



## Journal Article

# Mathematical Marriages: Intercourse between Mathematics and Semiotic Choice

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## DISCUSSION PAPER

### MATHEMATICAL MARRIAGES: INTERCOURSE BETWEEN MATHEMATICS AND SEMIOTIC CHOICE

Roy Wagner

**Abstract:** This paper examines the interaction between semiotic choices and the presentation and solution of a family of contemporary mathematical problems centred around the so called 'stable marriage problem'. I investigate how a socially restrictive choice of signs impacts mathematical production both in terms of problem formation and of solutions. I further note how the choice of gendered language ends up constructing a reality, which duplicates the very structural framework that it imported into mathematical analysis in the first place. I go on to point out some semiotic lines of flight from this interlocking grip of mathematics and gendered language.

**Keywords:** mathematics, semiotics, language, stable marriage, gender role stereotypes

#### Introduction

The production of mathematics is, to a large extent, a semiotic activity. A lot of what a mathematician does is to manipulate symbols and words. Nevertheless, mathematics is the one domain of knowledge that is most exposed to a myth of pure and ideal transcendence beyond the symbols in which it is engraved.

The purpose of this essay is to analyze how language may shape mathematical considerations. In order to make the analysis concrete and relevant, we focus on a set of specific mathematical problems, related to the 'stable marriage problem', and on the role of gendered language in the shaping of its mathematical research.

The path followed by this text requires a close and attentive analysis of detail. I tried to find a decent balance between methodological rigour and non-specialist accessibility. I hope that the objectives of this text are not lost in technical detail, and that at the same time these objectives are convincingly laid out for the professional practitioner.

The study of gender role discourse in scientific texts is a relatively recent, but already established trend. Prominent examples include (Cohn, 1987; Keller, 1992; Martin, 1996 and Potter, 2001). The reading of a text in this framework consists of a two-pronged argument. First it demonstrates how gender role stereotypes are reproduced by the scientific text, and then it investigates the ways in which these stereotypes constrain scientific production.

This essay too will begin by applying this same framework. We will show how stereotypes are mathematically reproduced, but more importantly we will show how gender biased language may restrict the possibilities which a mathematician has for doing mathematics. In this text I would like to suggest that a certain restrictive choice of signs can limit and confuse mathematical research programs, that a certain discourse may foreclose or impose various mathematical questions and results, and that a biased symbolic fabric may weave its bias through mathematics into a produced reality.

But my aspirations don't end there. I further hope that my contribution may exceed the application of the already established feminist-discursive critical framework to yet another scientific domain. I hope to expand the horizon of existing critique by relating it more explicitly to a post-structural semiotic framework. The claims I wish to highlight are that what various formations of discourse open or foreclose is never clear and distinct; that there is no predetermined saturation of

possibilities laid out by a given text or choice of sign; that the internal logic of a certain discursive realm, having an initial restrictive effect, may proceed in unexpected ways towards positions, which undermine the very commitments openly declared. This tension between the restrictive effect of linguistic choices and the power of the sign to break with any supposed context is, I believe, a tension that merits scientific, ethical and political attention. To an extent, it forces the latter two on the former.

I should clarify that this text does not presume to study the relations between gender and mathematics in general. It is restricted to a single family of problems and includes no institutional analyses (the sociological evidence brought up is anecdotal). This essay is intended as a case study of how choice of signs may affect mathematical problem formation and solution. It is about the power of the sign over mathematical production, and about how the 'outside' of mathematics ends up affecting its 'inside' to the point where this 'inside/outside' distinction is severely undermined. This text does, however, contribute an important case study to feminist critiques of mathematics, which often suggest 'big picture' narratives without a careful study of the details and plurality of practiced mathematical languages and modes of production (e.g. Shulman, 1996).

My attempt here is to alert practitioners to the ethical and political rapport which language may have with the production of mathematical results. My attempt here is to alert the critical scholar's attention to the underdetermination of this ethical and political rapport.

### **The Stable Marriage Problem**

Let's begin this discussion with two compact formal statements of the problem addressed by Hall's theorem (Hall 1935), as presented in a contemporary textbook (do not be put off by the unfamiliar technical terminology, it will all be explained soon enough). Here is how the problem appears in (Wilson 1996: 112):

if there is a finite set of girls, each of whom knows several boys, under what conditions can all the girls marry the boys in such a way that each girl marries a boy she knows?

And here is how the same problem appears on the next page:

if  $G = G(V_1, V_2)$  is a bipartite graph, when does there exist a complete matching from  $V_1$  to  $V_2$  in  $G$ ?

The latter formulation is a technical question about *graphs*, which are simply configurations of points, along with lines that connect some pairs of these points. This question dates back to the 1930s (or even earlier, according to some authors). The former formulation, which very often accompanies the more abstract one, is referred to (Weyl, 1949). In this formulation the points in the configuration become boys (elements of the set  $V_1$ ) and girls (elements of the set  $V_2$ ), and the lines connecting boys and girls become relations of acquaintance. The *marriage problem* is to find whether there exists a *complete matching*, that is a monogamous heterosexual marriage arrangement, which includes all girls, and which matches each girl to a boy she knows.

Our task in this paper is to outline the interrelations between mathematical discourse and gender role stereotypes in the context of this problem, and to explore their mutual impact. We will, however, focus not on Hall's solution to the above question, but on a more recent version of the problem, introduced in Gale and Shapley (1962), which includes a component of personal preferences. Let us review this version in more detail before we survey the use of gender role language in various textbooks and papers studying this problem over the last 30

years. Our initial presentation will be borrowed from one of the most important texts presenting the problem (Knuth, 1997: 1).

Let  $H$  and  $F$  be two finite sets of  $n$  elements.  $H$  is the set of men  $A_1, A_2, \dots, A_n$  and  $F$  is the set of women  $a_1, a_2, \dots, a_n$ . A matching is ... a set of  $n$  monogamous marriages between the men and the women ... . Suppose that each man has an order of preference for the women and each woman an order of preference for the men ... . A matching is *unstable* if a man  $A$  and a woman  $a$ , not married to each other, mutually prefer each other to their spouses. This 'liaison dangereuse' occurs when:

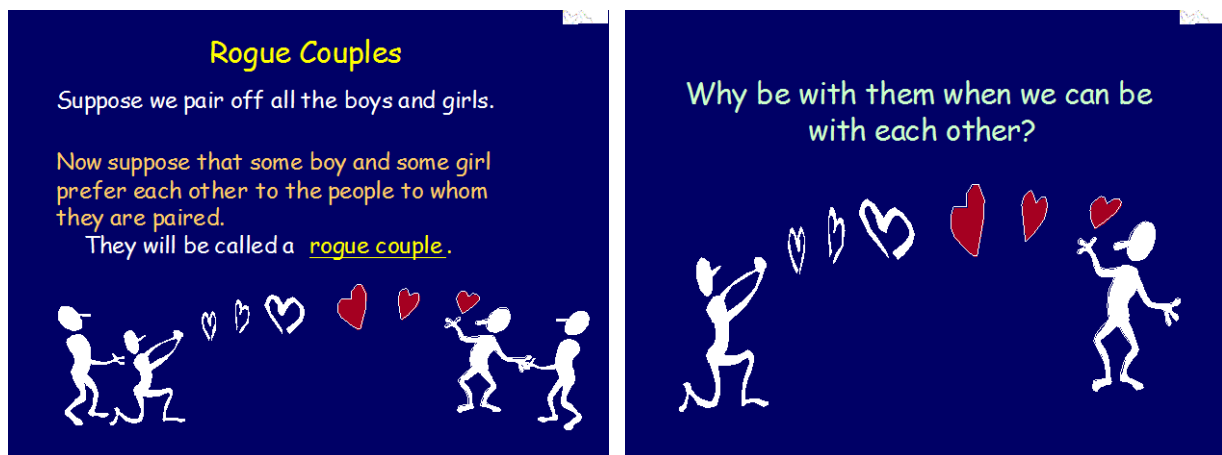
- $A$  is married to  $b$ ;
- $a$  is married to  $B$ ;
- $A$  prefers  $a$  to  $b$ ;
- $a$  prefers  $A$  to  $B$ .

(The opinions of  $b$  and  $B$  are irrelevant here.) A matching is *stable* if this situation does not occur.

To complement the formal setting by a slightly more accessible one, we borrow some of Steven Rudich's powerpoint presentation slides (Figures 1 & 2), actually in use in his contemporary computer science undergraduate classes (Rudich, 2003). The situation which Knuth terms 'unstable' is presented in Rudich's slides as the existence of a 'rogue couple'.

#### Figures 1 & 2

Reprinted with permission from Steven Rudich 'The Mathematics of 1950s Dating: Who Wins the Battle of the Sexes?', available at <http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15251/discretemath/Lectures/dating.ppt>



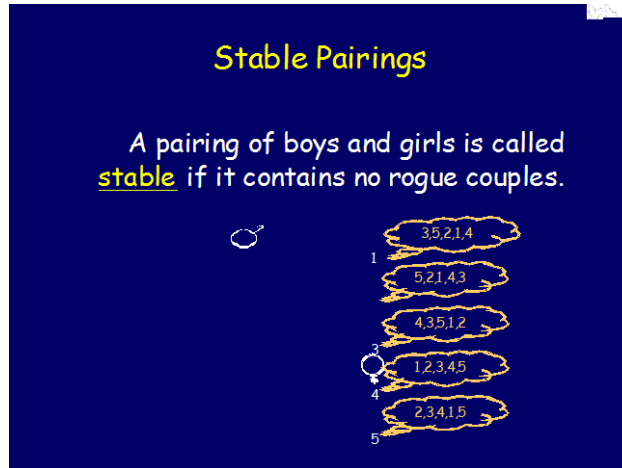
The next slide (Figure 3) is a diagram representing 5 boys and 5 girls together with each person's ranking of members of the opposite sex, and an instance of a stable matching (here 'pairing').

#### Figure 3

Reprinted with permission from Steven Rudich 'The Mathematics of 1950s Dating: Who Wins the Battle of the Sexes?', available at <http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15251/discretemath/Lectures/dating.ppt>

## Stable Pairings

A pairing of boys and girls is called **stable** if it contains no rogue couples.



You can verify that any girl and boy not married to each other (e.g. boy 1 and girl 2) do not both prefer each other over their actual spouses. Indeed, while boy 1 actually does prefer girl 2 over his actual wife (girl 5), girl 2 prefers her assigned husband (boy 2) over boy 1. Therefore, boy 1 and girl 2 do not form a rogue couple, and according to the rules of the problem, they will not break their marriages in order to marry each other. You can go on verifying that in the above example there is indeed no rogue couple.

It is not clear a-priori whether any system of preferences admits a stable matching. Gale and Shapley proved that such a matching does indeed always exist, and supplied a simple *algorithm* (mechanizable process) for producing a stable matching. Before commenting on their algorithm, it is important to emphasize that this algorithm is not the only method for producing a stable matching. It is, however, historically the first to have appeared, and the most widely reproduced in the literature (although, I believe, not the simplest available).

A compact description of the process of generating a stable matching can be found in (Bollobás, 1988: 86), where it is referred to as 'simply the codification of the rules of old-fashioned etiquette: every boy proposes to his highest preference and every girl refuses all but her best proposal.' Each rejected boy continues to propose to his next highest preferences, and each girl continues refusing all but her highest preference among the boys who actually propose to her at any given time, possibly rejecting a boy whose proposal she had previously accepted. 'This goes on until no changes [new proposals] occur; then every girl marries her only proposer she has not yet refused.'

A more graphic description of this process is available in Figures 4 & 5, again, from Rudich (2003).

### Figures 4 & 5

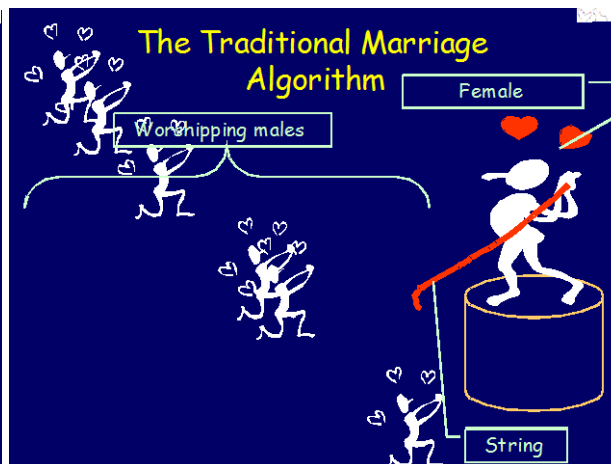
Reprinted with permission from Steven Rudich 'The Mathematics of 1950s Dating: Who Wins the Battle of the Sexes?', available at <http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15251/discretemath/Lectures/dating.ppt>

### Traditional Marriage Algorithm

For each day that some boy gets a  $\emptyset$  No  $\emptyset$  Do:

- ≠ Morning
  - ⊆ Each girl stands on her balcony
  - ⊆ Each boy proposes under the balcony of the best girl whom he has not yet crossed off
- ≠ Afternoon (for those girls with at least one suitor)
  - ⊆ To today's best suitor:  $\emptyset$  Maybe, come back tomorrow  $\emptyset$
  - ⊆ To any others:  $\emptyset$  No, I will never marry you  $\emptyset$
- ≠ Evening
  - ⊆ Any rejected boy crosses the girl off his list

If no boys get a  $\emptyset$  No  $\emptyset$   
 Each girl marries the boy to whom she just said  $\emptyset$  Maybe  $\emptyset$



Gale and Shapley proved that this iterative process must converge to a one-to-one matching, and that this matching is indeed stable in the sense defined above (no rogue couples).

### Mathematical Reproduction of Gender Role Stereotypes

As promised in the introduction, the first issue we shall study is the reproduction of gender role stereotypes in the mathematical literature. This can be found in abundance in the texts under review. However, we should not be too naïve in tracking down such reproduction. What I would like to avoid is the latent hypothesis that there exists a clear and distinct set of stereotypes to be reproduced.

In the dozen-or-so textbooks, monographs and research papers, which I surveyed for the purposes of this analysis, I found substantial variety concerning notation, terminology, and focus on men or women (most texts in fact focus more on what women do in the process of the matching algorithm). Rudich's slides above, for instance, are exceptional in the way they put women 'on a pedestal' (as we shall see below, it is a deliberately ironic choice). But there are also many points which are common to all or almost all the texts which I surveyed.

The first and most obvious common point is that throughout the literature it is the boys who propose. A typical statement in this context is 'We will adopt the traditional approach describing the men as 'suitors' in a 'courtship' process, but analogous results may be obtained by reversing the roles of the sexes' (Gusfield and Irving, 1989: 8). The problems are symmetric – the roles of men and women may be interchanged – but the literature consistently opts to solicit the men to propose.

The only exception that I found to this rule (excluding texts which do not quote the matrimonial imagery at all) is a text which does use matrimonial language, but which refuses to assign genders: 'We leave the reader to assign the sexes of  $V_1$  and  $V_2$ ' (Asratian et al. 1998: 67). I believe that this 'gender neutral' presentation demonstrates precisely the intricacy of gender role stereotypes. Gender role stereotypes do not disappear when gender assignment disappears. Rather, it is the encapsulation of gender roles into two fixed and distinct positions which entrenches the text in a stereotype. To resist the stereotype it is not enough to allow its reversal.

Another trend which binds the various presentations of the problem regards the active/passive division. Overall, men are predominantly active – they propose. Women are predominantly re-active – they reject or accept. Once the girls have reacted, men find themselves in a quantitatively secondary passive position – rejected or engaged (this twofold hierarchy is confirmed by an exhaustive verb

count in the various texts). There is perhaps a problem in suggesting that this picture is stereotypic, since men find themselves both active and passive. One may respond that the picture is indeed stereotypic because it assigns men an *initiatory* active position, which renders their subsequent re-inscription into a passive role less important. One can also suggest that the two male positions build on two complementing stereotypes: the docile passive woman, who should respond only when approached, and the manipulating witch, who employs passive-aggressive means of domination (these two positions are traced by Simone de Beauvoir in [Beauvoir 1989] and by Catherine Clément, in [Cixous and Clément 1986]).

However, this reconfirmation of the stereotype poses a further, more sinister problem. Does it not perhaps become too easy to label practically any text as stereotypic, if we allow for ever-more-sophisticated articulations of the stereotype? Rather than seeking a way out of the matrix, do we not elaborate the matrix of stereotypes into an a-priori all encompassing entity?

It is indeed never completely clear whether any specific portrayal has genuinely escaped a stereotype, or merely redeployed it differently within a larger stereotypic framework. The question is, therefore, not whether any particular forms of a stereotype are reproduced or not. Avoiding the stereotype, I believe, consists of allowing agents to move across positions in a structural matrix so as to manipulate its boundaries (or, if we are more optimistic, building escape hatches into the matrix). Stereotyping discourses, like homophobia according to David Halperin, 'contain no fixed propositional content' (Halperin, 1995: 33). It is the immobility which they impose on a restricted set of relative articulations that is stereotypic. To paraphrase Foucault, resisting a stereotype would then consist 'of the art of not being [stereotyped] in a certain way and at a certain price' (Foucault, 1996: 384).

A particularly disturbing feature in the various narrations of the algorithm is that in none of the surveyed texts do the women ever say 'yes' to the marriage proposals. Their replies are either a definite 'no', a deferring 'maybe', or a silence which is interpreted as provisional or permanent consent. At the end of the algorithm all the final silences and 'maybes' are automatically consummated in marriages. A woman, according to the mathematical literature, needn't say 'yes'. It suffices that she does not explicitly refuse.

A final anecdote will shed some light on the way that stereotypes are integrated into the mathematical discourse, and on the decentralized distribution of intent in their reproduction. A colleague of mine told me about an instructor, who, in presenting a variant of the marriage problem, introduced into the problem the character of a girl who dates a large number of boys. The students in the classroom opted to refer to this girl as 'slut'. Another instructor, having heard the story, went on to use the term in his own classes as well.

#### **Do Gender Roles Constrain the Formation and Solution of Problems?**

The result of the Gale-Shapley algorithm is very strongly biased in favour of the proposing side (which is a euphemism for 'men'). In fact, given a system of preferences, the Gale-Shapley algorithm produces the stable matching which is worst for all women and best for all men. This does not mean that each man gets his first choice and every woman her last – indeed, such a matching needn't be stable nor even monogamous. What *female pessimality* and *male optimality* mean is that no stable matching exists, where some man is assigned a spouse whom he prefers to the one assigned by the Gale-Shapley algorithm; on the other hand, no stable matching exists, which assigns some woman a spouse less desirable to her than the spouse assigned her by the Gale-Shapley algorithm.

As we noted above, the stable marriage problem is symmetric (the sides can be exchanged without changing the problem). Gender roles discourse however is asymmetric, as is Gale and Shapley's algorithm and its result. Should we conjecture that a link exists between these two forms of asymmetry? Below I will present some circumstantial evidence suggesting that gender role stereotypes reproduced in theoretical deliberations substantially affected mathematical results. But before I do so, the relation between science and language, which I would like to propose, must be better articulated.

I do not believe that we should describe the relation between languages we use and sciences we produce as causal. In fact, I do not even believe that we should claim a correlation between key features of a scientific product and of the language in which it is in-scribed. My disbelief, however, does not follow from a conviction that such claims are false. My objection is that I know of no methodological framework that can properly assert causality or correlation between scientific claims and their language. First, I see no methodological scheme which can convincingly separate the two strata (scientific 'content' and underlying language). Second, even if we could effect such a separation, I doubt that we could come up with a corpus of texts that would supply proper control over the variables we wished to correlate. Finally, and most importantly, the implication of causality would assume a linear narrative which does not adequately account for the complex relations between language and scientific texts.

What is then the value of the circumstantial evidence which I will bring up? My goal is to make room for ways of thinking which allow for the possibility that language makes an impact on scientific (specifically, mathematical) production. What is urgent is to make the ethical rather than rational decision, which renders thinkable the impact of discursive constraints (such as gender role stereotypes) over the emerging forms of science (such as biased solutions of the marriage problem). Even more urgent is our willingness to endorse the political and ethical repercussions of such openness: that scientists must assume responsibility for their use of language. Less possible and also less urgent for human kind, however, is to decide the precise determination of the ontology behind this interaction (actually, there are some theoretical texts which suggest such frameworks without succumbing to naïve forms of idealism or structuralism; one can cite here among others the work of Cavailles [1970; see also Webb, 2006], Derrida's [1989] *Origin of Geometry*, and perhaps even my own [Wagner, 2007 and 2008]).

Let's review, then, the evidence which I propose for thinking the impact of gender role discourse on the mathematical analysis of the marriage problem. The fact that the Gale-Shapley algorithm is male-optimal is noted in Gale and Shapley's original paper from 1962. It took, however, an additional 9 years for the observation that the algorithm is female-pessimal to reach the literature (McVitie and Wilson 1971). Both facts are just as easy to observe. Should we interpret this temporal gap as reflecting a foreclosure on stating the question from the women's point of view?

The Gale-Shapley algorithm is indeed strongly biased in favour of the proposers. Today, however, there are several algorithms for producing stable matchings, which generate results of a more balanced kind. It is interesting to observe that the first steps in this direction were quoted by a subsequent author with no reference to a publication (Selkow, quoted in Knuth [1997: 51]; Conway's median observation, quoted in Knuth [1997: 56]). Should we interpret this deferral of publication as indicating a lack of interest in generating balanced algorithms?

The first kind of more balanced algorithms start with the Gale-Shapley algorithm, and manipulate it by exchanging spouses in controlled ways. In the presentation of these algorithms the elements of analysis undergo various transformations. The unit of analysis eventually corresponds to the symmetric notion of *possible pair* rather than to men or women as in the original algorithm (Gusfield and Irving,



1989; Vande Vate, 1989; Teo and Sethuraman, 1998). Can the shift towards a symmetric unit of analysis account for the emergence of more balanced algorithms?

The second kind of more balanced algorithms emerges from Roth and Vande Vate (1990), further developed in Ma (1996) and in Romero-Medina (2005). The basic step in these versions is applying the Gale-Shapley algorithm in a way that allows all players to assume the proposing and reactive roles. The algorithm and its analysis are (in my opinion) simpler than Gale and Shapley's algorithm: people enter a room one at a time. Each new entrant is matched to their highest preference among all those in the room willing to accept the new entrant (either because they are single or because they prefer the new entrant over their actual partner). Now, the matching of the new entrant may have separated someone in the room from their spouse. Such newly single persons are then matched as if they have just entered the room one by one. This goes on until no more couples in the room are broken. Then a new person enters the room, and so on, until everyone is matched.

This algorithm is practically identical to the Gale-Shapley algorithm, except that it conducts the proposal sessions individually, rather than in men/women blocks. Nevertheless, when it came into the theory (in 1990), this minor variation was quite a novelty – so much so that it solved an open problem raised by Knuth 14 years earlier. Should we explain this long delay by an adherence to gender role stereotypes?

These instances of circumstantial evidence can be confronted with various critiques. Obviously, one could offer alternative explanations to the theoretic developments which would have nothing to do with gender or language. I do not claim that such explanations are necessarily invalid. My objective is to open up the *possibility* that gender role discourse could have had crucial impact on the development of the theory. I will therefore only comment specifically on one possible objection.

One could claim that more balanced algorithms emerged later in the theory simply because such algorithms are more difficult to produce (Luce Irigaray's [1985] *Fluid Mechanics* encountered a similar critique, which claimed that the underdevelopment of fluid mechanics had nothing to do with language and gender, but simply with the sheer complexity of the subject). For some – but not all – of the more balanced algorithms this claim is indeed true. In fact, one version of the problem of finding a balanced stable matching, the so called *sex-equal matching problem*, is known to be NP-hard (Kato, 1993) (NP-hardness is a technical substitute for the vague notion of not being practically computable). However, this excess of difficulty could itself be an effect of language. We know, for instance, that geometric problems may change their degree of difficulty if presented in terms of calculus or classical geometry. Even the notion of NP-hardness itself depends on a linguistic construct in the framework of a Turing Machine, which shouldn't monopolize our notion of hardness of computation; indeed, increasing doubts are cast on whether Turing machines do indeed exhaust our horizons of computation.<sup>1</sup>

Before we move on with the argument, I would like to note a paper dealing with another combinatorial problem, the so called *ménage* problem. The abstract reads: 'The *ménage* problem asks for the number of ways of seating  $n$  couples at a circular table, with men and women alternating, so that no one sits next to his or her partner. We present a straight-forward solution to this problem. What distinguishes our approach is that we do not seat the ladies first' (Bogart & Doyle 1985). The authors claim, with a certain reservation, that 'of all the ways in which sexism has held back the advance of mathematics', the fact that all previous algorithms seated the ladies first 'may well be the most peculiar'. It seems that at least some mathematicians directly involved with such subject matter acknowledge the constraint imposed by stereotypes on mathematical discovery, at least as a peculiarity.

## Beyond Gender: Ideological Commitments and Problem Formation

The mathematical presentation of results around the marriage problem is replete with explicit, uncriticized, ideological commitments. One author goes so far as formulating the following theorem: 'A male-denumerable society is espousable if and only if it is good' (Nash-Williams, 1978). Here *espousable* means that all men can be married (given the combinatorial restrictions of the problem). For our purposes the term *denumerable* can be ignored. The term *good* refers to a technically defined mathematical condition, which, for purposes of a compact statement, the author substitutes with the simple adjective.

One may claim that once the meaning of the terms is known, the supposedly ideological statement ('it is good for all man to marry') disappears. But Nash-Williams could have opted to call his technical condition *bad*, or to phrase his theorem as a necessary and sufficient condition for a male-denumerable society *not* to be espousable. The ideological choices made in the formulation of this statement shouldn't be ignored.

But the main ideological commitment imposed on the entire theory of stable matching is, of course, the notion of stability – preventing the occurrence of 'rogue couples', not married to each other but preferring each other over their assigned spouses. This notion is imposed on the discourse as an absolute ideal. Typical quotes include: 'A pairing is doomed if it contains a rogue couple' (Rudich, 2003); 'The idea of stable matching is inspired by the search for an idyllic society of married couples' (Asratian et al. 1998: 67); and 'from the practical and algorithmic standpoint [some specific] results are negative, since they do not lead to acceptable ways to produce desirable stable matchings using the Gale-Shapley algorithm' (Gusfield and Irving, 1989: xv).

Stability, however, is not obviously relevant to the theory from the point of view of applications. The most widely discussed applications of stable matching schemes are the matching of residents to hospitals and of students to schools. If we consider information gaps (people don't know each other's preferences), change of mind and hesitation (preferences are contingent, change over time and may depend on what is learned over time of others' preferences), contractual obligations and practical impediments (which may prevent 'rogue couples' from 'eloping', even if such couples do exist), stability may become irrelevant.

The actual relevance of stability has only been considered critically since the mid 80s. Roth and Sotomayor (1990) survey some aspects of the relevance of stability. To be fair, their analysis is lucid enough to raise many pertinent objections, including some of those mentioned above. They even write (page 156) 'at least one of the authors would feel very differently about the theory presented here if the weight of the empirical evidence were different'. But as is often the case in neo-liberal economic analysis, the interpretation of empirical evidence allows ideological commitments to gain the upper hand. It was in fact the very evidence raised by Roth and Sotomayor, which evinced my suspicion that the success or failure of a matching scheme in contemporary applications may not depend heavily on stability, but rather on other factors (and that before going into a critique of the very definition of 'success', which affects the analysis of how a successful scheme is to be generated).

I will not ponder here over the details of the debate, because it is outside the scope of this essay. I would only like to indicate that the quest for stable solutions is an overarching characteristic of contemporary game-theory and economics. Many questions related to the production of *mobility* – which is often marked as a desirable social goal – are foreclosed by contemporary game-theoretic and economic frameworks.

We will end this section with one more example of the way in which ideological commitments constrain the formation and solution of problems. 'One of the main

difficulties with using the men-optimal stable matching mechanism', explain the authors of a research paper, 'is that it is manipulable by the women: some women can intentionally misrepresent their preferences to obtain a better stable partner' (Teo et al. 1999: 430). Rather than describe preference list manipulation as a way of turning an unbalanced algorithm into a more balanced one, the literature condemns this opportunity as 'cheating', namely as something which should be prevented.

### **A False Sense of Reality**

Before we go on to comment on some complementing aspects of the semiotic potential of the interaction between mathematical texts and gender role stereotypes, I would like to include a brief comment on the relation between the mathematical text, ideological commitments, and a supposed underlying reality.

The Gale-Shapley algorithm was introduced in 1962. It had, however, in fact already been in use for 11 years by that time. Since 1951 residents (formerly interns) have been assigned to American hospitals via a centralized application of a variant of the Gale-Shapley algorithm. That this was the case was only observed by Roth in the mid '80s. One might suspect that the notion of *stability* served as an a-posteriori justification for the coincidence between an unbalanced assignment scheme and a biased mathematical discourse, both emerging from interrelated ideological commitments. To quote Emily Martin quoting David Harvey, perhaps 'what we have here is an implanting of social imagery on representations ... so as to lay a firm basis for reimporting exactly that same imagery as natural explanations of social phenomena' (Martin, 1996: 113).

It would be extremely uncharitable to assume that any of the cited researchers genuinely believes that the stable marriage model is a perfectly adequate description of social reality. Nevertheless, the literature does contain statements which indicate that the analysis and its results are taken seriously: 'Societal habits thus favour men' (West, 1996: 177), and 'We shall give a result which perhaps demonstrates an effect of complete inequality of the sexes' (Asratian et al. 1998: 70). When Rudich states that 'The largest, most successful dating service in the world uses a computer to run TMA' (TMA here is the Traditional Marriage Algorithm, which is another title for the Gale-Shapley algorithm), he ignores the many practical details which undermine the relevance of the mathematical presentation to the actual implementation he mentions. When one reads that 'The algorithm is the codification of old fashioned etiquette' (Bollobás, 1998: 86), one reads a rather doubtful and revisionist social history.

The gap between the model and the supposed reality is not ignored in the literature. One can read that 'We will sometimes speak about courtship, but never of dependent children or mid-life crises' (Roth and Sotomayor, 1990: 1) and that 'This makes the (somewhat unrealistic) assumption that it is always better to be married (to an acquaintance) than to stay single' (Bollobás, 1998: 85). Many texts consider the entire marriage terminology as 'frivolous' (Wilson, 1996: 113).

This, however, does not prevent the same texts from considering the job-assignment formulation of the same problem as 'more serious', even though this version too abstracts and ideologizes a myriad of factors which drive mathematical analysis away from any social reality which it purports to seriously emulate. To quote Carol Cohn, 'There is no reality behind the words. Or, rather, the "reality" of which they speak is itself a world of abstraction' (Cohn, 1987: 22).

### **Semiotic Drift**

As final evidence to the power of gender role discourse over mathematical production, here is one more anecdote. A colleague of mine reports a seminar,

where a speaker presented a variant of the marriage problem with infinitely many people. One of the seminar attendants complained that the matrimonial terminology was disturbing or distracting, and asked for a more abstract presentation. My colleague reports that the speaker did give it a try, but constantly found himself reverting to the use of matrimonial terminology.

So suppose we finally accept that a certain system of signs may make a certain impact on a certain discourse. Should we banish this system of signs from that discourse? Should we segregate gender-talk from science in order to 'purify' science from the undue influence of ideological commitments?

The first objection to such a stance is that the desired 'purity' is unattainable. Our language is imbued with gender, whether we like it or not. Some theoreticians may go as far as claiming that every discourse which operates a binarism operates a gendered structure.

But the desire for a purified scientific language encounters a further, more radical problem. This desire to purify science from linguistically imposed bias relies on an assumption that behind the various ways of representing scientific or mathematical problems there is an objective and originally present problem – that our presentation of a problem covers over a 'real' problem. I would rather promote a stance according to which the scientific or mathematical problem is, precisely, the various ways in which it is re-presented; that mathematical problems are not simply covered by the symbolic fabrics and practices into which they are sewn, but that they are actually spun from those very symbolic fabrics and practices, and that they consist of quilting together various symbolic and practical fabrics. Accepting such a stance implicates the scientific and mathematical author in a responsibility for the ideological commitments and opportunities in which scientific production is immersed. Scientists should no longer pretend to be able to discard all such constraints and opportunities by claiming that they are but frivolous surface effects. There is no essence to a mathematical problem other than an essential dependence on the texts and practices through which the problem is articulated.

I believe that most practicing mathematicians would not object to the claim that the contingent representation of a problem both constrains and enables its further development and analysis. Rephrasing problems and re-setting different problems in each other's terms are important tasks for the mathematician (Knuth's book quoted above, for instance, focuses on the relations between the stable marriage problem and other important problems in the theory of algorithms). I also believe that most mathematicians would agree that this is still the case even when doing mathematics in highly professional abstract symbolism – a practice which invariably depends on intricate impure mixtures of formal manipulations and metaphorical intuitions, and never reaches a purely formal level. Many mathematicians would, however, object to an erasure of the myth of ideal underlying mathematical problem, as well as to the ethical burden that such erasure imposes upon the symbol-manipulating researcher. The debate is therefore predominantly an ethico-ontological one, and cannot be reduced to the framework of an ontic rational choice.

There is another objection to the purification of scientific language from gender role discourse. This further objection is that language-puritanism stands in direct contrast to the voices of feminist critique of science. If anything, contemporary feminist critique seems to encourage implicating science with as many points of view as possible in order to expand its 'objectivity'. This includes, obviously, gendered points of view (e.g. Harding, 1993).

To all the objections above, I would like to add a further objection, derived from post-structural semiotics. When one considers the sign's force of holding on to connotations and ideological commitments (which affects symbolic production regardless of whether the researcher is interested in that force), one should

also recall that this is but one manifestation of the sign's radical potential. In the words of Derrida 'a sign carries with it a force that breaks with its context' (Derrida, 1988: 9). The sign is so materially constrained that it is impossible to force it to hold on to any determination of meaning, context, or rules of manipulation. And this freedom, which inheres in signs, is precisely that which constitutes its ecstatic force.

The matrimonial imagery is a strong mnemonic device for researchers and students alike (which is why even researchers who can't be reproached for sexism often use it). Students tend to remember the marriage problem long after their course is over. One may conjecture that the reinforcing interference between gender role stereotypes and mathematical language is the result of both systems being extreme forms of hierarchic and binary languages.

But, while it is indeed true that there is a lot of hierarchy and binarism in mathematical structures, reducing mathematical language to such effects is far from accurate. As mathematician John Conway asserted in a Cambridge lecture, set theory is a fine language, but not all mathematicians must work inside it.

An alternative view of mathematical language is presented by Derrida and Kristeva. In her early work Kristeva sets the task of 'finding out how texts ... have brought into ideology those reformations of the signifier, which they only, together with logico-mathematical work, can produce' (Kristeva, 1969: 287). In an interview Derrida reflects this stance: 'The effective progress of mathematical notation thus goes along with the deconstruction of metaphysics, with the profound renewal of mathematics itself and the concept of science for which mathematics has always been the model' (Derrida, 1981: 35). Mathematics, according to this view, is transformative, because it is an instance of a symbolic practice, whose semiotic substance encourages it to manipulate and undermine ideological commitments.

To substantiate this position I will take the reader through some examples of semiotic-mathematical transformations and their interaction with ideological commitments. As we shall see, this interaction often is conservative and mutually reinforcing, but it can also transform, manipulate and undermine the ideological commitments of the discourses with which mathematical production interacts. The interaction between mathematics and its signs cannot be reduced to a straightforward, conservative and reactionary imposition.

The variations of the marriage problems include what we might call 'alternative families'. For example, instead of a matching based on a gender division, we may be given a set of people each of whom can be matched to any other. The literature never refers to this as the 'homosexual marriage problem'. Rather, it is called the 'roommates problem', and even 'A unisex problem: stability of roommates' (Knuth, 1997: 64). The one exception is Rudich's powerpoint presentation, which describes this situation as 'bisexual dating'. The role of this scenario in Rudich's presentation, however, is to demonstrate a case where a stable matching is unattainable. Since stability is strongly marked as an essential ideal in Rudich's presentation ('a pairing is doomed if it contains a rogue couple'), bisexual dating seems to be negatively marked.

The example above demonstrates how, when the discourse encounters an opportunity to endorse progressive aspects of sexuality, it tries to clean itself up and revert to gender neutral language. Conservative sexuality discourse is strong enough to prevent the mathematical setting from reflecting unorthodox possibilities, even when such possibilities make mathematically-metaphorical sense.

The same phenomenon is repeated when other combinations are suggested. For instance when triples, rather than couples are to be matched, the three roles in most presentations are either men, women and children, or men, women and dogs

(e.g. Knuth 1997, 64). This choice reiterates both normative family stereotypes and the position of women in such families, while foreclosing the horizon of polyamorous relationships. Yet another instance for this form of puritan-conservatism is the construct theoretically referred to as 'chains' or 'rotations'. This term refers to a set of couples, who exchange spouses according to a certain rule. The literature never describes this situation as 'swinging' or 'open relationships'. Rather, at that point the texts opt to revert to a technical gender-free language.

But non-orthodox sexuality does manage to occasionally impose itself on mathematical discourse, as the following quote demonstrates (the context is an attempt to relate, or 'reduce', one variant of the marriage problem to another): 'Such a reduction might involve turning each person into two persons, one male and one female (splitting one's personality into its animus and anima)' (Gusfield and Irving, 1989: 221). The splitting, which mathematically makes perfect sense, imposes at least a moderate form of trans- or bi-sexuality on the underlying notion of gender.

One of the most important variations of the marriage problem deals with a situation where each man may marry several women. Some authors refer to this variant as the harem problem (e.g. Wilson, 1996: 114). It is interesting to note that this text presents the marriage problem as a problem about women marrying men – but when the harem problem is introduced in an exercise, the men turn out to be the ones who marry many women.

Most authors, however, shun the use of marriage terminology for this variant. 'It is convenient to use politically correct terms', writes Bollobás (1998), and turns to the student-college formulation of the problem, where each college should be matched to several students. But this attempt to clean up the mathematical discourse from reactionary gender imagery ends up generating progressive opportunities for cross-gendering. In translating the man-woman matching problem to a student-school matching problem, it is not clear which should play whose role. Thus, Teo et al. (1999) assigns the women to the role of students, whereas Immerlica and Mahdian (2005) makes the opposite choice. A person reading both papers is confronted with the necessity to allow a simultaneous double role allocation for men and women (which is a much more intricate endeavour to sustain than simply choosing between a given allocation of position and its inversion), if this reader is interested in retaining the contents of both papers at once.

On some occasions, the gender imagery is confronted with other metaphorical images, which may or may not reinforce gender role stereotypes. Thus, when Knuth (1997) demonstrates links between the stable marriage problem and other algorithmic problems (related to path finding and hash tables), the transformations rely on such statements as 'The city ... plays the role of the woman' and 'The cell that each [man] occupies corresponds to the number of his girlfriend' (Knuth, 1997: 35, 40). The image of the woman as a place which confines the man correlates nicely with the ideological link between women and domesticity.

But such interactions between different problems do not necessarily work out so elegantly oppressive. Roth and Sotomayor (1990) regard the problem from an economic point of view, and introduce the term 'marriage market'. This commodification of the matrimonial situation is reflected in another paper by such statements as 'The cost for man  $i$  to be married with woman  $j$  is  $x(i, j)$  and the cost for woman  $j$  to be married with man  $i$  is  $y(j, i)$ ' (Dzierzawa & Oméro, 2000: 322). Then, following its own statistical physics inspired logic, the paper goes on to transform cost to energy, as men and women become species of particles. The effects of this semiotic line of flight cannot be confined or anticipated in advance.

As a somewhat optimistic final statement, I will include the following quote: 'The man optimal matching corresponds to the minimal P-set ... . Mathematically

there is no problem with this, but the reader may have to make a psychological transformation to avoid unconsciously identifying (male) dominance with maximality' (Gusfield and Irving, 1989: 75). Regardless of what a P-set is, this quote demonstrates how a perfectly viable mathematical transformation might impose a psychological re-evaluation of ideological commitments.

But how seriously should we take these few mathematical lines of flight from imposing gender role stereotypes? There is something a little too easy about such transformations. The fact that it is so easy for Knuth to write 'It suffices to exchange the role of the men and the women and apply the theorem' (Knuth, 1997: 57) indicates how detached this scenario is from an actual reform of extra-mathematical discourse. The situation is akin to the one where Lacan states that 'One could, at a pinch, write  $xRy$ , and say  $x$  is a man,  $y$  is a woman, and  $R$  is the sexual relationship' (Lacan, 1998: 35), in order to demonstrate how detached mathematical symbolism is from the limitations of his analytic situation, where a sexual relationship cannot exist.

We could claim that the above effects of ideological replication, constraints, and semiotic drift are confined to mathematical texts, and therefore have nothing to do with social change. We could then infer that we needn't bother at all with the languages that science uses, because these languages are restricted to a confined textual or discursive 'reservation'. If, however, we believe that there is some (however small) interaction between mathematical language and social reality (if only because it is spoken by real, at least somewhat socially functional people), then the semiotic drift imposed on gender role stereotypes by mathematical transformations should not be ignored.

Lacan, whom we have just quoted as pointing out the irrelevance of mathematical manipulations, is in fact the very theoretician whom we can quote as endorsing (at least in his earlier work) the possibility of therapy through an exploration of the symbolic realm. For earlier Lacan psychoanalytic therapy can be read as consisting of exhausting the potential of symbolic transformations. A patient (little Hans) may be encouraged to develop 'all the possible permutations of a limited number of signifiers ... . We see here that ... man can find a solution to the impossible by exhausting all possible forms of the impossibilities that are encountered when the solution is put into the form of a signifying equation' (Lacan, 2006: 432).

Perhaps, then, we should encourage mathematics to explore conceptions that are feminist or queer. Perhaps we should encourage social and exact scientists to carry their latent and explicit ideological commitments through mathematics' obscure transformations. Perhaps this would lead us towards new semiotic possibilities for confronting the impossible impasses in our ways of speaking gender and/or science. Perhaps encouraging signifiers to cut across discursive systems where they do not, supposedly, 'belong', does have some therapeutic potential for our contemporary social malaises.

## Notes

This paper is part of the author's PhD thesis, entitled *Post-Structural Readings of a Logico-mathematical Text*, written in Tel-Aviv University's Cohn Institute for the History and Philosophy of Science and Ideas under Prof. Adi Ophir and Prof. Anat Biletzki (the thesis focuses on a textual analysis of Gödel's proofs of his first incompleteness theorem). I would like to thank both my advisors, as well as Prof. Sandra Harding, Jay Lemke and the anonymous referee for some good advice concerning this paper.

I would like to quote one of the referees, Jay Lemke, who wrote: 'The domains where [the] issues [focused on in this paper] seem to arise are game theory and its relationship to some branches of neo-liberal economics ... . [These] intersections with game theory are prominent in terms of conflating social with formal definitions of "optimality" of various kinds. I would be very interested to read some case studies from these domains where both the bias is documented, and where the formal analyses suggest unorthodox critiques of neo-liberal assumptions and values ... . I would much rather have seen a longer paper that... went on to confirm its thesis with another example, perhaps from neo-liberal economics' use of game theory, whose generalizability would be more transparent. Enough so to send dozens of scholars off in search of still further such instances. Perhaps this small match is enough to light such a fire.' Acknowledging the risk that this 'small match' might not be 'enough', I include the above quotation so as to encourage 'a more substantial kindling' of such project, which, albeit not quite the project I am engaged in, I join Lemke in the hope of inciting.

1. I am not suggesting here that the Turing Machine framework constrains some non-sexist approaches to the marriage problem because it is a sexist construct. Sexist language is not the only possible road to sexist results. Even if the Turing Machine is, as some researchers claim, a non-heterosexual construct, this is not enough to guarantee that it would yield non-sexist mathematical knowledge. Trying to think outside the sexist box is only one step towards doing better mathematics, but there would always be other constraints which must be questioned in order to continue improve our science, genderwise or otherwise. Taking responsibility, I suggest, means to keep questioning the relations between the constraints we impose on signs and our science's prescriptive and normative results. The attitude which says: 'even if we try our best, there might always emerge unexpected problems, so why fuss about the constraints we put on signs?' is, I maintain, copping out.

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