Survival to adulthood and the growth drag of pollution

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Survival to Adulthood and the Growth Drag of Pollution

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Abstract

Environmental pollution adversely affects children’s probability to survive to adulthood, reduces thus parental expenditures on child quality and increases the number of births necessary to achieve a desired family size. We argue that this mechanism will be intensified by economic inequality because wealthier households live in cleaner areas. This is the key mechanism through which environmental conditions may impose a growth drag on the economy. Moreover, the adverse effect of inequality and pollution on children’s health may be amplified, if the population group that is least affected decides about tax-financed abatement measures. Our theory provides a candidate explanation for (1) the observed positive correlation between inequality and the concentration of pollutants at the local level, and (2) the hump-shaped evolution of child mortality ratios between cleaner and more polluted areas during the course of economic development.

Keywords: Endogenous Growth, Endogenous Fertility, Inequality, Mortality, Pollution

JEL: O10, Q50, I10

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1. INTRODUCTION

The transition from stagnation to growth originated by the Industrial Revolution induced an immense degradation of the environment and in terms of morbidity and mortality adverse effects on individuals’ health state. Moreover, this take-off in income per capita growth has been accompanied by a massive shift in demographic variables, the demographic transition. The hereby induced decline in fertility rates which followed an initial increase in fertility rates was mainly the result of increasing parental expenditures for their offspring’s human capital that ultimately paved the way for sustained economic growth in per capita terms (Galor and Weil, 2000 and Galor, 2011).

In this paper, we argue that increasing expenditures on education are positively associated to children’s probability to survive to adulthood. The probability to survive to adulthood depends positively on the stage of economic development and disposable incomes of households but is adversely affected by environmental pollution. An increase in the probability to survive to adulthood reduces the number of births necessary to achieve a desired family size and leaves more resources available for educating the surviving children. Thus, economic development may be conducive for children’s survival probabilities, but may also generate via pollution an adverse impact on children’s probability to survive to adulthood. In this context, economic inequality is not only decisive for human capital investment and the emergence of differential fertility between richer and poorer households (de la Croix and Doepke, 2003), but also for agents’ exposure to environmental pollution because wealthier households live in cleaner areas. The exposure to pollutants triggers again children’s probability to survive to adulthood and the willingness of parents to invest in education. This is the key mechanism and the novelty of our approach through which environmental conditions may impose a growth drag on the economy. Moreover, we argue that the adverse effect of inequality and pollution on children’s health may be amplified, if the population group that is least affected decides about the level of tax-financed abatement measures, since this population group may prefer the lowest tax rate which increases the pollution stock. Thus, political inequality interacts with economic inequality and health through residential sorting. The described mechanism is important since it allows us to develop a candidate explanation for: (1) The observed positive cross-country correlation between economic inequality and pollution at the local level as it is illustrated by Figure 1(a), and (2) the hump-shaped evolution of
child mortality ratios between areas that are subject to different degrees of environmental pollution as illustrated in Figure 2.

The nexus between inequality, health, and residential exposure to pollutants has been documented impressively by economic historians. Szreter (1997) argues “...there is indeed something intrinsically dangerous and socially destabilizing in the wake of economic growth...”. He motivates his statement by the following observations: (1) Local authorities were failing the management of their environments, and, (2) as a consequence of it wealthier citizens moved to the periphery of the cities.

In earlier stages of economic development, urban populations often had no access to clean water, and no facilities for disposal of human and other wastes. Pollution from smoke and other factory discharges contaminated the atmosphere and the environment. In this line of argumentation Hassan (1985) documented the significance of water as an industrial raw material: Fresh water was used for commercial purposes while the new entrepreneurial class saw no point in spending money for sanitation and sewage treatment plants which had no obvious commercial benefit. As we remarked above residential assignments to
pollution are non random but depend on economic wealth.\textsuperscript{1} Thus, exposure to pollution caused by production and population density is and was endogenous although the germ theory of disease was generally accepted only after 1870.\textsuperscript{2}

The coexistence of economic growth, mortality and morbidity increase has been regarded as something of a puzzle by economic historians - such as Haines and Kintner (2000), Schofield and Reher (1991), and Fogel (1997). However, Szreter (1988) argued already that the key to understanding the mortality transition in England lies in local politics. Only after political reform (Reform Acts), political inequality (and potentially income inequality) was reduced, and issues as sanitation, public health and the fragile onset of abatement measures in the production sector became topics of high(er) priority in the agendas of (local) policies. Obviously, budgetary constraints are always a limitation for public expenditures, but contrary to earlier periods the phase of the Industrial Revolution was marked by though moderate but still notable increases in per capita incomes.

In this paper, we capture the effect of political inequality in the following way: Depending on their level of wealth, population groups are allocated to different locations characterized by distinct levels of exposure to pollutants, in the sense that wealthier households live in cleaner areas - for example due to differences in the proximity to emission sources. As a consequence, preferred levels of tax-financed abatement measures differ between population groups. The adverse effect of inequality and pollution on children’s health can then be amplified, if political institutions are biased towards the rich, i.e. the population group that is least affected decides about the level of tax-financed abatement measures. Thus and even though there is not necessarily a direct link from inequality to health, economic inequality interacts with social segregation and political inequalities which translate into inequalities in health (Deaton, 2003; Alesina, Baqir, and Easterly, 1999).\textsuperscript{3}

Moreover, our paper provides a candidate explanation for the positive cross-country correlation between inequality and average pollution at the local level captured by the concentration of particulate matter in the air (see Figure 1(a)).\textsuperscript{4} Figure 1(b) and (c) support the

\textsuperscript{1}See for example Tiebout (1956) and Roback (1982).
\textsuperscript{2}Theories before the germ theory like the "miasma" theory (Deaton, 2003) stressed the relevance of a clean environment for health.
\textsuperscript{3}Examples for this mechanism comprise the British Reform Act in the 19th century and the Civil Rights Act in the US (Donohue and Heckman 1991, and Card and Krueger 1993).
\textsuperscript{4}Particulate matter concentrations refer to fine suspended particulates less than 10 microns in diameter (PM10) that are capable of penetrating deep into the respiratory tract and causing significant health damage. Data for countries and aggregates for regions and income groups are urban-population weighted PM10 levels in residential areas of cities with more than 100,000 residents. The estimates represent the average annual exposure level of the average urban resident to outdoor particulate matter. The
arguments of our theory in the sense that pollution interferes with the quality quantity trade-off; see also Figure 1(d) which illustrates the quality quantity trade-off as such. We emphasize the exposure of children to environmental pollution since they tend to be more vulnerable as a result of early life exposure leading directly to increased child mortality or indirectly through changes in birth outcomes that translate into higher mortality risks later in life (Schwartz, 2004). Furthermore, children’s survival probabilities are, as described above, directly linked to economic development through population growth rates and human capital investment.

The initially adverse impact of economic development on children’s survival probabilities is impressively documented by Figure 2. In the nineteenth century, child mortality rates in cities relative to rural areas increased in Sweden and Norway rapidly to a 1.6 ratio at the end of the century and experienced a decline to a ratio smaller than one during the first quarter of the 20th century only. In this paper, we explain the hump-shaped evolution of child mortality differentials by differences in the residential exposure to pollutants. Moreover, poorer households living in more polluted areas exhibit lower human capital endowments compared to richer households living in less polluted areas. Thus poorer households invest less in child quality while their children face a lower probability to survive to adulthood. Consequently, mortality differentials increase if production and pollution increase. In later phases of economic development, the mortality differentials close again because even poorer households exhibit higher incomes and the economy may adopt endogenously environmental abatement measures. Nevertheless, we show that initial inequality increases mortality differentials and shapes its transition since more households are exposed to a higher level of pollutants.

Recent empirical findings (Chay and Greenstone, 2003; Currie and Neidell, 2005; Currie, et al., 2009; Ebenstein, 2012) support convincingly the link between exposure to pollution state of a country’s technology and pollution controls is an important determinant of particulate matter concentrations (see World Bank Indicators, 2013).

5In fact pollution affects both conception and fetal deaths (Buck Louis et al., 2009; Sanders and Stoecker, 2011). Recent empirical evidence shows that for birth outcomes the period of exposure is relevant (see Salam et al., 2005): the first trimester is the period during which the neural tube is transformed into the brain and spinal cord and many other organs experience rapid development (de Graaf-Peters and Hadders-Algra, 2006; Cunningham et al. 2010).

6Because of data availability, we show the case of Sweden and Norway, here. Bairoch (1988) discusses very similar developments for other European countries. Szreter (1997) documents that rapid urbanization associated with the Industrial Revolution induced higher mortality rates in cities than in the countryside, and a decline in overall life expectancy. In addition Hainse (2004) and Komlos (1998) provide evidence for increased morbidity over the same period of time indicating that physical height of soldiers declined during the 19th century in the U.S. as well as England and the Netherlands.
tants and birth outcomes at present times, which is still significant but obviously less pronounced compared to earlier stages of economic development.\footnote{Chay and Greenstone (2003) provide evidence for the impact of air pollution on infant mortality in the U.S. during the recession period 1981-82 and conclude that a 1-percent reduction in total suspended particulates results in a 0.35-percent decline in infant mortality at the county level. Currie and Neidell (2005) report that reductions in carbon monoxide during the 90s saved around 1000 infant lives in California. Currie, Neidell, and Schmieder (2009) find that a one-unit change in mean carbon monoxide during the first two weeks after birth increases the risk of infant mortality by 2.5%. Moreover, the fifteen-year decline in CO from 1989 to 2003 translates into $720 million in lifetime earnings from improvements in birth weight and $2.2 billion from the reduction in infant mortality for the 2003 US birth cohort. For an extensive overview, see Graff Zivin and Neidell (2013).} In addition, air and water pollution seems to become a topic of major concern in fast growing developing countries like China. Berkeley Earth’s scientific director, Richard Muller, stated that breathing in Beijing’s air is the equivalent to smoking forty cigarettes a day and calculated that air pollution caused 1.6m deaths a year in China or 17% of the total (The Economist, 2015).

The frame, we present here is able to replicate the historical development path in accordance to empirical patterns: First, in early stages of economic development there is no abatement since the marginal benefit of abating is low and expenditure shares on private consumption are due to a hierarchy of needs high. Second, there is a slow take-off in terms of income per capita growth. Third, the pollution stock increases through economic development and is positively associated to inequality. Fourth, the evolution of the populations’ growth rate and the evolution of children’s regional mortality differentials is

Figure 2: Ratio of child mortality rates in urban and rural regions (Bairoch, 1988)
hump-shaped.
The remainder of the paper is organized as follows: In the next section, we relate our paper to the literature in more detail. In Section 3, we introduce our overlapping generations framework. In Section 4, we discuss heterogeneities with respect to agents’ human capital endowments and with respect to their exposure to pollutants. Section 5 performs numerical experiments dealing with the (long-run) effects of different amounts of initial inequality and explores the interaction between inequality, exposure to pollutants and preferences for tax-financed abatement measures. Finally, Section 6 concludes.

2. RELATION TO THE LITERATURE

The economic literature models the interaction between the economic sphere and the environment by recognizing that environmental pollution is a by-product of the production or consumption processes which adversely affects individuals’ utility or causes detrimental productivity effects. Usually the literature and also this paper analysis the regulation of a single pollutant and abstracts from complementary or substitutive relationships between different kinds of pollutants; related to the latter, see Moslener and Requate (2007;2009). Sustainability in a broader sense requires then a balanced growth path being compatible with non-declining environmental quality such that in general pollution approaches a finite steady state level (see for example Ordas Criado et al., 2011; Brock and Taylor, 2005 and 2010; Bovenberg and Smulders, 1995). Especially the introduction of environmental concerns into agents’ utility function turns out to be a powerful tool since it facilitates the analysis of endogenous policy interventions which affect both the transition and the steady state levels of an economy. In particular, models of endogenous growth allow for a reasonable analysis of the interaction between growth and the environment since the long-run growth rate may be altered by policy interventions. 

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8Pollution generating a negative externality on agents’ welfare has been analyzed within a Ramsey framework by van der Ploeg and Withagen (1991), Gradus and Smulders (1993), Beltratti (1996) or Xepapadeas (1997, Chapter 3). In this framework, a social planner may reduce the environmental impact of human activities by implementing an emission tax or the allocation of resources to abatement measures in order to reduce emissions. For further details see also Xepppadeas (2005).

9Aghion and Howitt (1998) introduced environmental concerns into a model of purposeful investment in R&D and show that unlimited growth is compatible with non-declining environmental quality, if the elasticity of marginal utility is sufficiently high while Grimaud (1999) suggests policy instruments necessary to implement the socially optimal path derived by Aghion and Howitt into the decentralized economy. In the 2000s, models with polluting natural resource use have been introduced for example by Schou (2000;2002) or Grimaud and Rouge (2005) who consider human capital and R&D-based growth models where the pollution flow adversely affects utility or productivity. Hart (2004) analysis an endogenous growth model where pollution can be reduced by green research. Related to the latter approach is
In this paper, we consider an endogenous growth model where human capital is the major driver of economic growth, i.e. we take a development economic perspective. Moreover, we consider in contrast to the above mentioned literature not a Ramsey household but heterogeneous OLG households that are alive for two periods. This households are compared to the dynastic Ramsey household selfish, but derive utility out of pay-off relevant variables of their offspring (impure altruism), for example the health status, education, and next period’s level of environmental quality. OLG models with endogenous human capital are the workhorse in the literature of inequality and education. A seminal paper in this literature is Glomm and Ravikumar (1992) where human capital among individuals is unevenly distributed and subject to spillovers from parents to children and from the social environment to children. By doing so we do not challenge social planner solutions in Ramsey frameworks but we emphasize the role of potentially myopic and socially biased policy instruments in view of income heterogeneities and socially segregated societies. Of course, OLG models have been applied to environmental aspects as well. Mariani, Perez-Baharona and Raffin (2010) and Varvarigos (2010) are probably the first papers relating life expectancy and environmental quality to poverty traps. This paper complements their work in the sense that we relate children’s probability to survive to adulthood and economic inequality to environmental pollution and prospects of future growth. The distinguishing feature of our framework is that we do not focus on longevity but on children’s probability to survive to adulthood. Second, for the economy as a whole the transition process is not subject to global indeterminacy. Finally, we consider agents that are heterogeneous with respect to their human capital endowments. So far endogenous population dynamics have been linked to our best knowledge to the depletion

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10 In other words, parents do not care about their descendants utility but receive a warm glow of giving (Andreoni, 1989). Related to this context is Becker’s view that parents’ expenditures on children are motivated by the desire for having “higher quality” children (Becker, 1960).

11 For some important applications, see for example de la Croix and Doepke (2003, 2004), Benabou (1996), Aghion and Howitt (1998), and Galor and Zeira (1993).

12 Mariani et al. (2010), inspired by John and Pecchenino (1994), analyze the interrelationship between pollution, longevity and environmental poverty traps. Related to this paper is Ono and Maeda (2001) and Jouvet et al. (2010). Varvarigos (2010) considers endogenous longevity, based on the seminal work by Chakraborty (2004). There, longevity depends on tax-financed pollution abatement measures and public health expenditures. An interesting result is that the distribution of public expenditures affects the likelihood that an economy can avoid a poverty trap.
of non-renewable resources only.\textsuperscript{13} With respect to the channel connecting fertility and health our work is also related to Strulik (2004;2008). The difference is that we allow for inequality, pollution, and endogenous growth.

A further advantage of OLG models is that they open an elegant way for the implementation of polit-economic processes like tax-financed policy interventions in a heterogeneous agent setting. In this framework, we analyze the factors that influence agents’ preferences for tax-financed abatement measures, such that our work is also related to a growing body of literature which emphasizes the role of institutions for prospects of future economic development.\textsuperscript{14} This literature emphasizes that existing institutions and policies are not necessarily efficient but the outcome of conflicting preferences over certain public affairs. The presence of social conflict implies that different social groups formulate different preferences about political outcomes, with the implication that the political outcome is always optimal for some but never for all agents. Depending on political institutions, the most powerful social group will implement its most preferred policy. Thus for a democracy, the median voter may serve as a reasonable approximation, but since political institutions changed during the course of economic development, we will conduct several experiments capturing different degrees of enfranchisement in which either the richest or the poorest social group will manage to set their preferences into action.

In the framework we present here, abatement measures emerge endogenously during the course of economic development and are aimed at an improvement in environmental quality and children’s health. Therefore our paper is also related to a very important strand of the literature dealing with the evaluation of such policy interventions, i.e. the value of a statistical life (VSL). The VSL is a prospective measure and in general defined as the willingness to pay for small reductions in the risk of death. From a conceptual point of view the VSL is thus deeply rooted in the public finance literature (for more details, see Viscusi; 2000).\textsuperscript{15} VSL estimates are usually related to individuals’ willingness to pay for reductions in their own probability of death. Our framework in turn deals with children’s prospects to survive to adulthood and related investment of parents in their children’s

\textsuperscript{13}See for example Bretschger (2013), Paretto and Valente (2015), and Schaefer (2014) as an probably incomplete list of references. As regards the adverse impact of population growth on the environment during earlier stages of economic development our paper is related to Cronshaw and Requate (1997).

\textsuperscript{14}See for example Acemoglu and Robinson (2009), ch. 4 for a nice introduction into the material.

\textsuperscript{15}Estimates for people’s willingness to trade off wealth for a reduction in the probability of death guide the evaluation of environmental policies, health and public safety in travel etc. (Ashenfelter; 2006). For example, the U.S. Environmental Protection Agency (EPA) or the Department of Transportation base policy evaluations on VSL measures, see Blomquist (2004). For an early discussion, see Schelling (1968).
education and health. In the literature, VSL estimates for children are sizable but limited in number and subject to rather high variations.\textsuperscript{16} Nevertheless, looking at estimates of values for children’s health indicate that parents behave considerably altruistic with respect to their children’s health which is the relevant finding for the modeling purposes here.\textsuperscript{17} In this paper, we apply standard tools of the public finance literature. This means that agents do not only consider the trade off between wealth and children’s mortality risks (VSL) but formulate preferences about the entire trade off between cost and benefits of pollution abatement measures which also includes the improvement in the future quality of the environment.

3. THE MODEL

3.1. Human Activities and Pollution

In this setting, time is discrete, indexed by $t$ and ranges from 0 to $\infty$. A large number of firms produce aggregate output, $Y_t$, using a constant returns to scale technology of Cobb-Douglas type, where $K_t$ denotes aggregate physical capital and $L_t$ aggregate effective labor allocated to production, such that

$$Y_t = AK_t^\alpha L_t^{1-\alpha},$$

(1)

with $A > 0$, $\alpha \in (0,1)$.

The environmental impact of human activity can be captured by population size, affluence and technology.\textsuperscript{18} Here, production generates emissions, $E_t$, which may be attenuated by

\textsuperscript{16}Carlin and Sandy (1991) analyze mothers’ use of child safety seats for their children. They report that their estimate of mothers’ VSL for their children who are under the age of 5 years is approximately 87% of Blomquist’s (1979) estimate of VSL for adult drivers based on use and non-use of seat belts. Blomquist, Miller, and Levy (1996) analyze motorists’ use of safety equipment. Their best estimate of VSL for children less than 5 years of age is about 32% greater than the best estimate of VSL for adults ($2.8$ million). Jenkins, Owens, and Wiggins (2001) estimate parents’ VSL for their bicycling children as approximately $2.9$ million and $4.3$ million for bicycling adults. Mount et al. (2001) estimate VSLs for adults and children. Their estimate for children of $7.3$ million is slightly greater than the estimate for adults of $7.2$ million.

\textsuperscript{17}Agee and Crocker (2001) estimate that parents value their children’s health twice as much as their own health. In their study, the risk associated to smoking would be of acute episodes of respiratory attacks and of chronic diseases which develop later in children’s lives. The effects from smoking to children’s health are certainly comparable to environmental pollutants in the air which are considered in this paper. Liu et al. (2000) who find that for comparable colds a mother’s willingness to pay to prevent her child’s cold is approximately twice her willingness to pay to prevent her own cold. Similarly, Dickie and Ulery (2001) find that parents value their children’s health about twice as much as their own health.

\textsuperscript{18}This refers to the so-called IPAT-identity, where the impact is reflected by population size, affluence and technology.
abatement measures, $M_t$, financed by a proportional tax, $0 \leq \tau_t < 1$, on households’ income, and may be dampened by the compatibility of the technology with the environment reflected by $\Pi_t$, such that the level of emissions in period $t$ is given by

$$E_t = \Pi_t^{-1}(b_2Y_t - b_3M_t) = \Pi_t^{-1}(b_2 - b_3\tau_t)Y_t,$$

(2)

with $b_2, b_3, \Pi_t > 0$, $M_t = \tau_tY_t$ and $0 \leq \tau_t < 1$.

Note that the tax rate, $\tau_t$, is endogenous, not necessarily positive, and it depends on the preferences of a pivotal agent.\textsuperscript{19} For simplicity we assume a one-to-one relationship between the skill level of the working force, i.e. aggregate human capital, $H_t$, and $\Pi_t$, such that $\Pi_t^{-1} = H_t^{-1}$.\textsuperscript{20} Moreover, the environment is adversely affected by population size, $N_t$, which captures the adverse effect of population density and congestion on the environment. Finally, the environment regenerates at a constant rate, $b_1$, per period of time. As regards the evolution of the stock of pollutants, we assume in spirit of John and Pecchenino (1994) a standard and simple accumulation law which assures analytical tractability

$$P_{t+1} = (1-b_1)P_t + E_t + b_4\Pi_t^{-1}N_t = (1-b_1)P_t + (b_2 - b_3\tau_t)\frac{Y_t}{H_t} + b_4\frac{N_t}{H_t},$$

(3)

with $0 < b_1 < 1$, $b_2, b_3, b_4 > 0$ and $0 \leq \tau_t < 1$.

### 3.2. Households

Consider an economy populated by a continuum of overlapping generations and a large number of households indexed by $i$. Households live for two periods: childhood and adulthood. All economically relevant decisions are made in the adult period of life. Adult households care about the number of children, $n^i_t$, they wish to raise, and quality per child, $q^i_t$, reflecting their health state, $s^i_t$, and the level of human capital per child, $h^i_{t+1}$.\textsuperscript{21} Human capital per child depends on education, $e^i_t$, the parental level of human capital, $h^i_t$ and the level of pollutants per child, $E^i_t$.
where $\eta \in (0,1)$ reflects the impact of education on the level of human capital. $\nu \in (0,1)$ denotes the intergenerational transmission of human capital or the intergenerational persistence between parental human capital and the level of human capital per child. The parameter $\varepsilon > 0$ will allow for $e_i^t = 0$.

The health state, $s^i_t$, of a child born in household $i$ governs the probability, $\pi^i_t$, to survive childhood. $s^i_t$ is determined by an intrinsic component, $d^i_t$, which is endogenous to the household and captures expenditures on health and nutrition per child. In addition, $s^i_t$, is also determined by an extrinsic component, $\bar{\pi}_t$, which is exogenous to the household and reflects the state of development as well as the quality of the environment.\footnote{The disaggregation of survival probabilities into an extrinsic and an intrinsic component stems from biology. The intrinsic component is for example nutrition while the extrinsic component is reflected by the natural environment, for example temperature. For further details see Strulik (2008).} Thus,

$$\pi^i_t = \min \{ 1; s^i_t(d^i_t, \bar{\pi}_t) \},$$

with

$$s^i_t = \lambda(d^i_t)^{\bar{\pi}_0}, \text{ if } d^i_t \leq \bar{d},$$

$$s^i_t = \lambda(d^i_t)^{\bar{\pi}_1}, \text{ if } d^i_t > \bar{d},$$

where $\lambda > 0$ represents a productivity parameter, $\bar{\pi}_0 > 0$ is a constant parameter and $\bar{d} \geq 1$ denotes a critical threshold level.

Eqs. (6) and (7) capture the fact that improvements in the extrinsic component enhance the productivity of the intrinsic component only if the intrinsic component is above a critical threshold level, $\bar{d}$. Intuitively, only if calorie intake is above a critical threshold, improvements in the economic environment increase the productivity of $d^i_t$.\footnote{Note that $\frac{\partial \pi^i_t}{\partial d^i_t} > 0$ only if $d^i_t > 1$ since $\bar{\pi} < 1$, such that we impose without further loss of generality $\bar{d} \geq 1$, see (6) and (7).} The extrinsic component, $\bar{\pi}_t$, is positively affected by the state of economic development reflected by the average stock of human capital, $\bar{h}_t$, but it is adversely affected by environmental pollution, $P_t$, such that

$$\bar{\pi}_t = \bar{\pi}_t(\bar{h}_t, P_t),$$

$$h^i_{t+1} = (\varepsilon + e^i_t)^\eta(h^i_t)^\nu(\bar{h}_t)^{1-\nu},$$

where $\eta \in (0,1)$ reflects the impact of education on the level of human capital. $\nu \in (0,1)$ denotes the intergenerational transmission of human capital or the intergenerational persistence between parental human capital and the level of human capital per child. The parameter $\varepsilon > 0$ will allow for $e^i_t = 0$. The health state, $s^i_t$, of a child born in household $i$ governs the probability, $\pi^i_t$, to survive childhood. $s^i_t$ is determined by an intrinsic component, $d^i_t$, which is endogenous to the household and captures expenditures on health and nutrition per child. In addition, $s^i_t$, is also determined by an extrinsic component, $\bar{\pi}_t$, which is exogenous to the household and reflects the state of development as well as the quality of the environment.\footnote{The disaggregation of survival probabilities into an extrinsic and an intrinsic component stems from biology. The intrinsic component is for example nutrition while the extrinsic component is reflected by the natural environment, for example temperature. For further details see Strulik (2008).} Thus,
with $\bar{\pi}_t(0, P_t) = 0$, $\lim_{P_t \to \infty} \bar{\pi}_t(\bar{h}_t, P_t) = 0$, $\bar{\pi}(\bar{h}_t, 0) < \infty$ and $\lim_{\bar{h}_t \to \infty} \bar{\pi}_t(\bar{h}_t, P_*) = \bar{\pi}_* < \infty$.\footnote{This assumption states that human capital is essential for the extrinsic survival component and assures the existence of a stationary value as $\bar{h}$ grows at a constant rate along the balanced growth path whose existence in turn requires that the pollution stock reaches a constant value.} Reasonably, health and human capital per child affect quality per child in a complementary way, such that

$$q^i_t = (s^i_t)(\bar{h}^t_{t+1})^\beta \quad \gamma, \beta > 0, \quad (9)$$

with $q^i_t = 0$ if $s^i_t = 0$, but in light of (4), $q^i_t > 0$ if $e^i_t = 0$. In addition to $n^i_t$ and $q^i_t$, adult agents care about their own level of consumption, $c^i_t$, above subsistence needs, $\bar{c}$, and the amount of bequests per child, $b^i_t$.\footnote{Note that the introduction of capital accumulation through bequests does not affect the qualitative results of our theory but it improves the calibration of the numerical experiments. In addition, it implements a Malthusian relationship between income and fertility relevant in earlier stages of economic development into the model.} To the contrary, agents experience a disutility from the future level of pollution, $P_{t+1}$, such that preferences of a member $i$ of generation $t$ that is born in $t-1$ are specified as

$$u^i_t = \ln(c^i_t - \bar{c}) + \gamma \ln(n^i_t q^i_t) + \rho \ln b^i_t - \mu P_{t+1}, \quad (10)$$

with $\beta, \gamma, \mu, \rho, \bar{c} > 0$.\footnote{The presence of $P_{t+1}$ in the parental utility function captures some degree of altruism with respect to the preservation of the environment for the next generation. We omit $P_t$ as a direct argument in the parental utility function for convenience since it would not alter the optimization problem with respect to $\tau_t$ in the political process, because $\tau_t$ can only affect $P_{t+1}$ but never $P_t$, see also Eq. (3). Moreover, note also that $q_t$ is already a function of $P_t$. Constant marginal disutility from pollution assures analytical tractability. A concave argument is not necessarily compatible with a utility maximum. A quadratic argument is analytically much more difficult to handle and leads qualitatively to the same results. The difference between the linear and the quadratic case is that abatement measures will be more attractive at lower levels of pollution, in the former case, while abatement is less attractive at higher levels compared to the quadratic formulation. Proofs are available upon request.}

Now several remarks are at order: (i) the presence of subsistence consumption, $\bar{c} > 0$, introduces a hierarchy of needs in the sense that increasing incomes lower the importance of expenditures on consumption while expenditure shares for the other arguments in the utility function increase. This concerns on the one hand the preferences for the number of children and their quality and on the other hand the willingness to pay for environmental abatement measures in the political process which will be introduced in the following subsection. (ii) Contrary to consumption, education is a luxury good, since $\varepsilon > 0$ - see (4). (iii) As, $\pi^i_t = \pi^i_t(s^i_t)$, the appearance of $n^i_t q^i_t$ in the parental utility function implies that parents derive utility out of the number of surviving offspring, such that agents are forced to increase fertility whenever child mortality is high in order to achieve their desired
family size and vice versa. If $\pi^i_t = 1$ parents derive utility out of an increasing health state of their children.\textsuperscript{27} (iv) Environmental pollution causes a negative externality to agent’s welfare.

We denote post-tax variables by “ˆ”, the wage rate per efficient unit of labor and the return on capital by $w_t$ and $R_t$, respectively, such that the budget constraint of an agent $i$ endowed with one unit of time, human capital, $h^i_t$, and assets, $b^i_{t-1}$, reads as\textsuperscript{28}

$$\hat{y}^i_t = (1 - \tau_t)(w_i h^i_t + R_t \hat{b}^i_{t-1}) = (\hat{w}_t h^i_t z + \hat{w}_t \bar{h}_t e^i_t + b^i_t + d^i_t) n^i_t + c^i_t, \quad \tau_t \in [0, 1).$$  \hspace{1cm} (11)

Child rearing costs are captured by: first, forgone wage earnings, $\hat{w}_t h^i_t z n^i_t$, with $0 < z < 1$ denoting the time share necessary to raise one child to adulthood. Second, expenditures for education, $\hat{w}_t \bar{h}_t e^i_t n^i_t$, where education is provided by an educational sector employing teacher equipped with the average level of human capital, $\bar{h}_t$. Third, expenditures on nutrition, $d^i_t$, and the level of bequests per child, $b^i_t$. The subsequent lemma summarizes households’ optimal decisions.

**Lemma 1 (Households’ decisions)**

Adult agents maximize lifetime utility as given by (10) subject to the budget constraint (11), and the evolution of human capital per child (4), while ignoring their impact on the evolution of the aggregate pollution stock. Denote by $x^i_t = \frac{h^i_t}{\bar{h}_t}$ household $i$’s level of human capital relative to the average, $\bar{h}_t$, then there exists a threshold level of relative human capital

$$\tilde{x}_t = \frac{(1 - \pi_t) \gamma - \rho}{\gamma \beta \eta z} \varepsilon, \quad \text{implying that } e^i_t = 0, \text{ if } x^i_t \leq \tilde{x}_t \text{ and } e^i_t > 0, \text{ if } x^i_t > \tilde{x}_t, \text{ such that optimal decisions are given by}$$

\textsuperscript{27}We could also associate the health state of children, explicitly, to morbidity and further to productivity in human capital accumulation and labor productivity when adult. This would however increase notational complexity of this paper substantially. Further below, we will see that health improvements will increase human capital accumulation, such that, moreover, the implementation of morbidity would not yield any further gain in insights.

\textsuperscript{28}The difference between $b^i_t$ and $\hat{b}^i_t$ is that the latter contains redistributed wealth of children that did not survive childhood.
(i) if \( x^i_t > \bar{x}_t \) and thus \( e^i_t > 0 \):

\[
\begin{align*}
  c^i_t &= \frac{1}{1 + \gamma} \left[ \hat{y}^i_t + \gamma \bar{c} \right], \\
  n^i_t &= \frac{\gamma}{1 + \gamma} \frac{\hat{y}^i_t - \bar{c}}{\hat{w}_t \bar{z} + h_i e^i_t + b^i_t + d^i_t}, \\
  e^i_t &= \frac{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho \varepsilon}{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho}, \\
  d^i_t &= \frac{\gamma \bar{\pi}_t (z - \frac{z}{x^i_t})}{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho \hat{w}_t h^i_t}, \\
  b^i_t &= \frac{\rho z}{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho \hat{w}_t h^i_t},
\end{align*}
\]

with \( \gamma(1 - \beta \eta - \bar{\pi}_t) - \rho > 0 \).\(^{29}\)

(ii) if \( x^i_t \leq \bar{x}_t \) and thus \( e^i_t = 0 \):

\[
\begin{align*}
  c^i_t &= \frac{1}{1 + \gamma} \left[ \hat{y}^i_t + \gamma \bar{c} \right], \\
  n^i_t &= \frac{\gamma}{1 + \gamma} \frac{\hat{y}^i_t - \bar{c}}{\hat{w}_t \bar{z} + b^i_t + d^i_t}, \\
  b^i_t &= \frac{\rho z}{\gamma(1 - \bar{\pi}_t) - \rho \hat{w}_t h^i_t}, \\
  d^i_t &= \frac{\gamma \bar{\pi}_t z}{\gamma(1 - \bar{\pi}_t) - \rho \hat{w}_t h^i_t},
\end{align*}
\]

with \( \gamma(1 - \bar{\pi}_t) - \rho > 0 \).

Households spent a fraction \( \frac{1}{1 + \gamma} \) of their post-tax income on consumption. The remaining part, \( \frac{\gamma}{1 + \gamma} \), is spent on child rearing. Fertility, \( n^i_t \), is positively related to disposable incomes, but it is negatively related to forgone wage earnings per child, \( zw_i h^i_t \), and negatively associated to expenditures on child quality as captured by \( e^i_t, d^i_t \), as well as the level of bequests, \( b^i_t \), per child. This variables depend, for their part, positively on the level of parental relative human capital, \( x^i_t \), and positively on the extrinsic component of children’s survival probability, \( \bar{\pi}_t \). This means that a favorable environment which increases the number of surviving offspring reduces the desired level of fertility. Consequently, more resources are available for education, nutrition and bequests.

\( e^i_t > 0 \) requires that parents’ relative human capital stock, \( x^i_t \), exceeds \( \bar{x}_t \) as determined by (12). Obviously, the critical threshold, \( \bar{x}_t \), is declining in the extrinsic survival component, \( \bar{\pi}_t \).

\(^{29}\)The non-negativity constraint \( \gamma(1 - \beta \eta - \bar{\pi}_t) - \rho > 0 \) is a common feature in models dealing with the quality-quantity trade-off: the weight of utility attached to the pure presence of children, \( \gamma \), should exceed the weight of children’s quality components in the parental utility function.
If relative human capital of household \( i \) falls short of \( \tilde{x}_t \) it follows that \( e^i_t = 0 \) and fertility is at the highest feasible value while the remaining child quality components, \( b^i_t \) and \( d^i_t \), reach their lowest possible values, see Lemma 1, item (ii).

At the beginning of the second period of life (adulthood), bequests of children that didn’t survive to adulthood are equally redistributed within the family among the surviving offspring. Thus, wealth per adult at the beginning of period \( t + 1 \) is

\[
\tilde{b}^i_t = \frac{b^i_t}{\pi^i_t},
\]

At this point several points are worth being noticed: (i) for low levels of income, agents devote relatively more resources to consumption in order to cover subsistence needs while expenditures for fertility and child quality are low. During earlier stages of economic development this mechanism is reinforced due to a low \( \tilde{h}_t \) and thus a low \( \pi^i_t \). Consequently, probabilities to survive to adulthood are low. Especially the critical threshold, \( \tilde{x}_t \), is relatively high such that depending on the distribution of human capital only few households invest in education for their offspring. Consequently, income gains at this stage of economic development are channeled towards an increase in fertility while the incentives to invest in child quality are low. (ii) Agents characterized by \( x^i_t > \tilde{x} \) are willing to invest in education for their offspring and contribute, by doing so, to a slow increase in the average stock of human capital, \( \bar{h}_t \). In this early phase of economic development, capital accumulation fueled by bequests constitutes the major source of aggregate output growth. The increase in \( \bar{h}_t \) enhances the extrinsic survival probability of children, \( \bar{\pi}_t \), while the increase in production depletes environmental quality reflected by an increase in \( P_t \). Thus, the increase in production induces an offsetting effect on \( \bar{\pi}_t \). (iii) Those dynasties which invest in education benefit from an increase in wage incomes. Due to \( \varepsilon > 0 \) and a low extrinsic survival component, expenditures on education tend to be relatively low while the increase in wage incomes allows for higher expenditures on health and nutrition enhancing children’s prospects to survive to adulthood. Thus population growth must increase. Ultimately, the risk not to survive to adulthood may play a declining role as the economy develops, potentially accelerated by the implementation of abatement measures. In view of declining mortality risks, parents reduce fertility in order to achieve their desired family size and allocate more resources towards child quality. Hence, population growth peaks, declines towards its steady state value and follows the, for industrialized countries, well documented hump-shaped pattern. (iv) Obviously, one could also argue...
that pollution lowers agents’ productivity in human capital accumulation and production. We abstract from these effects just for notational convenience, since \( \bar{\pi}_t \) affects the level of \( e_i^t \) and thus human capital per worker, positively, such that we capture the qualitative direction of detrimental health effects of pollution on human capital accumulation while we are downsizing its quantitative size - see also footnote 27.

3.3. Pollution Abatement Measures

We now introduce the endogenous emergence of abatement measures during the course of economic development. Using the language of the more recent contributions to the literature on institutions (see for example Acemoglu and Robinson, 2009): Institutions are not necessarily efficient but the solution to and the canalization of social conflicts between citizens. In economic terms, the state maximizes lifetime utility of a pivotal social group or a pivotal agent given their optimal decisions. Now, the implementation of such measures obviously depends on whose preferences are considered as being pivotal. Let’s appoint an agent, \( i \), as the pivotal agent \( p \). Then, the government in power sets a tax rate \( 0 \leq \tau_t < 1 \) maximizing the pivotal agent’s lifetime utility (10) given her optimal decisions as specified by Lemma 1 and by allowing for the evolution of the pollution stock (3). The question about whose preferences should be considered as representative depends on political institutions. In a democracy, we should consider the median-voter as the decisive agent. If, in turn, the political system is biased towards the rich or the poor, the median voter should be considered as a theoretical benchmark, rather.

Whether or not the position of the pivotal agent in the income distribution matters for the implementation of pollution abatement, depends, as we will see in the subsequent proposition, on the existence on subsistence consumption, \( \bar{c} > 0 \).

Proposition 1 (Preferred tax rate)

The government maximizes lifetime utility, (10), of a pivotal agent, \( p \), given optimal decisions as specified by Lemma 1 and the evolution of the pollution stock as defined by (3), such that

\[
\max_{0 \leq \tau_t < 1} u^p_t \rightarrow \tau^p_t. \tag{23}
\]

(i) If \( \bar{c} > 0 \) the preferred tax rate of the pivotal agent reads

\[
\tau^p_t = \frac{(2y^p_t - \bar{c})b_3\mu Y_t - [(1 + \gamma(\bar{\pi}_t + \rho))y^p_t H_t] - \sqrt{\Psi}}{2b_3\mu y^p_t Y_t}, \tag{24}
\]

\(^{30}\)For a proof, see Appendix A.1.
with \(\Psi = [1 + \gamma (\bar{\pi}_t + \rho)]^2 (y^P_t)^2 H_t^2 + 2b_3 \mu \bar{c} y^P_t H_t Y_t [1 + \gamma (2 - \bar{\pi}_t - \rho)] + b_2^2 \mu^2 \bar{c}^2 Y_t^2 > 0\) since \(2 - \bar{\pi}_t - \rho > 0\) under reasonable parameter restrictions. Moreover, the preferred tax rate is increasing in income, \(y^P_t\), but declines in the level of subsistence needs, \(\bar{c}\), i.e. \(\frac{\partial \tau^p_t}{\partial y^p_t} > 0\) and \(\frac{\partial \tau^p_t}{\partial \bar{c}} < 0\). Furthermore, \(\tau^p_t = 0\), if \(y^P_t \leq \bar{y}_t\), with \(\bar{y}_t > 0\) if \(\frac{H_t}{Y_t} < \frac{b_3 \mu}{1 + (\bar{\pi}_t + \rho)}\).

(ii) If \(\bar{c} = 0\) the preferred tax rate reads as
\[
\tau^p_{t, \bar{c}=0} = 1 - \frac{1 + \gamma (\bar{\pi}_t + \rho) H_t}{b_3 \mu} \frac{H_t}{Y_t}
\] (25)
and it is independent from the level of income of the pivotal agent, i.e. \(\frac{\partial \tau^p_t}{\partial y^p_t} = 0\), such that \(\tau_t = \tau^i_t = \tau^p_{t, \bar{c}=0}\) for all \(i\). Moreover, \(\tau_t = 0\), if \(\frac{H_t}{Y_t} < \frac{b_3 \mu}{1 + (\bar{\pi}_t + \rho)}\).

(iii) For \(\bar{c} \geq 0\), i.e. disregarded the existence of subsistence consumption, capital accumulation increases the preferred tax rate, while human capital accumulation reduces the preferred tax rate, such that \(\frac{\partial \tau^p_t}{\partial K_t} > 0\) and \(\frac{\partial \tau^p_t}{\partial H_t} < 0\).

Due to the existence of subsistence consumption, \(\bar{c} > 0\), the level of the tax rate, \(\tau^P_t\), depends positively on the pivotal agent’s income, \(y^P_t\) (item (i)). Thus environmental preferences are subject to a hierarchy of needs, in the sense that richer agents prefer more abatement. On the other hand richer agents live in less polluted areas which may reduce their willingness to pay, since the extrinsic component of their children’s survival probabilities is higher. We will come back to the role of different exposures to pollutants further below. If \(\bar{c} = 0\), the preferred tax rate would be the same for all agents unless there is again no other source of heterogeneity (item(ii)) like different exposures to pollutants inducing different levels of \(\bar{\pi}_t\). Disregarded the existence of subsistence consumption, capital accumulation increases the preferred tax rate since it increases the pollution stock and thus the marginal benefit from taxation. On the other hand, human capital accumulation increases production and pollution as well but increases also the productivity of abatement measures. Since the latter overcompensates the former, the tax rate is declining in \(H_t\) (item (iii)).

4. INEQUALITY AND REGIONAL SURVIVAL DIFFERENTIALS

In this section, we allow for residential differences in the exposure to pollutants.\(^{31}\) Consider a population that inhabits two areas denoted by \(A\) and \(B\). For example due to a closer

\(^{31}\)The formal description of the general equilibrium can be found in Appendix A.4.
residential proximity to emission sources, children born in region B suffer compared to region A’s children a higher impact of environmental pollution on the extrinsic component of their survival probability.

Lemma 2 (Residential differences in the exposure to pollutants)

Residents of region B are more exposed to pollutants than region A-agents, such that \( \bar{\pi}_A > \bar{\pi}_B \).

Since the exposure to pollutants will be negatively associated to housing prices we can expect that richer agents live in healthier areas.\(^{32}\) In order to keep the model tractable, we implement a shortcut of residential sorting in the sense that agents are allocated to region A or B according to their relative level of human capital, \( x_i^t \): Agents with relative human capital, \( x_i^t \), above some threshold level, \( \hat{x} \), live in region A while type-B agents are characterized by \( x_i^t \leq \hat{x} \).\(^{33}\) The following proposition summarizes the evolution of relative human capital, \( x_i^jt_j, j = A, B \), in the two regions.\(^{34}\)

Proposition 2 (Evolution of relative human capital in region A and B)

(i) Relative human capital of population group A, \( x_i^{A,t} \), evolves according to

\[
x_{i+1}^A = \left( \frac{zx_i^A - \varepsilon}{z - \varepsilon} \right)^{\eta} \left( x_i^A \right)^{\nu},
\]

with a stationary and stable solution at \( x_i^{A,t} = x_i^{A,*} = 1 \) for all \( t \) and \( i \) for which \( x_i^{A,t} > \hat{x} \).

(ii) Relative human capital of population group B, \( x_i^{B,t} \), evolves according to

\[
x_{i+1}^B = \left( \frac{zx_i^B - \varepsilon}{z - \varepsilon} \right)^{\eta} \left( \frac{1 - \beta(\eta - \rho - \bar{\pi}_A) - \rho - \bar{\pi}_B}{1 - \beta(\eta - \rho - \bar{\pi}_B)} \right)^{\eta} \left( x_i^B \right)^{\nu},
\]

where \( x_i^{B,t} \leq \hat{x} \). Moreover,

\(^{32}\)Gayer et al. (2000;2002) analyze the housing market surrounding superfund sites in Grand Rapids, Michigan and confirm that less risk sells for higher prices. Moreover they find that risk premiums are substantial: After the release of the EPA Remedial Investigation, premiums for safer locations imply values of statistical cancer of approximately $4.3-5.0 million in 2000 dollars.

\(^{33}\)An explicit modeling approach of residential sorting would require a lifetime utility arbitrage condition between the two regions and the explicit modeling of housing prices. This would expand the structure of the model drastically without delivering further insights, since welfare is positively affected by wage incomes while its growth rate depends on human capital accumulation. Moreover, one could argue that residential sorting is driven by assets, rather. Note, that this would not affect our results, since assets per household are determined by the level of bequests per child, i.e. \( b_i^t \) which in turn depend on \( x_i^t \) and post-tax income, see Lemma 1.

\(^{34}\)The proof can be found in Appendix A.2..
(a) $x_{i+1}^{i,B}$ is affected by the extrinsic components of children’s survival probability in the two regions, $\bar{\pi}_j^i$, $j = A, B$. Thus, there exists a stable conditional steady state $x_{i}^{i,B}|_{\bar{\pi}_j^i}$ in each period, $t$.

(b) In light of Lemma 2, it follows that $\frac{1 - \beta \eta - \rho - \bar{\pi}_j^i}{1 - \beta \eta - \rho - \bar{\pi}_j^i} < 1$, such that the $x^{i,B}$-locus is below the $x^{i,A}$-locus and $x_{i}^{i,B}|_{\bar{\pi}_j^i} < 1$.

(c) If the $\frac{\bar{\pi}_j^A}{\bar{\pi}_j^B}$ ratio shrinks (increases) during the transition towards $\bar{\pi}_j^A$ with $\bar{\pi}_j^A > \bar{\pi}_j^B$, the $x^{i,B}$-locus moves upwards (downwards), where $x_{i}^{i,B} < x_{i}^{i,A}$. If $\frac{\bar{\pi}_j^A}{\bar{\pi}_j^B}$ exceeds a critical threshold level, the $B$-locus is always below the 45-degree line and relative human capital endowments in region $B$ approach zero within finite time.

We present the reasoning of Proposition 2 graphically in Figure 3. The evolution of $x_{i}^{i,j}$, $j = A, B$ follows the $A$- or the $B$- locus as described by (26) and (27). In Figure 3, we fixed the threshold level of relative human capital, $\hat{x}$, to the conditional steady state of population group $B$, $x_{i}^{i,B}|_{\bar{\pi}_j^i}$, such that the evolution of relative human capital follows the solid gray line. The assumption that agents with average human capital live in region $A$ is not harmful: if they were allocated to the $B$-region, $x_{i}^{i,B}$ would evolve

---

Note also that, both, the $A$ and the $B$-locus exhibit an unstable steady state close to the origin because of $\varepsilon > 0$. Empirically the region to the left of this steady state is irrelevant since it exceeds the maximal possible number of children over the life course by far.
according to (26) and $x_{i,A}^{t}$ would then be positively influenced by the survival differential between the $A$ and the $B$ region.

Now, the location of the $B$-locus is conditional on the state of the extrinsic components of children’s survival probabilities, $\tilde{\pi}_i^j$, $j = A, B$, which approach a constant value in the long-run, $\tilde{\pi}_i^j$, see also eq. (8), such that the $B$-locus is stationary in the long run, too. Due to a closer proximity of region-$B$ agents to emission sources, the $B$-locus is always below the $A$-locus (item (ii)(b)). If the ratio between the two declines (increases) during the transition, the $B$-locus moves upwards (downwards) and the extrinsic survival component of children living in region $B$ catches up (declines) relative to region $A$. Moreover, if the difference between $\tilde{\pi}_i^A$ and $\tilde{\pi}_i^B$ is sufficiently large, the $B$-locus may even be located below the 45-degree line such that relative human capital of this population group would approach zero within finite time (item (ii)(c)).

Given Lemma 2, type-$B$ agents exhibit a higher fertility and lower investment in education per child (see Lemma 1). Furthermore, the forces of the quality quantity trade-off are amplified via a below average level of human capital, i.e. $x_{i,B}^{t,B} < 1$ for all agents $i$ living in region $B$. Therefore, relative human capital $x_{i,B}^{t,B}$ is evolving at a slower pace over time compared to region $A$. Moreover, in the long-run, region $B$ will be characterized by a level of human capital endowment below the average, $x_{i,B}^{t,*} = x_{i,A}^{t,*} = 1$.

At this point it is also worth emphasizing a crucial feature of our theory: The difference between the long-run values in the extrinsic survival components between the two regions is responsible for long-run differences in relative human capital endowments. This implication holds even though children’s survival probabilities, $\pi_{i,j}^t$, have approached one. Mortality differentials between both population groups are of no concern in the long-run, if the conditional steady state of population group $B$ is increasing during the transition and survival probabilities are equal to 1 within finite time in both regions. Nevertheless, differences in the extrinsic components of the survival probabilities persist in the just described sense and translate into different expenditures in child quality. Thus environmental conditions, which are exogenous at the household level, affect the decisions to invest in child quality although survival probabilities are high. The magnitude of the effect depends obviously on the long-run difference between $\tilde{\pi}_i^A$ and $\tilde{\pi}_i^B$ which, in turn, is determined by the amount of abatement measures.$^{36}$

Now, the magnitude of the preferred tax rate depends also on the location of the pivotal

$^{36}$This effect will be discussed more in detail in Section 5.
agent, i.e. $j = A, B$.\textsuperscript{37}

**Proposition 3 (Effect of $\bar{\pi}_t$ on the preferred tax rate)**

The preferred tax rate of a pivotal agent in region $j = A, B$, $\tau_{p,j}^t$, is inversely related to the extrinsic component of children’s survival probability, $\bar{\pi}_t$, i.e.

$$\frac{\partial \tau_{p}^t}{\partial \bar{\pi}_t} < 0,$$

(28)

given the sufficient condition

$$\frac{1 + \gamma (\bar{\pi}_t + \rho)}{b_3 \mu} y_t^p \frac{H_t}{Y_t} > \bar{c}.$$  

(29)

The last proposition states that agents exposed to less favorable environmental conditions prefer (c.p.) a higher tax rate than their counterparts living in cleaner areas given that their income, $y_t^i$, is sufficiently high. As $\bar{\pi}_t$ as well as $\frac{H_t}{Y_t}$ are constant in the long-run, the sufficient condition (29) assuring $\frac{\partial \tau_{p}^t}{\partial \bar{\pi}_t} < 0$ will be fulfilled for all income classes within finite time. Nevertheless, owed to the hierarchy of needs, agents living in region $A$ may prefer a higher tax rate than their poorer counterparts living in region $B$, if the magnitude of $\frac{\partial \tau_{p}^t}{\partial \bar{\pi}_t}$ is small. If region-$B$ agents are comparatively wealthy and/or exposed to relatively unfavorable environmental conditions they prefer a higher tax rate than agents living in region $A$.

### 5. NUMERICAL EXPERIMENTS

Before we turn to the numerical evaluation of our model, we present the underlying set of parameters and a sketch of the numerical method in the next subsection. In our first experiment (Section 5.2.), we abstract from differences in regional exposures to pollutants in order to analyze the link between children’s survival probabilities, inequality, and pollution in isolation. In Section 5.3., we introduce regional survival differentials of children into the model and conduct the following numerical experiments: First, we investigate changes in the initial distribution of the population over the regions, due to different amounts of initial inequality, while population group $A$ decides about the tax rate. Second, we examine the long-run effects of changes in initial inequality on the long-run performance of the economy, given that either population group $A$’s or $B$’s preferred tax rate is implemented.

\textsuperscript{37}For the proof, see Appendix A.3.
5.1. Calibration and Method

We choose parameters of the model such that the balanced growth path of the model fits to empirical observations of the US economy and United Nations long-run projections. One period in our model has a length of 30 years. We fix the capital income share in the production of \( Y_t \), \( \alpha \), at 0.3. Moreover, capital depreciates within 30 years entirely, i.e. \( \delta = 1 \). As regards child-rearing time, we fix the time share necessary to raise one child to adulthood, \( z \), at 0.072 which implies opportunity cost about 15 percent of parents’ time endowment (see de la Croix and Doepke, 2003 and Knowles, 1999). The remaining parameters are fixed in an iterative way. In order to match a long-run interest rate of

<table>
<thead>
<tr>
<th></th>
<th>( \alpha = 0.3; \delta = 1; A = 3.25 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology</td>
<td></td>
</tr>
<tr>
<td>Pollution</td>
<td>( b_1 = 0.85; b_2 = 1.1; b_3 = 1.365; b_4 = 0.05 )</td>
</tr>
<tr>
<td>Human capital</td>
<td>( B = 3.3; \eta = 0.17; \nu = 0.65; \varepsilon = 0.05 )</td>
</tr>
<tr>
<td>Preferences</td>
<td>( \rho = 0.4; \gamma = 0.95; \beta = 0.625; \mu = 0.8; \bar{c} = 0.15 )</td>
</tr>
<tr>
<td>Survival</td>
<td>( \psi_0 = 0.375; \psi_1 = 0.05; \psi_1^A = 0.05; \psi_1^B = 0.055; \Lambda = 0.18; \lambda = 1000 )</td>
</tr>
<tr>
<td>Child-rearing</td>
<td>( z = 0.072 )</td>
</tr>
</tbody>
</table>

Table 1: Parameters

4% per year and an investment share in the vicinity of 14% fitting the 10 year average of US private fixed capital formation as a share of GDP, we set \( \rho = 0.4, \bar{c} = 0.15 \), and \( A = 3.25 \). The long-run growth rate per year should be around 2% while long-run population growth is zero. This implies \( \gamma = 0.95, \beta = 0.625 \) and \( B = 3.3 \), while \( \varepsilon = 0.05, \eta = 0.17 \) and \( \nu = 0.6 \) are chosen such that the implied fertility differential between the wealthiest and the poorest households does not exceed three surviving children and the average expenditure share for education is in between 5-7%. Since, we consider the evolution of a single pollution stock and abstract, therefore, from any interaction between pollutants, we assume a rather risk averse calibration as far as the absorptive capacity of the environment is concerned and set \( b_1 = 0.85 \). As regards the extrinsic component of children’s survival probability, we specify \( \bar{\pi}_t^j \) as follows

\[
\bar{\pi}_t^j = \max \left\{ \psi_0 \frac{\bar{h}_t}{1 + \bar{h}_t} - \psi_1^j P_t^2; 0 \right\} \quad j = A, B, \quad (30)
\]

with \( \psi_1^A < \psi_1^B \).\(^{38}\)

\(^{38}\)Obviously, (30) satisfies the assumptions regarding eq. (8), in the sense that \( \bar{\pi}_t^j \) is concave in \( \bar{h}_t \), with \( \bar{\pi}_t^j = 0 \), if \( \bar{h}_t = 0 \) and \( \lim_{\bar{h}_t \to \infty} < \infty \). Moreover, the ratio \( \frac{\bar{\pi}_t^A}{\bar{\pi}_t^B} \) declines towards \( \frac{\psi_0 - \psi_1^A P_t^2}{\psi_0 - \psi_1^B P_t^2} \). Thus, the \( x_{t+1}^A \)-locus moves upwards during the transition but stays always below the \( x_{t+1}^B \)-locus as \( \bar{\pi}_t^B > \bar{\pi}_t^A \), see also Proposition 2.
In order to take account for the observation of an environmental Kuznets curve, which is apparently more realistic as far as local pollutants are concerned, we set $b_2 = 1.1$, $b_3 = 1.365$, $b_4 = 0.05$ and $\mu = 0.8$, $\psi_0 = 0.75$, $\psi_1 = 0.05$ for no regional differences, but $\psi_1^A = 0.05$, $\psi_1^B = 0.055$ in the presence of regional differences. This also implies an upper boundary for the pollution tax, in the long-run, of appr. 0.14 compatible with the afore mentioned long-run interest rate of 4% and the investment share of 14%. In order to get a reasonable fit of the transition period we set $\bar{\lambda} = 0.18$ and $\lambda = 1000$.

For the simulation of the model we generate a large number of households, $N_0 = 1000$, and draw for each of them an initial level of human capital, $h_{0i}^t$, from a log-normal distribution, $F(\mu_h, 0, \sigma_h, 0)$. Given the initial distribution of wealth, i.e. $\tilde{b}_0 = \tilde{b}_0$, the initial capital stock, $K_0$, is known. For a given $P_0$, the only unknown is aggregate labor supply to production, $L_0$, which in turn depends on households’s decisions. We therefore estimate labor supply by means of the delta method using the moments of the log-normal distribution and households’ optimal decisions as specified by Lemma 1, the tax rate (24) and factor prices as determined by (54) and (55). Now, the equilibrium solutions for our artificial sample of households in $t = 0$ are known and the state of the economy for the next period is obtained from the equilibrium conditions as described in Section A.4.. The next step of the iteration starts again with the delta method. The procedure continues until a stopping criterion $\bar{\varepsilon} = 10^{-5}$ between two iteration steps is reached implying that the economy is sufficiently close to its steady state.

5.2. Transitory Dynamics and Initial Inequality

For the moment, we abstract from differences in regional exposures to pollutants and analyze the interrelationship between children’s health, inequality, and pollution in isolation. Under this circumstances, inequality in terms of initial human capital endowments has no long-run effect. The evolution of relative human capital follows (26) and converges thus to $x^*_i = 1$ for all $i$ and $t$. The results are depicted in Figure 4. Note also that growth rates of variables between two periods denoted by $g$ are adjusted to their 30 years average. Thus

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39Since we focus rather on local pollution, the emergence of a Kuznets curve may seems to be a reasonable fit. For a detailed discussion on the Kuznets curve, see Dasgupta et al. (2002). Note also that the qualitative results of the paper are independent from the emergence of an environmental Kuznets curve.

40In the baseline scenario, we set $\mu_h = 0.4$ and $\sigma_h = 0.5$.

41We assume that initial wealth is equally distributed in order to avoid an arbitrary source of heterogeneity. After the first period, differences in wage incomes will generate inequality in wealth as well.

42See for example Oehlert (1992).
a growth rate of 0.01 between two periods has to be interpreted as the yearly average growth rate of 1% over 30 years.

So far, our model is able to capture several empirical regularities of economic development: a hump-shaped evolution of the (net-) population’s growth rate, a slow take-off, i.e. increasing growth rates in output per capita, an environmental Kuznets curve, increasing expenditures on abatement measures, and increasing survival probabilities of children.

Initially, the net population’s growth rate is low, due to low survival probabilities of children and high expenditure shares on parental consumption. In this stage, low survival probabilities of children are owed to a low extrinsic component of children’s survival probability, $\bar{\pi}_t$, caused by a low stage of economic development reflected by the average stock of human capital, $\bar{h}_t$. A low $\bar{\pi}_t$ in conjunction with a low relative human capital...
endowment, $x^i_t$, for the majority of households, induces parents to allocate few resources to education, health and bequests (see Lemma 1). Some households are even at the corner solution $e^i_t = 0$. Thus, the growth rates of human capital and output per capita ($g_h$ and $g_{Y/N}$) are low. A slowly increasing level of aggregate production - fueled mainly by capital accumulation - increases pollution. At the same time and depending on the distribution of human capital some households invest in human capital, if their relative human capital stock is above the critical threshold $\bar{x}$ as determined by (12). By doing so they contribute to an increase in the average level of human capital, $\bar{h}_t$. If the latter effect overcompensates the adverse effect of increasing pollution on the extrinsic survival component, $\bar{\pi}_t$ starts to increase. The increase in $\bar{\pi}_t$ strengthens the forces of the quality quantity trade-off, i.e. average expenditures on health and bequests increase while expenditures on education might still be low due to $\varepsilon > 0$. Moreover, households’ income increase such that the expenditure share on consumption shrinks and the expenditure share for offspring increases. The latter and the increase in child survival probabilities contribute to an increase in net-population growth.

An increase in the level of human activities captured by population size and the level of aggregate production induces a fast increase in the pollution stock, but the preferred tax rate of the pivotal agent is still zero since her income is below the critical income level, $\tilde{y}$, see Proposition 1, item (i). Increasing incomes and an increasing accumulation of physical capital induce an increase in the marginal benefit from taxation, such that the preferred tax rate of the pivotal agent is eventually positive and increasing over time. Over time, the probability to survive childhood approaches one such that the forces responsible for an increase in the net-population’s growth rate disappear. Further income gains are devoted to child quality. Thus, net-population growth reaches a maximum and starts to decline towards its long-run value. Declining population growth and increasing expenditures on child quality fuel further increases in output growth per capita.

Higher initial inequality (dashed line)$^{43}$ means that more households are characterized by a lower level of relative human capital, $x^i_t$, which implies also that more households are below the critical threshold level, $\bar{x}_t$, necessary for $e^i_t > 0$, see also (12). Consequently, average expenditures on child quality are, compared to the baseline scenario, reduced. Therefore, average human capital evolves at a lower pace implying a lower level of the

$^{43}$We keep the mean of the initial distribution of human capital constant.
extrinsic component of children’s survival probability, \( \bar{\pi}_t \). Due to lower survival probabilities of children and lower expenditure shares for fertility, the net-population’s growth rate falls also short of the reference scenario, in this early stage of economic development.\(^{44}\) Moreover, as the pollution stock increases above the reference level, the pivotal agent prefers a higher tax rate during the transition to the long-run equilibrium. After survival probabilities have reached one, net-population growth starts to decline. Since, however, more agents are characterized by a low \( x_i^t \) and \( \bar{\pi}_t \), fertility declines as compared to the reference scenario at a higher level towards its long-run value. Thus, output per capita is also converging at a lower growth rate towards its long-run value.

5.3. Regional Survival Differentials

Let’s consider now a population which inhabits two regions, \( A \) and \( B \). Population group \( B \) suffers a higher exposure to pollutants than population group \( A \). Therefore, initial inequality in human capital endowments triggers the distribution of the population over the two regions and affects the evolution of the economy not only during the transition but also in the long-run (see Figure 3). In the following two experiments, we analyze therefore: (1) changes in the initial distribution of the population over the two regions given that population group \( A \) decides about the tax rate, and, (2) the long-run effects of initial inequality given that either population group \( A \)’s or population group \( B \)’s preferred tax rate is implemented.

(1) Changes in initial inequality while group \( A \) decides about the tax rate

We change the initial distribution of the population over the two regions by changing the initial distribution of relative human capital while we keep the mean of the distribution constant. Due to a closer proximity to emission sources of population group \( B \), we have \( \bar{\pi}_t^A > \bar{\pi}_t^B \). Moreover, the two population groups converge to different steady states characterized by \( \bar{\pi}_*^A = 1 > \bar{\pi}_*^B \) (see Proposition 2), such that the initial distribution of the population over the two regions will alter the steady-state of the economy.

As type-\( A \) agents are equipped with a higher relative human capital stock than type-\( B \) agents, type-\( B \) agents invest less in child quality and exhibit a higher number of births.\(^{44}\) Further increases in initial inequality could also induce a delay in the implementation of abatement measures because the pivotal agent’s income surpasses the critical threshold level, \( \tilde{y} \), later.
Figure 5: Evolution of the mortality differential between region A and B. Baseline: solid line; increased initial inequality: dashed line.

Lower expenditures on health and nutrition in conjunction with a lower external component of children’s survival probability induce a slower increase in children’s survival probabilities in region B as compared to region A. Consequently, the mortality differential between region B and A as expressed by \((1 - \pi_A^t) - (1 - \pi_B^t) = \pi_A^t - \pi_B^t\) must increase. Since expenditures on health are subject to declining marginal returns and since \(\bar{\pi}_j^t, j = A, B\) is concave in \(\bar{h}_t\), mortality differentials must decline again, eventually accelerated if region A reaches the upper boundary of one (see solid line in Figure 5). Thus our model is able to trace the hump-shaped evolution of mortality differentials between regions characterized by different degrees of environmental degradation as it has been illustrated by Figure 2.

Higher initial inequality (dashed line in Figure 5) implies that more households are living in region B, such that more households are characterized by a relatively low \(x_i^t\) and a higher exposure of their children to pollutants compared to the baseline scenario. Consequently, more households face lower survival probabilities of their children and exhibit lower expenditures for child quality. Due to the slower increase in children’s survival probabilities, the peak of the net-population’s growth rate is delayed. Moreover, average expenditures on child quality are also reduced compared to the baseline scenario. Thus higher initial inequality increases the mortality differential between both regions and population growth declines, compared to the reference scenario, at a higher level towards a
Figure 6: Long-run effects of initial inequality, while either population group A (gray lines) or B (black lines) decides about the tax rate

higher steady state value. The latter in combination with lower expenditures for education reduces the long-run growth rate of the economy.\textsuperscript{45}

(2) Long-run effects of initial inequality, while either population group A or B decides about the tax rate

Again we increase initial inequality in human capital while we keep the mean of the distribution constant. In Figure 6 we plot the stationary long-run values of the tax rate, the extrinsic component of the survival probability and the pollution stock against the increase in initial inequality. There we consider two scenarios in the sense that either population group A (gray line) or B (black line) is decisive for the tax rate, for example because either the median-voter is located in one or the other region, or political institutions are biased towards richer or poorer agents. Apparently population group B which is more affected by pollution prefers a higher tax rate in the long-run (\(\tau_B^* > \tau_A^*\), since in light of Prop. 3: \(\frac{\partial \tau_j^*}{\partial \bar{\pi}_j^*} < 0\)).\textsuperscript{46}

Higher initial and thus long-run inequality induces a higher pollution stock because the economy accumulates human capital at a slower pace, such that the pivotal agent prefers

\textsuperscript{45}A larger population going along with a lower accumulation of human capital increases in addition the pollution stock, such that higher inequality induces a higher preferred tax rate in the long-run. The respective long-run values are in the baseline scenario: \(g_{X*} = 0.005; g_{Y/X*} = 0.022; \tau_A^* = 0.115\) and for the high-inequality scenario: \(g_{X*} = 0.0062; g_{Y/X*} = 0.020; \tau_A^* = 0.122\).

\textsuperscript{46}Note also that the sufficient condition (29) may be violated in earlier stages of economic development. In this case population group A would prefer a higher tax. Nevertheless this is only a transitory phenomenon because as has been pointed out, (29) will be met within finite time.
a higher tax rate. Therefore the extrinsic components of children’s survival probabilities are lower. A lower $\bar{\pi}^j$, $j = A, B$ is, apart from higher inequality, the second channel responsible for reduced human capital accumulation (and increased population growth). Hence, the long-run growth rate is inversely related to initial inequality while the long run pollution stock is positively associated to initial inequality which matches the observation related to Figure 1. The long-run effects are weakened, though, if the population group that is most affected by pollutants decides about the tax rate, since this group prefers the highest tax rate. Thus the connection between inequality, pollution can be mitigated by institutional reforms.

6. SUMMARY AND CONCLUSIONS

We argued that increasing expenditures on education are positively associated to children’s probability to survive to adulthood. The probability to survive to adulthood depends positively on the stage of economic development and disposable incomes of households but it is adversely affected by environmental pollution. In this context, economic inequality is not only decisive for human capital investment and the emergence of differential fertility between richer and poorer households (de la Croix and Doepke, 2003), but also for agents’ exposure to environmental pollution because wealthier households live in cleaner areas. The exposure to pollutants triggers again children’s probability to survive to adulthood and the willingness of parents to invest in education. This was the key mechanism and the novelty of our approach through which environmental conditions may impose a growth drag on the economy.

Moreover, we analyzed how initial conditions in terms of higher initial inequality affect the interaction between economic development, degradation of the environment, parental decisions to invest in child quality, and their willingness to pay for tax-financed abatement measures. We demonstrated that higher initial inequality lowers average expenditures on child quality, in terms of education and health, such that the growth rate of human capital is reduced. Thus, the pollution stock and the tax rate increase which reinforces the negative effect of inequality on child quality and thus increases the number of births with a lower survival probability per child.

Finally, the adverse effect of inequality and pollution on children’s health can then be amplified, if political institutions are biased towards the rich, i.e. the population group that is least affected decides about the level of tax-financed abatement measures. The
described mechanism provides a candidate explanation for: (1) The observed positive cross-country correlation between economic inequality and pollution at the local level, and (2) the hump-shaped evolution of child mortality ratios between areas that are subject to different degrees of environmental pollution. In addition, our research suggests a channel through which inequality translates into health differentials even though there is not necessarily a direct link from inequality to health, but as implied by our theory: Economic inequality interacts with social segregation and political inequalities which translate into inequalities in health (Deaton, 2003; Alesina, Baqir, and Easterly, 1999).

We strongly believe that our research has important implications for economic policies along the following dimensions: The interplay between economic development, degradation of the environment and political institutions is not only interesting from a historic and thus academic perspective but applies also for fast developing countries like China in the present.47 Moreover, the question through which channels economic inequality translates via political institutions into health differentials deserves attention of policy makers in developed and developing countries alike.

REFERENCES


47Breathing in Beijing’s air is the equivalent to smoking forty cigarettes a day, The Economist (2015).


MATHEMATICAL APPENDIX

A.1. Proof of Proposition 1

Plugging optimal decisions (13)-(17) and the evolution of the pollution stock, (3), into
(10), we obtain as an first-order condition
\[
\frac{\partial \tau^p_t}{\partial \pi_t} = \left\{ \mu b_3 Y_t \gamma \tau^p_t + [(1 + \gamma(\bar{\pi}_t + \rho) - \mu(2b_3 - \bar{c})) H_t y^p_t] \gamma \tau^p_t \right. \\
\left. -[1 + \gamma(\bar{\pi}_t + \rho)] H_t y^p_t + [(\bar{\pi}_t + \rho) - 1] \gamma \bar{c} + b_3 \mu (y^p_t - \bar{c}) Y_t \right\} \\
\left(1 - \tau^p_t \right) H_t [(1 - \tau^p_t) y^p_t - \bar{c}] \right)^{-1} = 0,
\]
(31)
such that
\[
\tau^{p}_{t_{1,2}} = \frac{(2y^p_t - \bar{c})b_3 \mu Y_t - [(1 + \gamma(\bar{\pi}_t + \rho)) y^p_t H_t] \pm \sqrt{\Psi}}{2b_3 \mu y^p_t Y_t},
\]
(32)
with \( \Psi = [1 + \gamma(\bar{\pi}_t + \rho)]^2 (y^p_t)^2 H_t^2 + 2b_3 \mu \bar{c} y^p_t H_t [1 + \gamma(2 - \bar{\pi}_t - \rho)] + b_3^2 \mu^2 \bar{c}^2 Y_t^2 \). Note that \( \Psi > 0 \), since \( 2 - \bar{\pi}_t - \rho > 0 \) under reasonable parameter restrictions. In addition \( \frac{\partial^2 u^p_t}{\partial (\tau^p_t)^2} < 0 \), if and only if
\[
\tau^p_t = \frac{(2y^p_t - \bar{c})b_3 \mu Y_t - [(1 + \gamma(\bar{\pi}_t + \rho)) y^p_t H_t] - \sqrt{\Psi}}{2b_3 \mu y^p_t Y_t}. \quad 48
\]
(33)
Derivation of partial derivatives:

(1) An increase in subsistence consumption reduces the preferred tax, since
\[
\frac{\partial \tau^p_t}{\partial \bar{c}} = -\frac{1}{2y^p_t} - \frac{1}{2} \left( \sqrt{\Psi} \right)^{-\frac{1}{2}} \frac{\partial \Psi}{\partial y^p_t}
\]
(34)
and
\[
\frac{\partial \Psi}{\partial \bar{c}} = 2b_3 y^p_t \mu H_t [1 + \gamma(2 - \bar{\pi}_t - \rho)] + 2b_3^2 \mu^2 Y_t \bar{c} > 0,
\]
(35)
it follows that \( \frac{\partial \tau^p_t}{\partial \bar{c}} < 0 \).

(2) An increase in the pivotal agent’s income, \( y^p_t \), increases the preference for \( \tau^p_t \), because
\[
\frac{\partial \tau^p_t}{\partial y^p_t} = \frac{2b_3 \mu Y_t - (1 + \gamma(\bar{\pi} + \rho)) H_t - \frac{1}{2} \Psi^{-\frac{1}{2}} \frac{\partial \Psi}{\partial y^p_t}}{2b_3 \mu Y_t y^p_t} \\
- \frac{(2y^p_t - \bar{c})b_3 \mu Y_t - [(1 + \gamma(\bar{\pi}_t + \rho)) y^p_t H_t] - \Psi^{\frac{1}{2}}}{2b_3 \mu y^p_t (y^p_t)^2} \\
= -\frac{1}{2} \Psi^{-\frac{1}{2}} \frac{\partial \Psi}{\partial \tau^p_t} y^p_t + \bar{c} b_3 \mu Y_t + \Psi^{\frac{1}{2}} \geq 0.
\]
(36)
Noting \( \Psi \) implies that \( \Psi^{\frac{1}{2}} \geq 0 \), such that \( \frac{\partial \tau^p_t}{\partial y^p_t} > 0 \).

48Proof available upon request.
(3) Noting (24), it follows that \( \tau_t^p = 0 \), if \( \tilde{y}_t = y_t^p \) with
\[
\tilde{y}_t = \frac{c[-\gamma(1 - \rho - \pi_t)H_t - b_3\mu Y_t]}{[1 + \gamma(\pi_t + \rho)]H_t - b_3\mu Y_t}.
\]
(38)
Since the nominator of the last expression is negative, because \((1 - \rho - \pi_t) > 0\) under reasonable parameter restrictions, it follows that \( \tilde{y} > 0 \), if \([1 + \gamma(\pi_t + \rho)]H_t < b_3\mu Y_t \), which implies
\[
\frac{H_t}{Y_t} < \frac{b_3\mu}{1 + \gamma(\pi_t + \rho)}.
\]
(39)
(1), (2) and (3) verify item (i). Item (ii) follows from (33), for \( \tilde{c} = 0 \).

Item (iii) can be verified as follows:

(4) An increase in aggregate human capital reduces the preferred tax rate
\[
\frac{\partial \tau_t^p}{\partial K_t} = \frac{(2y_t^p - \tilde{c})b_3\mu \frac{\partial y_t^p}{\partial Y_t} - \frac{1}{2} \Psi^{\frac{1}{2}} \frac{\partial \Psi}{\partial Y_t} \frac{\partial y_t^p}{\partial Y_t}}{2b_3\mu y_t^p Y_t}
\]
\[
- \frac{(2y_t^p - \tilde{c})b_3\mu y_t^p - [(1 + \gamma(\pi_t + \rho))y_t^p H_t] - \Psi^{\frac{1}{2}} \frac{\partial Y_t}{\partial Y_t} \geq 0}
\]
(40)
such that \( \frac{\partial \tau_t^p}{\partial K_t} \geq 0 \), if
\[
-\frac{1}{2} \Psi^{\frac{1}{2}} \frac{\partial \Psi}{\partial Y_t} Y_t + [(1 + \gamma(\pi_t + \rho))y_t^p H_t] + \Psi^{\frac{1}{2}} \geq 0.
\]
(41)
Noting now the definition of \( \Psi \), it can be shown that \( \Psi^{\frac{1}{2}} > \frac{1}{2} \Psi^{\frac{1}{2}} \frac{\partial \Psi}{\partial Y_t} Y_t \), such that \( \frac{\partial \tau_t^p}{\partial K_t} > 0 \).

(5) An increase in aggregate human capital reduces the preferred tax rate
\[
\frac{\partial \tau_t^p}{\partial H_t} = \frac{(2y_t^p - \tilde{c})b_3\mu \frac{\partial y_t^p}{\partial Y_t} - [(1 + \gamma(\pi_t + \rho))y_t^p Y_t]}{2b_3\mu y_t^p Y_t^2}
\]
\[
- \frac{(2y_t^p - \tilde{c})b_3\mu y_t^p - [(1 + \gamma(\pi_t + \rho))y_t^p H_t] - \Psi^{\frac{1}{2}} \frac{\partial Y_t}{\partial H_t}}{2b_3\mu y_t^p Y_t^2}
\]
(42)
Reshufffing terms yields
\[
\frac{\partial \tau_t^p}{\partial H_t} = -[(1 + \gamma(\pi_t + \rho))y_t^p Y_t] \frac{H_t}{\frac{\partial Y_t}{\partial H_t} (1 - \alpha) + \frac{\partial Y_t}{\partial Y_t}} - \frac{1}{2} \Psi^{\frac{1}{2}} \frac{\partial \Psi}{\partial Y_t} Y_t
\]
\[
+ [(1 + \gamma(\pi_t + \rho))y_t^p H_t] + \Psi^{\frac{1}{2}}.
\]
(43)
Noting now the definition of \( \Psi \), we obtain
\[
\Psi^{\frac{1}{2}} < \frac{1}{2} \Psi^{\frac{1}{2}} \left( \frac{\partial \Psi}{\partial H_t} Y_t - \frac{\partial \Psi}{\partial Y_t} \right).
\]
(44)
As \( \frac{Y_t}{H_t} = \frac{H_t}{1 - \alpha} \) and \( \alpha \in (0, 1) \) it follows that \( \frac{\partial \tau_t^p}{\partial H_t} < 0 \).
A.2. Proof of Proposition 2

(1) Individual human capital evolves according to (4). Dividing (4) by $\bar{h}_t$ and defining $x_i^t = \frac{h_i^t}{\bar{h}_t}$, we obtain: $\frac{h_i^{t+1}}{\bar{h}_t} = (\varepsilon + e_i^t)^\eta (x_i^t)^\nu$. Thus

$$x_i^{t+1} = (\varepsilon + e_i^t)^\eta (x_i^t)^\nu \frac{\bar{h}_t}{\bar{h}_{t+1}}.$$  \hspace{1cm} (45)

Noting further (15), and that $\varepsilon + e_i^t = \frac{\gamma \beta \eta [zx_i^t - \varepsilon]}{\gamma (1 - \beta \eta - \bar{\pi}_t) - \rho}$ yields

$$\frac{\bar{h}_{t+1}}{\bar{h}_t} = \left( \frac{\gamma \beta \eta [zx_i^t - \varepsilon]}{(1 - \beta \eta - \bar{\pi}_t) - \rho} \right)^\eta (x_i^t)^\nu.$$  \hspace{1cm} (46)

Since $x_i^t = 1$ for agents endowed with average human capital, $x_i^{t+1}$ writes in light of the definition of $\varepsilon + e_i^t$ and (46) as

$$x_i^{t+1} = \left( \frac{zx_i^t - \varepsilon}{z - \varepsilon} \right)^\eta (x_i^t)^\nu.$$  \hspace{1cm} (47)

(2) Given that agents with average human capital are subject to $\bar{\pi}_t^A$ while agents in region $B$ are exposed to $\bar{\pi}_t^B$ with $\bar{\pi}_t^A \neq \bar{\pi}_t^B$, relative human capital of agents in region $A$ evolves again according to

$$x_{i,A}^{t+1} = \left( \frac{zx_{i,A}^t - \varepsilon}{z - \varepsilon} \right)^\eta (x_{i,A}^t)^\nu.$$  \hspace{1cm} (48)

Observing (46) and noting further that $\varepsilon + e_{i,B}^t = \frac{\gamma \beta \eta [zx_{i,B}^t - \varepsilon]}{(1 - \beta \eta - \bar{\pi}_t^A) - \rho}$, yields

$$x_{i,B}^{t+1} = \left( \frac{zx_{i,B}^t - \varepsilon}{z - \varepsilon} \right)^\eta \left( \frac{(1 - \beta \eta - \bar{\pi}_t^A) - \rho}{(1 - \beta \eta - \bar{\pi}_t^B) - \rho} \right)^\eta (x_{i,B}^t)^\nu.$$  \hspace{1cm} (49)

(3) We verify existence and stability of stationary solutions for the $A$- and the $B$-locus and begin the proof with the $A$-locus.\footnote{Remember, that the scenario without residential differences in the exposure to pollutant corresponds entirely to region $A$’s dynamics.} Relative human capital evolves according to (48). Note that

(a) $x_{i+1}^{i,A} = 0$, if $x_i^{i,A} = \frac{\varepsilon}{z}$.\footnote{There is a meaningless solution for $x_i^{i,A} = 0$.}

(b) there exists a meaningful solution at $x_i^{i,A} = x_i^* = 1$. Since

$$\frac{\partial x_{i+1}^{i,A}}{\partial x_i^{i,A}} = \left( \frac{zx_i^{i,A} - \varepsilon}{z - \varepsilon} \right)^\eta \left[ \frac{2\eta (x_i^{i,A})^\nu}{zx_i^{i,A} - \varepsilon} + \nu (x_i^{i,A})^\nu - 1 \right] > 0$$  \hspace{1cm} (50)
because \(zx_i^* - \varepsilon > 0\) and \(z - \varepsilon > 0\) for economically meaningful solutions. It follows immediately that the stationary solution \(x_i^* = 1\) is stable, if \(|\frac{\partial x_i^*}{\partial (x_i^*)} \big|_{x_i^* = 1}\) < 1. Thus, we obtain for \(x_i^* = 1\): \(\frac{\partial x_i^*}{\partial (x_i^*)} \big|_{x_i^* = 1} = -\frac{z(\eta + \nu - \varepsilon)}{z - \varepsilon}\), which is smaller than 1 if \(z(\eta + \nu - 1) - \varepsilon(\nu - 1) < 0\). Since \(z > \varepsilon\) and \(\nu, \eta, z, \varepsilon > 0\) it follows that the above inequality holds if and only if \(\nu + \eta < 1\). Thus, if \(\nu + \eta < 1\), the \(A\)-locus intersects the 45°-line at \(x_i^* = 1\) from above. From (a), we know that the \(A\)-locus intersects the abscissa at \(\xi\) which implies the existence of a second unstable steady state between \(\xi\) and 1.\(^{51}\) The only difference between the \(A\)- and the \(B\)-locus is the appearance of the factor \(\left(\frac{\gamma(1 - \beta \eta - \varepsilon)}{\gamma(1 - \beta \eta - \varepsilon^2)}\right)^{-\beta}\) in the latter, which is smaller than 1, since \(\tilde{\pi}_t^A > \tilde{\pi}_t^B\). It is trivial to see that this induces a downward shift of the \(B\)-locus relative to the \(A\)-locus. Thus, the (conditional) stable stationary solution of the \(B\)-locus is smaller than 1. If, moreover, the difference between \(\tilde{\pi}_t^A\) and \(\tilde{\pi}_t^B\) exceeds a critical threshold, the \(B\)-locus is always below the 45-degree line, such that relative human capital of type-\(B\) agents converges to zero.

### A.3. Proof of Proposition 3

From Proposition 1, item (ii), it follows outright that \(\frac{\partial \tau_p^c}{\partial \pi_t} < 0\). If \(c > 0\), the sign of \(\frac{\partial \tau_p^c}{\partial \pi_t}\) depends on the stage of economic development, the distribution of human capital, and the position of the pivotal agent in the income distribution:

\[
\frac{\partial \tau_p^c}{\partial \pi_t} = -\frac{\gamma y_t^p H_t}{2b_3 y_t^3 Y_t} - \frac{1}{2} \frac{\sqrt{\Psi} \frac{1}{2} \tau_t^c}{2b_3 y_t^3 Y_t} \quad (51)
\]

Thus, a sufficient condition for \(\frac{\partial \tau_p^c}{\partial \pi_t} > 0\) is \(\frac{\partial c}{\partial \pi_t} > 0\), which is the case if

\[
2\gamma [1 + \gamma (\tilde{\pi}_t + \rho)] (y_t^p)^2 H_t^2 - 2b_3 \gamma y_t^p H_t Y_t \tilde{c} > 0, \quad (52)
\]

\[
\frac{1 + \gamma (\tilde{\pi}_t + \rho) y_t^p H_t}{b_3 y_t^3 Y_t} > \tilde{c} \quad (53)
\]

The right-hand side of (53) is constant. On the other hand, \(\tilde{\pi}_t\) and \(\frac{H_t}{Y_t}\) are constant in steady state, while \(y_t^p\) is increasing during the transition and growing at a constant growth rate in steady state, it follows that there exists for a given stage of economic development

\(^{51}\) A more rigorous proof requires to verify monotonicity: Define \(G' = \frac{\partial x_i^*}{\partial x_i^*} \) and \(G'' = \frac{\partial^2 x_i^*}{\partial x_i^*^2}\), the curvature of the \(A\)-locus can now be expressed by \(\frac{G''}{G'} = \frac{zz_{x_i^* - \varepsilon} + z_{x_i^* - \varepsilon^2}}{(\nu - 1) \varepsilon^2 + 2\varepsilon(\eta - \nu - 1) + z^2 (x_i^* - \delta^2)}[(\eta + \nu - (\eta + \nu)]\). The nominator is always positive, since \(zz_{x_i^* - \varepsilon} - \varepsilon > 0\). Since \(G' > 0\), \(\frac{G''}{G'}\) is negative if \(G'' < 0\). \(\frac{G''}{G'}\) is negative if \(\nu + \eta < 1\). Which proofs concavity of the \(A\)-locus, i.e. the existence of at most two steady states.
expressed by $H_t$ and $Y_t$ a threshold income of the pivotal agent, $y^p_t$, such that $y^p_t \geq \bar{y}_t$ implies $\frac{\partial \tau_t}{\partial \bar{\pi}_t} < 0$.

A.4. Equilibrium

Noting that in period $t = 0$, population size, $N_0$, equals the number of households (dynasties), an equilibrium can be defined as follows: Given a large number of households $i \in [1, ..., N_0]$ in period $t = 0$, an initial distribution of individual human capital that determines for each agent, $h^i_0 > 0$, and thus the average stock of human capital, $\bar{h}_0 > 0$. Given as well an initial distribution of individual wealth that determines, $b^i_0 > 0$, and thus $K_0 > 0$, and given an initial stock of pollution $P_0 > 0$, an equilibrium consists of a sequence of aggregate quantities $\{K_t, K_{t+1}, L_t, P_t, P_{t+1}, N_t, N_{t+1}, H_t, H_{t+1}, \bar{h}_t, \bar{h}_{t+1}\}_{t=0}^{\infty}$, a sequence of factor prices and tax rates $\{w_t, R_t, \tau_t\}_{t=0}^{\infty}$, with $\tau_t \geq 0$, and a sequence of optimal decisions $\{c^i_t, n^i_t, e^i_t, d^i_t, b^i_t\}_{t=0}^{\infty}$ with $e_t \geq 0$ that maximize lifetime utility (10), subject to (4) and (11). Perfect competition implies that equilibrium rates of reward read as

$$ w_t = (1 - \alpha)Ak_t^{\alpha}, $$

$$ R_t = r_t + \delta = \alpha Ak_t^{\alpha - 1}, $$

where $0 \leq \delta \leq 1$ is the depreciation rate of physical capital and $k_t = \frac{K_t}{L_t}$ represents capital per efficient unit of labor. Time devoted to child rearing and education is not available for production, such that

$$ L_t = \sum_{i=1}^{N_0} N^i_t \left[(1 - zn^i_t)h^i_t - e^i_t n^i_t \bar{h}_t\right]. $$

The size of a type-$i \in [1, ..., N_0]$ household denoted by $N^i_t$ with relative human capital endowment, $x^i_t$, evolves from one period to another according to $N^i_{t+1} = \pi^i_t n^i_t N^i_t$. Hence population size in $t + 1$ is obtained as

$$ N_{t+1} = \sum_{i=0}^{N_0} N^i_{t+1}. $$

Average human capital in $t$ is given by $\bar{h}_t = \sum_{i=1}^{N_0} \frac{N^i_t}{N_t} h^i_t$ and evolves according to

$$ \frac{\bar{h}_{t+1}}{\bar{h}_t} = \frac{\sum_{i=1}^{N_0} \frac{N^i_{t+1}}{N_{t+1}} h^i_{t+1}}{\sum_{i=1}^{N_0} \frac{N^i_t}{N_t} h^i_t}. $$
In light of (4) and noting that a household endowed with average human capital is characterized by $x_i^t = 1$, the evolution of relative human capital of household $i$ is governed by

$$x_{i+1}^t = \left( \frac{zx_i^t - \varepsilon}{z - \varepsilon} \right)^\eta (x_i^t)^\nu. \quad \text{(59)}$$

The stock of aggregate capital in the subsequent period is determined by the sum of wealth per child surviving to adulthood

$$K_{t+1} = \sum_{i=1}^{N_0} N_{i+1}^t \tilde{b}_i^t. \quad \text{(60)}$$

Finally, the tax rate is obtained from (24) or (25) and the level of the pollution stock in the next period, $P_{t+1}$, is determined by (3). 

\[52\] For the derivation of (59), see Appendix A.2., item (1).
B.1. Optimal decisions of household $i$

Maximizing (10) subject to (11) by recognizing (4) but ignoring the impact on $P_{t+1}$ yields the following set of first-order conditions with $\lambda_t$ denoting the shadow price,

$$\frac{1}{c^i_t - \bar{c}} = \lambda^i_t,$$  \hspace{1cm} (61)

$$\frac{\gamma}{n^i_t} = \lambda^i_t [\hat{w}_i h^i_t z + \hat{w}_i h_i e^i_t + b^i_t + d^i_t],$$  \hspace{1cm} (62)

$$\frac{\gamma \beta \eta}{e^i_t + \bar{c}} = \lambda^i_t \hat{w}_i h_i n^i_t,$$  \hspace{1cm} (63)

$$\frac{\rho}{b^i_t} = \lambda^i_t n^i_t,$$  \hspace{1cm} (64)

$$\frac{\gamma \hat{\pi}^i_t}{d^i_t} = \lambda^i_t n^i_t.$$  \hspace{1cm} (65)

Equating (63) and (62) yields

$$e^i_t = \frac{\beta \eta}{1 - \beta \eta} \left( z x^i_t + \frac{b^i_t + d^i_t}{\hat{w}_i h_i} \right) - \frac{\varepsilon}{1 - \beta \eta}. \hspace{1cm} (66)$$

Equating (64) and (62) and recognizing that $x^i_t = \frac{b^i_t}{\bar{c}}$ yields together with (66)

$$\frac{\rho}{b^i_t} = \frac{\gamma}{\hat{w}_i h_i (z + \frac{c^i_t}{x^i_t} + \frac{b^i_t + d^i_t}{\hat{w}_i h_i})}, \hspace{1cm} (67)$$

$$\Rightarrow b^i_t = \frac{\rho}{\gamma (1 - \beta \eta) - \rho} \left( z - \frac{\varepsilon}{x^i_t} \hat{w}_i h^i_t + d^i_t \right). \hspace{1cm} (68)$$

Equating (65) and (62) using (66) and (68) yields

$$d^i_t = \frac{\gamma \hat{\pi}^i_t (z - \frac{\varepsilon}{x^i_t})}{\gamma (1 - \beta \eta - \hat{\pi}^i_t) - \rho} \hat{w}_i h^i_t, \hspace{1cm} (69)$$

which implies with (68) and (69)

$$b^i_t = \frac{\rho (z - \frac{\varepsilon}{x^i_t})}{\gamma (1 - \beta \eta - \hat{\pi}^i_t) - \rho} \hat{w}_i h^i_t, \hspace{1cm} (70)$$

$$e^i_t = \frac{\gamma \beta \eta z x^i_t - (\gamma (1 - \hat{\pi}^i_t) - \rho) \varepsilon}{\gamma (1 - \beta \eta - \hat{\pi}^i_t) - \rho}. \hspace{1cm} (71)$$

The threshold level of relative human capital which ensures positive expenditures on education is obtained from $e^i_t = 0$ which implies in light of the last expression that

$$\gamma \beta \eta z x^i_t = [(1 - \hat{\pi}^i_t) \gamma - \rho] \varepsilon \hspace{1cm} (72)$$

$$\Rightarrow \hat{x}_t = \frac{[(1 - \hat{\pi}^i_t) \gamma - \rho] \varepsilon}{\gamma \beta \eta z}, \hspace{1cm} (73)$$

with $x^i_t \leq \tilde{x}_t \rightarrow e^i_t = 0$ which implies item (ii).
B.2. Proof of Proposition 1

Plugging optimal decisions (13)-(17) and the evolution of the pollution stock, (3), into (10), we obtain as an first-order condition

\[ \frac{\partial u_p^p}{\partial \tau_i^p} = \left\{ \mu b_3 y_i y_i^p \tau_i^2 + [(1 + \gamma(\bar{\pi}_t + \rho) - \mu(2b_3 - \bar{c})) H_t y_i^p] \tau_i^p \right\} \]

\[-[1 + \gamma(\bar{\pi}_t + \rho)] H_t y_i^p + [(\bar{\pi}_t + \rho) - 1] \gamma \bar{c} + b_3 \mu(y_i^p - \bar{c}) Y_t \}

\[ \left\{ (1 - \tau_i^p) H_t [(1 - \tau_i^p) y_i^p - \bar{c}] \right\}^{-1} = 0, \]  

(74)

such that

\[ \tau_{t,1,2}^p = \frac{(2y_i^p - \bar{c})b_3 \mu Y_t - [(1 + \gamma(\bar{\pi}_t + \rho)) y_i^p H_t] \pm \sqrt{\Psi}}{2b_3 \mu y_i^p Y_t}, \]  

(75)

with \( \Psi = [1 + \gamma(\bar{\pi}_t + \rho)]^2 (y_i^p)^2 H_t^2 + 2b_3 \mu \bar{c} y_i^p H_t Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)] + b_3^2 \mu^2 \bar{c}^2 Y_t^2 \). Note that \( \Psi > 0 \), since \( 2 - \bar{\pi}_t - \rho > 0 \) under reasonable parameter restrictions. In addition \( \frac{\partial^2 u_p^p}{\partial (\tau_i^p)^2} < 0 \), if and only if

\[ \tau_i^p = \frac{(2y_i^p - \bar{c})b_3 \mu Y_t - [(1 + \gamma(\bar{\pi}_t + \rho)) y_i^p H_t] - \sqrt{\Psi}}{2b_3 \mu y_i^p Y_t} \]  

(76)

(1) An increase in subsistence consumption reduces the preferred tax, since

\[ \frac{\partial \tau_i^p}{\partial c} = -\frac{1}{2y_i^p} - \frac{1}{2} \left( \frac{\sqrt{\Psi}}{2} \right)^{-\frac{1}{2}} \frac{\partial \sqrt{\Psi}}{\partial c} \]  

(77)

and

\[ \frac{\partial \Psi}{\partial c} = 2b_3 y_i^p \mu H_t Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)] + 2b_3^2 \mu^2 Y_t \bar{c} > 0 \]  

(78)

it follows that \( \frac{\partial \tau_i^p}{\partial c} < 0 \).

(2) An increase in household income, \( y_i^p \), increases the preference for \( \tau_i^p \), because

\[ \frac{\partial \tau_i^p}{\partial y_i^p} = \frac{2b_3 \mu Y_t - (1 + \gamma(\bar{\pi}_t + \rho)) H_t - \frac{1}{2} \Psi^{\frac{1}{2}} \frac{\partial \Psi}{\partial y_i^p}}{2b_3 \mu y_i^p} \]  

\[-(2y_i^p - \bar{c})b_3 \mu Y_t - [(1 + \gamma(\bar{\pi}_t + \rho)) y_i^p H_t] - \Psi^{\frac{1}{2}} \]  

\[ \frac{2b_3 \mu Y_t (y_i^p)^2}{2b_3 \mu y_i^p} \]  

\[ = 2b_3 \mu Y_t y_i^p - \gamma(\bar{\pi}_t + \rho) H_t y_i^p - \frac{1}{2} \Psi^{\frac{1}{2}} \frac{\partial \Psi}{\partial y_i^p} \]  

\[-(2y_i^p - \bar{c})b_3 \mu Y_t + [(1 + \gamma(\bar{\pi}_t + \rho)) y_i^p H_t] + \Psi^{\frac{1}{2}} \]  

\[ = -\frac{1}{2} \Psi^{\frac{1}{2}} \frac{\partial \Psi}{\partial y_i^p} y_i^p + \bar{c} b_3 \mu Y_t + 1 + \Psi^{\frac{1}{2}} 0. \]  

(79)

(80)

(81)

53Proof available upon request.
Noting $\Psi$ and observing that $\Psi > \frac{1}{2} \Psi^{-\frac{1}{2}} \frac{\partial \Psi}{\partial y_t^p}$, such that

$$[1 + \gamma(\bar{\pi}_t + \rho)]^2(y_t^p)^2 H_t^2 + 2\bar{c}b_3\mu y_t^p H_t Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)] + \bar{c}^2 b_3^2 \mu^2 Y_t^2 > \frac{1}{2} \left[2y_t^p H_t^2[1 + \gamma(\bar{\pi}_t + \rho)]^2 + 2\bar{c}b_3\mu H_t Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)]\right] y_t^p$$

(82)

$$\Rightarrow \bar{c}b_3\mu y_t^p H_t Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)] + \bar{c}^2 b_3^2 \mu^2 Y_t^2,$$

(83)

it follows that $\frac{\partial \tau_t^p}{\partial K_t} > 0$.

(3) Noting (24), it follows that $\tau_t^p = 0$, if $\tilde{y}_t = y_t^p$ with

$$\tilde{y}_t = \frac{\bar{c}[1 - \gamma(1 - \rho - \bar{\pi}_t) H_t - b_3\mu Y_t]}{[1 + \gamma(\bar{\pi}_t + \rho)] H_t - b_3\mu Y_t}.$$  

(84)

Since the nominator of the last expression is negative, because $(1 - \rho - \bar{\pi}_t) > 0$ under reasonable parameter restrictions, it follows that $\tilde{y} > 0$, if $[1 + \gamma(\bar{\pi}_t + \rho)] H_t < b_3\mu Y_t$ which implies

$$\frac{H_t}{Y_t} < \frac{b_3\mu}{1 + \gamma(\bar{\pi}_t + \rho)}.$$  

(85)

(1),(2) and (3) verify item (i). Item (ii) follows from (76), for $\bar{c} = 0$.

Item (iii) can be verified by

(4) An increase in capital accumulation increases the preferred tax rate

$$\frac{\partial \tau_t^p}{\partial K_t} = \frac{(2y_t^p - \bar{c})b_3\mu \frac{\partial y_t^p}{\partial K_t}}{2b_3\mu y_t^p H_t} - \frac{\frac{1}{2} \Psi^{-\frac{1}{2}} \frac{\partial \Psi}{\partial Y_t}}{2b_3\mu y_t^p H_t Y_t^2}$$

implying that

$$-\frac{1}{2} \Psi^{-\frac{1}{2}} \frac{\partial \Psi}{\partial Y_t} Y_t + [(1 + \gamma(\bar{\pi}_t + \rho)) y_t^p H_t] + \Psi^\frac{1}{2}.$$  

(87)

Noting now the definition of $\Psi$, it can be shown that $\Psi^\frac{1}{2} > \frac{1}{2} \Psi^{-\frac{1}{2}} \frac{\partial \Psi}{\partial Y_t} Y_t$, since

$$\Psi > \frac{1}{2} \frac{\partial \Psi}{\partial Y_t} Y_t$$

(88)

implies

$$[1 + \gamma(\bar{\pi}_t + \rho)]^2(y_t^p)^2 H_t^2 + 2\bar{c}b_3\mu y_t^p H_t Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)] + \bar{c}^2 b_3^2 \mu^2 Y_t^2 > \frac{1}{2} [2\bar{c}b_3\mu y_t^p H_t Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)] + 2\bar{c}^2 b_3^2 \mu^2 Y_t^2] Y_t,$$

(89)

$$\Rightarrow [1 + \gamma(\bar{\pi}_t + \rho)]^2(y_t^p)^2 H_t^2 + \bar{c}b_3\mu y_t^p H_t Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)] > 0,$$

(90)

such that $\frac{\partial \tau_t^p}{\partial K_t} > 0$.  

48
(5) An increase in aggregate human capital reduces the preferred tax rate

\[
\frac{\partial \tau^p}{\partial H_t} = \frac{(2y^p_t - \bar{c})b_3\mu \frac{\partial Y_t}{\partial H_t} - [(1 + \gamma(\bar{\pi}_t + \rho))y^p_t] - \frac{1}{2} \Psi^{\frac{1}{2}} \left( \frac{\partial \Psi}{\partial H_t} + \frac{\partial \Psi}{\partial Y_t} \frac{\partial Y_t}{\partial H_t} \right)}{2b_3\mu y^p_t Y_t^2} - \frac{(2y^p_t - \bar{c})b_3\mu Y_t - [(1 + \gamma(\bar{\pi}_t + \rho))y^p_t H_t] - \Psi^{\frac{1}{2}} \frac{\partial Y_t}{\partial H_t}}{2b_3\mu y^p_t Y_t^2}.
\]

(91)

Reshuffling terms yields

\[
\frac{\partial \tau^p}{\partial H_t} = - \frac{[(1 + \gamma(\bar{\pi}_t + \rho))y^p_t} Y_t \frac{\partial Y_t}{\partial H_t} - \frac{1}{2} \Psi^{\frac{1}{2}} \left( \frac{\partial \Psi}{\partial Y_t} \right) Y_t - \Psi^{\frac{1}{2}} \frac{\partial Y_t}{\partial H_t} + \Psi^{\frac{1}{2}}.
\]

(92)

As \( \frac{Y_t}{H_t} = \frac{H_t}{1-\alpha} \), we obtain

\[
\frac{\partial \tau^p}{\partial H_t} = - [(1 + \gamma(\bar{\pi}_t + \rho))y^p_t] \frac{H_t}{1-\alpha} - \frac{1}{2} \Psi^{\frac{1}{2}} \left( \frac{\partial \Psi}{\partial H_t} \frac{H_t}{1-\alpha} + \frac{\partial \Psi}{\partial Y_t} \right) + [(1 + \gamma(\bar{\pi}_t + \rho))y^p_t H_t] + \Psi^{\frac{1}{2}}.
\]

(93)

Noting now the definition of \( \Psi \), we obtain

\[
\Psi^{\frac{1}{2}} < \frac{1}{2} \Psi^{\frac{1}{2}} \left( \frac{\partial \Psi}{\partial H_t} \frac{H_t}{1-\alpha} + \frac{\partial \Psi}{\partial Y_t} \right),
\]

(94)

since \( \alpha \in (0,1) \) it follows that

\[
[1 + \gamma(\bar{\pi}_t + \rho)]^2 (y^p_t)^2 \frac{H^2_t}{1-\alpha} + \bar{c}b_3\mu y^p_t \frac{H_t}{1-\alpha} Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)] > \psi > [1 + \gamma(\bar{\pi}_t + \rho)]^2 (y^p_t)^2 H^2_t + \bar{c}b_3\mu y^p_t H_t Y_t[1 + \gamma(2 - \bar{\pi}_t - \rho)].
\]

(95)

(96)

For the same reasoning, we yield

\[
[(1 + \gamma(\bar{\pi}_t + \rho))y^p_t H_t] \frac{H_t}{1-\alpha} > [1 + \gamma(\bar{\pi}_t + \rho)]y^p_t H_t,
\]

(97)

such that \( \frac{\partial \tau^p}{\partial H_t} < 0. \)
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