New Type of Three-Axis Hall Sensor Designed for High-Accuracy Magnetic Field Measurements

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Keep your eyes on the stars, and your feet on the ground.

- Theodore Roosevelt

To my parents
Abstract

Hall sensors (1D and 3D) are routinely used in magnetic field measurements of beamline magnets, insertion devices, and detector magnets at research institutes and accelerator facilities. These measurements are a high-end application of Hall sensors, demanding high accuracies up to $10^{-4}$ (at 1 T level) or even beyond. While this is consistently achieved with uniaxial (1D) Hall sensors in a single-component magnetic field volume, the measurement of all three components of a magnetic field, simultaneously to high accuracy with Hall sensors, remains a challenge. None of the commercially available three-axis (3D) Hall sensors proclaims similar measurement accuracy to 1D Hall sensors. Currently, 3D Hall sensors suffer from either, or a combination, of the following: large spatial distribution between sensors’ active areas; high signal noise; cross-sensitivity among measurement axes due to angular errors or due the planar Hall effect (PHE); the inability to measure at a single point in space and time.

A new type of three-axis Hall sensor is proposed, consisting of three pairs of uniaxial Hall sensors in a very small active volume. Due to its unique configuration, the new sensor can address current three-axis Hall sensor limitations — it provides: a high spatial resolution of $30 \mu m \times 30 \mu m \times 1 \mu m$ for each field component; the full field vector measurements practically at a single point in space and time; and compensation of the planar Hall effect as well as loop-induced voltages by the pairs of 1D Hall sensors.

The feasibility of the proposed sensor has been proven in a prototype with an active volume as small as $200 \mu m \times 200 \mu m \times 200 \mu m$ and outer dimensions of $4 mm \times 4 mm \times 4 mm$. The prototype was fabricated from six silicon(Si)-doped gallium arsenide (GaAs) Hall sensors that were each glued by epoxy onto MACOR support cuboids and subsequently assembled using precision assembly tools designed especially for this purpose. The accuracy of the dimensions of the 1D Hall sensors and the MACOR cuboids lay well within the design tolerances of 10 µm. The added thickness from epoxy was below 15 µm. A low fabrication yield of $\sim 50\%$ was mainly due to detachment of soldered wires during assembly steps.

Characterization and calibration of the six Hall sensors was done in a calibration magnet with field homogeneity of at least $10^{-4}$ T in a volume of 10 mm radius. A non-magnetic piezoelectric rotation system was employed with two orthogonal rotation axes to mimic any field direction to the 3D Hall sensor.

A characterization scheme based on harmonic analysis of the Hall voltage dur-
ing rotation has been developed in order to determine angular errors among sensors and the surface normal vector of each Hall sensor. Angular errors among individual Hall sensors in the prototype were found to lie between $0.1^\circ$ and $0.5^\circ$, with two outliers at $0.01^\circ$ and $1.0^\circ$. As expected, they lay above the tolerable error of $0.006^\circ$ for non-corrected measurements. Nevertheless, the PHE compensation by the pairs proved to be effective with a reduction of more than 35 times, resulting in a remnant maximum error of $< 0.5$ Gauss at $1\,T$ or $0.5 \cdot 10^{-4}$.

Calibration of each sensor was done against NMR reference probes over the field range of $\pm 1.5\,T$. Because the gap of the calibration magnet was too small to encompass both the Hall sensor rotation system and the NMR probe, calibration was done in two steps: first the calibration magnet was calibrated (field against magnet current), and subsequently the Hall sensors were calibrated (Hall voltage against magnet current). The repeatability of calibration of the field against the magnet current was $0.1$ Gauss. The calibration results of each sensor were described with a polynomial function of up to fifth order governing the offset, sensitivity, and non-linearity of each sensor.

Field reconstruction was done through the knowledge of the sensors’ surface normal vectors, thereby taking the sensors’ non-orthogonalities stemming from the assembly into account, and through the coefficients of the polynomial functions of the calibration process.

First test measurements were performed in a superconducting solenoid detector magnet as well as in an undulator. To determine the absolute accuracy of the prototype 3D Hall sensor, the sensor’s reconstructed total magnetic field, inside a homogeneous and constant magnetic field of $\sim 1\,T$ and of various directions, was recorded. The reconstructed magnetic field value was constant up to $8$ Gauss or $8 \cdot 10^{-4}$.

The hypothesis for not having achieved a better accuracy, was a long-term instability of the Hall voltages. After applying a bias current in series to all six sensors, the Hall voltages drifted for several hours before stabilizing to Hall voltages with a peak-to-peak fluctuation of $0.1\%$. Ambient temperature fluctuations, change in magnetic field (value or direction), non-constant bias current, $1/f$ noise, noise currents in cabling, packaging-induced stress, could sequentially all be eliminated as causes. An indication for solving the long-term instability as well as the initial drift, was found in passivation of the Hall sensors with silicon nitride ($\text{SiN}_x$).

In this research work, the state-of-the-art in three-axis Hall sensors got advanced by designing, realizing, and characterizing a novel three-axis Hall sensor which has a sub-millimetre active volume, allows determination of the full magnetic field vector at the same point at the same time, while compensating for the planar Hall effect and angular offsets. The combination of these features in one
three-axis Hall sensor is unprecedented. The idea of placing six uniaxial Hall sensors in a very small volume that forms a three-axis Hall sensor was made possible by the original way of assembly. The prototype confirmed that, without instability of the Hall voltages, the target measurement accuracy can be achieved and surpassed.
Zusammenfassung

Hall-Sensoren (1D und 3D) werden routinemässig zu Magnetfeldmessungen von Magneten für Strahlführungslinien, Insertion Devices und Detektormagneten in Forschungsinstituten und Beschleunigeranlagen verwendet. Diese Messungen sind eine hochwertige Anwendung von Hall-Sensoren, welche hohe Genauigkeiten bis zu $10^{-4}$ (auf einem Niveau von 1 T) oder sogar darüber hinaus fordern. Während dies konsequent mit einachsigen (1D) Hall-Sensoren in einem einkomponentigen Magnetfeldvolumen erreicht wird, bleibt die Messung aller drei Komponenten eines Magnetfeldes, gleichzeitig zu einer hohen Genauigkeit mit Hall-Sensoren, eine Herausforderung. Keine der kommerziell erhältlichen Drei-Achsen (3D) Hall-Sensoren bietet eine ähnliche Messgenauigkeit wie 1D Hall-Sensoren. Heutige 3D Hall-Sensoren leiden an einer, oder einer Kombination, der folgenden Möglichkeiten: grosse räumliche Verteilung zwischen den aktiven Sensorbereichen; starkes Signalrauschen; Querempfindlichkeit bei den Messachsen aufgrund von Winkelfehlern oder aufgrund des planaren Hall-Effekts (PHE); die Unfähigkeit, an einem einzelnen Punkt in Raum und Zeit zu messen.

Eine neue Art des Drei-Achsen Hall-Sensors wird vorgeschlagen, der aus drei Paaren von einachsigen Hall-Sensoren in einem sehr kleinen aktiven Volumen besteht. Aufgrund seiner einzigartigen Konfiguration kann der neue Sensor Beschränkungen aktueller Drei-Achsen Hall-Sensoren angehen — er bietet: eine hohe räumliche Auflösung von $30 \mu m \times 30 \mu m \times 1 \mu m$ für jede Feldkomponente; die komplette Feldvektor-Messung praktisch an einem einzigen Punkt in Raum und Zeit; und Kompensation des planaren Hall-Effekts sowie schleifeninduzierte Spannungen durch die Paare von 1D Hall-Sensoren.

Die Realisierbarkeit des vorgeschlagenen Sensors hat sich in einem Prototypen mit einem Aktivvolumen so gering wie $200 \mu m \times 200 \mu m \times 200 \mu m$ und äußeren Abmessungen von $4 \text{ mm} \times 4 \text{ mm} \times 4 \text{ mm}$ bewährt. Der Prototyp wurde aus sechs Silizium (Si) dotierten Galliumarsenid (GaAs) Hall-Sensoren hergestellt, die jeweils mit Epoxidharz auf MACOR Auflageblocks geklebt wurden und anschliessend mit Präzisionswerkzeug, welches speziell für diesen Zweck entwickelt wurde, zusammengestellt. Die Genauigkeit der Abmessungen der 1D Hall-Sensoren und der MACOR Auflageblocks lag gut innerhalb der Konstruktionstoleranzen von $10 \mu m$. Die zusätzliche Dicke der Epoxidschicht betrug weniger als $15 \mu m$. Eine geringe Herstellungsausbeute von $\sim 50\%$ ist hauptsächlich auf Ablösung der gelöteten Drähte während der Montageschritte zurückzuführen.
Charakterisierung und Kalibrierung der sechs Hall-Sensoren wurde ausgeführt in einem Eichmagneten mit Feldhomogenität von mindestens $10^{-4}$ T in einem Volumen mit 10 mm Radius. Ein nichtmagnetisches piezoelektrisches Rotationssystem mit zwei orthogonalen Drehachsen wurde eingesetzt, womit sich jede Feldrichtung zum 3D Hall-Sensor imitieren lässt.

Ein Charakterisierungsschema basierend auf der harmonischen Analyse der Hall-Spannung während der Rotation wurde entwickelt, um Winkelfehler zwischen den Sensoren festzustellen und um die Normalvektoren jedes Hall-Sensors zu bestimmen. Winkelfehler zwischen den einzelnen Hall-Sensoren im Prototyp wurden bestimmt und lagen zwischen 0.1° und 0.5° bei zwei Ausreißern von 0.01° und 1.0°. Wie erwartet, lagen sie über dem zulässigen Fehler von 0.006° für nicht-korrigierte Messungen. Nichtdestotrotz hat sich die Kompensation des PHE durch die Paare, mit einer Reduktion von mehr als 35 mal, was einem restlichen maximalen Fehler von $< 0.5$ Gauss bei 1 T oder $0.5 \cdot 10^{-4}$ entspricht, als wirksam erwiesen.

Jeder Sensor wurde gegen NMR Referenzproben in einem Feldbereich von $\pm 1.5$ T kalibriert. Da der Spalt des Eichmagnets zu klein war, um sowohl das Rotationssystem als die NMR-Sonde zu umfassen, wurde die Kalibrierung in zwei Schritten getätigt: zuerst wurde der Eichmagnet kalibriert (Magnetfeld gegen Magnetstrom), und anschliessend wurden die Hall-Sensoren kalibriert (Hall-Spannung gegen Magnetstrom). Die Wiederholbarkeit der Feldkalibrierung gegen den Magnetstrom betrug 0.1 Gauss. Die Kalibrierungsergebnisse jedes Sensors wurde mit einem Polynom bis fünfter Ordnung beschrieben, mit welchem Nullpunktverschiebung, Empfindlichkeit und Nichtlinearität jedes Sensors gedeckt wurde.

Die Feldrekonstruktion geschah durch Kenntnis der Normalvektoren der Sensoren, somit werden die Nicht-Orthogonalitäten aus dem Zusammenbauschritt miteinbezogen, und durch die Koeffizienten der Polynome des Kalibrierungs-vorgangs.

Erste Testmessungen wurden in einem supraleitenden Detektormagnet sowie in einem Undulator durchgeführt. Um die absolute Genauigkeit des Prototyps des 3D Hall-Sensors zu bestimmen, wurde das rekonstruierte Gesamtfeld des Sensors in einem homogenen und konstanten Magnetfeld von $\sim 1$ T und bei verschiedenen Feldorientierungen aufgezeichnet. Die rekonstruierten Magnetfeldwerte waren bis auf 8 Gauss konstant, oder $8 \cdot 10^{-4}$.

Laut Hypothese war die Genauigkeit nicht besser wegen der Langzeitinstabilität der Hall-Spannungen. Nach Anlegen eines Stroms auf die reihengeschalteten sechs Sensoren, drifteten die Hall-Spannungen über mehrere Stunden, bevor eine Stabilisierung der Hall-Spannungen mit einer Spitze-Spitze-Schwankung von 0.1% erreicht wurde. Umgebungstemperaturschwankungen, Änderung
des Magnetfeldes (Wert oder Richtung), nicht-konstanter Strom, 1/f-Rauschen, Rauschströme im Kabel, verpackungsinduzierter Stress, konnten der Reihe nach alle als Ursachen beseitigt werden. Ein Hinweis zur Beseitigung der Langzeitinstabilität sowie den Anfangsdrift wurde in Passivierung der Hall-Sensoren mit Siliziumnitrid (SiNx) gefunden.

Contents

Abstract vii

Zusammenfassung xi

List of Symbols and Abbreviations xix

1 Introduction 1

2 The Hall Sensor for Magnetic Field Measurements 5
   2.1 Working Principle of Hall Sensors 5
      2.1.1 The Hall Effect: A Simplified Description 5
      2.1.2 Second-Order Effects: A Qualitative Description 8
      2.1.3 An Accurate Description 10
   2.2 Basic Characteristics of Hall Sensors 12
      2.2.1 Material Properties 13
      2.2.2 Geometry Influence 15
      2.2.3 Hall Sensor Parameters 17
   2.3 Magnetic Field Measurements of Magnets with Hall Sensors 19
      2.3.1 The ”Zoo” of Magnets 19
      2.3.2 Field Mapping 21
   2.4 Hall Sensor Calibration 21

3 Design of a Novel Three-Axis Hall Sensor: The ”Hallcube” 29
   3.1 Two Concepts 29
      3.1.1 The Truncated Pyramids Design 29
      3.1.2 The Cuboids Design 30
   3.2 Design of the Uniaxial Hall Sensor Components 33
      3.2.1 Geometry of the Sensing Part 33
      3.2.2 Electrical Connections 33
      3.2.3 Materials Selection 35
   3.3 Thermal Analysis 41
   3.4 Measurement Error Description 45
      3.4.1 Angular Errors between Measurement Axes 45
      3.4.2 Planar Hall Effect Compensation with a Pair of Hall Plates 46
      3.4.3 Spatial Distribution 48
Contents

7.3.2 Measurement Setup ........................................ 129
7.3.3 Results and Analysis ........................................ 132

8 Conclusion ......................................................... 139

Appendices .......................................................... 143

A Process Runsheets ............................................... 143
B COBRA Measurement Scheme ................................. 145
C COBRA Measurement Results at THETA = 45°, 90°, and 135° ... 147
D Positions of Rotation Stages for Pairwise Hall Sensor Calibration . 151

Bibliography .......................................................... 153

Acknowledgement .................................................. 161

Publications ........................................................ 163

Curriculum vitae ................................................... 165
## List of symbols

All parameters are given in SI units

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>m$^2$</td>
<td>Area</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>rad</td>
<td>Rotation angle of rotation stage alpha</td>
</tr>
<tr>
<td>$\alpha_T$</td>
<td>%K$^{-1}$</td>
<td>Temperature coefficient</td>
</tr>
<tr>
<td>$\beta$</td>
<td>rad</td>
<td>Rotation angle of rotation stage beta</td>
</tr>
<tr>
<td>$B$</td>
<td>T</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>$c$</td>
<td>ms$^{-1}$</td>
<td>Speed of light</td>
</tr>
<tr>
<td>$c_i$</td>
<td>TV$^{-1}$</td>
<td>Calibration coefficients</td>
</tr>
<tr>
<td>$d$</td>
<td>m</td>
<td>Spacing between Hall sensors</td>
</tr>
<tr>
<td>$e$</td>
<td>C</td>
<td>Elementary electric charge</td>
</tr>
<tr>
<td>$E$</td>
<td>NC$^{-1}$</td>
<td>Electric field</td>
</tr>
<tr>
<td>$E_g$</td>
<td>eV</td>
<td>Bandgap</td>
</tr>
<tr>
<td>$F$</td>
<td>N</td>
<td>Force</td>
</tr>
<tr>
<td>$g$</td>
<td>m</td>
<td>Gap height of insertion device</td>
</tr>
<tr>
<td>$\gamma$</td>
<td></td>
<td>Relativistic Lorentz factor</td>
</tr>
<tr>
<td>$G_H$</td>
<td></td>
<td>Geometrical correction factor</td>
</tr>
<tr>
<td>$I$</td>
<td>A</td>
<td>Electric current</td>
</tr>
<tr>
<td>$J$</td>
<td>Am$^{-2}$</td>
<td>Current density</td>
</tr>
<tr>
<td>$k$</td>
<td>eVK$^{-1}$</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>$K$</td>
<td></td>
<td>Insertion device parameter</td>
</tr>
<tr>
<td>$K_s$</td>
<td>s$^3$kg$^{-1}$m$^{-3}$</td>
<td>Kinetic coefficients</td>
</tr>
<tr>
<td>$k_{th}$</td>
<td>Wm$^{-1}$K$^{-1}$</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>$l$</td>
<td>m</td>
<td>Length of semiconductor plate</td>
</tr>
<tr>
<td>$\lambda_{ID}$</td>
<td>m</td>
<td>Insertion device period</td>
</tr>
<tr>
<td>$m^*$</td>
<td>kg</td>
<td>Effective mass</td>
</tr>
<tr>
<td>$m_e$</td>
<td>kg</td>
<td>Electron mass</td>
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<tr>
<td>$\mu_{n,p}$</td>
<td>m$^2$V$^{-1}$s$^{-1}$</td>
<td>Electron, hole mobility</td>
</tr>
<tr>
<td>$\mu_H$</td>
<td>m$^2$V$^{-1}$s$^{-1}$</td>
<td>Hall mobility</td>
</tr>
<tr>
<td>$n$</td>
<td>m$^{-3}$</td>
<td>Unit normal vector of 1D Hall sensor</td>
</tr>
<tr>
<td>$n$</td>
<td>m$^{-3}$</td>
<td>Charge carrier density (electrons)</td>
</tr>
<tr>
<td>$n_s$</td>
<td>m$^{-2}$</td>
<td>Sheet carrier density (electrons)</td>
</tr>
<tr>
<td>$p$</td>
<td>m$^{-3}$</td>
<td>Charge carrier density (holes)</td>
</tr>
<tr>
<td>$P$</td>
<td>W</td>
<td>Electric power</td>
</tr>
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### Contents

- $P_H$: m$^3$A$^{-1}$s$^{-1}$T$^{-1}$, Planar Hall coefficient
- $R$: Ω, Resistance
- $R_H$: m$^3$A$^{-1}$s$^{-1}$, Hall coefficient
- $R_{th}$: KW$^{-1}$, Thermal resistance
- $\rho$: Ωm, Resistivity
- $\rho_s$: Ω, Sheet resistivity
- $s$: m, Width of electrical contact pad
- $\sigma_{n,p}$: Ω$^{-1}$m$^{-1}$, Electron, hole electrical conductivity
- $t$: m, Thickness of semiconductor plate active layer
- $T$: K, Temperature
- $\tau$: s, Relaxation time
- $\theta_H$: rad, Hall angle
- $v$: ms$^{-1}$, Velocity
- $v_d$: ms$^{-1}$, Drift velocity
- $V_H$: V, Hall voltage
- $V_{PH}$: V, Planar Hall voltage
- $w$: m, Width of semiconductor plate
- $Y_{lm}^*$: Spherical harmonics
## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>1D</td>
<td>One-dimensional</td>
</tr>
<tr>
<td>2DEG</td>
<td>Two-dimensional electron gas</td>
</tr>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog-to-digital converter</td>
</tr>
<tr>
<td>AlGaAs</td>
<td>Aluminium gallium arsenide</td>
</tr>
<tr>
<td>Au</td>
<td>Gold</td>
</tr>
<tr>
<td>BB</td>
<td>Blue-bottom</td>
</tr>
<tr>
<td>BT</td>
<td>Blue-top</td>
</tr>
<tr>
<td>COBRA</td>
<td>Constant bending radius</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary metal-oxide semiconductor</td>
</tr>
<tr>
<td>CNC</td>
<td>Computer numerical control</td>
</tr>
<tr>
<td>Cr</td>
<td>Chromium</td>
</tr>
<tr>
<td>DC</td>
<td>Direct current</td>
</tr>
<tr>
<td>DVM</td>
<td>Digital voltmeter</td>
</tr>
<tr>
<td>ETH</td>
<td>Eidgenössische Technische Hochschule</td>
</tr>
<tr>
<td>FEL</td>
<td>Free electron laser</td>
</tr>
<tr>
<td>FIRST</td>
<td>Frontiers In Research: Space and Time</td>
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<tr>
<td>GB</td>
<td>Green-bottom</td>
</tr>
<tr>
<td>GT</td>
<td>Green-top</td>
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<tr>
<td>GaAs</td>
<td>Gallium arsenide</td>
</tr>
<tr>
<td>Ge</td>
<td>Germanium</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>Water</td>
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<tr>
<td>H$_2$O$_2$</td>
<td>Hydrogen peroxide</td>
</tr>
<tr>
<td>H$_2$SO$_4$</td>
<td>Sulfuric acid</td>
</tr>
<tr>
<td>HMDS</td>
<td>Hexamethyldisilazane</td>
</tr>
<tr>
<td>IC</td>
<td>Integrated circuit</td>
</tr>
<tr>
<td>ID</td>
<td>Insertion device</td>
</tr>
<tr>
<td>InAs</td>
<td>Indium arsenide</td>
</tr>
<tr>
<td>InSb</td>
<td>Indium antimonide</td>
</tr>
<tr>
<td>IPA</td>
<td>Isopropanol</td>
</tr>
<tr>
<td>KTI</td>
<td>Kommission für Technologie und Innovation</td>
</tr>
<tr>
<td>MBE</td>
<td>Molecular beam epitaxy</td>
</tr>
<tr>
<td>MEG</td>
<td>Mu to e gamma</td>
</tr>
<tr>
<td>Ni</td>
<td>Nickel</td>
</tr>
<tr>
<td>NL</td>
<td>Non-linearity</td>
</tr>
<tr>
<td>NMR</td>
<td>Nuclear magnetic resonance</td>
</tr>
<tr>
<td>PECVD</td>
<td>Plasma-enhanced chemical vapor deposition</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>PEEK</td>
<td>Polyether ether ketone</td>
</tr>
<tr>
<td>Pt</td>
<td>Platinum</td>
</tr>
<tr>
<td>PMMA</td>
<td>Polymethyl methacrylate</td>
</tr>
<tr>
<td>PSI</td>
<td>Paul Scherrer Institute</td>
</tr>
<tr>
<td>RB</td>
<td>Red-bottom</td>
</tr>
<tr>
<td>RMS</td>
<td>Root mean square</td>
</tr>
<tr>
<td>RT</td>
<td>Red-top</td>
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<tr>
<td>Si</td>
<td>Silicon</td>
</tr>
<tr>
<td>SiN</td>
<td>Silicon nitride</td>
</tr>
<tr>
<td>SI</td>
<td>Semi-insulating</td>
</tr>
<tr>
<td>SLS</td>
<td>Swiss light source</td>
</tr>
<tr>
<td>SµS</td>
<td>Swiss muon source</td>
</tr>
<tr>
<td>TTL</td>
<td>Transistor-transistor logic</td>
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1 Introduction

I am in some respects distinctly handicapped in all my scientific endeavors, being unskilful of hand and slow of apprehension. On the other hand, I am very persistent, and fond of wrestling with a difficult problem in my own slow way; any success I may have attained is to be attributed to these two qualities [1].

- Edwin H. Hall

In 1879 the PhD student Edwin Hall discovered “a new action of the magnet on electric currents”. He measured a transverse voltage on a gold foil in the presence of a magnetic field perpendicular to the current flow in the foil [2]. At the time, the electron had not yet been discovered by Thomson and electricity was believed to be the movement of a continuous fluid. Hall made his discovery in search of a contradiction to Maxwell’s notion that a magnet acts on a conductor carrying a current but not on the current itself. He questioned this statement given that “a wire bearing no current is in general not affected by a magnet and a wire bearing a current is affected exactly in proportion to the strength of the current”. To test his hypothesis that a magnet indeed acts on the current itself, he suggested that the resistance of a current-carrying conductor must increase inside a magnet because the current should be drawn to one end. The experiments he conducted failed to demonstrate the anticipated increase in resistance, however. Without giving up, he considered that if electricity is assumed to be an incompressible fluid, it might not be possible to force the current to one side of the conductor. In that case, the anticipated effect of a magnet on a current-carrying conductor would not be an increase of resistance, but a potential difference across the current direction, caused by induced stress from electricity pressing towards one side of the conductor. His efforts and persistence were rewarded when he measured the sought-after potential difference on a thin gold foil. In Hall’s honour, this potential difference became known as the Hall voltage and the effect that causes it became known as the Hall effect. Hall continued to study electrical phenomena during his life and published coefficients governing the strength of the Hall effect in different metals. In 1954, Goldberg and Davis reported on a “new galvanomagnetic effect”, they had discovered that a voltage across the direction of current flow is also generated when the magnetic field is not orthogonal to the current-voltage plane but is in that plane. It became known as the planar Hall effect.
1 Introduction

The Hall effect in metals like in the gold foil from Edwin Hall’s experiment is very small and had found no practical application. The technical utilization of the Hall effect started only with the discovery of the high mobility III-V compounds indium arsenide and indium antimonide at the research laboratory of Siemens-Schuckert AG in 1952. Around the same time, Integrated Circuit (IC) technology was invented which progressed from the sixties onwards. For Hall sensors, this technology facilitated the integration of electronic circuitry on the same chip as the Hall sensor. It also brought about the vertical Hall sensor \[3\] which unlike conventional (horizontal) Hall sensors generates a Hall voltage from a magnetic field parallel to the chip surface and not orthogonal to it. This enabled the development of integrated circuit three-axis Hall sensors. Nowadays, the Hall effect and Hall effect devices are used in a range of applications like in solid-state physics for carrier mobility and concentration measurements, position and motion sensing, current measurements, switches, and the most obvious application is magnetic field measurements.

Hall sensors are used in magnetic field measurements of beamline magnets, insertion devices, and detector magnets at e.g. research institutes and accelerator facilities. These measurements are a high-end application of Hall sensors demanding high accuracies, $10^{-4}$ at 1 T has become a norm. For some of these magnet measurements, 1D Hall sensors are sufficient, such as for the measurement of the symmetry planes of magnets which predominantly have only a single field component. Three-axis magnetic field measurements are however needed in some cases, in general whenever the magnetic field volume of interest is not a single-component field volume. Magnetic field measurements with 1D Hall sensors have reached a state of the art. By controlling the alignment of the sensor as well as repeated thorough calibration against non-linearity effects, temperature dependence, and offset, a measurement accuracy of 100 µT, or $10^{-4}$ for 1 T fields, can be routinely achieved \[4\],\[5\],\[6\],\[7\]. However, the same can not be said of three-axis field measurements. None of today’s commercially available three-axis Hall sensors offers the same accuracy that can be reached with single-axis Hall sensors. The reasons for this are poor spatial resolution, high noise, cross-sensitivity between measurement axes, or the lack of possibility to measure in a single point in space and time. For this particular reason, there is a clearly defined need for a high-accuracy three-axis magnetic field sensor in accelerator facilities such as synchrotron light sources and Free Electron Lasers (FELs), but also in the research community in general.

Three-axis Hall sensors can be classified into two general types \[8\]:

(a) The most straightforward way to measure all three field components simultaneously, is by combining three individual and orthogonally arranged 1D horizontal Hall sensors. The main intention with this type of three-axis Hall sensor is
that the high sensitivity and excellent precision of the 1D Hall sensor can simply be extended to three directions. The major drawback is that cross-sensitivity among measurement axes, and hence measurement errors, due to the planar Hall effect and due to non-orthogonality among the individual Hall sensors can be high. Another drawback is the large spatial distribution of the single elements. Although adept placement of three sensors could reduce the distance to a minimum of 250 µm [9], reduction to zero is not possible, preventing measurements of all field components at the same point at the same time.

(b) The second type of 3D Hall sensors is IC Hall sensors realized by CMOS technology. They incorporate a three-axis Hall sensor on a single chip: either monolithic [10], or by integrating two or more vertical and horizontal Hall sensors [11]. Monolithic devices have potentially smaller active volumes due to the reduced number of contacts since they have only one active region to measure all three field components. However, the performance of monolithic devices is lowered by the difference in sensitivity to the three field components as well as large cross-sensitivity among them [10],[12],[13]. Vertical Hall sensors have inferior characteristics such as higher offset and lower sensitivity compared to horizontal Hall sensors [14]. The spinning-current technique [15], which compensates for planar Hall voltages as well as offset voltages, is less effective when applied to vertical Hall sensors due to the more complex current flow in these devices [16]. Attempts to improve the performance of vertical Hall devices, by optimizing their symmetry, have recently resulted in low offset vertical Hall devices [17],[18]. Although the technology has been applied in a 2D device [19], the achieved accuracy is of the order of a percent and it is yet to result in a 3D device. Instead, a different type of sensor was pursued in the form of a hexagonal prism [20]. Isotropic sensitivity to all three field components was achieved but with a thickness of 525 µm, its active volume is big and, the obtained accuracy is again of the order of a percent. Also, the three field components cannot be measured at the same time.

Given the high accuracy that can be achieved with 1D horizontal Hall sensors for room-temperature magnet measurements, the goal of extending this technology to a 3D Hall sensor, without loss of measurement accuracy to any field direction, is pursued in this thesis. It aspires the development of a new type of three-axis Hall sensor which consists solely of 1D horizontal Hall sensors, yet overcomes the drawbacks of large active volume as well as cross-sensitivity among measurement axes due to angular errors and the planar Hall effect. The design should focus on simplicity of a 3D sensor that is inherently precise due to the superiority of horizontal Hall sensors, and hence can do without the added complication of compensation electronics. The accuracy should be enforced by (1) the high precision of the superior 1D Hall sensors compared to IC Hall sensors,
1 Introduction

and (2) the translation of high precision into high accuracy by a newly developed, dedicated and comprehensive 3D characterization and calibration scheme. The design should foresee the feasibility in manufacturing of the sensor without compromising on achieving a sub-millimetre active volume and outside sensor dimensions in the millimetre-range. The proof of principle of the sensor is then to be demonstrated with a prototype version.

To achieve the aforementioned objectives, the research presented in this thesis was for the most part carried out in the Magnet Section of the Paul Scherrer Institute (PSI) in Villigen. The Magnet Section has a long-term experience in high-accuracy magnet measurements with commercial 1D Hall sensors. Its temperature-controlled Hall sensor measurement lab facilitates state-of-the-art field-mapping with an automated five-axis measurement machine that slides on compressed air pads on a precisely machined 3.3 m × 2.0 m × 0.5 m granite block. The same lab contains a dipole magnet with a highly homogeneous field for calibration of the Hall sensors in a range of ±2 T. Here, all the testing, characterization, calibration, as well as the design of Hall sensors presented in this thesis were carried out (see respectively Chapters 7, 5, 6, and 3). The fabrication of the Hall sensors was done mainly at the Eidgenössische Technische Hochschule Zürich, in particular at the Solid State Physics Laboratory and FIRST lab. The first test magnet measurements with the fabricated prototype three-axis Hall sensor (“Hallcube”) were performed in a superconducting detector magnet for the Mu to E Gamma (MEG) experiment in collaboration with the MEG team at PSI; and in a laser heater undulator for the Swiss Free Electron Laser (SwissFEL) in collaboration with the Insertion Devices Group at PSI (see Chapter 7). In Chapter 2, the relevant principles of 1D horizontal Hall sensors for magnet measurements are given in a nutshell, to lay the foundation for the rest of this thesis.

The work presented in this thesis was multidisciplinary — in general spanning the design of the sensor and tooling, chip fabrication and processing, assembly, mathematical modelling for the sensor characterization, coding, calibration, field measurements, and analysis of the results. These, unless duly accredited to others within the thesis, were carried out by the author.
2 The Hall Sensor for Magnetic Field Measurements

Somewhere, something incredible is waiting to be known.

- Carl Sagan

2.1 Working Principle of Hall Sensors

In this section, the basic working principle and properties of Hall sensors are given. The focus here lies on the application of uniaxial horizontal Hall sensors in magnetic field measurements. For a more general or complete description of Hall sensors, including other applications, or for an in-depth coverage of galvanomagnetic effects and semiconductor physics, the reader is referred to [21], [22], [23], [24], [25].

2.1.1 The Hall Effect: A Simplified Description

To understand the Hall effect, consider the simplified conditions of a homogeneous and isothermal current-carrying semiconductor plate. The semiconductor is assumed to be extrinsic (n-type or p-type) so that only one type of charge carriers (electrons or holes) has to be considered. To simplify matters further, assume that these charge carriers in the semiconductor undergo only instantaneous collisions and that long-range interactions between charge carriers and ions can be neglected as well as interactions among the charge carriers themselves (Drude model). By applying an electric field $E$ across the length of the semiconductor plate, in-between collisions the electric force $F_E$ causes a directed motion of the charge carriers and a current density $J$ is established. The average net velocity of charge carriers in the direction of an applied electric field is the drift velocity and for an n-type semiconductor is given by

$$v_d = \nu = -\mu_n E$$

(2.1)

with $\mu_n$ the electron mobility. The minus sign takes care that the drift velocity of electrons is in opposite direction to $E$ and $J$. For a p-type semiconductor, the drift velocity of holes is in the same direction as $J$ and $E$, i.e. $v_d = \mu_p E$ with $\mu_p$ the hole mobility.

In the presence of a magnetic field, the magnetic force $F_B$ acts on the charge carriers as well. The average force over many collisions of the charge carriers
Figure 2.1: Hole motion and current deflection effect in an infinite p-type semiconductor under influence of transversal electric and magnetic fields.

with the lattice can be written as \( qv_d \times B \). This term is added to the electric force and the total force on a charge carrier in the presence of both an electric and a magnetic field is the Lorentz force,

\[
F = q(E + v_d \times B),
\tag{2.2}
\]

with \( q \) the charge of electrons \((-e)\) or holes \((e)\). The direction of the magnetic force is independent of the sign of the charge carriers and is only governed by the vectors \( E \) and \( B \). For transversal electric and magnetic fields, a free charge carrier follows a cycloid path motion in a direction orthogonal to both the electric and magnetic field. Due to loss of velocity at collisions, the path of motion of charge carriers in a semiconductor is made up of many cycloid segments which leads to an overall motion along a straight line that makes an angle \( \theta_H \) to the applied electric field \( E \). In other words, the current density vector \( J \) is rotated by \( \theta_H \) with respect to the vector \( E \), see Fig. 2.1 for a sketch of this current deflection effect for holes. \( \theta_H \) is called the Hall angle and is given by

\[
tan \theta_H = \frac{I_x}{I_z}. \tag{2.3}
\]

For an n-type and p-type semiconductor this gives respectively \( tan \theta_H = \mu_n B \) and \( tan \theta_H = -\mu_p B \), i.e. the rotation direction depends on the sign of the charge carriers [22]. This effect only occurs in infinite semiconductors. If the semiconductor has finite dimensions in the direction of the magnetic force, no net current can flow in that direction as it must be confined within the boundaries of the semi-
2.1 Working Principle of Hall Sensors

To understand the Hall effect in that case, consider a long rectangular semiconductor plate, but of finite width, or \( l \gg w \), on which the fields \( E_z \) and \( B_y \) act, see Fig. [2.2] for the case of an n-type semiconductor. Since the magnetic force \( qv_d \times B \) is in the direction of the confined width, charge carriers accumulate on the side of the semiconductor plate, causing a charge separation over its width. The electric field \( E_H \) across the width that is established by this, prevents further charge separation and in steady-state the electric force \( qE_H \) balances the magnetic force so that the charge carriers follow the same trajectory in-between collisions as in the absence of a magnetic field:

\[
E_H = -v_d \times B. \tag{2.4}
\]

The occurrence of a transversal electric field, the so-called Hall field \( E_H \), is characteristic for the Hall effect. The total electric field is the sum of \( E_H \) and the original electric field \( E \) applied across the length of the semiconductor. Therefore, the total electric field vector is rotated with respect to the current density vector. Like in the current deflection effect, this angle is called the Hall angle and is given by

\[
\tan \theta_H = \frac{E_x}{E_z}. \tag{2.5}
\]

For an n-type and p-type semiconductor this gives respectively \( \tan \theta_H = -\mu_n B \) and \( \tan \theta_H = \mu_p B \).

The size of the transverse field \( E_x \) is expected to be proportional to the applied magnetic field and current. The proportionality factor is called the Hall coefficient:

\[
R_H = \frac{E_x}{J B}. \tag{2.6}
\]

The transversal field is \( E_x = -E_H = v_d B \) for electrons or \( E_x = E_H = v_d B \) for holes, \( v_d \) being the electron or hole drift velocity. The current density can be replaced by the relation \( J = nq_v \), where \( n \) is the carrier density. The resulting Hall coefficient is then

\[
R_H = 1/nq, \tag{2.7}
\]

or \( R_H = -1/ne \) for electrons and \( R_H = 1/ne \) for holes. An equivalent expression for the Hall coefficient from Eq. [2.6] is obtained by inserting \( E_x = -\mu_n E B \) for electrons or \( E_x = \mu_p E B \) for holes and \( J = \sigma E \), where \( \sigma \) is the material conductivity. The resulting Hall coefficient then becomes \( R_H = -\mu_n/\sigma \) for electrons or \( R_H = \mu_p/\sigma \) for holes.

The voltage measurable across the width of the plate, is known as the Hall voltage:

\[
V_H = -\int_{-w/2}^{w/2} E_H \cdot dx = \int_{-w/2}^{w/2} (v_d \times B) \cdot dx. \tag{2.8}
\]
The drift velocity $v_d$ can be replaced by the current density from the relation $J = nqv_d$. Note that the vector $J \times B$ points in the $-x$ direction and hence,

$$V_H = -\frac{1}{nq} JBw = -\frac{1}{nq} \frac{I}{wt} Bw = -\frac{1}{nq} \frac{I}{t} B = -R_H \frac{I}{t} B,$$  

(2.9)

where the current density of the semiconductor of thickness $t$ and width $w$ was replaced by the bias current, $I = J\, wt$.

In case of mixed conduction, the current densities of the different types of charge carriers must be summed to obtain the total current density $J$. For electron and hole conduction, it can be shown [21] that the Hall coefficient becomes

$$R_H = \frac{1}{e} \frac{n_p \mu_p^2 - n_n \mu_n^2}{(n_p \mu_p + n_n \mu_n)^2},$$  

(2.10)

with $n_n$ and $n_p$ respectively the electron and hole carrier density. In the limits $n_n \gg n_p$ or $n_p \gg n_n$, the Hall coefficients given above for n-type and p-type semiconductors are obtained again. For an intrinsic semiconductor, $n_n = n_p$ and in the case of equal electron and hole mobilities, the Hall coefficient, and thus the Hall voltage, is zero — the to the same side deflected electrons and holes do not produce a Hall field. However, in most cases $\mu_n > \mu_p$. Also, electron and hole densities can be modulated by doping the semiconductor with impurities.

### 2.1.2 Second-Order Effects: A Qualitative Description

In a semiconductor with more than one type of charge carrier, the charge carriers typically have different mobilities and thus drift velocities. The consequence is that the Hall coefficient is dependent on the applied magnetic field. Eq. 2.10 independent on magnetic field, only applies in the weak field limit of $\mu^2 B^2 \ll 1$. Also in a semiconductor with only one type of charge carrier, the Hall coefficient depends on applied magnetic field if the relaxation time of the carriers varies with energy [26]. The simplified Drude model, in which a single average drift velocity for all charge carriers is assumed, cannot account for this fact. Eq. 2.7 for the Hall coefficient only applies in the strong field limit of $\mu^2 B^2 \gg 1$. In the weak field limit, the Hall coefficient is also independent on magnetic field and is given by $R_H = A/qn$ with $A = \langle \tau^2 \rangle / \langle \tau \rangle^2$ a on scattering mechanism dependent constant whose value is of the order of $\sim 1$ [22]. For metals and degenerate semiconductors, $A = 1$.

A spread in drift velocity is also the origin of two other second order effects that occur in Hall sensors: the physical magneto-resistance effect and the planar Hall effect.
2.1 Working Principle of Hall Sensors

Physical Magneto-resistance Effect

The physical magneto-resistance effect is the effect of an increase in material resistivity with increasing magnetic field. In a long semiconductor plate of finite width, see Fig. 2.2, the charge carrier trajectories in the presence of an orthogonal magnetic field are the same as in absence of that field due to the established Hall field which compensates the magnetic force. But, this is only on average true — the established Hall field compensated the magnetic field as if all charge carriers would have the same velocity. In reality, the individual carriers have different velocities. Those carriers with lower velocity are more affected by the Hall field than by the magnetic field while those carriers with higher velocity are more affected by the magnetic field than by the Hall field. Due to the drift velocity spread, a microscopic current deflection effect takes place. The carriers with higher and lower velocity than the average are deflected to opposite directions, to the $+x$ and $-x$ direction in Fig. 2.2. Hence, the contribution of the carriers with higher and lower velocities to the conductivity is reduced.

Planar Hall Effect

So far, only magnetic fields directed orthogonal to a Hall sensor were considered. The Hall voltage in the Hall sensor of Fig. 2.2 is the integral of the Hall field $E_H$ over the width of the Hall sensor, see Eq. 2.8. Since the carriers drift in the $z$ direction, the Hall voltage is zero if the $B_y$ component of the magnetic field vector is zero. If the magnetic field vector makes an angle $\phi$ to the $y$-axis, the Hall voltage would be $V_H = -R_H \frac{1}{2} B \cos \phi$. Nevertheless, the occurrence of a voltage in the case of a magnetic field directed in plane to a Hall sensor ($B_y = 0$) was observed in 1954 [27]. It was found that the generated voltage was proportional to $B_y \sin 2\theta$, where $\theta$ represents the angle between the current vector and the magnetic field vector in the Hall sensor’s plane, $B_p$. In other words, the voltage displays a double angular dependence and the voltage is zero when the magnetic field vector is directed along or orthogonal to the current vector, i.e. the $z$ or $x$ direction in Fig. 2.2. This effect became known as the planar Hall effect, whose name has been coined by analogy to the “ordinary” Hall effect. The effect can be qualitatively understood in the same manner as the physical magneto-resistance effect. If the magnetic field vector is in the $xz$ plane, but not along the $z$ direction, a Hall field that balances the magnetic force, is established in the $y$ direction. Following the same reasoning above for the magneto-resistance effect, the spread in drift velocity causes carriers that are faster or slower than average to be deflected in the $+y$ or $-y$ direction. In turn, these carriers, as soon as they move in the $y$ direction, experience a magnetic force directing them in the $xz$ plane. The carrier deflection is (partly) in the $z$ direction, resulting in a voltage measurement.
2 The Hall Sensor for Magnetic Field Measurements

(x-direction) only if a magnetic field component in the z direction exists. Hence, for the planar Hall voltage to occur, the in-plane magnetic field vector must have both $B_z$ and $B_x$ vector components. A net voltage occurs because carriers moving in $+x$ direction have higher than average mobilities and the carriers that move in $-x$ direction have lower than average mobilities (or vice versa).

2.1.3 An Accurate Description

It was seen in Section 2.1.1 that in the presence of a magnetic field, the current vector and the total electric field vector no longer coincide. To specify fully the electric field, therefore, in general three components must be defined [28]:

$$E = \rho_B J - R_H (J \times B) + P_H (J \cdot B) B. \quad (2.11)$$

The component of $E$ parallel to the current direction is related to the resistivity in the presence of a magnetic field (magneto-resistance effect). The component orthogonal to the current vector and orthogonal to the plane containing the current vector and the magnetic field vector is the Hall electric field. The remaining component is orthogonal to the current vector but is in the same plane as the current vector and the magnetic field vector. The occurrence of this component is due to the planar Hall effect. An accurate analysis of these galvanomagnetic phenomena in semiconductors can be obtained from transport theory by solving the kinetic Boltzmann equation under the relaxation time approximation. The Boltzmann equation relates disturbances in the carrier distribution function, caused by external fields, to the equilibrium-restoring effect of scattering [23]. Only the relevant results, without derivation, of such analysis will be summarized here. For further details and a derivation of the solution to the Boltzmann equation, the reader is referred to [22] or [23].

Consider a homogeneous and isothermal semiconductor on which an electric field $E$ and a magnetic field $B$ act. In the case that there is only one type of charge carrier (strongly extrinsic semiconductor in which the minority carriers are neglected) whose effective mass is isotropic, the solution for the current density in the semiconductor is [22]

$$J = q^2 K_1 E + \frac{q^3}{m^*} K_2 (E \times B) + \frac{q^4}{m^{*2}} K_3 B (E \cdot B). \quad (2.12)$$

In this equation, $m^*$ is the scalar effective mass of the charge carriers and $K_s$ are the kinetic coefficients in the presence of a magnetic field which are given by

$$K_s = \frac{n}{m^*} \left\langle \frac{\tau^s}{1 + \mu^2 B^2} \right\rangle,$$  (2.13)
with $\tau$ the relaxation time. In the case of more than one type of charge carrier, the current densities for each type of carrier must be summed. The case of anisotropic effective mass of the charge carriers, in which the kinetic coefficients are tensors, is not considered here.

Since for Hall sensors the most interesting parameter is the Hall voltage, it is useful to rewrite Eq. 2.12 for the current density into an equation for the electric field whose $x$-component when integrated over the width of the Hall plate, gives the Hall voltage. The inverse of Eq. 2.12 is the same equation as stated in the beginning of this section,

$$E = \rho_B J - R_H (J \times B) + P_H (J \cdot B) B,$$  \hspace{1cm} (2.14)

and whose coefficients are now given by [23]

\begin{align*}
\rho_B &= \frac{1}{q^2 K_1 \left(1 + \frac{q^2}{m^* K_1^2} \right)^2 B^2}, \quad (2.15a) \\
R_H &= -\frac{q^2 K_3}{q^2 K_1 \left(1 + \frac{q^2}{m^* K_1^2} \right)^2 B^2}, \quad (2.15b) \\
P_H &= \frac{\left( \frac{q}{m^* K_1} \right)^2 - \frac{q^2 K_3}{m^* K_1}}{q^2 K_1 \left(1 + \frac{q^2}{m^* K_1^2} \right)^2 B^2 \left(1 + \frac{q^2}{m^* K_1^2} \right)^2 B^2}. \quad (2.15c)
\end{align*}

\hspace{1cm} (2.15d)

In the case of current density $J_z$ and magnetic field $B_y$, the same result for the Hall voltage as in Eq. 2.9 is obtained. The Hall coefficient $R_H$ however, is given by Eq. 2.15b with $K_s$ according to Eq. 2.13 With Eq. 2.13 and $\mu = e\tau/m^*$, $R_H$ can be rewritten as [22]

$$R_H = \frac{1}{nq} \frac{\left( \frac{\mu^2}{1 + \mu^2 B^2} \right)}{\left( \frac{\mu}{1 + \mu^2 B^2} \right)^2 B^2 + \left( \frac{\mu}{1 + \mu^2 B^2} \right)^2 B^2}. \quad (2.16)$$

If there is more than one type of charge carrier, the current densities for each charge carrier must be summed. The Hall coefficient in that case becomes [22]

$$R_H = \frac{\sum_a q_a n_a \left( \frac{\mu_a}{1 + \mu_a^2 B^2} \right)}{\left( \sum_a q_a n_a \left( \frac{\mu_a}{1 + \mu_a^2 B^2} \right) \right)^2 + \left( \sum_a q_a n_a \left( \frac{\mu_a}{1 + \mu_a^2 B^2} \right) \right)^2 B^2}. \quad (2.17)$$
The Hall coefficient, as well as the resistivity (Eq. 2.15a for one type of charge carrier), depend on magnetic field. The $V_H B$-characteristic is not perfectly linear opposite to what was to be expected from the simplified equation for the Hall voltage derived in Section 2.1.1. In the weak-field region, Eq. 2.16 is approximated by $R_H = \frac{A}{q n} \left(1 - a \mu_d^2 B^2\right)$ and in the strong-field region it is approximated by $R_H = \frac{1}{q n} \left(1 + a' \mu_d^2 B^2\right)$ with $\mu_d$ the drift mobility and $A$, $a$, and $a'$ on scattering mechanism dependent constants [22].

In a homogeneous and isothermal Hall sensor of finite width and thickness, see Fig. 2.2, the current must be confined within the Hall sensors’ boundaries so the only net current that can flow is along its length dimension, hence $J = (0, 0, J_z)$. The output voltage of the Hall sensor in a magnetic field $B$ of arbitrary direction, is then obtained by integrating the $x$ component of the electric field in Eq. 2.14 over the width $w$ of the Hall sensor. The result is the sum of the Hall voltage generated by the Hall effect and a planar Hall voltage generated by the planar Hall effect:

$$V_{out} = V_H + V_{PH} = -R_H \frac{I}{t} B_y + P_{PH} \frac{I}{t} B_x B_z.$$ (2.18)

For a general coordinate system, this equation can be rewritten in the form

$$V_{out} = V_H + V_{PH} = -R_H \frac{I}{t} B_n + P_{PH} \frac{I}{2t} B_p^2 \sin 2\theta.$$ (2.19)

where $B_n$ and $B_p$ are respectively the magnetic field component orthogonal and in-plane to the Hall sensor, and $\theta$ is the angle between the current vector and $B_p$. Note the factor $\frac{1}{2}$ between the second term in Eq. 2.18 and Eq. 2.19 which stems from $B_x B_z = \frac{1}{2} B_p^2 \sin 2\theta$.

### 2.2 Basic Characteristics of Hall Sensors

The characteristics of a Hall sensor are mainly governed by the material the Hall sensor is made of. The magnitude of the Hall voltage that can be achieved, the Hall sensor’s electrical resistance, the linearity of the Hall voltage with magnetic field, or the dependence of the Hall voltage on temperature, are all material dependent and often a compromise has to be found between desired characteristics. Apart from material choice, also the geometry of a Hall sensor affects its characteristics. Obviously the cross-sectional area and length are decisive about the electrical resistance and, from Eq. 2.19 a Hall sensor should preferably be thin to maximize the Hall voltage. Apart from its thickness, the geometrical shape of a Hall sensor affects the maximum Hall voltage and the Hall voltage linearity with magnetic field.
2.2 Basic Characteristics of Hall Sensors

2.2.1 Material Properties

Most modern Hall sensors are made from thin-film semiconducting material that was deposited onto a non-conducting substrate, called a wafer. The term thin film generally refers to a layer with a thickness of a single atomic layer up to a few micrometers. During the deposition process, impurities can be deliberately added to the thin film. This process is called doping and by doing so, the electron or hole concentration in the semiconductor can be increased which alters its characteristics. For example, the carrier concentration of pure, intrinsic (i.e. undoped) semiconductors, especially those with a small bandgap, has a strong temperature dependence:

\[ n \propto \exp \left( -\frac{E_g}{2kT} \right) \tag{2.20} \]

with \( E_g \) the bandgap, \( k \) the Boltzmann constant, and \( T \) the temperature. The temperature sensitivity is reduced by doping the semiconductor. Above the freeze-out temperature, nearly all impurity atoms are ionized and their electrons or holes contribute to the carrier concentration. If the carrier concentration by doping exceeds the intrinsic carrier concentration, the carrier concentration remains nearly constant with increasing temperature. This range is called the extrinsic or saturation range. Only at high enough temperatures, the intrinsic carrier concentration catches up and the temperature dependence of the carrier concentration is again according to Eq. 2.20.

A disadvantage of doping is that by increasing the carrier concentration, the Hall coefficient is reduced proportionally. Also the mobility of an extrinsic semiconductor is lower than that of the same semiconductor material in pure form. Semiconductors with high mobility charge carriers are most suitable for the exploitation of the Hall effect because of the occurrence of large Hall angles and thereby pronounced Hall effect. For that reason, Hall sensors are mostly made from n-type semiconductor materials where electrons are the dominant carriers which have higher mobilities than holes. Some common semiconductors used for the fabrication of Hall sensors and their properties are listed in Table 2.1. The values given in the table are indicative as they vary with dopant concentration.

According to Eq. 2.9 the active layer thickness, the bias current, and the Hall coefficient determine the magnitude of the Hall voltage in a given magnetic field orthogonal to the Hall sensor. In Section 2.1.1 it was deduced that the Hall coefficient of an n-type semiconductor is to first approximation given by \( R_H = -1/en \) or, equivalently, \( R_H = -\mu_n/\sigma \). The Hall coefficient is inversely proportional to the carrier concentration so one might conclude that to attain the maximum Hall voltage: the Hall sensor should be thin, have a low carrier concentration, and the applied bias current to the Hall sensor should be high. The first two requirements result in a Hall sensor of high resistance, and together with the re-
The Hall Sensor for Magnetic Field Measurements

Table 2.1: Material properties at 300 K of semiconductors used typically for Hall sensors [29].

<table>
<thead>
<tr>
<th>Material</th>
<th>$n$ [cm$^{-3}$]</th>
<th>$\mu_n$ [cm$^2$V$^{-1}$s$^{-1}$]</th>
<th>$E_g$ [eV]</th>
<th>$R_H$ [cm$^2$C$^{-1}$]</th>
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<tr>
<td>Ge</td>
<td>$2.4 \cdot 10^{13}$</td>
<td>3900</td>
<td>0.67</td>
<td>-90000</td>
</tr>
<tr>
<td>Si</td>
<td>$2.5 \cdot 10^{15}$</td>
<td>1300</td>
<td>1.12</td>
<td>-2500</td>
</tr>
<tr>
<td>GaAs</td>
<td>$3.0 \cdot 10^{15}$</td>
<td>6400</td>
<td>1.42</td>
<td>-2100</td>
</tr>
<tr>
<td>InAs</td>
<td>$5.0 \cdot 10^{16}$</td>
<td>22000</td>
<td>0.36</td>
<td>-125</td>
</tr>
<tr>
<td>InSb</td>
<td>$9.0 \cdot 10^{16}$</td>
<td>70000</td>
<td>0.17</td>
<td>-70</td>
</tr>
</tbody>
</table>

Requirement of high bias current, in a high power dissipation. The maximum Hall voltage of a Hall sensor is thus limited by the permissible increase in temperature of the semiconductor due to heat loss. Suppose the maximum permissible power dissipation in a Hall plate with resistance $R$ is $P_{\text{max}} = I_{\text{max}}^2 R$. In that case, the maximum Hall voltage that can be generated in the Hall plate is

$$V_{H,\text{max}} = \frac{|R_H|}{t} \sqrt{\frac{P_{\text{max}}}{R} B} = \sqrt{\frac{R_H^2 P_{\text{max}} \sigma w t}{l^2}} B, \quad (2.21)$$

where $\sigma = \frac{1}{R \omega t}$ is the conductivity of the Hall plate of thickness $t$, width $w$, and length $l$. For an n-type material, $|R_H| = \mu_n / \sigma$, and hence,

$$V_{H,\text{max}} = \sqrt{\frac{P_{\text{max}} w}{l t}} \sqrt{\mu_n |R_H| B}. \quad (2.22)$$

Therefore, the maximum Hall voltage that can be attained for a certain $P_{\text{max}}$, depends on the product $\mu_n R_H$.

Let’s consider two types of semiconductors with high values of $\mu_n R_H$. The first type are the III-V compound semiconductors indium arsenide (InAs) and indium antimonide (InSb) with very high electron mobilities. Due to their small bandgap on the other hand, they have a large intrinsic carrier concentration at room temperature. Thus, even for high purity materials, the Hall coefficient of these materials is low [21]. The other type are materials like silicon (Si) and germanium (Ge) which have relatively low electron mobilities but, because also their electron densities at room temperature are much smaller, a large Hall coefficient is achieved in pure materials [21]. Although with both materials high values of $\mu R_H$ are achieved, the first type, having large electron mobilities but relatively small Hall coefficients, is advantageous. The large Hall coefficients of the second type of material, prerequisites high purity materials. The lower the electron concentration, the larger the relative influence of uncontrollably occurring surface
2.2 Basic Characteristics of Hall Sensors

Charge impurities on the Hall sensor sensitivity \([21],[23]\). The advantage of Si is the possibility to integrate e.g. an amplifier, temperature sensor, or analog-to-digital converter (ADC) on chip with the Hall element, creating an integrated circuit (IC) Hall sensor.

2.2.2 Geometry Influence

So far, only infinitely long and infinitely wide Hall sensors were considered. Hall sensors are typically four terminal devices — two bias current contacts and two contacts to measure the Hall voltage. In principle, any slab of conducting material with four terminals could function as a Hall sensor. In an infinitely long Hall plate but of finite width, or \(l/w \gg 1\), an electric field is established transversal to the bias current direction in the Hall plate, which is measurable as the Hall voltage. If the width of the Hall plate is infinite, or \(l/w \ll 1\), the Hall effect manifests itself as a current deflection effect. It can be understood that for realistic devices for which neither \(l/w \gg 1\) nor \(l/w \ll 1\) holds, the expected Hall field is neither maximum nor zero.

In the description of the ideal case of an infinitely long Hall sensor, the effect of the bias contacts could be ignored. Realistic devices are of finite width and length with reasonably sized bias contacts for practical reasons of easier wire bonding/soldering as well as avoiding locally large current densities that would occur in the vicinity of point-like contacts. The effect of the bias contacts is that they will short-circuit the Hall electric field at both ends of the Hall sensor. No Hall field can develop in the vicinity of the bias current contacts and current deflection takes place in these areas. This has two consequences: first, a reduction of the Hall voltage from the theoretical value for an infinitely long plate, and second, an increase of resistance with magnetic field. This increase in resistance is due to the longer current trajectory in presence of a magnetic field and is called the geometrical magneto-resistance effect.

A finite geometry weakens the Hall voltage but it enhances the geometrical magneto-resistance effect. In the middle of a long semiconductor plate, the current vector is parallel to the longitudinal extension of the plate, as in the case of absence of a magnetic field. The Hall voltage is highest in this middle section and no increase of resistance takes place. Near the bias current contacts, where no Hall field establishes and current deflection takes place, the current path is longer than in absence of a magnetic field which is noticeable as an increase in resistance. The shorter the Hall sensor, the more pronounced the current deflection effect, the smaller the attainable Hall voltage, and the larger the geometrical magneto-resistance effect.

For a voltage-biased Hall sensor, the magneto-resistance effect directly affects the linearity of the \(V_HB\)-characteristic in Eq. 2.9 because the current in the Hall
sensor decreases with increasing magnetic field and resistance. To avoid this non-linearity, Hall sensors are typically current-biased by a constant current source. In that case, the magneto-resistance effect needs not to be taken into account.

From the above considerations it follows that to maximize the Hall voltage, a Hall sensor should preferably have a large length to width ratio \( l/w \). The attainable Hall voltage for a Hall sensor of finite dimensions can be expressed as

\[
V_H = - \frac{R_H}{I} f B G_H(l/w, \theta_H) = V_H^\infty G_H(l/w, \theta_H) \tag{2.23}
\]

where \( G_H \) is a geometrical correction factor, \( 0 < G_H < 1 \), and \( V_H^\infty \) is the Hall voltage of an ideal infinitely long Hall plate \( (l/w \to \infty) \). The geometrical correction factor approaches unity with increasing \( l/w \) ratio and Hall angle \( \theta_H \). The dependency on the Hall angle is explained by considering that the larger the Hall angle is, the larger the Hall effect and the longer is the section of current lines that are parallel to the applied electric field along the length of the Hall sensor. Thus the geometrical correction factor introduces a non-linearity in the Hall voltage dependency on the magnetic field. The smaller the geometry influence on the Hall voltage, the better the linearity of the Hall voltage to the magnetic field, the \( V_H B \)-characteristic. The geometrical correction factor was calculated by [30] for a rectangular Hall plate for various \( l/w \) ratios and Hall angles. Below \( l/w \) values of 1.5, the geometrical correction factor decreases rapidly and values around \( l/w = 2 \) are suggested for which \( G_H > 0.9 \).

A useful geometry for Hall sensors is a cruciform geometry. The advantage of the cruciform Hall sensor was demonstrated in a comparative study between rectangular and cruciform Hall plates by [31]. They showed that a high geometrical factor close to 1 can be achieved with a cruciform shape, despite large contacts. For example, a symmetric cruciform Hall plate with length-to-width ratio of its limbs of \( l/w = 1 \) and a large contact pad width \( s \) for which \( s/l = 1 \), is equivalent to a rectangular Hall plate with \( l/w = 2.721 \) and \( s/l = 0.013 \). The cruciform, with larger contact widths, is practically much easier to realize. Additionally, compared to a rectangular Hall plate, the cruciform has a smaller, symmetric, and well-defined active area which makes it ideal for point-like measurements and hence, field mapping. For these reasons, the cruciform geometry is most prevalent in Hall plates. The correction factor for cruciform samples with length-to-width ratio \( l/w \) of its limbs was calculated by [32] with an accuracy to within 0.5% for \( l/w \geq 0.38 \):

\[
G_H(l/w, B) = 1 - 1.045 \frac{\theta_H}{\tan \theta_H} e^{-\pi l/w}. \tag{2.24}
\]

For weak magnetic fields, the correction factor can be expressed as a quadratic
2.2 Basic Characteristics of Hall Sensors

The function of the Hall angle \( \theta_H \) [23],

\[
G_H = \left( 1 - 1.045e^{-\pi l/w} \right) \left[ 1 + \frac{1.045e^{-\pi l/w}}{3(1 - 1.045e^{-\pi l/w})} \theta_H^2 \right]. \tag{2.25}
\]

Hence for weak magnetic fields, the geometrical correction factor exhibits a quadratic dependence on magnetic field.

2.2.3 Hall Sensor Parameters

To summarize, the most important Hall sensor parameters for a magnetic field measurement application will be given below.

Non-Linearity

Because of the physical and geometrical magneto-resistance effect, Hall sensors are typically current-biased with a constant current source. The non-linearity of the \( V_H B \)-characteristic of a current-biased Hall sensor is due to a magnetic field dependence of the Hall coefficient and of the geometrical coefficient associated with the finite geometry of the Hall sensor. For weak fields, the geometrical correction factor, Eq. 2.25 and the Hall coefficient, Eq. 2.17 have a quadratic magnetic field dependence that is of opposite sign and therefore compensates to a certain extend. This can be used to devise a Hall sensor where the two non-linearity effects cancel out [33]. In general, linearity is highest for Hall sensors with geometries featuring a large \( l/w \) ratio or cruciform shape and which are made of materials with predominantly one type of charge carrier.

The non-linearity can be defined in different ways, e.g. as the maximum deviation \( \Delta V_H \) between the measured Hall voltage and the ideal linear Hall voltage dependence on magnetic field, given in percentage of the maximum measured Hall voltage \( V_H \):

\[
NL = \frac{\Delta V_H}{V_H} \% . \tag{2.26}
\]

Besides the non-linearity, the magnetic field range over which the non-linearity has been measured should always be specified, since the non-linearity increases with magnetic field. Current-biased Hall sensors with a linearity error as low as \( \pm 0.03\% \) in the magnetic field range \( < 1 \text{T} \) could be realized from GaAs with cruciform geometry [34]. The Hall sensor’s non-linearity is, just like its offset, taken into account by the calibration of the Hall sensor, e.g. as the second and higher order coefficients of a polynomial function.
Sensitivity

The sensitivity of a current-biased Hall sensor is defined as the Hall voltage divided by the magnitude of the orthogonal magnetic field $B_n$ and by the bias current $I$:

$$S = \frac{V_H}{B_n I}. \quad (2.27)$$

A high sensitivity is obtained with a thin semiconductor that has a high Hall coefficient. The sensitivity is limited by the permissible dissipated power in the Hall sensor. Since the response of a Hall sensor is never perfectly linear with magnetic field, in practice the sensitivity will be defined from a linear fit to a set of measured values $(B_n, V_H)$.

Offset

The offset is the Hall sensor’s output voltage in absence of a magnetic field. The origin of an offset voltage is a lateral misalignment of the Hall voltage contacts or an electrical asymmetry within the semiconductor stemming from e.g. material inhomogeneities or stress. Usually the offset is given, instead of in voltage, as a more meaningful magnetic-field-equivalent value. Offset reduction can be achieved by several techniques, most popular is the so-called spinning-current technique [35] with an external or integrated circuit. For the Hall sensor application of magnetic field measurements, active offset reduction is typically not necessary because the magnetic field reconstruction from a Hall voltage measurement is done offline. The offset is taken into account by the calibration of the Hall sensor, e.g. as the zeroth coefficient of a polynomial function.

Temperature Coefficient

The temperature coefficient $\alpha_T$ of a current-biased Hall sensor is given as the relative change of Hall voltage with temperature $T$, given in $\%K^{-1}$ or $\%C^{-1}$:

$$\alpha_T = \frac{\Delta V_H}{V_H \Delta T}. \quad (2.28)$$

Typical values for $\alpha_T$ in the range $0^\circ C - 100^\circ C$ for III-V materials are [36]: $-0.07\%K^{-1}$ (InAs), $-1.5\%K^{-1}$ (InSb), $-0.04\%K^{-1}$ (GaAs), and for silicon [23]: $-0.08\%K^{-1}$. The temperature sensitivity of Hall sensors has to be taken into account in high-precision magnetic field measurements. For example by calibrating a Hall sensor at different temperatures and using this calibration data in the magnetic field reconstruction. Alternatively, magnetic field measurements should be performed in a temperature-controlled room. If the Hall sensor’s temperature
2.3 Magnetic Field Measurements of Magnets with Hall Sensors

A high-end and high-accuracy application of Hall sensors, is in magnetic field measurements of detector magnets in particle physics experiments and of beamline magnets for particle accelerators. Here, Hall sensors are routinely used and are with reason one of the most important measurement tools: they are cheap, relatively easy to use, are applicable in a broad magnetic field range (from below milliTesla to a few Tesla), work in a non-homogeneous field, do not influence the magnetic field they are supposed to measure, and have a small sensing area which allows point measurements. The latter makes Hall sensors indispensable in magnetic field mapping where the magnetic field is measured at different points inside a magnet to create a magnetic field map of the magnet. Point measurements with a Hall sensor within the magnetic field area of interest are very useful to study the effect of the magnet on the particle beam. From the measured field map, beam relevant magnet characteristics can be obtained such as field shape, field integral, effective field length, homogeneity, etc.; and it can be assessed whether the field quality of the magnet meets its specifications. Field maps can also be used for beam ray-tracing simulations.

2.3.1 The “Zoo” of Magnets

Magnets and magnet systems at particle accelerator facilities can be divided in three groups: beamline magnets, insertion devices, and detector magnets. These magnets can be electromagnets (air-cooled or water-cooled), permanent magnets, and superconducting magnets. The size of a magnet (system) ranges from a few centimetres to several meters and it weighs from under a kilo to hundreds of tons. The magnetic field area of interest of a magnet, is the area that the beam traverses, i.e. the aperture of the magnet where the vacuum chamber is installed. The magnet aperture and vacuum chamber are typically straight or slightly curved.

Beamline magnets are mainly used to steer or focus the particle beam. Types of beamline magnets, each with a different function in the beamline, are: solenoids, dipoles, quadrupoles, sextupoles, octupoles, etc. Schematic drawings of a dipole and a quadrupole magnet with their characteristic magnetic field lines are given in Fig. 2.3a and 2.3b. Higher order multipole magnets are simply a continuation of these figures. As can be seen from Fig. 2.3 on the symmetry planes of a magnet, only one field component exists. 1D Hall sensors are thus perfectly suitable for mapping the mid-plane of a magnet. The further off the mid-plane, the stronger the influence of the side-components.
Figure 2.3: Examples of common magnets and magnet systems in a particle accelerator: two types of beamline magnets and an insertion device. It can be seen from the drawn field lines, that along a mid-plane the magnetic field has only one component.

Insertion devices are used in synchrotron light sources where they oscillate the beam and thereby create synchrotron radiation. There are two types of insertion devices, undulators and wigglers, which are classified by the level of chromaticity of the radiation they produce. Simply stated, an insertion device consists of an array of magnets with alternating polarity which facilitates the oscillation of the beam. In Fig. 2.3c, a schematic drawing of an insertion device and its magnetic field lines is shown. Again, on the mid-plane only one field component exists. The spacing between alternate north and south poles can be few millimetres only, resulting in magnetic fields of very high gradients. In reality, only a perfect insertion device has a mid-plane on which only one field component exists. One of the reasons for a performing a (3D) magnetic field measurement
of an insertion device, is to "create" a magnetic mid-plane by minimizing the measured side-components of the field via pole shimming.

Detector magnets can be complex magnet systems consisting of several (often solenoid) magnets.

2.3.2 Field Mapping

Magnetic field mapping requires the precise and controllable positioning of the Hall sensor in a magnet. This can be done with a multi-axes positioning bench with an arm carrying the Hall sensor. The arm should be thin enough to enable field mapping in the desired area of the magnet aperture. The arm should also be long enough so that the Hall sensor’s measurement range along the beam axis spans over the full length of the magnet and beyond (to measure magnet stray field). Minimally three translational axes to position the Hall sensor in $x$, $y$, and $z$ (beam) direction are required. Additional rotation axes are desirable, e.g. to align the Hall sensor orthogonally to the magnetic field.

Field mapping can be done on-the-fly, or (slower) by a stop-and-go method. Typically, voltage-to-field reconstruction is done offline after the measurement is completed. The measured voltages at each spatial position are translated to magnetic field values by the calibration coefficients of the Hall sensor.

One criterion for measurement accuracy requirement, is the good-field region of magnets which is often $10^{-4}$ and hence, the measurement precision should be of that order too. With high precision measurement electronics, careful alignment of the Hall sensor, stabilization or control of Hall sensor environment temperature, and a repeated thorough calibration of the Hall sensor, a measurement accuracy of $100 \mu T$, or $10^{-4}$ for 1 T fields, can be routinely achieved [4],[5],[6],[7].

2.4 Hall Sensor Calibration

During the calibration of a Hall sensor, its output voltages are translated into magnetic field values. The calibration process controls the accuracy that the Hall sensor will achieve. A precise Hall sensor in combination with careful calibration, results in the optimal accuracy of the Hall sensor. Since Hall sensors have a non-linear dependency on magnetic field, they should be calibrated over the full magnetic field range of interest. The dependency of the Hall voltage on temperature is dealt with by either maintaining a constant ambient temperature during calibration and measurement, or by taking the temperature coefficient into account during calibration and measurement.

High accuracy of single-axis Hall sensors of better than $10^{-4}$ for 1 T fields is achieved by calibration at regular time intervals in a homogeneous field against nuclear magnetic resonance (NMR) probes with a Teslameter. NMR probes are
2 The Hall Sensor for Magnetic Field Measurements

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>SBV 585-S1</th>
<th>HE244</th>
<th>LHP-MU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semiconductor</td>
<td>InAs</td>
<td>GaAs</td>
<td>unknown</td>
</tr>
<tr>
<td>Active area [mm²]</td>
<td>0.15 × 0.15</td>
<td>unknown</td>
<td>0.10 × 0.10</td>
</tr>
<tr>
<td>Nominal bias current $I_n$ [mA]</td>
<td>100</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>Maximum bias current $I_{max}$ [mA]</td>
<td>200</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Typical supply side resistance $R$ [Ω]</td>
<td>2.5</td>
<td>500</td>
<td>4.0</td>
</tr>
<tr>
<td>Temp. coeff. $\alpha_T$ [%/°C] ($T \sim 25$ °C)</td>
<td>0.007</td>
<td>0.015</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 2.2: Specifications of the Siemens SBV 585-S1, Hoeben HE244, and Arepoc LHP-MU commercial Hall sensors.

the reference system offering a measurement range to over 13 T, an absolute accuracy of 5 ppm, and an insensitivity to temperature fluctuations and small angular misalignments of the probes [37]. To maintain a high accuracy, it is advisable to apply the same current source and voltmeter during calibration and measurement.

As an example, the calibration of three commercial 1D Hall sensors will be presented, a Siemens SBV 585-S1, a Hoeben HE244, and an Arepoc LHP-MU. Some specifications of these sensors are listed in Table 2.2. The Hall sensors are supplied with a direct current (DC) from constant current sources. For the SBV 585-S1 sensor, the current is 150 mA from a North Hills CS-I2-2. The HE244 and LHP-MU are supplied with a bias current of respectively 0.5 mA and 40 mA by a Keithley 6221. Their Hall voltages are measured by a 8½-digit HP/Agilent 3458 digital voltmeter (now Keysight Technologies). One sensor is calibrated at a time in a Bruker B-E 30hf dipole calibration magnet. The sensor is placed together with a Metrolab NMR probe in the centre and they are positioned equidistant from the mid-plane of the magnet so that the NMR probe and the Hall sensor are subjected to the same magnetic field. Five NMR probes (Metrolab probe number 1-5) in combination with a Metrolab PT2025 Teslameter are employed to cover a magnetic field range up to 2 T. The five NMR probes are placed together on an aluminium support in the calibration magnet and each probe at a time can be slid into the central position of the magnet. The Hall probes reside on an approximately 2 m long and up to 2 cm diameter arm of an automated five-axis positioning bench which slides on compressed air pads over a machined 3.3 m × 2.0 m × 0.5 m granite block. With this measuring machine the Hall sensor has three translational and two rotational degrees of freedom and is placed in the centre of the magnet orthogonally to the magnetic field.

The magnet current is set from -96 A to 96 A in steps of 1 A, covering a magnetic field range of approximately -2 T to 2 T. The magnet currents -2 A to 2 A
2.4 Hall Sensor Calibration

are omitted as the magnetic field is too weak for NMR frequency locking. At each magnet current and after the magnetic field has stabilized (controlled by the NMR probe), the Hall voltage is measured 100 times at a integration time of the voltmeter of 20 ms to filter out 50 Hz noise. Before and after the Hall voltage measurement, the magnetic field from the NMR probe is read and if the difference between the two measurements is less than 5 µT, the averaged value of the 100 voltage measurements and the magnetic field value from NMR are stored in a file. At the end of the current range, the Hall sensor’s offset value at zero magnetic field is measured in a zero-Gauss chamber.

The easiest way to describe the non-linearity in the Hall voltage dependency of the magnetic field is a polynomial,

\[ B = c_0 + c_1 V_H + c_2 V_H^2 + \ldots + c_n V_H^n. \] (2.29)

The calibration coefficients \( c_i \) are obtained from a least-squares (polynomial) fit to the Hall voltage and magnetic field measurement data. A higher order \( n \) reduces the error of the fit to the data but above a certain order, the improvement is insignificant. This order is different for each Hall sensor depending on its linearity. The differences in non-linearity of the SBV 585-S1, HE244, and LHP-MU are visible in respectively Figs. 2.4a, 2.5a, and 2.6a. In these graphs the difference between the magnetic field as measured by the NMR probes and the magnetic field from the polynomial fit, \( B_{fit} \), with two coefficients \( (c_0 \text{ and } c_1) \) is plotted against the magnetic field as measured by the NMR probes, \( B_{NMR} \). In the same graphs the maximum error of the polynomial fit, \( MaxErr \), and the root mean square error of the fit, \( RMS \), are displayed in Gauss. \( MaxErr \) is the maximum difference that occurs between the magnetic field value from the polynomial fit and from the NMR probe reading. \( RMS \) is defined as

\[ \sqrt{\frac{\sum_i (B_{i,NMR} - B_{i,fit})^2}{npt}}, \] (2.30)

with \( npt \) the number of measured points. The non-linearity \( NL \) is defined as the maximum difference between \( B_{NMR} \) and \( B_{fit} \) that occurs, given in percentage of the maximum measured magnetic field \( B_{max} \) which is 2 T in this case:

\[ NL = \frac{B_{NMR} - B_{fit}}{B_{max}} \times 100\%. \] (2.31)

\(^1\) Although generally banned from the scientific community for not being an SI-unit (Nikola Tesla received the honour), the Gauss unit is still in use in the magnetic measurement community because of its convenience. In this thesis, it shall appear as well and the forgiving reader is reminded that 1 Gauss = 10^{-4} T.
2 The Hall Sensor for Magnetic Field Measurements

(a) Non-linearity of the Siemens SBV 585-S1 sensor.

(b) Rest signal after fitting an 8th order polynomial to the calibration data.

Figure 2.4: Calibration results of the Siemens SBV 585-S1 sensor at $I_{bias} = 150$ mA.
2.4 Hall Sensor Calibration

(a) Non-linearity of the Hoeben HE244 sensor.

(b) Rest signal after fitting a 12\textsuperscript{th} order polynomial to the calibration data.

Figure 2.5: Calibration results of the Hoeben HE244 sensor at $I_{\text{bias}} = 0.5\, \text{mA}$.  

\begin{align*}
\text{Max}\, \text{Err} &= 1.18128\times10^{-01} \\
\text{RMS} &= 7.55935\times10^{-02}
\end{align*}

\begin{align*}
B_{\text{cal}} - B_{\text{ref}} &= \sum (c_i \cdot V) \\
B_{\text{ref}} &= G \\
(V) &= \text{mV}
\end{align*}

\begin{align*}
c_0 &= 0.317503\times10^{-01} \\
c_1 &= 0.208088\times10^{-03}
\end{align*}
2 The Hall Sensor for Magnetic Field Measurements

(a) Non-linearity of the Arepoc LHP-MU sensor.

(b) Rest signal after fitting an 18th order polynomial to the calibration data.

Figure 2.6: Calibration results of the Arepoc LHP-MU sensor at $I_{bias} = 40$ mA.
2.4 Hall Sensor Calibration

The deduced non-linearity from the graphs in Figs. 2.4a, 2.5a, and 2.6a is 0.13% for the SBV585-S1, 0.06% for the HE244, and 0.65% for the LHP-MU sensor.

The non-linearity is governed by the higher polynomial coefficients. The remaining difference between $B_{NMR}$ and $B_{fit}$ after taking the full set of coefficients, is plotted against $B_{NMR}$ in Figs. 2.4b, 2.5b, and 2.6b for the three Hall sensors. This difference is small, it lies within ±0.5 Gauss for all three Hall sensors. The level of the remaining difference is approximately the maximum accuracy that can be achieved from field reconstruction of a measured Hall voltage by applying Eq. 2.29 with the full set of coefficients. The coefficients $c_i$ of the polynomial fit with unit Gauss · mV$^{-1}$ are listed in a table on the right-hand side of each of the graphs. With these coefficients, in combination with Eq. 2.29, the magnetic field in Gauss can be reconstructed from a measured Hall voltage in mV. The coefficient $c_0$ is the offset of a Hall sensor in Gauss. The sensitivity of a Hall sensor is given by $1/c_1$ in mV · Gauss$^{-1}$ or by $10^4/c_1$ in mV · T$^{-1}$. 

3 Design of a Novel Three-Axis Hall Sensor: The “Hallcube”

Design is not just what it looks like and feels like. Design is how it works.

- Steve Jobs

A novel design for a three-axis Hall sensor will be presented. It is a combination of six 1D Hall sensors in such way that each Hall sensor forms one face of a sub-millimetre-sized cube. Pairs of sensors from opposite faces are then sensitive to the same component of the magnetic field vector. Such a geometry has several advantages. First, the high sensitivity and precision of 1D Hall sensors is exploited. The double readings for each field component by the pairs enhance the precision. Second, the averaged values of the pairs for all three field components can be assigned to the same point in space, at the centre of the cube. This allows for measurement of the full field vector at that point at the same time. And last, with appropriate electrical connections of the 1D Hall sensors from opposing cube faces, described in Section 3.2.2 and 3.4.2, the undesired planar Hall effect as well as loop-induced voltages can be practically cancelled out.

3.1 Two Concepts

Simply assembling six Hall sensors onto a cube is not satisfactory as it results in a large spatial distribution of the Hall sensors, hardly below a few millimetres. Two different designs that miniaturize the active volume to $200 \, \mu\text{m} \times 200 \, \mu\text{m} \times 200 \, \mu\text{m}$ are proposed [38].

3.1.1 The Truncated Pyramids Design

The initial design is shown in Fig. 3.1 (concept by V. Vranković, PSI). Six 1D Hall sensors each reside on top of a truncated pyramid support (e.g. from ceramic) with a $4 \, \text{mm} \times 4 \, \text{mm}$ base and $1.9 \, \text{mm}$ height including the Hall sensor, see Fig. 3.1b. The Hall sensors themselves are also of a truncated pyramid shape with their sensing part located at the top surface of dimensions $100 \, \mu\text{m} \times 100 \, \mu\text{m}$. The design of the 1D Hall sensors is shown in Fig. 3.1c with a magnification of the
cruciform sensing part (black) with Ohmic contacts (yellow). The six truncated pyramids with their Hall sensors on top are then assembled into a cube. The result is an inner hollow cubical volume of dimensions $200\,\mu\text{m} \times 200\,\mu\text{m} \times 200\,\mu\text{m}$. The six 1D Hall sensors form the faces of this inner cube and their active areas of $30\,\mu\text{m} \times 30\,\mu\text{m}$ are in the centre of each face. For clarity, in Fig. 3.1a the pairs of Hall sensors that are sensitive to the same magnetic field component are shown in the same colour (red, green, or blue). Electrical connections to the Ohmic contacts of the 1D Hall sensors can be established by metallization of channels along the pyramidal facets (not depicted).

There are few challenges in the realization of this design, namely: fabrication of the miniature Hall sensors which need to be of pyramid shape, precise fabrication of the base pyramids, handling and mounting of these Hall sensors to their pyramid supports, and metallization which is not confined to a plane. A feasible fabrication step would be to process Hall sensors on standard rectangular die, attach each die to the top surface of a truncated pyramid, and by laser ablation remove excess die material as well as create pyramidal side walls of the die.

### 3.1.2 The Cuboids Design

A simpler to realize design is shown in Fig. 3.2. It consists of six 1D Hall sensors, each located on a cuboid support (e.g. from ceramic) with a $2.1\,\text{mm} \times 2.1\,\text{mm}$ base and $1.9\,\text{mm}$ height including the Hall sensor, see Fig. 3.2a. When assembled, these six building blocks form an inner hollow cubical volume of 1D Hall sensors with dimensions $200\,\mu\text{m} \times 200\,\mu\text{m} \times 200\,\mu\text{m}$. The six sensors form the faces of this inner cube, with their active areas are at the centre of each face. In Fig. 3.2b the design of the 1D Hall sensor is shown. The cruciform sensing part with an active area of $30\,\mu\text{m} \times 30\,\mu\text{m}$ is located at the corner of the die, in the centre of a $200\,\mu\text{m} \times 200\,\mu\text{m} \times 200\,\mu\text{m}$ area. Three sensors are designed as shown in Fig. 3.2b and the other three are their mirror images. Fig. 3.2c and Fig. 3.2d show the assembly of two halves with three building blocks. A magnification of the sensing part of the bottom half assembly, which forms half of the inner cube, is shown in Fig. 3.2e. The complete three-axis Hall sensor is shown in Fig. 3.2f and will be denoted “Hallcube” hereafter. For clarity, again the pairs of Hall sensors that are sensitive to the same magnetic field component are shown in the same colour (red, green, or blue). The channel cutouts in the cuboid supports are designed to carry out the wiring (not depicted). The inner volume of $200\,\mu\text{m} \times 200\,\mu\text{m} \times 200\,\mu\text{m}$ is formed by the $200\,\mu\text{m}$ difference in height and width of the building blocks. This is more clearly demonstrated in a cross section of the Hallcube, see Fig. 3.2g.

Both concepts for a three-axis Hall sensor described above employ the same-sized cruciform sensing parts, share the same inner active volume and outer dimensions. The latter design is pursued due to its simpler 1D Hall sensor handling,
3.1 Two Concepts

(a) Three-axis Hall sensor made from assembly of six truncated pyramids, each supporting a 1D Hall sensor.

(b) Design of a single truncated support pyramid e.g. from ceramic (grey shades) with Hall sensor on top (blue), dimensions in µm.

(c) Design of the uniaxial Hall sensor (top view) with magnified sensing part in black and Ohmic contacts in yellow.

Figure 3.1: First conceptual design of the three-axis Hall sensor.
3 Design of a Novel Three-Axis Hall Sensor: The “Hallcube”

(a) Building blocks: cuboid support with channel cutout (grey) and 1D Hall sensors (blue).

(b) Design of the 1D Hall sensor with active area (black) and Ohmic contacts (yellow).

(c) Assembly of top three building blocks.

(d) Assembly of bottom three building blocks.

(e) Magnification of active areas.

(f) Three-axis Hall sensor made from assembly of all six building blocks.

(g) Cross section of the three-axis Hall sensor through a mid-plane (without wire channels).

Figure 3.2: Revised design of the three-axis Hall sensor.
3.2 Design of the Uniaxial Hall Sensor Components

3.2.1 Geometry of the Sensing Part

The design of the Hall plates is shown in Fig. 3.2b. The sensing part of the Hall plate (shown in black) is a cruciform semiconductor of thickness in the µm range. There are several advantages of a cruciform Hall sensor. It combines wide contacts (low resistance) with a small geometrical correction and hence, an improved linearity of the $V_{HI}B$-characteristic (Section 2.2.2). Furthermore, cruciform Hall sensors have a well-defined, small and symmetric active area making them ideal for point-like measurements and hence, field mapping. And, the cruciform is invariant under rotation by $\pi/2$ which will be exploited in the Hallcube to cancel out the planar Hall effect.

In the current design, the limbs of the cruciform structure have an equal length and width of 30 µm. Thus the active area, at the centre of the cruciform, is small with 30 µm $\times$ 30 µm. The active area is in the centre of a 200 µm $\times$ 200 µm area at the corner of the Hall plate. The width of the metal contacts to the cruciform (shown in yellow) is designed to slightly exceed the width of the cruciform’s limbs by 5 µm, see the magnified area in Fig. 3.2b. This will ensure that the contacts overlap the full width of the limbs of the cruciform structure even in case of any slight misalignment of the electrical contacts. Hence, offset voltages induced by misaligned contacts are minimized. The upper and left Ohmic contact to the cruciform in Fig. 3.2b start out with a width of 30 µm and are 25 µm from the edge of the Hall plate. The contacts are extended outwards to gain a larger contact area for easier wire soldering or bonding.

3.2.2 Electrical Connections

The six individual Hall plates in the Hallcube form three pairs, each pair dedicated to measure one field component. The electrical connections to each pair of Hall plates can be made strategically such to counteract two effects.

The first is the planar Hall effect. It is well known that because of the double angular dependence of the planar Hall effect (see Eq. 2.19), the generated planar Hall voltage can be compensated for by averaging the output of two Hall plates placed in parallel and rotated in plane 90° to each other [39], [40]. Therefore, the electrical connections to two Hall plates that form a pair should be made such that their bias current vectors are defined to be 90° rotated with respect to each
3 Design of a Novel Three-Axis Hall Sensor: The "Hallcube"

other. In that case, the planar Hall voltages in pairs of Hall plates have opposite signs and are cancelled out by averaging.

The second effect is an unwanted induced voltage between the Hall voltage contacts. Such an induced voltage $V_{\text{ind}}$ occurs in a loop during on-the-fly magnetic measurements due to a change of magnetic flux. If the magnetic field vector is orthogonal to the enclosed surface of the loop with area $A$, the induced voltage is given by

$$V_{\text{ind}} = A \frac{dB}{dt}. \quad (3.1)$$

The change in magnetic field depends on the measurement speed and the field gradient. For example, in case of a 50 Tm$^{-1}$ gradient field, a measurement speed of 50 mm s$^{-1}$, and a loop area of 1 mm$^2$, the induced voltage would be about 2.5 µV. Typically, for 1D Hall sensors with large contact pads, loops are minimized by soldering a twisted-pair wire to the two opposite contact pads for the Hall voltage. The wire soldered to one of the contact pads is laid flat over the cruciform sensing part where it meets, and is twisted with, the wire soldered to the other contact pad. In that case, practically no loop exists. This scheme is not possible in the Hall plates of the designed Hallcube. The realization of the inner closed active volume of 200 µm × 200 µm × 200 µm necessitates the outward extended contact areas over the Hall plate’s surface. The consequence of these extended contact areas is a loop area of about 1 mm$^2$ between the Hall voltage contact pads. However, the induced voltages from these loops can be compensated by each pair of Hall plates from opposing cube faces. Namely, by choosing appropriately the positive and negative connections for the bias current in the pair of Hall plates, the induced voltages in pairs of Hall plates have opposite signs and hence, are cancelled out by averaging.

The compensation of the planar Hall voltage and loop-induced voltage by a pair of opposite Hall plates is clarified in Fig. 3.3. The blue pair of Hall plates are denoted BB (for "blue bottom" Hall plate) and BT (for "blue top" Hall plate), respectively. To compensate for the planar Hall effect the contact pads for the bias current are chosen such that the current vector in the two Hall plates is orthogonal. This is indicated by the red arrows. The current (arrow) direction is freely selectable, only orthogonality is necessary for the planar Hall voltage compensation. The other two contact pads on each Hall plate are for the Hall voltage readout by a voltmeter. $V_+$ denotes the positive input and $V_-$ the negative input of a differential signal voltmeter. The output voltage of the differential voltmeter is therefore $V_+ - V_-$, which can be a positive or negative number. In the example in Fig. 3.3 the magnetic field vector points into the page which results in $V_+ - V_- > 0$. As can be confirmed by Fig. 3.3 the current direction was chosen such that if the flux of the given field changes, the induced voltages measured
3.2 Design of the Uniaxial Hall Sensor Components

(a) Pair of Hall plates from opposite cube faces denoted BB (“blue bottom”) and BT (“blue top”).

(b) Magnification of the region with active areas.

Figure 3.3: Compensation of the planar Hall voltage and loop-induced voltage by a pair (in this case blue) of Hall plates.

in the BB Hall plate and the BT Hall plate, have opposite sign. Hence again by averaging the output voltages of the two Hall plates that form a pair, also the loop-induced voltage is cancelled out.

3.2.3 Materials Selection

Hall Plates

Choosing the right material for a Hall sensor involves compromises. For example, it was seen in Section 2.2.1 that to achieve the highest possible Hall voltage, the product $\mu R_H$ should be maximized. However, to achieve high linearity and low temperature sensitivity in a Hall sensor, the material should be rendered more metal-like, hence adding additional charge carriers by heavily doping it (which reduces $R_H$ and also mobility is negatively affected by heavy doping). In general, semiconductors with a wide bandgap are more temperature insensitive, and are so in a wider range of temperatures.

It was seen in Section 2.4 that the calibration procedure takes well care of non-linearities and offset voltages. Achieving a high linearity by sacrificing on sensitivity to magnetic field, is therefore not essential. To achieve a low temperature sensitivity, yet maintain a high magnetic field sensitivity, makes GaAs a suitable material. The electron mobility of pure (intrinsic) GaAs lies with 8500 cm$^2$V$^{-1}$s$^{-1}$
3 Design of a Novel Three-Axis Hall Sensor: The ’Hallcube’

Figure 3.4: Sketch of the heterostructure and the design of a Hall bar test structure (design by C. Rössler, ETH Zürich).

between the mobility of Si (1400 cm²V⁻¹s⁻¹) and the very high mobilities observed in pure InAs (30000 cm²V⁻¹s⁻¹) and InSb (77000 cm²V⁻¹s⁻¹) \cite{41}. An outstanding property of GaAs is its relatively large bandgap and therefore low temperature sensitivity, see Section 2.2.1. Thus, even with moderate doping, low temperature sensitivities can be achieved while the sensitivity to the Hall effect remains high and hence, moderate supply currents are feasible.

Hall sensor test samples were fabricated from three GaAs wafers, each with a different active layer: a GaAs – Al₀.₃Ga₀.₇As heterostructure, a moderately doped GaAs layer, and a heavily doped GaAs layer. The active layers were grown by molecular beam epitaxy (MBE) on semi-insulating (SI) GaAs substrates by C. Reichl and others at the Solid State Physics Laboratory at the ETH Zürich.

The first of the three considered materials is not very common in Hall sensors for magnetic measurements. The GaAs – Al₀.₃Ga₀.₇As heterostructure is a 100 nm thin layer of Al₀.₃Ga₀.₇As, i.e. 30% of the gallium atoms are replaced by aluminium atoms, grown on top of a semi-insulating GaAs substrate. The bandgap of Al₀.₃Ga₀.₇As is 1.9 eV, compared to 1.5 eV for GaAs. The band-bending from joining these materials results in a quantum-well in which the electrons from the doped Si-atoms are confined. The electrons are free to move in two directions but are confined in the third dimension, and are therefore referred to as a two-dimensional electron gas (2DEG). The ionized dopant atoms are outside of the well in the Al₀.₃Ga₀.₇As. This so-called modulation doping reduces electron-ion scattering. Because of the two-dimensional confinement and the reduced ion scattering, very high electron mobilities are achieved in heterojunction devices, especially at very low temperatures. At higher temperatures, the mobility becomes limited by phonon scattering. Even though the operating temperature of the designed Hallcube is room temperature, and hence the mobility becomes limited by phonon scattering, it is interesting to find out how a Hall sensor of this material would perform. A sketch of the wafer material and of the test samples with a Hall bar structure are shown in Fig. 3.4 The material and Hall sensors fab-
3.2 Design of the Uniaxial Hall Sensor Components

Fabricated from it will be denoted by HPF1. The design and the photolithographic mask for the HPF1 Hall sensors was by C. Rössler, ETH Zürich.

The moderately and heavily doped GaAs materials have a Si-doped active layer of about 1 µm thickness, grown on top of a semi-insulating GaAs substrate. A sketch of the wafer material and the design of the cruciform Hall sensor test structures is given in Fig. 3.5. The moderately and heavily doped GaAs material and the Hall sensors fabricated from it will be denoted respectively HPF2 and HPF3.

The Hall bar mesa for the HPF1 Hall sensor and the cruciform mesa for the HPF2 and HPF3 Hall sensors were obtained by standard photolithography using AZ5234 resist (HPF1 sensor) and S1828 resist (HPF2 and HPF3 sensors), followed by wet etch in $\text{H}_2\text{O} : \text{H}_2\text{SO}_4 : \text{H}_2\text{O}_2$ (100:3:3). A thicker photoresist was chosen for the HPF2 and HPF3 sensors because of the larger etch depth that needs to be reached, see also Section 4.1.5. Ohmic contacts were realized by alloying evaporated Au/Ge/Ni. The fabrication process for the HPF1 and the HPF2 and HPF3 sensors is summarized in Table A.1 and A.2 of Appendix A.

The different Hall plates were characterized and calibrated in a Bruker B-E 30hf dipole calibration magnet. The calibration procedure is identical to what was described in Section 2.4. The Hall plate bias current was supplied by a Keithley 6221 constant current source and its Hall voltage was read by an 8½-digit HP/Agilent 3458A digital voltmeter. The obtained Hall sensor characteristics at room temperature are summarized in Table 3.1 — sensitivity to the Hall effect, internal resistance between the two bias current contacts (at $B = 0$ T), applied bias current, non-linearity over the range $B = 0 − 2$ T, and the temperature coefficient of the Hall voltage (around room temperature, at $B = 1$ T).

![Figure 3.5: Sketch of the wafer materials and the design of Hall sensor test structure for the Si-doped GaAS samples.](image)
Characteristic parameters of the HPF1, HPF2, and HPF3 wafer materials such as the carrier density, the mobility, and the resistivity can be estimated from:

\[
\rho [\Omega \text{cm}] = \frac{R [\Omega] w [\text{cm}] t [\text{cm}]}{l [\text{cm}]} \quad (3.2)
\]

\[
\mu_H [\text{cm}^2 \text{V}^{-1} \text{S}^{-1}] = 10^4 \frac{S [\text{VA}^{-1} \text{T}^{-1}]}{R [\Omega]} \frac{l [\text{cm}]}{w [\text{cm}]} \quad (3.3)
\]

\[
n_H [\text{cm}^{-3}] = 6.25 \cdot 10^{14} \frac{1}{S [\text{VA}^{-1} \text{T}^{-1}]} \frac{l [\text{cm}]}{t [\text{cm}]} \quad (3.4)
\]

These relations follow directly from Eq. 2.9 with \(\mu_H = |R_H \sigma|\) and \(n_H = 1/|eR_H|\) [42]. \(\mu_H\) denotes the Hall mobility and only equals the electron mobility if the charge carriers in the GaAs materials are solely electrons which all have the same magnitude of velocity in the direction of the current vector. The same applies to \(n_H\), the Hall concentration: \(n_H = n_e\) only if the carrier density is made up of electrons of the same velocity. The results at room temperature are summarized in Table 3.2. Since the active layer of the HPF1 material consists of a 2DEG, it is meaningful to specify a two-dimensional sheet carrier density \(n_s\) and sheet resistivity \(\rho_s\). For the HPF2 and HPF3 materials, these can be converted to the three-dimensional values via \(n_H = n_s / t\) and \(\rho = \rho_s t\), with \(t\) the active layer thickness which is listed in Table 3.2 (except for the HPF1 material). The listed material parameters are obtained from measurements with the Hall sensors fabricated from these materials, see Table 3.1. The values for the resistance to be inserted in Eq. 3.2 and 3.3 should be the resistance of the semiconductor material’s active layer of \(w\) and length \(l\) between the two current contacts. This was only measured correctly for the the HPF1 material facilitated by the Hall bar structure. In the Hall bar structure of Fig. 3.4, the resistance of the active layer between the two bias current contacts (left middle and right middle) was determined from

<table>
<thead>
<tr>
<th>Type</th>
<th>(S [\text{VA}^{-1} \text{T}^{-1}])</th>
<th>(R [\Omega]) (B = 0 \text{T})</th>
<th>(I_{\text{bias}} [\text{mA}])</th>
<th>(NL [%]) (B = 0 - 2.0 \text{T},) (T \sim 25^\circ \text{C})</th>
<th>(\alpha_T [% / ^\circ \text{C}]) (B = 1.0 \text{T},) (T \approx 25 \pm 4^\circ \text{C})</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPF1</td>
<td>1600</td>
<td>42.5k</td>
<td>0.05</td>
<td>0.8-1.2</td>
<td>0.3</td>
</tr>
<tr>
<td>HPF2</td>
<td>70</td>
<td>600</td>
<td>1</td>
<td>0.1-0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>HPF3</td>
<td>2</td>
<td>30</td>
<td>10</td>
<td>0.015</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Table 3.1: Main characteristics of Hall sensors fabricated from different Si-doped GaAs materials (see also Table 3.2). The HPF2 and HPF3 Hall sensors are of the cruciform structure in Fig. 3.5. the HPF1 Hall sensor is of the Hall bar structure in Fig. 3.4.
3.2 Design of the Uniaxial Hall Sensor Components

<table>
<thead>
<tr>
<th>Type</th>
<th>( t ) [( \mu \text{m} )]</th>
<th>( n_s ) ( [\text{cm}^{-2}] )</th>
<th>( \mu_{\text{H}} ) ( [\text{cm}^{-2}\text{V}^{-1}\text{s}^{-1}] )</th>
<th>( \rho_s ) [( \Omega )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPF1</td>
<td>—</td>
<td>( 3.6 \cdot 10^{11} )</td>
<td>7800</td>
<td>2240</td>
</tr>
<tr>
<td>HPF2</td>
<td>1</td>
<td>( 8.6 \cdot 10^{12} )</td>
<td>3600</td>
<td>200</td>
</tr>
<tr>
<td>HPF3</td>
<td>1</td>
<td>( 3.1 \cdot 10^{14} )</td>
<td>2000</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 3.2: Main characteristic properties of different Si-doped GaAs materials fabricated by MBE. Average Hall mobilities and sheet carrier densities were measured at room temperature on a fabricated Hall bar test structure.

the voltage \( V \) between the bottom two contacts for a given bias current \( I \) and in absence of a magnetic field. In that case, \( R = \frac{V}{I} \). For the HPF2 and HPF3 materials, the resistance of the active layer between two opposite bias current contacts in Fig. 3.5 was determined from the voltage \( V \) between these contacts for a given bias current \( I \) and in absence of a magnetic field. Hence, the resulting resistance is slightly larger since it includes the resistance of the bias contacts which are, however, expected to be small.

From the results in Table 3.1 it is once more clear that Hall plates that excel in one characteristic, have poor results in another. The HPF1 sensors feature very high sensitivity but poor linearity and high temperature sensitivity. Another disadvantage is the high impedance and corresponding high sensitivity to noise of the HPF1 sensor. This becomes especially a problem when the Hall sensor cabling is long. Hall sensor cables for magnetic field measurements are typically few meters long to facilitate on-the-fly measurements. Note that the high internal resistance of HPF1 is only partly due to the material specific high sheet resistivity (Table 3.2). It is also partly due to dimensions of the Hall bar structure with a length and width of respectively 1900 \( \mu \text{m} \) and 100 \( \mu \text{m} \). For a cruciform structure similar to HPF2 and HPF3 with a length/width ratio of 3, the expected resistance for the HPF1 sensor would be about six times smaller than reported in Table 3.1.

The HPF3 sensor has very good linearity and low temperature sensitivity, but its sensitivity to the Hall effect is rather low due to the high carrier concentration. A high supply current of 10 mA was chosen to achieve a good sensitivity of 20 mVT\(^{-1}\). To achieve the same sensitivity as the HPF1 and HPF2 sensors, a bias current of 40 mA must be supplied. For the cruciform structure width an active layer thickness of 1 \( \mu \text{m} \), these high currents result in current densities of around \( 10^5 \text{ Acm}^{-2} \) and failure rates become high. In Hall plates based on the HPF3 material, failure mechanisms included sudden high offset voltages of a few mV, indicating micro-ruptures in the semiconductor material, and electromigration effects with visible dendrite structures on the semiconductor surface.
A high sensitivity of the Hall sensor to the magnetic field is desired. The minimum required sensitivity is determined primarily by the measurement electronics for the Hall voltage reading, the voltmeter. High precision electronics should be used to exploit the high precision of the Hall sensor itself. The Hall sensors in this thesis, unless stated otherwise, are supplied by a Keithley 6221 constant current source with an accuracy in the range $10^{-4}$ [43]. Their Hall voltages are measured with an $8\frac{1}{2}$-digit HP/Agilent digital voltmeter (DVM) [44]. The peak-to-peak voltage of a short-circuited DVM is less than 1 µV and the rms voltage is, depending on settings for range and auto-zeroing, maximum $\pm 0.3$ µV. Therefore, if the Hall sensor is to resolve 0.1 Gauss, the sensor needs to have a sensitivity of at least $100$ mV T$^{-1}$. In that case, the measurement error for a single point is $\pm 0.05$ Gauss but for multiple measurements it will be better than $\pm 0.03$ Gauss. The permissible bias current to the sensor should be high enough to reach the required sensitivity.

The HPF2 sensor is a good compromise between high sensitivity at low bias current, and good linearity and temperature sensitivity characteristics. The material for the Hallcube is chosen to be similar, with slightly lower dopant concentration by a factor of about 1.3 to push its sensitivity towards $100$ mV T$^{-1}$ for 1 mA bias current. This material will be denoted by HPF5 and the fabrication of the prototype Hallcube based on this material is described in the next chapter.

### Cuboid Supports

The material for the cuboids should be a non-magnetic, non-conductive (no Eddy currents), rigid material that is machinable to tight tolerances. Apart from electrical insulating properties, thermal and optical insulating properties of the material are desirable. The Hall sensor active areas are encapsulated by the cuboids which shield them from environment temperature fluctuations and light. The material’s coefficient of thermal expansion should match that of GaAs, $5.75 \mu m \, m^{-1} \, ^{\circ}C^{-1}$ (room temperature) [25].

Polyether ether ketone (PEEK), sapphire, and MACOR (trademark for a machinable glass ceramic made of silica and metal oxides) were considered as suitable materials. PEEK is a robust thermoplastic and cheap compared to MACOR and sapphire. Being a plastic, its linear coefficient of thermal expansion is highest of the three materials, $30 \mu m \, m^{-1} \, ^{\circ}C^{-1}$ (20°C − 100°C) [45], and is about 5 times more than that of GaAs.

The linear expansion coefficient for sapphire is closest to that of GaAs. It lies, depending on orientation, in the range $5.0 − 5.8 \mu m \, m^{-1} \, ^{\circ}C^{-1}$ (20°C − 50°C)

---

1The skipping of index number 4 is solely for historical event reasons: HPF4 was designated to a wafer whose properties were similar to HPF3 and hence, was not pursued for the Hallcube prototype.
A disadvantage of sapphire is that, because it is extremely hard, machining it with diamond tools is expensive. Another disadvantage is that it would require a coating since it is transparent to light.

MACOR was chosen as the material for the cuboid supports. MACOR is machinable with standard metal working tools and machining tolerances are tight — up to 13 µm are listed by the manufacturer [47]. The achieved accuracy in cuboid dimensions fabricated was even higher, see Section 4.2. Its linear coefficient of thermal expansion coefficient is 9.0 µm m⁻¹ °C⁻¹ (25 °C – 300 °C) [47], which is close to that of GaAs.

### 3.3 Thermal Analysis

When supplying the Hallcube with a bias current, heat is generated mainly at the active areas of the Hall sensors which reside inside the small hollow volume at the centre of the cube. The Hall sensors are encapsulated by the MACOR cuboids which act as thermal insulators with a thermal conductivity of 1.46 W m⁻¹ °C⁻¹ [47], compared to 46 W m⁻¹ °C⁻¹ of GaAs [25]. This is desirable because it shields the Hall sensors from environment temperature fluctuations. However, the heat inside the hollow volume must be manageable and not lead to too high temperatures.

To establish a crude approximation of the elevated temperature inside the hollow volume, consider first the simplified case of steady-state thermal conduction through a single Hall cuboid. The problem is analogous to the problem of thermal conduction through a wall and the following equation, derived from Fourier’s law, can be applied:

$$\Delta T = T_i - T_a = PR_{th}$$  \hspace{1cm} (3.5)

with $T_i - T_a$ the temperature difference between the inside temperature of the hollow volume (one side of the wall) and the ambient temperature (the other side of the wall). $P = I_{bias}^2 R$ is the dissipated power and $R_{th}$ is the thermal resistance. The thermal resistance is the sum of the thermal resistances of the 500 µm thick GaAs layer and the 1400 µm thick MACOR block, $R_{th} = R_{th}^{GaAs} + R_{th}^{MACOR}$. The thermal resistance for each material is given by

$$R_{th} = \frac{t}{k_{th}A}$$  \hspace{1cm} (3.6)

with $t$ the material thickness along the direction of heat conduction, $A$ the area across the direction of heat conduction, and $k_{th}$ the material’s thermal conductivity. Inserting the Hall plate’s and cuboid’s thermal conductivities and surface areas of 2.1 mm × 2.1 mm into Eq. 3.6 and the sum of the resulting thermal resistances into Eq. 3.5 along with $R \approx 700 \Omega$, the temperature difference is calculated
3 Design of a Novel Three-Axis Hall Sensor: The "Hallcube"

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ [Ω$^{-1}$m$^{-1}$]</th>
<th>$k_{th}$ [Wm$^{-1}$°C$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MACOR</td>
<td>$\infty$</td>
<td>1.46</td>
</tr>
<tr>
<td>Epoxy</td>
<td>$\infty$</td>
<td>0.3</td>
</tr>
<tr>
<td>Si-GaAs</td>
<td>$\infty$</td>
<td>46</td>
</tr>
<tr>
<td>Si-doped GaAs</td>
<td>$4 \cdot 10^{-4}$</td>
<td>46</td>
</tr>
<tr>
<td>Gold</td>
<td>$2.137 \cdot 10^{-8}$</td>
<td>315</td>
</tr>
</tbody>
</table>

Table 3.3: Electrical resistivities and thermal conductivities in the ANSYS model of the Hallcube.

as a function of bias current:

$$T_i - T_a = I^2 R \left( R_{th}^{GaAs} + R_{th}^{MACOR} \right) = I^2 \cdot 700 \, \Omega \cdot (2.46 \, ^{°}CW^{-1} + 217.44 \, ^{°}CW^{-1}) \quad (3.7)$$

Suppose the MACOR outer surface is maintained at ambient (room) temperature ($T_a$). Imposing an upper limit to $T_i$ of 20 °C above room temperature, limits the bias current to a maximum of about 10 mA. Since most of the heat is generated at the corner of each of the Hall plates in the 200 µm $\times$ 200 µm $\times$ 200 µm volume, a very conservative approach is to take for the area in Eq. 3.6 not the full Hall plate and MACOR surface areas but an area of only 100 µm $\times$ 100 µm, which is roughly twice the cruciform’s area. Under that condition and for the same temperature limit, the bias current is limited to 0.5 mA.

A thermal model of the Hallcube was calculated in ANSYS Workbench (courtesy of A. Anghel, PSI). In the 1:1 model, the following simplifications were introduced: soldered wires were disregarded; the annealed Au/Ge/Ni metal contacts were modeled as a 200 nm gold layer on top of the Si-doped GaAs; the epoxy layer was modeled as a 15 µm rectangular layer. The electrical and thermal conductivities assigned to the materials in the model calculation are listed in Table 3.3 A thermal calculation for the Hallcube was made for an ambient temperature of 22 °C, body-to-air convection rate of 5 Wm$^{-2}$°C$^{-1}$, and for a bias current of 1 mA in all six Hall sensors.

The results of this calculation are summarized in Fig. 3.6. The highest temperature occurs at the active area of the Hall sensor whose temperature is about 0.4 °C higher than the outside temperature of the MACOR support cuboids and about 14 °C above the ambient temperature of 22 °C. This value is tolerable. For Hall sensors, the nominal bias current is typically defined as that current for which the semiconductor has an overtemperature of 15 °C in free air convection [21],[36], and the maximum Hall sensor bias current is typically 1.2 – 1.5 times higher than
3.3 Thermal Analysis

(a) Thermal analysis result, full model.

(b) Thermal analysis result, full model, showing one out of six Hall cuboids.
3 Design of a Novel Three-Axis Hall Sensor: The “Hallcube”

(c) Magnification of the active area of one of the six Hall sensors including the mesh.

(d) Current density in the active area region of one of the six Hall sensors.

Figure 3.6: Thermal model of the Hallcube at 1 mA bias current, calculated in Ansys Workbench (courtesy of Alexander Anghel, PSI).
3.4 Measurement Error Description

The current density in the active area region is locally high where there is a sharp angle in the transition between the metal contacts and the cruciform active area, see Fig 3.6d. The maximum current density that occurs is $2.8 \cdot 10^4 \text{Acm}^{-2}$. The actual current density values could be a few orders of magnitude lower because the annealed Au/Ge/Ni contacts on GaAs have been replaced by just Au contacts on top of GaAs.

3.4 Measurement Error Description

Three-axis Hall sensors made of conventional Hall plates face three main accuracy-limiting factors: angular errors between their measurement axes, the planar Hall effect, and the spatial distribution between their sensing areas. In relation to these factors, what are the requirements to a three-axis Hall sensor as the one proposed in Section 3.1.2?

The measured voltage of a homogeneous and isothermal Hall plate driven at a constant current $I$ in a magnetic field of magnitude $B$ is the sum of the Hall voltage generated by the Hall effect and a planar Hall voltage generated by the planar Hall effect (given by Eq. 2.19 and repeated here for easy reference):

$$V_{out} = V_H + V_{PH} = \frac{R_H}{t} IB_n + \frac{P_H}{2t} IB_p \sin 2\theta$$  \hspace{0.5cm} (3.8)

where $t$ is the Hall plate’s active layer thickness, $R_H$ is the Hall coefficient and $P_H$, by analogy, is the planar Hall coefficient. $B_n$ is the magnetic field component normal to the Hall plate and $B_p$ is the field component that is in plane with the Hall plate and is at an angle $\theta$ with the Hall current vector.

3.4.1 Angular Errors between Measurement Axes

Ideally, in a three-axis Hall sensor there should be no cross-sensitivity between the measurement axes. Field component measurements along one measurement axis should be independent of the values of field components along the other measurement axes. Ignoring the planar Hall effect for the moment, this only holds true when the three measurement axes of the three-axis Hall sensor are perfectly orthogonal or, in other words, when the field vector can be decomposed orthogonally along the three measurement axes. Imposing a target upper limit of $10^{-4}$ in cross-sensitivity implies that the maximum allowed angular error in the orthogonality between measurement axes is 0.1 mrad or 0.006°. This is technologically extremely hard if not impossible to achieve at this scale in a three-axis Hall sensor made up of individual Hall plates. However, this does not invalidate the concept. For example, current three-axis IC Hall sensors, despite their single-chip
configuration, claim the non-orthogonality between the measurement axes to be no better than $< 0.5^\circ$ [15]. The remedy to overcome too large angular errors is to effectively reduce them by a suitable calibration method. Even if the set criterion of $0.006^\circ$ is out of reach, it is beneficial to minimize the angular errors because for small angular errors, the correction will be linear.

3.4.2 Planar Hall Effect Compensation with a Pair of Hall Plates

Cross-sensitivity between the measurement axes also arises from the planar Hall effect. Due to the double angular dependence of the planar Hall effect, the generated planar Hall voltage can be compensated for by averaging the output of two Hall plates placed in parallel and rotated by $90^\circ$ with respect to each other, such that their current vectors are orthogonal. The cancellation is complete if the planar Hall coefficients of the two plates are equal, the rotation angle between the two plates is $90^\circ$ and the values of the $B_p$ components in both plates are the same. Consider the realistic case of two Hall plates with slightly different planar Hall coefficients $P_{H1}$ and $P_{H2}$ and in-plane field components $B_{p1}$ and $B_{p2}$. Let $\theta$ be the angle between $B_{p1}$ and the Hall current vector of Hall plate 1 and let $\phi$ be the rotation angle between the two plates. The average of the planar Hall voltages is then

$$\frac{1}{2}(V_{PH1} + V_{PH2}) = \frac{1}{2} \left( \frac{P_{H1}}{2t} IB_{p1}^2 \sin 2\theta + \frac{P_{H2}}{2t} IB_{p2}^2 \sin 2(\theta - \phi) \right)$$

$$\equiv \frac{1}{2} \left( A_1 \sin 2\theta + A_2 \sin 2(\theta - \phi) \right)$$

$$= \frac{1}{2} \left( (A_1 + A_2 \cos 2\phi) \sin 2\theta + (-A_2 \sin 2\phi) \cos 2\theta \right)$$

(3.9)

where $A_1 = \frac{P_{H1}}{2t} IB_{p1}^2$ and $A_2 = \frac{P_{H2}}{2t} IB_{p2}^2$ are the planar Hall voltage amplitudes of the individual Hall plates. Eq. [3.9] can be rewritten in the form

$$\frac{1}{2}(V_{PH1} + V_{PH2}) = C \sin(2\theta + \gamma)$$

(3.10)

where $C = \frac{1}{2} \left( \sqrt{(A_1 + A_2 \cos 2\phi)^2 + (-A_2 \sin 2\phi)^2} \right)$ is the amplitude of the average planar Hall voltage of the pair of Hall plates and $\gamma = \arctan((-A_2 \sin 2\phi) / (A_1 + A_2 \cos 2\phi))$. Working out the amplitude $C$ gives

$$C = \frac{1}{2} \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos 2\phi}$$

(3.11)

In the ideal case where $A_1 = A_2 \equiv A$ and $\phi = \pi/2$ the amplitude $C$ is zero and the planar Hall voltage is fully cancelled out by the two plates.
3.4 Measurement Error Description

Rotation Angle Error

Assume the ideal case except for the angle $\phi$ which due to misalignment is given a small error $\epsilon$, hence $\phi = \pi/2 + \epsilon$. Then,

$$C = \frac{1}{2} \sqrt{2A^2(1 + \cos(\pi + 2\epsilon))} = \frac{1}{2} A \sqrt{2(1 - \cos 2\epsilon)}$$

$$\approx \frac{1}{2} A \sqrt{2(1 - (1 - 2\epsilon^2))} = A\epsilon \quad (3.12)$$

Therefore the ratio $C/A$, between the amplitude of the average planar Hall voltages of two plates, and the amplitude of the planar Hall voltage of a single plate, is $\epsilon$. For small angular errors, this means that per degree angular error in the placement of the two plates, 1.7% of the planar Hall voltage is not cancelled out.

Unequal Planar Hall Coefficients

Next, assume the ideal case except for the planar Hall coefficients of two plates not being exactly equal. This can occur because no two Hall plates are identical and $P_H$, like $R_H$, in Eq. $3.8$ is an on material dependent constant. In this case the amplitude $C$ becomes

$$C = \frac{1}{2} |A_1 - A_2| = \frac{1}{2} t |P_{H1} - P_{H2}| IB_{ip}^2$$

$$(3.13)$$

and the ratio $C/A$, between the amplitude of the average planar Hall voltages of two plates, and the amplitude of the planar Hall voltage of a single plate, is $\frac{1}{2} |P_{H1} - P_{H2}|/P_{H1}$. This means that for 1% difference between the two planar Hall coefficients $P_{H1}$ and $P_{H2}$, 0.5% of the planar Hall voltage is not cancelled out.

Unequal In-Plane Magnetic Field Component

Finally, assume the ideal case except that the in-plane field component is not equal for both plates. Then the amplitude $C$ is

$$C = \frac{1}{2} t |B_{p1}^2 - B_{p2}^2| IP_H$$

$$(3.14)$$

and the ratio $C/A$, between the amplitude of the average planar Hall voltages of two plates, and the amplitude of the planar Hall voltage of a single plate, is $\frac{1}{2} |B_{p1}^2 - B_{p2}^2|/B_{p1}^2$. For small percentage differences in the field components, the effect on the planar Hall voltage compensation is one to one: that same percent-
Design of a Novel Three-Axis Hall Sensor: The “Hallcube”

The planar Hall voltage is not cancelled out. There are two reasons why the in-plane field component is different for two Hall plates. The first is a non-parallelism between the plates. If the angle between the plates’ normal vectors is $\zeta$, then the largest percentage difference in in-plane field components that occurs is $|B_p - B_p \cos \zeta| / B_p = 1 - \cos \zeta$. The second cause for a different in-plane field component for two Hall plates is their spacing in a non-homogeneous field. The larger their spacing $d$ and the higher the field gradient $\nabla B_p$ over this distance, the larger the difference:

$$|B_{p1} - B_{p2}| = \nabla B_p d$$

(3.15)

3.4.3 Spatial Distribution

The spatial distribution of active areas within a three-axis Hall sensor would be ideally zero, meaning that the full magnetic field vector is measured at a point. In the design presented in Section 3.1.2, the active areas of the three orthogonal pairs of Hall plates are distributed over a cube of size $200 \mu m \times 200 \mu m \times 200 \mu m$. But, by employing a pair of Hall sensors for each of the three field components, the Hall voltages can be assigned practically to a single point, making the spatial distribution virtually zero. In reality however, the measurement axes of the six individual Hall plates will not cross at a single point, due to alignment errors in the fabrication process. Therefore, the averaged output voltages of the three pairs of Hall plates are not truly assigned to a point but rather three proximate points.

In a non-homogeneous field, the field vector is different at the separate locations of the individual Hall plates. The assigned average of the magnetic field measured by a pair of Hall plates to the midpoint between them only deviates from the real field at that midpoint when the field distribution is non-linear. The smaller the distance between the pair of Hall plates, the smaller the error. In a quadratic field distribution the maximum error that is made by taking the field between two plates spaced a distance $d$ apart to be linear is $a(d/2)^2$, where $a$ is the quadratic coefficient of the field distribution. For a spacing as small as $200 \mu m$ and a tolerable error of $10^{-4}$ T, fields up to $10000$ T m$^{-2}$ induce less than the tolerable error.
4 Device Fabrication

The way to get started is to quit talking and begin doing.

- Walt Disney

The fabrication process involves three main steps. First, the individual Hall plates and support cuboids are fabricated. In a second step, each of the Hall plates is mounted on a cuboid support. Finally, six Hall cuboids are wired and assembled into the complete device. In a perfect assembly, the inner hollow volume forms a perfect cube of dimensions 200 µm × 200 µm × 200 µm, in which the three measurement axes are orthogonal and cross at a single point. To minimize limitations in measurement accuracy due to angular and alignment errors, tight dimensional tolerances were striven for in both the Hall plate and support cuboid dimensions. At this point, for the prototype, the tolerance was 10 µm. The process flow for the fabrication of the individual Hall plates is shown in Fig. 4.1a-h as a cross-sectional view through the four contact pads. The process flow for the assembly steps is show in Fig. 4.1i-k.

4.1 Process Technology for Hall Plates Fabrication

The wafer material, which was used for the fabrication of the individual Hall plates, was a 1 µm thick Si-doped GaAs layer on a 500 µm semi-insulating GaAs substrate. The active layer was grown by molecular beam epitaxy (MBE) by C. Reichl and others at the Solid State Physics Laboratory at the ETH Zürich. The average carrier density of the layer at room temperature is about \(6.3 \times 10^{16} \text{ cm}^{-3}\) and the electron mobility is about 4300 cm\(^2\)V\(^{-1}\)s\(^{-1}\), see also Section 5.2.3. The material and Hall sensors fabricated from it will be denoted by HPF5. The cruciform mesa plus contact areas of the Hall plates were realized by photolithography and wet chemical etch. Processing was done on chip-level (not wafer-level) — the wafer was scribed by a hand-held diamond scriber pen and subsequently cleaved into individual die of roughly 6 mm × 6 mm.

4.1.1 Lithography

The individual 6 mm × 6 mm die were cleaned in an ultrasonic bath in acetone and subsequently isopropanol (IPA), rinsed in H\(_2\)O and blow-dried with nitro-

\(^1\)This chapter is partly reproduced from [8] with permission from Elsevier B.V.
Figure 4.1: Flowchart of the fabrication process.
(a) GaAs wafer with active layer and spin-coated photoresist (not to scale)
(b) Patterned photoresist (photolithography)
(c) Pattern transfer
(d) Photolithography (photoresist not to scale), post-exposure image reversal bake
(e) Flood exposure
(f) Development
(g) Metallization (not to scale)
(h) Lift-off, annealing, dicing
(i) Hall plate to MACOR bonding
Figure 4.1: Flowchart of the fabrication process, continued.
(j) Assembly of two Hall plate-MACOR cuboids
(k) Complete device assembly: front view, top view and side view
4 Device Fabrication

Figure 4.2: Optical microscope image of the cruciform mesa plus contact areas for two mirror image Hall sensors (before metallization).

gen. S1828 photoresist was applied to a die at 70 Hz with 15 s ramp time and 50 s total time and baked for 2 min at 100 °C. The resist was patterned by mask exposure for 20 s at 350 W (Karl Süss MJB 3) followed by development in MF26A for 40 s.

4.1.2 Mesa Etch

The pattern was transferred to the active layer by a wet chemical etch in H₂O:H₂SO₄:H₂O₂ (100:3:3) for 10 minutes at an etch rate of ~ 150 nm/min. The etch process was stopped by a rinse in H₂O. The resist was removed in a final cleaning step in acetone and IPA, rinsed in H₂O and blow-dried with nitrogen. A section of a die with two cruciform mesa can be seen in Fig. 4.2.

4.1.3 Ohmic Contacts

Ohmic contacts were realized by alloying evaporated Au/Ge/Ni which is a well-established and widely used Ohmic contact to n-GaAs. They have low resistivity [48], [49] and are found to be non-magnetic [50], [51]. Before the metallization step, the contact areas on the mesa are patterned by photolithography. An image reversal process is applied in order to obtain an undercut profile. For this, AZ5214 photoresist is applied at 70 Hz with 15 s ramp time and 50 s total time and baked for 2 min at 90 °C. The resist is exposed everywhere except at the desired contact areas by mask exposure for 2 s at 350 W (Karl Süss MJB 3). Then the resist is image reversal baked at 115 °C for 2 minutes. A subsequent flood exposure for 20 s without a mask makes the so far unexposed areas developable and development is done for 90 s in AZ326MIF. After patterning, Ohmic contacts were realized by thermal evaporation (courtesy of H. Scherrer and S. Tiegermann, ETH Zürich) of Ge (18 nm)/Au (50 nm)/Ge (18 nm)/Au (50 nm)/Ni (40 nm)/Au (100 nm), a lift-off process in acetone, and annealing under nitrogen flow at 600 mbar for 2 minutes at 120 °C and 20 s at 450 °C.
4.1 Process Technology for Hall Plates Fabrication

(a) Close-up of one scribed (horizontal) and one scribed and cleaved (vertical) edge of test wafer material indicating the dimensional error.

(b) Scribed and cleaved Hall plate.

Figure 4.3: Dicing with precision diamond scriber tool.

4.1.4 Dicing

For the photolithography step, a photomask was designed such to incorporate the structures for four Hall plates on a single die. Since each Hall plate is designed to be 2.1 mm × 2.1 mm, this leaves a large border of nearly a millimetre around the structures on each 6 mm × 6 mm die. This was done purposefully for two reasons. First, the wafer can be quickly scribed and cleaved into die to be processed since they will be diced precisely into their designed dimensions after processing. The second reason is to remove the elevated structures at the edges of the die. During the photoresist spin-coating, resist piles up at the edges of the die and this resist is, due to its thickness, underexposed and not fully developed and hence, the material underneath is not etched away. These unwanted edge structures, also called edge beads, are diced away.

Dicing of the 6 mm × 6 mm die into four 2.1 mm × 2.1 mm die was done with a precision diamond scriber and break tool at the FIRST lab of the ETH Zürich, in collaboration with and by courtesy of E. Gini. For this purpose 5 µm wide marker lines were included in the photomask to define the lines along which the wafer material is to be scribed under the microscope of the diamond scriber tool. The error in dimensions of the die that resulted from this dicing method was less than 5 µm deviation from the nominal value, see Fig. 4.3.

4.1.5 Failure Analysis

During the fabrication process for the Hall plates, following problems may occur and following improvements in the fabrication process were identified.
A thick photoresist S1828 has been chosen to overcome resist adhesion problems with thin resist during wet etch. Employing a thin resist such as AZ5214 leads to poor mesa results, from rough side-walls (insufficient adhesion) to complete degradation of the mesa (no adhesion). Fig. 4.4 shows the result of one of the test structures with an etched mesa under poor resist adhesion. At the edges of the mesa, the resist is not sticking and etchant can crawl underneath resulting in non-uniform active areas. Fig. 4.5 shows the result of a mesa where the photoresist released itself completely during wet etch. Very rough side-walls, triangular shaped mesa and eventually an etched away active layer are the consequences. These problems are resolved by employing a thick resist such as S1828 or by reducing the spinning speed with which the thinner resist AZ5214 is applied. The loss in resolution from a thicker photoresist is not relevant for Hall sensors because, while smooth side-walls and symmetry are important for their performance (i.e. linearity and offset), the exact dimensions of the mesa are not. Minor adhesion problems, leading to rough side-walls of the mesa, may also oc-
cur with thick photoresist which is above a certain age. Before photoresist replacement is necessary, the adhesion can be sufficiently enhanced by a pre-bake of the wafer material at 100 °C to remove moisture and/or by applying a primer such as TI-prime or HMDS.

The error resulting from the scribe/cleaver dicing method in Hall plate dimensions is smaller than 5 µm. Although accurate, the method is slow compared to a dicing saw because every scribed line is manually adjusted and cleaved. About every one in tenth die is not cleaved properly and one or more of its sides are not sharp and vertical. This effect is expected to diminish for thinner wafer materials. The Hallcube could be fabricated with 200 µm thick Hall plates (instead of 500 µm) and MACOR cuboids of height 1700 µm (instead of 1400 µm).

4.2 Cuboid Supports

The MACOR support cuboids were made by CNC machining at the University of Basel (courtesy of P. Reimann). The critical overall dimensions of the cuboids were controlled and lay well within the tolerances of 10 µm. The batch of 16 cuboids had an average width, length, and height of 2.103 mm, 2.101 mm, and 1.400 mm respectively with standard deviations of respectively 5 µm, 0.3 µm, and 0.5 µm, see Fig. 4.7 and Table 4.1. The larger error in one dimension (width) compared to the other dimensions is ascribed to the manufacturing process. A longer slab of material is machined to two dimensions (in this case length and height) and subsequently cut along the width into the separate cuboids.

4.3 Assembly

A low error of less than 5 µm was achieved in the nominal dimensions of the Hall plates as well as the support cuboids. But for the ultimate angular and alignment errors between the six Hall plates, the assembly is a decisive and therefore
4 Device Fabrication

![Figure 4.7: Cuboid dimensions (nominal values).](image)

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<th>h [mm]</th>
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<tr>
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</tbody>
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Table 4.1: Survey with micromaster capaµsystem IP54 0-30 mm.

a critical step in the fabrication process. The assembly of the Hallcube was
done in three sub-steps, using alignment tools that were designed especially for
each assembly step (construction design of the author’s concept by S. Sidorov,
PSI). First, six individual Hall plates were each mounted on a MACOR cuboid.
Next, two Hall cuboids were mounted together and finally, the complete
assembly of $3 \times 2$ Hall cuboids was done.

The Hall plates were each mounted on their MACOR support cuboid using
epoxy (without fillers) which added up to 15 µm to the overall height dimen-
sion. The accuracy of the outer dimensions of the Hallcube is irrelevant and only
the inner active volume of $200 \mu m \times 200 \mu m \times 200 \mu m$ is important. Therefore,
each cuboid and Hall plate has only two out of four side surfaces that need to be
aligned after assembly, namely those adjacent to the corner with the cruciform
active area. For this task, an alignment tool was made with which the Hall plate
and the MACOR support cuboid can be held at right angle and clamped together
during the curing of the epoxy in an oven at 70 °C for several hours.

In the next step, electrical connections were made to three out of six Hall plates
(2, 4 and 6 in Fig. 4.1) by soldering $\sim 15$ cm long 100 µm diameter copper
4.3 Assembly

(a) Second assembly step.

(b) Third and final assembly step.

Figure 4.8: Schematic drawing of (parts of) the assembly tools (picture courtesy of S. Sidorov, PSI).

enamelled wires to their contact pads. Then, in a second alignment tool, two cuboids with Hall plates were glued together with epoxy and cured in an oven at 70 °C for several hours. In this assembly step, one common side surface needs to be aligned and a 200 µm step in height of the assembled pair needs to be assured. Therefore, the second alignment tool was made such that each cuboid with Hall plate can be pushed from top and sides into a right angle on a horizontal base plate with a 200 µm elevated step, see Fig. 4.8a for a schematic drawing. After assembly, the total height of the three pairs deviated from the nominal value (4000 µm) by 16 µm, 28 µm, and 30 µm which is ascribed mainly to the glue thickness.

In the final assembly step, electrical connections were made to the remaining three Hall plates by soldering 100 µm copper enamelled wires to their contact pads. After that, the three pairs of cuboids plus Hall plates were glued together with epoxy into the complete device. Since in this step the inner closed volume is formed, the aligned surfaces are inaccessible and the three pairs of cuboids plus Hall plates are aligned to their outside surfaces. This is done in a third alignment tool where the three pairs are pushed into a right angle on a flat horizontal base during the curing of epoxy in an oven at 70 °C for several hours. A clamping of the Hall cuboids from the top was foreseen but omitted to avoid force on the soldered wires before the epoxy was cured. The final assembly step is schematically represented in Fig. 4.8b and the result of the assembled prototype Hallcube in its final assembly tool is shown in Fig. 4.9.
4 Device Fabrication

Figure 4.9: The prototype Hallcube in its final assembly tool.

4.3.1 Electrical Connections

As foreseen in the design (see Section 3.2.2), the electrical connections were made carefully, such to counteract two effects. The first is the planar Hall effect. To compensate for it, the bias current directions in pairs of Hall plates on opposing cube faces are defined to be 90° rotated with respect to each other. In that case, the planar Hall voltages in pairs of Hall plates have opposite sign and cancel out by averaging.

The second effect is an unwanted induced voltage between the Hall voltage contacts. Loops outside of the Hall plate are avoided by using twisted-pair wires. The loop on the Hall plate itself has an area of about 1 mm². The positive and negative bias current connections on each Hall plate were chosen such that the induced voltages in two Hall plates from opposing cube faces have opposite sign, and hence cancel out by averaging.

The 6 × 4 copper enameled wires were further connected to a wrapped bundle of seven ~ 8 m long and ~ 1 mm diameter coax cables. The 12 wires for the Hall voltage readout were connected to six such coax cables — two wires from the $V_+$ and $V_-$ contacts of each Hall plate were connected to the core and shield of each coax cable. The remaining six wires for the bias current were serially connected and the resulting single $I_+$ and $I_-$ pair of wires was connected to the core and shield of one coax cable. The other end of each coax cable was soldered to a bipolar LEMO plug. These plugs were subsequently connected to the current supply and voltmeters.

4.3.2 Failure Analysis

The yield during assembly is only about 50%, due to detachment of soldered wires during assembly. Dimensions for assembly are tight, with contact pad dimension of 300 µm × 300 µm, MACOR channel width of 400 µm and height of
4.3 Assembly

600 µm, and four soldered wires each of diameter 100 µm. Due to generation of large inductive loops, (non-insulated) wire bonding is not a preferred solution. The yield may be increased by improving the contact pads, e.g., by thickening the top gold layer or adding a copper layer. Alternatively, a slightly larger channel width of the MACOR support cuboids is feasible.

The thickness of epoxy of up to 15 µm leads to a corresponding maximum misalignment between individual Hall plates within the $200 \times 200 \times 200$ µm active volume. Therefore, the magnetic field components cannot truly be assigned to a single point, but to three points separated in space by up to 15 µm. This separation can obviously be reduced by choosing a thinner glue, provided there is no loss in mechanical stability. Alternatively, if the added overall height to a cuboid plus Hall plate from the glue thickness can be made reproducible, the MACOR cuboids can be fabricated with a height that is reduced by this amount.
5 Characterization

[...] there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns – the ones we don’t know we don’t know.

- Donald Rumsfeld

It was explained in Section 2.4 that a uniaxial Hall sensor can be calibrated inside a homogeneous magnetic field, oriented orthogonal to the Hall sensor. By recording the Hall voltages at different known magnitudes of the magnetic field, one obtains the specific calibration coefficients for such a Hall sensor. To characterize and also to calibrate a three-axis Hall sensor on the other hand, it is necessary to assess its response in a homogeneous magnetic field of any orientation, not just at different magnitude. For this task, a rotary positioning system with two orthogonal rotation axes was employed.

5.1 Measurement System

To assess different field directions to the Hallcube in a homogeneous field, a system of two non-magnetic piezoelectric nano rotary positioning stages (SmarAct SR2013-S-NM), denoted $\alpha$ and $\beta$ rotation stage respectively, was used. The rotator configuration with Hallcube is shown schematically in Fig. 5.1. The six constituent Hall plates in the Hallcube are denoted RB, RT, GB, GT, BB, and BT respectively. The $\beta$ rotation stage is connected to the $\alpha$ rotation stage by an L-shaped bracket so that the $\alpha$ rotation axis and the $\beta$ rotation axis are perpendicular. The Hallcube is mounted on the $\beta$ rotation stage via an aluminium support piece. As visible in Fig. 5.1, the support piece (construction design of the author’s concept by S. Sidorov, PSI) features four holes. Two diagonal threaded holes were used to fix the Hallcube inside the support piece from top with a rubber-softened aluminium fixing plate. The remaining two diagonal holes were used to fix the support piece onto the $\beta$ rotation stage disk as well as to fix the wires above the Hallcube with a bridge-shaped aluminium clamp. Fixing the wires prevents them from twisting and possibly breaking near the uniaxial Hall sensors.

The Hallcube revolves around the $\beta$-axis which in turn revolves around the $\alpha$-axis. The whole rotation system was designed such that the Hallcube rotates on its spot, in other words the centre of the Hallcube is at the crossing of the $\alpha$-axis
5 Characterization

Figure 5.1: Rotation system based on two SmarAct SR2013-S-NM piezoelectric nano rotary positioning stages (picture courtesy of S. Sidorov, PSI).

with the $\beta$-axis. With such a configuration, inside a fixed homogeneous magnetic field parallel to the $y$-axis in Fig. 5.1b any magnetic field vector direction to the Hallcube can be mimicked by rotation of the Hallcube about the $\alpha$ and $\beta$ rotation axes.

Fig. 5.1b shows the rotation system in the $\alpha = +45^\circ$ and $\beta = 0^\circ$ position (its "home position"). For both rotation stages of the system, a positive rotation angle corresponds to a counterclockwise rotation as indicated by the arrows around the rotation axes in Fig. 5.1b. The local coordinate system of the Hallcube is chosen such that in the home position, the $y$-axis is along the $\beta$-axis, the $z$-axis is along the $\alpha$-axis, and the $x$-axis points in the direction $\hat{y} \times \hat{z}$, see Fig. 5.1b.

The two SmarAct SR-2013-S-NM rotators have a movement resolution of less than 3 µrad and a sensor resolution better than 25 µrad. Therefore, the rotation of a Hall sensor into a given measurement position can be accomplished with very high precision. However, this is only true if the rotation axes of the two rotary positioners are perpendicular. This will not be the case if the rotation axis of a rotary positioner is not normal to its rotating disk or if the L-bracket is imperfect. The L-bracket has been investigated with a CNC measuring machine (by courtesy of M. Dänzer, PSI) where it was found that the bracket angle deviated from 90° by less than 0.01°. The two rotary positioners have been investigated with a digital indicator (Mitutoyo) of 1 µm resolution. The errors were up to 20 mrad and were subsequently corrected by SmarAct to below 1 mrad.
5.1 Measurement System

The rotator configuration was placed in the centre of a Bruker B-E 30hf dipole calibration magnet with 38 mm gap, large enough to encompass the system. A schematic drawing of the calibration magnet with the rotation system is given in Fig. 5.2. The rotation system was supported by an aluminium arm and block which was screwed onto the return yoke of the magnet (construction design of the author’s concept by H. Jegge, PSI). The arm contains a lever-screw mechanism with which the tilt angle of the rotation system inside the magnet could be manually adjusted. In the vicinity of the Hallcube, at approximately 1 cm spacing, a temperature sensor was placed to monitor the temperature inside the magnet gap.

The bias current (DC) was supplied to all six individual Hall sensors in series by a Keithley 6221 constant current source. The individual Hall sensors’ Hall voltages were measured using six 8½-digit HP/Agilent 3458A digital voltmeters (DVMs). The DVMs, rotation stages, magnet power supply, temperature sensor, and NMR Teslameter (Metrolab PT2025) were connected to the network. Data acquisition and controls of these devices were done by the EPICS control system (implementation by courtesy of P. Chevtsov, PSI) using the Channel Access (CA) protocol in Python scripts. Several Python scripts were written for different measurement schemes to run autonomously. Measurement of the Hall voltages was
5 Characterization

always done in a stop-and-go method using the internal trigger of the DVMs: a trigger of each DVM was activated only after a given rotation system position was reached. Unless stated otherwise, at every trigger 100 voltage samples, each acquired at 20 ms integration time, were taken. The average, peak-to-peak, and rms of the 100 voltage samples of each of the six DVMs were written to a file together with the $\alpha$ and $\beta$ motor positions, temperature, magnet current, and NMR probe reading.

5.2 Characteristics of the Constituent 1D Hall Sensors

5.2.1 Current-Voltage Characteristic

To obtain the current-voltage characteristic (IV-characteristic), the voltage drop over the six uniaxial Hall sensors with direct bias current supplied in series, was measured with a DVM. The bias current was increased from 1 µA to 0.7 mA and subsequently decreased from 0.7 mA to 1 µA. The bias current was monitored by measuring the voltage drop over a temperature-stable resistor of 20 Ω, placed in series to the six Hall sensors, on a second DVM. The magnet current was zero and hence, apart from a small remnant field, no magnetic field was applied. At every current step, 500 voltage samples, each acquired at 20 ms integration time, were taken. The resulting IV-characteristic is shown in Fig. 5.3a where each data point is the average value of 500 voltage samples. The difference in the curve between increase and decrease of the current is visible in Fig. 5.3b where the ratio of the voltage drop over the Hallcube, $V_{\text{bias (sensor)}}$, and the 20 Ω resistor, $V_{\text{bias (20 Ω)}}$, is plotted in percentage of the average ratio over the full current range. By dividing the Hallcube bias voltage with the voltage drop over the 20 Ω resistor, possible current fluctuations of the Keithley 6221 are eliminated. The curve in Fig. 5.3b displays a difference in Hallcube bias voltage between the current increase direction (upper and black part of the curve) and the current decrease direction (lower and orange-black part of the curve). The total resistance of the Hallcube appears to be unstable. This is also seen by a drop in the curve between the 0.36 mA and 0.38 mA bias current measurement points. Between those two data points lies a one hour measurement break. The effect could be due to a temperature difference of the constituent 1D Hall sensors at various currents. On the decreasing current part (orange-black) the temperature at each point is higher than on the increasing current part if it were heated up by the current ramp-up. The ratio of the measured bias voltages in percentage rises to 4.5% for 1 µA bias current (outside the range of the graph) which is due to an unreliable voltage drop measurement over the 20 Ω resistor at the low current end.

The Hall voltage output of the RB sensor versus bias current has been measured at a magnetic field of $\sim$ 1 T. As before, the bias current is supplied in
5.2 Characteristics of the Constituent 1D Hall Sensors

(a) Bias voltage of the Hallcube as a function of direct bias current.

(b) Ratio of the measured voltage drop over the Hallcube and a temperature-stable 20\,\Omega resistor, in percentage of the average ratio over the full current range.

Figure 5.3: IV-characteristic of the Hallcube with direct bias current applied to the constituent uniaxial Hall sensors in series.
Figure 5.4: Hall voltage of the RB sensor versus bias current at \( \sim 1 \) T field (red curve, left scale). The black curve (right scale) shows the peak-to-peak measurement error of the Hall voltage, given in percent of the average Hall voltage at each bias current.

series to all six uniaxial Hall sensors within the Hallcube. The bias current was increased from 1 \( \mu \)A to 0.6 mA and at every current step 500 Hall voltage samples, each acquired at 20 ms integration time, were taken. The result is shown as the red curve in Fig. 5.4 where each data point is the average value of 500 voltage samples. The peak-to-peak error of 500 samples, given in percentage of the average Hall voltage at each bias current, is plotted in black in the same graph. The relative peak-to-peak error decreases to below \( 10^{-4} \) for bias currents above 0.1 mA. The best results can therefore be accomplished with \( I > 0.1 \) mA and the higher the bias current, the higher the signal-to-noise ratio. For testing the prototype Hallcube, a low bias current of \( I = 0.1 \) mA was chosen.

5.2.2 Hall Sensor Parameters

The obtained characteristics of the six individual Hall sensors, at room temperature and at 0.1 mA bias current, are summarized in Table 5.1: sensitivity to the Hall effect, offset voltage, internal resistance between the two bias current contacts (at \( B = 0 \) T), non-linearity in the magnetic field range \( 0 - 2 \) T, and the temperature coefficient of the Hall voltage (around room temperature, at \( B = 1 \) T). The measured values of the six Hall sensors lie within the ranges that are given in Table 5.1.
Table 5.1: Main characteristics of the six Hall sensors fabricated from HPF5 material.

<table>
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<tr>
<th>$S, [VA^{-1}T^{-1}]$</th>
<th>Offset $[\mu V]$ $(I = 0.1\ mA)$</th>
<th>$R, [\Omega]$ $(B = 0\ T)$</th>
<th>$NL, [%]$ $(B = 0 - 2.0\ T)$, $T \sim 25\ ^\circ\ C$</th>
<th>$\alpha_T, [%/\ ^\circ\ C]$ $B = 1.0\ T$, $T \approx 25 \pm 4\ ^\circ\ C$</th>
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5.2.3 Material Properties

The main characteristics of the HPF5 material at room temperature are listed in Table 5.2, the active layer thickness, the sheet carrier density, the mobility, and the sheet resistivity. Their average values were estimated from Eq. 3.2-3.4 and measurements with the six Hall sensors fabricated from these materials, see Table 5.1.

<table>
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<tr>
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<th>$n_s, [cm^{-2}]$</th>
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<th>$\rho_s, [\Omega]$</th>
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Table 5.2: Main characteristic properties of the HPF5 material.

5.3 PHE Compensation

Planar Hall voltage compensation by the pairs of Hall plates in the Hallcube can be demonstrated by rotation in a homogeneous field. A pair of Hall plates is rotated in such a way that the magnetic field and the Hall plates are coplanar. For the three pairs of Hall plates, this is achieved in the following rotation scheme (refer to Fig. 5.1b with the magnetic field parallel to the $y$-axis):

Red pair: $\alpha = -135^\circ \rightarrow 225^\circ$ (full rotation), $\beta = 0^\circ$ or $\pm 180^\circ$
Green pair: $\alpha = -45^\circ$ or $135^\circ$, $\beta = -180^\circ \rightarrow 180^\circ$ (full rotation)
Blue pair: $\alpha = -135^\circ \rightarrow 225^\circ$ (full rotation), $\beta = \pm 90^\circ$

The results of the three measurements, carried out at $I = 0.1\ mA$ and $B \sim 1\ T$, are shown in Figs. 5.5, 5.6, and 5.7.

If the magnetic field vector were exactly coplanar with the Hall plates in each of the three measurements, the output voltage signals would be generated by the planar Hall effect, and there would be no contribution from the ordinary Hall effect since no field component orthogonal to the Hall plates exists. In a non-perfect system, a small orthogonal component of the magnetic field vector to the Hall
5 Characterization

Figure 5.5: Planar Hall voltage measurement with GB (left graph) and GT (right graph) Hall plates for β rotation by 360° at α = −45°.
5.3 PHE Compensation

Figure 5.6: Planar Hall voltage measurement with BB (left graph) and BT (right graph) Hall plates for $\alpha$ rotation by 360° at $\beta = -90$. 

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Figure 5.6: Planar Hall voltage measurement with BB (left graph) and BT (right graph) Hall plates for $\alpha$ rotation by 360° at $\beta = -90$. 

69
Figure 5.7: Planar Hall voltage measurement with RB (left graph) and RT (right graph) Hall plates for a rotation by 360° at β = 0.
5.3 PHE Compensation

plates exists. Even for small angular errors, i.e. small orthogonal field components, the planar Hall voltage of a Hall plate due to the in-plane field component is overshadowed by the Hall voltage due the orthogonal field component because the Hall plate’s sensitivity to the Hall effect is much larger than its sensitivity to the planar Hall effect.

The planar Hall voltage signal can be extracted from harmonic analysis of the output voltage signals in Fig. 5.5, 5.6, and 5.7. For full $\beta$ rotation of a pair of Hall plates, the Fourier series of the voltage signal of a Hall plate in that pair is given by

$$V(\beta) = \frac{a_0}{2} + a_1 \cos(1\beta) + b_1 \sin(1\beta) + a_2 \cos(2\beta) + b_2 \sin(2\beta) + \ldots \quad (5.1)$$

$$= A_0 + A_1 \cos(1\beta - p_1) + A_2 \cos(2\beta - p_2) + \ldots \quad (5.2)$$

with $A_i$ and $p_i$ the amplitude and phase of the $i$th harmonic term. The analogous equation for full $\alpha$-rotation holds. The harmonic coefficients $A_n$ and $p_n$ are listed up to 8th order below the graphs in Figs. 5.5, 5.6, and 5.7. Also listed are the rms error of the Fourier series to describe the voltage signal and the relative error (rms error divided by the highest amplitude). Recalling the double angular dependence of the planar Hall voltage on the magnetic field rotation angle, the second harmonic term in the harmonic series is the planar Hall effect contribution to the output voltage. Apart from that, all Hall plates show a zeroth harmonic (constant term) and first harmonic. The first harmonic is due to the ordinary Hall effect and stems from a magnetic field component that is orthogonal to the Hall plate and rotates around an axis that is coplanar with the Hall plate. Hence, the first harmonic term can only occur in the case that the Hall plate normal vector is not perfectly parallel to the rotation axis ($\alpha$-axis or $\beta$-axis) and wobbles around it. The zeroth harmonic is a constant term and apart from a Hall plate offset voltage, a voltage term that remains constant during rotation can only occur if a magnetic field component parallel to the rotation axis exists.

As an illustration, the planar Hall voltage measurement with one of the red Hall plates under exaggerated angular errors of the magnetic field vector $B$ and the Hall plate normal vector to the $\alpha$-axis, is sketched in Fig. 5.8. The Hall plate normal vector makes an angle $\theta_p$ and $B$ makes an angle $\pi/2 - \theta_B$ to the rotation axis. $B_{nr}$ and $B_{pr}$ are the components of $B$ that are respectively normal and parallel to the rotation axis. In this example case, the zeroth harmonic term of a full $\alpha$ rotation stems from the Hall voltage generated by the field $B_{pr} \cos \theta_p$, which is the magnitude of the component of $B_{pr}$ that is normal to the Hall plate. The first harmonic term stems from the Hall voltage generated by the field $B_{nr} \sin \theta_p$, which is the magnitude of the component of $B_{nr}$ that is normal to the Hall plate. The magnetic field component of interest is coplanar with the Hall plate and generates
the planar Hall voltage which occurs as the second harmonic term in the voltage signal decomposition. The magnitude of the coplanar magnetic field component during rotation is $B_{nr} \cos \theta_p$. In summary,

\begin{align*}
A_0 &= f_H (B_{pr} \cos \theta_p) = f_H (B \sin \theta_B \cos \theta_p) \tag{5.3a} \\
A_1 &= f_H (B_{nr} \sin \theta_p) = f_H (B \cos \theta_B \sin \theta_p) \tag{5.3b} \\
A_2 &= f_{PH} (B_{nr} \cos \theta_p) = f_{PH} (B \cos \theta_B \cos \theta_p) \tag{5.3c} \\
B_{nr}^2 + B_{pr}^2 &= B^2 \tag{5.3d}
\end{align*}

where $f_H$ and $f_{PH}$ are functions governing the sensitivity of the Hall plates to the Hall and planar Hall effect.

The expected angular errors for $\theta_B$ and $\theta_p$ are small, not more than a few degrees. Therefore, the measurement of the planar Hall voltage is to first order correct under the estimation

\begin{align*}
A_0 &\approx f_H (B \sin \theta_B) \tag{5.4a} \\
A_1 &\approx f_H (B \sin \theta_p) \tag{5.4b} \\
A_2 &\approx f_{PH} (B) \tag{5.4c}
\end{align*}

The largest first harmonic amplitude occurs in Hall plate BB. The amplitude is about $230 \mu$V. Since at 0.1 mA bias current the sensitivity to the Hall effect is approximately $10 \text{ mV/T}$, this voltage amplitude corresponds to a field component of 230 Gauss (assuming linear field dependence of the Hall voltage). The corresponding angular error for that particular plate is, estimated from Eq. \ref{5.4c}, $\theta_p = 1.3^\circ$. The zeroth harmonic varies among the probes and is $200 \mu$V at maximum. Thus, ignoring offset voltages, the angle $\theta_B$ can be estimated to be of the order of a degree at most.

The sensitivities of the six individual Hall plates to the Hall effect are of the order $100 \text{ VA}^{-1}\text{T}^{-1}$. At 0.1 mA bias current and in a coplanar magnetic field of 1 T, the planar Hall voltage amplitudes of the Hall plates are in the range $17 - 18 \mu$V. Hence, without compensation, the sensitivity of a Hall plate to the planar Hall...
5.3 PHE Compensation

Figure 5.9: Planar Hall voltage compensation with the green pair of parallel Hall plates rotated in plane 90° to each other. The two dashed curves are the planar Hall voltages of the individual Hall plates respectively, measured at 0.1 mA bias current and at 1 T. The solid curve is their calculated average.

Figure 5.9: Planar Hall voltage compensation with the green pair of parallel Hall plates rotated in plane 90° to each other. The two dashed curves are the planar Hall voltages of the individual Hall plates respectively, measured at 0.1 mA bias current and at 1 T. The solid curve is their calculated average.

effect is already about 500 times smaller than its sensitivity to the Hall effect. Therefore, without compensation the expected error would be about 20 Gauss in a 1 T field, or 0.2%. In the error analysis of Section 3.4, it was shown that the successful compensation of the planar Hall effect by a pair of Hall plates depends on two things — the error in 90° rotation angle between the two plates, and the inequality of the planar Hall voltage amplitude between the two plates. The largest error in the rotation angle \( \phi \) occurs in the red pair where \( \phi = 88.76° \) (thus an error of 1.24°). Nevertheless, the planar Hall voltage compensation is least effective for the green pair because besides an angular error in \( \phi \) of 0.66°, the Hall plates in the green pair have the largest difference in planar Hall voltage amplitudes. The result of the planar Hall voltage compensation for the green pair is shown in Fig. 5.9 for full \( \beta \) rotation. The two dashed curves are the planar Hall voltages measured by the two Hall plates individually and the solid curve is obtained by simply averaging these two output voltages. From this it can be deduced that the pairs’ remnant planar Hall voltage amplitude after compensation is below 0.5 \( \mu \)V \( \cong \) 0.5 Gauss for a 1 T in-plane field (maximum error). Therefore, by using pairs of Hall plates, the planar Hall voltage could be reduced about 35 times for the worst aligned pair and can almost certainly be further reduced by better alignment.
5 Characterization

5.4 Angular Errors

5.4.1 Simple Identification

The angular errors among all individual Hall sensors were found by three rotations of the Hallcube in a magnetic field of about 1 T: rotation by 360° around each of its three local coordinate axes x, y, and z. In the first estimation, it is assumed that the α-axis and β-axes are orthogonal, and the magnetic field vector points along the β-axes when α = 45° (home position). Hall sensor GT (see Fig. 5.1b) is taken as a reference to which the angles are defined. The result of rotation around the z-axis, with α from −135° to 225° and β = 0°, is shown in Figs. 5.10 and 5.11 together with the resulting harmonic decomposition of the voltage signal. The rotary system with Hallcube orientation is graphically represented at the bottom of the graphs. In this representation, the rotating disks of the α and β rotation stage are indicated as two dotted rings, and the rotation arrow indicates the rotation stage (either α or β) that was used for rotation of the Hallcube. From the phase difference of the first harmonic term between the green and blue Hall sensors, the angles between them can be deduced. The angles between the green and red Hall sensors are found in an analogous manner from the result of a 360° rotation around the x-axis, with α from −135° to 225° and β = −90°, see Figs. 5.12 and 5.13. Finally, from a 360° rotation around the y-axis, with β from −180° to 180° and α = −45°, see Figs. 5.14 and 5.15, the angles among the blue and red Hall sensors can be deduced.

The deduced angular errors (deviations from 90°) are summarized in Fig. 5.16. Because of the angular accuracy of the rotation stages, the listed values for the angular errors sum up to zero, as they should in theory. To visualize the small angular deviations from 90° (given in text), they are graphically exaggerated five times. The orthogonality errors lie in the range between 0.1° and 0.5° with two outliers of 0.01° and 1.0°. The three pairs of Hall cuboids that resulted from the second assembly step, see Fig. 4.11, have orthogonality errors between 0.2° and 0.3°. Thus it is plausible that larger errors are due to the third assembly step. In the third assembly step, a clamping of the Hall cuboids from the top was foreseen but omitted to avoid force on the soldered wires before the epoxy was cured. To be conclusive, one needs to observe the errors in additional assemblies.

With current assembly tools, all angular errors can be brought below 0.5°. This is more than sufficient to compensate effectively for the planar Hall voltage, but it is not sufficient to reach below 10^{−4} in cross-sensitivity. With an improvement of tooling, one might hope to reduce the angular errors induced during assembly even further. Nevertheless, the maximum tolerable error of 0.006° is out of reach on such a small-scale device with such assembly methods. Therefore it is important to eliminate measurement errors stemming from non-orthogonalities among
5.4 Angular Errors

Figure 5.10: Hall voltage measurement with GB (left graph) and GT (right graph). Hall plates for α rotation by 360° at β = 0°.

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5 Characterization

Figure 5.11: Hall voltage measurement with BB (left graph) and BT (right graph)
Hall plates for $\alpha$ rotation by 360° at $\beta = 0°$. 
5.4 Angular Errors

Figure 5.12: Hall voltage measurement with GB (left graph) and GT (right graph). Hall plates for \(\alpha\) rotation by 360° at \(\beta = -90°\).
Figure 5.13: Hall voltage measurement with RB (left graph) and RT (right graph) Hall plates for a rotation by 360° at $\beta = -90°$. 
5.4 Angular Errors

Figure 5.14: Hall voltage measurement with BB (left graph) and BT (right graph).
Hall plates for $\beta$ rotation by 360° at $\alpha = -45^\circ$. 

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5 Characterization

Figure 5.15: Hall voltage measurement with RB (left graph) and RT (right graph). Hall plates for \( \beta \) rotation by 360° at \( \alpha = -45° \).
5.4 Angular Errors

Figure 5.16: Estimation of the angular errors (deviations from 90°) among individual Hall sensors. To visualize the small angular errors (text), they are graphically exaggerated five times.

Hall plates. This will be done by identifying the relative surface normal vectors of all six Hall plates and, after calibration, by applying the knowledge of the surface normals in the field reconstruction (Chapter 6).

5.4.2 Identification of the Hall Sensors’ Normal Vectors

The surface normal vector of each Hall sensor can be described by two angles, φ and θ. The local coordinate system of the Hallcube will be adopted again, where the y-axis is along the β-axis, the z-axis is along the α-axis, and the x-axis in the $\hat{y} \times \hat{z}$ direction. The angle φ is defined as the angle between the surface normal vector and the y-axis, and θ is the angle between the projected surface normal vector onto the xz-plane and the negative x-axis, see Fig. 5.17. The surface normal vector of each of the six Hall sensors is then of the form

$$n = (n_x, n_y, n_z) = (-\sin \phi \cos \theta, \cos \phi, \sin \phi \sin \theta). \hspace{1cm} (5.5)$$

The angles φ and θ that define the surface normal vector of each Hall sensor, can be determined from the phase of the first harmonic term of the harmonic decomposition in Section 5.4.1. Given a phase $p_1$ of the first harmonic term in the harmonic analysis of the Hall voltages from a full α rotation at $\beta = 0°$, it follows from Fig. 5.17 that

$$-p_1(\beta = 0°) = \arctan\left(\frac{-x_p}{y_p}\right) = \arctan\left(\frac{\sin \phi \cos \theta}{\cos \phi}\right) = \arctan(\tan \phi \cos \theta). \hspace{1cm} (5.6)$$

The left-hand side is the negative phase $p_1$ of the first harmonic which originates from the fact that a clockwise rotation about the α-axis corresponds to negative rotation angles. On the other hand, given a phase $p_1$ of the first harmonic term in
5 Characterization

Figure 5.17: Coordinate system to define the normal vectors of the Hall sensors by the angles $\theta$ and $\phi$.

the harmonic analysis of the Hall voltages from a full $\alpha$ rotation at $\beta = -90^\circ$, it follows that

$$-p_1(\beta = -90^\circ) = \arctan\left(\frac{z_p}{y_p}\right) = \arctan\left(\frac{\sin\phi\sin\theta}{\cos\phi}\right) = \arctan(\tan\phi\sin\theta)$$

From Eqs. 5.6 and 5.7 the angles $\theta$ and $\phi$ can be deduced as follows:

$$\theta = \arctan\left(\frac{z_p/y_p}{-x_p/y_p}\right) = \arctan\left(\frac{\tan(-p_1(\beta = -90^\circ))}{\tan(-p_1(\beta = 0^\circ))}\right)$$

(5.8a)

$$\phi = \arctan\left(\frac{-x_p/y_p}{\cos\theta}\right) \text{ or } \phi = \arctan\left(\frac{z_p/y_p}{\sin\theta}\right)$$

(5.8b)

There are in total four solutions for $(\phi, \theta)$. If $(\phi = a, \theta = b)$ is a solution then $(\phi = a - 180, \theta = b)$, $(\phi = -a, \theta = b - 180)$, and $(\phi = 180 - a, \theta = b - 180)$ are solutions as well. Since the expected angular errors are small (not more than a few degrees), two out of four solutions can be eliminated and the remaining two solutions are in fact identical.

To compensate for an angular error in the magnetic field direction, the phase $p_1$ from two additional full $\alpha$ rotations is taken into account, at $\beta = 90^\circ$ and
\[ \beta = 180^\circ. \] In that case, one can write
\[ \theta = \arctan \left( \frac{z_p / y_p}{-x_p / y_p} \right) = \arctan \left( \frac{\tan \left[ (\beta = -90^\circ) - p_1(\beta = 90^\circ) \right] / 2}{\tan \left[ (\beta = 0^\circ) - p_1(\beta = 180^\circ) \right] / 2} \right) \]
\[ (5.9a) \]

\[ \phi = \arctan \left( \frac{-x_p / y_p}{\cos \theta} \right) \quad \text{or} \quad \phi = \arctan \left( \frac{z/y}{\sin \theta} \right) \]
\[ (5.9b) \]

whose values are independent on an angle of the magnetic field vector with respect to the \( y \)-axis.

The resulting angles \( \phi \) and \( \theta \) for the six Hall sensors, obtained from this method are:
\[ \phi_{GB} = 178.888, \theta_{GB} = -45.383 \]
\[ \phi_{GT} = -0.717, \theta_{GT} = -86.719 \]
\[ \phi_{BB} = -91.060, \theta_{BB} = 0.442 \]
\[ \phi_{BT} = 89.526, \theta_{BT} = 0.317 \]
\[ \phi_{RB} = -90.795, \theta_{RB} = -89.787 \]
\[ \phi_{RT} = 89.491, \theta_{RT} = -89.842 \]

With the found angles \( \phi \) and \( \theta \), the surface normal vector of each Hall sensor directs inwards to the centre of the hollow cube. The sign of the GB, BB, and RB normal vector is changed, such that the case \( B \cdot n > 0 \), for any Hall sensor results in a positive Hall voltage of that sensor. For the green sensors, a magnetic field in the \( y \) direction results in a positive Hall voltage and makes the dot product with the green sensors’ normal vectors positive. For the blue sensors, a magnetic field in the \( -x \) direction results in a positive Hall voltage and makes the dot product with the blue sensors’ normal vectors positive. For the red sensors, a magnetic field in the \( -z \) direction results in a positive Hall voltage and makes the dot product with the red sensors’ normal vectors positive. Therefore, the surface normals are defined as:
\[ n_{GB} = [- \sin(178.888) \cos(45.383), \cos(178.889), - \sin(178.888) \sin(45.383)] \]
\[ n_{GT} = [\sin(0.717) \cos(86.719), \cos(0.717), \sin(0.717) \sin(86.719)] \]
\[ n_{BB} = [- \sin(91.060) \cos(0.442), \cos(91.060), - \sin(91.060) \sin(0.442)] \]
\[ n_{BT} = [- \sin(89.526) \cos(0.317), \cos(89.526), \sin(89.526) \sin(0.317)] \]
\[ n_{RB} = [- \sin(90.795) \cos(89.787), \cos(90.795), \sin(90.795) \sin(89.787)] \]
\[ n_{RT} = [- \sin(89.491) \cos(89.842), \cos(89.491), - \sin(89.491) \sin(89.842)] \]
\[ (5.10) \]
5 Characterization

Figure 5.18: Third harmonic amplitude of GB and GT sensors at each $\beta$ position, obtained from harmonic analysis over $2\pi$ rotation of $\alpha$.

5.5 Higher Harmonics

In the harmonic analysis of the voltage output signal of the six Hall sensors during a $360^\circ$ rotation over a given rotation axis (Sections 5.3 and 5.4), only the harmonic terms up to second order have been discussed. Looking at the list of harmonic coefficients below each of the graphs in Figs. 5.5-5.7 and Figs. 5.10-5.15, it can be seen that harmonic terms up to third order contribute significantly to the voltage output signal. The origin of the third order harmonic term is not fully understood. Its induction by angular errors of the rotation system could be eliminated by a simulation of an imperfect rotation system based on a calculation using rotation matrices. The existence of Hall voltage dependency on the product of all three magnetic fields components, $B_x \cdot B_y \cdot B_z$, has been observed by [52]. For the HPF5 Hall sensors, however, this dependency could not be reproduced. Instead, the third order harmonic term is independent on the separate field components $B_x$, $B_y$, and $B_z$, and is constant for a constant magnitude of the magnetic field, $B$. This is shown for the Hall sensors GB and GT, whose third harmonic from a full $\alpha$ rotation is plotted at different $\beta$ angles. The fact that the observed amplitude of the third order harmonic term is constant for different orientations of a magnetic field vector with constant magnitude, indicates that the third order harmonic term could stem from a non-linearity in the Hall voltage dependency on the magnetic field. This can at least partially account for the amplitude of the third order harmonic term as will be shown in Section 6.2 of the next chapter.
5.6 Long-term Stability Problems

A poor long-term stability of the Hall voltages of the individual Hall sensors in the prototype Hallcube has been detected. The Hall voltages of all six uniaxial Hall sensors have been measured over days during which fluctuations of these voltages have been observed. A result of such a measurement at 0.1 mA bias current and at a constant magnetic field of about 1 T, is given in Fig. 5.19. The Hallcube is positioned at $\alpha = -9.736^\circ$ and $\beta = 45^\circ$ so that each of the six sensors is subjected to approximately the same magnetic field orthogonal to its sensor surface. The Hall voltages in the figure are given in percentage of their average value over the time of measurement. The grey curve is the result of a temperature measurement in the vicinity of the Hallcube (right scale). The black curve is the magnetic field value measured by a reference NMR probe and is also given in percentage of its average value over the time of measurement. The peak-to-peak fluctuation of the Hall voltages about their average values is up to 0.1% from several measurements over multiple days. Furthermore, it was observed that when applying a bias current to the sensors after a longer, several hours, period of no applied bias current, the Hall voltages drifted for several hours before stabilizing to an average value with a peak-to-peak fluctuation of up to 0.1%.

The following causes of this long-term instability were hypothesized: fluctu-
5 Characterization

ations of the magnetic field (value or direction), temperature, humidity or bias current; low-frequency 1/f noise of the Hall sensors; noise-currents in the cabling; internal stresses due to packaging [23]; surface state effects.

As can be seen in Fig. 5.19, the Hall voltages of the uniaxial Hall sensors follow more or less the same trend over time. Because of this, low-frequency noise of the Hall sensors is highly unlikely to be the cause as it would not affect the sensors in an identical manner. It is also visible in Fig. 5.19 that there is no visible correlation between the Hall voltage fluctuations and the ambient temperature. The magnetic field was constant to within 0.3 Gauss and is also ruled out as the cause. If a fluctuation of the direction of the magnetic field were to explain the fluctuation in the Hall voltages, it would need to be as high as \(\pm 20^\circ\) which is unsubstantial. Noise-currents in the cabling are unlikely to cause low-frequency fluctuations of the Hall voltage. To be certain, the Hallcube was biased by two Keithley 6221 constant current sources, each delivering 0.1 mA to three sensors in series. The correlation between the curves of the Hall voltages remained and therefore cannot be due to noise-currents in the cabling.

To find out whether the bias current from the Keithley 6221 constant current source itself fluctuates, the voltage drop over the six Hall sensors was monitored on one of the six DVMs. Also the voltage drop over a 20 \(\Omega\) temperature-stable resistor was monitored on a second DVM. From this measurement it was found that the bias current is constant to within \(\pm 40\) ppm and can be eliminated as a cause for Hall voltage fluctuation. However, it was observed that the voltage drop over the six Hall sensors in series followed the same trend as the fluctuation of the Hall voltages over time. Since the voltage drop over the 20 \(\Omega\) temperature-stable resistor has proven the bias current to be constant, the total resistance of the Hall sensors must fluctuate. The most plausible reason for this to happen is a change in temperature which affects the carrier concentration in the semiconductor and thereby its resistance and the Hall voltage. It would also be expected to affect all six sensors in the same manner because they reside in a closed and small volume. However, no relation between the ambient temperature outside the Hallcube and its Hall voltages was found. Apart from that, the temperature coefficient of the HPF5 Hall sensors that would explain the measured amount of change in Hall voltages would need to be 0.2 \(\%K^{-1}\), which is high and unreported on for GaAs with a doping level as the HPF5 material.

In order to be certain to eliminate temperature as the cause, and to test the packaging-induced stress hypothesis, the long-term stability on single bare HPF5 Hall sensors have been investigated. One sensor was glued with epoxy onto a piece of MACOR to resemble the condition of the Hall sensors within the prototype Hallcube. A second sensor was lightly taped to a piece of MACOR. Both sensors demonstrated similar behaviour to the sensors in the prototype Hallcube:
5.6 Long-term Stability Problems

Figure 5.20: Long-term measurement of a single SiN passivated HPF5 Hall sensor at 0.1 mA bias current and 1.0 T.

the Hall voltage drifts for several hours before reaching an equilibrium around which the Hall voltage fluctuates. No reduction in fluctuation nor relation to the ambient temperature could be observed.

Finally, a single bare HPF5 sensor featuring a SiN passivation layer was tested. The passivation layer was applied in a final chip-processing step by plasma-enhanced chemical vapour deposition (PECVD) at the FIRST lab of the ETH Zürich, in collaboration with and by courtesy of P. Burkard. The thickness of the deposited layer was examined with an ellipsometer and was found to be \(\sim 260\) nm. The HPF5 sensors that were passivated had contacts made from Au/Ge/Pt instead of Au/Ge/Ni, i.e. nickel was replaced by platinum. The layer thicknesses of the Au/Ge/Pt contact materials as well as the fabrication and annealing procedure were identical to that for the Au/Ge/Ni contacts. The reason for the different contacts is that this particular sensor was made for contact testing purposes in addition to testing the effect of passivation. To study the long-term Hall voltage stability of this passivated sensor, it was lightly taped to a piece of MACOR and its Hall voltage was recorded over 12 hours in a constant magnetic field of about 1 T. The Hall voltage recording started immediately after the bias current of 0.1 mA was applied. The result of the Hall voltage, given in percentage of its average value over the time of measurement, is shown in Fig. 5.20. No initial drift nor fluctuation of the Hall voltage is observed. Hence, plausible causes for the Hall voltage instability are humidity fluctuations and surface effects, despite the enclosed volume in which the six sensors reside. With passiv-
5 Characterization

The peak-to-peak fluctuation from Fig. 5.20 is 0.01%. The question whether the nickel in the Au/Ge/Ni contacts takes any part in the long-term instability of the Hall voltage, could be negated with a third test sensor — a bare and unpassivated HPF5 Hall sensor with Au/Ge/Cr contacts, i.e. nickel was replaced by chromium.
6 Calibration

The pessimist complains about the wind; the optimist expects it to change; the realist adjusts the sails.

- William A. Ward

The calibration of a three-axis Hall sensor cannot be done in the same, straightforward, manner as for single-axis Hall sensors like described in Section 2.4. The multiple single-axis Hall sensors in a three-axis Hall sensor cannot be simultaneously positioned orthogonally to the magnetic field. Since the angular errors in the prototype Hallcube are small, pairs of Hall sensors could be calibrated simultaneously to save time. In any case, in contrast to single-axis Hall sensor calibrations, a precise and dedicated rotation system is required for three-axis Hall sensor calibrations — either a system that can rotate the magnetic field over two orthogonal axes, or a system similar to the one described in Chapter 5 which, instead of the field, rotates the three-axis Hall sensor.

Simply calibrating all individual Hall sensors inside a three-axis Hall sensor and reconstructing the magnetic field vector from their calibration coefficients is adequate as long as the angular errors of the Hall sensors are taken into account. If not, the accuracy of a three-axis Hall sensor would be limited depending on the amount of cross-sensitivity between the Hall sensors due to angular errors and the planar Hall effect. An alternative, elaborate 3D calibration scheme was developed by [52],[53],[54]. In this scheme a three-axis Hall sensor is rotated in a given number of steps around an axis that is perpendicular to the magnetic field vector, as well as around an axis that is along the magnetic field vector, thereby covering a full sphere rotation. At each angular step, the Hall voltages are recorded as well as the magnetic field from a reference probe (e.g. NMR). The resulting large data set is decomposed into orthogonal functions — spherical harmonics for the angular dependence and Chebyshev polynomials for the magnetic field and temperature dependence. Since the known galvanomagnetic effects generating a Hall voltage can be ascribed to a magnetic field component orthogonal to the Hall sensor surface (Hall effect) and an in-plane magnetic field (planar Hall effect), since the planar Hall voltages of the Hallcube are almost fully compensated for, and since the angular errors of the Hall sensors within the Hallcube are known, such a calibration scheme leads to an over-determined system and is unnecessarily time-consuming. The notion of a 3D Hall effect in [53] by the observation of an existing $Y_3^2$ spherical harmonic in the Hall voltage signal decomposition,
could partially be reproduced. The $Y^2_3$ spherical harmonic implies a Hall voltage dependency on the product $B_x \cdot B_y \cdot B_z$ \[52\]. The third order harmonic term in the harmonic analysis in Section \[5.5\] however, is independent on $\beta$ rotation angle (see Fig. \[5.18\]). From the forthcoming calibration results, it is plausible that the third harmonic term observed with the prototype Hallcube can at least partly be ascribed to the non-linearity of the individual Hall sensors.

For three-axis Hall sensors with sufficient compensation of the planar Hall voltage (sufficient in the sense that the remnant planar Hall voltage induces less than the acceptable error), a simpler calibration scheme is proposed which is divided in two parts: finding the angular errors among the individual Hall sensors (according to the characterization scheme of \[5.4\]) and calibration of the individual Hall sensors analogous to a single-axis Hall sensor calibration. Finally, reconstruction of the magnetic field values will take into account both informations.

The rotation system that was employed for the characterization of the Hallcube (see Section \[5.1\]) would be equally useful as a positioning tool for the individual Hall sensors during their calibration. Employing instead a system in which the field is rotated around the sensors, would require an in comparison costly rotation of a (few tons) magnet, or a superconducting 3D Helmholtz coil. The disadvantage of the system for Hall sensor rotation on the other hand, is that even though it is small enough to fit inside a 38 mm gap dipole magnet, it is too large for an NMR probe to be placed in the magnet centre as well. A magnet that can encompass the rotation system as well as an NMR probe which are both placed at its centre and equidistant from its mid-plane, would need to have a gap of at least 65 mm. Such a magnet would have to be large in order to obtain the same homogeneity as a 38 mm gap magnet.

### 6.1 Measurement Setup

With the current system, there are two calibration options. The first option is to calibrate the Hallcube against the NMR probe by placing them both on the mid-plane and equidistant from the centre. In this option, the smallest possible distance between the NMR probe and Hallcube is 2 cm. If the NMR probe and Hallcube are placed on the mid-plane and 1 cm from the centre of the magnet, the magnetic field they experience should theoretically be the same. To know whether that is the case and whether the homogeneity at 1 cm radius from the magnet centre is large enough, requires a mapping of the Bruker B-E 30hf calibration magnet. In the magnetic field range $0.7 - 2.1$ T, the required homogeneity for the NMR probe to lock is $250 - 600$ ppm/cm. The NMR probe has an active volume (diameter $\times$ length) of $4$ mm $\times 4.5$ mm, hence the maximum required homogeneity is about $10^{-4}$. 
6.1 Measurement Setup

The second option is to calibrate the calibration magnet (magnetic field against magnet current) with an NMR probe in its centre, and subsequently calibrating the Hallcube in the magnet centre (Hall voltage against magnet current). Such a calibration scheme is usually not advised since one has to blindly rely on the magnet current delivered by the power supply. Whether this scheme is applicable, depends on the reliability of the power supply and the field reproducibility during cycling of the magnet.

6.1.1 Mapping the Calibration Magnet

The Bruker B-E 30hf calibration magnet has been mapped with a Siemens SBV 585-S1 Hall sensor (see Section 2.4 for details of the sensor). Its 150 mA direct bias current was supplied by a North Hills CS-I2-2 current source and its Hall voltage output was read by a 8½-digit HP/Agilent 3458A digital voltmeter (DVM) at 20 ms integration time. Before mapping the magnet, its geometrical centre has been determined with a FARO arm. From the geometrical centre, measurement

![Figure 6.1: Profile plot of the calibration magnet's magnetic field B_y versus z position, at x = 0 and y = 0. The field values are measured at different magnet currents and are given in percentage of the field value at the centre of the magnet, B_0.](image-url)
in the xz-plane was done on-the-fly, in both directions from -70 mm to 70 mm with a measurement step size of 2 mm. The xz-plane was measured at different y positions, -14, -10, 0, 10, and 14 mm. Mapping was repeated at magnet currents 19, 38, 42, 46, 50, 54, 58, 63, 68, 75, 84, and 96 A.

The result of the magnetic field measurement on the mid-plane, along the z-axis (x = 0 and y = 0) is shown in Fig. 6.1. In this figure, the field profiles obtained at different magnet currents are displayed in different colour. The field values are given in percentage of the field value at the geometrical centre, $B_0$.

From the graph it is obvious that at higher magnet currents, the magnet poles become saturated and the homogeneous region becomes smaller and smaller. Away from the centre, the magnetic field at higher magnet currents first increases before dropping. This indicates the presence of an air slit in the centre of the magnet pole which is in effect similar to shimming the outer pole rim and is done to enlarge the homogeneous region. At high currents, the onset of saturation effects takes place in the centre where there is least iron. The region of $10^{-4}$ homogeneity at
96 A spans about ±10 mm in z. The homogeneous area at this current is shown in a contour plot, see Fig. 6.2. In the figure, the given dimensions on the x and z scale are in mm. The contour lines represent equal magnetic field values, in percentage of $B_0$. The homogeneous regions with homogeneity $1 \cdot 10^{-4}$, $2 \cdot 10^{-4}$, and $5 \cdot 10^{-4}$ are indicated in magenta.

At a magnet current of 96 A and a magnetic field of about 2 T, the homogeneous region would be just about large enough to encompass both the NMR probe and the Hallcube. However, in order for the Hallcube and the NMR probe to experience the same magnetic field, they must be placed at exactly the same distance from the magnetic centre of the magnet. This is not as trivial as placing the Hallcube and NMR probe at the centre and equidistant above and below the mid-plane, which is easily determined. Misalignments in the lateral placement readily result in too large differences in magnetic field experienced by the NMR probe and the Hallcube. Namely, at higher magnet currents the magnetic field around the centre increases more or less quadratically.

6.1.2 Calibration of the Calibration Magnet

As an alternative calibration method, the reliability on magnet current as a measure for magnetic field of the calibration magnet is tested. The magnetic field versus magnet current was measured by NMR probes in the range -38 A to 38 A. The difference in magnetic field values between four sequential runs is shown in Fig. 6.3. In the figure, the three curves are the difference of each run to the first run. The maximum difference that occurs is about 0.25 Gauss. The maximum difference among all runs except the first run is however, less than 0.1 Gauss. Since at the second run the magnet has already been cycled once (during the first run), one can conclude that the magnetic field at a given magnet current is highly reproducible, provided the magnet current is always ramped in the same way. This has been confirmed by a measurement scheme in which the magnet was first pre-cycled a few times. Then, even if few days passed between two measurements, the difference in field values was found to be less than 0.2 Gauss over the full magnet current range. Since the method has proven to be reliable and is providing the opportunity to calibrate the Hallcube in the centre of the calibration magnet, where the homogeneity level is highest, it has been adapted as calibration method.

The calibration magnet was calibrated over the current range from -96 A to 96 A. Beforehand, the magnet was ramped down to -96 A and then three times pre-cycled, $3 \times (-96 \text{ A} \rightarrow 96 \text{ A} \rightarrow -96 \text{ A})$, with a waiting time of 300 s at the maximum currents. This was done to erase its “memory” of previous ramping and a remnant field. After pre-cycling the magnet, the current was increased from -96 A to 96 A in 1 A current steps. At each current step, the NMR probe
6 Calibration

Figure 6.3: Difference between 4 sequential NMR probe measurements of the magnetic field in the centre of the calibration magnet, for a magnet current range from \(-38\, \text{A}\) to \(-16\, \text{A}\), \(+16\, \text{A}\) to \(+38\, \text{A}\), \(38\, \text{A}\) to \(+16\, \text{A}\), and \(-16\, \text{A}\) to \(-38\, \text{A}\). The three curves are the difference in measured field values between the second and first run, between the third and first run, and between the fourth and first run.

The NMR probes measure only the total magnetic field, not its direction, and the magnetic field has a positive value regardless of the magnet current. The sign of the magnetic field is allocated to the calibration data based on the physical convention that a magnetic field vector points from north to south pole, and by the sign of the Hall voltage. In the case presented in Fig. 3.3, where the magnetic field vector points into the paper, the Hall voltage \(V_H = V_+ - V_-\) would be positive for both Hall sensors BB and BT. When the magnet current is negative, the top pole of the calibration magnet is the south pole and the bottom pole is the north pole, which corresponds to the magnetic field vector pointing in the \(+y\) direction. The calibration positions were chosen such that the GB, BB, and RB Hall sensors were at \(+y\) and the GT, BT, and RT sensors were at \(-y\) position (refer to Fig. 5.1b). Hence, in these positions, the case in Fig. 3.3 corresponds to a magnetic field in the \(+y\) direction and thus to a negative magnet current. The positive and negative voltage and current connections in all three pairs (red, green, and blue) are identical to the blue pair shown in Fig. 3.3. Therefore, in the calibration data of...
the magnet calibration, the sign of the magnetic field should be assigned negative when the magnet current has a value > 0. This is not influenced by the hysteresis of the magnet, since the magnet current range −1 A to 1 A within which the magnetic field changes direction, was omitted from the calibration.

6.2 Calibration of the Individual Uniaxial Hall Sensors

The calibration of the individual Hall sensors was done in pairs. A pair of Hall sensors (red, green, or blue) was aligned to the magnetic field vector such that the two Hall sensors have more or less the same angular error to the field. The parallelism between the two sensors has to be good enough for this scheme. The maximum non-parallelism occurs for the green probes, the angle between them is 0.75°. Thus, by correct alignment in the magnetic field, the field component perpendicular to the sensors’ surfaces, is about $B \cos(0.75°/2)$. This corresponds to an acceptable error of $0.2 \cdot 10^{-4}$.

The calibration position, in which two sensors of a pair experience the maximum field component along their surface normals, was found by slight rotation in the calibration magnet at a magnetic field of about 1 T. For the green pair, the calibration position was sought in two steps. First, by manually modifying the tilt angle of the arm that supports the rotation system (see Fig. 5.2). The position midway between the positions of maximum Hall voltage for each Hall sensor was fixed. In the second step, a rotation around the $\alpha$-axis by the $\alpha$ rotation stage was performed. The Hall voltage of one sensor was recorded over five angular steps, $-2°; -1°; -0°; 1°; 2°$, about the starting position which was $\alpha = 45°$. The extremum of the quadratic fit was set to be the new starting angle after which the process was repeated two more times. The same was done for the other Hall sensor of the pair and the midway position was selected as the calibration position of the $\alpha$ rotation stage. The $\beta$ rotation stage merely revolves the Hall sensors around their surface normal and was kept at the position $\beta = 0°$.

The calibration position of the red and blue pair was sought by three times repeated rotation by $\pm 2°$ around the $\alpha$-axis by the $\alpha$ rotation stage, alternated with a $\pm 2°$ rotation around the $\beta$-axis by the $\beta$ rotation stage. The starting positions for the blue pair were $\alpha = -45°$ and $\beta = 0°$, and the starting positions for the red pair were $\alpha = -45°$ and $\beta = 90°$. The results of the angular positions are listed in Appendix D.

Once the calibration positions were found, the calibration magnet was ramped to -96 A and precycled three times following the same scheme as during its calibration. The Hall voltages of a pair of Hall sensors were recorded in the magnet current range from -60 A to -2A and 60 A to 2 A. The maximum magnet current of 60 A corresponds to a magnetic field of about 1.5 T. At each magnet current, the
waiting time for field stabilization was identical to the waiting time in the magnet calibration scheme. After stabilization at each current, 100 measurements of the Hall voltages of two sensors were taken by two DVMs at an integration time of 20 ms. The green, blue, and red pair of Hall sensors were calibrated successively at a constant bias current of 0.1 mA (all six sensors in series). The temperature inside the calibration magnet during calibration fluctuated between 30 °C and 31 °C.

After the calibration of the magnetic field against magnet current and the calibration of the Hall voltages of all uniaxial sensors against magnet current, the correlation between the magnetic field and Hall voltage is obtained. For each Hall sensor, the relationship between the magnetic field and the Hall voltage is described in a polynomial function by a least-squares fit to the calibration data. The method was described in Section 2.4 and the reader is referred back to that section for understanding the forthcoming results and notations. The calibration results for the Hall sensors GB, GT, BB, BT, RB, and RT are shown in Figs. 6.4-6.9. In each figure, the top graph displays the non-linearity of the Hall sensor, obtained from a least-squares polynomial fit to the calibration data with two coefficients. The Hall sensors demonstrate a very high linearity between 0.03% and 0.05% over the magnet field range -1.5 T to 1.5 T, which is ascribed to their cruciform geometry and the moderate mobility of the Si-doped GaAs. The bottom graphs display the remaining magnetic field after a polynomial fit with the full set of coefficients that are relevant for each Hall sensor. The rms error of the fit and the level of the remaining field are the noise of the Hall sensor. The rms error of the fit is at least a factor 2 higher in comparison to three different commercial Hall sensors (see Figs. 2.4b, 2.5b, and 2.6b). This is due to the fact that the signal-to-noise ratio of the HPF5 Hall sensors at 0.1 mA bias current is lower than the signal-to-noise ratio of the commercial Hall sensors. The rms error and peak-to-peak error of 100 Hall voltage measurements that were taken at each magnet current, lie respectively in the range 0.15 – 0.20 µV and 0.6 – 0.8 µV for the six HPF5 Hall sensors. At a given bias current, these values are independent of the magnetic field, i.e. strength of the Hall voltage. By increasing the bias current, the rms error and peak-to-peak error of the Hall voltage measurements increases but less than linearly. From Fig. 5.4 in Section 5.2.1 it is observed that by increasing the bias current from 0.1 mA to 0.5 mA, the relative rms error and peak-to-peak error can be reduced almost two times. Apart from the low bias current that was applied, also the poor long-term stability of the Hall voltages, described in Section 5.6, negatively affects the signal-to-noise ratio.

The set of polynomial functions with which the magnetic field orthogonal to a
6.2 Calibration of the Individual Uniaxial Hall Sensors

(a) Non-linearity of the GB sensor.

(b) Rest signal after fitting a 5th order polynomial to the calibration data.

Figure 6.4: Calibration results of the GB sensor at $I_{bias} = 0.1$ mA.
6 Calibration

(a) Non-linearity of the GT sensor.

(b) Rest signal after fitting a 3rd order polynomial to the calibration data.

Figure 6.5: Calibration results of the GT sensor at $I_{bias} = 0.1$ mA.
6.2 Calibration of the Individual Uniaxial Hall Sensors

(a) Non-linearity of the BB sensor.

(b) Rest signal after fitting a 4th order polynomial to the calibration data.

Figure 6.6: Calibration results of the BB sensor at $I_{\text{bias}} = 0.1$ mA.
6 Calibration

(a) Non-linearity of the BT sensor.

(b) Rest signal after fitting a 4th order polynomial to the calibration data.

Figure 6.7: Calibration results of the BT sensor at $I_{bias} = 0.1$ mA.
6.2 Calibration of the Individual Uniaxial Hall Sensors

(a) Non-linearity of the RB sensor.

(b) Rest signal after fitting a 3rd order polynomial to the calibration data.

Figure 6.8: Calibration results of the RB sensor at $I_{bias} = 0.1 \, mA$. 
6 Calibration

(a) Non-linearity of the RT sensor.

(b) Rest signal after fitting a 3rd order polynomial to the calibration data.

Figure 6.9: Calibration results of the RT sensor at $I_{bias} = 0.1 \text{ mA}$. 

102
6.2 Calibration of the Individual Uniaxial Hall Sensors

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>GB</th>
<th>BB</th>
<th>RB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>$-0.243106 \cdot 10^2$</td>
<td>$-0.802773 \cdot 10^2$</td>
<td>$0.320435 \cdot 10^2$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$0.931210 \cdot 10^3$</td>
<td>$0.992845 \cdot 10^3$</td>
<td>$0.995100 \cdot 10^3$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$-0.263749 \cdot 10^{-1}$</td>
<td>$-0.112983 \cdot 10^{-2}$</td>
<td>$0.451926 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$c_3$</td>
<td>$0.112905 \cdot 10^{-2}$</td>
<td>$-0.234133 \cdot 10^{-2}$</td>
<td>$-0.355269 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$c_4$</td>
<td>$0.224658 \cdot 10^{-4}$</td>
<td>$0.362539 \cdot 10^{-4}$</td>
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</tr>
<tr>
<td>$c_5$</td>
<td>$-0.453260 \cdot 10^{-5}$</td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>GT</th>
<th>BT</th>
<th>RT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>$0.237683 \cdot 10^1$</td>
<td>$0.609747 \cdot 10^2$</td>
<td>$0.597070 \cdot 10^2$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$0.972585 \cdot 10^3$</td>
<td>$0.996867 \cdot 10^3$</td>
<td>$0.969184 \cdot 10^3$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$-0.164937 \cdot 10^{-1}$</td>
<td>$-0.374505 \cdot 10^{-2}$</td>
<td>$0.767894 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$c_3$</td>
<td>$-0.277857 \cdot 10^{-2}$</td>
<td>$-0.118037 \cdot 10^{-2}$</td>
<td>$-0.214503 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$c_4$</td>
<td></td>
<td>$0.991697 \cdot 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>$c_5$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Calibration coefficients $c_i$ of the individual Hall sensors for $I_{bias} = 0.1$ mA, units are Gauss mV$^{-i}$.

Hall sensor, $B_n$, can be reconstructed from its Hall voltage measurement, are:

\[
B_n^{GB} = c_0^{GB} + c_1^{GB} V_{GB} + c_2^{GB} V_{GB}^2 + c_3^{GB} V_{GB}^3 + c_4^{GB} V_{GB}^4 + c_5^{GB} V_{GB}^5
\]  
(6.1a)

\[
B_n^{GT} = c_0^{GT} + c_1^{GT} V_{GT} + c_2^{GT} V_{GT}^2 + c_3^{GT} V_{GT}^3
\]  
(6.1b)

\[
B_n^{BB} = c_0^{BB} + c_1^{BB} V_{BB} + c_2^{BB} V_{BB}^2 + c_3^{BB} V_{BB}^3 + c_4^{BB} V_{BB}^4
\]  
(6.1c)

\[
B_n^{BT} = c_0^{BT} + c_1^{BT} V_{BT} + c_2^{BT} V_{BT}^2 + c_3^{BT} V_{BT}^3 + c_4^{BT} V_{BT}^4
\]  
(6.1d)

\[
B_n^{RB} = c_0^{RB} + c_1^{RB} V_{RB} + c_2^{RB} V_{RB}^2 + c_3^{RB} V_{RB}^3
\]  
(6.1e)

\[
B_n^{RT} = c_0^{RT} + c_1^{RT} V_{RT} + c_2^{RT} V_{RT}^2 + c_3^{RT} V_{RT}^3
\]  
(6.1f)

In principle, these equations are valid only within the range of calibration, i.e. for magnetic fields up to 1.5 T. Since the HPF5 Hall sensors are so linear, the possible error for results outside of the calibration range is expected to be small. The calibration coefficients belonging to Eqs. 6.1 are listed in Table 6.1 with units Gauss mV$^{-i}$ for the $i^{th}$ calibration coefficient. The field-equivalent offsets of the sensors (coefficients $c_0$) are between 2.4 Gauss and 80.3 Gauss. The sensitivities of the Hall sensors ($1/c_1$) are between 10.05 mVT$^{-1}$ and 10.74 mVT$^{-1}$ at 0.1 mA bias current.

In Section 5.5, a third order harmonic term was observed in the harmonic ana-
6 Calibration

Analysis of the Hall voltages from a rotation of the Hallcube relative to the magnetic field. From the inverse of Eqs. [6.1] the expected third harmonic term due to the Hall sensor’s non-linearity can be estimated. The inverse equation is obtained by reversing the least-squares polynomial fit, i.e. by fitting the measured Hall voltages to the measured magnetic field values. The inverse equation for each Hall sensor is of the form

\[ V_H = c'_0 + c'_1 B_n + c'_2 B_n^2 + c'_3 B_n^3 + ..., \]  

(6.2)

with \( B_n \) the magnetic field component orthogonal to the sensor. The rotation of a Hall sensor in a magnetic field is equivalent to rotation of the magnetic field around the Hall sensor. Let’s assume a rotation of the magnetic field vector over an angle \( \theta \) around an axis that is orthogonal to the Hall sensor’s normal vector. In that case the magnetic field component orthogonal to the Hall sensor is \( B_n = B \cos \theta \). Inserting this into Eq. (6.2) the fourth term in case of \( B = 1 \text{T} \) becomes

\[ c'_3 B_n^3 = c'_3 (B \cos \theta)^3 = \frac{c'_3}{4} \cos(3\theta) + \frac{3}{4} \cos \theta. \]  

(6.3)

Hence, in the harmonic analysis of the Hall voltage signal from a 360° rotation in a constant magnetic field of about 1 T field, the amplitude of the third harmonic term is expected to be \( c'_3 / 4 \). The third coefficient for the GB and GT Hall sensors is found to be \( c'^{GB}_3 = -0.150336 \cdot 10^{-14} \text{mV Gauss}^{-3} \) and \( c'^{GT}_3 = 0.31134 \cdot 10^{-14} \text{mV Gauss}^{-3} \). Hence, for a field of 1 T (10000 Gauss), the expected third harmonic amplitude would be 1.5 µV/4 for Hall sensor GB and 3.1 µV/4 for Hall sensor GT. The observed values in Fig. 5.18 are about 3-5 times higher than what would be expected from the Hall sensors’ non-linearities. The reason is unclear. Further study of the effects observed by [52] would be necessary to understand and prove their origin.

6.3 Magnetic Field Vector Reconstruction

The directions of the surface normals of the six Hall sensors were defined in Section 5.4 such that when a magnetic field vector points in the same direction as the surface normal, it induces a positive Hall voltage in the Hall sensor. The Hall effect in any Hall sensor is due to an orthogonal magnetic field component. The magnetic field measured by each of the six Hall sensors is therefore equal to the dot product between the Hall sensor’s surface normal vector \( n \) and any given magnetic field vector \( B \), where \( n \) and \( B_n(V) \) are according to respectively Eq.
This is not entirely true since the contribution of the planar Hall voltage has been ignored. However, the planar Hall voltages are nearly fully compensated by summing the Hall voltages of two sensors that form a pair. There is no reason to believe that the compensation will not work as well by summing the magnetic field values obtained from these Hall voltages by the calibration coefficients, e.g. $B_{n}^{GB} + B_{n}^{GT}$ and equivalent for the other two pairs. The planar Hall effect increases the Hall voltage reading for one sensor and reduces it for the other sensor in a pair. Therefore, also the resulting magnetic field from the calibration coefficients is higher for one sensor and lower for the other and, cancelled out by averaging. This works because the Hall voltage dependency on orthogonal magnetic field is very linear for the HPF5 Hall sensors but even if that were not the case, the planar Hall voltage sensitivity to the magnetic field is orders of magnitude smaller than the Hall voltage sensitivity to the magnetic field and locally the function $B_{n}(V_{H})$ is linear within a small voltage range. The previous reasoning results in three equations,

\begin{align*}
B \cdot (n_{GB} + n_{GT}) &= B_{n}^{GB}(V_{GB}) + B_{n}^{GT}(V_{GT}), & (6.4a) \\
B \cdot (n_{BB} + n_{BT}) &= B_{n}^{BB}(V_{BB}) + B_{n}^{BT}(V_{BT}), & (6.4b) \\
B \cdot (n_{RB} + n_{RT}) &= B_{n}^{RB}(V_{RB}) + B_{n}^{RT}(V_{RT}). & (6.4c)
\end{align*}

Writing $n_{GB} = (n_{x,GB}, n_{y,GB}, n_{z,GB})$ (and equivalent for the other 5 Hall sensors) and $B = (B_{x}, B_{y}, B_{z})$, the components of the magnetic field vector, $B_{x}$, $B_{y}$, and $B_{z}$, are now found:

\begin{align*}
B_{x} = \frac{\text{det}
\begin{pmatrix}
B_{n}^{GB}(V_{GB}) & n_{y,GB} + n_{y,GT} & n_{z,GB} + n_{z,GT} \\
B_{n}^{GT}(V_{GT}) & n_{y,GB} + n_{y,GT} & n_{z,GB} + n_{z,GT} \\
B_{n}^{BB}(V_{BB}) & n_{y,BB} + n_{y,BT} & n_{z,BB} + n_{z,BT} \\
B_{n}^{BT}(V_{BT}) & n_{y,BB} + n_{y,BT} & n_{z,BB} + n_{z,BT} \\
B_{n}^{RB}(V_{RB}) & n_{y,GB} + n_{y,GT} & n_{z,GB} + n_{z,GT} \\
B_{n}^{RT}(V_{RT}) & n_{y,GB} + n_{y,GT} & n_{z,GB} + n_{z,GT}
\end{pmatrix}}{\text{det}
\begin{pmatrix}
n_{x,GB} + n_{x,GT} & n_{y,GB} + n_{y,GT} & n_{z,GB} + n_{z,GT} \\
n_{x,BB} + n_{x,BT} & n_{y,BB} + n_{y,BT} & n_{z,BB} + n_{z,BT} \\
n_{x,RB} + n_{x,RT} & n_{y,GB} + n_{y,GT} & n_{z,GB} + n_{z,GT}
\end{pmatrix}} & (6.6a)
\end{align*}
6 Calibration

\[ B_y = \frac{\det \begin{pmatrix} n_{x,GB} + n_{x,GT} & B_n^{GB} (V_{GB}) + B_n^{GT} (V_{GT}) & n_{z,GB} + n_{z,GT} \\ n_{x,BB} + n_{x,BT} & B_n^{BB} (V_{BB}) + B_n^{BT} (V_{BT}) & n_{z,BB} + n_{z,BT} \\ n_{x,RB} + n_{x,RT} & B_n^{RB} (V_{RB}) + B_n^{RT} (V_{RT}) & n_{z,RB} + n_{z,RT} \end{pmatrix}}{\det \begin{pmatrix} n_{x,GB} + n_{x,GT} & n_{y,GB} + n_{y,GT} & n_{z,GB} + n_{z,GT} \\ n_{x,BB} + n_{x,BT} & n_{y,BB} + n_{y,BT} & n_{z,BB} + n_{z,BT} \\ n_{x,RB} + n_{x,RT} & n_{y,RB} + n_{y,RT} & n_{z,RB} + n_{z,RT} \end{pmatrix}} \] (6.6b)

\[ B_z = \frac{\det \begin{pmatrix} n_{x,GB} + n_{x,GT} & n_{y,GB} + n_{y,GT} & B_n^{GB} (V_{GB}) + B_n^{GT} (V_{GT}) \\ n_{x,BB} + n_{x,BT} & n_{y,BB} + n_{y,BT} & B_n^{BB} (V_{BB}) + B_n^{BT} (V_{BT}) \\ n_{x,RB} + n_{x,RT} & n_{y,RB} + n_{y,RT} & B_n^{RB} (V_{RB}) + B_n^{RT} (V_{RT}) \end{pmatrix}}{\det \begin{pmatrix} n_{x,GB} + n_{x,GT} & n_{y,GB} + n_{y,GT} & n_{z,GB} + n_{z,GT} \\ n_{x,BB} + n_{x,BT} & n_{y,BB} + n_{y,BT} & n_{z,BB} + n_{z,BT} \\ n_{x,RB} + n_{x,RT} & n_{y,RB} + n_{y,RT} & n_{z,RB} + n_{z,RT} \end{pmatrix}} \] (6.6c)

6.4 Errors of measurement

With the calibration coefficients (Table 6.1), the normal vectors of the uniaxial Hall sensors (Eq. 5.10), and the developed field reconstruction method (Eq. 6.6), the Hallcube is equipped to fulfil its purpose: magnetic field measurements. From the six output voltages of the Hallcube in any magnetic field, the three magnetic field components of that field can now be deduced.

The known error sources during Hall voltage measurement and calibration of the uniaxial Hall sensors, are summarized in Table 6.2. They are grouped into error sources from the electronic devices (the Keithley 6221 current source and the 8½-digit HP/Agilent 3458 digital voltmeters (DVMs)), during calibration, and from the HPF5 uniaxial Hall sensors themselves.

The error sources listed for the Hall sensors are valid for a bias current of 0.1 mA and at a magnetic field of 1 T. Obviously, the relative error contribution from the DVMs reduces for higher bias currents. The short-circuited DVM error as well the Hall voltage error are given for a DVM integration time setting of 20 ms. The rms error of the short-circuited DVM is dependent on the settings for the range and auto-zeroing. The lower limit is valid for the 0.1 mV and 1 mV range with auto-zeroing activated. The error in magnetic field to Hall sensor non-orthogonality is given for the worst of the three pairs in the prototype Hallcube. The error values given in round brackets are the relative errors in ppm calculated for the prototype Hallcube at 0.1 mA bias current and 1 T magnetic field.

The largest error stems from the long-term instability of the Hall voltages of the individual Hall sensors in the Hallcube. As was discussed in Section 5.6
6.4 Errors of measurement

<table>
<thead>
<tr>
<th>Error source</th>
<th>Value</th>
<th>Unit</th>
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<tbody>
<tr>
<td><strong>Electronic devices</strong></td>
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<tr>
<td>Short-circuited DVM rms / peak-to-peak</td>
<td>0.1 – 0.3 / &lt; 1 (10 – 30 / &lt; 100)</td>
<td>[µV] [ppm]</td>
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<td>Keithley 6221, 0.1 mA DC (peak-to-peak)</td>
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<td>[ppm]</td>
</tr>
<tr>
<td><strong>Calibration method (range -1.5 T to 1.5 T)</strong></td>
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<td></td>
</tr>
<tr>
<td>NMR precision / absolute accuracy</td>
<td>1 / 5</td>
<td>[ppm]</td>
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<tr>
<td>Magnetic field to magnet current calibration</td>
<td>0.2 (20)</td>
<td>[ppm] [Gauss]</td>
</tr>
<tr>
<td>Magnetic field to Hall sensor non-orthogonality</td>
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<td>[ppm] [deg]</td>
</tr>
<tr>
<td>Air temperature fluctuation during calibration</td>
<td>30 – 31 (300)</td>
<td>[ppm] [°C]</td>
</tr>
<tr>
<td><strong>HPF5 Hall sensors (I_{bias} = 0.1 mA, B = 1 T)</strong></td>
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<td></td>
</tr>
<tr>
<td>PHE compensation (@ 1 T in-plane field)</td>
<td>&lt; 0.5 ( &lt; 50)</td>
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<td>[ppm]</td>
</tr>
<tr>
<td>V_H peak-to-peak, long-term (days)</td>
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<td>[ppm]</td>
</tr>
<tr>
<td>α_T (@ T ~ 25 °C)</td>
<td>~ −300</td>
<td>ppm/°C</td>
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Table 6.2: List of known measurement error sources. In round brackets are given the relative errors in ppm, calculated for the prototype Hallcube at 0.1 mA bias current and 1 T magnetic field.

This error can likely be reduced by a factor of 10 when these Hall sensors are passivated. Since this error is apparent only during long-term measurements (the short-term error is low), magnet measurements lasting shortly don’t have the absolute accuracy that was striven for but they still have a high relative accuracy. This means, for example, that the distribution of a measured field can be correct but the field values that make up the distribution are offset.
7 Test Measurements with the Prototype Hallcube

It doesn’t matter how beautiful your theory is, it doesn’t matter how smart you are. If it doesn’t agree with experiment, it’s wrong.

- Richard P. Feynman

First test measurements with the Hallcube were performed within its predominant application: the measurement of magnets. The Hallcube was applied in a magnetic field measurement of the COBRA detector magnet (Section 7.2) and the SwissFEL laser heater U50 undulator (Section 7.3), both located at the Paul Scherrer Institute (PSI). However, it is not straightforward to analyse the obtained measurement results in terms of absolute accuracy. The NMR probe, used as reference during calibration, has a relatively large active volume (diameter × length) of 4 mm × 4.5 mm, works only in a uniform magnetic field, and measures only the field magnitude, not its direction. Therefore, the NMR probes could not be used as a reference during these measurements. In the analysis and graphical representation of the measurement results of the COBRA magnet and the laser heater U50 undulator, Sections 7.2.3 and 7.3.3, the measurement errors are disregarded and not specified. Rather than focusing on the absolute accuracy of the measured field values, the performance of the Hallcube is tested through the repeatability of measurement, its ability to distinguish weak field components in the presence of strong field components, and whether its measured field components are according to the physical description of a magnetic field (Maxwell’s equations). On the other hand, it is important to assess the measurement accuracy of the Hallcube. A straightforward way to do so is inside a homogeneous field, e.g. once again inside the calibration magnet.

7.1 Homogeneous Magnetic Field Measurement

A reliable assessment of the accuracy of the Hallcube is the reconstruction of the modulus (magnitude) of a magnetic field vector under different angles of incidence. Regardless of the orientation of the magnetic field vector to the Hallcube, its magnitude \( B = |\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} \) is constant. Rotation of a constant magnitude field vector around the Hallcube is equivalent to rotation of the Hallcube in a constant homogeneous field. To mimic different magnetic field vector directions and check the constancy of the total magnetic field measured by the
Hallcube, the same measurement system that was used during the characterization and calibration of the Hallcube, see Section 5.1, was used here. The great advantage of this method to assess the accuracy of the Hallcube, is that it is independent on imperfections of the piezoelectric rotation system or on alignment of the Hallcube with respect to the field. Also, since the measurement takes place in the homogeneous calibration magnet, the measured total field can be referenced to the measurement result of the highly accurate NMR probe. The magnetic field inside the calibration magnet, measured by the reference NMR probe, was 9985.2 Gauss. The bias current of the Hallcube was 0.1 mA. The $\beta$ positions were set from $\beta = -180^\circ$ to $\beta = 180^\circ$ in steps of 15$^\circ$. At each $\beta$ position, a full $\alpha$ rotation from $\alpha = -135^\circ$ to $\alpha = 225^\circ$ was performed in steps of 15$^\circ$ at which the six Hall voltages of the Hallcube were measured.

Let’s consider first the results of the rotation about one axis only, e.g. the $\alpha$-axis. The position of the $\beta$ rotation stage is chosen such that all six Hall sensors during a rotation experience a relatively large field component along their surface normal, hence all six sensors generate a relatively large Hall voltage. The surface normals of the green sensors are perpendicular to the $\alpha$-axis, regardless of the $\beta$ position. The surface normals of both the red and blue sensors would point equally and at maximum orthogonality to the $\alpha$-axis at $\beta = 45^\circ$ (or equivalent). In that case, their graphs for measured field component versus $\alpha$ angle would overlap. For better visibility of all three field components, the measured field components at $\beta = -165^\circ$ (equivalent to 15$^\circ$) are considered. The result of the field component measurement by the red, green, and blue pairs of Hall sensors in their local coordinate system is shown in Fig. 7.1a. The square root of the sum of squares of these values, is the black curve in the same graph. The black curve is replotted on a different scale where the total magnetic field is given in percentage of its average value, see Fig. 7.1b. The maximum deviation of the total field from its average value is $+0.03\%$ and $-0.02\%$. For comparison, in the same graph the grey curve represents total field reconstruction obtained by three Hall plates only (BB, GT, and RB) and without taking into account their angular errors. In other words, the field reconstruction was done solely by the calibration coefficients, e.g.

$$B_x = c_0^{BB} + c_1^{BB} V_{BB} + c_2^{BB} V_{BB}^2 + ... \quad (7.1)$$

and analogous for $B_y$ and $B_z$. The same reconstruction can be done for the other three Hall plates: BT, GB, and RT. The Hall plates BT, GB, and RT are located at respectively $+x$, $+y$, and $+z$ in the local coordinate system of the Hallcube whereas the Hall plates BB, GT, and RB are located at respectively $-x$, $-y$, and $-z$, see Fig. 5.1b. The result of the total field reconstruction for all $\beta$ and $\alpha$ positions with three Hall plates and without correcting for their angular errors, is given in a contour plot in Fig. 7.3a for the Hall plates BT, GB, and RT and in Fig. 7.3b for
7.1 Homogeneous Magnetic Field Measurement

(a) Field component measurement at Hallcube bias current 0.1 mA and for full $\alpha$ rotation in $15^\circ$ steps, at $\beta = -165^\circ$ in local coordinate system of the Hallcube: $B_x$ (BB and BT sensors), $B_y$ (GB and GT sensors), and $B_z$ (RB and RT sensors). The resulting magnitude of the magnetic field vector is shown in black, see also the graph below.

(b) Close-up of the magnitude of the magnetic field vector (black curve), now given in percentage of its average value. The grey curve, for comparison, is the total field reconstruction obtained by the Hall plates BB, GT, and RB only and without taking into account their angular errors, it is also given in percentage of the average total field value.

Figure 7.1: Measurement of the total magnetic field vector and its magnitude by rotation of the Hallcube in a homogeneous field of about 1 T.
the Hall plates BB, GT, and RB. The $\alpha$ positions are given on the horizontal axis and the $\beta$ positions are given on the vertical axis.

In case of no correction for the planar Hall voltage or angular errors, the reconstructed total magnetic field, which should be constant, is obviously not constant. The reconstructed total magnetic field has a peak-to-peak difference of 88 Gauss when the reconstruction is done by the Hall plates BT, GB, and RT, see Fig. 7.3a and 189 Gauss when the field reconstruction is done by the Hall plates BB, GT, and RB. The reason for the larger peak-to-peak difference for reconstruction with BB, GT, and RB is the large angular error of \( \sim 1^\circ \) that occurs between the Hall plates BB and GT, see Fig. 5.16. In Figs. 7.3a and 7.3b it can be seen that the total magnetic field has a double angular dependency on the $\alpha$ rotation angle for a given $\beta$ position. The occurrence of double angular dependence should not be confused with the planar Hall effect. It was shown in Section 5.3 that the uncompensated planar Hall voltage has a maximum amplitude of about 20 Gauss. Here, the visible double angular dependence is a direct consequence of the angular errors among the individual Hall sensors. The double angular dependency is easily explained in a two-dimensional system with two Hall plates, see Fig. 7.2. Imagine the magnetic field vector $\mathbf{B}$ to be rotated around an axis perpendicular to the plane (into the paper) and imagine the planar Hall effect can be neglected. If the two Hall plates are perpendicular, the field component generating a Hall voltage in Hall plate 1 is $B_1 = B \sin \alpha$ and the field component generating a Hall voltage in Hall plate 2 is $B_2 = B \cos \alpha$. The magnitude of the reconstructed field would thus be perfect,

$$\sqrt{B_1^2 + B_2^2} = \sqrt{B^2 \sin^2 \alpha + B^2 \cos^2 \alpha} = B. \quad (7.2)$$

If Hall plate 2 is given a small angular error $\xi$, the field component generating its Hall voltage would, instead, be $B_2 = B \cos(\alpha + \xi)$. In that case, the magnitude of the reconstructed field becomes

$$\sqrt{B_1^2 + B_2^2} = \sqrt{B^2 \sin^2 \alpha + B^2 \cos^2(\alpha + \xi)} = B \sqrt{1 - \sin(2\alpha + \xi) \sin \xi}, \quad (7.3)$$
7.1 Homogeneous Magnetic Field Measurement

e.g. it displays a double angular dependency on the rotation angle $\alpha$. The double angular dependence on $\alpha$ in Fig. 7.3b is largest at the positions $\beta = 0^\circ$ and $\beta = \pm 180^\circ$. This is to be expected because at these $\beta$ positions, the large $\sim 1^\circ$ angular error between Hall plates BB and GT (see Fig. 5.16) is dominant and causes the double angular dependence. On the contrary, at the positions $\beta = \pm 90^\circ$ the angular error between the Hall plates GT and RB is dominant but is only $\sim 0.1^\circ$. In Fig. 7.3a it can be seen that the double angular dependence on $\alpha$ is largest at the positions $\beta = 45^\circ$ and $\beta = -135^\circ$. This can as well be explained based on the angular errors in Fig. 5.16. At $\beta = 0^\circ$ and $\beta = \pm 90^\circ$, the angular errors between GB and BT respectively between GB and RT are dominant. Both are of the same size of $\sim 0.3^\circ$. The double angular dependence at $\beta = 90^\circ$ and $\beta = -90^\circ$ is of opposite sign as it must be, due to the $180^\circ$ phase shift. Therefore, at $\beta = 45^\circ$, inbetween the positions $\beta = 0^\circ$ and $\beta = 90^\circ$, both angular errors contribute to the double angular dependence and enhance it whereas at $\beta = -45^\circ$ the double angular dependence is diminished.

In the next step, the angular errors of the Hall sensors will be taken into account via the reconstruction scheme of Section 6.3 and the Hall sensors’ normal vectors that were found in Section 5.4.2. First the field reconstruction will again be done on three Hall sensors only and hence, without compensating for the planar Hall effect. For the field reconstruction with the three Hall plates BT, GB, and RT, this is done by setting $B_{n}^{BB}$, $B_{n}^{GT}$, $B_{n}^{RB}$, $n_{BB}$, $n_{GT}$, and $n_{RB}$ in Eqs. 6.6 to zero and vice versa for the field reconstruction with the three Hall plates BB, GT, and RB. The results are shown in a contour plot, see Figs. 7.3c and 7.3e for the total field reconstruction with BT, GB, and RT and Figs. 7.3d and 7.3f for the total field reconstruction with BB, GT, and RB. The peak-to-peak magnetic field difference is 23 Gauss in Fig. 7.3c and 21 Gauss in Fig. 7.3d. Compared to the peak-to-peak difference of 88 Gauss in Fig. 7.3a and 189 Gauss in Fig. 7.3b it can be concluded that the angular correction method is successful. Both sets of three Hall sensors have the same value in peak-to-peak field which is to be expected if the angular errors for each set are corrected for. The remaining $\sim 20$ Gauss fluctuation in the total magnetic field displays a double angular dependency which is due to the planar Hall effect.

Finally, the total field reconstruction will be done with the Hallcube (all six constituent Hall sensors) via the reconstruction scheme of Section 6.3 and the Hall sensors’ normal vectors that were found in Section 5.4.2. Now, the angular errors of the Hall sensors as well as the planar Hall voltages are corrected for. The result of the total field reconstruction over all $\alpha$ and $\beta$ rotation angles is represented in a contour plot in Fig. 7.4. The total field reconstruction for any field direction to the Hallcube is accurate to within $+8$ Gauss and $-4$ Gauss, or $0.08\%$, and is 8 times larger than the targeted accuracy of $10^{-4}$ at the 1 T level. The reason that the
7 Test Measurements with the Prototype Hallcube

(a) Total field reconstruction from BT (+x), GB (+y), and RT (+z) Hall sensors without angular error correction.

(b) Total field reconstruction from BB (−x), GT (−y), and RB (−z) Hall sensors without angular error correction.

(c) Total field reconstruction from BT (+x), GB (+y), and RT (+z) Hall sensors with angular error correction.

(d) Total field reconstruction from BB (−x), GT (−y), and RB (−z) Hall sensors with angular error correction.

(e) Repetition of of (c) with adapted range.

(f) Repetition of of (d) with adapted range.

Figure 7.3: Full sphere rotation, total field reconstruction with three Hall plates at every α and β position. The field measured by a reference NMR probe was 9985.2 Gauss
accuracy is not better can be either that the Hall sensor normals are wrong or that the calibration coefficients are wrong. Also the long-term instability of the Hall sensors can be a cause. The total measurement time was just under 20 hours with a 9 hour break between the positions $\beta = -15^\circ$ and $\beta = 0^\circ$. The measurement started at $\beta = -180^\circ$ and ended at $\beta = 180^\circ$. In Fig. 7.4 it can be seen that the peak-to-peak error at each $\beta$ position is only about 6 Gauss and the average total magnetic field gradually increases with $\beta$ position. This increase can be explained by the long-term instability of the Hall voltages which over a time of 20 hours is similar to the obtained accuracy of 0.08%, see Fig. 5.19. The measurement at one $\beta$ position lasts only about 20 min and the peak-to-peak error of 6 Gauss in these measurements is too large to be explained by Hall voltage fluctuations. Apart from that, the total field for each $\beta$ position in Fig. 7.4 has the same shape: higher at the end positions of $\alpha$ and lower in the middle. However, it is very possible that the calibration coefficients are wrong due the long-term instability of the Hall voltages. The calibration procedure described in Section 6.2 lasted in total just under 30 hours. It is plausible that with resolved long-term instability of the Hall sensors, the accuracy will improve to the level of the Hall voltage stability of the tested passivated Hall sensor which was 0.01% (see Section 5.6).

Figure 7.4: Total magnetic field measured with the prototype Hallcube at 0.1 mA bias current (serial, DC) at various magnetic field directions, mimicked by rotation of the Hallcube in a homogeneous and constant field. The field measured by an NMR reference probe was 9985.2 Gauss.
7 Test Measurements with the Prototype Hallcube

7.2 Solenoid Magnetic Field Measurement

A solenoid magnet is basically a current-carrying coil wound helically around a cylinder. The main magnetic field component of a solenoid is along the solenoid axis and its integral is proportional to the Ampere-turns. Because the flux lines must close on themselves, a radial field component exists as well, being strongest at the ends of the solenoid. In a perfectly symmetric solenoid, no azimuthal field component exists.

7.2.1 The COBRA Detector Magnet

The COnstant Bending RAdius detector magnet was built for the MEG experiment (Mu to E Gamma) at PSI. This experiment is running since 2008 with the goal to search for the lepton flavour violating decay of a muon into an electron and a photon. The COBRA magnet is a superconducting solenoid magnet with a room temperature bore and two resistive coils for stray field reduction. The main superconducting magnet consists of 5 coils: one central coil (inner diameter 699.1 mm), two gradient coils (809.1 mm), and two end coils (919.1 mm). A schematic drawing of the COBRA magnet is given in Fig. 7.5. The COBRA magnet was designed to provide a gradient field (central field is 1.27 T) such that decayed positrons follow trajectories of constant projected bending radius and increasing axial pitch that depend only on the particle’s momentum, not its emission angle [55]. A three-dimensional magnetic field map of the COBRA magnet is needed for a precise reconstruction of a three-dimensional positron track in the gradient field.

7.2.2 Measurement Setup

The COBRA magnet consists of different bore radii (300 mm – 410 mm) and is accessible from one end only. To be able to measure its full volume in situ, a dedicated portable field mapper was developed which incorporates the $\alpha$ and $\beta$ rotation system described in Section 5.1 and with which the full volume can be accessed [56]. The magnetic field measurement machine consists of a granite bench with a linear stepper motor that can cover a range of 2000 mm, about three quarters of the length of COBRA (z-axis). This motor drives the Hall sensor measurement arm, a more than 2 m long carbon fibre tube, into the COBRA magnet. To measure the full volume of the COBRA magnet, two additional (non-magnetic) piezomotors were installed at the front end of the carbon fibre tube, see Fig. 7.6. One piezomotor, the PHI motor, is mounted on the end of the carbon fiber tube with its rotation axis pointing along the axis of the carbon fibre tube, the z-axis of COBRA. The other piezomotor, the R motor, is connected to the PHI motor.
Figure 7.5: Schematic drawing of the COBRA magnet: front view (left) and side view (right) (picture courtesy of W. Ootani, PSI).

by an arm of length \( L_{PHI} \) that is orthogonal to the \( PHI \) motor rotation axis. The rotation axis of the \( R \) motor is parallel to the \( z \)-axis. The Hallcube resides on the \( \alpha \) and \( \beta \) piezomotor system whose \( \alpha \) motor is connected to the \( R \) motor by an arm of length \( L_R \) that is orthogonal to the \( R \) motor rotation axis. The rotation axis of the \( \alpha \) motor is parallel to the \( z \)-axis of COBRA as well. In this system, the \( R \) motor acts as a joint and defines the measurement radius inside COBRA. The \( PHI \) motor defines the azimuthal angle. The Hallcube was mounted on the \( \beta \) motor in the same support as was used during its characterization and calibration (see Fig. 5.1). The \( \alpha \) and \( \beta \) motors are redundant in the measurements with the Hallcube and are kept at fixed position during measurement. The \( \alpha \) motor was not used to correct the Hallcube position for different \( PHI \) and \( R \) motor positions and instead these positions were corrected for mathematically after measurement.

Prior to the measurement of COBRA, a survey of the system with a laser tracking system has been carried out to align the \( PHI \), \( R \) and \( \alpha \) motor axes to be parallel to the \( z \)-axis of the COBRA magnet (courtesy of the Survey Group, PSI). The rotation planes of the \( PHI \) motor and \( R \) motor were aligned orthogonal to the COBRA \( z \)-axis. During survey, the distance between the \( PHI \) motor and \( R \) motor rotation axes as well as the distance between the \( R \) motor and \( \alpha \) motor rotation axes was found to be \( L_{PHI} = 179.792 \) mm and \( L_R = 179.825 \) mm respectively.

Four planes of COBRA were measured, the \( x = 0 \) plane, \( y = 0 \) plane, \( x = y \) plane and \( x = -y \) plane which corresponds to a step size in azimuthal angle \( PHI \).
of 45°. The measurement radius $R$ was increased in steps of 15 mm up to 270 mm. The $R$ motor angle setting corresponding to radius $R$ is deduced from Fig. 7.6 as well as the initial PHI motor angle setting corresponding to Hallcube position $y = 0, x = -R$. They are respectively,

$$ R_{\text{ANG}} = \arccos \frac{L_{\text{PHI}}^2 + L_R^2 - R^2}{2L_{\text{PHI}}L_R}, \quad (7.4a) $$

$$ \text{PHI}_{\text{start}} = \arccos \frac{R^2 + L_{\text{PHI}}^2 - L_R^2}{2L_{\text{PHI}}R}. \quad (7.4b) $$

For example, at $R = 15$ mm the motor angle settings are $R_{\text{ANG}} = 4.781$ and $\text{PHI}_{\text{start}}(y = 0, x = -R) = 87.735°$. For clarity, since the PHI values are not integer numbers, the angle THETA is introduced to identify the planes, where (see Fig. 7.6)

$$ \text{THETA} = 180 + \text{PHI}_{\text{start}} - \text{PHI}. \quad (7.4c) $$
Hence, for the given example, at $R = 15\,\text{mm}$ and $PHI = PHI_{\text{start}}$, $\theta = 180^\circ$. The $R$ motor and $PHI$ motor have a movement resolution of $0.09^\circ$. A list of all set angular values and actual (encoder) values for all radii $R$ in steps of $15\,\text{mm}$ and $PHI$ values of the four planes is given in Appendix B.

Measurement along $z$ is done at a speed of $50\,\text{mm/s}$ from $z = 207.9\,\text{mm}$ to $z = 2387.9\,\text{mm}$ with $z_0 = 1297.9\,\text{mm}$ the geometrical centre in the local coordinate system of the COBRA magnet. The six individual Hall sensors in the Hallcube were supplied with a direct bias current of $0.1\,\text{mA}$ in series by a Keithley 6221 current source. The output voltages of the six sensors were measured by six $8\frac{1}{2}$-digit HP/Agilent 3458A digital voltmeters (DVMs). An integration time of $20\,\text{ms}$ was chosen to suppress $50\,\text{Hz}$ noise. Along $z$, $1001$ measurement points were taken via external transistor-transistor logic (TTL) trigger to all six voltmeters simultaneously (serial connection). This corresponds to a measurement spacing of about $2\,\text{mm}$.

The COBRA magnet could be measured only at $18\%$ of its nominal current ($48.0\,\text{A}$ on the normal conducting coils and $53.9\,\text{A}$ on the superconducting coils). The shutdown period of the Swiss Muon Source (SμS) facility at PSI, of which the MEG experiment is part, is the only time slot when the COBRA magnet is accessible. During the measurement time slot, the magnetic field of COBRA at nominal current would have been too high and disturbed the work during shutdown at the muon source facility.

### 7.2.3 Results and Analysis

The total measurement of the COBRA magnet lasted $9\frac{1}{2}$ hours plus $4\frac{1}{2}$ the next day. During both days, the temperature inside the COBRA magnet increased during the measurement from $22.4\,^\circ\text{C}$ to $23.5\,^\circ\text{C}$ (power dissipation of the normal conducting coils). The obtained measurement data were re-sampled to an equidistant spacing of $2\,\text{mm}$ by a 5-point quadratic fit interpolation. The surface normal vectors of all six constituent uniaxial Hall sensors are known and hence, with the developed calibration scheme, the three field components can be deduced at each measurement point. However, all results are obtained in the local coordinate system of the Hallcube which is meaningless because the orientation of the local coordinate system changes for different $PHI$ and $R_{\text{ANG}}$ motor positions. The resulting field components at any given $PHI$ and $R_{\text{ANG}}$ position are brought into a global coordinate system defined at $PHI = 0$ and $R_{\text{ANG}} = 0$. This is done by applying a backward rotation $-PHI$ and $-R_{\text{ANG}}$ on the measured field components $B_x$, $B_y$, and $B_z$ in the local coordinate system.

A contour plot of equal magnetic field values measured on the mid-plane (at $y = 0$) of the main magnetic field component $B_z$, and the side-component $B_x$, is shown in Fig. 7.7. Fig. 7.7a is the contour plot of the main magnetic field compon-
ent $B_z$ whose value inside the solenoid is smallest in the centre and increases with increasing radius ($x$ on the mid-plane). Fig. 7.7b is the contour plot of the side-component $B_x$ whose value is, as expected, antisymmetric in $x$ and $z$ and rises towards the entrance and exit of the magnet. The largest values occur between the superconducting coils in the centre of the magnet.

To analyse these results, let’s first consider the measurement data on the axis of the COBRA magnet, i.e. at $R = 0$. There are four measured lines along $z$ at $R = 0$, namely at the angles $THETA = 0^\circ, 90^\circ, 180^\circ, 270^\circ$. The result of four measurements of the main component $B_z$ is given in Fig. 7.8a. The extremum of the main field component and its field integral from each of the four curves, are given in the insert of the graph. The maximum differences between the four measurements are respectively 0.0121\% for the extremum and 0.0102\% for the field integral. In the absolute term this corresponds to $\pm 0.1$ Gauss at 0.19 T. The result of four measurements of the $B_x$ component and four measurements of the $B_y$ component is given in Fig. 7.8b. On the central line of the COBRA magnet, $x, y = 0$, the components $B_z$ and $B_y$ should not exist. Their field profile resembles the field profile of the main component $B_z$ which suggests that these field components are not real but that they result from an angle of the Hallcube to $B_z$. This is not surprising, since the orientation of the global coordinate system of the Hallcube (defined as the local coordinate system of the Hallcube at $PHI = 0$ and $R_{ANG} = 0$) with respect to the COBRA coordinate system is unknown, it must first be deduced. Since at the time of measurement, no accurate tool to align the Hallcube's coordinate system to COBRA’s coordinate system was available, the alignment was mimicked by a mathematical manipulation of the measurement data. Three rotation matrices causing a pitch, yaw, and roll were applied to the magnetic field components in the local coordinate system of the Hallcube. The pitch, yaw, and roll angles were optimized until the $B_x$ and $B_y$ field components were at minimum on the central line ($x, y = 0$). The result of optimized Hallcube rotation angles $yaw = 0.34^\circ, pitch = -0.33^\circ, roll = 1.68^\circ$, is shown in Fig. 7.9. The remaining non-zero values (especially on the negative $z$-axis) suggest that either the $z$-axis does not coincide with the magnetic axis of the COBRA magnet or the environment fields are such. The difference among four curves of $B_x$ and four curves of $B_y$ could be due to a change in $PHI$ plane angle with rotation, so that the $z$-axis is different among the four $THETA$-positions. Alternatively, the difference among the curves could be due to the Hallcube yielding different results in the global coordinate system depending on the orientation of its local coordinate system.

The measurement results, after alignment correction, will be given in the more logical coordinate system in which $B_z$ is the axial and main magnetic field component, $B_r$ is the radial field component, and $B_\phi$ is the azimuthal field compon-
7.2 Solenoid Magnetic Field Measurement

(a) Contour plot of equal $B_z$ component field values, in percentage of the measured extremum of 2276.0 Gauss.

(b) Contour plot of equal $B_x$ component field values, in percentage of the measured extremum of -372.6 Gauss.

Figure 7.7: Contour plots of equal magnetic field values measured on the mid-plane ($y = 0$) of the COBRA magnet.
7 Test Measurements with the Prototype Hallcube

(a) Measurements of the main field component $B_z$ on axis and at four different THETA-positions (overlapping curves). The resulting extremum of the field component and the field integral along $z$ are given in the table insert.

(b) $B_x$ and $B_y$ side-component measurement. Their non-zero values and resemblance to $B_z$ field shape, indicate the angular error of the Hallcube to $B_z$.

Figure 7.8: Measurement of the main and side-components at $R = 0$ and THETA = 0°, 90°, 180°, 270° and in the local coordinate system of the Hallcube.
7.2 Solenoid Magnetic Field Measurement

(a) Remaining side-component field values after optimization of the rotation angles of the Hallcube ($yaw = 0.34^\circ$, $pitch = -0.33^\circ$, $roll = 1.68^\circ$).

(b) Magnification of the remaining side-field component values.

Figure 7.9: Optimization of the rotation angles of the Hallcube.
7 Test Measurements with the Prototype Hallcube

(a) Axial field component $B_z$ along $z$. The different curves are measurement results at different measurement radii.

(b) Radial field component $B_r$ along $z$ and at different radii.
7.2 Solenoid Magnetic Field Measurement

ent. The corrected measurement results at THETA = 0° and at different radii are shown in Fig. 7.10. The results obtained at the other THETA positions, THETA = 45°, 90°, and 135°, are similar and are given in Appendix C. The different curves (colours) in each graph are the measurement results obtained at different radii in the COBRA magnet bore. For each radius, two curves are plotted corresponding to the two different measurement positions on the measured plane defined by the THETA angle. For example, at THETA = 0 (the horizontal mid-plane), the two measurement positions for each radius $R$ are $x = -R$ and $x = R$. In Fig. 7.10a, the result of the $B_z$ measurement is shown and in Fig. 7.10b the result of the $B_r$ measurement at different radii. Note that at the z-positions where $dB_z/dr = 0$, also $dB_r/dz = 0$ in agreement with $\nabla \times B = 0$ following from Maxwell’s equation. The azimuthal component $B_\phi$ is non-zero on the negative z-axis and has a small value of about 4 Gauss, see Fig. 7.10c. Its sign is opposite for the $x = -R$ and $x = R$ measurements. That actually means that the azimuthal component in each case is pointing in the $+y$ direction. This suggests the existence of a stray/environment field measurable at the negative z-axis of the COBRA magnet. The stray field is measurable also at THETA = 135° and, to less extent, at THETA = 90° but is negligible at THETA = 45°, see Figs. C.1c, C.2c, C.3c in Appendix C. This suggests a stray field in a direction nearly parallel to the THETA = 45° plane.

Figure 7.10: Measurement results at THETA = 0°, after alignment correction.
7 Test Measurements with the Prototype Hallcube

Figure 7.11: Schematic view of a planar insertion device with electron trajectory. $\lambda_{ID}$ and $g$ are the device period and gap respectively. The light and dark grey blocks represent the periodic structure of dipole magnets with alternating polarity.

7.3 Periodic Magnetic Field Measurement

A different type of magnetic device, used in synchrotrons and free electron lasers, are insertion devices (ID) of which there are two classes — wigglers and undulators. These devices consist of a linear sequence of dipole magnets (usually small blocks of permanent magnets) of alternating polarity and thereby, producing a periodic magnetic field variation. This causes a beam of electrons traversing the structure to ”wiggle” or “undulate” around a straight line. The principle is schematically illustrated in Fig. 7.11. By undergoing oscillations, the electrons emit synchrotron radiation in a narrow cone of natural opening angle $1/\gamma$ with $\gamma$ being the relativistic Lorentz factor [57]. The cone axis is along the instantaneous tangent to the particle trajectory. If the angular deflection of the beam is smaller or close to the natural emission angle $1/\gamma$, the emitted radiation along the trajectory overlaps and interferes, resulting in a nearly monochromatic spectrum. Insertion devices with such characteristics are referred to as undulators. Otherwise, if the angular deflection of the beam is large, interference effects can be neglected and the spectrum is quasi-continuous. In that case the insertion device is referred to as wiggler [57],[58]. The general term insertion device stems from the fact that these devices are “inserted” in straight sections of the beam track and, ideally, the beam entering the insertion device and the beam exiting the insertion device, are on the same track as if unaffected by the device.

The main magnetic field component in an insertion device is the vertical magnetic field component $B_y$ which varies periodically with $z$ position, i.e. along the length of the insertion device. Since the curl of the magnetic field must be zero, off the symmetry plane $y = 0$ a longitudinal field component $B_z$ must exist and $dB_y/dz = dB_z/dy$. Typically, the width in $x$ of the alternating polarity dipole structures is large enough that the $B_x$ field component near the axis of the insertion device can be ignored. For a planar device the magnetic field is assumed to be sinusoidal with period $\lambda_{ID}$. With that ansatz, the Laplace equation is solved
and the resulting idealized field is \[ B = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 0 \\ -B_0 \cosh \left( \frac{2\pi y}{\lambda_{ID}} \right) \sin \left( \frac{2\pi z}{\lambda_{ID}} \right) \\ -B_0 \sinh \left( \frac{2\pi y}{\lambda_{ID}} \right) \cos \left( \frac{2\pi z}{\lambda_{ID}} \right) \end{pmatrix} \] (7.5)

where \( B_0 \) is the peak field at the beam axis. \( B_0 \) decreases with gap \( g \) according to \[ B_0 = \frac{B_{pt}}{\cosh \left( \frac{\pi g}{\lambda_{ID}} \right)} \] (7.6)

where \( B_{pt} \) is the pole tip field at \( y = g/2 \). On the symmetry plane \( y = 0 \) only \( B_y \) exists and hence,

\[ B = -B_0 \sin \left( \frac{2\pi z}{\lambda_{ID}} \right) \hat{y}. \] (7.7)

The equations of motion for an electron in an insertion device can be deduced from the Lorentz force on that electron,

\[ F = m_e \gamma \frac{dv}{dt} = -e(v \times B) \] (7.8)

with \( \gamma \) the relativistic Lorentz factor. On the horizontal symmetry plane, under the assumption that \( B_x \) and the vertical motion are negligible (\( v_y = 0 \) and \( \dot{v}_y = 0 \)), this results in two coupled equations \[ \frac{d^2x}{dt^2} = \frac{e}{\gamma m_e} B_y \frac{dz}{dt}, \] (7.9a)
\[ \frac{d^2z}{dt^2} = -\frac{e}{\gamma m_e} B_y \frac{dx}{dt}. \] (7.9b)

A first order solution is obtained by considering \( v_x \ll v_z \) and \( v_z \approx v = \text{constant} \) such that \( \frac{d^2z}{dt^2} = 0 \) and thus only Eq. 7.9a is relevant. A change of variables from time to electron position then gives the first order expression of the electron motion \[ \frac{d^2x}{dz^2} = \frac{eB_y}{\gamma m_e \gamma} = \frac{eB_0}{\gamma m_e \gamma} \sin \left( \frac{2\pi z}{\lambda_{ID}} \right) \] (7.10)

which can be solved by integration. Integrating once gives the horizontal angular deflection of the electrons from the \( z \)-axis whose peak value for relativistic electrons with \( v \approx c \) is \[ \theta_{max} \approx \left| \frac{dx}{dz} \right|_{max} = \frac{eB_0 \lambda_{ID}}{2\pi \gamma m_e c} = \frac{K}{\gamma}, \] (7.11)
7 Test Measurements with the Prototype Hallcube

where

\[ K \equiv \frac{eB_0\lambda_u}{2\pi m_e c} = 0.9337B_0[T]\lambda_{ID}[cm]. \quad (7.12) \]

The introduced dimensionless quantity \( K \) is known as the wiggler or undulator parameter. It scales the maximum angular deviation of the electrons with respect to the axis, \( \theta_{\text{max}} \), to the natural opening angle of radiation emitted by relativistic electrons in a magnetic field, \( 1/\gamma \) \[57\]. For wigglers, \( K \gg 1 \) and for undulators, \( K \leq 1 \) but the condition is not so strict and \( K \)-values up to 3 can still fall in the latter category.

From the perspective of application of the Hallcube, what makes insertion devices special compared to ordinary electromagnets is the periodic variation of their magnetic field over very small distances. Periods can be as small as \(< 10 \text{ mm} \) \[60\] and the maximum field, depending on period, can be a few Tesla with the trend of reducing the insertion device gap size \[61, 62\]. The magnetic field gradient and non-linearity are very high in these extreme cases. The 4 mm \( \times \) 4 mm \( \times \) 4 mm outer dimensions of the prototype Hallcube limit its application to insertion devices with a gap size of 4.5 mm and larger. Smaller gap insertion devices are only measurable by opening their gap and mathematically scaling the results to those desired at smaller gap.

7.3.1 Laser Heater Undulator for the SwissFEL

The Swiss Free Electron Laser (SwissFEL) is a hard X-ray free electron laser that is under construction and planned to be finished in 2016 at PSI \[63\]. The SwissFEL injector facility will include a laser heater system to controllably increase the uncorrelated energy spread of the electron beam from 1 keV or less to a few keV \[64\]. This is done to reduce micro-bunching instabilities of the cold beams which cause an emittance and energy spread growth during the acceleration and compression stages beyond the tolerable level \[64, 65\]. In the laser heater system the electron beam energy spread is controllably increased by a resonant interaction of the beam with a transversally polarized laser beam inside an undulator \[64\], the SwissFEL laser heater U50 undulator.

The SwissFEL laser heater U50 undulator is shown in Fig. \[7.12\]. It consists of a planar Halbach type array of permanent magnets with eight full periods of length \( \lambda_{ID} = 50 \text{ mm} \). The gap is variable from 20 mm to 40 mm and for the smallest gap, the peak field on the mid-plane is \( B_0 \sim 0.6 \text{ T} \) \[64\].

The main field component, \( B_y \), is periodic along \( z \) and is a symmetric function in \( y \) with extrema on the mid-plane \( (y = 0) \), see Eq \[7.5\]. This magnetic field distribution is non-linear and with high gradients. The high gradients (in \( z \)) and the non-linearity (in \( y \)) affect respectively the effectiveness of the planar Hall voltage compensation and the accuracy of the voltage interpolation of the Hallcube, as
7.3 Periodic Magnetic Field Measurement

was discussed in Chapter 3.4. It is important to find out if the resulting measurement error is still below the striven value of $10^{-4}$ at the 1 T level.

Let’s consider first the planar Hall effect compensation. The field gradient is highest at the crossings of the periodic field with the z-axis. The gradient can be found by differentiation of Eq. 7.7 which, with $B_0 = 0.6$ T and $\lambda_{ID} = 50 \cdot 10^{-3}$ m, gives $75$ T/m. The largest difference in in-plane field component between two Hall sensors occurs when one Hall sensor is exactly at the position along the z-axis where $B_y = 0$. With the spatial separation between two Hall sensors of 200 µm, the other Hall sensor’s in-plane field component is then 150 Gauss. However, as was demonstrated in Chapter 5.3 the Hall sensors’ sensitivity to the planar Hall effect is about 500-600 times weaker than their sensitivity to the Hall effect which means that an in-plane field component of 150 Gauss actually results in a measurement error of only $< 0.3$ Gauss. And, since the output voltage of pairs of Hall sensors are averaged, this error is halved to $< 0.15$ Gauss.

Next, consider the error stemming from linear interpolation of the non-linear field distribution. The non-linearity in $y$ is highest around the extrema along the z-axis, see Eq. 7.5 for $B_y$. Therefore, also the error in interpolation is largest around the extrema, namely when the two Hall sensors of the pair that are sensitive to the $B_y$ field component are equidistant from the mid-plane — one Hall sensor is 100 µm above and the other is 100 µm below the mid-plane. The relative error between the linearly interpolated field and the true field value at $y = 0$ is,

$$
\frac{1}{B_y(y = 0)} \left( B_y(y = 0) - \frac{B_y(y = +0.1 \text{ mm}) + B_y(y = -0.1 \text{ mm})}{2} \right) = \\
\frac{B_0 \cosh \left( \frac{2\pi \cdot 0.1}{\lambda_{ID}} \right) - B_0}{B_0} = \cosh \left( \frac{2\pi \cdot 0.1}{\lambda_{ID}} \right) - 1. \quad (7.13)
$$

If this error is to be $\leq 10^{-4}$ the condition on the period length of the insertion device is

$$
\lambda_{ID} \geq 44.43 \text{ mm} \quad (7.14)
$$

This holds for the SwissFEL laser heater U50 undulator.

7.3.2 Measurement Setup

The measurement bench of the U50 undulator, see Fig. 7.12 was placed in a temperature controlled measurement hutch at the Swiss Light Source (SLS) Insertion Devices Group in which the temperature varied between 22.7°C and 22.8°C. The measurement bench consisted of several alignment stages and one Heidenhain motorized stage with linear encoder for motion in $z$. Positioning of the Hallcube in $x$ and $y$ was done with two manually operable Oriental motor Vexta stepper
motor stages. Turning the motor axis by two full turns translates in a linear motion of 1 mm. The estimated error in manual positioning was 10° which corresponds to 14 µm. Alignment of the Hallcube could be done with a 2-axis manual Kohzu goniometer stage. This was used to water-level the support holder of the Hallcube.

The support holder for the Hallcube is shown in Fig. 7.13 (construction design of the author’s concept by courtesy of S. Sidorov, PSI). It was made from polymethyl methacrylate (PMMA) and was designed to fit the existing measurement bench of the undulator. It contains channels on the top and bottom side to carry out the wires from the Hallcube. The base plate on which the Hallcube resides is 0.5 mm thick, making the total height of the section that travelled inside the gap of the undulator 4.5 mm. A horizontal and vertical cavity were included to place a Wyler 0.1 mm/m precise spirit level.

First, the Hallcube was aligned to the magnetic axis of the undulator which was then defined as the z-axis ($x = 0$ and $y = 0$). The horizontal alignment was done by placing the Hallcube in the geometrical horizontal centre of the undulator. A precise magnetic alignment was not necessary because the main field component $B_y$ around the centre is independent on $x$ position and $B_x$ can be neglected. The vertical alignment was done magnetically because $B_z$ and also $B_y$ are dependent on vertical position. Multiple scans in $z$ at different $y$ positions were made to find the minimum absolute field value for $B_z$. After the central line at $x = 0$ and $y = 0$ was defined, field measurement scans along this line were repeated from the minimum undulator gap of 20 mm to the maximum gap at 40 mm in steps of 2 mm gap height. And, at the minimum undulator gap of 20 mm, the field was scanned along $z$ at different $x$ and $y$ Hallcube positions: $x$ at -0.5, 0, +0.5 mm (1 turn of the stepper motor axis) and $y$ at -0.125, 0, +0.125 mm (1/4 turn of the stepper motor axis).

The measurement of the COBRA magnet, described in Section 7.2.2 above, was performed at a scanning speed of 50 mm/s and DVM integration time of 20 ms.
7.3 Periodic Magnetic Field Measurement

The Hall voltage is therefore integrated over a path length of 1 mm which is acceptable for most electromagnets with moderate gradually changing field gradients such as COBRA. For the SwissFEL laser heater U50 undulator with a period length of 50 mm in which the field gradient changes between +0.6 T and -0.6 T, integrating the Hall voltage over 1 mm path length is no longer acceptable. Maintaining a DVM integration time of 20 ms to suppress 50 Hz noise, the scanning speed must be reduced. Let’s put a stringent condition that the integrated field over 20 ms assigned to the path middle should differ no more than 0.1 Gauss from the true value at the path middle. Let $\Delta$ be the integration path length and let $s$ be the measurement position along $z$. On the undulator axis, the difference between the true field at $s$ and the integrated field assigned to $s$ is

$$B_{z=s} - B_{\text{avg}} = B_0 \sin \left( \frac{2\pi s}{\lambda_{ID}} \right) - \frac{1}{\Delta} \int_{s - \frac{\Delta}{2}}^{s + \frac{\Delta}{2}} B_0 \sin \left( \frac{2\pi z}{\lambda_{ID}} \right) dz =$$

$$B_0 \sin \left( \frac{2\pi s}{\lambda_{ID}} \right) \left[ 1 - \frac{\lambda_{ID}}{\pi \Delta} \sin \left( \frac{\pi \Delta}{\lambda_{ID}} \right) \right]. \quad (7.15)$$

The difference is maximum for $s = \lambda_{ID}/4$ and with $B_0 = 0.6$ T = 6000 Gauss and $\lambda_{ID} = 50$ mm, $\Delta$ [mm] must fulfill the condition

$$B_{z=s} - B_{\text{avg}} = 6000 \left[ 1 - \frac{50}{\pi \Delta} \sin \left( \frac{\pi \Delta}{50} \right) \right] < 0.1 \text{ Gauss} \quad (7.16)$$

Solving numerically gives $\Delta < 0.16$ mm. Hence, the maximum scanning speed along $z$ is 8 mm/s.

Measurement along $z$ was done at a low speed of 2.1 mm/s from $z = 0$ mm to $z = 560$ mm. The six individual Hall sensors in the Hallcube were supplied
with a direct bias current of 0.1 mA in series by a Keithley 6221 current source. The output voltages of the six sensors were measured by six DVMs. A DVM integration time of 20 ms was chosen to suppress 50 Hz noise. Along each line in \( z \), at every 0.112 mm a measurement point was taken by external TTL trigger to all six DVMs simultaneously (serial connection). Therefore, over the full scanning length of 560 mm, 5000 points were measured (the \( z = 0 \) position was not recorded). This is the maximum number of points that could be stored in the internal buffer of each DVM. The trigger signal was given to the DVMs as soon as the \( z \) motor had reached or passed its trigger position which was monitored by the motor’s linear encoder. The maximum jitter was 1 ms which at a driving speed of 2.1 mm/s corresponds to 2.1 µm.

### 7.3.3 Results and Analysis

Because of the high gradient fields up to 75 T/m over short length, it is important that during measurement along \( z \) the encoder position and trigger signal are reliable. The maximum jitter of 2.1 µm at 2.1 mm/s driving speed would induce an error of about ±1.5 Gauss in the main \( B_y \) field component measurement. The actual error was found to be about twice this value as demonstrated in Fig. 7.14 which shows the difference between two subsequent measurements of the \( B_y \) field component along \( z \), on the mid-plane, and at 20 cm gap height.

As in the COBRA measurements, the local coordinate system of the Hallcube is aligned to the coordinate system of the U50 laser heater undulator after the measurement is completed. This is done by applying a pitch, yaw, and roll rotation on the measured magnetic field components in the local coordinate system of the Hallcube. The pitch, yaw, and roll angles were found through optimization by minimizing the \( B_z \) and \( B_x \) side-component fields on the central line of the undulator. Fig. 7.15a shows the measurement results of all three field components along the central line prior to coordinate system correction. The existing \( B_x \) and \( B_z \) field components are in phase with the dominant \( B_y \) component and are due to an angular error of the Hallcube to \( B_y \). After optimization and correction, the resulting side-components are shown in Fig. 7.15b.

The remaining non-zero \( B_x \) side-component is no longer due to an angular error of the Hallcube to \( B_y \), since the characteristic field profile of \( B_y \) is not present. Instead, it could be caused by the imperfect, non-parallel pole surfaces of the undulator. The \( B_z \) component should be non-zero only off the mid-plane. From the result in Fig. 7.15b it can be concluded that the Hallcube is only on the mid-plane along the first half of the undulator length and deviates from the mid-plane afterwards. This is due either to misalignment of the poles of the undulator along its length, or due to misalignment of the measurement bench (\( z \) motor range) along the undulator.
7.3 Periodic Magnetic Field Measurement

From the different measurements in \( x \) at \(-0.5, 0, +0.5\) mm and \( y \) at \(-0.125, 0, +0.125\) mm at 20 mm gap height, the derivatives of the field components can be computed. Assuming that the measured field components are assigned to the same point, at the centre of the Hallcube, it should hold that \( \nabla \cdot B = 0 \). This is indeed the case as seen in the results of \( dB_x/dx, dB_y/dy, dB_z/dz \), and their sum in Fig. 7.16a. The result of \( dB_y/dz \) and \( dB_z/dy \) on the measurement data, and their subtraction is shown in Fig. 7.16b. Hence, also \( dB_y/dz = dB_z/dy \) holds true as it should which means that the measurement results of the undulator field from the Hallcube are sensible.

Unlike the measurement results in the calibration magnet with an NMR probe as a reference, absolute accuracy of the measurement results obtained in the SwissFEL laser heater U50 undulator, cannot be assessed. A comparison of results will be done by means of the K-value which has been measured with a commercial SENIS H3A03S three-axis Hall sensor (consisting of three orthogonally arranged 1D Hall sensors, see [66] for details) by the Insertion Devices Group, and it has been modeled by its manufacturer, Daresbury Laboratory, with RADIA software [64]. The measurement data with the SENIS commercial Hall sensor

Figure 7.14: Repeatability of measurement: difference between two subsequent measurements of \( B_y \) along \( z \), on the mid-plane and at 20 cm gap height.
7 Test Measurements with the Prototype Hallcube

(a) Measurement of the undulator’s main ($B_y$) and side-component fields ($B_x$ and $B_z$) along $z$, on the mid-plane, and at 20 cm gap height. The measurement results are obtained in the local coordinate system of the Hallcube.

(b) Remaining side-component field values after optimization of the rotation angles of the Hallcube ($yaw = 0.48^\circ$, $pitch = 2.04^\circ$, $roll = 0.44^\circ$).

Figure 7.15: Results before and after optimization and correction for the rotation angles of the Hallcube.
7.3 Periodic Magnetic Field Measurement

(a) Derivatives of the measured field components $dB_x/dx$, $dB_y/dy$, and $dB_z/dz$ along $z$. Their sum disappears in accordance with $\nabla \cdot \mathbf{B} = 0$.

(b) Result of $dB_y/dz$ and $dB_z/dy$ on the measurement data, and their subtraction showing that also $dB_y/dz = dB_z/dy$ holds true.

Figure 7.16: Assessing the validity of the measurement data by testing their agreement with the electromagnetic relations.
7 Test Measurements with the Prototype Hallcube

(a) $K$-values versus gap height from maximum field measurement with the Hallcube, the SENIS H3A03S commercial Hall sensor (measurement off the mid-plane, data courtesy of M. Calvi, PSI), and simulated $K$-values in RADIA (from [51]) with exponential fit.

(b) Differences among the $K$-value measurements in percentage versus gap height.

Figure 7.17: $K$-values and differences in $K$-values of two different Hall sensor measurements and RADIA simulated data.
were obtained without careful alignment of the sensor the mid-plane. Hence, the results are obtained slightly off the mid-plane in presence of a $B_z$ component.

The $K$-value is obtained at different gap heights from measurement of the $B_y$ component. It is computed from Eq. 7.12 using for $B_0$ the maximum measured $B_y$ along $z$, and $\lambda = 50$ mm. The resulting $K$-values from the measurement results of the Hallcube, the SENIS H3A03S, and from the RADIA simulation are shown in Fig. 7.17a for different gap heights. The difference among the $K$-values measurements is given in percentage in Fig. 7.17b. Differences between the measured $K$-values are up to about a percent, with a difference in maximum measured $B_y$ of up to about 25 Gauss.
8 Conclusion

Now this is not the end. It is not even the beginning of the end. But it is, perhaps, the end of the beginning.

- Sir Winston Churchill

Hall sensors have come a long way from the thin gold foil in Edwin Hall’s experiment. They have reached a state-of-the-art where accuracies of $10^{-4}$ at the 1 T level are consistently achieved and limited mainly by the necessary electronics to the sensor (current source, voltmeter). While this is true for uniaxial horizontal Hall sensors, to the knowledge of the author no 3D Hall sensor, which can measure the full field vector, is as accurate as these 1D Hall sensors.

The work reported on in this thesis covered the invention of a new type of three-axis Hall sensor: its design, fabrication, characterization, calibration, field reconstruction, and first test measurements. The aim of the novel design for a three-axis Hall sensor was to contribute to the improvements in Hall sensor technology and push the accuracy of three-axis magnetic field measurements with Hall sensors to the next level — by trying something new based on something known and good: the uniaxial Hall sensor in a plane. Has the aim been achieved? Yes and no.

The single prototype has confirmed the validity of the design concept and has a high potential to achieve and surpass the goals it purposed to fulfil. Passivation likely has stopped the prototype from reaching its full potential. Stability of the Hall voltage was enhanced 10 times from an unpassivated to a passivated sensor. It is to be expected that the same order of magnitude in accuracy improvement is due for a 3D Hall sensor built from passivated 1D Hall sensors. To prove this, time was unfortunately lacking, but time will tell.

Characterization, calibration, and magnetic field reconstruction have been successful: the planar Hall effect compensation by the pairs was so successful, despite in-plane rotation angle errors of up to $1.2^\circ$, that no further correction methods in the reconstruction mathematics needed to be implemented. From the very beginning it was clear that angular error tolerances of $0.006^\circ$ were as good as impossible to achieve. Yet, despite the angular errors exceeding this limit, they could be taken into account and corrected for. For the prototype it could not be estimated how large the remaining measurement error due to angular errors among the sensors was, because the measurements were masked by the dominating long-term instability of the Hall voltages.
Apart from passivation, one of the first next steps in the further development of the 3D Hall sensor should be packaging and yield improvement. The disappointing $\sim 50\%$ yield, mainly due to detachment of soldered wires, should be increased by the proposed relaxing of the tight contact pad, wire diameter, and channel width dimensions. This should be easy to implement without modifying the outer or inner cube dimensions. The wires should be bundled and have a common exit from e.g. a moulded package.

A useful add-on to the current Hall sensor’s design would be an incorporated temperature sensor. The temperature sensitivity of the 3D Hall sensor was $\sim 0.03\%K^{-1}$ around room temperature. Measurements have been performed in a temperature-controlled environment where a temperature sensor, incorporated in the sensor and its calibration, was in principle not essential. For other measurement environments, temperature conditions may be less ideal and a temperature sensor will prove necessary. The hollow cubical volume renders ideally for that! A temperature sensor placed in the hollow volume would measure the temperature exactly in the right spot: inside the active volume of the 3D Hall sensor. However, in its current design, the active volume is an enclosed volume leaving no gap to carry out wires of a potential temperature sensor. Without modification to the inner active volume, the temperature sensor can only be placed outside the active volume, for example in a cavity in the MACOR underneath the backside of one or more 1D Hall sensors.

As desired, electronics surrounding the sensor was kept simple. Yet, it was monstrous with seven devices: six voltmeters and a current source. It is possible to half the number of voltmeters by reading Hall voltages of pairs of Hall sensors instead of connecting each Hall sensor to its own voltmeter. However, by doing so, the knowledge about the relative angles among all six sensors would partially become useless. Such an approach is in contradiction with striving for highest accuracy. The number of voltmeters could also be reduced by means of a switch board. This approach, in turn, abandons one of the highlights of the developed sensor: the measurement of all three field components at the same point, at the same time. In this light, one of the next goals should be the development of electronics accompanying the 3D Hall sensor — forming a complete measurement unit. The six voltmeters, and preferably also the current source, could all be merged in a single electronic device with six ADC’s.

An unresolved, and unaddressed issue, is the sensor alignment before measurement. Without alignment, the measured field components are correct but they are given in the local coordinate system of the Hall sensor. The pre-knowledge of the magnet’s magnetic field shape, makes possible a post-measurement translation from local to global coordinate system, as was done in the test measurements with the prototype 3D Hall sensor. Even if the obtained results are correct, this
should not be the method of choice. The alignment system used at magnet measurement labs in research and accelerator facilities, is typically dedicated to alignment of a 1D Hall sensor—alignment in one plane (mainly the horizontal) only. This is insufficient for alignment of the prototype 3D Hall sensor presented in this thesis, as well as for any other 3D Hall sensor. Aligning only one measurement axis, leaves the other two measurement axes undefined. The development of a practical and precise alignment mechanism is an unattended task but should not be neglected within high-accuracy magnet measurement labs or 3D Hall sensor manufacturers.

In the author’s opinion, based on the results of the 3D Hall sensor prototype, there is no doubt that sensors of this new type have the ability for very accurate determination of the magnetic field vector. All the possible applications for these sensors are hard to imagine at this moment but are surely not confined solely to measurements of magnets in the research institutes and accelerator facilities. As is so often the case, the product itself will define its usage. Along this thought it must be noted that it would be a pity if the 3D Hall sensor prototype remains just that: a prototype and the object of a PhD thesis. The sensor should be “out there”: to be improved, further developed, and most of all to be used by whoever requires high-accuracy full magnetic field vector knowledge.
## Appendices

### A Process Runsheets

<table>
<thead>
<tr>
<th>Process step</th>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cleaning</strong></td>
<td>Acetone, ultrasonic bath</td>
<td>1 @ 25 @ 10 [min @ °C @ % power]</td>
</tr>
<tr>
<td>IPA</td>
<td>1</td>
<td>[min]</td>
</tr>
<tr>
<td>H₂O rinse</td>
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<td>[min]</td>
</tr>
<tr>
<td>N₂ blow-dry</td>
<td></td>
<td></td>
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<tr>
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<td>AZ5214 resist</td>
<td>4200/280/50 [rpm/rpms⁻¹/s]</td>
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<td>[s @ W]</td>
</tr>
<tr>
<td>AZ326MIF development</td>
<td>30</td>
<td>[s]</td>
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<tr>
<td>H₂O rinse</td>
<td>40</td>
<td>[s]</td>
</tr>
<tr>
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<td>H₂O:H₂SO₄:H₂O₂ 100:3:3</td>
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<td>[s]</td>
</tr>
<tr>
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<td>[s]</td>
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<td>[s @ °C]</td>
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Table A.1: Chip-level fabrication process for HPF1 Hall sensors.
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<td>[s]</td>
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Table A.2: Chip-level fabrication process for HPF2, HPF3, and HPF5 Hall sensors.
### B COBRA Measurement Scheme

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Table B.1: Nominal values of the COBRA field mapper $R_{\text{ANG}}$ and $PHI$ angles and deviations from the set values.
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\textbf{C  COBRA Measurement Results at THETA = 45^\circ, 90^\circ, and 135^\circ}
\end{center}

(a) Axial field component $B_z$ along $z$. The different curves are measurement results at different measurement radii.

(b) Radial field component $B_r$ along $z$ and at different radii.
Figure C.1: Measurement results at $THETA = 45^\circ$, after alignment correction.

(c) Azimuthal field component $B_\phi$ along $z$ and at different radii.

(a) Axial field component $B_z$ along $z$. The different curves are measurement results at different measurement radii.
(b) Radial field component $B_r$ along $z$ and at different radii.

(c) Azimuthal field component $B_\phi$ along $z$ and at different radii.

Figure C.2: Measurement results at $\theta = 90^\circ$, after alignment correction.
(a) Axial field component $B_z$ along $z$. The different curves are measurement results at different measurement radii.

(b) Radial field component $B_r$ along $z$ and at different radii.
Figure C.3: Measurement results at $\text{THETA} = 135^\circ$, after alignment correction.

### D Positions of Rotation Stages for Pairwise Hall Sensor Calibration

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Table D.1: Positions of $\alpha$ and $\beta$ rotation stages at maximum Hall voltage.
Bibliography


Acknowledgement

First of all, I would like to thank Professor Christofer Hierold for the opportunity to conduct my PhD research under his supervision. Although I have not been a "full" member of the Micro and Nanosystems Group, since my office and main work space was at the Magnet Section of the Paul Scherrer Institute, I have experienced the research environment of his group, which is a great one! I thank Professor Hierold for his support and the freedom he gave me to conduct the research independently and in my own way. I appreciated very much his constructive criticism and dedicated and precise questions during my Friday Talk sessions in his group. I thank the co-referees of my PhD thesis, Professor Jens Gobrecht and Dr. Stéphane Sanfilippo. Professor Gobrecht has connected me to various people that have helped me during my research. I was given access to his laboratory facilities and could count on the help of several of his current and former team members. I thank Dr. Stéphane Sanfilippo, my boss at PSI during my PhD, for his help and support during my PhD. Also he has let me conduct my research with freedom which I appreciated a lot. I could always count on his support if I needed help from the Magnet Section team members for technical design drawings or machining of pieces. And, he has always reserved a measurement time slot for me, even in the busiest of magnet measurement periods.

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Thanks to the best mama for checking the German abstract on spelling. Which brings me to the last but not least thank you: to my family and friends in the north of The Netherlands and Germany, who saw less and less of me as the PhD research entered its final phase. Now, we shall catch up on that!

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Publications

Reviewed Articles
A1 V. Vranković, C. Wouters, R. Thermer, M. Zsiga, P. Chevtsov, W. Hugentobler, T. Höwler, R. Sawada, K. Leki, T. Iwamoto, and W. Ootani, "In situ magnetic field measurement of the COBRA solenoid detector magnet with the prototype of a newly developed 3D Hall sensor – Hallcube.", Submitted for publication.


Pending Patent Applications

Talks

T3 C. Wouters, “A brand-new Hall sensor for 3D magnetic field measurements”, *GFA and SwissFEL Accelerator Seminar*, June 22, PSI Villigen, Switzerland, 2014.

**Poster Presentations**


# Curriculum vitae

## Personal Details

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<th>Silke Christina Wouters</th>
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<tbody>
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<td>Citizenship</td>
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## Education

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<tr>
<td>09/2006 – 03/2010</td>
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## Work Experience

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<tr>
<td>03/2011 – 09/2011</td>
<td>Paul Scherrer Institute, Villigen, Switzerland</td>
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## Language

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