Doctoral Thesis

Development of a novel low-energy, high-brightness μbeam line

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Development of a novel low-energy, high-brightness $\mu^+$ beam line

A thesis submitted to attain the degree of
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(Dr. sc. ETH Zurich)

presented by

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Abstract

Development of a novel low-energy, high-brightness $\mu^+$ beam line

by Andreas EGGENBERGER

Low-energy muon ($\mu^+$) beams possess a relatively poor phase space quality. Improving the beam brightness is of great benefit for various precision experiments in the field of particle physics, atomic physics, and for solid state investigations using $\mu$SR techniques. This thesis reports on the ongoing developments at ETH Zurich and PSI for realizing a new $\mu^+$ beam cooling scheme. With this scheme, the phase space of a standard surface muon beam will be reduced by $O(10^{10})$, with an efficiency above $O(10^{-3})$, which results in an increase of brightness of seven orders of magnitude.

The principle of this scheme is to stop and manipulate muons in cryogenic helium gas. The muon swarm, which stops in a sizable volume, is successively compressed into a small, "point-like" volume by making use of E- and B-fields and gas density gradients. Then, the compressed muon swarm is extracted into vacuum through a small orifice, and delivered to the experiments.

Phase space compression is achieved in three consecutive stages: Transverse compression (perpendicular to the incoming beam axis), longitudinal compression (along the beam axis), and extraction into vacuum. In an initial phase of the development, these stages can be tested separately. This thesis reports on the separate tests of the longitudinal and transverse compression stages.

The construction of the transverse compression target has been achieved meeting several challenging requirements. The target is at cryogenic temperatures with a stable vertical gas density gradient. Inside the gas, high electric fields compress the stopped muon swarm in vertical direction. Efficient transverse compression was observed in 2015, and data analysis is ongoing.

In 2014 and 2015, longitudinal compression was measured to occur with a high efficiency of $(90 \pm 13)\%$ within $2.5\,\mu$s. Additionally, in 2015, we also demonstrated that the muons can be moved from stage to stage. The measured data agree well with preliminary simulations, demonstrating that no unknown effect hinders the realization of the complete beam line.
Zusammenfassung

Development of a novel low-energy, high-brightness $\mu^+$ beam line

von Andreas EGGENBERGER


In 2014 und 2015 wurde gemessen, dass die longitudinale Kompression mit einer grossen Effizienz von $(90 \pm 13)\%$ innerhalb von 2.5 $\mu$s erreicht wird. Zusätzlich konnten wir 2015 zeigen, dass die Myonen von einer Stufe zur nächsten gebracht werden können. Die gemessenen Daten stimmen gut mit vorläufigen Simulationen überein, was beweist, dass keine unbekannten Effekte der Realisierung der kompletten Strahllinie im Wege stehen.
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This thesis would not have been possible without the support of many people, whom I would like to thank herewith.

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1 Summary

Since its discovery 80 years ago, the muon has been playing a unique role in particle physics. Its application in physics ranges from large-scale tomography to sensitive probes of sub-µm thin films, and from precision tests of the fundamental theory of quantum electrodynamics to the most precise determination of physical constants. Furthermore, muons may serve as a window to study the behavior of antimatter and they hint towards physics beyond the standard model [1–6]. Common to most experiments is the requirement for a high-brightness, low-energy muon beam.

Conventional surface muon beams are produced by pions decaying at rest on the surface of a production target. The emerging muons have an energy close to 4.1 MeV but a relatively poor phase space quality. In order to improve the phase space, a fast cooling scheme is needed because of the short lifetime of the muon: $\tau_\mu = 2.2 \text{ µs}$. Well-established conventional cooling schemes used for stable particles such as electron or stochastic cooling cannot be applied in this case. A newly proposed scheme to efficiently cool the positive muon ($\mu^+$) beam is currently under investigation [7], which decreases the phase space of the initial $\mu^+$ swarm by a factor $\mathcal{O}(10^{10})$ with an efficiency of $\mathcal{O}(10^{-3})$. In this thesis, the ongoing efforts at the Eidgenössische Technische Hochschule Zürich (ETH Zurich) and the Paul Scherrer Institut (PSI) in Villigen towards the realization of the phase space compression scheme are described.

The basic idea of the phase space compression is to stop a regular surface $\mu^+$ beam in a cryogenic gas target, where high electric and magnetic fields are applied such that the stopped $\mu^+$ swarm is successively compressed: first in transverse direction (perpendicular to the incoming beam axis), then in longitudinal direction (along the beam axis), and finally, the compressed swarm is extracted from a 1 mm diameter orifice into vacuum.

This novel beam line, as shown in Fig. 1.1, consists of several stages: a transverse stage (where compression perpendicular to the incoming beam axis occurs), a longitudinal stage (where compression along the initial beam axis occurs), and an extraction stage where the $\mu^+$ is extracted from the He gas into vacuum. In a first phase of the development, these three stages can be tested separately.

Longitudinal compression was tested and improved in three different beam times. Feasibility of the longitudinal compression was demonstrated and a high
compression efficiency of \((90 \pm 13)\%\) after 2.5 \(\mu\)s in 8.5 mbar He gas was extracted. Note that this efficiency does not account for the muon decay. The measured time spectra, which have been used to expose the \(\mu^+\) swarm compression in the gas, agree well with simulations based on Geant4 [8], extended to low energy (eV regime). In the beam time in 2015, an additional, vertical component was added to the electric field required for the compression. Together with the magnetic field, this leads to a horizontal drift of the \(\mu^+\) in \(+x\)-direction, necessary to move the \(\mu^+\) from stage to stage. This drift has been successfully demonstrated.

We also developed the challenging cryogenic setup needed for the transverse compression. The cryogenic target has a gradient in temperature from 6.1 K to 18.6 K and sustains a large electric field of 1.4 kV/cm in 5 mbar He gas.

During a beam time in 2015, we succeeded to steer the \(\mu^+\) stopped in the cryogenic gas in various directions by adjusting the strength of the electric fields and the gas density gradient. The general behavior observed confirms the simulations. We have clear indications that we see a very efficient \(\mu^+\) drift accompanied by \(\mu^+\) swarm compression. Further data analysis is still ongoing.

Additionally, we also investigated the existence of the density gradient required for transverse compression in 2013 [9]. This was achieved by means of a neutron radiography experiment on the strongly neutron absorbing \(^3\)He isotope. The measured gradient is in good agreement with theoretical predictions, showing no sign of convection that could hinder the compression.
Chapter 1. Summary

A significant part of the work for this thesis was spent for constructing the transverse compression stage, which faces challenging requirements: High electric and magnetic fields have to be established in a few mbar He gas at cryogenic temperatures, with various geometrical and mechanical constraints.

We are currently analyzing the data of the transverse compression stage, to be compared with Geant4 simulations extended to low energy [10].

The present thesis is organized as follows:

Chapter 2 starts by providing some basic background information about the muon, with focus on the production of muon beams. At the end, some examples of applications with muons and muonium are presented.

Chapter 3 introduces the working principle of the proposed phase space compression scheme. It starts by introducing the relevant physics required to understand how the compression scheme works. This is followed by the description of the compression scheme, which is divided into three stages: transverse compression, longitudinal compression and extraction into vacuum. The chapter concludes with some remarks about muon loss mechanisms that could reduce the efficiency of the compression, a description of the so-called "runaway effect" which is a necessary prerequisite for efficient compression, and the derivation of the achievable phase space compression factor.

Chapter 4 is dedicated to the longitudinal compression stage. Starting with a brief review of a preliminary test in 2011, the principle to detect the $\mu^+$ swarm compression is described. An overview of the target construction for the 2014 experiment is given, followed by the analysis and discussion of the measured data. Based on the results of the 2014 beam time, we improved the target in the following year, and measured the longitudinal compression again in 2015. The analysis of these measurements conclude Chapter 4.

In the last Chapter 5, the preliminary results for the transverse compression are presented. We could show that the $\mu^+$ swarm stopped in the cryogenic gas target can indeed be compressed in transverse direction, however, no quantification of the efficiency can be made yet. This is followed by a more detailed description of the transverse compression target, with focus on various important technicalities that needed to be considered in order to construct the target.

Chapter 6 concludes this thesis by summarizing the main achievements, and by giving an outlook for the next steps towards realization of the full phase space compression beam line.
2 Muons and muon beams

2.1 Muons in the picture of particle physics

If one were to introduce the muon in one sentence, its properties could be summarized as written in Ref. [3], p. 1:

Muons are unstable elementary particles of two charge types (positive $\mu^+$ and negative $\mu^-$) having a spin of 1/2, an unusual mass intermediate between the proton mass and the electron mass ($1/9m_p$, 207$m_e$) and 2.2 $\mu$s lifetime.

The muon was first discovered in 1936 by Anderson and Neddermeyer as part of cosmic rays [11, 12], however, 80 years later, it still represents a highly active field of research. In the context of modern particle physics, the muon is a 2\textsuperscript{nd} generation lepton (the $e^\pm$ and the $\tau^\pm$ represent the first and third generation, respectively). It is a point-like particle as far as we know today, with its stringent limits on the size of the muon coming from the measurement of the muon $(g - 2)$ [5], or from the upper limit upon flavor non-conserving decays [13]. Its lifetime of 2.2 $\mu$s makes the muon a convenient probe for many studies.

Muons are produced in the upper atmosphere by cosmic ray interactions and successive pion decays, but they can also be produced at particle accelerator facilities. While the muons produced in the atmosphere have high energies exceeding $\mathcal{O}$(TeV) but a low flux, the latter have lower energies but high intensities.

At accelerators, charged pions are typically produced via the reactions

\[
p + p \rightarrow p + n + \pi^+ \qquad p + p \rightarrow p + p + \pi^- \qquad p + n \rightarrow p + p + \pi^- \qquad p + n \rightarrow n + n + \pi^+.
\]

The $\pi^\pm$ decay with a lifetime of $\tau_\pi = 26$ ns almost 100% into $\mu^\pm$ through the weak interaction [14]:

\[
\pi^+ \rightarrow \mu^+ + \nu_\mu \\
\pi^- \rightarrow \mu^- + \bar{\nu}_\mu.
\]
Chapter 2. Muons and muon beams

Figure 2.1: Pion ($\pi^+$) decay into a muon ($\mu^+$) and a neutrino ($\nu_\mu$). Because $\nu_\mu$ are left-handed, conservation of momentum and angular momentum implies that the $\mu^+$ helicity is -1 in the pion rest frame.

Because this is a two-body decay, and due to the parity-violation of the weak interaction allowing only negative helicity neutrinos, the $\mu^+$ ($\mu^-$) spin is aligned anti-parallel (parallel) to its momentum in the pion rest frame, as shown in Fig. 2.1.

2.2 Muon decay

Muons almost exclusively decay through the decay channel

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu,$$

which is a three-body decay.\(^1\) Therefore, the decay positron is not mono-energetic. Its energy spectrum peaks at $E_{e}^{\text{max}} \approx 52.8 \text{ MeV} (= m_\mu c^2 / 2)$. The spectrum is known as the Michel spectrum, and the parameters describing it give insight into the underlying physics (see [15, 16] for more details).

Parity violation in the weak decay causes neutrinos to have left-handed helicity, i.e., the spin is anti-parallel to the momentum, and anti-neutrinos to have right-handed helicity. This implies that the direction of the decay positron is asymmetrically distributed with respect to the muon spin. The angular distribution of the decay positron $e^+$ with energy $E_e$ is given in Ref. [3]:

$$N_e(E_e) \propto 1 - A(E_e) \cos \theta_e$$

(2.1)

$$A(E_e) = \frac{E_{e}^{\text{max}} - 2E_e}{3E_{e}^{\text{max}} - 2E_e},$$

(2.2)

where $\theta_e$ is the angle between the spin of the $\mu^+$ and the positron momentum. The angular distribution of the emitted positron for various positron energies is sketched in Fig. 2.2.

\(^1\)To facilitate the discussion, we focus here on the positive muon $\mu^+$, however, the discussion is analogously for $\mu^-$ with the opposite sign in Eq. (2.2).
2.3. Muon production at accelerator facilities

The \( \mu^+ \) produced at meson facilities can be divided in surface, cloud and decay channel muons [3]. Surface muons are produced by pions stopping close to the surface in a production target. These \( \mu^+ \) are (almost) mono-energetic with a momentum of 29.8 MeV/c (4.1 MeV energy) and 100% longitudinally polarized [18]. The number of negative surface muons is strongly reduced compared to the positive muons because the \( \pi^- \) stopping at the surface of the production target undergo nuclear capture. On the other hand, cloud muons originate from pions leaving the production target and decaying in flight. They typically have a much lower polarization, which depends on the their energy, but both charge states (\( \mu^\pm \)) can be selected. Lastly, a decay channel muon beam is obtained by transporting the pions produced in a production target into a decay channel where they decay. Backward or forward decay muons can be selected and transported to the experiments. Due to this configuration, the muon beam is typically highly polarized and is “free” of electron contamination. A comprehensive discussion can be found in [3].

At the Paul Scherrer Institute in Villigen, Switzerland, the pions are produced from a high-intensity proton beam (2.2-2.4 mA proton current) which hits the pion production target. It is a rotating graphite target where roughly \( 10^{16} \) protons/s are

![Figure 2.2: Angular distribution of the emitted positron from the \( \mu^+ \) decay with respect to the \( \mu^+ \) spin direction. The angular distribution is given for various positron energies. Reproduced from [17].](image)
impering. With an efficiency of $O(10^{-6})$, surface $\mu^+$ are produced, collected and guided along various beam lines. The emerging muons are collected and guided along the beam lines by means of bending and focusing magnets towards the experimental sites. Figure 2.3 presents an overview of the beam lines at PSI. Our experiment was conducted at the $\pi E1$ beam line where muon momenta between 10 and 500 MeV/c can be selected [21]. If even lower energies are required, as for example for muon spin rotation/relaxation studies, see Sec. 2.4.1, the $\mu^+$ have to be moderated.

---

2Depending on the direction of the extraction, the flux of surface $\mu^+$ varies. Ongoing studies on the optimization of the target geometry are presented in [20].
2.3. Muon production at accelerator facilities

2.3.1 Low-energy muons

Muons with momenta lower than 29.8 MeV/c arise from muons emitted at some depth from the production target surface. However, the obtainable muon rates are decreasing drastically with decreasing momentum, as illustrated in Fig. 2.4. The figure shows the rates of $\mu^+$ and $\mu^-$, respectively, which can be obtained at the $\pi E1$ beam line at PSI per mA proton current. To obtain $\mu^+$ with even lower energy at reasonable muon rates, a moderation scheme is required. In the following, three methods are presented which can be used to moderate the $\mu^+$ down to energies in the keV regime.

Muon cooling via Mu formation and ionization

One possible method to obtain low-energy muons is to first slow down a surface $\mu^+$ beam in a hot tungsten foil, or Si powder, or an aerogel. These materials act first as moderators and then as converters of $\mu^+$ into muonium (Mu=$\mu^+e^-$) atoms. Muonium is the bound state of an electron and the positive muon, analogously to the hydrogen atom. A fraction of these $\mu^+$ is thus emitted from the converter surface as Mu atom with energies in the eV regime. Afterwards, the ionization of muonium
Figure 2.5: $\mu^+$ cooling by means of Mu formation and subsequent ionization is achieved by moderating an incoming $\mu^+$ beam in a suited material (hot tungsten foil, Si powder or aerogel), where also Mu formation occurs. The Mu diffuse out of the moderator with eV energies and are ionized by laser irradiation.

atoms is induced by synchronous irradiation of the Mu with two lasers [22], which is sketched in Fig. 2.5. A drawback of this method is that the polarization of the produced $\mu^+$ is only 50%.

This scheme is exploited to generate a slow muon beam line at the J-PARC facility in Japan [23]. The slow muons are intended to be used for a muon $(g - 2)$ experiment, which might give insights into the $\sim 3\sigma$ discrepancy between Standard Model predictions and the measurement of the muon anomalous magnetic at Brookhaven National Laboratories (BNL) in the USA [5]. The thermal muonium is emitted into vacuum from silica aerogel, onto which a muon beam of 23 MeV/c momentum is impinging. About half the $\mu^+$ are stopped within the 300 $\mu$m aerogel target. The reported vacuum yield from silica aerogel is about 3%. In order to achieve the design goal of an overall cold muon production efficiency of 1%, the laser ionization efficiency needs to be several 10%, which is currently being studied [24, 25].

Frictional cooling

The energy evolution of the stopping power of $\mu^+$ in matter has a positive slope for $\mu^+$ kinetic energies below $\sim 10$ keV, as sketched on the top left in Fig. 2.6. This can be exploited to cool $\mu^+$ by sending them sequentially through matter and afterwards re-accelerating them by means of an electric field [26]. Because of the positive slope of the $dE_{\text{kin}}/dx$ curve, the energy loss in the foil for particles with higher energy is larger than the energy loss for particles with lower energy. Therefore, the longitudinal energy spread (along the accelerating field) converges to an equilibrium energy, and thus, phase space compression in longitudinal direction occurs, as illustrated in Fig. 2.6 (Top Right).
2.3. Muon production at accelerator facilities

Figure 2.6: (Top Left) The energy loss in matter has a positive slope for kinetic energies below 10 keV, i.e., the energy loss in this range is larger for $\mu^+$ with higher energies than for $\mu^+$ with lower energies. (Top Right) Frictional cooling in longitudinal direction with an electric field between the foils. The initial energy spread $\delta E_{\text{kin}}^{\text{initial}}$ is reduced to $\delta E_{\text{kin}}^{\text{final}}$. The $\mu^+$ kinetic energy loss in the foils is partly compensated by the acceleration between the foils due to the electric field. (Bottom) Frictional cooling in transverse direction, showing a muon trajectory with decreasing transverse energy (smaller spiraling "radius" in the B-field).

Phase space compression for muons has been achieved by applying a well-defined electric field between a stack of foils along a magnetic field [27]. The principle of the experiment is similar to Fig. 2.6 (Top Right and Bottom). In each foil of thickness $\Delta x_{\text{foil}}$, the muons lose an average energy $\Delta E_{\text{kin}} = (dE_{\text{kin}}/dx)\Delta x_{\text{foil}}$ due to frictional forces. The energy loss along the E-field direction can be partially compensated from the electric field between the foils: $\Delta E_{\text{kin}} = qE\Delta x_{\text{acc}}$, thus the longitudinal phase space is reduced. Additionally, since the frictional forces act against the momentum of the particle, and whilst the acceleration is along just one direction, the transverse phase space could also be reduced. This is schematically sketched in Fig. 2.6 (Bottom) by the muon trajectory with a decreasing spiraling trajectory.
Chapter 2. Muons and muon beams

The high-intensity, low-energy muon beam line at PSI

At PSI, there exists a dedicated low-energy $\mu^+$ beam line (LE-$\mu^+$, LEM) with tunable energies between 0.5 and 30 keV, especially designed and optimized for materials science [28, 29]. The working principle of the LEM beam line is to moderate a surface $\mu^+$ beam in a 125 $\mu$m thick Ag foil, which is at a few kV. The backside of this foil is coated with a solid rare gas film of 100 nm thickness. About $10^{-4}$ of the incoming $\mu^+$ exit the downstream side of the moderator with an energy of a few eV. The reason for this is a rather large threshold for energy loss in these materials, and once the muon energy is below this threshold, they can diffuse out of these materials almost unhindered [30]. The $\mu^+$ leaving the moderator at eV energies are accelerated to energies of a few keV. The overall efficiency for moderating and collimating the $\mu^+$ to a size of $3 \times 3$ cm$^2$ is $10^{-4}$ to $10^{-5}$, as summarized in Table 2.1.

Table 2.1: Selected properties of the $\mu$E4 LEM beam line at PSI as given in [28]. The muon rates are given for 28 MeV/c muon momentum, a primary proton beam of 1.8 mA, incident on the 4 cm thick production target "E", and the rates are measured on a $3 \times 3$ cm$^2$ area.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta p/p$ (FWHM)</td>
<td>4.5% - 7.5%</td>
</tr>
<tr>
<td>$\mu^+$ intensity</td>
<td>$1.8 \times 10^8$/s</td>
</tr>
<tr>
<td>Low-energy $\mu^+$ rate at sample</td>
<td>7000/s</td>
</tr>
<tr>
<td>$x$-$y$ beam spot (FWHM) at sample</td>
<td>$4.0 \times 2.5$ cm$^2$</td>
</tr>
<tr>
<td>$x$-$y$ beam divergence (FWHM) at sample</td>
<td>$150 \times 450$ mr$^2$</td>
</tr>
</tbody>
</table>

2.4 Research with muons and muonium

In the following, one application and two experiments that could benefit greatly from our beam line are briefly presented. However, this represents only a small fraction of possible experiments that could profit from a high-brightness $\mu^+$ beam. Examples of other experiments are measurements of the muon anomalous magnetic moment $a_\mu = (g - 2)/2$, and the search for a muon electric dipole moment [23, 31]. A review of precision muon physics has recently been published [6].

2.4.1 $\mu$SR

One of the applications that could directly benefit from an improved beam line is the well-established technique of $\mu$SR, which stands for muon spin rotation, or muon
2.4. Research with muons and muonium

Spin relaxation, or muon spin resonance, depending on the application. It is a technique commonly used to probe mainly magnetic properties of matter. Extensive literature on this topic exists for example in [2, 32–34], and only the very simplest case of muon spin rotation is mentioned briefly in the following. Polarized $\mu^+$ are implanted into a material. The muon spin will perform a precession around the local magnetic field, which consists of an internal field $B_{\text{int}}$ and an applied external field $B_{\text{ext}}$: $B_{\text{local}} = B_{\text{int}} + B_{\text{ext}}$. The precession of the muon spin is measured by detecting the direction of the emitted positrons from muon decays. In general, one counts the number of detected positrons $N(t)$ at a fixed position. The time spectrum follows the decrease given by the $\mu^+$ decay ($e^{-t/\tau}$) modulated at a precession frequency $\omega_{\mu}$ as

$$N(t) = N_0 e^{-t/\tau} (1 + A_{\mu} \cos(\omega_{\mu} t + \phi)),$$

where $N_0$ is a normalization constant (usually a fit parameter), $\tau$ the muon lifetime, $A_{\mu}$ the muon decay asymmetry, $\omega_{\mu} = \gamma_{\mu} B_{\text{local}}$ (here, $\gamma_{\mu} \approx 859$ MHz/T is the gyromagnetic ratio) is the spin precession frequency in radians, and $\phi$ is a phase related to the positron detector position.

A typical $\mu$SR time spectrum is shown in Fig. 2.7, where the exponential muon decay spectrum $N_0 e^{-t/\tau}$ is modulated by $A_{\mu} \cos(\omega_{\mu} t + \phi)$ due to the local magnetic field $B_{\text{local}}$. The data presented in this figure was simulated with $B_{\text{ext}} = 100$ G.

For bulk material investigations, one usually choses the highest possible $\mu^+$ flux.
and disregards the energy of the $\mu^+$ as long as the material is homogeneous, since the exact stopping distribution of the $\mu^+$ is of no importance. Low-energy $\mu^+$ have to be used to investigate thin films of nm scales [2]. The implantation depth of the $\mu^+$ can be chosen by varying the $\mu^+$ kinetic energy. For example, the range of a 15 keV $\mu^+$ in Cu is about 70 nm.

Our novel beam line will be able to deliver mono-energetic $\mu^+$ with energies in the keV regime as needed for the thin film studies. Different to existing beam lines [28], our beam line will deliver $\mu^+$ with a beam size of $O(100)\ \mu m$ diameter, thus opening the way for investigation of a large number of materials which can be produced in a very small quantity only.

Additionally, with an estimated efficiency of the phase space compression of order $10^{-3}$ and a very narrow energy resolution, it would increase the intensity of $\mu^+$ impinging on a $\mu$SR target by an order of magnitude.

2.4.2 Measuring antimatter gravity with muonium

A high brightness $\mu^+$ beam would also allow for obtaining a high-quality muonium beam. The $\mu^+$ from our novel beam line at very low energy and small transverse size could be used to produce a Mu beam of high quality for next generation experiments based on Mu.

One possibility is to produce Mu by stopping a low-energy $\mu^+$ beam in superfluid He (SFHe). The Mu formation rate in SFHe has been demonstrated to be $O(90)\%$ [35, 36]. After their formation, the Mu diffuse to the surface where emission from the SFHe occurs because Mu has a negative affinity (positive chemical potential) in SFHe. Emission of Mu from SFHe has not yet been verified but there are predictions for H, D, T which should also hold true for Mu after appropriate scaling. According to these predictions, the muon should be emitted from the surface with 23 meV kinetic energy with a transverse energy given by the temperature of the SFHe [37]. In this way, a mono-energetic $\mu^+$ beam can be realized with very small divergence.

This Mu beam could be used to measure the gravitational interaction of antimatter. So far, no direct measurement of antimatter gravity experiments has been performed, although indirect tests suggest that it does not differ from that of ordinary matter [39]. However, a direct test using an antimatter system is highly desirable. For charged antimatter such as antiprotons or positrons, any residual electric fields would prevent a measurement, thus an electrically neutral system such as antihydrogen or muonium is required. Several antihydrogen experiments are ongoing, for example by the AEgIS [40], GBAR [41], or ALPHA [38] collaboration.

\par

\footnotetext[3]{It should be mentioned here that the ALPHA collaboration sets a limit on the ratio of the gravitational mass $M_g$ of antihydrogen to the inertial mass $M$ of hydrogen: $110 > M_g/M > -65$ [38].}
2.4. Research with muons and muonium

Since Mu consists of 99.5% antimatter \( m_\mu \approx 207m_e \), it will ultimately be a measurement of the gravitational acceleration \( \bar{g} \) of antimatter. The other peculiarity of gravity tests with Mu is that the gravitational interaction of second generation leptons could be tested for the first time, which distinguishes the Mu measurements from the antihydrogen measurements.

A proposed scheme to measure \( \bar{g} \) is to use a Mach-Zehnder interferometer [4, 42, 43]. The Mu beam is sent through three small gratings with a pitch length of about 100 nm, separated by a few cm but very precisely aligned parallel to each other, as shown in Fig. 2.8. The first two gratings are responsible for the interference pattern, whereas the last grating is moved to scan the resulting pattern. This experiment would ultimately lead to a statistical sensitivity on measuring \( \bar{g} \) of \( S \approx 0.3 \) g per \( \sqrt{\#\text{days}} \), assuming a suitable beam delivering \( 10^5 \) Mu/s [4]. Thus, the sign of \( \bar{g} \) would be determined within the first day of measurement, and a sensitivity of 3% is achieved after 100 days of data acquisition.

2.4.3 Muonium spectroscopy: Hyperfine splitting and 1S-2S transition measurements

Muonium is an atomic system composed of a positive muon and an electron. This purely leptonic system is free from the uncertainties related to the finite size and complex structure of the nucleus. Even though the lifetime of this system is limited, it offers an interesting possibility to test bound-state QED [44] and to deduce fundamental constants.

In fact, Mu spectroscopy of the 1S-2S transition and the hyperfine splitting leads to the best verification of charge equality in the first two generations of particles,
to a test of the gravitation interaction of antimatter, to an improved determination of the muon mass, and to the most precise value of the muon-to-electron magnetic moment ratio [45].

A novel generation of 1S-2S Mu spectroscopy experiment can be conceived to improve the measurements from the present 4 ppt accuracy [46]. These experiments can capitalize on the to-date availability of high-power continuous wave (cw) lasers. However, the use of cw lasers requires a Mu “cloud” in vacuum significantly smaller compared to previous experiments. Thus, an improvement of the Mu source quality that can be achieved with the beam line described previously (Sec. 2.4.2) opens the way for next generation Mu experiments [47].
3 Working principle of the novel $\mu^+$ beam line

In this chapter, the working principle of the phase space compression scheme, which is at the core of the novel $\mu^+$ beam line, is explained. First, the basic concept is introduced to give the reader an overview of the distinct features of the compression scheme. Then, the drift velocity vector of a charged particle in gas is presented, and the physical processes occurring when the $\mu^+$ are stopped in helium gas are briefly discussed. The novel high-brightness beam is produced by compressing a standard $\mu^+$ beam. This is achieved in various stages within a helium gas target: first, transverse compression occurs, followed by longitudinal compression and extraction into vacuum. The working principle of these stages is central to this chapter. It is concluded by discussing the mechanisms leading to losses of $\mu^+$ during compression, which reduce the overall efficiency of the beam line. Moreover, the "runaway effect" - crucial for the longitudinal compression stage - is presented, and in the end, a short derivation of the achievable reduction in phase space is given.

3.1 The basic concept

The new concept [7] for phase space compression of a $\mu^+$ beam is based on a position-dependent muon drift velocity vector ($\vec{v}_D$) in a gas. A conventional surface $\mu^+$ beam is stopped in a helium gas target which is placed inside a strong longitudinal (along the incoming beam axis) magnetic field. High electric fields are applied across the target to steer the $\mu^+$. Due to different density regimes in the helium gas, caused by temperature gradients, the $\mu^+$ drift along different directions depending on their position. In the experiments described in this thesis, there is usually only one $\mu^+$ at the time in the compression target, because the incoming $\mu^+$ flux is a few 10 kHz and the $\mu^+$ lifetime is 2.2 $\mu$s. Thus, the concept of a $\mu^+$ "swarm" refers to the hypothetical accumulation of muons in the target, if it were not for the $\mu^+$ decay.

As shown in Fig. 3.1, the compression of the stopped muon swarm occurs in three sequential stages: first, using high E- and B-fields and a temperature gradient
Figure 3.1: Scheme of the muon beam compression concept. Muons from a surface $\mu^+$ beam are stopped inside a cryogenic He gas target. In the first stage, a stationary gas density gradient and high E- and B-fields compress the muon beam transversely ($y$-direction). The $\mu^+$ then drift into a second stage at room temperature where longitudinal ($z$-direction) compression occurs. Successively, the $\mu^+$ are drifted towards a small orifice, where vacuum extraction occurs. The approximate dimensions of the target are $10 \times 5 \times 50 \text{ cm}^3$.

At cryogenic temperatures, transverse compression (perpendicular to the incoming beam and B-field axis) occurs. After transverse compression, the $\mu^+$ drift towards a second stage which is at room temperature. There, the density is constant and the electric field is applied in such a way that the $\mu^+$ swarm is compressed in longitudinal direction. After these compressions, the $\mu^+$ swarm is brought towards the third stage where the $\mu^+$ are extracted through a 1 mm hole into vacuum. The extracted $\mu^+$ can then be re-accelerated by means of a pulsed electric field, and sent to an experiment. This pulsed E-field provides the time information which allows to have a tagged $\mu^+$ beam.

In this process, a continuous surface muon beam of 29.8 MeV/c momentum (kinetic energy of 4.1 MeV), 3% momentum spread and 100 mm$^2$ beam spot area is transformed into a beam of eV energy and 1 mm radius. As detailed later, this results in a phase space reduction of 10 orders of magnitude with an efficiency of $10^{-3}$. Therefore, the brightness of the output beam is increased by 7 orders of magnitude. The full compression process occurs in about 10 µs.
3.1. The basic concept

This compression scheme, consisting of the three stages, has the advantage that the various stages can - in a first step - be tested separately, thus simplifying the experimental development significantly. A scheme of the target is shown in Fig. 3.1, and the different stages are described in more detail in the next sections.

3.1.1 The drift velocity vector

The working principle of our novel beam line can be understood by considering the drift velocity vector \( \vec{v}_D \) of the \( \mu^+ \) in gas. In the presence of electric and magnetic fields, the motion of a \( \mu^+ \) in gas (this is of course true for any ion) can be described by the equation:

\[
\frac{d\vec{v}}{dt} = e\vec{E} + e(\vec{v} \times \vec{B}) - K\vec{v},
\]

(3.1)

where \( m \) and \( e = +|e| \) are the mass and charge of the \( \mu^+ \), \( \vec{v} \) its instantaneous velocity and \( K \) describes a frictional force proportional to \( \vec{v} \) that is caused by collisions with the gas atoms. It turns out that the ratio \( m/K \) has the dimension of a characteristic time, thus we can define \( \tau_c \equiv m/K \), where \( \tau_c \) is the mean free time between collisions [48, 49]. For \( t \gg \tau_c \), a steady state is achieved where \( d\vec{v}/dt = 0 \) and Eq. (3.1) becomes

\[
\frac{e}{m} \vec{E} = \frac{1}{\tau_c} \langle \vec{v} \rangle - \frac{e}{m} (\langle \vec{v} \rangle \times \vec{B}).
\]

(3.2)

In order to solve this equation, we introduce the cyclotron frequency as \( \omega = (e/m)B \) and define the mobility \( \mu \) of the muons in the gas as \( \mu \equiv (e/m)\tau_c \). The stationary solution \( \vec{v}_D = \langle \vec{v} \rangle \) of the above equation is called the drift velocity vector. Solving Eq. (3.2), one obtains the drift velocity vector \( \vec{v}_D \) of charged particles in gas, in the presence of electric and magnetic fields [49]:

\[
\vec{v}_D = \frac{\mu E}{1 + \omega^2 \tau_c^2} \left[ \hat{E} + \omega \tau_c \left( \hat{E} \times \hat{B} \right) + \omega^2 \tau_c^2 (\hat{E} \cdot \hat{B}) \hat{B} \right],
\]

(3.3)

where \( \hat{E} \) and \( \hat{B} \) are the unit vectors along \( \vec{E} \) and \( \vec{B} \), respectively.

The drift velocity vector consists thus of three terms, each along a different direction, and each contributing with a different weight: \( 1 \), \( \omega \tau_c \) and \( \omega^2 \tau_c^2 \), respectively. For a constant B-field (as is our case), the cyclotron frequency \( \omega \) stays constant within our setup. However, \( \tau_c \) is position dependent as it depends on the number density \( n = n(x, y, z) \) through the equation \( 1/\tau_c = n\sigma \), where \( \sigma \) is the total elastic cross section. This implies that \( K \propto \sigma \) justifying the intuitive interpretation of \( K \) being a frictional force. By choosing \( \tau_c \), as well as \( \vec{E} \) and \( \vec{B} \), appropriately, each of the three terms in the brackets of Eq. (3.3) can be made dominant. This means that \( \vec{v}_D \) can point in different directions at different positions: \( \vec{v}_D = \vec{v}_D(x, y, z) \).
3.1.2 Muons stopping in gas

When $\mu^+$ are stopping in gas, several energy loss processes occur until the $\mu^+$ are thermalized. A very rough classification of these different processes can be made by distinguishing three energy regimes, each of which defined by a different dominant energy loss process. Note that most subtleties of the energy loss are omitted here for simplicity, but more details can be found for instance in Refs. [3, 50–52]. The three processes are also given in Fig. 3.2, which shows the energy loss $dE_{\text{kin}}/(ndx)$ along a fixed direction $x$ in helium gas. The orange dashed line corresponds to the inelastic process when the $\mu^+$ ionizes or excites the He atom, the dotted blue line represents the energy loss due to charge-exchange processes (electron capture and loss), and the solid black line represents the elastic collisions at low energy. In the following, these three processes are briefly described.

- Muons with energies above a few tens of keV loose their energy mainly through inelastic collisions with electrons of the gas atoms, which get ionized or excited. The energy loss as a function of the $\mu^+$ energy is well described by the Bethe formula, which states (neglecting relativistic, density-effect, shell and Barkas corrections) [52]:

$$-\left(\frac{dE_{\text{kin}}}{dx}\right) = \frac{4\pi e^4 z_p^2 Z_T n}{m_e v^2} \ln \left(\frac{2m_e v^2}{I_T}\right), \quad (3.4)$$

where $n$ is the number density of the target, $z_p$ the charge of the projectile, $Z_T$ the atomic number of the target, $v$ the projectile velocity, $e$ and $m_e$ the charge and mass of the electron, and $I_T$ the mean excitation energy of the electrons in the target.

The energy loss from 4 MeV down to energies of around 30 keV takes about 10 ns at 1 atm pressure, quite independently of the gas [50]. It has been demonstrated that the stopping power for $\mu^+$ in gases agrees well with the stopping power for protons at the same energies under the assumption of velocity scaling. The stopping power of a proton $-\frac{dE}{dx}_{\text{proton}}(E_p)$ at energy $E_p$ is the same as for a $\mu^+$ at the same velocity [52]:

$$-\left.\frac{dE_{\text{kin}}}{dx}\right|_{\mu^+} (E_{\mu^+}) = -\left.\frac{dE_{\text{kin}}}{dx}\right|_{\text{proton}} \left(\frac{m_p}{m_\mu} E_{\text{proton}}\right), \quad (3.5)$$

where $m_p$ and $m_\mu$ are the proton and muon mass, respectively. Therefore, the cross section for $\mu^+$ can be inferred from SRIM [53] data and its parametrization. However, a thorough theoretical description for the energy loss of $\mu^+$ in the 10 keV range is, to the best of our knowledge, still missing, which was
3.1. The basic concept

Figure 3.2: Stopping power $dE_{\text{kin}}/(ndx)$ of the $\mu^+$ as a function of kinetic energy. The data for elastic collisions is listed in [54], whereas for the charge-exchange regime velocity-scaled cross sections from proton data were taken [55]. The inelastic collisions are scaled proton data from the SRIM data [53] (available in Geant4).

also pointed out in [27]. As soon as the $\mu^+$ reaches energies around 1 keV, another process becomes important for the energy loss: The so-called “charge-exchange”.

• The charge-exchange regime contributes significantly to the energy range between $\sim 30$ keV and $O(100)$ eV. In this regime, a series of electron capture (1) and subsequent electron loss (2) processes take place:

  $$(1) \quad \mu^+ + \text{He} \rightarrow \text{Mu} + \text{He}^+$$

  $$(2) \quad \text{Mu} + \text{He} \rightarrow \mu^+ + \text{He} + e^-.$$  

The associated cross sections are denoted by $\sigma_{10}$ for electron capture (process (1)) and $\sigma_{01}$ for the electron loss (process (2)). Some cross sections for Mu formation and Mu ionization are given in Appendix A.

During the slowing down, a $\mu^+$ undergoes about 120 cyclic charge-exchange processes, which takes only 0.2 ns at 1 atm (again relatively independent of the nature of the gas) [50]. Thus, the $\mu^+$ spends about $10^{-12}$ s per cycle in the
electrically neutral bound state of Mu, which can cause losses in our phase space compression scheme (see also Sec. 3.5).

The ionization energy of He gas is 24.6 eV, whereas the Mu has a ionization energy of 13.6 eV [51, 56]. Hence, the total energy loss of the $\mu^+$ when forming muonium is 11 eV. In one full charge-exchange cycle of electron capture and subsequent loss, the $\mu^+$ therefore loses 24.6 eV.

In principle, Mu formation occurs down to $\mu^+$ energies equal to the ionization energy of He. The fraction of Mu which emerge from the charge-exchange process is determined by the competition between capture and ionization processes. If the threshold for Mu formation is low enough (as for example in Kr or Xe), 100% Mu formation is measured [50, 51]. Conversely, in He and Ne there is 0% and 5% Mu formation, respectively [50, 51].

• Below about 100 eV, elastic scattering ($\mu^+ + \text{He} \rightarrow \mu^+ + \text{He}$) becomes dominant. The $\mu^+$ does not form muonic atoms such as $\mu^-$ but spends the rest of its lifetime as free diamagnetic $\mu^+$. The thermalization down to $k_B T$ takes around 10 ns at room temperature and 1 atm gas pressure, but the slowing down time depends strongly on the gas in this case. Moreover, during the last two stages of the slowing down (cyclic charge-exchange and elastic collisions), the hyperfine interaction between the $\mu^+$ and the $e^-$ spins starts to become important, but little to no depolarization is measured in the charge-exchange regime at 1 atm pressure because the $\mu^+$ reside for only $O(1)$ ps in the neutral bound state of Mu per charge-exchange cycle [50].

### 3.2 Transverse compression

In this section, we consider the compression in transverse direction in more detail. The $\mu^+$ traveling along the $z$-direction enter the transverse compression target, which is at cryogenic temperatures and contains 5 mbar of He gas. A stationary density gradient is established in vertical direction by cooling the lower side of the target to 4 K while heating the upper side to 12 K, as shown in Fig. 3.3. Muons stopping in the lower part of the target, where the density is higher, have a smaller $\tau_e$ than the ones stopping in the upper part. A strong, constant electric field ($\vec{E} = 1/\sqrt{2}(1, 1, 0)$, $|\vec{E}| \approx 1.5$ kV/cm) is applied, perpendicular to the magnetic field ($\vec{B} = (0, 0, 5)$ T), which is along the $z$-axis. With this choice of $\vec{E}$ and $\vec{B}$, we get $\vec{E} \cdot \vec{B} = 0$ and thus Eq. (3.3) simplifies to

$$\vec{v}_D = \frac{\mu E}{1 + \omega^2 \tau_e^2} \left[ \hat{E} + \omega \tau_e \left( \hat{E} \times \hat{B} \right) \right]. \quad (3.6)$$
3.3. Longitudinal compression

The density in the target midplane is chosen such that $\vec{v}_D$ points in $x$-direction. Below this plane, the first term of Eq. (3.6) becomes dominant so that the drift occurs mainly in $\hat{E}$-direction. Above this plane, due to the larger temperatures, the collision frequency decreases and thus the second term becomes dominant and the $\mu^+$ travel mainly along the $\hat{E} \times \hat{B}$-direction. The simulated trajectories are shown in Fig. 3.3, where one sees that the muon swarm is compressed towards the "tip" (right corner of the triangular cross section) of the transverse compression stage. Depending on the initial vertical position of the $\mu^+$, it takes between 1 and 3 $\mu$s for them to drift into the tip.

In this stage, the $\mu^+$ beam, starting with a vertical extension of about 15 mm, is compressed down to 2 mm in vertical direction. The longitudinal dimension (in $z$-direction) of about 50 cm is not affected [7].

3.3 Longitudinal compression

Directly attached to the tip of the transverse compression stage follows the longitudinal stage, which is at room temperature. The transition between the transverse
Chapter 3. Working principle of the novel $\mu^+$ beam line

Figure 3.4: (Top) Expected compression of the $\mu^+$ swarm in the longitudinal compression stage. The electric field points towards the center of the target (at $z = 0$), compressing the $\mu^+$ swarm into a "point-like" swarm at $z = 0$. (Bottom) Due to the additional $E$-field in $+y$-direction, the $\mu^+$ experience a force in $\hat{E} \times \hat{B}$-direction, resulting in a drift towards the extraction orifice.

By choosing $E > 0$ for $z < 0$ and $E < 0$ for $z > 0$, the drift velocity $\vec{v}_D$ for $\mu^+$ points towards the center of the target at $z = 0$. As a result, the $\mu^+$ swarm is compressed in longitudinal direction, along the $z$-axis. This is schematically drawn in Fig. 3.4 (Top), which is a view from above onto the $zz$-plane.

Applying an additional electric field component along the vertical $y$-direction in
3.4 Extraction into vacuum

Following the longitudinal compression stage, there is another stage where a mixed compression in $y$- and $z$-direction is achieved. This compression, again at cold temperatures, is accompanied by a drift in $x$-direction so that the $\mu^+$ can be extracted into vacuum. In vacuum, the $\mu^+$ can be re-accelerated in $z$-direction, extracted from the B-field and sent towards an experiment.

The $\mu^+$ leaves the gas target through an orifice of 1 mm diameter. The gas flowing from the target into the vacuum has to be efficiently pumped away so that the $\mu^+$ passing the orifice is basically in vacuum. A scheme of the orifice region is shown in Fig. 3.5.

The He gas lost through the orifice has to be replaced. He gas is injected in the target very close to the orifice, as indicated in Fig. 3.5. The injected gas acts as a "barrier" for the gas which is in the target and flows "directly" through the orifice into the vacuum (see also Fig. 3.5 (Right)).
3.4.1 Extraction from the high B-field region

For the majority of the applications, the compressed $\mu^+$ beam has to exit the 5 T magnetic field region. In principle, the $\mu^+$ would follow the magnetic field lines, which means that the compressed $\mu^+$ swarm would increase in size and the phase space density would be deteriorated again. A termination of the magnetic field lines in an abrupt way (for example using ferromagnetic grids) is necessary such that the $\mu^+$ motion does not follow the field lines. This means that the $\mu^+$ have to be fast enough to not be able to follow the abruptly changing magnetic field lines.

A similar problem has been solved some years ago for a positron beam [57]. There, a field-terminating grid was used with a two-fold purpose. Firstly, it helped to decrease the transverse energy of the extracted beam. And secondly, it terminated the field lines non-adiabatically.

3.5 Muon loss mechanisms in the compression scheme

There are four main mechanisms for $\mu^+$ losses during our phase space compression scheme, which are described below and summarized in Table 3.1.

- The muons decay with a mean lifetime of 2.2 $\mu$s. To minimize these losses, a fast compression scheme is needed. According to preliminary simulations [7], our cooling scheme requires 8-10 $\mu$s while all other cooling schemes applicable for ions are in the millisecond time scale [58, 59]. Being fast is usually achieved by having low gas densities and high electric fields. However, this is competing with the wish to have a small stopping volume along the $z$-direction in the transverse compression stage, which would require high gas density.

- Another source of $\mu^+$ losses during the compression is formation of thermalized Mu, which means that the $\mu^+$ exits the charge-exchange regime as Mu. Although the formation of thermalized Mu in He gas is negligible, this situation changes if the He gas is contaminated. Depending on the contaminant, Mu formation can play a significant role (for example in H$_2$, the Mu formation rate is $(61 \pm 4)\%$ [51]). It could also happen that some sort of meta-stable $\mu^+X$-molecule is formed. Such impurities were an issue when we performed the first test of longitudinal compression in 2011. This is discussed in detail in Secs. 4.1 and 4.5.
3.5. Muon loss mechanisms in the compression scheme

Table 3.1: The most dominant mechanisms which lower the efficiency of phase space compression, and possible ways to avoid them.

<table>
<thead>
<tr>
<th>Loss mechanism</th>
<th>Caused by . . .</th>
<th>Can be minimized by . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon decay</td>
<td>Finite lifetime</td>
<td>Fast compression</td>
</tr>
<tr>
<td>Mu formation</td>
<td>In He, only a fraction &lt; 1% of ( \mu^+ ) thermalizes as Mu. Impurities like H may considerably increase this fraction</td>
<td>Avoidance of impurities in the gas</td>
</tr>
<tr>
<td>Temporary Mu formation in the slowing-down process</td>
<td>The Mu atom is not confined by the E- and B-fields and drifts into the walls</td>
<td>Appropriate (sufficiently large) target design</td>
</tr>
<tr>
<td>Stopping in the walls</td>
<td>Target misalignment with respect to the B-field axis, not well-defined electric fields</td>
<td>Careful target construction, precise simulation of the real electric fields</td>
</tr>
</tbody>
</table>

- Although only a small fraction <1% of the stopped muon form Mu, in the keV energy regime (for example during the slowing down process), each muon undergoes several charge-exchange processes. There, the \( \mu^+ \) are temporarily in the neutral bound state of Mu and thus cannot be confined in radial direction by the magnetic field. They may hit the wall if the ionization mechanism (Mu + He \( \rightarrow \) \( \mu^+ + \ldots \)) is not fast enough. Although the time the \( \mu^+ \) spends as Mu is of \( \mathcal{O}(0.1) \) ns (see Sec. 3.1.2), we find in Geant4 simulations that this is sufficient for a non-negligible fraction of the Mu to hit the walls of the target [10].

It was mentioned previously that the final state Mu formation in He gas is negligible. However, during the cyclic charge-exchange regime (100 eV to 1 keV), this is not the case. Knowing the electron capture cross section \( \sigma_{01}(E_{\text{kin}}) \) (which depends on the muon kinetic energy \( E_{\text{kin}} \)), and the electron loss cross section \( \sigma_{10}(E_{\text{kin}}) \), allows the calculation of the "effective charge" of the muon by

\[
Q_{\text{eff}}(E_{\text{kin}}) = \frac{\sigma_{01}(E_{\text{kin}})}{\sigma_{01}(E_{\text{kin}}) + \sigma_{10}(E_{\text{kin}})}.
\]

\( Q_{\text{eff}} \) can be interpreted as the probability to find the muon as a free \( \mu^+ \) ion, and \( 1 - Q_{\text{eff}} \) as the probability that the \( \mu^+ \) is in the neutral state of Mu. The charge-exchange cross sections \( \sigma_{01} \), \( \sigma_{10} \), and \( Q_{\text{eff}} \) are shown in Fig. 3.6 (see also [50] for more details). They are obtained by scaling proton data as presented in [55].

During the charge-exchange cycle, when the muon is in the neutral state, its
motion is not constrained by the E- and B-fields. Thus, the Mu may drift to a wall where the $\mu^+$ can become adsorbed or where the E-fields are not well defined. To avoid significant losses of $\mu^+$ due to these Mu hitting the wall of the target, one can adapt the geometry accordingly (i.e., make the targets larger). This competes with the requirement of a small target, given mainly by the need to have temperature gradients with small spatial extension and limitations due to electric breakdown which increases with the size of the E-field.

- Finally, $\mu^+$ losses can be due to misalignments of the target relative to the magnetic field axis (i.e., the beam axis). The magnetic field inside the 5 T magnet used for the experiment is sufficiently homogeneous such that careful alignment of the target with respect to the magnet should avoid problems for the longitudinal compression stage. However, for the transverse target, which is at cryogenic temperatures, this is not trivial because of the thermal contraction, causing the target to shift form its initial position while cooling it down.

### 3.6 Runaway effect

The aforementioned "runaway effect" was discovered when measuring the mobility of ions in He gas as a function of the reduced electric field $E/N$, where $N$ is the number density$^1$ of the helium gas [60]. It was measured [61], as well as calculated [62], that above a certain electric field strength the ions cannot lose enough momentum by elastic collisions to achieve a steady-state average velocity. Neglecting other energy loss mechanisms, this means that the ions (in our case the $\mu^+$) would be continuously accelerated and reach infinite velocity. In reality, the $\mu^+$ are accelerated up to a few keV energy where the stopping power of charge-exchange and He ionization and excitation becomes important (see Fig. 3.2).

A simplified derivation of the runaway effect is included in Appendix B; here, only the key result is presented. Changing to the picture of momentum-transfer cross sections, the equilibrium between collisional deceleration and field acceleration ($= eE/m$) can be written as (see Eq. (B.6))

\[ eE = M_r v_D N \tilde{v}_r \sigma_{tr}(\tilde{v}_r), \]

(3.9)

where $M_r = mM/(m + M)$ is the reduced mass of the atom-ion pair ($m$ is the ion mass, $M$ the atom mass), $v_D$ is the $\mu^+$ drift velocity along the electric field direction, $N$ the number density, $\tilde{v}_r$ the relative velocity between ion and atom and $\sigma_{tr} = \sigma_{tr}(\tilde{v}_r)$

---

$^1$In order to be consistent with literature, the capital letter "N" is used for the number density in the context of reduced electric fields, whereas otherwise the number density is called "n" throughout this thesis.
3.6. Runaway effect

Figure 3.6: (Top) The cross section $\sigma_{10}$ for Mu formation ($\mu^+ + \text{He} \rightarrow \mu + \text{He}^+$) by electron capture (black line) and $\sigma_{01}$ for Mu ionization ($\mu + \text{He} \rightarrow \mu^+ + \text{He} + e^-$) by electron loss (orange line) as a function of the $\mu^+$ kinetic energy. (Bottom) The effective charge of the muon as a function of its energy. It is defined as $Q_{\text{eff}} = \frac{\sigma_{01}}{\sigma_{01} + \sigma_{10}}$ and reflects the probability to have muons as free (unbound) muons.

is the momentum-transfer cross section. After some reformulations and substitutions (details in [63]), we obtain the condition for the onset of runaway:

$$\left( \frac{E}{N} \right) > 2E_{\text{kin}}\sigma_{\text{tr}}(\tau_r) \left( \frac{m}{m + M} \right)^{1/2}. \quad (3.10)$$

$$^2\sigma_{\text{tr}}(\tau_r) = \int_0^\pi (1 - \cos \theta)\sigma(\tau_r, \theta)\sin \theta d\theta$$ with $\sigma(\tau_r, \theta)$ being the differential cross section for elastic scattering at an angle $\theta$ for a mean kinetic energy $E_{\text{kin}} = 1/2M_r\tau_r^2$. 


Runaway is thus reached above a reduced electric field value $E/N$ expressed by Eq. (3.10), given usually in units of Td (Townsend), with $1 \text{Td} = 10^{-21} \text{Vm}^2$. It is reported by [60, 61] that runaway for protons in He gas occurs when the mass-scaled field strength is above 220 Td (see Appendix B), which translates for muons in helium gas to

$$E/N \gtrsim 36 \text{Td}.$$ 

For comparison, some typical values of the electric field strengths reached in the transverse and the longitudinal compression stages are

- **Transverse (10 K, 5 mbar):**
  $$\frac{E}{N} = \frac{1.5 \text{kV/cm}}{36.2 \cdot 10^{17} \text{ cm}^{-3}} = 41 \text{Td}$$

- **Longitudinal (293 K, 5 mbar):**
  $$\frac{E}{N} = \frac{60 \text{V/cm}}{1.24 \cdot 10^{17} \text{ cm}^{-3}} = 48 \text{Td}.$$ 

### 3.7 Phase space

A figure of merit of a beam is given by the 6-dimensional phase space occupied by the particles. Each particle is described by the "coordinate"

$$\{x, p_x, y, p_y, z, p_z\},$$

where $(x, y, z)$ represent the particle’s position and $(p_x, p_y, p_z)$ its momentum. Instead of the tuple $(z, p_z)$, a particle beam can be described in a given $xy$-plane along the $z$-propagation by

$$\{x, p_x, y, p_y, t, E\},$$

where $t$ is the time when the particle is arriving at the plane at $z$ and $E$ its energy. The phase space surface occupied by a beam is described (in an approximate way) by

$$P = \sigma_x \cdot \sigma_{p_x} \cdot \sigma_y \cdot \sigma_{p_y} \cdot \sigma_t \cdot \sigma_E,$$

where $\sigma$ are the corresponding standard deviations. It is the 6-dimensional surface that will be improved by 10 orders of magnitude by the use of our novel device compressing the muon beam.

To quantify the achievable phase space reduction with our beam line, we start to list the parameters of the beam entering the transverse compression stage. The values of the $\pi E3$ beam line at PSI can be taken as a reference for a "standard" muon beam line, which are listed in Table 3.2.

The energy uncertainty $\Delta E/E = 2\Delta p/p$ for this muon beam is thus 16% at FWHM. Since we are dealing with a continuous beam, in principle the times are stochastically distributed. However, a detector can be used to provide the
3.7. Phase space

Table 3.2: Properties of the \( \pi E3 \) beam line at PSI [64].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>28 MeV/c</td>
</tr>
<tr>
<td>Momentum acceptance (FWHM)</td>
<td>8%</td>
</tr>
<tr>
<td>Spot size (FWHM)</td>
<td>15 mm horizontal; 30 mm vertical</td>
</tr>
<tr>
<td>Divergence (FWHM)</td>
<td>80 mrad horizontal; 30 mrad vertical</td>
</tr>
</tbody>
</table>

Time information. We assume that the time can be deduced with a resolution of \( \Delta t = 1 \text{ ns} \).

After the compression, the \( \mu^+ \) exiting the orifice have a velocity of about 5 mm/\( \mu \text{s} \) in \( x \)-direction. Assuming that the exiting \( \mu^+ \) are pulsed (re-accelerated) with 1 MHz frequency, we can attribute an uncertainty (FWHM) of 5 mm in \( x \)-direction and 2 mm in \( y \)-direction to every \( \mu^+ \). We further assume that we re-accelerate the \( \mu^+ \) to 15 keV energy (which corresponds to a momentum of 1.7 MeV/c), and that the longitudinal kinetic energy uncertainty is increased to 20 eV after acceleration.

By comparing the phase space entering the transverse compression stage and the phase space at its output after acceleration to 15 keV, we obtain the following ratio:

\[
\frac{P_{\pi E3}}{P_{\mu \text{Cool}}} = \frac{(15 \text{ mm} \cdot 30 \text{ mm}) (80 \text{ mrad} \cdot 30 \text{ mrad}) (28 \text{ MeV/c})^2 \cdot 1 \text{ ns} \cdot (0.16 \cdot 4.1 \text{ MeV})}{(5 \text{ mm} \cdot 2 \text{ mm}) \left( \sqrt{\frac{1 \text{ eV}}{15 \text{ keV}}} \right)^2 \cdot (1.7 \text{ MeV/c})^2 \cdot 1 \text{ ns} \cdot (20 \text{ eV})} \\
\approx 1.4 \times 10^{10}.
\]  

This ratio demonstrates that, using our novel concept, we will be able to reduce the phase space of a standard muon beam by about 10 orders of magnitude. The exact value strongly depends on the re-acceleration scheme and extraction from the magnetic field. Assuming an efficiency for the phase space reduction of about \( 10^{-3} \), this would lead to an improvement of the brightness of standard \( \mu^+ \) beams by seven orders of magnitude.
4 Longitudinal compression tests

A major simplification for the demonstration of our phase space compression scheme is that the longitudinal compression stage - as well as the other two stages - can be tested separately. This chapter is dedicated to the tests of the longitudinal compression stage. We carried out three different experiments in 2011, 2014 and 2015, respectively, which are described in chronological order. In 2011, the focus was put on demonstrating the feasibility of the longitudinal compression stage, which is described in Sec. 4.1. Based on the conclusions of this beam time, the follow-up experiment in 2014 was designed. Problems encountered in 2011 were solved, which lead to a high compression efficiency. This is presented in detail in Secs. 4.2-4.5. The last experiment, conducted in 2015, was already designed with respect to the continuation of the beam line after the longitudinal compression stage, thus an additional drift of the muons towards the stage where later extraction into vacuum will occur was tested. The setup and analysis of the 2015 experiment is discussed in Sec. 4.7.

4.1 First demonstration of longitudinal compression in 2011

Longitudinal compression was first demonstrated in 2011 [65]. However, from the 2011 data, it was not possible to extract a value for the compression efficiency due to problems related to the setup. But we made sure to eradicate these issues after the first beam time, and in fact they did not occur anymore during the subsequent two beam times. The goal of this section is to introduce the most important concepts and to pave the way for the understanding of the various improvements and optimizations made to the setup in the subsequent years.

4.1.1 General approach to demonstrate longitudinal compression

The basic principle of the experiment is to stop $\mu^+$ in He gas and to attract them into the center of the target along the magnetic field lines by using an electric field.
Chapter 4. Longitudinal compression tests

Figure 4.1: Sketch of the experimental setup in 2011. The muons entering from the left are stopped in a few mbar He gas. The electric field, generated by an electric potential applied in the center of the target (“HV”), compresses the muons towards the center of the target. Two positron detectors (labeled “P1” and “P2”, respectively) detect the decay positrons as a function of time relative to the entrance detector. Reproduced from [65].

A time spectrum is constructed from the signals in the entrance detector and the positron counters, which are placed close to the compression region, to expose the $\mu^+$ swarm compression.

The experiment in 2011, as well as the other two in 2014 and 2015, respectively, was performed at the $\pi$E1 beam line at PSI. The principle of the experiment can be inferred from the setup given in Fig. 4.1. Muons with a momentum of about 10 MeV/c cross an entrance detector defining the starting time $t = 0$. The fraction of the $\mu^+$ stopped inside the 300 mm long target in $\sim$10 mbar helium gas is about 1%. The stopped $\mu^+$ swarm is compressed by the electric field according to the last term in Eq. (3.3). By plotting the number of positrons detected in the positron counters P1 and P2 as a function of time (relative to the time $t = 0$ given by the entrance detector (D1) placed in front of the target), one can observe that – in the case of compression – the number of decay positrons in the center of the target increases with time, compared to the case of no compression.

This is especially visible when the counts of the time spectra are multiplied with $\exp(t/2198)$, where $t$ is the time (in ns) when the positron is detected, in order to cancel the decrease caused by the muon decay. This multiplication yields, assuming no compression, a flat spectrum. If the muons are attracted close to the positron detector, the time spectrum shows an increase of events. If the electric field is reversed,
4.1. First demonstration of longitudinal compression in 2011

![Diagram of lifetime correction](image)

**Figure 4.2:** Illustration of the "lifetime correction" applied to a typical decay spectrum. On the left, a sketch of a raw decay spectrum is shown, whereas by multiplying the counts in the histogram with $e^{\exp(t/\tau)}$ (where $t$ is in ns and $\tau = 2198$ ns is the $\mu^+$ lifetime), the spectrum on the right is obtained. As can be seen, subtle changes at late times are well visible when correcting for the muon decay. To simplify the notation, the vertical axes in the "lifetime corrected" spectra are labeled with $N^*$ in this thesis.

The time spectrum shows a decrease with time. This is illustrated in Fig. 4.2, which emphasizes that looking at so-called "lifetime corrected" time spectra makes it easier to see changes in the positron decay rate at later times. To simplify the notation, the vertical axis of lifetime corrected time spectra are labeled $N^*$ throughout this thesis, or normalized $N^*$ if the time spectra are additionally normalized (usually to the bin containing $t = 0$).

The longitudinal compression target was constructed using four printed circuit boards (PCBs) with electrodes on them, glued together with Araldite glue to contain the helium gas in which the $\mu^+$ are stopped. An electric potential is applied in the center of the target to electrodes on the lateral target walls. The electric field is then generated by a series of such electrodes, connected by resistors acting as voltage dividers, defining a V-shaped potential (see also top of Fig. 4.1). Two positron counters (P1, P2) were placed at the minimum of the electric potential, detecting the decay positrons. Brass pieces placed close to the positron detectors acted as shielding in order to narrow the acceptance of the positron counters, and hence to improve the sensitivity for compression.

### 4.1.2 Results of 2011

The results of the compression experiment in 2011 are shown in Fig. 4.3. It can be seen that when no electric potential is applied (red squares), the number of detected positrons stays constant with time, whereas for a negative potential, the muon distribution is compressed towards the center of the target. Accordingly, when applying a positive electric potential in the target center, the muons are “pushed” out...
Chapter 4. Longitudinal compression tests

Figure 4.3: Measured (points) and simulated (continuous lines) positron time spectra, where the muon lifetime was removed by multiplying with \( \exp(t/2198) \) (see also Fig. 4.2). Adapted from [65].

of the center of the target, yielding a small decrease in the number of detected decay positrons. The peak around 50 ns is given by the in-flight decay of \( \mu^+ \) which do not stop in the gas target (muons with an initial momentum of 10 MeV/c have a velocity of about 30 mm/ns). Their possible decay in front (i.e., in the acceptance region) of P1 and P2 gives rise to this prompt peak around 10 ns.

The continuous lines in this figure are obtained from a Geant4 simulation. However, a feature had to be implemented in the simulation in order to be able to reproduce the measured data. In the experiment, there were some impurities in the helium gas, mainly from outgassing of the Araldite glue. These impurities can capture a low-energy muon, which forms either an electrically neutral bound state or an ion with small mobility. These particles cannot be steered efficiently by the electric field and therefore account for background. This is illustrated in Fig. 4.4, where a simulation was run for the case of pure He gas (solid lines) and the case when a "chemical capture" rate is implemented due to contaminants in the He gas. It is clearly visible that without impurities, the compression would continue up to 2 \( \mu s \) or more, whereas due to the capture of the \( \mu^+ \) by contaminants in the gas, the compression is finished after about 0.5 \( \mu s \), decreasing the compression efficiency.

Compression efficiency in 2011

The conclusion of the 2011 experiment was that longitudinal compression is feasible, and that the underlying mechanisms are understood. By introducing an impurity cutoff in the simulations, the simulated time spectra matched the measured data well, however, extracting an efficiency is problematic.

The next step was to improve the setup such that the compression efficiency can be determined, which was the goal of the experiment performed in 2014. As
4.1. First demonstration of longitudinal compression in 2011

![Figure 4.4: Simulated positron time spectra without (continuous lines) and with (dashed lines) chemical absorption, which is due to impurities in the helium gas (adapted from [65]). The capture rate implemented in the simulation to fit the data was $R = 40 \times 10^6 \text{s}^{-1}$ for $\mu^+$ below 10 eV.]

will be detailed later, reliable extraction of the efficiency is possible only if the positron counts for a positive, i.e., repulsive, potential (green curve in Fig. 4.3) approaches zero at late times. Since this is clearly not the case, there is a large flat background preventing a meaningful statement about the compression efficiency. This background arises not only from $\mu^+$ being captured by impurities, but also from $\mu^+$ being adsorbed on either the lateral walls, the upstream detector, or the downstream wall.

### 4.1.3 Problems in 2011 and improvements for 2014

Several problems were discovered during the experiment and the analysis of the 2011 data. Table 4.1 summarizes these issues, and illustrates how they were improved for the 2014 beam time.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impurities due to outgassing of the Araldite glue and the PCBs</td>
<td>Use glass plates instead of PCBs, and a special low-outgassing glue</td>
</tr>
<tr>
<td>Small number of observables</td>
<td>Add more scintillators (21+2)</td>
</tr>
<tr>
<td>Limited resolution</td>
<td>Have scintillators read out in coincidence</td>
</tr>
<tr>
<td>Large background</td>
<td>Add more shielding; align target to the magnetic field carefully; avoid impurities</td>
</tr>
</tbody>
</table>

Table 4.1: Improvements in the 2014 experiment compared to 2011.
4.2 Longitudinal compression 2014

An improved setup was constructed for the beam time in 2014. The main focus was put on a high sensitivity for compression, and on the detection of the muon movement along the incoming beam axis (z-axis). To avoid problems encountered in 2011 caused by impurities in the helium gas, a cleaner gas target was realized by changing the design and materials of the target.

The main principle of the 2014 design was similar to the 2011 design, and a sketch showing the most important features is displayed in Fig. 4.5. As in 2011, the muons first crossed an entrance detector, labeled D1 in Fig. 4.5, giving the start time \( t = 0 \). They then crossed a thin foil of about 2 \( \mu \text{m} \) Mylar, covered with 0.4 \( \mu \text{m} \) aluminum on both sides, that prevented the helium gas from entering the beam line vacuum.

![Diagram of the target](image)

**Figure 4.5:** Schematic view of the target used to test longitudinal compression in 2014 (not to scale). The muons pass through the entrance detector D1 and are stopped inside 5 mbar He gas. Applying a negative electric potential to the lateral walls in the middle of the target attracts the \( \mu^+ \) swarm towards the center. In order to observe the temporal evolution of the compression, several scintillators, which detect the decay positrons, are mounted above the target.

The actual target was made of four 3 mm thick glass plates (see Fig. 4.6 (a)), onto which electrodes were sputtered to define the electric field. Each metallic line was set at different voltage to define a constant electric field. This has was achieved by soldering resistors from line to line, which acted as voltage dividers. The glass plates were arranged such that the He gas volume had a cross section of \( 12 \times 12 \text{ mm}^2 \),

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1One appropriate combination for sputtering electrodes is first a 5 nm thin Ti layer directly on the glass plates, followed by 100 nm Cu and 50 nm Ni, covered with about 35 nm Au to prevent oxidation.
4.2. Longitudinal compression 2014

with a length of about 300 mm. In Fig. 4.6 (b), the special arrangement of the glass plates is shown. It allowed all the soldering to be done outside the He gas volume, such that there was minimal influence on the purity of the He gas. The four glass plates were glued together using the Stycast 1266 two-component epoxy, which is low outgassing and therefore suited for applications requiring high gas purity. A small He gas flow through the target additionally ensured low contaminants in the helium gas.

The helium gas pressure was measured before and after the target with pressure gauges. The entrance detector, located about 500 mm upstream the target center, was made from a 55 µm thick plastic scintillator. 21 scintillators along the z-axis, with cross section of $6.5 \times 5 \text{ mm}^2$, were mounted with their respective center at $z = -100 + 10 \cdot (j - 1) \text{ mm}$, where $j = 1, \ldots, 21$ and $y = 11.5 \text{ mm}$, covering the full target in x-direction. On the opposite side, there were two smaller scintillators, T1, and T2, respectively, with cross section of $3 \times 3 \text{ mm}^2$. These two scintillators were located at $z = 0 \text{ mm}$ and at $y = -10.5 \text{ mm}$ and $y = -19.5 \text{ mm}$, respectively.

All scintillators were read out with silicon photo-multipliers (SiPM), and were shielded with brass collimators. A photograph of the scintillators with some of the SiPMs already mounted is shown in Fig. 4.6 (f). The two scintillators, T1 and T2, could be read out in coincidence to improve the geometrical resolution and therefore the sensitivity for compression.

The electric field is created by applying an electric potential at the central metallic stripe ($z = 0$). Two different active lengths were tested in our experiment, by grounding the stripes either at $z = \pm 104 \text{ mm}$ for the long cell, or at $z = \pm 51 \text{ mm}$ for the short cell, respectively. This results in an electric field along the z-axis. In order to have fast compression and to be above the runaway threshold (see Sec. 3.6), a high electric field of at least $E/N \approx 36 \text{ Td}$ is required. This corresponds to a potential applied at the center of the target exceeding 550 V in the long cell with $p = 11 \text{ mbar}$.

4.2.1 Muon rates and stopping efficiency

As mentioned before, all three experiments were performed at the πE1 beam line at PSI (see also Sec. 2.3). This beam line delivers approximately $2 \times 10^6 \text{ µ}^+ / \text{s}$ with a momentum $p = 28 \text{ MeV/c}$ (surface muons), for an average proton beam current of 2.2 mA. Tuning the beam to a muon momentum of 10 MeV/c yielded about 20-22 kHz $\text{µ}^+$ on the 12 mm diameter entrance detector in a 5 T magnetic field.

The stopping efficiency in a few mbar helium gas is rather low: Depending on the precise momentum of the beam, only about 1% of the muons are actually stopped within a 30 cm long cell containing 10 mbar helium.

The pileup correction (described in the next paragraph) reduced the number of potentially good events by about 25%. Considering all these factors, and also including the geometrical detection efficiency of the scintillators, yields an actual event rate
Figure 4.6: Photos of the longitudinal target used in 2014: (a) The four glass plates with electrodes and the HV connectors. (b) Assembly of the four glass plates. Note that the soldering is done outside the He volume. (c) Bar of 21 scintillators in the brass collimator. (d) Detectors and brass collimators mounted inside the casing. (e) Longitudinal target with open casing mounted into beam line. The $\mu^+$ are coming from the left. On the right, the target is partially inside the 5 T solenoid. (f) Scintillators (white) read out by SiPMs mounted on PCBs.
of detected positrons between 50 and 100 events/s, summed over all detectors. Typically, one measurement run lasted 2 hours, i.e., \(2 \times 10^8 \mu^+\) at the entrance detector and \(\mathcal{O}(10^5)\) good events in the positron counters.

### 4.2.2 Data taking and pileup correction

In the experiments in 2014 and 2015, TDC (time to digital converter) counts were used which provide only time information of the signals above thresholds. The measured time spectra were corrected for pileup. The basic idea of the pileup correction is as follows: When a muon crosses the entrance detector it opens a gate of 12 µs. Within this gate, a detector signal in at least one of the positron detectors has to be recorded. If this is the case, the event is accepted, and the corresponding time of the signal is written to the hard disk. Of course, there can be several hits coming from this muon, for example if the positron crosses two scintillators. Various possibilities are summarized in Fig. 4.7.

![Figure 4.7: Different event characteristics: Good events have only one hit in the entrance detector within the 12 µs time gate and at least one hit in one of the other scintillators. If there is only a hit in the entrance but none in the other scintillators, or the other way round, there will be no event recorded. If there are two hits in the entrance detector within a 12 µs time window, the corresponding data is rejected.](image)

However, if a second muon crosses the entrance detector when a gate is already open, the attribution of the decay positron to the muon is ambiguous. This means it is not possible to claim that the positron that was detected first originates from the muon that crossed the entrance detector first. Hence, these events should be rejected, which is exactly what the pileup rejection did. If the entrance detector sees two events within the 12 µs gate it rejects both of them. Otherwise an uncorrelated,
constant (i.e., flat) background would be introduced in the data (for non-lifetime corrected time spectra). The effect is nicely seen in Fig. 4.8, where the flat background turns into an exponential background due to the lifetime correction.

4.3 Compression in 2014 from time spectra

4.3.1 A simple way to extract the compression efficiency

In this section, we present an approximated determination of the compression efficiency. This determination relies neither on simulations of the $\mu^+$ beam compression nor on simulations of the background. To calculate the compression efficiency, we introduce the variables as depicted in Fig. 4.9. The sketch in this figure shows a typical time spectrum for negative applied high voltage (HV) (compression; black), no HV (orange), and positive HV (“decompression”; blue), similar to the measured spectrum shown in Fig. 4.3. The two variables $a$ and $b$ are the heights of the count level at time $t \approx 2.5$ $\mu$s, from which the level for positive HV (called $c$) has been subtracted. There are two reasons why $t \approx 2.5$ $\mu$s was chosen to determine $a$ and $b$. Firstly, it has to be ensured that for various experimental conditions (different target lengths,
4.3. Compression in 2014 from time spectra

gas pressures, electric field strengths etc.) the compression is finished, and thus the length of the time interval chosen to determine $a$ and $b$ does not influence their respective values significantly. And secondly, the full phase space compression should occur within 8 $\mu$s. The $\mu^+$ passes through all the other compression and transition stages within 5-6 $\mu$s, depending on the local gas density in which it is stopped in the transverse compression stage [7]. Thus we are only interested in the muons which are compressed into the center of the longitudinal target within about 2.5 $\mu$s.

![Figure 4.9: Cartoon of the time spectra for negative (black), zero (orange) and positive (blue) electric potential. The variables $a$ and $b$ are used to compute the compression efficiency.](image)

In order to extract the compression efficiency, we need to know $a$, and $b$, but also the geometrical acceptance of the detectors considered. As an example, Fig. 4.10 shows the geometrical acceptance of coincidence detection in T1 and T2. In other words, the histogram in Fig. 4.10 shows the decay position of the muon for every positron detected in coincidence. For this simulation, $10^7$ muons have been generated at rest which decay homogeneously along the target in the interval $z = [-50, 50]$ mm. The FWHM of the peak is measured to be $8.5 \pm 0.5$ mm. This value has to be compared with the length of the active region of the target which is 208 mm for the long cell and 102 mm for the short cell.

For 100% compression efficiency, i.e., if all muons inside the active volume were compressed into the center of the target, we would observe an increase in detected positrons by a factor $f_{\text{ideal}}$. The measured compression efficiency $\varepsilon_{\text{time}}^{(0)}$, obtained by analyzing the time spectra, can be extracted in an approximated way using the relation

$$
\varepsilon_{\text{time}}^{(0)} = \frac{a/b}{f_{\text{ideal}}},
$$

(4.1)
Figure 4.10: The acceptance of the central detectors T1 and T2 in coincidence as a function of the \( \mu^+ \) decay position, assuming a homogeneous \( \mu^+ \) distribution, as simulated with Geant4. The full width at half maximum (FWHM) is 8.5 ± 0.5 mm, while the active target spans over 208 mm and 102 mm for the long and short cell, respectively.

where \( a \) and \( b \) are the measured count levels at late times as defined in Fig. 4.9, while the factor \( f_{\text{ideal}} \) is extracted from Geant4 simulations as follows. Note that the above formula is only meaningful if the assumption \( a, b \gg c \) holds, which means that the background (\( c \)) is small. Otherwise, substantial background corrections would be necessary.

The factor \( f_{\text{ideal}} \) is obtained by forming the ratio between number of counts in the considered detector for two situations:

- All \( \mu^+ \) decay homogeneously distributed along \( z \in [-104, 104] \) mm
- The same number of \( \mu^+ \) decay within \( z \in [-0.5, 0.5] \) mm.

Therefore, using a the simulation, we obtain \( f_{\text{ideal}} \) according to

\[
f_{\text{ideal}} = \frac{N_{\mu^+}^{\mu^+ \text{ decaying at } z \in [-0.5, 0.5]}}{N_{\mu^+}^{\mu^+ \text{ decaying at } z \in [-104, 104]}},
\]

(4.2)

where \( N_{\mu^+} \) is the number of positron counts in the detector. In the estimate of the compression efficiency, we assume a homogeneous (or linear, i.e., with a slope along \( z \)) distribution of the muon stop probability along \( z \) in the active region.

We have made various systematic studies to determine the uncertainty of \( f_{\text{ideal}} \). Uncertainties of the T1 and T2 position in \( y \)-direction, uncertainty of the final size
Figure 4.11: The variation of $f_{\text{ideal}}$ as a function of minimum deposited energy in each detector T1 and T2 for various final swarm sizes (colors) and detector positions (shapes), for the short cell. The round circles show the design position, whereas the triangles, diamonds and crosses show an offset of the scintillators. The coloring represents the final $\mu^+$ swarm size for all detector positions: 1 mm wide swarm along z-direction (black), 3 mm wide swarm (orange) and 1 mm wide swarm with an offset of 1 mm in z-direction, thus its center being at $z = -1$ mm (blue). Only events depositing an energy larger than "Energy Cut" in each detector are considered. The horizontal line is the average of $f_{\text{ideal}}$, the hatched band depicts its standard deviation.

of the muon swarm, and possible misalignments between scintillators and E-field have been taken into account. $f_{\text{ideal}}$ has been computed for variations of these parameters including their dependence on the energy cuts in the detectors. For minimal ionizing particles, one would expect about 0.6 MeV energy deposition in a 3 mm thick scintillator, however, not all positrons will cross the full 3 mm of the scintillator as they cross it under different angles. Unfortunately, we have no calibration which relates the deposited energy to the detection thresholds chosen in the experiment, thus the dependence of $f_{\text{ideal}}$ on the deposited energy in the detectors T1 and T2 needs to be investigated. These studies are presented in Fig. 4.11, in which $f_{\text{ideal}}$ is calculated for various energy cuts on the deposited energy in each of the two detectors T1 and T2. The colors represent the final $\mu^+$ swarm distribution for the case of a
Table 4.2: For the runs which meet the runaway condition, we list the results of the data analysis: The geometrical factor $f_{\text{ideal}}$, the ratio $a/b$ and the compression efficiency $\varepsilon^{(0)}_{\text{time}}$ according to Eq. (4.1).

<table>
<thead>
<tr>
<th>p [mbar]</th>
<th>E [V/cm]</th>
<th>E/N [Td]</th>
<th>$f_{\text{ideal}}$</th>
<th>$a/b$</th>
<th>$\varepsilon^{(0)}_{\text{time}}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-48</td>
<td>38</td>
<td>21.6 ± 1.6</td>
<td>13.9 ± 1.7</td>
<td>64 ± 10</td>
</tr>
<tr>
<td>11</td>
<td>-90</td>
<td>33</td>
<td>10.6 ± 0.8</td>
<td>10.2 ± 0.7</td>
<td>97 ± 10</td>
</tr>
<tr>
<td>5</td>
<td>-90</td>
<td>72</td>
<td>10.6 ± 0.8</td>
<td>6.8 ± 0.8</td>
<td>64 ± 9</td>
</tr>
<tr>
<td>5</td>
<td>-60</td>
<td>48</td>
<td>10.6 ± 0.8</td>
<td>8.5 ± 1.0</td>
<td>80 ± 11</td>
</tr>
</tbody>
</table>

1 mm wide swarm (black), a 3 mm wide swarm (orange) and a 1 mm wide swarm centered at $z = -1$ mm (i.e., with a 1 mm offset in $z$-direction). The full circles show the case of the design position for T1 and T2, whereas the other shapes (triangles, diamonds, crosses) represent a misalignment as given in the legend of the figure. For these cases, the same color scheme was applied for the final state $\mu^+$ swarm width. The case of the energy cut being at zero is not reasonable and therefore not taken into account when calculating the average of $f_{\text{ideal}}$. Using only the values for the case where T1 and T2 are placed at their design position (full circles in Fig. 4.11), we obtain $f_{\text{ideal}} = 10.6 \pm 0.8$ for the short cell, and $f_{\text{ideal}} = 21.6 \pm 1.6$ for the long cell.\(^2\) In Fig. 4.11, the value of $f_{\text{ideal}}$ for the short cell is marked by the horizontal line and its standard deviation is represented by the hatched band.

Two time spectra showing the three cases where a negative, a positive, and no electric potential was applied in 11 mbar (Top) and 5 mbar (Bottom) helium gas, respectively, is shown in Fig. 4.12. The ratio $a/b$ is obtained from the data by fitting a constant between $t = 2.5 \mu s$ and $t = 6 \mu s$. The results of the fit are listed in Table 4.2. By using Eq. (4.1), the compression efficiency $\varepsilon^{(0)}_{\text{time}}$ can be calculated. The error on the compression efficiency is calculated by Gaussian error propagation of the uncertainties obtained when fitting $a$ and $b$, and $f_{\text{ideal}}$. Note that this efficiency does not include the losses caused by muon decay which need to be considered separately.

In order to fit $a$ and $b$ to the data, it is necessary that the compression is terminated, i.e., in the lifetime compensated time spectra constant values are reached. In the computation of the efficiency, we thus consider only the experimental situations with sufficiently high reduced electric fields $E/N$ such that compression is finished (runaway conditions). A direct comparison between a situation in which runaway

\(^2\)The simulation was performed for the short target cell, thus the factor $f_{\text{ideal}}$ needs to be scaled by length for the long target cell, i.e., $f_{\text{ideal}}$ needs to be multiplied with $208/102 = 2.04$. 

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4.3. Compression in 2014 from time spectra

Figure 4.12: Time spectra of the T1 and T2 coincidence detectors for the short cell, with 11 mbar (Top) and 5 mbar (Bottom) He gas inside the target. A potential of -450 V was applied (black), yielding $E/N = 33$ Td and $E/N = 72$ Td, respectively. The cases for no potential (orange) and a positive, repulsive potential of +450 V (blue) are also shown. About $1.7 \times 10^8$ pileup-corrected entrance detector trigger events contributed to the spectrum in the case of 11 mbar and -450 V, and $2.6 \times 10^8$ events in the case of 5 mbar and -450 V.
conditions are met and one in which $E/N$ is too low is shown in Fig. 4.13. For a sufficiently high reduced electric field of $E/N = 72$ Td, the compression is terminated (orange), whereas for $E/N = 21$ Td the accumulation of muons in the center of the target still continues at late times (blue).

4.3.2 Diffusive motion of the $\mu^+$ into the active region

In order to determine the compression efficiency more accurately, the diffusion of the $\mu^+$ in the He gas has to be taken into account. The diffusive motion increases the effective active region, because some $\mu^+$ stopping outside it can diffuse into that region and hence be accelerated towards the center of the target. This increases the compression efficiency. To account for this diffusive motion, we can modify Eq. (4.1) by introducing a diffusion parameter $D$:

$$\varepsilon_{\text{time}}^{(1)} = \frac{a/b}{f_{\text{ideal}}(1 + D)}.$$  

(4.3)
4.3. Compression in 2014 from time spectra

In the experiment, we observe no evidence for diffusion. In contradiction to that, the parameter $D$ has been estimated by means of a Geant4 simulation, and detailed studies of the implementation of the diffusion processes are still ongoing. A preliminary investigation indicates that $D \approx 20 - 30\%$ for the long cell [10]. In order to take this into account, we assign an uncertainty to $D$ and assume $D = (0^{+20}_{-10})\%$. The results (more precisely: the uncertainties) presented in Table 4.2 have therefore to be corrected according to Eq. (4.3), which leads to the results presented in Table 4.3. In order to eliminate this uncertainty related with the diffusion process, in the 2015 beam time, we added just outside the active region a counteracting field, so that diffusion of $\mu^+$ into the active region is suppressed.

As can be seen in Table 4.3, the measured efficiency in a target with 5 mbar He gas is smaller than for a target with 11 mbar. This could be related to $\mu^+$ which have temporarily formed neutral Mu atoms and thus are not confined in radial direction by the magnetic field. Hence, they may hit the wall and be adsorbed, reducing the compression efficiency because they cannot be moved into the center of the target. These wall losses are responsible for the fact that $\varepsilon_{\text{time}}(5 \text{ mbar})$ tends to be smaller than $\varepsilon_{\text{time}}(11 \text{ mbar})$, because for higher pressure, the probability to ionize a Mu atom is higher (larger collision rate). The magnitude of the wall loss effect strongly depends on the geometry of the target, as well as on the muonium formation and ionization cross section.

Because of these effects, which are not well understood, we give a lower limit on the compression efficiency of $\varepsilon_{\text{time}}^{(1)} > 44\%$ (90\% C.L.), measured in 5 mbar He gas. Detailed simulations are ongoing which will help to improve the accuracy of the value for $\varepsilon_{\text{time}}$ further [10].

Table 4.3: Measured longitudinal compression efficiencies without ($\varepsilon_{\text{time}}^{(0)}$) and with ($\varepsilon_{\text{time}}^{(1)}$) a correction for the diffusive motion according to Eq. (4.3).

<table>
<thead>
<tr>
<th>p [mbar]</th>
<th>E [V/cm]</th>
<th>E/N [Td]</th>
<th>cell type</th>
<th>$\varepsilon_{\text{time}}^{(0)}$ [%]</th>
<th>$\varepsilon_{\text{time}}^{(1)}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-48</td>
<td>38</td>
<td>long</td>
<td>64 ± 10</td>
<td>64^{+10}_{-16}</td>
</tr>
<tr>
<td>11</td>
<td>-90</td>
<td>33</td>
<td>short</td>
<td>97 ± 10</td>
<td>97^{+10}_{-22}</td>
</tr>
<tr>
<td>5</td>
<td>-90</td>
<td>72</td>
<td>short</td>
<td>64 ± 9</td>
<td>64^{+9}_{-16}</td>
</tr>
<tr>
<td>5</td>
<td>-60</td>
<td>48</td>
<td>short</td>
<td>80 ± 11</td>
<td>80^{+11}_{-20}</td>
</tr>
</tbody>
</table>
4.3.3 Direct comparison with simulations

As a final result, we show a simulated time spectrum in Fig. 4.14, along with the corresponding measurement of the long target cell with 5 mbar He gas and an applied electric potential of -500 V. In the Geant4 simulation, a $\mu^+$ beam with $p = 11.55 \text{ MeV}/c$ momentum and a relative uncertainty on the momentum of $\Delta p / p = 3\%$ was propagated through the entrance detector and the target window foil. The normalization of the simulation to the data is done for late times between 3 µs and 4 µs. Note that for the simulation, the standard Geant4 package was used with extensions to include low-energy processes. A thorough investigation of the simulations is ongoing [10]. However, the good agreement between simulation and measured data is a strong indication that compression is very efficient and no unknown effects seem to appear.

Figure 4.14: Preliminary comparison between measured (orange dot) and simulated (black line) time spectra for 5 mbar He, -500 V and the long cell ($\pm 104 \text{ mm active region}$). The simulation was normalized to the data between $t = 3$ and $t = 4 \mu$s. Simulation data by courtesy of I. Belosevic.
4.4 Compression efficiency from the space distribution along $z$

In the 2014 setup, 21 scintillators as depicted in Fig. 4.5 were placed along the active region of the target. These detectors can be used to reconstruct the position of the $\mu^+$ swarm at various times. Fig. 4.15 shows a raw space distribution as measured with these detectors. Plotted is the number of counts in the different detectors for two time windows: $t \in [100, 200]$ ns (black) and $t \in [2500, 2600]$ ns (orange) for a negative (attractive) potential.

As visible, more events are measured in the central detectors at late times, which indicates that $\mu^+$ have been compressed into the center. Quantification of the compression efficiency from this space distribution is more challenging and less accurate than the results extracted from the time spectra (Sec. 4.3). This is due to the smaller resolution of the individual scintillators S1 to S21 compared with the T1 and T2 detectors in coincidence, and further due to larger background effects. Nevertheless, in this section, we try to deduce an approximate value for the compression efficiency from these measurements.

Figure 4.15: Counts for the 21 scintillators placed along $z$ for two time windows $t \in [100, 200]$ ns (black) and $t \in [2500, 2600]$ ns (orange). These count distributions correspond to the $\mu^+$ swarm distribution at the given times (corrected for the muon decay as usual), convoluted with the detector resolution.
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Interpretation of the spectra:

- It is important to note that the distribution plotted in Fig. 4.15 is not the number of $\mu^+$ vs. $z$, but the number of detected positrons vs. detector. The names of the detectors are indicated on top of the graph and their position ranges from $z = -100, -90, \ldots, 90, 100$ mm. The width of the peak at late times is therefore a convolution of the detectors’ geometrical acceptances and the $\mu^+$ space distribution (see also Sec. 4.4.1).

- At early times $t \in [100, 200]$ ns, the $\mu^+$ distribution has to be approximately flat as confirmed by simulations of the stop distribution at low gas pressure and by comparing measured stop distributions obtained with different gas pressures. Therefore, the increase of counts in the scintillators S1 and S2, as well as S20 and S21, cannot be related to the stop distribution in the active region, but rather to $\mu^+$ stops downstream and upstream of the active region, which are sufficiently frequent to produce some signal at the periphery of the active region. This is a background effect that we need to subtract. More details are given in Sec. 4.4.2.

4.4.1 Geometrical acceptance of the 21 scintillators

Figure 4.16 (Top) shows the simulated detector acceptance for every third of the 21 scintillators. In these histograms, the initial $z$-positions of the decay positrons hitting the scintillator is displayed. For the simulation, $10^6 \mu^+$ were generated and decayed homogeneously distributed along the $z$-axis in the region $z \in [-150, 150]$ mm. The FWHM of the single scintillator acceptance is measured to be around 25 mm. This value has to be compared with the FWHM for the two coincidence detectors T1 and T2 derived earlier (Sec. 4.3), which was found to be $8.5 \pm 0.5$ mm. Because of this larger acceptance, the resolution of the 21 scintillators is lower.

The lower plot in Fig. 4.16 shows the number of detected positrons in each detector for $2 \times 10^5 \mu^+$ generated uniformly at $z \in [-1.5, 1.5]$ mm. It demonstrates that even a fully compressed $\mu^+$ swarm will give rise to a relatively wide distribution in counts between the various scintillators. The peak shown in Fig. 4.16 corresponds thus to the simulation of 100% compression efficiency without background.

4.4.2 Background subtraction

As mentioned previously, the increased number of detected positrons of the stop distribution at the beginning (S1, S2) and the end (S20, S21) of the active region cannot arise from $\mu^+$ stopped inside the active region. In fact, Geant4 simulations confirm that in the active region and at low He gas pressure, an approximately homogeneous $\mu^+$ stop distribution is expected. Figure 4.17 shows a stop distribution for a
4.4. Compression efficiency from the space distribution along $z$

**Figure 4.16:** (Top) Simulated geometrical acceptance for every third of the 21 detectors. Shown is the position of the muon decay for a detected positron in the corresponding detector ("S1", "S4" etc.). (Bottom) Simulation of the detected space distribution along $z$ for $2 \times 10^5 \mu^+$ decay at $z \in [-1.5, 1.5]$ mm.
Figure 4.17: Simulated stop distributions in 5 mbar He gas along $z$ for different initial $\mu^+$ beam momenta. It is approximately flat within the active volume, i.e., $z \in [-104, 104]$ mm.

$\mu^+$ beam crossing the entrance detector (55 $\mu$m plastic scintillator at $z = -400$ mm), the gas target window (2 $\mu$m Mylar foil covered with 0.4 $\mu$m Al on both sides at $z = -206$ mm), and then stopping in 5 mbar He gas. The initial momentum of the beam was chosen between 11.55 MeV/c and 12.30 MeV/c, as labeled in the figure, with $\Delta p/p = 3\%$ momentum bite. To support the validity of this simulation, we have performed a time-of-flight (ToF) measurement using an empty cell.

Figure 4.18 shows the time spectra of various scintillators. The kinetic energy in the empty target extracted from this ToF measurement is $E_{\text{kin}} = 66 \pm 2$ keV. This has been calculated via the equation

$$E_{\text{kin}} = \frac{1}{2}mv^2 = \frac{1}{2}m \left( \frac{d}{\Delta t} \right)^2,$$

where $m$ is the muon mass, $v = d/\Delta t$ the velocity, $d = 200$ mm the distance between the first and the last scintillator and $\Delta t$ is the time between the two maxima of the time spectra for S1 and S21, respectively. The given uncertainty on $E_{\text{kin}}$ stems solely from the uncertainty of the fit on $t$.

The measured $E_{\text{kin}}$ from the ToF is well in agreement with the average kinetic energy in the target obtained from a Geant4 simulation when starting with an initial $\mu^+$ beam momentum of 11.55 MeV/c with $\Delta p/p = 3\%$ momentum bite. After
4.4. Compression efficiency from the space distribution along \( z \)

![Figure 4.18: Time spectra measured in five detectors (S1, S6, S11, S16, and S21, respectively) for an empty gas target. The position of the maxima, extracted by fitting a Landau distribution, is used to extract the average kinetic energy in the empty target.](image)

passing through the entrance detector and the target window foil, the average kinetic energy is \( E_{\text{kin}} = 65.6 \pm 1.3 \) keV. The small difference observed from the nominal beam line momentum of 11.2 MeV/c can be related to the inaccuracy of the beam line momentum (scaling of the magnets, hysteresis effects etc.), to the uncertainty in the thickness of the foil and entrance detector, and to the calculation of \( E_{\text{kin}} \) using the maximum of a fitted Landau distribution.

Therefore, we have indirectly demonstrated that the increase of the counts in the outermost scintillators is not caused by an inhomogeneous stopping distribution in the gas. A measurement accomplished with the empty cell has been performed to understand this increase, see Fig. 4.19 (a). In this case, there are no \( \mu^+ \) stopping in front of the detectors. Nevertheless, there are signals at late times induced from \( \mu^+ \) which stopped either upstream (entrance detector, Mylar foil) or downstream the active region. These \( \mu^+ \) may produce signals in the scintillators S1 to S21 with decreasing probability close to the target center.

The empty cell measurement presented in Fig. 4.19 (a), corresponding to a background measurement, is scaled and subtracted from the measured compression spectra. Scaling is done with the goal to minimize the residuals between a linear function and the initial stop distribution whereof the scaled background contribution has been subtracted. The scaling factor is the parameter which was varied.
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Figure 4.19: (a) Initial stop distribution along z-axis for the long cell with 5 mbar He gas pressure. The blue line represents the count distribution of an empty target (background events). This histogram results from subtracting the blue curve from the black curve so that the flatest distribution is obtained. This flatness is evaluated using a \( \chi^2 \)-test comparing the result of this subtraction to a linear function. (b) Approximately flat \( \mu^+ \) stop distribution along the z-axis in the gas after subtracting the background contribution. (c) Initial (\( t \in [100, 200] \) ns, black) and final (\( t \in [2500, 2600] \) ns, orange) distribution of the counts along the z-axis. (d) Same space distribution of the counts as in (c) after subtracting the background contribution.
4.4. Compression efficiency from the space distribution along $z$

The justification for this procedure is given in the discussion above, in which we concluded that the $\mu^+$ stop distribution is approximately homogeneous along $z$. The resulting initial stop distribution after background subtraction is shown in Fig. 4.19 (b).

4.4.3 Extracting the compression efficiency

Figure 4.19 (c) illustrates that, although background subtraction was performed as described in the previous section, there is still a considerable background at late times (in the orange histogram) for the outer detectors. The most likely source for these counts are $\mu^+$ stops in the inactive region. In the following, a simulation for various compression efficiencies is presented, which takes the detector hits due to $\mu^+$ decays outside the active region into account. In this way, for every assumed compression efficiency, we obtain a histogram similar to the one shown in Fig. 4.19 (d). By performing a $\chi^2$-test comparing the experimentally obtained histogram with the simulated histograms for various assumed compression efficiencies, it is possible to extract limits on the compression efficiency.

The muon distribution for an assumed compression efficiency of 100% (orange) and 70% (shaded blue), respectively, is sketched in Fig. 4.20. In the case of 100% compression efficiency, a constant number $N_0$ of $\mu^+$ decay outside the active region (neglecting the effect of diffusion), whereas all the other $\mu^+$ are within 3 mm of the active region.
center of the target.\(^3\) Hence, in the peak we find \(N_{\text{center}} = 100\% \cdot 208/3 \cdot N_0\), where the factor \(208/3\) is the active length divided by the peak width. For the case in which only 70% compression efficiency is assumed, the number of \(\mu^+\) decays outside the active region remains the same. But now only \(0.7 \cdot 208/3 \cdot N_0\) muons decay in the central peak, and additionally \(0.3 \cdot N_0\) muons decay homogeneously distributed along the active region. The outcome in terms of detected positron count distribution of such a simulation is shown in Fig. 4.21 in orange for 100% compression efficiency and in blue for 70% compression efficiency, respectively.

Histograms similar to those shown in Fig. 4.21 were obtained from simulations for various compression efficiencies \(\varepsilon_{\text{sim}}\), which we denote with \(H_{\text{sim}}(\varepsilon_{\text{sim}})\). These histograms were then fitted to the experimentally measured space distribution with the fit function

\[
H_{\text{total}}(\varepsilon_{\text{sim}}) = w_1 H_{\text{const}}^{\text{bg}} + w_2 H_{\text{tilt}}^{\text{bg}} + w_3 H_{\text{sim}}(\varepsilon_{\text{sim}}),
\]

where \(w_i\) are the three fit parameters, \(H_{\text{const}}^{\text{bg}}\) is a constant histogram, and \(H_{\text{tilt}}^{\text{bg}}\) is a histogram with filling linear in \(z\). The decomposition of \(H_{\text{total}}(\varepsilon_{\text{sim}})\) as in Eq. (4.5) is schematically shown in Fig. 4.22. The histogram \(H_{\text{tilt}}^{\text{bg}}\) takes into account an asymmetric stop distribution or different contributions to the background from the upstream.

\(^3\)The actual width of the central peak does not matter: We have no sensitivity for variations of the peak width up to a width of 10 mm, see Fig. 4.16.
and downstream part of the target. Its slope was determined by fitting the outermost four bins on each side of the background subtracted histogram simultaneously (i.e., Fig. 4.19 (b)) with a linear function (for both gas pressures separately). The determination of the slope was performed before fitting $H_{total}(e^{sim})$ with Eq. (4.5), and the constant in the linear fit of $H_{tilt}^{bg}$ to the background subtracted histogram was set to zero. Hence, the outermost bin of the histogram $H_{tilt}^{bg}$ is empty. Once the slope of $H_{tilt}^{bg}$ has been determined, the weight $w_2$ was set to 1. So to be more precise, only $w_1$ and $w_3$ were free fit parameters in Eq. (4.5). The numerical values of $w_1$ and $w_3$ depend on the total number of entries in the histograms $H_{const}^{bg}$ and $H_{sim}(e^{sim})$.

The $\chi^2$ obtained from the fit of $H_{total}(e^{sim})$ to the measured data gives insight into the compatibility of the histograms [66]. This $\chi^2$-test is available from the ROOT data analysis framework [67]. The extracted $\chi^2$, with $\chi^2_{red} = \chi^2$/n.d.f. and n.d.f = 19 degrees of freedom, is plotted in Fig. 4.23 (Top) as a function of the assumed $e^{sim}$. It is a measure how well the two histograms (the measured data and the simulation $H_{total}$) agree with each other. Values for $\chi^2_{red}$ larger than 1 indicate that there is a discrepancy between our simplified model and the measured data, and is partly due to small errors on the simulated histogram because of large statistics in the simulation. As can be seen, we have not much sensitivity to extract a value for $e_{space}$, but a lower limit can be given. The low sensitivity can already be anticipated by looking at Fig. 4.21. If the histogram $H_{sim}(70\%)$, which represents the expected space distribution in the case of 70% compression efficiency, is scaled appropriately, it will not differ significantly from $H_{sim}(100\%)$, i.e., the case of 100% compression efficiency.

Usually, the $\chi^2$ as a function of the test parameter can be approximated by a second order polynomial. In such a case, the $\pm n\sigma$ standard deviation can be obtained graphically by extracting the values of the test parameter at $\chi^2_{min} \pm n$, in which $\chi^2_{min}$ denotes the minimum of the parabola (note that this is not the reduced $\chi^2$) [68, 69]. However, for our case, we set a limit on the efficiency and require $e < 100\%$, thus we have not the full parabola. Nevertheless, the $1\sigma$ uncertainties can be found by the method just explained, and we obtain $e_{space}^{(0)} > 60\%$, and
Figure 4.23: (Top) The $\chi^2$ between measurements (after background subtraction) and $\mathcal{H}_{\text{total}}(\varepsilon_{\text{sim}})$ as function of $\varepsilon_{\text{sim}}$. The limited sensitivity allows only to extract lower limits for the efficiency. We find $\varepsilon_{\text{sim}} > 60\%$ and $\varepsilon_{\text{sim}} > 65\%$ for 5 and 11 mbar, respectively (90% C.L.). (Bottom) The histogram $\mathcal{H}_{\text{total}}$ according to Eq. (4.5) for $\varepsilon_{\text{sim}} = 80\%$. 
4.4. Compression efficiency from the space distribution along $z$

$\varepsilon_{\text{space}}^{(0)} > 65\%$, for both 5 mbar and 11 mbar, respectively, (90% C.L.).\footnote{Since we are dealing with discrete values of $\varepsilon_{\text{sim}}$, the standard deviation was extracted by requiring $\chi^2(\varepsilon_{1\sigma}) \geq \chi^2_{\text{min}} + 1$, where $\chi^2(\varepsilon_{1\sigma})$ is the $\chi^2$ for the largest $\varepsilon_{\text{sim}}$ still satisfying the inequality.} This efficiency is very well in agreement with the results $\varepsilon_{\text{time}}^{(0)}$ obtained from the time spectra presented in Sec. 4.3.

For comparison, in the lower panel of Fig. 4.23 the measured space distribution for 5 mbar He gas in the long target cell is plotted together with $\mathcal{H}_{\text{total}}(\varepsilon_{\text{sim}})$ for $\varepsilon_{\text{sim}} = 80\%$ (orange). Additionally, the three contributions $\mathcal{H}_{\text{const}}, \mathcal{H}_{\text{tilt}}$, and $\mathcal{H}_{\text{sim}}(\varepsilon_{\text{sim}})$, weighted appropriately, are also shown in blue. The good agreement between a simple simulation and the measured data, together with the consistency between the compression efficiencies extracted from the time spectra and the space distribution, represents a major step towards the realization of the proposed scheme for phase space compression.

4.4.4 Diffusive motion of the $\mu^+$ obtained from the experiment

The diffusion of stopped $\mu^+$ in the gas where no electric field is present allows them to enter the active region and, subsequently, be accelerated towards the center of the target. This diffusion would manifest itself in the data by an increased number of detected positrons at later times. Indeed, the integral number of counts between $t \in [100, 200]$ ns and $t \in [2.5, 2.6]$ $\mu$s increases by $(6.8 \pm 1.6)\%$ (averaged for 5 and 11 mbar). However, the geometrical acceptance for a muon swarm located in the center of the target is also about 8% higher, compared to a swarm which extends over the full active region, thus no net diffusion effect was measured.

For the same reasoning as in Sec. 4.3.2, we assume a diffusion of $D = (0^{+20}_{-0})\%$, to account for the discrepancy between $D_{\text{exp}} \approx 0$ and $D_{\text{sim}} \approx 20 – 30\%$. The lower limits on the compression efficiencies $\varepsilon_{\text{space}}^{(0)}$ extracted from the space distribution are corrected with $D$ using Eq. (4.3), and we obtain, again with 90% confidence,

\[
\varepsilon_{\text{space}}^{(1)}(5 \text{ mbar}) > 55\%
\]
\[
\varepsilon_{\text{space}}^{(1)}(11 \text{ mbar}) > 60\%
\]

Several factors contribute to the discrepancy between $D_{\text{sim}}$ and $D_{\text{exp}}$. Firstly, it is not possible to eliminate all background events in the data, which decreases the calculated contribution of $\mu^+$ diffusing into the active region. Examples for such backgrounds could be the underestimation of wall stops, or a general flat background arising from electrically neutral bound states of $\mu^+$ hitting the lateral walls during compression. Secondly, $D_{\text{sim}}$ depends sensitively on the energy of the $\mu^+$ when entering the target. However, in the simulation the $\mu^+$ are generated at rest,
homogeneously distributed in z-direction. For $\mu^+$ at rest, no significant diffusion into the active volume is observed. The diffusion depends sensitively on the $\mu^+$ cross sections in the keV regime (i.e., for $\mu^+$ which are not at rest), which are still investigated [10]. And thirdly, the electric field close to the border of the active region is not precisely known, whereas in the simulation, it is perfectly defined. Further studies of the diffusive motion are ongoing and once a full simulation is established, this issue will also be addressed.

4.5 Compression with impurities in the helium gas

The following section investigates the effect of gas impurities on the compression efficiency. In the 2011 experiment, it was claimed that impurities in the He gas were the reason that the experimentally measured compression stops already at times $t \approx 500$ ns [65]. At that point, there was a disagreement between theory (simulation) and data. The measured time spectra (see Fig. 4.24) showed that compression is already finished around 500 ns (for 12 mbar He). However, in the simulations, compression continued for times larger than 2 $\mu$s.

To overcome this discrepancy between simulation and measurement, a “chemical capture” effect was introduced into the simulation. It was assumed that for energies below a cutoff energy of 10 eV, the muons interact with impurities to form muonic ions or to replace a proton from such a molecule. To account for this chemical capture, a constant loss rate $R$ was introduced for muons below this cutoff energy. The rate was fitted to the spectrum and the result was that $R = 40 \times 10^6$ s$^{-1}$ for energies smaller than 10 eV.

In 2014, we injected impurities into the longitudinal gas target in a controlled way in order to prove the impurity hypothesis introduced in 2011 to explain the data. As shown in Fig. 4.25, we measured the time spectrum using the coincidence detectors (T1, T2) for various admixtures of $H_2$ and $O_2$ contaminants. The solid black line shows the compression in 5 mbar He gas for -500 V and no impurities. The dotted blue and orange lines represent the time spectra where 0.01 mbar (blue) and 0.02 mbar (orange) of $H_2$ were added into the He gas. The dashed green line shows the case where 0.01 mbar $O_2$ was introduced into the He gas.

Because the number of detected positrons in the coincidence detectors (T1, T2) decreases considerably when impurities are introduced in the He gas, we have proven that the impurities are considerably reducing the compression efficiency. Thus, the purity of our gas must have contaminants concentrations much smaller than 1‰. This impurity concentration will play an even more important role when the total setup is realized because of the longer times needed for the $\mu^+$ to go along the various stages (8-10 $\mu$s). In the following section, a simple model is developed describing this $\mu^+$ chemical capture.
4.5. Compression with impurities in the helium gas

4.5.1 Collision rate and polarizability

A muon passing by a neutral system (atom or molecule) induces a rearrangement of charges in the system. The ease of this rearrangement is quantified using the polarizability $\alpha$ of the system. In leading order, for large distances between the $\mu^+$ and the neutral system, the rearrangement of the charges corresponds to an electric dipole. This induced dipole is related to the polarizability of the atom or molecule by

$$d_{\text{ind}} = \alpha E', \quad (4.6)$$

where $E'$ is the strength of the electric field produced by the muon at the position of the neutral system. The potential energy of this dipole in the electric field is

$$U_{\text{pot}} = -d_{\text{ind}} \cdot E' = -\alpha (E')^2. \quad (4.7)$$

Therefore, the attractive potential between a $\mu^+$ and a polarizable neutral system can be described as

$$U_{\text{pot}} = -\frac{\alpha e^2}{r^4} \quad (4.9)$$

because $E' = e/r^2$ for a point-like charge.

Following a classical trajectory description as presented in Appendix C or Refs. [70, 71], the knowledge of this potential can be used to compute the cross

![Figure 4.24](image-url)

**Figure 4.24:** Simulated time spectrum with (dashed lines) and without (solid lines) chemical capture for the 2011 setup (reproduced from [65]). If one allows for chemical capture of the $\mu^+$, the time spectra saturates at early times.
section for the formation of a bound system. This cross section reads as

$$\sigma(v) = \frac{\pi}{v} \sqrt{\frac{4e^2\alpha}{M_r}}, \quad (4.10)$$

where $M_r$ is reduced mass of the $\mu^+$ and the contaminant, which can be approximated for simplicity by the $\mu^+$ mass, and $v$ is the $\mu^+$ velocity, which is much larger than the one of the heavy, thermal contaminant. Note that the cross section of Eq. (4.10) is larger than the cross section for forming a bound state, which is what is actually measured in the experiment. The cross section of Eq. (4.10) only describes the case where the $\mu^+$ is attracted towards the contaminant, but it does not necessarily have to form a bound state. Nevertheless, we approximate the chemical capture formation rate $R$ by

$$R = \frac{\pi}{v} \sqrt{\frac{4e^2\alpha}{M_r}} vn = n \sqrt{\frac{4\pi^2e^2}{M_r}} \sqrt{\alpha} = nk\sqrt{\alpha}, \quad (4.11)$$

where $n$ is the gas number density ($n = 1.3 \times 10^{17} \text{ cm}^{-3}$ for 5 mbar).
4.5. Compression with impurities in the helium gas

4.5.2 Model to describe the muon "chemical capture"

Assuming that no impurities are in the gas, the change in the number of $\mu^+$ inside the acceptance region of the detector is given simply by the flux

$$dN_{\text{pure}} = \rho v_D dt,$$  

where $\rho$ is the density of $\mu^+$ and $v_D$ their drift velocity (we assume a one-dimensional case here for simplicity). It was verified in a simulation that the average $v_D$ does not change by more than a factor of 2 between $t = 1 \mu s$ and $t = 3 \mu s$, which is sufficient for our level of approximation. Integration of Eq. (4.12), assuming $\rho = \rho_0 = \text{constant}$, leads to the number of $\mu^+$ in the central part of the target:

$$N_{\text{pure}}(t) = \rho_0 v_D t + \tilde{N}_0.$$  

The integration constant $\tilde{N}_0$ represents the initial amount of stopped $\mu^+$ before the compression starts. Saturation effects as visible in previously shown time spectra (e.g., Fig. 4.12) occur due to the fact that we are dealing with an active region of finite size.

When impurities are present, the density $\rho$ of free $\mu^+$ (not bound by chemical capture) subject to the compression changes with time as $d\rho = -R \rho dt$. Integrating yields $\rho(t) = \rho_0 e^{-Rt}$. Of course, this is only valid for the lifetime-compensated case.

Equation (4.12) thus is - when impurities are present - modified to

$$dN_{\text{imp}}(t) = \rho(t) v_D dt.$$  

Its solution reads

$$N_{\text{imp}}(t) = N_0 - \frac{\rho_0 v_D}{R} \cdot e^{-Rt}.$$  

For $t = 0$ we get: $N_{\text{imp}}(t = 0) = N_0 - \rho_0 v_D / R = \frac{1}{R} N_{\text{pure}}(t = 0) = \tilde{N}_0$. This defines the relation between $N_0$ and $\tilde{N}_0$ and allows to rewrite $N_{\text{imp}}(t)$:

$$N_{\text{imp}}(t) = \tilde{N}_0 - \frac{\rho_0 v_D}{R} \left( e^{-Rt} - 1 \right).$$  

The ratio of muons compressed into the center with and without impurities after subtraction of the initial number of muons is

$$\frac{N_{\text{imp}}(t) - \tilde{N}_0}{N_{\text{pure}}(t) - \tilde{N}_0} = -\frac{\rho_0 v_D}{R} \left( e^{-Rt} - 1 \right) = -\frac{1}{R} \left( e^{-Rt} - 1 \right).$$  

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This ratio corresponds to the ratio of compression efficiencies measured with and without impurities at a given time $t$:

$$\frac{N_{\text{imp}}(t) - \tilde{N}_0}{N_{\text{pure}}(t) - \tilde{N}_0} = \frac{\varepsilon_{\text{imp}}(t)}{\varepsilon_{\text{pure}}(t)}.$$  \hfill (4.18)

Figure 4.26 shows this ratio for $t = 2 \mu s$ and the data already presented in Fig. 4.25. It represents the decrease of the compression efficiency as a function of the contaminants concentration. A fit of the data yields $(Rt)_{\text{fit}} = 135 \pm 10$ for $p = 1$ mbar, where the error is solely due to the uncertainty on the fit value. By using $t = 2 \mu s$ from the fit, we obtain

$$R = 67 \pm 5 \times 10^6 \text{ s}^{-1}.$$  \hfill (4.19)

This number has to be compared with the value that can be calculated with Eq. (4.11) (see also Appendix C):

$$R = 166 \times 10^6 \text{ s}^{-1} \quad \text{(for 1 mbar H}_2\text{)}$$  \hfill (4.20)

$$R = 231 \times 10^6 \text{ s}^{-1} \quad \text{(for 1 mbar O}_2\text{)}$$  \hfill (4.21)
for \( p = 1 \) mbar and the polarizabilities for \( \text{H}_2 \) and \( \text{O}_2 \), which are known from literature [72]:

\[
\begin{align*}
\alpha(\text{H}_2) &= 0.818 \times 10^{-24} \text{ cm}^3 \\
\alpha(\text{O}_2) &= 1.570 \times 10^{-24} \text{ cm}^3
\end{align*}
\]

Therefore, our simple model correctly predicts the \( \mu^+ \) chemical capture rate for \( \text{H}_2 \) within a factor of 3. Our value is smaller than the theory value, however, this comparison is only qualitative, with several limitations. On the one hand, the experimentally measured rates being deduced at times \( t = 2 \) \( \mu \)s when saturation effects already occur are slightly underestimated. Furthermore, the cross sections calculated from theory are orbiting cross sections. Even if the \( \mu^+ \) orbits towards the contaminant, it does not necessarily form a bound state and become captured. And finally, the rate computed from theory (see Appendix C) is based on a classical approximation.

In summary, we have demonstrated that gas impurities strongly affect our compression efficiency. Assuming that we tolerate a decrease of the efficiency due to chemical capture of only 10\%, we found with our simple model describing the decrease in compression efficiency that for the longitudinal compression stage, the \( \text{H}_2 \) concentration must be below 0.002 mbar. Assuming a pressure of 5 mbar He gas, this corresponds to 400 ppm. However, it is important to note that this requirement is increased in the setup when all stages are connected because the \( \mu^+ \) are drifting in the gas for 8-10 \( \mu \)s. By scaling with that time (factor 4-5) and because we have possibly underestimated the "capture rate" due to saturation effects in the experiment, we find that in the final setup, we have to have a gas with contaminants concentrations below the 400 ppm \( \times \frac{2 \mu \text{s}}{10 \mu \text{s}} \times \frac{1}{3} \approx 25 \text{ ppm} \) level. The additional factor of 3 accounts for the above explained saturation effects. Moreover, contaminants other than H might have significantly larger capture cross sections, setting more stringent limits on the tolerable contaminants concentration. However, this exceeds the scope of this thesis and is not considered any further.

### 4.6 Conclusions of the 2014 tests

The experiment in 2014 was significantly improved compared to the first one in 2011. The major conclusion is that the compression efficiency of the longitudinal compression stage is \( \varepsilon^{(1)}_{\text{time}}(5 \text{ mbar}) > 44\% \) (90\% C.L.) and \( \varepsilon^{(1)}_{\text{time}}(11 \text{ mbar}) = (97^{+10}_{-22})\% \) for 5 and 11 mbar, respectively, when calculated from the time spectra. The effect of \( \mu^+ \) diffusing into the active volume limits the precision of the statements about the compression efficiency and requires a thorough simulation. Therefore, for the 2015
setup, the electric field has been designed such that no $\mu^+$ can diffuse into the active volume, which will simplify data analysis considerably (see next section).

The space distribution of the positron counts in the various detectors has been analyzed at different times and allowed us to deduce a $\mu^+$ swarm compression with efficiencies of $\varepsilon_{\text{space}}^{(1)} > 55\%$ and $\varepsilon_{\text{space}}^{(1)} > 60\%$ (90\% C.L.) for 5 and 11 mbar, respectively, in good agreement with the values extracted from the time spectra.

Additionally, we verified the impurity hypothesis formulated in 2011 to account for the faster saturation of compression measured experimentally than predicted from simulations. By introducing a certain amount of contaminants in the He gas in a controlled way, we could demonstrate that the compression efficiency is indeed strongly reduced due to chemical capture effects. Considering the full compression target, the contamination in the He gas should stay below the 25 ppm level in order to not significantly reduce the efficiency of the phase space compression scheme.
4.7 Longitudinal compression 2015

After the successful experiments in 2014, and the obtained high compression efficiency $\varepsilon^{(1)} > 44\%$ (90% C.L.), the goal for a repetition of the longitudinal compression experiment in 2015 was twofold. First, the diffusion processes from the inactive region into the active region, complicating data analysis and the determination of the compression efficiency, should be suppressed. And second, an additional vertical electric field was applied, giving rise to a drift of the $\mu^+$ in $+x$-direction by means of the $\hat{E} \times \hat{B}$-component of Eq. (3.3). This drift serves to move the $\mu^+$ towards the next stage where the extraction into vacuum will occur. Therefore, the transition between the longitudinal and the extraction stage was tested.

4.7.1 The 2015 target design

The new target realized for the 2015 beam time has two additional features compared to the 2014 setup:

- It eliminates the diffusion of $\mu^+$ at late times into the active region. To do so, additional electrode strips are added at the periphery of the target to define an electric field $E_{\text{out}}$ pointing outwards as shown in the upper panel of Fig. 4.27.

- It adds a vertical electric field component $E_y$ which causes a drift in $+x$-direction. This was achieved by using tilted electrodes at the vertical, lateral side walls. The resulting electric field is sketched in the lower panel of Fig. 4.27, and a simulation of the electric field obtained using the COMSOL multiphysics tool [73] is present in Fig. 4.28.

The target has been realized using a Kapton foil lined with many electrodes, as shown in Fig. 4.29. Appropriately folding the Kapton foil is used to generate a volume having electric field lines in $y$- and $z$-direction. How the Kapton foil is folded is sketched on the magnification on the right hand side of Fig. 4.29.

The Kapton foil is glued into a U-shaped support and a cover. The internal part of the U-shaped profile is made of an insulator. It is then hermetically sealed so that outside the target there is air at ambient pressure, whereas inside we put about 5 mbar He gas. Folding the Kapton foil gives rise to the three-dimensional electrode arrangement and the corresponding E-field as shown in Fig. 4.28.

The photos presented in Fig. 4.30 show in detail the Kapton foil glued into the U-shape before being sealed to the beam line. The advantage of the folding procedure is that the amount of glue being in contact with He gas is minimized. Furthermore, the number of HV connectors needed to establish the electric field, and the complexity of the arrangement of the voltage dividers to define the required fields, are
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Figure 4.27: Electric field configuration in the longitudinal target of 2015. The red arrows indicate the additional field component added in the 2015 setup as compared to 2014. (Top) $E_{\text{out}}$ is added to prevent $\mu^+$ from diffusing into the active area. (Bottom) Applying an additional field in $y$-direction moves the $\mu^+$ towards the extraction stage due to the $E \times B$-drift.

Figure 4.28: Cut in the $yz$-plane of a 3-D simulation from the COMSOL multiphysics tool where the electric potential (color) and the electric field (arrows) are depicted. In black are the (here transparent) electrodes with a $30^\circ$ angle relative to the $z$-axis.
Figure 4.29: Flat Kapton foil with soldered resistors and electrodes. The equipotential electrodes (red) mark the borders of the active region. Below the photo is a sketch of the electric potential. The magnification on the right hand side shows how the Kapton foil is folded and glued into the U-shape support structure (see also Fig. 4.30).
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Figure 4.30: (Top Left) Photo showing the U-shaped profile into which the Kapton foil is glued. (Top Right) The brass shielding into which the U-shaped profile is mounted. (Bottom) Almost finished target: resistors on the Kapton foil, the electrodes to apply electric potential - altogether surrounded by massive brass shielding to absorb the $e^+$ from $\mu^+$ decay.

optimized. When the Kapton foil is folded, all four walls of the target are connected so that by defining the electric potential at one wall, the electric potentials on all the other walls are automatically defined. Again, two different lengths of the active region could be chosen by setting the appropriate electrodes to ground, but due to failure of an electrode, only the data measured with the long cell can be evaluated.

The electric field strength for the long cell is $E=(HV_1[V]/9.85) \text{ V/cm}$. The electrodes on the Kapton were designed in such a way that the $y$-component is exactly a factor of 2 larger than the $z$-component: $E_y = 2E_z$. The electric potential is always applied at the central electrode (labeled HV$_1$ in Fig. 4.29), whereas several connectors can be used to define the ground position. The electric field $E_{out}$ is generated by additionally applying a negative potential at the electrodes HV$_2$, preventing stopped $\mu^+$ from diffusing into the active region. The insert in Fig. 4.29 shows the pattern of electrodes in the central part, which creates the vertical component of the electric fields.
4.7. Longitudinal compression 2015

Figure 4.31: Cross section of the longitudinal target 2015 at $z = 0$. The seven scintillators B11 to B17 are dedicated to detect the $E \times B$-drift, whereas B3 and T3 in coincidence are used to expose the longitudinal compression. Massive brass shielding around the target reduces the background significantly and increases the spatial detector resolution. To allow the $\mu^+$ to drift a longer distance in $+x$-direction, the $\mu^+$ beam was injected off-axis with its center at $x = -5$ mm.

This U-shaped target is then enclosed in a brass support into which the scintillators are mounted. The same principle as in 2014 was used to detect the decay positrons: Scintillating bars, attached to SiPMs, are installed at dedicated positions. In order to improve the resolution of the detectors, the U-shape is surrounded by massive brass shielding, which is drawn in yellow in Fig. 4.31. This figure shows a cross section through the target in the center at $z = 0$ mm. The Kapton foil (red) is glued into the U-shaped support structure (gray), wrapped in a thin PVC insulator (green) to avoid electric field distortions. This is then embedded into the brass collimators (yellow). Shown in blue are the scintillators: B11 to B17 are used to detect the $\mu^+$ drift into $+x$-direction, whereas the scintillators Bi and Ti (with $i = 1, \ldots, 5$) detect the compression (only B3 and T3 are visible in Fig. 4.31). They are detecting coincidence events which narrows the geometrical acceptance in $z$-direction and hence improves the resolution, as concluded in 2014.

Another feature introduced in 2015 is the off-axis injection of the $\mu^+$ beam. Due to the additional vertical field component, the $\mu^+$ will drift in $+x$-direction. The off-axis injection allows the $\mu^+$ to drift for a longer time before hitting the lateral side wall. The point of injection at $x = -5$ mm is also marked in Fig. 4.31.
4.7.2 Geometrical acceptance in 2015

With a Geant4 simulation, the geometrical acceptance of the central detectors T3 and B3 in coincidence was determined,\(^5\) analogously to 2014 and as already presented in Sec. 4.3.1. Plots of this acceptance for some variations of the detector placements are shown in Fig. 4.32.

To compute \(f_{\text{ideal}}\) (see Eq. (4.2)), the length of the active region must be determined. Because we have strongly slanted electrodes, this length depends on the vertical position of the beam. To account for possible beam misalignments, we attribute an uncertainty of ±6 mm to this length. The uncertainty on the geometrical acceptance due to a similar sensitivity analysis as already performed for the 2014 setup is estimated to be 10%. This is slightly larger than in 2014 (there, the uncertainty on \(f_{\text{ideal}}\) was about 8%), which is due to the longer scintillators, and that not all possible misalignments were accounted for in simulations. Thus, we find \(f_{\text{ideal}} = 12.2 \pm 1.5\) by adding the independent uncertainties in quadrature. The measured FWHM of the geometrical acceptance is 10.5 ± 1.0 mm, i.e., about 2 mm larger than in 2014.

The geometrical acceptance for the 7 drift detectors is shown in Fig. 4.33. Knowing the geometrical acceptance is the key to demonstrate the \(\bar{E} \times \bar{B}\)-drift: An increase in detector counts with time indicates a movement of the (compressed) \(\mu^+\) swarm towards a region of higher detector acceptance, as discussed in more detail in Sec. 4.9.

\(^5\)The length of the active volume is not fixed in 2015 but depends on the vertical position \(y\). Thus the average length of the active region is used when calculating \(f_{\text{ideal}}\): \(L_{\text{avg}} = 2 \times 86.5\) mm.
4.8 Compression in 2015

Recalling the discussion in Sec. 3.6, the runaway condition is fulfilled if
\[ E/N \gtrsim 36 \text{Td} \]. In the 2015 experiment, it was possible to reach \( E/N = 29 \text{Td} \)
(8 mbar, 600 V). If a larger electric potential was applied, electric discharges started
to occur. Since runaway conditions are not met, it has to be judged from each individual
time spectrum whether the compression is finished. If the time spectrum
becomes flat at a certain time (as for example at \( t \approx 5 \mu\text{s} \) for the case of the black curve in Fig. 4.34, representing the situation where \( E/N = 29 \text{Td} \)), the compression
is finished, and hence the compression efficiency at that time can be calculated.
However, if the time spectrum continues to increase for later times and never
becomes flat, as in the case of the orange histogram in Fig. 4.34 (representing the situation where \( E/N = 19 \text{Td} \)), compression is ongoing for times up to \( t = 10 \mu\text{s} \). In
this case we can only extract the percentage of \( \mu^\pm \) compressed into the target center
at a certain time. This means only a lower limit on \( \varepsilon_{\text{time}} \) can be given.
In order to calculate the compression efficiency $\varepsilon_{\text{time}}$ we recall the formula presented in Eq. (4.3): $\varepsilon_{\text{time}}^{(1)} = (a/b)/(f_{\text{ideal}}(1 + D))$. Because of the additional electric field preventing $\mu^+$ from diffusing into the active region, $D = 0$, and thus

$$\varepsilon_{\text{time}}^{(1)} = \frac{a/b}{f_{\text{ideal}}}.$$  

(4.22)

This allows a straightforward calculation of the compression efficiency without requiring knowledge about diffusion processes obtained from a simulation. The results presented in Table 4.4 summarize the compression efficiency for two different reduced electric fields $E/N$. Because the compression is not finished after 2.5 $\mu$s, the values of $a$ and $b$ were determined between $t = 5$ $\mu$s and $t = 7.5$ $\mu$s. The averaged compression efficiency after 5 $\mu$s for $E/N = 29$ Td is $\varepsilon_{\text{time}}^{(1)} = (107 \pm 16)$%.

**Compression efficiency for conditions below the runaway threshold**

If runaway conditions are not met, we can determine a lower limit of $\varepsilon_{\text{time}}^{(1)}$ at a certain time $t$. Here, we used $t \in [2, 2.5]$ $\mu$s. As can be seen from the time spectra (Fig. 4.34), most of the $\mu^+$ have been compressed into the center at this time. The compression
4.8. Compression in 2015

Table 4.4: Compression efficiency $\epsilon_{\text{time}}^{(1)}$ calculated from Eq. (4.22) for $t \in [5, 7.5] \mu s$ for the experiment in 2015, for various reduced electric field strengths.

| p [mbar] | E [V/cm] | E/N [Td] | $f_{\text{ideal}}$ | a/b | $\epsilon_{\text{time}}^{(1)}$ [%] |
|----------|----------|----------|-----------------|-----|----------------|--------|
| 8.5      | 61       | 29       | 12.2 ± 1.5      | 12.3±1.4 | 100 ±17       |
| 8.5      | 61       | 29       | 12.2 ± 1.5      | 14.4±1.8 | 117 ±21       |
| 8.5      | 51       | 24       | 12.2 ± 1.5      | 10.5±1.3 | 86 ±15        |
| 8.5      | 51       | 24       | 12.2 ± 1.5      | 12.1±1.4 | 99 ±17        |
| 10.5     | 61       | 23       | 12.2 ± 1.5      | 12.1±1.5 | 99 ±17        |

Efficiencies at $t = 2.5 \mu s$ are presented in Fig. 4.35, where the compression efficiency is plotted as a function of $E/N$, according to Eq. (4.22). We obtained about 50% compression efficiency at $t = 2.5 \mu s$ already for $E/N = 22 \text{Td}$, which translates into an electric field of 55 V/cm at 10 mbar He gas pressure. Extrapolating linearly in Fig. 4.35 yields 100% compression efficiency at $t = 2.5 \mu s$ for a reduced electric field of $E/N = 30.9 \pm 5.5 \text{Td}$, which is in agreement with the calculated value for the onset of runaway (Sec. 3.6). However, due to electric discharges in the target cell, it was not possible to reach values $E/N \geq 30 \text{Td}$.

Finally, we can extract the average of the compression efficiency after 2.5 $\mu s$, for the two measurements which were closest to runaway condition ($E/N = 29 \text{Td}$):

$$\epsilon_{\text{time}}^{(1)} (t = 2.5 \mu s) = 90 \pm 13\%.$$
4.9 \( \hat{E} \times \hat{B} \)-drift in 2015

The major advancement compared to the experiment in 2014 is the addition of a vertical electric field component \( E_y \). This results in an \( \hat{E} \times \hat{B} \)-drift towards the final stage of the full compression (see Figs. 3.1 and 3.5), where extraction into vacuum will occur.

To detect this drift, seven dedicated scintillators (B11, ..., B17) were mounted along the \( x \)-direction at \( z = 0 \), as shown in Fig. 4.31. Because these drift detectors are placed at \( z = 0 \), they are also sensitive to the longitudinal compression. Therefore, the time spectrum will, for early times, be dominated by the compression. However, once the compression is finished, the drift detectors will be sensitive to the muon drift in \( +x \)-direction.

The expected structure of the time spectra is sketched in Fig. 4.36, together with a schematic of the detector position. At early times, there is an increase in all detectors due to compression in \( z \)-direction, then the drift in \( +x \)-direction becomes visible. For the drift detectors located at \( x < 0 \), the number of detected positrons will decrease, because the drift towards larger \( x \) moves the \( \mu^+ \) to a region with lower geometrical

![Figure 4.36: (Left) Schematic view of the longitudinal target used in 2015, which emphasizes the position of the drift detectors. The dashed line represents a trajectory of a \( \mu^+ \) which, after longitudinal compression, slowly moves in \( +x \)-direction due to the \( \hat{E} \times \hat{B} \)-drift. (Right) Expected time spectra for the drift detectors. Compression dominates at early times, but as soon as the \( \mu^+ \) swarm is compressed into the center of the target, the drift becomes visible.](image-url)
acceptance. The opposite happens for the drift detectors located at $x > 0$. For those detectors, the $\mu^+$ drift into the region of higher acceptance, yielding an increase in positron counts.

In order to demonstrate that the $\mu^+$ actually drift in $+x$-direction, we show the measured time spectra ($p = 8.5$ mbar, $HV_1 = -600$ V) for three scintillators in Fig. 4.37: The scintillator centered at $x = -20$ mm (B12), the central one at $x = 0$ (B14), and the one centered at $x = +20$ mm (B16), respectively (see also Fig. 4.36 for the labeling convention).

The time spectra presented in Fig. 4.37 are, as usual, corrected for the $\mu^+$ decay. Additionally, they are normalized to 1 at the bin containing $t = 0$ to emphasize the effect of the drift. As anticipated, the scintillator centered at $x = +20$ mm (B16) detects more and more positrons, whereas for the scintillator at $x = -20$ mm (B12), the count rate starts decreasing when the longitudinal compression is not dominating anymore after about 1 $\mu$s. This proves that the $\mu^+$ indeed are drifting in $+x$-direction, demonstrating the capability to bring the $\mu^+$ towards the prospective point of extraction into vacuum.

**Figure 4.37:** The drift in $+x$-direction manifests itself after the dominating contribution of compression is finished, which is around 1 $\mu$s. The time spectra were recorded with 8.5 mbar He gas and $HV_1 = -600$ V, and they have been normalized to 1 at the bin containing $t = 0$. 

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Because the increase in detector counts at early times is dominated by the muon swarm compression, it would be helpful to normalize the spectra to a drift scintillator which sits centrally above the incoming muon beam. Such a detector would only see the increase in positron counts which is due to the compression, i.e., due to the accumulation of muons in the center of the target. But this is only true as long as the muons do not drift into regions where their acceptance changes significantly. At late times, the muons might drift out of the high acceptance region of this detector, which would then introduce a bias in the normalization. Therefore, it is crucial to investigate the geometrical acceptance of the drift detectors in more detail.

If the time spectra are normalized to the central drift detectors, assuming that they are not sensitive to the position of the $\mu^+$ in horizontal $x$-direction, then the contribution to the time spectrum arising from the compression is eliminated. More precisely, we normalize the time spectra of the lateral detectors (B12, B16) to the sum of the three central detectors (B13, B14, B15). The geometrical acceptance of the sum of these detectors depends only weakly on the $x$-position, as shown in Fig. 4.38 (blue). Hence, this sum is basically insensitive to the $\mu^+$ motion and can be used to normalize the time spectra.

With a minimizer package, it is also possible to find weights $w_i$, $i = 13, 14, 15$ for each acceptance histogram of the detectors $B_i$, $i = 13, 14, 15$, in order to further minimize the dependence of the acceptance of the sum on the $\mu^+$ decay position. This is shown in orange in Fig. 4.38, and the weights found from this procedure are used for further analysis. The weights obtained from this procedure are $w_{13} = 0.30$, $w_{14} = 0.09$, and $w_{15} = 0.60$. Several combinations of weights $w_i$ yield similarly flat acceptance histograms, with no significant effect on the extracted drift velocity.

The procedure described above can be interpreted as constructing an artificial "detector" which is only sensitive to compression. By normalizing to this "detector", the effect of longitudinal compression is strongly suppressed in the time spectra. This is shown in Fig. 4.39, which contains the same data as already shown in Fig. 4.37 for the scintillators B12 (orange), B14 (blue), and B16 (black), respectively.

**Drift velocity**

Once the effect of the compression is removed from the time spectra, the drift velocity $v_D$ in $x$-direction can be extracted. Figure 4.40 illustrates a simple way to extract the drift velocity from the measured drift spectra (after removing the compression effect). An increase of the number of counts in the time spectra (left) corresponds to a drift of the $\mu^+$ into a region of larger detection efficiency (right). The drift velocity

---

6 The minimizer is the class "TFractionFitter" of the ROOT-framework. It finds weights $w_i$ to $w_3$ such that $\sum_{i=1}^3 w_i H_i$ is fitted to a uniform histogram ($H_i$ are the histograms of the drift detectors) by means of a standard likelihood fit.
can be extracted in an approximated way as

$$v_D = \frac{\Delta x (c_2 - c_1)}{\Delta t (c'_2 - c'_1)/c'_1},$$

(4.23)

where $c_1$ and $c_2$ correspond to the detected positron "counts" at times $t_1$ and $t_2$ and $c'_1$ and $c'_2$ are the detector acceptances for $\mu^+$ decaying at $x_1$ and $x_2$, respectively. Note that Eq. (4.23) assumes a linear dependence of the time spectrum on $t$ and the detector acceptance region on $x$, whose validity is approximately guaranteed for small $\Delta x$, as shown in Fig. 4.40.

The drift velocity extracted using Eq. (4.23) and the measured time spectra combined with the simulated space-resolved detector efficiency are presented in Table 4.5 for various experimental conditions. Averaging the velocities $v_D$ for $E_y = 0.12$ kV/cm (i.e., $E_z = 0.06$ kV/cm) and $p = 8.5$ mbar results in $v_D = 1.1 \pm 0.2$ mm/μs. The uncertainty is obtained from the uncertainty on the

Figure 4.38: Geant4 simulation of the geometrical acceptance as a function of the muon decay position in $x$-direction for the case of a simple sum of the central 3 scintillators B13, B14 and B15 (blue) and for the case of a weighted sum with weights $w_{13}$, $w_{14}$ and $w_{15}$ (orange). The geometrical acceptance of the weighted sum shows weaker dependence on the $\mu^+$ decay position, which is the reason why this sum is used for further analysis. The simulation included all the brass collimation, all scintillators and the magnetic field.
Chapter 4. Longitudinal compression tests

Figure 4.39: The same time spectra for the drift detectors as in Fig. 4.37, but normalized to the weighted sum of the central detectors. With this normalization, the muon drift away from the scintillator B12 towards B16 is much more pronounced than in Fig. 4.37.

fit, not taking into account several systematic effects. Examples for such systematic effects which affect the measured drift velocity are the linearization of the detector acceptances or saturation effects in the time spectra arising from target misalignments and straggling in the gas.

In the last column of this table, the drift velocities $v_{B14}^D$ are presented. These were obtained by normalizing the time spectra not to a sum of the central three detectors, but only to B14. It turns out that the hereby obtained drift velocities agree within their uncertainty with $v_D$, but tend to have larger differences between B12 and B16. The reason for this is that the geometrical acceptance of B14 is less homogeneous than for the weighted sum, presented in Fig. 4.38.

It is now interesting to compare these results with simulations which include the elastic collision of the $\mu^+$ in the He gas. Figure 4.41 shows the average drift position of $\mu^+$ versus time in a 5 T magnetic field in 8.5 mbar He gas for various electric field strengths. From this plot, it can be deduced that the drift velocity for $E_y = 0.12$ kV/cm is a bit below $v_{D}^{\text{sim}} \lesssim 2.4$ mm/µs (green triangles), which is about a factor of two larger than the drift velocity extracted from the experiment of $v_D = 1.1 \pm 0.2$ mm/µs. This discrepancy is most likely due to limitations arising...
Figure 4.40: Schematic to extract the drift velocity: An increase in detected positron "counts" from \( c_1 \) to \( c_2 \) within a time \( \Delta t \) in the time spectra (left) corresponds to a translation of the \( \mu^+ \) swarm by \( \Delta x \) into a region of higher acceptance of the corresponding detector (right). A linear dependence of both the time spectrum on \( t \) and the simulated detector acceptance on \( x \) is assumed, which is assured by choosing \( \Delta t \) and \( \Delta x \) accordingly small. The drift velocity is then calculated using Eq. (4.23).

Table 4.5: Drift velocity \( v_D \) determined using Eq. (4.23) for the scintillators B12 and B16, respectively. The last column contains the drift velocity \( v_D^{B14} \) where the time spectra were only normalized to the detector B14 and not the weighted sum of Fig. 4.38. The indicated uncertainties arise solely from the fit and do not take into account any systematic effects.

<table>
<thead>
<tr>
<th>( p ) [mbar]</th>
<th>( E_y ) [kV/cm]</th>
<th>scintillator</th>
<th>( v_D ) [mm/( \mu s )]</th>
<th>( v_D^{B14} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5</td>
<td>0.12</td>
<td>B12</td>
<td>1.05 ± 0.18</td>
<td>1.15 ± 0.20</td>
</tr>
<tr>
<td>8.5</td>
<td>0.12</td>
<td>B16</td>
<td>1.07 ± 0.30</td>
<td>0.91 ± 0.34</td>
</tr>
<tr>
<td>10.5</td>
<td>0.12</td>
<td>B12</td>
<td>0.72 ± 0.22</td>
<td>0.86 ± 0.25</td>
</tr>
<tr>
<td>10.5</td>
<td>0.121</td>
<td>B16</td>
<td>0.34 ± 0.30</td>
<td>0.15 ± 0.34</td>
</tr>
<tr>
<td>8.5</td>
<td>0.10</td>
<td>B12</td>
<td>0.95 ± 0.19</td>
<td>0.71 ± 0.25</td>
</tr>
<tr>
<td>8.5</td>
<td>0.10</td>
<td>B16</td>
<td>1.07 ± 0.32</td>
<td>1.47 ± 0.42</td>
</tr>
<tr>
<td>8.5</td>
<td>0.12</td>
<td>B12</td>
<td>1.24 ± 0.23</td>
<td>1.14 ± 0.28</td>
</tr>
<tr>
<td>8.5</td>
<td>0.12</td>
<td>B16</td>
<td>1.23 ± 0.44</td>
<td>1.43 ± 0.53</td>
</tr>
</tbody>
</table>
Figure 4.41: Simulations of the drift distance $\Delta x$ as a function of time $\Delta t$ for various electric field strengths in a 5 T magnetic field and 8.5 mbar He gas. The drift velocity $v_D = \Delta x/\Delta t$ can be extracted from this plot, and is in the case of $E_y = 0.12$ kV/cm about $v_D = 2.4$ mm/$\mu$s, about a factor of two larger than the measured drift velocity (Table 4.5). Note that in the simulation the $\mu^+$ were generated at rest (courtesy I. Belosevic).

from the simple way of extracting the drift velocity from the measured time spectra, such as assuming linear relations of the geometrical acceptances or neglecting initial beam distribution. It is interesting to note that $v_D$ is almost independent from the He gas pressure for low pressures in the simulation. This can also be seen for example when calculating the drift velocity in vacuum, which is given in Ref. [49]:

$$
\vec{v}^{\text{vacuum}}_D = \frac{\vec{E} \times \vec{B}}{B^2}.
$$

Plugging in the values of $\vec{E} = (0, 0.12, 0.06)$ kV/cm and $\vec{B} = (0, 0, 5)$ T, we also obtain $v_D^{\text{vacuum}} = 2.4$ mm/$\mu$s.
In the three beam times in 2011, 2014, and 2015, several improvements of the longitudinal compression stage have been accomplished, and are listed in what follows. However, it is important to note that when testing longitudinal compression, the \( \mu^+ \) beam is entering along the \( z \)-direction into the target. This is different from the final setup, where the \( \mu^+ \) are entering from the transverse compression stage at low energies. Apart from the low stopping efficiency in few mbar He gas, we do not expect any implications on the compression by injecting the muons longitudinally.

- In 2011, the effect of longitudinal compression has been exposed for the first time. However, the compression and quantification of its efficiency were limited by problems with impurities and possible misalignments of the \( \mu^+ \) beam.

- In 2014, both these problems were solved, and the space resolution of the detectors was improved. The effect of the impurities on the compression efficiency was studied by introducing in a controlled way impurities into the gas target. The impurity hypothesis formulated in 2011 was confirmed. Moreover, several additional detectors were implemented to study the time evolution of the space-resolved \( \mu^+ \) swarm. The extracted compression efficiency for 5 mbar He gas was was above 44% after \( t = 2.5 \mu s \) (90% C.L., with no \( \mu^+ \)-decay related efficiency accounted for), and for 11 mbar it was \( (97^{+10}_{-22}) \% \). The quantification of the compression efficiency of the 2014 data was limited by non-negligible diffusion effects, and possibly unaccounted background. These effects can be estimated with simulations but prompted us to construct a new target free of this issue for the 2015 beam time.

- In 2015, we measured longitudinal compression without the issues from diffusion. Furthermore, we added a vertical electric field to induce a drift of the \( \mu^+ \) swarm in \( x \)-direction. The results for the longitudinal compression tests are:
  - We extracted a compression efficiency after 2.5 \( \mu s \) from the time spectra of \( (90 \pm 13)\% \) for a cell with active length of \( 2 \times 86 \) mm, a pressure of \( p = 8.5 \) mbar and \( E/N = 29 \) Td. Moreover, the compression efficiency increases to \( (107 \pm 16)\% \) after 5 \( \mu s \), and it drops at smaller \( E/N \approx 24 \) Td to 50\% (at \( t = 2.5 \mu s \)).
  - Impurity effects have been suppressed, quantified, and understood.
  - Besides compression, a drift in \( +x \)-direction has been induced with a lower limit on the velocity of \( v_D = 1.1 \pm 0.2 \) mm/\( \mu s \).
  - An overall good agreement between measurements and Geant4 simulation has been observed.
5 Transverse compression tests

The last part of this thesis covers the first stage of the compression scheme: the transverse compression. The requirement of cryogenic temperatures, a static vertical density gradient, and high electric and magnetic fields inside the gas makes the transverse target difficult to build. In 2014, a first attempt was made to measure transverse compression. However, due to a cold leak and problems with the stability of the electric field, it was not possible to see compression. In 2015, much effort was put into refining the design and the choice of materials, which was rewarded by the successful demonstration of transverse compression in December 2015.

This chapter starts with the general idea of demonstrating transverse compression, followed by the discussion of the results in Sec. 5.1. Then, the experimental design is explained in some more detail in Sec. 5.2. Thereafter, in Sec. 5.3 follows the description of a dedicated experiment that demonstrated the feasibility of establishing a vertical density gradient in the target by means of neutron radiography, which was performed in 2013. This is followed by selected details about the choice of materials and the construction of the transverse compression target in Sec. 5.4.

General idea to demonstrate transverse compression

The $\mu^+$ with an initial momentum of 11 MeV/c enter the 25 cm long transverse compression target after passing through an entrance detector giving the start time $t = 0$. A significant fraction of the $\mu^+$ is stopped in the He gas of about 5 mbar at cryogenic temperatures with an average temperature of 8 K. This corresponds to an average gas number density of $n \approx 4.5 \times 10^{18}$ cm$^{-3}$. A high electric field $\vec{E} = (1, 1, 0)$ kV/cm is applied perpendicular to the magnetic field $\vec{B} = (0, 0, 5)$ T. The target is sketched in Fig. 5.1, where some expected $\mu^+$ trajectories are illustrated. A vertical temperature gradient is created across the target by keeping the lower part of the target at 4 K (6 K was reached in the experiment) while heating the upper part to about 12 K (18 K in the experiment). This results in a gas density gradient which is responsible for the position-dependence of the drift velocity vector $\vec{v}_D$, see Eq. (3.3). Thus, $\mu^+$ stopping in the lower part of the gas target drift upwards, whereas $\mu^+$ stopping in the upper part drift down towards the tip of the target.
Figure 5.1: Sketch of the transverse target mounted on the cold finger. The He gas cell where the $\mu^+$ are stopped is drawn in yellow. The black lines represent some muon trajectories which start on the left and drift to the right while the swarm is compressed in vertical direction. By means of the scintillators A1-A4 (blue), the $\mu^+$ movement is detected as the swarm is compressed towards the tip of the triangular target. Two sapphire plates are used to define the upper and lower temperature of the gas target.

Note that the absolute temperature per se is not crucial for demonstrating transverse compression only. As long as the ratio of the density exceeds a factor of 3, the same compression efficiency can be achieved because the helium gas pressure inside the target can be adjusted correspondingly. However, when the transverse compression stage is connected to the longitudinal stage, the pressure in both compression stages will be the same. The optimal pressure of 5 mbar is given by the competing desire for high pressure yielding large stopping power in the transverse target and the desire for low pressure in the longitudinal target allowing fast compression.
Chapter 5. Transverse compression tests

Figure 5.2: Measured raw time spectra in various scintillators surrounding the gas target, after correcting for the muon decay. The positron count rate in A4 decreases at late times, whereas in A1 it increases. This indicates a $\mu^+$ swarm movement away from A4 towards A1, which is placed close to the tip of the transverse compression target.

Scintillators A1-A4 placed around the target detect the decay positron of the muon at time $t$ relative to the entrance detector. The time spectra of the positrons in the various scintillators expose the compression of the $\mu^+$ swarm. Such measured time spectra are shown in Fig. 5.2 for the three scintillators A1, A3 and A4 given in Fig. 5.1. It can be observed that the count rate in the detector A4 decreases with time (blue diamonds), indicating that the $\mu^+$ drift away from the scintillator A4. The number of detected positrons in detector A3 (orange circles) is first increasing and after 0.5 $\mu$s it decreases, which reflects the fact that the $\mu^+$ first approach and then move away from the A3 scintillator, i.e., they pass by it. The increase in detected positrons in scintillator A1 (black crosses) clearly shows that the $\mu^+$ are moving towards a higher acceptance region of this detector, which is placed at the tip of the target where, in the final setup, the longitudinal compression stage will be.

However, even without a density gradient, there exist certain combinations of gas densities and electric field strengths which allow the $\mu^+$ swarm to approach the tip of the target, without actually being compressed in vertical direction, i.e., by undergoing a pure drift. The details of how to distinguish between drift and actual compression will be explained in Sec. 5.1.2.
Chapter 5. Transverse compression tests

5.1 Test of transverse compression - experiment and first results

5.1.1 Simulated geometrical acceptance

In order to understand how the $\mu^+$ swarm is moving inside the transverse target cell, the geometrical acceptances of the various scintillators have to be known. For this reason, a simulation with G4Beamline [74] was performed.

The simulation was run with $10^7 \mu^+$ distributed homogeneously across the $xy$-plane in the center of the target ($z = 0$ mm). The result of this simulation for three distinct detectors is shown in Fig. 5.3, where the initial position of the detected positron is filled in a two-dimensional histogram. Therefore, these graphs display the space-resolved probability to detect the positron emitted by the $\mu^+$ at a certain position ($x,y$) in a given scintillator.

5.1.2 Expected behavior

Two separate measurements were performed in which the $\mu^+$ beam entered through either of the two openings "1" or "2" in the aperture in front of the target (see Fig. 5.3 (d)). By comparing the time spectra obtained in several scintillators for the two initial beam positions, but otherwise identical experimental conditions, the distinction between compression and drift can be demonstrated. To have a pure drift (no compression), we performed measurements without the density gradient, i.e., the upper part of the target was not heated.

The expected $\mu^+$ trajectories and time spectra are qualitatively shown in Fig. 5.4 for two different initial positions of the $\mu^+$ beam ("1": upper position; "2": lower position) and three different $E/N$ conditions: The first condition (blue) leading to the desired muon swarm compression, the second (orange) for a drift in $x$-direction without compression, and the third (black) for a drift along the electric field. Unfortunately, it was not possible during the experiment to obtain a drift perpendicular to the electric field (in $\hat{E} \times \hat{B}$-direction). The electric field required for this case has to be larger than 1.5 kV/cm, but electric discharges started to occur for the given pressure and temperature above a field strength of 1.4 kV/cm.

First results from preliminary simulations

Preliminary simulations with Geant4, with a self-developed extension to low energy (eV regime), were made to simulate the muon trajectories in the gas [10]. For this purpose, the $\mu^+$ start at rest on the $xy$-plane in the center of the target, at the
5.1. Test of transverse compression - experiment and first results

Figure 5.3: (a, b, c) Simulated geometrical acceptance of the three scintillators A3, A1, and A4. Each simulation is run for $10^7$ events distributed homogeneously in the $xy$-plane in the center of the target ($z = 0$). (d) Target geometry and sketch of the detector placement for the upgraded transverse compression test. The orange structure on top of the target represents a brass collimator which absorbs the positrons from $\mu^+$ decays. "1" and "2" indicate the two initial muon beam positions which are defined in the experiment by an upstream aperture.
Chapter 5. Transverse compression tests

Figure 5.4: (Top) Muon trajectories for three different conditions such that either ideal compression, only drift in $x$-direction, or drift along the electric field occurs. The two circles indicate the initial $\mu^+$ position that can be selected in the experiment. (Middle) Corresponding time spectra for the three scintillators A1, A3, and A4 for $\mu^+$ starting in the upper aperture opening. (Bottom) Same as (Middle), but for $\mu^+$ starting from the lower opening. The blue lines always represent the case of compression, the orange lines the case of a drift in $x$-direction, and the black lines the case of a drift along the electric field. Note that the dimensions of the scintillators are not to scale.
5.1. Test of transverse compression - experiment and first results

Figure 5.5: (a) Simulated $\mu^+$ trajectories if a temperature gradient is applied between $T_{up} = 18.6$ K and $T_{low} = 6.1$ K in 8.6 mbar He gas and an electric field of $E = 1.5$ kV/cm. (b) The case for no temperature gradient ($T = 5.4$ K), with an electric field of $E = 1.1$ kV/cm and $p = 3.6$ mbar. (c) Same situation as (b) but with a higher electric field of $E = 1.4$ kV/cm. The black circles denote the starting positions of the $\mu^+$ defined in the experiment by the aperture in front of the target. Courtesy of I. Belosevic [10].

positions “1” or “2”, respectively, defined by the aperture (see Fig. 5.3). In coordinates, this translates to $x = -15$ mm, $y = \pm 4.5$ mm for the central position of the generated $\mu^+$ swarm. Only a few $\mu^+$ were simulated, as this simulation is very time consuming if large statistics is required. The case for ideal compression (temperature gradient between 6.1 K and 18.6 K, $E = 1.5$ kV/cm) is shown in Fig. 5.5 (a), whereas the case without a temperature gradient (i.e., no compression, only drift) is shown in Fig. 5.5 (b) and (c). The only difference between these two figures is the electric field strength: For $E = 1.1$ kV/cm, the $\mu^+$ drift is along the electric field, whereas for $E = 1.4$ kV/cm, the $\mu^+$ drift in horizontal direction.

First results from the experiment

The conditions for $E/N$ and temperature gradients used in the simulations of Fig. 5.5 have been reproduced experimentally. The measured time spectra for three scintillators are shown in Fig. 5.6. The upper row corresponds to the case where the $\mu^+$ beam entered the target through the upper aperture opening (“1”), whereas the bottom row shows the results for the $\mu^+$ beam entering through the lower aperture opening (“2”).

These measurements confirm the expected behavior, which has already been shown qualitatively in Fig. 5.4. Notably for scintillator A1, the increase in detected positrons is about the same for both initial $\mu^+$ beam positions (roughly a factor of 10). But the time spectrum becomes flat at different times for the two initial beam
Chapter 5. Transverse compression tests

\[ E = 1.5 \, \text{kV/cm}, \, T_{\text{up}} = 18.6 \, \text{K}, \, T_{\text{low}} = 6.1 \, \text{K}, \, p = 8.6 \, \text{mbar} \]

\[ E' = 1.1 \, \text{kV/cm}, \, T = 5.3 \, \text{K}, \, p = 3.6 \, \text{mbar} \]

\[ E'' = 1.4 \, \text{kV/cm}, \, T = 5.3 \, \text{K}, \, p = 3.6 \, \text{mbar} \]

**upper aperture position**

![Graphs showing normalized counts over time for different fields and temperatures.](image)

**lower aperture position**

![Graphs showing normalized counts over time for different fields and temperatures.](image)

**Figure 5.6:** (Top) Measured time spectra for three different scintillators in the case where the \( \mu^+ \) are entering the upper aperture opening. The blue diamonds show the case of compression: \( T_{\text{up}} = 18.6 \, \text{K}, \, T_{\text{low}} = 6.1 \, \text{K} \) with 8.6 mbar gas pressure, and \( E = 1.5 \, \text{kV/cm} \). The black crosses and the orange circles show the case for no temperature gradient and two different field strengths (\( E = 1.1 \, \text{kV/cm} \) and \( E = 1.4 \, \text{kV/cm} \), respectively). (Bottom) Same as (Top) but with the \( \mu^+ \) initial position "2". All time spectra are normalized to 1 at \( t = 100 \, \text{ns} \) to ease comparison.
5.2. Experimental setup of the transverse compression stage

Positions. For $\mu^+$ entering the upper opening in the aperture, the time spectrum becomes flat around 2 $\mu\text{s}$, whereas it becomes flat only after 3.5 $\mu\text{s}$ if the $\mu^+$ enter the lower aperture opening. This trend was confirmed by simulations which predict drift times of about 1.7 $\mu\text{s}$ and 3.0 $\mu\text{s}$, respectively. The different drift velocity arises from different gas densities sampled by the $\mu^+$. At higher temperatures, the density is lower and thus the velocity larger.

Detailed analysis is ongoing to also quantitatively evaluate the measured data [10]. However, already with this rough analysis, we clearly see the drift in $x$-direction accompanied with muon swarm compression in vertical $y$-direction. Transverse compression has thus been demonstrated.

5.2 Experimental setup of the transverse compression stage

5.2.1 First tests in 2014

A first attempt to test transverse compression was made in 2014. However, technical problems with the target prevented successful data taking:

- Leakage: There was a small cold leak in the target which appeared at temperatures below 100 K.
- High voltage: Electrical contact broke between the electrodes sputtered onto sapphire and the wires soldered onto them at cryogenic temperatures.

Despite these two problems that prevented successful data taking in 2014, many components worked well during this first, preliminary test: The temperature reached the required values of roughly 6 and 18 K, providing a density gradient exceeding a factor of 3, inside a 5 T magnetic field. All the scintillators worked outstandingly well in the cold, as well as the transition between the scintillators and the wavelength shifting fibers, which guided the scintillation light to the SiPMs at warm temperatures. And we managed to steer the $\mu^+$ beam correctly into the cryogenic target.

5.2.2 Experimental target in 2015

In order to establish the vertical gas density gradient, which is crucial to compress the $\mu^+$ swarm, we need to cool the lower part of the transverse target to approximately 4 K while keeping the upper part at around 12 K. We chose crystalline sapphire ($\text{Al}_2\text{O}_3$) as material for the base and top plate, because it is a very good thermal conductor ($\sim 1000 \text{ W/(m·K)}$ at 10 K [75]) and an outstanding electrical insulator. The
lateral side walls of the target are made out of 50 µm thick Kapton foil, as indicated in Fig. 5.1. The thermal conductivity of Kapton is low (∼ 0.02 W/(m·K) at 10 K [76]), which is crucial for maintaining the temperature gradient.

In a dedicated experiment, we have verified that the permeability of He gas through Kapton is sufficiently low at cold temperatures. This was accomplished by measuring the leak rate of helium gas through Kapton foils of various thicknesses at temperatures between 300 K and 150 K. The permeability decreases exponentially with decreasing temperature, until the leak rate of a few mbar He gas through the Kapton foils is not measurable anymore. More details can be found in [77].

The need of high electric fields inside the gas did not allow to use metallic parts to seal the target and to confine the helium gas. Therefore, the approach of gluing the target was chosen. This, however, is difficult because the glue has to keep the target leak tight at cryogenic temperatures and it has to deal with the mechanical stress induced by the different thermal expansion coefficients of the various materials used (for more details see Sec. 5.4.1). Well-known cryogenic epoxies are the various kinds of Stycast, from which the model 1266 was chosen. Some of its properties are presented in Appendix E. After many attempts on the optimal gluing strategy, a method was found to reproducibly construct leak-tight targets which stayed leak-tight even after several cooling cycles. A description of the steps to glue the target is given in Appendix F. In summary, the first step is to glue the Kapton foil onto the sapphire plates. This has to be done very precisely to have the electrodes at the proper position. In a second step, the side walls defining the shape of the target, as well as the end caps used for tightening, are glued onto the Kapton foil. A photo of this step is shown in Fig. 5.7 (Left). Then, the target is "closed" by gluing the upper sapphire plate onto the already fixed side walls. If everything is built with sufficient precision, the Kapton will wrap nicely around the end caps and can be tightened easily. Finally, a mounting for the temperature sensor, the heat pad, and the gas tubes are glued to the target.

The electric field inside the target is defined by lining the Kapton foil which encloses the gas with several longitudinal metallic strips. The strips are set at different electric potential with the help of four points defining four potentials and three voltage dividers as shown in Fig. 5.7 (Right). The resistors of the voltage divider are placed outside the gas region to avoid discharges due to sharp edges caused by soldering the resistors. Therefore, as visible on the photo in Fig. 5.7, the voltage dividers are placed upstream of the target entrance window.

The transverse compression target is placed inside a 5 T solenoid magnet with a 20 cm diameter bore hole. It is mounted onto a cold finger, coming from the cryostat and reaching into the magnet bore, which is illustrated in the sketch presented in Fig. 5.8, showing the most important features of the setup. This cold finger is approximately 1 m long in order to position the target into the center of the magnet,
5.2. Experimental setup of the transverse compression stage

Figure 5.7: (Left) Lined Kapton during the glueing process. (Right) Scheme of the HV connections in the transverse target to define the required E-field. Four different electric potentials are applied at the positions labeled "HV". The electric field is then defined by the resistors acting as voltage dividers.

Figure 5.8: Sketch of the experimental setup of the transverse compression stage. The target sits on a 1 m long cold finger, cooled to 4 K. The isolation vacuum tube is inside a 5 T solenoid with a 20 cm bore hole. Note that the dimensions are not to scale.
where the magnetic field is most homogeneous. The cold finger is surrounded by a thermal shield and the vacuum tube. A photo of the transverse target mounted on the cold finger is shown in Fig. 5.9 (Top). Note that the scintillators, HV cables and collimators are removed for the sake of clarity. In Fig. 5.9 (Bottom), the mounted target (back view) is shown, with the scintillators, positron shielding, and HV connectors.

The decay positrons are detected by scintillators. These scintillators are glued with optical cement\(^1\) to wavelength shifting fibers\(^2\), which guide the light produced

\(^1\)Optical cement is a somewhat colloquial word for a glue with appropriate refraction index and transmission used to glue scintillators, fibers and similar optical components.

\(^2\)Wavelength shifting fibers (WSF) are scintillating fibers which have the highest absorption efficiency at a slightly shorter wavelength compared to the emission wavelength. This means they absorb for example blue light and emit green light, which is then guided along the fiber.
in the scintillators from the cold region into the warmer region at the back of the cryostat. A special feedthrough for the fibers has been developed to extract the fiber light from vacuum into air, where the SiPMs detect the light.

The thermal flux into the target produced by the HV cables was minimized by using steel wires, and by thermally coupling them to the first stage of the cryostat. Similarly, the gas connection made from a 5 mm inner diameter steel tube, was also thermally coupled to that stage.

The long horizontal vacuum tube of the cryostat was directly connected to the beam line vacuum tube. A 25 µm thin Kapton foil with 40 mm diameter at the end of the cryostat vacuum tube was sufficient to have vacuum inside the cryostat even if it was not connected to the beam line. This allowed to cool down the target already before mounting it to the beam line vacuum. Therefore, we could quickly switch between the longitudinal and transverse target during the beam time.

The $\mu^+$ coming from the accelerator thus had to pass through this foil before passing another 2 µm thin Mylar foil, coated with aluminum, which was as part of the thermal shield. After this foil, an aperture with two openings (5 mm diameter) was placed on the cold finger at a well defined position relative to the target, which collimated and defined the initial transverse position of the muon beam in the target.

Target alignment to the beam axis is challenging because the B-field lines relative to the bore hole mechanics are not well defined and because the "target-cold finger" assembly moves by about 1 cm while cooling down. The actual position of the target relative to the beam is thus not known. Therefore, we used three dedicated scintillators downstream of the target as shown in Fig. 5.10 to center the muon beam on the target axis. The intensity in these three scintillators was used to align the target and the $\mu^+$ beam by moving the solenoid and the cryostat.

![Figure 5.10](image-url): (Left) Sketch of the transverse target showing the three scintillators downstream the target used to detect $\mu^+$ passing through the target. These scintillators are used to align the beam. (Right) Photo showing the inside of the target and the three scintillators.
5.3 Neutron radiography of a stationary gas density gradient

It is not straightforward to measure the temperature distribution inside a gas target of only a few centimeters height. A dedicated experiment was conducted at the MORPHEUS beam line at PSI in 2013 to measure the density gradient by means of neutron radiography [9]. For this purpose, the strongly neutron absorbing isotope $^3$He was filled into a metallic gas target. The bottom and top plates of this target were made of copper, whereas the lateral side walls consisted of very thin stainless steel walls to minimize heat flux through the walls. Two cells were used, one with a rectangular cross section of $40 \times 40 \text{mm}^2$, and a triangular cell with a height of 40 mm. Both cells were 200 mm long.

A vertical (in $y$-direction) temperature gradient was established by heating the upper plate to $T_{\text{up}} = 26 \text{ K}$ while keeping the bottom plate at $T_{\text{low}} = 7 \text{ K}$. The temperature gradient leads to a density gradient, such that the number density $n$ in the gas becomes a function of the vertical position $y$ in the target:

$$n(y) = \frac{p}{k_B T(y)},$$

(5.1)

where $p$ is the gas pressure and $k_B$ the Boltzmann constant $k_B = 1.380 \times 10^{-23} \text{ J/K}$.

Neglecting beam divergences, the transmitted neutron intensity $I(y)$ through a $L = 200 \text{ mm long} \ ^3$He gas cell can be written in a one-dimensional model as

$$I(y) = I_0 e^{-\sigma Ln(y)},$$

(5.2)

where $I_0$ is the transmitted intensity through an empty cell, and $\sigma$ the neutron absorption cross section for $^3$He gas (which depends on the neutron wavelength).

The principle of the neutron radiography experiment is shown in Fig. 5.11, where the position-dependent intensity of transmitted neutrons depends on the $^3$He gas density $n(y)$. A two-dimensional image plate was placed behind the experimental setup to detect the transmitted neutron intensity $I(x, y)$. The irradiated image plate could be read out with a dedicated scanner, which revealed information about the incoming neutron intensity. By means of photostimulated luminescence, which is linearly proportional to the number of incident neutrons, each pixel $(x, y)$ with size of $0.1 \times 0.1 \text{ mm}^2$, is a direct measure of the number of transmitted neutrons [78]. Using Eq. (5.2), the spatial density distribution inside the $^3$He gas cell can be calculated.

In Fig. 5.12, the two-dimensional density distribution for the triangular gas cell is shown. The upper copper plate was heated to 26 K, whereas the bottom plate was kept at 7 K, resulting in a temperature ratio of 3.7. It can be seen from this figure that
5.3. Neutron radiography of a stationary gas density gradient

Figure 5.11: Scheme of the neutron radiography principle used to measure the gas density gradient. The transmitted neutron intensity (after the neutrons passed through the $^3$He gas cell) is a function of the gas number density $n$. This scheme was used to measure the vertical ($y$-direction) density (temperature) gradient needed for the transverse compression.

Figure 5.12: Gas density measured for a triangular gas cell filled with $^3$He gas at 5 mbar pressure and with a temperature gradient between 7 K and 26 K. The feature around $x = y = 30$ is due to a vacuum flange of the cryostat absorbing the neutrons.
the achieved density ratio also exceeded a factor of 3 as required for the transverse compression stage.

The measured density distribution is well described by theoretical predictions. Our values for the parameterization of the thermal conductivity of $^3$He gas, obtained from fitting the measured data, agree well with the theoretical values given in Ref. [79]. The good agreement between experimental data and theoretical prediction leads to the conclusion that no significant convection effect, which could destroy the stationary density gradient, occurs in our setup, even for a triangular geometry. A detailed analysis can be found in [9, 80].

5.4 Construction of the transverse compression target

This section is intended to give an overview of various aspects which were considered when constructing the transverse compression target. Finding a way to satisfy the demands from physical and geometrical restrictions represented a significant part of the work done in this thesis.

5.4.1 Overcoming mechanical stress due to mismatching thermal contraction coefficients

Materials generally shrink in size when they are cooled down. The total length contraction $\Delta L$ of a material with initial length $L$ by cooling it from room temperature $T_0$ to cryogenic temperatures $T_1$ is given by

$$\Delta L = \int_{T_0}^{T_1} \alpha(T) dT,$$

(5.3)

where $\alpha(T)$ is the thermal expansion coefficient. Because the transverse target is made from various materials, it is extremely important to choose materials with matching thermal expansion coefficients. During some preliminary tests, it was found that even a difference in thermal expansion of $\Delta L/L = 10^{-4}$, which is $\Delta L = 1 \mu m$ for $L(T_0) = 1 cm$, is sufficient to cause the breaking of a glue joint. Another aspect that needs to be considered is that the two plates defining the upper and lower temperature have to be electrically insulating, while on the same time they have to be very good thermal conductors. Our choice was to use single crystal sapphire plates. To avoid differences in length contraction when sustaining the two plates, we used thin frames made from laser-cut aluminum oxide ($Al_2O_3$) ceramics, which is supposed to have similar thermal contraction behavior as the crystalline
A version (sapphire). The end cap frames were made by PVC (see Fig. 5.7 (Right)). To avoid problems related with the different thermal expansion, the plastic frames were not glued on the sapphire but they were "floating", held only by the Kapton foil. This foil was used everywhere to contain the He gas. Its flexibility allows to compensate for the various expansion coefficients.

In Fig. 5.13, it is visible that the PVC end cap is indeed further upstream than the sapphire ends. This construction can easily deal with all the different thermal expansion coefficients, and not much stress is applied onto the glue joints where the side walls are glued onto the Kapton.

Sapphire crystal has a much smaller integral thermal contraction than copper. For copper, it is about $33.7 \times 10^{-4}$, whereas for sapphire, it is $7.9 \times 10^{-4}$ [76]. Over the full length of the target, which is 25 cm, this results in a difference of 0.6 mm that could cause serious stress on the materials. Therefore, the sapphire could not be screwed onto the cold finger but was clamped onto it as shown in Fig. 5.14. A thin indium sheet (100 µm thin) was put between the sapphire and the copper at position "1" indicated in Fig. 5.14 to optimize the thermal contact, and to absorb some of the stress caused by the thermal contraction.

### 5.4.2 Thermal optimizations of the heat flux

One objective was to minimize the thermal flux from the (warmer) top sapphire plate to the bottom plate. The heat flux through the helium gas cannot be reduced,
Chapter 5. Transverse compression tests

Figure 5.14: Close-up of the mounted transverse target. The metal piece in the foreground is the He gas pillar. The numbers "1"-"3" denote the interfaces discussed in Sec. 5.4.3.

therefore, the heat flux through the side walls and the lateral wall has to be minimized. Assuming a constant thermal conductivity $\kappa$, the heat flow is given by

$$\dot{Q} = \kappa A \Delta T / \Delta x,$$

(5.4)

where $\Delta T$ is the temperature difference, $A$ the area perpendicular to the heat flux and $\Delta x$ the distance over which the heat is transported. Table 5.1 reports some approximate values of $\kappa$ for some materials used for the transverse target.$^3$ A graph showing $\kappa(T)$ for selected materials between 1 K and 300 K is given in Appendix G. Using the values in Table 5.1, the total heat flux $\dot{Q}$ through the target under the assumption of $\Delta T = 12$ K ($T_{\text{low}} = 6$ K and $T_{\text{up}} = 18$ K) can be calculated and amounts to about 1 W. It is evident from this table that in our optimized geometry, the dominating contribution to the thermal flux is the He gas. This allows us to make a simulation of the temperature distribution inside the target even if the thermal conductivity of the other materials is not precisely known (see Sec. 5.4.4).

$^3$There are no precise values of the thermal conductivity for $\text{Al}_2\text{O}_3$ between 5 and 20 K. However, as visible in Fig. 1 of Ref. [81], there is evidence that the thermal conductivity drops well below $10^{-2}$ W/(cm·K) for 99.6% pure $\text{Al}_2\text{O}_3$ around 10 K. Since we used ceramics with lower purity (96% $\text{Al}_2\text{O}_3$), the thermal conductivity is also lower.
5.4. Construction of the transverse compression target

Table 5.1: Thermal conductivity $\kappa$ for the materials used in the transverse target design, and heat flow $\dot{Q}$ through our target, from the upper sapphire plate to the lower plate.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\kappa$ at 10 K [mW/(cm·K)]</th>
<th>area $A$ [cm$^2$]</th>
<th>$\dot{Q}_{\text{tot}}$ for $\Delta T=12$ K [mW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>He gas</td>
<td>0.17 [82, 83]</td>
<td>100</td>
<td>800</td>
</tr>
<tr>
<td>Kapton</td>
<td>0.3 [84, 85]</td>
<td>$\lesssim 0.1$</td>
<td>$&lt;1$</td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td>&lt;10 [81]</td>
<td>0.3</td>
<td>180</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>980</td>
</tr>
</tbody>
</table>

5.4.3 Temperature losses through Kapton and Stycast glue

Knowledge of the gas temperature inside the target is crucial to understand the $\mu^+$ swarm compression. Unfortunately, we cannot measure the temperature of the gas by inserting temperature sensors into it because of the problems with electric breakdowns. Thus, we have to deduce the temperature of the gas by measuring the temperature of the top sapphire and at the cold finger. Because of several thermal transitions (numbers "1"-"3" in Fig. 5.14), there is a temperature increase between the cold finger and the internal part of the Kapton foil defining the temperature of the gas at the bottom of the target. It has to be made sure that by measuring the temperature of the cold finger, we know what the temperature of the Kapton foil inside the target is (at position "3"). We assume that the gas in the vicinity of the Kapton foil has the same temperature as the foil.

In order to measure the temperature loss, a setup as sketched in Fig. 5.15 was used. The top sapphire plate was heated with various powers and its steady-state temperature was measured for two cases: (a) the plate is thermally coupled to the cold finger through a thin indium sheet, and (b) through a layer of Kapton and glue (and the indium sheet). In such a way, the thermal resistivity through the Kapton foil and the glue can be approximately inferred. The thickness of the glue layer is typically 10-20 µm.

The measured values are summarized in Table 5.2. Additionally, a long-term (40 hours) measurement was performed to check whether the thermal conductivity changes with time, which was not the case.

\footnote{This does not reflect reality insofar as in the real setup, the sapphire will always be coupled to the cold finger through the indium sheet, and the Kapton is glued on top of the sapphire and never coupled to the indium, but the transitions are nevertheless the same.}
Chapter 5. Transverse compression tests

Figure 5.15: Sketch of the setup used to measure the temperature loss through a thin layer of Kapton and glue. Note that the thickness of the layers is exaggerated in the picture. The Kapton foil is 50 µm thick, and the glue layer was estimated to be 10-20 µm thick.

It turns out that the temperature difference between $T_{\text{sapphire}}$ and $T_{\text{copper}}$ is 100 mK if there is no Kapton and glue layer, and it is about 350 mK if both the Kapton and glue layer are present. A very conservative estimate is therefore, that if the measured temperature on the cold finger is $T_{\text{copper}}$, then the temperature of the gas inside the target is maximally $T_{\text{gas}} = T_{\text{copper}} + 0.5$ K.

5.4.4 Simulating the temperature distribution in the vicinity of the side walls

Because the heat flow through the He gas is dominating over all other contributions to the heat flux (see Sec. 5.4.2), we can simulate the temperature distribution inside the transverse target, even though the thermal conductivity $\kappa(T)$ of the other materials are not well known. For simplicity, we assume that $\kappa(T)$ of sapphire is the same as of copper, which is approximately true between 10 K and 20 K (see Appendix G). The thermal conductivities for copper and Kapton are analytically known [76], for the others we assumed the following: $\kappa_{\text{PVC}} = \kappa_{\text{Al}_2\text{O}_3} = 0.1$ W/(m·K) [87, 88], and $\kappa_{\text{Stycast}} = 0.6$ W/(m·K) [87]. The temperature loss through the Kapton foil and the glue (see Sec. 5.4.3) was implemented in COMSOL using a "thin thermally resistive layer" between Kapton foil and sapphire (position "1" in Fig. 5.16 for the bottom sapphire, and at the analogue position for the top sapphire). The thermal resistivity was chosen to match the measured values given in Table 5.2.

The resulting temperature distribution is shown in Fig. 5.16. The left plot displays a cut in the $zy$-plane at $x = -15$ mm, i.e., 5 mm from the lateral back wall. As visible, the deviation from the ideal temperature distribution close to the side walls is limited to a range of only about 5 mm. Therefore, even though the end caps are
5.4. Construction of the transverse compression target

Table 5.2: Average temperatures measured at the sapphire and the copper cold finger of Fig. 5.15. The uncertainty of the measured temperatures is ±5 mK [86].

<table>
<thead>
<tr>
<th>$P_{\text{heating}}$</th>
<th>temperature</th>
<th>without Kapton</th>
<th>with Kapton &amp; glue</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 mW</td>
<td>$T_{\text{sapphire}}$ [K]</td>
<td>6.336</td>
<td>6.385</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{copper}}$ [K]</td>
<td>6.291</td>
<td>6.299</td>
</tr>
<tr>
<td></td>
<td>$\Delta T$ [K]</td>
<td>0.045</td>
<td>0.086</td>
</tr>
<tr>
<td>875 mW</td>
<td>$T_{\text{sapphire}}$ [K]</td>
<td>8.504</td>
<td>8.633</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{copper}}$ [K]</td>
<td>8.402</td>
<td>8.298</td>
</tr>
<tr>
<td></td>
<td>$\Delta T$ [K]</td>
<td>0.102</td>
<td>0.335</td>
</tr>
</tbody>
</table>

floating, which implies a small gas layer close to the end caps which is not directly in contact with the sapphire plates, the distortion of the temperature affects only a small extension in $z$-direction. In the analysis of the compression data, this small effect can be neglected. Moreover, to avoid any possible influences of this effect on the measured time spectra, we position the positron detectors several centimeters away from the end caps. Fig. 5.16 (Right) shows the temperature distribution for a cut in the $xy$-plane at $z = 70$ mm, i.e., 70 mm upstream the center of the target.

Figure 5.16: COMSOL simulations of the temperature distribution inside the target. (Left) Cut in the $zy$-plane at $x = -15$ mm (i.e., 5 mm from the lateral back wall). The end cap made of PVC and the support frame made of $\text{Al}_2\text{O}_3$ have little effect on the gas temperature in the region above the sapphire. (Right) Cut in the $xy$-plane at $z = 70$ mm.
5.5 Conclusions of the transverse compression tests

Regarding the transverse compression stage, we obtained the following results:

1. Using a simplified metallic target, we have proven the feasibility to establish a gradient in density in the He gas. This was demonstrated using neutron radiography on $^3$He gas. The measured density distribution agrees well with the prediction from theory, showing no signs of convection in the gas [9].

2. We have developed a challenging gas target to demonstrate transverse compression. The target must fulfill several requirements:
   - work at cryogenic temperatures
   - sustain a temperature gradient between $T_{\text{low}} \approx 6$ K and $T_{\text{high}} \approx 18$ K
   - stay helium leak tight after several thermal cooling cycles
   - sustain large electric fields $E \approx 1.4$ kV/cm in low pressure helium gas without breakdown

3. We have designed and developed a complete setup to test transverse compression, starting from all the cryogenic parts which include the gas target, the last part of the beam line, the entrance detector, positron detectors and DAQ.

4. We have demonstrated transverse compression with high efficiency. Analysis of the data is still ongoing and a simulation of the full compression scheme is being developed. Preliminary evaluations show that the measurements agree well with the simulations.
6 Conclusions

Within the course of this thesis, the path for realizing the novel high-brilliance, low-energy $\mu^+$ beam line has been paved. In two beam times at the Paul Scherrer Institute, the longitudinal and transverse stage have been tested separately.

Longitudinal compression was demonstrated with an efficiency of $(90 \pm 13)\%$ after 2.5 $\mu$s, if the muon decay is not taken into account. The effect of impurities on the muon swarm compression has been investigated. The level of contaminants in the He gas, mostly H, has to be <25 ppm, in order to avoid significant losses due to chemical capture.

By adding a transverse ($y$-direction) field component to the longitudinal ($z$-direction) electric field, we could verify experimentally that the $\mu^+$ can be drifted in $x$-direction, thus steered from one compression stage to the next. This is needed later on in the full setup, in order to move the $\mu^+$ towards the orifice where extraction into vacuum occurs.

In this thesis, the very challenging cryogenic target for the transverse compression stage has been developed. It could sustain a stationary, vertical gas temperature gradient between 6 and 18 K, a high electric field of 1.4 kV/cm in 5 mbar He gas, and it did not degrade in performance, even after several thermal cycles. During a beam time in 2015, we could demonstrate transverse compression. We have been able to differentiate between a simple drift and compression in transverse direction by measuring time spectra with various detectors for several experimental conditions (different E-field strengths and directions, gradients in temperature and gas pressures). We have strong evidence that the $\mu^+$ swarm compression, as well as the drift towards the next compression stage, is very efficient. Data analysis to quantify the efficiency is ongoing, together with the development of a full simulation.

The current design of the transverse and longitudinal compression stage allows to continue the development without major conceptional changes. The next step towards the realization of the beam line is to realize the transition between the transverse and longitudinal stages, such that combined transverse and longitudinal compression can be tested. Then, the $\mu^+$ will enter the longitudinal stage coming from the transverse stage, having low energy, in comparison to the test experiments where the $\mu^+$ beam entered the longitudinal target along the $z$-axis.
Chapter 6. Conclusions

Although mostly a mechanical and thermal problem, it might turn out to be challenging to minimize the distance between these two stages in order to optimize the $\mu^+$ transmission efficiency.

Another challenging aspect that needs to be addressed is the extraction into vacuum. We are preparing tests to study the gas density and the gas flows in the orifice region. In parallel, we are starting to investigate the injection and extraction of the $\mu^+$ from the high magnetic field. But having proven the feasibility of transverse and longitudinal compression, there seems to be no impediment towards the realization of the full phase space compression scheme.
Cross sections for muonium formation in H$_2$ and O$_2$ can be calculated from the cross sections for protons at the same velocity by means of “velocity-scaling”: According to [52], the stopping power of a $\mu^+$ is the same as for a proton with the same velocity (see also Eq. (3.5)). In Ref. [55], the cross sections of H$^+$ + H$_2$ → H + . . . and H$^+$ + O$_2$ → H + . . . are given. The phenomenological formula according to [55] is given by:

$$\sigma_{qq'} = \sigma_0 [f(E_1) + a_7 f(E_1/a_8)]$$  (A.1)

where $\sigma_0 = 10^{-16}$ cm$^2$, $E_1 = E_0 - E_t$ with $E_0$ the projectile energy and $E_t$ the threshold energy for the process and $(q,q')$ are the initial and final charge states of the projectile. The function $f(E)$ is given by

$$f(E) = \frac{a_1(E/E_R)^{a_2}}{1 + (E/a_3)^{a_2+a_4} + (E/a_5)^{a_2+a_6}},$$  (A.2)

where $a_i$ are parameters fitted to measurements and $E_R=25.00$ keV is the Rydberg energy of 13.6 eV, multiplied by the ratio of the atomic hydrogen mass to the electron mass. $\sigma_{10}$ thus describes the single-electron capture cross section of H$^+$: H$^+$ + H$_2$ → H + . . . For He and Ne, this formula has to be slightly adapted for the case of $\sigma_{10}$ (the other $(q,q')$-combinations need no modification):

$$\sigma_{10} = \sigma_0 \frac{a_1(E/E_R)^{a_2}}{1 + (E/a_3)^{a_2+a_4} + (E/a_5)^{a_2+a_6} + (E/a_7)^{a_2+a_8}}$$  (A.3)

The parameters $a_1$ to $a_8$ as well as $E_t$ are given in Table A.1, and the corresponding graphs showing the cross section for H$^+$ + H$_2$ → H + . . . and H$^+$ + O$_2$ → H + . . . are presented in Fig. A.1.
Appendix A. Cross sections for muonium formation

Table A.1: Parameters used to calculate the cross section ($\sigma_{10}$) for hydrogen formation $H^+ + \ldots \rightarrow H + \ldots$ in various gases using Eq. (A.3), taken from [55]. If the values $a_7$ and $a_8$ are not given, the cross section is evaluated using only the first term in Eq. (A.1).

<table>
<thead>
<tr>
<th>target</th>
<th>$E_t$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>$a_7$</th>
<th>$a_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>0.011</td>
<td>2.69</td>
<td>0.75</td>
<td>34</td>
<td>1.99</td>
<td>73</td>
<td>4.52</td>
<td>5.6</td>
</tr>
<tr>
<td>H$_2$</td>
<td>$1.84 \times 10^{-3}$</td>
<td>53.6</td>
<td>0.84</td>
<td>7.75</td>
<td>1.22</td>
<td>35.4</td>
<td>4.97</td>
<td>...</td>
</tr>
<tr>
<td>O$_2$</td>
<td>$-1.5 \times 10^{-3}$</td>
<td>$3.86 \times 10^5$</td>
<td>1.6</td>
<td>0.069</td>
<td>0.33</td>
<td>7.86</td>
<td>3.92</td>
<td>$3.2 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Figure A.1: Cross section for charge exchange rate of $H^+ + H_2 \rightarrow H + \ldots$ (left) and $H^+ + O_2 \rightarrow H + \ldots$ (right) as function of the projectile energy. Reproduced from [55].
B Runaway effect

When charged particles in a gas are continuously accelerated by an electric field, the so-called "runaway" effect can occur. Runaway takes place when the reduced electric field \( E/N \), with \( E \) being the electric field strength, and \( N \) the number density of gas atoms, is so high that the energy loss of ions due to collisions with the gas atoms cannot compensate the energy gained by the electric field. Therefore, the average energy of the ions increases continuously. This effect has been described theoretically [60] and verified experimentally with protons in He gas [61]. A thorough theoretical derivation of the runaway effect can be found in [49, 63], whereas here only a brief summary is presented.

We consider an elastic collision between an ion with mass \( m \) and a gas molecule having mass \( M \). The relative momentum of an ion-neutral collision is \( M_r \vec{v}_r \), where \( M_r = \frac{mM}{m+M} \) is the reduced mass and \( \vec{v}_r \) is the ion-neutral relative velocity. The momentum loss of the ion can be split into a component along the electric field and a component perpendicular to it. For sufficiently strong electric fields, the average momentum component transverse to the electric field averages to zero, leaving only a momentum loss parallel to the field:

\[
\Delta(\!\!\! M_r \vec{v}_r \!\!\!) = \langle M_r \vec{v}_r (1 - \cos \theta) \rangle, \tag{B.1}
\]

where \( \theta \) is the angle between the relative velocities before and after the collision. If we are only interested in the momentum loss along the electric field, we can replace \( \langle \vec{v}_r \rangle \equiv \vec{v}_D \) by the drift velocity \( v_D \), which is the average velocity along the electric field, i.e., the quantity of interest. Thus, we write for the average momentum loss along the electric field direction:

\[
M_r v_D (1 - \cos \theta). \tag{B.2}
\]

The average number of collisions of an ion per unit time into the angle between \( \theta \) and \( \theta + d\theta \) is

\[
N \pi_r \sigma(\pi_r, \theta) \sin \theta d\theta, \tag{B.3}
\]

with \( \pi_r \) being the mean relative speed between the ion and the gas atoms, \( \sigma(\pi_r, \theta) \)
the differential cross section for elastic collisions, and $N$ the gas number density.

Multiplication of Eq. (B.2) with Eq. (B.3), and integration over all deflection angles $\theta$, yields the momentum loss $\Delta P$:

$$\Delta P = M_r v_D N \tau_r 2\pi \int_0^\pi (1 - \cos \theta) \sigma(\tau_r, \theta) \sin \theta d\theta. \quad (B.4)$$

The integral

$$\sigma_{tr}(\tau_r) \equiv 2\pi \int_0^\pi (1 - \cos \theta) \sigma(\tau_r, \theta) \sin \theta d\theta, \quad (B.5)$$

is called the momentum-transfer or transport cross section $\sigma_{tr}$.

In steady-state conditions, the momentum gained by the electric field per unit time, $eE$, equals the momentum lost due to collisions per unit time. Therefore, we can write for steady-state conditions

$$eE = M_r v_D N \tau_r \sigma_{tr}(\tau_r). \quad (B.6)$$

The remaining task is to relate $v_D$ to $\tau_r$. We write for the mean squared relative velocity

$$\bar{v}^2 = (\vec{v} - \vec{V})^2 = \bar{v}^2 + \bar{V}^2; \quad (B.7)$$

where $\vec{v}$ is the ion velocity and $\vec{V}$ is the neutral velocity. The cross term averages to zero. By combining the equations of momentum and energy balance, as is derived extensively in Ref. [63], we find for the ion energy

$$\frac{1}{2} m \bar{v}^2 = \frac{1}{2} M \bar{V}^2 + \frac{1}{2} m v_D^2 + \frac{1}{2} M v_D^2. \quad (B.8)$$

The first term is equal to $3kT/2$ and is the thermal energy of the ion acquired by collisions with the gas atoms. The next term is the part of the kinetic energy obtained from the electric field, visible as drift motion. The last part represents the random part of the field energy. Light ions ($m \ll M$) have most of their field energy as random motion, whereas for heavy ions, most energy is contained in the drift motion. Two assumptions are made at this point. Firstly, we can interchange mean squared velocities with squared mean velocities. Secondly, we assume that $\bar{V}^2 = 0$, i.e., thermal contributions are negligible. Solving Eq. (B.8) for $v_D$ under these two assumptions and using Eq. (B.7) yields the relation between $v_D$ and $\tau_r$:

$$v_D = \tau_r \left( \frac{m}{m + M} \right)^{1/2}. \quad (B.9)$$

$1$The force on an ion is $eE$ which must be equal to the momentum gained from the field per unit time, according to Newton’s second law: $F = dp/dt$
Appendix B. Runaway effect

The above assumptions, together with the fact that we are using mean velocities rather than considering the full distribution, introduces an error of $O(1)$ [49]. We define the mean relative kinetic energy as

$$E_{\text{kin}} \equiv \frac{1}{2} M_r \left( \frac{v^2 + V^2}{v_r^2} \right) = \frac{1}{2} M_r \bar{v}_r^2,$$  \hspace{1cm} (B.10)

where for the second equality both our assumptions were used. By using Eqs. (B.9) and (B.10), the energy balance equation (B.6) becomes

$$\epsilon E = 2E_{\text{kin}} N \left( \frac{m}{m + M} \right)^{1/2} \sigma_{\text{tr}}(\bar{v}_r).$$ \hspace{1cm} (B.11)

As mentioned before, runaway occurs if the energy gained by the electric field between collisions ($E$) is larger than the average energy lost per collision. This condition can thus be written as

$$\left( \frac{E}{N} \right) > 2E_{\text{kin}} \sigma_{\text{tr}}(\bar{v}_r) \left( \frac{m}{m + M} \right)^{1/2}.$$ \hspace{1cm} (B.12)

It is reported by [60] and [61] that runaway occurs for protons in He when the mass-scaled field strength $\left( \frac{m + M}{m} \right)^{1/2} (E/N)$ is above 220 Td, i.e.,

$$\left( \frac{E}{N} \right) > 220 \text{ Td} \left( \frac{m}{m + M} \right)^{1/2}.$$ \hspace{1cm} (B.13)

For muons in helium gas, the mass-scale factor $\left( \frac{m + M}{m} \right)^{1/2} \approx 6$, and the condition for runaway translates into

$$E/N \gtrsim 36 \text{ Td}. \hspace{1cm} (B.14)$$
C Chemical capture rate based on atomic polarizabilities

The goal of this appendix is to derive the cross sections and "chemical capture" rates for muons at contaminants. We compute these quantities assuming an interaction between the $\mu^+$ and the contaminant to be given by the interaction between the electric field of the charged particle and the induced dipole of the contaminant. This potential takes the form of a polarization potential

$$U_{\text{pol}}(r) = -\frac{e^2 \alpha}{2r^4},$$  \hspace{1cm} (C.1)

where $\alpha$ is the polarizability of the neutral system, and $r$ the distance between the $\mu^+$ and the neutral system. Using a classical trajectory approach, starting from this potential, the cross section $\sigma$, and the corresponding collision rate $R$ can be derived.

Consider a $\mu^+$ colliding with another particle (that could be an atom or a molecule which we assume without internal structure). The dynamics of the two colliding particles moving under the action of the interaction potential $U(r)$ can be described using classical trajectories. The trajectories of this potential have first been described by Langevin [70], and a reprint of typical orbits is shown in Fig. C.1. The trajectories are defined by two parameters: The relative velocity $v$ between the $\mu^+$ and the neutral particle, and the impact parameter $b$. As shown in Fig. C.1, there exists a critical value $b_0$ for which the incoming $\mu^+$ performs a spiraling motion towards the center, until some repulsive force stops it. The critical parameter is given by [71]

$$b_0 = \left( \frac{4e^2 \alpha}{Mr v^2} \right)^{1/4},$$  \hspace{1cm} (C.2)

where $Mr$ is the reduced mass of the system. We assume that for $b < b_0$ the $\mu^+$ undergoes chemical capture, and thus the capture cross section is

$$\sigma(v) = \pi b_0^2 = \frac{\pi}{v} \sqrt{\frac{4e^2 \alpha}{Mr}}.$$  \hspace{1cm} (C.3)
Figure C.1: Classical trajectories as a function of the impact parameter $b$. Trajectories with $b < b_0$, $b_0$ being a critical impact parameter, enter the reaction sphere and "fall" towards the center. These curves are modified from [70], reproduced from [71].
Appendix C. Chemical capture rate based on atomic polarizabilities

The collision rate \( R = \sigma n v \) thus becomes

\[
R = \frac{\pi}{v} \sqrt{\frac{4e^2\alpha}{M_r}} vn = n\sqrt{\frac{4\pi^2e^2\alpha}{M_r}}. \tag{C.4}
\]

In the following, we derive the numerical factors used in Sec. 4.5. Starting from Eq. (C.4) and expressing the elementary charge in cgs units\(^1\) we get:

\[
R = n\sqrt{\frac{4\pi^2e^2\alpha}{M_r}} \tag{C.5}
\]

\[
= n\sqrt{\frac{4\pi^2(4.803 \times 10^{-10}\text{gcm}^3/\text{s})^2 \times \alpha[\text{cm}^3]}{1.883 \times 10^{-25}\text{g}}} \tag{C.6}
\]

\[
= n\sqrt{4.83654 \times 10^7 \cdot \sqrt{\alpha[\text{cm}^3]} \text{cm}^3/\text{s}} \tag{C.7}
\]

\[
= n[1/\text{cm}^3]\sqrt{\alpha[\text{cm}^3]} \cdot 6954 \text{ s}^{-1}. \tag{C.8}
\]

Expressing the polarizability in units of \(10^{-24} \text{ cm}^3\) (1 mbar = \(2.65 \times 10^{16} \text{ cm}^{-3}\) at room temperature), yields

\[
R = p[\text{mbar}]\sqrt{\alpha[10^{-24}\text{cm}^{-3}]} \cdot 1.84 \times 10^8 \text{ s}^{-1}. \tag{C.9}
\]

Using the polarizability values \(\alpha(H_2) = 0.818 \times 10^{-24} \text{ cm}^3\) and \(\alpha(O_2) = 1.570 \times 10^{-24} \text{ cm}^3\) [72], we find thus

\[
R_{H_2}(n) = n[1/\text{cm}^3] \cdot 6.290 \times 10^{-9} \text{ s}^{-1} = p[\text{mbar}] \cdot 1.67 \times 10^8 \text{ s}^{-1} \tag{C.10}
\]

\[
R_{O_2}(n) = n[1/\text{cm}^3] \cdot 8.714 \times 10^{-9} \text{ s}^{-1} = p[\text{mbar}] \cdot 2.31 \times 10^8 \text{ s}^{-1}. \tag{C.11}
\]

These rates can be used to calculate the cross section \(\sigma\)

\[
\sigma = \frac{R}{vn} = \frac{\sqrt{\alpha[\text{cm}^3]}6954}{v[m/s]} \text{ cm}^2 \tag{C.12}
\]

which is plotted versus the muon kinetic energy in Fig. C.2.

---

\(^1\) \(e=4.803 \times 10^{-10}\) statC, i.e., 1 statC = \(\frac{1 m\cdot A\cdot s}{10 C} \approx 3.335 \times 10^{-10} \text{ C}\)
Figure C.2: Cross section $\mu^+ + H_2 \rightarrow \text{"chemical capture"}$ (black) and $\mu^+ + O_2 \rightarrow \text{"chemical capture"}$ (red) calculated according to Eq. (C.12).
D  Fit results 2014

All longitudinal compression data of 2014 have been fitted with a constant function at times between 2.5 µs and 6 µs. The fit results are presented in Table D.1, where the columns “amplitude (−HV)” corresponds to the count level when a negative potential was applied, and correspondingly for the column “amplitude (+HV)” or “amplitude (0 HV)”. The levels a and b, respectively, are the levels after the subtraction of the “+HV” amplitude, as illustrated in Fig. 4.9. The ”+HV” run is always the most positive potential for the corresponding pressure and cell configuration. For example, for the long cell at 5 mbar and -200 V, the ”+HV” corresponds to +500 V. The fits were performed using a weighted log-likelihood method within the ROOT-framework.

The comment in the last row of the table has the following meaning. Results labeled “good” are used for the analysis, whereas the other data sets are excluded due to the following reasoning: “no runaway” means that there is no stable flat top, i.e., the reduced electric field $E/N$ is not above the threshold for runaway condition. The two runs with “rising BG” mean that there was no pileup correction implemented (see also Sec. 4.2.1), i.e., there is an uncorrelated background that rises in the lifetime corrected time spectra preventing a reasonable fit.
Appendix D: Fit results 2014

Table D.1: Compression results from 2014. The fit was performed for time $t \in [2.5, 6] \mu s$ and time spectra measured in the detectors $T_1$ and $T_2$ in coincidence.

<table>
<thead>
<tr>
<th>HV</th>
<th>$p$</th>
<th>$a/\phi$</th>
<th>$\phi$</th>
<th>$a$</th>
<th>(0, H)</th>
<th>(0, H$^+$)</th>
<th>(0, H$^-$)</th>
<th>$d$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>550</td>
<td>5</td>
<td>30.63</td>
<td>8.49 ± 1.00</td>
<td>30.63</td>
<td>8.49 ± 1.00</td>
<td>30.63</td>
<td>8.49 ± 1.00</td>
<td>30.63</td>
<td>8.49 ± 1.00</td>
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<tr>
<td>450</td>
<td>5</td>
<td>6.82 ± 0.79</td>
<td>4.94 ± 0.47</td>
<td>4.94 ± 0.47</td>
<td>4.94 ± 0.47</td>
<td>4.94 ± 0.47</td>
<td>4.94 ± 0.47</td>
<td>4.94 ± 0.47</td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>5</td>
<td>6.22 ± 0.58</td>
<td>4.47 ± 0.38</td>
<td>4.47 ± 0.38</td>
<td>4.47 ± 0.38</td>
<td>4.47 ± 0.38</td>
<td>4.47 ± 0.38</td>
<td>4.47 ± 0.38</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>5</td>
<td>5.62 ± 0.49</td>
<td>3.94 ± 0.33</td>
<td>3.94 ± 0.33</td>
<td>3.94 ± 0.33</td>
<td>3.94 ± 0.33</td>
<td>3.94 ± 0.33</td>
<td>3.94 ± 0.33</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>5</td>
<td>5.02 ± 0.41</td>
<td>3.37 ± 0.27</td>
<td>3.37 ± 0.27</td>
<td>3.37 ± 0.27</td>
<td>3.37 ± 0.27</td>
<td>3.37 ± 0.27</td>
<td>3.37 ± 0.27</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>5</td>
<td>4.42 ± 0.35</td>
<td>2.84 ± 0.24</td>
<td>2.84 ± 0.24</td>
<td>2.84 ± 0.24</td>
<td>2.84 ± 0.24</td>
<td>2.84 ± 0.24</td>
<td>2.84 ± 0.24</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>5</td>
<td>3.82 ± 0.29</td>
<td>2.26 ± 0.19</td>
<td>2.26 ± 0.19</td>
<td>2.26 ± 0.19</td>
<td>2.26 ± 0.19</td>
<td>2.26 ± 0.19</td>
<td>2.26 ± 0.19</td>
<td></td>
</tr>
</tbody>
</table>

*con_junction*
E Properties of Stycast epoxy

Stycast® is a two-component epoxy made by Emerson & Cuming™, suited for cryogenic applications. Two different models have been tested for the construction of the transverse target: Stycast 1266 and Stycast 2850 FT with catalyst 24 LV. They are both room-temperature curing and electrically insulating. The main difference is that Stycast 2850 is filled with graphite particles which improves thermal conductivity, and its thermal expansion coefficient is matched to the expansion coefficient of brass. But due to the graphite filling, it is not an ideal epoxy to hermetically seal a target. Therefore, all cryogenic glueing was done using the transparent, low-viscosity Stycast 1266. Mentioned below are some of the properties of the Stycast 1266.

E.1 General properties

Many properties are listed in the data sheet [89]. Stycast 1266 has a mixing ratio of 28 parts of Part B per 100 parts of Part A, measured by weight. A handy quantity is 5 g of Part A mixed with 1.4 g of Part B. The working life time is between 30 and 60 minutes. Curing times are 8-16 h at room temperature (25° C) or 1-2 h at 65° C. In order to obtain optimal results, the mixed epoxy should be degassed before applying.

E.2 Cryogenic properties

At cryogenic temperatures \( T = 4.2 \, \text{K} \), the fracture stress of Stycast 1266 is about 18 kg/mm\(^2\), the Young’s modulus is between 435 kg/mm\(^2\) and 467 kg/mm\(^2\), and the critical stress 11.8 kg/mm\(^2\), as reported in [90]. Note that these properties are a few times higher at cryogenic temperatures than at room temperature. The integral thermal contraction between 296 K and 4 K is 1.15%, i.e., \( L_4 = 0.9885 \times L_{296} \), where \( L_4 \) and \( L_{296} \) are the length at a temperature of \( T = 4 \, \text{K} \) and \( T = 296 \, \text{K} \), respectively [91]. A plot of the linear expansion coefficient is shown in Ref. [87] (Fig. 3.17 therein). Unfortunately, no conclusive values of the thermal conductivity of Stycast
1266 around 10 K have been found. However, in Ref. [87], the thermal conductivity at 1 K is given as 0.039 W/(m·K), which is four times higher than for the Stycast 2850 FT and about 20 times higher than 100 mbar He gas at 4 K [82].
F Glueing the transverse target

The target for the transverse compression stage consists of a top and a bottom sapphire plate, the Kapton foil with electrodes, the side walls made from Al₂O₃ ceramics, and the end caps with PVC frames and Kapton foils. In order to obtain a He tight target at cryogenic temperatures, the thermal expansion coefficients of the various materials have to match each other. Sealing of the target happens at the transition between two Kapton foils, and between the PVC end cap and the Kapton foil. Both the Kapton and the PVC are "flexible" enough to deal with the different thermal expansion coefficient of the respective material and the glue.

For these cryogenic applications, the Stycast 1266 two-component epoxy is well suited. It has low viscosity and therefore fills small gaps easily. More information can be found in Appendix E, where some properties of the Stycast epoxy are listed. The transverse target is glued in three steps, which are described in the following.

Step 1: Glueing Kapton to the sapphire

In order to obtain a reproducible leak tight target, it is essential that a mask is used to position the sapphire relative to the side walls and the Kapton foil. Alignment of the foil to the sapphire is best practiced before glueing, and then the foil is fixed with some tape at one side to the mask. This is shown in Fig. F.1, where the Kapton foil is already glued onto the sapphire plates, and the side walls and end caps are also glued onto the Kapton foil. This step is the most important one, since it defines the exact geometry of the transverse target. Also, a misalignment of ~ 1 mm makes it impossible to seal the target hermetically. Therefore, the two sapphires are put into the mask and the top sapphire is fixed with the spacers indicated in Fig. F.2, in order to not allow it any freedom for movements. Particular care has to be taken that the superfluous glue coming out between the Kapton and the sapphire (at position "(a)" in Fig. F.2) does not touch the mask. Because the thermal contact between the Kapton and the sapphire should be as good as possible, there must not be any air bubbles enclosed in the glue between Kapton and sapphire. Eventual bubbles can be removed by "massaging" them out by hands. The amount of glue used to glue the Kapton foils to the sapphire is about 1 g of glue per sapphire plate. It does not
matter whether the end caps are glued onto the Kapton “lying” on their side (as in Fig. F.1) or standing upright (as in Fig. F.2). The important point is that they are positioned such that - when later on the target is “folded” into the triangular shape - the HV connectors are precisely at the desired position in the corner of the cell. Otherwise, the electric field is not correct, which cannot be adjusted anymore. Once the side walls and end caps are in position, a lot of weights has to be put onto the Kapton foil to press the Kapton to the sapphire while the glue is curing.

Step 2: Closing the target

The glue has to cure for about 16 hours at room temperature before proceeding with the next step, in which the top sapphire with the Kapton foil glued to it is lifted out of the PVC mask and the Kapton foil is folded around the triangular side walls. A photo of the partly closed target is shown in Fig. F.3. The target is sealed hermetically at the interface between the Kapton foils in the tip and around the PVC end caps.

Step 3: Adding gas connections and temperature sensor mountings

The final step is to glue the gas tubes into the end caps. This can only be done after the target has been mounted to the copper base plate. Additionally, some mechanics for the temperature sensors on top of the target has to be glued onto the sapphire, as well as the heat pad. These items should also be pressed against the sapphire while curing the glue.
Appendix F. Glueing the transverse target

Figure F.2: Sketch of the first step when glueing the transverse target used in 2015.

Figure F.3: Step 2 of the glueing. The top plate of the sapphire is put onto the side walls. The PVC end caps tighten the target, whereas the Al$_2$O$_3$ ceramic side walls define the target geometry.
G  Thermal conductivity coefficients

![Figure G.1: Thermal conductivity of selected materials. Reproduced from [75].](image)

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Temperature gradient in the target for various heating powers

The measured temperatures of the top and bottom sapphire plate, respectively, as a function of the applied power dissipated in the heat pad mounted on top of the target, are presented in Table H.1. Approximately 5 mbar He gas were inside the target. No magnetic field was applied, and electric discharge tests were performed during the measurements, which distort the measured temperatures slightly as they can heat up the He gas inside the target. Therefore, the reported values should only be taken as benchmark values.

Table H.1: The measurements were performed with about 5 mbar He gas inside the target and no magnetic field. Discharge tests were performed during the measurements.

<table>
<thead>
<tr>
<th>$P_{heating}$ [mW]</th>
<th>$T_{bottom}$ [K]</th>
<th>$T_{top}$ [K]</th>
<th>$\Delta T$ [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.8</td>
<td>8.5</td>
<td>3.74</td>
</tr>
<tr>
<td>49.75</td>
<td>5.2</td>
<td>12.5</td>
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</tr>
<tr>
<td>49.75</td>
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<td>7</td>
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<td>93.48</td>
<td>5.5</td>
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<tr>
<td>193.28</td>
<td>6.1</td>
<td>19.7</td>
<td>13.5</td>
</tr>
</tbody>
</table>

Similar measurements as presented in Table H.1 were also performed with a different, but almost identical, target, and they are presented in Table H.2. For these measurements, a new base plate was used, i.e., the coupling between the cold finger and the base plate is different. The main difference was that for some measurements
Appendix H. Temperature gradient in the target for various heating powers

A magnetic field of 5 T was applied, and the vacuum tube of the cryostat was inside the bore hole of the magnet, slightly reducing the incident thermal radiation.

Table H.2: The He gas pressure inside the target was about 6 mbar for all the measurements. Discharge tests were performed during the measurements.

<table>
<thead>
<tr>
<th>B [T]</th>
<th>P_{heating} [mW]</th>
<th>T_{bottom} [K]</th>
<th>T_{top} [K]</th>
<th>ΔT [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>260</td>
<td>6.3</td>
<td>19.6</td>
<td>13.3</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>4.7</td>
<td>7.8</td>
<td>3.1</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>4.8</td>
<td>8.5</td>
<td>3.7</td>
</tr>
<tr>
<td>0</td>
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<td>5.0</td>
<td>11.5</td>
<td>6.5</td>
</tr>
<tr>
<td>0</td>
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<td>13.8</td>
<td>8.6</td>
</tr>
<tr>
<td>0</td>
<td>110</td>
<td>5.5</td>
<td>15.5</td>
<td>10</td>
</tr>
<tr>
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<td>170</td>
<td>6.0</td>
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<td>12.2</td>
</tr>
<tr>
<td>5</td>
<td>210</td>
<td>6.2</td>
<td>19.7</td>
<td>13.5</td>
</tr>
</tbody>
</table>

It has to be noted that for both measurements, the front of the vacuum tube was sealed with a glass window, and not the full thermal shield was mounted compared to the situation during the beam time. This leads to a higher overall temperature compared to the minimal achievable temperature.
## Example of $\pi E1$ beam line settings

**Table I.1: Setting of active beam line elements in $\pi E1$ on 10\textsuperscript{th} Dec 2015. The setting is optimized for $\mu^+$ with a momentum of 10.1 MeV/c.**

<table>
<thead>
<tr>
<th>Element</th>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
<th>Factor</th>
<th>Comment</th>
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</thead>
<tbody>
<tr>
<td>magnet</td>
<td>QTH51</td>
<td>-15.25</td>
<td>A</td>
<td>0.5</td>
<td>scalevalue 20%</td>
</tr>
<tr>
<td>magnet</td>
<td>QTH52</td>
<td>7.29</td>
<td>A</td>
<td>0.5</td>
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Appendix I. Example of $\pi E_1$ beam line settings

Figure I.1: Beam profile of the $\pi E_1$ beam line. Courtesy of C. Petitjean.
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