seems to have some verbs relating to the Subject (**note** and **prove**), and some to the Agent (**replace**). However, it also contains temporal and locative elements. The noting is articulated as having occurred in the past. This is a psychological and relative past. If for instance, I skip the text's discussion of recursive relations, and move directly to the main argument (as in fact I first did), then my reading Subject and Gödel's writing Subject get out of synch, and manifest a temporal relativity, whereas the mathematical Subject is supposed to be free of psychological temporality. Gödel's Subject's present-perfect **have** is my Subject's future-simple **will**. This de-synchronisation is in fact explicitly supported by the text, which states that the discussion of recursive functions is **a parenthetic consideration that for the present has nothing to do with the formal system *P*** [1931, 157]. The discrepancy between Gödel's Person and mine has seeped through to create a discrepancy between Gödel's Subject and mine. The embodied Person is allowed to carry the Subject along with it.

On the other hand, the locative preposition **in** is, as far as the text is concerned, an absolute disembodied location, which is invariant to any Subject and Agent (I do not claim that it is an absolute invariant, but only that the text articulates it as such). That this locative be a disembodied invariant is demonstrated by the text's claim that such manipulations can be mechanised. However, even Rotman allows a restrictedly embodied manifestation of locality into the mathematical code, as provided by the *zero* point of reference. **The point to be made here about "0" is that in practice its two senses — Coded number and metaCoded origin — are inextricable from each other: in the course of manipulating the number sign, the meta-sense is always present shadowing it, being**

**part of another layer of meaning which adjoins and penetrates the formal layer available in the code** [Rotman1, 75].

The three examples above *are* analysed according to Rotman's articulation, however problematic and obscure the analysis turned out. There is, however, nothing *in* the text to impose such analysis. Gödel's verbs and pronoun **we** do not seem to distinguish between the Person, Subject and Agent, but rather to bundle them seamlessly together. We saw that a single verb can order into action all three characters, and that their realms of authority are not clearly distinct. On top of this obscurity, a writer who were to take as a point of departure Foucault's examples quoted above, would likely end up with a somewhat different character distribution.

I find no explicit textual evidence that such triple articulation permeates the text. And yet, we cannot ignore the fact that we *can* indeed *rewrite* the text so as to conform to Rotman's grammar. The fact of this capacity must not be dismissed, because it is still a fact *of* the text. It is still a fact of the text that such a reading can be sustained, and as such this fact may very well be relevant to to the question of how the text works. This fact can contribute to our understanding of how the text interacts with possible readers, even though it is no more a fact *of* the text than the effect of an *outside* intervention. The fact that a structure, which is not *in* the text, can be imposed *upon* the text, that such imposition does effect some changes, but does not entirely break the text down, and that such imposition can appear, at least at first sight, quite convincing — these facts testify to a critical feature of mathematical texts: openness[4]. This is the same sort of openness, or rather tolerance, which allows the extraction from mathematical texts of platonist, logicist, formalist and intuitionist positions — even though these positions needn't be discoverable from analysing mathematical texts according to terms they explicitly state.

[4]An entirely different question is *for what purpose* we should want to impose upon a text Rotman's (or any other) articulation. Rotman uses his imposed structure to argue for a radically finitist view, which I respect, and, with some modifications, endorse. Rotman's rearticulation has its merits. But it is not 'discovered' within the text.

What we impose on the text from the outside is not without precedent inside. The roles suggested by Foucault and Rotman, as well as platonist, logicist, formalist and intuitionist metaphysics/epistemologies are not completely foreign to the text. They are not arbitrary chance constructs that have nothing to do with the mathematical text. They can all be supported to a small extent by evidence which is excavated from within the text. Indeed, If this had not been the case, no one would have been able to successfully impose these constructs on mathematical texts, or at least not in ways which appear convincing to the extent that they do. The mathematical text communicates with various structures, which are not completely foreign to it, but which are not properly endorsed by it either. This is precisely how a mathematical text can signify: by communicating with its outside, by incorporating inside a residue of its outside, by denying its outside the clear and distinct metaphysical status of outside.

This diagnosis of openness lies well within the range spanned from Barthes' and Foucault's notions of authorship in [Barthes2] and [Foucault2], through Eco's conception of *The Open Work* [Eco] to the Derridian maxim that **there is no outside-text** [Derrida, 158]. The novelty is in demonstrating that mathematical texts are not outside this range. Without such openness, without the text's incorporation of externally imposed enunciative positions, the text would simply not communicate with 'foreign' readers and hence soon fail to signify. But imposing upon **we** the three (Person, Subject, Agent) who are one is still awkward and contrived, and it is the interpretive gap between the text and the reconstruction that I seek to bring up and explore. Facing **we** with the question **What is thy name?** one should not confine to the asylum the possibility of **saying, My name is Legion: for we are many** (Mark 5, 9).

## 4. When, where and how we do it

Here is what we learnt from analysing the enunciative position's actions: that the text tolerates, at a price, divisions which are imposed on it, and allows trace evidence of such divisions to be excavated from its inside. This openness to an outside (which undermines a clear inside/outside division, because it blurs the line between imposing something from the outside and discovering it inside) is a crucial constituent of the text's semiotic capacity, but leaves us without a concrete analysis of the text's subject position. We shall therefore try to determine this position via the adverbials and modalities interacting with this position.

Temporality makes its mark on the text in various forms. Temporality appears in formal manipulations, which may take place one **after** another, as in: **we must therefore (1) eliminate the abbreviations and (2) add the omitted parentheses** [1931, 156]. Temporality also appears along the course of the argument, as in: **We have noted that $x\mathcal{B}y$ is a recursive relation** [1934, 359], and in **As will be shown later, however, the converse does not hold** [1931, 173]. Finally, temporality intervenes in reviewing and interpreting the text: **such a proposition involves no faulty circularity, for initially it [only] asserts that a certain well-defined formula ... is unprovable. Only subsequently (and so to speak by chance) does it turn out that this formula is precisely the one by which the proposition itself was expressed** [1931, 152].

But to appreciate the nature of this temporality, an important clue is provided by the transmutation of 'outside world' temporality into 'mathematical' temporality. This happens in the following passage: **Suppose that on 4 May 1934, $A$ makes the single statement, "Every statement which $A$ makes on 4 May 1934 is false." This statement clearly cannot be true. Also it cannot be false, since the only way for it to be false is for $A$ to have made a true statement in the time specified and in that time he made only the single statement** [1934, 362].

This form of the liar paradox is then translated into mathematical form. To do that, Gödel first constructs an explicit enumeration of all formulas in his formal system. Then a formula is constructed claiming that formula number so-and-so has a certain property $F$ (interpreted as being false). The construction is so manipulated that the number of that formula is the very number so-and-so mentioned inside the formula. The formula therefore says of itself that it has property $F$ (namely, is false). But where in the transition from **4 May 1934** to the self-referential statement did temporality disappear?

Temporality has been converted into enumeration. The date **4 May 1934** served only to designate $A$'s claim. This designation device is transformed into Gödel's enumeration. The form of temporality acknowledged here by the text is so reductive, it is not even properly serial. The numbers conferred upon formulas have nothing to do with the order in which they appear. Time is but a pool of events and designations, which excludes co-occurrence, but which does not display any equivalent of duration (some possible exceptions will be treated below).

But the most interesting aspect is not the articulation of time into a set of mere mutual exclusions, but the ambiguous relation between the enunciative position and this temporality. On the one hand, the enunciative position is within time, as the above quotes explicitly mark. However, the same subject has the capacity to review the entire temporal pool. What **will be shown** and what **we have noted** is readily available for the current moment of the discussion. Whatever happened **initially** and **subsequently** in the quotation standing four paragraphs above is bundled together in a contemporary moment where the proposition is self-referential, but **involves no faulty circularity**. Even hypothetical moments of an unspecified time can all be bundled up and reviewed together: for any relevant system of axioms $\kappa$, **there always are propositions** ... **that are undecidable** ... **as soon as** some specific proposition **is not** $\kappa$**-PROVABLE** [1931, 195].

The enunciative position, which is within time, and at the same time can observe time and even form it by the operation of enumerating its minimal elements — this complex position contributes to the complex process of semiosis. First, temporality as a set of mutual exclusions registers the distinct elements (formal propositions) studied by the mathematical text. Second, temporality as a sequence of past and future occurrences helps order and narrate the text for the reader. Finally, by taking a stance which can inspect all these different times Gödel counters the grave accusation that his argument is groundless, and shows that his **proposition involves no faulty circularity**. In so far as temporality is reduced to the articulation of mutual exclusive elements, there is a sort of circularity (self reference) — the referent formula and referee formula are one and the same. But in so far as temporality has a before and an after, in so far as **initially it [only] asserts** ... and **Only subsequently (and so to speak by chance) does it turn out** ... — in that sense we obtain a non-circular turn of events.

The extraction of meaning from the text depends here on instilling a difference across the self-referring instance. The fact that the order of arguing and reading is not absolute, but revisable by any reader, does not weaken the argument that depends on its distinct temporal sequencing. The fact that one has to adopt an omni-temporal point of view in order to complete the story does not weaken it either. But the enunciative position must be able to endorse *all* these conceptions of temporality in order to put Gödel's argument together. The argument's validity depends on a multiplicity within the enunciative position. If the enunciative position were severed into several positions, each endorsing only one conception of time, the argument will simply not stick together.

Some researchers insist on discarding the narrative, and reading the proof 'purely formally', without retaining the notions of reference and expression operative in the proof. This reading is of course tenable, but leaves the reader in a senseless position. In this reading all that Gödel's theorem provides us with is a couple of highly complex arithmetic propositions concerning some very big numbers. Unless these numbers are allowed to refer to formulas, there is no incompleteness theorem. But we may go even further: if we refuse to let signs refer, all we have is a bunch of symbols that obey some combinatorial restrictions — not even an arithmetical statement. At any rate, I am interested here in following Gödel's (widely acclaimed as brilliant) texts, not the dull reconstructions that some authors would like to impose on them.

The notion of space in the text is usually confined to a relative location in a sequence. This relative, disembodied, notion of space is further abstracted by formulations such as: **a formula will be a finite sequence of natural numbers** which is accompanied by the footnote: **That is, a number-theoretic function**

**defined on an initial segment of the natural numbers. (Numbers, of course, cannot be arranged in a spatial order)** [1931, 147]. The spatial concept of **sequence** is here reduced to a function providing a 'location name' for every element in the de-spatialised sequence (an ordinal place-name assigned to each element in the sequence). The underlying spatial configuration is dismissed and marked impossible.

Spatial notions, however, are rarely related to the speaking subject. Two of the few exception are: **This makes no difficulty in principle. However, in order not to run into formulas of entirely unmanageable lengths** ... **the construction of the undecidable proposition would have to be slightly modified** [1931, 149] and **The proof that** so-and-so holds **is too long to give here** [1934, 359]. Here the physical limitations of space, and likely also time (or perhaps spatialised-time), suddenly intervene.

The discrepancy between this last notion of space-time and the ones observed immediately before are better accounted for by an analysis of modality in the text. The text appears to distinguish difficulty **in principle** from **actual** difficulties. Consider, for instance, that **we always understand by "formula of** the formal system ***PM*** **" a formula written without abbreviations (that is, without the use of definitions). It is well known that [in *PM*] definitions serve only to abbreviate notations and therefore are dispensable in principle** [1931, 147]. It is obvious that abbreviative definitions cannot be dispensed with, if mathematics is something which is to be done by physically and culturally constrained humans of a kind we tend to meet. The above statement, therefore, serves, rather than to make a claim, to articulate the field where the argument is to apply. The argument is to apply in the realm of **in principle**[5]. One should bear in mind that in practice mathematicians hardly ever write proofs in as precisely articulated formal systems as ***PM*** at all.

Similar considerations apply to the already quoted statement, which appears on the same page: **It would be very easy (although somewhat cumbersome) to**

[5]This realm opens up a discussion of the multitude of conceptual layers operative in the proof, especially around different notions of the term meaning. This analysis is conducted in [Wagner3]

**actually write down this formula**. Easy, perhaps — but not humanly feasible. Today it may be feasible to program a computer to output such a formula, but extremely cruel, and quite likely unfeasible, to expect a human to examine the output. Either way, this option was not available to Gödel. Again, we have a statement which operates in the language game of the **in principle** grammar, where things can be done which cannot *actually* be done.

We may appear to have found a textual support for the Agent vs. Subject-Person division. It is the disembodied, unlimitedly industrious Agent, who can do whatever can be done **in principle**. The Person, perhaps even the Subject, are excluded from this capacity. But whereas we do have two distinct modalities in the text, there is no indication that the enunciative position is in fact divided according to these modalities[6]. Consider for instance the statement (also on the same page) **it can be shown that the notions "formula", "proof array", and "provable formula" can be defined in the system *PM*; that is, we can, for example, find a formula** $F(v)$ **of *PM* ... such that** $F(v)$ **... says:** $v$ **is a provable formula.** The first **can** will in fact be achieved in the text. But the next **can** is precisely the kind which can only be achieved **in principle**, and the last **can** falls somewhere in the middle, depending on what aspect of **find**ing is at stake. The enunciator must endorse both modalities if it is to attain both human accessibility (actual capacity) and the mathematical ideal (**in principle** capacity). The text does not show any sign that the enunciator relates differently to these seamlessly interwoven layers of capacity — and yet these layers are distinguished by the text itself, when it briefly insists on the **in principle** reservation.

What I have been trying to show is that the texts make sense and lend themselves to the extraction of meanings by, on the one hand, supporting, to a certain extent, divisions of the speaker, while, on the other hand, refusing to be pinned

[6]Claiming that any grammatical division (or even that any modal division) necessarily entails a division of the subject position leads to grotesque results. The imperative and indicative moods are distinct, but they do not necessarily require distinct subject positions to function properly. Indeed, Austin's *How to Do Things with Words* ends up collapsing the distinctions between indicatives and performatives.

down to such divisions. The enunciative position has been shown to be distributed across various roles — but without actually grounding any stable textual articulation of such roles; it has been shown to endorse various incompatible modalities of temporality, spatiality and capacity — and then bind them together into intelligibility by avoiding clear distinctions and explicit syntheses. The text relies on many diverse positions to explain itself, to make the reader understand; but the text cannot commit itself to any single position, or set these positions as entirely apart. In doing that it would simply fall apart. The text manages to produce its meaning by distributing differences across the repeated enunciative stance. The semiotic machine embodied by the morpheme **we** is, like Benveniste's **I**, a blurring interface that ties a plurality of readers, writers and their forms of being to texts in ways that make sense. Regardless of popular belief, mathematical texts, such as these, do not make sense by pure formal clarity. They make sense by having it both (and more) ways.

## 5. Who do we do it with? (a slight detour)

Whoever **we** are, they can't stand alone. **I use *I* only when I am speaking to someone who will be a *you* in my address. It is this condition of dialogue that is constitutive of *person*, for it implies that reciprocally *I* becomes *you* in the address of the one who in his turn designates himself as *I*** ... **This polarity of persons is the fundamental condition in language, of which the process of communication, in which we share, is only a pragmatic consequence** [Benveniste, 224–225].

A polar position to one of the texts' enunciative position is established in the very first lines of the introduction to the 1931 text. **The development of mathematics toward greater precision**, Gödel explains, **has led, as is well known, to the formalisation of large tracts of it, so that one can prove any theorem using nothing but a few mechanical rules. The most comprehensive formal systems that have been set up hitherto are the system *Principia mathematica (PM)* on the one hand and the Zermelo-Fraenkel axiom**

**system of set theory on the other. These two systems are so comprehensive that in them all methods of proof are formalised, that is reduced to a few axioms and rules of inference. One might therefore conjecture that these axioms and rules of inference are sufficient to decide *any* mathematical question that can at all be formally expressed in these systems. It will be shown below that this is not the case, that on the contrary there are in the two systems mentioned relatively simple problems in the theory of integers that cannot be decided on the basis of the axioms** [1931, 145].

The footnotes to the third sentence acknowledge Whitehead and Russell, Fraenkel, von Neumann and Hilbert and Bernays as authors of formal systems. These various authors are invoked as if they belong to a single unified front, binding together the fundamentally logicist agenda of Russell, Hilbert's finitism, Zermelo's realist point of view and von-Neumann's pragmatist formalism, subjugating all to an ideological field which is negatively marked by the adjective **mechanical**. This adjective, along with the verb **reduce**, suggests a non-voluntary, non-subjective and impoverished mathematical practice. Indeed, at the time of writing the paper, and despite technological advances, machines had already had the kind of connotation to be canonically presented in Chaplin's *Modern Times* (1936). The term **mechanical** manipulates the reader to conflate the scientific connotation of 'mechanics' — Newtonian-mechanical absolute determinism, recently rendered defunct by relativity and quantum theory — with the more recent problematic technological connotation of automatic production. At the same time, the use of the term **mechanical** in the context of formalism is far from obvious. A mechanical approach will only be established a few years later by Church and Turing.

In the above quote a position of 'they' has been set-up and characterised. 'They' are ascribed a mighty achievement. Due to 'their' work, **one can prove any theorem using nothing but a few mechanical rules**. An unsuspecting reader may be lured to believe that a certain saturation has been achieved within **large tracts** of mathematics, whereby given a theorem, a proof can (mechanically!) be produced. And indeed, those readers who swallow the bait[7] **might therefore conjecture that these axioms and rules of inference are sufficient to decide *any* mathematical question that can at all be formally expressed in these systems** [1931, 145].

Note that the constative assertion is reduced by the hypothetical modal **might**. The text, however, does not yet clarify to the possibly misled reader that the formalist project allows to transcribe and verify by **few mechanical rules** only those proofs which are *already discovered*. Unproven and unrefuted theorems remain unproven and unrefuted regardless of the formalist project. The modal shift from constative to hypothetical serves to muddle the reader, who is now watching the conjecture he was manipulated into forming subjected to the doubt inscribed in a **might**. And indeed, **It will be shown below that this is not the case, that on the contrary there are in the two systems mentioned relatively simple problems in the theory of integers that cannot be decided on the basis of the axioms** [1931, 145]. But is it an oversight that an explicit definition of undecidability (there exists a statement such that neither it nor its negation can be proved) is postponed to the bottom of the next page? The specialist may have known exactly which undecidability Gödel was talking about. Other mathematicians were probably still kept in suspense.

The rhetorical structure we have just reviewed is standard. A picture is painted, then questioned, and finally announced invalid. The enunciative position in the text takes advantage of this bold manoeuvre. For the **It will be shown** to emerge both as a legitimate offspring and a parricidal revolutionary with respect to a certain lineage, a paternal position has to be, in the space of a few lines, both established and denounced. This position is established by forcing together Russell, Hilbert, Fraenkel and von Neumann into a straw-coalition, ignoring the fact that formalism

[7]The translation appears to be more misleading than the original, which speaks of *Beweismethoden* (proof methods) rather than theorems. But even the original claim is stronger than what was considered as commonly accepted at the time, and indeed, Gödel himself casts a doubt on this claim at the end of the paper [1931, 195].

was extremely young[8], highly unstable, and far from resting on a consensus among the contributors named above. This paternal position immediately falls victim to a parricide performed by branding this straw-formalism with the suspect mark of mechanism, and assigning to it the ambiguous and misleading formulation **axioms and rules of inference are sufficient to decide *any* mathematical question**, which formalists would not have dared to claim as achieved.

The positioning accomplished here relies on the ability of an established rhetorical structure to impose itself on a discursive field which is still underdetermined and emerging. This move provides the rhetorical support (which is, of course, not the only support) for putting it, later in the text, that **The solution suggested by Whitehead and Russell, that a proposition cannot say something about itself, is too drastic** [1934, 362].

The tension between belonging to, and breaking away from 'them', which gives rise to the text's impersonal **it** and collective **we** enunciative positions, is particularly manifest in the manoeuvres around the **proposition that says about itself that it is not provable [in *PM*]**, which, as a footnote we have already quoted explains, **involves no faulty circularity, for initially it [only] asserts that a certain well-defined formula ... is unprovable. Only subsequently (and so to speak by chance) does it turn out that this formula is precisely the one by which the proposition itself was expressed** [1931, 151]. The enunciative position endorses here at once the aversion from and attraction to self-reference. No one would deny that the self-referential proposition was constructed with the explicit intention of emulating self-reference inside a formal system. And yet the mere possibility of telling a revisionist history, obviously false *as a history*, relating the accidental genesis of this self-reference, is sufficient to allow contemporary logic both to embrace Gödel's form of self reference[9], and to be revolutionised by it. The

[8]Merely 13 years passed from Hilbert's 1918 formulation; see Kleene's introduction to Gödel's 1931 paper in [Gödel, 126].

[9]There are, of course, logicians who altogether deny that Gödel's proposition is self-referential. But I don't believe anyone would deny that self-reference was a **leading principle** in the construction of the proposition.

text remains formalist. But into this **mechanic** realm of formalism quietly sneaks an element of **so to speak** ... **chance**.

But all this rhetorical mechanics and revisionist historical articulation of meta-mathematical positions relates to the preamble to the introduction, and, for fear of being accused of lurking in the margins, we must move back into the core of the mathematical text.

## 6. Who does it to us?

A formalist 'they' position was set-up in order to simultaneously counter and produce the text's enunciative **we** position. But this 'they' position is far from exhausting the stance polar to the enunciative **we**. In order to complete the articulation of this polarity, let us track down the few occasions where **we** is demoted from the subject position to the object position **us**.

First, we must acknowledge how rare this move is. In both texts there are only nine occurrences of **us**, of which three appear in the inclusive imperative **let us**. Nevertheless, these occurrences are revealing of the presence of the enunciative position in the much more frequent impersonal and passive constructions[10].

We are told that **if a formal decision** ... **of the SENTENTIAL FORMULA** 17Gen$r$ ... **is presented to us, we can actually give** ... **a PROOF of** Neg(17Gen$r$) [1931, 177]. Ignoring the meaning of 17Gen$r$, we observe that polar to **we** a position which may present **us** with mathematical offerings is established. Confronting this position, **we** becomes a reactive (**giv**ing once **present**ed), rather than an initiating instance.

But this polar position is not confined to a **giv**ing role. **We** can **have before us a proposition that says about itself that it is not provable** [1931, 149–151]

[10]I should have, perhaps, analysed the impersonal and passive constructions in the text. Indeed, Foucault writes that the subjective formation **is situated at the level of 'it is said'** [Foucault1, 122]. However, since the relation between some of these constructions and the enunciative position is debatable (for instance, in statements of the form 'it follows that...'), I preferred to focus on the textual occasions where the voice emanating from the enunciative position is explicitly marked as such (lest I be be accused of hearing voices).

— the polar position can speak. In fact, more than just speaking, it can modify the capacities of the enunciative position. A certain proof, for example, **allows us to actually derive a contradiction** ... **once a PROOF of** $w$ ... **is given** [1931, 195] (without getting into what it is, such a **PROOF of** $w$ is supposed not to exist).

**We** is therefore a subject position in both senses of the word: subject of and subject to. However, the **proposition that says**, which we **have before us**, is explicitly produced by **we** in the text. Even the hypothetical object **PROOF of** $w$ above, if provided at all, must necessarily be provided by someone who can present proofs, and therefore share **we**'s enunciative position. **We**, it seems, plays a game of catch with itself, or rather, as is now obvious, with themselves.

**We** have many voices, many positions from which **we** speak. **We** speak to and of ourselves and our creations, who in turn, as the above quotes demonstrate, do not hesitate to speak back and challenge. In some sense (a sense I wish to impose) **we** are none but Gödel and I, the reader, in the most intimate moment of reading — intimate but not private, because I know many others have done it with him too, and because, to a certain extent, here I am doing it with him in public. Perhaps these so many discursive partners are the cause of this text being infested with so much meaning. Having intercourse with so many codes (encoded and decoded by so many partners), it is bound to say many things. **We**, which is supposed to establish a single common denominator, ends up, it seems to me, forcing the text open to many different entangled codes and complicit partners. So many different entangling codes indeed, that it is no longer clear that one could speak of *any* individual code at all.

But that is a matter for separate discussion. I would like to conclude this discussion with the words by which Barthes described the ambiguous enunciative position in Balzac's short story *Sarrasine*. **Who is speaking?** he asks. **Is it a scientific voice** ... **Is it a phenomenalist voice naming what he sees**? ... **Here it is impossible to attribute an origin, a point of view, to the statement. Now, this impossibility is one of the ways in which the plural nature**

**of a text can be appreciated ... it may happen that in the classic text, always haunted by the appropriation of speech, the voice gets lost, as though it had leaked out through a hole in the discourse. The best way to conceive the classical plural is then to listen to the text as an iridescent exchange carried on by multiple voices, on different wavelengths and subject from time to time to a sudden *dissolve*, leaving a gap which enables the utterance to shift from one point of view to another, without warning: the writing is set up across this tonal instability (which in the modern text becomes atonality), which makes it a glistening texture of ephemeral origins** [Barthes1, 41–42].

I could use Foucault's words to conclude this section. **In the proposed analysis, instead of referring back to *the* synthesis or *the* unifying function of *a* subject, the various enunciative modalities manifest his dispersion. To the various statuses, the various sites, the various positions that he can occupy or be given when making a discourse. To the discontinuity of the planes from which he speaks. And if these planes are linked by a system of relations, this system is not established by the synthetic activity of a consciousness identical with itself, dumb and anterior to all speech, but by the specificity of a discursive practice. I shall abandon any attempt, therefore, to see discourse as a phenomenon of expression — the verbal translation of a previously established synthesis; instead, I shall look for a field of regularity for various positions of subjectivity. Thus conceived, discourse is not the majestic unfolding manifestation of a thinking, knowing, speaking subject, but, on the contrary, a totality, in which the dispersion of the subject and his discontinuity with himself may be determined** [Foucault1, 54–55]. I would like to endorse this quote, but I am not sure that the terms **regularity**, **totality**, and **may be determined** are pronounced here with the cautious and critical tone in which I would like to have heard them[11]. I am worried that this quote underplays the capacity of a textual

[11]They most likely are so pronounced in other parts of *The Archeology*.

**we** to disperse and integrate, regularise and breach, totalise and re-open, determine and render undecidable. If we do not underplay these capacities, we can perhaps acknowledge not only the rigorous aspect of mathematics, but also its disruptive creative force. And so I shall not be satisfied before I come again (now in a voice drenched with the fluid complexity of so many enunciative distributions compacted into a brief contractive exhale: **we** — no longer opposing a trinity, but embracing its charge, embracing it *too*) to exclaim **My name is Lesion: for we are many**.

## 7. Appendix: a brief sketch of Gödel's argument

Gödel's argument concerns a standard formal system (based on Russell and Whitehead's *Principia Mathematica*) with a fixed set of symbols for logical operators (such as conjunction, negation, existence quantifier, etc.), functions, constants and variables. It is crucial that the formal system can represent the universal quantifier ('for all'), negation, and the natural numbers. Explicit and finitely verifiable syntactic criteria determine whether a given sequence of symbols is a legitimate formal expression, or in Gödel's terminology, a *formula*[12]. Finally, an explicit set of syntactic rules decide whether a sequence of formulas constitutes a proof.

Gödel's argument proves that as long as the formal system is consistent (there is no formula such that both it and its negation are provable)[13] there exists a formula in the language, such that neither this formula, nor its negation are provable. Such formulas are called *undecidable.* The scope of the argument was shown by Gödel to cover not just one specific formal system, but to rule over a wide variety of formal systems, which include all 'mainstream' languages which can represent natural numbers.

The first component in the argument is a method for translating any finite sequence of the formal language's symbols into a number. This translation method, which we will not review here, has the following properties:

[12] *Formula* here should be thought of as a proposition or statement, rather than as a formula for computing or constructing something.

[13] There is a delicate reservation here around the notion of $\omega$-inconsistency, but we won't go into it as it is not considered in this paper.

(1) No two symbol sequences correspond to the same number
(2) Given a symbol sequence, its number can be computed by a finite mechanisable procedure
(3) Given a number, the symbol sequence which corresponds to it can be computed by a finite mechanisable procedure

Note that this enumeration covers all symbol sequences, and includes those which make up legitimate formulas and legitimate proofs according to the system's syntactic rules.

The next component of the argument is to prove that various formal relations between formulas can be translated into arithmetic relations between the numbers representing these formulas. Arithmetic relations refer here to relations based on arithmetic operations that can be expressed by the limited vocabulary of Gödel's formal language. For instance, the relation "The symbol sequence numbered $x$ proves the formula numbered $y$" can be translated into an arithmetic relation between the numbers $x$ and $y$, which can be expressed in the formal language. We will denote here this formal relation by $P(x, y)$. In fact, Gödel demonstrates that symbol sequence number $x$ proves formula number $y$ if and only if the relation $P(x, y)$ can be proved in the formal system; moreover, symbol sequence number $x$ fails to prove formula number $y$ if and only if the relation $\neg P(x, y)$ (the negation of $P(x, y)$) can be proved in the formal system.

Via a clever construction Gödel produces a number $g$, such that the following formal sequence:

$$\forall x(\neg P(x, g)),$$

which reads: 'for every $x$, the symbol sequence numbered $x$ does not prove the formula numbered $g$', is numbered $g$. Therefore $g$ is the number of the formula which claims that no number $x$ corresponds to a proof of the formula numbered $g$. Simply put, formula number $g$ states that formula number $g$ is unprovable. The negation of formula number $g$ would say, then, that formula number $g$ is provable.

It is now easy to show that, unless we have an inconsistency, formula number $g$ cannot be proved.

- Suppose formula number $g$ could be proved.
- Then there would exist a symbol sequence which would be a proof of $g$. Let its number be $y$.
- As a result, it would be possible to prove $P(y, g)$.
- On the other hand, if we could prove formula number $g$ (namely that for all $x$, $\neg P(x, g)$), we could prove the particular case where $x$ is the above $y$, and prove $\neg P(y, g)$.
- But the last two conclusions are inconsistent.

Now we turn to showing that the negation of formula number $g$ cannot be proved.

- Suppose we could prove the negation of formula number $g$.
- As stated above, this would mean that formula number $g$ would be provable.
- But we have already shown above that this would yield an inconsistency.

Note that this argument relied on a semantic move ("this would mean that..."), based on our interpretation of formula number $g$. This is the so called *semantic argument*. Since it is not relevant to this paper, we omit the summary of the formally rigorous *syntactic argument*, and set aside the issue of inconsistency vs. $\omega$-inconsistency that emerges from the syntactic argument.

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