Is the Environment Compatible with Growth? Adopting an Integrated Framework

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Is the Environment Compatible with Growth? Adopting an Integrated Framework

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Is the Environment Compatible with Growth? Adopting an Integrated Framework

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Abstract

The paper develops an integrated baseline model to assess the trade-offs between the natural environment and economic growth. Consumption growth is considered under welfare and sustainability aspects. The framework features capital accumulation and the sectoral structure of the economy as key elements to cope with resource scarcity and pollution. Model extensions varying the number of sectors and inputs, changing central functional forms, and introducing poor input substitution and population growth are presented. The setup highlights the dual role of used inputs as a source of environmental problems and a part of the solution; it also discusses uncertainty and momentum effects. The paper concludes that the environment and economic growth can be compatible but that small deviations from the optimal paths may entail unsustainable development. Critical issues for sustainability are insufficient foresight, increasing damage intensity, and suboptimal policy making while population growth and poor input substitution are not necessarily precarious for future development.

Keywords: Natural environment, endogenous growth, multisector model, poor substitution, population growth.

JEL Classification: Q43, O47, Q56, O41

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1 Introduction

The natural environment and the economy form two systems which are closely interlinked. However, they have very distinct characteristics and are guided by different mechanisms. The question about the systems' boundaries has attracted most attention in the literature. Natural scientists have stressed that the environment has clear limits: The surface of the planet is given, important resource stocks are exhaustible and increasingly depleted, other resources regenerate according to biological laws but cannot grow beyond a certain threshold. Ecological systems tend to stationary equilibria but never to unlimited growth. On the contrary, economic growth has often been characterized by exponential functions for key macroeconomic variables like income and consumption, abstracting from any physical limits. In many models, economic development with ever increasing income is predicted, even in the long run. Hence one question appears to be obvious: How can a finite planet potentially host an infinite world economy?

The message of the "limits to growth" proponents (Meadows et al. 1972) was that bounded resource supply will ultimately limit total world income. Following the Malthusian arguments which focused on limited food supply it was argued that ongoing economic growth is not compatible with limited natural resources such as fossil fuels. Contradicting this claim, the economic discipline stressed the importance of man-made inputs as a substitute for natural inputs and the ability of markets to cope with scarcities in general. It argued that the efficiency of the market system dealing with shortages would also apply to natural scarcities. The first wave of capital-resource models focused on physical capital as a substitute for exhaustible resources and featured capital accumulation as the driver of economic growth (Dasgupta and Heal 1974, Solow 1974, Stiglitz 1974). The subsequent discussion between the economists and the ecologists centered around the extent of substitution possibilities between the different inputs\(^2\) and the role of (exogenous) technical progress, raising total factor productivity.\(^3\) Another debated issue was the dependence of capital on materials which are limited in supply, restricting capital accumulation in the long run. Under the heading of "dematerialization" of the economy it was concluded that ongoing capital accumulation will have to increasingly rely on capital types which can be decoupled from materials, such as human and knowledge capital. Recently, there has also been a renewed interest in the link between economic growth and individual well-being, which should also be addressed from the viewpoint of economics.

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\(^2\)The classical one-sector model with capital and resources as inputs requires good input substitution (i.e. the substitution elasticity exceeding unity) for consumption being sustained in the long run, see Dasgupta and Heal (1974).

\(^3\)Stiglitz (1974) shows in a classical capital resource model with an input substitution elasticity of unity (a moderately optimistic assumption) that consumption can be sustained in the long run if the rate of (exogenous) resource-augmenting progress is sufficiently high.
The early (but not broadly acknowledged) contribution of Suzuki (1976) opened a research avenue focusing on a broad definition of capital, distinguishing between physical and knowledge capital. Every capital type is accumulated via endogenous investments, according to costs and benefits. Interpreting capital in a broad way, returns to overall capital may become constant so that the incentives for capital accumulation do not vanish over time like in the seminal Ramsey growth model. So-called "new growth theory" (Romer 1990, Rebelo 1991) was developed independently of Suzuki-type resource economics but applied a very similar setup to mainstream growth theory, identifying the main drivers of economic development in the long run. It has significantly enriched the study of possible trade-offs between environment and growth. Using the new dynamic setup, various contributions have added the important aspects of environmental pollution (Bovenberg and Smulders 1995 and 1996), natural resource scarcity (Scholz and Ziemse 1999, Barbier 1999, Schou 2000, Groth and Schou 2002, Grimaud and Rouge 2003, Peretto and Valente 2015), directed technical change (Smulders and de Nooij 2003, di Maria and Valente 2008, and Grimaud and Rouge 2008, Pittel and Bretschger 2010, Acemoglu et al. 2012), and poor input substitution (Bretschger 1998, Bretschger and Smulders 2012, Bretschger 2015). In these papers, it is analyzed under which conditions the economy can still grow in the long run, despite the boundaries imposed by limited natural resource supply.

Climate change and optimal climate policy pose another challenge for endogenous growth theory in the context of environmental restrictions. The accumulation of greenhouse gases causes damages to the economy which may occur more or less regularly or may hit in the form of larger climate shocks. In analogy to the analysis of exhaustible resources the private sector and environmental policy can provide compensating forces in the form of capital investments, emission abatement, and advancement of clean and renewable energies. Policy has a much bigger importance compared to resource scarcity because climate change arises due to negative externalities constituting a large market failure. The tradeoffs between economic growth and the environment have to be studied in a framework including stock pollution; contributions to this literature include numerical simulation models (Nordhaus and Boyer 2000, Stern 2007) and theoretical papers (Michel and Rotillon 1995, Bretschger and Valente 2011, Bretschger et al. 2011, Van der Ploeg and Withagen 2014, Bretschger and Suphaphiphat 2014, Bretschger and Karydas 2016, and Bretschger and Vinogradova 2016). These papers derive the optimal responses to the climate challenge in different model setups featuring various forms of environmental impacts and technological options available to the society.

Economic contributions dealing with the trade-offs between environment and growth

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\[ Y' > 0, \quad Y''(K_t) < 0, \quad \text{and } Y'(\infty) = 0, \] which imposes an upper limit to \( Y \) and thus limits to growth due to ever decreasing marginal returns to capital.
should provide a representation of the balance between environmental restrictions, such as resource scarcity and pollution, and counteracting economic forces, e.g. in the form of investment and abatement. Limited resource supply and the pollution of the air, the atmosphere, the soils, and the water are the main reasons why environment and growth may not be compatible. Natural resource inputs will decrease in the future because of their scarcity or because of their polluting impact. As a consequence, if current welfare levels should be preserved or increased, the economy needs to substitute man-made inputs for natural resources. The input which is accumulable is capital, in its different forms of physical, human and knowledge capital. Hence, the effects of capital accumulation and the incentives for savings lie at the heart of substitution of scarce natural resources. Importantly, limits to growth are present in these theories as a possibility, but usually not as a necessity. Among the central reasons for an optimal growth slowdown or optimal negative growth are the impatience of individuals leading to too low savings, inappropriate or lacking environmental policies, too low abatement efficiency, and decreasing returns to accumulable inputs. It may be an optimal outcome to slow down or even to revert the growth process when the underlying conditions, including resource scarcity and climate change, are not favorable for further development. The case of "optimal degrowth" can thus also be studied using standard economic analysis.

Based on a normative assessment of economic growth under environmental restrictions the present paper develops a baseline model integrating resource scarcity, climate change, and endogenous growth in a single tractable framework. It provides closed-form analytical solutions and discusses the intuition behind the results. The model includes the recent advances of endogenous growth theory and focuses on the endogenous accumulation and depreciation of capital as key elements for the study of sustainability. Capital is built up in a separate sector of the economy, which is a central feature of the baseline model. The sectoral structure of the economy and structural change are shown to be important for the substitution of natural inputs and for pollution mitigation. Labor input is analyzed in separate later sections of the paper. The paper discusses many possible model extensions of the baseline model and shows the consequences. In each case I derive how the economic effects of environmental restrictions critically depend on the assumed functional relationships. Because the field is very broad the paper has to present a selection which aims to complement the already existing surveys of the literature which are provided in Smulders (1999), Brock and Taylor (2005), and Xepapadeas (2005).

The basic question whether nature and growth are compatible or not will be answered, not unexpectedly, by the usual economic phrase "it depends." For the non-specialist it

5In the traditional capital-resource models decreasing returns to capital cause a slowdown of voluntary savings and investments preventing the economy from reaching a sustainable path while enforced constant savings on a level given by the so-called "Hartwick rule" ensure constant consumption in the long run.
might be a bit surprising how many different model variants can be used to address the fundamental questions about sustainability. And there is even a wider variety of literature on the topic which cannot be included due to space constraints. Nevertheless, the paper at hand aims to determine in precise terms on which determinant and to which extent the general conclusions apply. Moreover, it will provide a general assessment of the critical issues affecting the outcome in a positive or in a negative way.

The remainder of the paper is organized as follows. In section 2 I provide the normative background for an assessment of economic growth. Section 3 develops and discusses the baseline model and extensions. In section 4, poor input substitution is introduced in the analysis. Section 5 adds additional sustainability topics and section 6 concludes.

2 Desirable growth

Whether economic growth is desirable or not was subject of a public debate which has recently gained broader attention, at least in the developed countries. Given the possible attractiveness of "degrowth" (Kallis et al. 2012) it appears advisable to provide a short welfare-based foundation of the subsequently studied mechanics of dynamic economic systems. Focusing on the normative aspect of economic growth may help to build bridges between the mainstream economic view and its critiques. I start by specifying the term "growth" and by limiting the case of the paper to what I call "desirable" growth.

For policy makers and the public, economic growth is usually defined by the annual growth rate of GDP, the statistically determined income of each country. It is well known that the GDP ignores the natural environment and other central determinants of well-being such as distribution, security, and health. In order to get a useful policy guideline serving the purpose growth must be defined in a broader sense. The final aim of all economic activities is to satisfy the needs of the economic agents; hence, one needs to define economic growth as the growth of well-being, which is captured by utility functions. That individual utility remains constant or increases over time is a broadly applicable criterion to define sustainable states in economics (Asheim et al. 2001).

When formulating concrete models one has to limit the number of arguments of the utility function in a sensible way. I first consider the function

$$U_t = U(C_t, E_t, X_t)$$  \hspace{1cm} (1)

where individual utility $U$ is determined by consumption goods summarized by $C$, ecosystem services $E$, and other factors such as security and health, summarized in $X$; $t$ is the time index. $U$ is assumed to grow in all the three arguments of the function. \(^7\) Growth of utility

\[^6\]Stiglitz et al. (2010)

\[^7\]Because income distribution is not a focus $U$ is taken to be the utility of a representative agent.
is given by taking log differentials of Eq. (1)

$$\hat{U}_t = \theta_C \hat{C}_t + \theta_E \hat{E}_t + \theta_X \hat{X}_t$$

(2)

where the $\theta$s are utility elasticities derived from the functional form of $U$ showing the relative valuation of the arguments by individuals. Development characterized by $\hat{U}_t \geq 0$ is labelled "sustainable" growth, because future individuals and generations are in an equal or a better position compared to today. Following the logic of economics growth is desirable if it is compatible with sustainable development in the form of constant or increasing overall well-being over the very long run. Specifically, provided that $\hat{U}_t \geq 0$ can be achieved due to $\hat{C}_t > 0$ we speak of "desirable" growth because increasing consumption promotes well-being over time. Looking at the compatibility of environment and growth we can see from Eq. (1) that $\hat{U}_t \geq 0 \iff \theta_C \hat{C}_t + \theta_E \hat{E}_t + \theta_X \hat{X}_t \geq 0$. This says that a society which is satisfied with a given level of consumption ($\hat{C}_t = 0$) can concentrate on the natural environment and other factors ($\hat{E}_t, \hat{X}_t \geq 0$) when it aims to achieve constant or increasing well-being. Conversely, a society that wishes to have positive consumption growth ($\hat{C}_t > 0$) has to take care that any intended or unintended decrease of nature and other factors ($\hat{E}_t, \hat{X}_t < 0$) is sufficiently compensated or overcompensated by rising consumption possibilities now and in the future. It might hold true that it becomes increasingly difficult or even impossible to substitute $C$ for fading $E$ and $X$ when these are on a sufficiently low level. In Eq. (2) this would appear in the form of high values for $\theta_E, \theta_X$ and low $\theta_C$ so that further reductions in $E$ and $X$ would have a high negative effect on welfare while an increase in $C$ would not add much to it.

The purpose of the welfare growth decomposition in Eq. (2) is to show that consumption growth is only one among several options to increase well-being, the final goal of economic activities and policies. But note that the decomposition into the different components does not reflect possible causal relationships between the arguments, which will be the focus in the rest of the paper. In Eq. (2) it is not specified whether the different arguments are substitutes or complements or if they develop independently. Important examples of links between consumption and the environment will be analyzed below with the help of concrete models. Specifically, I will study the case where $\hat{C}_t$ depends on resource scarcity and the pollution of the environment, in particular climate change. I will study the important class of models where the environment limits consumption growth but when it happens we have $\hat{C}_t \geq 0 \iff \hat{U}_t \geq 0$ so that consumption growth is indeed desirable. Put differently, environmental restrictions will be assumed to hit the production of final output but not to appear separately in the utility function.

In reality there are various interdependencies in the utility functions between individuals, leading to phenomena such as envy, status-seeking, and conspicuous consumption.
These might affect the perceived welfare (or happiness) of an individual considerably but can be separated from the main threats to the economy imposed by natural resource scarcity, the topic of the paper at hand. A broader definition of utility might include further elements like status, inequality, health, or security but this is not the focus of this paper (which is about the natural environment).

It should be noted that a negative growth rate of consumption, $\dot{C}_t < 0$, ("degrowth") is compatible with constant or increasing well-being, $\dot{U}_t > 0$, if the other determinants of utility provide sufficient compensation. Applied to Eq. (2) it means that a decrease in consumption must be compensated by an increase in ecological services or other factors such as health and security. Relating my analysis to recent development of the world economy I rather look at the opposite case, which is that a decrease in natural resource stocks or in ecological services must be compensated by an increase in man-made inputs such as capital allowing for consumption and utility of an average individual to be constant or increasing over time.

Accumulable capital is defined in a broad sense, including physical, human, and knowledge capital. There is a direct effect of capital accumulation on the marginal productivity of capital which has to be analyzed when discussing long-run growth. In addition, there are many indirect effects, in particular related to the natural environment. First, natural resources are used as an input in combination with capital so that their use might be affected by capital accumulation. Second, part of output created by capital accumulation might be used for pollution abatement. Third, environmental pollution, possibly increased by economic growth, may destroy part of the capital stock such that, in each time period, we have additional capital due to investments but also destroyed capital due to environmental damages. As a consequence, to determine whether an economy is growing or shrinking, we have to look at the net capital accumulation, which is the approach taken in the following.\footnote{The procedure to concentrate on net capital accumulation is comparable to the approach of so-called "genuine savings" often used in applied work.}

Capital investments are financed through savings which compete with different types of expenditures such as consumption and pollution abatement. Savings result from the intertemporal optimization of the households; individuals are willing to hold assets when the asset return at least compensates for their impatience, usually expressed by the discount rate. Provided that the net return is larger than the discount rate, capital is built up and positive growth becomes feasible based on individual decisions. For the firms it is optimal to employ capital up to the point where the rental rate equals the marginal productivity of capital. The main conflict between the natural environment and economic growth arises when net productivity of capital is negatively affected by environmental restrictions. These restrictions are given by limited natural resource stocks, affecting capital accumulation, and
limited absorptive capacities of natural sinks, affecting capital depreciation. They reduce economic growth and may even revert positive consumption growth into negative growth. But provided that the marginal productivity of capital can be kept at a sufficiently high level we still get positive consumption growth.

Degrowth of consumption is optimal from today’s perspective when net marginal productivity of capital is at a too low level. Specifically I will show below that impatience can even cause decreasing welfare over time, $\hat{U}_t < 0$. This case is thus a regular outcome of an economic model with optimizing agents, there is no such thing that automatically drives a market economy to positive consumption growth. However, optimal from today’s perspective is not necessarily sustainable, as I will illustrate in a figure in the next section. The result $\hat{U}_t < 0$ is a non-sustainable outcome harming future generations, providing them with a lower level of well-being compared to the current generation. It has therefore to be questioned under which conditions degrowth is in the interest of future generations.

To further determine this and its links to the natural environment I turn to net capital accumulation in the next section.

3 Baseline Model

3.1 Building the Framework

Key elements The baseline model of this paper integrates several central features. First, endogenous capital accumulation is introduced as a key driver of development and hence as an important element of sustainability. Capital is interpreted in a broad sense, including human and knowledge capital, so that not only investments in physical capital but also education and innovation are considered to play a role when coping with natural resource scarcity. Second, capital is produced in a separate sector in the economy, expressing the fact that used inputs and input intensities differ between the capital and the consumer sector. This has consequences for the prospects of long-run growth because these depend on specific input conditions in the sector driving growth which is the capital sector. Third, the threats to sustainability stem from natural resource scarcity on the one hand and from pollution and climate change on the other. While most theory papers focus on one of the two aspects the baseline model unifies the two basic environmental restrictions in a single framework. Fourth, having more than one sector opens the door for analyzing structural change, which may supplement input substitution as a mechanism for coping with decreasing resource input. Fifth, labor plays a dual role for sustainability as a user of natural resources and a producer of possible substitutes; this is considered in separate sections of the paper.

I start by combining capital accumulation, resource scarcity, and stock pollution in a single model to derive a first set of results on the trade-off between the environment and
economic growth. It is the aim to present a comprehensive framework which is still tractable and instructive. Then, I extend the approach in order to include different assumptions on used inputs and on substitution between inputs. I will show how the conclusions with respect to sustainability depend critically on the assumed parameter values and functional relationships.

**Formal setup** In the tradition of capital resource models (Dasgupta and Heal 1974, Solow 1974, Stiglitz 1974) final output $Y$ is produced by capital $K$ and natural resources $R$ according to

$$Y_t = A(\epsilon_t K_t)^\alpha R_t^{1-\alpha}, \quad (3)$$

where $t$ is the time index, $A$ a parameter for total factor productivity, and $\epsilon_t \in [0, 1]$ the share of capital used for final goods. While the earlier literature relied on one-sector models with $\epsilon_t = 1$ this paper stresses the importance of having multiple sectors and thus assuming $\epsilon_t < 1$. Labor input will be added separately and discussed below. First-order conditions of profit maximization in the $Y$-sector yield $\alpha/(1 - \alpha) = p_K \epsilon K / p_R R$ where $p_K$ is the rental price of capital. $R$ is a natural resource which is extracted from resource stock $S$ and regenerated by nature at a rate $\zeta$, hence

$$\dot{S}_t = \zeta S_t - R_t \quad (4)$$

The resource stock is growing, remaining constant, or shrinking depending on $\dot{S}_t \geq 0 \Leftrightarrow \zeta S_t \geq R_t$. In the case of a renewable resource with a sustainable harvest rate we get $\dot{S}_t = 0$ so that $\zeta = R_t / S_t$ and $R_t = constant$. For nonrenewable resources we have $\zeta = 0$ so that $\dot{S}_t \leq 0$ and $R_t > 0$ cannot be constant for ever.\(^{10}\)

Capital goods are manufactured in a separate sector which is intensive in the use of capital as an input. In the baseline model capital is assumed to be the only input for simplicity\(^{12}\) so that investments $I$ read

$$I_t = B(1 - \epsilon_t) K_t \quad (5)$$

with $B > 0$. The rate of capital depreciation is endogenous and given by $D$ denoting the share of capital lost, $D_t \in (0, 1)$,\(^{13}\) then, capital accumulation becomes

$$\dot{K}_t = I_t - D_t = B(1 - \epsilon_t) K_t - D_t K_t. \quad (6)$$

\(^9\)A will be endogenized below.

\(^{10}\)Regeneration can also be presented as a function, e.g. the often-used logistic function. For the purpose of this paper the introduction of a constant parameter is sufficient.

\(^{11}\)Following mainstream resource literature I ignore the effects of endogenous discoveries of new resource stocks for the sake of brevity. Part of the literature includes - for simplicity - a further special case with $\zeta = 0$ and $R_t \simeq constant$ when $S_t$ is very large compared to $R_t$, like in the case of coal.

\(^{12}\)The assumption will be relaxed later in order to include resources and labor.

\(^{13}\)If depreciation is different for physical capital compared to human or knowledge capital one can insert a parameter for the ratio of physical capital to total capital, see Bretschger and Karydas (2016).
Pollution stock $P$ increases with polluting economic activities, in particular with the use of natural resources, and may decay at a specific rate; hence I write

$$
\dot{P}_t = \phi R_t - \omega P_t,
$$

where $\phi \geq 0$ and $\omega \in [0,1]$ is the decay of the pollution stock; $\omega = 1$ is equivalent to a full decay in $t$ representing the case of flow pollution. Provided that $\omega < 1$, less than the whole pollution stock vanishes in $t$ so that the economy encounters a stock pollution problem when $\omega = 0$ there is no natural decay.

The economy is negatively affected by pollution flow and/or pollution stock which may damage consumption, productivity, and/or capital stock. I consider the case of pollution stock $P$, e.g. the stock of greenhouse gases, harming capital according to

$$
D_t = h_t P_t
$$

where $h$ is a function for impact intensity, which may depend on used resource stock $\hat{S}_t = S_0 - S_t$ and possibly other factors. I assume that resources extracted first are relatively dirty compared to resources used at later stages, which relates to the transition from coal to gas and shale gas, so that $\partial h / \partial \hat{S} < 0$.

Turning to consumption $C_t$, equilibrium on consumer goods markets ensures $Y_t = C_t$ and $p_Y = p_C$. The social planner maximizes utility given by a function of the familiar CRRA type, reading

$$
\max_{C_t, c_t} \int_0^\infty C_t^{1-\sigma} - \frac{1}{1-\sigma} e^{-\rho t} dt
$$

where $\sigma$ is the coefficient of relative risk aversion (equal to the inverse of the intertemporal consumption substitution elasticity). Because I focus on resource scarcity and climate change I do not add a separate argument for ecosystem services $E$ in the utility function like in Eq. (1). As a matter of fact, the remaining stock of exhaustible resources and the state of the atmosphere do not primarily affect individual utility but rather harm production and the capital stock.

**Assumptions** For the integrated baseline model I consider a combination of the most imminent threats to sustainability. I make a number of rather extreme assumptions to show the trade-off between environment and growth most clearly:

- Resource $R$ is exhaustible, $\zeta = 0$, 

---

14. This applies to many aspects of regional or local pollution of the air and water.
15. Like in the case of the climate problem and the pollution of soils, oceans, and other ecosystems.
16. The use of a quality argument to motivate the shape of a function is in analogy to the old Ricardian motivation of decreasing returns to land when the most fertile grounds are cultivated first.
17. This functional form encompasses the case of log utility often used in climate economic models as a special case ($\sigma = 1$).
18. Ecosystem services would appear as an important argument of the utility function if the amenity of the landscape or regional pollution were the focus of the study.
• there is no decay of pollution stock, \( \omega = 0 \),
• climate impact is given by \( h_t = \frac{\eta}{S_t} \), with \( \eta > 0 \),
• there is no emission mitigation technology available.

Under these assumptions, I get \( \dot{P}_t = \phi R_t = -\phi \dot{S}_t \) leading to \( P_t = P_0 + \phi (S_0 - S_t) \) so that with \( P_0 = 0 \) I get \( D_t = hP_t = \eta \phi \).

### 3.2 Social Optimum

I first analyze an optimal path for the economy to discuss the model characteristics with respect to the optimality of economic growth. The social planner problem is to maximize utility subject to the restriction given in Eq. (6), using (7) and (8), given initial capital stock \( K_0 \) and resource stock \( S_0 \). Thus, the planner has to choose optimal consumption, resource extraction, and optimal sectoral capital allocation at each point in time.

The Hamiltonian of this problem reads

\[
H = \frac{C^{1-\sigma} - 1}{1 - \sigma} + \mu_Y[A(\epsilon K)^\alpha R^{1-\alpha} - C] + \mu_K[B(1-\epsilon)K - \eta \phi K] - \mu_S R
\]  

(10)

The optimization problem includes three control variables, \( C, R, \) and \( \epsilon \), and two state variables, \( K \) and \( S \). The first-order conditions of the problem are given by

\[
H_C = 0 : C^{-\sigma} - \mu_Y = 0
\]  

(11)

\[
H_\epsilon = 0 : \mu_Y \alpha A \epsilon^{-1} K^\alpha R^{1-\alpha} - \mu_K BK = 0
\]  

(12)

\[
H_R = 0 : \mu_Y (1-\alpha) A(\epsilon K) K^{\alpha} R^{-\alpha} - \mu_S = 0
\]  

(13)

\[
H_K = \rho \mu_K - \dot{\mu}_K : \mu_Y \alpha A \epsilon^{-1} K^\alpha R^{1-\alpha} + \mu_K B(1-\epsilon) - \mu_K \eta \phi = \rho \mu_K - \dot{\mu}_K
\]  

(14)

\[
H_S = \rho \mu_S - \dot{\mu}_S : 0 = \rho \mu_S - \dot{\mu}_S
\]  

(15)

These five conditions are supplemented by appropriate transversality conditions for resource and capital stock, ensuring that the values of the stocks approach zero in the very long run. Manipulating and combining Eqs. (11)-(15) I get for the consumption growth rate in equilibrium, see the Appendix

\[
\dot{C} = \frac{\alpha (B - \eta \phi) - \rho}{\sigma}
\]  

(16)

which depends on constant parameters and hence is itself a constant so that Eq. (16) reflects consumption development on a balanced growth path. Consumption, output, and capital grow at constant albeit different rates; sectoral capital allocation \( (\epsilon_t) \) is constant.

---

\(^{19}\) One can well think of alternative impact functions; the version adopted here is convenient, allowing for a balanced growth path. Possible extensions are discussed below.
The socially optimal growth rate increases in the productivity of capital in the capital sector $B$ and the output elasticity of capital in final goods production $\alpha$. It decreases with growing impact intensity $\eta$, pollution intensity $\phi$, discount rate $\rho$, and relative risk aversion $\sigma$.

Consumption growth is constant in equilibrium; it can be positive or negative. Put differently, depending on the parameters from technology and preferences it might be optimal for society to have a growing or a shrinking economy. The baseline economy exhibits a trade-off between consumption growth and the natural environment when $\alpha(B - \eta\phi) < \rho$ which says that capital productivity $B$ is too low to compensate for pollution-induced capital damages $\eta\phi$ and the impatience of the households $\rho$. This is the case of optimal economic degrowth, which means that capital accumulation is not attractive enough for society so that it rather chooses to run down capital stock in order to increase welfare. On the contrary, $\alpha(B - \eta\phi) > \rho$ represents the case when capital productivity is sufficiently high so that society wishes to continuously invest in further capital yielding positive consumption growth in the long run. It does not hold that the essential use of exhaustible resources or climate change *per se* cause "limits to growth" in this approach.

Based on Eq. (16) Figure 1 depicts the (log of) consumption in the $\ln C_t$ space as a function of time which is given by the difference between the positive impact of capital productivity $B$ and the negative impact of pollution induced depreciation and the impatience of the households. The figure distinguishes between the two cases of optimum positive growth (Figure 1a above) and optimum negative growth (Figure 1b below).
3.3 Optimal Policy

**Environmental Policy** The social planner chooses the welfare maximizing saving, resource extraction rate, and sectoral capital allocation. Because there are no externalities involved in resource extraction, private and social optimum only diverge because of the negative externalities of pollution. Atomistic and selfish agents do not consider pollution when taking investment decisions. For a firm $i$, $i = 1, 2.., N$, the (private) marginal productivity...
of capital is $B_i$ and not $B_i - D_i = B_i - \phi_i \eta$ like in the social optimum, so that aggregating over all firms yields the marginal productivity of capital as $B$ which is higher than the social optimum given by $B - \phi \eta$, see Eq. (6), (7), and (8). Consequently, investments and growth become excessive i.e. too high compared to social optimum. An optimal policy restoring welfare maximum then suggests using more capital in final goods production entailing higher current consumption and lower consumption growth compared to the market equilibrium. However, the model does not point at a possibility of an extensive growth causing some kind of "limits to growth" in the long run. For this to emerge we need more drastic threats to sustainability like convex environmental damage functions or climate tipping points, see below. In the present case, standard environmental economics prescribes reducing emissions either by taxing resources à la Pigou or to attach a price to pollution through a permit market. There is room for an additional welfare-improving policy if society aims to correct the individual discount rates for ethical reasons (see Stern 2007 for a motivation). Then, resource use may be slowed down by appropriate resource conservation policy so that the growth rate of consumer goods increases, shifting consumption from the present to the future.

The non-specialist reader might ask where resource scarcity is reflected in the formula for consumption growth in Eq. (16). While climate change visibly pops up in the term $\eta \phi$ slowing down the growth rate it is not immediately clear how the increasing depletion of resource stocks enters the optimal growth rate. Firms use resources up to the point where their market price $p_R$ equals their marginal productivity, which can be extracted from Eq. (13); expressed in growth rates I then get, using Eq. (15), $\dot{\mu}_Y + \dot{p}_R = \dot{\mu}_S = \rho$. From Eq. (11) I have $\dot{\mu}_Y = -\sigma \dot{C}$ so that combining yields $\dot{p}_R = \sigma \dot{C} + \rho = r$ with $r$ denoting the interest rate in the economy. This is the famous "Hotelling" rule, saying that resource prices grow at a rate which equals the interest rate; the extracted resource quantity can be shown to grow with minus the interest rate. Resources are depleted faster when individuals are more impatient. As a consequence, the negative effect of the discount rate on consumption growth in Eq. (16) is capturing two different effects: the impatience of the households and decreasing resource use over time. Important features of this kind of resource extraction are that the planner optimizes over an infinite time horizon, which is a very optimistic assumption for any kind of policy or market framework, and that there is perfect foresight about the future, another heroic assumption. I will get back to this issue at the final assessment of the growth-environment trade-off.

Speaking about difficulties, the implementation of an optimal environmental policy is a delicate issue in reality, at the national level but especially at the international level, where free-riding is a convenient option for many countries. In fact, the baseline model does not include a real abatement policy so far. In a model extension one can add a
special abatement technology and an associated policy by assuming that part of capital or output is used to provide protective measures against emissions, see e.g. Bretschger and Valente (2011) and Bretschger and Suphaphiphat (2014). The impact of the abatement policy depends on the size of the expenditures allocated to the task and the efficiency of the abatement technology. If we assume abatement efficiency is given by the parameter \( \nu \), we get in Eq. (7) with \( \omega = 0 \) that pollution stock changes according to \( \dot{P}_t = (\phi - \nu) R_t \). Optimal growth then reads, based on Eq. (16), \( \dot{C} = \frac{\alpha [B - \eta (\Phi - \nu)] - \rho}{\sigma} \) which improves the chances that the welfare optimum is characterized by positive consumption growth (we have \( \eta \nu \) affecting \( \dot{C} \) positively). Efficiency of pollution abatement \( \nu \) may play a special role for policy, to which I turn next.

**Technology Policy**

Given the existence of an abatement technology, policy may aim to improve its efficiency by allocating some expenditures to research for that purpose. Assuming there exists a (costly) way to improve the technology (i.e. the efficiency of reducing emissions) to a welfare-optimal efficiency level \( \tilde{\nu} > \nu \) (equalling costs and benefits of technology-improving efforts), optimal growth becomes \( \dot{C}_{opt} = \frac{\alpha [B - \eta (\Phi - \tilde{\nu})] - \rho}{\sigma} \). This model result is especially interesting because it shows the possible links between optimal policies, maximizing welfare from today’s perspective, \( \dot{C}_{opt} \), and sustainable policies, \( \dot{C} > 0 \), allowing for constant or increasing consumption in the long run.

Depending on the parameter values of the model we get three possible scenarios for the effects of technology policy. It may occur that (1) the economy is sustainable and policy improves welfare development, that (2) the economy is not sustainable and policy achieves both sustainability and efficiency, or that (3) even technology policy is not strong enough to bring about positive consumption growth. Table 1 provides a summary of the scenarios.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Technology policy</th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \dot{C}_{opt} &gt; 0 )</td>
<td>( \dot{C} &gt; 0 )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( \dot{C}_{opt} &gt; 0 )</td>
<td>( \dot{C} &lt; 0 )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( \dot{C}_{opt} &lt; 0 )</td>
<td>( \dot{C} &lt; 0 )</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Long-run development and policy

Figure 2 shows the three different cases graphically. In all three cases technology policy is suitable to improve welfare. The figure visualizes that, in scenario 1, the economy is sustainable already without such a policy, but policy can still improve overall welfare. Interesting is scenario 2 where technology policy allows for both welfare improvement and the transition from a nonsustainable to a sustainable economy at the same time. For
scenario 3, additional policies have to be implemented to achieve a sustainable state because even with technology policy the current generation opts for degrowth that is a declining consumption profile over time. In Michel and Rotillon (1995) a support of investments results as an optimal policy because they assume that the linear technology for capital is not given but results out of a learning process. Then, because learning entails positive spillovers, the social planner has to insure the optimum by considering not only the negative spillovers of pollution but also the positive spillovers by learning.

The abatement function \(-\nu R_t\) is linear in \(R\) which is a special but convenient case. One might also assume that abatement is nonlinear i.e. becomes more difficult with increasing \(R\) but, on the other hand, there may also be learning effects in abatement activities. Hence, the linear case may be seen as a middle case accommodating both effects.

### 3.4 Model Extensions

**Resources in Capital Sector** The production function of the capital sector may be extended in order to include exhaustible resource as inputs, see Groth and Schou (2002) and Groth (2007).\(^\text{20}\) Indeed, fossil fuels and scarce minerals are also used to produce machines and even research and education may use some fossil fuels (directly e.g. via heating or indirectly e.g. via computers employed). This obviously puts an additional burden on the capital build-up which is the growth engine of the economy because shrinking resource input threatens the profitability of investments. I abstract from pollution in the following.

\(^{20}\)The authors also consider labor input which I treat separately below.
and replace Eqs. (3) and (5) by writing final output as

\[ Y_t = A_t (\epsilon_t K_t) \alpha (\pi_t R_t)^{1-\alpha} \]  

(17)

and investments according to

\[ \dot{K}_t = I_t = B(1 - \epsilon_t)K_t [(1 - \pi_t)R_t]^\gamma \]  

(18)

with \(0 < \pi, \gamma < 1\). Despite the linear relation between capital and investments in the capital sector, the economy cannot produce optimal positive growth rates under these conditions. The reason is that the capital sector exhibits decreasing returns to the producible input \(K\) because resource input \(R\) is shrinking over time. The conclusion is similar to the first generation of capital resource models: With an exhaustible resource and an elasticity of input substitution of unity, positive consumption growth cannot be sustained.

Varying the conditions slightly offers a more optimistic outlook, however. First, one could consider increasing returns in final goods production and an output elasticity of capital exceeding unity. Is it realistic? There are two ways of motivating the assumption. First, one may think of endogenizing \(A\) by positive learning effects. A simple spillover function would read \(A_t = K_t^\psi (\psi > 0)\) which would raise total returns to capital in final output production to \(\alpha + \psi\) where possibly \(\alpha + \psi > 1\). An alternative with a similar goal was provided in the early contribution by Suzuki (1976), where \(A\) is knowledge capital accumulated in the same way as physical capital \(K\) so that again returns to all accumulable inputs in final output production may become unity or even exceed unity.\(^{21}\) If it is larger than unity, sustained growth becomes feasible because the increasing costs of producing capital are accommodated by increasing returns when using the capital. If the increasing returns to capital are due to the intentional accumulation of knowledge capital, market equilibrium entails ongoing growth (Groth 2007), if it is due to positive externalities policy has to support investments to achieve that goal.

The introduction of resource-augmenting technical progress alleviates the situation, because then resource input, measured in efficiency units, might be constant or even rise over time. One possibility is to introduce two types of technical progress, one directed at resource efficiency, the other at capital efficiency. Smulders and de Nooij (2003), Di Maria and Valente (2008), and Pittel and Bretschger (2010) show in models with directed technical change that increasing resource scarcity induces innovation efforts to be directed towards resource efficiency so that this scenario appears to be realistic. Similarly, the substitution of renewable resources for exhaustible resource would lead to an outcome like in the baseline model, because capital input in the investment sector would be multiplied by constant

\(^{21}\) Suzuki presents a one-sector model but the reasoning continues to hold here.
instead of shrinking resource input. Another model variant is to include population growth, to which I turn now.\footnote{Groth (2007) shows that for final output given by $Y = Y(K_t, L_t, R_t)$ increasing returns to the capital-cum-labour input combined with positive population growth is required to offset the effect of decreasing resource use.}

Adding labor input I am now turning to the introduction of constant labor input in the baseline model; population growth will be analyzed below. Taking the setup of the last subsection but replacing resources in the capital sector by labor, the model includes the two sectors according to

$$Y_t = A(\epsilon_t K_t)^\alpha (\varphi L_t)^\beta R_t^{1-\alpha-\beta}$$

(19)

and

$$K_t = I_t = B(1 - \epsilon_t)K_t [(1 - \varphi) L_t]^{\gamma'}$$

(20)

with $0 < \beta, \gamma' < 1$. A balanced growth path would prescribe constant labor input in the capital sector ($\varphi \text{ const}$) so that we come back to the constant returns to $K$, enabling constant capital accumulation and sustained growth. This is very much in conformity with the baseline model and those variants of the fully endogenous growth models without exhaustible resources. Here it becomes possible to change consumption growth without decreasing the pace of resource use, simply by increasing the labor share allocated to the capital sector $(1 - \varphi)$, see Grimaud and Rouge (2003) for a similar setup featuring endogenous innovation.

Core growth sector It is debatable whether the necessity to include exhaustible resources in the capital sector continues to hold when capital is interpreted as knowledge capital. On the one hand, energies and minerals are also used in the research labs. On the other, the input used most intensively is labor, specifically the human brain or, put differently, skilled labor. If we assume that resources are an essential input in the production of physical capital according to Eq. (18) we could consider adding a separate sector producing knowledge capital. As soon as there are only renewable inputs included in the core sector, we are back to endogenous growth, because the core sector never stops producing additional knowledge capital which offsets the decreasing capital returns in the (physical) capital sector, see Rebelo (1991).

Impact functions I have assumed in the baseline model that the environmental impact $h$ is decreasing in used resource stock in a linear way, i.e. with a constant impact intensity $\eta$. It is straightforward to see that an increasing and/or a nonlinear function, e.g. an impact intensity increasing as a quadratic function of the pollution stock, changes the results significantly. If climate damages grow disproportionately relative to capital the economy of the baseline model will no longer follow the balanced growth path. If depreciation rates grow faster than investments the build-up of net capital stops in finite time, which brings
consumption growth to an end and might even turn it into negative rates (Bretscherger and Valente 2011). One can think of various possible forces restoring growth, for example positive spillovers raising total factor productivity $A_t$, directed technical change, or increasing returns to overall capital. But the case of the impact functions shows how strongly the changes of environmental damages over time affect the characteristics of economic growth, a point which I will take up in the conclusions.

**Other Assumptions** It is immediately evident that the strict assumptions used for the baseline model can be modified. In particular, I had posited that the resource is exhaustible ($\zeta = 0$) and there is no decay of pollution stock ($\omega = 0$). Provided that the resource is fully or partly renewable the trade-off between environment and economic growth is relaxed. This is the main driver behind the efforts to direct the energy future towards renewable energies. Moreover, the larger is the natural decay of the pollution stock, the less stringent have to be the optimal abatement policies.

### 4 Poor input substitution

**4.1 Changing the Framework**

**Central Assumption** It has been argued by ecologists that capital resource models are probably too optimistic when assuming a Cobb Douglas production function for final output, using an input substitution elasticity of unity. Indeed, in a one-sector economy and without technical progress economic growth is not feasible in the long run when the input substitution is poor (Dasgupta and Heal 1974). Empirical studies provide mixed results but often find elasticities of input substitution to be lower than unity, see Bretschger (2015) and the literature cited therein. In multisector economies the growth prospects with poor input substitution look different, however, because sectoral change constitutes an additional mechanism to cope with exhaustible resources, see Bretschger (1998), Bretschger and Smulders (2012).

**Setup** I will now analyze the impact of poor input substitution in the baseline economy, abstracting from pollution as in the last subsection.\(^{23}\) I rewrite the production of final output as

\[
Y_t = A_t \left[ \alpha (\epsilon_t K_t)^{\frac{\kappa - 1}{\kappa}} + (1 - \alpha) R_t^{\frac{\kappa - 1}{\kappa}} \right]^{\frac{\kappa}{\kappa - 1}}
\]

(21)

where the elasticity of substitution between $K$ and $R$ is given by $\kappa \leq 1$ while investments are still

\[
\dot{K}_t = I_t = B(1 - \epsilon_t)K_t.
\]

(22)

As already used in the previous section I assume a learning spillover from the capital build-up to total factor productivity, i.e. following Bretschger and Smulders (2012)\textsuperscript{24} I write $A_t = K_t^\delta$ with $\delta > 0$, where $A$ is given for a single firm.\textsuperscript{25} Following Eq. (21), profit maximization in the final goods sector yields

$$\frac{\epsilon_t K_t}{R_t} = \left( \frac{\alpha}{1-\alpha} \right)^\kappa \left( \frac{p_{Kt}}{p_{Rt}} \right)^{-\kappa}. \quad (23)$$

It proves to be convenient to look at the cost share of capital in final output production, defined by $d_t = \frac{p_{Kt} \hat{r}_t K_t}{p_{yt} Y_t}$. Based on Eq. (23) we obtain

$$\frac{d_t}{1-d_t} = \left( \frac{\alpha}{1-\alpha} \right)^\kappa \left( \frac{p_{Kt}}{p_{Rt}} \right)^{1-\kappa}. \quad (24)$$

With increasing resource scarcity we get $\hat{p}_{Rt} > 0$ so that $p_{Kt}/p_{Rt}$ decreases over time and so does $d$ following Eq. (24) due to poor input substitution ($\kappa < 1$). This yields a potential drag on capital accumulation because there is less and less compensation for investments available in the course of time. With given rental prices for capital, when the capital share $d$ approximates zero, investments and growth come to a complete stop, see the first-wave capital resource models of Dasgupta and Heal (1974) and Solow (1974) featuring an economy with a single sector. However, in a multisector approach, a countereffect to decreasing accumulation is active in the form of sectoral change, shifting capital from the consumer to the capital sector and reducing capital prices in the economy. Hence $\epsilon_t$ is no longer a constant but now reflects sectoral change. Capital inflow in the capital sector increases capital growth.

By adding first order conditions for utility maximization in decentralized equilibrium one can write two differential equations for $d$ and $g \equiv \dot{K}$ to describe the dynamics of the system, see Bretschger and Smulders (2012) for the details. Figure 3 shows a phase diagram in $g-d$ space for the case $\kappa < 1$. Any path converging to $d = 1$ as well as $g = 0$ and $d = 0$ must be ruled out since it violates the transversality conditions of the model; the economy jumps on the saddle path, which lies between the $\dot{d} = 0$ and $\dot{g} = 0$ loci, and asymptotically approaches long-run equilibrium. The optimal path leads the economy through continuous sectoral change which is characterized by a reallocation of capital from the consumer to the capital sector. Capital is steadily accumulated and resources are increasingly depleted. In the long run, $\epsilon$ approximates zero so that capital growth converges to productivity $B$; consumption growth depends on the same $B$ as well as on spillover intensity $\delta$ and the preference parameters $\sigma$ and $\rho$ like in the standard model. Positive consumption growth

\textsuperscript{24}Contrary to the simpler setup used here they assume capital to be an aggregate of heterogenous capital varieties and capital productivity to rise with positive research investments.

\textsuperscript{25}The spillover can be omitted when adopting the setup of Bretschger (2013) where final output is assembled from heterogenous input varieties like in Romer (1990).
remains feasible despite poor input substitution, provided the discount rate does not exceed a critical value; a finding which is qualitatively very similar to the baseline model.

**Figure 3: Poor input substitution, phase diagram**

**Scarcity Paradox** The model with poor input substitution confirms that a resource-depleting economy must overcompensate decreasing natural resource use by man-made capital if it aims at further growth. According to the logic of the model, inputs are reallocated to the sectors building capital and knowledge which are driving the growth process in the long run. When this mechanism is applied to a cross-section of country with different levels of resource use one can state that a resource-poor economy has to build more capital in order to arrive at equal or higher income levels compared to resource-rich economies. In many industrialized countries this has actually happened in the past. Many regions with only low local stocks of materials and with unfavorable natural conditions host economies with a strong capital sector and high living standards, maybe higher than in resource-rich countries. We can call this a “scarcity paradox”: When the economy is not supplied abundantly by natural resources but mainly has to build on man-made inputs it may, under favorable conditions, develop very successfully in the long run, while a resource-rich economy may invest less in capital which induces lower growth of the economy.
4.2 Population Growth

According to the analysis so far, capital build-up, substitution, and sectoral change have proved to be powerful tools of the economy to cope with the restrictions imposed by the natural environment. But there is one domain where natural scientists forcefully challenged the economic reasoning: population growth. It has been argued that the planet cannot afford hosting a significantly higher human population (Ehrlich 1968), a concern which is often shared in the public. In an economic growth model, with a growing population, different forces are in play. First, a higher population means that more mouths have to be fed and that potentially more natural resources are used. Second, a higher labor force entails a lower endowment of physical capital per workplace which reduces physical capital productivity. These effects imply a drag on per capita growth, similar to the older Malthusian predictions.\(^{26}\) Third, however, a growing labor force enlarges the set of productive inputs and may help exploiting the full benefits of capital accumulation. For example, in the baseline model extended by labor in the capital sector, population growth accelerates capital accumulation \textit{ceteris paribus}, because more inputs are available to produce capital over time. These effects of population growth support consumption growth, so that the net effect needs to be determined with the help of concrete models.\(^{27}\)

To this end I build on the previous subsection with poor input substitution, which is already a challenge for a growing economy, and interpret capital as knowledge capital, which is non-rival in use and can therefore be shared by all the agents. I rewrite final output as a function of knowledge \(K\) and intermediate input \(X\):\(^{28}\)

\[
Y_t = K_t^\tau X = K_t^\tau \left[ \bar{\kappa} (\alpha_t L_t)^{\frac{\epsilon}{\kappa}} + (1 - \bar{\kappa}) (\beta_t R_t)^{\frac{\epsilon - 1}{\kappa}} \right]^{\frac{\tau}{\epsilon - 1}} \tag{25}
\]

where \(0 < \tau < 1\) and the elasticity of substitution between labor \(L\) and capital \(R\) in intermediate production is given by \(\kappa \leq 1\). Investments in research use labor, resources, and knowledge as inputs and are written as

\[
\hat{K}_t = I_t = [(1 - \alpha_t) L_t]^{\frac{\epsilon}{\kappa}} [(1 - \beta_t) R_t]^{1 - \frac{\epsilon}{\kappa}} K_t. \tag{26}
\]

By defining \(\hat{K}_t = \hat{g}_t\) and \([(1 - \alpha_t) L_t]^{\frac{\epsilon}{\kappa}} = L_{gt}^\epsilon, [(1 - \beta_t) R_t]^{1 - \frac{\epsilon}{\kappa}} = R_{gt}^{1 - \frac{\epsilon}{\kappa}}\) and taking log differentials I obtain

\[
\hat{g}_t = \xi \hat{L}_{gt} + (1 - \xi) \hat{R}_{gt} \tag{27}
\]

\(^{26}\)Shortages of food or land do not stand longer in the foreground of the debate because past development has shown that people can live in very dense areas and that the agricultural sector has become very productive.

\(^{27}\)As usual in growth modeling, labor force and population are given by the same variable; hence, unemployment and the inactive population are disregarded.

\(^{28}\)The procedure is adopted from Bretschger (2013) which uses an increasing-in-varieties approach but the basic model elements are represented here.

\(^{29}\)The assumption reflects the idea of positive spillovers of Romer (1990): the creation of new ideas entails an internal return for the research lab and an external return augmenting the entire knowledge stock \(K\) benefitting all the agents equally.
which allows discussing the main model effects. Exhaustible resource depletion implies $\dot{R}_{gt} < 0$ so that capital growth can only be sustained with positive labor force growth $\dot{L}_{gt} > 0$, which may arise for two different reasons in the model. First, poor input substitution in final goods production depresses demand for labor in $Y$-production, releasing it to the capital sector which raises $L_g$ over time.\footnote{One could imagine distinguishing different labor types and requiring some education effort for sectoral labor reallocation but the model uses only one labor type for simplicity.} Second, families’ preferences for offspring (and decreasing mortality due to improved health care) increase total population size, which is favorable for growth because labor is an essential input in the research lab. The introduction of different types of labor and an education sector refine the analysis but do not change the fact that additional labor is favoring labor-intensive activities such as research.

Where are the negative effects of population growth present in this setup? Resource depletion is not intensified by a larger population in this model because resource use is fully governed by the Hotelling rule which I described in the last section. The price path for resources emerging from intertemporal optimization of households determines resource use and prevents an overuse relative to the optimum. Moreover, as capital is knowledge capital here, there is no negative effect of population growth on capital productivity. Knowledge capital is a public good which can be shared by every economic agent in the same way, also by the additional population.

For population growth to become endogenous one has to determine fertility and mortality rates out of the model. The most dominant characteristic of recent years has been so-called "demographic transition," reflecting the empirical fact that fertility rates are decreasing with rising income.\footnote{This can be integrated in the present approach, see Bretschger (2013), where it is also shown that with a transition to renewable energies the economy enters a phase of stable population and endogenous constant consumption growth.} The result of population growth helping economic growth, at least in a transition phase, is a quite strong counterposition to the widespread public concern of assigning world "overpopulation" to have a negative impact. The approach features the positive impact of population growth in a knowledge-based economy which is often neglected. Of course, several model assumptions are quite optimistic, like the validity of the Hotelling rule and the absence of physical capital. But the result of a positive effect of population growth also emerges in a broader class of models of the Suzuki-type, including physical capital, see Groth (2007).

## 5 Broadening the Scope

The baseline model and its extensions cover a broad range of sustainability problems exhibiting possible trade-offs between the environment and growth. But they abstract from other important issues such as uncertainty, indeterminacy, and pollution lags. These topics
will be briefly discussed in this section.

5.1 Uncertainty

Ecological systems are very complex by nature. Hence, the effects they impose on the economy are often uncertain. For climate change many effects have to be inferred from complex climate models, giving clear qualitative guidelines but naturally offering a range of quantitative predictions and leaving some room for interpretation and policy implementation. Future climate shocks cannot be predicted with precision but their possible occurrence has to be fully taken into account in today’s decisions. In similar way, resource extraction, ecosystem services, and human health are important for economic development but may involve a high degree of uncertainty.

Environmental shocks take many different forms, ranking from continuous small-scale events to larger disasters up to so-called tipping points, causing huge damages to the world economy, possibly destroying the whole capital stock. Adding uncertainty to our model can be done by extending the capital accumulation process (Bretscher and Vinogradova 2016). Looking at pollution-induced damages to capital, depreciation \( D \) used in the baseline model becomes a stochastic process \( j \) or a sum of different processes \( j = 1, 2, \ldots J \). Change of capital is then written as

\[
\begin{align*}
    dK_t &= [(B(1 - \epsilon_t)K_t)dt - \sum_{j=1}^{J} \gamma_{jt}(P_t, K_t)dq_{jt}] \\
    \text{(28)}
\end{align*}
\]

with \( dt \) denoting the change over time as in the baseline model and \( dq_{jt} \) labeling the increment of stochastic process \( j \). In case of an environmental shock, an amount \( \gamma_{jt} \in (0, K_t) \) of the existing capital stock is destroyed. Depending on the assumptions, damage size and/or arrival rate can be endogenously determined in the model. Often-used specifications for the stochastic part are the Poisson and the Wiener process; they may also be combined. The natural assumption is that shocks are recurring over time. The more severe tipping points can be captured by a Poisson process with a high capital destruction rate in case of an event, possibly pushing the economy to an absorbing state without the possibility to recover afterwards.

Similar to the deterministic models of growth with environmental restrictions, agents prefer to implement optimal investment plans and abatement policies in case of environmental shocks. Including uncertainty into the analysis has different effects. On the one hand, capital constitutes a buffer against environmental disasters, on the other it may also contribute to the underlying pollution problem if it induces higher resource use or is itself polluting. Abatement policy reduces current consumption but dampens the shocks, benefiting agents who have a preference for consumption smoothing according to the usual
assumptions. It results from these kind of models that optimal environmental policies become more stringent once environmental shocks are taken into account, because it is valuable to the economy to reduce uncertain the size and the frequency of environmental shocks.

5.2 Momentum

Comparing the agreed temperature goals of global climate policy with the predicted emission paths of the different countries there is a striking gap. Current policy efforts will have to increase sharply in the coming years should the emission targets effectively be reached. It may become feasible to convince voters and governments to reduce carbon emissions at a faster pace, but it has proved to be difficult in the past. The political task would become much easier if so-called "momentum effects" or "speeding moments" supported the process. In general, the terms label a situation with non-linear development where a single event can trigger a major move of the economy without causing too high costs. A prominent case for such a development is given when the expectations of market participants play a distinct role (Smulders and van der Meijden 2016, Bretschger and Schäfer 2016).

Models including the possibility of expectation-driven equilibria assume multiple steady states and an indeterminacy of long-run equilibrium. In case of an indeterminacy, pure expectations determine the equilibrium selection process determining long-run growth. Environmental policy may affect the process by shifting the region where expectations matter and by providing a coordination of expectations e.g. by consistent policy announcements.

5.3 Pollution delay

For most cases of regional pollution the damages to the economy occur with no or minimal time delay after the emissions have taken place. Smog in the cities harms the population on a daily basis; it is an example of flow pollution with $\omega = 1$ in Eq. (7) of the baseline model. For other pollution types things look different. Especially in the case of climate change, there is a major time lag between the emissions and the damages to the economy, usually assumed to amount to several decades or even centuries. These delays in the carbon cycle can be integrated in macroeconomic climate models, see Gerlagh and Liski (2016) and Bretschger and Karydas (2016), where the authors assume pollution-induced damages to capital accumulation as in the baseline model. It allows us to study how climate change and abatement policy affect the growth rate and the transition of the economy towards long-run steady state. If the emissions diffusion process is not fully understood, carbon policies, investments, and resource extraction cannot be expected to be on an optimal level.
6 Conclusions

The present paper has addressed the question whether the natural environment and economic growth are compatible. The answer was, not surprisingly, that a generally valid answer cannot be given. The normative assessment yields that consumption growth is not a necessary element of welfare improvement. But looking at the recent development of the world economy, economic growth is an empirical fact and often even a target of economic policy. The question then is how consumption growth can be positive in the future given the presence of environmental restrictions such as resource scarcity and pollution damages. In the paper I have developed a baseline model looking at resource scarcity, climate change, and endogenous growth in a single framework. The model includes the recent advances of endogenous growth theory and provides closed-form analytical solutions. It is shown how the effects of environmental restrictions for the economy critically depend on the assumed inputs, sectors, and functional relationships. The baseline model and the various extensions are useful to point at the critical issues in the context. Following the analysis of the paper, several issues were classified as critical, while others - possibly of great public concern - could be identified as less critical.

A first critical issue related to sustainable resource use is the lacking foresight of market participants. The Hotelling rule resulting from the intertemporal optimization requires a very long or even an infinite time horizon for optimal resource depletion. This assumes not only a substantial interest in future development (including future generations) but also basic issues like long-run ownership guarantees for resource stock, which are often not given in resource-extracting regions. A too rapid resource extraction may induce a conflict between economic growth and the environment at a later stage of development with only low resource stocks left. A second critical issue associated with pollution is the possibility of (highly) nonlinear damage functions. If the negative effects of accumulated pollution stocks grow rapidly with increasing emissions, and are probably coupled with uncertainty and delays in the emission cycles, environmental damages may not be properly included in current decision and thus pose problems for economic development at later stages. A third complex of critical issues centers around the efforts to build capital and invent new technologies at a rate which is high enough to compensate for fading natural resource use. The social planner solution of the baseline yields optimal capital accumulation but myopic behavior of investors or nonlinearities in technical development might lead to deviations from the optimal path. A related problem is the speed of developing clean and renewable technologies as a substitute for exhaustible and polluting resources. Markets may be efficient in pushing the technological limits and expectation-driven investments may support the transition. But policy will also be needed to guide the markets, which
leads to a fourth broad critical topic for sustainability: policy making. Environmental policies are difficult to implement on a national scale because the costs are often highly visible and concentrated on certain groups of the population while the benefits are more diffuse. It is often repeated that environmental policy measures have a regressive impact on income distribution even when the empirical evidence is at least very mixed. Most difficult is international policy coordination, however. There are huge incentives for individual countries for free riding when global environmental problems have to be solved. In the absence of a global legal authority it is the task of international treaties to provide policy solutions, in particular to allocate burden-sharing rules. This has proven to be difficult in many cases but can be achieved as shown with the Montreal protocol aimed at preserving the Ozone layer.

There are several issues which appear, based on the present analysis, to be less critical for the trade-offs between the environment and growth. The first is the fact that some energies and resources are used as inputs in the capital and knowledge sector of the economy. In the model this can cause a major growth slowdown. But in reality the dominant factor in knowledge production is labor; sectoral energy input can most probably be provided by renewable energy sources. Hence, it is a feasible task to keep the knowledge sector running, in the worst case with increased public support. A second less dramatic problem is population growth. Demographic transition, urbanization, and high productivity of agricultural and industry will help avoid global food shortages and unbearable space constraints. With unchanged pollution per capita the overall pollution problem will rise, of course. However, current population growth is highest in countries with low resource use per capita. Accordingly, it remains to be a major task to change the behavior in industrialized and emerging economies (with moderate or zero population growth). A last issue which emerged from traditional capital resource models to be unfavorable for sustainability is poor input substitution. It turned out that allowing for sectoral change, endogenous innovation, and endogenous population size that development can be sustainable, even when input substitution itself is not very flexible.

Many topics are left for future research. The combination of the existing capital resource models with the analysis of uncertainty offers a broad variety of additional research questions. Uncertainty in the political sector and its impact on investment is another aspect which deserves more attention in future work. Finally, all strands of literature can be tested and applied by using quantitative analysis, picking realistic parameter values for the models and deriving prescriptions for environmental policies.
References


7 Appendix

Optimal consumption growth in the baseline model is derived as follows. Log differentiating Eq. (11) yields

\[ -\sigma \dot{C} = \dot{\mu}_Y, \]  

(A.1)

rewriting Eq. (12) gives

\[ \frac{\mu_Y}{\mu_K} = \frac{BK}{\alpha A e^{\alpha - 1} K^{\alpha - 1}}, \]  

(A.2)

and by inserting Eq. (A.2) in Eq. (14) I obtain

\[ \rho - \dot{\mu}_K = B e + B (1 - \epsilon) - \eta \phi. \]  

(A.3)

Using Eq. (15) and Eq. (13) I write

\[ \dot{\mu}_S = \rho = \dot{\mu}_Y + \alpha (\dot{K} - \dot{R}) \]  

(A.4)

which is combined with Eq. (A.1) so that

\[ \rho = -\sigma \dot{C} + \alpha (\dot{K} - \dot{R}). \]  

(A.5)

Taking the log differential of Eq. (A.2)

\[ \dot{\mu}_K + \dot{K} = \dot{\mu}_Y + \alpha \dot{K} + (1 - \alpha) \dot{R} \]  

(A.6)

and combining with Eq. (A.3) gives

\[ \dot{\mu}_Y = (1 - \alpha) \dot{K} - (1 - \alpha) \dot{R} + \rho - B + \eta \phi \]  

(A.7)

which I combine with Eq. (A.4) to have

\[ -\alpha (\dot{K} - \dot{R}) = (1 - \alpha) (\dot{K} - \dot{R}) - B + \eta \phi \]  

(A.8)

yielding

\[ B - \eta \phi = \dot{K} - \dot{R} \]  

(A.9)

which is inserted in Eq. (A.5)

\[ \sigma \dot{C} = \alpha (B - \eta \phi) - \rho \]  

(A.10)

to finally give the expression for consumption growth of the baseline model in the main text.
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