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LARS-based PC-NARX surrogate models for computing seismic fragility curves

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Earthquake engineering aims at assessing the performance of structures and infrastructures w.r.t recorded or potential quakes.

Due to uncertainties in the localization, magnitude, structural behaviour and resistance, etc. probabilistic approaches are commonly used.

Fragility curves

For a given performance criterion $g \leq g_{adm}$, the fragility curve represents the conditional probability of failure given an intensity measure $IM$:

$$Frag(IM) = P(g > g_{adm} | IM)$$

Example

- $g = \max_k \max_{t_i \in [0,T]} |\delta^k_{t_i}|$ ($k$-th interstorey drift)

- $IM$: peak ground acceleration (PGA), pseudo-spectral acceleration (PSa), cumulative absolute velocity (CAV), etc.
Fragility curves

Classical approach

- Select a set of ground motions (recorded / synthetic)
- Compute the transient structural response (finite element analysis)
- Assume a parametric shape for the fragility curve, e.g. a lognormal shape:

\[ Frag(IM) = \mathbb{P}(\Delta \geq \delta_{adm} | IM) = \Phi \left( \frac{\ln IM - \alpha}{\beta} \right) \]

- Fit the parameters \((\alpha, \beta)\)

Limitations

- Predefined shape of the curve
- Subject to epistemic uncertainties when the number of ground motions is small
Fragility curves

Classical approach

- Select a set of ground motions (recorded / synthetic)
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- Assume a parametric shape for the fragility curve, e.g. a lognormal shape:

\[
Frag(IM) = \mathbb{P}(\Delta \geq \delta_{adm} \mid IM) = \Phi\left(\frac{\ln IM - \alpha}{\beta}\right)
\]

- Fit the parameters \((\alpha, \beta)\)

New proposal

- Use non parametric statistics for the fragility curves

  Mai, Konakli & Sudret (2015)

- Use surrogate models of the transient analysis based on polynomial chaos expansions
Outline

1 Introduction

2 PC-NARX model for time-dependent systems
   Reminder on polynomial chaos expansion
   NARX model
   Calibration of a PC-NARX model

3 Non parametric fragility curves
Polynomial chaos expansions


Problem statement

- Consider a computational model $\xi \in D_{\Xi} \subset \mathbb{R}^M \mapsto y = \mathcal{M}(\xi)$, where the uncertain input parameters are modelled by a random vector $\Xi \sim f_{\Xi}$.
- The polynomial chaos expansion of $Y = \mathcal{M}(\Xi)$ reads:

$$Y(\xi) = \sum_{\alpha \in \mathbb{N}^M} y_{\alpha} \psi_{\alpha}(\xi)$$

where:

- $\xi = \{\xi_1, \ldots, \xi_M\}$ is the vector of uncertain input parameters
- $\psi_{\alpha}(\xi)$ are multivariate orthogonal polynomials obtained by tensor product:

$$\psi_{\alpha}(\xi) = \prod_{i=1}^{M} \psi_{\alpha_i}^{(i)}(\xi_i)$$

- $y_{\alpha}$’s are expansion coefficients
PC-NARX model for time-dependent systems

Reminder on polynomial chaos expansion

PCEs for time-dependent problems

Time-frozen PCE

**Naive idea:** Solve a PCE at each time instant independently

\[ Y(t, \xi) = \sum_{\alpha \in \mathbb{N}^M} y_\alpha(t) \psi_\alpha(\xi) \]

Example: Quarter car model

\[ x(t) = A \sin(\omega t) \]

Reference

Time-frozen PCE

Does not work!
PCEs for time-dependent problems

**Time-frozen PCE**

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**Example: Quarter car model**

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Reference

Time-frozen PCE

Does not work!
Nonlinear AutoRegressive with eXogenous input model

Deterministic NARX model

Based on a time-dependent input excitation $x(t)$ and corresponding system response $y(t)$, the dynamics is captured through:

$$y(t) = \mathcal{F}(x(t), \ldots, x(t - n_x), y(t - 1), \ldots, y(t - n_y)) + \epsilon_t$$

where:

- $z(t) = (x(t), \ldots, x(t - n_x), y(t - 1), \ldots, y(t - n_y))^T$ is the vector of current and past values
- $n_x$ and $n_y$ denote the maximum input and output time lags
- $\epsilon_t \sim \mathcal{N}(0, \sigma^2_{\epsilon}(t))$ is the residual error
- $\mathcal{F}()$ is a functional of NARX terms, usually linear-in-parameters:

$$y(t) = \sum_{i=1}^{n_g} \vartheta_i \ g_i(z(t)) + \epsilon_t$$
Computational model with uncertainties

\[ y(t, \xi_x, \xi_s) \overset{\text{def}}{=} M(x(t, \xi_x), \xi_s) \]

- \( \xi_x \): uncertainty in the input excitation
- \( \xi_s \): uncertainty in the system

PC-NARX expansion

\[ y(t, \xi) = \sum_{i=1}^{n_g} \vartheta_i(\xi) g_i(z(t)) + \epsilon_g(t, \xi) \quad \xi = (\xi_x, \xi_s) \]

The NARX stochastic coefficients \( \vartheta_i(\xi) \) are represented by PCEs:

\[ \vartheta_i(\xi) = \sum_{\alpha \in A_i} \vartheta_{i, \alpha} \psi_\alpha(\xi) \]
**PC-NARX model**

Spiridonakos et al., 2015a, 2015b

Computational model with uncertainties

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\[ \vartheta_i(\xi) = \sum_{\alpha \in A_i} \vartheta_{i,\alpha} \psi_{\alpha}(\xi) \]
PC-NARX model

\[ y(t, \xi) = \sum_{i=1}^{n_g} \sum_{\alpha \in A_i} \theta_{i,\alpha} \psi_{\alpha}(\xi) g_i(z(t)) + \epsilon(t, \xi) \]

Interpretations

- PC-NARX is a NARX model in which each (random) coefficient is expanded as a PCE
- Compared to time-frozen PCE, a specific dynamics of the random coefficients is imposed
Outline

1. Introduction

2. PC-NARX model for time-dependent systems
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   Calibration of a PC-NARX model

3. Non parametric fragility curves
Experimental design

Data

- $N$ realizations of the input excitation, cast as
  \[(x_k[1], \ldots, x_k[T])^T, \ k = 1, \ldots, N\] (T time instants)
- The corresponding system response computed by a simulator, cast as
  \[(y_k[1], \ldots, y_k[T])^T\]

Example: quarter car model
Deterministic NARX calibration using LAR

For a particular realization $\xi_k$

- Select NARX model (candidate terms):

$$z(t) = (x(t), \ldots, x(t - n_x), y(t - 1), \ldots, y(t - n_y))^T$$

$$\phi(t) = \{g_i(z(t), \ i = 1, \ldots, n_g)\}^T$$

- Use least angle regression (LAR) to select the best explanatory subset of terms

  \textit{Efron et al., 2004}

- Compute the coefficients $\varphi_k$ by ordinary least-squares

\textbf{Prediction error} (of model $\#k$ on trajectory $l$)

$$\epsilon_{\#k}^l = \frac{\sum_{t=1}^{T} (y(t, \xi_l) - \hat{y}_{\#k}(t, \xi_l))^2}{\sum_{t=1}^{T} (y(t, \xi_l) - \bar{y}(t, \xi_l))^2}$$
Common NARX basis

Premise

To expand the NARX coefficients onto a PC basis, it is necessary to have a common NARX model for all trajectories.

Procedure

- Select $K \leq N$ trajectories ("NARX learning set"), e.g. with the strongest non-linear behaviour (peak displacement, velocities, etc.)

- Determine the sparse deterministic NARX models for realizations $k = 1, \ldots, K$, which leads to $P \leq K$ different possible models called #1, \ldots, #P.

- Compute the NARX coefficients of the $N$ trajectories, for each model #p, and evaluate an average error:

$$
\varepsilon_p = \frac{1}{N} \sum_{k=1}^{N} \varepsilon_{#p}
$$

- Select the final best NARX model that minimizes $\varepsilon_p$. 
PCE of the NARX coefficients

PCE calibration

- Once a common NARX basis has been found, $N$ realizations of the NARX coefficients are available:

$$\mathcal{E} \mathcal{D} = \{ \vartheta_{i,k}, i = 1, \ldots, n_g; k = 1, \ldots, N \}$$

- $n_g$ different sparse PC expansions are built from this experimental design, using least-angle regression (LAR) 
  \cite{BlatmanSudret2011}

\[ \vartheta_i(\xi) = \sum_{\alpha \in A_i} \vartheta_{i,\alpha} \psi_{\alpha}(\xi) \]

PC-NARX prediction

- For a new realization of the input parameters $\xi_0$, the NARX coefficients are first evaluated from PCEs

- Then they are plugged into the NARX model
Outline

1. Introduction
2. PC-NARX model for time-dependent systems
3. Non parametric fragility curves
Steel frame

- 2D steel frame submitted to synthetic ground motions
- Synthetic earthquakes generated in time domain

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>C.o.V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_y$ (MPa)</td>
<td>Lognormal</td>
<td>264.2878</td>
<td>18.5</td>
<td>0.07</td>
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<tr>
<td>$E_0$ (MPa)</td>
<td>Lognormal</td>
<td>210000</td>
<td>630</td>
<td>0.03</td>
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</table>
Stochastic ground motion

Stochastic excitation

- Obtained by a modulated filtered white noise process

\[ x(t) = q(t, \alpha) \sum_{i=1}^{n} s_i(t, \lambda(t_i)) \cdot \xi_i \quad \xi_i \sim \mathcal{N}(0, 1) \]

- Parameters of the filter \( \lambda = (\omega_{mid}, \omega', \zeta_f)^T \) are calibrated on recorded signals

- Global parameters (Arias intensity \( I_a \), duration \( D_{5-95} \), strong phase peak \( t_{mid} \)) are transformed into the parameters \( \alpha \) of the modulation function \( q(t, \alpha) \) (e.g. gamma distribution)
### Stochastic ground motion

#### Parameters of the excitation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Support</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_a$ (s.g)</td>
<td>Lognormal</td>
<td>$(0, +\infty)$</td>
<td>0.0468</td>
<td>0.164</td>
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<tr>
<td>$D_{5-95}$ (s)</td>
<td>Beta</td>
<td>[5, 45]</td>
<td>17.3</td>
<td>9.31</td>
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<tr>
<td>$t_{mid}$ (s)</td>
<td>Beta</td>
<td>[0.5, 40]</td>
<td>12.4</td>
<td>7.44</td>
</tr>
<tr>
<td>$\omega_{mid}/2\pi$ (Hz)</td>
<td>Gamma</td>
<td>$(0, +\infty)$</td>
<td>5.87</td>
<td>3.11</td>
</tr>
<tr>
<td>$\omega'/2\pi$ (Hz)</td>
<td>Two-sided exponential</td>
<td>[-2, 0.5]</td>
<td>-0.089</td>
<td>0.185</td>
</tr>
<tr>
<td>$\zeta_f$ (.)</td>
<td>Beta</td>
<td>[0.02, 1]</td>
<td>0.213</td>
<td>0.143</td>
</tr>
</tbody>
</table>
Non parametric fragility curves

Two trajectories (first floor displacement)

- Reference solution: Monte Carlo sampling of 10,000 non linear transient analyses
- PC-NARX: 300 samples

![Graph showing two trajectories with reference and PC-NARX lines.](image-url)
Statistics of the first floor drift

- Reference solution: Monte Carlo sampling of 10,000 non linear transient analyses
- PC-NARX: 300 samples
Post-processing: fragility curves

Reminder

Fragility curves represent the **conditional probability** that the maximal response $\Delta$ attains or exceeds the admissible threshold $\delta_0$, given the earthquake peak ground acceleration (PGA):

$$\text{Frag}(PGA; \delta_o) = P[\Delta \geq \delta_0 | PGA]$$

Non-parametric approaches

- **Kernel-smoothing**: non parametric representations of the conditional PDF of $\Delta|PGA$ are used
- It requires a **large number of simulations**, thus the use of PC-NARX surrogates

Mai, Konakli & Sudret (2015)
Maximal first floor displacement and fragility curve

- Reference solution: Monte Carlo sampling of 10,000 non linear transient analyses
- PC-NARX: 300 samples

Fragility curves
Conclusions

- Polynomial chaos expansions are facing challenging issues when modelling time-dependent systems such as arising in structural dynamics.

- A non-intrusive approach based on NARX models (from structural identification) and sparse PCE is proposed.

- The accuracy is remarkable on the statistical moments (mean/std. deviation), PDF of the maximum output, but also on particular trajectories.

- The method was successfully used for computing fragility curves in earthquake engineering applications.
Questions ?

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The Uncertainty Quantification Laboratory
www.uqlab.com

Thank you very much for your attention !