Doctoral Thesis

Simulation of Power System Dynamics using Dynamic Phasor Models

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Simulation of Power System Dynamics using Dynamic Phasor Models

A dissertation submitted to the
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presented by
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Abstract

Computer simulations of electric power systems are an essential part of planning, design and operation in the power industry. Due to the increased loading of the power systems, the stability has become a concern, so that programs for the analysis of the transient behavior of power systems have become an integral part of system design and control, in order to maximize the ability of a system to withstand the impact of severe disturbances. Generally as large systems are under consideration, simplifying assumptions are often made to facilitate an efficient simulation of such large power systems with the so called Transient Stability Programs (TSP). In the TSP, the fast electromagnetic transients are neglected and it is assumed that the power transfer takes place at the system frequency. The focus in these programs are more on the slower electromechanical transients, which have more a system wide effect. However, recent blackouts due to increasingly sensitive operating conditions, have created a need for more detailed and comprehensive studies. Such detailed studies including the fast electromagnetic transients are done with so called Electromagnetic Transient Programs (EMTP). In contrast to the electromechanical transients, the electromagnetic transients have more a local character so that only a small part of the complete power system is usually studied with the EMTP. The simulation of the complete power system with the EMTP is computationally inefficient, since too small simulation step sizes are employed for the calculation of the fast electromagnetic transients. Hence, the combined simulation of the electromagnetic and electromechanical transients is a challenging task.

The aim of this dissertation is to fill the gap between the TSP and EMTP by developing a new simulation tool based on the dynamic phasor representation of the power system, which facilitates the combined
simulation of the electromagnetic and electromechanical transients in an accurate, efficient and systematic way. For this purpose, a general and systematic simulation framework was developed for the simulation of power system transients with the dynamic phasor models of major power system components. The accuracy and computational efficiency of the dynamic phasor model representation were compared with other traditionally used power system representations. Furthermore, new numerical integration algorithms were developed for the accurate and efficient simulation of systems represented by dynamic phasors. The developed prototype of the new simulation tool was implemented in the commercially used power system analysis program NEPLAN [1].
Kurzfassung


Somit kann man sagen, dass die kombinierte Simulation von elektromagnetischen und elektromechanischen Ausgleichsvorgängen noch immer eine grosse Herausforderung darstellt. Das Ziel dieser Dissertation ist die existerende Lücke zwischen den TSP und EMTP zu schliessen,
Chapter 1

Introduction

The electric power system is today regarded as the most important of all infrastructure systems built by mankind [2]. Also, other systems such as telecommunication, transportation, water supply, etc. are more or less dependent on a reliable supply of electric power for their proper operation. Consequently, the requirements on the security and reliability of the electric power system are very high. Due to the increased loading of the power systems, the stability has become a concern. This is of course accentuated by the recent blackouts in the USA and Europe. In order to achieve the required high level security and reliability, the power system must be analyzed in detail during planning and operation. The need for simulations in the field of power systems arose already at the beginning of the 20th century from the impracticability of performing system wide experiments. For this reason, various analytical tools have been developed to study the dynamic behavior of power systems. Programs for the analysis of the transient behavior of power systems have become an integral part of system design and control, in order to maximize the ability of a system to withstand the impact of a severe disturbance.

Time-domain simulation programs are an integral part of the power system analysis tools. With the time-domain simulation programs the dynamic response of a power system to disturbances or to changes in the system state can be computed by using the appropriate mathematical models and numerical algorithms.

The dynamic phenomena in power systems can be classified into differ-
Chapter 1. Introduction

Figure 1.1: Time frame of power system transients

ent time ranges [3]. As depicted in Figure 1.1, the time range of the power system transients varies from microseconds (Lightning) to hours and days (Long term dynamics). The overall time range of power system transients is generally classified into fast electromagnetic transients and slow electromechanical transients. Thus, the complete power system can be seen as a coupled electromechanical and electromagnetic system with a wide range of time constants. There are fast electromagnetic transients due to the interaction between the magnetic fields of inductances and electrical fields of capacitances. Besides, there are also slower electromechanical transients due to the interaction between the mechanical energy stored in the rotating machines. Generally, different time-domain simulation tools are used for studying the different dynamic phenomena in power systems.

Electromagnetic transient phenomena are usually triggered by changes in the network configuration. Such changes may be due to the closing or opening action of circuit breakers or power electronic equipment, or by equipment failure or faults. These phenomena are more local in character. This fast electromagnetic transients are typically studied with the Electromagnetic Transients Programs (EMTP) [4]. As fast dynam-
ics are of concern, the simulation step size is of the order of tens of microseconds but can even be smaller depending on the type of electromagnetic phenomenon being studied. Due to the long time constants associated with the dynamics of power plants, such as generators and turbines, simplified models of such devices are often sufficient for the time frame of typical electromagnetic transients studies.

Electromechanical transients are slower transients that are due to the interaction between the mechanical energy stored in the rotating machines and the electrical energy stored in the electrical network. A mismatch between the mechanical energy and the electrical energy involves the oscillation of machine rotors because of an unbalance between turbine and generator torques. The analysis of this class of transients is known as transient stability simulation. Such studies are done with the so called transient stability programs (TSP) (e.g. SIMPOW [5], EUROSTAG [6]). An important assumption for this type of analysis is that the exchange of energy between generators and other dynamic equipment takes place with the electric network remaining at power frequency (system frequency). With this assumption, electromagnetic transients cannot be properly represented and are neglected during the simulation process. This is achieved by using the steady-state phasor representation of the electrical network quantities at the system frequency. This approach is referred to as the quasi-steady state approach in the literature. The omission of the fast electromagnetic transients allows the transient stability simulation programs the use of larger step sizes. These transient stability programs usually include models of different power system components that are appropriate for phenomena that have characteristic time constants that are about a hundred milliseconds or larger. With these simulation programs, very large power systems could be studied, e.g. interconnected system of Western Europe - UCTE. The phenomena of concern in these studies are quite often system wide, and the complete system needs in many cases to be included for meaningful analysis.

For a long period of time, the electromagnetic transients programs and transient stability programs have not been unified in a common simulation program. Reasons for the separate use of these programs are:

- The objectives of electromagnetic transients analysis and of transient stability analysis are different. In the electromagnetic transients analysis one investigates fast transient phenomena that usu-
ally decay rapidly and have no impact on the slower system dynamics. In the transient stability analysis the aim is to study the impact of a severe disturbance on the whole power system.

- It is technically possible to perform a transient stability analysis also in an EMTP. But the required simulation time in an EMTP would increase drastically due to the following reasons:
  
  - the longer time span of the transient stability simulations compared with the electromagnetic transients analysis.
  - the use of much smaller integration step sizes in an EMTP compared with the traditional transient stability programs.
  - the size of the power system to be simulated in transient stability simulations is much larger than that is typically used in electromagnetic transients analysis. Because of the local character of electromagnetic transients due to the higher damping, the size of the power system is much smaller compared with the transient stability analysis.

- Both program types generally use different system representations for the modelling of the components and also different integration methods to facilitate an efficient simulation of the transient phenomena under consideration.

Because of the above mentioned problems, the combined simulation of the electromagnetic and electromechanical transients has always been a challenging task.

There are different approaches to overcome this problem. In the simulation program SIMPOW [7], the full network equations and machine equations are described in the DQ0 reference frame which is rotating with the system frequency. This representation of the system equations in the DQ0 reference frame increases the simulation efficiency significantly under balanced conditions, since the variations in the DQ0 transformed quantities are much slower than the instantaneous ABC quantities. SIMPOW has two simulation modes, one for electromagnetic and another for electromechanical transients. There is also the possibility to switch between these modes during the simulation. Such changes in the simulation mode become clearly visible with jumps in the state variables whenever the program switches from the EMTP-mode to the transient stability mode. These artificial jumps may cause e.g.
a trip of a protective relay, thereby altering the system configuration incorrectly.

Other approaches combine the electromagnetic and electromechanical simulations by representing the full network and machine differential equations without making the quasi-steady state approximation and simulate the complete system in the EMTP mode. This is achieved by using an efficient solution algorithm that is accurate and numerically stable for a wide range of frequencies. Efforts in this direction have been reported in [8, 9, 10]. Some important issues related to the work in [9, 10] will be discussed in Chapter 3.

Recent developments, particularly the introduction of more power electronics based equipment e.g. HVDC and FACTS components, also increase the need for the detailed time-domain simulation of such devices, since the transient stability programs based on fundamental frequency phasor modelling techniques cannot directly represent the faster transients characterizing the HVDC and FACTS systems. One approach to overcome this problem, is to model and simulate some parts of the system with a detailed full time domain representation and the rest of the system in the quasi-steady state representation and interface the different modelling approaches appropriately. In [11, 12], this approach was used to simulate the HVDC and FACTS transient/dynamic behavior in a power system based on an interactive execution of a TSP and an EMTP.

On the other hand efforts have also been made to decrease the lack of accuracy of the fundamental frequency phasor models of the FACTS. For this purpose, the concept of generalized averaging method, also referred to as dynamic phasors approach, was proposed in [13] to model power electronics based equipment. The main idea behind this method is to represent the periodical or nearly periodical system quantities not by their instantaneous values but by their time varying Fourier coefficients (dynamic phasors). The variations of the time varying Fourier coefficients are much slower than the original instantaneous values. The application of this method was then extended to model FACTS devices [14, 15, 16] to increase the accuracy of the fundamental frequency phasor models. The same approach has also been applied to model electrical machines under unbalanced conditions [17, 18, 19].

As mentioned previously, the need for a simulation tool which is able to simulate the electromagnetic and electromechanical transients in an
efficient and accurate way is the main incentive of this research project. Hence, the objective of this thesis is to develop a systematic concept for the combined simulation of electromagnetic and electromechanical transient phenomena. To achieve this goal, a new simulation tool will be developed which is based on the dynamic phasors representation of the power system. A general and systematic simulation framework will be developed including the appropriate numerical methods to compute the system response represented by dynamic phasors efficiently. The dynamic phasor models of the major power system components will be developed and implemented in the new simulation tool. The new system representation with dynamic phasors will be compared systematically with other commonly used system representations in the power system analysis in terms of accuracy and computational efficiency.

1.1 Outline of the Thesis

Following the introduction, in Chapter 2, a general and systematic simulation framework will be developed for the simulation of power system transients, which will then be also used throughout the thesis.

Chapter 3 gives an overview of the different system representations used in the area of power systems analysis. For the theoretical assessment of the simulation performance of these modelling techniques, the different system representations will be examined in the frequency domain.

In Chapter 4, the dynamic phasor models of major power system components will be derived and their simulation performance will be compared to other commonly used model/system representations in terms of accuracy and computational efficiency.

In Chapter 5 the dynamic phasor model of the doubly-fed induction generator (DFIG) will be derived. The derived model will then be used to study the dynamic response of the DFIG to balanced and unbalanced voltage sags.

Chapter 6, the dynamic phasor model of Thyristor-Controlled Series Capacitor is derived based on previous work done by Mattavelli et al. [14] and the accuracy and simulation efficiency of the model is compared with the detailed time domain model and other existing fundamental frequency models.
1.2 Contributions

The focus in Chapter 7 is on the derivation of methods suitable for the numerical integration of systems represented by dynamic phasors. This is achieved investigating the numerical integration techniques in the frequency domain.

Finally Chapter 8 concludes the thesis by summarizing and discussing the most important achievements and suggesting possible future work.

1.2 Contributions

The main contributions of this PhD studies research work can be summarized as follows:

- A new simulation tool has been developed for the combined simulation of electromagnetic and electromechanical transients, which is based on the dynamic phasor representation of the power system.

- Dynamic phasor models of major power system components, including the DFIG, have been derived and implemented.

- A systematic comparison of the dynamic phasor approach has been made with the existing power system representations regarding accuracy and computational efficiency.

- A new numerical integration method based on the trapezoidal method has been developed, which is optimized for the numerical integration of systems represented by dynamic phasors.

1.3 List of Publications

Publications related to the Topic of the Thesis


Chapter 1. Introduction

Frame in Power System Dynamic Simulations” Presented at the 7th IEE International Conference on Advances in Powers System Control, Operation and Management, 30 October - 2 November, Hong Kong, 2006.


Other Publications in the Project


Chapter 2

Simulation Framework

In this chapter, a general and systematic simulation framework is developed for the simulation of power system transients. The chapter starts with an introduction to the Differential Switched Algebraic State-Reset representation, which is used as a systematic and general mathematical representation for the hybrid dynamic behavior of power systems. Based on this representation a modular and flexible simulation environment will be developed. Important procedures of the simulation process and some implemented tools facilitating the model development for the modeler will be covered in more detail. The chapter ends with a power system case study showing the capabilities of the implemented simulation environment.

2.1 Introduction

The dynamic behavior of physical systems are often studied with appropriate simulation programs. Simulation programs consist of appropriate mathematical models describing the relationship between the quantities that can be observed in the system as mathematical relations and numerical methods that are used for the numerical calculation of the complex system behavior where a closed form analytic solution can not be specified. In the power system transient analysis, component models can be described by Ordinary Differential Equations (ODEs), Differential-Algebraic Equations (DAEs) or Partial Differential Equations (PDEs)
depending on the nature of the study. For example, if the purpose of the study is more local and the interest is on the distribution of electromagnetic fields along a transmission line in a power system, then a distributed parameter representation with Partial Differential Equations will be used as a mathematical representation for the transmission line model. If the purpose of the study is the interconnection of the transmission line with other components in the system, where interaction between electromagnetic fields of different components is of concern, a lumped parameter representation (R,L,C) with Ordinary Differential Equations would be sufficient as a mathematical representation. If our concern is on slower and more system wide electromechanical transients, even the electromagnetic transients of the transmission line can be neglected and its behavior can be described by simple algebraic equations.

Furthermore, the choice of an adequate mathematical model representation also depends on the characteristics of the system under investigation and thus of the components which build up the system. For example, electrical power system is composed by several components that form a large-scaled system. There are large generation units such as hydro plants, thermal plants having slow continuous dynamic behavior with large time constants in order of a few seconds. But there are also controllers such as Automatic Voltage Regulators (AVR) showing fast continuous dynamics with time constants in order of fifty milliseconds. Components such as tap changing transformers and protection devices exhibit also discrete dynamics due to discrete behavior e.g. transformer tap positions and protection relay logics. As the whole power system is interconnected, there is a permanent interaction between these continuous and discrete dynamics. The mathematical representation of the models should capture both continuous and discrete nature of the components.

Another important criterion in the selection of the mathematical representation of a system is the linearity. If all the operators in a mathematical model present linearity, the resulting mathematical model is defined as linear otherwise nonlinear. There are approaches where the nonlinear system behavior is approximated by piecewise linear system behavior. In such cases computationally efficient methods and algorithms for linear systems are applied to compute the system behavior. The efficiency of these piecewise linear approximations depend on the degree of the system nonlinearities. Also in power systems there are many sources of nonlinearities. For example some nonlinearities are inherent to compo-
2.2 Hybrid System Representation

Components, like magnetic saturation of iron core or other nonlinearities are in the mathematical relation between the quantities of the system e.g. the relation between active power, voltage magnitude and voltage angle:

\[ P = \frac{V_1 V_2}{X} \sin (\theta_1 - \theta_2). \]

Accordingly power systems can be described as a large-scale, nonlinear system with fast/slow continuous and discrete dynamics. This permanent interaction between continuous and discrete dynamics makes power systems an important class of hybrid systems. The used mathematical representation should be capable of capturing all these characteristics of the power system in a mathematical formulation. Our focus in the following section will be on the mathematical system representation proposed in [20] and [21], which captures all the important aspects of power systems. This representation will be then used as system/model representation throughout the thesis.

2.2 Hybrid System Representation

An adequate mathematical representation for power systems should capture the nonlinear continuous and discrete dynamics. Such systems, where the system behavior is governed by both discrete and continuous states, are called hybrid systems. In hybrid systems, there is such a strong coupling between these discrete and continuous behavior of the system, so that they must be analyzed simultaneously.

Hybrid systems are characterized by:

- continuous and discrete states
- continuous dynamics
- discrete events
- mappings that define the evolution of discrete states at events

Conceptually, a hybrid system can be thought as a directed graph as illustrated in Figure 2.1, where the nodes describe the continuous dynamic system behavior at a given mode and the edges show the conditions and directions of the transition from one system mode to another. If one transition condition is fulfilled, the system jumps from one mode
Chapter 2. Simulation Framework

Figure 2.1: Directed graph view of a hybrid system

to an other and remains there, till another transition condition is fulfilled, which takes the system to one other mode. The jumping reflects the influence of the discrete event behavior and is dependent both on trigger condition and the discrete state evolution mapping.

In power systems, the continuous system behavior at different modes can be described by Differential-Algebraic Equations (DAEs). The discrete events or switching actions inherent in power systems force the system to jump to another system mode as illustrated in Figure 2.1, where the continuous behavior of the system is described by another set of DAEs. This jump to new set of DAEs can be caused due to a change in a discrete state variable e.g. tap position of a tap changing transformer.

As proposed in [20] and [21], such a hybrid system behavior can be modeled by a set of $\text{Differential, Switched Algebraic and State Reset}$ equations (DSAR) as given in equations (2.1) - (2.5).

\[
\begin{align*}
\dot{x} & = f(x, y, z, \lambda) \quad (2.1) \\
\dot{z} & = 0 \quad (2.2) \\
0 & = g^{(0)}(x, y, z) \quad (2.3) \\
0 & = \begin{cases} 
g^{(i^-)}(x, y, z, \lambda) & y_{s,i} < 0 \\
g^{(i^+)}(x, y, z, \lambda) & y_{s,i} > 0 
\end{cases} \quad i = 1, \ldots, s \quad (2.4) \\
z^+ & = h_j(x^-, y^-, z^-, \lambda) \quad y_{r,j} = 0 \quad j = 1, \ldots, r \quad (2.5)
\end{align*}
\]
It can easily be seen, that (2.1) describes the differential equations, (2.3) and (2.4) the so called switched algebraic equations and (2.5) the state reset equations, where

- $x$ are continuous dynamic states
- $z$ are discrete states
- $y$ are algebraic states
- $\lambda$ are parameters

of the system. The superscript $-$ stands for pre-event values and $+$ for post event values. At the beginning, the system behavior is described by the DAE given in (2.1) and (2.3). A transition to another set of DAE takes place if the corresponding transition condition is fulfilled. Such transition conditions are checked by means of so called event variables $y_s$ and $y_r$. The $y_s$ determine the switching events and $y_r$ state reset events. An event is triggered by an element of $y_s$ changing sign and/or an element of $y_r$ passing through zero. By switching events, which are caused by $y_s$ sign changes, the functional description of the system is changed from $g^{(i^-)}(x, y)$ to $g^{(i^+)}(x, y)$. If we look at the formulation (2.2) and (2.5), we see that the discrete states $z$ are constant between events ($\dot{z} = 0$) and at state reset events caused by $y_r$ the values of the discrete states change according to the state reset functions $h_j$. At state reset events, the values of dynamic states $x$ are continuous, which is most often the case in physical systems. The equations (2.1)-(2.5) capture all the important aspects of a hybrid system.

Large system models are most effectively built using a modular representation. In a modular representation, the components of the system are grouped together as subsystems, and then the subsystems are combined to build up the complete system. A modular representation can be designed in a causal or non-causal manner.

In causal modelling, the model behavior is described by a rigid input/output relation (e.g. $y = f(u)$). Causal modelling is like a method computing output values ($y$) by operating on input values ($u$). Some variables of the model are defined as inputs and some as outputs. The output ($y$) can be calculated if the input ($u$) is defined. Building systems by connecting such causal models means using the output of one model
as an input to another model. For example, the MATLAB based simulation program SIMULINK [22] uses the causal modelling approach.

The major difference between non-causal modelling and causal modelling is that there is no distinction between input and output variables in the non-causal representation. In non-causal modelling, variables are involved in equations that must be satisfied, to reflect the components behavior. Systems in non-causal modelling are built by simply connecting the interface variables of the models. The behavior of such an interconnected system is defined in terms of the signals that satisfy both the modules behaviors and the interconnection constraints induced by the interconnection architecture. One of the advantages of non-causal modelling is that it allows the modeler to use directly the model equations and variables, meaning that he does not need to specify directly input and output signals and their relation in an explicit form. Non-causal modelling also supports component hierarchies, allows the reuse of modelling knowledge in an object-oriented manner, which is the state of the art in today’s software development. For example, MODELICA [23] uses the non-causal modelling approach.

The DSAR structure allows also a modular and non-causal model representation, where the component model behavior is described by simply writing the equations and variables of the component model in the proposed form given in (2.1)-(2.5).

With minor modifications and extensions the following structure can be used for a modular component model representation.

\[
\begin{align*}
\dot{x}_k &= f_k(x_k, y_k, z_k, \lambda_k) \\
\dot{z}_k &= 0 \\
0 &= g_k(0)(x_k, y_k, z_k) \\
0 &= \begin{cases} g_k^{(i^-)}(x_k, y_k, z_k, \lambda_k) & y_{k,s}^i < 0 \\ g_k^{(i^+)}(x_k, y_k, z_k, \lambda_k) & y_{k,s}^i > 0 \end{cases} \quad i = 1, \ldots, s \\
z_k^+ &= h_j^k(x_k^-, y_k^-, z_k^-, \lambda_k) \quad y_{k,r}^j = 0 \quad j = 1, \ldots, r
\end{align*}
\]

with the following definitions

- \(x_k\) is a vector of continuous dynamic states of the \(k^{th}\) model
2.2. Hybrid System Representation

- \( z_k \) is a vector of discrete states of the \( k^{th} \) model
- \( y_{k,i} \) are internal algebraic states of the \( k^{th} \) model
- \( y_{k,s} \) are algebraic states determining the switching events of the \( k^{th} \) model
- \( y_{k,r} \) are algebraic states determining the state reset events of the \( k^{th} \) model
- \( y_{k,ext} \) are external algebraic states of the \( k^{th} \) model, which serve as interface variables to other models
- \( y_k \) is a vector of all algebraic states of the \( k^{th} \) model, which is union of \( y_{k,i}, y_{k,s}, y_{k,r} \) and \( y_{k,ext} \).
  \[
y_k = y_{k,i} \cup y_{k,s} \cup y_{k,r} \cup y_{k,ext}
\]
- \( \lambda_k \) is a vector of parameters of the \( k^{th} \) model
- \( f_k \) is a vector of differential equations of the \( k^{th} \) model
- \( g_k \) is a vector of algebraic equations of the \( k^{th} \) model
- \( h_k \) is a vector of state reset equations of the \( k^{th} \) model

Thus the model is described by defining the variables \( (x_k, y_k, z_k, \lambda_k) \) of the model and the equations \( (f_k, g_k, h_k) \) of the model which must be satisfied. But, if it comes to build a system or subsystem consisting of such models, the interface variables of the models must be linked together. Hence, the behavior of such an interconnected system is determined in terms of the signals or variables that satisfy both the model behaviors and the interconnection constraints induced by the link equations. To illustrate the interconnection or linking concept, we consider the system depicted in Figure 2.2 consisting of three models, each of them having 2 variables and 1 algebraic equation relating these 2 variables together. If we look only at the model equations, we have an under-determined system with 6 variables \( (y_{1,1}, y_{1,2}, y_{2,1}, y_{2,2}, y_{3,1}, y_{3,2}) \) and 3 equations \( (g_1, g_2, g_3) \).

\[
\begin{align*}
g_1(y_{1,1}, y_{1,2}) &= 0 \\
g_2(y_{2,1}, y_{2,2}) &= 0 \\
g_3(y_{3,1}, y_{3,2}) &= 0
\end{align*}
\]
The 3 missing equations, to determine the system completely, are given by the link equations

\[
\begin{align*}
y_{1,1} - y_{3,1} &= 0 \\
y_{1,2} - y_{2,1} &= 0 \\
y_{2,2} - y_{3,2} &= 0
\end{align*}
\]

which are shown as dotted connections in Figure 2.2. Now we have a system with 6 unknowns and 6 equations, which is a necessary condition for the solvability of the system. The link equations are simple linear algebraic equations. In a system consisting of \( m \) models, they can be formulated as

\[
c(y_{1,ext}, y_{2,ext}, \ldots, y_{m,ext}) = 0
\]

The structure of such a system can be illustrated in a two dimensional array as shown in Figure 2.3. The horizontal axis contains all the continuous dynamic states \( x \) and algebraic states \( y \). States belonging to the
same model are also grouped together. In the vertical axis, the model equations are listed. First the differential equations $f$, then the algebraic equations $g$ followed by the connection equations $c$ are listed. It can easily be seen that each model equation is only described by its own states e.g. $g_1(x_1, y_1)$. This can be seen as a block diagonal structure in the two dimensional representation. The overall dependence is finalized by the connection matrix, which here in this illustrative representation covers the entire range of $y$ variables. Normally only interface variables $y_{k,ext}$ of the models are involved in the link equations. This representation, shows a sparse structure. The shaded areas in the Figure show the maximum possible dependence of the model functions on the model variables meaning, if all $f$ and $g$ equations of one model will depend on all dynamic states and algebraic states $(x, y)$, which is normally not the case. Normally, these equations and also the connection equations are sparse, which is also reflected in the overall structure.

The sparsity of the overall system representation in Figure 2.3 becomes
also noticeable in the system matrices used during the numerical calculation of the system trajectory.

In this section, the focus was on the employed hybrid system representation DSAR, which captures all the important aspects of hybrid systems. The DSAR is general, systematic and allows a modular and object oriented system representation.

### 2.3 Simulation Process

In the previous sections, our focus was on a systematic and general system and model representation. This section deals with the implementation issues of the simulator based on the DSAR model representation.

If we consider typical trajectory of a hybrid system, which is depicted in Figure 2.4, we can describe simulation procedure roughly as follows. First step is the calculation of the initial values of the system variables \((x_0, y_0, z_0)\) at simulation start \((t = t_0)\). With the known initial values

\[
\begin{align*}
\dot{x} &= f(x, y, z) \\
0 &= g^{(0)}(x, y, z)
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= f(x, y, z) \\
0 &= g^{(1)}(x, y, z)
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= f(x, y, z) \\
0 &= g^{(2)}(x, y, z)
\end{align*}
\]

Discrete
\[
z^+ = h_1(x^-, y^-, z^-)
\]

Discrete
\[
z^+ = h_2(x^-, y^-, z^-)
\]

Figure 2.4: Illustrative trajectory of a hybrid system
of the variables, the continuous system trajectory can be calculated by using appropriate numerical integration techniques. This continuous system trajectory calculation lasts till an event forces the system to switch to another mode, where the system is described by another set of equations. This so called *event handling* is a crucial part of hybrid system simulation. After the correct event handling, the values of the system variables in the new mode are calculated at $t = t_1$ and these values are the initial conditions for the new continuous section. These procedures are repeated until the simulation end time is reached. Figure 2.5 shows the simplified flow chart of the overall simulation process, where $x^+_e, y^+_e, z^+_e$ denote the calculated post-event values of the system variables and $t_e$ the calculated event time.

In the following, the three major tasks of the simulation process will be discussed, namely:

**Initialization**

$x_0, y_0, z_0, t_0$

$t < t_{\text{end}}$ -> **End**

no

**Calculation of Continuous trajectory**

yes

**Event detected**

no

**Event handling and Reinitialization**

$x^+_e, y^+_e, z^+_e, t_e$

yes

Figure 2.5: Simplified flow chart of the overall simulation
Chapter 2. Simulation Framework

- Initialization
- Calculation of continuous system trajectory
- Event handling and Reinitialization

2.3.1 Initialization

Initialization procedure computes or specifies the initial values of the continuous dynamic states \( x_0 \) and algebraic states \( y_0 \), which are required to start the numerical integration at the very first time step. In the implemented simulation framework, there are 2 different initialization modes available:

- **Initialization with predefined dynamic states**
- **Initialization at steady-state**

In the **Initialization with predefined dynamic states** the user supplies the exact initial values of the dynamic states \( x = x_0 \) and the initial guess of the algebraic values \( y = \hat{y}_0 \) at \( t = t_0 \). In this case, as the dynamic states are predefined, the initial values of algebraic states \( y_0 \) must satisfy the algebraic equations \( g \) and the connection equations \( c \) at \( t = t_0 \) given as:

\[
0 = g(x_0, y) \\
0 = c(y)
\]

The solution of the this nonlinear equation in \( y \) with an iterative Newton method will provide us with the initial values of the algebraic variables \( y_0 \). In this case, the system is generally not in steady-state (\( \dot{x} \neq 0 \)).

In the **initialization mode at steady-state**, all derivatives of dynamic states are set to zero. This means a constraint on \( \dot{x} \) at \( t = t_0 \) is put namely \( \dot{x}|_{t=t_0} = 0 \). Now initial values \( x_0 \) are also unknown and the solution of the equation

\[
\dot{x}|_{t=t_0} = 0 = f(x, y) \\
0 = g(x, y) \\
0 = c(y)
\]
2.3. Simulation Process

after $x$ and $y$ gives the initial values for dynamic states $x = x_0$ and algebraic states $y = y_0$.

The numerical solution of these nonlinear algebraic equations are performed with the Newton-Raphson Method and will be treated in the next section, as such nonlinear equations and their numerical solution is also an important task during the calculation of the continuous system trajectory.

2.3.2 Calculation of Continuous Trajectory

Now, we will deal with the numerical computation of the continuous trajectory between events described by a system DAEs.

A system of nonlinear ordinary differential equations such as $\dot{x} = f(x, t)$ cannot be solved analytically but must be solved numerically [24]. The basic concept of a numerical solution algorithm is to approximate the true solution of $x(t)$ at a set of time points $t_0, t_1, ..., t_n$ by the calculated values $x_0, x_1, ..., x_n$. This approximation is done by advancing the solution from $t_n$ to $t_{n+1} = t_n + \Delta t_{n+1}$ with integration step size $\Delta t_{n+1}$ and calculating $x_{n+1}$ as a function of $\Delta t_{n+1}$, previously calculated states $x_n, ..., x_{n-m}$ and derivatives $\dot{x}(t_n), ..., \dot{x}(t_{n-m})$. A generalized formulation for such a numerical integration method is:

$$x_{n+1} = \Psi (\Delta t_{n+1}, [x_n, x_{n-1}, ..., f(x_{n+1}), f(x_n), ...])$$ (2.11)

In this formulation, the function $\Psi$ is called Discretization Function of the numerical integration method.

For the numerical solution of systems described by DAEs, Gear [25] proposed the simultaneous solution approach, where the differential and algebraic equations of the system are solved together. The used numerical integration method discretizes the differential equation at $t = t_{n+1}$ with the discretization function $\Psi$ as shown in equation (2.11) and converts it into a set of nonlinear algebraic equations, which must be satisfied at $t = t_{n+1}$. At $t = t_{n+1}$ the system variables must satisfy not only the discretized differential equations but also the algebraic equations $g$ and the algebraic connection equations $c$. At $t = t_{n+1}$, the set of equations, which have to be solved, is given as

$$x_{n+1} = \Psi (\Delta t_{n+1}, x_n, ..., x_{n-m}, f(x_{n+1}, y_{n+1}), f(x_n, y_n), ...)
0 = g(x_{n+1}, y_{n+1})
0 = c(y_{n+1})$$
Chapter 2. Simulation Framework

After bringing \( x_{n+1} \) to the right side of the equation, the Discretized System Function at \( t = t_{n+1} \) can be formulated as:

\[
0 = \Psi(\Delta t_{n+1}, x_n, ..., x_{n-m}, f(x_{n+1}, y_{n+1}), f(x_n, y_n), ...) - x_{n+1} \\
0 = g(x_{n+1}, y_{n+1}) \\
0 = c(y_{n+1})
\]

With \( \chi = [x_{n+1}y_{n+1}]^T \), a vector consisting of all unknown variables \( x_{n+1}, y_{n+1} \), the expression becomes

\[
F(\chi) = \begin{bmatrix}
\Psi(\Delta t_{n+1}, f(x_{n+1}, y_{n+1}), f(x_n, y_n), ..., - x_{n+1} \\
g(x_{n+1}, y_{n+1}) \\
c(y_{n+1})
\end{bmatrix} = 0 \quad (2.12)
\]

In this formulation, the history terms \( x_n, ..., x_{n-m}, y_n, ..., y_{n-m} \) and integration step size \( \Delta t_{n+1} \) are known, \( x_{n+1} \) and \( y_{n+1} \) are unknown. The complete set of discretized system equations \( F(\chi) \) at \( t = t_{n+1} \) is nonlinear and can be solved with Newton-Raphson algorithm. Newton-Raphson method is the standard method used for the numerical solution of nonlinear algebraic equations and is explained in many text books in more detail (e.g. [24]). The main idea of the method is illustrated in Figure 2.6 for a one dimensional case \( f(x) = 0 \). Starting with an initial guess \( x_0 \) which is reasonably close to the true solution, the function is approximated by its tangent line and the intercept of this tangent line is calculated \( x_1 \). This intercept is typically a better approximation to the function’s root than the original guess, and the method is iterated till \( |f(x_i)| \leq \varepsilon \). At each iteration \( x_{i+1} \) is calculated by

\[
x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}
\]

where \( i \) denotes the iteration counter. In the multidimensional case of the discretized system function, this is formulated as

\[
\chi^{i+1} = \chi^i - F^{-1}_\chi(\chi^i) F(\chi^i) \quad (2.13)
\]

If the convergence is reached, we have our solutions for \( x_{n+1} \) and \( y_{n+1} \), at \( t = t_{n+1} \). As shown in (2.13), for solving this set of nonlinear equations \( F(\chi) = 0 \), we need to evaluate \( F(\chi) \) (2.12) and the Jacobian \( F_\chi(\chi) \) (2.14) at each iteration.

\[
F_\chi(\chi) = \begin{bmatrix}
\Psi f_{x} - I & \Psi f_{y} \\
g_x & g_y \\
0 & c_y
\end{bmatrix} \quad (2.14)
\]
Independent of the used numerical integration method, a large set of non-linear algebraic equations (2.12) has to be solved iteratively at every simulation time step.

\( F(\chi) \) (2.12) is dependent on

- the used numerical integration method \( \Psi \).
- the model equations \( f \) and \( g \).
- the topology equations \( c \).

In \( F_\chi(\chi) \) (2.14) we need the partial derivatives of

- the used numerical integration method \( \Psi_f \).
- the models equations \( f_x, f_y, g_x \) and \( g_y \).
- the topology equations \( c_y \).

During the simulation process, functions of the numerical integration method (\( \Psi \) and \( \Psi_f \)), the model equations (\( f, g, f_x, f_y, g_x \) and \( g_y \)) and the topology equations (\( c, c_y \)) must be evaluated. With such an abstraction, the simulation framework can be structured in a hierarchical and object-oriented manner. Figure 2.7 shows the interface between the simulation kernel and simulated system model. It shows the details of...
the information exchange between the system model and the simulation kernel.

The choice of the most suitable numerical integration method depends highly on the characteristics of the simulated system. A combined simulation of transient and long term phenomena in power systems requires the solution of a large nonlinear stiff set of differential-algebraic equations. Stiff systems are systems exhibiting a wide-range of time varying dynamics from very fast to very slow dynamics. Besides accuracy and efficiency, the numerical stability of the used integration method plays a significant role in the simulation of stiff systems. The most commonly used numerical integration methods, namely the Backward Euler, the Trapezoidal method and Gear’s method, also known as Backward Differentiation Formulas (BDF), have been implemented, as they are mainly used for the solution of systems of stiff ordinary differential equations. The discretization functions $\Psi$ and their partial derivatives of the Backward Euler and the Trapezoidal method are given in table 2.1.

Details of the implemented numerical integration methods will be given in the next chapter. But some of the important properties of the used
2.3. Simulation Process

\[ x_{n+1} = \Psi(x_{n+1}, x_n, h_{n+1}) \]

<table>
<thead>
<tr>
<th>Method</th>
<th>( x_n + h_{n+1} f_{n+1} )</th>
<th>( h_{n+1} )</th>
<th>( f_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backward Euler</td>
<td>( x_n + h_{n+1} f_{n+1} )</td>
<td>( h_{n+1} )</td>
<td>( f_{n+1} )</td>
</tr>
<tr>
<td>Trapezoidal</td>
<td>( x_n + \frac{h_{n+1}}{2} [f_{n+1} + f_n] )</td>
<td>( \frac{h_{n+1}}{2} )</td>
<td>( \frac{1}{2} [f_{n+1} + f_n] )</td>
</tr>
</tbody>
</table>

Table 2.1: \( \Psi \), \( \Psi_f \) and \( \Psi_h \) of some implemented integration methods with \( f_n = f(x_n) \) and \( f_{n+1} = f(x_{n+1}) \)

methods can be summarized as follows. The Trapezoidal method is numerically stable, but can cause numerical oscillations after mode switchings. The Gear’s (BDF) method is a variable order method and thus its region of stability depends on the order of the method. The low (2. and 3.) order the Gear’s (BDF) methods can cause numerical damping to unstable modes of the physical system and can make them numerically stable. For higher (4., 5. and 6.) order Gear’s (BDF) methods the opposite phenomena is also possible namely that physically stable modes near to the imaginary axes become numerically unstable. But in general, its region of stability is suitable for numerical integration of stiff systems.

For an efficient and accurate numerical simulation of physical systems, methods and algorithms are required which reduce the overall simulation time. Such algorithms try to reduce the overall number of mathematical operations needed for the calculation of the approximated system trajectory. Mostly applied strategies to achieve this computational efficiency are as follows.

- **Automatic Step Size Control**

- **Application of Dishonest Newton Method** in the solution of (2.13) meaning keeping the Jacobian matrix \( F_\gamma(\gamma^i) \) constant over some iterations.

- **The usage of sparse matrix solution techniques** in the solution of (2.13).

It is desirable to use the largest possible integration step size \( \Delta t \) to advance the solution from \( t_n \) to \( t_{n+1} \) while keeping a pre-determined level of accuracy. The largest possible integration step size depends on the dynamics of the system and on the level of desired accuracy. If system quantities are varying rapidly the step size should be chosen
small enough to come up with these fast dynamics. Conversely, if the quantities are varying much slower the step size should be chosen large enough to speed up the simulation while keeping the level of accuracy. This automatic step size adjustment is mostly accomplished based on the local truncation error bounds which is a measure of the desired accuracy. Local truncation error $\epsilon_T$ of an integration method is defined as the error introduced at a single step and is measured as the difference between the true solution and the calculated solution meaning $\epsilon_T = x(t_{n+1}) - x_{n+1}$, where $x(t_{n+1})$ is the true solution and $x_{n+1}$ is the calculated solution by the integration method. This automatic step size control is a crucial requisite for an efficient numerical simulation.

As mentioned previously, independent of the used numerical integration method, the non-linear algebraic equation set (2.12) has to be solved iteratively at every simulation time step. Another commonly applied strategy is to reduce the number of operations during the solution of this nonlinear equation by employing the dishonest Newton method. In this method, the Jacobian $F_\chi$ is kept constant as long as the convergence rate is below some predefined bound (e.g. three iterations).

The iterative solution of (2.13) requires the solution of a large-scaled linear equation system $A \cdot x = b$. For the solution of such large-scaled linear equation systems, computationally efficient sparse matrix solution techniques are applied. In our implementation sparse matrix solution package UMFPACK [26] has been used for solving these linear equations. Usage details of the UMFPACK library can be found in [26].

All of these described strategies increasing the computational efficiency are implemented in the described simulation framework for the calculation of the continuous system trajectory.

2.3.3 Event Handling and Consistent Reinitialization

Simulation of hybrid systems is complicated, as the presence of discontinuities give rise to changes in the functional form of the system equations. In the DSAR representation, mode transitions are formulated with the equations (2.4) and (2.5). After such a change, the system is in a new continuous section, where continuous trajectory is computed by the adequate numerical integration as described in the previous sec-
2.3. Simulation Process

tion. When and how this transition from one continuous section to the other one takes place is the key issue in the following part.

Such changes in the functional form of the system equations or mode transitions are triggered by so called events. Such events can occur at a specified time in the future and are called time events. There are also events where the time of occurrence is not known right from the start. These events occur only if some conditions on continuous states are satisfied. These kind of events are referred as state events. For example the tripping of a relay protecting a device from over-current depends on the current through. Thus the time, when the state event occurs, is not known before and should be determined during the simulation process.

Accurate simulation requires state events to be located precisely and processed in strict time order as the discontinuities resulting from the occurrence of a state event can drastically change the future evolution of the overall system behavior. Important aspects of event handling are presented in [27]. In this section, the implemented event handling algorithm is described in more detail.

In general, event handling is done in two stages:

- **Event recognition**: At this stage, the task is only to detect, whether an event has occurred or not. The event recognition is warranted by watching the event variables $y_s$ and $y_r$ of each model during simulation process. A sign change of $y_s$ and/or zero crossing of $y_r$ should be recognized as an event. Different directions of the sign change (from + to - or from - to +) can cause different changes in the functional description.

- **Event location**: If an event has been detected, the simulation kernel must calculate the exact event time $t_e$, where the corresponding event variable becomes zero i.e. $y_s(t_e) = 0$ or $y_r(t_e) = 0$.

Figure 2.8 shows the typical trajectory of an event variable. To be able to recognize an event, the simulation engine must keep track of the event variables. After advancing the solution from $t_n$ to $t_{n+1}$, it should check whether the event variables have crossed zero or not. If an event is recognized, the exact time of the event $t_e$ must be calculated to make the transition to the new system mode at the correct time instance. After an event has been recognized some intermediate steps must be taken to accomplish the transition to the new system mode.
First step is to calculate the exact event time $t_e$. Let’s assume we are at $t = t_n$ and advance the solution by integrating with step size $\Delta t_{n+1}$ to $t = t_{n+1}$. We recognize that the event variable $y_e$ has changed sign in this interval. At $t = t_n$, $y_e$ was negative and at $t = t_{n+1}$ it becomes positive. To compute $t_e$, we make a small modification in our discretized system function $F(\chi)$ in (2.12). We reformulate our problem saying that at event time $t_e$ the event variable $y_e(t_e)$ becomes zero. Thus we add an additional constraint $y_e = 0$ to the discretized system function $F$ and augment the unknown variables $[x_{n+1} \, y_{n+1}]^T$ with $\Delta t_e$. The extended discretized system function is referred $F_e$ and is given in (2.15).

$$F_e(\chi_e) = \begin{pmatrix} \Psi(\Delta t_e, f(x_{n+1}, y_{n+1}), f(x_n, y_n), \ldots) - x_{n+1} \\ g(x_{n+1}, y_{n+1}) \\ c(y_{n+1}) \\ y_{e,n+1} \end{pmatrix} = 0$$

with

$$\chi_e = \begin{pmatrix} x_{n+1} \\ y_{n+1} \\ \Delta t_e \end{pmatrix}$$
2.3. Simulation Process

Jacobian of $F_e$ becomes (2.16)

$$F_{e,\chi}(\chi_e) = \begin{bmatrix}
(\Psi_f f_x - I) & (\Psi_f f_y) & \Psi_{\Delta t} \\
 g_x & g_y & 0 \\
 0 & c_y & 0 \\
 0 & 0 & 1 & \ldots & 0 & 0
\end{bmatrix}$$

The solution of this nonlinear equation set $F_e$ gives the correct step size $\Delta t_e$ to reach the exact event time $t_e$ and at the same time the values of the dynamic states $x^-$ and algebraic states $y^-$ just prior to the event. Thus, it is possible to accomplish the event handling only with minor modifications in the simulation framework.

$$F_e(\chi_e) = 0 \Rightarrow t_e = t_n + \Delta t_e, x^-(t_e), y^-(t_e)$$

The overall simulation flow chart is shown in Figure (2.9). It starts with the initialization and the initial values $x_0$ and $y_0$ are computed as described in section 2.3.1. After the initialization, we enter the actual simulation loop. We advance our solution in the continuous region according to Section 2.3.2 from $[x_n, y_n]$ to $[x_{n+1}, y_{n+1}]$ by applying the selected numerical integration method. With these calculated values, we enter the event recognition procedure, where every event variable $y_e$, which is a subset of the algebraic variables $y$, is checked whether they have changed sign or crossed zero from step $n$ to $n+1$. If an event has been recognized, an intermediate step is made and (2.15) is solved to determine the exact event time $t_e$. The solution of (2.15) gives also the pre-event values $x^-(t_e)$ and $y^-(t_e)$ and thus the current continuous section can be closed by storing these values $t_e$, $x^-(t_e)$ and $y^-(t_e)$. The switch to the new system mode takes place in three steps.

- As formulated in (2.1-2.5), the continuous dynamic states are continuous at events so that post-event values of the dynamic states can directly given as $x^+ = x^-$.

- Then, event values of the discrete states $z^+$ are computed by (2.5) at $t_e$ with the pre-event values $x^-$, $y^-$ and $z^-$. 

$$z^+ = h_j(x^-, y^-, z^-, \lambda)$$

- Then, with these post event values $x^+$ and $z^+$, equation (2.4) is solved for determining the post-event algebraic state values $y^+$.

$$g^{(i+)}(x^+, y^+, z^+, \lambda) = 0$$
Thus, all post-event values $x^+, y^+$ and $z^+$ are computed, which provide the new initial conditions for the next continuous section. The simulation loop lasts till the simulation time $t$ reaches $t_{end}$.

### 2.4 Automatic Code Generator

As discussed in the previous sections, the simulation kernel needs the models to be described in the DSAR structure, and for the numerical simulation each model has to provide the kernel with the model func-
2.4. Automatic Code Generator

The modeler simply writes the model equations in the required DSAR structure in a text file called *Symbolic Definition File* (SYMDEF), by defining the

- continuous dynamic states $x$
- discrete states $z$
- algebraic states $y$
- event variables $y_r$ and $y_s$
- differential equations $f$
Chapter 2. Simulation Framework

- switched-algebraic equations $g$
- state-reset equations $h$

The Automatic Code Generator processes the Symbolic Definition File of the model and creates, depending on the platform, the model’s MATLAB/C++ class source files by using the symbolic toolbox of MATLAB for symbolic manipulation and for the analytical calculation of the partial derivatives of the model functions. This process is depicted in Figure 2.10. The format of such a Symbolic Definition File and how the user formulates a model in a Symbolic Definition File will be shown in a simple example.

Examples of Symbolic Definition Files

As an example, we will write the models of some important components of the power system shown in Figure 2.11 in Symbolic Definition Files. The system comprises one dynamic load model (exponential recovery), one feeder, tap-changing transformer, 3 transmission lines and 4 nodes. The same system can be found also in [20]. First we will formulate

![Diagram of example power system](Figure 2.11: Example Power System)

<table>
<thead>
<tr>
<th>Component</th>
<th>Symbolic Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line12a</td>
<td>$R = 0 \quad X = 0.65$</td>
</tr>
<tr>
<td>Line12b</td>
<td>$R = 0 \quad X = 0.40625$</td>
</tr>
<tr>
<td>Line34</td>
<td>$R = 0 \quad X = 0.80$</td>
</tr>
<tr>
<td>Trafo</td>
<td>$V_{low} = 1.04 \quad N_{max} = 1.1 \quad T_{tap} = 20.0 \quad N_{step} = 0.0125$</td>
</tr>
<tr>
<td>Feeder</td>
<td>$</td>
</tr>
<tr>
<td>Load</td>
<td>$P_0 = 0.4 \quad Q_0 = 0.0 \quad T_p = 5 \quad T_q = 5 \quad A_s = 0 \quad A_t = 2 \quad B_s = 0 \quad B_t = 2$</td>
</tr>
</tbody>
</table>

the first order dynamic exponential recovery load model [28] with continuous dynamics. The dynamic behavior of the load model can be
described by the following set of Differential Algebraic Equations.

\[
\begin{align*}
\frac{dx_p}{dt} &= -\frac{x_p}{T_p} + P_0 (|V|^{\alpha_s} - |V|^{\alpha_t}) \quad (2.17) \\
\frac{dx_q}{dt} &= -\frac{x_q}{T_q} + Q_0 (|V|^{\beta_s} - |V|^{\beta_t}) \\
P_L &= \frac{x_p}{T_p} + P_0 |V|^{\alpha_t} = V_d I_d + V_q I_q \\
Q_L &= \frac{x_q}{T_q} + Q_0 |V|^{\beta_t} = V_d I_q - V_q I_d
\end{align*}
\]

where

- \(x_p\) ... Internal load state for active power [p.u.]
- \(x_q\) ... Internal load state for reactive power [p.u.]
- \(P_0\) ... Rated Load Active Power (at 1 p.u. voltage) in [p.u.]
- \(Q_0\) ... Rated Load Reactive Power (at 1 p.u. voltage) in [p.u.]
- \(\alpha_s\) ... Steady-state active power voltage dependency
- \(\alpha_t\) ... Transient active power voltage dependency
- \(\beta_s\) ... Steady-state reactive power voltage dependency
- \(\beta_t\) ... Transient reactive power voltage dependency
- \(T_p\) ... Active power recovery time constant in [s]
- \(T_q\) ... Reactive power recovery time constant in [s]
- \(V = V_d + j V_q\) ... Complex voltage of the load bus
- \(I = I_d + j I_q\) ... Complex load current
- \(P_L\) ... Active power consumed by the load
- \(Q_L\) ... Reactive power consumed by the load

For the given dynamic load model, the symbolic definition file shown in Figure 2.12. The symbolic definition file has 4 different sections depicted as gray shadings in 2.12.

- Section starting with the label **definitions**: contains all the variables and parameters of the model.
- In the section starting with the label **equations**: the first order ordinary differential equations \(f\) of the model are given, where the \([dt()]\) stands for the derivative operator.
• Section starting with the label *g_equations*: comprises all the switched-algebraic equations $g$ of the model. The $g$-equations must have unique names (e.g. $g_1, g_2, g_3$).

• And finally, section starting with the label *h_equations*: comprehends the state reset equations $h$ of the model.

```
%-----------------------
definitions:
%-----------------------
dynamic_states xp xq
discrete_states
external_states Vd Vq Id Iq
internal_states Vt Ps Pt Qs Qt PLoad QLoad
events
parameters P0 Q0 Tp Tq As At Bs Bt

%-----------------------

f_equations:
%-----------------------
dt(xp) = -xp/Tp + P0*Vt^As - P0*Vt^At
dt(xq) = -xq/Tq + Q0*Vt^Bs - Q0*Vt^Bt

%-----------------------

g_equations:
%-----------------------
g1 = Vt - (Vd^2+Vq^2)^(1/2)
g2 = (Vd*Id + Vq*Iq) - (xp/Tp + Pt)
g3 = (Vd*Iq - Vq*Id) - (xq/Tq + Qt)

%-----------------------

h_equations:
%-----------------------
```

Figure 2.12: Symbolic Definition File of dynamic load model

In the definitions section, different keywords are used to define the correctly the variables and parameters of the model, namely:

• continuous dynamic states ... $x$ ... [*dynamic_states*]
• discrete states ... $z$ ... [*discrete_states*]
• external states ... $y_{ext}$ ... [*external_states*]
• internal states ... $y_i$ ... [*internal_states*]
• event variables ... $y_r, y_s$ ... [*events*]
• parameters ... $\lambda$ ... [*parameters*]
2.4. Automatic Code Generator

The dynamic load model has only (slow) continuous dynamics. It does not have any discrete states ($z$). In the power systems, the tap changing transformer is an ideal example of a hybrid model with discrete states. Next, the tap changing transformer will be treated as an example for modelling discrete behavior in symbolic definition file.

The control logic of the tap changing transformer can be described as follows. As long as the voltage measured at the high-voltage end of the transformer is within the allowed deadband or the tap is at the upper limit, the timer is blocked. The timer will start to run if the voltage gets outside the deadband. If the timer reaches the time set for tap delaying, a tap change will occur and the timer will be reset but not necessarily blocked. Blocking and resetting of the timer takes place if the voltage moves back to within the deadband. The symbolic definition file of the tap changing transformer is shown in Figure 2.13.

The difference to the previous load model example is that there are discrete states (e.g. $N$ and timeron) in the tap changer control logic. The $+/−$ signs in front of event variable gives the direction of the sign change, when the event is triggered. A $+$ sign means, an event will be triggered if the event variable changes sign from $−$ to $+$. The state-reset equations $h$ calculate the new values of the discrete states depending on events. For example if the timer was on for $T_{tap}$ seconds, the event $t_{until\_tapchange}$ is triggered and the tap position $N$ is increased by $N_{step}$ and at the same time the timer is reset ($\text{timer}+ = 0$). With such a formulation, it is possible to capture other discrete and hybrid behavior inherent in the power systems (e.g. control logics of protection devices).

After the models are formulated in the symbolic definition files, they are processed by the automatic code generator and the source codes of the models are created as depicted in Figure 2.10. As a simulation example, the system in Figure 2.11 is simulated for 200 seconds. After 10 seconds, the system is subjected to a disturbance, namely the dashed transmission line (Line12b) is tripped. Figures 2.14(a)-2.14(d) show the trajectories of some selected variables of the system calculated by the used simulator. The data for the test system is given in Figure 2.11. At $t = 0$, the system is at steady state. The initial value of the tap position is set to 1.0375 ($N_0 = 1.0375$). At $t = 10$ seconds, Line12a is tripped. Right after the line tripping, the voltages start to drop. The measured voltage on the high-voltage end drops below 1.04 (Figure 2.14(a) $V_3 ≈ 0.98$), which is outside the deadband of the tap changer.
Chapter 2. Simulation Framework

Figure 2.13: Symbolic Definition File of tap changing transformer model
2.4. Automatic Code Generator

controller. Thus, the timer is reset and starts running (Figure 2.14(c) $\text{Timer on/off} = 1$). During the following 20 seconds ($t = 10 \rightarrow 30$) the voltage remains outside the deadband, so that at $t = 30$ seconds the tap position $N$ is incremented by $N_{\text{step}}$ (Figure 2.14(b)) and the timer is reset (Figure 2.14(d)). This operation causes only a minor increases in the voltage level, so that after 20 seconds, the tap position is incremented once more. This increment is applied till $t = 110$ at every 20 seconds. At $t = 110$, the tap position reaches its maximum allowed value $N_{\text{max}} = 1.1$ and this blocks the timer and resets it. The timer remains blocked till end of the simulation.

This example shows, that the proposed simulation framework is capable of simulating the combined continuous and discrete dynamics inherent in power systems. The described model examples of dynamic load model and tap changing transformer show, how such continuous and discrete behavior can be formulated in the so called symbolic definition files in a structural way.
2.5 Implementation Issues

In the previous sections 2.2-2.3, we focused on the modelling framework based on the DSAR hybrid system representation and important procedures of the simulation process. In following section, the focus will be on some implementational details of the simulator.

The described simulation framework has been implemented in MATLAB environment [22] and as a C++ Dynamic Link Library (DLL) in NEPLAN [1]. A DLL is a library that contains code and data that can be used by more than one program at the same time. By using DLL’s, a program can be modularized into separate components.

There are some important differences between the MATLAB implementation and C++ implementation regarding the used numerical integration methods, event handling algorithms and source codes of the models.

Numerical Integration

The MATLAB version uses the build-in numerical integration methods ode15s and ode23tb solvers [29]. The ode15s solver is a variable order solver based on the numerical differentiation formulas (NDFs). Optionally, it uses the backward differentiation formulas (BDFs, also known as Gear’s method). The ode23tb solver is an implementation of TR-BDF2, an implicit Runge-Kutta formula with a first stage that is a trapezoidal rule step and a second stage that is a backward differentiation formula of order two. Both methods allow the numerical integration of differential algebraic equations.

In the C++ version, numerical integration methods such as trapezoidal method and BDF method are implemented. Additionally so called matched versions of these methods are also derived and used for time domain simulation of power system transients with dynamic phasor models. Details of these methods will be discussed later.

Event Handling

The MATLAB version uses the build-in event handling mechanism [30]. The simulation framework provides a MATLAB interface function, which passes over the required information for event handling, such as:

- the value of the event variable at given time instant.
2.6 Summary

- the information whether or not the integration should stop when a sign change is observed in the event variable.
- the desired directionality of the zero crossings.

The \textbf{C++ version} uses the event handling algorithm described in Section 2.3.3. The main difference is in the determination of zero crossings, which is an integrated part of the numerical integration method in the C++ implementation.

\textbf{Source Code of the Models}

In both \textit{MATLAB} and \textit{C++ versions} are realized in an object-oriented manner. In Object-Oriented Programming, there are so called classes, which are the prototypes defining the state and behavior of a real-world objects. The mathematical models are mapped to \textit{MATLAB} or C++ classes with model parameters ($\lambda$) as properties of the class, model descriptor functions ($f, g, h$) and their partial derivatives ($f_x, f_y, g_x, g_y$) as methods of the class.

The simulation kernel in the MATLAB implementation uses directly the MATLAB code (M-files) of the model classes. In the C++ implementation, an intermediate step is taken and the C++ source codes of the model class is compiled into a Dynamic Link Library. The DLL of the model supplies all the required interface functions to the simulation kernel during the simulation process.

In both implementations the user can write the model’s MATLAB source file or C++ source file on his own or can also use the ACG tool which creates the corresponding source code automatically by processing the symbolic definition file of the model as described in Section 2.4.

\section*{2.6 Summary}

In this chapter, the focus was on the simulation framework, which has been used throughout the thesis for the simulation of the models and case studies.

The implemented simulation framework is based on non-causal modelling of systems and models represented by Differential Switched Algebraic State-Reset Equations (DSAR) [31], capturing mixed continuous and discrete dynamics. In this framework, every model is fully
described by the model equations given in the DSAR structure, and models are connected together with so-called topological links through their interface variables, which can be described by simple linear algebraic equations. In such a system, the models have to satisfy both the model equations and the algebraic topology equations.

Important sub-procedures of the simulation algorithm were discussed in detail such as initialization of the system, calculation of the continuous trajectory between events, and event recognition and event handling.

To facilitate the creation of models for the user, a tool called Automatic Code Generator has been developed, where the user simply writes the model equations and the Automatic Code Generator creates, depending on the platform, the model’s MATLAB/C++ class source files, which are then used by the simulation kernel for the calculation of the system trajectory. Later, a simple power system example was used to demonstrate how models are implemented in the simulation environment and the simulation example showed that simulator is capable of simulating the mixed continuous and discrete behavior of hybrid systems such as power systems.
Chapter 3

Modelling of Power Systems

So far, our focus was on the implemented simulation framework and the employed numerical integration techniques for the simulation of the transient behavior of power systems. These numerical integration techniques allow a time-efficient simulation of the system trajectory. Another important issue affecting the simulation efficiency is the selection of an adequate system representation for the modelling of the power system components. In this chapter, the main focus will be on the different system representations used in the area of power systems analysis.

3.1 Introduction

Besides numerical integration techniques, there are other important issues directly affecting the simulation performance. Important characteristics of power systems allow us to perform useful transformations or to make simplifying approximations, which can enormously increase the simulation efficiency by keeping the same degree of accuracy.

The complete power system can be seen as a coupled electromechanical and electromagnetic system with periodic or nearly periodic system quantities. There are fast dynamics due to the interaction between the magnetic fields of inductances and electrical fields of capacitances in power system components. These fast dynamics are referred as Electromagnetic Transients (EMT). There are also slower dynamics due to the interaction between the mechanical energy stored in
the rotating machines and the electrical energy stored in the electrical network. These slower dynamics are referred as Electromechanical Transients. Thus, the power system contains a wide range of time constants, due to the fast dynamics and slow dynamics. Hence, the combined simulation of both phenomena is a challenging task.

Electromagnetic Transients Programs (EMTP) [4] and ”EMTP”-like programs are generally used for a detailed simulation of such fast electromagnetic transients. Such detailed component models are generally described in the original three-phases (ABC).

One usually made assumption or approximation in power system modelling regards the different time scales of the electromagnetic and electromechanical phenomena. In most of the power system studies, the focus is on the slower electromechanical oscillations, so that the fast electromagnetic transients of the electrical network are neglected. In such studies, electrical network quantities such as voltages, currents and fluxes are represented by their steady-state values at the fundamental frequency.

Besides approximations regarding the different time scales of different phenomena, it is also common to employ different transformations or approximations leading to more simulation-efficient variable representations. In such cases the maximum occurring frequency in the system quantities is reduced by transformations or approximations, so that larger step sizes can be used during numerical integration of the system equations.

In the following, starting with the original three-phase ABC representation of the component models and system quantities, we will discuss other commonly used variable representation in DQ0 coordinates, which is based on an orthogonal transformation of the original three-phase quantities. Furthermore, we will introduce the dynamic phasors approach based on an adequate Fourier approximation of the system quantities and use this approach for modelling the power system components in the developed power system simulator.

The aim of this chapter is to give a general overview of the different variable representations used in power system modelling and also to compare their simulation performance under different system conditions. For the theoretical assessment of the simulation performance of these modelling techniques, we will examine them in the frequency domain.
3.2 ABC Three-Phase Representation

A diagram of these different modelling techniques is depicted in Figure 3.1.

![Diagram of ABC Three-Phase Representation](image)

Figure 3.1: Different variable representations used in power systems modelling

### 3.2 ABC Three-Phase Representation

The three-phase ABC model representation with lumped parameters provides the basis for other model representations. All electrical quantities of the electrical network such as voltages, currents etc. and all model equations are given in the three-phase (ABC) reference frame. Such models are commonly used in EMTP-like detailed time-domain simulations.

In the ABC reference frame any kind of equipment can be modelled easily, e.g. power electronic based equipment such as FACTs and HVDC. The main idea of such devices is to modify the electrical energy form by using power electronic devices such as thyristors, IGBTs etc. From the simulation point of view such a power electronic device can be seen as a periodically switched system, where the system behavior is differently described depending on the on/off status of the components. There are programs like [32], where such power electronic systems are simulated efficiently by representing the system as a piecewise linear switched system.
Nevertheless in large power systems the representation of network voltages and currents in the three-phase ABC reference frame would increase the computational burden due to the presence of AC phase quantities varying with the power frequency or system frequency (50, 60 Hz) even during steady state conditions.

To show the advantages and disadvantages of this variable representation regarding simulation performance, we take a general three-phase steady state AC current

$$\begin{bmatrix}
i_{a}(t) = I_{a} \cos(\omega t + \theta_{a}) \\
i_{b}(t) = I_{b} \cos(\omega t + \theta_{b}) \\
i_{c}(t) = I_{c} \cos(\omega t + \theta_{c})
\end{bmatrix}$$ (3.1)

with arbitrary magnitudes and phase offsets. As the current source is assumed to be constant, it can be interpreted as the steady-state value of a current quantity in the system.

Normally in power systems, the three phases of all electrical quantities such as voltages and currents have the important property of being balanced and being in positive sequence. This can be formulated as

$$\begin{bmatrix}
i_{a}(t) = I \cos(\omega t + \theta) \\
i_{b}(t) = I \cos(\omega t + \theta - \frac{2\pi}{3}) \\
i_{c}(t) = I \cos(\omega t + \theta + \frac{2\pi}{3})
\end{bmatrix}$$ (3.2)

With the method of Symmetrical components [33] however, it is possible to express a set of N unbalanced polyphase quantities as the sum of amplitude invariant N symmetrical sets of balanced quantities. In the case of the general three-phase quantity in (3.1), this can be expressed as

$$i_{abc}(t) = i_{abc,p}(t) + i_{abc,n}(t) + i_{abc,z}(t)$$ (3.3)

with

$$\begin{align*}
i_{abc,p}(t) &= I_{p} \begin{bmatrix}
\cos(\omega t + \theta_p) \\
\cos(\omega t + \theta_p - \frac{2\pi}{3}) \\
\cos(\omega t + \theta_p + \frac{2\pi}{3})
\end{bmatrix} \\
i_{abc,n}(t) &= I_{n} \begin{bmatrix}
\cos(\omega t + \theta_n) \\
\cos(\omega t + \theta_n + \frac{2\pi}{3}) \\
\cos(\omega t + \theta_n - \frac{2\pi}{3})
\end{bmatrix} \\
i_{abc,z}(t) &= I_{z} \begin{bmatrix}
\cos(\omega t + \theta_z) \\
\cos(\omega t + \theta_z) \\
\cos(\omega t + \theta_z)
\end{bmatrix}
\end{align*}$$
3.2. ABC Three-Phase Representation

Figure 3.2 depicts that with the method of symmetrical components, it is possible to express any set of unbalanced three-phase quantities (3.1) as the sum of three symmetrical sets of balanced phasors (3.3). One set of phasors has the same phase sequence as the system (positive sequence \( i_{abc,p} \)), the second set has the reverse phase rotation (negative sequence \( i_{abc,n} \)), and the third set all has the same phase (zero sequence \( i_{abc,z} \)).

Regardless if the quantities are balanced or unbalanced quantities, the efficiency of the simulation in the ABC reference frame suffers from the periodicity due to the presence of AC phase quantities as given in (3.1) and (3.2). The frequency spectrum of power system transients in the ABC reference frame contains frequency components which are centered around the system frequency \( \omega_s \). The half-bandwidth of these spectra are generally less than the system frequency. In signal processing such signals are called bandpass signals. The frequency content of such a bandpass signal is illustrated in Figure 3.3.

The used integration step size in numerical integration is bounded by the maximum step size, while the maximum possible step size for bandlimited signals is dictated by the Nyquist criterion with \( h_{\text{max}} = 1/(2f_{\text{max}}) \).
Thus, \( h_{\text{max}} \) gives the upper bound of the maximum applicable step size for a proper simulation. In the case of such bandpass signals the maximum occurring frequency is around the system frequency \( f_s \).

### 3.3 Baseband Representation

In the three-phase ABC representation of the system quantities and equations, the instantaneous signals have been used. These phase quantities have generally a bandpass characteristic as illustrates in Figure 3.3, where the frequency content of the power system transients are concentrated only around the system frequency. But in many applications (e.g. modulation techniques in communications) the bandpass signals are mostly represented by their analytic signal [34].

The basic idea of the analytic representation of bandpass signals is that the negative frequency components of the spectrum of a real-valued function do not contain any additional information about the signal so that the negative frequency components can be discarded without any loss of information. This is illustrated in Figure 3.4. Generally, the analytical signal is also frequency shifted to reduce the highest occurring frequency as illustrated in Figure 3.5.

Mathematically, the analytic signal of a real-valued bandpass signal \( x(t) \) is defined by:

\[
x_a(t) = x(t) + j \hat{x}(t)
\]
3.3. Baseband Representation

Figure 3.4: Spectrum of an analytical signal

\[ X_a(j\omega) \]

\[ X_{bb}(j\omega) = X_a(j\omega - j\omega_s) \]

Figure 3.5: Spectrum of a baseband signal

where \( \hat{x}(t) \) is the Hilbert transform of \( x(t) \). The Hilbert transform \( \hat{x}(t) = \mathcal{H}\{x(t)\} \) of function \( x(t) \) can be obtained by the convolution:

\[
\hat{x}(t) = h(t) * x(t) = \left( \frac{1}{\pi t} \right) * x(t)
\]

\[
= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(t-\tau)}{\tau} d\tau
\]

The Fourier transform of \( h(t) \) is given by

\[
\mathbf{H}(j\omega) = \mathcal{F}\{h(t)\} = \begin{cases} 
  j, & \text{for } \omega < 0, \\
  0, & \text{for } \omega = 0, \\
  -j, & \text{for } \omega > 0,
\end{cases}
\]

where \( \mathcal{F} \) denotes the Fourier transform. The frequency shift of the
analytical signal $x_a(t)$ is done by

$$x_{bb}(t) = x_a(t) e^{-j\omega_s t} \quad (3.4)$$

where the subscript $x_{bb}$ stands for the baseband representation of the original signal $x(t)$. The signal $x_{bb}$ has all the information about the original signal $x(t)$ which can easily be calculated as

$$x(t) = \Re \{ x_{bb}(t) e^{j\omega_s t} \} \quad (3.5)$$

With (3.5) and the derivative given as

$$\frac{dx(t)}{dt} = \Re \left\{ \frac{dx_{bb}(t)}{dt} e^{j\omega_s t} + j\omega_s x_{bb}(t) e^{j\omega_s t} \right\} \quad (3.6)$$

the system and component equations can be transformed into a baseband representation.

From a simulation performance point of view, there are advantages to use the baseband quantities for the representation of the system quantities. The reduction of the highest frequency makes it possible to use larger step sizes during numerical integration which makes the simulation of power system transients more efficient.

Theoretical details and advantages of this approach for the simulation of power system transients was first studied in [10]. Authors also used the baseband representation in the symmetrical components technique for analyzing balanced and unbalanced power system transients. In [9], the three phase representation was relaxed and the concept was generalized to overcome the three phase limitation. The approach was then employed for combined simulation of electromagnetic and electromechanical transients. In [10] and [9] the authors refer to the baseband quantities as dynamic phasors. In our context however, the definition of dynamic phasor is different and will be discussed in Section 3.5.

If the frequency content of the transients are concentrated only around the system frequency, the use of the baseband representation of the system quantities increases the simulation performance enormously. However, if the spectrum of the transients is not condensed around the system frequency, but also around other harmonics, this advantage disappears. As the frequency shift applied to the analytical signal is only $j\omega_s$, meaning $X_{bb}(j\omega) = X_a(j\omega - j\omega_s)$, the maximum frequency of the baseband signal with other harmonics will be higher than with only the spectrum content condensed around the system frequency as illustrated in Figure 3.6.
Another commonly used variable representation in power systems area is the DQ0 representation. The DQ0 or Park’s transformation is mainly employed in the derivation of model equations of electrical machines [33]. The DQ0 reference frame is generally referred as a reference frame rotating with the system frequency. There are some similarities between the DQ0 transformation and the baseband representation outlined in the previous section.

The purpose of the baseband representation of the ABC phase quantities is to reduce the maximum occurring frequency in the system quantities and hence to allow the use of larger step sizes during the simulation process. In a way, we take advantage of the periodic or nearly periodic behavior of the system quantities around system frequency. Till now the orthogonality of the selected variable representation was not of concern. The ABC phase quantities are not orthogonal, but an orthogonal representation of these quantities is also a desirable property.

The DQ0 transformation can be formulated in two steps. The first step is to obtain an orthogonal representation of the ABC phase quantities. As a second step a frequency shift is applied to obtain the baseband representation of the analytical orthogonal quantities.

Mathematically, the DQ0 or Park’s transformation is given by the fol-
following equation

\[
\begin{bmatrix}
    x_d \\
    x_q \\
    x_0
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
    \cos (\theta) & \cos (\theta - \frac{2\pi}{3}) & \cos (\theta + \frac{2\pi}{3}) \\
    -\sin (\theta) & -\sin (\theta - \frac{2\pi}{3}) & -\sin (\theta + \frac{2\pi}{3}) \\
    \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
    x_a \\
    x_b \\
    x_c
\end{bmatrix}
\]

\[T_{dq0}\] (3.7)

with \(\theta = \omega_s t = 2\pi f_s t\) and \(f_s\) is the system frequency.

The inverse transformation matrix is defined as

\[
\begin{bmatrix}
    x_a \\
    x_b \\
    x_c
\end{bmatrix} = \begin{bmatrix}
    \cos (\theta) & -\sin (\theta) & 1 \\
    \cos (\theta - \frac{2\pi}{3}) & -\sin (\theta - \frac{2\pi}{3}) & 1 \\
    \cos (\theta + \frac{2\pi}{3}) & -\sin (\theta + \frac{2\pi}{3}) & 1
\end{bmatrix}
\begin{bmatrix}
    x_d \\
    x_q \\
    x_0
\end{bmatrix}
\]

\[T_{abc}\] (3.8)

The derivatives of the time dependent transformation matrices \(T_{dq0}\) and \(T_{abc}\) can be given as

\[
\frac{dT_{dq0}}{dt} = -\omega_s (J \cdot T_{dq0})
\]

(3.9)

\[
\frac{dT_{abc}}{dt} = \omega_s (T_{abc} \cdot J)
\]

(3.10)

with

\[
J = \begin{bmatrix}
    0 & 1 & 0 \\
    -1 & 0 & 0 \\
    0 & 0 & 0
\end{bmatrix}
\]

(3.11)

The DQ0 transformation of the general three-phase constant AC current (3.1) can be expressed as the sum of the DQ0 transformations of the sequence components given in (3.3), namely as

\[
i_{dq0}(t) = i_{dq0,p}(t) + i_{dq0,n}(t) + i_{dq0,z}(t)
\]

(3.12)
with

\[ i_{dq0,p}(t) = I_p \begin{bmatrix} \cos(\theta_p) \\ \sin(\theta_p) \\ 0 \end{bmatrix} \] (3.13)

\[ i_{dq0,n}(t) = I_n \begin{bmatrix} \cos(2\omega_s t + \theta_n) \\ -\sin(2\omega_s t + \theta_n) \\ 0 \end{bmatrix} \] (3.14)

\[ i_{dq0,z}(t) = I_z \begin{bmatrix} 0 \\ 0 \\ \cos(\omega_s t + \theta_z) \end{bmatrix} \] (3.15)

After the analytical review of the DQ0 transformation and transformed general quantities, we will consider their simulation performance under different system conditions.

One of the advantages in analyzing the power system with symmetrical components in DQ0 reference frame is that under balanced steady-state operation the electrical quantities have constant values. If the power system is at balanced steady-state, the steady-state current contains only the positive sequence component \( i_{dq0,p} \), which is time independent and constant. For other modes of balanced operation, such as during electromechanical oscillations, these quantities vary slowly with time (maximum 2-3 Hz). For balanced operation, the frequency content of the transients will be centered around DC, which is also the aim of the aforementioned baseband representation. This leads also to faster simulation times under balanced conditions, as the variation of variables in the DQ0 reference frame are much slower than the original variables in the three-phase ABC reference frame or even constant at steady-state. Therefore in some programs (e.g. SIMPOW [7]) the DQ0 transformation is applied to all components in the system. All variables and equations of the models in the three-phase ABC reference frame are transformed to the DQ0 reference frame.

The DQ0 transformation is a single reference frame transformation as the reference frame rotates with system frequency. Therefore the simulations in the DQ0 reference frame will be efficient around system frequency, which is the case in balanced symmetrical systems.

But if there are unbalanced conditions in the system or other harmonics, this efficiency can decrease drastically. If the three-phase current
is unbalanced, it contains all three sequence components \(i_{dq0,p}, i_{dq0,n}\) and \(i_{dq0,z}\). Under unbalanced conditions, the frequency content of the transients will be centered not only around DC but also around \(2\omega_s\) due to negative sequence quantities and around \(\omega_s\) due to zero sequence quantities. As mentioned before, this decreases the simulation performance drastically as the maximum occurring frequency in the baseband signal is increased. The same disadvantage was also mentioned for the baseband represented signals in Section 3.3. Unbalanced conditions or asymmetrical components however do not cause such harmonics in the ABC reference frame. These harmonics with \(2\omega_s\) and \(\omega_s\) during unbalanced conditions are inherent to the DQ0 representation of positive, negative, and zero sequence quantities.

### 3.5 Dynamic Phasor Representation

Irrespective of the chosen variable representation, there is always a periodicity in the system quantities at steady-state. In the ABC representation the periodicity is with system frequency \(\omega_s\) regardless of the system conditions (balanced/unbalanced). In the DQ0 representation, depending on the sequence component of the signal, the negative sequence quantities have a periodicity with double system frequency \(2\omega_s\), zero sequence quantities a periodicity with system frequency \(\omega_s\). The positive sequence quantities are DC-quantities at steady-state, which brings significant advantages in simulating symmetrical power systems under balanced conditions. The disadvantages of the baseband representation and the DQ0 representation arise if the frequency content of the transformed quantities are not centered around a carrier frequency. In such cases, the baseband signals are not narrow banded but rather wide banded. In this section, the focus will be on an analytical tool trying to overcome this problem by a Fourier approximation of the system quantities.

The frequency spectrum of power system transients in the ABC reference frame mostly contains frequency components which are centered around system frequency \(\omega_s\). But the usage of power electronic based devices such as FACTs or HVDC give rise to other harmonics in the system due to the periodically switched mode of operation of these devices. In the DQ0 reference frame (even without power electronic based devices) high frequency components are not only centered around DC
but also around double system frequency depending on the sequence component (positive, negative or zero sequence). The half-band width of these spectra is generally less than the system frequency. If the spectrum of the transients is not condensed around the system frequency, but also around other harmonics, the advantages of baseband representation disappears, as the maximum frequency of baseband signal is also increased. Due to the Nyquist criterion, this has also a direct effect on the maximum permissible step size during simulation.

In such cases the spectrum of the power system transients can be approximated as illustrated in Figure 3.7. If the frequency contents of the transients are centered around $k\omega_s$ with their half bandwidth less than $\omega_s$, the overall spectrum is a disjunctive combination of the spectra around $k\omega_s$ ($\tilde{X}_k(j\omega)$).

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} \tilde{X}_k(j\omega)$$

As the spectral content $\tilde{X}_k(j\omega)$ is the frequency shifted version of the baseband signal $X_k(j\omega)$, meaning $\tilde{X}_k(j\omega) = X_k(j\omega - jk\omega_s)$, the overall spectrum can be expressed as the sum of frequency shifted spectra of analytical baseband signals.

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} X_k(j\omega - jk\omega_s)$$

With the inverse transformation from frequency domain to the time
domain, the signal \( x(t) \) becomes
\[
x(t) = \sum_{k=-\infty}^{+\infty} X_k(t) e^{j k \omega_s t}
\]
where \( X_k(t) \) is the analytical baseband signal. Thus the signal \( x(t) \) is fully defined by the baseband signal \( X_k(t) \).

This approach can be also seen as a modified form of the Fourier series representation of a periodic signal. A possibly complex valued periodic signal with period \( T \) [e.g. \( x(\tau) = x(\tau - T) \)] can be expressed with a Fourier series representation of the form given by
\[
x(\tau) = \sum_{k=-\infty}^{\infty} X_k e^{j k \omega_s \tau}
\]
where \( \omega_s = 2\pi/T \) and \( X_k \) is the \( k^{th} \) Fourier coefficient in complex form. It is important to note, that as the signal \( x(\tau) \) is periodic the Fourier coefficients \( X_k \) are time invariant.

In power system steady-state analysis, these Fourier coefficients \( X_k \) are also referred to as phasors. During transients, the system is not in a pure periodic but nearly periodic state. The idea is now to extend this approach to nearly periodic signals [13] and to approximate \( x(\tau) \) in the interval \( \tau \in (t-T,t] \) with a Fourier series representation of the form given in (3.16).
\[
x(\tau) = \sum_{k=-\infty}^{\infty} X_k(t) e^{j k \omega_s \tau}
\] (3.16)

In this representation, as the signal \( x(\tau) \) is nearly periodic and since the interval under consideration slides as a function of time, the Fourier coefficients \( X_k \) are time varying. This expression can also interpreted as an orthogonal signal expansion of the function \( x(t) \) with orthonormal basis \( e^{j k \omega_s t} \). The corresponding coordinate \( X_k(t) \) belonging to \( e^{j k \omega_s t} \), can be determined by the following inner product of the original signal with the corresponding basis signal \( e^{j k \omega_s t} \)
\[
X_k(t) = \left< x(t), e^{j k \omega_s t} \right> = \langle x(t) \rangle_k
\]
which is determined by
\[
X_k(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau) e^{-j k \omega_s \tau} d\tau \quad (3.17)
\]
3.5. Dynamic Phasor Representation

These time varying Fourier coefficients, which are equivalent to the analytical baseband signals in the previous representation, are referred to as dynamic phasors. Throughout the thesis, this methodology is called the Dynamic Phasors Approach.

In the dynamic phasors approach, we are interested in cases where only a few coefficients provide a good approximation of the original waveform.

\[ x(t) \approx \sum_{k \in K} X_k(t) e^{j k \omega_s t} \]  

(3.18)

where \( K \) is the set of Fourier coefficients approximating the original waveform. As notation, here lowercase letters \( x(\tau) \) are used for instantaneous variables and uppercase letters \( X_k(t) \) for dynamic phasors.

Some important properties of dynamic phasors are:

- The relation between the derivatives of \( x(\tau) \) and the derivatives of \( X_k(t) \), which is given in (3.19), where the time argument \( t \) has been omitted for clarity. This can easily be verified by differentiating the formula given in (3.16)

\[ \langle \frac{dx}{dt} \rangle_k = \frac{dX_k}{dt} + j k \omega_s X_k \]  

(3.19)

- The product of two time-domain variables equals a discrete time convolution of the two dynamic phasor sets of the variables, which is given in (3.20).

\[ \langle xy \rangle_k = \sum_{l=-\infty}^{\infty} (X_{k-l} Y_l) \]  

(3.20)

- For a complex valued signal \( x \), the relationship between \( X_k \) and \( X_{-k} \) is given as:

\[ X_{-k} = (X^*)_k \]  

(3.21)

where \( X^*_k \) means the conjugate complex of \( X_k \). For a real valued signal, this property becomes

\[ X_{-k} = X_k^* \]  

(3.22)
Chapter 3. Modelling of Power Systems

In the used simulation framework, the continuous dynamic behavior of the power system components is described by Differential Algebraic Equations

\[
\frac{dx}{dt} = f(x, y) \\
0 = g(x, y)
\] (3.23)

Using the appropriate approximations for the dynamic states \(x\) and algebraic states \(y\) in (3.16) and the properties (3.19-3.20), we can approximate the set of \(f\) and \(g\) equations of the model with a new set of equations and get the definition of the dynamic phasor model in a new set of functions \(F\) and \(G\) as

\[
\frac{dX_k}{dt} = F_k(X_k, Y_k) - jk \omega_s X_k \\
0 = G_k(X_k, Y_k)
\] (3.24)

where the dynamic phasors \(X_k\) become the new continuous dynamic states and \(Y_k\) the new algebraic states.

The dynamic phasors approach offers a number of advantages over conventional methods.

+ The selection of set \(K\) gives a wider bandwidth in the frequency domain than the traditional slow quasi-stationary assumptions used in Transient Stability Programs, where the electromagnetic transients are totally neglected.

+ As the dynamic phasors \(X_k\) are all narrow banded baseband signals, they can be numerically computed more efficiently.

+ The time domain simulations of such large systems with periodically switched power electronic based components not only feature a significant computational burden, but also give little insight into the problem sensitivities to design controllers or protection schemes. The dynamic phasors approach also allows an analytical insight into such problems, as it approximates a periodically switched system with a continuous system.

− One disadvantage of the dynamic phasors approach is that the number of variables and equations in the phasor dynamics approach (3.24) is higher than in the original equations (3.23), which
can make the simulation inefficient. But the efficiency gain due to slower variations in the Fourier coefficients is mostly higher than the efficiency loss due to the increased number of variables and equations.

The dynamic phasors approach is based on an adequate approximation of the system variables and equations, whereas the DQ0 reference frame representation or the analytical baseband representation are based on transformations. Thus great care must be taken in the selection of the approximating signal space $K$ to reflect the correct system behavior in the application of the dynamic phasor approach.

For example in the case of the DQ0 reference frame in (3.12), an appropriate selection for the approximation would be $K = \{0, 1, 2\}$ as unbalanced conditions induce terms with double system frequency ($k = 2$) in negative sequence and terms with system frequency ($k = 1$) in zero sequence.

In fact the dynamic phasors approach can be applied to both variable representations in ABC and in DQ0 as the dynamic phasors approach is a general analytical tool. One advantage of the DQ0 based dynamic phasor model is the achieved decoupling of the sequence quantities with $K = \{0, 1, 2\}$ which is an important property of the DQ0 transformation. In ABC-based dynamic phasor models, this is not the case.

The dynamic phasors approach has been successfully employed to model electric machines, FACTS devices and power converters [17, 18, 14, 15]. Our aim in this thesis is to use the dynamic phasor representation as a system representation in our simulator where the models are derived based on this approach and compare their simulation efficiency and accuracy to other commonly used modelling techniques.

### 3.6 Summary

In this chapter, we discussed different variable representations used in power systems modelling and power systems analysis. The main focus was on their advantages and disadvantages from the simulation performance point of view.
• **Three-phase ABC Representation**: The three-phase ABC representation provides a basis for the other derived variable representations. Important advantages and disadvantages are:

  + As instantaneous quantities are used, any kind of equipment can be modelled with this approach (e.g. detailed simulation power electronic based devices).
  + As it takes the true physical model the simulations are accurate.
  
  − For large power systems, the simulations can be inefficient as all components are modelled in detail and the instantaneous quantities are used for simulation.

• **Baseband ABC Representation**: In this representation, the bandpass real valued instantaneous quantities are transformed to baseband complex valued quantities, which can be simulated more efficiently i.e. with larger step sizes. Important advantages and disadvantages are:

  + As the frequency contents of the quantities are only shifted in the frequency domain, the models are still accurate.
  + The transformed quantities can be simulated more efficiently as the variations in the baseband signal are much slower than in the instantaneous quantities.
  
  − If the frequency content of the instantaneous values are not concentrated around the system frequency, this latter advantage disappears.

• **DQ0 Representation**: The aim of the DQ0 representation is to obtain an orthogonal and baseband representation of the original ABC quantities.

  + The models in DQ0 reference frame also give accurate results, since these models are derived by a transformation and not an approximation.
  
  + Simulations with these models are efficient if the system is in a balanced condition. In this case, only the positive sequence network is simulated. The variations of the DQ0 transformed positive sequence quantities are much slower than the instantaneous values of the quantities in ABC reference frame, so that they can be simulated with larger step sizes.
3.6. Summary

− However, if the system is in an unbalanced condition, negative and zero sequence components occur in the system, with $2\omega_s$ and $\omega_s$ oscillations. This makes the simulations inefficient as the step size must be kept small enough to come up with these oscillations.

− DQ0 transformation is a single reference frame transformation. Thus it is only efficient at fundamental frequency or near to fundamental frequency. If there are other harmonics in the system (e.g. due to FACTS devices), this efficiency will decrease drastically.

• Dynamic phasor models: In the dynamic phasors approach, the quantities are approximated with their time-varying Fourier coefficients also named dynamic phasors. As these dynamic phasors have a baseband characteristic, they can be computed also efficiently.

  + The accuracy of the simulation is dependent on the appropriate selection of Fourier coefficients $K$.

  + Simulations with phasor dynamics are efficient as we are simulating Fourier coefficients and their variations are much slower than the instantaneous values. Thus, simulations are faster.

  + Dynamic phasors approach applied to power electronic devices transform the periodically switched system into a continuous system, so that we gain more analytical insight (e.g. eigenvalue analysis).

  − The number of the variables and equations can increase depending on the selection of $K$. This can decrease simulation performance.

In the next chapter, we will derive the dynamic phasor models of some major components and will give a comparative assessment of the simulation performance of the described modelling techniques under balanced and unbalanced conditions. The comparison will regard the accuracy and efficiency under different operation conditions of the ABC, DQ0 and dynamic phasor models.
Chapter 4

Comparative Assessment of Different Modelling Techniques

In the previous chapter we focused on the commonly used system variable representations in power system modelling and also introduced the dynamic phasors approach. In this chapter, we will derive the dynamic phasor models of major power system components and compare their simulation performance with others in the ABC and DQ0 reference frame in terms of accuracy and computational efficiency.

4.1 Introduction

Chapter 3 dealt with the conventional system representations in power system modelling and analysis. Furthermore, the concept of dynamic phasors has been introduced where the instantaneous system quantities are represented by their time varying Fourier coefficients. The theoretical evaluation of these system representations regarding their simulation performance under different operating conditions has been discussed by looking at their characteristics in the frequency domain. The conclusions we arrived at were more of a qualitative nature. In this Chapter, the aim is to justify these qualitative conclusions with quantitative values by comparing the accuracy and computational efficiency
of these modelling techniques in a systematic way. The consistency of this systematic comparison is warranted by implementing the differently represented models of major power system components and simulating them under different operating conditions using the same simulation framework. Details of the used simulation environment were given in Chapter 2. Figure 4.1 illustrates the described aim of this Chapter.

![Diagram](image)

**Figure 4.1**: Systematic comparison of different modelling techniques

The Chapter is structured as follows. First of all, dynamic phasor models of major power system elements will be derived. In this Chapter, we will focus on the synchronous machine and the transmission line models for electromagnetic transients studies, since the used test case (Single Machine Infinite Bus-SMIB) contains these elements. However also dynamic phasor models of Thyristor-controlled Series Capacitor and Doubly-fed Induction machine are in the scope of the thesis. The dynamic phasor model of the synchronous machine will be derived in the conventionally used ABC and DQ0 reference frame.

Besides the detailed models of these components capturing fast electromagnetic and slow electromechanical transients, we will also derive the so called reduced order models capturing only the slower electromechanical transients. These reduced order models are equivalent to the
fundamental frequency models used in transient stability studies.

As a test case, the SMIB system will be used under different operating conditions namely, balanced and unbalanced conditions. Unbalanced faults (e.g. single-line to ground fault) and asymmetrical components will be treated as unbalanced conditions. Finally the simulation performance of these dynamic phasor models will be compared with those in ABC and DQ0.

### 4.2 Transmission Line Model

In this section, the dynamic phasor model of a simple transmission line will be derived, where the equivalent circuit of each phase is represented by series RL branches with different R-L values and no mutual couplings between the three phases. The dynamic phasor models will be based on model equations in the the original ABC phases and also in the DQ0 reference frame. Hence, we will briefly treat the transmission line model in the ABC and DQ0 reference frame and then accordingly derive the dynamic phasor models.

#### 4.2.1 Dynamic Phasor Model – ABC

The per unit model equations of the above mentioned transmission line in the original ABC phases yield

\[
\frac{d\psi_{abc}}{dt} = v_{abc} - R_{abc} i_{abc} \quad (4.1)
\]

\[
\psi_{abc} = L_{abc} i_{abc} \quad (4.2)
\]

with flux, voltage and current vectors

\[
\psi_{abc} = \begin{bmatrix} \psi_a(t) & \psi_b(t) & \psi_c(t) \end{bmatrix}^T
\]

\[
v_{abc} = \begin{bmatrix} v_a(t) & v_b(t) & v_c(t) \end{bmatrix}^T
\]

\[
i_{abc} = \begin{bmatrix} i_a(t) & i_b(t) & i_c(t) \end{bmatrix}^T
\]

The time argument \((t)\) has been dropped in the electrical quantities \(\psi_{abc}(t), v_{abc}(t)\) and \(i_{abc}(t)\). The square and diagonal inductance and
resistance matrices $R_{abc}$ and $L_{abc}$ are defined as

\[
R_{abc} = \text{diag} \begin{bmatrix} R_a & R_b & R_c \end{bmatrix}
\]
\[
L_{abc} = \text{diag} \begin{bmatrix} L_a & L_b & L_c \end{bmatrix}
\]

Unequal resistance and inductance values for each phase have been used to allow implementation of asymmetrical component models. For ease of formulation the mutual inductances have been ignored, but these can easily be included in the model. Equation (4.1) provides the basis of the model equations for the hereafter derived transmission line models.

Now, we want to derive the dynamic phasor model of the transmission line model given in (4.1). The first step is to decide on the approximating Fourier coefficients set for the behavior of the model under balanced and unbalanced conditions. In other words, we first define the set of dynamic phasor $K$ in (3.18) used for the approximation.

In the case of the transmission line model given in (4.1) an appropriate selection for $K$ would be $K = \{1\}$, since the currents and voltages in the ABC phases are assumed to be sinusoidal or nearly sinusoidal with system frequency. With this approximation and the properties (3.19-3.22), the dynamic phasor model of the transmission line in the ABC phases with $K = \{1\}$ from (4.1) can be derived as follows. With $K = \{1\}$, the model equation (4.1) becomes

\[
\langle \frac{d\psi_{abc}}{dt}\rangle_1 = \langle v_{abc}\rangle_1 - \langle R_{abc}i_{abc}\rangle_1
\]
\[
\langle \psi_{abc}\rangle_1 = \langle L_{abc}i_{abc}\rangle_1
\]

Applying the rules (3.19-3.22) for the derivative and product, yields

\[
\frac{d\langle \psi_{abc}\rangle_1}{dt} = \langle v_{abc}\rangle_1 - R_{abc} \langle i_{abc}\rangle_1 - j\omega_s \langle \psi_{abc}\rangle_1 \quad (4.3)
\]
\[
\langle \psi_{abc}\rangle_1 = L_{abc} \langle i_{abc}\rangle_1 \quad (4.4)
\]

The model given in (4.3-4.4) is the EMT equivalent dynamic phasor model capturing electromagnetic transients. In transient stability analysis, the network transients are commonly neglected. Neglecting the network transients would mean neglecting the variations in the dynamic phasor $\langle \psi_{abc}\rangle_1$ by setting $\frac{d\langle \psi_{abc}\rangle_1}{dt} = 0$. Thus, the model reflects only
4.2. Transmission Line Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Approximation $K = {\ldots}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{abc}$</td>
<td>${1}$</td>
</tr>
<tr>
<td>$v_{abc}$</td>
<td>${1}$</td>
</tr>
<tr>
<td>$i_{abc}$</td>
<td>${1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Approximation $K = {\ldots}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{abc}$</td>
<td>constant</td>
</tr>
<tr>
<td>$L_{abc}$</td>
<td>constant</td>
</tr>
</tbody>
</table>

Table 4.1: Appropriate approximation for variables and parameters of the transmission line model with DPA

the steady state impedance relation between voltage phasor $\langle v_{abc} \rangle_1$ and current phasor $\langle i_{abc} \rangle_1$, namely.

$$\langle v_{abc} \rangle_1 = \langle i_{abc} \rangle_1 (R_{abc} + j\omega_s L_{abc})$$

4.2.2 Dynamic Phasor Model – DQ0

Using the DQ0 or Park’s transformation, described in Section 3.4, the model equations (4.1) can be transformed into the following form using properties (3.7) and (3.9)

$$\frac{d\psi_{dq0}}{dt} = v_{dq0} - R_{dq0} i_{dq0} - \omega_s J \psi_{dq0}$$  \hspace{1cm} (4.5)

$$\psi_{dq0} = L_{dq0} i_{dq0}$$  \hspace{1cm} (4.6)

with flux, voltage and current vectors

$$\psi_{dq0} = T_{dq0} \psi_{abc} = [ \psi_d(t) \quad \psi_q(t) \quad \psi_0(t) ]^T$$

$$v_{dq0} = T_{dq0} v_{abc} = [ v_d(t) \quad v_q(t) \quad v_0(t) ]^T$$

$$i_{dq0} = T_{dq0} i_{abc} = [ i_d(t) \quad i_q(t) \quad i_0(t) ]^T$$

$J$ given in equation (3.11) and the time dependent and non-diagonal inductance and resistance matrices $L_{dq0}(\theta_s)$ and $R_{dq0}(\theta_s)$, with $\theta_s = \omega_s t$ and $\omega_s$ the system frequency. The expression for $L_{dq0}(\theta_s)$ is given as

$$L_{dq0}(\theta_s) = T_{dq0}(\theta_s) \cdot L_{abc} \cdot T_{abc}(\theta_s)$$

$$= \begin{bmatrix} L_{dq}(\theta_s) & L_{dq,0}(\theta_s) \\ L_{0,dq}(\theta_s) & L_0 \end{bmatrix}$$  \hspace{1cm} (4.7)
with the following definitions
\[
L_{dq} = \Re \{ \langle L_{dq} \rangle_0 + \langle L_{dq} \rangle_2 e^{j2\theta_s} \} \tag{4.8}
\]
\[
L_{dq,0} = \Re \{ \langle L_{dq,0} \rangle_1 e^{j\theta_s} \} \tag{4.9}
\]
\[
L_{0,dq} = \Re \{ \langle L_{0,dq} \rangle_1 e^{j\theta_s} \} \tag{4.10}
\]
\[
L_0 = L_0 \tag{4.11}
\]
\[
\langle L_{dq} \rangle_0 = \begin{bmatrix} L_\gamma & 0 \\ 0 & L_\gamma \end{bmatrix}, \langle L_{dq} \rangle_2 = \frac{1}{3} \begin{bmatrix} L_\alpha & jL_\alpha \\ jL_\alpha & -L_\alpha \end{bmatrix} \tag{4.12}
\]
\[
\langle L_{dq,0} \rangle_1 = \frac{1}{3} \begin{bmatrix} 2L_\beta \\ 2jL_\beta \end{bmatrix}, \langle L_{0,dq} \rangle_1 = \frac{1}{3} \begin{bmatrix} L_\beta & jL_\beta \end{bmatrix}, \langle L_0 \rangle_0 = L_\gamma \tag{4.13}
\]
\[
L_\gamma = \frac{1}{3} L_a + \frac{1}{3} L_b + \frac{1}{3} L_c \tag{4.14}
\]
\[
L_\alpha = \frac{2}{3} L_a - \frac{1}{3} L_b - \frac{1}{3} L_c = \frac{1}{3} \Re \left\{ L_a + L_b e^{j\frac{2\pi}{3}} + L_c e^{-j\frac{2\pi}{3}} \right\} \tag{4.15}
\]
\[
L_\beta = \frac{1}{3} \sqrt{3} (L_c - L_b) = \frac{1}{3} \Im \left\{ L_a + L_b e^{j\frac{2\pi}{3}} + L_c e^{-j\frac{2\pi}{3}} \right\} \tag{4.16}
\]

The resistance matrix \( R_{dq0} \) also has the same form.

After grouping the dq0 coordinates of the quantities \( x_{dq0} \) and parameters into \( x_{dq} \) and \( x_0 \) coordinates, the model equations become
\[
\frac{d\psi_{dq}}{dt} = v_{dq} - R_{dq} i_{dq} - R_{dq,0} i_0 - \omega_s J_2 \psi_{dq} \tag{4.17}
\]
\[
\frac{d\psi_0}{dt} = v_0 - R_{0,dq} i_{dq} - R_0 i_0 \tag{4.18}
\]
\[
\psi_{dq} = L_{dq} i_{dq} + L_{dq,0} i_0 \tag{4.19}
\]
\[
\psi_0 = L_{0,dq} i_{dq} + L_0 i_0 \tag{4.20}
\]

with
\[
J_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{4.21}
\]

It is important to note the periodic time dependence of \( L_{dq} \) due to the \( e^{j2\theta_s} \) term and of \( L_{dq,0}, L_{0,dq} \) due to the \( e^{j\theta_s} \) term in case of the DQ0 transformed asymmetrical transmission line model.

In the case of a symmetrical line with \( L_a = L_b = L_c, L_{dq0} \) becomes diagonal and time independent as \( L_\alpha = 0, L_\beta = 0 \) and \( L_\gamma = L_\alpha \), which
4.2. Transmission Line Model

significantly simplifies the model equation (4.12) in the DQ0 reference frame. Already this simple example shows that the DQ0 transformation is advantageous only in conjunction with symmetrical components since asymmetrical conditions in the components induce time dependent parameters.

In the case of a symmetrical transmission line in the DQ0 reference frame, the model inductance matrix \( L_{dq} \) is constant and diagonal, \( L_{dq,0} \) and \( L_{0,dq} \) are zero. Assuming an asymmetrical transmission line, these model inductance matrices will not be constant but time dependent due to the periodic terms with \( e^{j\theta_s}, e^{2j\theta_s} \) as stated in (4.12-4.15). In this case for an asymmetrical transmission line the appropriate approximation for the model parameters \( L_{dq}, R_{dq} \) will be \( K = \{0, 2\} \), for \( L_{dq,0}, L_{0,dq}, R_{dq,0}, R_{0,dq} \) it will be \( K = \{1\} \) when deriving the corresponding dynamic phasor model in the DQ0 reference frame.

As discussed in Section 3.4, in the case of unbalanced conditions, where all three sequence components (positive, negative and zero sequence) are present, the DQ0 transformed sequence components will appear with different frequencies in the DQ0 reference frame. Under unbalanced conditions, the frequency content of the positive sequence quantities will be centered around DC and that of the negative sequence quantities will be around \( 2\omega_s \). Both positive and negative sequence components will be observed in the \( d \) and \( q \) axis but at different frequencies (i.e. around 0 and \( 2\omega_s \)). The zero sequence components will only be observed in the \( \theta \) axis and at frequencies around \( \omega_s \).

Hence, in the case of unbalanced conditions, the appropriate set of approximating Fourier coefficients for the transformed phase quantities in the DQ0 frame are \( K = \{0, 2\} \) for the \( dq \)-coordinates and \( K = \{1\} \) for the \( \theta \)-coordinate.

Table 4.2 shows the appropriate set of approximating set of Fourier coefficients for the dynamic phasor transmission line model in the DQ0 reference frame. Since the phase quantities in the DQ0 reference frame are also real valued, the positive indexed dynamic phasors e.g. \( K = \{0, 1, 2\} \) contain all the information about these quantities (3.22).

With the approximations given in Table 4.2 for electrical quantities and model parameters, the dynamic phasor model of the transmission line
Table 4.2: Appropriate approximation for variables and parameters of the transmission line model with DPA in the DQ0 reference frame

in the DQ0 reference frame can be given as

\[
\frac{d\psi_{dq}}{dt}\bigg|_0 = \langle v_{dq} \rangle_0 - \langle R_{dq}i_{dq} \rangle_0 - \omega_s J_2 \langle \psi_{dq} \rangle_0
\]
\[
\frac{d\psi_{dq}}{dt}\bigg|_2 = \langle v_{dq} \rangle_2 - \langle R_{dq}i_{dq} \rangle_2 - \langle R_{dq,0}i_2 \rangle_2 - \omega_s J_2 \langle \psi_{dq} \rangle_2
\]
\[
\frac{d\psi_0}{dt}\bigg|_1 = \langle v_0 \rangle_1 - \langle R_{0,dq}i_{dq} \rangle_1 - \langle R_0i_0 \rangle_1
\]
\[
\langle \psi_{dq} \rangle_0 = \langle L_{dq}i_{dq} \rangle_0 + \langle L_{dq,0}i_0 \rangle_0
\]
\[
\langle \psi_{dq} \rangle_2 = \langle L_{dq}i_{dq} \rangle_2 + \langle L_{dq,0}i_2 \rangle_2
\]
\[
\langle \psi_0 \rangle_1 = \langle L_{0,dq}i_{dq} \rangle_1 + \langle L_0i_0 \rangle_1
\]

With the properties regarding the derivative, product and conjugate complex of dynamic phasors (3.19-3.22), the dynamic phasor model of
the transmission line yields

\[
\frac{d\langle \psi_{dq} \rangle_0}{dt} = \langle v_{dq} \rangle_0 - \langle R_{dq} \rangle_0 \langle i_{dq} \rangle_0 - \langle R_{dq} \rangle_2 \langle i_{dq} \rangle_2 \nonumber
- \langle R_{dq,0} \rangle_1 \langle i_0 \rangle_1 - \omega_s J_2 \langle \psi_{dq} \rangle_0 \quad (4.17)
\]

\[
\frac{d\langle \psi_{dq} \rangle_2}{dt} = \langle v_{dq} \rangle_2 - \langle R_{dq} \rangle_2 \langle i_{dq} \rangle_0 - \langle R_{dq} \rangle_0 \langle i_{dq} \rangle_2 \nonumber
- \langle R_{dq,0} \rangle_1 \langle i_0 \rangle_1 - \omega_s J_2 \langle \psi_{dq} \rangle_2 \quad (4.18)
\]

\[
\frac{d\langle \psi_0 \rangle_1}{dt} = \langle v_0 \rangle_1 - \langle R_{0, dq} \rangle_1 \langle i_{dq} \rangle_0 - \langle R_{0, dq} \rangle_1 \langle i_{dq} \rangle_2 \nonumber
- \langle R_0 \rangle_0 \langle i_0 \rangle_1 \quad (4.19)
\]

\[
\langle \psi_{dq} \rangle_0 = \langle L_{dq} \rangle_0 \langle i_{dq} \rangle_0 + \langle L_{dq} \rangle_2 \langle i_{dq} \rangle_2 \nonumber
+ \langle L_{dq,0} \rangle_1 \langle i_0 \rangle_1 \quad (4.20)
\]

\[
\langle \psi_{dq} \rangle_2 = \langle L_{dq} \rangle_2 \langle i_{dq} \rangle_0 + \langle L_{dq} \rangle_0 \langle i_{dq} \rangle_2 + \langle L_{dq,0} \rangle_1 \langle i_0 \rangle_1 \quad (4.21)
\]

\[
\langle \psi_0 \rangle_1 = \langle L_{0, dq} \rangle_1 \langle i_{dq} \rangle_0 + \langle L_{0, dq} \rangle_1 \langle i_{dq} \rangle_2 + \langle L_0 \rangle_0 \langle i_0 \rangle_1 \quad (4.22)
\]

The parameters \( \langle L_{dq} \rangle_k, \langle L_{dq,0} \rangle_k, \langle L_{0, dq} \rangle_k \) and \( \langle L_0 \rangle_k \) correspond to the expressions given in (4.8-4.11). Compared with the original model equations in the DQ0 reference frame, the dynamic phasor model inductances and resistances are constant in contrast to those represented with instantaneous quantities in the DQ0 reference frame. However, compared with dynamic phasor model equations in the ABC reference frame, the dynamic phasor model equations in the DQ0 reference frame are more complicated.

In the case of a symmetrical transmission line model, the parameters \( \langle L_{dq,0} \rangle_1, \langle L_{0, dq} \rangle_1 \) and \( \langle L_{dq} \rangle_2 \) become zero. This simplifies the model equations significantly.

\[
\frac{d\langle \psi_{dq} \rangle_0}{dt} = \langle v_{dq} \rangle_0 - \langle R_{dq} \rangle_0 \langle i_{dq} \rangle_0 - \omega_s J_2 \langle \psi_{dq} \rangle_0 \quad (4.23)
\]

\[
\frac{d\langle \psi_{dq} \rangle_2}{dt} = \langle v_{dq} \rangle_2 - \langle R_{dq} \rangle_2 \langle i_{dq} \rangle_0 - \omega_s J_2 \langle \psi_{dq} \rangle_2 \quad (4.24)
\]

\[
\frac{d\langle \psi_0 \rangle_1}{dt} = \langle v_0 \rangle_1 - \langle R_0 \rangle_0 \langle i_0 \rangle_1 \quad (4.25)
\]

\[
\langle \psi_{dq} \rangle_0 = \langle L_{dq} \rangle_0 \langle i_{dq} \rangle_0 \quad (4.26)
\]

\[
\langle \psi_{dq} \rangle_2 = \langle L_{dq} \rangle_0 \langle i_{dq} \rangle_2 \quad (4.27)
\]

\[
\langle \psi_0 \rangle_1 = \langle L_0 \rangle_0 \langle i_0 \rangle_1 \quad (4.28)
\]

The formulation of the asymmetrical transmission line dynamic phasor
model in the DQ0 components is computationally not that efficient due to the increased complexity.

The transmission line model defined with the equations (4.12-4.15) is the EMT equivalent model in the DQ0 reference frame with instantaneous values. The equivalent fundamental frequency models for transient stability studies assume generally that the fast electromagnetic transients have decayed and the electrical quantities have reached their steady state values. The fundamental frequency model derivation is generally accomplished by setting the derivative of the electrical quantities equal to zero if the steady state values of these quantities are DC quantities. Hence, this can only be applied to the equations (4.12-4.15) if and only if the system is under balanced conditions. Since the system is under balanced conditions, it will only contain positive sequence components, which get constant in the DQ0 reference frame at balanced steady state. For unbalanced steady state conditions, setting the derivative to zero will not be valid, since the negative sequence components will have oscillations with $2\omega_s$ and zero sequence components with $\omega_s$.

In contrast to the representations with the instantaneous values in the DQ0 reference frame, the steady state values of dynamic phasors in the DQ0 reference frame are constant even under unbalanced conditions. This allows a systematic derivation of reduced order models for transient stability studies, just by setting the derivatives of the dynamic phasors of the network quantities to zero. Hence the dynamic phasor model of the transmission line used for transient stability analysis where the electromagnetic transients are neglected, can be derived by setting the $\frac{d\langle \psi_{dq}\rangle_k}{dt} = \frac{d\langle \psi_0\rangle_k}{dt} = 0$ in the model equations (4.17-4.19).

4.3 Synchronous Machine

The modelling of synchronous machines has been a primary research topic in power system engineering for several years. Today there are a large number of different models used in different studies. Hence, the choice of the correct model highly depends on the frequency range of the study. In this section, we will base our dynamic phasor model on the the synchronous machine representation described in Chapter 3 in [33]. This representation is commonly used in electromagnetic transients studies and also in transient stability analysis. In this part of the chapter,
starting with the synchronous machine equations in the ABC and DQ0 reference frame, the dynamic phasor model of the synchronous machine will be derived in both reference frames. In [17] a similar derivation was made based on the space vector representation. Here we will use the DQ0 representation of the model equations.

4.3.1 Dynamic Phasor Model – ABC

The synchronous machine under consideration has one damper winding in the D-axis, two damper winding in the Q-axis and a field winding. The notation and formulae for describing the synchronous machine model are adopted from [33]. The following assumptions are made for the model derivation

- The stator currents are assumed to be positive when they flow out of the machine.
- The rotor currents are assumed to be positive when they flow into the machine.
- The direct axis (d-axis) is centered magnetically in the center of the north pole
- The quadrature axis (q-axis) leads the direct axis by 90° electrical degrees in the direction of rotation

The overall set of per unit model equations of the synchronous machine is generally divided into the voltage equations,

\[
\frac{1}{\omega_s} \frac{d\psi_s}{dt} = e_s + R_{ss} i_s \\
\frac{1}{\omega_s} \frac{d\psi_r}{dt} = e_r + R_{rr} i_r
\]  

(4.29)  

(4.30)

the flux linkages equations between the stator/rotor windings,

\[
\psi_s = L_{ss} (\theta_r) i_s + L_{sr} (\theta_r) i_r \\
\psi_r = L_{sr} (\theta_r) i_s + L_{rr} i_r
\]

(4.31)  

(4.32)
and the mechanical equations describing the effect of the unbalance between the electromagnetic torque and the mechanical torque.

\[
T_e = \frac{2\sqrt{3}}{9} [i_a (\psi_c - \psi_b) + i_b (\psi_a - \psi_c) + i_c (\psi_b - \psi_a)] \quad (4.33)
\]

\[
\frac{d\omega_r}{dt} = \frac{1}{2H} (T_m - T_e - K_D \omega_r) \quad (4.34)
\]

\[
\frac{1}{\omega_s} \frac{d\delta_r}{dt} = \omega_r - 1 \quad (4.35)
\]

with the following definitions:

- \(\psi\) ... the vector consisting of the per unit stator and rotor fluxes
- \(e\) ... the vector consisting of the per unit stator and rotor voltages
- \(i\) ... the vector consisting of the per unit stator and rotor currents
- \(T_e\) ... the per unit electrical torque
- \(T_m\) ... the per unit mechanical torque
- \(\omega_s\) ... the system frequency
- \(\omega_r\) ... the per unit rotor speed
- \(\delta_r\) ... the rotor angle
- \(\theta_r\) ... the angle by which the direct axis leads the A-phase winding in the direction of rotation \(\theta_r = \omega_s t + \delta_r\)
- \(R\) ... the diagonal per unit resistance matrix
- \(L\) ... the per unit inductance matrix
- \(H\) ... the per unit inertia constant
- \(K_D\) ... the per unit damping factor
Analytically one has

$$\psi = \begin{bmatrix} \psi_a & \psi_b & \psi_c & \psi_{fd} & \psi_{1d} & \psi_{1q} & \psi_{2q} \end{bmatrix}^T$$

$$i = \begin{bmatrix} -i_a & -i_b & -i_c & i_{fd} & i_{1d} & i_{1q} & i_{2q} \end{bmatrix}^T$$

$$e = \begin{bmatrix} e_a & e_b & e_c & e_{fd} & 0 & 0 & 0 \end{bmatrix}^T$$

$$R = \text{diag}\left[ \begin{array}{cccccc} R_a & R_a & R_a & R_{fd} & R_{1d} & R_{1q} & R_{2q} \end{array} \right]^T$$

The flux linkage for each generator winding is composed of a linkage due to the stator windings and a linkage due to the windings on the rotor. Hence, the instantaneous flux linkage can be divided into two parts.

$$\begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} L_{ss} \left( \theta_r \right) & L_{sr} \left( \theta_r \right) \\ L_{rs} \left( \theta_r \right) & L_{rr} \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix} \quad \text{(4.36)}$$

with $L_{ss}$, $L_{rr}$ being the self inductances of the stator and rotor windings and $L_{sr}$, $L_{rs}$ the mutual inductances between the stator and rotor windings. The self inductances of the stator and rotor windings $L_{ss}$, $L_{rr}$ and the mutual inductances between the stator and rotor windings $L_{sr}$, $L_{rs}$ are defined as

$$L_{ss} = \begin{pmatrix} L_{a,a} & L_{a,b} & L_{a,c} \\ L_{b,a} & L_{b,b} & L_{b,c} \\ L_{c,a} & L_{c,b} & L_{c,c} \end{pmatrix}$$

$$L_{rr} = \begin{pmatrix} L_{f,f} & L_{f,1d} & L_{f,1q} & L_{f,2q} \\ L_{1d,f} & L_{1d,1d} & L_{1d,1q} & L_{1d,2q} \\ L_{1q,f} & L_{1q,1d} & L_{1q,1q} & L_{1q,2q} \\ L_{2q,f} & L_{2q,1d} & L_{2q,1q} & L_{2q,2q} \end{pmatrix}$$

$$L_{sr} = \begin{pmatrix} L_{a,f} & L_{a,1d} & L_{a,1q} & L_{a,2q} \\ L_{b,f} & L_{b,1d} & L_{b,1q} & L_{b,2q} \\ L_{c,f} & L_{c,1d} & L_{c,1q} & L_{c,2q} \end{pmatrix}$$

$$L_{rs} = \begin{pmatrix} L_{f,a} & L_{f,b} & L_{f,c} \\ L_{1d,a} & L_{1d,b} & L_{1d,c} \\ L_{1q,a} & L_{1q,b} & L_{1q,c} \\ L_{2q,a} & L_{2q,b} & L_{2q,c} \end{pmatrix}$$
where the individual inductances are defined as follows:

\[ L_{a,a} = L_{aa0} + L_{aa2} \cos (2\theta_r) \]
\[ L_{b,b} = L_{aa0} + L_{aa2} \cos \left(2\theta_r - \frac{4\pi}{3}\right) \]
\[ L_{c,c} = L_{aa0} + L_{aa2} \cos \left(2\theta_r + \frac{4\pi}{3}\right) \]
\[ L_{a,b} = L_{ba} = L_{ab0} + L_{aa2} \cos \left(\theta_r + \frac{\pi}{3}\right) \]
\[ L_{a,c} = L_{ca} = L_{ab0} + L_{aa2} \cos \left(\theta_r - \frac{\pi}{3}\right) \]
\[ L_{b,c} = L_{cb} = L_{ab0} + L_{aa2} \cos \left(2\theta_r - \pi\right) \]
\[ L_{f,f} = L_{f1d} \]
\[ L_{1d,1d} = L_{1d} \]
\[ L_{1q,1q} = L_{1q} \]
\[ L_{2q,2q} = L_{2q} \]
\[ L_{f,1d} = L_{f1d} \]
\[ L_{1q,2q} = L_{2q,1q} = L_{12q} \]
\[ L_{f,1q} = L_{f2q} = L_{1q,f} = L_{f,2q} = 0 \]
\[ L_{1d,1q} = L_{1d,2q} = L_{1d,2q} = L_{2q,1d} = 0 \]
\[ L_{a,f} = L_{f,a} = L_{f1d} \cos \left(\theta_r\right) \]
\[ L_{b,f} = L_{f,b} = L_{f1d} \cos \left(\theta_r - \frac{4\pi}{3}\right) \]
\[ L_{c,f} = L_{f,c} = L_{f1d} \cos \left(\theta_r + \frac{4\pi}{3}\right) \]
\[ L_{a,1d} = L_{1d,a} = L_{a1d} \cos \left(\theta_r\right) \]
\[ L_{b,1d} = L_{1d,b} = L_{a1d} \cos \left(\theta_r - \frac{2\pi}{3}\right) \]
\[ L_{c,1d} = L_{1d,c} = L_{a1d} \cos \left(\theta_r + \frac{2\pi}{3}\right) \]
\[ L_{a,1q} = L_{1q,a} = -L_{a1q} \sin \left(\theta_r\right) \]
\[ L_{b,1q} = L_{1q,b} = -L_{a1q} \sin \left(\theta_r - \frac{2\pi}{3}\right) \]
\[ L_{c,1q} = L_{1q,c} = -L_{a1q} \sin \left(\theta_r + \frac{2\pi}{3}\right) \]
4.3. Synchronous Machine

\[
L_{a,2q} = L_{2q,a} = -L_{a2q} \sin(\theta_r)
\]
\[
L_{b,2q} = L_{2q,b} = -L_{a2q} \sin\left(\theta_r - \frac{2\pi}{3}\right)
\]
\[
L_{c,2q} = L_{2q,c} = -L_{a2q} \sin\left(\theta_r + \frac{2\pi}{3}\right)
\]

Since the rotor is rotating with the angular velocity \(\omega_r\) with respect to stator, the self and mutual inductances are not constant and depend on the electric rotor position \(\theta_r(t)\) with respect to the winding of phase A. The stator self inductance matrix \(L_{ss}\) consists of a constant term and of a time dependent periodic term varying with \(2\theta_r\). The rotor/stator mutual inductance matrices \(L_{sr}\) and \(L_{rs}\) are also time dependent and periodic with \(\theta_r\). The rotor self inductance matrix \(L_{rr}\) is constant. Thus, the overall inductance matrix \(L\) can be expressed as

\[
L(\theta_r) = \Re \left\{ \left( \begin{array}{ccc}
L_{ss,0} + L_{ss,2} e^{j2\theta_r} & L_{sr,1} e^{j\theta_r} \\
L_{rs,1} e^{j\theta_r} & L_{rr,0}
\end{array} \right) \right\}
\]

with

\[
L_{ss,0} = \begin{bmatrix}
L_{aa0} & L_{ab0} & L_{ab0} \\
L_{ab0} & L_{aa0} & L_{ab0} \\
L_{ab0} & L_{ab0} & L_{aa0}
\end{bmatrix}
\]

\[
L_{rr,0} = \begin{bmatrix}
L_{ffd} & L_{f1d} & 0 & 0 \\
L_{f1d} & L_{11d} & 0 & 0 \\
0 & 0 & L_{11q} & L_{12q} \\
0 & 0 & L_{21q} & L_{22q}
\end{bmatrix}
\]

\[
L_{rs,1} = -L_{sr,1}^T = \begin{bmatrix}
L_{afd} & L_{afd} e^{-j\frac{2\pi}{3}} & L_{afd} e^{j\frac{2\pi}{3}} \\
L_{a1d} & L_{a1d} e^{-j\frac{2\pi}{3}} & L_{a1d} e^{j\frac{2\pi}{3}} \\
jL_{a1q} & jL_{a1q} e^{-j\frac{2\pi}{3}} & jL_{a1q} e^{j\frac{2\pi}{3}} \\
jL_{a2q} & jL_{a2q} e^{-j\frac{2\pi}{3}} & jL_{a2q} e^{j\frac{2\pi}{3}}
\end{bmatrix}
\]

\[
L_{ss,2} = L_{aa2} \begin{bmatrix}
1 & e^{-j\frac{2\pi}{3}} & e^{j\frac{2\pi}{3}} \\
e^{-j\frac{2\pi}{3}} & e^{j\frac{2\pi}{3}} & 1 \\
e^{j\frac{2\pi}{3}} & 1 & e^{-j\frac{2\pi}{3}}
\end{bmatrix}
\]

Thus, with the equations (4.29-4.35) and the position dependent inductance matrix \(L(\theta_r)\), the synchronous machine model equations with instantaneous variables in the ABC reference frame are defined.
Our aim is now to derive the dynamic phasor model based on this ABC reference frame representation. The derivation of the dynamic phasor model requires an appropriate selection of the approximating Fourier coefficients for the variables and parameters of the model. With $\omega_s$ set as the fundamental frequency for the dynamic phasor approximation and considering $\theta_r(t) = \theta_s(t) + \delta_r(t) = \omega_s t + \delta_r(t)$, the time dependent inductance matrices $L_{ss}(\theta_r), L_{sr}(\theta_r), L_{rs}(\theta_r)$ and $L_{rr}$ can be expressed as

\[
L_{ss}(\theta_r) = \Re \{ \langle L_{ss} \rangle_0 + \langle L_{ss} \rangle_2 e^{j2\theta_s} \}
\]

\[
L_{rs}(\theta_r) = \Re \{ \langle L_{sr} \rangle_1 e^{j\theta_s} \}
\]

\[
L_{sr}(\theta_r) = \Re \{ \langle L_{rs} \rangle_1 e^{j\theta_s} \}
\]

\[
L_{rr}(\theta_r) = \Re \{ \langle L_{rr} \rangle_0 \}
\]

with

\[
\langle L_{ss} \rangle_0 = L_{ss,0}, \quad \langle L_{ss} \rangle_2 = L_{ss,2} e^{j2\delta_r}
\]

\[
\langle L_{sr} \rangle_1 = L_{sr,1} e^{j\delta_r}, \quad \langle L_{rs} \rangle_1 = L_{rs,1} e^{j\delta_r}
\]

\[
\langle L_{rr} \rangle_0 = L_{rr}
\]

With these approximations, the variations in the inductance matrix $L$ with $\dot{\theta}_r(t)$ have been reduced to $\dot{\delta}_r(t)$ during transients. At steady state, the inductance matrix $L$ becomes constant in the dynamic phasor representation due to the constant rotor angle i.e. $\dot{\delta}_r(t) = 0$, whereas in the original three-phase ABC representation, the inductance matrix is still time dependent due to the synchronous speed $\dot{\theta}_r(t) = \omega_s$ even in steady state.

Since unbalanced conditions are considered, the appropriate set of approximating Fourier coefficients for stator quantities (denoted with the subscript s) is $K_s = \{1\}$ and for the rotor quantities (denoted with the subscript r) $K_r = \{0, 2\}$. Table 4.3 shows the overall set of approximating set of Fourier coefficients for the dynamic phasor model of the synchronous machine in the ABC reference frame.

With the approximations given in Table 4.3 for the model variables and parameters, and the properties (3.19-3.22), the dynamic phasor model of the synchronous machine in the ABC reference frame yields
4.3. Synchronous Machine

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Approximation K = {...}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_s, e_s, i_s$</td>
<td>${1}$</td>
</tr>
<tr>
<td>$\psi_r, e_r, i_r$</td>
<td>${0, 2}$</td>
</tr>
<tr>
<td>$T_e, \omega_r, \delta_r$</td>
<td>${0, 2}$</td>
</tr>
<tr>
<td>$T_m$</td>
<td>${0}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Approximation K = {...}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{ss}$</td>
<td>${0, 2}$</td>
</tr>
<tr>
<td>$L_{sr}, L_{rs}$</td>
<td>${1}$</td>
</tr>
<tr>
<td>$L_{rr}, R_{ss}, R_{rr}$</td>
<td>${0}$</td>
</tr>
</tbody>
</table>

Table 4.3: Appropriate approximation for variables and parameters of the synchronous machine model with DPA in the ABC reference frame

Voltage Equations:

\[
\frac{1}{\omega_s} \frac{d\langle \psi_s \rangle_1}{dt} = \langle e_s \rangle_1 + R_{ss} \langle i_s \rangle_1 - j \langle \psi_s \rangle_1 \tag{4.37}
\]

\[
\frac{1}{\omega_s} \frac{d\langle \psi_r \rangle_0}{dt} = \langle e_r \rangle_0 + R_{rr} \langle i_r \rangle_0 \tag{4.38}
\]

\[
\frac{1}{\omega_s} \frac{d\langle \psi_r \rangle_2}{dt} = \langle e_r \rangle_2 + R_{rr} \langle i_r \rangle_2 - j 2 \langle \psi_r \rangle_2 \tag{4.39}
\]

Flux Linkage Equations:

\[
\langle \psi_s \rangle_1 = \langle L_{ss} \rangle_0 \langle i_s \rangle_1 + \langle L_{ss} \rangle_2 \langle i_s \rangle_1^* + \langle L_{sr} \rangle_1 \langle i_r \rangle_0 + \langle L_{sr} \rangle_1^* \langle i_r \rangle_2 \tag{4.40}
\]

\[
\langle \psi_r \rangle_0 = \langle L_{rs} \rangle_1 \langle i_s \rangle_1^* + \langle L_{rs} \rangle_1^* \langle i_s \rangle_1 + \langle L_{rr} \rangle_0 \langle i_r \rangle_0 \tag{4.41}
\]

\[
\langle \psi_r \rangle_2 = \langle L_{rs} \rangle_1 \langle i_s \rangle_1 + \langle L_{rr} \rangle_0 \langle i_r \rangle_2 \tag{4.42}
\]

Mechanical Equations:

\[
T_e = \frac{2\sqrt{3}}{9} \left[ \langle i_a \rangle_1 \left( \langle \psi_c \rangle_1 - \langle \psi_b \rangle_1^* \right) + \langle i_a \rangle_1^* \left( \langle \psi_c \rangle_1 - \langle \psi_b \rangle_1 \right) \right.
\]

\[
+ \langle i_b \rangle_1 \left( \langle \psi_a \rangle_1 - \langle \psi_c \rangle_1^* \right) + \langle i_b \rangle_1^* \left( \langle \psi_a \rangle_1 - \langle \psi_c \rangle_1 \right) \]

\[
+ \langle i_c \rangle_1 \left( \langle \psi_b \rangle_1 - \langle \psi_a \rangle_1^* \right) + \langle i_c \rangle_1^* \left( \langle \psi_b \rangle_1 - \langle \psi_a \rangle_1 \right) \right] \tag{4.43}
\]
The dynamic phasor model described by the equations (4.37-4.47), captures both electromagnetic and electromechanical transients. In transient stability analysis, the electromagnetic network transients and the electromagnetic stator transients are generally disregarded. The equivalent dynamic phasor model for transient stability analysis can be derived by setting the stator flux transients to zero in (4.37) i.e \( \frac{d\psi_s}{dt} = 0 \).

### 4.3.2 Dynamic Phasor Model – DQ0

The model equations of the synchronous machine are usually given in the DQ0 reference frame rotating with the rotor speed \( \omega_r \). One of the main advantages in treating the synchronous machine in the DQ0 reference frame rotating with the rotor speed is that the transformed inductance matrices \( L_{ss}, L_{sr}, L_{rs} \) and \( L_{rr} \) yield constant terms. The same notation and formulae are used as in [33] for the model equations. The overall set of per unit model equations in the DQ0 reference frame are given by

**Voltage Equations:**

\[
\frac{1}{\omega_s} \frac{d\psi_{dq}}{dt} = e_{dq}^r + R_{dq} i_{dq}^r - \omega_r J_2 \psi_{dq} \quad (4.48)
\]

\[
\frac{1}{\omega_s} \frac{d\psi_0}{dt} = e_0^r + R_0 i_0^r \quad (4.49)
\]

\[
\frac{1}{\omega_s} \frac{d\psi_r}{dt} = e_r + R_{rr} i_r \quad (4.50)
\]
 Flux Linkage Equations:

\[
\psi_{dq} = L_{dq} i_{dq}^r + L_{dq,r} i_r \tag{4.51}
\]

\[
\psi_0 = L_0 i_0^r \tag{4.52}
\]

\[
\psi_r = L_{r,dq} i_{dq}^r + L_{rr} i_r \tag{4.53}
\]

Mechanical Equations:

\[
T_e = \psi_d i_q^r - \psi_q i_d^r \tag{4.54}
\]

\[
\frac{d\omega_r}{dt} = \frac{1}{2H} (T_m - T_e - K_D \omega_r) \tag{4.55}
\]

\[
\frac{1}{\omega_s} \frac{d\delta_r}{dt} = \omega_r - 1 \tag{4.56}
\]

with \( J_2 \) given in (4.16) and

\[
\psi = \begin{bmatrix} \psi_d & \psi_q & \psi_0 & \psi_{1d} & \psi_{1q} & \psi_{2q} \end{bmatrix}^T
\]

\[
i = \begin{bmatrix} -i_d^r & -i_q^r & -i_0^r & i_{fd} & i_{1d} & i_{1q} & i_{2q} \end{bmatrix}^T
\]

\[
e = \begin{bmatrix} e_d^r & e_q^r & e_0^r & e_{fd} & 0 & 0 & 0 \end{bmatrix}^T
\]

\[
R = \text{diag} \begin{bmatrix} R_a & R_a & R_a & R_{fd} & R_{1d} & R_{1q} & R_{2q} \end{bmatrix}^T
\]

\[
L_{dq} = \begin{bmatrix} L_d & 0 \\
0 & L_q \end{bmatrix}, L_{dq,r} = \begin{bmatrix} L_{a fd} & L_{a 1d} & 0 & 0 \\
0 & 0 & L_{a 1q} & L_{a 2q} \end{bmatrix}
\]

\[
L_{r,dq} = \begin{bmatrix} L_{a fd} & 0 \\
L_{a 1d} & 0 \\
0 & L_{a 1q} \\
0 & L_{a 2q} \end{bmatrix}, L_{rr} = \begin{bmatrix} L_{f fd} & L_{f 1d} & 0 & 0 \\
L_{f 1d} & L_{11d} & 0 & 0 \\
0 & 0 & L_{11q} & L_{12q} \\
0 & 0 & L_{12q} & L_{22q} \end{bmatrix}
\]

The stator voltages and currents in the DQ0 reference frame rotating with the rotor speed \( \omega_r \) are interfaced with the network equations represented in the DQ0 reference frame rotating with the synchronous speed \( \omega_s \) by the following transformation:

\[
\begin{bmatrix}
e_d^s \\
e_q^s \\
e_0^s
\end{bmatrix} = \begin{bmatrix}
\cos(\delta_r) & -\sin(\delta_r) & 0 \\
\sin(\delta_r) & \cos(\delta_r) & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
e_d^r \\
e_q^r \\
e_0^r
\end{bmatrix} \tag{4.57}
\]
When we derive the dynamic phasor model of the synchronous machine in the DQ0 reference frame, the d- and q-axis coordinates of the stator side electrical quantities are approximated by \( K = \{0, 2\} \) due to the positive \((k = 0)\) and negative \((k = 2)\) sequence components and the 0-axis coordinates by \( K = \{1\} \) due to the zero sequence component. The rotor side electrical quantities are approximated by \( K = \{0, 2\} \) as was the case also in the ABC reference frame. The appropriate set of the approximating Fourier coefficients for the variables and parameters of the dynamic phasor model of the synchronous machine in the DQ0 reference frame are given in Table 4.4. With the approximations in Table 4.4, the dynamic phasor model equations yield

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Approximation K = {...}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi_{dq}, i_{dq}, e_{dq} )</td>
<td>{0, 2}</td>
</tr>
<tr>
<td>( \psi_0, i_0, e_0 )</td>
<td>{1}</td>
</tr>
<tr>
<td>( \psi_r, i_r, e_r )</td>
<td>{0, 2}</td>
</tr>
<tr>
<td>( T_e, \omega_r, \delta_r )</td>
<td>{0, 2}</td>
</tr>
<tr>
<td>( T_m )</td>
<td>{0}</td>
</tr>
</tbody>
</table>

Table 4.4: Set of approximations for the variables and parameters of the synchronous machine model in the DQ0 reference frame

Voltage Equations:

\[
\frac{1}{\omega_s} \frac{d\langle \psi_{dq} \rangle_0}{dt} = \langle e_{dq}^r \rangle_0 + R_a \langle i_{dq}^r \rangle_0 \]
\[
+ J_2 \left( \langle \omega_r \rangle_0 \langle \psi_{dq} \rangle_0 + \langle \omega_r \rangle_2 \langle \psi_{dq} \rangle_2^* + \langle \omega_r \rangle_2^* \langle \psi_{dq} \rangle_2 \right) \quad (4.58)
\]

\[
\frac{1}{\omega_s} \frac{d\langle \psi_{dq} \rangle_2}{dt} = \langle e_{dq}^r \rangle_2 + R_a \langle i_{dq}^r \rangle_2 \]
\[
+ J_2 \left( \langle \omega_r \rangle_0 \langle \psi_{dq} \rangle_2 + \langle \omega_r \rangle_2 \langle \psi_{dq} \rangle_2^* \right) - j 2 \langle \psi_{dq} \rangle_2 \quad (4.59)
\]

\[
\frac{1}{\omega_s} \frac{d\langle \psi_0 \rangle_1}{dt} = \langle e_0^r \rangle_1 + R_a \langle i_0^r \rangle_1 - j \langle \psi_0 \rangle_1 \quad (4.60)
\]

\[
\frac{1}{\omega_s} \frac{d\langle \psi_r \rangle_0}{dt} = \langle e_r \rangle_0 + R_{rr} \langle i_r \rangle_0 \quad (4.61)
\]

\[
\frac{1}{\omega_s} \frac{d\langle \psi_r \rangle_2}{dt} = \langle e_r \rangle_2 + R_{rr} \langle i_r \rangle_2 - j 2 \langle \psi_r \rangle_2 \quad (4.62)
\]
4.3. Synchronous Machine

Flux Linkage Equations:

\[
\langle \psi_{dq} \rangle_0 = L_{dq} \langle i_{dq}^r \rangle_0 + L_{dq,r} \langle i_r \rangle_0 
\]

\[
\langle \psi_{dq} \rangle_2 = L_{dq} \langle i_{dq}^r \rangle_2 + L_{dq,r} \langle i_r \rangle_2 
\]

\[
\langle \psi_0 \rangle_1 = L_0 \langle i_0 \rangle_1 
\]

\[
\langle \psi_r \rangle_0 = L_{dq,r} \langle i_{dq}^r \rangle_0 + L_{rr} \langle i_r \rangle_0 
\]

\[
\langle \psi_r \rangle_2 = L_{dq,r} \langle i_{dq}^r \rangle_2 + L_{rr} \langle i_r \rangle_2 
\]

Mechanical Equations:

\[
\langle T_e \rangle_0 = \langle \psi_d \rangle_0 \langle i_d^r \rangle_0 + \langle \psi_d \rangle_2 \langle i_q^r \rangle_2 + \langle \psi_d \rangle_2^* \langle i_d^r \rangle_2 
- \langle \psi_q \rangle_0 \langle i_q^r \rangle_0 - \langle \psi_q \rangle_2 \langle i_d^r \rangle_2 - \langle \psi_q \rangle_2^* \langle i_q^r \rangle_2 
\]

\[
\langle T_e \rangle_2 = \langle \psi_d \rangle_0 \langle i_d^r \rangle_2 + \langle \psi_d \rangle_2 \langle i_q^r \rangle_0 
- \langle \psi_q \rangle_0 \langle i_q^r \rangle_2 - \langle \psi_q \rangle_2 \langle i_d^r \rangle_0 
\]

\[
\frac{d\langle \omega_r \rangle_0}{dt} = \frac{1}{2H} (T_m - \langle T_e \rangle_0 - K_D \langle \omega_r \rangle_0) 
\]

\[
\frac{d\langle \omega_r \rangle_2}{dt} = \frac{1}{2H} (-\langle T_e \rangle_2 - K_D \langle \omega_r \rangle_2) - j2\omega_s \langle \omega_r \rangle_2 
\]

\[
\frac{1}{\omega_s} \frac{d\langle \delta_r \rangle_0}{dt} = \langle \omega_r \rangle_0 - 1 
\]

\[
\frac{1}{\omega_s} \frac{d\langle \delta_r \rangle_2}{dt} = \langle \omega_r \rangle_2 - j2\langle \delta_r \rangle_2 
\]

The equivalent dynamic phasor model for transient stability analysis can be derived by setting the stator flux transients in the DQ0 reference frame to zero in (4.58-4.60) i.e \( \frac{d\langle \psi_{dq} \rangle_0}{dt} = \frac{d\langle \psi_{dq} \rangle_2}{dt} = \frac{d\langle \psi_0 \rangle_1}{dt} = 0 \).

In Section 2.4, the Automatic Code Generator tool was implemented to facilitate the creation of models for the user. This tool has been extended to facilitate also the creation of the dynamic phasor models. The so called Dynamic Phasor Model Creator takes the model definition file of the detailed time domain model in the ABC or DQ0 reference frame together with the appropriate set of Fourier coefficients for each state of the model (e.g. Table 4.4) and creates the model definition file of the corresponding dynamic phasor model. This procedure is illustrated in Figure 4.2. All dynamic phasor models of the components discussed in this Section have been created using the described tool.
4.4 Test Cases

In this Section, the aim is to compare the accuracy and computational efficiency of the different modelling techniques in a systematic way. For this purpose, the previously described models i.e.

- detailed time domain models in the ABC reference frame (EMT-ABC)
- detailed time domain models in the DQ0 reference frame (EMT-DQ0)
- dynamic phasor models in the ABC reference frame (DYNPH-EMT-ABC)
- dynamic phasor models in the DQ0 reference frame (DYNPH-EMT-ABC)

of major power system components have been implemented in the simulation framework described in Chapter 2. The labels in the parenthesis in the above list will be used as reference to the used modelling technique in the following figures containing the simulation results.

As test case, we have used the Single Machine Infinite Bus (SMIB) system described in [33]. Figure 4.3 shows the single line diagram of the test system. Simulations have been performed with the detailed models for simulating the electromagnetic transients and with the reduced order
models for transient stability analysis. Comparisons have been done regarding the accuracy and computational efficiency of these models. As a quantitative evaluation of the computational efficiency of these models, the overall CPU simulation times have been compared. The evaluation of the accuracy has been done in a more qualitative way by comparing the plots of the system quantities in the figures, which seemed to be enough for our purpose.

4.4.1 Unbalanced Faults

In this test case, a single-phase to ground fault occurs at the BUS2 end of the LINE3 at 0.011 seconds and is removed after 0.20 seconds by disconnecting the line LINE3. Simulations were obtained using Matlab 7.1 running on a Intel Pentium IV CPU with 3.80 GHz and 2 GB of RAM. The trapezoidal method with an absolute tolerance \( atol = 10^{-3} \) and relative tolerance \( rtol = 10^{-4} \) has been used for the numerical integration of the four test cases.

Detailed Models

Figure 4.4 shows the evolution of the dynamic phasor components \( \langle T_e \rangle_0 \) and \( \langle T_e \rangle_2 \) of the electrical torque. We observe that the per unit dynamic phasor component with \( k = 0 \), i.e. \( \langle T_e \rangle_0 \), captures the positive sequence electrical torque of \( T_e \). The dynamic phasor component with \( k = 2 \), i.e. \( \langle T_e \rangle_2 \), captures the negative sequence electrical torque of \( T_e \). This is only present during unbalanced conditions which is shown in the zoomed section of the Figure 4.4.

![Figure 4.3: Single line diagram of the SMIB system](image-url)
Figures 4.5-4.7 show the evolution of the per unit fault phase current $i_a$, per unit generator phase-A terminal current $i_a$ and per unit generator electrical torque $T_e$, after the unbalanced fault. In these Figures the corresponding instantaneous values of the quantities in the simulations with the dynamic phasor models are used. They can be computed by the following approximation:

$$x(t) \approx \Re \left\{ \sum_{k \in K} X_k(t) e^{j k \omega_s t} \right\}$$

where $K$ represents the set of the approximating Fourier coefficients used for the particular state or variable. For example the instantaneous value of the electrical torque $T_e$ simulated with the dynamic phasor models can be approximated by

$$T_e(t) \approx \langle T_e \rangle_0 + \Re \{ \langle T_e \rangle_2 \} \cos (2 \omega_s t) - \Im \{ \langle T_e \rangle_2 \} \sin (2 \omega_s t)$$

Table 4.5 shows the required CPU simulation times for the overall simulation with four different modelling techniques. Table 4.5 and Figures (4.5-4.7) allow a fair comparison of the four different modelling techniques, since simulations have been done with the same simulation framework using the same numerical integration technique with the same degree of numerical accuracy i.e. $atol = 10^{-3}$ and $rtol = 10^{-4}$.

<table>
<thead>
<tr>
<th>Modelling Technique</th>
<th>CPU Simulation Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMT-ABC</td>
<td>469.359</td>
</tr>
<tr>
<td>EMT-DQ0</td>
<td>70.468</td>
</tr>
<tr>
<td>DYNPH-EMT-ABC</td>
<td>11.688</td>
</tr>
<tr>
<td>DYNPH-EMT-DQ0</td>
<td>15.487</td>
</tr>
</tbody>
</table>

Table 4.5: CPU simulation times of the overall simulation with four different modelling techniques

If the four modelling techniques are compared regarding accuracy, all results in Figures (4.5-4.7) are in overall agreement during the whole simulation interval and also during the interval with the unbalanced single phase to ground fault, which is depicted in the zoomed section of the plots.

Figure 4.7 depicts the phase-A fault current $i_{a \text{fault}}$ during the fault simulated with four different modelling techniques. Generally the fault current has two distinct components [33], a fundamental frequency component with a subtransient and a transient period and an unidirectional
4.4. Test Cases

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.4}
\caption{Overall evolution of the dynamic phasors $K = \{0, 2\}$ of the per unit electrical torque $T_e$ of GEN1 after a single line to ground fault at BUS2 simulated with the detailed dynamic phasor models in the ABC reference frame. The zoomed section shows the evolution during the unbalanced fault.}
\end{figure}

component which decays exponentially in several cycles. The unidirectional component occurs due to fact that stator fluxes cannot change instantaneously, so that this produces the dc offset in the phase currents. This dc-component and the fundamental frequency component of the fault current is also evident in the simulations with the dynamic phasor models.

We can conclude that the accuracy of the modelling techniques are nearly the same in all four cases. But if the comparison is made regarding the computational efficiency as shown in Table 4.5, the dynamic phasor models show a better performance than their counter parts with
Figure 4.5: Overall evolution of the per unit field current $T_e$ after a single line to ground fault at BUS2 simulated with the four different detailed models. The zoomed section shows the evolution during the unbalanced fault.

the instantaneous values.

The instantaneous three-phase ABC models are computationally inefficient due to the permanent presence of the AC quantities even at steady state, so that the highest frequency is always around the system frequency $\omega_s$, which affects the admissible maximum integration step size.

The instantaneous three-phase DQ0 models are computationally more efficient than the instantaneous three-phase ABC models due to the dominating balanced operating conditions except during the unbalanced fault. After fast transients have decayed, the frequency content of the DQ0 transformed quantities will be located around DC during the bal-
4.4. Test Cases

Figure 4.6: Overall evolution of the per unit generator terminal current \(i_a\) after a single line to ground fault at BUS2 simulated with the four different detailed models. The zoomed section shows the evolution during the unbalanced fault.

anced conditions so that they can be integrated with larger step sizes increasing the simulation performance significantly. Due to the increased computational efficiency during balanced conditions, the instantaneous models in the DQ0 reference frame are 6 times faster than those in the ABC reference frame.

Both dynamic phasor models in the ABC and DQ0 reference frame are faster compared with the models with the instantaneous values. The variations in the dynamic phasors of the system quantities are much slower than in the instantaneous values of the quantities and they become constant at steady state. This fact does not depend on the selected reference frame. The selected reference has only an impact
on the selection of the approximating Fourier coefficients. For example in the DQ0 reference frame, the negative sequence quantities will be present with double system frequency in the d- and q-axis, which implies the selection of \( k = 2 \) for the approximation of the negative sequence quantities. In the ABC reference frame this is achieved with \( k = 1 \) for all positive, negative and zero sequence components. The decreased variations in the dynamic phasors allow the use of larger step sizes during numerical integration.

The minor performance difference between the dynamic phasor models in the ABC and DQ0 reference frame is ascribed to the increased number of variables in the DQ0 reference frame due to the approximating set of Fourier coefficients \( K = \{0, 1, 2\} \) in the network equations to account for unbalanced conditions.
Overall we can say that the simulations with the dynamic phasor models are much faster than with the detailed time domain models while maintaining nearly the same degree of accuracy.

**Reduced Order Models**

In this part, we will use

- reduced order dynamic phasor models in the ABC reference frame (DYNPH-RMS-ABC)
- reduced order dynamic phasor models in the DQ0 reference frame (DYNPH-RMS-DQ0)

for the simulation of the same test case. The labels in the parenthesis in the above list will be used as reference to the used modelling technique in the following figures containing the simulation results. Such reduced order models are generally used for transient stability studies where the electromagnetic transients in the electrical network are neglected as the focus is more on the slower electromechanical oscillations.

Figure 4.8 shows the evolution of dynamic phasors $\langle T_e \rangle_0$ and $\langle T_e \rangle_2$ simulated with the reduced order models in the ABC reference frame compared with those simulated with the detailed dynamic phasors models. As fast network electromagnetic transients are neglected, the reduced order dynamic phasor models only capture the slower electromechanical oscillations (here around 1 Hz), which are also observable in the evolution of the $\langle T_e \rangle_0$. The omission of the fast transients appears also in the overall simulation times which is given in Table 4.6.

Figure 4.9 depicts the phase-A fault current $i_{a fault}^{\text{fault}}$ during the fault simulated with detailed and reduced order models. Due to the omission of the stator flux changes in the reduced order dynamic phasor models, the dc-component component of the fault current is not captured in the simulations with the reduced order models, where the fundamental frequency is still evident.
Figure 4.8: Overall evolution of the dynamic phasors $K = \{0, 2\}$ of the per unit electrical torque $T_e$ of GEN1 after a single line to ground fault at BUS2 simulated with the detailed dynamic phasor models and the reduced order models in the ABC reference frame.

Table 4.6: CPU simulation times of the overall simulation with dynamic phasor models and reduced order dynamic phasor models
Figure 4.9: The evolution of the per unit phase-A fault current $i_a^{\text{fault}}$ during the unbalanced fault simulated with detailed dynamic phasor models and reduced order dynamic phasor models.
4.4.2 Asymmetrical Components

The aim is now to investigate the simulation performance of the four different modelling techniques under permanent unbalanced conditions e.g. due to the presence of asymmetrical components in the system. In this test case such an asymmetry in the components is achieved by changing the phase-A inductance of LINE1 from 0.15 pu to 0.75 pu right at the beginning of the simulation \((t = 0.0001\,\text{sec})\). Thus, the system is under unbalanced conditions during the whole simulation.

**Detailed Models**

Figure 4.10 shows the evolution of the per unit generator electrical torque \(T_e\) for the described scenario simulated with four different modelling techniques. We see again an overall agreement in all four results even in the zoomed section. If the CPU simulation times are compared, the detailed dynamic phasor models are significantly faster than the detailed models with the instantaneous values as given in Table 4.7. Under unbalanced conditions and even at steady state the instantaneous electrical torque \(T_e\) will not have only DC component but also a component which oscillates with \(2\omega_s\). The amplitude of the \(2\omega_s\) oscillation depends on the degree of the asymmetry. The detailed models with instantaneous values become computationally cumbersome due to these permanent oscillations. The performance loss of the DQ0 models with instantaneous values compared to the previous test case with an unbalanced fault is ascribed to the permanent existence of unbalanced conditions. In the previous test case, the unbalanced conditions took only 0.2 seconds and then the system was back again under balanced conditions for the rest of the simulation, whereas the DQ0 models with instantaneous values were computationally more efficient.

This drawback disappears if detailed dynamic phasor models are used instead. In simulations with the dynamic phasor models, only the envelope of these oscillations are simulated, which has much slower variations

<table>
<thead>
<tr>
<th>EMT-ABC</th>
<th>EMT-DQ0</th>
<th>DYNPH-EMT-ABC</th>
<th>DYNPH-EMT-DQ0</th>
</tr>
</thead>
<tbody>
<tr>
<td>416.359 [s]</td>
<td>187.25 [s]</td>
<td>3.859 [s]</td>
<td>4.75 [s]</td>
</tr>
</tbody>
</table>

Table 4.7: CPU simulation times of the overall simulation with four different modelling techniques
4.4. Test Cases

Figure 4.10: Overall evolution of the per unit field current $T_e$ after a single line to ground fault at BUS2 simulated with the four different detailed models. The zoomed section shows the evolution during the unbalanced fault.

Reduced Order Models

Figure 4.12 shows the evolution of dynamic phasors $\langle T_e \rangle_0$ and $\langle T_e \rangle_2$ simulated with the reduced order models in the ABC reference frame compared with those simulated with the detailed dynamic phasors models. The corresponding CPU simulation times are given in Table 4.8.

Both the detailed and reduced dynamic phasor models are able to simulate asymmetrical conditions in an efficient way. The degree of accuracy
Figure 4.11: Overall evolution of the dynamic phasors $K = \{0, 2\}$ of the per unit electrical torque $T_e$ of GEN1 after changing the phase-A inductance from 0.15 [pu] to 0.75 [pu] simulated with the detailed dynamic phasor models in the ABC reference frame.

Table 4.8: CPU simulation times of the overall simulation with dynamic phasor models and reduced order dynamic phasor models

<table>
<thead>
<tr>
<th>Method</th>
<th>RMS-ABC</th>
<th>RMS-DQ0</th>
<th>EMT-ABC</th>
<th>EMT-DQ0</th>
</tr>
</thead>
<tbody>
<tr>
<td>DYNPH</td>
<td>0.95 [s]</td>
<td>0.97 [s]</td>
<td>3.859 [s]</td>
<td>4.75 [s]</td>
</tr>
</tbody>
</table>

of the detailed dynamic phasor models is expectedly higher than the reduced dynamic phasor models.
Figure 4.12: The dynamic phasors $K = \{0, 2\}$ of the per unit electrical torque $T_e$ of GEN1 after changing the phase-A inductance from 0.15 [pu] to 0.75 [pu] simulated with the detailed dynamic phasor models and the reduced order models in the ABC reference frame.
4.5 Summary

The aim of this Chapter was to make a systematic and transparent comparison of the commonly used modelling techniques in the ABC and DQ0 reference frame with dynamic phasor models regarding their accuracy and efficiency. The consistency of this systematic comparison was warranted by implementing the differently represented models of major power system components and simulating them under different operating conditions using the same simulation framework.

For this purpose the dynamic phasor models of the synchronous machine and the transmission line has been derived based on their model equations in the ABC and DQ0 reference frame. The key point in the derivation of the dynamic phasor models was the appropriate selection of the Fourier coefficients set $K$ to approximate the model behavior. Depending on the used reference frame, we have used different sets of coefficients for the approximation of the system variables.

The focus was more on unbalanced operating conditions. We have first simulated a system with symmetrical components with an unbalanced fault. Later, we considered the same system with asymmetrical components. The two test cases have been simulated and the accuracy and efficiency of the different modelling techniques have been compared with each other. Simulation results have shown that the detailed dynamic phasor models are significantly more efficient than the detailed models with the instantaneous values, whilst keeping the same degree of accuracy under balanced and unbalanced conditions. Thereby the selected reference frame for the dynamic phasor models did not play a significant role in the simulation accuracy and efficiency. The key point for deriving dynamic phasor models in different reference frames was the appropriate selection of the Fourier coefficients set $K$.

Besides detailed electromagnetic simulations, we have also made simulations with the so called reduced order models for transient stability studies. Such models have been derived systematically from the detailed dynamic phasor models by simply neglecting the fast network transients. Also these reduced order models were able to simulate the unbalanced conditions in an efficient way.
Chapter 5

Dynamic Phasor Model of the DFIG

In this Chapter, the dynamic phasor model of the doubly-fed induction generator (DFIG) will be derived. The derived model will be then used to study the response of the DFIG to balanced and unbalanced voltage sags. The simulation performance of the derived model dynamic phasor model will be compared with the detailed model in terms of accuracy and computational efficiency.

5.1 Introduction

In recent years, wind power generation has become very significant in different countries around the world [35]. Due to the increasing wind power penetration, wind farms, like conventional power plants, should in the future be considered in the dynamic stability assessment of power systems.

Special rules and requirements have recently been set for wind power production, regarding for example the behavior of wind generators during unbalanced conditions [36]. In this respect, a major concern is the response to voltage sags that occur at the bus where the wind farm is connected to the network. So far, wind farms were most often disconnected if a voltage sag of a significant magnitude occurred. However,
new grid regulations impose that wind farms must contribute actively to grid stability.

To address the wind farm adequacy with respect to contingencies, the first step is to set up an accurate model of the wind turbines. In this Chapter, only the variable speed wind turbine with doubly-fed induction generator (DFIG) is considered, as it is expected to be the most common device used in future wind farms [37]. The DFIGs are more robust. With DFIGs, speed variation of 30% around synchronous speed can be obtained and also more energy captured from the wind.

Some commercial software packages are able to simulate the electromagnetic model of wind turbines (e.g. PSCAD, PLECS, Simpow, DigSilent, PSS/E, SimPowerSyms). A comparison of these models can be found in [38]. The detailed electromagnetic model is adequate for studying the behavior of a single machine, but the computational burden can become cumbersome for studying the behavior of a wind farm that is typically composed of tens of wind turbines.

On the other hand, electromechanical models are adequate to study the collective behavior of wind farms. A variety of simplified models for transient and voltage stability studies have been proposed in the literature [39, 40, 41]. However, electromechanical fundamental frequency models are not particularly suitable for studying unbalanced conditions.

Our aim in this Chapter is the use of the dynamic phasor approach for modeling and studying the behavior of DFIGs. As discussed in previous chapters, the dynamic phasors provide a suitable framework and are able to model accurately unbalanced conditions and, at the same time, are computationally more efficient than EMTP simulations.

Two studies will be carried out to investigate the wind turbine behavior during balanced and unbalanced conditions. These cases are briefly described below:

1. Balanced Voltage sag near to the DFIG bus.

2. Unbalanced Voltage sag near to the DFIG bus.

The responses of the electromagnetic and dynamic phasor models are compared and discussed for the two cases. The electromechanical fundamental frequency model behavior is also compared for the balanced voltage sag case.
The Chapter is organized as follows. In Section 5.2, starting from the detailed model equations for the induction machine and converter, the proposed DFIG model is derived using the dynamic phasors approach. Section 5.3 gives a comparative assessment of the derived models. Section 5.4 comprises simulations with the proposed dynamic phasor and detailed models following balanced and unbalanced voltage sags. Furthermore, the accuracy and efficiency of the different models are compared with each other.

5.2 DFIG Model

Wind turbines using a DFIG are composed of the following components:

- Wind turbine
- Induction machine
- AC/DC/AC (back-to-back) IGBT-based PWM converter
- Transformer

As shown in Figure 5.1, the stator of the induction machine is connected directly to the grid. The rotor of the induction machine is fed at variable frequency through the AC/DC/AC (back-to-back) converter. The grid-side of the converter is connected to the grid through a transformer. We start with a detailed description of the mathematical model of the doubly-fed induction generator. The detailed models of the DFIG components will be treated in the DQ0 reference frame. However, the zero sequence quantities are omitted as no neutral currents are possible due to the winding connections.

5.2.1 Drive Train Model

As our focus in this Chapter is more on the behavior of the DFIG in relation to voltage sags, all rotating masses, such as turbine, gearbox and shafts, are lumped together into a single equivalent mass. Elastic shafts and resulting torques are neglected. Thus, the model behavior of the drive train is given by the simple swing equation as

$$\frac{d\omega_r}{dt} = \frac{T_m - T_e}{2H} \quad (5.1)$$
Chapter 5. Dynamic Phasor Model of the DFIG

where $T_m$ is the mechanical torque extracted from the wind, $T_e$ is the electrical torque produced by the induction machine and $\omega_r$ is the rotor speed (frequency). $T_m$ is assumed to be constant in our case studies.

The dynamic phasor model equations of the drive train will be derived in the following subsection together with the induction machine equations.

5.2.2 Induction Machine

Detailed Time Domain Model

The starting point for the derivation of the dynamic phasor model of the wound rotor induction machine are the well-known 5th order model equations in the DQ-reference frame rotating at synchronous speed $\omega_s$ [33]. With the following definitions:

$$
\psi_{dq,s} = \begin{bmatrix} \psi_{ds} & \psi_{qs} \end{bmatrix}^T, \psi_{dq,r} = \begin{bmatrix} \psi_{dr} & \psi_{qr} \end{bmatrix}^T
$$

$$
i_{dq,s} = \begin{bmatrix} i_{ds} & i_{qs} \end{bmatrix}^T, i_{dq,r} = \begin{bmatrix} i_{dr} & i_{qr} \end{bmatrix}^T
$$

$$
v_{dq,s} = \begin{bmatrix} v_{ds} & v_{qs} \end{bmatrix}^T, v_{dq,r} = \begin{bmatrix} v_{dr} & v_{qr} \end{bmatrix}^T
$$

$$
R_{ss} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix}, \quad R_{rr} = \begin{bmatrix} R_r & 0 \\ 0 & R_r \end{bmatrix}
$$
The per unit model equations of the induction machine in the DQ-reference frame are given by Stator Voltage Equations:

\[ \frac{1}{\omega_s} \frac{d\psi_{dq,s}}{dt} = v_{dq,s} + R_{ss} i_{dq,s} + J_2 \psi_{dq,s} \] (5.2)

Rotor Voltage Equations:

\[ \frac{1}{\omega_s} \frac{d\psi_{dq,r}}{dt} = v_{dq,r} + R_{rr} i_{dr} + \sigma J_2 \psi_{dq,r} \] (5.3)

Stator and rotor flux linkage equations:

\[ \psi_{dq,s} = L_{ss} i_{dq,s} + L_{sr} i_{dq,r} \] (5.4)

\[ \psi_{dq,r} = L_{rs} i_{dq,s} + L_{rr} i_{dq,r} \] (5.5)

Torque equation:

\[ T_e = \psi_{qr} i_{dr} - \psi_{dr} i_{qr} \] (5.6)

Drive train equation:

\[ \frac{d\omega_r}{dt} = \frac{T_m - T_e}{2H} \] (5.7)

with \( J_2 \) given in (4.16). In the rotor voltage equations \( \sigma \) is referred to as the slip between the synchronous speed \( \omega_s \) and the rotor frequency \( \omega_r \) and is defined as

\[ \sigma = \frac{\omega_s - \omega_r}{\omega_s} \] (5.8)

The 5th order induction machine model defined by the equations (5.2-5.8) is commonly used in electromagnetic transients studies.

**Dynamic Phasor Model**

The key point in the derivation of the dynamic phasor models is the appropriate selection of a set of Fourier coefficients \( X_k \) in (3.16) for
an adequate approximation of the model behavior. Since unbalanced conditions are of concern, the system will contain both positive and negative sequence quantities which are respectively mapped as dc and second harmonic values in DQ-reference frame. Due to this fact, an appropriate selection for $K$ in (3.16) would be $K = \{0, 2\}$ for the model derivation, where dynamic phasors with $k = 0$ capture positive sequence quantities and the ones with $k = 2$ negative sequence quantities. Table 5.1 shows the set of approximating Fourier coefficients for the quantities of the induction machine. Using the properties for the derivative (3.19) and product (3.20) of dynamic phasors, and setting the fundamental frequency for the Fourier approximation $\omega = \omega_s$ in (3.19), the dynamic phasor model of the induction machine becomes:

**Stator voltage equations:**

\[
\frac{1}{\omega_s} \frac{d}{dt} \langle \psi_{dq,s} \rangle_0 = \langle v_{dq,s} \rangle_0 + R_{ss} \langle i_{dq,s} \rangle_0 + J_2 \langle \psi_{dq,s} \rangle_0 \quad (5.9)
\]

\[
\frac{1}{\omega_s} \frac{d}{dt} \langle \psi_{dq,s} \rangle_2 = \langle v_{dq,s} \rangle_2 + R_{ss} \langle i_{dq,s} \rangle_2 + J_2 \langle \psi_{dq,s} \rangle_2 - j 2 \langle \psi_{dq,s} \rangle_2 \quad (5.10)
\]

**Rotor Voltage Equations:**

\[
\frac{1}{\omega_s} \frac{d}{dt} \langle \psi_{dq,r} \rangle_0 = \langle v_{dq,r} \rangle_0 + R_{rr} \langle i_{dq,r} \rangle_0 + \sigma J_2 \langle \psi_{dq,r} \rangle_0 \quad (5.11)
\]

\[
\frac{1}{\omega_s} \frac{d}{dt} \langle \psi_{dq,r} \rangle_2 = \langle v_{dq,r} \rangle_2 + R_{rr} \langle i_{dq,r} \rangle_2 + \sigma J_2 \langle \psi_{dq,r} \rangle_2 - j 2 \langle \psi_{dq,r} \rangle_2 \quad (5.12)
\]
5.2. DFIG Model

Stator and rotor flux linkage equations:

\[
\langle \psi_{dq,s} \rangle_0 = L_{ss} \langle i_{dq,s} \rangle_0 + L_{sr} \langle i_{dq,r} \rangle_0 
\]

(5.13)

\[
\langle \psi_{dq,r} \rangle_0 = L_{rs} \langle i_{dq,s} \rangle_0 + L_{rr} \langle i_{dq,r} \rangle_0
\]

(5.14)

\[
\langle \psi_{dq,s} \rangle_2 = L_{ss} \langle i_{dq,s} \rangle_2 + L_{sr} \langle i_{dq,r} \rangle_2
\]

(5.15)

\[
\langle \psi_{dq,r} \rangle_2 = L_{rs} \langle i_{dq,s} \rangle_2 + L_{rr} \langle i_{dq,r} \rangle_2
\]

(5.16)

Torque and slip equation:

\[
\langle T_e \rangle_0 = \langle \psi_{qr} \rangle_0 \langle i_{dr} \rangle_0 + \langle \psi_{qr} \rangle_2 \langle i_{dr} \rangle_2 + \langle \psi_{qr} \rangle_2^* \langle i_{dr} \rangle_2
\]

(5.17)

\[
\langle T_e \rangle_2 = \langle \psi_{qr} \rangle_2 \langle i_{dr} \rangle_2 + \langle \psi_{qr} \rangle_0 \langle i_{dr} \rangle_2
\]

(5.18)

\[
\langle \sigma \rangle_0 = \frac{\omega_s - \langle \omega_r \rangle_0}{\omega_s}
\]

(5.19)

\[
\langle \sigma \rangle_2 = \frac{\omega_s - \langle \omega_r \rangle_2}{\omega_s}
\]

(5.20)

Drive Train equations:

\[
\frac{d\langle \omega_r \rangle_0}{dt} = \frac{\langle T_m \rangle_0 - \langle T_e \rangle_0}{2H}
\]

(5.21)

\[
\frac{d\langle \omega_r \rangle_2}{dt} = \frac{-\langle T_e \rangle_2}{2H} - j2\omega_s\langle \omega_r \rangle_2
\]

(5.22)

In symmetrical systems under balanced condition, the negative sequence quantities with \( k = 2 \) will disappear and simplify the model equations.

5.2.3 Converter Model

The converter model depicted in Figure 5.1 consists of two PWM modulated three-legged IGBT-based Voltage Source Converters (VSC) connected through a dc link. The converter model with IGBTs includes detailed representation of the power electronic IGBT converters which commonly use a high switching frequency e.g 1620 Hz. An acceptable accuracy using the detailed converter model can only be achieved by keeping the integration step size small enough. Hence such detailed models are well suited for observing harmonics and control system dynamic performance over relatively short periods of times (typically hundreds of milliseconds). Details of the IGBT-based converters are given in [42].
Average Model

The back-to-back voltage source converter (VSC) model used here is based on an average model of the VSC. This model omits the high frequent switching dynamics of the IGBTs, but preserves the dynamics resulting from the control system of the converter and the DC-link dynamics. This assumption is acceptable, since the PWM modulation frequency is much higher than the system frequency, so that the switching dynamics can be neglected. With these assumptions, the average VSC model is given by

\[
\begin{align*}
    v_{d(1,2)} &= m_{d(1,2)} v_{dc} \\
    v_{q(1,2)} &= m_{q(1,2)} v_{dc}
\end{align*}
\]  

(5.23)  

(5.24)

where \( m_d \) and \( m_q \) denote the modulation indices of the PWM and are the control variables of the converter. Furthermore, assuming that we have a lossless converter and a lossless dc-link, the converter dynamics can be described by the following equation:

\[
\frac{1}{\omega_s} \frac{dv_{dc}}{dt} = \frac{1}{C} i_{dc} = \frac{1}{C} \left( \frac{P_1 - P_2}{v_{dc}} \right) \\
= \frac{1}{C} (m_{d1} i_{d1} + m_{q1} i_{1q} - m_{d2} i_{d2} - m_{q2} i_{q2})
\]  

(5.25)

If the dc-link capacitor dynamics are neglected in (5.25), then the equation reflects the active power balance between rotor and grid side. A similar average model has been used for the modelling of the unified power flow controller (UPFC) in [15].

The rotor-side and grid-side controllers of the converter are standard PI controllers. The rotor-side controller \((m_{d1}, m_{q1})\) regulates the total active \((P_g)\) and reactive power \((Q_g)\) generated by the DFIG by controlling the rotor voltages \(v_{d,r} = m_{d1} v_{dc}\) and \(v_{q,r} = m_{q1} v_{dc}\) of the induction machine. The total active \((P_g)\) and reactive power \((Q_g)\) generated by the DFIG is illustrated in Figure 5.1. The rotor-side controller equations are given as:

\[
\frac{dx_{d1}}{dt} = K_{I,P_g} (P_{g,ref} - P_g) \\
    m_{d1} = x_{d1} + K_{P_g,v_{dc}} (P_{g,ref} - P_g)
\]  

(5.26)  

(5.27)

\[
\frac{dx_{q1}}{dt} = K_{I,Q_g} (Q_{g,ref} - Q_g) \\
    m_{q1} = x_{q1} + K_{P,Q_g} (Q_{g,ref} - Q_g)
\]  

(5.28)  

(5.29)
5.2. **DFIG Model**

with $K_I$ and $K_P$ being the corresponding integral and proportional gains of the PI-controllers.

The grid-side controller $(m_{d2}, m_{q2})$ regulates the dc-side voltage $v_{dc}$ and the $q$-component of the grid-side current $i_{q,g}$ by controlling the grid side voltages $v_{d,g} = m_{d,2}v_{dc}$ and $v_{q,g} = m_{q,2}v_{dc}$ of the converter. The grid-side controller equations are given as:

\[
\begin{align*}
\frac{dx_{d2}}{dt} &= K_{I,v_{dc}} (V_{dc,ref} - v_{dc}) \quad (5.30) \\
m_{d2} &= x_{d2} + K_{P,v_{dc}} (V_{dc,ref} - v_{dc}) \quad (5.31) \\
\frac{dx_{q2}}{dt} &= K_{I,i_{q,g}} (I_{Q,ref} - i_{q,g}) \quad (5.32) \\
m_{q2} &= x_{q2} + K_{P,i_{q,g}} (I_{Q,ref} - i_{q,g}) \quad (5.33)
\end{align*}
\]

With the lossless back to back voltage source converter (5.25), the rotor-side controller (5.26-5.29) and the grid-side controller (5.30-5.33) equations, the average model of the converter fully defined.

**Dynamic Phasor Model**

To derive the dynamic phasor models of the converter, we again have to specify the set of approximating Fourier coefficients for the converter model. Under unbalanced conditions on the ac-side the electrical power does not only have a dc-part but also a component varying with double system frequency due to the negative sequence quantities. Due to the power balance, these oscillations with double system frequency are also transferred to the dc-side. Thus, an appropriate selection for $K$ in (3.16) is $K = \{0, 2\}$, where $k = 0$ is for dc-quantities and $k = 2$ for the second harmonic due to the unbalanced conditions on the ac-side.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Approximation $K = {\ldots}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{dc}, i_{d1}, i_{q1}, i_{d2}, i_{q,g}$</td>
<td>${0, 2}$</td>
</tr>
<tr>
<td>$m_{d1}, m_{q1}, x_{d1}, x_{q1}, P_g, Q_g$</td>
<td>${0, 2}$</td>
</tr>
<tr>
<td>$m_{d2}, m_{q2}, x_{d2}, x_{q2}$</td>
<td>${0, 2}$</td>
</tr>
</tbody>
</table>

Table 5.2: Appropriate approximation for the quantities of the converter model including the controllers with DPA
With Table 5.2, the dynamic phasor model of the back-to-back voltage source converter is given as:

\[
\frac{C}{\omega_s} \frac{d\langle v_{dc}\rangle_0}{dt} = \langle m_{d1}\rangle_0 \langle i_{d1}\rangle_0 + \langle m_{d1}\rangle_2 \langle i_{d1}\rangle_2 + \langle m_{d1}\rangle_2^* \langle i_{d1}\rangle_2 + \\
\langle m_{q1}\rangle_0 \langle i_{q1}\rangle_0 + \langle m_{q1}\rangle_2 \langle i_{q1}\rangle_2 + \langle m_{q1}\rangle_2^* \langle i_{q1}\rangle_2 - \\
\langle m_{d2}\rangle_0 \langle i_{d2}\rangle_0 - \langle m_{d2}\rangle_2 \langle i_{d2}\rangle_2 - \langle m_{d2}\rangle_2^* \langle i_{d2}\rangle_2 - \\
\langle m_{q2}\rangle_0 \langle i_{q2}\rangle_0 - \langle m_{q2}\rangle_2 \langle i_{q2}\rangle_2 - \langle m_{q2}\rangle_2^* \langle i_{q2}\rangle_2 \tag{5.34}
\]

\[
\frac{C}{\omega_s} \frac{d\langle v_{dc}\rangle_2}{dt} = \langle m_{d1}\rangle_2 \langle i_{d1}\rangle_0 + \langle m_{d1}\rangle_0 \langle i_{d1}\rangle_2 + \\
\langle m_{q1}\rangle_2 \langle i_{q1}\rangle_0 + \langle m_{q1}\rangle_0 \langle i_{q1}\rangle_2 - \\
\langle m_{d2}\rangle_2 \langle i_{d2}\rangle_0 - \langle m_{d2}\rangle_0 \langle i_{d2}\rangle_2 - \\
\langle m_{q2}\rangle_2 \langle i_{q2}\rangle_0 - \langle m_{q2}\rangle_0 \langle i_{q2}\rangle_2 - j2C \langle v_{dc}\rangle_2 \tag{5.35}
\]

The dynamic phasor model of the rotor side controller yields:

\[
\frac{d\langle x_{d2}\rangle_0}{dt} = K_{I,v_{dc}} (V_{dc,ref} - \langle v_{dc}\rangle_0) \tag{5.36}
\]

\[
\frac{d\langle x_{d2}\rangle_2}{dt} = -K_{I,v_{dc}} \langle v_{dc}\rangle_2 - j2\omega_s \langle x_{d2}\rangle_2 \tag{5.37}
\]

\[
\langle m_{d2}\rangle_0 = \langle x_{d2}\rangle_0 + K_{P,v_{dc}} (V_{dc,ref} - \langle v_{dc}\rangle_0) \tag{5.38}
\]

\[
\langle m_{d2}\rangle_2 = \langle x_{d2}\rangle_2 - K_{P,v_{dc}} \langle v_{dc}\rangle_2 \tag{5.39}
\]

\[
\frac{d\langle x_{q2}\rangle_0}{dt} = K_{I,i_{q,g}} (I_{Q,ref} - \langle i_{q,g}\rangle_0) \tag{5.40}
\]

\[
\frac{d\langle x_{q2}\rangle_2}{dt} = -K_{I,i_{q,g}} \langle i_{q,g}\rangle_2 - j2\omega_s \langle x_{q2}\rangle_2 \tag{5.41}
\]

\[
\langle m_{q2}\rangle_0 = \langle x_{q2}\rangle_0 + K_{P,i_{q,g}} (I_{Q,ref} - \langle i_{q,g}\rangle_0) \tag{5.42}
\]

\[
\langle m_{q2}\rangle_2 = \langle x_{q2}\rangle_2 - K_{P,i_{q,g}} \langle i_{q,g}\rangle_2 \tag{5.43}
\]

The dynamic phasor model of the grid-side controller has the same structure as the rotor-side controller with the corresponding variables and parameters.
5.3 Comparative Assessment of Models

As also performed in Section 4.4, four different models are compared to study the dynamic behavior of the DFIG during balanced and unbalanced voltage sags. These four different models are as follows:

- Detailed Models
- Fundamental Frequency Models
- Dynamic Phasor Models
- Reduced Order Dynamic Phasor Models

**Detailed Models (EMT)**

As the detailed model of the induction machine, the full set of the 5th order induction machine model equations (5.2-5.8) is used, as it captures both the fast dynamics due to the electromagnetic transients and the slow dynamics due to electromechanical transients.

The average converter model (5.25) takes the electromagnetic dc-link dynamics into account but neglects the switching dynamics of the VSC. As transformer models, we use the transmission line model in the DQ0 reference frame discussed in Section 4.2.

**Fundamental Frequency Models (RMS)**

As these models neglect the electromagnetic transients and capture only the slow dynamics due to electromechanical transients, the fast electromagnetic stator flux changes are omitted. This is achieved by setting \( \frac{d}{dt} = 0 \) in (5.2). The rotor flux transients (5.3) are still considered. This leads to the well known 3rd order induction machine model used in transient stability studies.

For the converter, the dc-link dynamics are neglected by setting \( \frac{d}{dt} = 0 \) in (5.25) and the steady state value of the dc-capacitor voltage is used. The same also holds for the transformer model.

We should emphasize that setting \( \frac{d}{dt} = 0 \) in the DQ-reference for steady state operation in (5.2) and (5.25) is equivalent to omitting the negative
sequence quantities, since only the positive sequence components are described by DC quantities at steady state. Thus, these fundamental frequency models can only be used to simulate balanced conditions with symmetrical components, as they consider only the positive sequence quantities.

**Dynamic Phasor Models (DYNPH-EMT)**

Dynamic Phasor Models also capture both electromagnetic and electromechanical transients, similarly to the *detailed models*. This time however we do not simulate the instantaneous values of the system quantities $x(t)$, but rather their time-varying Fourier coefficients (dynamic phasors) $\langle x(t) \rangle_k$. The dynamic phasor model equations of the induction machine are given in (5.9-5.22) and of the converter in (5.34-5.43).

**Reduced Order Dynamic Phasor Models (DYNPH-RMS)**

Reduced Order Dynamic Phasor Models capture only the electromechanical transients. Stator flux electromagnetic transients of the induction machine are neglected by setting $\frac{d}{dt} = 0$ in (5.9-5.10).

The dc-link dynamics of the converter are also omitted by setting $\frac{d}{dt} = 0$ in (5.34-5.35).

In contrast to fundamental frequency models, setting $\frac{d}{dt} = 0$ in (5.9-5.10) and (5.34-5.35) means only neglecting the fast transients in the positive and negative sequence quantities. These models can still be used for unbalanced conditions and even for asymmetrical components, as they include positive ($k = 0$) and negative ($k = 2$) sequence quantities. For the balanced case fundamental frequency models and reduced order dynamic phasor models are equivalent.

**5.4 Test Cases**

The accuracy and efficiency of the described models under balanced and unbalanced conditions are compared in the following case study. The test case under consideration is depicted in Figure 5.2. It consists of one DFIG connected to an external grid through a transformer. The external grid is represented as a constant voltage source, which can
supply balanced and unbalanced voltages in three-phases (ABC). The per unit parameters of the system are given in Table 5.3.

<table>
<thead>
<tr>
<th>DFIG</th>
<th>Converter &amp; Controllers</th>
<th>TR-DFIG</th>
<th>TR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_s = 50 \text{[Hz]}$</td>
<td>$C = 0.5 \text{[pu]}$</td>
<td>$R = 0.003 \text{[pu]}$</td>
<td>$R = 0.015 \text{[pu]}$</td>
</tr>
<tr>
<td>$H = 3.00 \text{[s]}$</td>
<td>$K_{P,P_g} = 0.01$</td>
<td>$L = 0.300 \text{[pu]}$</td>
<td>$L = 0.150 \text{[pu]}$</td>
</tr>
<tr>
<td>$R_s = 0.01 \text{[pu]}$</td>
<td>$K_{I,P_g} = 0.04$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_r = 0.01 \text{[pu]}$</td>
<td>$K_{P,Q_g} = 0.005$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_s = 0.10 \text{[pu]}$</td>
<td>$K_{I,Q_g} = 5.000$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_r = 0.08 \text{[pu]}$</td>
<td>$K_{P,v_{dc}} = 0.002$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_m = 3.00 \text{[pu]}$</td>
<td>$K_{I,v_{dc}} = 0.050$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_{P,i_{q2}} = 2.50$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_{I,i_{q2}} = 500.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_{dc,ref} = 2.5[pu]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_{q,ref} = 0.0[pu]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$P_{g,ref} = 1.0[pu]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Q_{g,ref} = 0.0[pu]$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3: Parameters of the test system given on the system bases

Simulations were obtained using Matlab 7.1 running on an Intel Pentium IV CPU with 3.80 GHz and 2 GB of RAM. The trapezoidal method with an absolute tolerance $atol = 10^{-3}$ and relative tolerance $rtol = 10^{-4}$ has been used for the numerical integration of the test system.
5.4.1 Balanced Voltage Sag

In this case study, a 50% three-phase voltage sag is applied at 0.1 seconds and cleared after 3 cycles. The efficiency and the accuracy of the above mentioned four different models are examined.

Figure 5.3 and 5.4 depict the evolution of the machine electrical torque $T_e$ and the converter capacitor voltage $v_{dc}$ simulated with the detailed models (EMT) and detailed dynamic phasor models (DYNPH-EMT). Both results are in an overall agreement during the whole simulation interval. If the CPU simulation times in Table 5.4 are compared, the detailed dynamic phasor models (DYNPH-EMT) are faster than the detailed models with instantaneous values (EMT) while maintaining nearly the same degree of accuracy. Simulation results obtained with

<table>
<thead>
<tr>
<th>EMT</th>
<th>DYNPH-EMT</th>
<th>DYNPH-RMS</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.68 [s]</td>
<td>8.65 [s]</td>
<td>1.54 [s]</td>
<td>1.28 [s]</td>
</tr>
</tbody>
</table>

Table 5.4: CPU Simulation times in seconds - Balanced Voltage Sag

the reduced order dynamic phasor models (DYNPH-RMS) and fundamental frequency models (RMS) are given in Figure 5.5. The Figure shows the evolution of the dynamic phasors $\langle T_e \rangle_0$ and $\langle v_{dc} \rangle_0$ simulated with the detailed dynamic phasor models (DYNPH-EMT), with the reduced order dynamic phasor models (DYNPH-RMS) and the fundamental frequency models (RMS). As expected, due to the omission of the fast electromagnetic transients except for the rotor circuit of the induction machine, the simulations with RMS models and DYNPH-RMS models are faster than with the DYNPH-EMT models but also less accurate.
5.4. Test Cases

Figure 5.3: Evolution of the electrical torque $T_e$ after a 50% balanced voltage sag for 3 cycles computed with detailed models (EMT) and detailed dynamic phasor models (DYNPH-EMT).
Figure 5.4: Evolution of the converter capacitor voltage $V_{dc}$ after a 50% balanced voltage sag for 3 cycles computed with detailed models (EMT) and detailed dynamic phasor models (DYNPH-EMT).
Figure 5.5: Evolution of the electrical torque $T_e$ and dc-voltage $v_{dc}$ after a 50% balanced voltage sag for 3 cycles computed with detailed dynamic phasor models (DYNPH-EMT) and reduced order dynamic phasor models and reduced (DYNPH-RMS).
5.4.2 Unbalanced Voltage Sag

In this case study, a 50% one-phase voltage sag is applied at 0.1 seconds and cleared after 3 cycles. The same efficiency and accuracy comparison is done as in the test case with the balanced voltage sag. In this case study fundamental frequency models (RMS) are omitted, as explained in Section 5.3 they are not suitable for simulating unbalanced conditions.

The Figures 5.6 and 5.7 show the evolution of the machine electrical torque $T_e$ and the converter capacitor voltage $v_{dc}$ computed with the detailed models (EMT) and detailed dynamic phasor models (DYNPH-EMT). Table 5.5 shows the required CPU simulation times.

![Figure 5.6: Evolution of the electrical torque $T_e$ after a 50% unbalanced voltage sag for 3 cycles computed with detailed models (EMT) and detailed dynamic phasor models (DYNPH-EMT).](image-url)
5.4. Test Cases

Figure 5.7: Evolution of the converter capacitor voltage $v_{dc}$ after a 50% unbalanced voltage sag for 3 cycles computed with detailed models (EMT) and detailed dynamic phasor models (DYNPH-EMT).

Also in the unbalanced voltage sag case, an overall good match is observed between the dynamic phasor model results and detailed model results (Figures 5.6 and 5.7). Again, the dynamic phasor models show better performance than the detailed models in terms of CPU simulation time. As expected, the reduced order dynamic phasor models

<table>
<thead>
<tr>
<th>EMT</th>
<th>DYNPH-EMT</th>
<th>DYNPH-RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>47.26 [s]</td>
<td>11.78 [s]</td>
<td>3.54 [s]</td>
</tr>
</tbody>
</table>

Table 5.5: CPU Simulation times in seconds - Unbalanced Voltage Sag
show the shortest simulation time, but the accuracy is low as fast dynamics are neglected. In contrast to fundamental frequency models, with the reduced order dynamic phasor models we are able to simulate unbalanced conditions. Figure 5.9 shows the trajectory of the negative sequence electrical torque which is mapped through the dynamic phasor $\langle T_e \rangle_2$.

![Graph 1](image1.png)

![Graph 2](image2.png)

Figure 5.8: Dynamic Phasors $\langle T_e \rangle_0$ of the electrical torque and $\langle v_{dc} \rangle_0$ of the dc-voltage after a 50% unbalanced voltage sag for 3 cycles computed with detailed dynamic phasor models (DYNPH-EMT) and reduced order dynamic phasor models and reduced (DYNPH-RMS).
5.4. Test Cases

Figure 5.9: Real and imaginary part of the dynamic phasor $\langle T_e \rangle_2$ after a 50% unbalanced voltage sag for 3 cycles computed with detailed dynamic phasor models (DYNPH-EMT) and reduced order dynamic phasor models and reduced (DYNPH-RMS).
5.5 Summary

In this Chapter, the dynamic phasor model of the DFIG has been derived. The derived dynamic phasor model is then used to study the dynamic behavior of the DFIG in relation to balanced and unbalanced voltage sags and results have been compared with the conventional detailed DFIG model in terms of accuracy and computational efficiency. Simulations have showed that the dynamic phasor models were able to simulate the dynamic response of the DFIG to balanced and unbalanced voltage sags as accurately as the detailed time domain models while decreasing the required simulation time considerably. Simulations with dynamic phasor models were 4 times faster compared with the detailed time domain models. We should emphasize that an average model of the converter has been used in detailed time domain model. If we have used the detailed model of the converter with the IGBTs, the simulations would take much longer due to the high frequent switchings of the IGBTs.

Simulations have also been performed with the reduced order dynamic phasor models neglecting network transients. These models have been compared with the fundamental frequency models, which are commonly used for transient stability studies under balanced conditions. The reduced order dynamic phasor models were also able to simulate the unbalanced conditions efficiently.
Chapter 6

Dynamic Phasor Model of the TCSC

In this chapter, the dynamic phasor model of Thyristor-controlled Series Capacitor is derived based on previous work done by Mattavelli et al. [14] and the accuracy and simulation efficiency of the model is compared with the detailed time domain model and other existing fundamental frequency models.

6.1 Introduction

Thyristor-controlled Series Capacitor (TCSC) is capacitive reactance compensator which consists of a series capacitor bank in parallel with a thyristor-controlled reactor (TCR) in order to provide a smoothly variable series capacitive reactance (see Figure (6.1)). The main control unit of the TCSC is the TCR, which is a static var controller providing fast control over the reactive power using power electronics. By means of the firing angle of the thyristors the effective inductive reactance can be controlled and causes rapid reactive power exchange between the TCR and the system. By controlling the TCSC reactance one can compensate the line impedance and thus allow increased power flow through the line. This fast control of line impedance can be used for damping power system oscillations, power flow regulations and mitigating subsynchronous resonance [43, 44, 45].
The major focus in this chapter is mainly on the accurate and simulation-efficient modelling of the TCSC’s dynamic response. In most of the Transient Stability Programs, the TCSC is modelled as a controlled variable impedance at fundamental frequency [43]. As mentioned in previous chapters, such fundamental frequency models assume that the electrical quantities of the components such as voltages and currents can be approximated by sinusoidal signal at fundamental frequency. These models give accurate results depending on the operating point. For example in the case of the TCSC, the validity of this assumption is only valid for a specified range of firing angle, which is the control variable of the TCR branch. When fundamental frequency phasor models do not give accurate results, a full time domain simulation might be needed. However a complete representation of a large power system in an electromagnetic transients program is very difficult and will not normally give additional information.

The main idea in the following is to use the dynamic phasors approach to improve the accuracy of the TCSC model. Major contributions in this direction were reported in [14]. In [14] Mattavelli et al. apply the dynamic phasors approach and extend the fundamental frequency model of the TCSC by taking into account also the effects of major harmonics in the fundamental frequency with a steady-state correction. Here, the dynamic phasor model will be derived by directly including the major harmonics in the model. To comply with the derivation in [14], the same variable and parameter names are used.

The chapter is organized as follows. First a simple TCSC circuit will be analyzed to explain the modes of operation and some important parameters. Afterwards, the steady state analysis of the TCSC quantities
will allow us to decide, which time varying Fourier coefficients should be included in our dynamic phasor model to improve the accuracy of the TCSC model. Then starting with the traditional fundamental frequency model, other more detailed dynamic phasor models will be derived including other harmonics. Finally, the accuracy of the derived TCSC dynamic phasor model will be compared with detailed time domain models and with the TCSC dynamic phasor model presented in [14].

6.2 Characteristics and Circuit Analysis of TCSC

In this section the functionality of the TCSC will be discussed in detail. As depicted in Figure 6.1, the TCSC has a hybrid characteristic due to the thyristors T1 and T2. The thyristor can be modelled as an ideal switch that closes if the voltage between anode and cathode is positive and a non-zero gate signal is applied. It opens again if the current through the thyristor recrosses zero. The switchings of the thyristors are alternated at the system frequency. Hence the time domain model equations of the per-phase TCSC can be given as

\[ C \frac{dv}{dt} = i_L - i \quad (6.1) \]
\[ L \frac{di}{dt} = q \cdot v \quad (6.2) \]

where \( q \) is a switching function that shows the thyristor status: \( q = 1 \) if one thyristor is conducting and \( q = 0 \) if both thyristors are blocking. The time argument \( t \) has been omitted for clarity.

In the following, we recall the mode of operation and some important parameters of the TCSC referring to the Figures 6.1-6.2(b) under the assumption of having a constant sinusoidal line current at the system frequency. Looking at the steady state and transient wave form of the quantities, we observe 5 intervals.

1. In the interval \([0 \leq \omega t \leq \alpha]\) both thyristors block as the gate signal \( g \) is zero. The angle \( \alpha \) is referred to as the firing angle and is the control variable of the TCSC. In this interval (I) the switching
Figure 6.2: (a) Steady-state and (b) Transient waveforms of $i_L(t)$, $v(t)$ and $i(t)$ over one period with reference conduction angle $\sigma_r = 66^\circ$, with $\hat{I}_L = \text{max}(i_L)$, $\hat{V} = \text{max}(v)$ and $\hat{I} = \text{max}(i)$ in the interval $0 \leq \omega t \leq 2\pi$. 
function \( q \) is zero and the model is described by:

\[
C \frac{dv}{dt} = i_L 
\]

\[
i = 0 
\]  

(6.3)  

(6.4)  

II At \( \omega t = \alpha \), a non-zero gate signal is \( g \) applied to the gate of thyristor T1. As the voltage between anode and cathode is positive, thyristor T1 starts conducting while T2 blocks. T1 conducts till the current through T1 recrosses zero, which happens at \( \omega t = \alpha + \sigma \). The angle \( \sigma \) is referred to as the conduction angle, as during this period the TCR branch conducts. In the interval (II) \( [\alpha \leq \omega t \leq \alpha + \sigma] \) the TCR branch conducts, meaning \( q = 1 \), and the model is described by:

\[
C \frac{dv}{dt} = i_L - i 
\]

\[
L \frac{di}{dt} = v 
\]

(6.5)  

(6.6)  

III At \( \omega t = \alpha + \sigma \) the current through T1 recrosses zero and T1 starts blocking again. Now in the interval \( [\alpha + \sigma \leq \omega t \leq \pi + \alpha] \) both thyristors block as in interval (I) and same equations also hold for interval (III). This lasts till \( \omega t = \pi + \alpha \), where the gate signal \( g \) of T2 becomes again nonzero.

IV At \( \omega t = \pi + \alpha \) a non-zero gate signal is \( g \) applied to the gate of thyristor T2. As the voltage between anode and cathode is positive, thyristor T2 starts conducting, while T1 blocks. T2 conducts till the current through T2 recrosses zero, which happens at \( \omega t = \pi + \alpha + \sigma \). In the interval (IV) \( [\pi + \alpha \leq \omega t \leq \pi + \alpha + \sigma] \), T2 conducts and same equations in interval (II) also hold for interval (IV).

V At \( \omega t = \pi + \alpha + \sigma \) the current through T2 recrosses zero and T2 starts blocking again. Now in the interval \( [\pi + \alpha + \sigma \leq \omega t \leq (2\pi)] \) both thyristors block as in interval (I & III) and same equations also hold for interval (V).

Figure 6.2(a) shows the steady state trajectory of the important quantities of the TCSC, namely line current \( i_L \), TCR current \( i \) and the
capacitor voltage $v$, under the assumption of a purely sinusoidal line current. At steady state, the conduction angle $\sigma$ equals to the reference conduction angle $\sigma_r$, which is given as $\sigma_r = \pi - 2\alpha$.

As illustrated in Figure 6.2(b), during non-steady-state the conduction angle $\sigma$ differs from its reference value $\sigma_r$. During transients $\sigma$ can be approximated by $\sigma = \sigma_r + 2\phi$, where $\phi$ is the angle difference between the peak value of the line current $i_L$ and the negative peak value of the TCR current $i$.

As we can see from the trajectories of the capacitor voltage and TCR current, these quantities are periodic with the system frequency but they are not purely sinusoidal. For example, at the operating point in the Figures 6.2(b) and 6.2(a) with $\sigma_r = 66^\circ \rightarrow \alpha = 57^\circ$, the assumption of having only fundamental frequency in the capacitor voltage is not valid any more. An appropriate approach to define roughly the range of validity of the fundamental frequency models would be to look at harmonic contents of these quantities in dependence of conduction angle at steady state.

### 6.3 Steady State Analysis

The steady state harmonic contents of the capacitor voltage $v$ and TCR current $i$ can be calculated with the steady state analytical expressions independence of the reference conduction angle $\sigma_r$. For example, the $k^{th}$ steady state harmonic content $V_k$ of the capacitor voltage $v$ yields

$$V_k(\sigma_r) = \frac{1}{T} \int_0^T v(\tau, \sigma_r) \cdot e^{-j k \omega_s \tau} d\tau$$

where the steady state capacitor voltage depends on the reference conduction angle $\sigma_r$. To calculate the analytical expression for $V_k$ in dependence of $\sigma_r$, the analytical expressions for the $v$ and $i$ in intervals (I-V) must be calculated.

Starting with interval (II) where the TCR branch is conducting, the TCSC model is given by the equations (6.5) and (6.6). There are two boundary conditions in this interval, namely the TCR branch current
must be zero at the beginning and at the end of the conducting interval. 

\[ i(\omega t = \alpha) = i \left( \frac{\pi}{2} - \frac{\sigma_r}{2} \right) = 0 \]

\[ i(\omega t = \tau) = i \left( \frac{\pi}{2} + \frac{\sigma_r}{2} \right) = 0 \]

The analytical solution of this initial value problem with linear differential equations can be calculated using Laplace transform.

\[ C \left[ sV(s) - v_0^+ \right] = I_L(s) - I(s) \]  \hspace{1cm} (6.8)

\[ L \left[ sI(s) - i_0^+ \right] = V(s) \]  \hspace{1cm} (6.9)

with \( v_0^+ = v(0^+) \) and \( i_0^+ = i(0^+) \). After replacing \( V(s) \) in (6.8) with (6.9) and \( v_0^+ \) with \( L \frac{di_0^+}{dt} \) the Laplace transform of the TCR current \( i \) becomes

\[ I(s) = I_L(s) - \frac{\omega_0^2}{s^2 + \omega_0^2} + i_0^+ \frac{s}{s^2 + \omega_0^2} + \frac{di_0^+}{dt} \frac{1}{s^2 + \omega_0^2} \]

with \( \omega_0 = \frac{1}{\sqrt{LC}} \). After applying a time shift of \( \frac{\pi}{2 \omega} \), where the line current has its peak value, the Laplace transform of the line current \( i_L \) becomes

\[ I_L(s) = \mathcal{L} \{ i \} = \mathcal{L} \left\{ \sin \left( \omega t + \frac{\pi}{2} \right) \right\} = \frac{s}{s^2 + \omega^2} \]

The inverse Laplace transformation of \( I(s) \) yields:

\[ i = \mathcal{L}^{-1} \{ I(s) \} = S \left( -\cos \left( \frac{\eta \omega t}{2} \right) + \cos \left( \omega t \right) \right) \]

\[ + i_0^+ \cos \left( \frac{\eta \omega t}{2} \right) + \frac{di_0^+}{dt} \frac{\sin \left( \frac{\eta \omega t}{2} \right)}{\eta \omega} \]  \hspace{1cm} (6.10)

with \( \eta = \frac{\omega_0}{\omega} \), \( S = \frac{\eta^2}{\eta^2 - 1} \) and two unknown variables \( i_0^+ \) and \( \frac{di_0^+}{dt} \). Considering the time shifted boundary conditions of the TCR current \( i(\frac{\sigma_r}{2 \omega}) = 0 \) and \( i(\frac{\sigma_r}{2 \omega}) = 0 \), \( i_0^+ \) and \( \frac{di_0^+}{dt} \) yield

\[ i_0^+ = S \left( 1 - \frac{\cos \left( \frac{\sigma_r}{2} \right)}{\cos \left( \frac{\eta \sigma_r}{2} \right) \eta} \right), \quad \frac{di_0^+}{dt} = 0 \]  \hspace{1cm} (6.11)

After replacing \( i_0^+ \), \( \frac{di_0^+}{dt} \) in (6.10) with (6.11) and shifting back to the time origin, the TCR current becomes

\[ i^{(II)} = S \left( \sin \left( \omega t \right) - \frac{\cos \left( \frac{\eta \omega t}{2} \right) \cos \left( \frac{\sigma_r}{2} \right)}{\cos \left( \frac{\eta \sigma_r}{2} \right) \eta} \right) \]  \hspace{1cm} (6.12)
With (6.5) the capacitor voltage \( v \) is given as

\[
v^{(II)} = SX_L \left( \cos(\omega t) + \frac{\eta \sin(\eta (\omega t - \frac{\pi}{2})) \cos(\frac{\sigma_r}{2})}{\cos(\eta \frac{\sigma_r}{2})} \right)
\] (6.13)

In the interval (I), the TCR branch is blocking and the model equations in this interval are given in (6.3-6.4) with the boundary condition regarding the continuity of the capacitor voltage \( v(\alpha^-) = v(\alpha^+) \), meaning the capacitor voltage value right at the end interval (I) is equal to the capacitor voltage value right at the beginning of interval (II). Repeating again the same procedure applied previously, \( v \) yields

\[
I_L(s) = C \left[ sV(s) - v_0^+ \right] \Rightarrow
\]

\[
V(s) = X_C \omega \frac{I_L(s)}{s} + \frac{v_0^+}{s} \Rightarrow
\]

\[
v^{(I)} = X_C - X_C \cos(\omega t) + v_0^+
\]

And with the boundary condition \( v^{(I)}(\frac{\pi}{2} - \frac{\sigma_r}{2}) = v^{(II)}(\frac{\pi}{2} - \frac{\sigma_r}{2}) \), \( v_0^+ \) yields

\[
v_0^+ = X_C \sin(\frac{\sigma_r}{2}) - X_C + S X_L \left( \eta \cos(\frac{\sigma_r}{2}) \tan(\eta \frac{\sigma_r}{2}) - \sin(\frac{\sigma_r}{2}) \right)
\]

Hence the TCR branch current \( i \) and capacitor voltage \( v \) in interval (I) can be expressed as

\[
i^{(I)} = 0
\]

\[
v^{(I)} = -X_C \cos(\omega t) + S \left( X_C \sin(\frac{\sigma_r}{2}) - X_L \eta \tan(\eta \frac{\sigma_r}{2}) \cos(\frac{\sigma_r}{2}) \right)
\]

The steady state expressions for \( i \) and \( v \) in all 5 intervals are given in Table 6.1 and 6.2. With the given steady state expressions, the \( k^{th} \) harmonic content \( V_k \) in (6.7) yields

\[
V_k = \frac{1}{T} \left[ \int_{0}^{\frac{\pi}{\omega}} v^{(I)}(\tau, \sigma_r)e^{-jk \omega_s \tau} d\tau + \int_{\frac{\pi}{\omega}}^{\frac{\pi - \alpha}{\omega}} v^{(II)}(\tau, \sigma_r)e^{-jk \omega_s \tau} d\tau + \int_{\frac{\pi + \alpha}{\omega}}^{\frac{\pi + \alpha}{\omega}} v^{(III)}(\tau, \sigma_r)e^{-jk \omega_s \tau} d\tau + \int_{\frac{\pi - \alpha}{\omega}}^{\frac{\pi - \alpha}{\omega}} v^{(IV)}(\tau, \sigma_r)e^{-jk \omega_s \tau} d\tau + \int_{\frac{\pi + \alpha}{\omega}}^{\frac{\pi + \alpha}{\omega}} v^{(V)}(\tau, \sigma_r)e^{-jk \omega_s \tau} d\tau \right]
\]
6.3. Steady State Analysis

\[
\int_{-\frac{\alpha}{\omega}}^{\frac{\pi}{\omega}} v^{(IV)}(\tau, \sigma_r) e^{-j k \omega_s \tau} d\tau + \int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} v^{(V)}(\tau, \sigma_r) e^{-j k \omega_s \tau} d\tau
\]

\[
V_k = \frac{4 C_k}{\pi k} \left[ \frac{X_C^2}{2(X_C - X_L)} \left( \sin \left( \frac{\sigma_r(k+1)}{2} \right) + \sin \left( \frac{\sigma_r(k-1)}{2} \right) \right) \right] + \frac{X_C^2 \cos \left( \frac{\sigma_r}{2} \right) \cos \left( k \frac{\sigma_r}{2} \right) \left( k \tan \left( k \frac{\sigma_r}{2} \right) - \eta \tan \left( \eta \frac{\sigma_r}{2} \right) \right)}{(X_C - X_L) (\eta^2 - k^2)}
\]

(6.14)

with \( C_k = (-1)^{\frac{k-1}{2}} \). The fundamental frequency content of the capacitor voltage \( V_1 \) results in

\[
V_1 = \frac{1}{\pi} \left[ -X_C \pi + \frac{X_C^2 (\sin(\sigma_r) + \sigma_r)}{(X_C - X_L)} + \frac{4 X_C^2 \cos^2 \left( \frac{\sigma_r}{2} \right) \left( \tan \left( \frac{\sigma_r}{2} \right) - \eta \tan \left( \eta \frac{\sigma_r}{2} \right) \right)}{(X_C - X_L) (\eta^2 - 1)} \right]
\]

(6.15)

**Harmonic Distortion** of \( k^{th} \) harmonic \( (\rho_k) \) is defined as.

\[
\rho_k = \frac{|V_k|}{|V_1|}
\]

(6.16)

As given in (6.14) and (6.15), \( V_k \) and \( V_1 \) are dependent on the reference conduction angle \( \sigma_r \). Figure 6.3 shows the evolution of \( \rho_k \) in dependence of \( \sigma_r \) for different harmonics \( k \).

For conduction angle values \( 0^\circ - 40^\circ \) the participation of higher harmonics in steady-state capacitor voltage \( v \) quite low. For \( \sigma_r = 40^\circ \) the harmonic distortion factor \( \rho_3 \) is only around 3.8% and \( \rho_5 \) around 1.87%. This means, \( v \) is well approximated by the fundamental phasor component \( V_1 \).

For conduction angle values \( 40^\circ - 180^\circ \) the participation of higher harmonics become significant. For example for \( \sigma_r = 66^\circ \), which is also the operating point illustrated in Figures 6.2(a) and 6.2(b), \( \rho_3 \) is around 17.24% and \( \rho_5 \) around 5.6%. In this case, not only the fundamental frequency but also other harmonics should be taken into account.
### Table 6.1: Steady-state TCR branch current $i_T(t)$ over a period

<table>
<thead>
<tr>
<th>Range</th>
<th>$i_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega t \in [0, \alpha)$</td>
<td>$i_T^{(I)}(t) = 0$</td>
</tr>
<tr>
<td>$\omega t \in [\alpha, \pi - \alpha)$</td>
<td>$i_T^{(II)}(t) = \frac{X_C}{X_C - X_L} \left( \sin(\omega t) - \frac{\cos(\rho(\omega t - \frac{\pi}{2})) \cos(\frac{\sigma r}{2})}{\cos(\rho \frac{\sigma r}{2})} \right)$</td>
</tr>
<tr>
<td>$\omega t \in [\pi - \alpha, \pi + \alpha)$</td>
<td>$i_T^{(III)}(t) = 0$</td>
</tr>
<tr>
<td>$\omega t \in [\pi + \alpha, 2\pi - \alpha)$</td>
<td>$i_T^{(IV)}(t) = \frac{X_C}{X_C - X_L} \left( \sin(\omega t) + \frac{\cos(\rho(\omega t - \frac{3\pi}{2})) \cos(\frac{\sigma r}{2})}{\cos(\rho \frac{\sigma r}{2})} \right)$</td>
</tr>
<tr>
<td>$\omega t \in [2\pi - \alpha, 2\pi)$</td>
<td>$i_T^{(V)}(t) = 0$</td>
</tr>
</tbody>
</table>
6.3. Steady State Analysis

<table>
<thead>
<tr>
<th>$v^{(interval)}(t)$</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v(I)(t) = \left[-X_C \cos(\omega t) + \left(\frac{X_C^2 \sin(\frac{\sigma s}{2}) - \rho X_C X_L \cos(\frac{\sigma s}{2}) \tan(\rho \frac{\omega}{2})}{X_C - X_L}\right)\right]$</td>
<td>$\omega t \in [0, \alpha]$</td>
</tr>
<tr>
<td>$v(II)(t) = \left[\frac{X_C X_L}{X_C - X_L} \left(\cos(\omega t) + \frac{\rho \cos(\frac{\sigma s}{2}) \sin(\rho(\omega t - \frac{\alpha}{2}))}{\cos(\rho \frac{\omega}{2})}\right)\right]$</td>
<td>$\omega t \in [\alpha, \pi - \alpha]$</td>
</tr>
<tr>
<td>$v(III)(t) = \left[-X_C \cos(\omega t) + \left(\frac{-X_C^2 \sin(\frac{\sigma s}{2}) + \rho X_C X_L \cos(\frac{\sigma s}{2}) \tan(\rho \frac{\omega}{2})}{X_C - X_L}\right)\right]$</td>
<td>$\omega t \in [\pi - \alpha, \pi + \alpha]$</td>
</tr>
<tr>
<td>$v(IV)(t) = \left[\frac{X_C X_L}{X_C - X_L} \left(\cos(\omega t) - \frac{\rho \cos(\frac{\sigma s}{2}) \sin(\rho(\omega t - \frac{3\pi}{2}))}{\cos(\rho \frac{\omega}{2})}\right)\right]$</td>
<td>$\omega t \in [\pi + \alpha, 2\pi - \alpha]$</td>
</tr>
<tr>
<td>$v(V)(t) = \left[-X_C \cos(\omega t) + \left(\frac{X_C^2 \sin(\frac{\sigma s}{2}) - \rho X_C X_L \cos(\frac{\sigma s}{2}) \tan(\rho \frac{\omega}{2})}{X_C - X_L}\right)\right]$</td>
<td>$\omega t \in [2\pi - \alpha, 2\pi]$</td>
</tr>
</tbody>
</table>

Table 6.2: Steady-state capacitor voltage $v^*(t)$ over a period
6.4 Fundamental Frequency Dynamic Phasor Model

Now, in this section the fundamental frequency TCSC model will be derived using the dynamic phasors approach. As mentioned also in Section 6.3, in the fundamental frequency model, we assume that all electrical quantities are well approximated by their fundamental frequencies meaning

\[ i(t) \approx \Re\{I_1(t) e^{j\omega_s t}\} \]
\[ v(t) \approx \Re\{V_1(t) e^{j\omega_s t}\} \]

With these assumptions the equations (6.1) and (6.2) become

\[
C \frac{dV_1}{dt} = I_L - I_1 - j\omega_s C V_1 \quad (6.17)
\]
\[
L \frac{dI_1}{dt} = \langle q v \rangle_1 - j\omega_s L I_1 \quad (6.18)
\]

With properties (3.20) and 3.22, the term \( \langle q v \rangle_1 \) is expressed as:

\[ \langle q v \rangle_1 = Q_0 V_1 + Q_2 V_{-1} = Q_0 V_1 + Q_2 V_1^* \]

With equation (3.17), the coefficients \( Q_0 \) and \( Q_2 \) of the switching func-
6.4. Fundamental Frequency Dynamic Phasor Model

\[
\omega t \xi - \sigma r \frac{2}{2} \xi + \sigma r + 2 \phi
\]

Figure 6.4: Switching function - \(q(t)\)

tion \(q\), shown in Figure 6.4, yield

\[
Q_0 = \langle q \rangle_0 = \frac{\sigma}{\pi}
\]

\[
Q_2 = \langle q \rangle_2 = \frac{\sin(\sigma)}{\pi} e^{-j 2(\xi + \phi)}
\]

General term for \(Q_k\) is given as expressed

\[
Q_k = \langle q \rangle_k = \frac{\sin \left(\frac{k \sigma}{2}\right)}{\frac{k \pi}{2}} e^{-jk(\xi + \phi)} \quad (6.19)
\]

which leads to the final expression for

\[
\langle q v \rangle_1 = \frac{1}{\pi} \left( V_1 \sigma + V_1^* \sin(\sigma) e^{-2j(\xi + \phi)} \right)
\]

At steady-state, with \(\frac{dI_1}{dt} = 0\), \(\phi = 0\), \(\sigma = \sigma_r\) and \(\xi = \psi - \pi/2\), the equation (6.18) leads to standard equivalent inductance \(L_{eq}\) of the TCR.

\[
L_{eq} = \frac{V_1}{j \omega_s I_1} = L \frac{\pi}{\sigma_r - \sin(\sigma_r)}
\]

The derived fundamental frequency models has been implemented in the MATLAB simulation framework described in chapter 2. The detailed time domain simulation are done with PLECS. PLECS [32] is a toolbox for high-speed simulations of electrical and power electronic circuits under MATLAB/Simulink. To test the dynamic behavior of the TCSC,
we used the same test circuit as in [14] shown in Figure 6.5 with the same data: \( C = 176.84 \mu F \), \( L = 6.8 mH \), \( i_L = \hat{I}_L \cdot \sin(\omega t) \), \( \hat{I}_L = 1 kA \), \( \omega = 2\pi f \), \( f = 60 Hz \). The dynamic behavior of the TCSC for changes in the line current magnitude \( \hat{I}_L \) and reference conduction angle \( \sigma_r \) is tested with the following event sequence given in Table 6.3. The dynamic response of capacitor voltage \( v \) to the given event sequence in Table 6.3 is depicted in Figure 6.6. The response of the fundamental frequency model to the same sequence of events is shown in Figure 6.7 denoted as \( V_1 \). It is also possible to measure the fundamental frequency of the instantaneous capacitor voltage \( v \) depicted in Figure 6.6 by performing a Fourier analysis of the capacitor voltage over a running window of one cycle of the fundamental frequency of the signal. This measured fundamental frequency of the capacitor voltage is denoted as \( V_{1m} \) in Figure 6.7. Looking carefully to the different intervals, where we have different operating points regarding reference conduction angle \( \sigma_r \) and line current \( \hat{I}_L \), we make following conclusions.

- In the intervals where the reference conduction angle is 66°, the fundamental frequency model does not give satisfactory results.

<table>
<thead>
<tr>
<th>Interval</th>
<th>( t_{from}[s] )</th>
<th>( t_{to}[s] )</th>
<th>( \hat{I}_L[kA] )</th>
<th>( \alpha[^\circ] )</th>
<th>( \sigma_r[^\circ] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.0</td>
<td>0.5</td>
<td>1.00</td>
<td>57</td>
<td>66</td>
</tr>
<tr>
<td>II</td>
<td>0.5</td>
<td>1.0</td>
<td>1.25</td>
<td>57</td>
<td>66</td>
</tr>
<tr>
<td>III</td>
<td>1.0</td>
<td>1.5</td>
<td>1.00</td>
<td>57</td>
<td>66</td>
</tr>
<tr>
<td>IV</td>
<td>1.5</td>
<td>2.0</td>
<td>1.00</td>
<td>70</td>
<td>40</td>
</tr>
<tr>
<td>V</td>
<td>2.0</td>
<td>3.0</td>
<td>1.00</td>
<td>57</td>
<td>66</td>
</tr>
</tbody>
</table>

Table 6.3: Sequence of events

Figure 6.5: Test case for TCSC
6.4. Fundamental Frequency Dynamic Phasor Model

Figure 6.6: Capacitor voltage $v$ of the TCSC simulated with PLECS

Figure 6.7: Measured fundamental frequency component $|V_1|^m$ of capacitor voltage of the TCSC simulated with PLECS compared with $|V_1|$ of the dynamic phasor fundamental frequency model
even at steady state. This conclusion was determined during the steady state analysis in the previous section.

- Also the dynamic response in this intervals with $\sigma_r = 66^\circ$ to changes in line current amplitude is not accurate regarding the time constants. The detailed model ($V_1^{m}$) shows a slower dynamic response to line current amplitude changes in this interval than the fundamental frequency model.

+ In the interval $\sigma_r = 44^\circ$, the fundamental frequency model gives satisfactory results at steady state.

- But the dynamic response in this interval with $\sigma_r = 40^\circ$ to changes in line current amplitude is also not accurate regarding the time constants. The detailed model ($V_1^{m}$) shows a slower dynamic response to line current amplitude changes in this interval than the fundamental frequency model ($V_1$).

The fundamental frequency TCSC model gives satisfactory results only if the conduction angle is small. For higher values of $\sigma_r$, the model can not reflect the participation of higher harmonics even at steady-state. Because of that, we aim to improve our model by including other major harmonics in the approximation the TCSC dynamic behavior.

After an eigenvalue and participation factor analysis the $4^{th}$ order model can be reduced to a $2^{nd}$ order model, as the TCR current dynamic phasor $I_1$ has much faster decaying dynamics than the capacitor voltage dynamic phasor $V_1$. Figure 6.8 shows the comparison of the simulation results with $4^{th}$ order model ($|V_1|$) and reduced $2^{nd}$ order model ($|V_1^r|$). As both results are in an overall agreement, the neglect of the TCR current fast dynamics is justified.

### 6.5 Improved Dynamic Phasor Model

In the derivation of the fundamental frequency model, the assumption was made that the all electrical quantities are well approximated by their fundamental frequencies. As illustrated in Figure 6.3, the participation of the higher harmonics increases in the interval $40^\circ \leq \sigma_r \leq 180^\circ$. Especially the $3^{rd}$ and $5^{th}$ harmonics become perceivable. Accordingly
6.5. Improved Dynamic Phasor Model

![Graph showing capacitor voltage of the TCSC]

Figure 6.8: Capacitor voltage of the TCSC

our assumption on the capacitor voltage changes and we assume that the capacitor voltage is well approximated by

$$v(t) \approx \Re \{ V_1(t) e^{j \omega_s t} + V_3(t) e^{j 3 \omega_s t} + V_5(t) e^{j 5 \omega_s t} \}$$

With these assumptions in (6.1-6.2) the dynamic phasor model of the TCSC becomes

\[
\begin{align*}
C \frac{dV_1}{dt} &= I_L - I_1 - j \omega_s CV_1 \\
L \frac{dI_1}{dt} &= \langle q v \rangle_1 - j \omega_s LI_1 \\
C \frac{dV_3}{dt} &= -I_3 - j 3 \omega_s CV_3 \\
L \frac{dI_3}{dt} &= \langle q v \rangle_3 - j 3 \omega_s LI_3 \\
C \frac{dV_5}{dt} &= -I_5 - j 5 \omega_s CV_5 \\
L \frac{dI_5}{dt} &= \langle q v \rangle_5 - j 5 \omega_s LI_5
\end{align*}
\] (6.20)

As the line current is presumed to be sinusoidal at system frequency its harmonic contents at $3\omega_s$ and $5\omega_s$ are zero, meaning $\langle I_L \rangle_3 = 0$, $\langle I_L \rangle_5 = 0$. With properties (3.20), (3.22) and the assumption on the capacitor
voltage \( v \) the terms \( \langle q\ v \rangle_1, \langle q\ v \rangle_3 \) and \( \langle q\ v \rangle_5 \) are expressed as:

\[
\langle q\ v \rangle_1 = Q_0 V_1 + Q_2 V_1^* + Q_4^* V_3 + Q_4 V_3^* + Q_6^* V_5 + Q_6 V_5^*
\]

\[
\langle q\ v \rangle_3 = Q_2 V_1 + Q_4 V_1^* + Q_0 V_3 + Q_6 V_3^* + Q_8^* V_5 + Q_8 V_5^*
\]

\[
\langle q\ v \rangle_5 = Q_4 V_1 + Q_6 V_1^* + Q_2 V_3 + Q_8 V_3^* + Q_0 V_5 + Q_{10} V_5^*
\]

(6.21)

The dynamic phasors \( Q_k \) of the switching function \( q \) are defined as (6.19). Hence the dynamic phasor model is fully described in terms of the time varying Fourier coefficients of the capacitor voltage \( v \), TCR current \( i \) and switching function \( q \). As applied in the case of fundamental frequency model, the order of the dynamic phasor model can be reduced by neglecting the fast dynamics of the TCR current meaning \( \frac{dI_1}{dt} = 0, \frac{dI_3}{dt} = 0 \) and \( \frac{dI_5}{dt} = 0 \). The same test circuit and same sequence of events have been used to assess the accuracy of the improved model with 3\(^{rd}\) and 5\(^{th}\) harmonics included.

Figures 6.9(a)-6.9(c) show the dynamic response of the TCSC dynamic phasor model. Variables \( V_1, V_3 \) and \( V_5 \) denote the capacitor voltage dynamic phasors calculated by the model, where variables \( V_1^m, V_3^m \) and \( V_5^m \) denote measured Fourier coefficients of the instantaneous capacitor voltage from the detailed time domain simulation. Figure 6.10 shows the simulated envelope of the capacitor voltage, denoted as \( V_{135} = V_1 + V_3 + V_5 \). Again looking at the different intervals we come to the following conclusions.

1. In all intervals the improved dynamic phasor model gives satisfactory results at steady state. In the case with the fundamental frequency model this was achieved only for small conduction angles \( (0^\circ \leq \sigma_r \leq 40^\circ) \).

2. Also the dynamic response in all intervals to changes in line current amplitude is accurate regarding the time constants. Both the detailed model \( (V_1^m, V_3^m, V_5^m) \) and the dynamic phasor model \( (V_1, V_3, V_5) \) show similar dynamic response to line current amplitude changes.

The improved dynamic phasor model with 3\(^{rd}\) and 5\(^{th}\) harmonics gives better results in all sections (I-V), as it considers the participation of higher harmonics as well.

In [14], Mattavelli et al. include also the 3\(^{rd}\) and 5\(^{th}\) harmonics in the model derivation. They include the effects of these harmonics in the
Figure 6.9: Measured fundamental frequency component, 3\textsuperscript{rd} and 5\textsuperscript{th} harmonics (|V_1|^m, |V_3|^m, |V_5|^m) of capacitor voltage of the TCSC simulated with PLECS compared with the |V_1|, |V_3| and |V_5| of the improved dynamic phasor model
Chapter 6. Dynamic Phasor Model of the TCSC

Figure 6.10: Capacitor voltage $v$ versus the calculated envelope with the dynamic phasor model $V_{135} = V_1 + V_3 + V_5$

fundamental frequency meaning

$$ C \frac{dV_1}{dt} = I_L - I_1 - j \omega_s C V_1 $$

$$ L \frac{dI_1}{dt} = \langle qv \rangle_1 - j \omega_s L I_1 $$

with

$$ \langle qv \rangle_1 = Q_0 V_1 + Q_2 V_1^* + Q_2^* V_3 + Q_4 V_3^* + Q_4^* V_5 + Q_6 V_5^* $$

But as the Transient Stability Programs can only handle the fundamental frequency of the quantities, $V_3$ and $V_5$ are approximated by the fundamental frequency $V_1$ and the corresponding harmonic distortion factors $\rho_k$.

$$ V_3 = |V_1| \rho_3(\sigma) e^{-j3\psi} $$

$$ V_5 = |V_1| \rho_5(\sigma) e^{-j5\psi} $$

with $\psi = \angle V_1^*$ and $\rho_k$ given in 6.16. With these substitutions $\langle qv \rangle_1$ becomes a function of $V_1$, $Q_k \rho_k$ and $\sigma$. In this way the fundamental frequency model is improved including the effects of higher harmonics.

Results achieved for these test cases in [14] and in this chapter are in overall agreement.
6.6 Test Case

To test the improved TCSC dynamic phasor model, Two Area Test System in [33] has been slightly modified as shown in Figure 6.11. BUS8 has been split into BUS8 and BUS12 and in the line between TCSC has been placed between these nodes. Model Parameters of the system are taken from [33]. For the TCSC, same parameter set is used as in the previous Section.

![Figure 6.11: Single line diagram of the modified Two Area System](image)

Three simulations have been carried out. First one is the detailed EMT simulation, where the component models of the two area system has been implemented and simulated with PLECS [32]. Second simulation contains the fundamental frequency models of all components including the fundamental frequency model of the TCSC. The simulations with the fundamental frequency models of the components has been carried out in the implemented MATLAB simulation framework. Finally, in the third simulation, also the fundamental frequency models has been used except for the TCSC. The fundamental frequency TCSC model has been replaced by the improved dynamic phasor model.

In test studies carried out, the simulation starts from steady state operation. At $t = 0.2$ seconds, the firing angle of the TCSC is changed from $68.3^\circ$ to $59^\circ$. The firing angle is set back to its original value after 0.5 seconds at $t = 0.7$ seconds.

The evolution of electrical torque $T_e$ of synchronous machine GEN1 for all three simulations is depicted in Figure 6.12. The results of EMT simulation and the one with improved dynamic model of the TCSC are in overall agreement. Simulation results with the fundamental frequency model of the TCSC differ from the others.
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If CPU simulation times are compared, the simulation with the improved dynamic phasor TCSC model is much faster than with the detailed EMT models by having nearly the same degree of accuracy.
Chapter 7

Optimization of Numerical Integration Methods for the Dynamic Phasors

The numerical calculation of the system trajectory is a major part in the simulation of dynamical systems. Various numerical integration methods are applied for the calculation of the trajectory of dynamic systems. Regardless of the selected variable representation, in most of the cases, numerical integration techniques such as forward-Euler, backward-Euler, trapezoidal or Gear’s method are employed. This chapter focuses on the derivation of methods suitable for the numerical integration of systems represented by dynamic phasors by investigating the numerical integration techniques in the frequency domain.

7.1 Introduction

Various numerical integration methods are applied for the numerical calculation of power system dynamics. Regardless of the selected variable representation, which have been discussed in Chapter 3, in most of the cases, numerical integration techniques such as forward-Euler, backward-Euler, trapezoidal, Gear’s method are employed. Generally, in three phase instantaneous value representation for modelling of power systems, the integration step size is very limited. Even if fast transients
have decayed, the sinusoidal AC quantities of the electrical grid with the system frequency $f_s$ cannot be integrated with the maximum possible integration step size $h_{max} = \frac{1}{2f_s}$, due to the lack of accuracy of the used numerical integration methods at the system frequency. The most commonly used integration methods, such as forward-Euler, backward-Euler, trapezoidal, Gear’s method, are optimal for the numerical integration of low frequency signals.

To overcome this problem in [46] the authors introduced another integration method adapted to the periodic steady state at $f_s$. This method allows the use of larger step sizes for the numerical integration of three phase instantaneous electrical AC quantities.

Another approach is to use the so called root-matching method [47]. Numerical integration techniques generally discretize a continuous system $H(s)$ by mapping it into an equivalent discrete time system $H(z)$. The optimum discretization method should match the poles, zeros and final value of the difference equation to those of the actual continuous system. If these conditions are fulfilled, the difference equations are stable regardless of the step size, if the actual continuous system is also stable. This root-matching approach can be interpreted as an adjustment of the discretization (numerical integration) method to the eigenvalues of the system. This method has been successfully applied for the discretization of the RL, RC, etc. branch elements in EMTP instead of the traditionally employed trapezoidal discretization method.

Since we are using the dynamic phasor representation for power system modelling and simulation, our aim in this Chapter is to optimize the numerical integration method for variables represented by dynamic phasors. For instance, let us assume a linear time invariant dynamic system modelled as a set of first-order differential equations

$$\dot{x}(t) = A x(t) + B u(t)$$  \hspace{1cm} (7.1)

with an arbitrary sinusoidal forcing or source function $u(t)$ with frequency $k f_s$ and low frequency state $x(t)$. Assuming that the source function can be approximated by $u(t) \approx U_k(t) e^{j k \omega_s t}$, the dynamic phasor model of the system becomes

$$\dot{X}_k(t) = (A - j k \omega_s I) X_k(t) + B U_k(t)$$  \hspace{1cm} (7.2)

$X_k$ and $U_k$ are the new variables of the system and $I$ is the identity matrix. The eigenvalues of the new system modelled with dynamic phasors
7.2. Linear Multi-step Methods

are shifted by $-j k \omega_s$. This frequency shift gives rise to faster oscillations in the step response, which decay in stable systems, depending on the real part of the eigenvalues. These fast oscillations are inherent to dynamic phasor models. As mentioned previously, in the root matching method, the poles, zeros and final value of the discrete time system are matched to those of the actual continuous system. The idea now is to relax this condition of root-matching and to match the numerical integration not exactly to the eigenvalues of the dynamic phasors but approximately to the oscillatory frequencies $k \omega_s$ of the dynamic phasors $X_k$.

The Chapter is organized as follows. First a general overview of the numerical integration techniques will be given with the major focus on the linear multi-step methods. This is followed by the investigation of the numerical integration techniques in the frequency domain, which allows us to derive the desired frequency matched discretization methods. The accuracy and efficiency of these derived methods will be compared with the traditional numerical integration methods in a simple power systems example modelled with dynamic phasors.

7.2 Linear Multi-step Methods

Numerical integration techniques generally discretize a continuous system $H(s)$ by mapping it into an equivalent discrete time system $H(z)$ by using the discretization function (2.11) $\Psi$. For example a first order nonlinear ordinary differential equation of the form

$$\dot{x}(t) = f(x, t)$$

is mapped into a discrete time system by using a discretization function generally given as

$$x_{n+1} = \Psi (h_{n+1}, [x_n, x_{n-1}, \ldots], [f(x_{n+1}), f(x_n), \ldots])$$

The discretization function uses the previously computed values of the state variables and their derivatives to compute approximately the current value of the state variables. An integration method defined in equation (2.11) must satisfy certain criteria concerning:

- Numerical Accuracy
Numerical Accuracy: An integration method approximates the value at \( x(t_{n+1}) \) by the given function \( \Psi \) in equation (2.11). The error produced at each step can be defined as \( |x(t_{n+1}) - x_{n+1}| \), under the assumption that the numerical solution \( x_n \) at \( t_n \) coincides with the real solution \( x(t_n) \). This error introduced by advancing the solution from \( t_n \) to \( t_{n+1} \) in a single step is called also the local truncation error \( \varepsilon_l \). Numerical accuracy ensures that this error remains bounded during the simulation. Especially for the stationary solution the local truncation error should become zero.

Numerical Stability: The local error \( \varepsilon_l \) incurred at each step should not propagate to future times. Otherwise the error would increase and the numerical solution would diverge.

Numerical Efficiency: Major points influencing efficiency of an numerical integration algorithm are:

- Number of computational operations in function \( \Psi \) at each step.
- Number of executions of \( \Psi \) during the simulation, which mainly depends on the used step size \( h \).
- Memory needed to store the precalculated values needed for the evaluation of \( \Psi \).

After outlining some important criteria for practical numerical integration methods, we will focus on an important class of these methods, namely the linear multi-step methods. The most commonly employed numerical integration techniques such as the forward-Euler, backward-Euler, trapezoidal rule, Gear’s method belong to this family of numerical integration methods [24]. A linear multi-step method’s discretization function \( \Psi \) is a linear combination of previously computed values of the state variables \( x \) and their derivatives \( \dot{x} \).

A multi-step integration method of degree \( p \) has the form:

\[
x(t_{n+1}) \approx \sum_{i=0}^{p} a_i x(t_n - i h) + \sum_{i=-1}^{p} b_i \dot{x}(t_n - i h)
\] (7.3)
Methods with \( b_{-1} = 0 \) are called *explicit* methods, and methods with \( b_{-1} \neq 0 \) are called *implicit* methods. Several methods can be derived by pre-defining some of the coefficients in (7.3). For example by setting \( a_1 = a_2 = a_3 = \ldots a_p = 0 \), we will get Adam’s Methods.

- **Adam’s Bashforth Methods** \((b_{-1} = 0, p = k - 1)\)

  \[
x_{n+1} = a_0 x_n + \sum_{i=0}^{p} b_i f(x_{n-i})
  \]

- **Adam’s Moulton Methods** \((b_{-1} \neq 0, p = k - 2)\)

  \[
x_{n+1} = a_0 x_n + \sum_{i=-1}^{p} b_i f(x_{n-i})
  \]

where \( k \) is defined as the order of the numerical integration method. Adam’s Moulton Methods are implicit methods and therefore they have to be solved iteratively at every integration step. Iterative methods generally require an initial guess for the solution of nonlinear equations. A good initial guess reduces the number of required iterations for convergence which directly affects the computational efficiency of the method. Such an initial guess e.g. can be calculated by an explicit Adam’s Bashforth method of the same order.

\[
x_{n+1}^p = a_{0,p} x_n + b_{0,p} f(x_n) + b_{1,p} f(x_{n-1})
\]

with the coefficients \( a_{0,p} = 1 \), \( b_{0,p} = \frac{3}{2} \) and \( b_{1,p} = -\frac{1}{2} \). This process is often referred to as a predictor-corrector approach, since Adam’s Bashforth method predicts the solution and Adam’s Moulton method corrects the solution. The predictor-corrector approach is commonly employed in conjunction with local truncation error estimation, which is then used for an adaptive step-size selection. If a predictor-corrector approach with same orders \( k \) is used the local truncation error can be approximated as \( \varepsilon_l(t) = C \| x^p - x \| \), with \( x^p \) the predictor, \( x \) the corrector solution and \( C \) a constant dependent on the order of used method.

Our focus in this section will be on the most frequently used 2\(^{nd} \) order Adam’s Moulton method, known also as trapezoidal method, on which we will base our new frequency matched methods. The trapezoidal method is also commonly used in the simulation of electromagnetic transients [4]. As our aim is to derive a method which is matched to the
oscillatory frequencies $k\omega_s$ of the dynamic phasors $X_k$, it is convenient to investigate the numerical integration techniques in the frequency domain.

In the case of $2^{nd}$ order Adam’s Moulton method the time domain approximation of $x(t+h)$ is given by the following general expression.

$$x(t + h) \approx a_0 x(t) + b_{-1} \dot{x}(t + h) + b_0 \dot{x}(t)$$  \hspace{1cm} (7.5)

or in discrete form

$$x_{n+1} = a_0 x_n + b_{-1} f(x_{n+1}) + b_0 f(x_n)$$  \hspace{1cm} (7.6)

In the case of the trapezoidal method the coefficients are $a_0 = 1$ and $b_0 = b_{-1} = \frac{h}{2}$. In the following we want to focus on the error analysis in the frequency domain and numerical stability of the trapezoidal method.

### 7.2.1 Error Analysis in Frequency Domain

The generalized local truncation error of the $2^{nd}$ order Adam’s Moulton method is given as

$$\varepsilon_l(t + h) = x(t + h) - [a_0 x(t) + b_{-1} \dot{x}(t + h) + b_0 \dot{x}(t)]$$  \hspace{1cm} (7.7)

The Laplace transformation of the local truncation error yields

$$\Sigma_l(s) = X(s) \left[ e^{sh} - a_0 - b_{-1}se^{sh} - b_0 s \right] E_l(s)$$  \hspace{1cm} (7.8)

Generally the numerical integration methods are optimized for DC stationary solutions meaning that the local truncation error becomes zero at $s = 0$ ($E_l(0) = 0$). This is also observable in Figure 7.1, which depicts the frequency response of the local error function $E_l(s)$ for the trapezoidal method. The derivation of the trapezoidal method can also be accomplished in the frequency domain. The coefficients $a_0$, $b_{-1}$ and $b_0$ in the general formulation of the $2^{nd}$ order Adam’s Moulton can also be determined from the local error function in the frequency domain $E_l(s)$ in such a way, so that the local error function $E_l(s)$ has a triple root at DC ($s = 0$). This leads to the solution of the following three
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The solution yields the well known coefficients of the trapezoidal method namely \( a_0 = 1 \), \( b_{-1} = \frac{h}{2} \) and \( b_0 = \frac{h}{2} \). Such an investigation of the local truncation error in the frequency domain opens new possibilities to derive new numerical integration methods which are not optimized at DC but at also other frequencies.

7.2.2 Numerical Stability Analysis

Numerical Stability is one of the most important properties of the numerical integration methods. Numerical stability ensures that the local truncation error remains bounded in subsequent integration steps. The standard method for testing numerical stability is to investigate how the eigenvalues of the continuous time system (s) are mapped to the eigenvalues of the discrete time system (z). Numerical stable methods map the stable eigenvalues of the continuous time system with \( \Re \{ s \} < 0 \) to eigenvalues in discrete time system with \( |z| < 1 \). For the 2\(^{nd}\) order Adam’s Moulton method, this can be calculated by replacing \( z = e^{s \cdot h} \).
in (7.8) and solving $E_l(s) = 0$ in $z$ yielding

$$ z = \frac{a_0 + b_0 s}{1 - b_{-1} s} $$

Thus, the stability condition becomes

$$ |z| = \left| \frac{a_0 + b_0 s}{1 - b_{-1} s} \right| < 1 \quad (7.12) $$

In case of the trapezoidal method with pure real coefficients and $s = \sigma + j\omega$ the stability condition yields

$$ (a_0 + b_0 s)(a_0 + b_0 s)^* < (1 - b_{-1} s)(1 - b_{-1} s)^* $$

$$ 1 + \frac{h^2}{4} |s|^2 + h \Re \{s\} < 1 + \frac{h^2}{4} |s|^2 - h \Re \{s\} $$

$$ \Re \{s\} < 0 $$

$$ \sigma < 0 $$

This means, that stable continuous time systems with $\Re \{s\} < 0$ are mapped directly to stable discrete time systems with $|z| < 1$ regardless of the selected step size $h$. Such numerical integration methods mapping stable continuous time systems to stable discrete time systems are called absolute stable methods or A-stable methods. Figure 7.2 shows $|z|$ as a function of step size $h$ and $\Re \{s\}$. We also observe that $0 < |z| < 1$, which is the condition for numerical stability. After the local truncation error analysis in the frequency domain and the stability region analysis of the trapezoidal method, the focus in the following sections will be on the frequency matched methods and their error and stability region analysis.

### 7.3 Frequency Matched Trapezoidal Method for Real Signals

If the trapezoidal method is used for the numerical integration of sinusoidal electrical AC quantities with system frequency (e.g. 50 Hz), they cannot be integrated with the maximum possible integration step size due to the lack of accuracy of the trapezoidal methods at the system frequency. As Figure 7.1 shows, the local truncation of the trapezoidal
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![Figure 7.2: Region of Stability (|z| < 1) of the Trapezoidal Method as a function of the step size $h$ and the real part of the eigenvalue $\sigma = \Re\{s\}$](image)

method at 50 Hz is much higher than at DC frequencies, so that the step size must be kept small enough to achieve the same degree of accuracy at 50 Hz.

To overcome this problem, in [46] the authors propose the use of a new numerical integration method which is adapted to the periodic steady state of the electrical AC quantities. With this method, it is possible to compute the sinusoidal AC state variables with angular frequency $\omega_s$ at steady state exactly without introducing a local truncation error regardless of the selected step size $h$. This is achieved by investigating the local truncation error $\varepsilon_t(t)$ in the frequency domain. The investigation of the local error function in the frequency domain opens new possibilities to derive new numerical integration methods yielding minimum local truncation error not only at DC but also at other frequencies. The coefficients $a_0$, $b_{-1}$ and $b_0$ can be calculated in such a way that the roots of $E_t(s)$ in (7.8) are placed at frequencies other than DC. As stated in [46], the coefficients $a_0$, $b_{-1}$ and $b_0$ can be calculated in such a way that the roots of $E_t(s)$ are placed at $-j\omega_s$, 0 and $j\omega_s$. The new coefficients $a_0$, $b_{-1}$ and $b_0$ are determined by solving the following
three equations.

\[ E_l(0) = 1 - a_0 = 0 \]  \hspace{1cm} (7.13)

\[ E_l(j \omega_s) = e^{j \omega_s h} - a_0 - j b_{-1} \omega_s e^{j \omega_s h} - j b_0 \omega_s = 0 \]  \hspace{1cm} (7.14)

\[ E_l(-j \omega_s) = e^{-j \omega_s h} - a_0 + j b_{-1} \omega_s e^{-j \omega_s h} + j b_0 \omega_s = 0 \]  \hspace{1cm} (7.15)

The solution of (7.13-7.15) yields

\[ a_0 = 1 \]  \hspace{1cm} (7.16)

\[ b_0 = \frac{j}{\omega_s} \frac{1 - e^{j \omega_s h}}{1 + e^{j \omega_s h}} = \frac{1}{\omega_s} \tan \left( \frac{\omega_s h}{2} \right) \]  \hspace{1cm} (7.17)

\[ b_{-1} = \frac{1}{\omega_s} \tan \left( \frac{\omega_s h}{2} \right) \]  \hspace{1cm} (7.18)

With equation (7.6) and the new coefficients \( a_0 \), \( b_{-1} \) and \( b_0 \) the frequency matched trapezoidal method is fully defined. In the following, a local truncation error analysis will be performed on the new method in the frequency domain.

### 7.3.1 Error Analysis in Frequency Domain

Figure 7.3 shows the frequency response of the local truncation error for the frequency matched trapezoidal method for real variables at \( \omega_s = 2 \pi 50 \text{ s}^{-1} \) for different step sizes \( h \). The desired frequency response of

![Figure 7.3: Error frequency response \( E_l(s) \) of Matched Trapezoidal for real variables](image)
the local error function with three roots at $-j\omega_s$, 0 and $j\omega_s$ is observable. The local truncation error is zero at the matched frequencies regardless of the used step size, so that DC quantities and quantities with 50 Hz are computed without any error, which is the case in a power system at steady state without additional harmonics.

During electromechanical transients, however the frequency content of the state variables of the electromechanical system will not be exactly at the DC but in a narrow bandwidth around DC with a half-bandwidth of typically 2-3 Hz. Figure 7.4 shows the error frequency response of the trapezoidal method (denoted with the abbreviation TR) and of the frequency matched trapezoidal method for real signals (denoted with the abbreviation TR-MR). The frequency matched method shows a

![Figure 7.4: Error frequency response $E_l(s)$ of trapezoidal and frequency matched trapezoidal method for real signals with $h = 0.001$ and $\omega_s = 2\pi 50$](image)

lack of accuracy around DC compared with the original method, hence the same degree of accuracy can only be achieved with smaller step sizes. However around the matched oscillatory frequency (50 Hz), the frequency matched method is much more accurate than the original trapezoidal method. As also reported in [46], in a system with a three-phase instantaneous value representation, the electrical AC quantities can be computed more efficiently with the frequency matched trapezoidal method than with the trapezoidal method.

After analyzing the accuracy of the frequency matched method and comparing it with the trapezoidal method, in the following, we will investigate the numerical stability of the matched method.
7.3.2 Numerical Stability Analysis

As discussed in Section 7.2.2, the general condition for numerical stability of the 2\textsuperscript{nd} order Adam’s Moulton method is given in (7.12). In the case of the frequency matched trapezoidal method, the coefficients are also real valued. Thus the stability condition becomes

\[
(a_0 + b_0 s) (a_0 + b_0 s)^* < (1 - b_{-1} s) (1 - b_{-1} s)^*
\]
\[
a_0^2 + b_0^2 |s|^2 + 2 b_0 \Re \{s\} < a_0^2 + b_{-1} |s|^2 - 2 b_{-1} \Re \{s\}
\]
\[
b_0 \Re \{s\} < 0
\]
\[
b_0 \sigma < 0
\]

For stable systems (\(\sigma < 0\)) the condition for numerical stability becomes

\[
\frac{1}{\omega_s} \tan \left( \frac{\omega_s h}{2} \right) > 0
\]
\[
k \pi < \frac{\omega_s h}{2} < (2k + 1) \frac{\pi}{2}
\]
\[
k \frac{f_s}{2} < h < \frac{2k + 1}{2 f_s}
\]

with \(k \in \mathbb{N}\). The expression for \(k = 0\), namely \(h < \frac{1}{2 f_s}\), reflects the Nyquist sampling theorem.

Figure 7.5 depicts \(|z|\) as a function of step size \(h\) and \(\Re \{s\}\). In contrast to the trapezoidal method, the stability range of the frequency matched trapezoidal method for real signals is limited. We observe that \(|z| < 1\) is only valid for step sizes \((h)\) fulfilling the condition 7.19. However, the step size can be increased till \(h_{\text{max}} = \frac{1}{2 f_s}\) as dictated by the Nyquist criterion. If the used step size is outside of the valid stability region (e.g. \(\frac{1}{2 f_s} < h < \frac{1}{f_s}\)), the stable continuous system is mapped to an unstable discrete time system. Thus the matched trapezoidal method for real signals is not an A-stable method.

The described method has been successfully employed for the simulation of power system transients represented with three-phase instantaneous values, where the steady state is described by the periodic AC quantities [46]. The same method can also be applied to systems represented by dynamic phasors. In contrast to the three-phase instantaneous values, the dynamic phasor quantities are constant at steady state. Thus, the
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Figure 7.5: Region of Stability ($|z| < 1$) of the Trapezoidal Method matched at $-j\omega_s$, 0 and $j\omega_s$ as a function of the step size $h$ and the real part of the eigenvalue $\sigma = \Re\{s\}$

The traditional trapezoidal method (and also other linear multi step methods such as backward Euler or Gear’s Method) can be used for the numerical integration of dynamic phasor quantities. The dynamic phasor representation is a baseband representation so that DC optimized (matched) numerical integration methods are accurate enough after fast transients have decayed. However as discussed previously, during fast transients with high frequencies, the trapezoidal method gets inaccurate and inefficient. The main idea of using such a frequency matched numerical integration is to increase the simulation accuracy and efficiency during the fast transients inherent to the dynamic phasors approach due to the shift of the eigenvalue’s oscillatory frequency as mentioned in Section 7.1.

The decreased accuracy of the frequency matched method around DC is not that critical, if it is used in conjunction with three-phase instantaneous values, as they are not DC at steady state but rather periodic with system frequency $f_s$. However, the use of the frequency matched method in conjunction with dynamic phasors requires a high accuracy around oscillatory frequencies $k\omega_s$ and DC.
In the following Section, we will derive another frequency matched method based again on the trapezoidal method, which is more accurate and numerically A-stable.

7.4 Frequency Matched Trapezoidal Method for Complex Signals

In the previous Section, a method was derived, which is optimized for the numerical integration of real sinusoidal signals with frequency $f_s$ or frequencies around $f_s$. However as discussed in Section 3.3, real bandpass signals can also be represented by their analytical counterparts, as they contain all the information about the original bandpass signal. The major advantage of using an analytical or complex signal is that the bandwidth of the original signal is reduced, as only positive (or negative) frequency components of the real signal are considered.

The frequency matched trapezoidal method described in the previous Section was derived for the numerical integration of periodic real bandpass signals. In this case the derivation of the frequency matched trapezoidal method was performed by placing two of the three roots of the local error in frequency domain at $-j\omega_s$ and $j\omega_s$ as real valued sinusoidal signals’ spectra with angular frequency $\omega_s$ are located at these frequencies. The representation of the real bandpass signals by their analytical counterparts brings one additional freedom in the placing of the roots, as the complex signal has only frequency component at positive or negative frequency. Hence, only one root has to be placed at the frequency $j\omega_s$ or $-j\omega_s$, the remaining two roots can be placed at DC.

In the following, the new method will be derived by using the same methodology as in previous sections, namely calculation of the coefficients $a_0, b_{-1}$ and $b_0$ in such a way that the two roots of $E_l(s)$ in (7.8) are placed at DC and one at $j\omega_s$. The new coefficients $a_0, b_{-1}$ and $b_0$ are determined by solving the the following three equations.

\[
E_l(0) = 1 - a_0 = 0 \tag{7.20}
\]
\[
\frac{dE_l(0)}{ds} = h - b_{-1} - b_0 = 0 \tag{7.21}
\]
\[
E_l(j \omega_s) = e^{j\omega_s h} - a_0 - j \omega_s e^{j\omega_s h} b_{-1} - j \omega_s b_0 = 0 \tag{7.22}
\]
7.4. Frequency Matched TR-Method for Complex Signals

The solution of (7.20-7.22) yields

\[ a_0 = 1 \] (7.23)

\[ b_0 = \frac{h}{2} + j \left( \frac{1}{\omega_s} - \frac{h}{2} \cot \left( \frac{\omega_s h}{2} \right) \right) \] (7.24)

\[ b_{-1} = \frac{h}{2} - j \left( \frac{1}{\omega_s} - \frac{h}{2} \cot \left( \frac{\omega_s h}{2} \right) \right) \] (7.25)

As a next step, an error analysis will be performed on the new frequency matched method and will be compared with to antecedently discussed methods.

7.4.1 Error Analysis in Frequency Domain

Figure 7.6 pictures the frequency response of the local truncation error for the frequency matched trapezoidal method for complex variables at \( \omega_s = 2 \pi \frac{50}{50} \text{s}^{-1} \) for different step sizes \( h \). The desired frequency response of the local error function with three roots double root at DC and a single root \( j \omega_s \) is viewable. The local truncation error is zero at the matched frequencies regardless of the used step size, so that DC quantities and complex periodic quantities with 50 Hz are computed without any error. Figure 7.7 shows the error frequency response of the trapezoidal method (denoted with the abbreviation TR), the frequency response of the local error function with three roots double root at DC and a single root \( j \omega_s \) is viewable.
matched trapezoidal method for real signals (denoted with the abbreviation **TR-MR**) and the frequency matched trapezoidal method for complex signals (denoted with the abbreviation **TR-MC**). As expected,

\[ |E_l(j\omega)| \]

![Figure 7.7: Error frequency response](image)

Figure 7.7: Error frequency response \( E_l(s) \) of trapezoidal, frequency matched trapezoidal methods for real and complex variables with \( h = 0.001 \) and \( \omega_s = 2\pi \times 50 \)

placing two roots at DC in the error frequency response \( E_l(s) \) increased the accuracy of the method for complex signals compared to the one for real signals. The frequency matched method for complex signals is more accurate than the matched method for real signals around DC and also around \( j\omega_s \).

Used in conjunction with predictor-corrector approach, the coefficients of the explicit predictor method (7.4) can be determined in the same way by placing two roots at DC and one root at \( j\omega_s \) yielding

\[
\begin{align*}
a_{0,p} &= 1 \\
b_{0,p} &= \frac{h}{2} + \frac{\sin(\omega_s h)}{\omega_s} + j \left( \frac{h}{2} \left( \frac{\sin(\omega_s h)}{1 - \cos(\omega_s h)} \right) - \frac{\cos(\omega_s h)}{\omega_s} \right) \\
b_{1,p} &= -\frac{h}{2} \frac{\sin(\omega_s h)}{\omega_s} + j \left( \frac{\cos(\omega_s h)}{\omega_s} - \frac{h}{2} \left( \frac{\sin(\omega_s h)}{1 - \cos(\omega_s h)} \right) \right)
\end{align*}
\]

### 7.4.2 Numerical Stability Analysis

Same procedures as in previous sections will be applied to define the region of stability of the new method. In the case of the frequency
7.4. Frequency Matched TR-Method for Complex Signals

matched trapezoidal method, the coefficients $b_0$ and $b_{-1}$ are complex valued but are also conjugate complex, meaning $b_{-1} = b_0^*$. With (7.23-7.25), the stability condition becomes

$$(a_0 + b_0 s) (a_0 + b_0 s)^* < (1 - b_{-1} s) (1 - b_{-1} s)^*$$

$$a_0^2 + |b_0 s|^2 + 2 \Re \{b_0 s\} < a_0^2 + |b_{-1} s|^2 - 2 \Re \{b_{-1} s\}$$

$$\Re \{b_0 s\} < -\Re \{b_{-1} s\}$$

$$\sigma < 0$$

As in the case of trapezoidal method, this method is also an A-stable method mapping stable continuous time systems ($\Re \{s\} < 0$) directly to stable discrete time systems ($|z| < 1$) irrespective of the selected step size $h$. Figure 7.8 depicts $|z|$ for the new method as a function of step size $h$ and $\Re \{s\}$.

Figure 7.8: Region of Stability ($|z| < 1$) of the Trapezoidal Method matched at 0, 0 and $j\omega_s$ as a function of the step size $h$ and the real part of the eigenvalue $\sigma = \Re \{s\}$

The described frequency matched method for complex signal is numerically stable and has also higher DC accuracy compared with the one for real signals. Thus, it can also be used in conjunction with the dynamic phasors.
In the next section, the accuracy and efficiency of these derived frequency matched methods will be compared to the traditional trapezoidal method in two simple examples.

7.5 Test Cases

In the previous sections the focus was on the derivation of the frequency matched methods. The aim of this section is to compare the accuracy and efficiency of these methods with each other and verify the previously made conclusions by simulations. In this section, the derived numerical integration methods will be used for the simulation of two test systems. As a first example, a system of ordinary differential equations with known analytic solution will be solved numerically by using the discussed numerical integration methods. Second example is the Single Machine Infinite Bus system, which has been also used in Section 4.4.

All three methods use a variable step-size selection algorithm based on the described local truncation error estimation with predictor-corrector approach. Simulations were obtained using Matlab 7.1 running on a Intel Pentium IV CPU with 3.80 GHz and 2 GB of RAM.

7.5.1 Example with Analytic Solution

In this case study, the traditional trapezoidal method, frequency matched trapezoidal methods for real signals and for complex signal will be used for the numerical integration of a 4\textsuperscript{th} order ordinary differential equation system, whose analytic solution is also known. The results of the three integration methods will be compared regarding accuracy and computational efficiency. The differential equation system under consideration is given as

\[
\begin{align*}
\dot{x}_1 &= -\sigma_1 x_1 - \omega_1 x_2 \\
\dot{x}_2 &= -\sigma_1 x_2 + \omega_1 x_1 \\
\dot{x}_3 &= -\sigma_2 x_3 - \omega_2 x_4 \\
\dot{x}_4 &= -\sigma_2 x_4 + \omega_2 x_3 \\
y_1 &= x_1 + x_3 \\
y_2 &= x_2 + x_4
\end{align*}
\]
with the parameter $\omega_1 = 2\pi 3$, $\omega_2 = 2\pi 60$, $\sigma_1 = 3$, $\sigma_2 = 6$ and the initial values $x_1(0) = 1$, $x_2(0) = 0$, $x_3(0) = 1$, $x_4(0) = 0$. The analytic solution of the initial value problem yields
\[
\begin{align*}
x_1 &= \cos(\omega_1 t) e^{-\sigma_1 t} \\
x_2 &= \sin(\omega_1 t) e^{-\sigma_1 t} \\
x_3 &= \cos(\omega_2 t) e^{-\sigma_2 t} \\
x_4 &= \sin(\omega_2 t) e^{-\sigma_2 t}
\end{align*}
\]

The system is integrated using the TR, TR-MR (matched at $-j\omega_2$, 0 and $j\omega_2$) and TR-MC (matched at $0^2$ and $-j\omega_2$). Figure 7.9 shows the results of the analytic solution and the numerical solution computed with three methods discussed in previous sections in the overall simulation interval $0 < t < 1.4$ and in a zoomed section with the required CPU simulation times. In the zoomed section we observe that the TR method has lack of accuracy during high frequency oscillations with $\omega_2 = 2\pi 60$, where the frequency matched methods (TR-MR,TR-MC) give almost the accurate results during this fast transients. But if the fast transients have decayed and the system behavior is governed by low frequency transients with $\omega_1 = 2\pi 3$, TR-MR method has less accurate results. However, the TR-MC method computes the results with high degree of accuracy during fast and slow transients. Comparison of the overall CPU simulation times of all three integration methods shows that the TR-MC is also the fastest method for this simple test case.

### 7.5.2 SMIB system

The Single Machine Infinite Bus (SMIB) system will be simulated as a simple power system example modelled dynamic phasors approach. The components of the system are modelled by dynamic phasors and are represented in the original three phases (ABC). Same test case has been also used in Section 4.4 for the simulation of unbalanced and asymmetrical conditions with dynamic phasors. The simulated system has overall 40 differential and 150 algebraic variables. In this test case, a single-phase to ground fault occurs at the BUS2 end of the LINE3 at 0.1 seconds and is removed after 0.20 seconds by disconnecting the line. It is important to mention, that the approximating set of Fourier coefficients is selected as $K = \{1\}$ for the transmission line model and $K = \{0, 1, 2\}$ for the synchronous machine model, as unbalanced conditions are of
The dynamic behavior of the test case is simulated by using the trapezoidal method and frequency matched trapezoidal methods for real and complex signals by using the same degree of accuracy for the three methods (error tolerance $10^{-3}$).

First simulation is done by using the traditional trapezoidal method (TR) for all dynamic phasors $X_k$. In the second simulation, the dynamic phasors with $k = 0$ are discretized using TR and with $k > 0$ using TR-MR with the frequencies matched to $-j k \omega_s$, 0 and $j k \omega_s$. In the third simulation, the dynamic phasors with $k = 0$ are discretized using TR and with $k > 0$ using TR-MC with the frequencies matched to $-j k \omega_s$ and 0$^2$ (with double root at 0).
7.5. Test Cases

Figure 7.10: Evolution of the positive sequence electrical torque $T_{e,0}$ computed with TR, TR-MR and TR-MC (all with $\varepsilon_{tol} = 10^{-3}$) compared with the detailed simulation (TR with $\varepsilon_{tol} = 10^{-6}$) and required CPU simulation times

<table>
<thead>
<tr>
<th></th>
<th>DT</th>
<th>TR</th>
<th>TR-MR</th>
<th>TR-MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time [s]</td>
<td>186.125</td>
<td>25.410</td>
<td>49.750</td>
<td>26.045</td>
</tr>
</tbody>
</table>

Figure 7.10 shows the evolution of the positive sequence electrical torque $T_{e,0}$ computed with TR, TR-MR and TR-MC compared with the detailed simulation (DT), which is computed with TR method by keeping the error tolerance quite low ($10^{-6}$), which ensures an accurate simulation. The table in Figure 7.10 shows also the required CPU simulation times with different integration methods.

In the zoomed section in Figure 7.10 with fast transients around $\omega_s$, the higher accuracy of the TR-MR and TR-MC methods compared with TR
method is observable, where the TR-MR and TR-MC methods are in overall agreement with the detailed simulation results. The increased accuracy and efficiency of the TR-MC method at DC (also at $\omega_s$) becomes noticeable in the comparison of the overall CPU simulation times. TR-MC method is two times faster than the TR-MR method due to its increased accuracy also for DC frequencies and as fast as the TR method in this test case.
7.6 Summary

In this Chapter, a numerical integration method based on the trapezoidal method is derived for the simulation of systems modelled with dynamic phasors by minimizing the local truncation error around the oscillatory frequency ($k\omega_s$) of the dynamic phasors ($X_k$). This was achieved by investigating the local truncation error of the trapezoidal method in the frequency domain and placing the roots of the error function at the desired matched frequencies $-jk\omega_s$, 0 and $jk\omega_s$ so that the real sinusoidal signals at $k\omega_s$ can be computed without any error regardless of the selected step size. The use of complex/analytic signal representation facilitated the improvement of the numerical accuracy, efficiency and stability of the derived method by placing a double root at 0 and only one root at $jk\omega_s$ or $-jk\omega_s$ compared with frequency matched methods used for real signals.

The frequency matched method shows a better performance regarding accuracy and efficiency during fast transients with the frequencies around $k\omega_s$ than the trapezoidal method. During slow transients with low frequencies, the frequency matched trapezoidal method for complex signals is computationally more efficient the trapezoidal method for real signals, due to its decreased error around DC. Besides its accuracy and efficiency, another important property of the frequency matched method for complex signals is its numerical A-stability, where the frequency matched method for real signals does not have this property.
Chapter 8

Conclusions and Outlook

In this thesis, a new simulation tool has been developed and implemented which enables the combined simulation of the electromagnetic and electromechanical transients by using the dynamic phasor representation of the power system and its components. The main idea behind the dynamic phasors approach is to represent the system quantities by their time varying Fourier coefficients. The system quantities have generally a bandpass characteristic with their frequency contents centered around the system frequency or multiples of the system frequency, whereas the time varying Fourier coefficients are baseband/lowpass quantities around DC. Thus the frequency content of the simulated quantities is decreased, which allows the use of larger step sizes during simulation process and thus increases the simulation speed.

For this purpose, a systematic and general simulation framework based on the Differential Switched Algebraic State-Reset (DSAR) representation of hybrid systems, has been developed facilitating a modular and flexible model development. Based on this general simulation framework, the dynamic phasor models of major power system components, e.g. synchronous machines, doubly-fed induction generator (DFIG), thyristor-controlled series capacitor (TCSC), etc. have been developed and implemented with the new simulation tool. With the dynamic phasors approach, besides the fundamental frequency component also other harmonics can be included in the model derivation. Such harmonics occur for example in the TCSC model equations due to the periodical switchings of the thyristors or also in the electrical machine equations.
in the DQ0 reference frame due to the negative and zero sequence components during unbalanced conditions. The simulation performance of these models regarding accuracy and efficiency has been compared systematically to that traditionally achieved with electromagnetic transients programs. Simulations have shown that the dynamic phasor models have nearly the same degree of accuracy compared with the detailed time-domain models if appropriate approximations are made in the model derivation. In terms of computational speed, the dynamic phasor models proved to be more efficient than the detailed time-domain models. Simulations with dynamic phasors were 5-10 time faster than with the detailed time domain models, by maintaining a high degree of accuracy.

The systematic derivation and formulation of the dynamic phasor approach based on their original detailed time-domain models allow also some model simplification depending on the nature of study. In this way, reduced order dynamic phasor models have been derived which are in a way equivalent to the fundamental frequency models used in transient stability programs neglecting the fast network-side fast electromagnetic transients. With such reduced order dynamic phasor models it was possible also to include other harmonics in the model derivation like in the detailed dynamic phasor models but neglect their fast electromagnetic transients. With this systematic and appropriate order reduction, we were able to simulate unbalanced conditions even with fundamental frequency equivalent models efficiently.

Numerical integration techniques and algorithms play a significant role in the simulation of dynamical systems such as power systems. Therefore efforts also have been made on the derivation of methods suitable for the numerical integration of systems represented by dynamic phasors by investigating the numerical integration techniques in the frequency domain. The derived frequency-matched trapezoidal method for complex signals increased the accuracy and the efficiency of the simulations with systems represented by dynamic phasors compared with the commonly used trapezoidal method.

The prototype of the simulation tool has been implemented in MATLAB. The basic steps have been taken to implement the developed models and algorithms also in the commercial simulation program NEPLAN [1] for power system analysis. In NEPLAN, the dynamic phasor models in the ABC reference frame are implemented.
Outlook

The main purpose of the thesis was to develop an appropriate prototype of a new simulation tool for the simulation of power system dynamics with the dynamic phasor models of the power system components. Next step will be to complete the started integration of the developed simulation framework and the derived dynamic phasor models in NEPLAN, which is a full commercial software program for power system analysis.

The future work in the scope of this thesis can include the following subjects.

- In terms of existing models, the existing library of dynamic phasor models should be extended by other commonly used power system components such as other FACTS devices (SVC, HVDC, UPFC, etc.), detailed transformer models etc. In the derived DFIG dynamic phasor model, our focus was more on the component itself and not on the complete wind turbine including also the controls (e.g. pitch control) or protections (e.g. crow bar protection). The derived DFIG model must be augmented by the commonly used control and protection models.

- As mentioned in Chapter 1, one commonly applied approach to combine the accuracy of the EMTP and the computational efficiency of the TSP is to model and simulate some parts of the system with a detailed full time domain representation (EMTP) and the rest of the system in the quasi-steady state fundamental frequency representation (TSP) and interface the different modelling approaches appropriately. The drawback of this approach is that only the fundamental frequency components can be interfaced in this algorithm and other harmonics are omitted. This idea can be improved by using the dynamic phasor models instead of the fundamental frequency models in this approach as also other harmonics can also be included in the dynamic phasor models.

- In terms of the numerical algorithms optimized for dynamic phasors, the combined usage of the here derived frequency matched trapezoidal method for complex signals with traditionally used methods such as the Trapezoidal, Gear’s method etc. can further improve the simulation performance. The frequency matched
trapezoidal method for complex signals can be employed right after large disturbances in the system and can be used for the numerical integration of the fast oscillations inherent to the dynamic phasor models. But if these fast oscillations have decayed one could switch back to the traditional methods which are optimized for the integration of signals with frequencies near to DC.

- In terms of the numerical algorithms of the simulation framework, the general simulation and modelling framework developed here is based on the fully implicit and nonlinear representation of the system and model equations, so that also the overall numerical integration is based on implicit and iterative methods. Advantages of linear systems can be incorporated in the simulation process by employing computationally more efficient methods for the linear part of the system.
Bibliography


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