



Working Paper

## Testing Temporal Disaggregation

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Testing Temporal Disaggregation

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# Testing temporal disaggregation\*

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## Abstract

Economists and econometricians very often work with data which has been temporally disaggregated prior to use. Hence, the quality of the disaggregation clearly affects the quality of the analyses. Building on Chow and Lin's (1971) disaggregation model this paper proposes a new estimation approach and a specification test which assesses the quality of the disaggregation model. An advantage of the proposal is that estimation and testing can both be pursued using the aggregated data while the standard method requires a mixture of high and low frequency data. A small simulation study shows that the test indeed provides useful information.

*JEL classification:* F31, F47, C53

*Keywords:* temporal disaggregation, restricted ARMA

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# 1 Introduction

Quantitative economic analysis very often has to rely on data whose observation frequency is systematically lower than desired. For example, economic activity which is commonly expressed as the flow of value added is generated continuously. However, as it would require enormous resources to actually observe this process, most countries use annual estimates of economic activity as the basis of their statistics. In contrast, many other variables such as money stock and interest rates are available at a far higher frequency (and can often also be observed more accurately). Nevertheless, researchers, policy makers and the public, all have genuine interest in high frequency information on low frequency data for efficient and timely decision making. Therefore, statistical offices all around the world work on providing temporarily disaggregated data to serve this aim. Statisticians at the European Commission have even developed a free software tool for conducting disaggregation. This software is called ECOTRIM and is available upon request from Roberto Barcellan, European Commission, Statistical Office Directorate C -Unit C2, Jean Monnet Building BECH B3/398, L-2920 Luxembourg.

It is evident that the quality of the disaggregation is very important for the users. Unfortunately, an according assessment is in general haunted by the fact that the true high frequency data is unobservable. This paper proposes a statistical test that is consistent with a very popular disaggregation procedure due to Chow and Lin (1971), henceforth CL. This test is thus meant to make an informed choice between modelling alternatives. An advantage of the proposal is that estimation and testing can both be pursued using the aggregated data while the standard method requires switching between high and low frequency data.

The next section reviews the disaggregation approach in question, reasons why this model is still very attractive despite its numerous alternatives and sets thus the framework of the analysis. The third section describes the new estimation approach while the fourth outlines the testing strategy and provides simulation result. Finally, conclusions are drawn.

## 2 Chow&Lin revisited

### 2.1 The basic model

The following data generating process for the high frequency data is supposed.

$$Y_h = \beta X_h + U_h \tag{1}$$

$$U_h \sim (0, \Sigma_h)$$

The endogenous variable,  $Y_h = (y_{h,1}, y_{h,2}, \dots, y_{h,T})'$ , is thus assumed to depend on an exogenous variable, or a set of exogenous variables,  $X_h = (x_{h,1}, x_{h,2}, \dots, x_{h,T})'$ , and an innovation process,  $U_h$ . The subscript  $h$  indicates high frequency. The general idea is to use high frequency information on  $x$  and an estimate of  $\Sigma_h$  to obtain estimates for  $y_{h,t}$  which is not directly observable. In order to estimate  $y_{h,t}$ , CL suggest a particular structure for  $\Sigma_h$ . Denoting  $U_h = (u_{h,1}, u_{h,2}, \dots, u_{h,T})'$  they suggest to consider the stationary process

$$u_{h,t} = \rho u_{h,t-1} + \epsilon_{h,t} \tag{2}$$

$$|\rho| < 1 \tag{3}$$

$$\epsilon_{h,t} \sim i.i.d.(0, \sigma_h^2).$$

It is worthwhile to notice that (1) is suitable for both, stationary and non-stationary variables  $Y_h$  and  $X_h$  as long as (3) holds. If  $y_{h,t}$  and  $x_{h,t}$  were both nonstationary, then under (3), they are cointegrated in the sense of Engle and Granger (1987). Due to the fact that cointegration is now a well understood property of many fundamental economic relationships (1) represents a very attractive approach to the disaggregation of low frequency data. Furthermore, if (1) is a cointegration relationship, forecasts of  $y_{h,t}$  based on  $x_{h,t}$  will in general outperform forecasts which are not based on cointegration relations at least at longer horizons. In contrast, the proposals by Fernandez (1981) and Litterman (1983) suggest nonstationary processes in (2) which generally result in ‘smoother’ high frequency estimates at the expense of forecasting performance. The latter property is important because many disaggregation exercises serve the provision of early estimates of high frequency information on  $y$  such as quarterly GDP estimates. These are usually constructed on forecasted values of  $y$  and hence large forecast errors imply according revisions later on. For the purpose of the current paper it shall be assumed that the variables  $x$  and  $y$  are both integrated of the same order. This order may be one or zero.

In the CL approach (1) is transformed into a low frequency model with observable data series  $y_{l,t}, x_{l,t}$  and the error process  $\varepsilon_{l,t}$ . In the following, the focus will be on the temporal aggregation which is accomplished by pre-multiplying (1) by a matrix  $C_m$  of dimension  $(T/m \times T)$  where

$$C_m = \begin{pmatrix} 1_{1 \times m} & 0_{1 \times m} & \cdots & 0_{1 \times m} \\ 0_{1 \times m} & 1_{1 \times m} & \cdots & 0_{1 \times m} \\ \vdots & & \ddots & \vdots \\ 0_{1 \times m} & \cdots & \cdots & 1_{1 \times m} \end{pmatrix}$$

and  $m$  is the number of high frequency observations that are temporally aggregated to yield the low frequency data. For example, if temporal aggregation of quarterly to annual data is considered,  $m = 4$  would be chosen. Although the test to be proposed in the following can be applied to various values of  $m$ , we are going to look at the case  $m = 4$  only. Further special cases can be dealt with along similar lines.

After temporal aggregation (1) mutates to

$$Y_l = \beta X_l + U_l \tag{4}$$

$$Y_l = C_m Y_h$$

$$X_l = C_m X_h$$

$$U_l = C_m U_h$$

and thus  $Y_l = (y_{l,1}, y_{l,2}, \dots, y_{l,\tau}, \dots)'$ ,  $X_l = (x_{l,1}, \dots, x_{l,\tau}, \dots)'$ , and  $U_l = (u_{l,1}, \dots, u_{l,\tau}, \dots)'$  are temporal aggregates. It is important to notice that the aggregation only affects the parameters of the error process while the parameter characterising the linear relationship between dependent and independent variable is still completely described by  $\beta$ . The latter is thus independent of  $C_m$  for all  $m$ . A general treatment of linear aggregation and its implications for multivariate autoregressive models is provided by Marcellino (1999), for example.

## 2.2 Estimation

Chow and Lin (1971) suggest to estimate  $\rho$  and  $\beta$  subject to the aggregation constraint (4).

They proposed the following feasible generalised least squares (GLS) estimate:

$$\hat{\beta}_{CL} = (X_l' \Sigma_l^{-1} X_l)^{-1} X_l' \Sigma_l^{-1} Y_l \tag{5}$$

$$\Sigma_l = E(U_l U_l')$$

$$\hat{U}_l = \left[ C_l - X_l (X_l' \Sigma_l^{-1} X_l)^{-1} X_l' \Sigma_l^{-1} \right] Y_l$$

The key element in the estimation is the matrix  $\Sigma_l$ , which however, can be obtained by considering that

$$\begin{aligned}\Sigma_l &= E(U_l U_l') \\ &= E(C_m U_h U_h' C_m') \\ &= C_m \Sigma_h C_m'.\end{aligned}$$

Since  $\Sigma_h$  is known by assumption, and  $C_m$  by construction  $\Sigma_l$  is also identified up to  $\rho$ . In general, as Marcellino (1999) has shown, the elements of  $\Sigma_l$  are nonlinear functions of  $\rho$  and  $m$ . Notice, however, that estimation is feasible by a sequence of linear regressions where only an initial value for  $\rho$  is required. Subsequent updates of  $\rho$  can be based on a regression of  $\hat{u}_{h,t}$  on  $\hat{u}_{h,t-1}$  where  $\hat{u}_{h,t}$  is the estimated high frequency residual. The details of this calculation are not repeated here. The reader is referred to Chow and Lin (1971), p. 373. The resulting estimates for  $Y_h$  have desirable properties such as being BLUE (cf. CL). However, I would like to highlight the fact that the CL regression procedure requires repeated switching between the level of aggregation of the observable data and the level of the estimated high frequency data. One contribution of this paper is to provide an alternative that makes the switching redundant.

As has been noted before, once assumption (2) has been made, the opportunities for checking the appropriateness of the disaggregation procedure are rather limited. The main reason is that the implied low frequency structure of the variance-covariance matrix needs to be imposed on the data in order to obtain the estimates. This paper suggests to explicitly scrutinise this aspect by testing whether or not the implicit nonlinear restrictions on the parameters of the low frequency residual process are statistically acceptable or not.

### 3 An alternative to the Chow&Lin estimation approach

Before doing so, an alternative estimation procedure is described in the following. It permits the formulation of a general model against which the temporally aggregated model appears as a restricted version. That allows the definition of a likelihood ratio test to check appropriateness of the temporal aggregation.

I start by repeating that the aggregation restriction can alternatively be expressed as a restriction on the error process  $U_l$ . Following Wei (1990), temporal aggregation of an autore-



gressive process  $U_h$  of order one results in a new error process which can be described as an autoregressive–moving average process. The order of the autoregressive as well as the moving average component is one ( $ARMA(1, 1)$ ) and the components will be labelled  $\rho^*$ , and  $\phi^*$  respectively. Given the fact that  $\beta$  is invariant to  $C_m$  the aggregation restriction is thus a restriction on the parameters of the low frequency error process,  $U_l$ .

The following lines exemplify the approach for  $m = 4$ . A general approach is dealt with in the appendix. The basic idea is to express  $\rho^*$  and  $\phi^*$  in terms of  $\rho$  and  $m$ . Define  $z_{h,t} = y_{h,t} - \beta x_{h,t}$  and calculate  $z_{l,\tau} = z_{h,t} + z_{h,t-1} + z_{h,t-2} + z_{h,t-3}$ . After some algebra, one obtains

$$z_{l,\tau} = \rho^4 z_{l,\tau-1} + u_{l,\tau}^*, \quad (6)$$

where

$$\begin{aligned} u_{l,\tau} = & \epsilon_{h,t} + (1 + \rho)\epsilon_{h,t-1} + (1 + \rho + \rho^2)\epsilon_{h,t-2} + (1 + \rho + \rho^2 + \rho^3)\epsilon_{h,t-3} \\ & + (\rho + \rho^2 + \rho^3)\epsilon_{h,t-4} + (\rho^2 + \rho^3)\epsilon_{h,t-5} + \rho^3\epsilon_{h,t-6} \end{aligned} \quad (7)$$

and hence  $\rho^* = \rho^4$ . According to Wei (1990, p. 409)  $u_{l,\tau}$  has an  $MA(1)$  structure

$$\begin{aligned} u_{l,\tau} &= \phi^* \varepsilon_{l,\tau-1} + \varepsilon_{l,\tau} \\ \varepsilon_{l,\tau} &\sim i.i.d.(0, \sigma_l^2) \end{aligned}$$

with

$$E(u_{l,\tau}, u_{l,\tau-s}) = \begin{cases} (1 + \phi^{*2})\sigma_l^2 & \text{for } s = 0, \\ \phi^* \sigma_l^2 & \text{for } s \pm 1, \\ 0 & \text{else.} \end{cases} \quad (8)$$

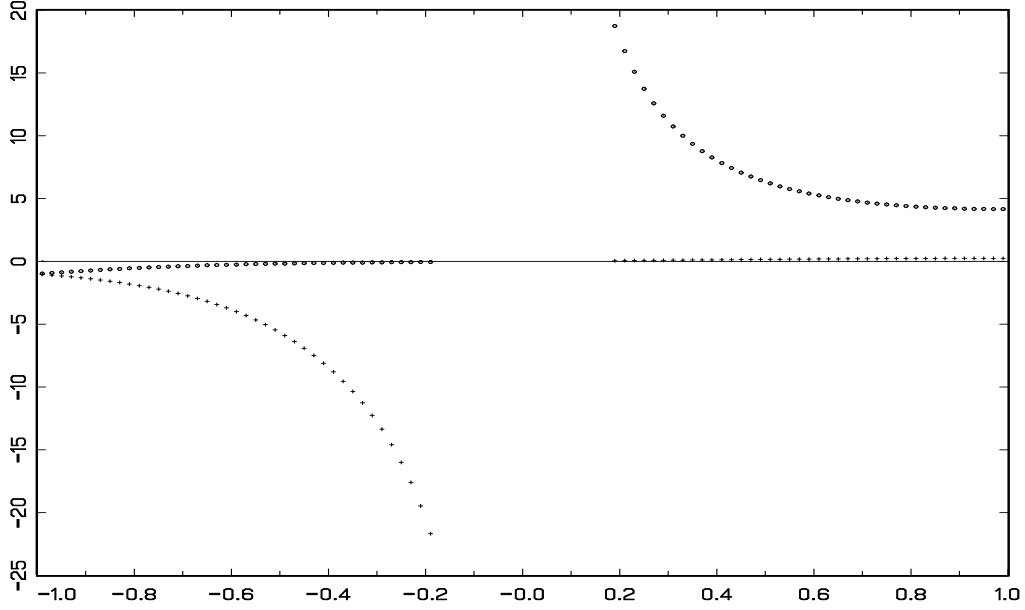
The first and second line can alternatively be expressed as

$$\begin{aligned} (1 + \phi^{*2})\sigma_l^2 &= \sigma_h^2 S_0 = \sigma_h^2 (4 + 6\rho + 8\rho^2 + 8\rho^3 + 8\rho^4 + 6\rho^5 + 4\rho^6) \\ \phi^* \sigma_l^2 &= \sigma_h^2 S_1 = \sigma_h^2 (\rho + 2\rho^2 + 4\rho^3 + 2\rho^4 + \rho^5) \end{aligned}$$

where it has been made use of (7) and (8). The solution for  $\phi^*$  can consequently be given as

$$\phi^* = \frac{S_0}{2S_1} \pm \sqrt{\frac{S_0^2}{4S_1^2} - 1}. \quad (9)$$

Notice that the square root term is always positive which can be conjectured from its monotonicity in  $\rho$  and looking at the limiting cases of  $\rho \rightarrow 0$  and  $\rho \rightarrow \pm 1$  respectively. As it turns out only one of the solutions yields an invertible  $MA$  representation. Furthermore, the whole expression will be dominated by the first term since  $|\frac{S_0}{2S_1}| > \sqrt{\frac{S_0^2}{4S_1^2} - 1}$  given  $\rho \neq 0$ . The first



**Figure 1:** The parameter  $\phi^*$  (vertical axis) as a function of  $\rho$  (horizontal axis).

term can also be written as  $\frac{1}{2\rho} \frac{S_0}{(1+2\rho+4\rho^2+2\rho^3+\rho^4)}$  telling that  $\rho$  and  $\phi^*$  will always have the same sign. The variance of the aggregated error process can be calculated as  $\sigma_l^2 = S_0/(1 + \phi^{*2})\sigma_h^2$ . It implies that  $\phi^*$  can finally be identified as the choice out of the two possible options that always ensures a non degenerate variance. Noticing that  $\sigma_l^2$  approaches zero for  $\rho \rightarrow 0, \rho > 0$  and  $\phi^* = \frac{S_0}{2S_1} + \sqrt{\frac{S_0^2}{4S_1^2} - 1}$ , it is reasonable to consider the resulting invertible *MA* coefficient as the true coefficient of the aggregated process. This completes the characterisation of the aggregated model. Figure 1 illustrates the results by depicting the relation between  $\rho$  and  $\phi^*$ . As a notational convention I will provide the value of  $m$  (in parentheses) if convenient because  $S_0, S_1, \phi^*$ , and  $\rho^*$  all depend on  $m$ .

Finally, (9) implies an important restriction of the alternative approach. It is given by the impossibility to calculate  $\phi^*$  for  $\rho = 0$ . It can therefore be expected that values of  $\rho$  close to zero may cause serious trouble. Feasible approaches to estimate the model parameters are the Kalman filter and various numerical optimisation methods. Using standard notation,  $L$  is the lag operator with  $Lx_t = x_{t-1}$ , and we set up a model as in (4), but consider the *ARMA*(1, 1) error process

$$(1 - \rho L)u_{l,\tau} = (1 + \phi L)\varepsilon_{l,\tau} \tag{10}$$

where  $\varrho = \rho^*$  and  $\phi = \phi^*$  when the aggregation restriction is in place. The corresponding empirical model is denoted  $ARMA^*(1,1)$ . Furthermore, I introduce the parameter vectors  $\theta = \begin{pmatrix} \varrho \\ \phi \end{pmatrix}$  and  $\theta^*(m) = \begin{pmatrix} \rho^*(m) \\ \phi^*(m) \end{pmatrix}$  to obtain handy expressions for future use.

## 4 Testing disaggregation

Once estimated, the parameters of the model allow the calculation of a maximum likelihood value that can be obtained from filtering the data with the derived  $ARMA^*(1,1)$ . Based on those values and on the corresponding likelihood values of alternative models of an unrestricted model, straightforward likelihood-ratio tests can be computed.

What hypothesis should be tested depends naturally on the final objective of the analysis. If the researcher wants to know whether or not a given data set can be regarded aggregated in the way Chow and Lin (1971) presumed, from quarterly to an annual level, say, the following steps of analysis could be considered. First, test whether or not the aggregated process follows an  $ARMA(1,1)$  model. This can be done by standard tests in a general-to-specific setting, for example. In the simulation study below, tests of significance of the  $AR(1)$  and the  $MA(1)$  components are looked at individually. Given, that the process is indeed an  $ARMA(1,1)$ , the aggregation restriction might be tested against the hypothesis of an unrestricted  $ARMA(1,1)$  process. The latter test will be in the focus of the following exercise.

Two remarks are in order. Firstly, it will not be the objective of the study to discuss the implicit complications due to the multi-step testing. It is always assumed that no particular test decision has been made at a previous stage of the analysis when applying a test. In other words, only “ideal” conditions are supposed. Secondly, while a researcher might also be interested in the question whether or not a series is aggregated from, say, quarterly to annual rather than from monthly to annual levels, the corresponding testing problem is only briefly addressed. The reason is the following. Testing, for example,  $H_0 : m = 4$  against  $H_1 : m = 12$  results in a non-nested testing problem. As of today, no satisfactory solution for that is available. The existing answers are rather demanding and would take this work too far away from its core. Instead, an indirect testing strategy is pursued. The null hypothesis  $H_0 : m = 4$  will be tested against an  $ARMA(1,1)$  of arbitrary (albeit stationary and invertible) parametrisation. Thus,

a – as it turns out – rather local power analysis is conducted. Most importantly, however, the size of the basic test is scrutinised in detail.

The principal simulation approach uses a moderate sample size of  $T/m = 100$  although a limited investigation also addresses asymptotic properties.

#### 4.1 Aggregation restriction

The question in the centre of this paper is whether or not an observed  $ARMA(1,1)$  structure arises from the aggregation of the data. This question can be answered by considering

$$H_0^1 : \theta = \theta^*(m) \text{ vs. } H_1^1 : \theta \text{ arbitrary}, \quad (11)$$

given  $m > 1$ . Here, the decision rule is to accept the disaggregation procedure if  $H_0^1$  cannot be rejected. For the current purpose it appears reasonable to test  $H_0^1$  against some other, general  $ARMA(1,1)$  process. In particular, data will be generated by  $ARMA(1,1)$  processes (ref. (10)) where either  $\rho$  or  $\phi$  systematically vary unidirectionally.

#### 4.2 Model selection

In applied problems, the researcher may face a model selection problem. If, for example, the data is generated at the level of the observable data frequency, no  $MA$  effect should be present. Accordingly, if there was no  $AR$  effect, the CL method would not be applicable and a linear interpolation would be as good. Hence, both these hypotheses could be tested and the corresponding pairs of hypotheses are:

$$H_0^2 : \phi = 0 \text{ vs. } H_3^1 : |\phi| < 1, \quad (12)$$

$$H_0^3 : \rho = 0 \text{ vs. } H_2^2 : |\rho| < 1. \quad (13)$$

#### 4.3 Simulation study

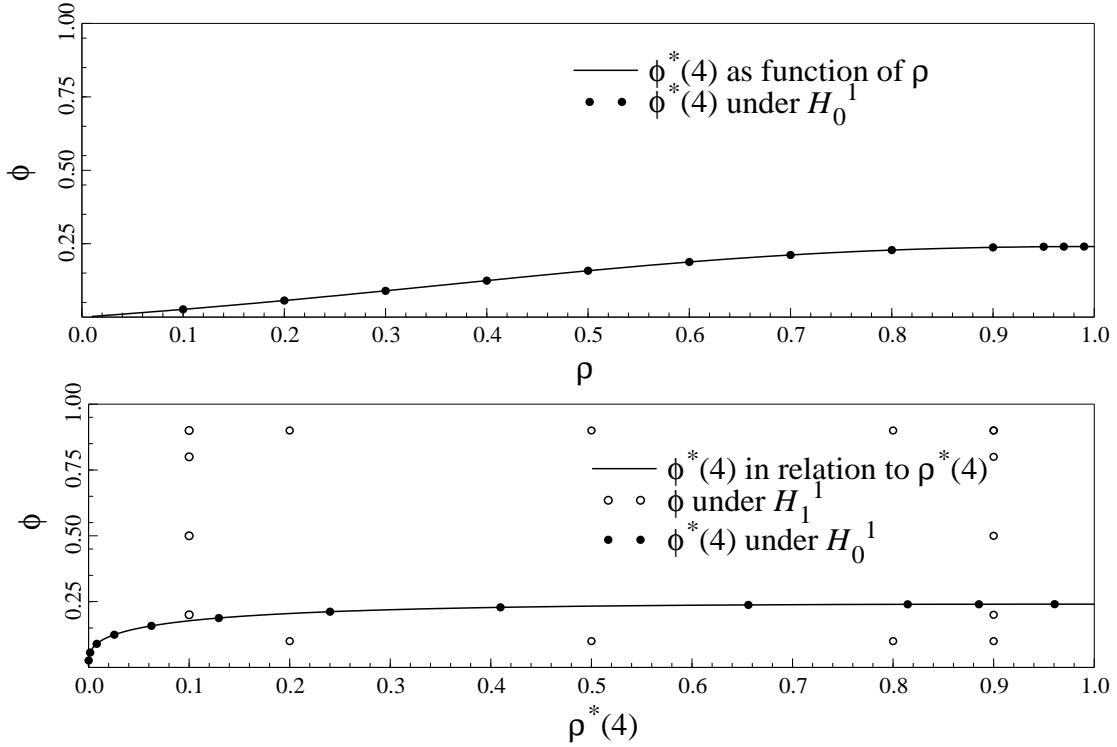
In this section the results of a limited simulation study are presented. I combine the analysis of hypotheses (12) through (13) by the following strategy. The data is filtered by an  $AR(1)$ , an  $MA(1)$ ,  $ARMA^*(1,1)$  and an unrestricted  $ARMA(1,1)$  process. A  $\sim$  on top of the estimates indicates the means of the estimators over the random draws. The test results may then be

interpreted in the following way. Rejecting  $H_0^2$  and  $H_0^3$  indicate the possibility of an aggregation effect *i.e.* the observed data may have been generated at a lower frequency and the presence of the  $ARMA(1, 1)$  structure may be due to subsequent temporal aggregation. Significance of  $H_0^1$  indicates that the presence of a moving average effect cannot be attributed to the aggregation of the data. In such a case the CL procedure should not be applied. On the other hand, if  $H_0^2$  and  $H_0^3$  are rejected while  $H_0^1$  is not, the data can be regarded aggregated and disaggregation by the CL method is appropriate. In order to obtain a clear view on the capabilities of the testing procedure in this paper each test is performed independently. In an alternative analysis yet to come the result of the combined tests are going to be discussed. Besides the test statistics the average coefficient estimates are reported.

The first simulation regards the size of the test of  $H_0^1$ . Data is generated according to (4) and (10) with  $\varrho = \rho^*(4)$ ,  $\phi = \phi^*(4)$  and  $x_{l,\tau}$ ,  $\varepsilon_{l,\tau}$  as independent pseudo normal variables. The empirical size for various choices of  $\rho$ , and  $\sigma_h^2$  is compared to its nominal level which is set to .05. In each draw  $x_{l,\tau}$  is a zero mean random normal variable with variance  $\sigma_h^2$ . For each draw 1000 (pre-sample) values of  $y_{l,t}$  are discarded to reduce the dependency on starting values. The means of testing is a standard likelihood-ratio test being chi-square distributed with one degree of freedom under the null hypothesis and under the assumption of independent tests. In addition, a general model selection criteria, the Akaike (Akaike, 1969) criterion, is used to discriminate between possible  $m$ .

Figure 2 summarises the simulation setup. The size of  $H_0$  is investigated along the solid line in Figure 2. This line represents the possible values of  $\phi^*(4)$  given  $\rho$  and  $\rho^*(4)$ . The solid circles signify the points at which the size will be investigated. Furthermore,  $ARMA(1, 1)$  processes are generated under  $H_1^1$  with choices of  $\varrho$  and  $\phi$  represented by the circles off the solid line. Thus, these test result indicate the power of the test against arbitrarily chosen alternatives in the admissible parameter space.

In order to assess the test procedure tables 1 and 2 give the corresponding rejection frequencies as well as the population means and standard deviations of the estimates for  $\varrho$  and  $\beta$ . Note however, for the  $ARMA^*(1, 1)$  models the reported  $\varrho$  correspond to the value in (2). Finally, the opportunity to discriminate between various alternative disaggregate levels is scrutinised by generating data under  $H_1^1 : m = 2$  and  $m = 12$ .



**Figure 2:** Summary of first simulation experiment in  $(\rho, \phi)$  and in  $(\rho^*(4), \phi)$  space.

The calculations were performed using an *Ox3.40* program (Doornik and Ooms, 1993) making use of its standard *ARFIMA* package. It is capable of filtering data by various definitions of ARFIMA models and of calculating the according likelihood function. In combination with a simple hill climbing algorithm estimates for  $\rho$  and  $\phi^*$  can be obtained.

#### 4.3.1 Disaggregation restriction

The simulation results indicate that the empirical size of the test of  $H_0^1$  has a tendency to exceed its nominal level. While the nominal level has been fixed at .05, the empirical size is in the range of .061 – .117 (see table 1). The over rejection may be a result of the small sample size. In addition, the choice of the model parameters seems to be related to excess rejections. In particular, the smaller  $\rho$  the more readily the empirical size is above its nominal level. For values  $\rho \geq .5$  the test size appears satisfactory, however. Equivalently, the convergence rate of the test size with respect to  $T$  appears rather low for  $\rho$  close to zero. Using  $T/m = 1000$ , the empirical size remains significantly above its nominal level for  $\rho < 0.2$ .

**Table 1:** Parameter estimation and empirical test size in a small sample simulation

$$y_{h,t} = 1 + 1.5x_{h,t} + u_{h,t}, T/m = 100$$

$$x_{h,t} \sim N(0, \sigma_h^2)$$

$$u_{h,t} = \rho u_{h,t-1} + \epsilon_t$$

$$\epsilon_t \sim N(0, \sigma_h^2)$$

$$m = 4, \text{ ref.: equation (4)}$$

$\sigma_h^2$	$\rho$	model	$H_0$	%	$\hat{\rho}$	$\hat{\sigma}_\rho$	$\hat{\phi}$	$\hat{\sigma}_\phi$	$\hat{\beta}$	$\hat{\sigma}_\beta$
1	.2	AR(1)	$H_0^3$	10.1	.24	.47			1.51	.23
		MA(1)	$H_0^2$	8.7			.06	.11	1.51	.23
		ARMA*(1,1)	$H_0^1$	11.7	.20	.18			1.51	.23
		ARMA(1,1)			-.01	.80	.07	.59	1.51	.24
	.5	AR(1)	$H_0^3$	10.3	.64	.19			1.52	.31
		MA(1)	$H_0^2$	7.1			.22	.10	1.52	.31
		ARMA*(1,1)	$H_0^1$	7.7	.47	.15			1.52	.31
		ARMA(1,1)			.02	.73	.21	.42	1.51	.31
	.8	AR(1)	$H_0^3$	35.3	.86	.03			1.52	.40
		MA(1)	$H_0^2$	56.8			.55	.08	1.51	.41
		ARMA*(1,1)	$H_0^1$	6.1	.79	.05			1.52	.39
		ARMA(1,1)			.73	.25	.27	.19	1.51	.40
.95	AR(1)	$H_0^3$	62.6	.96	.01			1.51	.41	
	MA(1)	$H_0^2$	100.0			.77	.07	1.50	.53	
	ARMA*(1,1)	$H_0^1$	6.1	.94	.02			1.51	.40	
	ARMA(1,1)			.94	.02	.27	.12	1.51	.40	
4	.8	AR(1)	$H_0^3$	35.3	.86	.03			1.57	1.60
		MA(1)	$H_0^2$	56.8			.55	.08	1.56	1.65
		ARMA*(1,1)	$H_0^1$	6.1	.79	.05			1.57	1.58
		ARMA(1,1)			.73	.25	.27	.19	1.56	1.59

ARMA\*(1,1) denotes the restricted ARMA(1,1) model ( $m = 4$ ). The column headed % signifies the rejection frequency of the hypotheses to the left in percentage points. Columns ' $\hat{\rho}$ ' (' $\hat{\sigma}_\rho$ '), ' $\hat{\phi}$ ' (' $\hat{\sigma}_\phi$ '), ' $\hat{\beta}$ ' (' $\hat{\sigma}_\beta$ ') report the average (standard deviation) of the estimated model parameters. Note: Estimates for  $\rho$  can be compared to the true parameter only for ARMA\*(1,1). A comparison to  $\varrho = \rho^*(4)$  should otherwise be made. The empirical size of the test of  $H_0^1$  exceeds its nominal level especially for  $\rho$  close to zero.

**Table 2:** Empirical power against arbitrary alternative hypotheses

$$y_t = 1 + 1.5x_t + u_t, t = 1, 2, \dots, 100$$

$$x_t \sim N(0, 1)$$

$$(1 - \rho L)u_t = (1 + \phi L)\varepsilon_t$$

$$\varepsilon_t \sim N(0, 1)$$

$\phi$	$\varrho$	model	$H_0$	%	$\hat{\varrho}$	$\hat{\sigma}_{\hat{\varrho}}$	$\hat{\phi}$	$\hat{\sigma}_{\hat{\phi}}$	$\hat{\beta}$	$\hat{\sigma}_{\hat{\beta}}$
.1	.2	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	14.4	.58	.03			1.50	.03
		<i>ARMA</i> (1, 1)			.61	.22	.11	.11	1.50	.03
	.5	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	65.7	.79	.01			1.50	.02
		<i>ARMA</i> (1, 1)			.84	.02	.10	.05	1.50	.03
	.8	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	97.6	.93	.01			1.50	.02
		<i>ARMA</i> (1, 1)			.94	.01	.10	.04	1.50	.02
	.9	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	99.1	.97	.00			1.50	.02
		<i>ARMA</i> (1, 1)			.97	.00	.10	.03	1.50	.02
.2	.1	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	6.7	.58	.03			1.50	.03
		<i>ARMA</i> (1, 1)			.38	.41	.20	.10	1.50	.03
.5		<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	100.0	.76	.01			1.50	.02
		<i>ARMA</i> (1, 1)			.52	.18	.50	.05	1.50	.02
.8		<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	100.0	.81	.01			1.50	.02
		<i>ARMA</i> (1, 1)			.55	.07	.80	.02	1.50	.01
.9		<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	100.0	.82	.01			1.50	.02
		<i>ARMA</i> (1, 1)			.55	.06	.90	.01	1.50	.01
.9	.2	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	100.0	.84	.01			1.50	.02
		<i>ARMA</i> (1, 1)			.67	.03	.90	.01	1.50	.01
	.5	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	100.0	.91	.01			1.50	.02
		<i>ARMA</i> (1, 1)			.84	.01	.90	.01	1.50	.01
	.8	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	100.0	.97	.00			1.50	.02
		<i>ARMA</i> (1, 1)			.94	.01	.90	.01	1.50	.01
	.9	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	100.0	.98	.00			1.50	.02
		<i>ARMA</i> (1, 1)			.97	.00	.90	.01	1.50	.01
.2	.9	<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	23.1	.97	.00			1.50	.02
		<i>ARMA</i> (1, 1)			.97	.00	.20	.03	1.50	.02
.5		<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	100.0	.98	.00			1.50	.02
		<i>ARMA</i> (1, 1)			.97	.00	.50	.03	1.50	.02
.8		<i>ARMA</i> <sup>*</sup> (1, 1)	$H_0^1$	100.0	.98	.00			1.50	.02
		<i>ARMA</i> (1, 1)			.97	.00	.80	.02	1.50	.01

*ARMA*<sup>\*</sup>(1, 1) denotes the restricted *ARMA*(1, 1) model ( $m = 4$ ). The column headed % signifies the rejection frequency of the hypotheses to the left in percentage points. Columns ' $\hat{\varrho}$ ' (' $\hat{\sigma}_{\hat{\varrho}}$ '), ' $\hat{\phi}$ ' (' $\hat{\sigma}_{\hat{\phi}}$ '), ' $\hat{\beta}$ ' (' $\hat{\sigma}_{\hat{\beta}}$ ') report the average (standard deviation) of the estimated model parameters. The values of  $\phi$  and  $\varrho$  are only provided if they change from line to line. The results show that the power of the test rapidly increases with the distance between  $\theta$  and  $\theta^*(4)$ . The increase is more rapid in the direction of  $\phi$ .



The accuracy of the point estimates of the  $\beta$  parameter appears to be unaffected by the choice of  $\rho$ . This can be conjectured from the reported standard deviations. There is no situation where the  $t$ -ratio comes close to 2, it is always much larger. Quite reasonably, the variance depends on the underlying variance of the innovation process and the variance of  $x_{l,\tau}$ .

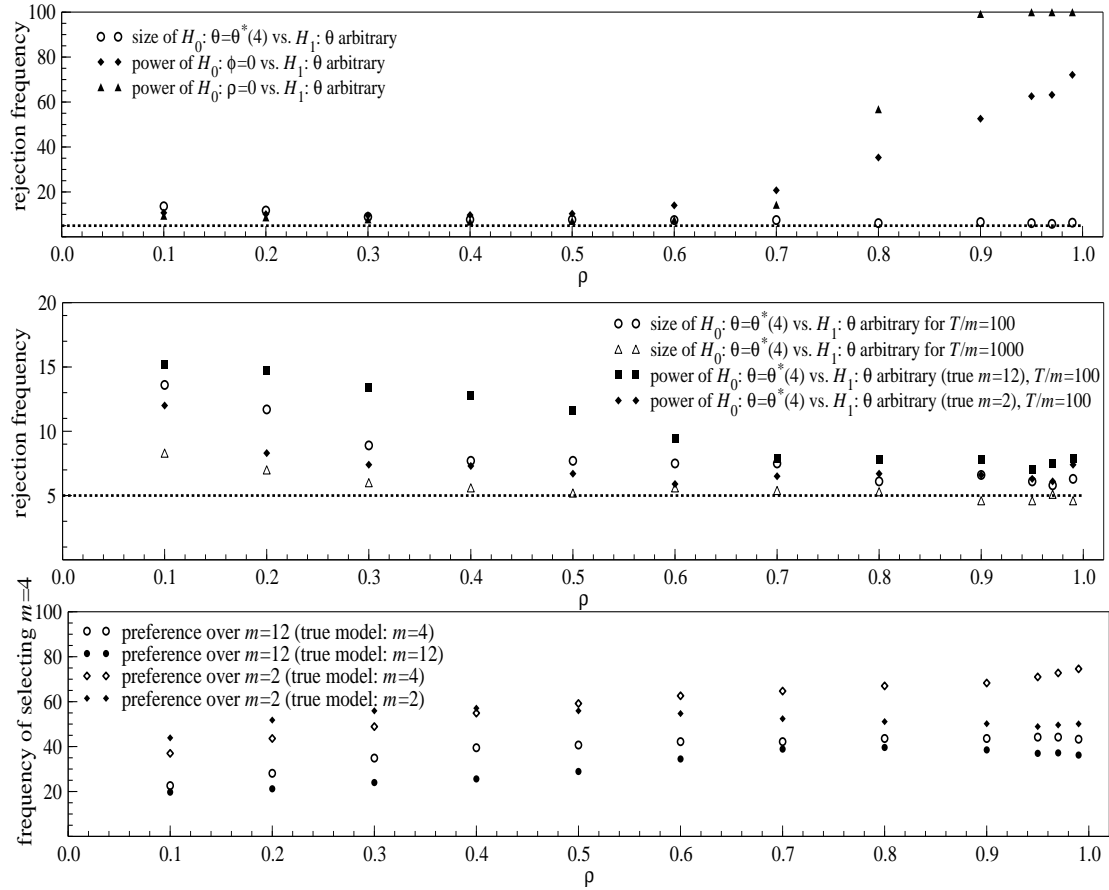
In sum, when  $H_0^1$  is true, the performance of the test appears more or less appropriate. Problems arise when the autoregressive parameter becomes small. In such a situation, however, the intra-annual information is not very important in the sense that a straightforward linear interpolation would result in very similar disaggregated data estimates compared to a disaggregation which is based on a small  $\rho$ .

When regarding the power of the test against arbitrary alternatives data is generated as an  $ARMA(1, 1)$  process with various autoregressive and moving average parameters. The results are collected in table 2. In general, the difference between the true autoregressive parameter and the supposed  $\rho^*(4)$  seems to be less important for obtaining a high rejection frequency than the difference for the moving average component. This can be seen from the first five rows in table 2 where the power is indeed larger the larger  $\rho$ . However, while 1 is only approached for  $\rho$  close to 1, for  $\phi > .2$  the empirical power is always 1.

As expected the point estimates of  $\beta$  are again very reliably estimated. The estimated variance of the estimates does hardly vary at all across models and is relatively small. Hence, there is a good chance to correctly determine the related series. This leads to the conclusion that independent of whether or not the true disaggregated model can be identified forecasting, or nowcasting on the basis of the true related series is, in principle, feasible.

### 4.3.2 Model selection

So far, the aggregation restriction  $m = 4$  has been tested. If a researcher was moreover also interested in distinguishing between various possible disaggregation methods, *i.e.* the decision about the most appropriate disaggregation level, an according choice of  $m$  has to be made. Obviously, the pure disaggregation test is not applicable because the alternative hypothesis does not nest the null. This paper looks in the remainder at a multi-step model selection strategy to identify the correct  $m$ . The first step is to ensure that indeed, an  $ARMA(1, 1)$  process describes the observations appropriately. However, instead of a thorough general-to-specific approach, a

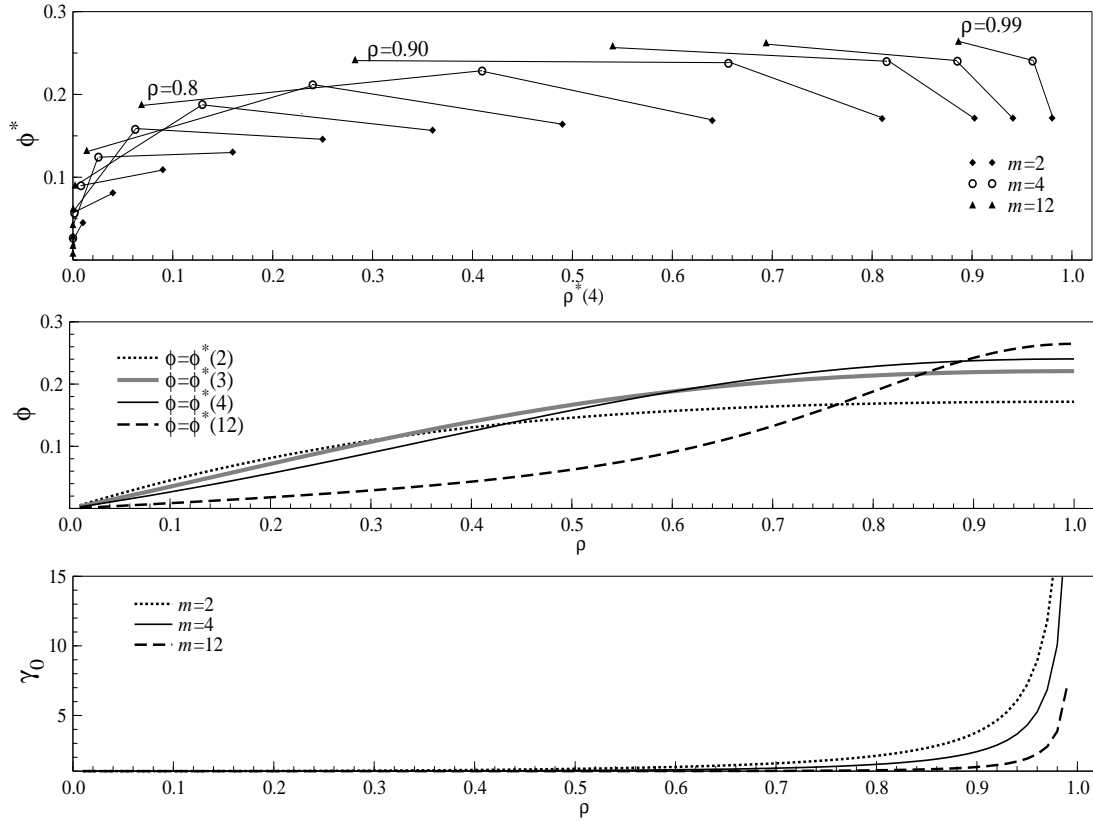


**Figure 3:** Top and middle panel: Size and power of the likelihood ratio test against nested alternatives, bottom panel: selection between non-nested models.

simple check of the significance of the  $AR$  and the  $MA$  parameter is pursued.

Secondly, in order to discriminate between different possible  $m$ , disaggregation tests are performed with the various candidates of  $m$  in the null and arbitrary  $ARMA(1,1)$  processes in the alternative. Ideally, if the data was generated with  $m = 4$ , for example, then the null hypotheses  $m = 2$ , or  $m = 12$  should be rejected while the null hypotheses  $m = 4$  should not. Likewise, if the data was generated with  $m = 12$ , the null  $m = 4$  should be rejected and so forth.

Figure 4 illustrates the choices of  $\phi^*$  and  $\rho^*$  for various  $m$ . It is interesting to notice that regardless of the actual choice of  $m$  the implicit  $ARMA(1,1)$  parameters are close to each other for a considerable range of  $\rho$ . In fact, all possible pair of lines have at least one point in common. This implies that there are always some  $\rho$  for which the observable data could have been generated at two possible higher frequencies. In other words, the aggregated data



**Figure 4:** Top panel: the simulation experiment in the  $(\rho^*, \phi^*)$  space, the lines connect pairs of  $\rho^*$  and  $\phi^*$  generated with  $\rho$  fixed yet varying  $m$ , middle panel: values of  $\phi^*$  for popular choices of  $m$  (semi-annual to annual, monthly to quarterly, quarterly to annual, monthly to annual aggregates), bottom panel: autocovariance for  $\sigma_l^2 = 1$  and various  $m$ .

information does not suffice to identify  $m$ . Moreover, given a certain variance of the aggregated process, the variance of the innovation process of the disaggregated model does not vary much with  $m$  for  $\rho < .8$  (see bottom panel of Fig. 4). A reasonable *a priori* presumption therefore is that the discrimination on the basis of this indirect method might not be very powerful. Thus, in an additional exercise the general model selection criterion due to Akaike (1969) will be employed.

The simulation results are graphically depicted in Figure 3. Its top panel reports the power of the first step of the model selection procedure. It shows that the probability to choose the  $ARMA(1, 1)$  correctly increases with increasing  $\rho$ . This result does not come as a surprise given

the fact that both,  $\phi^*(m)$  and  $\rho^*(m)$ , rise when  $\rho$  approaches one.

At the second step of the model selection procedure it should be decided what particular  $m$  led to the observation of an  $ARMA(1, 1)$  process. The middle panel of the figure shows that the corresponding indirect conjecture is ridden with low power of the disaggregation test against local alternatives. Notice that an  $ARMA(1, 1)$  process owing to  $m = 12$  is observationally fairly close a process obtained with  $m = 4$  (cf. Figure 4, top panel). However, at least the power exceeds the corresponding size of the test implying that the test works in the appropriate direction. In the bottom panel it is tested if it was possible to gain from using the Akaike information criterion. The picture is drawn such that the probability of a correct choice is positive when the filled symbol is above its empty counterpart. As it turns out, this is not always the case when the objective is to discriminate between  $m = 4$  and  $m = 2$ . When choosing between  $m = 4$  and  $m = 12$ , the Akaike criterion delivers some useful information. With the indirect test method, it seems to be easier to discriminate between  $m = 4$  and  $m = 2$  when compared to choose between  $m = 4$  and  $m = 12$ . The details are given in the middle panel.

## 5 Summary and conclusions

This paper suggests an alternative estimation of the established CL disaggregation approach. In addition to simplifying the analysis by allowing a one-step-estimation on the basis of the aggregated data, it permits to perform various specification tests. In particular, the aggregation restriction can directly be tested. A simulation study reveals that the test indeed has power against alternative data generating processes. However, the use of the test is partly limited due to excessive size in small samples and low power against local alternatives. Future research might be devoted to correct for the small sample effects and to the development of superior model selection strategies. In particular, testing against non-nesting alternatives might prove useful. Nevertheless, the new estimation and testing approach may provide an additional valuable tool for disaggregation model selection in the presence of modelling alternatives.

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## A Appendix

In this appendix I outline the derivation of  $\phi^*$  as a function of  $m$  given the aggregation problem described in the main text. Firstly, I write (6) for general  $m$  and then show how  $S_0$  and  $S_1$  result. The value of  $\phi^*$  then results by applying (9). Unfortunately, I only can guess but have no proof for  $\phi^*$  being a real valued function for an arbitrary choice of  $m$ . On the other hand, for any given  $m$  it is easy to check whether or not  $\phi^*$  is real. For most applications  $m$  will not exceed 12, and for each of the corresponding  $\phi^*$  a real value is obtained.

The starting point is the notion that any  $z_{h,t}$  can be given as

$$z_{h,t} = \rho^n z_{h,t-n} + \rho^{n-1} \epsilon_{h,t-n+1} + \rho^{n-2} \epsilon_{h,t-n+2} + \dots + \epsilon_{h,t} \quad (\text{A.1})$$

which implies for temporal aggregation over  $m$  periods,

$$\begin{aligned}
z_{h,t} + z_{h,t-1} + \dots + z_{h,t-m+1} &= \rho^m z_{h,t-m} + \rho^{m-1} \epsilon_{h,t-m+1} + \rho^{m-2} \epsilon_{h,t-m+2} + \dots + \epsilon_{h,t} \\
&+ \rho^m z_{h,t-m-1} + \rho^{m-1} \epsilon_{h,t-m} + \rho^{m-2} \epsilon_{h,t-m+1} + \dots + \epsilon_{h,t-1} \\
&\vdots \\
&+ \rho^m z_{h,t-2cm} + \rho^{m-1} \epsilon_{h,t-2m+2} + \rho^{m-2} \epsilon_{h,t-2m+3} + \dots \\
&\quad + \epsilon_{h,t-m+1} \\
&= \rho^m (z_{h,t} + z_{h,t-1} + \dots + z_{h,t-m+1}) + u_{l,\tau}.
\end{aligned}$$

The aggregated process can now more compactly be written as

$$z_{l,\tau} = \rho^m z_{l,\tau-1} + u_{l,\tau} \quad (\text{A.2})$$

where the error term  $u_{l,\tau}$  is the sum of the elements of the  $(m \times m)$  matrix  $\Phi_\tau$ :

$$\Phi_\tau = [\epsilon_t \ \epsilon_{t-1} \ \dots \ \epsilon_{t-m+1}]' \otimes [(\rho L)^{m-1} \ (\rho L)^{m-2} \ \dots \ (\rho L)^0] \quad (\text{A.3})$$

which makes use of the lag operator,  $L$ , with  $L^i x_t = x_{t-i}$ . It is instructive to expand  $\Phi_\tau$ :

$$\Phi_\tau = \begin{pmatrix} \rho^{m-1} \epsilon_{h,t-m+1} & \rho^{m-2} \epsilon_{h,t-m+2} & \rho^{m-3} \epsilon_{h,t-m+3} & \dots & \rho^0 \epsilon_{h,t} \\ \rho^{m-1} \epsilon_{h,t-m} & \rho^{m-2} \epsilon_{h,t-m+1} & \rho^{m-3} \epsilon_{h,t-m+2} & \dots & \rho^0 \epsilon_{h,t-1} \\ \rho^{m-1} \epsilon_{h,t-m-1} & \rho^{m-2} \epsilon_{h,t-m} & \rho^{m-3} \epsilon_{h,t-m+1} & \dots & \rho^0 \epsilon_{h,t-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{m-1} \epsilon_{h,t-2m+1} & \rho^{m-2} \epsilon_{h,t-2m+2} & \rho^{m-3} \epsilon_{h,t-2m+3} & \dots & \rho^0 \epsilon_{h,t-m+1} \end{pmatrix} \quad (\text{A.4})$$

This matrix has an interesting structure. In particular, the innovations with identical time subscripts are to be found along the diagonals. Thus, the variance of  $u_{l,\tau}$  is the sum of the squared sums of the diagonal elements. At the same time the power to which  $\rho$  is raised is the same in each column. Therefore, every secondary diagonal can be regarded a truncated version of the main diagonal with respect to the power coefficients. The following auxiliary matrices and operator are useful in finding handy expressions. Let me use the operator *diag* which stacks

the main diagonal of a symmetric matrix into a vector. Hence,

$$\begin{aligned}
\Psi &\equiv \mathbf{1}_{m \times 1} \otimes [\rho^{m-1} \rho^{m-2} \dots \rho^0] \\
diag(\Psi) &= [\rho^{m-1} \rho^{m-2} \rho^{m-3} \dots \rho^0]' \\
&= \psi' \\
H &\equiv \begin{pmatrix} 1 & 1 & \dots & 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 1 & 1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 1 & 1 & \dots & 0 & 0 \\ 0 & 0 & \ddots & 1 & 1 & 1 & \ddots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 & 1 \end{pmatrix}
\end{aligned}$$

where  $diag(\Psi)$  and  $H$  have dimensions  $(m \times 1)$  and  $(m \times 2m - 1)$  respectively, and  $\mathbf{1}_{m \times 1}$  is a  $(m \times 1)$  vector of ones. Notice that  $H$  is essentially a matrix of  $m$  rows of a  $m$  dimensional column vector of ones within a  $(m \times 2m - 1)$  matrix of zeros where in each successive row the vector of ones is shifted one column to the right. The product  $\psi H$  now conveniently collects the  $2m - 1$  sums of the diagonal elements of  $\Phi_\tau$  in a  $(1 \times 2m - 1)$  vector omitting for the sake of simplicity the innovation terms. The variance of  $u_{l,\tau}$  can now be obtained as

$$\begin{aligned}
S_0 &\equiv \psi H H' \psi' \\
E(u_{l,\tau} u_{l,\tau}) &= \sigma_h^2 S_0
\end{aligned}$$

which makes use of the *i.i.d.* property of the  $\epsilon_t$ .

For deriving  $S_1$ , decompose  $H = (h_1, \mathbf{1}_{m \times 1}, h_2)$  where  $h_1$  and  $h_2$  are  $(m \times m - 1)$  matrices collecting the sums of the diagonal elements below and above the main diagonal respectively. Consider now  $\Phi_{\tau-1} = L^m \Phi_\tau$  whose sum of elements define  $u_{l,\tau-1}$ . The value of  $S_1$  is linear in the covariance between  $u_{l,\tau}$  and  $u_{l,\tau-1}$ . Therefore, we need to multiply the sums of the elements above the main diagonal of the matrix  $\Phi_{\tau-1}$  with the sums of the elements below the main diagonal of the matrix  $\Phi_\tau$  diagonal by diagonal. With the aid of  $h_1$  and  $h_2$  one can write

$$\begin{aligned}
S_1 &= \psi h_1 h_2' \psi' \\
E(u_{l,\tau} u_{l,\tau-1}) &= \sigma_h^2 S_1.
\end{aligned}$$

Figure 4 depicts the values of  $\phi^*$  which are obtained for various choices of  $\rho$  and sets them in relation to the hypotheses tested in the simulation study.