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Author(s):

Müller, Christian

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Christian Müller

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Swiss Federal Institute of Technology Zurich

On a Remedy for Temporal Aggregation Effects *

Christian Müller

Konjunkturforschungsstelle (KOF) an der
Eidgenössischen Technischen Hochschule Zürich

CH-8092 Zürich, Switzerland

Tel.: ++41-(0)1-632 46 24

Fax: ++41-(0)1-632 12 18

Email: christian.mueller@kof.gess.ethz.ch

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Abstract

Aggregating time series confounds parameters across time and equations (see Working (1960), Marcellino (1999)). We discuss what one can expect from rule-of-the-thumb prescriptions to remedy some of the unwanted effects by choosing appropriately among different possible aggregation strategies.

JEL classification: C32, C43

Keywords: cointegration, aggregation, time series

1 Introduction

Economic data such as interest rates or money stock are often aggregated over time before they are used in statistical analysis. In fact, most macro-economic data bases do not even provide the original, say daily data on interest rates rather than monthly or quarterly aggregates. Consequently these aggregates are used simply for practical reasons.

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The aggregation has its price, though, and this has been intensely studied e.g. by Working (1960), Tiao (1972), Weiss (1984), Lütkepohl (1984), Granger and Siklos (1995), Mamingi (1996), or Breitung and Swanson (1998) to name but a few. Marcellino (1999) has provided a concise approach towards multivariate ARMA processes including explicit formulas for the relationship between the original and the transformed time series.

Looking from the other side, e.g. Hallerbach (2000) has suggested a filter for removing some of the side effects of temporal aggregation. In a simulation study Haug (2002) raises the point that various tests for cointegration lead to different conclusions depending on the innovation properties of the disaggregated process.

In this paper we illustrate that selected negative side effects of temporal aggregation can be avoided by applying the appropriate aggregation strategy. This proves useful if one can make a choice among alternatives of aggregation. We will look at the implications of those choices by selecting a particular data generating process (DGP) which posses the property of cointegration as do many macroeconomic time series. The focus will be on univariate analysis which should mimic the situation of a researcher who is interested in some quick and basic analysis of a hypothesised relationship.

2 Data Generating Process and Aggregation Methods

2.1 Aggregation Strategies

Define the n -dimensional process $x = \{x_t\}_{t=0}^{\infty}$ which evolves according to (1)

$$G(L)x_t = S(L)\epsilon_t \tag{1}$$

where $G(L) = I - G_1L - G_2L^2 - \dots - G_gL^g$, $S(L) = I - S_1L - S_2L^2 - \dots - S_sL^s$. Here, L is the lag operator, the G_i , S_i are $n \times n$ coefficient matrices, and ϵ_t is a white noise error

term, $\epsilon_t \sim WN(0, \Sigma)$.

We distinguish two aggregation strategies. The first is called skip sampling, or equivalently point-in-time sampling or systematic sampling. The new label will be used for ease of referencing because we can now refer to a handy skip parameter a which defines the new process

$$x^* = \{x_\tau^*\}_{\tau=0}^\infty = \{x_{a\tau}\}_{\tau=0}^\infty \quad (2)$$

The second method is average sampling which is ruled by the parameter m :

$$x^+ = \{x_\tau^+\}_{\tau=0}^\infty = \left\{ \sum_{i=0}^{m-1} x_{t+i} \right\}_{t=0}^\infty \quad (3)$$

Naturally, (2) and (3) can be combined to yield various mixed aggregates.

2.2 The DGP

To keep things simple we will investigate a very elementary data generating process:

$$\Delta x_{1,t} = \epsilon_{1,t} \quad (4)$$

$$\Delta x_{2,t} = \alpha(x_{2,t-1} - x_{1,t-1}) + \epsilon_{2,t} \quad (5)$$

which is a special case of (1) and written in the error correction representation with $\Delta = (1 - L)$ and $\alpha = -1$. The disturbances $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are uncorrelated and assumed to be drawn from a distribution with mean zero and variance σ_1^2 and σ_2^2 respectively. The term $(x_{2,t-1} - x_{1,t-1})$ is labelled error correction term, and after temporal aggregation we will denote the process by $x_\tau(a, m)$.

It is possible to show that this process maintains its error correction representation even after aggregation with various a and m . Some more effects of aggregation are summarized in table 1.

Table 1: Aggregation Strategies their Implications for the DGP

Effect on	$x_{1,t}$		$x_{2,t}$
	$a > 1$	$m > 1$	
Representation	preserved	preserved	
Correlations			
ec-term, $e_{2,\tau}$	0	x	$a > 1$
$e_{1,\tau}, e_{2,\tau}$	x	x	
$e_{i,\tau}, e_{i,\tau\pm 1}$	0	x/0	
Representation	preserved	preserved	
Correlations			
ec-term, $e_{2\tau}$	0	x	$m > 1$
$e_{1,\tau}, e_{2,\tau}$	x	x	
$e_{i,\tau}, e_{i,\tau\pm 1}$	0	x	

* The x indicate non-zero effects, x/0 stands for a non-zero correlation in the $x_{1,\tau}$ process only.

Table 1 immediately suggests that there will be problems when applying ordinary least squares (OLS) for estimating α in a single equation approach. Nevertheless, OLS is very popular because of its simplicity. Estimation also very often is conducted to obtain a quick answer to seemingly simple question like significance of explanatory variables. Thus, if one has the suspicion that a certain linear combination of variables should be stationary and cointegrated, OLS might be applied to (5). Another question might be what the value of α is because that provides information of how long it takes before deviations from the long-run relationship vanish.

However, as soon as $m > 1$ the error correction term will be correlated with the innovation process and thus lead to biased OLS estimates. The reason for this phenomenon is, roughly speaking, the memory of the process which in our case is even infinite. Therefore,

an easy prescription for dealing with this hassle might be to apply rather large skip steps in order to partly decouple the process from its own past.

As table 1 shows, this would not change the representation of the process and thus preserve the information we are interested in. In fact, occasionally in applied work (see e.g. Wolters, Teräsvirta and Lütkepohl (1998)) it is recommended to use large a in order to shorten the memory of the innovation process. It could likewise be asked if this might also help coping with the OLS bias.

2.3 Remedying Temporal Aggregation - A small simulation study

In what follows we will simulate $x_\tau(a, m)$ for various a and m to check if a systematic variation of a helps to improve OLS estimation. To that aim we run regressions on (5) and perform t -tests on the estimates for α .

We choose a and m such that realistic constellations are covered. For example, daily data on interest rates are available and we want to aggregate the observations up to a monthly or to quarterly level. Thus, we could average over seven days, or skip sample every seventh day to obtain weekly data. From these we could again average or skip sample to obtain monthly or even quarterly data. More formally, starting with daily observations for $m = 7$ and $a = 4$, or $m = 4$ and $a = 7$ temporal aggregation leads to data at monthly frequency. However, we could likewise average over the whole month (28 days) or pick every 28th observation only. That way we can check whether or not large a improve the inference compared to aggregation with large m .

First, we address the size of the standard t -test. Thus, the first hypothesis to be investigated is $H_0^1 : \alpha = -1$. The results are the subject of table 2.

It turns out that starting with some a, m further temporal aggregation by skip sampling

Table 2: Empirical rejection frequencies of $H_0^1 : \alpha = -1$

$T = 100$		averaging parameter (m)							
R=1000		1	2	3	4	7	14	28	84
	1	.049	.758	.943	.982	.991	.995	.995	.996
	2	.074	.330	.530	.580	.722	.958	.782	-
skip	3	.049	.235	.363	.387	.480	-	.575	-
parameter (a)	4	.059	.161	.238	.282	.353	-	-	-
	7	.051	.120	.163	.193	.236	-	.282	-
	14	.070	.092	.099	.236	.140	-	-	-
	16	.056	-	-	-	-	-	-	.165
	28	.049	.069	.070	-	.100	-	-	-
	84	.052	-	-	-	-	-	-	-

The nominal significance level, ζ , is 0.05.

leads to an improvement of the size. As an example consider column three where we start with averaging over four consecutive periods. Instead of rejecting the null hypothesis in five out of one hundred throws, it is rejected in roughly 98 cases. In order to aggregate in total over 28 periods we could go from there either to column seven ($m = 28, a = 1$) or to row five of the same column ($m = 4, a = 7$). It turns out that indeed in the latter case the empirical size of .19 is much more reasonable than the .995 of the first option. Overall it can be confirmed that for all combinations of m and a with fixed values of $\bar{c} = am$ the empirical size is better for those situations in which $a > m$ and sometimes more or less even matches the theoretical one ($m = 2, a = 28$).

So far we looked at the size. The other side of the coin is the power of the test, however. Now, instead of investigating the power against local alternatives, we focus on a more interesting hypothesis which is no cointegration. We therefore put forth the

hypothesis $H_0^2 : \alpha = 0$. Rejecting H_0^2 leads to the conclusion that $x_{1,\tau}$ and $x_{2,\tau}$ are cointegrated which actually is the case.

Table 3: Empirical rejection frequencies of $H_0^1 : \alpha = 0$

T=100		averaging parameter (m)							
R=1000		1	2	3	4	7	14	28	84
	1	1	1	1	1	1	1	.999	.999
	2	1	1	1	.998	.981	.942	.806	-
skip	3	1	1	.994	.967	.880	-	.694	-
parameter (a)	4	1	.993	.957	.908	.774	-	-	-
	7	1	.921	.767	.680	.525	-	.347	-
	14	.946	.670	.508	.525	.284	-	-	-
	16	.930	-	-	-	-	-	-	.171
	28	.778	.393	.265	-	.174	-	-	-
	84	.324	-	-	-	-	-	-	-

$\zeta = 0.05$

The simulation design is as before. Summarizing the outcomes displayed in table 3 we find that the power of the test is generally higher if $m > a$ for any \bar{c} considered. In this setting, it can also be argued that the difference in power is larger the larger $|m - a|$. Picking again the examples $m = 4, a = 7$ vs. $m = 28, a = 1$, and $m = 2, a = 28$ vs. $m = 28, a = 2$ the empirical powers are .680 vs. .999, and .393 vs. .806 respectively.

Remarks

1. Although not discussed, the correlation between regressor and error term could be accounted for by including $\Delta x_{1,\tau}$ as a regressor. The estimate of the coefficient on this regressor will be significant and therefore be biased, too.

2. The appropriate representation of $x_\tau(a, m)$, $a > 1, m > 1$ is a multivariate one. Therefore, even recurring to a univariate ARMA model for $x_{2,\tau}$ does generally not solve the problems showed in the simulation.
3. Even though the process might be regarded very special, due to the results by Marcellino (1999) it cannot be expected that the OLS deficiencies are limited to it. Instead, the distortions in size and power are functions of the specific original DGP.

3 Summary

Data used in empirical work has often to be temporally aggregated. In some cases, there can be made a choice between skip sampling and averaging. We asked what option should be recommended for quick and easy OLS analysis of a simple cointegrated process.

The answer to this question is twofold. First there are sizeable gains in terms of avoiding errors of type one. Thus, if the focus is on the size of a t -test, skip sampling with large a should be preferred.

On the other hand, doing so deteriorates the power of the test. Therefore, one has to weigh both effects when finally deciding about the aggregation method.

For this special DGP the tradeoff seems to be in favour of skip sampling, a further theoretical analysis for more general cases appears desirable, however.

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