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A Meso-Scale Framework to Support Urban Planning

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A MESO-SCALE FRAMEWORK TO SUPPORT URBAN PLANNING

to Busbus

Understanding the evolution and causalities behind the emergence of residential and business districts is of high importance in urban planning to reduce and cope with uncertainties. This thesis proposes a planning support system that has the capabilities to support and guide the urban planning practice from an urban economics perspective, but is not restricted to it.

This research originates from the observation that urban planning support systems generally either have a very simplistic view on urban systems and an engineering perspective on them, or are very detailed and try to thoroughly model many properties of the urban complex system. Both ways of modelling and simulating cities provide valuable insights into urban systems, but are either too general or too detailed and computationally expensive in certain planning stages.

The proposed framework aims to close this gap by working in the meso-scale, which provides the possibility to use strengths from both approaches. This is possible by relaxing the rigid assumptions of the simple models, and abstracting properties from the comprehensive models. Providing a framework in the meso-scale allows to study phenomena that do not occur in the macro-scale of simple models, or would need an aggregation of the result in the micro-scale. The aggregation is theoretically possible to do, but is a very time intensive and tedious task. The meso-scale framework focuses on the phenomena in the meso-scale directly.

The meso-scale does not focus on providing too much detail to the urban planners and decision makers. They can use it as a planning tool, that supports them by sketching possible future states of the system. It allows them to gain experience artificially by exploring the possible impacts and implications different actions and interventions might have, without the distraction of too much detail.

This thesis solves core challenges that come with modelling in the meso-scale. It shows how such a framework can be used as a decision and solution finding support tool in different planning stages for urban systems.

The framework was applied in real world scenarios, with authorities from the city of Zürich, Switzerland, and has shown its value in early stages of planning process in several stakeholder workshops. Finally, the framework developed in this research can also be used to support urban planners and decision makers in the process of exploring the solution space of possible interventions and how these desired states can be reached.

Die Stadtplanung befasst sich mit der Schaffung sinnvoller räumlicher Organisation verschiedener Aktivitäten in einer Stadt, sowie mit der Konfiguration urbaner Räume. Ein wichtiger Punkt in diesem Zusammenhang ist die Anordnung von Wohn- und Geschäftsvierteln. Man sollte diese nicht als abgeschlossene Einheiten betrachten, sondern als Teile eines Ganzen, die untereinander interagieren. Es ist fast unmöglich, die resultierenden Wechselwirkungen umfassend zu erfassen und zu verstehen. Aus diesem Grund sind Entscheidungen in der Stadtplanung oftmals mit grossen Ungewissheiten verbunden. In diesem Kontext entwickelt die vorliegende Forschungsarbeit ein Modell, welches Planungsgremien bei der Entscheidungsfindung für komplexe stadtplanerische Aufgaben unterstützen und begleiten kann.

Diese Methode wird mittels eines Planungsuntertützungs-Tools umgesetzt, welches sein Wurzeln in der räumlichen Ökonomie hat, diese aber aus einer anderen Perspektive betrachtet; nämlich aus jener der Komplexitätsforschung. Dieser Ansatz ist an sich nicht neu. Es bestehen schon verschieden Ansätze, wie dieser Perspektivenwechsel umgesetzt werden kann. Die dazu verwendeten Modelle haben aber oftmals das Problem, dass deren Anwendung relativ zeitintensiv ist und es dadurch nicht möglich ist, mehrere Planungsvarianten in relativ kurzer Zeit zu evaluieren. Genau das ist aber aus Sicht des Autors eine wichtige Eigenschaft, die Planungsunterstützungs-Tools erfüllen sollten. Einerseits können dadurch verschiedenen Optionen durchgespielt und evaluiert werden, andererseits erlaubt es virtuell Erfahrungen über das Verhalten des untersuchten urbanen Systems zu sammeln. Wie aus der Literatur zum Stand der Forschung hervorgeht, ist Erfahrung eine wichtige Eigenschaft in der Erarbeitung von Lösungen. Normalerweise ist es aber ein langwieriger Prozess diese zu sammeln, was auch in der Stadtplanung gilt. Es kann Jahre dauern, bis Auswirkungen einer Entscheidung oder eines Projektes ersichtlich werden. Das Aneignen von Erfahrungen anhand eines Planungs-Tools kann in diesem Kontext sehr wertvoll sein.

Diese Forschungsarbeit erarbeitet die Grundlagen, damit diese Ziele erreicht werden können. Hierfür wird ein neuer Modelltyp entwickelt, der die komplexen Wechselwirkungen zwischen verschiedenen urbanen Akteuren abbilden kann. Im Rahmen dieser Arbeit sind es die Wechselwirkungen von Firmen und Haushalten. Das entwickelte Modell kann aber vom Aufbau her auch in anderen Szenarien verwendet werden. Es ist das Herzstück des vorgeschlagenen Planungsunterstützungs-Tools. Im zweiten Teil dieser Arbeit wird an drei verschiedenen Fallbeispielen gezeigt wie das Tool verwendet werden kann. Das erste Fallbeispiel zeigt die Verwendung des Tools als Analyseinstrument. Diese Analyse wurde in Zusammenarbeit mit der Stadt Zürich durchgeführt und untersucht die sozio-ökonomischen Effekte einer Fussgängerbrücke auf das umliegende Stadtgebiet.

Das zweite Fallbeispiel beschreibt die Erfahrungen von Studenten, welche das Tool im Rahmen ihrer Diplomarbeiten verwendet haben. Es wird belegt, dass die Studenten es nicht gewohnt sind solche Tools zu verwenden und im Nachhinein anders an die Problemstellung ihrer Diplomarbeit herangehen würden.

Das letzte Fallbeispiel beschreibt die Möglichkeiten des Tools wie es zusammen mit Methoden des maschinellen Lernens verwendet werden kann. Das Beispiel zeigt auf, wie das Tool verwendet werden kann, wenn man eine Stadt in eine vordefinierte Richtung entwickeln will, die Wege dahin aber unbekannt sind. My answer on both counts is no. This different approach is not just the use of computers. It is economics done differently, economics based on different concerns, an economics where the problems are different and the very idea of a solution is different. — W. Brian Arthur [1]

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ACRONYMS

- ABM Agent-Based Modelling
- CA Cellular Automata
- CPT Central Place Theory
- EQ Equilibrium Model
- HB Main Station
- MC Monte Carlo
- PCA Principle Component Analysis

Part I

INTRODUCTION & RELATED WORK

INTRODUCTION

As cities grow larger, they experience a complexity that is hardly manageable with traditional urban planning approaches and tools. A crucial part of managing cities is the emergence of patterns of land-use, for example the development of residential and business locations. Understanding the evolution and the causalities behind the emergence of these patterns is of high importance. It allows the planners to act and react before the appearance of any criticality, thus leading to a more semi-resilient city, where negative effects are absorbed and positive development is fostered.

Figure 1 depicts the size of the largest cities in the last five millennia, according to Chandler [2]. It shows that it is the first time in the last five thousand years, that we experience cities with the tremendous size of modern urban regions. The largest cities before the industrial revolution had just around one million inhabitants. According to City Population [3] it would rank behind 500th place nowadays. The growth of today's mega-regions has not stopped yet, especially in developing countries. UNFPA predicts that the current urban population will double within the next fifteen years and will reach eighty percent of the worlds population by the year 2030 [4].

The extreme sizes of modern urban agglomerations has a big influence on the national, but also on the global economy [5]. The cities with the most economic activity in 2008 accounted for more than forty percent of the global economic activity, by housing only 17% of the world's population [6]. With the increasing amount of people living in urban agglomerations, they are of even greater importance to the world's economy.

The importance of urbanisations to a nation's economies is strongly depicted in the developing world. Research reveals that the developing countries with the most rapid urbanisation are generally also the ones with the strongest and most stable economies [5].

The importance of cities to the national and the global economy demands for adaptive-resilient cities; resilient in the way that no negative externalities can lead to a breakdown of the urban system, and adaptive so that it can react to these changes and externalities in a good way. Bettencourt et al. [7] show that many properties and dynamics in cities follow super-linear scaling laws, for example, with increasing size the per capita productivity of inhabitants increases. However, not only positive properties get more efficient, negative properties also follow super-linear scaling laws. For example, per-



Figure 1: The number of inhabitants in largest cities of their time according to Chandler [2] and Brinkhoff [3]. Plotted in semi-logarithmic scale.

capita crime rate is generally higher in larger cities, compared to smaller cities.

Positive, but also negative, properties often experience self-reinforcing effects. Agglomeration economies and higher efficiencies of infrastructure foster an adaptive-resilient city in a virtuous circle, whereas large population numbers can also lead to traffic congestion, segregation, and more pollution (vicious circle). One of the main tasks for urban planners and decision makers is to foster virtuous circles and oppress vicious circles.

In his seminal paper, Lee describes that "...city planning emphasized the need for comprehensive thinking and a comprehensive master plan in order to guide metropolitan growth, prevent large-scale inefficiencies and negative neighborhood effects, and preserve open space." [8].

This statement might not be perfectly valid anymore in modern planning. Modern planning consists of larger groups of decision makers and planners, and might also involve other stakeholders, such as community representatives [9]. Modern urban planning is no longer about defining a master plan, it is an iterative and many-stage process that plans, projects a desired future, and redefines both over time [10].

Lee points out an important fact about urban planning, metropolitan systems can only be *guided* by authorities. It is hardly possible to steer urban systems and to force them into a desired direction of development, and can only be achieved with big effort. Systems with many actors involved – of which urban systems are no exception – organise themselves within the given boundaries. This leads often to unexpected outcomes and may even lead to near critical states that can easily destabilise the system if an unexpected externality occurs, especially when the rules by the authorities are too restrictive. In order to guide the metropolitan growth and development in a good and resilient way, it is of high importance to understand the basic dynamics of the urban system. This demands for analysis methods and tools that take self-organising into account and make the planners aware of possible impacts and consequences certain decisions may have. Understanding the impacts of decisions allows for an opportunity to guide the urban system in an efficient way. Often only small interventions are needed in a self-organising system to lead it to the desired outcome [11, 12].

Urban planners can help cities develop virtuous cycles by guiding and supporting the self-organisataion process. However the proper tools and theoretical background are need to support them in this process.

1.1 MOTIVATION & RESEARCH OBJECTIVES

In the course of my PhD, I took part in several workshops with a focus on city planning. The workshops had different foci and different people took part. Some of the workshops were held exclusively with city authorities, others had a broader spectrum of participants. In most of the workshops, one or more digital decision and planning support tools were used.

I observed during these workshops, that digital tools can support the planning process, but also divert from the initial topic. I was part of a workshop, in which initially the building volumes of new building developments were the focus. At first, they were the main topic of the discussions, but soon the topic changed to the looks of the buildings and to random details. The details were only added by the designer of the visualisations to make them look more realistic. For a significant time of the workshop, these details were the topic, even though it was mentioned several times that they are just there as examples. The stakeholders in the workshop were distracted by details which added no additional information to the initial focus, and pretend a level of detail that was not available at that stage of the planning discussion.

Another observation I made during some of the workshops is that urban planners and decision makers often find it interesting to see the results of socio-economic simulations. However, they are most often interested in different variants of the simulation and want to see results from different prediction scenarios. Most of the modelling and simulation tools I encountered during the workshops were not able to provide this. The reasons are diverse and the high computational they often have is just the most prominent one.

In this regard, I started to doubt on the usability of modelling and simulation in the context of supporting urban planning. Most of the tools and frameworks I encountered until then were either to expen-

6 INTRODUCTION

sive in many ways, or provided a level of detail that was diverting workshop participants more than helping them in finding and exploring solutions. These observations lead to the principle question of this thesis:

Can modelling and simulation support urban planners and decision makers finding solutions to reach a certain goal?

Modern planning practice is not a task that is carried out by a single person. Thus, the question is

How?

modelling and simulation can support and guide planning discussions and help planning committees in an efficient way. The tool must respond quickly and offer different possibilities on how the goal might be achieved; all during discussions and a flexibility to test different scenarios immediately. It should not be a tool that depicts a general state, it should rather depict states that directly result from the current state. The tool must have a sound foundation, and incorporate most recent findings in social system dynamics. Urban economics offers a strong foundation to do this research. It is a field of microeconomics that proposes many thorough explanations on how certain patterns within cities emerge.

The traditional models of urban economics work within restrictive assumptions and are therefore rather limiting especially within the field of complexity science. The fundamental economic theories are however important to lay the initial groundwork for this research and should not be neglected. This leads to the research work done in the first part of this thesis:

How can neoclassical urban economic models be adapted to reflect the perspective of complexity science?

Existing models have been developed within this scope, but are limited in their capacity to only answer "what-if" type questions. The user has to input the intervention and then the model gives feedback on possible outcomes. This research aims to go a step further and explores possibilities that can provide urban planners and decision makers with a scenario in which they can ask "how-to" type of questions. They can input the goal they want to reach and the framework proposes different options to reach it. This leads to the last objective of this research, which is formulated in the overarching research question:

How can models be used to describe possible solutions to reach a certain goal?

1.2 THESIS ORGANIZATION

The rest of the thesis is organised in the following. Chapter 2 discusses the context of the here described research. It provides the theoretical background and relevant context to support the proposed research outlined in this thesis.

Chapter 3 introduces the different parts of the planning support framework developed in this research and comprehensively discusses the methodology behind the different parts. The framework validation is presented in chapter 4 and illustrates the application of the model for both, theoretical and real world use-cases. The validation process demonstrates that the framework can support the planning process, and that it is capable of investigating traditional spatial economics models from a new perspective.

Chapter 5 focuses on the use of the planning support framework applied to different case studies. The chapter exemplifies how the model can be used in different stages of the urban planning and decision making process. It shows how the framework was applied in a workshop with authorities from the city of Zürich, Switzerland, how the framework can be used to sketch possible future states of a city, and how it can support the planning process in finding solutions to certain planning problems.

Chapter 6 discusses the findings and primary conclusions from this investigation.

The thesis combines work in the fields of urban planning support systems and urban economics. It does this by the means of applying methodologies from computational science and complexity science.

This chapter describes the theoretical background of the thesis and highlights the most important contributions in the different fields in the context of the topic.

Already in 1961, Jacobs argued in her seminal book *The Death and Life of Great American Cities* [13] that it is not enough to simply study the locations of things in cities [14]. She states that the evolving network of relationships and interactions have to be considered as the cement that holds cities together. Despite Jacobs's argumentation, many scientific fields considered cities as an agglomeration of many locations implementing activities. The only network that was somehow taken into account was the transportation network, often to add a spatial dimension to traditional approaches.

Section 2.1 introduces the core contributions of spatial and *urban economics* to urban research. Urban economics describes and theorises how patterns and locations of activities emerge; using precise mathematical descriptions. The section will show some examples when these models and theories were applied in planning practices in the Netherlands. Dutch planners already consulted regional and urban economics theories in the mid of the 20th century [15, 16], when the they were just emerging.

These early models and theories of urban economics were developed, when modern tools and methods were not available. It led to a top-down view on urban systems, with many strong assumptions. This changed with the increasing availability of computing power and has changed how cities are perceived. This has enabled scientists to use more dynamic model assumptions and study cities the way Jacobs suggested. Section 2.2 introduces this modern perspective: *complexity science*. It provides an overview of what complexity science is, and how it enables researchers to understand the dynamics of cities better.

The last section 2.3 emphasizes the possibilities complexity science and urban economics have to support urban planning, and points out a few properties that complex models must exhibit to support urban planning.

2.1 URBAN ECONOMICS

The discipline of urban economics is an intersection of geography and economics, more specifically of microeconomics [17–19]. Microeconomics is the study of decision making of households and firms. It investigates the decision making process when households and firms have to compete for limited resources. Households try to maximize their utility within this competition, whereas firms base their choices to maximize profit. Geography is the study of where activities occur in a spatial setting. Urban economics combines the two and studies the location of activities, assuming utility maximizing households and profit maximizing firms, and the spatial structure that emerges from this interaction [17–19].

This section introduces the most important works in the field of urban economics within the context of this research. It focuses on the core models of urban economics. I depict how the models developed and how the different core models build on the previous ones. The section does not provide a complete overview of theories developed and their successors, my goal is rather to point out the main ideas behind the core models. For a more thorough overview of the models and their extensions I suggest the books of O'Sullivan [17] and Brakman et. al [15] as starting point.

Section 2.1.1 introduces the very first work in regional economics and the works that directly conclude from that model and lay the foundation of urban economics. Regional economics has similar research goals as urban economics, but does not only focus on urban systems, it also focuses on wider systems, including multiple cities or also rural areas. It is a more general theory than urban economics, which focuses on urban systems only. Section 2.1.2 then introduces theories that have a different perspective on the founding model and focus on the location of centers, rather than assuming them to have a fixed location.

2.1.1 Location of Activities

The foundation of neoclassical urban economics was laid by the German economist Johann Heinrich von Thünen. He imagined "...a very large town at the center of a fertile plain, which is crossed by no navigable river or canal. Throughout the plain the soil is capable of cultivation and of the same fertility. Far from the town, the plain turns into an uncultivated wilderness, which cuts off all communication between this state and the outside world." [20] (Translation taken from Portugali [21]). He assumes that there is no additional city on the plain, and that the large town has to provide the rural areas with manufactured goods, who in return have to provide the city with provisions. The manufactured goods are all produced in close vicinity of the central market and form the central city.

The study of von Thünen focuses on the question of how agriculture will arrange around the central place, and where which type of agriculture will be located. One of the key assumption in his theory is perfect rationality. Von Thünen assumes that the producers of agricultural goods decide with perfect rationality, meaning that they select their location and production only on two factors, transport cost and price. If a good is expensive to transport or can only be consumed very fresh, it must be produced closer to the city, whereas goods that can be transported more cheaply can also be transported from further distances.

Von Thünen assumes linear transport cost for the goods produced in rural parts of the plain and that each product can be transported as the crow flies. The profit a producer makes with his product is then the market value of the good minus the transport cost. With the different transport costs and market values of the goods, the profit curves have different shapes. Figure 2 depicts the profit curves of three different products. By analysing the profit curves, the producers of provisions choose the goods they produce rationally. Assuming this perfect rationality of the farmers, agriculture arranges in concentric rings around the central city. Each of these rings produces only one type of goods. The lower part of figure 2 depicts the emergent pattern.

Von Thünen will never be aware that he built the foundation of a new economic field: Neoclassical Urban Economics. More than a century after his seminal work, locational theorists were inspired by his ideas and adapted them to an urban setting, calling it bid-rent theory. The works of Alonso, Muth, and Mills [22–24] generalize the different agricultural sectors to just one sector, but describe the central city in more detail. Instead of assuming that housing, factories and offices are located in the close vicinity of the central market without any specific order, they investigate the patterns of where they are located and the drivers behind this organisation. The well founded assumptions in this model are that offices need to be close to each other because of the importance of information exchange. Manufacturing firms orient towards the central market, and proximity to each other is of less importance. Households are oriented towards offices and manufacturing firms, because both provide them with income possibilities.

In a monocentric setting, the emergent pattern is similar to the isolated state, just with different types of land-use. The entities with the highest value generation, depending on the distance from the center (*bid-rent curves*), at one location overbid the other entities. The different shapes of the bid-rent curves are determined by transport cost of the entities and scaled by the value generation. Offices have a high



Figure 2: Ring pattern of von Thünen's isolated state and how it emerges from the isolated state theory.

transport cost because exchange of information is key, and if they have to travel longer distances to exchange information they suffer from high "production" cost. Manufacturing firms mainly exchange goods on the central market with customers, but do not require to be in close vicinity of other manufacturing firms, the interaction between them is neglected in this theory. Thus transportation cost is not a big matter of expense, since they only have to deliver their goods to the central market. The cost of commuting is lower for households than it is for either manufacturing firms and offices to do business. Thus they overbid the households on the locations near the city center and push them away from it.

This leads to a concentric ring pattern around the center of the city, where in each ring the same type of entity is present. The equilibrium state with these assumptions is very similar to the ones depicted in figure 2, but *Land use 1* changes to *Office, Land use 2* to *Manufacturing, Land use 3* to *Residents,* and *Land use 4* to *Agriculture*.

This key model of urban economics gives many insights into the patterns of cities and provides well founded explanations. There exist many extensions to this model , of which I will not give a complete overview since extensive reviews already exist, e. g. Edwards [18]. The successors of the Alonso-Muth-Mills model focus on different topics and provide possible explanations for many of the patterns that are

and were present in cities. The foci of the model adaptations range from explanations between income and location, to the influence of traffic networks on the urban patterns, but also racial segregation [17].

2.1.2 Location of centers

The theories discussed in section 2.1.1 can also be considered from a different perspective. Instead of asking what the patterns of activities are around a monocentric center, one can also ask how the centers (markets) have to be arranged to provide all inhabitants with the needed goods and services. The initial works with this perspective were introduced by Christaller [25] and Lösch [26], and consolidate to the *Central Place Theory*.

The central place theory assumes a featureless, large plain. Christaller assumes a central large city, that provides all the services households need. The households are uniformly distributed on the landscape, have all equal needs and fixed locations. Christaller, in his base model, then divides the services into different categories, where each category has a different service level. The service level describes the level of periodicity a service has, e. g. services that are demanded on a daily basis versus services that have only an episodic demand are in different categories. Since the households are equally distributed over the plain, some can not afford to travel to the central market on a daily basis. Thus the services with a short periodicity must be provided closer to the residents. This leads to a system of hierarchical cities and towns, were only a single large city is present in the system and many smaller ones are geometrically distributed on the plain. This leads to the hexagonal pattern depicted in figure 3.

Lösch [26] does not assume that the cities are fixed at locations and does not derive them geometrically. He assumes that all the services are free floating on the plain and arrange themselves to serve all needs. Initially, he starts with only one service provider per category and assumes that the whole plain is filled with households. He adds additional service providers and arranges them so that they can serve as many households as possible. This step is repeated until all the households have access to all the needed services. This procedure leads to a similar result as the theory developed by Christaller.

The central place theory was applied in real world scenarios [15]. In the early forties of the 20th century, a polder was reclaimed in the Netherlands. It had many properties that were very similar to the assumptions of the central place theory. The farmers on the land are approximately uniformly distributed and it can be assumed that they have similar preferences. The land is flat and almost homogeneous. The polder was mainly meant to host agriculture, but authorities also wanted to establish a few villages and a town on the polder. They designed the arrangements and the sizes of the agglomeration closely



Figure 3: The emergent pattern of the central place theory. All agglomerations serve all service categories up to a certain level, whereas the city hosts all services. The hexagonal shapes define the particular service boundary according to the agglomeration type.

to the results of the central place theory. It seems that the estimates of the authorities, using the central place theory, were very close to the demand. After half of a century, the numbers of inhabitants for the villages is very close to the numbers predicted by theory. Only the city in the center is more than twice as big as the central place theory suggested.

Brakman et al. [15] point out that the possible reasons for the extreme difference of the central place theory's prediction and reality is due to the underestimated agglomeration forces larger cities have. Other reasons could also be that the transport cost decreased considerably in the fifty years after planning the polder, or that agriculture became less important.

One of the main weaknesses, according to Krugman [27], of the central place theory is that it is not a theory that mainstream economists would consider a complete model. It is lacking an interesting aggregate story that emerges from carefully described individual behaviour. Krugman proposed a new theory and model of urban systems with the core-periphery model [28], which builds the foundation of geographical economics. Geographical economics does not invent a new perception of how urban systems should be modeled and theorised, but puts different concepts from economic theory into a single analytical framework¹. "What was missing before [geographical

¹ The framework of geographical economics also opened the door for other scientific fields, and interdisciplinary research. The nature of the framework, with its aggregation of different economic theories, allows also to implement theories from non-economic fields [15].

economics] was a general equilibrium framework with imperfect competition connecting these various insights and allowing for a detailed study of their interactions." [29].

The base model of geographical economics consists of two regions which are connected, but some distance apart from each other. There are two types of industries in the model, present in each region: food production and manufacturing firms. The consumers of the goods are also the workers in both industries. There are two types of workers, skilled and unskilled. The skilled workers earn their money in the manufacturing firms, the unskilled in food production. It is not possible for the workers to change to the other industry. The skilled workers have the possibility to change their location, which is not possible for the unskilled workers because agricultural land is immobile. The consumers consume goods from both regions, and prefer a high variety for their consumption which means that they prefer a little of many products to a lot of one product. The manufactured products from the other region have some transportation cost, which makes them more expensive for the further away region, but the manufacturing firms experience scaling effects, which means that they have lower production costs per unit if they have more workers. I will not go more into detail of this model, because these detailed properties have no direct impact on this thesis. For a more detailed review of the model, I refer to Brakman et al. [15].

The base model with only two regions is simple enough to analytically explore the influence of different parameter settings. This is not the case anymore if more regions are added. The first core-periphery model with multiple regions was introduced by Krugman and called the racetrack economy [27]. He arranged a number of cities in a circle and connected a region only with its direct neighbors. The only possibility to explore the parameter space of the racetrack economy, was by solving the model numerically. Solving the model numerically made it possible to show the effects of scaling laws on agglomeration with multiple regions, which would not have been possible analytically. If transport cost is not extreme, or two regions are not too far away from each other, the scaling effects have a high impact on the attraction regions have to skilled workers. In a racetrack economy of twelve regions, in most of the cases only a few regions have skilled workers and the others only contain food production. These results are an example for the strength of agglomeration effects (agglomeration economies) and show that they are an important property in how agglomerations (cities) evolve.

What highlights the model for this work – even though it focuses on the location of cities – is that it was, to my knowledge, one of the first (economic) models including a spatial dimension that was no longer analytically solvable for all cases. This accounts especially for the extended model with multiple locations. The decreasing computing cost made it possible to solve the model numerically. Krugman used computers to explore different outcomes when running parametric analysis and sensitivity studies for various initial conditions.

Computer driven models opened a new door for many new application of models. In the time that the above models were developed, "cities were first formally considered as 'systems', defined as distinct collections of interacting entities, usually in equilibrium, but with explicit functions that could enable their control, often in analogy to processes of planning and management." [14] This means that they were considered to be organized from a top-down perspective, where each of the elements in the system has its clear role and thus is can be clearly defined and controllable. Additionally, together with the assumption of a central controller ("invisible hand") that steers the system, it was assumed that cities are closed systems, with negligible influence from the outside.

This view of cities changed already during the time the above models were formulated. The next section discusses the opportunities the decreasing computing cost made available for modelling and simulation of urban systems, and how it made it possible to look at cities from a different perspective.

2.2 URBAN COMPLEXITY SCIENCE

The neoclassical models described in section 2.1 have very strong assumptions and have been found to be wanting within the field of Urban Complexity Science [14]. Cities are not in a state of equilibrium, they are in a constant state of change. They are in fact far away from equilibrium. The assumption of the existence of a central controller seems to be too strong too. Cities are not centrally ordered, they evolve with decisions made by many individuals and experience only occasional actions of a central authority.

The neoclassical models assume that the urban system is a closed system, not interacting with other cities or the world's economy. But as mentioned in the introduction, the twenty largest cities by economic activity account for more than forty percent of the world's economic activity [6]. Already this property illustrates that the cities are not closed off from the wider world.

Another rigid assumption of neoclassical economics is that they assume the absolute rationality of the households and firms, without any memory of the past and the influence of previous decisions. Both assumptions limit the potential of such models. Recent literature shows that, for example, location choice of households is not solely rational from an economic perspective [30].

With the rise of complexity science, the view on cities changed. Most of the strong assumptions of neoclassical models were dropped, mainly due to a change in perspective. The before assumed top down view on cities was exchanged with a bottom up perspective. For example, neoclassical models assume equilibrium and investigate and describe the (equilibrium) behaviour of individuals that leads to a certain pattern. This has led to the development of models that represent individuals as sole systems that interact with each other and can base their behaviour on past experiences and feedback on it. By focusing on the behaviour of individuals, instead of trying to find the behaviour that leads to a certain pattern, we can study questions of how systems develop and individuals adapt over time.

Arthur exemplifies this with his very early model of a stock market [1]. He takes a stock market model introduced by Lucas [31] as a basis and changes it to reflect the different perspectives complexity science has on systems. To ease the analysis of his equation based model, Lucas assumed that all individuals (investors) have the same behaviour. Arthur changed this by using computer programs to run the model. It allowed him to implement rules so that investors could adapt their strategies during a simulation run, based on the past. Lucas showed mathematically how the price of a stock changes over time according to its earnings. In contrast, the approach Arthur took on the problem gave him the possibility to observe real-world phenomena, such as small crashes and bubbles. Standard economics cannot show these effects.

What I have to emphasize here is that complexity science does not replace classical methodologies, it is a science that looks at things from a different perspective and studies various dynamics. As the example above shows, it focuses on how entities in the system change their behaviour and adapt under certain conditions. It allows us to study systems and explore the adaptation of different entities within it and further, how they react to various system changes.

The rest of this section discusses the properties complex systems have, i. e. what properties define a system as a complex system (subsection 2.2.1), and shows that urban agglomerations and cities exhibit most of these properties. Subsection 2.2.2 introduces methods to explore complex systems and some of the strengths and weaknesses of such approaches.

2.2.1 Properties of Complex Systems

There is not a strict definition for what constitutes a complex system. However most complexity scientists agree that a complex system must exhibit most of the properties described in this subsection. The characteristics are very general and describe most natural systems as complex systems. Cities are no exception to this. I will highlight the different characteristics with examples of dynamics and decision making in cities. The list of of characteristics I present here are loosely taken from Johnson [12], but most of literature agrees on them. THE SYSTEM CONSISTS OF MANY INTERACTING ENTITIES. All entities have a role in the system of interest. In an urban system, the entities can be, among other things, single people, households, or even problems and decisions [32]. This is mainly defined by the scope and scale of the system. For example in a traffic simulation, where the collective behaviour of the system is of interest, the actors are the drivers of different vehicles, or the vehicles themselves [33]. The entities (or agents) in traffic simulation interact with each other through different means. They interact because of physical closeness in the traffic. For example, the back driver has to stop when the front driver does so, if he wants to prevent an accident. The agents might also interact through the radio or phone. In this way they can spread information over further distances and change the behaviour of others also over distance.

The interaction of the agents is expressed through a network, which might change over time. It defines the level of interaction between the different agents and how information is carried through the system. This makes networks an integral part of complexity science. For many complexity scientists the study of complexity is synonymous to the study of networks and individual entities [12].

THE INDIVIDUAL ENTITIES' BEHAVIOUR IS AFFECTED BY MEM-ORY OR "FEEDBACK". They can learn from the outcome of previous decisions and make their present decisions using this knowledge, but also feedback on their current behaviour and information on the system itself helps them adapt their behaviour.

A famous example for this is the El Farol's Bar problem [34]. There is a finite number of people that enjoy going to the El Farol's bar for the weekly concert on Thursdays. However, people find it only more enjoyable than staying at home if less than sixty percent of the total number of people go to the bar. In the simplest case, people do not exchange any information and can only know the amount of people at the bar if they decide to go to the concert, or in retrospect. The agents have to make their decision weekly using the knowledge and experiences of the past. The main problem that arises within this relatively simple problem is how agents choose their strategy. What works best in long term, is when the agents have non-uniform strategies on how they adapt their decisions. Otherwise all agents would behave the same all the time and no satisfaction can be reached [21]. When the agents have different decision strategies, the system fluctuates a little around an optimal state in which most of the people are satisfied most of the time, after a certain initial phase.

Extended versions of the bar problem do also include feedback between the agents. Circles of friends have the possibility to interact with each other and make decisions together, this is again an interac-
tion network that allows the agents to interact with each other and gather additional information.

THE SYSTEM IS "OPEN". In nature, a system is rarely closed, actually the only pure closed system is the universe as a whole [12]. An "open" system can be influenced by other entities and by the environment. A closed system assumes that the environment does not change and that the interaction with external entities is negligible. As mentioned earlier in this chapter, cities have a high interaction level with other cities and the global economy. Actually they account for a big fraction of the wold's economic activity themselves. News from outside the assumed system can, for example, influence the economy of a city in a great deal.

THE SYSTEM IS DYNAMIC. A city is evolving and changing all the time. It does not stay in one state, or develop towards a state. The development of an urban agglomeration reflects the interaction of many agents, and the adaptation of the agents to the current state of the system. Since the adaptation of the agents is an iterative process and often not perfectly rational, the system never absolutely stabilises to one state.

Dynamic systems can also be characterized by an oscillatory behaviour, where the system of interest experiences more ordered states followed by near chaotic states. A good example for this are traffic jams. They appear when certain conditions are met, but also disappear again. The exact time and location of traffic jams is almost unpredictable.

THE SYSTEM'S BEHAVIOUR IS NON-LINEAR. A common perception of system behaviour is proportionality cause and effect; large stimuli cause large effects, whereas small stimuli cause only small effects. This is only true, when the system of interest is linear. This property is often observed in systems that are close to an equilibrium state. However, in systems that are far from equilibrium and dynamic, this assumption has to be dropped. Far-from-equilibrium states often result in surprising patterns.

Neoclassical economics, systems are regarded linear. It is assumed that there is one equilibrium state to which the systems always develops towards, independent of the initial condition. The left image of figure 4 depicts a linear function, with only one minimum. The layperson can interpret the figure in that sense, that he imagines that a small marble is put on the plain (initial condition) and gravity pulls it to the front corner where it will not further move (equilibrium state). The marble will always end up at the same location, wherever it is put at the beginning or pushed from its equilibrium state.



Figure 4: Illustration of linear and non-linear functions. Left is a linear function with only one minimum in the limited area, right a non-linear function with multiple local minima.

Non-linear dynamics are different. When a marble is put on the right image in figure 4, it does not always end up at the same location. Additionally, when it is just slightly pushed, or close to a ridge, it can end at a completely different state. The same holds true, if the system is close to equilibrium, it can not be assumed that a system ends in a similar state that it is pushed away from.

For cities, the "landscape" in which it develops is not fixed, also the "landscape" changes over time. Factors that change the system can be technological changes, but also political, economic, or social shifts.

THE SYSTEM HAS NO CENTRAL CONTROLLER. In neoclassical economics, the presence of an "invisible hand" or a central controller is often implied. The theory assumes that the market regulates itself, as if a higher authority would steer the system. This thought goes back to Adam Smith [35] and was adopted by many economists [36]. This is definitely not the case for cities, and also a limitation in neoclassical economics [1, 36]. Most systems are the product of decisions and actions of individual agents. Cities only experience occasional actions from a central controller. The main force that defines the state of a city is the inhabitants that live in it and adapt to the environment they are in and interact with.

An example of the *self-organization* process of city inhabitants is when congested roads are widened. This often does not lead to less congestion. Travelers normally try to avoid congestion and some choose different roads or times to travel. By widening the congested street, the incentive to change the strategy and use the street also in peak hours changes. After some time the widened streets suffer from the same amount of congestion as before [17]. The central controller decided to widen the street, but the inhabitants of the city can adapt to changes in the environment. They change their behaviour to the new circumstances and the implications of the intervention might be different to the initial aim.

2.2.2 Investigating Complex Urban Systems

The investigation of complex urban systems demands for different analysis methods, compared to the old ones. In the last years, two main directions have emerged in complexity research: complexity theories and complexity models [21].

Complexity theories explore the dynamics and properties of complex systems.

Modern complexity theories often use big data sets [37] as a starting point or to validate the theories, but it should not be mistaken with Big Data analytics. Big Data analytics gained a lot of attention in the recent years because it first helps to cope with the increasing amount of data that is produced every year, and secondly it has the capabilities to reveal patterns in data sets that are not obvious. It also allows to process large data streams and offers opportunities to increase the decision making process [38].

Complexity theories, in contrast to Big Data analytics, focuses on describing dynamics and properties of complex systems. It is close to Big Data analytics in the sense that it often uses similar types of data as a basis and uses information that is produced by Big Data analytics. However, instead on focusing on data processing and pattern recognition, complexity theorists investigate the possible causes of the emerging patterns or describe laws that are present in the system and the data. McCahill et al. [39], for example, use census data and aerial photographs to investigate the impact of parking provision on the use of cars per person. They show that parking provision is the main cause for car usage by applying a method normally used in epidemiology. Another example is the seminal paper by Bettencourt et al. [40]. They define a general, universal law that is valid for cities, independently of their cultural or historical background. The data sets available for the investigated cities were versatile and covered data sets ranging from wages to crime rate. All of the considered features of the cities follow scaling laws. This means that by doubling the size of a city, for example, the crime rate increases super-linearly. They are also able to provide a sound theory that provides an explanation of what causes these scaling properties of cities. Big Data analysis traditionally does not provide possible explanations for emergent patterns. Although Big Data analytics focuses on the algorithms and patterns, it does not aim to develop the theories which explain the causes of such patterns.

This thesis focuses on the second direction of complexity science and investigates complexity models. Complexity models explore the phenomena and properties that characterize complex systems. Even though they are simplifications of the real world, they test hypothesises and theories of complex and complicated systems, without interfering with reality. The researcher has the possibilities to investigate the changes and adaptations of individuals. It would hardly be possible to study the effects on the behaviour of people by randomly closing streets in a city, but these explorations are possible by using the above models. Of course, models are only of value if the dynamics they implement are based on well founded theories or good data about the system of interest [41].

The most prominent classes of complexity models of cities are Agent-Based Modelling (ABM) and Cellular Automata (CA). Both belong to the class of micro-scale models, in the sense that the modeler only has to implement the behaviour of small scale elements. No assumptions have to be made about the overall or aggregated behaviour of the individual entities of the system. To study the state of the system, the agents iteratively interact with each other and the environment, and adapt their behaviour accordingly. The patterns that emerge from the aggregated behaviour and interaction of the individuals can be studied to verify theories from complexity science. In contrast to the equation based approaches from neoclassical economics, the system is analysed by aggregating the emergent behaviour, and not by describing a behaviour that leads to a certain pattern. This is why ABM and CA are considered bottom-up modelling approaches².

The rest of this section describes the three most prominent modelling approaches in urban complexity sciences in the context of urban planning. First I will discuss Equilibrium Models (EQs), how they still can be applied in a complexity setting and under which conditions. Then CA are introduced and the context they are best used in. The third paragraph will discuss the weaknesses and strengths of ABMs.

At the end of this section, I will introduce a fourth modelling concept that has recently been suggested by Arthur [42] and which was selected as the basis for the modelling approach used in this thesis: *meso-scale modelling*.

EQUILIBRIUM MODELS It might sound surprising to those working outside of complexity science that EQs³ are used to simulate urban

Generally the equation based models exhibit phases of near chaotic behaviour and more steady states in their development process, but stay within the vicinity of the

² Some scientist do not agree that CA are pure bottom-up approaches, e.g. Portugali [21].

³ EQs are a special type of equation based models, which often never reach an equilibrium state, but develop towards a steady state around which they fluctuate. They do not develop to a fixed equilibrium state, but to a fixed set of states that that alternate constantly in the same order. Equation based models of complex systems have their roots in synergetics, introduced by Haken [43]. They are generally a set of differential equations that describe the system, and theoretically compute any future state of the system, when the current state and all its parameters are known.

systems also with a complexity perspective, even though complexity models drop by definition the equilibrium assumption. EQs assume that the model aims to reach a certain state that is in equilibrium and not changes anymore.

The changes in cities occur at different speeds. For example infrastructure changes very slowly, under normal circumstances. Whereas local traffic changes its state very quickly. The variation of how fast things change in cities can be used to apply EQ to make predictions. When fast changes are of interest, it can be assumed that the slow changing properties are fixed and define the environment in which the fast changing properties develop [44]. It is assumed that fast changes find an equilibrium state within the time frame given by the slow changes.

It is also possible to simulate slow change with EQs, but only if the fast changes are not modelled explicitly in the model framework or by the use of concepts from synergetics [43]. The outcome of this long-term predictions is then a single state to which the system develops, without explicitly defining the time horizon of when this state might be reached [44]. The results of these simulations should be interpreted as a direction to which the system would develop to, assuming that no externalities occur.

This property also makes EQ superior to other approaches, in the context of interpreting results. EQ, by definition, search for an equilibrium state starting from the initial parameters. The system does not change anymore, as soon as it reaches this state. This property makes the interpretation of the results more intuitive, especially when long-term predictions are calculated. It is not clear with ABM or CA when to stop the dynamic process of the simulation.

For short-term predictions, ABM and CA directly reflect a development in time, e.g. one simulation iteration reflects one year of development, and the dynamic nature of them does not influence the process of result interpretation.

A main advantage of EQs, compared to ABM and CA, is its speed in finding a solution and the stable solution within its equilibrium. The short response time of EQ allow to directly use them within planning discussions, and can facilitate the decision process [45]. Changing parameters in the model can immediately be simulated and give an immediate response. Thus directly and efficiently answering "what-if" type of questions within the given assumptions.

An extended discussion about simulating fast and slow changes of cities with equilibrium models, as well as an example of a framework can be found in the book "The New Science of Cities" by Batty [14].

targeted steady state. Since equation based models exhibit similar properties as EQs in the context of this research work, only with more dynamic solutions, I will not differentiate between the two in the rest of this thesis, and use EQs and equation based models as synonyms.

CELLULAR AUTOMATA CA are "... discrete spatio-temporal dynamic systems based on local rules" [46]. They sub-divide the spatial realm of the system of interest into equally sized cells, where each of the cells have a certain state. By letting the system computationally evolve, the cells can change their state, following terse, predefined rules. In the simplest case, the state of the cells is strictly dependent on the previous states of the direct neighbor cells. If we assume that cell in a one-dimensional realm can only have two states, then with only eight transition rules, the whole framework is defined. This value changes to 256 rules, if the system is two dimensional and all eight direct neighbors of a cell define the next state of the current cell.

Cellular Automata might be the simplest framework that can demonstrate complex behaviour [41]. CA makes it possible to have extremely complex behaviour of the system and demonstrate emergence; all following very simple rules.

These strict, but simple rules were often relaxed. This allows, for example, to take Tobler's first law of geography into account: "*everything is related to everything else, but near things are more related than distant things.*" [47]. Also the number of states is often increased in more recent CA models, that allows for example to not only have the states of developed and undeveloped cells, but also gives the possibility to simulate different kinds of land-uses, e. g. see Clarke [41]. Other relaxations of the transition rules include more probabilistic and stochastic properties, such as the transition potential of a single grid cell. For a thorough review of different relaxations and a classification of them, see Santé et al. [48].

The nature of CA, with the sub-division of space, makes them intuitively attractive to use for spatial simulation; the regular lattice of CA can be directly mapped onto a raster of geographical systems. They are the simplest urban development framework, because they entirely merge the population with the environment [14]. Additionally, spatial CA often exhibit self-similarity, a property often found in cities and large urban regions. Self-similarity describes the property, that similar patterns are found in a system in many scales. For example, when looking at modern cities as a whole, it often consists of one large center and a few sub-centers, a similar pattern that is also described by the Central Place theory, further elaborated in section 2.1.2. This pattern is often also observed when only a smaller part of the city is looked at, and repeats itself at different scales.

The number of transition rules increases dramatically by expanding the regions of interaction of each cell, and increasing the number of states. The only way to test these more complex CA with real world cases is by calibrating the models using historic data about the development of the urban area of interest. The emerging transition rules are often not easy to link to a theoretical background, which makes it hard to understand the causalities within these types of urban simulations [21].

AGENT-BASED MODELLING ABM are a computational modelling concept where the interactions between, and the behaviour of many individuals is modelled. In contrast to standard EQs, which often are described as a set of equations and model the system as a whole, ABMs focus on single individuals and their behaviour in the environment. The atomic agents live in the simulation environment and try to fulfill certain goals, e.g. they try to optimize their utility from an economic perspective.

This modelling type mimics the actions and interactions of single agents to predict the overall behaviour and patterns of the system. Agents can be of different abstraction levels. They can be single people, cars, or aggregations of those, but also larger scale quantities, such as countries. The main idea behind ABMs is that there are many individual agents that interact with each other and are part of a bigger environment. These agents (i) have to be specified at specific model scales (granularity) and types; (ii) consist of decision-making heuristics; (iii) may have the ability to learn or have behavioural rules implemented; (iv) consist of procedures for engagement, e.g. walking; and (v) exist in an environment that also interacts with them [41].

Some of these properties are also part of the key features of complex systems (see section 2.2.1). ABMs inarguably already implement many of the properties of complex systems by nature. But more importantly, they have the potential to implement most of the key features complex systems have that are not already included by definition. The bottom-up perspective of ABMs allows to test the impact of changes of individual behaviour on the whole systems aggregated behaviour, without making large scale assumptions.

Although ABMs offer several advantages, they also have their disadvantages. Often, modellers tend to build a model that tries to serve "too many purposes at the same time – too many variables and too much detail. Adding more components to a system generates the illusion of refinements added and uncertainties eliminated, but every component adds less that is known than not know." [8]. This statement points out an important fact about ABMs, they give the modeller many degrees of freedom to model a system, that it easily can happen that the model tries to implement too many things, especially in the context of urban modelling.

To calibrate urban ABMs numerous data sources have been used, where surveys and census data are the most prominent ones. Large scale urban ABMs need a lot of data to be calibrated and validated. This is one of the main critiques on ABMs. The Calibration and validation process of such models is rather difficult, if not impossible [41].

MESO-SCALE MODELLING The use of the above described modelling techniques had a profound impact on our understanding of human systems. EQs offered a way to formulate urban systems as a whole and investigate the micro-level behaviour that produces the emergent patterns and thus is a study of the system as a whole. The equilibrium properties are an elegant way to make the models more analysable in a mathematical context [1].

Agent-based Models and Cellular Automata, together with the new tools given by decreasing computational costs, changed the perception of urban systems. They made it possible to study the reaction of individuals to the patterns they created from their behaviour.

CA are best to investigate spatially distributed processes, such as spread and dispersal, when geometry, scale and the basic behaviour of the system is known. ABM require fewer assumptions on the system's behaviour and geometry, and also investigate the behaviour of individuals and their interaction among each other and the environment [41]. Certain approaches also exist to combine the two, using the strength of CAs in regional interaction, with the strengths of ABMs with weaker restrictions on regional interactions [49].

The three modelling techniques capture effects on either the microor the macro-scale. With the equilibrium assumption, it is assumed that the system orders itself given a certain amount of time, for example, traffic stabilizes at some speed. In the micro-scale, when looking at individuals and how they react to the system, the strong equilibrium assumption is implicitly dropped. The system does not aim to find an equilibrium state and stays dynamic. But with the micro-scale view, only the behaviour of single individuals is reflected and their behaviour has to be aggregated to depict the overall flow of traffic, for example, and how jams travel through the traffic network.

These interesting effects are neither directly covered by macro-scale nor by micro-scale models. They happen in a scale between, that is, on the meso-level.

In complexity economics, the meso-level is a step away from neoclassical economics, dropping the equilibrium assumption. Arthur [1] points out that in fact neoclassical economics is a special case of nonequilibrium economics, i.e. complexity economics. The meso-scale does not model single agents and their behaviour in the system, it models groups of agents in a probabilistic and deterministic way. Deterministic in the sense that it normally has the same sequence of systems states for the same initial conditions. This property, for example, is not given for standard discrete-choice models that ABMs often implement.

In discrete choice models in ABM modelling, e.g. FaLC [50] a commercial integrated land-use and transport model, agents calculate the utilities from all possible choices and from their past decisions, and then randomly choose one of the options weighted by the utility value. A single result from the this type of ABMs is not very significant, becuase fo the randomness, and it shows only one state from many possibilities. To circumvent this problem, ABMs are run many times to get probability distributions of the outcomes.

In meso-scale modelling, this is different. A land-use model in meso-scale directly works with the weighted utilities. Zünd et al. [51] use the weighted utilities of the location choices directly as probability distributions and move groups of agents directly according the given weights. This makes the resulting patterns of residential and business locations probabilistic and, since no randomness is involved, it becomes deterministic. To get the result of the model for one set of parameters, a meso-scale model has to do only one simulation run.

The aggregate behaviour of individuals reduces the degree of detail. It is no longer possible to observe the individual change of behaviour, only the change of the behaviour of groups.

This makes it also more complicated to implement meso-scale models. Traditional CA and ABM are relatively straightforward, however meso-scale models more complex to implement.

All of the above described modelling techniques have their advantages and disadvantage, and no single model type fits all needs. The technique selected should consider the task and the dynamics one wants to observe and study.

2.3 PLANNING & COMPLEXITY MODELS

This section describes the benefits and challenges of complexity models to support decision making and urban planning practice. The rapidly increasing sizes of modern cities have made decision making a cumbersome task, of which a big part is acting and reacting to avoid large scale inefficiencies [8].

Lee pointed this out about a half a century ago, but this has become even more extreme with modern cities. Understanding the impacts of planning decisions is an almost impossible task with the large amount of actors in modern cities. Lai and Han point out that the uncertainty reduces when as much information as possible is used [32]. However this is a notion that comes from the classical perspective on systems. Complexity theory has a contradictory view on the relation between uncertainty and information. It theorises that, similar to the information theory by Shannon [52], more information increases the uncertainty about (urban) complex systems [21].

In this regard, modelling and simulation should be regarded as tools to explore different outcomes, and not server as pure information generation machines. It provides the planners and the decision makers with ideas of what the implications of certain actions might be and a virtual planning experience [49]. Already neoclassical models have been used in this context. For example, in the Netherlands, they have been used in the decision making process to support and guide planning discussions [16]. Models can be a good tool to explore possible outcomes and to set the boundaries of possible outcomes of an intervention [14]. The boundaries can set the frame by excluding improbable outcomes, thus guide the discussion and decision process.

Self-organization and non-linearity effects are more distinctive, in larger (urban) systems. More individuals are part of the whole system and more possibilities to trigger cascading effects are present, particularly with modern technology and cheap communication. Systems with a high degree of connectivity are prone to failures, because the information can spread easily through the communication network and this can lead to an devaluation of the system and the utility of the individuals [53]. Small actions can have large impacts in complex systems. The self-immolation of a street vendor sparked big protests in Tunisia, and eventually led to a destabilisation of the Arab World [54].

Complex systems are by definition not controllable. Being able to control a city is only possible with high cost and lots of energy, if not impossible. A city cannot be steered like a car, which turns left if the steering wheel is turned in that direction. It is almost impossible to exactly predict the development process of cities, and what impact interventions might have. The only possibility to have certain amount of control over a complex urban system, is by guiding it. Biological systems are good examples for this. It is not possible to store all information and construction plans in each cell. Biology makes use of the self-organization and guides the process, without forcing control on it. Forceful control would destroy the organism [11, 36].

To be able to guide the self-organization process in cities, it is important to have a good understanding of the dynamics within the city. It is crucial to understand the impacts of decisions, and have reliable models that are able to identify the range of possible outcomes and development directions. The crucial part of complexity models is that they model the smallest reasonable entity in the system, and the interaction between them. It is more simple to train the single entities and observe the interaction among them, then building a theory of the whole system.

The following subsections discuss different properties that models should have to support urban planning practice and to provide the possibility to obtain virtual planning experience. First I elaborate on the level of detail of models and at which stage in the planning process they are most applicable. The second subsection discusses the solution process of complexity models with non-linear dynamics. The last subsection discusses the time scales in which models work, i. e. in short or long-term dynamics.

2.3.1 Consolidation vs. Exploration

Early models of urban systems were criticised to be either too 'simple' or not 'simple' enough [8, 55]. This led to a long period of reflection, extension, and rework of the existing models and their structure to make models more applicable and relevant for policy making [44]. From this process, two main modelling styles emerged, comprehensive modelling and exploratory modelling.

Supported by the decreasing computational costs, it was possible to disaggregate the early simple models into many sub-systems, and to add new sectors at a higher level of differentiation of activities; even down to the level of individual people. It is a natural process, to make models more comprehensive, especially if their goal is to support urban planning. Decision makers and urban planners demand for a level of detail that up until now only these large-scale models can provide. Large-scale in the current context means that, even though a model may consist of many small micro-models, it is complicated to set up, computationally expensive to run, and often, whole teams of analysts have to work on it along with all the sub-models. In many cases it would not be possible for an individual to maintain and set up the model [14].

In contrast to comprehensive models, exploratory models simplify the scope compared to neoclassical models. The focus of these models is not to describe the urban system thoroughly but to describe singular effects.

Due to their simplicity, exploratory tools are easy to set up and have normally a low computational cost. Thus they have a fast response time. The power of exploratory models is within the exploratory capabilities. They allow two ways of interaction with the model. The first, more often used scenario is that they allow to directly interact with them during discussions. Ideas can directly be implemented in the model and help to guide and inform the discussion real-time. The direct interaction with the model allows to formulate "what-if" type of questions, and to get a direct feedback [14]. This process can be described as sketching. Sketching in the context of arts is a process that is generally not a precise description of a solution, it is a method that allows to explore the general concept behind ideas and gives a first impression on how the result might look. The usage of simple, nonprecise models exhibit exactly this property and provide the planner with rough insights into what the consequences of certain interventions on a specific property might be.

The second way of using exploratory models, is to generate many simulations with different parameters. The resulting set of simulation results allows then to cluster them and to (i) evaluate the model, and (ii) to develop different scenarios to guide the urban system from the current state to a desired future state. This is the approach this thesis will later explore and evaluate. This modelling approach supports the solution process and helps to answer "how-to" type of questions. This way of exploring the possible states of a system belongs to the family of Monte Carlo (MC) simulations, introduced by Fermi, Ulam, and von Neumann [56].

Both large-scale and exploration models can help to reduce uncertainty in urban planning and to obtain virtual planning experience, but at different stages. In mechanical engineering, models are used in every development step. This is generally not the case in urban planning. In an early stage of the design process, simple models are used in mechanical engineering to explore many possible solutions and to get a rough estimation of their properties. The more the design process develops, the more complex and computationally expensive the models become [57]. The different models used to, for example, simulate the airflow around aircrafts are all based on the same theory and follow the same basic inputs.

This is hardly possible in the simulation of cities. Most of the introduced modelling techniques (see section 2.2.2) are very different from each other. EQ is a macro-model that describes equilibrium behaviour to reach a certain pattern; CA simulates the changes of land-use in a grid, depending on the states of the neighbors; and ABM simulates the interaction of many individuals and is often a comprehensive model. To allow similar behaviour, but with lower computational cost, the meso-scale model of Zünd et al. [51] can be based on similar data as traditional ABM land-use models, and the implementation of the different "agents" is also close to implementations of ABMs. The mesoscale models, together with an ABM, can first sketch the different possibilities of the future development of a city. After this first rough exploration a more detailed model can be used, since the basic direction of future development and goals are set by the first stage. With the combination of the two model types, this is all done on the basis of the same data sets and very similar implementations from a modelers point of view.

2.3.2 "Natural" Development Process

One of the main properties of complex systems is that they exhibit non-linear dynamics. Going back to the examples depicted in figure 4, complex urban systems develop within a landscape that is similar to the right image, but with many more dimensions.

Imagine that a drop of water is poured at one location of the hilly landscape. It will flow down the hill, until it stops at the bottom of the valley to where it is funneled. The process of flowing to this location is what I mean by "natural" development process. It describes the development process of the system towards its most natural stable state. Figure 5 depicts such a process. The water is dropped at the initial condition and develops along the red line, until it stabilizes at the bottom of the valley. The stabilization means that it does not further develop anymore and is at an equilibrium state.

To find the most "natural valley" is a well known problem in optimisation to find a local optimum. Non-convex optimization does cope with problems that are non-linear and whose problem landscapes may look similar to the one depicted in figure 5. Often methods in optimization calculate the direction of steepest descent of the function to optimize. They step some distance into this direction and reevaluate the steepest descent at the new location. If the problem landscape meets some requirements, it is possible to find the local minimum for any initial condition.

The development process of a complex urban systems can be described in the same perspective as the drop of water above. The drop of water moves according to the forces that act on it, which is only gravity and friction, and which I neglect in this example. With gravity being the only force that pulls on the drop, it moves into the direction in which the gravitational force is strongest. It is the direction of steepest descent, or in other words the direction of least resistance. This behaviour is often observed in natural systems. They develop into the direction of least resistance or the direction of maximal profits. Self-organising systems are no exception. The individuals adapt their behaviour as long as their maximum utility is not met. They adapt according to the knowledge they gained in the past and what behaviour looks most promising in their current state of knowledge. Following the concept of computational equivalence [58], concepts that can be observed in one system are often also present in other systems. Thus one principle assumption I make in this thesis is that complex urban systems develop into the direction of least resistance, given by the circumstances and the initial state of the given city.

Simulations are never continuous processes, they always calculate the current state of the system and the direction of the next step it makes; then it takes a step of certain size into this direction and redoes the process iteratively. The step size is a crucial part of the development process because it decides how far the system develops into one direction without reevaluation. Depending on the landscape the system develops and the step size, it can happen that the funnel of the next iteration step is a different one than the initial condition was in and the stable state it would now develop to is a different one. Therefore the time step has to be chosen very carefully!

But this property also allows for exploration of the states of the model and to find more stable and resilient states. Self-annealing techniques first have a big step size, and slowly decrease it over time. In optimisation, annealing methods are a probabilistic technique to find the global optima [59]. It takes the assumption from thermodynamics, that when a system has high energy first and is cooled down slowly,

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Figure 5: The development process of a drop of water from its initial condition. It flows down the hill until it stops at the bottom of a funnel. This process is depicted by the red line, which shows the path the drop of water would take.

it finds a more structured (stable) state than if it would be cooled down immediately. In the context of urban simulation, this allows to find possible stable and resilient states that are possible to reach with the current parameter set.

The "natural" state from an initial condition and the more resilient state found by the annealing technique are most probably in different funnels in the problem landscape. By analysing the funnels of the two solutions, it is possible to find ways the system has to be guided to develop into a desired direction. This role of investing energy to surmount the hills between the two funnels can be the role of a central controller. With this analysis, the central authority can find out the actions that have to be taken to change the current setting into a more preferred one.

2.3.3 Time Scale

Early models assumed that urban systems are equilibrium seeking and always developing towards an equilibrium state if they are pushed away from it. If the initial equilibrium was no longer reachable, for example due to drastic changes in the environment, it was assumed that only one new equilibrium state was targeted. One of the main reasons that this strong assumption was made, is that urban systems undergo changes in different time scales. The most visible ones are slow changes, the change in infrastructure. Infrastructure changes very slowly and many cities show similar infrastructure patterns. This led to the believe that such clear evidence of equilibrium should be the main focus in urban modelling and simulation [44]. But there are more rapid changes. Some change the usage of the infrastructure and the behaviour of actors in the city, and are not immediately visible. The changes cities experience are subdivided into three classes by Wegener et al. [60]: slow, medium speed, and fast processes. Wegener et al. describe the slow changes as the building of the infrastructure, as mentioned above. The medium speed processes are demographic changes, influencing the households and the overall population; economic changes, such as the employment status of residents; and technological changes, such as transport network equipment. The fastest changes in urban systems summarize to mobility: the usage of the infrastructure for housing or working, or traffic.

Modern large-scale models cope with the multiple time scale mostly in two ways. They either handle the slow changes as external factors, that have to be input between two simulation iterations [50], or they run the models in different stages. In these stages, first a stable state is developed for fast changes by assuming that the slower changes are fixed. Only after the fast changes have reached a stable state, the slower changes are simulated [61].

Exploratory models traditionally only focus on a single time scale. Generally speaking, the goal is to investigate only one effect or property, not to model the whole urban system comprehensively. Traditionally, CA model slow changing dynamics of cities. For example the CLUE-S model uses CA to simulate the land-use development of an island in the Philippines, without taking fast changes into account [62]. Batty uses urban EQs to simulate the first and second location of residents and the modes of transport they use. He makes the assumption that slower changes are fixed and undergo no development process. The model assumes that the fast changes will find an equilibrium state within the current state of the environment [44].

This way of thinking fits well together with the "natural" development described in section 2.3.2. Batty assumes that the problem landscape (i. e. the environment and infrastructure) does not change and that the fast changes develop to a local equilibrium. This is actually also how the RELU-TRAN model works for the fast dynamics. It fixes the environment and other slow changes and then finds a local equilibrium from the previous state, by also using local-equilibrium algorithms from convex optimization [61]. Then the slower changes are developed for one time step, which is equal to changing the problem landscape, and the fast changes find a new local equilibrium in the new setting.

Except for the above described time properties, modellers develop two types of models, the first are models that follow a time line, e.g. each iteration reflects a year of development. Other models do not directly reflect a development within a certain time frame. The idea behind these models is to find states of the urban system toward which it develops. This property is often present in simpler models, because it drops many assumptions about governing and other external factors. The first type of models are often large scale models that are directly used for short term planning and are of a very high level detail (comprehensive models), e. g. UrbanSim [63] or FaLC [50].

The model-framework developed in this thesis is of the latter and its results reflect the direction to which a city would develop toward, starting from an initial condition. In its general version, it does not assume anything about the infrastructure. Although it is comprised of a "natural" development process, it never finds an equilibrium state; only if the annealing mechanisms are used. This is the case, because the general version of the model does not assume anything about the infrastructure and develops it on the fly when the simulation is running. The constant change of basic infrastructure changes the problem landscape while the fast changes are developing.

2.4 SUMMARY

Urban planning is a task with many uncertainties involved. It is crucial to understand the impacts of a decision and subsequent consequences.

One way to decrease the uncertainties involved in the planning processes and increase planning experience is to use modelling and simulations. It can be used to guide and support discussions, and to explore possible impacts of interventions to a complex urban system. Additionally they provide the opportunity to gain planning experience in an artificial setting, which can be important, and can otherwise not easily be obtained in a feasible period of time.

Early models developed in the field of urban economics were quickly used for large scale planning scenarios in certain countries. They helped, for example, to guide the process of laying out and estimating the sizes of villages and a city on a polder in the Netherlands. Even though the early spatial and urban economic models make very strong assumptions on the environment and the system itself, it was possible to use them, because the properties of the polder were closely approximated by the model assumptions.

Generally, it is more difficult to apply neoclassical urban economics models in planning discussions than in the example above. This is due to the difference between the model assumptions and the given reality. In the example above, the polder exhibits many properties that are assumed by the model. Additionally, planners often demand a level of detail that is hardly possible to depict in the macroscopic view of neoclassical urban economic models.

With the decreasing computing costs in the last decades, it has become possible to model and simulate urban systems from a new perspective. It is no longer required that models must be analytically solvable to provide insights into the interplay of different dynamics and how they impact each other. This novel perspective also provides greater flexibility to drop most of the strong assumptions neoclassical models make. One of the key contributions of complexity science to modelling of urban systems is that it is now possible to focus on different smaller entities in the urban realm and the interactions between them, rather than having a systems view on the whole urban system. The focus is no longer to find a sound theory that explains the emergence of patterns, but is now a study of individual behaviour and how certain patterns emerge.

This perspective has led to comprehensive models, which heavily rely on training data. This is an advantage, on the one hand, because the behaviour of the individual entities relies on real world data. On the other hand it is almost impossible to get data with good quality for all implemented properties, that also reflects similar points in time and location and are not biased by preprocessing. The interpretation of large scale models must often be done with care. Comprehensiveness generally involves many parameters and it is often not clear what parameter leads to what outcome.

This is one of the main critiques large-scale models face, especially if they are used to predict future development of urban systems. Another critique of large scale models is that they are too detailed, especially for certain planning scenarios. Often in planning practice, it is not of interest what, for example, the behaviour of a single agent is. Especially when the planners and decision makers are interested in the development of land use in different parts of an urban system. It is unprofitable to have a higher level of detail then the task asks for. Higher levels of detail generally introduce more uncertainties and unknowns, but pretend a reduction of them. The uncertainties propagate also into the result, when they are aggregated. Aggregation of the results happens when the focus is on a different scale than initially intended for the model.

This thesis proposes a modelling framework that works on the meso-scale. It is the scale that is often missing, particularly if macro-scale models are not capable to reflect the desired dynamics, and when large-scale micro-models are too detailed or too expensive.

Part II

METHODOLOGY

This chapter describes the different elements of the framework. First, section 3.1 introduces the core of the framework comprehensively and describes the technology used to implement it.¹ Section 3.2 introduces an interface that makes it possible to ask the meso-scale model "whatif" type of questions, by sketching possible future states of the urban system, and allows to make many additional analyses on the results. It can serve as a support tool in decision finding, help to resolve conflicts between opinions that differ from one another [10], and provides the possibility to obtain virtual planning experience. Section 3.3 introduces the use of MC simulations to help the urban planners and decision makers in finding solutions and possible actions to guide the urban system to a certain state. It describes the methodology that can be used to answer "how-to" type of questions.

NOTATION Lower case letters indicate scalars. Lower case letters in bold indicate vectors and capital letters are matrices. Subscripts indicate sub-elements of the corresponding vector or matrix, if not noted otherwise. Averages are indicated with an overbar, e.g. \bar{w} means the average income per person. The hat indicates the sum of a property acting on an area, e.g. \hat{p}_i is the total number of residents working in sub-area i.

3.1 MESO-SCALE MODEL

This section describes the core of the framework I propose: the mesoscale model. The meso-scale is a scale that allows to investigate urban development in the large scale, without the computational overhead of traditional large-scale models. It has a high enough level of detail to reflect dynamics that can not be depicted in macro-scale and equilibrium simulations; such as self-organisation dynamics, feedback loops and individual interaction between the entities. The main drawback of meso-scale modelling is the loss of detail. Experience from different workshop shows that it is better to have a model that directly works in the scale that is in focus. A resolution that is much higher than the scale of interest confuse more then it helps in the understanding and distracts participants from the main questions, without adding additional value.

¹ Section 3.1 is a revised and updated version of a section of a paper that has been published in *Simulation*, 2016, Vol. 92.3, pp. 295–306 [51].



Figure 6: Simple circular economic flow.

The model simulates the spatial interaction of residential locations and business locations. It comprises two parts (or *sub-models*): an economic sub-model, instances of which are distributed in space by a spatial distance sub-model. The interaction is modelled through groups of households who base their preferences and decisions on the two sub-models. In each iteration, households evaluate their current utility on different properties of the two sub-models. Households serve as an abstraction for different resident types, which can be single person households, but also multi-people households. In the here described modelling approach, a households stands for one unit that needs an own space of residence.

The economic sub-model follows the concept of the simple circular economic flow for two sectors [64] depicted in figure 6. The two sectors are households and firms. The households spend all their money on consumption (*Goods*), which becomes the income of the firms (*Spending*). Since households work at these firms (*Workforce*), the income of the firms is distributed again to the households (*Income*) and thus closes the circular flow of income.

To simulate the evolution of business centers and dwelling areas, the spatial distance sub-model records the pairwise effective distances among locations in a city. The area of interest is divided into different zones. Within each zone I presume that households and workplaces are homogeneously distributed, and that a single point at the centroid of the zone models the entire zone. Figure 7 shows an example of an area divided into eight different sub-areas. From this division of the urban space, I construct a simple travel network with which the spatial distance sub-model works (I use the term "travel network" to refer to all modes of transit between nodes). Each of the nodes in the right panel of figure 7 represents one sub-area of the simulation model and holds one instance of the economic sub-model. The nodes all contain an instance of the circular flow model which are linked through the network and can also interact with each other through the travel network. This means that all the sub-areas can contain households and firms, which interact locally with each other, but households of all areas also interact with firms at all locations, and vice-versa.



Figure 7: Example of an area. The left panel shows the area divided into grid cells where the grid cells with a bullet are of interest. The right panel shows the extracted simple travel network.

The interaction between residential and business locations works through households, who have a location of residence and a location where they work; *primary* and *secondary locations*, respectively. During the iterative process of running the simulation the households evaluate the current situation in the whole area, using the two submodels. Following this evaluation, they change their primary and secondary location accordingly. Thus different business and dwelling areas evolve and disappear in the simulated area over time. I do not model firms explicitly, for the current model I assume that companies are where people work and are therefore given implicitly by the distribution of the households' work places.

The different households are not treated as atomic entities, but rather in groups of households. The location choices are calculated in a heuristic manner for these groups, and the households redistributed accordingly. This modelling decision reflects a maximum likelihood integration scheme that works with probabilities. To get the same outcome in an ABM, multiple runs would be needed if we used a discrete location choice scheme.

I chose the simple circular economic flow and not a more sophisticated model because it provides the most basic ideas about spending and income of households, and is the simplest model that represents the ideas of urban economics. This makes the simulation very traceable and allows influences of the parameters to be reliably and clearly analysed. More importantly, the simplest possible model allows to evaluate the feasibility of the approach: good predictions with the minimal model will likely be due to the modeling approach, not the specific sub-models used.

3.1.1 Spending behaviour

In the following, I describe how money spending of households within the urban realm is modelled. I use the concept of potentials, known from particle simulation for continuous materials [65], to model the



Figure 8: Attractiveness of three sub-areas (c_1, c_2, c_3) of different sizes to a one dimensional realm. x is a possible residential location. The location of the sub-areas is indicated by the dashed lines. The thicker red curve is the sum of all the attractiveness curves.

strength of the attractiveness between subareas. I model attractiveness as a Gaussian probability density function with a standard deviation σ placed at the center of each sub-area. The attractiveness Φ of one region to a region with a distance d to it is described as what is called the *kernel function*:

$$\Phi_{\sigma}(d) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{d^2}{2\sigma^2}}$$
(1)

 σ expresses the sensitivity of households to distance travelled. A low value of σ expresses high sensitivity to travel distance. The value of the sensitivity describes a probability that expresses the willingness of people to spend money at distance. σ says that if the area is homogeneous, about two thirds of the trips are within that value. The trips define where the households spend money. The density function is scaled according to the size of the firms in this sub-area, i. e. by the number of households working in this sub-area. Thus the density function defines the strength of a sub-area to attract money. This allows to mimic agglomeration economics. A one dimensional example is depicted in figure 8. It shows the strength to attract money from a location x to three different sub-areas (c_1 , c_2 , c_3) and the overall site.

Adding up the values of the individual density functions with respect to one location results in the total influences at this location. The individual influences of the different areas can then be expressed as a fraction of the total amount. In figure 8 this is the ratio between a companies attractiveness to the value of the top curve (red), e. g. the ratio of the value of the curve of company 2 (green) and the red curve at location x. I use these fractions to describe the amount of the total money households in the current sub-area spend at which locations. The flow of money $f_{j,i}$ from sub-area j to sub-area i is thus defined as:

$$f_{j,i} = m_j \frac{s_i \Phi(d_{j,i})}{\sum_{k=1}^{N} s_k \Phi(d_{j,k})}$$
(2)

Where s_i is the number of households working in area i, N the total number of sub-areas, m_j the total available money in area j, d the distance between the sub-areas and Φ the density function.

I chose to use potential modelling techniques for the interaction between the different sub-areas, because I think it is superior to traditional gravity models as used by, e. g. Wilson [66]. The kernel function Φ can be adapted, and thus there is a higher potential for configuration space explorations, that is, through model variants. In the described case, some additional experiments with a hat function

 $\wedge_{j,i} = \max(\mathfrak{m}_j - |\sigma d_{j,i}|, 0) \tag{3}$

as a density function were executed. But \land as a kernel function provided no additional value, on the contrary. The main drawback of the hat function is that it is finite. This means that, outside the radius of interaction, households have no interaction with a possibly large business center. This is not a desired property, because it would strictly limit the radius of interaction. The Gaussian density does not have this property since it has an infinite radius of interaction.

Figure 9 depicts an example of the spending heuristics described above with Φ as the kernel function. The grid cell containing the bullet point is the current sub-area of interest. The colors define the size of the fraction the other sub-areas attract from that area.

3.1.2 Income

The money earned by a sub-area is distributed among all the households working in this sub-area. Since the households working in one sub-area may live in other sub-areas, this ensures that the money flows back to the households who will spend it again in the next iteration.

In the current model I am indifferent among worker types and treat them as one group, only described by their number. Following this concept, I add up the total amount of earned money *w* of one sub-area. This is all the money which was spent in this sub-area but earned from all sub-areas. First I calculate the money flowing to subarea i from sub-area j.

$$\hat{w}_{j,i} = \frac{1}{p_{i,j}} \sum_{k=1}^{N} f_{k,j}$$
(4)



Figure 9: Amount of attraction of the different sub-areas on one sub-area in the north (grid cell containing the bullet point). The visualisation is made with real data of the density of companies in the city of Zürich. Bright red depicts a small value, whereas dark red is a high value. All cells with a value larger than a tenth of the maximum are shown. The city center is just above the lake at the bottom center. The bullet point is close to a sub-center of Zürich. The cell with the highest value (dark red) is at a location, which was mostly industrial and other businesses when the data was collected. $f_{k,j}$ is defined by equation 2. $p_{i,j}$ is the fraction of workers in region j dwelling in region i.

To get the total amount of money one sub-area has as income, I add up all the \hat{w} .

$$w_{i} = \sum_{k=1}^{N} \hat{w}_{k,i} \tag{5}$$

This is the sum of all the money households earn at their workplaces and is equal to the total available money in sub-area i, i. e.

 $m_i = w_i \tag{6}$

3.1.3 Travel Network

To be able to reflect an urban system, using the simple circular economic flow, I introduce a spatial dimension to it. This means, that the interaction between the different actors suffers from transport cost.

The distance measure I use in the current version of the model is a simplified travel network. Households are only allowed to travel through sub-areas when they are occupied by households or companies, an example is depicted in figure 7. This simplification assumes, that streets are only present, where households are present and that empty spaces are placeholders for natural obstacles within the simulated urban area. Within these restrictions, I assume that agents, represented by households, travel with the shortest possible path. They do not suffer from congestion or other external costs.

3.1.4 Location Choice

Location choice decisions for primary and secondary locations – subarea of residence and work, respectively – is decided upon the values of the money flows described in the previous sections, density of a sub-region and the distance between sub-areas.

PRIMARY LOCATION There are two location choices a household has to make, the first is to find a sub-area for residence. The households evaluate sub-areas upon two properties. The first is the ratio ρ_i between the occupancy of a sub-area and the total available space for dwelling and working m_i .

$$\rho_{i} = \frac{r_{i} + a_{i}}{c_{i}} \tag{7}$$

 r_i is the number of households at sub-area i and a_i the number of workplaces. This is an abstract, yet powerful, approach to model rent prices without explicitly taking them into account and adding a level of complexity which is not desired in the current model. This

abstraction of rent prices reflects the basic concepts of supply and demand.

The second property on which households decide their sub-area of residence is the distance $d_{i,j}$ from their current working location at sub-area i to the potential sub-area j for dwelling. The distance is defined as the shortest path in the network described in section 3.1.3.

Bigman and Fofack [67] describe that elasticity increases with the distance a property has from the optimal value. I apply this concept to the distance $d_{i,j}$ and the occupancy ratio ρ_i by calculating the property-wise utility with a negative exponential, i. e., the utility derived from the distance is:

$$u_{d_{i,j}} = e^{-\sigma_d d_{i,j}} \tag{8}$$

and for the occupancy ratio:

$$\mathfrak{u}_{\rho_i} = e^{-\sigma_\rho \rho_i} \tag{9}$$

 σ_d and sigma_{ρ} are scaling parameters that help to reduce the side effects that result from the different metrics of d and ρ , respectively. I combine the two properties on which households base their decisions with a Cobb-Douglas function [68] to get the total utility of one subarea for a resident. I use the Cobb-Douglas function, because of its properties, such as the direct relation of the exponent to the elasticity of the corresponding base, and the elasticity of substitution.

$$U_{i,j}^{R} = u_{d_{i,j}}^{\alpha_{r}} u_{\rho_{i}}^{(1-\alpha_{r})}$$

$$\tag{10}$$

The $U_{i,j}^R$ is the total utility a resident has who works at location j and evaluates to move to location i. α_r is the elasticity of $u_{d_{i,j}}$ relative to u_{ρ_i} and has to fulfill the following property: $0 \leq \alpha_r \leq 1$.

SECONDARY LOCATION The secondary location choice a household has to make is where it will choose to work. This decision depends on two properties. The first is the distance $d_{i,j}$ from their current location of residence i. It is calculated in the same way as in the primary location choice described before.

The other property that has an influence to the secondary location decision is the per-capita income \bar{w}_i of a worker at any sub-area i.

$$\bar{w}_i = \frac{1}{\hat{p}_i} \sum_{k=1}^N f_{k,i} \tag{11}$$

 $f_{k,i}$ are the values from equation 2 and \hat{p}_i is the total number households employed in region i. This implies that households are *eager*; they seek to maximize their income based on current conditions. Similar to the primary location choice, I apply an exponential function to model the increase of elasticity with distance from the optimal value on d and \bar{w} :

$$u_{\hat{w}_i} = e^{\sigma_{\bar{w}}\bar{w}_i} \tag{12}$$

 u_d is calculated in the same way as in equation 8. $\sigma_{\bar{w}}$ is a scaling parameter that removes side effects that result from the different metrics of u_d and \bar{w} . To get the total utility, I again use a Cobb-Douglas function to combine the two utilities:

$$U_{i,j}^{W} = u_{d_{i,j}}^{\alpha_{w}} u_{\bar{w}_{i}}^{(1-\alpha_{w})}$$
(13)

This is the total utility a household has if he works at location j but has his residence at location i. α_w has the same properties as α_r .

3.1.5 Integration Scheme

The background research on equation based models (see chapter 2) shows a pattern of detailed attention to the mathematical rationale and description of models, and to the analysis of results. In comparison, solution processes often receive scant coverage. For example, Krugman has for his famous core-periphery model only a short paragraph on numerically solving in a whole book: *"These equations are easily solved on the computer ..., I simply started with an initial guess at w and then cycled (with some damping) over (15)-(17) until convergence."* [69]. Krugman is actually exceptionally detailed in comparison to many studies: normally nothing is mentioned, excepting perhaps some initial conditions for the sensitivity analysis. Yet, complex non-linear models have complex solution landscapes and are sensitive to both solving processes and initial starting points (see section 2.3 for a more comprehensive discussion).

ABM presents a related problem in that, that solutions may be sensitive to the algorithms used for both agent decisions and combining agent actions. However, they have an intuitive appeal as they appear to simply develop "naturally"– every agent follows its implemented logic and the system's behaviour is simulated by changing each individuals state at every simulation (integration) step.

In my view, there should be more focus on solving, because it can change the outcome of the model dramatically. Figure 10 depicts two different runs of the here described model with exactly the same parameter set, except for the different constant integration step sizes (β_t in equation 15). The images depict the results after only six integration steps and already show a big difference in the results.

When we think of the solution space of a model as a non-convex, higher dimensional landscape in which the solver of an equation system for equilibrium models should find the "nearest" local minimum,



Figure 10: The panels depict two intermediate results of the model of the density distribution of workers plus households in a squared landscape. The model is run with the exact same parameters, but two different integration step sizes of 0.1 and 0.8, left and right panel, respectively. The darker a grid cell is, the higher the density is at this location.

that is, is the local equilibrium to which a system is most likely to develop (see section 2.3.2 for an in-depth discussion). This problem is also very well known in numerical optimization, when a search of the solution space is of interest, e.g., the natural stable configuration of a molecule from an initial state [70]. Especially for models that describe the development process of cities, I think it is crucial that the system evolves in a natural manner and does not arbitrarily jump around chaotically in the solution space. An example of the difference is depicted in figure 10. The small integration step ($\beta_t = 0.1$) lets the system develop slowly into the direction of least resistance, whereas the larger integration step ($\beta_t = 0.8$) allows much more changes between two states. Because of the non-convex nature of the solution landscape, it might end up in a different area of the solution landscape that would naturally evolve from a very different initial configuration. It does thus not reflect the natural development process, but may allow many, very different potential states of the whole system. Clearly, choosing an appropriate integration step is an important choice and one likely linked to anticipated rates of change in a region.

The here described meso-scale model is technically located between agent based models and equilibrium models. Internally I handle the agents as stocks which have to be redistributed, by flowing through a network. For this, I use the calculated utilities to build the matrices J^R and J^W , which are then used for the integration scheme. From now on, I will not differentiate between the two matrices, because the following steps have to be taken for both of them. The only difference in the calculate the matrix J^W directly from the utility matrix of a sub-area, in this step, the households money does not have to be changed, because the households stay residing in the same area. In

the case of J^R the money of the households has to be redistributed. I calculate this matrix not directly by using the utility matrix of a certain sub-area, but take the households that work at a certain area and use the aggregated utilities of them. When the households are then redistributed, the money has to be redistributed as well.

The matrix J can be seen as the Jacobian matrix of a numerical time integration. J has to ensure that the number of households does not change during the time steps. This is ensured by doing a column-wise normalization of the values of U.

$$J_{i,j} = \frac{U_{i,j}}{\|U_{:,j}\|_1}$$
(14)

where $||U_{:,j}||_1$ is the 1-norm of the column j of the utility matrix U. This allows integration in one calculation step, but with the idea of the natural development stepping of agent based models. I use for this an adapted explicit Euler method:

$$\mathbf{d}_{t+1} = \beta_t \mathbf{J} \mathbf{d}_t + (1 - \beta_t) \mathbf{d}_t \tag{15}$$

 d_t is the distribution of either households or workplaces at time t within the simulated area, and β_t the step size at time t, with $\beta_t \in [0, 1]$. Using this method, the system can be solved very efficiently and with a low computational cost.²

LIMITED CAPACITY TIME INTEGRATION The matrix J can also be understood as the households preferences of primary or secondary location. The algorithm I introduce in the following part recursively adapts the matrix J, until all households are distributed in the simulated area and the resulting distribution is within the boundary conditions given by the sizes of the sub-areas. The logic behind the algorithm is that households want to go to their most preferred location, but if there is no capacity at this location, they go to their second most preferred location, and so on.

Let **c** be the vector containing the total available space for each subarea.

$$\mathbf{c} = \begin{pmatrix} c_1 & c_2 & \cdots & c_N \end{pmatrix}^{\mathrm{T}}$$
(16)

² The described integration process, with the introduced model dynamics, of the model framework is very similar to the process of calculating the distribution over states from an initial configuration in Markov chains, but with a transition matrix that changes with the state distributions. The sub-areas can be considered as the states, and the matrix \hat{J} as the transition matrix, with $\hat{J} = \beta_t J + (1 - \beta_t)I$. Compared to traditional Markov chain simulations, the initial state is not atomic, but already a distribution among the different states (sub-areas).

where N is the number of regions. I distribute the households according to equation 15. If the following holds

$$\mathbf{s} = \mathbf{c} - \mathbf{d}_{t+1}, \mathbf{s}_i \ge 0, \forall i \in [1, 2, \dots, N]$$
⁽¹⁷⁾

the stopping criterion is already met and the next integration step can be taken. If Equation 17 is fulfilled, enough room is available in all sub-areas to distribute the households according to their preferences. If not, J has to be adapted so it can fulfill the available space conditions.

First I introduce the vector \mathbf{d}_{rest} and set its values equal to \mathbf{d}_t . Additionally I define the vector \mathbf{d}_{sol} to store the temporary distribution of households that is within the boundaries, but does not contain all the households yet. The values are set according to the following rule:

$$\mathbf{d}_{sol,i} \leftarrow \mathbf{d}_{sol,i} + \min(\mathbf{J}_{i,:} \mathbf{d}_{rest}, \mathbf{c}_i)$$
(18)

The households which were not distributed yet, are stored in d_{rest} :

$$\mathbf{d}_{rest} \leftarrow \sum_{j=0}^{N} g(J_{j,i}, \mathbf{d}_{rest}, \mathbf{c}_{j}, \mathbf{s}_{j})$$
(19)

with

$$g(J_{j,i}, \mathbf{d}_{rest}, \mathbf{c}_j, \mathbf{s}_j) = \begin{cases} 0 & \text{if } \mathbf{s}_j \ge 0\\ (1 - \frac{\mathbf{c}_j}{|\mathbf{s}_j|}) J_{j,i} \mathbf{d}_{rest} & \text{if } \mathbf{s}_j < 0 \end{cases}$$

Since it is not possible that households change their primary and secondary locations to locations that have no space available, matrix J is adapted accordingly. I assume that the weights for the different locations stay the same relative to each other, but that the ones that are full are no longer available.

$$J_{i,j} = \begin{cases} J_{i,j} & \text{if } \mathbf{s}_j \ge 0\\ 0 & \text{if } \mathbf{s}_j < 0 \end{cases}$$
(20)

The resulting matrix is then column-wise normed, to ensure that the total number of households does not change. After this step, the available space vector \mathbf{c} is adapted that it reflects the rest of the available capacities. If there still are households to distribute, the recursive algorithm starts again from equation 17.

LINEAR PROGRAMMING It is also possible to solve the limited capacity redistribution with linear programming. The way that the system then redistributes the households does not follow discrete location choice concepts of agent based models any more, but is formulated as an optimization problem.

First the integration step matrix with the included time step size has to be rewritten:

$$\mathbf{d}_{t+1} = (\beta_t \mathbf{J} + (1 - \beta_t) \mathbf{I}) \mathbf{d}_t$$

= $\mathbf{\hat{J}} \mathbf{d}_t$ (21)

The problem can now be formulated as a search for matrix J* which needs to be as similar as possible to \hat{J} . This is possible by mimicking the dot product, which is the sum of the element-wise product of the matrices. Maximizing this value is similar to minimizing the squared distance between the matrices. This follows from the properties of \hat{J} and J* and can then be solved as a linear programming optimization problem:

$$\begin{array}{ll} \max & \sum_{u=1}^{N} \sum_{\nu=1}^{N} \hat{J}_{u,\nu} J_{u,\nu}^{*} \\ \text{s.t.} & J^{*} \mathbf{d}_{t} \leqslant \mathbf{c} \\ & \| - J^{*} \|_{\max} \leqslant \mathbf{0} \\ & \| J_{:,u}^{*} \|_{1} = 1, \ u = 1, \dots, N \end{array}$$

The first condition ensures that the number of households in one sub-area is not larger than the maximal size. The second condition ensures that all values in the adapted matrix J* are non-negative, a property needed to ensure consistency in the model. The third condition ensures that all the columns of the resulting matrix J* are normalized, to ensure that the total number of households stays the same after every time step.

I have tested this variant of redistributing households, but not extensively. It is mentioned here for completeness. Several tests show that it speeds up the simulation in some cases, especially if the fraction of free space in the whole area is very small. But I decided against using linear programming, because location choice is not a strictly rational process. The dynamics of discrete location choice reflects this property better.

3.1.6 Implementation

The first prototypes of the model were implemented using scripting languages, i. e. Matlab [71] and Python in combination with the SciPy library [72]. The two languages and the corresponding libraries are

a natural choice for to develop the system, because most of the calculation steps described in section 3.1 can be expressed as matrix operations. Both languages provide a good framework for prototyping computational intensive frameworks, even though they are slower than compiled languages, such as C++. Matlab and Python are superior to compiled languages in early development stages of computational intensive frameworks, as they allow for faster programming prototyping process and have many advantages in cross-platform development.

The version of the model framework mainly used in this thesis is implemented in C++. It allows to implement the model framework in a computationally efficient way and to easily implement the model using modular programming concepts, which make it more flexible to different use cases. Additionally, there exist very efficient libraries that support matrix operations and parallelize them automatically. The core of the model is implemented using the Armadillo linear algebra library [73]. It is an interface that aims for a good balance between ease of use and speed. It works as a layer between well known linear algebra libraries and the user code. In the case of the current implementation, Armadillo connects to OpenBLAS [74], an optimized and open source version of the BLAS library [75]. One of the main differences between BLAS and OpenBLAS is that the latter automatically distributes the computations among the CPUs of the system, without any additional input needed by the programmer. The implementation does not use one of the often hardware-specifically optimised libraries offered by commercial companies, e.g. Intel MKL [76], because they are not open source, and often not applicable to all machines, nor freely available.

The Armadillo library would also allow to do some of the computations on GPU by using the NVBLAS runtime library, which is part of the CUDA Toolkit [77]. Using the GPU to do the core calculations of the model was also tested. But for the lack of proper hardware and the reasons mentioned above, the simulation runs that are discussed in this thesis are using OpenBLAS as the linear algebra backend.

The core parts of the implementation in C++ are shown in Appendix A, along with the minimal setup needed to run the model.

3.2 THE MODEL AS SKETCHING TOOL

Modelling and simulation is an imprecise science in the context of predicting future states of complex systems. As comprehensively described in chapter 2, it is a science that copes with many uncertainties and tries to predict future patterns of an unpredictable system. Even though a complex urban system is by definition unpredictable, modelling and simulation can provide insights and support to urban planners in several ways.

Modelling and simulation can be used to obtain artificial planning experience [49]. With this perspective, simulation is not about predicting the future comprehensively and with high precision. Simulations can be regarded as tools that enable planners and decision makers to artificially experience how certain actions and interventions might influence urban systems, all within a short period of time. It helps planners and decision makers to gain experience about the urban system they plan for.

The ambiguity of the model described in section 3.1, which is given due to its abstraction, allows urban planners and decision makers to interpret and reinterpret the solutions it provides. This has been shown to be an important tool in design. However, a framework is needed that has the right level of detail to not hinder creativity and solution finding [78].

A tool that provides the basic outline of the impact and implications an intervention and action can have, can be understood as a sketching tool. The Oxford English Dictionary defines sketch as: "A rough drawing or delineation of something, giving the outlines or prominent features without the detail...; a rough draught or design." [79].

The definition of sketching contains a visual representation of the result. Thus it is important that the model has a visual interface to visualise the input data, as well as the simulation results. To make the interpretation of the data more ascertainable, it is important that the different calculations of the model can be analysed. All this should be possible within the same interface or software [44].

To allow the urban planners to sketch possible urban scenarios, the model is connected to QGIS [80]. QGIS serves as a user interface and analysis tool, and enables the user to explore the impacts and consequences of different scenarios by asking "what-if" type of questions, thus sketching possible futures of the urban system. It is an open source geographic information system, with the possibilitity to develop plug-ins for it. QGIS already has many information visualisation tools implemented and enables to change geographic data. This makes it a good choice to use it in the current context.

Geographical data has to be prepared in a format that is readable in QGIS to be able to work with the model. Figure 11 depicts an imported geometric shape of the city of Zürich inside QGIS. Streets define the borders of the sub-areas; train tracks and natural obstacles are cut out. These steps can all be done using QGIS, no additional software is needed if the raw data is available. The basic shape of the city of Zürich in figure 11, and the street network to subdivide it, is freely available from OpenStreetMap [81].

When the geometric data is set up, only the main properties of the geometric shapes have to bee defined: the number of households and the number of workers at each sub-area.



Figure 11: QGIS with exemplary geometric data of the city of Zürich on top of a map [82]. The different sub-areas are defined by the streets that surround them. Train tracks and natural obstacles are cut out.

To make the setup more flexible for different tasks, QGIS is not directly connected to the model. QGIS communicates with the model through a middleware called LUCI [83]. This flexibility enables that the user can run the simulation on a local machine, but also on computationally more powerful machines. In either setup, the interface still resides on the local machine.

Using LUCI as a middleware, additionally provides the possibility to run multiple instances of the model in parallel. It makes it possible to run multiple simulations at the same time without any additional control mechanisms needed by the user. The connection to the middleware makes it also possible to integrate the model framework into larger systems, with multiple models and views of different types, as described by Treyer et al. [84].

With everything set up, it is possible to change the geometry, and other parameters within QGIS. It is possible to test different types of interventions and action to the urban system and directly see the possible impact they have on the primary and secondary location choices of households. With the described setup, it is possible to investigate the consequences and impacts of actions, ranging from adding and removing street network links, to changes in travel cost, and the construction of new office and residential buildings.

The results of the simulation runs are automatically loaded into QGIS, as soon as they are finished. This allows to compare the differ-
ent simulation results directly and also produce comparative statistics. Chapter 5 contains case studies which depict the sketching approach to explore the impact of different design proposals and interventions in the cities of Basel and Zürich, Switzerland.

3.3 FINDING SOLUTIONS

The setup described in section 3.2 allows the urban planners and decision makers to sketch possible futures of an urban area. It provides them with the possibility to investigate how interventions might impact the patterns of the urban area and lets them explore different possibilities. In this way, the model can support the urban planning process by guiding the discussions, and predict possible future states if specific actions are under discussion. It enables to ask "what-if" type of questions.

But what if an urban planner wants to, for example, strengthen the interaction of the city with a certain sub-area? One possibility is to explore many possibilities with the sketching approach discussed in section 3.2. But this way, it is almost impossible to test all possible solutions, it would even be a tedious task to test a big amount of possible solutions.

As mentioned in section 2.3 already small actions might trigger the desired effects, and possibly no big effort is needed. The huge amount of possible interventions makes it impossible to think of all possible solutions, especially because some might not be obvious. But knowing some small guiding actions that lead to the desired outcome may save a lot of effort and many resources. Compared to the paradigm described in section 3.2, the planners want to develop the urban system into a certain direction and ask themselves how this goal can be reached; this is a "how-to" type of question.

A solution to this problem is to do MC simulations of the model. MC simulations sample all the parameters from predefined probability distributions and then run the model with this sampled parameter set. This allows to automatically generate hundreds of different simulation results and, in the context of this thesis, hundreds of sketches of possible future patterns of residential and business districts.

It is not an easy task to get an overview over the generated simulations. The number of sketches is generally too high to sort and compare them manually. To simplify the task for urban planners, the simulation results are clustered using a hierarchical clustering method. The hierarchical clustering used in this research first assumes that each simulation result is a cluster by itself. Then it gradually merges sub-clusters, if the distances between the centroids of them are below a certain threshold. This procedure runs until all results are merged into a single cluster.



Figure 12: Dendrogram of a hierarchical clustering of the data of one MC simulation run. The distance depicts the distance between the centroids of the different sub-clusters and serves as a measure for when a larger cluster has to be divided into smaller ones.

The decision to use hierarchical clustering is that it produces all levels of resolution automatically and additionally also sets the clusters into relation to each other. Figure 12 depicts the dendrogram of the hierarchical clustering of a MC simulation run. The visualisation of the clustering depicts the distances between different sub-clusters and single simulation results. At the bottom, all simulation results are a cluster by themselves. By increasing the distance, the lines merge to one line, this means that the two sub-clusters were merged into one cluster.

Hierarchical clustering heavily depends on the distance between the simulation results. The result of one simulation run is normally very high dimensional, thus some preprocessing of the data is needed. The problem comes from the fact that with high dimensionality, many distance measures have a non-desired behaviour [85].

To reduce the dimensionality of the data, Principle Component Analysis (PCA) is applied to it. PCA reduces the number of a higher dimensional data set, but keeps as much information about the data as possible. It uses the data points to search for the principle directions of the data point distribution. The resulting orthogonal coordinate system uses the eigenvectors of the data distribution to align its axis and orders them according to the values of the eigenvalues in descending order. This means that the first dimension in the new coordinate system is the direction that contains the most information. In the case of the simulation runs for this research, PCA is able to reduce the dimensionality of the data more than two orders of magnitude



Primary principle axis

Figure 13: A scatter plot of one MC simulation of the model, showing the distribution along the two main principle components. The simulation results are the same as depicted in figure 12. The colors each depict one of the four clusters produced at this distance level. The green hexagon marks the current, real state, that was also added to the data before clustering.

with still containing 90% of the initial information. In the case of the city of Zürich, as depicted in figure 11, the dimensionality of the simulation results can be reduced from more than 5 thousand dimensions to 10, by keeping a little more than 90% of the information.

The hierarchical clustering of the simulation results offers novel opportunities how urban planners can work with modelling and simulation. By clustering groups of similar results into clusters, it is possible to focus only on the results that are of interest or investigate the properties of clusters that sketch an undesired pattern. This enables the urban planner to understand the dynamics of the urban system better and to understand possible chances and pitfalls a specific urban state offers. It gives the planner the possibility to first get a high level view on different states of an urban area and to study the desired states in more detail; with possibly generating additional simulation results within a certain sub-space of the parameter space.

The combination of MC simulations and hierarchical clustering of the results provides a framework to answer "how-to" type of questions. When data is available, the user of the framework can also add current situation to the MC simulation results and the clustering process. This enables to compare the current situation with the results and find the cluster the urban system is currently in. Choosing the right clustering level – defined by the distance in figure 12 – the data

is divided into the implied number of clusters. Figure 13 depicts a case in which the chosen distance produces four clusters; the green hexagon depicts the data point of the current situation within the light blue cluster. If we assume that the desired state of the urban system is in the red cluster, then all the simulation results' parameters in that cluster sketch possible solutions to reach the desired state. This process already allows to eliminate many possible interventions which are clustered into the other three clusters, and suggests different possible interventions of how the desired state of the urban system can be reached. It helps to filter out the interventions and actions that might generate false outcomes.

Important to note is that the distance parameter optimally has to be chosen in such a way that the current state of the system and the desired state are in different clusters. This makes it possible to get different proposals on how a certain stage can be reached. If the current situation and the desired state are in the same clusters, the only proposal on how to reach the desired state is the single simulation result that exactly depicts it.

The implementation of the clustering algorithms is programmed in Python using the SciPy library [72] for visualisation and the hierarchical clustering. The PCA is used from Scikit-learn [86]. There would also exist an implementation from SciPy, but that implementation the variables to unit variance. This is not a desired property in the research conducted in this thesis, because the different dimensions of the simulation results have all the same metric, and PCA should also take this into account. If the different dimensions are scaled to unit variance, too much weight would be given to the small varying zones.

3.4 SUMMARY

This chapter introduced the different parts of the framework that is developed to support urban planning and decision making tasks. The core of the framework, the meso-scale model, has a strong foundation in urban economics. It does not serve as a comprehensive model, it rather focuses on a single task, exploring the urban system from an urban economics perspective.

Modelling in the meso-scale, but still reflecting paradigms from complexity science, comes with some challenges. These challenges are tackled using methods from optimisation, probability theory and computational science in general.

Using modelling and simulation to explore possible future states of urban systems can be viewed as sketching. It gives the urban planners the opportunity to experiment with different interventions and actions to the urban system, and provides them with feedback of the possible implications the interventions and actions might have. To be able to sketch possible future states of an urban system, a visual interface is important. The model is connected to an open source GIS software, that allows to interact with the model, set up the input data, and allows to visualise and analyse the simulation results.

It is also possible to use the model to offer solutions to urban planners. This is especially useful, if the urban planners and decision makers want to reach a certain target. It gives them the opportunity to explore different ways how this goal can be reached. This can be very useful and save resources, because of the complex nature of urban systems, already small actions can guide it to the desired target. It also can suggest, possibly counter-intuitive, solutions that might have never come up in a manual exploration of impacts and consequences.

In this chapter, the core model of the framework and some of its capabilities are explored. I will present different properties of the model and show how it can be used in theoretical and be prepared for the use in practical applications.

Section 4.1 explores the capabilities of the meso-scale model introduced in section 3.1 in the context of spatial models of neoclassical economics. It is adapted in such a way that it reflects the basic assumptions of the different core models introduced in section 2.1. This allows to test the generality of the model in a theoretical context and to illustrate and validate the dynamics of the model. The validation does not explicitly validate the assumptions made in the neoclassical models, nor does it discuss the assumptions made in them. The validation is to test the meso-scale model and its capabilities to produce similar emergent patterns the neoclassical models have as a solution. The meso-scale model is set up to reflect the base assumptions of them.

The neoclassical models I compare the introduced model with are of three very different types. First (subsection 4.1.1) the meso-scale model is evaluated taking assumptions of the core-periphery model introduced by Krugman [87]. This model had a slightly different focus when it was developed then the meso-scale model. It is a simple model that focuses on the location and number of cities, rather than on the emergent patterns of single cities. However, it makes assumptions that make it a good choice to compare with, mainly the assumption that households reside at the same location they work in.

In Subsection 4.1.2, the model is compared to the Central Place Theory (CPT). The realm of the CPT is a two-dimensional space which assumes that the households have a fixed location and firms arrange themselves to serve the needs of the households in a profitable way.

Subsection 4.1.3 compares a model that has an opposite focus to the CPT: the monocentric city model. It assumes a single market at the center of the model realm and that the households and firms arrange themselves around it to maximise their utilities. Households optimise their utility according to rent prices and travel cost, whereas firms want to optimise their profit.

The second section (4.2) of this chapter evaluates the meso-scale model through calibration. Techniques from MC simulations are used to find the parameter set that produces the current distribution of households and firms in the city of Zürich, Switzerland. The parame-

ter set that produces the closest solution to the current distribution is then validated by comparing it to official data.

4.1 MODEL GENERALITY

This chapter compares the model described in section 3.1 with three core models from neoclassical spatial economics. It shows that the model has the capabilities to reproduce the results of to the equation based models, but also offers new ways to study the assumed dynamics taking the perspective of complexity science. It shows that the meso-scale model implements some generality and allows to study different types of systems with possibly opposing assumptions.

4.1.1 Racetrack Economy

The first neoclassical model the meso-scale model is compared with is also the most recent of the three: the core-periphery model. The assumptions of the core-periphery model, in the context of this comparison, are that the model realm is one-dimensional with periodic boundary conditions and that the households reside at the same location they are employed in.

The core-periphery model is the founding model of geographical economics and was introduced by Krugman [28]. The initial version of the core-periphery model focuses on two locations that can house two production industries: manufacturing and agriculture. Manufacturing is location independent and is able to change the location of production, whereas agriculture is fixed to a location due to its need of immovable land.

The core-periphery model is a theoretical approach to explain how a country can become differentiated into an industrialized "core" and an agricultural "periphery", endogenously. It is not a novel way of modeling spatial economics, but rather puts different known concepts from economic theory into a single analytical framework.

It shows that manufactures cluster to experience economics of scale. The model introduced in this thesis (see section 3.1) is a model that focuses mainly on firms that have a direct interaction with their customers, i.e. only forward-linkages are implemented. In contrast to this, the core-periphery implements both, forward and backward-linkages, since it also implements interactions between different companies and industries. This property is not present in the current version of the meso-scale model. Additionally, the core-periphery model assumes that each of the regions manufacturers produce a single product that is different from the goods produced in the other regions. The households have the preference to maximise the utility of their shopping bag, which is maximising the variety of goods. They try to find a good combination of expensive goods from further away

| Parameter | Value | Description |
|------------------|-------|---|
| σ _{d1} | 100 | Distance scaling primary location choice. |
| $\sigma_{ ho}$ | 100 | Density scaling primary location choice. |
| $\alpha_{\rm r}$ | 0 | Weighting factor primary location choice. |
| σ_{d_2} | 100 | Distance scaling secondary location choice. |
| $\sigma_{ar{w}}$ | 100 | Income scaling secondary location choice. |
| α_w | 1 | Weighting factor secondary location choice. |
| σ | 500 | Kernel width. |
| βo | 1 | Initial step size. |
| α_{β} | 0.999 | Step size reduction factor. |
| N | 5000 | Number of iterations. |

Table 1: Basic parameter set used to simulate the emergent patterns of the racetrack economy. The step-size is decreasing over time to reach an equilibrium state in the end.

and local goods that are cheaper. The higher price of goods produced further away is mainly depending on transport cost. The meso-scale model described in this thesis does not differentiate between different types of goods, it aggregates all goods to a single category. This chapter shows that business centers emerge, when these simplifications are made.

The meso-scale model is compared with an advanced version of the core-periphery model, which increases the number of locations to twelve [27]. The twelve regions are uniformly distributed along a circular pattern, thus the name *racetrack* economy. The two panels in figure 14 depict the geometric shape of the racetrack economy used for the simulation, the colors can be ignored at the moment. The different locations (squares) have all the same distances to their neighbors. In the example depicted in figure 14, the locations have a edge length of 100 units. This yields a distance of roughly 141 units between the centers of the regions, which is the distance the meso-scale model assumes between two neighbors.

In the core-periphery framework, workers choose their location of work according to the per-capita income of people at the different locations. They do not take commuting into account in their secondary location choice, because their household is assumed to move to the location of work. This leads to the parameter set depicted in table 1.

The parameters σ_{ρ} and σ_{d_2} do not impact the simulation in the current setup, because they are neutralised by the weighting factors of the total total utility, α_r and α_w , respectively. The α values are set to reflect the earlier mentioned dynamics for the location choices. The initial step size is chosen as being one, because in this case, it is



Figure 14: Result of a simulation run mimicking the core-periphery model. The left panel depicts the distribution of the households in the simulation realm, the right for the workers.

not important to find the "natural" state from the initial state, but a state that accounts for more generality. The number of iterations of the simulation is 5000, this ensures that the simulation runs until the step size is less than 0.01.

Initially, the households are equally distributed among the different regions, i. e. all regions contain the same amount of households. This initial setup would lead to no changes in the model realm, because all regions would have exactly the same properties. To solve this problem, Krugman adds small variances to the number of households per region. I do the same for the initial setup of the input geometry.

Using the above described setup, the final state of the model is the same as in the core-periphery model when transport cost is low. The number of agglomerations is equal to one and all the households that are able to change their location are located at that region. Figure 14 depicts the simulation result with the parameters from table 1. On the left panel, the distribution of the households is depicted, and the right panel shows the distribution of where people work.

One of the main differences between the core-periphery model as it is described by Krugman [87] and the meso-scale model as it is described in this thesis, is the solution process and development. In the case of the core-periphery model, the single agglomeration slowly emerges, with phases where multiple smaller agglomerations occur in the model realm. But the final stable solution is a single large agglomeration, containing all manufactures and households. In the case of the meso-scale model, the single large agglomeration also slowly emerges, but no smaller agglomerations occur in the development process. All households that move away from their area move to the region that is finally the single large agglomeration; Krugman calls it the industrialised "core". When the transport cost is increased in the core-periphery model, multiple agglomerations emerge as the stable solution with only agriculture around them. This effect is also present in the meso-scale model. When the transport cost is increased (mimicked by decreasing the kernel width σ) also multiple main agglomerations emerge. Simulation results depicting the impact of different kernel widths can be found in appendix **B**.

4.1.2 Central Places

One of the core theories of spatial economics is the theory of central places. Its focus is on how centers should be organised that they are able to serve the inhabitants of each region. The founding theory of this perspective is the central place theory, independently introduced by Christaller [25] and Lösch [26]. They approached the problem of arranging centers from different perspectives, but with similar assumptions and outcomes.

Both assume that a featureless plain consists of uniformly distributed, immobile households. The households have different needs that must be fulfilled, whereas these needs have different periodicity. This means that some have first level periodicity, because they are needed with higher frequency, such as bread. Other products have higher level of periodicity, because they are needed less frequently, e.g. new cars. The assumption is that there exist different sizes of centers that serve goods up to a certain level of periodicity, e.g. cities serve all levels, whereas villages only serve lower level goods. Goods with a higher level of periodicity need a larger catchment area to be profitable for the firms. The resulting pattern for both approaches is depicted in figure 3.

The context in which these theories have been developed, and the main ideas behind them is more comprehensively discussed in section 2.1.2.

Running the meso-scale model, introduced in section 3.1, mimicking the assumptions made by the theories of central places is similar to the approach that Lösch took. He assumes first that the simulation realm does not contain any centers. He then adds them one by one and arranges them to cover as much of the area as possible, until all households are within the catchment area of every product level.

The meso-scale model works a little bit differently. The different centers are not added manually to the model, but they emerge during the simulation. What has to mentioned at this point is, that the meso-scale model in its current state does not differentiate between different goods, all are regarded as the same. This property is very different to the assumptions made in the central place theory, with its assumption of different goods that have different levels of periodicity. Following will show that also with this aggregation of all goods to a

| Parameter | Value | Description |
|------------------|-------|---|
| σ_{d_1} | 0 | Distance scaling primary location choice. |
| $\sigma_{ ho}$ | 0 | Density scaling primary location choice. |
| $\alpha_{\rm r}$ | 0.5 | Weighting factor primary location choice. |
| σ_{d_2} | о | Distance scaling secondary location choice. |
| $\sigma_{ar{w}}$ | 100 | Income scaling secondary location choice. |
| α_w | 1 | Weighting factor secondary location choice. |
| σ | 300 | Kernel width. |
| βo | 0.25 | Initial step size. |
| α_{β} | 1 | Step size reduction factor. |
| Ν | 40 | Number of iterations. |

Table 2: Basic parameter set used to simulate the emergent patterns of the theory of central places.

single goods type, patterns emerge that are similar to the results of the central place theory.

It is not possible with the meso-scale model to have an infinite continuous plain, thus the simulations are run on a squared grid with one hundred sub-zones per axis, i. e. 10'000 sub-zones in total. The grid is set up in such a way that every grid cell could house all workers. The only factor that decides about the location of business centers is the per-capita income at a certain sub-zone. The step size of the simulation is set in such a way that the emergence of the different centers can be observed, and is not reduced during the simulation. The kernel width is set so that it is a little more than covering two neighboring sub-zones in all directions. The full parameter set is depicted in table 2.

The result of the simulation run is depicted in the top panel of figure 15. After forty iterations multiple centers of different sizes emerged. In the case of the depicted result, I take the assumption that centers are defined of neighboring clusters which house more workers than the lowest category described in the legend. This leads to three sizes of centers. More than 60'000 households find work at the location of the central cluster. The next smaller business clusters are the eight clusters that are arranged in a circular shape around the center of the grid. These clusters each house 18'000 workers. The smallest centers depicted in the top panel of figure 15 are the business centers close to the corners. These business clusters each house more than 16'000 workers. In total, the main centers of the simulation result house more than 1/30 of all workers, covering only 176 out of 10'000 sub-zones, i.e. roughly 1/60 of the total area.



Figure 15: Result of two simulation runs mimicking the central place theory. The panels depict two simulation results with the same parameter sets (see table 2) in a squared and a round "world", top and bottom, respectively. The colors depict the number of workers at one sub-area.

What can be noticed, when the distributions of centers in figure 15 is compared to the pattern in figure 3, is that the hexagonal pattern is not present in the simulation results. An explanation for this might be the geometric form of the grid. To see if this has a direct impact on the form of the distribution of centers, a simulation on a grid in a round shape was done, using the same parameter set described in table 2. The result is depicted in the bottom panel of figure 15.

Also in the simulation run with a circular grid shape, multiple centers of different sizes and shapes emerge. The clusters are of similar sizes and distributions as the ones in the case of the squared grid. The only exception is that there are large centers of similar total sizes, but with very different densities. The business cluster located at the center of the grid comprises of 16 grid cells, housing more than 22'000 workers, whereas the larger centers toward the border in the north, west, east, and south cover a much larger area, but with a smaller density. These four business clusters comprise of twice the number of grid cells, i. e. 32 grid cells, and each serves as secondary location for more than 23'000 households. Only these five main business centers house more than 1/10 of the total business activities, covering only 144 grid cell, or about 1/65 of the total area. All the centers in the simulation realm house about a fourth of all the work places, within an area that covers nearly 1/25 of the total area.

The change of the form of the grid does not impact the geometric arrangement of the clusters, still the hexagonal can not be reproduced. This could also be from the fact that the grid-cells are quadratic. A further possible explanation for the produced pattern might be that the used grid is not infinite, and the boundaries introduce a bias to the emergent pattern. This property is also indicated when subsequent states are compared. A solution to this problem could be to use periodic boundary conditions, but these would need further investigation, which is beyond the focus of the here described investigation.

The results depicted in figure 15 are only snapshots of the simulation. Because the step size β is not changed in the course of the simulation, the model does never reach a stable state. However, the pattern of few large centers and multiple smaller ones can be observed at any state of the simulation, except for the first few iterations after initialisation. A few of the other states are depicted in appendix B.

What is the most surprising result of this validation, is that business clusters are not emerging at single locations. The businesses also locate close to high value locations. This is mainly surprising, because there are no dynamics present in the meso-scale model that enforce such behaviour, the attraction of a single subarea has no impact to the attraction of neighboring areas. Still, areas with high attractiveness to spend money in, also seem to increase the attractiveness of neighboring subareas.

4.1.3 Monocentric City

The model of the isolated state is regarded as the founding model of neoclassical spatial economics. As described in section 2.1.1, it assumes a state that is cut off from the outside world and focuses on the agricultural land use around a central city. This model also laid the foundation of neoclassical urban economics. Instead of assuming an isolated state with hinterland, neoclassical urban economics focuses on the distribution of activities within an urban agglomeration. In the case of the monocentric city, it additional assumes that the city has no interaction with the outside.

It theorises about the location and distribution of different land uses. The founding model of neoclassical urban economics uses bidrent curves to describe the location of offices, manufactures, and households. The curves are similar to the bid-rent curves that describe the land use around the single city in the isolated state (see figure 2). The different actors in the model have different goals, for example, households try to maximise their utility with their location choice, whereas firms base their location choice to maximize profit. In the basic model, this leads to a pattern, where offices are close to the center and households locate themselves around the central market. This leads to a monocentric city with decreasing density or rent prices with distance from the center in the basic case.

The assumption of this founding model of urban economics is that there is a central market at the center of the realm, and all goods have to be sold at this location. The meso-scale model described in section 3.1 does not have the possibility to fix a market, but directly models the interaction between companies and households. Additionally, the meso-scale model has only a very general company type. It does not differentiate between manufacture and offices.

In this section, I show that the meso-scale model is able to mimic assumptions made in the monocentric city model and that it produces a similar emergent pattern to the pattern of the monocentric city model with these assumptions. The parameter set is depicted in table 3.

One of the key assumptions of the monocentric city model is that the firms base their location decision on maximising their profit. This can be reflected in the meso-scale model by setting the secondary location of households only according to the per-capita income of a region, i.e. $\alpha_w = 1$.

This is different for the primary location decision of households. They base their primary location decisions on maximising the utility, which depends on the commuting distance to the work place and the rent price. Thus the weight of the primary location α_r is set to weight both factors the same.

The monocentric city model, in its basic version, assumes that only one market is present in the whole urban area. This means that all

| Parameter | Value | Description |
|----------------------------|--------|---|
| σ_{d_1} | 100 | Distance scaling primary location choice. |
| $\sigma_{ ho}$ | 100 | Density scaling primary location choice. |
| $\alpha_{\rm r}$ | 0.5 | Weighting factor primary location choice. |
| σ_{d_2} | 0 | Distance scaling secondary location choice. |
| $\sigma_{ar{\mathcal{W}}}$ | 100 | Income scaling secondary location choice. |
| $lpha_w$ | 1 | Weighting factor secondary location choice. |
| σ | 10'000 | Kernel width. |
| βo | 0.3 | Initial step size. |
| α_{β} | 1 | Step size reduction factor. |
| Ν | 50 | Number of iterations. |

Table 3: Basic parameter reflecting the assumptions of the monocentric city model.

households have to spend their money at the same location. This can be translated to the meso-scale model that the kernel width σ has a very large value that does not favor closer companies to further away ones.

The number of iterations are chosen by trial and error, which showed that after fifty iterations a stable state is reached that only changes negligibly, also when no reduction of the step size is set. The geometry of the simulation consists of a squared grid with 100 grid cells per dimension, i.e. 10'000 grid cells in total. The grid cells have enough space to house all the households and workers at once. The width of a single quadratic grid cell is 100.

The pattern that emerges is very similar to the analytical result of the monocentric city, as depicted in figure 16. Around the center, a business district emerges that houses all workers. The business center pushes most of the households outside of the city center. Thus the households build a ring around the business district.

The center of the monocentric city is highly dense. This comes from the fact that the secondary location is only dependent on the percapita income of the workers. They do not take into account the commuting cost from home to work in their secondary location choice. Commuting is only taken into account, when the primary location is chosen. The primary location choice is dependent on two opposing factors that pull the residents towards the center and push it away from the center, i. e. travel cost vs. rent price.

The emerged pattern is depicted as a cross-section in figure 17. A main property that is depicted in this figure is that the border between business district and the residential district is not strict. The transition from one to the other is fluent, which is different to the



Figure 16: Stable state of a simulation run, set up to reflect the assumptions from the monocentric city model. The upper panel depicts the distribution of the households, the bottom panel the distribution of where people work. Only the fraction of the whole simulation realm is shown where households and workers cluster. The rest of the grid-cells is almost empty.



Figure 17: Cross-section of the stable state depicted in figure 16. The crosssection is chosen to go through the center and from east to west. Any other cross-section through the center looks similar, due to the point symmetric property of the result.

strict analytically solved approach used in the neoclassical model of the monocentric city. This effect most likely comes from the fact that the meso-scale model distributes the agents in a probabilistic way, which also moves residents to places that might not have the maximal utility value.

Another property that catches the eye is the relativ high density in the business district, compared to the densities in the residential ring. This effect is in accordance with the analytical solution of the monocentric city. The decrease of a single land use, e.g. housing, in terms of number of entities per area is almost linear in the non-border areas, as it is suggested by the neoclassical model.

4.2 VALIDATION THROUGH CALIBRATION

This section investigates the core model of the framework, that was introduced in section 3.1, in the context of its usability for real world scenarios. In contrast to the validations performed in section 4.1, there is no data available to directly compare the outcomes of the meso-scale model.

Traditionally, prediction models are evaluated using historical data. For example, Verburg et al. [88] evaluate a version of the CLUE-S model by simulating the change in land-use of an island in the Philippines between two different points in time. The initial parameters are calculated using a regression analysis and the model is used to simulate the development process within ten years. The predicted result and the real land use are then compared.

This is not a valid strategy for the meso-scale model introduced in this thesis. The model assumes that the emergence of patterns follows self-organising dynamics, and that the infrastructure is provided by the authorities. One of the main ideas why the model was developed, is that urban planners and decision makers are able to sketch possible futures of the urban system and provide the needed infrastructure to guide the urban system into a desired direction.

Following this idea, traditional validation methods of prediction models are not applicable. It would only be possible, if the assumption is made that all interventions in the urban system were made following the earlier mentioned basic idea of self-organisation and the according interventions. Since this would be a strong assumption that can not be validate by itself, the model is not compared to historical data in the traditional way.

Despite the above mentioned reasons, the analysis using historical data is not be possible for many cities. The city of Zürich, for example, has good data available for only the last few years. This time range is to short to evaluate the changes in the patterns of primary and secondary locations, since the adaptation process takes much longer. For the city of Zürich, Switzerland, for example, about 11 percent of the residents changed their primary location in 2016 [89]. This fraction is to small if only a few years of data is available.

In addition to the above mentioned points, it can not be assumed that the authorities perform a single action and then wait until the city has reorganised itself, also if the assumption is made, that the city organises itself into a stable state and not constantly changes. The meso-scale model itself does not directly take time into account. This makes it impossible to find the state of the system, at which the new interventions would be applied. The meso-scale model sketches possible future states, within a certain scenario. It provides possible states towards which the urban system would develop to if certain actions are taken. For these reasons, it does not make sense to compare the model with historical development processes and states.

To validate the model for real world use cases and to ensure the significance of it, a different approach to the traditional methods is taken. The validation of the meso-scale model is performed by using a MC simulation. The setup of the MC is described in subsection 4.2.1. In section 4.2.2, the results of the MC simulation are compared with the real distribution in the city of Zürich, Switzerland. The parameter from the simulation that is closest to the real distribution of Zürich is validated in section 4.2.3.

4.2.1 Setup

A MC simulation runs many instances of a model with different parameter sets. Compared to an absolutely random parameter set generation, a MC samples the different parameters according to a distribution and thus gives some control over the parameter space. The dif-

| Parameter | Range | Sample Distribution |
|----------------------------|--|--|
| σ _{d1} | 0 - 100 | Uniform |
| $\sigma_{ ho}$ | 0 - 100 | Uniform |
| $\alpha_{\rm r}$ | 0 - 1 | Uniform |
| σ_{d_2} | 0 - 100 | Uniform |
| $\sigma_{ar{\mathcal{W}}}$ | 0 - 100 | Uniform |
| α_w | 0 - 1 | Uniform |
| σ | $0 - \infty$ | Weibull with $\kappa = 1.5$, $\lambda = 1200$ |
| βo | 0.1, 1 | Uniform |
| α_{β} | 0.99 | |
| N | $\log_{\alpha_{\beta}}(0.001/\beta_{0})$ | |

Table 4: The distributions the parameters are sampled from for the Monte Carlo simulation. The Weibull function was chosen to sample σ , since it allows to sample extreme values, with most samples having a kernel width that have a decreasing influence with distance within the city.

ferent distributions from which the parameters are drawn that form one parameter set are depicted in table 4.

The scaling factors σ_{d_1} , σ_{ρ} , σ_{d_2} , and $\sigma_{\bar{w}}$ are drawn from a uniform distribution, with the range of zero to one hundred. Larger values than 100 have been shown to have no big effect anymore and can introduce numerical errors to the simulation that are not negligible. Because of this, the upper limit is chosen to not be more than 100. The weighting factors α_r and α_w are uniformly drawn from their full range, as per definition (see section 3.1) they are only valid in the range between zero and one.

The kernel width is sampled from a Weibull distribution. The Weibull distribution is depending on two parameters, the first (κ) defines the shape of the distribution. With the chosen value of $\kappa = 1.5$ the shape of the probability density function covers a large range with similar probability, but also allows to draw extreme samples outside this range; all larger than zero. The Weibull distribution is mainly chosen because of its flexibility, by depending only on two parameters. Especially the shape parameter κ allows to adapt the shape of the curve. This is a desired property, because the drawn samples should most of the time have a value that defines a kernel width which is clearly smaller than the width of the whole simulation realm.

The initial step size is randomly drawn from a uniform distribution in the range of 0.1 and one. This allows to also test different time integration speeds. The lower boundary is set to be 0.1, because this forces the model to undergo at least few iterations, and not just stay in its initial state. The step size is reduced steadily. The goal of the MC simulation is to find the closest state to the real state, not one that is depending on the initial condition of the simulation. For this reason, the reduction factor is set to a high value. The Number of iterations N is caluclated depending on the initial step size β_0 and the step size reduction factor α_{β} . A single simulation in the MC simulation runs as long as the maximum possible difference between two iterations is larger than 0.1 percent.

For each run in the MC simulation, the same geometry is used. It is defined by the basic shape of Zürich and the street network. The street network defines the border between two neighbouring subareas. Additionally, natural obstacles and train tracks are cut out. Figure 11 depicts the basic geometry.

The number of primary and secondary locations in the geometry is according to the data that was provided by the Federal Statistics office of Switzerland [90] and the Statistics Office of the city of Zürich [91]. To ensure that the simulations run properly, and that free space is available at the beginning in all subareas, each area is set to have space for additional ten percent of the total land use it has in reality.

4.2.2 Simulation results

This section investigates the results of the MC simulation and compares the results with the real world distributions of primary and secondary locations of households. This is an important step to check the plausibility of the meso-scale model. If the model is not able to reproduce states of the urban system that are similar to the real state, it is not capable to serve as a sketching tool in planning scenarios.

I do not make the assumption that the MC simulation finds a parameter set that perfectly reproduces the current pattern. This can not be possible for several reasons. For example, the model abstracts the real world and excludes properties that are also considered in location choice. For example the topology at the different location, neighborhood status, and social networks [30]. The location choices in the model are purely by means of economic factors.

Another property, that it can not be assumed that the current pattern of residential and business centers can be perfectly reproduced, is that the current pattern might not be a pattern that reflects the desired state of the actors. It might be that the law prohibits different land uses at certain locations. These locations would possibly experience different land use, if location choice dynamics were purely selforganising. Such an similar dynamics within the urban system are not considered in the meso-scale model.

The MC simulation produced more than three hundred results. To be able to compare the results, each of them is mapped to a vector. Every odd entry in the vector corresponds to the number of house-



Figure 18: Scatter plot of the results of a MC simulation to find the parameter set that produces the best result in reproducing the real world pattern of business and residential centers. The points are plotted along two axis with high variance.

holds in one subarea. Every even entry corresponds to the number of workers in one subarea.

Even though the number of results is much smaller than the number of dimensions in one vector, I apply the PCA to the results. The results should be regarded with care. Because the resulting principle components might not reflect the real principle components of the distribution. But PCA does find directions of high variation, also when the sample size is small. This helps to reduce the number of dimensions of the vectors, by still containing most of the stored information.

Figure 18 depicts the primary and secondary axis that result from this transformation. It shows that the relatively strict geometry that is used in the MC simulation does not restrict the single simulation runs. It could have been that the relative strict geometry (see subsection 4.2.1) would force the simulation results to end up in always the same states, independent of the parameter set. But figure 18 shows that this is not the case.

To find the result from the MC simulation that is closest to the real world pattern of business and residential districts, the real world pattern is also mapped to a vector, in the same way as described for the simulation results above.

This way, the most similar simulation result is also the result that maps to the most similar vector. Using this method, the different results from the MC simulation can be compared. Figure 19 depicts the best and the worst fits, together with the real distribution. The figure only depicts the center of the city of Zürich, Switzerland, because depicting the whole simulated area would make it impossible to point out the main differences between the different settings, due to the high resolution of the subdivision of the geometry. The interested reader can find the full figures for the here depicted results in appendix B.

The left column of figure 19 depicts the pattern households form. The main difference between the real pattern and the pattern, produced by the best fit is that in the depicted area slightly more households are present for the simulation result. On the other hand, the number of firms is smaller. Generally, the patterns are very similar and vary only slightly. In both cases, the center is mainly dominated by firms. Households are pushed away from the center. Only a small fraction of households can afford to reside in the vicinity of the main business district. The small differences between the simulated result and the real pattern comes most probably from the earlier mentioned abstractions, and are within a tolerable range.

The pattern produced in the worst fit solution has a very different pattern to the real pattern. The households reside in the center of the city, whereas firms are distributed more homogeneously in the urban area. An explanation for this behaviour could be the very small kernel width drawn for this particular simulation. It is about a fifth of the amount of the kernel width drawn for the best fitting solution. Because of this property, the firms have a strong decrease in attraction with distance, thus have to spread better to reach enough customers. The agglomeration effects of cumulative kernel strengths is possibly not strong enough to overcome the fast decrease of it.

What generally can be seen in the simulation results, but also in the real pattern, is that the households are distributed much more uniformly along the whole city, whereas firms tend to cluster at certain locations quite strongly. This leads to locations that contain almost no firms.

The legends in figure 19 depict this strongly. For the households, the maximum value of households per km² is in the lower range of the second group of workers. The maximum density of workers in the simulation, as well in the real distribution, is almost five times as big as the maximum density of households. But this number is only reached with the worst fit solution, the best fit solution has even smaller densities of households.

4.2.3 Parameter Set Validation

The last step of evaluating the model through calibration is to compare the parameter set of the best fitting result with the available data from statistics offices and research publications. If the parameters are in a meaningful range around the value official data and current research suggest, it is a strong indication that the meso-scale model has some truth in it, and is not only a toy model.



Figure 19: The best and worst solutions from the MC simulation are depicted. The top panel is the real distribution. The middle panel depicts the best fitting solution, and the bottom panel the worst fitting solution. The left column shows the results for the households, the right column the results for the workers. Only the center of Zürich is depicted, for the rest of Zürich, the differences between the results have similar characteristics.

| Parameter | Value | Description |
|----------------------------|-------|---|
| σ _{d1} | 17.6 | Distance scaling primary location choice. |
| $\sigma_{ ho}$ | 0.314 | Density scaling primary location choice. |
| $\alpha_{\rm r}$ | 0.300 | Weighting factor primary location choice. |
| σ_{d_2} | 21.5 | Distance scaling secondary location choice. |
| $\sigma_{ar{\mathcal{W}}}$ | 6.23 | Income scaling secondary location choice. |
| α_w | 0.207 | Weighting factor secondary location choice. |
| σ | 2095 | Kernel width. |
| βo | 0.981 | Initial step size. |
| α_{β} | 0.99 | Step size reduction factor. |
| N | 685 | Number of iterations. |

Table 5: The parameter set that produces the best fit solution in the MC simulation.

The parameter set that produced the closest distribution to the real distribution of households' primary and secondary locations is depicted in table 5.

The scaling factor each utility function has can not directly be compared to data in the literature or from statistics offices. But the relative relation they have to each other can be evaluated. The slopes of the possible utility values, with the parameters from table 5 are depicted in figure 20.

It catches the eye that in the left panel of figure 20 the utility decreases much faster with increasing distance than for increasing densities, which serve as abstraction for rent prices. The value of σ_{d_1} has a much higher value than the scaling factor σ_{ρ} . Also the weighting α_{r} weights the distance more in the location choice than the density at the locations. This means that the households are less elastic towards distance than the rent prices in their primary location choice. This agrees with the findings of Bürgle [92] and Schirmer et al. [93], who investigated the primary location choice of households in the greater Zürich area, Switzerland.

I did not find anything in the literature or in any data sets that allows to validate the parameters for the secondary location choice. But the values seem coherent, when they are compared with the parameters for the utility of primary locations. Especially the parameters that define the utility of the distance measure, are in the same order of magnitude for both utility function, i.e. $\sigma_{d_1} \sim \sigma_{d_2}$ and $\alpha_r \sim \alpha_w$.

The parameters for the utility of the income $u_{\bar{w}}$ can only be evaluated indirectly. The work of Schirmer et al. [93] does not directly focus on the secondary location choice, but their regression model includes a ratio of rent price to income. This is one of the most signifi-



Figure 20: The left panel depicts the slope of the utility function for primary locations, the right panel for the secondary locations.

cant factors in their result, thus the income can play an important role in the primary, also in the secondary location choice. For the magnitude of the two scaling values σ_{ρ} and $\sigma_{\bar{w}}$, and the difference they have, I can not find any data or literature to compare with. But the total weight of them in their respective utility agrees with findings in literature [30, 92, 93].

The kernel width defines the amount with which the attraction of one area decreases with distance. Because of the symmetric properties of the kernel function, it can also be viewed from a different perspective. It can be viewed as the defining parameter of the distribution of trips to spend money, by distance, as more comprehensively explained in section 3.1. When looking at the kernel width this way, it can be compared to official data about trips of people in the city of Zürich, Switzerland.

A rough estimate of the distance people travel can be calculated from different publications from statistics offices and public transport companies [94–97]. These publications are not from the same years, so the result should not be regarded as absolutely exact, but it allows to estimate the dimension of the average travel distance people do to spend money. This is meaningful, because the publications were all published within a short period of time of six years.

From the publications it is possible to conclude that the average distance a person travels to spend money in the city of Zürich is about 2.6 kilometers. This number comes from the average time a person travels of twelve minutes, the average travel speed of about 15 kilometers per hour for non on foot travel, its fraction in the whole trip of about sixty percent, the average speed on foot in Zürich, and its fraction of the whole trip.

The result of the MC calculation that produces the solution that is closest to the real distribution has a kernel width σ of 2095 meters. σ defines the distance within a little more than two thirds of the trips happen, if the firms were uniformly distributed over the landscape. According to the assumptions in the meso-scale model, this means that in average, half of the trips are below a distance of 1.6 kilometers.

This value is much smaller than the average travel distance estimated from official data. But the assumption of this value is different, it is set with the assumption that all firms are uniformly distributed over the city. In the real world, this is not the case. Thus it makes sense that the real trip distance is further than the kernel width.

When looking at the actual travel distances of the households in the simulation result, it shows that it has a value of 2.4 kilometers, with a standard deviation of about 590 meters. This value is very close to the real value of 2.6 kilometers.

The validation of the different parameters of the best fitting simulation result shows that the model must implement some truth in it. They all have values that are in a meaningful range when compared to different official publications and the literature. This indicates that the validity of the model for real world use cases is strong enough that it can be used for scenario sketching.

4.3 SUMMARY

This chapter validated the meso-scale model, the core of the here proposed framework, in two ways. First the meso-scale model was evaluated by comparing it with models from neoclassical spatial economics. The second part explored the validity of the model in the context of real world applicability.

The comparison of the meso-scale model was performed with three core models of neoclassical economics: the racetrack economy, the theory of central places, and the monocentric city model. These model all have different strong assumptions that allow to evaluated different setups of the model. In the racetrack economy, it is assumed that the primary and secondary location of households is the same location. This location is chosen only by the location choice of the secondary location. This allows to test the dynamics of the model in the context of agglomeration building. The model realm does not reflect a single city, but a set of many locations that are possible locations of cities. The meso-scale model was able to reproduce different patterns of the racetrack economy, mainly depending on transport cost. The goal of this validation was mainly to show that the emergent patterns of the racetrack economy can be reproduced with the mesoscale model. The discussion does not focus on the context in which the core-periphery model was developed directly, it is about the core dynamics of how people choose their primary and secondary locations.

The second core model of spatial economics the meso-scale model was compared with is the CPT. The focus of the CPT is similar to the one of the racetrack economy, but the assumptions are very different. The CPT assumes that the households are uniformly distributed in a featureless plain. The cities (agglomerations) should now be placed in such a way that they can serve as many different people as possible. In this context, the agglomerations are only depending on the number of workers at that place, since the households stay fixed at their initial location. The meso-scale model was able to produce similar patterns as the CPT suggests analytically. However, the meso-scale model is dynamic and workers base their work location also on the per capita income at different locations and are not perfectly rational. This leads to a dynamic result which never reaches a fixed state.

The last neoclassical model the meso-scale model was compared with, is the urban version of the founding model of neoclassical spatial economics: the monocentric city model. The assumptions in this model are weaker than in the other models. The model assumes that firms and households compete for their locations against each other. The resulting pattern is a ring pattern, with a big business district in the center. The meso-scale model was able reproduce the patterns, even as a stable state. The meso-scale model was set up to exhibit dynamic behaviour, but reached a certain state with the pattern of the monocentric city model, which only changed marginally over time when it was reached.

The last part of this chapter focused on the validity of the model to reflect real world dynamics. To validate the model, and if its dynamics have some truth in them, techniques from MC simulations were used. In the MC simulation, hundreds of results were produced, each corresponding to a parameter set. The results showed that the model is able to reproduce the current real distribution of workers and residents very closely. However, also very different patterns can be produced. This indicates that the model is not overfitted to reflect only dynamics of Zürich, Switzerland.

The parameter set that produced the closest pattern to the real pattern comprises of parameters that have meaningful values. For example, the model estimated that the households travel within a radius of 2.4 kilometers in average to spend money, whereas the real value according to official data is roughly 2.6 kilometers. This indicates that the model contains some truth. Thus it is applicable to help planners to sketch possible states of a city in a meaningful way. Part III

CASE STUDIES

This chapter describes three use cases of the framework described in chapter 3. The case studies are considerably different from each other and exemplify different stages in which the framework can be used in planning practice.

In the first case study (section 5.1), the core model of the framework is applied as an analysis tool. The investigation explores the impact on the economic relationship between two districts in Zürich, Switzerland, if a pedestrian and bicycle bridge is added to connect them. The case study was carried out for a workshop in collaboration with authorities of the city of Zürich, Switzerland. The model was one of several analysis tools that were applied in this workshop. The other tools are not described, because they analyse the setting at a different scale.

Section 5.2 explores the usability of the framework as a sketching tool. It describes a use case in which students had to use the framework. They had to explore different configurations and possible interventions to a large industrial building in the center of the city of Basel, Switzerland. The students applied the sketching approach of the framework in the context of their diploma thesis. It was a good test case to test if the tool meets the goals and it is a good opportunity to simulate how the model might be used in real planning scenarios.

The case study described in section 5.3 is a case study that exemplifies the use of the framework as a solution finding tool. Along with the feasibility check of the proposed methodology to find solutions, the section also shows the level of impact different actions can have to an urban system. Not only infrastructural interventions are explored, also the implications of economic and behavioral changes are investigated.

5.1 ADDING A BRIDGE: NEGRELLISTEG

This section describes a use case in which the meso-scale model is used to analyse the impact of an infrastructural intervention to the urban system. The model was used as one of the analysis tools in a joint project with the civil engineering department of the city of Zürich, Switzerland [98]. The project focused on the socio-economic impact a pedestrian and bicycle bridge in the center of Zürich has to the urban setting.



Figure 21: The site of the planned pedestrian and bicycle bridge in the center of Zürich, Switzerland. The blue box indicates the planned bridge called Negrellisteg. The only ways to currently go from one bridge head to the other is by taking the underbridge in the west or going through the main station (HB) in the east.

Currently, the two districts the bridge¹ is planned to connect, are quite disconnected from each other. They are north and south of a train track field, as depicted in figure 21. The Negrellisteg would have a length of about 200 meters and spans the full width of the train track field. The location is depicted as a blue box in figure 21. Currently, if a person would want to walk from the north side of the planned bridge to the southern side, the walk would be much longer. The shortest way to walk from one side to the other is by taking an underbridge in the west. The walking distance taking the underbridge is about four times longer than the direct distance. Another option to get from one side to another is going through the Main Station (HB). The distance taking this route is even longer. It is almost one kilometer long. This means that with the average walking speed of people in Zürich of 1.4 meters per second, it takes about ten to twelve minutes to walk from one side to the other.

This gap has been a known missing link in the pedestrian and bicycle network in the city for a longer time. It came more into focus again when the new housing and business developments on both sides of the tracks were planned. These developments replace storage buildings and industrial wasteland that served as an additional boundary and helped to obfuscate the missing link. However, the recent developments make the absence of the link between the two districts more apparent.

¹ The project is called Negrellisteg.

The planned bridge crosses a distance of about 200 meters, with not many possibilities for pillars. This makes it a costly project with an estimated cost of a few dozens of millions Swiss francs. The main interest of the civil engineering department of the city of Zürich is to understand what effects the bridge has to the dynamics and patterns in the vicinity of the bridge. The goal of the department is to increase the interaction between the two districts and to find out, if the multiple millions would pay off by increasing the livability for the inhabitants. Additionally, they are looking for investors. The analysis helps them to understand who the main beneficiaries, in economical terms, of the project would be. Thus it indicates them with possible people and companies they could approach.

The geometry which is analysed with the meso-scale model is the geometry depicted in figure 11. I also used a cropped version of the geometry, because one of the main interests of the civil engineering department was to find out the values of interaction of all areas with all other areas. This produces an output of a size that was not manageable anymore with Quantum GIS [80]. The cropped geometry contains the area around the Negrellisteg in the center and all subareas that are no further away than 1.5 kilometers. The cropping of the geometry is feasible in that sense, that the bridge is planned for pedestrians and bicycles. The average distance that people travel with these modes of transport are generally smaller than the extend of the cropped geometry.

The data to set up the initial distribution of primary and secondary locations of households is made according to data provided by the Swiss statistics office [90] and the statistics office of the city of Zürich [91]. Additionally to the available data, also the estimated number of workers and households for the new developments north and south of the train track field was added. The analysis includes them, because they are either already in construction, or will be in the near future. In the here presented analysis, the strength of abstraction is exhibited. The meso-scale model developed in this thesis does only have one type of business, and does not differentiate between the different business type. This abstraction is often criticised, but in the here described case study, it is a very useful property. It is not known yet, what types of companies will be at the new developments. At this stage, only the size of the provided space for companies is defined. This prevents that additional assumptions have to be made, which would introduce an additional level of uncertainty.

The parameter set that was used is the one that resulted from the calibration described in section 4.2. The simulation was run with the above described basic geometry, and an adapted version with the Negrellisteg. Except for adding the bridge, no values are changed in the second geometry.

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One of the key questions that was evaluated in this collaboration with the civil engineering department of the city of Zürich, was if the effect of the bridge is strong enough so that the two districts interact more with each other. To analyse this property, the meso-scale model is used with a special focus on the strength of attractiveness the different subareas have on each other. As more comprehensively explained in section 3.1, the strength of a kernel on the different areas defines how attractive it is to that subarea, i.e. the fraction it has in the sum of all kernels that reach to this subarea.

Figure 22 depicts the strength of interaction of three different locations with all subareas. The images show that the level of interaction increases significantly between the two districts. This is also the case for locations that are not in the closer vicinity of the bridge, as the bottom panel depicts.

Clearly visible in the left panels is also the boundary the train tracks introduce in the attraction levels of the two districts. Without the Negrellisteg, most of the more attractive subareas are on the same side of the train tracks as the subarea that is looked at. This changes significantly when the Negrellisteg is added. The subareas with high attractive levels are more distributed and also on the other side of the train tracks. Especially apparent is this for the northern district. The two locations depicted in the middle and bottom panels in figure 22 have a few very strong attraction points in the southern district when the bridge is added.

The study of the different single households and the change in attraction they experience when the bridge is added is clearly indicated. The meso-scale model indicates that the interaction level between the two districts would increase significantly, if the bridge is added.

The above analysis was done using the cropped version of the geometry, because of storing all the attractiveness values for all regions. This would have led to an output of a size that would not have been manageable anymore using Quantum GIS [80]. But the aggregated result was also made with the whole city. This means that the output is the sum of all potentials, all subareas have on all subareas. Figure 23 depicts the subareas in the center of the city of Zürich and how they change their overall attractiveness. The rest of the subareas in Zürich only change negligibly.

The biggest influence in the economic attractiveness of different subareas is happening around the bridge heads. Most of the subareas close to the bridge gain in attractiveness. The changes are not extreme, no subarea gains extremely. What is surprising, is that also further apart subareas gain attraction and that all the subareas that lose attractiveness are further away. It is mainly surprising if the urban system is regarded in the traditional sense. In the complexity perspective, with which the model was developed (see chapter 2 and



Figure 22: Interaction level of three subareas with their neighborhood. The red bullets indicate the location and the colors the strength of the interaction the location with the red bullet has with each area. The interaction level is normalised, so that the highest value is equal to one. The left panels depict the situation without the bridge, the right panels with the Negrellisteg.



Figure 23: Change of overall attractiveness of the different subareas in the center of Zürich when the Negrellisteg is added.

section 3.1) it is not surprising that local interventions can have impact in further away subareas.

An interesting property in the result is also that the passage at the HB gains attractiveness. Intuitively, I would have assumed that the passage's value decreases when the Negrellisteg is added. With the bridge, less people need to travel through the passage. Nevertheless, the attractiveness increases. The same effect is also present in a subarea close to the underbridge west of the Negrellisteg. Also this area gains attractiveness, even though it is close to an already existing pathway across the train track field that is one out of two options to cross it. Interestingly, the subarea right next to it loses attractiveness.

The collaboration with the civil engineering department of the city of Zürich led to fruitful results. It offered the possibility to work with real data on real projects. The collaboration also gave insights into what urban planners really need for their work and what types of results they expect from the tools. The collaboration was a good way to test the model and validate that a need exists for urban economic models, even if they just produce qualitative results like the mesoscale model developed within this thesis.

5.2 CHANGING THE CENTER: POSTREITERGEBÄUDE

Section 3.2 describes a setup of the framework when it is used as a sketching tool. The setup should help urban planners and decision makers to sketch the possible impacts of different interventions to the urban system and provide the possibility to gain virtual planning experience.


Figure 24: Geometry used for the simulation, indicated by the blue subareas. The lot of the Postreitergebäude is marked yellow. The simulation does not take the whole city of Basel, Switzerland, into account. This is due to computational limitations of personal computers and the desired resolution. The map in the background is from Stamen [82].

This section describes the results from a thesis elective course in which students used the framework to sketch the possible impacts of their planned designs. Seven students chose to take the thesis elective offered. It is a good number to understand how people use react to the framework and if it is a feasible tool to use in planning practice to ask "what-if" type of questions. It also gave me the chance to closely support them during their thesis work.

The diploma thesis in the architecture department of ETH Zurich simulates a real planning process. The students get a task and a site they have to plan a project for. The students who took the discussed thesis elective course had to change the use of the Postreitergebäude. It is a large building on top of the train track field in the vicinity of the main station Basel, Switzerland. The building covers the area of more than 10'000 m² and has multiple levels. Initially, it was built in the seventies of the twentieth century as an extension for postal services. However, the post recently underwent a reorganisation, during which the Postreitergebäude lost its role as a distribution center. Since then, it is only partially in use. The future of the building is currently unclear and discussions are ongoing of what should happen with it. It is unclear whether it should be teared down or converted to another use. The location of the Postreitergebäude is depicted by the yellow area in figure 24.

The task of the students of the thesis elective course was to use the framework to find an optimal configuration of functions to meet their goals. In the process of evaluating their proposals, they had to adapt the geometry depicted in figure 24 according to their planned inter-

vention and had to vary some of the values. The goal was that they get an idea of the influence different land uses within their projects have. However, I did not provide them with strict guidelines how they should apply the framework and how the analysis in detail has to be carried out. I only introduced them to the capabilities of the framework in a general manner and showed them how they can work with it. This was done on purpose to see how they react to the framework, what types of analysis they would do, and at which stage of the planning process they will use it.

To not lose much time on installing the framework, I provided the students with an image of a virtual machine, in which the whole framework is properly installed and running. The image comprises of Ubuntu [99] as the operating system and the setup described in section 3.2, including Quantum GIS [80] as the user interface, Luci as the middleware [83], and the C++ implementation of the meso-scale model. As virtualisation technology, Oracle VM VirtualBox is used [100].

Figure 24 depicts the area with which the simulations are run. It does not cover the whole city of Basel, Switzerland, and is quite small. There are two main reasons for this. The first and most important one is the computational cost. The virtual machine already introduces some computational overhead. It can not be assumed that all students have access to powerful machines, thus the geometry had to be restricted to cover only a fraction of the whole city of Basel, Switzerland. Another reason for the restriction of the geometry was the available data. Only data from the city of Basel was available, of which the border is just south of the simulated area. The data of the village just south of the city was not available. If more of the city of Basel would have been taken into account, too much weight would have been given to the northern part. This would have introduced a bias to the simulation that would make it not feasible anymore. The compromise between the available data and a simulation realm that enables all students to run the simulations on their computers led to the geometry depicted in figure 24. It has a high enough resolution that single building blocks can be analysed and focuses on the area around the Postreitergebäude.

Out of the seven students that chose the thesis elective course, only one used the framework in an early stage of the diploma work. He planned a partial deconstruction of the building that is replaced with new building parts. The resulting complex of buildings would have a new area of about 80'000 m². Parts of it would serve as an innovation center, a museum, and a library. The question he then asked is, if it makes sense to also add some housing opportunities to the building block. To evaluate this, he developed five alternatives, each with different amounts of housing and office space. The different options are equally distributed: the first option has no housing space, and all $80'000 \text{ m}^2$ are used for offices; the second option uses $20'000 \text{ m}^2$ for housing, and the rest for offices; up to the fifth option, in which all of the available space is used for housing.

The five variants were then simulated, using the earlier described setup. The student compared the different simulation results, with a focus on the closer vicinity of the Postreitergebäude. The results clearly depict that a mixture of housing and offices would lead to the largest revaluation of the district, with a ratio of one unit of housing per ten units of office space. Too much housing would lead to a devaluation of the closer vicinity around the building and also the interaction with the building would only be minimal.

The other students only analysed their project goals after they had developed their project. In these cases, the framework did not serve as a sketching tool to explore different scenarios, but as an analysis tool that sketches the impact of an already planned project.

One student, for example, empties the whole Postreitergebäude and wants to change it to a public space. To be able to fulfill this plan, he needs to rebuild a few houses on the southern side of the building. This gives him the opportunity to densify the rebuilt buildings. He uses this as one of the alternatives for the simulations. North of the Postreitergebäude, a new public space occurs according to his proposal. The students' plan is to densify also the buildings around this public space, with housing and also shopping opportunities. This scenario is another alternative he tested in the simulations.

Initially, the student planned to also remove the bridge that crosses the train tracks right next to the Postreitergebäude. Even though he did not decide to do this in the end, he used this scenario also in the economic evaluation provided by the framework.

The results were not very different from each other, but showed one very interesting fact. Three of the variants he was running in his evaluation showed that there is a high attraction point for primary locations on the southern side of the Postreitergebäude. However, two very different interventions did not show this property. The first was the one were the bridge and the Postreitergebäude are present, and the northern and the southern part were densified. The second result, that showed a very similar pattern, is when nothing is changed, and the Postreitergebäude and the bridge are deconstructed.

This is a very good example to show that very different interventions can lead to similar results. Removing the link between the two sides of the train tracks can have the same effect as densifying both sides and keeping the connection between the two. This is also one point, where the limitation of the framework is depicted. It is important to have urban planners and decision makers present in planning discussions, also when the proposed framework is used. Even though the two results are very similar, it could be that they have very different implications that are not covered by the framework. Thus it is important to regard the sketching framework as a sketching framework that only focuses on a limited amount of properties and does not cover all properties comprehensively. However, it allows to test different scenarios and to analyse them within the scope of the framework.

The setup of the framework that was provided to the students, allowed also to analyse the effects of different interventions around the Postreitergebäude for specific business categories. Not in the sense the multiple business categories could be simulated in parallel, but the focus of the framework could be set to reflect only one type of business. The categorisation used to choose between different business categories is the NOGA² categorisation used in Switzerland. Each business type belongs to one of 21 main categories. These main categories are also subdivided into smaller groups, but the students were only provided with the main categorisation.

One of the students used this categorisation to study the impact of two possibilities on the simulated system. Besides the basic simulation that is run without any changes to the geometry, he added 200 workers to the Postreitergebäude and simulated the effects on commerce, financial services, and education. These are the business types he planned to put into his design proposal.

He concludes from the simulation results that commerce has the biggest influence on the attractiveness of the Postreitergebäude. Similar effects are present for educational services, but less strong. The exploration of the effects when financial services are in focus led to surprising results. Adding financial services to the building increases the attractiveness of the district south of the building for financial services. However, the effect on the building itself is negligible.

The use of the framework in a thesis elective course made it possible to use the framework in an almost traditional urban planning setting. The scenario the students had to work on is a real scenario that is currently in discussion in the city of Basel, Switzerland. The feedback of the students was mainly that they were surprised that some of the projects had a big impact on the wider urban area, whereas other interventions did only change the urban state a little.

Another feedback was that they learned to also think of the influence the wider surrounding subareas have to the building, and vice versa. For some of them it was a new experience, to not only focus on the project itself.

Most of them criticised the framework to give them a lot of additional work. This shows that, even though the framework responds fast, it is a tedious task to sketch the influence and impact of multiple different design proposals. Especially if it has to be done by hand and for the multiple variants.

² Nomenclature Générale des Activités économiques

The students were not used to use such a tool. This can be observed by the stage of the design process they used the model. Only one out of seven student used the framework to explore different use cases in their design proposal. The others used the framework later to justify their design decisions. However, most of the students would in retrospective use the framework in an earlier stage of the design and planning process.

5.3 FINDING SOLUTIONS

This section discusses a case study where the full framework is used in the way described in section 3.3. It describes the whole process of how the framework can be used if a certain goal is defined for the urban system, but the solution to reach it is not known.

To be able to produce many solutions with the MC simulation, a lower resolution subdivision is used compared to the before described analyses. The amount of subareas is reduced from more than 2500 to about six hundred. The basic shape of the city of Zürich is subdivided into hexagonal cells, that have an edge length of 400 meters. Since not the real street networks serves as the border between the different subareas, which would provide additional information to the simulation, the hexagonal pattern minimises a directional bias that would be stronger with other grid shapes. The resulting geometry is depicted in figure 25. The figure also depicts the densities for the workers and households in the respective subareas according to the data from the Swiss statistics office [90] and the statistics office of the city of Zürich [91]. The upper panel depicts the distributions of the households, the bottom panel the distribution of workers.

The first section 5.3.1 of this chapter mimics an urban planning scenario. It uses the introduced framework to find possible infrastructural interventions that can lead to the desired state of the urban system.

In section 5.3.2 the results from the calibration are used to explore behavioural and economic changes, and their influence on the pattern of primary and secondary locations of households.

The interpretation of the results in the following sections are all made within the assumptions of the framework, described in chapter 3. They do not involve any topological information, except for the one provided by the basic geometry used, and also construction laws and politics are not taken into account. The feasibility of the solutions is also not discussed here. The model suggests projects within the capabilities it has and implements. It helps to understand and produce sketches of possible interventions to the urban system that may lead to certain outcomes.



Figure 25: The basic geometry of the city of Zürich is depicted. It serves as the input that is used in the MC runs to propose solutions to urban planners and decision makers. The top panel depicts additionally the distribution of households, the bottom panel the distribution of workers, in units per km².

The colors of the clusters a purely meant as a visual aid and do not have any additional meaning. They are chosen in a way that the colors are as differentiable as possible.

5.3.1 Infrastructural Interventions

This section exemplifies how the framework can be used in an urban planning scenario. The example is purely intellectual and follows the methodology that is described in section 3.3. The goal of the here described example is to reduce the traffic in the travel network. The assumption is, that the traffic is reduced, when the average travel distances of households is decreased. I am aware that this must not necessarily be the case for the whole network. Still, the goal of this example is to find infrastructural interventions that help to reduce the average travel distance of people.

The framework, i. e. the model, does not explicitly implement traffic. Thus the goal has to be refined to meet the properties of the model. One of the key assumptions of the model is an economic adaptation of the first law of Tobler, which states that *"everything is related to everything else, but near things are more related than distant things"* [47]. The model takes this law and adds the idea of agglomeration economics to it. From a households perspective, agglomeration economics states that business agglomeration have a higher attraction to households if their size is bigger. They can even be more attractive to close, but small centers. For the problem the here described case study explores, this means that large business centers have to be decreased and better distributed within the urban area.

The intervention to the city should be minor. The only types of interventions that are allowed in this thought experiment are to maximally add three things to the city, either new links to the travel network or additional buildings.

To get enough proposals of possible interventions, the MC simulation was run for about three weeks on four personal computers, and produced about 3200 results. The results are processed in the way described in section 3.3. Not all areas are used in the clustering, only the subareas of interest. This allows to reduce the numbers of dimensions further, and reduces the errors in the clustering method that are introduced by subareas which are not directly of interest. The clustering was only done using the 46 subareas that are not in the lowest category in the bottom panel of figure 25.

The resulting scatter plot with the two main principle directions is depicted in figure 26. The two principle directions contain about two thirds of the variation of the MC simulation results.

Already the scatter plot depicts, that most of the proposed interventions to the urban realm only have a minor influence on the emerging patterns of primary and secondary locations of households. This is



Figure 26: The scatter plot along the two main principle directions is depicted. The colors depict the four clusters for the chosen distance in the hierarchical clustering (see figure 27). The green hexagon depicts the location of the real pattern within the produced results.



Simulation results

Figure 27: The dendrogram of the hierarchical clustering is depicted. The distance that defines the colors in figure 26 is depicted by the dashed horizontal line. There are four clusters. The smallest of them is hardly visible on the left.

also depicted in the dendrogram shown in figure 27. There are two main clusters, of which the smaller only contains two solutions, and the rest is in the other side of the node.

The distribution of the results along the principle directions, and also the dendrogram helps to understand the distribution of the results. However, it does not show how different the patterns and the impact of the different proposed interventions are, and how the different solutions look like. This is depicted in figure 28. It shows the average difference from the calibrated version, for each subarea and cluster. The standard deviation within the clusters for single subareas are very small, thus are neglected in the here shown case. The distance in the dendrogram is chosen in a way that four clusters are defined.

Figure 28 depicts the average difference from the reference solution of the MC simulation results in one cluster. From figure 26 it is known that the calibrated solution, without any interventions is in the purple cluster. Most of the simulation results in this cluster produced very similar patterns to the reference solution. So this cluster can be neglected. In the current case this eliminates most of the proposed solutions and leaves only 140 as possible proposals.



Figure 28: The average absolute difference for each subarea and cluster, compared to the calibrated result. It allows the planners to select the cluster that contains the desired patterns. The colors in the central edges indicate the cluster the solutions belong to and correspond to the colors in figures 26 and 27.

The patterns of the dark yellow and the gray clusters only differ slightly from the reference pattern. Thus I also neglect the solutions that lead to these patterns. The only cluster that contains solutions that produce obviously different patterns to the reference pattern in the desired form, is the green cluster. It contains only two solutions. According to the hierarchical clustering in figure 27, they have an average distance of more than seven thousand units to all other results.

The solutions in the green cluster change the pattern of primary and secondary locations drastically, compared to the reference solution. They lead to patterns in which the households are more dominant in the sub-centers and firms are more homogeneously distributed, with a peak in the center of the city. The figures that depict the absolute patterns of these two solutions can be found in appendix C.

Both of the simulation results propose an additional link to the traffic network. Additionally one of the proposals also proposes to add a building. The two proposals are depicted in figure 29. The first result proposes a new link that connects the north with the south side of the Limmat river and a new building in the north with a size of 160 units (yellow in figure 29). The second solution proposes a single link in the south, that would serve as a fast link between the two sides of the lake (green in figure 29). Both of the links are pure links between two subareas, i. e. they only connect the two subareas depicted in the figure that are at the ends of the links. The subareas that are crossed by the link can not access it directly.

The calibrated parameter set used for this case study that is closest to the real distribution has an average travel distance of households to spend money of about 2.1 kilometers, with a standard deviation of 555 meters. In the first proposed intervention, the average travel distance reduces to 2 kilometers in average with a standard deviation of 405 meters. The second proposal produces a pattern in which the average travel distance of households is reduced to 2 kilometers too and with even a smaller variance of 390 meters.

Adding only one link to the travel network can thus reduce the variance in travel distances significantly and the average travel distance a little. This indicates that, first, households travel smaller distances, and second and more importantly, the travel distances are more homogeneous among the households in the city.

Interestingly, the second variant that is proposed by the framework, shares some similarities with a project that was suggested and evaluated in Zürich in the seventies of the twentieth century [101], and discussed for several decades. The city planned a tunnel that crosses the lake at a similar location as the proposed link crosses the lake. A big difference between the proposed link and the tunnel that was evaluated, is that the proposed link connects to much further points in the city. The tunnel of the real proposal would have connected mainly the two sides of the lake directly and would have served only



Figure 29: The two proposed interventions from the MC simulation are depicted. The first proposal suggest an additional link to the travel network (yellow rectangle) and a new building in the north with a size of 160 units of space (yellow hexagon). The second proposal suggests a single link that spans from east to west and crosses the lake (green rectangle). Both variants guide the system to a state in which the average travel distance of households is decreased.

motorised private transport. The project of the tunnel through the lake was finally rejected in favor of an alternative project.

After using the framework to explore very different types of possible interventions to reach a goal, the planner can now use the sketching approach described in section 3.2. They could use it to further narrow down the interventions. The framework proposed in the here described case two possible solutions that work within the very abstract assumptions of the model. The sketching approach now gives the possibility to the planner to vary them in a way that might be technically and/or politically more feasible.

The solutions to reach a goal are meant to serve as starting points in the general planning process and help to exclude many of the possible interventions. It enables the decision makers and urban planners to focus on solutions that show the desired effects, without the distraction by solutions that have different and maybe undesired impacts.

5.3.2 Economic Incentives

The intellectual game described in section 5.3.1 exemplifies how the proposed framework can be used in the context of urban planning. It shows that infrastructural interventions to the city can change the pattern. However, the impact of infrastructure on the pattern of primary and secondary locations is generally little. Most of the results from the MC simulation produced similar patterns to the reference pattern, as can be seen in figures 26 and 27.

When also behavioural and economic parameters can be changed, the diversity of results is much higher. In the case of the meso-scale model, the behavioural and economic parameters are the parameters that are set before a simulation is started and generated by the sampling of the MC simulation in the calibration step. The resulting patterns show a much higher diversity compared to the results when only infrastructure can be added, as figure 30 and figure 31 depict.

The clustering depicted in figure 30 and figure 31 clusters according to the same subareas as described in section 5.3.1. Only the distance in the hierarchical clustering is chosen differently. The number of clusters would otherwise be too big to show it in a readable way in this publication. The chosen distance that produces the nine clusters is eight thousand. This produces nine clusters that show significant differences between each other and the reference pattern. Figure 32 depicts the average difference per subarea and cluster. The figures with the absolute values are provided in appendix C. As in section 5.3.1, the standard deviation within a cluster is not significant, only for the gray cluster, which contains almost 75 percent of the results and has a variance per subarea that is not negligible.

The high variance of the values in the ninth cluster is also visible in the boxplots in figure 33. The parameters of the results that are in



Figure 30: The scatter plot along the two main principle directions is depicted for the results of the calibration runs. The colors depict the clusters for a distance of eleven thousend in the hierarchical clustering (see figure 31). The green hexagon depicts the pattern extracted from official data.



Figure 31: The dendrogram of the hierarchical clustering is depicted. The distance that defines the colors in figure 30 is defined by the distance of 8'000, depicted by the dashed horizontal line. With this distance, 9 clusters are defined. They are highlighted by the different colors.



Figure 32: The average absolute difference for each subarea and cluster, compared to the calibrated result. The colored squares at the bottom right edges indicate the cluster and correspond to the colors in figure 30 and figure 31.

this cluster show no clear pattern. They span the whole range of the respective parameter. Only the kernel width σ has a clear tendency to be in the range of between one thousand and two thousand meters.

The cluster that catches the eye most when looking at the parameters, is cluster 5. It contains only four results which is only a small fraction of the total amount of results. All four values share the property that the secondary location choice is depending mostly on the per-capita income, which itself has a very high elasticity. This means that distance has almost no influence into the secondary location choice and the differences in the utilities between the different subareas in terms of per-capita income are also not big. The primary location choice has similar properties, but not so extreme. The main influence into the location utility for residence is depending mainly on the occupation ratio, but has an influence also on the distance to the secondary location. These properties lead to a pattern, where the workplaces are uniformly distributed over the whole city. The households concentrate in the center of the city, where most of the space is provided. This is clearly visible in figure 44 and figure 45 in appendix C, cluster 5. The results in this cluster reflect the emergent pattern of primary and secondary locations, when the commuting distance is the less important factor in the location choice, and rent prices and per-capita income are valued more, with relatively small agglomeration effects.

The cluster with the results that differ most from the other clusters is cluster number 1, as depicted in figure 33. Visually, the differences between this cluster and the rest of the cluster is not very clear. Closer investigate reveals that all the results in cluster number 1 have a very distinct pattern compared to the other clusters. They exhibit many small business districts that are in close vicinity from each other. It is not only a uniform distribution of firms, but a pattern with many small centers. This fits very well with the small value of the kernel density. This means that the catchment area of single companies is very small, i. e. the attractiveness of a business center decrease strongly with distance. The other parameters of the cluster do not exhibit a noticeable range. The results in this cluster can be viewed as the emergent patterns, when transport cost would be increased dramatically, but all other parameters would not be changed.

Clusters 2, 3, and 4 have very similar patterns, mainly differing in the pattern of the primary locations. Cluster 3 contains the reference pattern and results that are similar to it. It is also the cluster, in which the difference is biggest to the other two, in terms of the pattern and in terms of parameters. In the third cluster, the primary location choice is mainly dependent on the distance to the secondary location, whereas the other clusters have a mostly equal proportion of distance and the density ratio. What makes the three cluster differ from each other, is that the households mainly agglomerate dif-



Figure 33: The range of the parameters per cluster is depicted. The red line indicates the mean value. The box ranges from the lower to the upper quartile and the whiskers show the range of all values. The colors at the bottom indicate the cluster the boxplots show the value from, the cluster sizes are shown on top of the figure.

ferently and how this effects the secondary locations. In the second cluster, the residents agglomerate around the multiple centers of the city, but are almost not present at the centers. In the third cluster, the residents are also pushed away from the centers, but less strong. The distribution of the households is more uniform than in cluster two and four. The fourth cluster pushes the households even more to the periphery. They mostly choose the west of the city, the south east, and the north east as their primary locations. All clusters have a small decrease of workers in the smaller business districts, in favor of the geometric center of the city. Some of the households change their secondary location more to the center, compared to the reference pattern. These three clusters contain results that are very similar to the real distribution, but with some small changes in the utility functions of the households. These small differences in the utility functions of the households are a higher elasticity towards rent prices (clusters two and four), or small changes in the transport cost.

Cluster 6 exhibits similar properties in the pattern as cluster 2, but the pattern is more fine grained. The central business district is less distinct from the rest of the city. The pattern of primary and secondary locations is much more complex than in cluster 2. The agglomerations are not very distinguishable from each other, and interwoven with each other. The business districts form a pattern, where multiple arms stretch out from the center, and connect again with each other at the periphery. The households locate between these business arms, but they also intermix with them. The uniform distribution of the firms is strongly interlinked with the small value for the kernel density, which is also observable in cluster number 1. Compared to cluster 1, the weights of the utility function for the secondary location is more distinct. It has a value that gives similar weight to both, the per-capita income and the commuting distance from the primary location. But the scaling values for the per-capita income per subarea is very high, which means that its utility decreases fast with lower incomes. The patterns in cluster number 6 emerge, when the incentive of households is low to travel far to spend money and the location utility function of primary and secondary location is well-balanced.

Cluster seven and eight contain results that have similar parameter sets. The main difference between the two is once more the width of the kernel. In cluster seven, all sixty results have a kernel width that is very small, all in a range of a few dozen meters. The kernel widths in cluster eight is much higher, but also in a small range, relatively speaking. In what they both also differ, but not as clearly, is the scaling factor σ_{d_2} and the distribution of the secondary location utility weight. In cluster eight, σ_{d_2} is much more distinct and has a smaller value. In cluster seven, the distribution is wider, without an apparent value. The secondary location weight is, on the other hand, more distinct in cluster number seven, with a value that slightly has a higher focus on the per-capita income.

Cluster seven has a very similar pattern of the primary locations of households as the pattern depicted in clusters one, six, and nine. But the pattern of secondary locations is very similar to the emergent pattern of cluster number eight. Cluster number eight depicts a pattern of primary locations that is similar to the pattern that cluster number five has. The two clusters generally depict emergent patterns that are set to have different levels of incentives for households to travel certain distances to spend money.

Generally speaking, parameter σ that defines the width of the kernel, is most different between the nine clusters. It influences the size and distribution of the business centers. Another factor that has a big influence on the emergent pattern, and thus to which cluster the result belongs to, is the weighting value α_w . It defines the impact of the two utilities, per-capita income and commuting distance, on the total utility for the secondary location choice.

The parameters that define the clusters have mainly an influence on the secondary location choice. This makes sense in the here carried out exploration, because the focus of it was solely on the existing business centers of the city of Zürich and the change in the number of workers. It was neglecting all other subareas in the clustering process. The clusters and other results would be very different, if other properties were used in the clustering process.

5.4 SUMMARY

This chapter exemplified how the framework described in this thesis can be applied in planning practice. It shows three different ways, how the whole framework, or parts of it, can be used to in different planning scenarios.

In the first section 5.1 of this chapter, a case study is discussed in which the model was used in an actual collaboration with authorities of the city of Zürich, Switzerland. The case study uses the core meso-scale model of the framework to analyse the economic and behavioural impacts of an intervention to the urban system. The authorities of the city were interested in the change of interaction between two currently almost separated districts if an additional pedestrian bridge is added. The two districts are geographically located in close vicinity to each other, but separated by train track fields. In the current situation, the currently existing possibilities to go from one district to the other are very far. Thus the interaction between the two districts is currently almost non-existent.

The construction of the bridge costs several dozens of millions. To find possible investors to build the bridge, the city of Zürich was also interested to find out if the bridge would add economic gain to the land owners and who would profit from it.

The meso-scale model shows that the interaction level between the two districts would increase significantly. This is also valid for the wider area. Not only in the close vicinity of the bridge would benefit from it. However, the analysis shows that the economic benefit would mostly be local. The main beneficiaries, in terms of land value increase and attraction, are the plots that are located around the bridge heads. The most surprising result is, that also the stores in the HB would experience positive effects from the bridge, even though they are currently located at one of the main passages that enable people to cross the train tracks.

The second section in this chapter, section 5.2, describes the use of the framework, when it is used as a sketching tool. To simulate a real world scenario, I offered a thesis elective course for diploma students, in which they used the framework for their diploma thesis. Within the general given task, the students had to analyse different variants that conform to their planned design. The task was to explore different mixtures of land use within their project and figure out the best ratio that helps them to reach their goals for their design.

Most of the students used the framework in a late stage of the design process. They used it mainly as a tool to justify their proposed designs. However, one of the students used the framework in an early stage. He explored different ratios of workers and households to find out what would lead to the desired effects. He based the configuration of the space of his diploma thesis on the findings he concluded by the use of the framework. Generally speaking, the students were surprised by the influence their designs had on the plots around the location of the given task. Some of their proposals had a big impact on the close vicinity. Other projects, of which the students expected a large impact, had only a minor impacts to the surrounding. Most of the students were not used to think about the impact of their designs to the wider surrounding area. They concluded that they would in retrospect use the framework already in a very early stage of the design and planning process, to understand the urban setting around the project better.

The last section, section 5.3, consists of two parts. The first part exemplifies how the framework can be used to find the types of infrastructural solutions that can help to reach a certain goal. It shows that MC simulations in combination with clustering can help the urban planners and decision makers to find very different solutions that all sketch the desired future. The goal of the example is to reduce the average travel distance of people in the city. The framework proposes two solutions to reach this goal. The first proposal is a single link that connects the east with the west in the south of the city. It suggests a link that crosses the lake. This proposal is very similar to an actual proposal that was discussed for several decades in the city of Zürich, Switzerland. The second proposal the framework suggests, is to add a link that connects the north of the city with a subarea west of the center. Additionally it proposes to add a building with 160 units of space to the very north of the city. Both proposals should not be seen as absolute solutions, but should be regarded as starting points. If they would be used in a real scenario, the planners could use the two proposal as general ideas of how possible interventions might look like and at what they can focus. The proposed solutions need to be refined to meet technical and political feasibility, because these are properties the framework does not focus on. The main gain planners have by using modelling and simulation in this way is, that they can exclude plans that are less likely to reach a certain goal and supports them to find infrastructural interventions that have a higher chance to lead to the desired state.

The second part of section 5.3 investigates the differences of the patterns of primary and secondary locations of households in the context of economic and behavioral changes. It uses the results from the MC simulation used for the calibration to understand the impact of changes in behaviour and economic incentives. The differences in the emergent patterns are much more significant and the analysis shows that the model is much more sensitive to changes in these parameters than to geometrical parameters, i. e. to changes in infrastructure.

Part IV

DISCUSSION & CONCLUSION

This chapter concludes the thesis. First, a summary of the carried out research and its context is provided in section 6.1. Section 6.2 discusses the findings of this thesis in relation to the research objectives defined in chapter 1. The last section of this chapter, section 6.3, concludes the accomplishments achieved in this research and section 6.4 posits future research directions.

6.1 THESIS SUMMARY

The carried out research in this thesis combines the two fields of urban economics and urban planning support systems. It does this by the means of applying methodologies from computational science and complexity science.

The core of the framework has its roots in urban and spatial economics. Urban economics describes and theorises about the location and patterns of activities in the urban realm, using precise mathematical descriptions. It provides a well founded basis for modelling and simulation of urban systems.

The early models of urban and spatial economics were already well developed when modern tools and methods became available. Due to the computational restrictions, this led to a top-down view on how systems were perceived. Top-down in this context means that the urban system was regarded as a system, in which all parts have their role. Another strong assumption in neoclassical urban economics is the assumption of equilibrium. This assumes that a system has a fixed state, towards which it always develops. If externalities push the system away from this state, it tries to reach this state again.

When observing urban systems, these assumptions of neoclassical models have been found to be wanting. Urban systems are never in an equilibrium state; the interaction of its inhabitants and other actors force it into a state of constant change. This change is not ordered, and experiences phases of near chaotic behaviour. No central controller or invisible hand steers the system and its actors. The central authorities of a city also do not have this role. They can enforce some rules on the actors in the system, and may occasionally take some actions. However, these actions are often slow and the actors adapt to it quickly and possibly in unexpected ways.

This way of theorising about systems of any kind has its roots in complexity science. Complexity science is a science that drops many of the assumptions from neoclassical sciences, and changes the per-

spective on systems. It regards them from a bottom-up perspective, which means that a system consists of many smaller units that interact with each other and are not steered by a central control. The interaction between these atomic units then defines the system's state. From an economic perspective, this means that not all actors are necessarily satisfied at one point in time, but rather change and adapt their behaviour constantly to optimise their utility. In reality, the behaviour and choices of inhabitants and other actors in urban systems are not strictly rational in the economic sense. This introduces a level of uncertainty to the dynamics and emergent patterns of urban systems that makes it a difficult task to foresee the impact and implications of actions to it, if not impossible. This is also one of the main reasons, neoclassical economics are not often used in planning practices. The strict assumptions they take often restrict their applicability for real world use cases, thus only few examples are known where they have been successfully applied.

The proposed framework is developed from this context. Existing models and frameworks can simulate urban systems from the perspective of complexity science. However, in general, these frameworks introduce a level of detail and computational overhead that is not needed in many stages of urban planning and decision making processes; or on the other hand, there are existing models that are too abstract and complicated to set up. The core of the proposed framework aims to reduce the abstraction level of the simple models, but does not assume a level of detail of large scale models, which are generally very data hungry. The core model of the framework thus simulates in the meso-scale. From the perspective of the modeller, the model acts like a large scale model, where each agent and actor can be modelled, but in the background, it works similarly to the simple and fast models.

In order to simulate atomic agents, that are not perfectly rational, the core model builds a heuristic for each group of agents, according to which they interact with the other agents and the environment. Basically, the core of the framework works directly with probabilities, whereas large scale models have to run multiple times with a high cost to produce probability distributions of, for example, primary and secondary locations, on which the planners and decision makers can base their decisions on. When single simulation runs are used as final results of large scale models, the informative value of the large scale frameworks is relatively low.

Moving away from the high resolution methods for complex systems introduces additional challenges to the framework, which must be resolved. One such challenge is to simulate the models in a "natural" way. This means that the simulation evolves in a way which would be the development process that is most natural from an initial condition. The proposed framework copes with this challenge by using techniques known from optimisation, in which the goal is to find the local optimum from an initial state. The proposed framework does not explicitly optimise any property, it lets the model evolve according to the above mentioned heuristic, by using techniques from optimisation.

The proposed framework provides support in different stages of the planning and decision making processes. Planners and decision makers can get support already in early and very general planning stages. The proposed framework can make recommendations for possible interventions, particularly when the planners and decision makers want to develop an urban system into a certain direction. The quantity of possible interventions that can be applied to the urban system is normally so large, that it is not possible to think of all of them. Additionally, since the urban system is a picture-perfect example of a complex system, the implications and impacts are hardly foreseeable. This is not only a challenge, but also provides additional opportunities. Even small changes to the system might lead it to the desired state, naturally, without using much energy or force.

When used in this context, the framework uses concepts from Monte Carlo simulations and machine learning to explore the solution space, and proposes possible interventions of different magnitudes which could lead the urban system to the desired state.

The proposed framework also provides the possibility to be used in a slightly different context. It can be used as a sketching tool in which the planners can roughly sketch the intervention or action, and explore the impact it may have. This exploration can be carried out for several different alternatives. The different alternatives can then easily be compared to each other, which helps to narrow down the alternative with the desired implications and impact, and exclude the alternatives that have undesired effects.

Last, but not least, the framework may support planners and decision makers in doing a more detailed analysis of a planned project. Theoretically this would already be possible in the other use cases of the framework, but for larger geometries and with many alternatives, the output increases to an amount that is hardly manageable with currently available software.

Using the framework as an economic and behavioural analysis tool enables the planners and decision makers to investigate different properties. These properties range from the levels of interaction between different subareas in the city to the amount of people that commute between areas. The framework can also estimate how land values might change, as well as estimate the pressure on housing and work place per subarea, among others.

The research carried out in this thesis developed and proposes a framework that has the capabilities to support urban planning and decision making, at different stages and helping to better understand the impact and implications different actions and interventions might have.

The use of the proposed framework is not limited to the context of urban planning and decision making. Many of the dynamics and properties the core model contains are rooted in urban and spatial economics, but with a complexity perspective. This enables researchers to explore models from neoclassical economics in the context of complexity science, and study the emergent patterns of activities in a spatial setting in which many of the strong assumptions neoclassical economics had to make can be dropped.

The proposed framework has a strong foundation in spatial and urban economics, but it is not restricted to it. The part of the core model that enables the evolutionary process, does not directly depend on the economic features, it only needs a heuristic on how to evolve. The heuristic is calculated separately form the time integration in order to consider other use cases outside of, or in combination with spatial and urban economics.

6.2 RESEARCH OBJECTIVES

This section discusses the methods and results that have been applied within the context of the research objectives described in section 1.1.

The overarching aim of this research is to test if modelling and simulation can support planners in the planning process by providing them with solutions to a problem that lead to certain states of the urban system. This question can be answered with yes and no. No in the regard that it is not possible with current knowledge and technology to comprehensively model the whole urban system, taking all dynamics, influences and other properties into account. Especially because current literature mostly agrees that urban systems are complex systems, with phases in which the urban system is close to chaotic behaviour and phases in which it is almost in equilibrium states. These phase changes are hard to foresee and can be very unpredictable. Thus, with current knowledge, there cannot exist a tool that predicts the possible impacts and implications of interventions and actions precisely. This implies that there can not exist any tool that can provide planners and decision makers with proposals of how the urban setting might be changed to develop into a desired direction.

On the other hand, the question can be answered with yes, if the assumption on the precision of the expected results is relaxed, and not interpreted as absolute. The results of simulations are only meaningful within the assumptions of the simulation model. When the model does reflect dynamics in the granularity that provides the necessary information to fulfill a task, it does not introduce additional uncertainties that pretend a precision that is not available. The models of this category are models that should be regarded as sketches of a system, outlining the prominent features without the details. In this respect, simulation can provide great value to the understanding of the impacts and implications certain actions have, but only within the assumptions and implemented dynamics of the model. In the model and framework proposed in this thesis, the focus of the model is economic. It uses well founded theories from neoclassical urban and spatial economics to sketch and predict the probabilities of where certain activities are and may be located.

Neoclassical urban economics is a science that precisely describes the behaviour of agents that leads to certain patterns. This is not a desired property when it is used in planning. In reality, agents adapt their behaviour and, economically speaking, try to maximise their overall utility constantly. To meet these properties, the core model of the framework is built to reflect many of the properties of complexity science. In this context, complexity science theorises about systems that change and adapt, and consist of many actors that interact with each other. In order to build the core model in this way, techniques from different computational and mathematical fields are used, ranging from Markov chains, and probability theory, to optimisation methods.

The core model of the proposed framework can simulate real world distributions reasonably, and give the planners and decision makers an idea how a city might develop in the future, and especially how the patterns could change with certain interventions and actions.

When the simulation results are regarded as sketches of possible future states, modelling and simulation can provide a positive answer to the overarching research objective of this thesis. Additionally, when the model is developed to produce reasonable results, which is indicated for the core model of the proposed framework, it can be run with various setups. From the results, it is then possible to select which sketch the desired outcome. Planners and decision makers can then select the simulation results which support their proposals for which interventions and actions can lead to the desired outcomes. This is the solution to the overarching research objective that is suggested in this thesis.

The drawback of this method is that in this way, it needs some time to pre-calculate the interventions and actions it proposes, which can take up to several days. However, this step is not meant to be interactive. It should provide planners and decision makers with general possibilities to intervene into the urban system, and provide them the possible impact of the intervention.

The proposed interventions should be viewed as a general direction of how the interventions and their outcomes might look like. The quantity of possible solutions is so large, that it is not computationally feasible to test all of them. Therefore, the planners and decision makers can use the framework after this step to further explore variations of the proposed actions. Thus enabling them to make the solutions more feasible, for example, in a political sense. To do this, the framework provides an interface to change the urban system and run the simulations. Running a single simulation is almost interactive, and does not require much time to calculate, particularly if the subdivision of the urban realm has a good resolution for the calculation. This gives the possibility to use the tool directly in planning discussions or workshops. The proposals that have been produce in a "how-to" scenario, can be further used in a "what-if" scenario, in which the interaction between the planners and the framework can be directly used in discussions to justify ideas and variations of different options.

6.3 CONCLUSION

Managing and planning cities has never been a simple task. Urban planning is a many-stage process that involves resolving inherent conflicts between many different interest groups and their representatives. With the rapid increase of sizes of cities, also the number of stakeholders has increased. This makes it an even more complex task in modern cities that is hardly manageable with traditional methods.

The research carried out in this thesis proposes a framework that has the capability to support urban planners and decision makers explore the emergent patterns of residential and business locations as well as the interaction between them. It supports them to appreciate the complexity an urban system has and helps to explore the possible impacts certain actions might have.

Workshops with the authorities from the city of Zürich, Switzerland, and discussions with Chinese authorities at an exhibition in China indicate the enormous need for tools that sketch possible future developments. Often the impact of certain plans is hard – if not impossible – to understand and may have side effects that lead the system to a state that contradicts with the desired outcome in the worst case.

This thesis investigates how planners and decision makers can be supported to better justify and understand the implications of certain actions and interventions to the urban system. It proposes a framework that has the capabilities to fulfill not only this task, but also help planners find different interventions and actions. It enables the planners and decision makers to first define a goal they want to reach, and then proposes possible variations. It enables a workflow in which the user is first provided with different options on how a goal can be reached, asking "how-to" type of questions. The different proposals can then be further investigated and adapted to meet, for example, political consent. This stage of the planning process is known as asking "what-if" type of questions. The proposed options from the first step are adapted and analysed. It may be, that already small adaptations to the proposed options may lead to a different pattern of residential and business locations and the interactions between them. The proposed framework can indicate if this would be the case, thus advancing the planning and decision making process, already in early stages of the process.

6.4 OUTLOOK

Even though this thesis establishes a framework that supports urban planning and decision making processes, and has been successfully tested in real and semi-real use cases, there is great potential for further exploration.

1. The core model of the proposed framework is an approach to implement ideas from neoclassical urban economics with the perspective of complexity science. It does this by weakening some of the assumptions made in the neoclassical case, but abstracts other properties. For example, the model used in this thesis only assumes one type of business. This is an abstraction of the urban system that is not done in neoclassical urban models. An important factor in all models introduced in section 2.1 are that different types of industry branches are assumed. This is therefore an opportunity for further investigation, especially because neoclassical economics assumes that the interaction between different companies and business types is one of the main properties that lets non-manufacturing and non-retail firms agglomerate.

The same homogenising property is used for households. Even though the core model of neoclassical economics do not focus on the location choice of different household types, recent literature indicates that the different types of possible neighborhoods have an influence on the primary location choice of households [102].

These properties would also help urban planners and decision makers to better understand the development of possible patterns in the urban system, because the agglomerations would also indicate the different types of businesses. This thesis did not focus on these properties and extend the model in this regard, because the general framework is of interest, and the traceability of the initial model would have been decreased. Nevertheless, this is a direction that offers potential for further research.

2. One of the important properties of complex systems is that some actors have a memory and adapt their behaviour according to the knowledge gained. This is even more present in urban systems, where the actors are people. This property is only implicitly applied in the core model of the proposed framework with the annealing mechanism.

Modelling memory explicitly in the proposed framework could improve the generality of the model. It would extend the core model of the framework also to other use cases, for example to study behaviour of actors in complex systems, and how it changes over time.

3. The annealing mechanism from point 2 forces the model to find a steady state that can be easily understood and analysed. But as mentioned multiple times in this thesis, complex systems experience constant change, they are never in a "final" state.

In this regard, additional research is needed to make the dynamic results more comprehensible, especially in the context of decision support systems. The dynamic property should further developed by finding ways to also visualise them in a clear and understandable way.

- 4. The model framework has only been extensively tested with data from the city of Zürich, Switzerland, and some tests have been carried out with low resolution and estimated data of Singapore, which is not discussed in this thesis due to data restrictions, but mentioned for completeness. More evaluations and use cases were not possible due to difficulties of the data gathering process and the available data in general. More research would be needed to further evaluate the feasibility of the proposed framework, and, more importantly, to further test the generality of the core model and the framework.
- 5. Visual language is important in urban decision support systems. It allows for stakeholders of different fields to find a common language, which is particularly important because urban planning and decision making often involves participants and interest groups with different expertises and backgrounds.

The proposed framework implements an interface when it is used in "what-if" type of discussions. The framework does not provide an interactive interface for the "how-to" scenario. This would provide additional value to the exploration of the proposed solutions and give the planners and decision makers the chance to better select solutions with the desired effects.

6. The framework, as it is proposed in this thesis, focuses on the primary and secondary locations of households. In this thesis, it stands as a single tool. This is not an inherent property of the model. Integrating it into a larger planning support system would provides additional information on the proposed solutions. It would increase the information the urban planners and

decision makers would get from the framework, and could help to further justify the given solutions.

Part V

APPENDIX


This appendix describes the software architecture and core parts of the C++ implementation of the meso-scale model. The software architecture follows the basic idea of the Fortran implementation of CMA-ES developed by Hansen [103]. This style of implementation proofed to be very flexible in terms of using the system in different use cases.

The whole implementation consists of three main classes, of which two are abstract classes. These two abstract classes *DataReader* and *DataWriter* give the user the flexibility to implement own input and output formats. The implementations of the input reader must provide some predefined functions, in order that the main simulation can understand the input data. The core of the program consists of the class *EcoSim* that implements the simulation. When the class is set up, the main function to run the simulation is the *iterate* function. The output writer uses the current *EcoSim* object to gather all the data and write it to a desired output and format.

The setup of the whole simulation consists only of a few commands, as depicted in listing 1.

The core of the model implementation, the class *EcoSim*, has one main function in which the calculations happen. The rest of the functions provide an interface to the outside, where the different calculated variables can be accessed through. The main function is the function *iterate* which takes an integer as an input. This integer defines the number of iterations the simulation will run until it breaks.

Listing 1: Commands to setup a simulation

```
/* Initialise the input reader, here a reader is used that takes
    a GeoJSON file as an input, and parses it. The input reader
    must inherit from DataReader. */
GeojsonReader reader(&inStr);
/* Initialise the simulation, with the input reader as input. */
EcoSim sim(&reader);
/* Initialise the output writer. In this case it produces a
    GeoJSON file as the output. The output writer must inherit
    from DataWriter. */
GeojsonWriter writer(&sim);
```

When it breaks, the current simulation state is stored and can be continued when the function *iterate* is called again.

This allows the user to, for example, store states of the simulation results or to process the current state in any other way, and continue with it afterwards.

Most of the calculations introduced in section 3.1 can be expressed as matrix calculations. For this reason, I implemented the numerical part of the simulation using the Armadillo linear algebra library [73]. It provides the user with an easy to use interface for linear algebra calculations, that can be connected to different, highly optimised linear algebra backends. This way of implementing the numerical part, gives the possibility to use different commercial implementations of linear algebra libraries, GPU enabled libraries, but also open source libraries (see section 3.1.6).

For each iteration of the simulation, first the utilities of the workers and households of each subarea are calculated and stored in matrices. Together with the current distribution of primary and secondary locations, these matrices are aggregated to the new transformation matrix, as if now space constraints were present.

To ensure that the space constraints are fulfilled, the transformation matrix has to be adapted. This process is depicted in listing 2 and is explained comprehensively in section 3.1.

After the new transformation matrix is calculated, the transformations are applied to the primary and secondary locations of households. The last step the simulation performs in one iteration is the recalculation of the money flows between the different subareas. After this step, the simulation continues with the next iteration. Listing 2: Implementation of the limited capacity distribution

```
void EcoSim::calcRedistributionMatrix(mat* initDist, vec* agents,
     vec* capacities) {
    /* Calculate the new distribtion, without space constraints.
        */
    vec tmp = (*initDist)*(*agents);
    /* Check if all constraints are fulfilled, if so, the
        function returns */
   vec a = (*capacities) - tmp;
    if (all(a >= 0)) {
        return;
    }
    /* Setup temporary matrices */
    mat toD = *initDist;
    int nrA = capacities->n_elem;
    /* Extract the subareas that are already full, or still have
        space with the current transformation matrix */
    uvec al0, age0;
    al0 = find(a<0);
    age0 = find(a>0);
    /* Adapt to transformation matrix so that no more people than
         possible are moved to the different subareas. */
   mat dt;
   dt = toD;
    dt.rows(al0) = (mat(al0.n_elem, nrA, fill::ones) +
            repmat((a.elem(al0))/tmp.elem(al0), 1, nrA)) %
            toD.rows(al0);
    initDist = &dt;
   vec geFac = sum(toD - dt, 0).t();
    initDist->rows(age0) = initDist->rows(age0) +
        normalise(initDist->rows(age0), 1, 0) %
        repmat(geFac.t(), age0.n_elem, 1);
    /* Proceed the iterative adaptation of the transformation
       matrix. */
    this->calcRedistribution(initDist, agents, capacities);
};
```

This appendix depicts additional results of the different evaluations performed in chapter 4. They are mentioned in the main text, but no figures are provided in the main body of it.

B.1 RACETRACK ECONOMY

This section provides additional results to the evaluation of the mesoscale model on the racetrack economy. It provides additional results of simulation runs with different transportation costs, i. e. with different kernel sizes of the centers.

The complete base parameters of the model runs in section 4.1.1 is shown in the following table:

| Parameter | Value | Description |
|----------------------------|-------|---|
| σ_{d_1} | 100 | Distance scaling primary location choice. |
| $\sigma_{ ho}$ | 100 | Density scaling primary location choice. |
| $\alpha_{\rm r}$ | 0 | Weighting factor secondary location choice. |
| σ_{d_2} | 100 | Distance scaling secondary location choice. |
| $\sigma_{ar{\mathcal{W}}}$ | 100 | Income scaling secondary location choice. |
| α_w | 1 | Weighting factor secondary location choice. |
| σ | 500 | Kernel width. |
| βo | 1 | Initial step size. |
| α_{β} | 0.999 | Step size reduction factor. |
| Ν | 5000 | Number of iterations. |

The input of the geometry of the model landscape is the same as used in section 4.1.1. There are twelve equidistant regions which are only connected to their direct neighbors. This forms a one-dimensional simulation realm with periodic boundary conditions. The limitation of the capacity of all the regions is set that it is possible to house all activities in one region. The distance between two neighboring regions is about 141 units, with a regions edge length of 100 units.

The travel cost of residents is implied by the kernel width σ . It describes the distribution size of the travel distances of households, if the firms were equally distributed throughout the model realm. Changing σ mimics changes in travel cost.

In the following, the simulation results of changes of σ are depicted. Decreasing the kernel width has the effect that multiple centers emerge.



The above figure is the same simulation result that is already depicted in section 4.1.1, with $\sigma = 500$. In the following pictures, the left panel depicts the distribution of the housholds, i. e. the number of households per region. The right panel depicts the number of households working in that region.



When the kernel width decreases to $\sigma = 250$ the result is still the same as above, but the location of the single agglomeration is at a different location.



With $\sigma = 125$, no single agglomeration emerges anymore, but three of them. They are all of equal size and have all the same periphery size.



The number of agglomerations in the realm increases when σ is further decreased to 60. The five agglomerations are all of similar size. This is surprising, because the distance between the centers of the regions are more than double the kernel size.



When the kernel width is much smaller then the distance between two neighboring regions, only a few regions are left that house no households and workers.

B.2 CENTRAL PLACES

This section provides the interested reader with additional results of the two simulation runs performed in section 4.1.2. These two simulation results were performed in such that they do not develop to an equilibrium state, but rather change they state continuously.

The parameter set used for the simulations is shown in the following table:

| Parameter | Value | Description |
|----------------------------|-------|---|
| σ_{d_1} | 0 | Distance scaling primary location choice. |
| $\sigma_{ ho}$ | 0 | Density scaling primary location choice. |
| $\alpha_{\rm r}$ | 0.5 | Weighting factor secondary location choice. |
| σ_{d_2} | 0 | Distance scaling secondary location choice. |
| $\sigma_{ar{\mathcal{W}}}$ | 100 | Income scaling secondary location choice. |
| α_w | 1 | Weighting factor secondary location choice. |
| σ | 300 | Kernel width. |
| βo | 0.25 | Initial step size. |
| α_{β} | 1 | Step size reduction factor. |
| Ν | 40 | Number of iterations. |

The households are equally distributed in the landscape and do not change their primary location during the simulation runs. The secondary location of the households is changed in every iteration step, depending only on the per-capita income of workers at a certain location.

The following figure shows the state of the system after 30 iterations. As mentioned in section 4.1.2, especially at the beginning, the boundary seems to have a big influence on the emergent pattern. The first agglomerations start to develop close to the corners and along the boundaries.



The following figure depicts the state of the system after 35 iterations. It can be observed that the agglomerations start to develop towards the center of the grid and get more distinct.



The following figure depicts the state of the system after 30 iterations, with a round grid. Similar effects as with the squared grid can be observed. The agglomerations first appear in the vicinity of the boundaries, and develop toward the center.



After the 30th iteration, a large, highly dense business cluster has emerged at center of the grid. As depicted in the lower panel of figure 15, this is not a final state, but develops afterwards to a polycentric pattern.



B.3 FULL CALIBRATION RESULTS

This section provides the full images of the calibration results performed in section 4.2. It provides the full overview of the three patterns that are discussed in that section, i. e. the real pattern according to the data provided by the Swiss statistics office [90] and the statistics office of the city of Zürich [91], the pattern of the best fitting result from the MC simulation, and the pattern of the worst fitting result.

The color classification is chosen differently than in subsection 4.2.2, because the agglomerations in the sub-centers of the city are smaller, thus would not be clearly distinguishable with the dominant center of the city. The classification is set in such a way, that the number of subareas of one color is the same for all colors.

Figure 34 depicts the densities of workers and households according to the official data, top and bottom, respectively. It also serves as the defining figure for the classification, only in this figure the number of areas with one color is the same for all colors. Figure 35 depicts the densities for the result of the MC simulation that matches the real distribution most closely. Figure 36 depicts the densities of the result with the biggest difference to the real densities.

The following table shows the different parameter sets for the best and worst fit solutions. What caches the eye is the low weight the distance gets in the primary location choice of households in the worst fit solution. Together with the small kernel density, this could be a reason for the emergent pattern.

| Parameter | Best fit | Worst fit |
|----------------------------|----------|-----------|
| σ_{d_1} | 17.6 | 46.5 |
| $\sigma_{ ho}$ | 0.314 | 8.11 |
| $\alpha_{\rm r}$ | 0.300 | 0.036 |
| σ_{d_2} | 21.5 | 1.74 |
| $\sigma_{ar{\mathcal{W}}}$ | 6.23 | 39.6 |
| α_w | 0.207 | 0.855 |
| σ | 2095 | 465 |
| βo | 0.981 | 0.990 |
| α_{β} | 0.99 | 0.99 |

In the best fit solution, the parameters are more homogeneous. For example, the distance has similar weight for both, the primary and secondary location choice.



Figure 34: Distribution of households and workers, top and bottom, respectively. The number are according to official data. The subareas are colored in quantiles, i.e. the colors contain the same number of subareas. The units of the legend are entities per km².



Figure 35: Distribution of households and workers, top and bottom, respectively, according to the simulation run that produces the closest fit to the real distribution. The subareas are colored in the same range as in figure 34. The units of the legend are entities per km².



Figure 36: Distribution of households and workers, top and bottom, respectively, according to the simulation run that produces the worst fit to the real distribution. The subareas are colored in the same range as in figure 34. The units of the legend are entities per km².

This appendix depicts the full distributions of the two MC results that could lead to the desired state described in section 5.3. The goal of the intellectual game is to have a city in which the business centers are less distinct and the land uses more mixed. This would lead to a reduction of the use of the transport networks, because the house-holds would not need to travel so far to buy goods, and the average commuting distances would decrease too.

In the first case study, only infrastructural interventions are allowed. Changes in travel cost or other incentives to reduce the travel distances are not allowed in the first example.

Figure 37 depicts the real distribution according to data from the Swiss statistics office [90] and the statistics office of the city of Zürich [91].

Figure 38 and figure 39 depict the patterns that result from the respective interventions, as depicted in figure 29.

The two proposed interventions lead to a better distribution of firms. The number of subareas that are in the second highest range, with density of 2927 to 9110 workers per km², increases significantly. The number of subareas that are in the highest category, ranging from 9110 to 82210 workers per km², stay almost the same, but are more similar to a monocentric city pattern. The values of the subareas differ quite a bit for the highest category, which can not be seen in the figures. The real pattern has in the center five subareas that contain more than sixty thousand workers per km². The area with the highest density of workers in the patterns of the proposed solutions are about 33 thousand workers per km². The two subareas with these values are not in vicinity with each other, one is in the center, the other more in the west.

The households also change their primary location. Because the firms generally distribute better in the city, the utility of moving to one of the sub-centers increases. But this effect is not as drastic as the new pattern of firms in the city.

Figure 44 and figure 45 depict the average results from the clusters made using the calibration results from section 5.3.2. For convenience, the other figures from the section are provided too.



Figure 37: Distribution of households and workers, top and bottom, respectively. The numbers are according to official data. The subareas are colored in quantiles, i. e. the colors contain the same number of subareas. The units of the legend are entities per km².



Figure 38: Distribution of households and workers, top and bottom, respectively. The numbers are from the first result in the green cluster. The subareas are colored the same way as in figure 37. The units of the legend are entities per km².



Figure 39: Distribution of households and workers, top and bottom, respectively. The numbers are from the second result in the green cluster. The subareas are colored the same way as in figure 37. The units of the legend are entities per km².



Figure 40: The scatter plot along the two main principle directions is depicted for the results of the calibration runs. The colors depict the clusters for a distance of eleven thousend in the hierarchical clustering (see figure 41). The green hexagon depicts the real pattern.



Figure 41: The dendrogram of the hierarchical clustering is depicted. The distance that defines the colors in figure 40 is defined by the distance of 8'000, depicted by the dashed horizontal line. With this distance, 9 clusters are defined.



Figure 42: The average absolute difference for each subarea and cluster, compared to the calibrated result. The colors in the central edges indicate the cluster the solutions belong to.



Figure 43: The range of the parameters per cluster is depicted. The red line indicates the mean value, the box ranges from the lower to the upper quartile, and the whiskers show the range of the values. The colors at the bottom indicate the cluster the boxplots show the value from, the cluster sizes are shown on top of the figure.



Figure 44: Pattern of the distribution of households in the city. The nine figures depict the average value of the subareas of the corresponding clusters from section 5.3.2.



Figure 45: Pattern of the distribution of workers in the city. The nine figures depict the average value of the subareas of the corresponding clusters from section 5.3.2.

- [1] W. B. Arthur. *Complexity and the Economy*. Oxford University Press, 2015.
- [2] T. Chandler. *Four Thousand Years of Urban Growth: An Historical Census*. St. David's University Press, 1987.
- [3] T. Brinkhoff. *Major Agglomerations of the World*. Last accessed: April 25, 2016. URL: http://www.citypopulation.de/.
- [4] UNFPA. State of World Population 2007. 2007.
- [5] UN-Habitat. "Economic Role of Cities." In: Global Urban Economic dialogue series (2012).
- [6] R. Florida, T. Gulden, and C. Mellander. *The Rise of the Mega-Region*. Working Paper Series in Economics and Institutions of Innovation 129. Royal Institute of Technology, CESIS - Centre of Excellence for Science and Innovation Studies, 2008.
- [7] L. Bettencourt and G. West. "A unified theory of urban living." In: *Nature* 467.7318 (2010), pp. 912–3.
- [8] D. B. Lee. "Requiem for Large-Scale Models." In: Journal for the American Institute of Planners 39.3 (1973), pp. 163–78.
- [9] M. Batty. "Evolving a Plan: Design and Planning with Complexity." In: *Complexity, Cognition, Urban Planning and Design*. Springer, 2016, pp. 21–42.
- [10] J. Portugali and E. Stolk. Complexity, Cognition, Urban Planning and Design: Post-Proceedings of the 2nd Delft International Conference. Springer, 2016.
- [11] A. Mikhailov. "Artificial life: an engineering perspective." In: Evolution of Dynamical Structures in Complex Systems. Springer, 1992, pp. 301–312.
- [12] N. F. Johnson. *Simply Complexity: A Clear Guide to Complexity Theory*. Oneworld, 2009.
- [13] J. Jacobs. *The life and death of great American cities*. 1961.
- [14] M. Batty. *The new science of cities*. MIT Press, 2013.
- [15] S. Brakman, H. Garretsen, and C. van Marrewijk. *The New Introduction to Geographical Economics*. Cambridge University Press, 2009.
- [16] E. Koomen and J. B. Beurden. *Land-Use Modelling in Planning Practice*. GeoJournal Library. Springer, 2011.
- [17] A. O'Sullivan. Urban Economics. McGraw-Hill Companies, Incorporated, 2009.

- [18] M. E. Edwards. *Regional and urban economics and economic development: theory and methods*. Auerbach Publications, 2007.
- [19] S. N. Durlauf, L. Blume, et al. *The new Palgrave dictionary of economics*. Vol. 6. Palgrave Macmillan Basingstoke, 2008.
- [20] J. H. von Thünen. Der isolierte Staat in Beziehung auf Landwirtschaft und Nationalokönomie: Zweite vermehrte und verbesserte Auflage.
 G. B. Leopold's Universitäts-Buchhandlung, 1942.
- [21] J. Portugali. "Complexity, Cognition and the City (Understanding Complex Systems)." In: J. Artificial Societies and Social Simulation 15.2 (2012).
- [22] W. Alonso. *Location and land use: toward a general theory of land rent*. Publication of the Joint Center for Urban Studies. Harvard University Press, 1964.
- [23] R. F. Muth. *Cities and housing*. University of Chicago Press, 1969.
- [24] E. S. Mills. "An Aggregative Model of Resource Allocation in a Metropolitan Area." English. In: *The American Economic Review* 57.2 (1967), pp. 197–210.
- [25] W. Christaller. Die zentralen Orte in Süddeutschland: Eine ökonomischgeographische Untersuchung über die Gesetzmässigkeit der Verbreitung und Entwicklung der Siedlungen mit städtischen Funktionen. Wissenschaftliche Buchgesellschaft, 1933.
- [26] A. Lösch. *Die räumliche Ordnung der Wirtschaft: Eine Untersuchung über Standort, Wirtschaftsgebiete und internationalen Handel.* University of California, 1940.
- [27] P. Krugman. "On the number and location of cities." In: *European Economic Review* 37.2–3 (1993), pp. 293–298.
- [28] P. Krugman. "Increasing Returns and Economic Geography." In: *Journal of Political Economy* 99.3 (1991), pp. 483–99.
- [29] G. Ottaviano and J. F. Thisse. "Agglomeration and economic geography." In: *Handbook of regional and urban economics* 4 (2004), pp. 2563–2608.
- [30] P. Schirmer, M. van Eggermond, and K. W. Axhausen. "Measuring Location in Residential Location Choice: An Empirical Study on the Canton of Zurich." In: 13th International Conference on Computers in Urban Planning and Urban Management. Utrecht, 2013.
- [31] R. E. Lucas. "Asset prices in an exchange economy." In: *Econometrica: Journal of the Econometric Society* (1978), pp. 1429–1445.
- [32] S. Lai and H. Han. *Urban complexity and planning: Theories and computer simulations*. Ashgate Publishing, Ltd., 2014.

- [33] B. Raney, N. Cetin, A. Völlmy, M. Vrtic, K. Axhausen, and K. Nagel. "An agent-based microsimulation model of Swiss travel: First results." In: *Networks and Spatial Economics* 3.1 (2003), pp. 23–41.
- [34] W. B. Arthur. "Inductive reasoning and bounded rationality." In: *The American economic review* 84.2 (1994), pp. 406–411.
- [35] Adam Smith and M Garnier. *An Inquiry into the Nature and Causes of the Wealth of Nations*. T. Nelson, 1838.
- [36] D. Helbing. Social Self-Organization. Springer-Verlag, 2012.
- [37] M. Schläpfer, J. Lee, and L. Bettencourt. "Urban Skylines: building heights and shapes as measures of city size." In: arXiv preprint arXiv:1512.00946 (2015).
- [38] J. Kelly. "Computing, cognition and the future of knowing." In: *IBM Research: Cognitive Computing* (2015).
- [39] C. McCahill, N. Garrick, C. Atkinson-Palombo, and A. Polinski. "Effects of Parking Provision on Automobile Use in Cities: Inferring Causality." In: *Submitted to Transport Research Board* (2016).
- [40] L. Bettencourt, J. Lobo, D. Helbing, C. Kühnert, and G. West.
 "Growth, innovation, scaling, and the pace of life in cities." In: *Proceedings of the National Academy of Sciences* 104.17 (2007), pp. 7301–7306.
- [41] K. C. Clarke. "Cellular Automata and Agent-Based Models." In: *Handbook of Regional Science*. Springer, 2014, pp. 1217–1233.
- [42] W. B. Arthur. Complexity Economics: A Different Framework for Economic Thought. INET Research Notes 33. Institute for New Economic Thinking (INET), 2013.
- [43] H. Haken. "Advanced Synergetics." In: Springer, Berlin (1983).
- [44] M. Batty. "Visually-Driven Urban Simulation: exploring fast and slow change in residential location." In: *Environment and Planning A* 45.3 (2013), pp. 532–552.
- [45] J. M. Epstein. "Why model?" In: *Journal of Artificial Societies* and Social Simulation 11.4 (2008), p. 12.
- [46] H. J. Miller. "Geocomputation." In: The SAGE handbook of spatial analysis. Sage, London (2009), pp. 397–418.
- [47] W. Tobler. "A Computer Movie Simulating Urban Growth in the Detroit Region." In: *Economic Geography* 46.2 (1970), pp. 234– 240.
- [48] I. Santé, A. M. García, D. Miranda, and R. Crecente. "Cellular automata models for the simulation of real-world urban processes: A review and analysis." In: *Landscape and Urban Planning* 96.2 (2010), pp. 108–122.

- [49] J. Portugali. *Self-Organization and the City*. Springer Series in Synergetics. Springer, 2000.
- [50] B. R. Bodenmann and B. Vitins. *Implementation of a land use transport interaction model for experimental game simulations*. STRC, Ascona. 2012.
- [51] D. Zünd, R. Woodbury, and G. Schmitt. "Meso-scale modeling of residential and business locations." In: *Simulation* 92.3 (2016), pp. 295–306.
- [52] C. E. Shannon. "A mathematical theory of communication." In: *Bell system technical journal* 27.3 (1948), pp. 379–423.
- [53] D. Helbing. "Globally networked risks and how to respond." In: *Nature* 497.7447 (2013), pp. 51–59.
- [54] Inc. Wikimedia Foundation. *Arab Spring*. Last accessed: Mai 17, 2016. URL: https://en.wikipedia.org/wiki/Arab_Spring/.
- [55] Garry D Brewer. *Politicians, bureaucrats, and the consultant: A critique of urban problem solving*. Basic Books New York, 1973.
- [56] C. Andrieu, N. De Freitas, A. Doucet, and M. I. Jordan. "An introduction to MCMC for machine learning." In: *Machine learning* 50.1-2 (2003), pp. 5–43.
- [57] S. Sharma, E. B. Coetzee, M. H. Lowenberg, S. A. Neild, and B. Krauskopf. "Numerical continuation and bifurcation analysis in aircraft design: an industrial perspective." In: *Phil. Trans. R. Soc. A* 373.2051 (2015).
- [58] S. Wolfram. A new kind of science. Vol. 1. Wolfram media, 2002.
- [59] S. Kirkpatrick and M. P. Vecchi. "Optimization by simmulated annealing." In: *science* 220.4598 (1983), pp. 671–680.
- [60] M. Wegener, F. Gnad, and M. Vannahme. *The time scale of urban change*. IRPUD, 1983.
- [61] A. Anas. "The optimal pricing, finance and supply of urban transportation in general equilibrium: A theoretical exposition." In: *Economics of Transportation* 1.1–2 (2012), pp. 64–76.
- [62] P. H. Verburg, W. Soepboer, A. Veldkamp, R. Limpiada, and V. Espaldon. "Modeling the Spatial Dynamics of Regional Land Use: The CLUE-S Model." In: *Environmental Management* 30.3 (2002), pp. 391–405.
- [63] P. Waddell. "A behavioral simulation model for metropolitan policy analysis and planning: residential location and housing market components of UrbanSim." In: *Environment and Planning B: Planning and Design* 27.2 (2000), pp. 247–263.
- [64] J. Sloman. *Economics, 6th edition*. Prentice Hall, 1999.

- [65] P. Koumoutsakos. "Multiscale flow simulations using particles." In: Annual Review of Fluid Mechanics 37 (2005), pp. 457– 487.
- [66] A. G. Wilson. "A statistical theory of spatial distribution models." In: *Transportation Research* 1.3 (1967), pp. 253–269.
- [67] D. Bigman and H. Fofack. Geographical Targeting for Poverty Alleviation: Methodology and Applications. Banque mondiale études régionales et sectorielles. World Bank, 2000.
- [68] C. W. Cobb and P. H. Douglas. "A theory of production." In: *The American Economic Review* (1928), pp. 139–165.
- [69] P. R. Krugman. *Development, Geography, and Economic Theory*. The Ohlin lectures. MIT Press, 1997.
- [70] C. Voglis and I. E. Lagaris. "Towards 'Ideal Multistart'. A stochastic approach for locating the minima of a continuous function inside a bounded domain." In: *Applied Mathematics and Computation* 213.1 (2009), pp. 216–229.
- [71] MATLAB. *The MathWorks, Inc.* Natick, Massachusetts, United States.
- [72] E. Jones, T. Oliphant, P. Peterson, et al. *SciPy: Open source scientific tools for Python*. 2001–. URL: http://www.scipy.org/.
- [73] C. Sanderson and R. Curtin. "Armadillo: a template-based C++ library for linear algebra." In: *Journal of Open Source Software* 1 (2016), pp. 26–32.
- [74] Q. Wang, X. Zhang, Y. Zhang, and Q. Yi. "AUGEM: automatically generate high performance dense linear algebra kernels on x86 CPUs." In: *Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis.* ACM. 2013, p. 25.
- [75] C. L. Lawson, R. J. Hanson, D. R. Kincaid, and F. T. Krogh. "Basic linear algebra subprograms for Fortran usage." In: ACM *Transactions on Mathematical Software (TOMS)* 5.3 (1979), pp. 308– 323.
- [76] *Intel Math Kernel Library. Reference Manual.* Santa Clara, USA. Intel Corporation, 2009.
- [77] J. Nickolls, I. Buck, M. Garland, and K. Skadron. "Scalable parallel programming with CUDA." In: *Queue* 6.2 (2008), pp. 40– 53.
- [78] B. Tversky. "Lines: Orderly and Messy." In: Complexity, Cognition, Urban Planning and Design. Springer, 2016, pp. 237–250.
- [79] Oxford English Dictionary. The definitive record of the English language. Last accessed: July 5, 2016. URL: http://www.oed. com/.

- [80] QGIS Development Team. QGIS Geographic Information System. Open Source Geospatial Foundation. 2016. URL: http://qgis. osgeo.org/.
- [81] OpenStreetMap Foundation. OpenStreeMap. Last accessed: July 8, 2016. URL: http://www.openstreetmap.org/.
- [82] Stamen Design. Last accessed: July 8, 2016. Map tiles under CC by 3.0. Data by OpenStreetMap under CC by SA. URL: http: //maps.stamen.com/.
- [83] L. Treyer, B. Klein, R. König, and C. Meixner. *Lightweight urban computation interchange (LUCI) system*. Eidgenössische Technische Hochschule Zürich, 2015.
- [84] L. Treyer, B. Klein, R. König, and C. Meixner. "Lightweight urban computation interchange (LUCI): a system to couple heterogeneous simulations and views." In: *Spatial Information Research* 24.3 (2016), pp. 291–302.
- [85] C. C. Aggarwal, A. Hinneburg, and D. A. Keim. "On the surprising behavior of distance metrics in high dimensional space." In: *International Conference on Database Theory*. Springer. 2001, pp. 420–434.
- [86] F. Pedregosa et al. "Scikit-learn: Machine Learning in Python." In: *Journal of Machine Learning Research* 12 (2011), pp. 2825–2830.
- [87] P. R. Krugman. *Geography and Trade*. Gaston Eyskens Lecture Series. Leuven University Press, 1991.
- [88] P. H. Verburg, W. Soepboer, A. Veldkamp, R. Limpiada, V. Espaldon, and S. S. A. Mastura. "Modeling the spatial dynamics of regional land use: the CLUE-S model." In: *Environmental management* 30.3 (2002), pp. 391–405.
- [89] *Statistisches Jahrbuch der Stadt Zürich*. Statistik Stadt Zürich, 2016.
- [90] Bundesamt für Statistik BFS. Last accessed: August 2, 2016. URL: http://www.bfs.admin.ch/.
- [91] Statistik Stadt Zürich. Last accessed: August 2, 2016. URL: https: //www.stadt-zuerich.ch/.
- [92] M. Bürgle. "Residential location choice model for the Greater Zurich area." In: 6th Swiss Transport Research Conference, Ascona. 2006.
- [93] P. Schirmer, B. C. Belart, and K. W. Axhausen. Location Choice in the Greater Zurich Area-an Intermediate Report. ETH Zurich, Institute for Transport Planning and Systems, 2011.
- [94] Mobilität und Verkehr 2010. Bundesamt für Statistik BFS, 2010.
- [95] Analyse: Arbeiter in Bewegung. Stadt Zürich Statistik, 2013.

- [96] Servicequalität 2014. Verkehrsbetriebe Zürich, 2014.
- [97] *Hin und Zurück: Verkehrsströme in der Stadt Zürich*. Stadt Zürich Statistik, 2008.
- [98] Tiefbauamt der Stadt Zürich. Last accessed: August 11, 2016. URL: https://www.stadt-zuerich.ch/.
- [99] Canonical Ltd. *Ubuntu*. Last accessed: August 15, 2016. URL: http://www.ubuntu.com/.
- [100] Oracle. VirtualBox. Last accessed: August 15, 2016. URL: https: //www.virtualbox.org/.
- [101] G. Singer and H. Saxer. *Seetunnel Zürich: Besonderheiten und Probleme bei der Projektierung.* Schweizerische Bauzeitung, 1970.
- [102] P. M. Schirmer, M. A. B. van Eggermond, and K. W. Axhausen.
 "The role of location in residential location choice models: a review of literature." In: *Journal of Transport and Land Use* 7.2 (2014), pp. 3–21.
- [103] N. Hansen. "The CMA evolution strategy: a comparing review." In: Towards a new evolutionary computation. Advances on estimation of distribution algorithms. Ed. by J. A. Lozano, P. Larranaga, I. Inza, and E. Bengoetxea. Springer, 2006, pp. 75–102.