SELECTIVE MORPHING
THROUGH DISTRIBUTED COMPLIANCE
WITH VARIABLE STIFFNESS
BASED ON EMBEDDED BI-STABLE STRUCTURES

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presented by
IZABELA KATARZYNA KUDER
M.Sc., Technische Universität München
born on 21. November 1987
citizen of Poland

accepted on the recommendation of
Prof. Dr. Paolo Ermanni, examiner
Prof. Dr. Andres Arrieta, co-examiner
Prof. Dr. Sergio Pellegrino, co-examiner

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Morphing systems able to efficiently adjust their characteristics to resolve the conflicting demands of changing operating conditions offer great potential for enhanced performance and functionality. The main practical challenge consists in combining the desired compliance to accomplish radical reversible geometry modifications at reduced actuation effort with high stiffness necessary for operational reasons.

The present thesis addresses these contradictory requirements through a novel approach combining the proven conformal shape adaptation benefits of distributed compliance with purely elastic stiffness variability provided by monolithically embedded bi-stable laminates. Switching between the stable states of the internal elements allows the global stiffness to be modified on demand at reduced actuation effort, activating deformation modes not attainable with uniform distributed topologies.

First, a numerical design methodology is developed for thermally-induced bi-stable laminates to fulfil the envisaged function of variable stiffness components integrated in a larger substructure. Bi-stability assessment relies on two independent criteria: a controlled displacement cool-down test, and the characteristic strain energy profile featuring minima corresponding to the equilibrium shapes. Tailoring the spatial stacking sequence distribution permits controlling the resulting stiffness characteristics, while satisfying the embeddability objective of clamping two opposite edges without bi-stability loss.

Second, the feasibility of the proposed strategy is proved numerically and experimentally by means of a NACA 0012 aerofoil with two bi-stable elements as part of a straightforward topology. A simple mechanical test demonstrates
the stiffness variability of the prototype as a ratio of 2.47 between the stiff and flexible configurations. This is confirmed in two-way static aeroelastic simulations for passive shape adaptation under a range of aerodynamic conditions, as well as obtaining predictions of operating points triggering the stable state switch. The unfavourable morphed shape observed in the post-snap-through regime stresses the key role of the concurrent aero-structural approach in tailoring the deformation modes resulting from the mutual interactions between the selectively compliant structure and distributed loading. The aeroelastic analysis method is thus employed to develop and evaluate an improved morphing system serving as substantiation, featuring a modified topology, and skin extensibility introduced through corrugated portions.

Further, a multidisciplinary design and optimisation methodology is implemented to study the extent of shape adaptation achievable with particular actuation energy, while seeking to embed the bi-stable components in the variable internal layout for maximum impact. The ensuing shape-adaptive solution features two selective deformation modes of intentionally dissimilar response, addressing distinct operational scenarios. An aerodynamically efficient high-lift configuration achieves large geometric changes due to reduced actuation demands. This is only possible by virtue of the internally tailored compliance, arising from the stable state switch of the embedded bi-stable components. The other, stiff mode, targets increased aerodynamic loading. This stiffness selectivity permits tuning the actuation resistance of the distributed topology as required for efficient operation, providing a clear functional enhancement. High fidelity fluid-structure interaction simulations demonstrate dynamic adequacy of the design.

The results presented herein thus prove the feasibility and potential of the proposed morphing strategy in decoupling the conflicting requirements of morphing in an actuation-efficient manner.
Formvariable Systeme, die ihre Eigenschaften effizient anpassen können, um widersprüchliche Anforderungen wechselnder Flugbedingungen aufzulösen, bieten ein enormes Verbesserungspotential an Leistung und Funktionalität. Die grösste Herausforderung besteht darin, die für signifikante reversible Formveränderungen bei reduziertem Aktuationsaufwand gewünschte Nachgiebigkeit mit betriebsbedingt hoher Gesamtsteifigkeit zu vereinbaren.


Im ersten Teil dieser Arbeit wird eine numerische Entwurfsmethodik für thermisch induzierte bistabile Laminate entwickelt, um die vorgesehene Funktion als steifigkeitsverändernde, in einer grösseren Innenstruktur eingebaute Komponente zu erfüllen. Dabei wird Bistabilität durch zwei unabhängige Kriterien beurteilt: eine verschiebungskontrollierte Abkühlssimulation, sowie das charakteristische Dehnungsenergieprofil, dessen Minima den stabilen Zuständen entsprechen. Durch einen massgeschneiderten räumlichen Lagenaufbau können die resultierenden Steifigkeitseigenschaften gesteuert werden, wobei die Zielvorgabe der Einspannung zweier gegenüberliegenden Kanten ohne Bistabilitätsverlust eingehalten werden kann.


Die hier vorgestellten Ergebnisse beweisen somit die Machbarkeit und das Potenzial der vorgeschlagenen Formanpassungsstrategie, um die widersprüchlichen Anforderungen der Formvariabilität energieeffizient zu vereinbaren.
Acknowledgements

The present thesis summarises the results of my work as research assistant at the Laboratory of Composite Materials and Adaptive Structures of ETH Zürich. The research was supported by the Swiss National Science Foundation under project number 147059 Selective Compliance Aerofoils with Variable Stiffness Multi-Stable Elements for Morphing Applications.

First and foremost I would like to express my sincere gratitude to Professor Paolo Ermanni for providing me with the opportunity to work in this exciting field. The research presented in this thesis would not have been possible without his continuous support, scientific advice and guidance.

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# Nomenclature

## Acronyms

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<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>3AS</td>
<td>Active Aeroelastic Aircraft Structures</td>
</tr>
<tr>
<td>AR</td>
<td>aspect ratio</td>
</tr>
<tr>
<td>ATVA</td>
<td>Adaptive Tuned Vibration Absorption</td>
</tr>
<tr>
<td>ATW</td>
<td>Adaptive Torsion Wing</td>
</tr>
<tr>
<td>AVSEA</td>
<td>Active Variable Stiffness Elastic Actuator</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon Fibre Reinforced Polymer</td>
</tr>
<tr>
<td>CHEM</td>
<td>Cold Hibernated Elastic Memory</td>
</tr>
<tr>
<td>CLT</td>
<td>Classical Lamination Theory</td>
</tr>
<tr>
<td>CTE</td>
<td>coefficient of thermal expansion</td>
</tr>
<tr>
<td>DARPA</td>
<td>Defense Advanced Research Projects Agency</td>
</tr>
<tr>
<td>DE</td>
<td>Dielectric Elastomer</td>
</tr>
<tr>
<td>DMF</td>
<td>Dynamic-Modulus Foam</td>
</tr>
<tr>
<td>EAP</td>
<td>Electroactive Polymer</td>
</tr>
<tr>
<td>EBL</td>
<td>Electro-Bonded Laminate</td>
</tr>
<tr>
<td>EMC</td>
<td>Elastic Memory Composite</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>F²MC</td>
<td>Fluidic Flexible Matrix Composite</td>
</tr>
<tr>
<td>FishBAC</td>
<td>Fish Bone Active Camber</td>
</tr>
<tr>
<td>FMC</td>
<td>Flexible Matrix Composite</td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid-Structure Interaction</td>
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<tr>
<td>GFRP</td>
<td>Glass Fibre Reinforced Polymer</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$A_f$</td>
<td>austenitic finish temperature</td>
</tr>
<tr>
<td>$A_s$</td>
<td>austenitic start temperature</td>
</tr>
<tr>
<td>$BCs$</td>
<td>width of clamping surfaces/regions of boundary condition assignment within the symmetric region of a bi-stable component</td>
</tr>
<tr>
<td>$c$</td>
<td>chord length</td>
</tr>
<tr>
<td>$c_d$</td>
<td>two-dimensional (aerofoil) drag coefficient</td>
</tr>
<tr>
<td>$c_l$</td>
<td>two-dimensional (aerofoil) lift coefficient</td>
</tr>
<tr>
<td>$\Delta u_{1,2}^n$</td>
<td>relative axial displacement increment during increment $n$</td>
</tr>
<tr>
<td>$E$</td>
<td>isotropic Young’s modulus</td>
</tr>
<tr>
<td>$f$</td>
<td>fitness function</td>
</tr>
<tr>
<td>$F^n$</td>
<td>interpolated force value at the current increment $n$</td>
</tr>
<tr>
<td>$F^{n-1}$</td>
<td>interpolated force value at the preceding increment $n - 1$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$F_{\text{peak}}$</td>
<td>reaction force at response peak of the flat stable state in axial compression</td>
</tr>
<tr>
<td>$F_{R,1}$</td>
<td>reaction force in longitudinal direction of a bi-stable laminate</td>
</tr>
<tr>
<td>$F_{R,3}$</td>
<td>vertical reaction force</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>stiffness ratio between the flat and curved stable state for a small deviation from equilibrium</td>
</tr>
<tr>
<td>$k_t^n$</td>
<td>tangent stiffness value at the current increment $n$</td>
</tr>
<tr>
<td>$l$</td>
<td>bi-stable component length</td>
</tr>
<tr>
<td>$l_i$</td>
<td>length of intermediate lamination region of a bi-stable component</td>
</tr>
<tr>
<td>$l_s$</td>
<td>length of symmetric lamination region of a bi-stable component</td>
</tr>
<tr>
<td>$\bar{l}_s$</td>
<td>length of symmetric lamination region normalised with total length $l$ of a bi-stable component</td>
</tr>
<tr>
<td>$l_{us,c}$</td>
<td>length of central unsymmetric lamination region of a bi-stable component</td>
</tr>
<tr>
<td>$\bar{l}_{us,c}$</td>
<td>length of central unsymmetric lamination region normalised with total length $l$ of a bi-stable component</td>
</tr>
<tr>
<td>$l_{us,o}$</td>
<td>length of outer unsymmetric lamination region of a bi-stable component</td>
</tr>
<tr>
<td>$\bar{l}_{us,o}$</td>
<td>length of outer unsymmetric lamination region normalised with total length $l$ of a bi-stable component</td>
</tr>
<tr>
<td>$M_f$</td>
<td>martensitic finish temperature</td>
</tr>
<tr>
<td>$n$</td>
<td>current increment</td>
</tr>
<tr>
<td>$T_g$</td>
<td>glass transition temperature</td>
</tr>
<tr>
<td>$u_1$</td>
<td>imposed displacement in longitudinal direction of a bi-stable laminate</td>
</tr>
<tr>
<td>$u_3$</td>
<td>imposed vertical displacement</td>
</tr>
<tr>
<td>$u_{3,\text{TE}}$</td>
<td>vertical displacement of the trailing edge of a morphing aerofoil</td>
</tr>
<tr>
<td>$u_{\text{peak}}$</td>
<td>imposed displacement at response peak of the flat stable state in axial compression</td>
</tr>
<tr>
<td>$V_\infty$</td>
<td>freestream velocity</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$w$</td>
<td>bi-stable component width</td>
</tr>
<tr>
<td>$x_s$</td>
<td>position of boundary between symmetric and unsymmetric lamination regions of a bi-stable component</td>
</tr>
<tr>
<td>$\bar{x}_s$</td>
<td>position of boundary between symmetric and unsymmetric lamination regions of a bi-stable component normalised with laminate half-length $l/2$</td>
</tr>
<tr>
<td>$x_{us}$</td>
<td>position of boundary between unsymmetric lamination regions of a bi-stable component</td>
</tr>
<tr>
<td>$\bar{x}_{us}$</td>
<td>position of boundary between unsymmetric lamination regions of a bi-stable component normalised with laminate half-length $l/2$</td>
</tr>
<tr>
<td>$x_{us,i}$</td>
<td>position of boundary between the central unsymmetric and intermediate lamination regions of a bi-stable component</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angle of attack</td>
</tr>
<tr>
<td>$\nu$</td>
<td>isotropic Poisson’s ratio</td>
</tr>
<tr>
<td>$\theta_{\text{flex}}$</td>
<td>inclination angle formed by the symmetrically laminated clamping regions of the flexible equilibrium state with the horizontal</td>
</tr>
<tr>
<td>$\theta_{\text{stiff}}$</td>
<td>inclination angle formed by the symmetrically laminated clamping regions of the stiff equilibrium state with the horizontal</td>
</tr>
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CHAPTER 1

Introduction

1.1 Motivation: Morphing Challenge

The traditional aerospace development process, strongly shaped by comprehensive decisions made at the programme initiation in response to expected market needs and subject to stringent regulations, is often considered conservative due to substantial barriers discouraging radical change. Maintaining a commercially acceptable risk level encourages gradual innovation and optimisation within a well-explored space of largely established design solutions. The topology, shape, size and material of the final configuration emerge as a trade-off in terms of the achievable mass efficiency under an extensive requirements set encompassing performance and cost targets, functional, structural and manufacturing considerations, combined with the available technology \[1\]. This is accompanied by the necessity to demonstrate failure-free operation within the flight envelope during the entire design life cycle. Hence, designs with time invariant properties are generated, save for a limited adaptivity in shape provided by aerodynamic surfaces. The inevitable consequence is a sub-optimal response to the vast range of possible flight conditions, standing in marked contrast with observations of natural systems in the surrounding world, intelligently and dynamically adjusting their characteristics to the environment \[1\].

Birds’ wings are capable of smooth, free geometry modifications, with many adaptations generated passively to conform to rather than resisting the
aerodynamic loading [2, 3]. However complex these shape changes might be, the corresponding number of degrees of freedom remains limited, which greatly facilitates control. The compliant, highly anisotropic lifting surfaces are brought into motion through muscles found mostly outside of the wings and connected to moveable structural members [3]. Birds then produce thrust through flapping and rotating their wings. By contrast, conventional engineering design largely relies on Cayley’s paradigm of function separation [2, 4]. In other words, the individual tasks of a flight vehicle such as lift and thrust generation are assigned to different and at most weakly coupled subsystems, which to a substantial extent are treated separately in the course of the traditional development process. Such pragmatic boundaries allow for a manageable complexity level, yet confine the accessible design space and reduce the optimisation prospects [2]. With the contemporary trend towards smart systems and intensive function integration, this division becomes increasingly blurred.

In general, enhanced performance and functionality of lightweight structures can be achieved by intensive, highly integrated multidisciplinary optimisation of existing approaches or a paradigm shift yielding completely new design concepts. Recent developments in smart and adaptive technologies show great promise for the latter, especially regarding certification-free systems such as Unmanned Aerial Vehicles (UAVs) [5, 6].

Morphing, or changing geometry and characteristics to cover a wide variety of operating environments with high efficiency, has thus come into focus of intensified research efforts, particularly in the aerospace domain [7–22]. The potential of this strategy is also increasingly recognised by the wind energy community [23, 24], and even proposed for automotive applications [25]. Aiming at reduced costs of energy generation synonymous with upscaled wind turbines, innovative solutions are sought to alleviate blade loads, reducing the levels of stress and fatigue which the structures undergo during their lifetime. This constitutes a prerequisite for large-scale designs to become feasible [24, 26].

Despite a wide variety of approaches reported in published studies, the challenge still remains to propose a concept featuring clear system level advantages over conventional configurations on a sufficiently high Technology Readiness Level (TRL) [14, 27]. The main difficulty consists in reconciling the inherently conflicting requirements of realising large shape adaptation
with adequate load-bearing capacity, low mass [2, 28, 29] and justifiable complexity. This implies the necessity for substantial stiffness anisotropy, combining the desired flexibility to accomplish radical reversible geometry modifications at reduced actuation cost with the demand for high deformation resistance imposed by operational functions [12, 30, 31].

A possible decoupling strategy, referred to as the variable stiffness approach, entails providing a controllable, time-variant stiffness range [11, 12]. Rather than remaining fixed in operation, as is the case for conventional solutions, the deformation resistance is tuned to ensure maximum efficiency through real-time adaptation to changing external conditions. This technique underlies a substantial volume of published studies, detailed in Section 2.1, and can be implemented on a material level, through mechanism-based or semi-active means, or exploiting structural elasticity. The aforementioned definition, involving the time aspect, should be distinguished from the concept of composite materials with spatially variable fibre orientations described for example in [32–38]. Within the structural elasticity category of temporally variable stiffness strategies, bi-stable laminates designed to feature markedly different properties in each equilibrium configuration suggest themselves as promising variable stiffness components [39–42]. Indeed, energy input is required only to trigger the switch between the statically stable shapes, which are retained with no auxiliary external intervention [43]. Full configuration and thus stiffness property control of multi-stable structures can be obtained by means of the dynamic snap-through strategy which targets natural vibration modes for reversible actuation between equilibrium configurations [44–46]. Furthermore, the deflection reversibility associated with the purely elastic nature of multi-stability adds to the robustness and scalability of the approach.

In seeking to address the lightweight morphing challenge, strength and stiffness need to be clearly distinguished as dissimilar quantities. However non-trivial, providing both is indeed feasible, with material characteristics, geometry, and loading and boundary conditions providing three major techniques to implement flexibility [47]. This categorisation equally applies to the realisation of variable stiffness, as will be seen in Section 2.1.
1.2 Shape Adaptation in Aircraft: Classifications

Morphing concepts may be grouped according to a variety of criteria. A number of possible categorisations relevant in the context of the present work are outlined in the following, including a brief summary of their key characteristics.

1.2.1 Discrete and Continuous

On the most general level, Ajaj et al. [22] distinguish two broad categories: discrete or local, and continuous geometry modification on the system level. The former allows for limited performance enhancements through deployment in selected operational conditions, as is the case for established technologies such as high-lift devices or variable wing sweep. The latter, animal flight-like, sets out a future direction of seamless, distributed adaptability within the complete mission profile, characterised by intensive task integration and multi-functionality [22].

The distinctive features associated with discrete and continuous strategies direct attention to another very general classification into mechanism-based and compliant approaches, with hybrid concepts found at the intersection [22, 48, 49].

Mechanism-based systems involving rigid-body motion permit significant design simplifications through function separation of load transmission and motion [29]. Although the global load-bearing capacity remains unaffected, drawbacks in terms of significant mass penalties, added complexity, increased part count, and tribological aspects become unavoidable [29]. The most prominent examples include the mature technologies of flaps and slats, or retractable landing gears [19]. These are generally single-function devices augmenting performance within a limited scope or in respect of a particular aspect of the flight mission [22]. Specifically, high-lift systems enable low take-off and landing speeds, landing gear retraction offers drag reductions when flying faster [19]. Variable sweep provides a viable solution to resolve the conflict between supersonic and slow flight capabilities [19]. In spite of such disadvantages as added complexity and mass, and increased maintenance effort [13], this technology has been successfully implemented on such production aircraft as Sukhoi Su-17, Mikoyan-Gurevich MiG-23, Panavia Tornado, Grumman F-14 Tomcat, or Rockwell B-1 Lancer. Although the aforementioned practical examples provide certain performance augmentation
through changing geometry, the term *morphing* inherently arouses expectations of a more comprehensive, often also more exotic or sophisticated, adaptation capability. Published literature contains numerous examples of research efforts relying on mechanisms to implement such more radical shape modifications. Example cambering concepts include patents by McKinney [50], Berry [51], Piening and Monner [52], or the work by Monner et al. [53]. Mechanisms have also been extensively used for wing planform morphing (see for example [54–57]). A summary of mechanism-driven concepts in the context of variable stiffness is given in Section 2.1.2.2.

Compared to the application of mechanisms, *compliance*, or deformability based on reversible strains, offers stronger potential for satisfying the light weight objective of geometry adaptation [2]. This heading includes broadly understood compliant mechanisms and aeroelastic tailoring [48]. Compliance imposes special requirements not only on the internal layout, but also on the external cover, as do conformal shape modification strategies in general. The desired morphing skin thus needs to exhibit strongly anisotropic stiffness, low in plane to facilitate shape variation and simultaneously sufficiently high out-of-plane for the aerodynamic form to be preserved under operational loads [12, 31, 58–60]. When implemented in a multi-disciplinary context with intensive function integration, compliant systems promise augmented performance in multi-point terms through continuous shape adaptation throughout the complete mission. By way of illustration, spanwise camber morphing realising flight control through lift distribution enhances the aerodynamic efficiency in a continuous manner [61]. Section 1.3 elaborates on the compliance concept, providing a brief review of the relevant literature.

1.2.2 Morphing Scale and Geometry Parameters

Weisshaar [19] proposes to treat geometry adaptation according to its scale as *large* or *small*. If the wing planform is resized, the former criterion applies, whereas local reshaping such as cambering falls under the latter heading.

Proceeding to a greater detail level, the individual characteristics of the wing geometry may serve as a classification basis, leading to three main groups: *planform*, *out-of-plane* and *aerofoil* [13, 14]. Alternatively, these can be summarised as *in-plane* (planform) and *out-of-plane* modifications. Wing chord, span and sweep constitute the major dimensions of in-plane or plan-
form morphing. Out-of-plane variations encompass camber modifications in the span- and chordwise directions (twist, chord- and span-wise bending). If distinguished as a separate category, aerofoil property adjustment involves changes of no significant impact on wing camber [13].

Camber, or aerofoil curvature, is an effective means of controlling the lift generated by the wing profile, offering advantages compared to pitching. This is commonly exploited in conventional control surfaces and high-lift devices. Applied symmetrically on a wing, increased camber boosts the maximum lift coefficient and reduces the stall speed. Further, its asymmetric deployment provides roll control, whereas gradual spanwise variation allows for inducing twist. On the other hand, camber reduction becomes desirable at high speed to decrease the associated drag. Bolonkin and Gilyard [62] analytically prove the effectiveness of wing camber control in transport aircraft using the existing articulate surfaces in augmenting aerodynamic efficiency within the flight envelope. Similarly, advantages in terms of lift-to-drag ratios, buffet boundary and wing root bending moment are presented by Szodruch [63]. Considering the mission level, maximum endurance gains by virtue of improved lift-to-drag ratios are demonstrated to ensue from periodic camber control accompanied by appropriate throttle and elevator setting [64]. What is more, seamless spanwise variation of the chordwise curvature promises aerodynamic benefits such as reduced friction and gap drag, as well as smaller radar signature [65]. The advantages outlined above have thus rendered camber modification an active area of morphing research.

The possibility of replacing articulate control surfaces with continuous, shape-adaptive counterparts potentially permits evaluating the expected gains as well as providing a starting point for a wider application of the morphing technology. Nevertheless, a higher level, holistic system approach offers stronger potential if the envisioned advantages of shape adaptation are to be fully unleashed [22].

1.3 Conformal Shape Adaptation Based on Distributed Compliance

1.3.1 Distributed and Selective Compliance

The conformal shape adaptation of a complete system, implemented in a reversible and repeatable manner based on structural elasticity, lies at the heart
of the distributed compliance concept, promising performance superior to the approaches of localised compliance or rigid-body motion underlying conventional mechanisms [66]. As a derived enhancement, selective compliance targets low deformation resistance of a particular structural configuration while maintaining considerable stiffness of the remaining modes [67]. The result is controllable, load-independent kinematics, promising to reconcile the reduced actuation effort necessary to achieve the desired morphed shape with largely preserved load-bearing capacity.

The thinkable advantages encompass intensive function integration resulting in a reduced part count and thus decreased complexity and mass, implying lower manufacturing and assembly costs, and shortened lead times [47]. Maintenance benefits are therefore expected by virtue of the added robustness, which additionally arises from distributed actuation, obviating the need for traditional redundant solutions [49]. The simplification in terms of a smaller number of degrees of freedom, in turn, facilitates more comprehensive modelling and simulation. Hence a considerable extension of the available design space may ensue from even more intensive exploitation of the interactions across all the involved domains. Simultaneous topology optimisation of a compliant system including actuation, sensing and control clearly illustrates this point [68]. Furthermore, the absence of joints eliminates tribological problems, offering potential performance enhancements (precision in the absence of backlash or wear; no lubrication). Certain applications might also benefit from facilitated miniaturisation [47]. On the global level, seamless geometry variation brings evident aerodynamic advantages in terms of lower drag and higher efficiency, synonymous with reduced fuel consumption. The reason is not only the elimination of gap drag, but also a smoother pressure distribution without spikes and flow separation associated with sharp curvature changes inherent in articulate control surfaces [69, 70].

The aforementioned favourable aspects are nevertheless accompanied by severe challenges, with the recurring contradiction between realising repeatable, often non-linear deformations, and ensuring usable load transmission characteristics at the forefront. Uncertain long-term behaviour (fatigue, relaxation), achievable motion magnitude as well as public perception constitute further imaginable problem areas [47]. Moreover, appropriate flight control schemes accommodating multi-disciplinarity and non-rigid body dynamics become necessary [71, 72]. Insufficient technological maturity of such key
components as smart actuation, control systems, and advanced skins [49] as well as upscaling issues [22] add to the obstacles hampering the advent of flight-ready, large scale morphing vehicles.

1.3.2 Compliant Morphing Concepts

As early as the 1920s, Parker [73] clearly states the advantages of increasing lift through cambering rather than pitching in search of broadening the achievable speed range without the need for higher landing speeds. A compliant variable camber concept is proposed and patented [73, 74], intended for application in combination with a stiff lifting surface on a bi- or tri-plane. Whereas at high speeds the rib maintains a low-drag, uncambered configuration, higher angles of incidence during slow flight induce its passive cambering. This enhances lift without the need for articulate flap deployment.

Due to the expected benefits outlined in Sections 1.3.1 and 1.2.2, a large volume of published studies focus on compliant camber-morphing concepts. Saggere and Kota [75], and Lu and Kota [76, 77] develop procedures for generating compliant mechanisms approximating prescribed shape changes through the effect of distributing the actuator input. Only the topology itself is considered.

Santer and Pellegrino [78] devise a modified load-path-based parametrisation allowing for synthesising distributed compliant topologies with reduced number of design variables while ensuring feasible solutions. The method is employed to find the internal layout of a leading edge deforming into a prescribed morphed configuration under a specific aerodynamic pressure and internal actuation load.

Spadoni and Ruzzene [79] propose a chiral aerofoil substructure aimed at large compliant shape changes. The achievable global deformations are studied via weak static aeroelastic simulations.

Frank et al. [80] synthesise mechanisms based on both lumped and distributed compliance to realise camber morphing to a prescribed shape, assuming an unstretchable skin. A closed outer cover also characterises the belt-rib aerofoil [81], supported by hinged stiffening spokes. A selectively compliant internal structure of the extended concept, featuring a favoured deformation mode, arises from a dedicated modal method [67, 82, 83]. The belt design constitutes the primary source of the load-bearing capacity. When
considered as part of a wing, however, the belt-rib system is revealed to suffer from performance deterioration compared to 2D predictions owing to 3D effects [84].

Phase 2 of the Defense Advanced Research Projects Agency (DARPA) Smart Wing project [85–92] involves the development of a compliant trailing edge (TE) control surface, based on a honeycomb core covered with a continuous silicone skin. Actuation is produced by an ultrasonic motor and transmitted through an eccentric, attaining high rates of 20° deflections below 0.33 s [91]. Performance benefits accomplished through retrofitting conventional control surfaces with the smart solutions investigated within the programme are demonstrated on the TRL of five [92].

Patents by Kota [93], and Kota and Hetrick [94], concern compliant system-based shape adaptation of aerofoil-like structures and are commercialised through FlexSys Inc., a company developing seamless, conformal morphing solutions [95]. Flight tests of the Mission Adaptive Compliant Wing (MACW), featuring a TE flap relying on distributed compliance, prove distinct advantages in terms of an increased laminar flow regime as well as augmented range due to the optimisation of aerodynamic efficiency in operation via morphing [96].

The bio-inspired Fish Bone Active Camber (FishBAC) concept [97–102] relies on substantial compliance anisotropy, featuring low chordwise stiffness to facilitate cambering effectuated by a conventional pulley-tendon system. Orthogonal stringers attached to a thin beam placed along the chord line support a pre-tensioned, elastomeric skin. Wind tunnel experiments show a lift variation capacity of $\Delta c_l = 0.72$ at 20 m/s and zero incidence as well as a 20%–25% improvement in aerodynamic efficiency compared to an equivalent flapped configuration [99].

Molinari et al. [61, 103–110] develop a multidisciplinary, aero-servo-elastic methodology to demonstrate the viability of conformal shape adaptation based on distributed compliance combined with smart actuation. The simultaneous incorporation of aerodynamic and structural aspects into the optimisation process permits defining physically meaningful requirements derived from the mission profile, as well as exploiting aeroelastic amplification effects, while guaranteeing freedom of static and dynamic instabilities. An efficient, load-carrying configuration emerges, featuring a non-obvious topology obtained from compact, Voronoï-based parametrisation. Concur-
Currently considering smart Macro Fiber Composite (MFC) actuation allows for overcoming its inherent limitations in a multi-functional role including the contribution to the load-bearing capacity. Sufficient altitude control authority is achieved through the conformal lift variation to replace conventional ailerons. This is further presented for a tailless configuration [61]. The morphing wing and the underlying methodology are experimentally validated, with flight tests on a model aircraft confirming promising maturity [108–110]. Potential challenges still to be addressed include reliability [108].

A multidisciplinary approach involving 3D aerodynamic effects and structural constraints is also adopted by Previtali et al. [60, 111–116] to develop a shape-adaptive wing featuring a novel double-corrugated skin. The distributed layout of the component compliant ribs is determined by means of a modified ground structure approach. Numerical and experimental investigations prove that the achieved extent of morphing suffices for roll control comparable with articulate ailerons [113, 115].

1.4 Research Need
Shape-adaptive systems able to efficiently adjust their characteristics to resolve the conflicting demands of changing operating conditions offer great potential for augmented functionality and performance. The expected gains have motivated numerous studies reflected in the relevant review literature [7–22], resulting in a wide variety of morphing concepts. Despite the headway made to date, however, the associated TRL remains low and penalties in terms of mass, complexity and cost tend to outweigh the achieved advantages, hampering a wider application of this technology. Therefore, the quest for a novel approach able to fully unleash the potential of morphing still continues.

As outlined in the preceding section, several studies have already demonstrated the feasibility of conformal shape adaptation based on distributed compliance (see for example [61, 102, 106, 110, 113]). An important feature of this strategy is that the actuation energy necessary for morphing is expended not only to overcome the distributed aerodynamic loads, but also to perform the elastic work required to induce the reversible structural deformations. The latter is absent in the case of conventional control surfaces, deflected at hinges rather than through distributed straining of the internal
1.5 General Approach, Thesis Objectives and Outline

The goal of this thesis is to explore the novel shape adaptation concept outlined above, combining distributed compliance with stiffness variability. In particular, multi-stable laminates are studied in the function of monolithically embedded variable stiffness components, targeted at providing an enhanced morphing functionality.

Figure 1.1 highlights the key components of the proposed approach. The aim is to develop a morphing wing profile whose adaptation capabilities topology. Hence, energy saving potential emerges from a reduction in the second contribution. This can be achieved through introducing variable stiffness elements into such a compliant topology to lower the structural resistance on-demand.

Although the notion of stiffness variability underlies a large volume of published studies, not enough attention has been given to this solution on its own in view of the potential offered for morphing applications. In particular, the implications of combining the benefits of distributed compliance with temporally variable stiffness have not been determined yet. Further, investigating bi-stable laminates in the function of variable stiffness components constitutes a focus shift from the large deflection potential commonly exploited in literature [20, 117–123].

The present work thus seeks to address this research gap. The strategy consists in imparting stiffness variability to distributed compliance systems through monolithic integration of bi-stable components, as indicated in Figure 1.1. Selectively altering the internal stiffness distribution through stable state switch allows for activating desired global deformation modes with reduced actuation effort. These selective configurations can be tailored to address specific, dissimilar operational scenarios, thereby decoupling the conflicting demands of changing external conditions. Compared to time-invariant systems, activating particular deformation modes permits enhanced actuation efficiency, promising mass savings arising from a decreased amount of the necessary actuation material. Accordingly, the combination of distributed compliance with variable stiffness creates the novel functionality of selective structural behaviour offering interesting potential for addressing the morphing challenge.
emerge from the selective stiffness properties of bi-stable elements strategically positioned within the distributed compliant topology. Therefore, the central role of the bi-stable components is to provide distinct equilibrium states of high and low deformation resistance. The locally induced snap-through then leads to an internal stiffness redistribution of the aerofoil. The current stable shapes of the embedded laminates, corresponding to a particular stiffness property combination, thus govern the global structural response of the wing section. In other words, changing the equilibrium states of the internal elements permits modifying the deformation resistance on the aerofoil level, leading to selective global configurations adopted under external loading such as aerodynamic pressure or actuation. This results in a system able to exhibit multiple deformation modes whose characteristics can be tailored to address desired dissimilar operational scenarios, promising shape adaptation at reduced actuation effort. Further, the distributed compliant nature combined with stiffness variability means that the selective configurations arise from the mutual interplay between the aerodynamic pressure and the corresponding structural deformations. This creates a need for concurrently considering the aerodynamic and structural response in an aeroelastic analysis approach. Finally, rather than relying on external loads for passive shape adaptation, global actuation can be introduced to achieve controlled morphing benefiting from the stiffness selectivity of the
wing profile for enhanced efficiency.

The adopted methodology increases in complexity as the individual aspects presented in Figure 1.1 successively enter into consideration. Accordingly, the study begins from the most fundamental level of devising bi-stable laminates possessing the desired characteristics to determine the available design space in terms of stiffness variability and embeddability. Having acquired the ability to generate bi-stable components fulfilling the intended function, the investigation progresses to gaining first insights into the aerofoil response selectivity achievable with the pursued approach. To that end, first morphing concepts with a simple internal layout featuring embedded bi-stable elements are developed and assessed in respect of the resulting variability of the global deformation resistance. An aeroelastic simulation method is subsequently introduced to determine and evaluate the selective aerofoil configurations arising from the two-way interactions between aerodynamic pressure and the compliant structure exhibiting specific internal stiffness distributions. Finally, all the four aspects: bi-stable laminates embedded in a variable distributed topology, an aeroelastic analysis approach, and global actuation are simultaneously considered to design and optimise shape-adaptive wing profiles according to prescribed performance targets. This seeks to fully exploit the interactions and available capabilities associated with the contributing dimensions.

The structure of the present work, introduced in the following, reflects the individual objectives leading to the accomplishment of the thesis goal.

To begin with, theoretical dimensions of the research are laid out in Chapter 2, reviewing the state of the art of variable stiffness in the morphing context as well as elaborating on the multi-stability phenomenon.

Chapter 3 is concerned with bi-stable laminates as the fundamental building blocks of the concept, undertaking a design space exploration of this class of composite structures given the requirements of stiffness variability and monolithic integration within a larger shape-adaptive system. A systematic design improvement methodology based on two independent criteria for bi-stability assessment is presented, allowing for the development of bi-stable components fulfilling the specifications imposed by the envisaged function.

Chapter 4 provides a numerical and experimental demonstration of global stiffness variability of an adaptive wing section with a basic distributed topology featuring two embedded bi-stable laminates. A simple mechanical
load is used for this initial proof of concept.

The fifth chapter introduces a weakly coupled static aeroelastic simulation environment to emphasise the importance of concurrently considering the mutual interactions between structural deformations and aerodynamic loads in the case of highly compliant systems. The global stiffness variability and aerodynamic adequacy of the previously validated demonstrator are assessed regarding the passively morphed shapes adopted under pressure distributions corresponding to a range of aerodynamic conditions. An improved configuration is also developed based on this concurrent aeroelastic method.

The next chapter addresses the proposed morphing approach combining distributed compliance with stiffness variability through a holistic realisation in a multidisciplinary context. This constitutes a crucial step in managing the inherent complexity and contradictory requirements on a higher level as well as fully exploiting the interactions between the contributing fields. Accordingly, a more complex topology is introduced as well as extending the preceding purely passive consideration to globally controlled shape adaptation.

In Chapter 7, the final configuration to emerge from the concurrent methodology is presented and evaluated numerically using low and high-fidelity approaches.

The final chapter summarises the main findings and concludes on the study as well as laying out future research lines opened by the present work.

1.6 Original Contributions

This project provided an important opportunity to explore the novel morphing approach of distributed compliance augmented through stiffness selectivity realised by means of embedded bi-stable laminates, thereby contributing to the growing area of morphing research. The resulting original contributions are summarised in the following.

• design methodology for embeddable bi-stable laminates featuring variable stiffness characteristics
  – understanding of the available design space and parameter sensitivity of this class of composite structures
  – ability to develop bi-stable configurations fulfilling the requirements imposed by the envisaged function of variable stiffness
components monolithically embedded into a distributed compliant topology

- two independent numerical criteria for bi-stability assessment

• proof of concept of globally variable stiffness aerofoil based on integrated bi-stable laminates

- numerical and experimental demonstration of passive global stiffness variability
- prediction and assessment of selective deformation modes adopted under distributed aerodynamic loading due to the internal stiffness distribution governed by the equilibrium configurations of the embedded bi-stable components

• concurrent design and optimisation framework for morphing systems combining distributed compliance with selective stiffness through embedded bi-stable laminates

- versatile, computationally efficient representation of stiffness response of embeddable, variable stiffness bi-stable laminates as user-defined finite elements
- impact maximisation of the bi-stable laminates on the structural response
- ability to design selective, intentionally dissimilar deformation modes to meet physically meaningful performance targets

† aerodynamically favourable high-lift configuration with reduced actuation demands, only attainable by virtue of the internally tailored distributed compliance arising from the stable state switch

† stiff configuration, aimed at stable operation under increased aerodynamic loading and providing a smaller lift variation range

• morphing concept with distributed compliance augmented through variable stiffness provided by bi-stable laminates

- advantages in terms of predicted aerodynamic efficiency compared to pitching an equivalent rigid wing profile
- actuation energy savings compared to a uniform distributed compliant topology, or stiff configuration, offering potential for reduced amount and mass of actuation material, or overcoming the authority limitations of smart solutions
CHAPTER 2

State of the Art

2.1 Morphing Applications of Variable Stiffness

This section is based on the journal publication:

2.1.1 Variable Stiffness Methodology Classifications

The variable stiffness approach to morphing implies the capacity to dynamically adjust stiffness properties as required by optimum performance under changing operating conditions. In contrast to realising a passive system with spatially tailored stiffness characteristics, the current definition stresses the temporal aspect. Specifically, the stiffness modifications occur in time to continuously enhance the efficiency in operation, in addition to a particular spatial distribution of structural stiffness. This strategy entails selectively decreasing the deformation resistance to reduce the associated actuation effort, while providing sufficient overall rigidity to limit the structural deformations arising from the external loads acting on the system. Accordingly, two broad notions fall under the heading of stiffness adaptation. The first is
the reversible strain capacity whose extent depends on the elastic material constants. In trying to reconcile compliance with functional and load-carrying requirements, a substantial range of elastic moduli rather than only fixed values becomes highly advantageous. The second notion refers to the overall structural behaviour, including bending and torsional rigidity along with the corresponding geometric parameters. These two general directions, namely material properties and geometry, allow a variety of strategies to be accommodated under the heading of stiffness variation, remaining valid not only for morphing, but also in a diversity of other disciplines.

The challenge posed by the provision of variable stiffness related to material properties lies in the inherent contradiction between high modulus and reversible deformation capacity. Specifically, the elastic strain regime of materials characterised by substantial rigidity is generally very narrow and vice versa [124]. Further, whereas a high rate of stiffness change usually implies small achievable variation scope, as is the case in capacitive shunting of piezoelectric materials, rather slow reaction times are associated with substances with wide elastic modulus modification range, exemplified by thermo-reactive Shape Memory Polymers (SMPs) [125].

In the field of vibration control, Lee et al. [126] present an analogous classification of stiffness variability provision techniques: altering geometry or shape, and adapting material properties. The latter strategy has been highlighted as easily applicable and controllable in the case of smart materials such as ferromagnetic shape memory and piezoelectric materials, Shape Memory Alloys (SMAs), electro- and magneto-rheological fluids and elastomers, and viscoelastic solids. Similarly, Kornbluh et al. [127] refer to two categories: materials with inherently variable properties, responding for instance to thermal or electromagnetic stimuli, or materials performing as actuators to induce changes in viscoelastic characteristics. Further, intrinsically adaptive and active materials are distinguished. The first group, including SMPs, SMAs, and magneto- and electro-rheological fluids, also described as semi-active due to their low energy requirements, undergo mechanical property changes when subjected to external stimulation. Active materials, in turn, exemplified by piezoelectric or magnetostrictive ceramics and Electroactive Polymers (EAPs), transform such external energy forms as electric or thermal into mechanical strain energy and vice versa. A semi-active arrangement is also possible, as in the case of shunted piezoelectric
systems [127–130].

These two classifications are more meaningful in the context of vibration control – a discipline in which variable stiffness is a well-known concept [131–133]. The vibration reduction of helicopter blades by means of cyclic stiffness tuning at the blade root provides one of numerous illustrations [134]. Consequently, a whole gamut of pertinent strategies originate in the field of Adaptive Tuned Vibration Absorption (ATVA) [126, 135], aimed at accommodating a broader, time-variant spectrum of excitation frequencies by means of an adequate stiffness adaptation. Widely covered techniques include area moment of inertia modulation of a rotating beam [136], shunted piezoelectric materials [137, 138], a spring arrangement with piezoceramic actuators [139], application of electrorheological fluids [140–142] or magnetorheological fluids [143–147], magnetorheological elastomers [148, 149] or electrorheological elastomers [150], shape memory alloys [151, 152], and ferromagnetic shape memory alloys [153, 154]. With the exception of SMAs, the above strategies carry little relevance as stiffness tuning techniques pertinent to morphing and therefore are not discussed any further in the following.

The aforementioned classification of variable stiffness concepts into material property, and geometry or shape adjustment is too general to adequately reflect the existing tendencies in morphing research. As further categorisation, four broad directions are proposed herein: material tailoring, active mechanical design, semi-active techniques and exploitation of elastic structural behaviour, as schematically shown in Figure 2.1.

### 2.1.2 Existing Variable Stiffness Concepts

#### 2.1.2.1 Material Tailoring

Purposeful material tailoring constitutes a distinct tendency among existing variable stiffness solutions. This encompasses not only material selection but also careful composite system design and optimisation in search of pronounced authority over the ensuing stiffness characteristics. The most widely covered approaches exhibiting the strongest morphing potential involve SMPs, especially as a basis for composite materials, and Fluidic Flexible Matrix Composites (F²MCs). Other strategies include pneumatically adapted honeycombs and passive techniques. The variable stiffness characteristics of selected concepts are summarised in Table 2.1.
Figure 2.1: Proposed classification of variable stiffness material and structural concepts for morphing applications.
Table 2.1: Summary of stiffness variability data of selected material tailoring concepts.

<table>
<thead>
<tr>
<th>Material</th>
<th>Reference</th>
<th>Stiffness Variability</th>
<th>Conditions</th>
<th>E&lt;sub&gt;cold&lt;/sub&gt;/E&lt;sub&gt;hot&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape Memory Alloys</td>
<td>[155]</td>
<td></td>
<td>82°C - 38°C</td>
<td>4</td>
</tr>
<tr>
<td>Hot/Cold NiTiNOL</td>
<td>[156]</td>
<td></td>
<td></td>
<td>55 – 38°C</td>
</tr>
<tr>
<td>Shape Memory Polymers</td>
<td>[157]</td>
<td>polyurethane-based</td>
<td>326 – 517</td>
<td>482 – 38°C</td>
</tr>
<tr>
<td>Hot/Cold T &lt;sup&gt;g&lt;/sup&gt; = 55°C</td>
<td></td>
<td>Church Resilience</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Memory Composites</td>
<td>[158, 159]</td>
<td>polystyrene-based</td>
<td>20°C - 80°C</td>
<td>100</td>
</tr>
<tr>
<td>Hot/Cold T &lt;sup&gt;g&lt;/sup&gt; = 55°C</td>
<td></td>
<td>CTD-DP-5.1 bulk thermoset resin</td>
<td>23°C – 90°C</td>
<td>100</td>
</tr>
<tr>
<td>Elastic Memory Composites</td>
<td>[160]</td>
<td>reinforcement: carbon-fibre (T300)</td>
<td></td>
<td>79 – 23°C</td>
</tr>
<tr>
<td>Hot/Cold T &lt;sup&gt;g&lt;/sup&gt; = 55°C</td>
<td></td>
<td>resins: styrene-based Veriflex&lt;sup&gt;®&lt;/sup&gt; S, VF 62</td>
<td>35°C – 75°C</td>
<td>79 – 23°C</td>
</tr>
<tr>
<td>Elastic Memory Composites</td>
<td>[161–163]</td>
<td>constant-variable stiffness layer laminate</td>
<td></td>
<td>15 – 77</td>
</tr>
<tr>
<td>Hot/Cold T &lt;sup&gt;g&lt;/sup&gt; = 55°C</td>
<td></td>
<td>reinforcement: 1095-steel hexagonal elements</td>
<td>23°C – 90°C</td>
<td>15 – 77</td>
</tr>
<tr>
<td>Fluidic Flexible Matrix Composites</td>
<td>[164]</td>
<td>tube: ±35° carbon fibre, silicone matrix</td>
<td></td>
<td>21.6 – 25.1</td>
</tr>
<tr>
<td>Fluidic Flexible Matrix Composites</td>
<td>[164]</td>
<td>working fluid: water</td>
<td></td>
<td>21.6 – 25.1</td>
</tr>
<tr>
<td>Fluidic Flexible Matrix Composites</td>
<td>[164]</td>
<td>F&lt;sup&gt;®&lt;/sup&gt;MC sheet four ±35° carbon fibre/silicone matrix tubes</td>
<td></td>
<td>discrete: closed/open-valve</td>
</tr>
<tr>
<td>Fluidic Flexible Matrix Composites</td>
<td>[164]</td>
<td>sheet resin: silicone</td>
<td></td>
<td>discrete: closed/open-valve</td>
</tr>
</tbody>
</table>
2.1.2.1.1 **Shape Memory Materials**  A large and growing body of literature has investigated shape memory materials, comprising alloys, ceramics, and polymers, due to the potential associated with the Shape Memory Effect (SME). SMAs have been widely employed for actuation purposes in the large shape adaptation context [165–170]. Nevertheless, it is SMPs which have attracted the most attention with respect to variable stiffness applications in morphing. This is based on the large reversible strain capacity they offer, combined with the additional property enhancement possibility when reinforced to form shape or elastic memory composites.

**Shape Memory Alloys**  What initiated focused research activities into shape-memory phenomena was the work of Buehler et al. [171] from the Naval Ordnance Laboratories (NOL), who observed plastic deformation- and temperature-dependent phase transformations of TiNi alloys close to stoichiometric composition. The association of the discoverers became then integrated into the name NiTiNOL coined for the material. Apart from the nickel-titanium configuration, two other SMA types exist. These are copper- and iron-based, yet the latter has not found many applications to date [169, 172]. Another classification can be undertaken according to the activation method, including temperature, or (less common) magnetic field [169].

At the microscopic level, SMAs feature two main phases, corresponding to higher and lower temperatures and possessing dissimilar crystal structures. These are the usually cubic austenite and monoclinic, tetragonal or orthorhombic martensite, respectively. The diffusionless transformations between the two configurations occur by shear lattice distortion, providing the foundations for the shape memory effect, whose underlying mechanism takes the following theoretical course (Figure 2.2). Cooling below the martensitic finish temperature $M_f$ leads to a full phase transformation from austenite to twinned martensite (1 $\rightarrow$ 2 in Figure 2.2). When in the cold state, the application of mechanical loading (2 $\rightarrow$ 3) induces reorientation, or detwinning, of the martensite, permitting material deformation sustained on load removal (3 $\rightarrow$ 4). Finally, heating above austenitic finish temperature $A_f$ induces a reverse transition from detwinned martensite to austenite (4 $\rightarrow$ 5 $\rightarrow$ 1, 5: transition initiation at austenitic start temperature $A_s$), on completion of which the original shape is regained. Successive cooling below $M_f$ yields
2.1 Morphing Applications of Variable Stiffness 23
twinned martensite without any further form change. Hence the process, also
called one-way shape memory effect, can be repeated, with the austenitic
shape recovery occurring for the temperature of the detwinned material be-
ing elevated above $A_f$ [168, 169, 173]. In the absence of appropriate training
entailing thermal cycling, however, a certain amount of unrecovered strain
remains and the response changes with every process recurrence. Since
actuation purposes require minimisation of any such adverse phenomena,
sufficient response stabilisation in the form of 100–1000 cycles needs to be
performed [166].

A further behaviour type charac-
teristic of SMAs is a two-way ef-
fect, achievable through targeted
training. In addition to the reac-
quision of the austenitic shape
when heated above $A_f$, the deformed
martensitic configuration can be re-
covered upon cooling without the
need for load application [166, 172].
A triple SME has also been recently
demonstrated [174].

These solid-to-solid phase trans-
formations are associated with sig-
nificant reversible deformation ca-
pability and can therefore generate
considerable stresses under appropriate constraint conditions [166]. A fur-
ther important aspect in view of the variable stiffness context are temperature-
dependent elastic modulus values, decreasing with reduced temperature and
vice versa [175]. Early investigations revealed a 2.54 mm diameter NiTiNOL
rod to exhibit a highly-nonlinear stiffness profile ranging from about 21 GPa
to 83 GPa within a temperature range of 38°C to 82°C, respectively [155].

Fig. 2.2: Shape-memory effect in
stress-strain-temperature
space.

The advantages of SMAs include considerable energy density ensuing
from large reversible deformation and stress generation potential, making
them especially attractive for actuation purposes. NiTi-based systems feature
actuation stresses reaching 500 MPa, accompanied by recoverable strains
close to 7% [166, 169]. On the other hand, heat transfer considerations
owing to large heat capacity of these metallic materials limit the attainable
actuation frequency range, leading to slow system response [166, 173]. Another problem area is limited fatigue life, with a typical maximum of about $10^4$ cycles to failure [172].

SMAs have been widely considered for actuation purposes in the morphing context [165–170]. The thermally-dependent Young’s modulus of these smart materials, however, has not played a central role to date. Indeed, the alloys only display limited relevance in the variable stiffness context because of disadvantages such as thermal hysteresis or slow response time accompanying a modest stiffness adaptation range [151]. Last but not least, it is elevated temperatures to which increased elastic modulus values correspond.

Shape Memory Polymers When externally stimulated, SMPs undergo a shape change process leading to the acquisition and retention of a different, temporary configuration. The original form, or permanent shape, can subsequently be resumed in response to an analogous outside input due to the SME [169, 176, 177]. A cyclic application between the two states is nevertheless not readily possible, requiring targeted processing [176].

The shape memory effect can be traced back to the presence of two distinct domains forming the structure of SMPs on the molecular level. These are referred to as an elastic and a transition segment. Whereas the latter undergoes substantial stiffness variations when externally stimulated, the elastic component retains its high deformability irrespective of the outside input. Although it is thermal activation which finds the most frequent use [178], alternatives such as light [179] as well as electric [180], magnetic, or mechanical means [181, 182] do exist.

An important parameter of the thermally-triggered SME is the transition temperature $T_{\text{trans}}$ marking the boundary between the hard ($T < T_{\text{trans}}$) and soft ($T > T_{\text{trans}}$) polymer states. Depending on whether the transition segment has an amorphous or crystalline character, $T_{\text{trans}}$ corresponds to either the glass transition $T_g$ or melting temperature $T_m$, respectively [183]. During the initial programming process, heating to exceed $T_{\text{trans}}$ induces softening and compliance of the transition component, which therefore becomes easily deformable in response to an applied load. By virtue of its inherent compliance, the elastic domain follows the strain path. Subsequent cooling below $T_g$ preserves the desired temporary shape, allowing the transition component
2.1 Morphing Applications of Variable Stiffness

to harden in the deformed configuration, which prevents elastic recovery of the other segment. The final stage entails elevating the temperature above $T_{\text{trans}}$ once more. The resulting softening of the transition segment prompts the elastic domain to release the elastic energy stored due to the temporary shape. Hence the material can retain its permanent form in a manner reflecting the reverse of the programming process [169, 177, 184].

These transformations are accompanied by altered mechanical properties of the polymer, including its Young’s modulus, pointing to the variable stiffness potential of SMPs [177]. Elevated temperature ($T > T_{\text{trans}}$) can induce a rigidity change reaching 500 times [185], compared to a 3–4 times variation in the case of SMAs [186]. Whilst producing considerable strains when heated, SMPs display some stiffness below their characteristic transition temperature.

When viewed from the morphing perspective, the superiority of SMPs compared to SMAs lies in smaller density, larger reversible strains, ease of adaptation of their thermo-mechanical characteristics to suit particular needs, responsiveness to other than thermal stimulants. Further advantages of SMPs include reduced material, manufacturing, and processing costs accompanied by fabrication flexibility. On the other hand, the realisable actuation forces are much lower than in the case of SMAs, with the corresponding maximum stresses of a few MPa contrasting with at least 10 MPa for SMAs [169]. This, however, is of no primary importance in the variable stiffness context. Nevertheless, aspects such as the danger of micro-scale damage to shape memory polymers in the event of insufficient programming temperature as well as uncertain long-term durability and reliability should not be neglected [169].

When employed alone, SMPs offer substantial reversible strain potential reaching 200\% [181], yet a number of major disadvantages hamper their direct structural application. These include considerable temperature sensitivity hindering controllability, low stiffness at elevated temperatures implying small recovery forces as well as slow transformation rate, poor thermal conductivity, performance deterioration within the initial thermo-mechanical cycles, large thermal expansion coefficient, and lack of energetic efficiency [124, 182, 183]. Although an improvement of stiffness properties can be achieved through the application of reinforcement, it only comes at the expense of a reduction in reversible deformation by a factor of ten [124]. As a remedy for those limitations, SMP-based composite materials termed shape
or elastic memory composites have been developed to reconcile sufficient stiffness with controlled large deformation capacity.

Commercially available shape memory polymers and composites featured in variable stiffness morphing applications include Veriflex®, its composite Veritex™, and Essemplex™ thermoplastic resin offered by the Cornerstone Research Group [187]. Further, a thermo-responsive thermoset SMP resin family TEMBO® put on the market by Composite Technology Development [188] provides a basis for a range of elastic memory composites.

Numerous studies have demonstrated the considerable stiffness variability potential of pure SMPs. By way of illustration, Tobushi et al. [156], who have experimentally analysed a thin film made of a polyurethane series SMP, prove an elastic modulus ratio of 100 below and above the glass transition temperature $T_g = 328$ K of the material. When in the cold state, the substance displays high strain resistance. Under laboratory conditions, heating takes place as a result of the application of high-temperature compressed air spray to the specimen, with cooling being achieved by means of liquefied CO$_2$. The necessity for the programming process above $T_g$ to involve multiple cycles is recognised so as to ensure invariant temporary shape features in the case of cyclic operation.

Some conclusions about the mechanical properties related to the behaviour of shape memory polymers in shear, as would be the case for an aircraft skin, can be found in the work of Keihl et al. [157] on the example of polystyrene-based specimens. Characterised by very low stiffness at elevated temperatures and thus acting like a membrane, the material undergoes undesired out-of-plane folding ensuing from compressive stresses induced by the shear loading. Axial pre-strain and geometry tailoring are suggested as possible preventive measures delaying the instability onset. Furthermore, the properties of SMP are proved to be both time and temperature dependent, with morphing rate restrained by not only its direct proportionality to the power required, but also the polymer recovery time.

The limitation of a rather slow time response of SMPs, caused by the inherent difficulty of cooling occurring through natural convection, has also been observed by Lee et al. [126]. The study involves a hybrid cantilever beam composed of a conducting NiTi core surrounded by a shape memory polymer sleeve to fulfil the spring role of a semi-active vibration absorber. Induced by resistive heating in the form of a controlled flow of electric current
2.1 Morphing Applications of Variable Stiffness through the core, temperature changes between 25 °C and 90 °C arise in the SMP outer sleeve, leading to stiffness and thus frequency variations.

Wang and Brigham [189] draw attention to the potential energy savings and morphing shape accuracy improvements through the implementation of an optimisation process. The ultimate objective of the strategy is a precise determination of target regions for selective thermal stimulation and actuation of SMP-based morphing systems to obviate the need for highly inefficient activation of the entire structure.

**Elastic Memory Composites** The performance of shape memory polymers, particularly in terms of recovery force and stiffness, can be substantially enhanced as a result of an appropriate reinforcement strategy [158, 159]. Despite an order of magnitude loss in the reversible strain capacity compared to pure SMPs [124], an adequate range of adjustable stiffness characteristics is still preserved in the morphing context. The term Elastic Memory Composites (EMCs) has been coined to denote composite materials relying on shape memory polymer matrices [190]. In this regard, Perkins et al. [191] divide SMPs applicable as EMC resins into three main classes. These are completely cured thermosets, thermoplastics featuring shape memory deterioration throughout their lifetime, and partially cured resins continuing the curing process in operation and thus undergoing property variations as time progresses. Since advantages such as superior durability, mechanical characteristics, and good manufacturability are associated with thermoset matrices [158, 159, 192], the term EMCs is often understood as an unambiguous implication of a thermoset resin [192]. An example is the epoxy-based [193] TEMBO® SMP developed by the Composite Technology Development, Inc. [188] and employed in numerous studies [158, 159, 190, 192, 194, 195], lending itself to both standard (glass, carbon, Kevlar-fibre) and nano-reinforcements (nanoparticles) [192].

In the course of experimental testing of the thermoset CTD-DP-5.1 resin, Abrahamson et al. [158, 159] discover that the shape memory effect can be induced not only in the standard temperature-load process, but also through the application of a high mechanical strain alone, without the need for heating during the programming phase. Further, the occurrence of the shape-memory effect is observed at a temperature not coinciding with the $T_g$ of the resin,
leading to the recommendation that the SMP cycle be related to the actual temperature at which transition occurs. Arzberger et al. [192] highlight the development necessity for the existing heating methods (resistive/radiative) with respect to unsatisfactory efficiency and uncertain reliability, whilst identifying SMP foams and nano-composites as future directions.

The potential of EMCs has originally been recognised for deployable space applications [190]. For this purpose, the structure is initially heated above the glass transition temperature and deformed to acquire the condensed stowage form. Cooling to preserve the compacted configuration follows, with the original shape recovery initiated in the case of the temperature being elevated above $T_g$ once more. According to Tupper et al. [190], the deployment process can be reversible, a feature indispensable for morphing applications. Nevertheless, this aspect seems not to be followed any further in their work.

Using plain-weave T300 carbon fibre to reinforce a commercial thermoset matrix, namely the styrene-based Veriflex®, has been investigated by Lan et al. [196] in view of its suitability for a deployable hinge application. Compared to a pure SMP, a higher storage modulus is achieved, featuring a pronounced drop within the temperature region of 40°C to 80°C.

Chen et al. [160] present a variable stiffness tube concept with possible morphing skin relevance. Made of a shape-memory polymer composite comprising a thermoset SMP resin reinforced with carbon fibres, the tube is manufactured by means of wet filament winding. An experimental ratio of the attainable effective axial elastic moduli of 79 is found between the low and elevated temperature states, with the volume fraction and winding angle of the fibres as parameters permitting further tailoring this value.

Reliable and sufficiently precise characterisation of mechanical properties of EMCs with adequate performance prediction in view poses an additional challenge.

Hulse et al. [194] discuss the non-linear character of the longitudinal modulus of unidirectional carbon fibre-reinforced elastic memory composites, with growing strain accompanied by a linear increase in the elastic modulus. The phenomenon is primarily attributed to the non-linear nature of the reinforcement, with processing imperfections contributing to a lesser extent. Another non-linear feature of EMCs pertains to their bending response at temperatures exceeding $T_g$, including locally varying effects, as discussed by
Lake and Campbell [195].

Arzberger et al. [192] and Lake and Campbell [195] raise the issue of the significance of strategy development for heating as deployment initiator and controller. Aspects such as efficiency, mass penalty, integrability, controllability and reliability are of primary concern. Potential insufficiency of a simple resistive heating technique in larger applications is highlighted [195].

**Shape Memory Composite Topology Concepts**  McKnight and Henry [161], McKnight et al. [162], and Henry et al. [163] draw inspiration from biology to combine appropriate material properties and topology, developing a tunable rigidity composite material concept formed of constant and variable stiffness segments, as schematically shown in Figure 2.3. These serve load-carrying and adjustable connectivity purposes, respectively. In the high-resistance structural mode, rigid connection exists between both fractions of the composite. The application of a shape memory polymer as the embedding low-stiffness component allows for the second compliant morphing mode in which considerable softening of the SMP matrix creates the possibility of relative motion between the constant stiffness elements, enabling substantial reversible strains to be achieved. Measurements based on several sample laminate structures comprising high yield spring steel and Diaplex 5510 SMP as the constant and variable stiffness components, respectively, yield a cold state \( T = T_g - 20^\circ\text{C} \) elastic modulus \( E_{\text{cold}} \) of 8 GPa to 12 GPa and reversible deformations of 2% to 10% when heated \( T = T_g + 20^\circ\text{C} \). The accompanying stiffness changes \( E_{\text{cold}}/E_{\text{hot}} \) range from 15 to 77 times, with the former corresponding to \( E_{\text{cold}} = 12.3 \) GPa, \( E_{\text{hot}} = 0.79 \) GPa and the latter to \( E_{\text{cold}} = 10.8 \) GPa, \( E_{\text{hot}} = 0.14 \) GPa [161, 162]. Conducting three-dimensional finite-element analyses, Henry et al. [163] report the possibility of property enhancement to 30% strain and 30 GPa modulus in the elevated and normal temperature states, respectively, through careful material parameter tuning. On the other hand, such temperature-controlled stiffness change necessitates the integration of heating devices within the laminate, which may affect the stiffness properties achieved. McKnight et al. [162] quote a time period of 1 min–2 min required to reach the transition temperature, highlighting the cooling possibility as a major limiting factor.

A laminate configuration involving alternating constant and variable stiff-
ness layers has been patented by McKnight and Barvosa-Carter [197]. Shape memory polymers are only an example of the adjustable modulus material which can be accommodated by the generic variable stiffness laminate concept. Alternatives suggested include shape memory alloys, electro- or magneto-strictive materials, liquid crystal elastomers as well as electro- and magneto-rheological substances. Distinguished by varying control means, the individual tunable modulus materials also differ by the achievable time constant of stiffness change.

![Figure 2.3: Segmented reinforcement variable stiffness laminate concept of McKnight et al. [161, 162].](image)

A cellular architecture patent by Henry and McKnight [124] offers a trade-off solution between sufficient rigidity and elastic deformation, providing substantial geometry and stiffness variation. Finite-element analyses prove an auxetic topology with strain-relieving curved cell walls to display the best stability and performance in view of large in-plane strain capacity [124]. Fabrication and testing of specimens made of both pure shape memory polymer and its continuous carbon fibre reinforced composite are reported as well. The utilisation of the underlying shape memory effect consists in the initial cooling of the previously buckled cellular structure to achieve a condensed arrangement. Subsequent heating to exceed the glass transition temperature $T_g$ eventually results in an expansion corresponding to an area increase of about 130%, facilitated by the auxetic or negative Poisson’s effect. However, the cellular topology poses several major difficulties which balance its substantial deformation capacity [124], with the main deficiencies carried by the empty zones inside the structure generating the large spatial stretching. These are significantly reduced overall stiffness of the material and
deformation-dependent variation of its properties caused by material redistribution during straining. An additional limitation is the controllability of the buckled configurations responsible for the large area extension possibility.

The aforementioned cellular topology [124, 198] and segmented reinforcement [161, 162] approaches are combined to yield another variable stiffness composite material concept developed by McKnight and Henry [199]. The strategy seeks to permit considerable shear deformations (5% to 20%) leading to large area increases without the occurrence of out-of-plane buckling. Ligament-connected discrete platelets, capable of rotation under shear loading, are embedded in a shape memory polymer matrix. In the overall arrangement, such reinforced layers feature offsets with respect to one another, additionally alternating with pure SMP layers to form a laminate. Facilitating shear transfer, this configuration improves the overall stiffness. Larger shear strains are enabled through axial pre-strain of SMP matrix portions expected to undergo compression under the overall shear loading, which delays the onset of buckling. Experiments show good effective stiffness of the composite (4 GPa–8 GPa) in a low-temperature state. Simultaneously, substantial shear strains (10°) are accomplished when heated above the polymer glass transition temperature, with reversibility ensured through both the shape memory effect and the reinforcement-stored energy.

Property enhancement is also sought through the application of shape memory polymers within sandwich structures. Yin et al. [200] mention a sandwich skin composed of a honeycomb core and an SMP cover to enclose a morphing wing capable of a mechanically-driven chordwise extension. Wire springs arranged parallel to each other are embedded in the SMP to provide heating. No further details permitting the assessment of the method, including the skin design, are provided.

As part of the activities of the Cornerstone Research Group, Inc. [187], Perkins et al. [191, 201] and Reed et al. [202] investigate an adaptive UAV wing concept aimed at 80% lift increase through chordwise elongation. Initially, the interior of the wing is designed to be filled with Dynamic-Modulus Foam (DMF), comprising an SMP resin and displaying stiffness variability over a small temperature difference. Softening and expansion arise from thermal stimulation, followed by hardening to fill a prescribed volume. Both pure SMP and corrugated dynamic modulus composite skins are investigated. Whereas the former is characterised by usable fatigue life compromised by
excessive brittleness when hardened, the latter proves insufficiently smooth in the absence of very large forces. Further weaknesses of the concept include irreversibility of the chordwise extension process as well as problematic heating through skin-integrated nichrome wires demanding high voltages at 40 W for sufficiently uniform temperature distribution [191, 201]. A refined internal structure concept superseding the DMF involves a tongue-and-groove sliding rib mechanism combined with a profiled honeycomb [202]. Another SMP foam concept, referred to as Cold Hibernated Elastic Memory (CHEM) and primarily intended for deployable structures, is discussed by Sokolowski [203].

2.1.2.1.2 Pneumatic Artificial Muscles and Fluidic Flexible Matrix Composites  Pressurised/Pneumatic Artificial Muscles (PAMs), also referred to as McKibben muscles, find widespread use in robotics [204–214]. The underlying idea is linear force generation through internal pressurisation (with liquid or gas) of an elastomeric tube sheathed with helical fibres. Specifically, the fibres convert the pressure-induced radial contraction into axial expansion and vice versa [213]. The lightweight actuation potential offered by PAMs has also been recognised by the morphing community [215–218].

A related development, called Fluidic Flexible Matrix Composites (F²MCs), originates in the research of Philen et al. [164, 219, 220] and Shan et al. [221, 222]. The concept, inspired by the world of plants, focuses on the variable stiffness characteristics associated with pressurisation. Strongly anisotropic composite tubes made of a flexible matrix reinforced with several distinctly oriented fibre layers allow for coupling internal pressure effects with axial deformation (Figure 2.4). An intermediate elastomer liner is also present between the working medium and the laminate tube surface, inhibiting leakage at high internal pressures. Multiple composite cylinders of this type, embedded in a matrix, can subsequently form building blocks of an adaptive structure such as a plate or sheet, capable of tunable stiffness behaviour relying on controlled pressurisation levels of the component tubes. Further property adjustment can be conducted through the variation of matrix or fibre material, fibre angle, layer count, or liner properties. Thill et al. [12] indicate a superior force generation potential of F²MCs compared to PAMs, emphasising the distinctive feature of the PAM fibre angles exceeding 15°.
A control concept not followed in other work involves an electroosmotic actuation strategy [220]. Voltage variation applied to a charged permeable membrane is used for altering the tube pressure. More precisely, the presence of the potential difference causes ions to cross the membrane, inducing a fluid flow into or out of the tube, changing its internal pressure.

Further, Philen et al. [164, 219] and Shan et al. [221] seal one end of the tube and install an inlet valve at the other one to provide longitudinal stiffness control. The open and closed configurations correspond to easily deformable and stiff/strain-resistant states, respectively. An additional degree of freedom is provided by the bulk modulus of the working fluid, whose high value contributes to large stiffness. Experiments on a wet filament wound ±35° carbon fibre-silicone matrix tube containing water as working fluid demonstrate an effective modulus ratio of 25.1, corresponding to 109.3 MPa and 4.35 MPa in the closed and open valve configurations, respectively [164]. The measured value decreases to 21.6 for a sheet structure comprising multiple matrix-embedded tubes. A superior ratio of 56 is achieved by Shan et al. [221, 222] with a ±35° carbon fibre-silicone matrix tube containing a silicone liner. Conducting sensitivity studies into the effective modulus ratio, Chen
et al. [223] analytically prove the theoretical possibility of reaching 120 by increasing the fluid radius (volume), with other factors considered including fibre volume fraction, fluid bulk modulus, material properties of the inner liner and tube thickness.

Shan et al. [222] extend the studies of F$^2$MC tubes and sheets to a symmetric [+60/0/−60]s laminate, proving the attainability of as many as eight modulus values in one direction through different open/closed valve patterns. Furthermore, the incorporation of a shape memory polymer matrix is also considered as a means of performance enhancement [224].

Chen et al. [223] highlight the potential of F$^2$MCs for morphing skin applications due to the considerable out-of-plane stiffness with simultaneous tunability in the longitudinal direction through closing and opening the valve for high and low rigidity, respectively. Chou and Philen [225] conduct a theoretical analysis of the application of F$^2$MCs sheets as top and bottom skins of a trailing edge flap of a glider. The aim is to affect the lift force through control surface deflections governed by the tube pressurisation. Li et al. [226] utilise F$^2$MC layers as face sheets of an aluminium honeycomb sandwich structure divided into segments by means of an integrated valve system, demonstrating transverse stiffness adaptability reaching a ratio of sixty.

Further work of Philen [227] addresses the limitation of only two discrete stiffness states corresponding to the open and closed valve positions. A closed-loop control system is implemented to follow a prescribed force-displacement path representing changing stiffness values, permitting active authority over the rigidity level of an F$^2$MCs tube.

Within the extensive literature body dealing with F$^2$MCs [164, 219–223, 226–229], analytical models of increasing complexity are developed and validated, leading to a deeper understanding of the system variable stiffness behaviour under parameter variations and permitting performance assessment of particular designs.

Chen et al. [230] propose a silicone rubber skin containing a system of pneumatic muscle fibres with an outer diameter reduced to 4 mm. Transverse stiffness adaptability is achieved through internal pressure variation from 0 MPa to 0.4 MPa, with the realisable ratios depending on the distributed load exerted on the skin specimen and reaching a value of 120 under 540 Pa.

The concept of using pressurisation to tune bending stiffness has also been
examined by Tabata et al. [231] in the form of a pneumatic device made of two very thin (200 \( \mu \)m) silicone rubber layers. As previously, the flexural rigidity enhancement stems from the application of pressure associated with perpendicular expansion and longitudinal shortening.

Whilst offering a significant number of design degrees of freedom permitting the achievement of a desired stiffness modulation range together with adequate load-bearing capacity, fluidic flexible matrix composites carry several limitations. One major drawback is the reliance on pneumatic arrangements, indicating added complexity and reduced integrability. The implications of cyclic loading corresponding to normal operating conditions for the long-term performance need to be explored as well.

Finally, it is noteworthy that the term Flexible Matrix Composite (FMC) generally implies the application of a highly deformable elastomer or silicone matrix. In this context, Murray et al. [233] investigate a one-dimensional morphing skin concept. Out-of-plane load-carrying capacity is sought through positioning pre-tensioned fibres along the corresponding direction, with morphing designed to occur perpendicularly, aligned with the weaker axis. Further, Shan [229] studies the application of FMCs for a helicopter tail rotor drive-shaft of high stiffness in torsion and low in flexure.

**Pneumatic Sandwich Structures** Fluidic flexible matrix composites are not the only concept of rigidity control based on an integrated pneumatic arrangement.

Vos et al. [232, 234–236] incorporate pressurisable elements into honeycomb cells as a means of realising stiffness variability of the structure (Figure 2.5). Whereas a considerable strain capability characterises the soft state not involving any additional pressure application, inflation leads to enhanced rigidity and changed geometry due to perpendicular extension. Reversible strains reaching 54% longitudinally or 76% laterally can be achieved,
with the maximum load-bearing capacity depending on the pressurisation scheme, lying at 70 kPa in the case of a pressure differential of 40 kPa [234]. A wing section with an integrated pressure-controlled trailing edge flap is tested, with a pressure change within 40 kPa proved to permit a 5% camber variation, producing an increase of 0.3 in the lift coefficient [235].

2.1.2.1.3 Material Anisotropy  A number of studies associate stiffness variability with passive material anisotropy, established during the development process and thus unchangeable. Including only the spatial dimension, such understanding differs from the definition adopted herein, equally focussed on the temporal aspect. Consequently, the summary given in the following serves completeness purposes, with the major pertinent types of mechanisms categorised under two headings of variable stiffness composites and sandwich structures.

Variable Stiffness Composites  The term variable stiffness composite is frequently used to imply a spatially changing distribution of material thickness and fibre orientations, aimed at a particular deformation behaviour [34, 36, 237, 238]. The concept, initiated in the late 1980s/early 1990s [32, 239, 240], has since been a particular focus of the research group of Gürdal [33, 35–38, 237, 238, 241]. As already discussed, this definition refers to a passive, invariant system and thus differs from the understanding involving variation in time employed in the current work. A possible morphing skin application of such spatially tailored stiffness entails realising regions of selectively reduced and increased rigidity corresponding to localised actuation and load-bearing requirements, respectively [36]. The development and optimisation of such a structure, however, demands a truly integrated aeroservoelastic environment. This includes actuation arrangement and loads, along with viscous aerodynamic effects, leading to vast computational expense.

Sandwich Structures  Bubert et al. [216] and Vocke et al. [242] develop a sandwich morphing skin intended for spanwise wing tip shape adaptation. One-dimensional strain reaching 100% is demonstrated, accompanied by a 100% surface area enlargement. The corresponding load-bearing capacity of up to 9.58 kPa proves sufficient to limit out-of-plane deformations to a
2.1 Morphing Applications of Variable Stiffness

maximum of 2.5 mm. The cover sheets are made of an elastomeric matrix composite based on a silicone matrix reinforced with carbon fibres running chordwise (Figure 2.6). The morphing spanwise direction is dominated by the flexible matrix material, bearing a strong similarity to the concept of Murray et al. [233]. Zero Poisson’s ratio of the honeycomb core ensures straining along the desired direction only. To compensate for the deficient bending stiffness of the design, a system of parallel carbon fibre sliding rods is introduced. Morphing actuation relies on a pneumatic artificial muscle.

In the case of such large one-dimensional deflections as single-axis morphing, constraining the transverse contraction induced by the Poisson’s effect leads to increased stiffness in the morphing direction. This, in turn, causes the actuation effort to rise. The potential of zero Poisson’s ratio honeycomb materials in eliminating this adverse phenomenon has been recognised by Olympio and Gandhi [243].

With spanwise morphing applications in mind, Olympio and Gandhi [243] develop two honeycomb concepts exhibiting no Poisson’s effect, referred to as an accordion and a hybrid cellular structure (Figure 2.7). Both arrangements feature in-plane longitudinal rigidity and deformability comparable to conventional configurations. When constrained in the non-morphing direction, however, the designs do not suffer from the aforementioned increased resistance along the morphing axis associated with larger actuation effort. While the hybrid cellular honeycomb is revealed to underperform conventional configurations in terms of load-bearing capacity, this does not hold for the accordion arrangement with adequately thick continuous walls.
Figure 2.7: Zero Poisson’s effect honeycombs designed by Olympio and Gandhi [243]. Initial configuration in black, deformation in tension (blue arrow) indicated with a dashed yellow line (no net transverse contraction).

Olympio and Gandhi [244] also explore the potential of sandwich structures based on a cellular core and flexible cover for morphing skin applications. Tailoring the underlying cell parameters in search of low in-plane stiffness, high strains, and large out-of-plane rigidity leads to superior properties compared to the original material without the cellular layout. Another study by Olympio et al. [245] and Asheghian et al. [246] concerns shear morphing based on a cellular honeycomb core sandwich with pre-tensioned flexible face sheets and strain-relieved cells. The skin design aims at a pronounced extent of material property variability.

The idea of spatial tailoring underlying both variable stiffness composites and sandwich structures permits significant property enhancement compared to the characteristics of the individual component materials. Another important practical implication is a potential reduction in the actuation effort required. Difficulties are likely to arise, however, due to the insufficient in-service flexibility associated with the completion of the final configuration definition during the development process.

2.1.2.2 Mechanism-Driven Variable Stiffness

Numerous studies employ active design relying on mechanisms to realise the adjustment of overall wing rigidity based on internal structural changes. In spite of the disadvantages mentioned in Section 1.2, arguments put forward in support of mechanism-driven stiffness variability are reduced power consumption compared to actuated elastic structural deformations and the relative ease of maintaining adequate load-bearing capacity.
In the years 2002–2005, the European project Active Aeroelastic Aircraft Structures (3AS) provided an incentive for the development of novel approaches aimed at beneficial exploitation of aeroelastic effects, inherently related to torsional and bending stiffness. Two research directions carry particular relevance, namely variable stiffness attachments within all-movable control surfaces, and structural concepts of an active and passive nature, with the former pertaining to both the interior and exterior (skins) [247, 248].

Griffin and Hopkins [249] advance an idea of varying the stiffness of a primary aircraft structure according to external changes in dynamic pressure to permit the operation of an outboard aileron in its post-reversal regime without losing control authority. This seeks to enhance roll control due to the significantly intensified roll forces associated with exceeding the reversal dynamic pressure. The proposed implementation involves two spars with adaptable shear transfer in their shear webs, allowing for stiffness variation of the wing box. On crossing the reversal dynamic pressure, the flight control system would induce the flexible wing mode by disabling the spar stiffness, simultaneously switching to the opposite aileron output signal. Hence roll effectiveness would be maintained, amplified in this regime for growing dynamic pressure.

A related Variable Stiffness Spar (VSS) concept aimed at adjustable torsional stiffness of the wing and maximised roll rate has been developed by Chen et al. [250]. Requiring no modifications to the outer wing structure, the VSS constitutes a replacement for the shear web of the original spar. Subdivided into discrete segments through joints found at its wing rib attachments, the spar is designed to rotate by an angle of 90° via electrical actuation. This results in sinusoidal stiffness variation governed by the rotation angle. Whereas no bending stiffness corresponds to horizontally directed joints, with the individual segments decoupled, gradual rotation associated with increasing segment connectivity leads to growing stiffness. The maximum rigidity is achieved in the vertical joint orientation with full spar continuity.

Florance et al. [251] report wind-tunnel testing of a VSS spar variant with a rectangular cross-section. Incorporated into a 26% scaled half-span model of an F/A-18A, the VSS is found at a chordwise position of 60% and covers 58% of the wing span, as indicated in Figure 2.8. A reduction in the wing bending stiffness of 14% at the root and 9% at the tip is measured as a
result of the spar rotation from the vertical to the horizontal orientation (Figure 2.8), whereas the corresponding decrease in the torsional rigidity reaches 22% and 8.5%, respectively. Substantial implementation difficulties accompanying the wind-tunnel testing process are emphasised.

Nam et al. [252] extend the VSS to a torsion-free wing concept relying on a small torque box positioned at the centre of the wing profile, accompanied by two variable stiffness spars found in close proximity to the leading and trailing edge. The horizontal positions of these structural members correspond to the minimum achievable torsional rigidity of the wing. To counteract the mass penalty of mechanical actuation, SMA spars are proposed in lieu of their standard counterparts to provide increased specific stiffness when heated.

Amprikidis and Cooper [253–257], Amprikidis et al. [258], and Hodigere-Siddaramaiah and Cooper [259] investigate two approaches involving spar translation and rotation as a means of tuning the bending and torsional rigidity as well as shifting the shear centre to achieve controlled twist under aerodynamic loads. The first idea relies on translating at least one spar to alter the torsional stiffness and local shear centre location, permitting a 40% decrease in the torsion constant (moveable middle spar at outermost positions). In the other concept, stiffness adjustment arises from the rotation of two spars within a range of 90°.

Ajaj et al. [260–263] coin
the name Adaptive Torsion Wing (ATW) for the proposed variable torsional stiffness wing box, illustrated in Figure 2.9. Mechanically actuated chordwise translation of the front and rear spar webs permits decreasing the corresponding separation and thus area enclosed by a thin-walled closed wing box, reducing the torsional rigidity of the wing. This enables aerodynamic loads to induce additional twist and subsequently maintain the deformed configuration. Simultaneous motion of both spar webs is recommended to avoid excessive actuation effort and lowered divergence speed despite greater effectiveness associated with forward spar translation in producing twist and pitch.

Another concept advanced by Cooper et al. [257, 265, 266] and Amprikidis et al. [264] concerns an all-moving aircraft fin featuring controlled torsional stiffness and tunable fuselage attachment position (Figure 2.10). This seeks to prevent adverse aeroelastic phenomena as well as increasing the efficiency to approach a value of two. The vertical control surface is designed to include a single attachment, capable of chordwise position variation. Further, changing the length of the connection according to an inverse proportionality permits adjusting the torsional stiffness. Nevertheless, difficulties such as the onset of plastic deformations and undue free-play are reported.

In addition, Cooper et al. [265] present several variants of tunable torsional stiffness attachments intended for the vertical tail concept, including a pneumatic arrangement featuring two cylinders with pressure-controlled stiffness, two motor-driven variable length (and thus stiffness) plate springs attached to the fin shaft, a system of five spring elements in series capable of increasing rigidity through selective element locking, and a rotating rectangular beam with the resultant device stiffness governed by the area moment of inertia.
Runge et al. [267–269] suggest exploiting the energy of the aerodynamic loads acting on a wing profile to modify its internal structure. This is envisaged to allow for controlled shifting of the shear centre position equivalent to the authority over the induced twist angle, without having to resort to power-intensive actuation. The concept relies on closing and opening the individual internal aerofoil cells to affect the shear stress distributions, as schematically shown in Figure 2.11. Discontinuous inner webs are subject to compression exerted by adjoining elements described as a ‘clutch-like system’ [267]. In the presence of aerodynamic loads, provided the twist angle needs modification, the clutch arrangement is activated. This means causing the two components of the web to slide relative to each other to produce discontinuity, until a particular relative displacement corresponding to a desired twist angle has been achieved. Experimental testing is conducted featuring three structures with the number of such adaptive webs increasing from one to three.

Other strategies, distinguished by an early conceptual stage, include a truss structure composed of unit cells actively controlled by means of cable or tendon sets. Bending moment transmission becomes possible through compliant joins of moderate bending resistance [270]. What is more, the tunable rigidity of an Active Variable Stiffness Elastic Actuator (AVSEA) is achieved through altering the stiffness of a leaf spring [271].

2.1.2.3 Semi-Active Variable Stiffness Solutions

Semi-active variable stiffness techniques only require external energy for the activation of the adaptive arrangement, without the need for constant power supply to retain the new configuration. Hence actuation effort can be reduced, facilitating the application of smart solutions. The vast majority of the relevant studies rely on structural design integrating semi-actively activated elements, with Electro-Bonded Laminates (EBLs) as an example of
an emerging strategy.

### 2.1.2.3.1 Electro-Bonded Laminates

The variable stiffness concept involving electrostatic bonding has been investigated by Tabata et al. [231], Bergamini et al. [272–277] and Di Lillo et al. [278–280].

Tabata et al. [231] first present an idea of bending stiffness variation based on stacking multiple thin polyimide layers with embedded nickel electrodes. Increasing voltage leads to growing bending resistance due to friction between the plies produced by electrostatic forces.

Bergamini et al. [272–277] employ the concept to develop beams with variable bending rigidity relying on replacing conventional permanent adhesive with reversible electrostatic connections. The studies involve two different experimental beam structures. The first is a sandwich configuration comprising a carbon-filled elastomer core whose both sides are covered with polytetrafluoroethylene (PTFE), and two Carbon Fibre Reinforced Polymer (CFRP) face sheets, each featuring a polyvinylidene fluoride (PVDF) film on the core side. An electric field applied across the two interfacial dielectric layers allows for controlling the shear stress transfer between the core and the face sheets, which grows with increasing potential, leading to enhanced flexural rigidity of the beam. Force-displacement profiles obtained in cantilever bending demonstrate ineffectiveness of voltages below 500 V. As expected, maximum stiffness is associated with the largest applied potential of 3000 V, with an eighteenfold reduction in the deflection at the control point. This nonetheless corresponds to about a third of the effect of conventional bonding [272, 273, 277].

The other configuration tested is a Glass Fibre Reinforced Polymer (GFRP) I-beam reinforced with CFRP cover plates able to be electrostatically attached.
to the outer flange surfaces (Figure 2.12) [274, 276, 277]. The major benefit of energetic efficiency ensues from only two interfaces requiring activation through electrostatic bonding [275]. The implications of the contact area deviating from perfect planarity for the quality of the electrostatic connection and dielectric layer deterioration under cyclic loading are highlighted [277].

The findings of the aforementioned studies lay the foundations for the variable bending stiffness concept of EBLs [277–281]. The approach entails voltage-controlled, reversible initiation and deactivation of the interfacial shear stress transfer between the component layers of the laminate, corresponding to a conversion between stiff and soft states, respectively (Figure 2.13). The phenomenon relies on the presence of both interlaminar friction and electrostatic forces. The application of voltage induces an electrostatic fusion of the individual plies, which thus exhibit an altered second moment of area as a single structure. This results in enhanced flexural rigidity, rising according to the layer count squared [279].

Di Lillo et al. [280] measure electric properties of thin dielectric polymer films for potential application in EBLs under the expected working conditions (quasi-static high fields). The desired large values of both permittivity and dielectric strength are identified as intrinsically opposing features, impeding the optimisation possibilities of EBL materials. To address this contradiction, a multi-layer electric model is introduced in view of enhancing EBL performance through considering both insulating and dielectric characteristics of the component polymer films [278]. The resultant behaviour, including the attainable shear force values, is nevertheless strongly influenced by surface and polymer quality, electrode thickness as well as displaying unexpected tendencies under inverted polarisation. A very important finding is performance degradation caused by friction under repeated loading [279].
2.1.2.3.2 **Electro-Active Polymers**  Another electrically-controlled concept, characterised by not only very large strain but also variable stiffness capacity, are EAPs, with Dielectric Elastomers (DEs) as an example of the field-activated subclass [127]. Voltage application across compliant electrodes acting on the surface of the polymer dielectric leads to areal expansion accompanied by thickness reduction, with actuated strains in excess of 100% realisable with pre-stretched silicone films [282]. Kornbluh and Pelrine [283] patent the use of electroactive polymer transducers combined with non-feedback and closed-loop control strategies for the purpose of stiffness variation.

The introduction of a contact surface limiting the bending motion is proposed as a means of non-linear stiffness enhancement of multi-layered electroactive conducting polymer actuators in bending in artificial muscle applications [284].

2.1.2.3.3 **Globally Variable Stiffness of Multilayer Beams and Aerofoils**  There is a large volume of published studies dealing with concepts of overall variation of beam bending stiffness [125, 285–287]. These exploit the dependence of flexural rigidity on the material Young’s modulus, and the cross-sectional area moment of inertia. Most semi-active implementations rely on variable modulus elements, usually polymers, embedded in the system. Analogous applies to torsional rigidity or aerofoil stiffness tuning.

Gandhi and Kang [125] develop and test a beam concept capable of flexural rigidity variation ranging from two to four times. The structure possesses two additional stiff covers, one on the top and one on the bottom surface, each placed on an intermediate polymer layer containing an embedded heating blanket (Figure 2.14). Elevating the temperature of the two polymer layers above the glass transition causes the polymer shear modulus to drop, leading to the decoupling of the cover layers from the beam and a consequent reduction in the overall bending stiffness.
The idea is further investigated by Murray et al. [285], who perform sensitivity studies aimed at maximising the realisable bending stiffness variation range. High cover layer and polymer stiffnesses and considerable polymer layer thickness are identified as beneficial. Depending on the boundary conditions of a beam, the anticipated factor of flexural stiffness change is 70 (one end clamped, the other free) or even 130 (both ends simply supported).

Another follow-up study by Murray and Gandhi [286] is placed into morphing context, considering the actuation effort, energy consumption and cover layer strain magnitude aspects. Largest bending stiffness variability potential corresponding to a 95 % reduction through polymer layer heating is proved to result from extensionally stiff covers, thick polymer layers, and small beam slenderness ratio. This optimum simultaneously implies significantly decreased actuation requirements and thus actuator mass savings as well as the greatest energetic efficiency. Further, the decoupling technique permits large beam deformations in bending without cover layer failure due to a reduction of 70 %–85 % in the corresponding strains.

Gordon and Clark [287] propose a variable stiffness beam concept based on reversible, thermally-induced bonding of the component layers. Heating intermediate adhesive films by means of nichrome wires leads to disconnection of the individual plies, altering the area moment of inertia of the beam. Experiments demonstrate the capability of reducing the beam stiffness by an order of magnitude.

Raither et al. [288] examine the adaptive coupling stiffness potential of laminates incorporating intermediate polymeric layers, extending the preceding work of Marques [289]. Thermal stimulation is employed to tune the shear stress transfer between the component layers, arranged in a symmetric stacking sequence with a unidirectional middle layer and two outer angle plies, as
indicated in Figure 2.15. The inherent bending-twist coupling remains unaffected in the ‘cold’ state with deactivated polymer layers fully transmitting shear stresses. At elevated temperature exceeding the glass transition point, however, the polymer modulus drops, thereby permitting large shear deformations. The effectively disconnected laminate layers thus produce a reduction in the coupling stiffness, which reaches an order of magnitude.

A beam featuring variable torsional stiffness arises as a further concept by Raither et al. [290–292]. A component web of the thin-walled box section, made of thermo-responsive polymers or electro-bonded laminates, is designed to undergo shear modulus changes under external stimulation. This shifts the profile shear centre location, inducing global twist under transverse loading, assuming the coincidence of the shear centre and the centroid in the original configuration (Figure 2.16). Based on the premise of insignificant implications for the flexural rigidity of the beam, the method allows for controlling the coupling between bending and torsion through careful selection of the relevant geometrical and material parameters. Further, varying the web modulus between low and high values corresponds to opening and closing the cross-sectional profile, respectively, providing authority over its torsional stiffness.

\[ t_1 G_1 t_1 > G_2 t_2 \]
\[ G_1 t_1 = G_2 t_2 \]
\[ G_1 t_1 < G_2 t_2 \]

Figure 2.16: Underlying idea of variable bending-twist coupling (based on [291]).

Similarly, Rivas et al. [293] investigate the cross-sectional stiffness values and shear centre positions of a thin-walled box beam with one or both vertical
webs featuring thermally-controlled Young’s modulus.
As an extension of the adaptive beam concept, Raither et al. [292, 294, 295] proceed through an aerofoil to a wing exhibiting tunable torsional rigidity. Divided into three cells, the aerofoil includes three thermally stimulated polymer interfaces of variable modulus, indicated in Figure 2.17. Adjustable torsional rigidity of the wing is then achieved through heating the polymeric elements, which is equivalent to opening and closing the individual profile cells or their combinations by virtue of the thermally reduced shear modulus. Altered shear centre positions thus ensue, offering a means of controlling the lift force through the wing twist.

![Figure 2.17: Variable bending-twist coupling aerofoil developed by Raither et al. [295], with three interfaces enabling adaptivity.](image)

Rajamohan et al. [147] seek to exploit the advantages of high bandwidth and temperature insensitivity of magnetorheological fluids, incorporating this smart substance as an intermediate layer between elastic covers of a sandwich beam. Rising magnetic field intensity leads to increasing stiffness of the system, which also proves boundary condition dependent, reaching a maximum in a clamped-free configuration. Nevertheless, the state of matter of magnetorheological fluids brings the disadvantage of low integrability with the associated mass and size penalties. Further drawbacks include high losses, large power consumption, and a very limited scope of stiffness adjustment [127]. Consequently, there is every likelihood that their relevance will remain limited to the field of vibration control.

### 2.1.2.4 Structural Elasticity-Related Variable Stiffness

#### 2.1.2.4.1 Buckling Instability
A relatively infrequent approach to morphing, yet gaining increasing popularity in recent years [296], seeks to exploit the unique characteristics of structural instabilities. In particular, the associated stiffness reduction allows large displacements to be achieved without undue actuation effort.

Vos et al. [297–300] and Barrett et al. [301–304] explore the potential of
post-buckled pre-compressed (PBP) piezoelectric actuators for unmanned morphing applications in overcoming the stroke limitation inherent in piezoelectric elements. The underlying idea is the decreased passive stiffness of the actuator when loaded close to the perfect buckling value. The instability mechanism then permits deflection magnification, with the axial force amplifying any initial bending imperfections [300]. Advantages include mass and energy savings compared to traditional actuation systems, as well as higher bandwidth and work output [297, 300]. Barrett and Barnhart [304] extend the post-buckled pre-compression strategy to cancel the passive torsional stiffness of the piezoelectric actuator employed at the helicopter rotor root through careful structural design entailing two additional springs. Experiments reveal the potential for 3.8-fold pitch deflection amplification.

Similarly, Phani et al. [305] employ buckling as a means of magnifying the achievable deflections. A trailing edge concept composed of multiple frame members is developed, as schematically shown in Figure 2.18, with piezoelectrically-triggered instability of selected struts leading to an overall morphing effect.

Ursache et al. [306–308] advance a camber morphing concept through controlled instability under asymmetric actuated loading. To that end, an Euler strut located at the aerofoil camber line, extendable to a plate in a 3D wing case [308], is designed to adopt predefined buckling shapes. Turnock et al. [309] apply a similar approach to wing shape adaptation of autonomous underwater vehicles. Nevertheless, the rather low technology readiness level of the method is emphasised, involving a meticulous design of the post-buckled shapes of pre-loaded supporting members.

Runkel et al. [310] demonstrate the feasibility of exploiting unstable behaviour to achieve passive twisting of thin-walled composite beams under longitudinal loading. The concept relies on inducing shear buckling in response to external forces in one component web made of anisotropic composite material with tailored lay-up properties. The instability-triggered shifting of the profile shear centre and modified torsional stiffness thus result in
the desired beam twisting. The potential of the presented mechanism for five-fold reduction in torsional rigidity without structural failure is revealed.

Gano et al. [311] develop a UAV concept intended to morph between a mono- and a bi-plane configuration. Two lifting surfaces, upper and lower, are connected at the wing tips, as Figure 2.19 illustrates. The former is designed to undergo actuator-controlled buckling, thus exhibiting large elastic deformations due to instability.

Mierheim et al. [312] examine the design of appropriate support conditions for struts aimed at large deformations and length variations through controlled buckling under loading exceeding the critical value. Advantages such as mass savings, convenient integration directly in the load path, shortening of up to 43\%, and buckling load adjustment possibility are balanced with difficulties associated with the necessity of very precise loading prediction complicated by inherent uncertainties. Indeed, in trying to induce tailored instabilities, a potential prediction and control challenge arises from the sensitivity to initial imperfections [296].

Last but not least, elastic instability has also been applied across smaller scales within metamaterials to obtain particular global features such as auxetic behaviour and radical shape and volume changes [313–317].

2.1.2.4.2 Multi-Stability An in-depth discussion of multi-stability as the focus of the current thesis follows in Section 2.2.

2.2 Structural Multi-Stability
In the early 1980s, the deviation of the actual cool-down shapes of thin unsymmetric carbon fibre/epoxy laminates from the predictions of Classical Lamination Theory (CLT) [318, 319] was first explained by Hyer [320–322]. In consequence of the thermal expansion coefficient mismatch between the component plies, the initially flat structures develop curvatures on cooling from the elevated processing to ambient temperature. More precisely, negligible thermal strains parallel to the fibres (typically $\alpha_1 = -18.0 \times 10^{-9} \text{K}^{-1}$ for
a graphite-polymer composite) are accompanied by significant transverse expansion dictated by the matrix, comparable to aluminium ($\alpha_2 = 24.3 \times 10^{-6} \text{K}^{-1}$) [319]. While CLT indicates a single saddle configuration (Figure 2.20(a)), square $[0_n/90_n]_T$ composites in reality exhibit two stable cylindrical states at room temperature (Figure 2.20(b),(c)), provided a minimum side length-to-thickness ratio is exceeded.

![Figure 2.20: Cool-down shapes of an unsymmetric cross-ply laminate: (a) saddle shape predicted by CLT; (b),(c) two cylindrical equilibrium configurations.](image)

Hyer linked this phenomenon with geometrical non-linearity and developed a minimum total potential energy-based formulation including non-linear mid-plane strain-displacement approximations [321–323]. Analytical and experimental studies to follow have considered different stacking sequences [322, 324–326], temperature and imperfection effects on the shape bifurcation behaviour [327], or non-linear bending response [328]. These efforts have been accompanied with theoretical models of different complexity and abstraction level, focussing on the out-of-plane displacement distribution and curvatures to provide stable shape predictions [325, 326, 329–331]. With the advancement of Finite Element Analysis (FEA), simulation studies have proliferated from the mid-1990s [332, 333], facilitating the analysis of increasingly complex configurations.

### 2.2.1 Smart Features of Multi-Stable Structures

As early as the first contribution of Hyer dealing with bi-stability [320], a tentative possibility of exploiting this unusual characteristic was expressed. However, no particular applications were advanced at the time, as the couplings inherent in unsymmetric lay-ups, such as bending-extension in the cross-ply case, precluded their use as conventional engineering solutions. With technical progress and the advent of research into morphing and smart systems, however, the attention of the scientific community has directed
towards the unique features of multi-stable configurations and means of harnessing them for advanced purposes. These can be summarised as large reversible deflection capacity at modest actuation expense, response tailoring potential, as well as special stiffness characteristics, to be detailed in the following.

2.2.1.1 Energy-Efficiency and Actuation

The stable state switch occurs via a profoundly non-linear, dynamic snap-through phenomenon, triggered by an external load \([334–336]\) of sufficient magnitude to overcome the potential energy peak separating the individual equilibrium configurations. Hence large deflections are generated at modest actuation effort. This feature becomes particularly attractive when seeking to achieve efficient shape adaptation. Indeed, the only energy input required concerns the change of the equilibrium states, which are retained with no auxiliary external intervention \([43]\). Furthermore, the deflection reversibility associated with the purely elastic nature of multi-stability adds to the robustness of the approach, with large repeatable transitions between prescribed geometries occurring without the need for supplementary mechanisms. This opens up the possibility of passive designs, extracting energy from the external loads acting on the system, such as aerodynamic forces, to initiate the shape change \([39, 41]\). A further functionality enhancement emerges from integrating multi-stable structures with smart actuation. In this context, piezoelectrically-triggered snap-through has been especially widely employed, with a particular focus on MFCs \([43, 323, 337–345]\). The considerable relevance of MFCs as a means of inducing the stable state switch arises primarily from improved strain output and surface conformability compared to pure piezoelectric ceramics \([346, 347]\), while retaining the high frequency and low mass advantages \([348]\). Nevertheless, this comes at the expense of high driving voltages and hysteretic behaviour \([349]\). Further, authority and reversibility limitations have proved to accompany the transition between the equilibrium configurations realised in a quasi-static fashion. An improved strategy \([44–46, 350]\), based on studies considering the dynamics of the phenomenon \([351–356]\), entails tuning the exciting frequency provided by MFCs according to the first vibration modes of the stable states to exploit the dynamic reduction of structural stiffness at resonance for an enhanced actuation effect. In doing this, symmetry breaking such as a clamped-free
set-up of a piecewise symmetric-unsymmetric lay-up becomes necessary to avoid uncontrolled oscillations associated with equal modal frequencies of both equilibrium shapes.

In addition, a compromise between bi-stability preservation and adequate actuation authority arises. Whereas small specimens become mono-stable owing to the stiffness added by the bonded MFCs, the capabilities of MFCs prove insufficient to induce snap-through of larger laminates \[43, 342, 357\]. Further, reducing the component stiffness to allow for stable state switch under achievable voltages leads to poor load-bearing capacity \[358, 359\]. Proposed as a potential countermeasure, combined optimisation of the composite and piezoelectric layers seeks to reduce the necessary actuation voltage while tailoring the directional bending stiffness to ensure that the operational loads can be supported \[359\]. Furthermore, the dynamic resonant actuation strategy proves effective in accomplishing reversible snap-through under a range of aerodynamic conditions \[46\], offering the possibility of further enhancements in terms of energy consumption, and dynamically-induced, rather than permanent mechanical, stiffness reduction through a feedback control scheme \[360\].

Other actuation means studied include SMA wires \[323, 361\] or a combination of MFCs with SMAs for snap-through reversibility \[362\]. The advantages of SMAs in terms of substantial actuation energy densities and strains are nevertheless balanced by slow time response and hysteretic behaviour requiring complex control \[5, 363\].

An interesting approach presented by Li et al. \[364\] relies on residual stress relaxation through local heating of GFRP laminates by means of integrated electrothermal alloys. Reversible stable state switch arises from lowering the potential energy level of the structure, as opposed to overcoming its peak as is the case for MFC and SMA strategies.

From the morphing viewpoint, the rapid nature of the snap-through phenomenon and the associated high rate of motion following a relatively small perturbing input present a limitation to be overcome, for instance, through careful design combined with an appropriate control law. On the other hand, this characteristic can be used to advantage to achieve actuation or switching functionality. Reproducing an immediate closing act of a bio-inspired Venus flytrap robot \[365\] provides an interesting example in this context.
2.2.1.2 Property Tailoring

Exhibiting significant potential for far-reaching material property, lay-up, and manufacturing process tuning [366], laminates become particularly promising candidates for tailored multi-stable structures. The design space of the common thermally-induced bi-stable configuration of a uniform cross-ply \([0_m/90_n]\) planform is well understood [366–369], as are more general unsymmetric stacking sequences \([0 \pm \alpha/90 \pm \alpha]\) [370]. Discrete [371, 372] or continuous, tow-steered [373–375] spatial lay-up tailoring permits obtaining desired characteristics. In this context, a first step towards improved integrability emerges from a two-region symmetric-unsymmetric lamination enabling the symmetric portion to be constrained without bi-stability loss of the complete structure [371, 372].

Thin unsymmetric carbon fibre/epoxy laminates are not the only structures to possess more than one equilibrium shape. To impart this characteristic to symmetric lay-ups, selective fibre pre-stress preceding the curing process has been proposed [376, 377]. Alternatively, integrated metal layers [378, 379] or aluminium stripes [380] deliver a desired residual stress state emerging from the thermal expansion coefficient mismatch between metal and composite, further expanding the design space. Yet another concept relies on complementing a unidirectional laminate with transverse portions in varying patterns, while preserving the overall symmetry of the stacking sequence [381].

In addition, multi-stability arises from anti-symmetric lay-ups such as \([(\pm \alpha)_n], [(\pm \alpha)_n/0/(\pm \alpha)_n], \) or \([\pm \alpha_1/\pm \alpha_2/\pm \alpha_2/\pm \alpha_1]\) combined with the initial curvature of cylindrical shells [382–387]. If made of an isotropic material, such structures can acquire bi-stability through plastic deformation [388], which supplemented through corrugation-based anisotropy yields metal shells able to convert to a coiled, non-corrugated equilibrium configuration [389, 390].

Moreover, curvature tailoring of the base, stress-free state is proposed for the purpose of preserving multi-stability on clamping one edge of a shell [391].

The possibility of two stable saddle configurations is also demonstrated, achieved by pressing flat and bonding two originally saddle-shaped panels [392].
Strategies leading to more than two equilibrium configurations include merging smaller symmetric and unsymmetric unit shells into compound, strongly multi-stable surfaces [393, 394] or combining layup and curvature effects [395, 396]. As opposed to monolithic multi-stable structures, a discrete assembly of four identical, rectangular bi-stable elements with an unsymmetric lay-up has also been presented, leading to a tri-stable component possessing a plane, concave, and convex shape [397]. This unit cell provides the foundations for more complex layouts [398].

The initial tool-based curvature offers yet another degree of freedom, affecting the final cured curvature and snap-through load [399, 400].

2.2.1.3 Stiffness Variability

Depending on the structural requirements which a multi-stable component has to fulfil to perform a particular function within a larger morphing system, the variable stiffness characteristics can be viewed from different perspectives.

The current study aims at significantly different properties of each equilibrium configuration, related not only to the underlying material, but also to the resulting specific geometry. This purposeful response anisotropy can be achieved through targeted design of unsymmetrically laminated composites [39, 40, 401]. Supplying energy to trigger a change between the stable states thus allows different stiffness characteristics to be provided and maintained without the need for further external intervention. Based on a purely elastic mechanism, the stiffness variability approach pursued herein presents a passive, low actuation means to achieve structural adaptability, as opposed to active or semi-active methods such as shape-memory effect or mechanical arrangements. This can also be seen in the case of the tri-stable lattice proposed by Dai et al. [397], whose concave state features lower bending stiffness than the plane configuration.

An alternative approach focuses on the large displacement potential associated with multi-stability rather than a distinct response of the individual stable shapes. In this context, high stiffness of each equilibrium configuration becomes crucial to ensure sustaining operational loads without inadvertent snap-through. At the same time, energy-efficient transition between the stable states requires low deformation resistance in the actuation direction. Example studies seeking such anisotropy include bi-stable laminate optimisation in terms of piezoelectric actuation and bending stiffness in the loading
direction [359], or an adaptive air inlet by Daynes et al. [122].

Following the path of directional stiffness tailoring even further leads to the objective of zero deformation resistance along a particular axis, which minimises the specific actuation demands, as presented by Lachenal et al. [123, 402, 403] and Daynes et al. [404] for the case of twisting. Similarly, Guest et al. [405] develop a neutrally-stable metal shell characterised by no stiffness in twist.

Compared with the permanently, statically decreased structural resistance in the actuation direction outlined above, the resonant snap-through technique summarised in Section 2.2.1.1 augments the actuation authority through a dynamic reduction in stiffness [44, 46]. As a prerequisite, however, sufficient anisotropy between the stable states reflected in distinct corresponding natural frequencies becomes crucial to avoid continual snap-through.

### 2.2.2 Influence Factors on Thermally Induced Bi-Stability

When exploiting thermally-induced stresses to obtain multi-stable behaviour, size constraints arise from laminate thickness, which imposes a minimum edge length required for more than one equilibrium configuration to exist [366, 406]. Otherwise the composite remains monostable, with a thickness-to-width ratio exceeding the critical threshold.

What is more, thermal multi-stability is known to exhibit sensitivity to further external factors. These have been identified as temperature dependence and characterisation uncertainties of material properties (elastic moduli, coefficient of thermal expansion (CTE)), nominal ply thickness and curing temperature difference scattering as well as environmental effects comprising moisture absorption and ambient temperature changes [320, 342, 366, 370, 374, 407–410]. The final residual stresses are also affected by through-the-thickness cure non-uniformity, chemical contraction and tool effects [330, 408, 411, 412]. This becomes clear in that the composites do not reacquire the initial flat shape of zero curvature when reheated to the elevated processing temperature [408].

High temperature and moisture, which can influence bi-stability up to the point of its complete loss [119, 342], arise as potential limitations of the application of thermally-induced bi-stable laminates made of carbon fibre/epoxy. Since real operation as part of morphing systems would subject such
composite structures to diverse conditions, the hygroscopically-driven stable shape and especially snap-through load variations (curvature relaxation and critical load reduction due to moisture) acquire particular importance if the equilibrium configuration change is to be designed to occur at a particular load threshold. By contrast, no fatigue problems are reported [413]. Further, time- and temperature-dependent, hysteretic behaviour is observed in bi-stable antisymmetric thin-ply carbon/epoxy composites, which undergo material relaxation and a reduction in the stored strain energy [414]. Nevertheless, the present study at its conceptual level does not explicitly consider these adverse phenomena due to the existence of alternative ways of obtaining bi-stability, such as broadly understood pre-stressing methods [415]. By way of illustration, the approach of imparting bi-stability to symmetric laminates through uniaxial fibre pre-stress provides the advantage of reduced sensitivity to environmental effects, combined with higher snap-through loads and thus enhanced load-carrying potential [376, 377]. What is more, a carefully chosen resin system could mitigate the moisture absorption problem.

### 2.2.3 Characteristics of Equilibrium Path

In general, two types of elastic instability of structural systems subjected to conservative loading can be distinguished: buckling and snapping [416]. The difference becomes clear from the corresponding equilibrium path, comprising system equilibrium points representing a continuous deformation in a single degree of freedom \( q \) under a varying load parameter \( \lambda \), as schematically shown in Figure 2.21.

In both cases, critical points of neutral equilibrium (\( q_{cr} \)) characterised by singularity of the symmetric tangential stiffness matrix mark the boundary between stable path and the onset of instability [417]. Buckling occurs in a bifurcation or branching point (Figure 2.21(a)), from which two equilibrium paths emerge, implying a non-unique solution [418]. By contrast, bi-stability is associated with extrema of the equilibrium path, or limit points, possessing a single tangent perpendicular to the ordinate at \( \lambda_{cr} \) [419, 420] (Figure 2.21(b)). On exceeding the critical load in the vicinity of a limit point (\( \lambda > \lambda_{cr,1} \)), no equilibrium states are found, which does not generally apply to a bifurcation [416]. The absence of equilibrium points beyond the limit point leads to an unstable situation in which an infinitesimal load change triggers a finite deflection corresponding to the distance to the next equilib-
rium state. Since this dynamic snap-through phenomenon does not follow an equilibrium path, inertia forces corresponding to the vertical departure from the equilibrium path arise. Their work contributes to the kinetic energy, leading to an accelerated motion. The final equilibrium is achieved owing to the damping forces, which allow oscillations about the other stable state to decay [421]. Furthermore, whereas stability corresponds to a positive slope of the equilibrium path, instability manifests itself as a negative first derivative [421].

![Equilibrium paths](image)

**Figure 2.21**: Schematic equilibrium paths in terms of a load parameter $\lambda$ as function of a degree of freedom $q$. (a) Response with a bifurcation point BP with corresponding critical load factor $\lambda_{cr}$. (b) Response with limit points LP1, LP2 with corresponding critical load factors $\lambda_{cr,1}$, $\lambda_{cr,2}$ and displacements $q_{cr,1}$, $q_{cr,2}$.

### 2.2.4 Morphing Applications of Multi-Stable Structures

In recent years, the unique features inherent in elastic structural instability have been increasingly considered for smart and adaptive applications [296]. The energy-efficient large deformation capacity combined with far-reaching response tailoring possibility offered by multi-stable laminates have attracted particular attention as a potential approach to realise morphing in the aerospace [12, 14, 20, 21], wind energy [24] and automotive [25] context, extending even to bio-inspired applications [365, 422, 423].
Turnock et al. [309] consider equipping the wing trailing edge of an underwater glider with a bi-stable composite actuated by means of a push-pull system to fulfill a flap role. Daynes et al. [121] combine six bi-stable symmetric [0/90]s square laminates with 1.1% pre-strain applied to the 0° fibres to design a flap providing two equilibrium camber configurations, as schematically shown in Figure 2.22. Capable of a 10° downward deflection, the device is found to display sufficient load-carrying capability not to involuntarily snap through. The underlying aerodynamic simulations are inviscid.

Diaconu et al. [119, 424] advance three aerofoil application concepts of bi-stable composites for trailing edge, camber, and chordwise morphing. These involve combining two bi-stable elements to form a flap, positioning a bi-stable component horizontally along the chord, or inserting one vertically along the main spar (Figure 2.23), respectively. At the same time, attention is drawn to problematic areas including the load-bearing capacity without unwanted snap-through, response to practical environmental conditions such as moisture, connectivity issues ensuing from the necessity to allow for relative sliding of components, as well as implications for the overall torsional stiffness.

Mattioni et al. [118, 406, 425] study the possibility of variable wing sweep relying on the bi-stability of a curved spar. The member is connected to a second spar by means of a truss arrangement which acts as a rib structure,
without constraining the spar motion. However, the issue of appropriate skin integration is not considered. Additional two concepts presented include a bi-stable element employed as a winglet in one equilibrium configuration, or providing a spanwise wing extension enhancing lift in the other. Secondly, a bi-stable trailing edge flap composed of two bi-stable components is proposed [118, 406].

Schultz [117] joins two bi-stable convex rectangular elements of a $[0/90]_T$ lay-up into an aerofoil-type structure capable of considerable twist variation between its two equilibrium states, with the transformation triggered by means of MFC actuators.

Lachenal et al. [123, 426] present a bi-stable assembly of two pre-stressed flanges separated by stiff spokes (Figure 2.24), exhibiting substantial twisting capabilities when transitioning between its two equilibrium configurations. A particular antisymmetric flange stacking sequence $[(+\alpha)_2/0/(-\alpha)_2]$ is found to yield zero torsional stiffness along the twist axis [123]. This feature, corresponding to a constant strain energy level, is exploited in search of low actuation twist morphing of a wind turbine blade [402]. However, discontinuous skin performing relative motion as well as a system of links and bearings lead to increased complexity. The twisting concept is subsequently modified to an I-beam exhibiting a non-linear, both positive and negative stiffness response when switching between its two stable twisted states [427]. A further extension includes an aerofoil characterised by zero torsional stiffness and hence minimised actuation demands in twisting, with coincident shear and aerodynamic centres at quarter-chord [404].

Daynes et al. [122] design an adaptive air inlet changing between an open and closed equilibrium configuration (Figure 2.25). Bi-stability arises from longitudinally varying bending stiffness of a GFRP laminate combined with clamped boundary conditions. The capability of the structure to maintain its stable shapes under an aerodynamic pressure of $\pm 3$ kPa is demonstrated.
2.3 Conclusions

Last but not least, it should be remembered that multi-stability is not limited to tailored composite materials. A different strategy entails exploiting the snap-through behaviour of struts in larger arrangements to design systems possessing multiple equilibrium configurations. Example concepts include a compliant plate varying from flat to a range of cylindrical shapes \([428]\), or multi-stable space frames \([429]\).

2.3 Conclusions

The substantial volume of published studies involving the variable stiffness notion proves the potential and significance of this solution with regard to the implementation of large shape adaptations in an efficient manner. Promising results have been demonstrated not only analytically and numerically, but also through experiments. On the other hand, the vast majority of the proposed concepts suffer from low maturity and technological readiness, together with uncertain reliability and long-term behaviour under cyclic loading associated with typical operating conditions. When analysing the general directions set out in the relevant literature in view of their future morphing potential, it is therefore crucial not to neglect the underlying limitations.

Relying on an external stimulus, shape-memory polymer-based materials offer considerable reversible strain and stiffness tuning capacity. Substantial property tailoring flexibility based on the introduction of reinforcement combined with manufacturability and cost advantages add to the attractiveness of this class of concepts. Nevertheless, the most frequent thermal activation method gives rise to difficulties related to the problematic integrability of heating systems, excessive sensitivity to temperature changes synonymous with limited controllability, which also applies to process reversibility. Further drawbacks include low recovery forces as well as unsatisfactory rate of property adjustment dictated by heat exchange and cooling aspects. The latter acquires special importance if manoeuvrability in terms of rapid shape
changes is required.

Fluidic flexible matrix composites and sandwich structures feature auspicious adaptability based on a broad range of achievable rigidity levels whilst offering an adequate load-bearing capacity. The inclusion of pneumatic arrangements, however, is inseparably connected with mass penalties, poor integrability and uncertain reliability issues. Passive materials with strongly anisotropic stiffness properties, such as zero Poisson's ratio sandwich configurations or variable stiffness composites, do not offer the desired temporal modifiability in operation.

Perhaps the most serious potential disadvantages of mechanical design aimed at tuning the overall stiffness properties are increased complexity, mass and part count, as well as tribological considerations. This conventional approach is nevertheless associated with lower implementation risk than smart solutions, whose long-term behaviour under operating conditions of an air vehicle is largely unknown. Further, adequate load-bearing capacity can be realised with more ease.

Semi-active techniques combining purposeful structural design with smart strategies such as electro-bonded laminates offer the advantages of relatively low power consumption and pronounced tailoring flexibility. The readiness level achieved to date, however, is too low to allow for conclusive viability predictions. The main technical challenges yet to be resolved include insufficient shear stress transfer, uncertain long-term behaviour in service, very large voltages required and the associated certification issues.

Purposeful instability aimed at exploiting the associated stiffness reduction for augmented actuation is newly enjoying increasing acceptance as a viable option, despite the traditional distrust directed towards such phenomena. Apart from airworthiness regulations, thinkable difficulties include imperfection sensitivity, uncertainty, and precise prediction of loading conditions.

The potential of multi-stable structures for morphing applications has to date been mostly attributed to the large deflection capacity accompanied by the modest external energy input required to trigger the transition between the equilibrium configurations. This also implies response predictability in terms of autonomous switching between prescribed geometries without the need for additional guiding arrangements. The elastic nature of the phenomenon ensures repeatability and promises enhanced robustness. Further, composite laminates offer considerable freedom to tailor the multi-stable
response to particular requirements. Recognising these advantages, the present work shifts the focus to yet another unique feature: stiffness variability arising from distinct properties corresponding to each equilibrium configuration. Conceivable problems to be encountered in real operation include the environmental effects on thermally-induced multi-stability, and the provision of sufficient load-bearing capacity to prevent involuntary snap-through. The implications of the rapid nature of the stable state switch for the system dynamics and the requirements for the control law present a further consideration.

This chapter has given an account of existing research directions concerned with variable stiffness as a possible approach addressing the morphing challenge. The limitations underlying each category highlight the extreme difficulty of developing a shape-adaptive solution exhibiting sufficient advantages and maturity level to become more widely adopted. Nevertheless, the variable stiffness strategy does offer substantial potential, especially in view of the rapid technological progress likely to allow components such as smart actuators to mature in the near future. In this context, a holistic design and optimisation approach in an integrated, multidisciplinary environment incorporating all the contributing disciplines from the concept phase constitutes a prerequisite for full exploitation of the available capabilities.
CHAPTER 3

Design Space of Embeddable Variable Stiffness Bi-Stable Laminates

This chapter is based on the journal publication:

The morphing strategy combining distributed compliance with variable stiffness provided by bi-stable laminates imposes two crucial requirements on the multi-stable solutions acting in the envisaged role. First, monolithic embeddability inside a larger shape-adaptive system demands the capability of constraining two opposite edges of the element without bi-stability loss. Only in this manner can discrete connections such as bolts or rivets be avoided and composite continuity preserved, a crucial aspect in view of the load-carrying capacity and failure-resistance. Second, stiffness variability implies maximising the difference in structural properties between the equilibrium configurations. These two aspects represent a departure from the focus on the large deflection potential associated with multi-stability prevailing in existing studies. Exploring and determining the limits of the parameter space
3.1 Design and Parametrisation of Variable Stiffness Bi-Stable Laminates

available for multi-stable components subject to the two main requirements, together with the associated major trends, thus facilitates decision-making in the tailoring process of the proposed selectively compliant morphing systems. This constitutes a prerequisite for exploiting the available capabilities of the variable stiffness laminates to the full.

In this context, a thermally-induced bi-stable composite concept whose two opposite edges can be constrained is introduced in [40, 430]. The current chapter extends these preliminary results through systematic exploration of the design space of two most basic classes of such integrable bi-stable components by means of FEA.

Two criteria for determining the bi-stability of the investigated configurations are developed as crucial aspects in view of the objectives of the study. Based on two initial specimens, comparisons with experimental results are provided, validating the numerical set-up. Results of the systematic design space exploration are then presented and discussed. The findings provide new insights into the main trends in response of the bi-stable configurations under the variable stiffness and embeddability considerations. As a major output, a numerical design methodology is established for such a class of components to exhibit desired behaviour under specifications imposed by the function fulfilled within a larger morphing structure. This lays the foundations for the subsequent development of selectively compliant shape-adaptive systems according to the pursued strategy.

3.1 Design and Parametrisation of Embeddable Variable Stiffness Bi-Stable Laminates

The two classes of embeddable variable stiffness bi-stable elements considered herein involve a three- and a two-ply configuration, with the latter introduced in [40, 430]. Both rely on the mechanism of thermally-induced bi-stability due to unsymmetric lamination. Figure 3.1 presents the spatially tailored stacking sequence distribution.

The three-ply layout comprises five distinct sections, referred to as symmetric \( (l_s) \), outer unsymmetric \( (l_{us,o}) \), and central unsymmetric \( (l_{us,c}) \). Further, the two-ply concept features an additional narrow unidirectional (UD) intermediate region \( (l_i) \) found between the outer and central unsymmetric sections, serving fibre continuity purposes. The symmetric lamination extending from
Figure 3.1: Layout and parametrisation of embeddable bi-stable laminate configurations. BCs: Clamping surfaces/regions of boundary condition assignment.
the opposite short ends of the composite plates allows for constraining the clamping surfaces (marked as BCs in Figure 3.1), thus ensuring monolithic embeddability.

As Figure 3.2 illustrates, the proposed stacking sequence distribution results in a curved and a flat stable shape, respectively, of low and high compressive stiffness. Additionally, an inherent inclination of symmetrically laminated clamping regions (angles $\theta_1$ and $\theta_2$) formed by the equilibrium states with the horizontal becomes apparent, as defined in Figure 3.3.

Investigating the two most basic configurations fulfilling the prescribed objectives does not restrict the validity of the study to tailoring the two specific classes. The constraint of thermal bi-stability loss caused by excessive laminate thickness-to-width ratio combined with the thickness of the UD material used for fabrication suggest the selection of the lowest possible layer count to limit specimen size in view of the envisaged integration within a morphing UAV. Since multi-stability is a scalable phenomenon, reduced ply thickness or larger laminate dimensions greatly expand the available design space, offering the possibility of an increased layer count and thus more sophisticated composite designs. Gaining insight into the parameter sensitivity of the simplest structures constitutes a prerequisite for proceeding to a higher complexity level, greatly facilitating the decision-making in creating new bi-stable components fulfilling the prescribed objectives. Further, the underlying design methodology is independent of a particular laminate layout, which can be generated through extension of the most basic configurations. This will be demonstrated in Section 4.1, in which four- and five-ply elements are introduced.
3.2 Numerical Model for Design Space Exploration

The design space for the herein studied specimens is explored through finite element modelling performed by means of the Abaqus®/Standard software according to established procedures for this type of problems [367, 371, 372]. Structured discretisation with four-node quadrilateral shell elements with reduced integration (S4R) is employed with a fine mesh seed of 2 mm to 5 mm, depending to the overall specimen size and based on experience gained from previous convergence studies. A static non-linear analysis is used throughout. Further, constant numerical stabilisation between $10^{-7}$ and $10^{-6}$ is added where necessary for convergence. In these cases, the results are critically assessed in terms of both the associated viscous damping energy (ALLSD) compared to the strain energy [431], and the observed physical behaviour of the experimental specimens. The laminate stacking sequence is assigned by means of composite shell sections. The equilibrium configurations are subsequently obtained through simulating the manufacturing process with a total temperature difference $\Delta T = 140$ K corresponding to the employed autoclave cycle, according to the procedure detailed in Section 3.2.1.1.

The relevant design parameters are varied in two different ways. First, the overall laminate size $(w, l)$ is kept constant while changing the internal dimensions of the individual lamination regions $(l_{us,c}, l_{us,o}, l_s)$. A three-ply specimen $w=150$ mm wide and $l=320$ mm long, and a two-ply laminate with $w=70$ mm and $l=180$ mm are studied. As Table 3.1 presents, three subcases are considered. Each involves a constant contribution of a specific piece-wise layout portion while varying the two other sections through changing the
parameter controlling their boundary \((x_{us}, x_s)\) in 5 mm increments. The intermediate UD stripe of the two-ply design is assumed constant at \(l_i=5\) mm. Hence the role of the component lamination regions in the response of the composite structure can be determined.

Second, changes in the specimen aspect ratio (AR) understood as the length-to-width ratio are investigated through varying the laminate width given a specific length and internal layout \((x_{us}, x_s)\).

Table 3.1: Three distinct investigation categories underlying the internal layout dimension parameter studies.

<table>
<thead>
<tr>
<th>#</th>
<th>Central Unsymmetric</th>
<th>Outer Unsymmetric</th>
<th>Symmetric</th>
<th>Parameter Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(l_{us,c} = 2 \cdot x_{us})</td>
<td>(l_{us,o} = l_{us,c}/l)</td>
<td>(l_s = l/2 - x_s)</td>
<td>(x_{us})</td>
</tr>
<tr>
<td>2</td>
<td>const</td>
<td>(l_{us,o} = l_{us,c}/l)</td>
<td>(l_s = l/2 - x_s)</td>
<td>(x_s)</td>
</tr>
<tr>
<td>3</td>
<td>(x_{us})</td>
<td>const</td>
<td>(l_s = l/2 - x_s)</td>
<td>(x_{us})</td>
</tr>
</tbody>
</table>

The numerical analysis methodology underlying the parameter studies involves two main stages, detailed in the following two subsections. First, bi-stability of the configurations is established according to two criteria. Second, the stiffness variability of the design subset fulfilling the embeddability objective is explored.

### 3.2.1 Bi-Stability Criteria

The determination of whether a given parameter set results in a bi-stable configuration acquires vital importance when exploring the design space of the two laminate classes (see Figure 3.1). First, unconstrained bi-stability requires verification. To that end, two distinct criteria are employed: an imperfection cool-down and snap-through strain energy test. Both involve fixing all the six degrees of freedom of the centre of the composite plate, necessary to prevent rigid-body motion.

The bi-stable configuration subset fulfilling both criteria is subsequently tested for bi-stability preservation upon constraining narrow stripes of the symmetric regions found at the short edges (BCs). This is achieved by rotating the free clamping surfaces of the cool-down configuration from the original inclination (Figure 3.3) to the horizontal and fixing them in a fully flattened
position. Hence the embeddability study aims to achieve planar clamping (see Figure 3.4). Configurations retaining bi-stability when constrained in this manner are described as fulfilling the embeddability or in-plane clamping requirement. The longitudinally tailored stacking sequence distribution makes the corresponding curvature primarily responsible for maintaining bi-stability in the constrained configuration. The clamping surfaces also feature a minor transverse curvature (especially visible in Figure 3.2(d)), which nevertheless levels out in consequence of the application of the pursued boundary conditions. This certainly induces additional stresses in the structure, yet has not been found to play a significant part as regards the bi-stable response.

3.2.1.1 Imperfection Cool-Down Criterion

The imperfection cool-down criterion exploits the special character of the temperature-displacement path which a bi-stable structure follows while cooled from an elevated curing temperature, featuring a bifurcation into two equilibrium branches. Each corresponds to a final stable state and is accessible through the application of a specific out-of-plane displacement \([45]\).

The numerical realisation begins with a simulation of the curing process, which yields a particular shape whose corners exhibit a specific out-of-plane \((z)\) displacement. This value, multiplied by a problem-dependent factor (1.3 in the current case), then becomes an imperfection displacement imposed on the corners of the laminate during a repeated cool-down analysis. The factor is selected based on previous experience, the order of magnitude of the \(z\) displacements involved, and sensitivity studies such that the resulting deformation is sufficient to disturb the specimen from the corresponding equilibrium state without inducing excessive distortion. An unloading step follows (no temperature difference, no imposed displacement), allowing the structure to stabilise. The resulting final configuration, if the same as the one obtained directly from the curing simulation, is assumed stable and termed first stable shape. Further, the laminate is forced to adopt a distinct state through the application of an opposite imperfection displacement during the cool-down analysis. As before, the subsequent unloading step aims at verifying whether this configuration can be maintained, yielding the second stable shape. A positive outcome proves the unconstrained bi-stability of the design.
3.2 Numerical Model for Design Space Exploration

3.2.1.2 Snap-Through Strain Energy Criterion
The other independent bi-stability criterion utilises the knowledge that each equilibrium configuration of a multi-stable structure corresponds to a strain energy minimum. A numerical snap-through experiment is therefore devised to provide the strain energy history (ALLSE: recoverable strain energy [431]) as function of the out-of-plane displacement. The analysis starts from a particular stable state obtained from simulating the laminate manufacturing process. The resulting thermally-induced stress field of the structure is reflected in a specific non-zero strain energy level as the initial condition. All six degrees of freedom of the central node are constrained to preclude rigid-body motion. First, the stability of this original configuration can be additionally verified by perturbing the laminate corners in the direction of the current shape through a displacement-controlled deformation, increasing the corresponding strain energy. On load removal, the structure stabilises in the initial equilibrium shape, returning to the particular local strain energy minimum. Second, the laminate is coaxed into a potential second configuration through subjecting the corners to the opposite of their original transverse displacement\(^1\) multiplied by an adequate problem-dependent factor. Complete unloading of the specimen follows, verifying the stability of the newly adopted state. The bi-stability of the design manifests itself in the strain energy minimum reached at the end of the analysis being dissimilar to the one corresponding to the initial configuration.

3.2.2 Stiffness Response Quantification
To quantify and compare the stiffness response of the configuration subset satisfying the embeddability objective, a numerical displacement-controlled experiment is conducted (Figure 3.4). With the two opposite boundary condition surfaces clamped in-plane, a compressive displacement of \(u_1 = 4\,\text{mm}\) is imposed on the right surface and the corresponding reaction forces, \(F_{R,1}\), retrieved to provide force–displacement profiles.
In the next section, the results obtained based on the presented numerical methodology are first shown for two experimental specimens, followed by the findings of the parameter study.

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\(^1\) relative to the flat state prior to manufacturing simulation
3.3 Design Space Exploration: Results and Discussion

3.3.1 Specimen Analysis

The component analyses of the numerical model for design space exploration detailed above are first applied to two bi-stable specimens of dimensions given in Table 3.2, manufactured from a CFRP prepreg with an average ply thickness of 0.155 mm. Table 3.3 presents the nominal material properties employed in FEA.

Table 3.2: Geometric parameters of two experimental specimens used for analysis validation. Symbol definitions according to Figure 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>w/s/</th>
<th>x_s</th>
<th>x_us</th>
<th>x_us,i</th>
<th>BCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit/mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>Three-ply</td>
<td>150.00</td>
<td>320.00</td>
<td>106.67</td>
<td>53.33</td>
<td>−</td>
</tr>
<tr>
<td>Two-ply¹</td>
<td>64.00</td>
<td>222.00</td>
<td>80.00</td>
<td>65.00</td>
<td>60.00</td>
</tr>
</tbody>
</table>

Table 3.3: Nominal material properties of a unidirectional ply employed in FEA.

<table>
<thead>
<tr>
<th>Property</th>
<th>E_{11}</th>
<th>E_{22}</th>
<th>ν_{12}</th>
<th>G_{12} = G_{13}</th>
<th>α_{11}</th>
<th>α_{22} = α_{33}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit/MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>−</td>
<td>MPa</td>
<td>K^{-1}</td>
<td>K^{-1}</td>
</tr>
<tr>
<td>Value</td>
<td>161 000</td>
<td>10 000</td>
<td>0.3</td>
<td>4 400</td>
<td>−1.8 \times 10^{-8}</td>
<td>24.3 \times 10^{-6}</td>
</tr>
</tbody>
</table>

In the following, the strain energy profiles obtained from the numeri-

¹ This two-ply specimen was developed within the Master’s thesis of Waeber [430] and presented in [40].
3.3 Design Space Exploration: Results and Discussion

cal snap-through test are discussed. Next, experimental validation of the simulation models for stiffness response quantification in compression is presented.

3.3.1.1 Strain Energy Profiles

The results of the snap-through strain energy test of the two experimental specimens are shown in Figure 3.5. To ensure more reliable verification through starting-point independence of the set-up, the analysis commences from the first stable shape for the three-ply composite, yet from the second configuration in the two-ply case (denoted as zero point of vertical displacement at laminate corners in Figure 3.5). The clear minima of the resulting strain energy profiles, whose distinct values reflect the desired asymmetry, represent the statically stable states. Subjecting the laminate corners to an out-of-plane displacement consistent in direction with the given shape (upwards if the corners point upwards) leads to an unstable deformed state with increased strain energy. When unloaded, the structure follows the same path to return to the initial stable configuration. Imposing a displacement in the opposite direction to the natural corner orientation triggers snap-through to the other equilibrium shape, followed by stabilisation in the newly attained state on load removal. The associated strain energy peak becomes an important design factor, constituting a trade-off between reduced power requirements (mild transition profile) and load-bearing capacity precluding undesired snap-through under load (distinct peak).

The strain energy curves confirm the empirical observation of higher effort associated with stable state transition of the two-ply specimen. Further, the favoured, more stable configurations (flat for the three-ply and curved for the two-ply laminate) in each case feature a lower energy level than the other equilibrium shape. This confirms the informative value of the snap-through strain energy test in designing new bi-stable components.
Figure 3.5: Strain energy profiles of the two bi-stable specimens as function of the out-of-plane displacement of the laminate corners. Note the two strain energy minima corresponding to the equilibrium configurations as well as perturbed unstable states with increased energy levels.
3.3.1.2 Stiffness Response Validation

The numerical stiffness response quantification method was validated by testing the two specimens in compression by means of a Zwick/Roell Z005 machine, as detailed in [40, 430]. Figure 3.6 shows the test rig with the three-ply specimen. The results are presented in Figure 3.7. It can be seen that the representative stiffness profiles of the two laminate classes are captured sufficiently well for the purpose of the current study.

The most visible discrepancy concerns the flat stable shape of the two-ply specimen, which exhibits a profoundly non-linear response in compression involving snap-through to the curved equilibrium state (Figure 3.7(b)). The main sources of error might be summarised as increased sensitivity to imperfections such as uneven distribution of cured ply thickness, as well as the clamped boundary condition introduction method. First, unlike the three-ply structure, the two-ply configuration does not contain a unidirectional ply continuing through the laminate in the loading direction. This, combined with higher geometric differences between the stable states and stronger non-planarity of the clamping surfaces, leads to a far more anisotropic response. Hence existing imperfections are amplified, substantially affecting the actual behaviour as opposed to the idealised FEA model. Sensitivity to initial imperfections is a common characteristic of unstable phenomena such as buckling or snap-through, hindering highly precise predictions [296]. By contrast, the three-ply specimen exhibits a much smoother behaviour without compression-induced stable state switch. This, combined with less pronounced anisotropy and higher bending stiffness

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1 The specimen clamp jaws were prepared within the Master’s thesis of Waeber [430].
(proportional to the power of three of the laminate thickness), results in a higher tolerance to imperfections. Second, the large geometric change between the states in the two-ply case and clamping surface non-planarity visible in Figure 3.2(d) increase the implications of any misalignments during testing. This is accompanied by finite rigidity of the actual test rig on the one hand and modelling simplifications neglecting clamping pressure on the other. Moreover, the inability to achieve perfectly coincident numerical and real residual stress fields becomes yet another contribution to the overall sources of error.

![Graph showing comparison of numerical and experimental reaction force results, $F_{R,1}$, in axial compression for a displacement-controlled test ($u_1$) of both stable shapes of the experimental specimens. Linear regression for stiffness difference estimation, $\Delta K$, added for a 0.5 mm deviation from equilibrium.](image)

(a) Three-ply  
(b) Two-ply

Figure 3.7: Comparison of numerical and experimental reaction force results, $F_{R,1}$, in axial compression for a displacement-controlled test ($u_1$) of both stable shapes of the experimental specimens. Linear regression for stiffness difference estimation, $\Delta K$, added for a 0.5 mm deviation from equilibrium.

Finally, it is also noteworthy that snap-through phenomena pose an inherent simulation challenge, exhibiting substantial sensitivity to spatial and tem-

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1 The fabrication, testing and simulation validation of this two-ply specimen was performed within the Master’s thesis of Waeb [430] and published in [40]. The validation is repeated herein due to differences in computational set-up compared to [40, 430].
poral discretisation, analysis parameter selection, accompanied by artificial damping and energy conservation issues [432].

The linear regression data to the experimental results of Figure 3.7, summarised in Table 3.4, reveal that the continuity-preserving zero-degree middle layer of the three-ply specimen greatly reduces the achievable stiffness difference, $\Delta K$, between the two states to about two, compared to a value of forty-eight obtained in the two-ply case. Since the same external load not exceeding the snap-through threshold would induce a larger displacement of the curved shape compared to the flat one, the actual stiffness variability potential becomes even larger due to the lower slope of the force-displacement curves of the flexible configuration away from equilibrium. Considering Table 2.1, the two-ply design thus becomes a particularly promising variable stiffness component. This will be confirmed in Section 3.3.3, demonstrating further tailoring possibilities through the internal layout distribution.

**Table 3.4:** Summary of the stiffness variability of the experimental specimens in terms of the linear regression slopes of the experimental curves of Figure 3.7.

<table>
<thead>
<tr>
<th></th>
<th>Three-ply</th>
<th>Two-ply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiff</td>
<td>34.7</td>
<td>111.3</td>
</tr>
<tr>
<td>Flexible</td>
<td>15.4</td>
<td>2.3</td>
</tr>
<tr>
<td>Ratio</td>
<td>2</td>
<td>48</td>
</tr>
</tbody>
</table>

### 3.3.2 Three-Ply Configuration
#### 3.3.2.1 Sensitivity Studies of Internal Layout Dimensions

Of the total of seventy-eight configurations considered, twenty-four fulfil the imperfection cool-down test for bi-stability. The unsuccessful specimens feature an insufficient fraction of the outer unsymmetric region: $\tilde{l}_{us,o} < 0.13 - 0.20$, which grows as the total unsymmetric contribution, $\tilde{l}_{us,o} + \tilde{l}_{us,c}$, increases. This $\tilde{l}_{us,o}$ threshold characterises a number of limit cases with a positive outcome of the strain energy criterion, despite a bi-stability loss prediction by the imperfection cool-down method. Their common feature is a flat profile of the corresponding strain energy minimum illustrated in Figure 3.8(b), implying marginal stability as a possible explanation. Another reason might be the simulation difficulty of unstable, highly non-linear phenomena using
a general static stabilised step. Experience gained with this type of problems shows that under certain circumstances the analysis might fail to converge to a solution of the lowest strain energy, which provides motivation for employing two independent criteria for bi-stability assessment. Therefore, only configurations passing both tests are considered for the further bi-stability verification when clamped. The marginal case of Figure 3.8(b)

(a) Bi-stable: $\bar{x}_s = 0.78$, $\bar{x}_{us} = 0.38$, $\bar{l}_{us,o} = 0.20$.

(b) Marginal: $\bar{x}_s = 0.78$, $\bar{x}_{us} = 0.44$, $\bar{l}_{us,o} = 0.17$.

(c) Monostable: $\bar{x}_s = 0.72$, $\bar{x}_{us} = 0.53$, $\bar{l}_{us,o} = 0.09$.

Figure 3.8: Strain energy profiles for a bi-stable, limit case, and monostable configuration as function of vertical displacement of the laminate corners. First stable shape as starting point of the analysis. Strain energy minima indicated with arrows, state at maximum imposed displacement with a cross.
can be compared to two distinct minima exhibited by a bi-stable (Figure 3.8(a)) and only one such extremum characterising a monostable structure (Figure 3.8(c)). Figure 3.8 clearly shows this distinction by indicating the strain energy minima with arrows, while marking the perturbed states at the peak imposed displacement with crosses.

Another factor of interest is the ratio of the central to the outer unsymmetric regions, which ranges between 1.25 and 2.5 for bi-stable specimens (lower values for growing total unsymmetric lamination region).

The requirement of retaining bi-stability with the clamping surfaces constrained in a horizontal position reduces the subset to eleven layouts, whose details are provided in Table 3.5. Figure 3.9 shows selected results of the three groups of internal layout dimension parameter studies.

In the first investigation class (#1 in Table 3.1), the contribution of the central unsymmetric region ($\tilde{l}_{us,c}$) is increased at the cost of reducing the outer unsymmetric part ($\tilde{l}_{us,o}$), while keeping the symmetric section ($\tilde{l}_s$) constant. From the data in Figure 3.9(a) and Table 3.5, a clear trend of increasing stiffness difference can be observed. The larger the contribution of $\tilde{l}_{us,c}$, the higher the absolute stiffness of the flat state and the lower the stiffness of the curved configuration. No significant influence on the clamping region inclination emerges from the considered variation of the two unsymmetric sections.

The congruence of the curves in Figure 3.9(b) confirms the conclusion that it is the central unsymmetric fraction, $\tilde{l}_{us,c}$, which primarily controls the stiffness difference between the two stable shapes, or the stiffness variability, of the three-ply configuration. Indeed, the results correspond to the second parameter study group (#2 in Table 3.1), in which the central unsymmetric region remains unaltered, and the outer unsymmetric area expands as the symmetric section reduces. As Table 3.5 confirms, the linearised stiffness ratios for a small disturbance from equilibrium, $\Delta K$, occupy a limited range for $\tilde{l}_{us,c} = \text{const}$. For total unsymmetric lamination fractions $2 \cdot \tilde{l}_{us,o} + \tilde{l}_{us,c}$ falling below 0.71, however, an insignificant decrease in $\Delta K$ can be observed as $\tilde{l}_s$ reduces and $\tilde{l}_{us,o}$ grows, followed by the opposite on exceeding this total unsymmetric contribution. The reason are insignificant stiffness variations of the flat stable state. This phenomenon might be a combined effect of shape non-planarity associated with the growing clamping surface rotation to the horizontal and the increasing contribution of the 0-degree dominated
Table 3.5: Numerical results for the three-ply layout parameter sets fulfilling the bi-stability requirement of in-plane clamping. Order according to constant contribution of the central unsymmetric lamination region.

<table>
<thead>
<tr>
<th>$\bar{x}_s$</th>
<th>$\bar{x}_{us}$</th>
<th>$\bar{l}_s$</th>
<th>$\bar{l}_{us,o}$</th>
<th>$\bar{l}_{us,c}$</th>
<th>$l_{us,c}/l_{us,o}$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\Delta \theta$</th>
<th>$\Delta z$</th>
<th>$\Delta U$</th>
<th>$\Delta K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>°</td>
<td>°</td>
<td>°</td>
<td>mm</td>
<td>mm</td>
<td>mJ</td>
<td></td>
</tr>
<tr>
<td>0.63</td>
<td>0.31</td>
<td>0.19</td>
<td>0.16</td>
<td>0.31</td>
<td>2.00</td>
<td>4.52</td>
<td>-3.73</td>
<td>8.25</td>
<td>5.80</td>
<td>2.80</td>
<td>2.15</td>
</tr>
<tr>
<td>0.66</td>
<td>0.31</td>
<td>0.17</td>
<td>0.17</td>
<td>0.31</td>
<td>1.82</td>
<td>4.89</td>
<td>-4.35</td>
<td>9.24</td>
<td>6.56</td>
<td>2.16</td>
<td>2.10</td>
</tr>
<tr>
<td>0.69</td>
<td>0.31</td>
<td>0.16</td>
<td>0.19</td>
<td>0.31</td>
<td>1.67</td>
<td>5.30</td>
<td>-4.93</td>
<td>10.22</td>
<td>6.90</td>
<td>1.31</td>
<td>2.05</td>
</tr>
<tr>
<td>0.72</td>
<td>0.31</td>
<td>0.14</td>
<td>0.20</td>
<td>0.31</td>
<td>1.54</td>
<td>5.73</td>
<td>-5.29</td>
<td>11.03</td>
<td>7.08</td>
<td>0.07</td>
<td>2.01</td>
</tr>
<tr>
<td>0.75</td>
<td>0.31</td>
<td>0.13</td>
<td>0.22</td>
<td>0.31</td>
<td>1.43</td>
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<td>6.21</td>
<td>11.69</td>
<td>7.12</td>
<td>1.51</td>
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<td>0.14</td>
<td>0.19</td>
<td>0.34</td>
<td>1.83</td>
<td>5.74</td>
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<td>7.33</td>
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<tr>
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<td>0.20</td>
<td>0.38</td>
<td>1.85</td>
<td>6.76</td>
<td>-5.97</td>
<td>12.73</td>
<td>8.24</td>
<td>5.54</td>
<td>3.96</td>
</tr>
<tr>
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<td>0.38</td>
<td>0.09</td>
<td>0.22</td>
<td>0.38</td>
<td>1.71</td>
<td>7.31</td>
<td>-6.17</td>
<td>13.48</td>
<td>8.91</td>
<td>3.42</td>
<td>4.14</td>
</tr>
</tbody>
</table>

1: subscripts referring to first and second stable shapes, uniform convention irrespective of the corresponding configuration (flat/curved)
2: $\Delta \theta = |\theta_1 - \theta_2|$: difference in clamping surface inclination angles between stable shapes
3: $\Delta z$: distance between the centres of the two stable configurations clamped in-plane
4: $\Delta U = U_2 - U_1$: strain energy difference between equilibrium shapes
5: $\Delta K = K_1/K_2$: ratio of linear regression slopes of compressive force-displacement curves for small disturbance from equilibrium

† unique case of curved first and flat second stable shape; stiff-to-flexible ratio comparable with the remaining cases: $\Delta K^{-1} = K_2/K_1 = 1/0.47 = 2.13$
3.3 Design Space Exploration: Results and Discussion

3.3.1 Layout Dimensions Sensitivity Study

Figure 3.9: Selected results of three groups of the layout dimensions sensitivity study of the three-ply configuration. Laminate layout defined by $\bar{x}_s = \frac{2x_s}{l}$, $\bar{x}_{us} = \frac{2x_{us}}{l}$: Position fractions of boundaries between symmetric-outer unsymmetric, and outer unsymmetric-central unsymmetric region, respectively (Figure 3.1(a) and Table 3.1).

(a) Constant symmetric region:
\[ \tilde{l}_s = 0.14; \tilde{l}_{us,c} = \uparrow, \tilde{l}_{us,o} = \downarrow. \]
No change in total unsymmetric contribution.

(b) Constant central unsymmetric region:
\[ \tilde{l}_{us,c} = 0.31; \tilde{l}_{us,o} = \uparrow, \tilde{l}_s = \downarrow. \]

(c) Constant outer unsymmetric region:
\[ \tilde{l}_{us,o} = 0.22; \tilde{l}_{us,c} = \uparrow, \tilde{l}_s = \downarrow. \]
unsymmetric lamination. Further, as the symmetric part reduces to allow for a larger outer unsymmetric section, the clamping region inclination rises (Table 3.5). This causes a growing angle difference $\Delta \theta$ between both states, deteriorating the embeddability objective of planar integration pursued herein. Obviously, permitting the edges to form an angle with respect to the horizontal would require a redefinition of this requirement. Further, the difference between the two equilibrium configurations, measured in terms of the vertical distance $\Delta z$ between the respective plate centre positions, increases.

Figure 3.9(c) presents the third and final investigation case (#3 in Table 3.1), which involves shifting a constant outer unsymmetric contribution outwards, towards the short edges. An expansion of the central unsymmetric part is thus achieved through reducing the symmetric section. As before, increasing $\bar{l}_{us,c}$ leads to growing stiffness variability. This, however, is accompanied by rising clamping surface inclination magnitude of both states, an adverse effect in view of the embeddability objective. The difference between the centre points of the laminate $\Delta z$ whilst clamped in the two equilibrium configurations increases as well.

Given a fixed size of the three-ply specimen, a clear conclusion can therefore be drawn that what exerts the greatest impact on the stiffness response is the central unsymmetric region, whereas the embeddability is primarily controlled by the symmetric and outer unsymmetric sections. A growing contribution of the central unsymmetric area implies stiffness gain of the flat and reduction of the curved shape, enhancing the structural adaptability. This holds irrespective of whether $\bar{l}_{us,c}$ expands at the cost of diminishing the symmetric or outer unsymmetric part. Yet maximising the symmetric region (and minimising the outer unsymmetric) benefits embeddability understood as the minimisation of the clamping surface inclination. An optimum compromise is therefore required in this respect due to bi-stability loss associated with insufficient length of the outer region, $\bar{l}_{us,o}$. Finally, it can be observed that the three-ply configuration only offers modest potential for property tailoring. Specifically, the available stiffness ratios between the two equilibrium states range between two and four, with similarly low sensitivity of the clamping surface inclination.
### 3.3.2.2 Aspect Ratio Sensitivity Studies

The influence of the laminate AR, defined as a length-to-width ratio, is investigated based on a configuration of a fixed length \( l = 320 \text{ mm} \) and internal layout, \( \bar{x}_s = 0.78 \) and \( \bar{x}_{us} = 0.38 \). The overall width \( w \) of the composite is increased from 90 mm to 320 mm (square form), covering an AR range of 3.56 to 1.00.

Only five of the total of twenty-four designs with AR between 2.13 and 1.68 satisfy the in-plane clamping bi-stability requirement (laminate width-to-thickness ratios ranging from 323 to 344). By contrast, both the imperfection cool-down and snap-through strain energy criteria confirm that twenty-two configurations with \( AR \leq 2.91 \) are bi-stable when unconstrained. This indicates 2.91 as the maximum aspect ratio (minimum width) above which no free-edge bi-stable specimens exist given the laminate thickness investigated herein.

**Table 3.6**: Numerical results for strain energy difference \( \Delta U_{\text{peak-well}} \) between the peak separating the stable state minima and the second configuration well for the three-ply configuration aspect ratio sensitivity studies. Results of AR = 1.52 used for \( \Delta U_{\text{peak-well}} \) definition on the left.

<table>
<thead>
<tr>
<th>AR</th>
<th>( \Delta U_{\text{peak-well}} ) mJ</th>
<th>AR</th>
<th>( \Delta U_{\text{peak-well}} ) mJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.56</td>
<td>0.000(^{\ast})</td>
<td>1.52</td>
<td>3.733</td>
</tr>
<tr>
<td>3.20</td>
<td>0.000(^{\ast})</td>
<td>1.45</td>
<td>2.622</td>
</tr>
<tr>
<td>2.91</td>
<td>0.001(^{\dagger})</td>
<td>1.39</td>
<td>1.984</td>
</tr>
<tr>
<td>2.67</td>
<td>0.666</td>
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<td>2.46</td>
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<td>1.102</td>
</tr>
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<td>2.13</td>
<td>2.534</td>
<td>1.19</td>
<td>0.969</td>
</tr>
<tr>
<td>2.00</td>
<td>2.836</td>
<td>1.14</td>
<td>0.578</td>
</tr>
<tr>
<td>1.88</td>
<td>3.005</td>
<td>1.10</td>
<td>0.190(^{\dagger})</td>
</tr>
<tr>
<td>1.78</td>
<td>3.119</td>
<td>1.07</td>
<td>0.490(^{\dagger})</td>
</tr>
<tr>
<td>1.68</td>
<td>3.215</td>
<td>1.03</td>
<td>0.293(^{\dagger})</td>
</tr>
<tr>
<td>1.60</td>
<td>3.497</td>
<td>1.00</td>
<td>0.508(^{\dagger})</td>
</tr>
</tbody>
</table>

\(^{\ast}\) monostable

\(^{\dagger}\) limited accuracy of the results approaching the bi-stability loss due to increasing sensitivity to simulation parameters

Table 3.6 summarises the values of the strain energy difference between
the peak separating the equilibrium state minima and the well of the second configuration, whose existence the underlying snap-through strain energy criterion seeks to study. The initially growing values reflect a more pronounced peak featured by the strain energy profiles as the width of the composite increases, with AR falling to 1.52. In other words, changing between the two stable shapes requires a growing energy input. Further reducing the AR causes the peak to flatten and the separation between the two minima to diminish, leading towards mono-stability. This indicates the existence of an optimum aspect ratio in view of the desired bi-stability level.

Figure 3.10 confirms the expected result that increasing laminate width, or reducing AR, is only reflected in an insignificant growth of the absolute response of each stable state. Hence the overall aspect ratio of the specimen can be identified as a design factor in controlling bi-stability of both the unconstrained and the clamped configuration, with no significant implications for stiffness variability.

![Figure 3.10](image)

**Figure 3.10:** Compressive stiffness response trends for an increasing width of the three-ply laminate (decreasing aspect ratio, $AR = l/w$). The internal layout dimensions remain unaltered at $\bar{x}_s = 0.78$ and $\bar{x}_{us} = 0.38$.

3.3.3 Two-Ply Configuration

Of all the thirty-six layout cases considered, fourteen prove to maintain bi-stability on in-plane clamping (Table 3.7) and additional ten when uncon-
strained as per the imperfection cool-down test. Similarly to the three-ply design, the snap-through strain energy criterion proves less conservative, indicating bi-stability loss of only five free-edge configurations. As already mentioned, this discrepancy arises from the general difficulty of snap-through simulations using static procedures, combined with the sensitivity exhibited by limit cases to the problem-dependent magnitude of the imperfection displacement.

In contrast to the three-ply design, unconstrained bi-stability is only retained when exceeding a minimum ratio between the central and outer unsymmetric regions: \( l_{us,c}/l_{us,o} > 3.5 – 4.5 \) (dependent on the total unsymmetric region). In other words, this layout encourages long and slender designs. As expected, insufficient contribution of the symmetric lamination region \( \bar{l}_s < 0.11 \) induces a bi-stability loss in the constrained configuration.

The findings of the layout dimensions studies of the two-ply composite largely confirm the previously drawn conclusions, as Figure 3.11 illustrates based on selected results. When in the flat shape, however, the two-ply laminate is found to switch to the other stable configuration in consequence of the application of even a very small compressive displacement (Figure 3.7(b)). Nevertheless, low as this snap-through triggering displacement value might be, the element still carries significant loads before changing configurations. This becomes an additional design variable of considerable importance.

The central unsymmetric region retains its key role in controlling the stiffness behaviour (Figure 3.11). In addition, this parameter exerts decisive influence on the compressive displacement triggering the snap-through. Indeed, the longer the central unsymmetric region, the sharper the stiffness response peak, and hence the smaller the critical displacement (Figure 3.11(a) and (c)).

From the data in Table 3.7, it can be observed that the two-ply design features an inverse proportionality between the clamping surface angles of both stable states. Specifically, growing edge inclination of the first (curved) shape, \( |\theta_1| \), is associated with decreasing inclination of the second (flat) form, \( |\theta_2| \), assuming an invariant central unsymmetric or symmetric region. Considering a constant symmetric contribution, as the outer unsymmetric region expands (and central unsymmetric shrinks), \( |\theta_1| \) diminishes and \( |\theta_2| \) grows, yet the effective angle difference \( \Delta \theta \) decreases. The same observations hold for a constant central unsymmetric region, nonetheless with insignificant
Table 3.7: Numerical results for the two-ply layout parameter sets fulfilling the bi-stability requirement of in-plane clamping. Order according to constant contribution of the central unsymmetric lamination region, $l_{us,c} = \text{const.}$

<table>
<thead>
<tr>
<th>$\bar{x}_s$</th>
<th>$\bar{x}_{us}$</th>
<th>$\bar{I}_s$</th>
<th>$\bar{I}_{us,o}$</th>
<th>$\bar{I}_{us,c}$</th>
<th>$\bar{I}<em>{us,c}/\bar{I}</em>{us,o}$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\Delta \theta$</th>
<th>$\Delta z$</th>
<th>$\Delta U$</th>
<th>$F_{\text{peak}}$</th>
<th>$u_{\text{peak}}$</th>
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</thead>
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<tr>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>-</td>
<td>°</td>
<td>°</td>
<td>°</td>
<td>°</td>
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<td>0.28</td>
<td>0.03</td>
<td>0.33</td>
<td>12.00</td>
<td>11.93</td>
<td>0.03</td>
<td>11.90</td>
<td>3.37</td>
<td>3.56</td>
<td>87/46$^\dagger$</td>
<td>0.11/0.85$^\star$</td>
</tr>
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<td>0.06</td>
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<td>6.58</td>
<td>-5.97</td>
<td>12.54</td>
<td>3.63</td>
<td>3.62</td>
<td>48$^\dagger$</td>
<td>1.31$^\dagger$</td>
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<td>-0.05</td>
<td>14.33</td>
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<td>0.12/0.70$^\star$</td>
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<td>61$^\dagger$</td>
<td>1.06$^\dagger$</td>
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<td>-6.58</td>
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<td>12.45</td>
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<td>0.03</td>
<td>0.67</td>
<td>24.00</td>
<td>28.42</td>
<td>-2.51</td>
<td>30.94</td>
<td>1.11</td>
<td>4.32</td>
<td>186</td>
<td>0.08</td>
</tr>
</tbody>
</table>

$F_{\text{peak}}$: reaction force at response peak of the flat stable state in axial compression (preceding snap-through or buckling)

$u_{\text{peak}}$: imposed displacement at response peak, $F_{\text{peak}}$, of the flat stable state in axial compression

$^\dagger$ configurations with flat state buckling into a shape opposite to the curved stable state due to insufficient total unsymmetric lamination ($2 \cdot \bar{I}_{us,o} + \bar{I}_{us,c} \leq 0.5$)

$^\star$ buckling configurations exhibiting double force peak (very short $\bar{I}_{us,o}$)
3.3 Design Space Exploration: Results and Discussion

**Figure 3.11:** Selected results of three groups of the internal layout dimensions sensitivity study of the two-ply configuration. Laminate layout defined by $\bar{x}_s = 2x_s/l$, $\bar{x}_{us} = 2x_{us}/l$: Position fractions of boundaries between symmetric-outer unsymmetric, and outer unsymmetric-intermediate region, respectively (Figure 3.1(b) and Table 3.1).

(a) Constant symmetric region:
$\bar{l}_s = 0.14; \bar{l}_{us,c} = \uparrow, \bar{l}_{us,o} = \downarrow$.
No change in total unsymmetric contribution.

(b) Constant central unsymmetric region: $\bar{l}_{us,c} = 0.44; \bar{l}_{us,o} = \uparrow, \bar{l}_s = \downarrow$.

(c) Constant outer unsymmetric region:
$\bar{l}_{us,o} = 0.06; \bar{l}_{us,c} = \uparrow, \bar{l}_s = \downarrow$. 
implications for $\Delta \theta$. Further, decreasing $\overline{l}_{us,c}$ reduces the clamping surface inclination of the curved shape irrespective of which of the two other regions, $\overline{l}_{us,o}$ or $\overline{l}_s$, remains unaltered. Given a particular proportion of the symmetric lamination, the outer unsymmetric part thus provides primary control over the embeddability objective. Since improving $|\theta_1|$ deteriorates $|\theta_2|$ and vice versa, a trade-off solution becomes necessary. In this regard, it is important to note that the substantial compliance and greater stability of the flexible equilibrium configuration enhances its clamping tolerance. By contrast, the flat shape exhibits increased sensitivity to boundary condition introduction, as previously discussed in Section 3.3.1.2. The three-ply laminate behaves differently, possessing two relatively stiff stable states and hence requiring minimum $\Delta \theta$ for embeddability reasons. An additional objective therefore emerges in the two-ply case to keep the edge inclination of the flat configuration as small as possible.

Further, one of the more significant findings to emerge from this study is that the embeddability and critical displacement considerations require an optimum compromise with stiffness difference maximisation. Large central unsymmetric region improves the achievable stiffness range, which only comes at the expense of a very small compression extent necessary to trigger the snap-through. This conflicts with the desired low clamping surface angles associated with reduced $\overline{l}_{us,c}$ and increased $\overline{l}_{us,o}$, necessary to limit the edge inclination of the flexible shape.

What is more, the two-ply layout is most suitable for long and slender structures. As shown in Table 3.7, insufficient proportion of the total unsymmetric part, $2 \cdot \overline{l}_{us,o} + \overline{l}_{us,c}$, leads to specimen buckling rather than stable state switch from flat to curved predicted in compression. Given the constant component width, this indicates the need for an adequate aspect ratio of the unsymmetrically laminated region. On the other hand, such unstable behaviour is in practice strongly affected by imperfections and initial conditions. When embedding the fabricated composite in a larger morphing system, the response could be purposefully guided in the preferred direction of the curved equilibrium configuration to counteract the potential buckling behaviour.

Finally, the results obtained for the two-ply laminate indicate a far larger property tailoring range achievable with layout parameters compared to the three-ply case. More precisely, the compressive snap-through load varies
between approximately 50 N and 190 N, with the corresponding critical displacement reducing from about 1 mm to 0.08 mm. In the same way, considerable sensitivity of the clamping surface angles of each equilibrium configuration ensues, lying between six and twenty-eight degrees for the flexible, and zero and nine degrees for the stiff state.

Due to the largely expected response trends obtained as function of the overall aspect ratio, the results concerning the two-ply design are not reported herein.

Considered in the context of the intended application to morphing of small unmanned aircraft systems operating at low velocities (10 m/s to 20 m/s), the obtained reaction forces can be assessed as sufficient for this task. The composites primarily aim at providing global stiffness variability when embedded into a larger system, permitting the activation of a particular deformation mode rather than supporting major structural loads. Further, bi-stability as a scalable phenomenon enables the load-carrying characteristics to be tailored as function of the laminate thickness and size.

### 3.4 Conclusions

This chapter has introduced an experimentally validated numerical methodology devised to facilitate the development of bi-stable laminates acting as variable stiffness elements within a larger distributed morphing topology. Two independent criteria allowing for bi-stability assessment of diverse configurations are developed. The first considers the characteristic strain energy profile featuring minima corresponding to the equilibrium shapes. The second entails displacement-controlled cool-down tests. Given the requirements of monolithic embeddability and stiffness variability ensuing from the envisaged function, a systematic design space exploration of two classes of such composite structures is performed by varying the spatial distribution of the individual lamination regions and the overall aspect ratio of the components.

The central unsymmetrically laminated section is found to be primarily responsible for the desired stiffness variability. The embeddability, concerned with retaining bi-stability when constrained in-plane and dependent upon the inclination of the clamping surfaces to the horizontal, is mainly governed by the other two regions. Providing a large symmetric and central unsymmetric while targeting a reduced outer unsymmetric part becomes advantageous for
simultaneous stiffness variability and good embeddability of the three-ply design. However, bi-stability preservation imposes limits on fulfilling these objectives, dictating a lower bound for the length of the outer unsymmetric region and a maximum ratio between the central and outer unsymmetric contributions. On the other hand, the two-ply configuration benefits from preferably large central and moderate outer unsymmetric sections, with longer and more slender designs encouraging bi-stability. Further, much higher sensitivity to layout parameters in tailoring the stiffness variability and clamping surface inclination characterises the two-ply laminate compared to the three-ply element.

The new insights gained based on two most basic configurations greatly facilitate decision-making in the design process of embeddable bi-stable components for stiffness variability. As will be seen in the next chapter, the findings lay the foundations for generating modified stacking sequence distributions and performing initial sizing of cognate laminates, according to external dimensions imposed by the larger morphing system. The proposed methodology, independent of the details of a particular layout, provides a systematic framework for bi-stability assessment and final design refinements of new configurations to identify trade-off solutions under the prescribed requirements.
CHAPTER 4

Variable Stiffness Aerofoils with Integrated Bi-Stable Laminates: Proof of Concept

This chapter is based on the conference contribution:

and on the journal submission:

The final version of this paper has been published in Journal of Intelligent Material Systems and Structures, vol. 27(14), Aug. 2016 by SAGE Publications Ltd, All rights reserved. ©Izabela K. Kuder, 2016. It is available at: http://jim.sagepub.com/content/27/14/1949.
Relying upon the in-depth analysis of the embeddable variable stiffness bi-stable components provided in the preceding part, the present chapter progresses to exploring the capabilities of the proposed shape adaptation concept. A selectively compliant rib based on a 500 mm chord NACA 0012 with two monolithically embedded bi-stable composites is presented. To that end, four distinct thermally-induced bi-stable laminate configurations are first developed and evaluated. A high level of stiffness variability of the composites is achieved while ensuring the capability of two opposite edges to be constrained within a larger system. Initial positioning studies of the bi-stable components in the interior of the wing section, conducted by means of FEA, provide first insights into response trends and interactions. The conclusions lead to the selection of the most promising bi-stable layout to be eventually integrated into the NACA 0012-based morphing system. The global response of the aerofoil is assessed numerically based on a simple mechanical test. A particular focus is placed on the structural stiffness modification potential by virtue of the stable state changes of the component laminates. The final prototype manufacturing and testing provides an experimental validation of the concept and the numerical procedure employed. The results allow for assessing the feasibility of the proposed morphing approach of selective compliance based on stiffness variability provided by monolithically embedded bi-stable elements.

4.1 Thin-Ply Bi-stable Laminates

4.1.1 Bi-stable Laminate Design Requirements
As already mentioned, the design process of the bi-stable components is governed by two fundamental considerations arising from the intended function as variable stiffness elements within a selectively compliant morphing system. In the first place, the laminates need to exhibit maximised stiffness difference, herein quantified in axial compression. At the same time, maintaining an adequate bi-stability level has to be ensured when embedded in the larger distributed topology with two opposite edges constrained.

At the initial stage, a NACA 0012 profile with a chord length of 500 mm is selected as the morphing system. Given the external space delimitation imposed by the dimensions of the wing section, the integration task of several bi-stable composites thus imposes an additional requirement of size minimi-
sation of the candidate designs. A fixed width of \( w = 50 \text{ mm} \) is adopted, corresponding to the depth of the available NACA 0012 manufacturing mould, while seeking to generate the shortest feasible specimens. This has to be reconciled with the bi-stability loss associated with excessive laminate thickness relative to the edge length when exploiting thermally induced stresses to obtain the multi-stable behaviour, as previously explained in Section 2.2.2. To address this challenge, the M40J/736 prepreg developed by North Thin Ply Technology [433] is employed, featuring a nominal ply thickness of 30 \( \mu \text{m} \). Table 4.1 summarises the relevant material data of a single lamina as used for the FEA.

Table 4.1: Material properties of a lamina of M40J/736 prepreg [434–436] used in FEA.

<table>
<thead>
<tr>
<th>Property</th>
<th>( E_{11} )</th>
<th>( E_{22} )</th>
<th>( \nu_{12} )</th>
<th>( G_{12} = G_{13} )</th>
<th>( \alpha_{11} )</th>
<th>( \alpha_{22} = \alpha_{33} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>MPa</td>
<td>MPa</td>
<td>−</td>
<td>MPa</td>
<td>K(^{-1})</td>
<td>K(^{-1})</td>
</tr>
<tr>
<td>Value</td>
<td>222 000</td>
<td>7 010</td>
<td>0.314</td>
<td>4 661</td>
<td>(-1.8 \times 10^{-8} )</td>
<td>(24.3 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

\( \dagger \) Assumed based on Chapter 3 and [319].

4.1.2 Thin-Ply Bi-stable Laminate Characteristics

The final four embeddable variable stiffness bi-stable composite configurations are provided in Figure 4.1 in parametrised form, with Table 4.2 showing the relevant dimensions. Figure 4.2 presents the unconstrained cool-down shapes of the laminates: the curved state flexible in compression and the flat configuration, stiff under this type of loading. These are obtained from FEA through simulating the curing process with a total temperature difference \( \Delta T = 59 \text{ K} \) emerging from the recommended autoclave cycle with temperature dwells at 62°C and 80°C.

The specific spatial stacking sequence distribution devised to enable constraining the clamping surfaces (marked as BCs in Figure 4.1) without bi-stability loss inherently leads to the inclination of the short edges from the horizontal in at least one of the states (defined in Figure 4.2(i), clearly visible in Figure 4.2(a), (c) and (e)), which is considered to adversely affect the embeddability objective of planar integration. This motivates the search for multiple designs to be adopted for positioning parameter studies within the aerofoil. Indeed, each of the final configurations possesses distinct edge
inclination characteristics, dissimilar to the remaining designs, arising as a trade-off under the aforementioned considerations.

Figure 4.1: Final configurations of embeddable bi-stable laminates in parametrised form. BCs: Clamping surfaces/regions of boundary condition assignment.
4.1 Thin-Ply Bi-stable Laminates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>( w )</th>
<th>( l )</th>
<th>( x_s )</th>
<th>( x_{us} )</th>
<th>( x_{us,i} )</th>
<th>( BCs )</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>mm</td>
<td>50.0</td>
<td>110.0</td>
<td>40.0</td>
<td>30.5</td>
<td>27.5</td>
<td>2.0</td>
</tr>
<tr>
<td>#2</td>
<td>mm</td>
<td>50.0</td>
<td>110.0</td>
<td>40.0</td>
<td>33.5</td>
<td>30.5</td>
<td>2.0</td>
</tr>
<tr>
<td>#3</td>
<td>mm</td>
<td>50.0</td>
<td>95.0</td>
<td>34.0</td>
<td>20.0</td>
<td>—</td>
<td>2.0</td>
</tr>
<tr>
<td>#4</td>
<td>mm</td>
<td>50.0</td>
<td>95.0</td>
<td>37.0</td>
<td>22.0</td>
<td>—</td>
<td>2.0</td>
</tr>
</tbody>
</table>

As before, the pursued maximised stiffness variability of each laminate refers to the difference in response to imposed compressive displacement between the two equilibrium shapes. The maximum reaction force, \( F_{\text{peak}} \), and the corresponding critical displacement, \( u_{\text{peak}} \), triggering the snap-through or causing buckling, serve as representative quantities for the flat configuration. These, rather that the linearised slopes of the ensuing force-displacement curves, are considered to most appropriately characterise the structural response of interest in view of the purpose of the present study. Accordingly, the reaction force values of the curved shape, \( F_{\text{flex}} \), at 2 mm of compression are selected for comparison of the adaptability provided by a given composite due to the asymptotic character of the corresponding force-displacement profiles. Table 4.3 summarises the main characteristics of each of the four bi-stable composite designs, predicted by FEA.

Although the stiff state of specimen #3 sustains the highest compressive load before changing configuration to curved (4.9 N), an extremely low displacement triggers the snap-through. The remaining laminates withstand higher critical compression (Table 4.3), which only comes at the expense of lower load-bearing capacity of the stiff equilibrium state (#1: 3.5 N, #2: 2.8 N, #4: 3.6 N) and buckling in place of snap-through. This undesirable phenomenon has already been observed in Section 3.3.3 for two-ply specimens characterised by low aspect ratio of the total unsymmetric region. Figure 4.3 provides further clarification, showing the strain energy profile of the composite #1 clamped in plane as function of the transverse displacement at the centre of the laminate. The middle point of the initially stiff configuration is subjected to an imposed out-of-plane displacement in the snap-through and buckling directions, followed by an unloading step aimed at achieving equilibrium.
Figure 4.2: Unconstrained cool-down shapes of the four final bi-stable laminate designs considered for aerofoil integration (displacements magnified three times for clarity). Clamping surface inclination definition for a front view of both stable shapes of configuration #1.

As Figure 4.3 illustrates, a downward perturbation (snap-through direction) causes the structure to settle in the flexible stable state characterised by a lower strain energy level than the initial stiff equilibrium shape. By contrast, an upward transverse displacement (buckling direction) forces the laminate into an unstable buckling path. On removal of this perturbation, the structure returns to the initial condition of the stiff stable state, reaching the other
minimum characterised by a higher strain energy level. It can be seen that
no difference in the strain energy gradient exists for a very small disturbance
from the stiff stable state, or the initial condition of the test (corresponding
to the zero displacement in Figure 4.3). Hence neither path is preferred
and imperfections guide the response of the structure. These are associ-
ated with the combination of the unsymmetric character of the intermediate
lamination region of layouts #1 and #2 (R_i in Figure 4.1), the relatively
short central unsymmetric portion (R_{unsym,central} in Figure 4.1), along with
the specific geometry which ensues. The shortening of the specimen during
the compressive test intensifies the curvature development in the direction
of the current stiff shape. In addition, the relatively low length of the central
unsymmetric region, which controls the curvature under the loading consid-
ered, prevents the curvature reversal necessary for the switch to the opposite
flexible state. These geometric considerations thus render buckling rather
than snap-through the favoured path.

Table 4.3: Summary of the main characteristics of the four bi-stable laminate
configurations considered, obtained by means of FEA.

<table>
<thead>
<tr>
<th>Property</th>
<th>θ_{flex}</th>
<th>θ_{stiff}</th>
<th>F_{peak}</th>
<th>u_{peak}</th>
<th>F_{flex}</th>
<th>F_{peak}/F_{flex}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>°</td>
<td>°</td>
<td>N</td>
<td>mm</td>
<td>N</td>
<td>—</td>
</tr>
<tr>
<td>#1</td>
<td>6.3</td>
<td>-2.4</td>
<td>3.5</td>
<td>0.25</td>
<td>0.7</td>
<td>5.0</td>
</tr>
<tr>
<td>#2</td>
<td>5.3</td>
<td>-3.3</td>
<td>2.8</td>
<td>0.38</td>
<td>0.6</td>
<td>4.7</td>
</tr>
<tr>
<td>#3</td>
<td>-6.7</td>
<td>0.5</td>
<td>4.9</td>
<td>0.002</td>
<td>1.1</td>
<td>4.5</td>
</tr>
<tr>
<td>#4</td>
<td>-0.5</td>
<td>1.7</td>
<td>3.6</td>
<td>0.25</td>
<td>1.7</td>
<td>2.1</td>
</tr>
</tbody>
</table>

θ_{flex}, θ_{stiff}: inclination angles formed by the symmetrically
laminated clamping regions of the flexible and stiff equilibrium state, respectively, with the horizontal (Figure 4.2(i))
F_{peak}: reaction force at response peak preceding snap-
through or buckling (stiff state)
u_{peak}: compressive displacement at response peak F_{peak} (stiff
state)
F_{flex}: reference value of the asymptotic reaction force of the
flexible stable state in 2 mm compression

An insufficient R_{unsym,central} is a direct consequence of the size minimisation
goal given a prescribed element width of w = 50 mm. On the other hand,
the very small laminate thickness (0.12 mm in the four-ply case) suggests
the choice of shorter components to postpone the snap-through threshold. In
the case of the composite #4, the unidirectional mid-ply continuing through the laminate precludes the snap-through in compression.

![Figure 4.3: Strain energy profiles for snap-through and buckling of specimen #1 clamped in plane in the initially stiff state as function of the displacement at the centre of the laminate. Imposed transverse displacement in the buckling (upward) and snap-through (downward) directions followed by an unloading step aimed at achieving equilibrium. Two minima corresponding to the stiff and flexible stable configurations, with the former characterised by a higher strain energy level.](image)

All the four layouts are nevertheless considered for integration into the NACA 0012 due to the possibility of local actuation to induce the change of the equilibrium configurations, making the snap-through in compression a requirement only critical if a fully passive design were pursued. Moreover, it should be remembered that the idealised finite-element models do not reflect the actual imperfections, arising for instance from the manufacturing process. These, combined with the boundary conditions, would affect the real response of the laminates. As a potential remedy, the bi-stable elements could be embedded according to the desired orientation, guiding the structural
displacement in the snap-through rather than buckling direction. Furthermore, although the flat state of laminate #3 features the desired horizontal edge orientation, its curved form exhibits the highest clamping surface inclination of all the four designs (Figure 4.2(e)). Reducing the magnitude of the edge angle of the stiff configuration of specimen #1 leads to a higher value corresponding to its flexible shape, with the opposite applicable to composite #2. This inverse proportionality has already been observed for the basic two-ply layout in Section 3.3.3. Finally, the five-ply element #4 possesses the smallest stiffness difference between the two equilibrium configurations, permitting smooth transition between both states.

4.2 Variable Stiffness Aerofoil Based on Embedded Bi-Stable Components

4.2.1 Concept and Numerical Implementation

The present morphing aerofoil concept derives from a known internal topology \[31, 437, 438\] with a NACA 0012 shape of 500 mm chord length. Selecting a symmetric profile is envisaged to facilitate the preliminary assessment of the potential of the integrated bi-stable elements for shape adaptation through global stiffness variability.

The wing section features three compliant units in the chordwise direction (Figure 4.4), each able to accommodate diagonally positioned bi-stable components. The current equilibrium shapes of the composites govern the overall stiffness response of the structure and its final shape under load. The aerofoil is described as stiff, or exhibiting the stiff mode, if all the integrated bi-stable elements adopt their flat equilibrium configuration characterised by high stiffness. Similarly, the flexible mode refers to the wing profile with the embedded bi-stable laminates all in the curved, low-stiffness shape.

As Figure 4.4 illustrates, the selectively compliant rib features two bi-stable components with layout #3 introduced in Section 4.1, placed aft of the D-spar occupying 27.7% of the chord. This configuration results from preliminary studies into different bi-stable laminate arrangements inside the NACA 0012 profile, whose main insights are summarised in Section 4.2.2.

The modelling and numerical simulation are performed by means of dedicated Python scripts employing the Abaqus®/Standard FEA software. The structure is discretised with four-node quadrilateral shell elements with re-
duced integration (S4R), and subjected to static analysis steps including non-linear effects (Nlgeom). A regular global mesh with a maximum element side length of 5 mm is employed. Where necessary for convergence, numerical stabilisation between $10^{-7}$ and $10^{-6}$ is added and the corresponding simulations investigated in terms of both the associated viscous damping energy (ALLSD) compared to the strain energy [431], and the known physical behaviour of the bi-stable components.

**Figure 4.4:** Numerical set-up of the selectively compliant rib with boundary conditions for global stiffness response assessment. Three chordwise compliant units for bi-stable element integration.

Since the laminates considered herein rely on thermally-induced bi-stability, their fabrication process is simulated separately to obtain accurate cool-down shapes and corresponding residual stress fields. These are subsequently imported into the overall model of the wing section, with kinematic coupling of all degrees of freedom utilised to attach the bi-stable element edges to the relevant compliant unit flanges (Figure 4.4). The stress field import is always followed by a relaxation step with no external loads, aimed at achieving equilibrium of the complete aerofoil structure, as recommended in [431]. The leading edge D-spar is assumed infinitely rigid [31], and modelled as clamped.

The main finite element model comprises the stiff aerofoil mode, with the bi-stable elements in their flat equilibrium shape imported from the aforementioned separate simulation of the curing process. As already stated, a relaxation analysis step with no external loads follows, yielding a final configuration in equilibrium, ready to be subjected to further analysis under
specific loading. The flexible mode of the compliant wing section is obtained from the stiff configuration by applying two additional analysis steps. First, snap-through displacements of 2 mm are imposed on the centres of the stiff laminates in the direction normal to their plane and downward. Second, the boundary conditions are removed to obtain equilibrium with the components in the flexible stable state. It should be noted that the snap-through is accompanied by minor changes in the longitudinal dimensions of the laminates. Fixing all degrees of freedom of both clamping surfaces of the composites would normally prevent the switch between the equilibrium configurations when applying infinitely rigid clamping. In the current case, however, the kinematic coupling to the aerofoil combined with the finite stiffness of the complete structure allows the snap-through to take place. This is accompanied by an insignificant displacement of the trailing edge of the wing section, resulting from the laminates changing stable states from flat to curved.

The global stiffness response of each of the two modes of the morphing rib is quantified by means of a simple mechanical test. To that end, a vertical displacement \( u_3 \) of 15 mm is imposed aft of the rearmost web, approximately 55 mm from the trailing edge. The ensuing reaction force–displacement curves, \( F_{R,3}(u_3) \), provide a basis for estimating the stiffness ratio between the stiff and flexible configuration, permitting the quantification of the global stiffness variability of the wing-like structure.

It can be observed that the simple topology of the investigated concept possesses a discrete, rather than a continuous, stiffness variability character. Indeed, the two equilibrium configurations of a single bi-stable element exhibit two distinct property sets. Imparting a far larger complexity to the internal substructure, combined with integrating multiple bi-stable laminates equipped with localised actuators, would allow the stable states and thus the corresponding stiffness properties to be selectively modified through triggering the snap-through of the specific elements. Hence a larger global stiffness variability would result by virtue of a greater number of the combinations of the stable states, selectable in a controlled manner. What is more, the attainable range of stiffness characteristics could be further enhanced by tailoring the stiffness properties of the bi-stable components on an individual basis through a carefully designed layout according to the methodology introduced in Chapter 3. To that end, a sophisticated optimisation process would become necessary, concurrently considering the topology of the morphing aerofoil,
the number of the integrated bi-stable elements and their specifically tailored stiffness response. This, however, falls beyond the scope of this preliminary investigation, which seeks to demonstrate the feasibility and potential of the proposed morphing concept. A first step towards increased complexity will be presented in Chapter 6, dealing with a complete design methodology of shape-adaptive aerofoils based on embedded bi-stable laminates.

4.2.2 Preliminary Observations

The morphing configuration introduced in the preceding section arises from preliminary parameter studies1 into the implications of different bi-stable element combinations (designs #1–#4 detailed in Section 4.1) and positions within the generic three-unit substructure (Figure 4.4) for the resulting response to the prescribed load case. Investigating the arrangement of the laminates integrated into the rib seeks to maximise the global stiffness variability quantified as the difference in static response to vertical loading between the two aerofoil modes, as well as exploring the ensuing deformation patterns of the morphing system, and the laminate behaviour when constrained. Due to a limited number of configurations studied, no claim for completeness or exhaustiveness is made.

As expected, long compliant units implying large regions of unsupported skin lead to undesirable skin deformations under the given load case. Further, the reduced snap-through thresholds of the elements (Table 4.3) combined with relatively low stiffness of the complete morphing rib suggest the integration of only one bi-stable component per compliant unit. The laminate sensitivity is a direct consequence of the dimension minimisation, with 0.12 mm thickness of the four-ply configuration.

The observation from Section 3.3.3 is confirmed that the main impact on embeddability within a larger compliant system can be attributed to the clamping surface inclination of the flat stable state, whereas the inherent compliance of the curved form permits a larger angle tolerance. Hence the edge inclination of the flat shape should ideally approach zero. Accordingly, specimen #3 (Section 4.1) exhibits the most satisfactory performance. Although characterised by a reduced overall angle difference between the two

1 The parameter studies were performed within the Master's thesis of Rist [439].
states, layouts #1, #2 and #4 are found unable to perform their function optimally when embedded into the NACA 0012-based rib according to the stringent in-plane clamping objective for two main reasons. First, partial bi-stability loss on short edge rotation to the horizontal and subsequent coupling with the wing section is numerically predicted. This manifests itself in the element changing to the flexible, more stable, state during the aforementioned relaxation step following the stress field import. In other words, no equilibrium of the complete aerofoil with an element in the flat configuration is achieved. A conclusion can therefore be drawn that the rigorous objective of in-plane clamping greatly reduces the available design space. The findings are confirmed empirically (see Section 4.3) based on specimens #1 and #2, found unable to maintain the stiff state when constrained in a position enforced by the implemented clamping arrangement which precludes any integration freedom regarding the short edge inclination or distance (Figure 4.5(b)). This could be remedied with alternative practical solutions offering more embedding flexibility. Second, buckling instead of changing stable states is predicted for layouts #1, #2 and #4 under the given load case of vertical displacement imposed forward of the trailing edge. As already mentioned in Section 4.1.2, modified laminate orientation would provide a potential countermeasure, as would guiding the component response towards the desired curvature of the flexible equilibrium configuration by means of tailored boundary conditions. Nonetheless, the solutions outlined above are not investigated any further herein due to the satisfactory performance obtained with element #3, and the early stage of the current studies. Moreover, proceeding from the given elementary load case to a more realistic distributed loading would entail a potential redesign.

### 4.3 Experimental Validation

First experimental validation of the numerical set-up, based on a different demonstrator, is provided in [41]. The current prototype\(^1\) features a GFRP profile (Figure 4.5(a)) manufactured through hand lamination of 100 g/m\(^2\) glass fibre fabric with L-235 epoxy resin and 235 hardener, yielding a cured thickness of 0.6 mm. Further, special clamping devices (Figure 4.5(b)) are

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\(^1\) The manufacturing was performed within the Master’s thesis of Rist [439].
implemented, allowing for installing and dismounting the bi-stable elements rather than bonding them permanently.

Figure 4.5: (a) GFRP profile and (b) bi-stable laminate clamping devices of the demonstrator [439].

Consistent with the numerical test for global stiffness quantification introduced in Section 4.2.1, the prototype is tested under displacement control by means of a Zwick/Roell Z005 machine with a rate of 20 mm/min. The experimental set-up is presented in Figure 4.6, featuring the stiff aerofoil configuration with both integrated bi-stable laminates in the flat equilibrium shape.

Figure 4.6: Test rig for stiffness response assessment of the morphing rib.

Firstly, the morphing rib shown in Figure 4.6 is subjected to an upward loading \( u_3 \), positive \( z \)-direction in Figure 4.6) while in the stiff and flexi-
4.3 Experimental Validation

ble modes (Figure 4.7(a) and (b), respectively). The resulting deformed configurations at maximum imposed displacement of 15 mm, illustrated in Figure 4.7(c), coincide since the loading causes the bi-stable elements of the initially stiff aerofoil mode to change their equilibrium forms from flat to curved. The corresponding reaction force–displacement curves, $F_{R,3}(u_3)$, are presented in Figure 4.8, demonstrating good agreement between experiment and simulation, especially in terms of the stiffness response as the central characteristic. Beginning with both laminates set to the straight configuration (stiff aerofoil mode), a nearly linear response is initially observed, until the snap-through to the flexible shape occurs on reaching a critical load. Nevertheless, prediction discrepancy of this phenomenon becomes clear from Figure 4.8. Whereas during the experiment both composites change configurations simultaneously, simulation shows a slight delay of the aft element in adopting the curved form.

A number of reasons contribute to this inconsistency. Firstly, the morphing system features substantial sensitivity and low imperfection tolerance. The former has already been discussed as a direct consequence of the size minimisation of the bi-stable components, leading to very low snap-through thresholds. Further, extreme accuracy is required to fabricate the laminates and subsequently mount them precisely. Any resulting misalignments thus affect the final response, especially regarding the flat configuration. As elaborated in Section 3.3.1.2, the initial condition and imperfection sensitivity of the strongly non-linear snap-through phenomenon characterising the stiff equilibrium shape in compression causes difficulties in providing highly accurate predictions. Further, the static analysis employed herein does not reflect the inherent dynamics of the stable state switch, which is accompanied by minor vibrations of the morphing rib.
during testing. Hence the snap-through might be triggered in both elements simultaneously by virtue of their aforementioned sensitivity.

![Figure 4.8: Stiffness response of the stiff and flexible mode of the selectively compliant wing section obtained under imposed vertical displacement, $u_3$, from experiment and FEA.](image)

Considering the stiffness behaviour of the flexible aerofoil mode (both laminates in the curved state, Figure 4.8), excellent agreement is achieved, showing convergence with the post snap-through regime of the initially stiff configuration. Based on the slopes of the reaction force–displacement curves of the stiff and steady-state flexible modes of the selectively compliant system, a ratio of 2.47 emerges. Given the profile skin of 0.6 mm being five times as thick as the integrated bi-stable elements, this value proves the potential of the current approach for realising global stiffness variability, even inside a much stiffer structure.

The response of the stiff configuration of the morphing rib under load directed in the positive and negative $z$-direction (according to coordinate system definition from Figure 4.6) is shown in Figure 4.9, again validating the FEA predictions obtained. Loading the stiff mode in the positive $z$-direction leads to the snap-through of the integrated bi-stable elements, which are subjected to compression. By contrast, reversed loading, pointing along the negative $z$-axis, does not induce this phenomenon. Indeed, the flat equilibrium states are maintained by virtue of the acting tension.
Figure 4.9: Response of the stiff mode of the selectively compliant wing section to loading in the positive and negative $z$-direction (indicated with $+z$ and $-z$ in the legend, respectively; reference frame defined in Figure 4.6).

To fully exploit the chordwise space available, adding a third, longer bi-stable element (specimen #1 or #2) behind the two #3 configurations has also been investigated. Nevertheless, the performance has not been found satisfactory in terms of the structural response or the achievable stiffness variability, for reasons outlined in Section 4.2.2. Indeed, difficulties has been encountered in embedding the #2 layout into the rigid clamping arrangement while preserving the flat equilibrium shape. Although bonding the element to the mounting flanges under partial preservation of the original short edge inclination of the stiff configuration has restored the bi-stability when integrated, this solution has not been pursued any further due to the reduction in the laminate impact on the global behaviour as well as early buckling under load. The corresponding morphing rib with curved #2 element found in the rearmost compliant unit is shown in Figure 4.10 in the deformed state, following the application of a vertical displacement of $u_3 = 15$ mm.

Figure 4.11 provides insights into the impact of the number of embedded bi-stable elements on the response of the flexible aerofoil mode. To that end, the global stiffness quantification experiment is performed with the empty rib with the composites removed, and with the two- and three-laminate configurations. As expected in the case of the curved equilibrium shapes,
very good agreement between simulation and experiment is obtained.

![Deformed aerofoil featuring three integrated laminates in their curved states.](image)

**Figure 4.10:** Deformed aerofoil featuring three integrated laminates in their curved states.

![Graph showing static response of the selectively compliant aerofoil.](image)

**Figure 4.11:** Static response of the selectively compliant aerofoil in the three- and two-laminate configurations, and with no bi-stable elements.

### 4.4 Conclusions

A first proof of concept of the proposed morphing strategy based on distributed compliance augmented through monolithically integrated bi-stable components is introduced. Following a numerical design process conducted by means of dedicated Abaqus® Python scripts, a prototype is manufactured to physically demonstrate the herein pursued approach to realising shape adaptation.
Characterised by two bi-stable laminates of minimised dimensions integrated into the internal topology of a 500 mm chord NACA 0012 profile, the morphing system displays selective deformation modes and global stiffness variability owing to the purely elastic mechanism resulting from structural bi-stability. This relies on the tailored difference in properties between the two equilibrium configurations of the integrated bi-stable components. Specifically, an overall stiffness ratio of 2.47 between the stiff and flexible modes of the morphing rib is achieved in a simple mechanical test: a promising result in view of the thickness of the bi-stable elements being very small compared to the skin thickness of the wing section. Experimental testing of the manufactured prototype shows good agreement with numerical predictions, thereby validating the simulation methodology underlying the design process of the compliant system.

The promising results of this preliminary study confirm the feasibility and potential of the devised morphing strategy aimed at reconciling the conflicting requirements underlying the realisation of shape-adaptive structures through the provision of selective deformation modes.
CHAPTER 5

Variable Stiffness Aerofoils with Integrated Bi-Stable Laminates: Aeroelastic Study

This chapter is based on the conference contribution:

and on the journal publication:

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In the previous chapter, the feasibility of the proposed morphing strategy has been proved numerically and experimentally by means of a NACA 0012-based configuration with two embedded bi-stable laminates. However, the simple mechanical test employed in Section 4.3 for performance evaluation in terms of global stiffness variability does not reflect the real operating conditions, which would subject the shape-adaptive rib to distributed pressure loads. This, combined with the selectively compliant internal substructure of the wing section, creates the necessity for investigating the aerodynamic adequacy of the ensuing deformation modes. To that end, the validated finite element models are extended to include a weak static aeroelastic coupling with the aerofoil analysis code XFOIL. Studying the response of the prototype morphing system under simplified aerodynamic conditions representing a range of operating points provides insights into the aerodynamic characteristics of the resulting passively morphed shapes. The unfavourable deformed configuration observed in the post-snap-through regime stresses the utmost importance of purposeful design of the desired selective deformation modes in a concurrent aero-structural manner. The aeroelastic approach is therefore employed to develop and evaluate an improved shape-adaptive wing section. This constitutes a first enhancement towards a multidisciplinary methodology indispensable for fully exploiting the capabilities of the proposed morphing strategy.

5.1 Concurrent Aero-Structural Simulation Method

The specificity of selectively compliant morphing systems entails radical, geometrically non-linear changes of shape under aerodynamic loading. The deformed outer profile thus exerts a non-negligible effect on the surrounding fluid, creating the need for simultaneous consideration of the structural and aerodynamic responses in a two-way interaction. At the same time, the distributed loading governs the particular equilibrium configurations passively adopted by the embedded bi-stable laminates, thereby activating specific global deformation modes of the shape-adaptive wing section. In other words, the deformed equilibrium shapes of the complete system, driven by the mutual interplay between the compliant structure and the aerodynamic pressure distribution, can only be determined and evaluated in terms of their aerodynamic adequacy via a concurrent aero-structural approach. This presents a
prerequisite for purposeful design of smooth morphed configurations able to sustain prescribed load levels.

Accordingly, the validated FEA routines detailed in Section 4.2.1 are extended to incorporate a weak static aeroelastic coupling with the open-source aerofoil analysis software XFOIL, based on an inviscid panel method combined with an integral boundary layer [440, 441]. A schematic of the aero-structural approach is shown in Figure 5.1.

**Figure 5.1:** Schematic of the weak static aeroelastic coupling between Abaqus® and XFOIL.

Given an operating point represented by a freestream velocity\(^1\), \(V_\infty\), and angle of attack, \(\alpha\), the corresponding pressure distribution determined by XFOIL is linearly interpolated onto the mesh of the finite-element model in Abaqus®/Standard. The structural response is then determined by means of a static non-linear analysis step. Provided the stiff aerofoil mode experiences snap-through due to the external load level, numerical stabilisation between \(10^{-7}\) and \(10^{-6}\) is introduced to achieve convergence. The deformed

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\(^1\) The freestream velocity is converted to Reynolds and Mach numbers for XFOIL input.
coordinates of the wing profile obtained on completion of the FEA provide input to XFOIL, which calculates a new pressure distribution to be applied to the deformed configuration. The analysis continuation is conducted by means of the Abaqus® *RESTART option. The process continues until convergence, fulfilled once the changes in the vertical displacement of the trailing edge, \( u_{3,TE} \), fall below 0.1 mm. This value is based on experience gained in the course of the simulations, reflecting convergence of the corresponding aerodynamic coefficients as well. Lastly, the lift and drag predictions \((c_l\) and \(c_d)\) of the final deformation mode of the selectively compliant system are obtained from XFOIL.

As in the preceding studies (Section 4.2.1), the D-spar remains clamped during the simulation. Implying infinite torsional rigidity, this precludes static aeroelastic instabilities related to global flexibility in twist such as divergence [442]. Hence only local phenomena require consideration.

It is important to be aware of the limitations underlying the weak static aeroelastic approach employed herein. First, the determination of pressure distributions and aerodynamic coefficients is performed by means of XFOIL, a simple panel-based aerofoil analysis tool with an integral boundary layer formulation. In particular, the drag predictions provided by the software tend to be optimistic. Second, the weak character of the coupling implies that a particular aerodynamic load is ramped until convergence of the non-linear static analysis. Only then does the recalculation of the pressure distribution occur for the deformed configuration, which implies a low frequency of data exchange between XFOIL and Abaqus®. A certain response overshoot can thus be expected since the actual aerodynamic loads vary continuously as the deformation of the structure progresses. Further, the steady-state formulation neglects dynamic effects, which inherently accompany the highly non-linear and transient snap-through phenomenon. On the other hand, predicting details of the behaviour associated with the stable-state switch would require a high-fidelity simulation involving a strong coupling, or at least high-frequency exchange between the fluid and structural solvers. This, however, constitutes an undue effort given the purpose of the present preliminary study. More precisely, the focus lies herein on the pressure-induced activation of selective deformation modes of the morphing wing section by virtue of the distinct stiffness characteristics associated with the individual stable states of the embedded bi-stable components. A further
aim is to identify and understand the ensuing behaviour trends of the novel shape-adaptive concept. Therefore, the details of the snap-through are not of primary interest. Accurate predictions of the corresponding load threshold are not required at this early stage either. Last but not least, potential local oscillations or instabilities are beyond the capabilities of the static analysis method.

Despite the aforementioned simplifications, the current aero-structural approach provides a computationally efficient means of gaining understanding of the response of the selectively compliant system to simplified operating conditions represented by a pressure distribution. The determination of aerodynamic coefficients of the morphed configurations lays the foundations for purposeful design of specific deformation modes to fulfil particular requirements. In addition, the findings can facilitate further studies by providing critical velocity and angle of attack range to be verified by more advanced fluid-structure interaction simulations. Finally, the moderate computational effort makes the method appropriate for incorporation into a multidisciplinary optimisation process of such variable stiffness aerofoils based on integrated bi-stable elements. In this context, the static analysis presents a good compromise between accuracy, numerical effort and robustness as regards the solution controls.

5.2 Static Aeroelastic Evaluation: Prototype
The structural response of the stiff and flexible modes of the prototype design introduced in Section 4.2.1 is investigated under distributed loading corresponding to velocity sweeping from 29 m/s to 41 m/s for angles of attack ranging from $-6^\circ$ to $10^\circ$. Figure 5.2 presents the ensuing three distinct final deformed shapes of the stiff configuration as the initial condition. These are reflected in Figure 5.3, which quantifies the behaviour of the morphing system at selected aerodynamic points in terms of the vertical displacement of the trailing edge, $u_{3,TE}$. Considering growing velocities and angles of attack, it can be seen that the stiff aerofoil mode (Figure 5.2(a)) can passively sustain a range of aerodynamic conditions, with no snap-through of the embedded bi-stable laminates predicted, for example, for 35 m/s and four degrees of incidence.

Given a sufficiently high velocity and angle of attack, the forward laminate
switches to the curved shape (Figure 5.2(b)). As Figure 5.3 illustrates, the system stabilises in this mode under moderate pressure distributions, exhibiting a final vertical displacement of the TE of 5 mm to 8 mm. Further increasing the aerodynamic load causes the rear bi-stable element to buckle into a configuration opposite to the flexible equilibrium form (Figure 5.2(c)), inducing a step-wise growth of the trailing edge deflection clear from Figure 5.3. From this point onwards, only the flexible aerofoil mode exists. A growing curvature of the aft part of the wing section can be observed (Figure 5.2(c)) as the trailing edge deflects between 19 mm and 38 mm in the flexible morphed configuration (Figure 5.3). The latter value is found to mark the upper load-bearing limit, corresponding to 41 m/s and an angle of attack of six degrees. Specifically, the deformed bi-stable components start to come into contact with the profile skin within the upper incidence and velocity range. Hence no data are presented in Figure 5.3 for these extreme operating points. Analogous to the downward loading test described in Section 4.3 and shown in Figure 4.9, negative angles of attack subject the integrated laminates to tension, thereby stabilising the stiff mode of the selectively compliant rib (Figure 5.2(a)).

A comparison between the static aeroelastic responses of the two aerofoil modes, flexible and stiff, for growing velocities and constant angles of attack of five and six degrees is presented in Figure 5.4. As long as the conditions allow the stiff configuration to be preserved or only trigger the snap-through of the forward bi-stable element, the flexible mode of the wing section features increased TE deflections at the same operating points. The...
corresponding deformed shape is shown in Figure 5.5. This expected result, clear from Figure 5.4, demonstrates the global stiffness variability attained with the current concept. In the presence of a sufficiently high load level, however, the rear laminate of the initially stiff configuration is observed to buckle rather than switching to the curved equilibrium form (compare Figures 5.2(c) and 5.5). Nevertheless, this deviation is of no influence on the final morphed shape of the selectively compliant rib, coincident for the two modes. Past the snap-through threshold, the stiff configuration can no longer be sustained and hence only the flexible mode exists, which is reflected in the congruence of the result points.

![Figure 5.3](image_url)

**Figure 5.3:** Vertical displacement of the trailing edge for aeroelastic simulations commencing with the stiff aerofoil mode. Final state change of one or both embedded bi-stable components depending on the aerodynamic loading. The corresponding three deformation modes are provided in Figure 5.2.

Due to the passive nature of the shape adaptation, increasing aerodynamic loading implies growing structural deformations, clear from Figures 5.3 and 5.4. For basic quantification of the global structural adaptability arising from the variable stiffness, the corresponding vertical displacements of the trailing edge of the flexible and stiff modes can be compared at different operating points, as Figure 5.4 illustrates. The obtained ratios range between...
18 (29 m/s at 7°) and 101 (41 m/s at 2°). For the data presented in Figure 5.4, values between 20 (29 m/s at 6°) and 45 (35 m/s at 5°) arise. As expected, the highest ratios directly precede the snap-through threshold. The stable state switch of the forward laminate of the initially stiff configuration reduces the ratio to 2–3.

![Figure 5.4: Comparison of vertical displacements of the trailing edge obtained from simulations starting from the stiff and flexible aerofoil modes as initial conditions for angles of incidence of five and six degrees. Flexible: Flexible aerofoil mode with both embedded bi-stable elements in the curved equilibrium shape as initial condition. Stiff: Stiff aerofoil mode as initial condition, final stable state switch of one or two bi-stable components depending on aerodynamic loading.](image)

The activation of the flexible mode of the selectively compliant system through bi-stable element snap-through under distributed load alters the global stiffness of the structure, enabling the flow to modify the aerofoil shape with more ease. This becomes clear from the ensuing aerodynamic coefficients of the morphing wing section, shown in Figure 5.6.

As already mentioned, negative angles of attack and the resulting tension acting on the bi-stable laminates force the structure into the stiff mode. As soon as a positive incidence arises, however, the flexible configuration features larger trailing edge deflections than its stiff counterpart. This is accompanied by a development of an aerodynamically disadvantageous mor-
phed profile (Figure 5.5), reflected in a higher drag of the flexible aerofoil mode (Figure 5.6(a)). Nevertheless, this only holds until the aerodynamic loads trigger the state change of the bi-stable components of the stiff configuration from straight to curved, rendering the wing section flexible. In the presented case of a freestream velocity of 35 m/s, the snap-through of the first composite is predicted at an angle of attack of six degrees, followed by the buckling of the rear laminate at seven degrees. On exceeding this threshold, the properties of the stiff aerofoil mode begin to clearly deviate from those displayed by a rigid NACA 0012. Indeed, a two-step increase in drag is revealed (Figure 5.6(a)), corresponding to the two consecutive morphed shapes (snap-through of the forward and buckling of the aft bi-stable element, respectively). The former point, found at six degrees of incidence, exhibits lower drag compared to the flexible configuration in the same conditions. The reason is the ability of the rear laminate to still remain in the straight equilibrium state (Figure 5.2(b)), thereby limiting the development of the unfavourable deformed profile of the wing section. Increasing the angle of attack to seven degrees renders the corresponding pressure distribution too high for the stiff configuration to be passively sustained. Hence only the flexible aerofoil mode exists, which is reflected in the congruence of the aerodynamic coefficients obtained from simulations commencing with each of the two basic states of the compliant system. As Figure 5.6(b) indicates, the flexible configuration generates insignificantly higher lift than the stiff counterpart in the pre-snap-through range. This can be explained by a flow-induced increase in camber towards the rear of the wing profile within the moderate trailing edge deflection regime.

The presented results of the static aeroelastic analyses of the selectively compliant aerofoil clearly show that in spite of the promising stiffness variability achieved, the aerodynamic characteristics of the concept need further improvement. More precisely, a substantial drag penalty arises from the unfavourable morphed shape, which primarily results from the relatively stiff, inextensible skin possessing the

![Fig. 5.5: Overlaid initial and deformed flexible aerofoil mode under pressure distribution corresponding to 35 m/s at eight degrees of incidence.](image-url)
same material properties as the internal substructure. Combined with the vertical webs separating the compliant units, this leads to an upward rigid-body motion rather than a smooth rotation. Applying an integrated aero-structural approach to concept development thus becomes essential if global stiffness variability of the selectively compliant system is to be combined with desired aerodynamic performance of its specific deformation modes. This also requires simultaneously considering the distributed topology including bi-stable laminate positioning, and skin properties. A first extension towards such a multidisciplinary methodology provides an improved configuration, introduced in the following.

![Graphs showing drag and lift coefficients](image)

**Figure 5.6:** Aerodynamic coefficients obtained at 35 m/s from static aeroelastic simulations commencing with the stiff and flexible aerofoil modes, compared with a rigid NACA 0012.

### 5.3 Aerodynamically Improved Configuration

The preceding section has emphasised the vital role of a concurrent, aero-structural approach in developing aerodynamically favourable, selective deformation modes of the adaptive aerofoil, arising from the stiffness variability of the embedded bi-stable components. To substantiate this, the static aeroelastic simulation method is employed to perform further parametric studies in search of a configuration featuring enhanced passively morphed shapes as regards the aerodynamic coefficients compared to the initial concept. The names *improved* and *original* will thus be used to distinguish

![Graphs showing drag and lift coefficients](image)
between the new and prototype designs, respectively.

The 500 mm chord NACA 0012 is retained as the larger morphing system, including the two bi-stable elements selected for the original configuration (#3 in Figure 4.1). Considering a simple topology, the chordwise positions and orientation angles of the internal edges and the embedded laminates are varied, as schematically indicated in Figure 5.7. Since the inextensibility of the skin is identified as a primary reason for the aerodynamically unfavourable final shape of the original configuration, corrugated composite segments are introduced into the skin as further design parameters and means of tailoring the global response. These are represented through the homogenisation model of Kress and Winkler [443, 444]. A circular profile with a 2.5 mm radius is assumed, with a total thickness of 0.15 mm arising from three layers of glass fibre/epoxy fabric with properties reported in Table 5.1.

**Figure 5.7:** Design variables considered in the parametric studies in search of aerodynamically enhanced selective deformation modes of the morphing aerofoil.

**Table 5.1:** Material properties of a plain-weave glass fibre/epoxy composite [444] used in the corrugated laminate homogenisation model.

<table>
<thead>
<tr>
<th>Property</th>
<th>$E_{11} = E_{22}$</th>
<th>$E_{33}$</th>
<th>$G_{12} = G_{13}$</th>
<th>$G_{23}$</th>
<th>$\nu_{12}$</th>
<th>$\nu_{13} = \nu_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>21 191</td>
<td>13 000</td>
<td>4 400</td>
<td>5 040</td>
<td>0.10</td>
<td>0.29</td>
</tr>
</tbody>
</table>
5.3 Aerodynamically Improved Configuration

The parametric studies are based on a load case of a free-stream velocity of 35 m/s and an angle of attack of eight degrees. At this operating point, only the flexible mode of the original aerofoil exists, in which the adverse aerodynamic characteristics become especially pronounced. Specifically, a 21% increase in drag and a 3% reduction in lift compared to a rigid NACA 0012 under the same distributed loading are predicted. The investigations thus focus on the flexible configuration, seeking to demonstrate the ability to design desired deformed shapes by virtue of simultaneously considering the aerodynamic and structural aspects.

An improved design to emerge is shown in Figure 5.8. The bi-stable elements are found in the second and third chordwise units, whose lower skin includes corrugated segments. Unlike the prototype, the substructural members are inclined from the vertical. The rigid D-spar occupies 30% of the chord length.

![Figure 5.8](image)

**Figure 5.8:** Improved configuration of the selectively compliant aerofoil based on two integrated bi-stable laminates with schematic of the circular corrugation assumed for the homogenised model (not to scale).

The configuration is selected based on a predicted 34% decrease in lift and a 10% reduction in drag of the flexible mode compared to a rigid NACA 0012 under the baseline load case \((V_\infty = 35 \text{ m/s}, \alpha = 8^\circ)\). This is considered sufficient to prove the potential of tailoring the aerodynamic characteristics of the selectively compliant system through distributed topology combined with deformable skin, while maintaining global stiffness variability. The purely passive nature of the present study implies upward deformations of the wing section associated with positive angles of attack. The ensuing significant reduction in lift without drag penalty presents a potential solution for load alleviation [41]. The applicability of the approach, however, is
much broader. As will be seen in Chapters 6 and 7, effecting desired lift variations can be achieved by introducing a global actuation means, combined with a less constrained topology parametrisation. Hence the simple method presented herein lays the foundations for a fully integrated optimisation process under prescribed aerodynamic performance and stiffness variability objectives.

5.4 Static Aeroelastic Evaluation: Improved Configuration

The new configuration is subjected to static aeroelastic analyses according to the procedure adopted for the original design, detailed in Section 5.2.

The final morphed shapes emerging from the simulations commencing from the stiff mode are shown in Figure 5.9. Figure 5.10 provides a quantification of the structural response to aerodynamic loading in terms of the vertical displacement of the trailing edge. Able to passively sustain moderate pressure loads, the stiff configuration (Figure 5.9(a)) exhibits non-zero trailing edge displacements due to the presence of the compliant corrugated skin portions, which differs from the observed response of the original design (compare the stiff mode data shown in Figures 5.3 and 5.10). As before, increasing the velocity and angle of attack initially triggers the snap-through of the forward bi-stable element (Figure 5.9(b)). This first transition, however, is not accompanied by a distinct step-wise increase in the trailing edge displacement because of the deformability of the skin. As opposed to the original
configuration, the rear laminate shows the desired behaviour of snap-through to the curved state rather than buckling (Figure 5.9(c)), with the trailing edge displaced by 21.8 mm to 32.8 mm within the velocity and angle of attack range considered. What is more, the improved design sustains slightly higher aerodynamic loading before both the embedded bi-stable laminates switch states from flat to curved, which marks the onset of the regime of the exclusive existence of the flexible mode.

![Figure 5.10](image_url)

**Figure 5.10:** Vertical displacement of the trailing edge for aeroelastic simulations commencing with the stiff mode of the new configuration. Final snap-through of one or both embedded bi-stable components depending on the aerodynamic loading. The corresponding three deformation modes are provided in Figure 5.9.

Further, a smoother curvature of the profile is achieved under distributed loading, implying an aerodynamically more favourable morphed shape of the compliant system. This is illustrated in Figure 5.11, showing the overlaid initial and deformed flexible configuration.

The data in Figure 5.12 demonstrate the global stiffness variability of the improved selectively compliant morphing system. Increased trailing edge motion arising from reduced global stiffness of the flexible mode becomes clear, compared to the initially stiff configuration with at least one element in its flat equilibrium shape. At an angle of attack of four degrees, the embedded
bi-stable components remain in their stiff state. The corresponding trailing edge displacement thus only results from the extensibility of the corrugated skin portions. Although not accompanied by a marked step-wise increase in the TE deflection, the switch of the forward bi-stable component to the curved state corresponds to an insignificantly altered displacement pattern (7°: 33 m/s to 37 m/s, 8°: 31 m/s to 37 m/s). As before, the coincidence of the result points, such as at eight degrees of incidence and 39 m/s, reflects the region in which the aerodynamic loading triggers the snap-through of both bi-stable laminates, inducing the flexible aerofoil mode as the only attainable configuration. Quantifying the passive structural adaptability in terms of TE displacement ratios between the flexible and stiff modes at specific operating points leads to values of 1.3 to 1.4. Compared with the prototype (see Section 5.2, Figure 5.4), this constitutes a considerable impact reduction of the bi-stable laminates on the overall response, and results from the contribution of the extensible skin portions to the deformations. Hence a need arises for defining an additional objective of maximising the effect of the stable state switch on the morphing capacity, as will be demonstrated in Section 6.

The aerodynamic coefficients presented in Figure 5.13 for a freestream velocity of 37 m/s prove a performance enhancement associated with passive morphing under distributed loading compared to the prototype design. Further, a considerable adaptation potential of the concept becomes clear. It can be observed that a difference in characteristics between the negative and positive angle of attack range exists, demonstrating the stiffness directionality of the morphing wing section based on an initially symmetric profile. At negative incidence, the bi-stable laminates of the flexible aerofoil mode are forced into the flat equilibrium configuration by the acting tension, which occurs in full below $-2^\circ$. The coincidence of the corresponding data points confirms this phenomenon. Therefore, the negative incidence domain exhibits limited response adaptation, driven by
the downward motion of the structure by virtue of the compliance of the corrugated skin portions. As decreasing negative angles of attack induce larger downward cambering, the aerodynamic coefficients tend towards smaller drag and higher lift compared to a rigid NACA 0012. Considering the stiff aerofoil mode at positive incidence, the forward bi-stable element is predicted to adopt its curved configuration (Figure 5.9(b)) at an angle of attack of six degrees, followed by the snap-through of the rear laminate at $\alpha = 10^\circ$ (Figure 5.9(c)). This is accompanied by decreasing drag and lift compared to a rigid NACA 0012. As already mentioned, the extensibility of the corrugated skin segments enables the rear part of the aerofoil to move upwards, leading to a modification of the aerodynamic coefficients towards reduced drag and lift. The full snap-through of both bi-stable elements to the curved configuration can again be clearly seen from the coincidence of the result points at an angle of attack of ten degrees.

Figure 5.12: Comparison of vertical displacements of the trailing edge obtained from simulations starting from the stiff and flexible modes of the new configuration as the initial conditions for angles of incidence of four, seven and eight degrees. **Flexible:** Flexible aerofoil mode with both embedded bi-stable elements in the curved/flexible equilibrium shape as initial condition. **Stiff:** Stiff aerofoil mode as initial condition, final snap-through of one or two bi-stable components depending on aerodynamic loading.
Compared to the prototype, the new configuration features a smoother upward deformation profile, yielding improved characteristics in terms of lift variation and drag. The current study thus demonstrates the feasibility of adapting the aerodynamic response of the selectively compliant concept through tailoring the substructure with integrated bi-stable components, while ensuring a deformable skin. Accordingly, purposeful aero-structural design plays a key role in controlling the characteristics of passive morphing under simplified aerodynamic conditions. Hence foundations are laid for concurrent optimisation featuring a more advanced topology, modified skin stiffness including corrugated segments, combined with bi-stable element positioning aimed at maximum impact of the switch between the equilibrium configurations, or the corresponding stiffness variability. This will be introduced in the following chapter.

**Figure 5.13**: Aerodynamic coefficients obtained at 37 m/s from static aeroelastic simulations commencing with the stiff and flexible modes of the new configuration, compared with a rigid NACA 0012.

### 5.5 Conclusions

A static aeroelastic approach is introduced as a prerequisite for predicting the selective deformation modes arising from the stiffness variability of the proposed morphing concept.

The response of the prototype to distributed aerodynamic loading is first studied. Global stiffness variability is demonstrated in terms of larger passive
shape adaptation of the flexible mode compared to the initially stiff configuration with at least one of the embedded bi-stable elements maintaining the flat equilibrium state. Nevertheless, a large drag penalty compared to a rigid NACA 0012 is found to accompany the snap-through-induced selective morphing, arising from the unfavourable deformed profile dictated by the inextensible skin. The concurrent aero-structural simulation method is thus employed to develop an enhanced configuration, featuring a modified topology, and corrugated skin segments. The improved aerodynamic characteristics of the passively morphed shapes of the new wing section, combined with the maintained objective of the global stiffness variability, prove the ability to control the aerodynamic response of the selectively compliant system. The results thus confirm the potential of the proposed design methodology simultaneously considering the static aeroelastic behaviour, internal topology, bi-stable element positioning, and the necessary skin extensibility. Hence foundations are laid for a purposeful design of selective deformation modes adopted by the variable stiffness aerofoil under specific aerodynamic conditions.
CHAPTER 6

Multidisciplinary Design and Optimisation Approach

This chapter is based on the journal publication:

The preceding chapter has stressed the key role of considering the mutual interplay between structural deformations and aerodynamic loads in correctly capturing the ensuing selective deformation modes of the compliant system. This constitutes a prerequisite for an intentional design of the deformed configurations adopted in operation in response to the distributed loading. It has also been seen, however, that introducing skin extensibility through corrugated segments for enhanced aerodynamic characteristics has reduced the impact of the stiffness variability provided by the bi-stable components on the global response. Further, only passive morphing has been studied. These two main aspects are therefore addressed in the present chapter.

A concurrent design and optimisation approach is presented, considering
the optimal positioning of the bi-stable components within the structure while assessing the actuation energy required for morphing under aerodynamic loading. Compared to a time-invariant system, activating specific deformation modes permits decreasing the amount of actuation energy, and hence the amount of actuation material to be carried. The integrated framework complements the existing aero-structural analysis method through a more complex, compliant topology incorporating the variable stiffness bi-stable laminates, and global actuation.

In the second part of the chapter, the corresponding design objectives are elaborated, accompanied by details of the optimisation set-up. The problem formulation aims at controlled adaptability through tailored deformation modes targeting different flight conditions.

6.1 Concurrent Aero-Structural Design and Optimisation Framework

Chapters 4 and 5 have provided a basic, numerical and experimental proof of concept for passive morphing of a NACA 0012 configuration with two bi-stable laminates integrated into a straightforward topology. Global stiffness variability ensuing from the distinct properties of the embedded bi-stable components has been demonstrated in a simple mechanical test and under distributed aerodynamic loading, in weakly coupled static aeroelastic analyses.

The prior purely passive concept is extended herein to controlled shape adaptation effected on the wing section level. The morphing extent achievable with specific actuation energy is considered, as well as examining the positioning of the bi-stable components within the variable internal layout for maximum impact. The purpose is to fully exploit the available characteristics of the bi-stable laminates employed in the variable stiffness function. Further, combining the benefits of distributed compliance and stiffness variability aims at high actuation efficiency. Specifically, switching between the stable states of the internal elements leads to on-demand stiffness adaptation, corresponding to the activation of particular deformation modes. Tailoring these to optimally respond to different flight conditions thus permits controlled morphing relying on the deliberate configuration selectivity. Compared to a time-invariant system, providing specific deformation modes allows for
enhanced actuation efficiency, with decreased amount of the necessary actuation material promising mass savings.

In order to simultaneously incorporate all the aforementioned dimensions from the very outset of the development process, a concurrent design and optimisation framework is implemented. Similar approaches have been proposed and successfully employed by Molinari et al. [106, 110] and Previtali et al. [113, 115].

As schematically shown in Figure 6.1, the integrated framework complements the existing aeroelastic analysis method introduced in Section 5.1 through a more complex, distributed topology in which the variable stiffness bi-stable laminates are embedded, corrugated skin portion allowing for cambering, and smart actuation inducing the geometry variation of the wing profile. The current configuration features a parametrised layout contained within the retained chord length of \( c = 500 \text{ mm} \), with a rigid D-spar occupying the forward 28\% of \( c \). The total breadth of the morphing system equals 50 mm, accommodating 40 mm-wide bi-stable components.

The members of the internal substructure are deliberately replaced with bi-stable elements to maximise the impact of the provided stiffness variability on the on-demand selectively compliant behaviour of the aerofoil. This is envisaged to provide the enhanced functionality of reducing the deformation resistance as required for minimum actuation effort. Specifically, the elastic work contribution associated with reversible straining of the internal substructure characteristic of distributed compliance can be decreased when needed via the stable state switch of the integrated bi-stable components. Accordingly, the total number of the embedded laminates is not prescribed. Each edge, defined by two parametrisation points marked as red dots in Figure 6.1, can be specified as either a standard, uniform member, or a bi-stable element. This design freedom relies on a versatile and computationally efficient representation of the embeddable variable stiffness bi-stable components as user-defined elements (UELs), delivering an accurate representation of the overall structural response.

The following sections elaborate on the individual components of the concurrent methodology.
**Figure 6.1:** Schematic of the main components of the multidisciplinary design and optimisation framework.
6.1.1 Distributed Topology with Corrugated Skin Portion

The interior of the NACA 0012 profile is parametrised to yield a distributed compliant topology, seeking to facilitate camber morphing as well as guiding aerodynamic loads to achieve a favourable deformed shape. Indeed, the internal stiffness distribution of selectively compliant systems plays a vital role in the global response, governing the outer aerofoil shape under loading, and thus the resulting aerodynamic performance.

Eight points are used to define the layout (marked as red dots in Figure 6.1), with all but two constrained to lie on the aerofoil surface. This simplification follows from initial experimentation combined with the size of the system, aiming at substructural members likely to feature sufficient length to represent the particular bi-stable configurations employed herein (see Section 6.1.4.2). The topology is then found using the Python Triangle function, based on the Delaunay triangulator and two-dimensional mesh generator by Shewchuk [445]. The reason for selecting such a straightforward approach is to limit the problem complexity at the initial stage, preclude intersections and guarantee connectivity, as well as obtaining direct control over the resulting edge count and lengths. The former determines the total of discrete parameters controlling the properties of a particular member, which can act either as a bi-stable laminate or a standard, uniform component of the internal layout. The latter is of importance in the assignment of bi-stable characteristics, as will be detailed in Section 6.1.4.2.

Furthermore, the outer skin possesses a corrugated portion (Figure 6.1) of a simple circular form with a radius of 7.5 mm, providing the necessary in-plane extensibility for cambering. This is represented through the homogenisation model of Kress and Winkler [443, 444], assuming three layers of glass fibre/epoxy fabric with a total thickness of 0.18 mm. The length of the corrugated segment is fixed as 75 mm to ensure ample room for compression required by shape adaptation within the validity range of the underlying linear model. The relatively low bending stiffness of the corrugated laminate renders a simultaneous optimisation with the internal substructure necessary to avoid non-smooth deformations. A more complex alternative to be considered at further stages of the study is to extend the framework for spatially tailored skin stiffness or highly anisotropic corrugation topologies [60, 112].
6.1 Concurrent Aero-Structural Design and Optimisation Framework

6.1.2 Global Actuation

To induce a global downward deformation for increased lift, Macro Fiber Composite (MFC) actuators in a unimorph configuration are assumed to act over the top surface of the morphing system, as indicated in Figure 6.1. Modelling is performed by means of thermal analogy (see for example [342, 446]). Due to the relatively large dimensions of the design, the properties of a single crystal MFC (PMN-PT fibre) according to Park and Kim [447] are employed in the optimisation. Compared to a standard, PZT-5A-based actuator, a single crystal MFC is predicted to offer five times higher piezoelectric strain constants and enhanced surface conformability [447]. Although not yet mature for direct application, this technology allows for a clearer demonstration of the advantages of the proposed approach, aiding quantification of the morphing characteristics of the individuals generated during the optimisation process. Further, as will be seen in Section 7.1, the significant potential of the presented concept in terms of global stiffness variability and the ensuing actuation benefits of the flexible deformation mode remain valid for standard MFCs (available technology). Finally, other smart or conventional actuation means could be equally incorporated into the concurrent framework.

Constant global actuation corresponding to a 1500 V input is employed, aimed at drawing comparisons between different topologies. Specifically, the resulting lift variation serves as a means of performance quantification of a given design. An actuation voltage of 1500 V yields a free strain of \(7003 \times 10^{-6}\) for a single crystal MFC, and \(1350 \times 10^{-6}\) for a standard MFC, given the free-strain-per-volt values of \(4.6688 \times 10^{-6} \text{V}^{-1}\) and \(0.9 \times 10^{-6} \text{V}^{-1}\), respectively, used in the thermal analogy and based on the data from [342, 447].

As already seen in Sections 4.3, 5.2 and 5.4, the bi-stable elements exhibit a distinct response in compression and tension. Whereas the flexible equilibrium configuration is strengthened by the former, the latter causes a switch to the stiff stable state. Further, the pronounced variable stiffness response occurs in compression. This has significant consequences for positioning the laminates within the distributed layout. As previously mentioned, the aim is to take maximum advantage of the available variable stiffness properties in reducing the actuation work against the compliant structure. To that end, the global actuation needs to subject the embedded bi-stable components
to compression, strengthening the flexible state while remaining below the
snap-through threshold of the stiff shape. If close to unloaded, the laminates
offer no advantage in terms of the energy required for morphing since their
variable stiffness properties are not exploited in selectively reducing the
global deformation resistance. Such a situation corresponds to a uniform
internal topology and does not lead to dissimilar aerofoil configurations.
This stands in contrast to the intentional deformation mode selectivity of the
wing profile pursued herein, emphasising the necessity to simultaneously
consider the global actuation and bi-stable laminate positioning within a
variable internal layout during the design and optimisation process.

Last but not least, employing a constant actuation area aims at reducing the
complexity of the problem at this preliminary stage, seeking to evaluate the
proposed morphing concept under a limited number of design parameters.
Simultaneously considering the spatial distribution of the actuation within
the optimisation constitutes a viable follow-up direction, promising a more
effective exploitation of the available actuation capabilities.

6.1.3 Concurrent Aero-Structural Analysis
The weakly coupled static aeroelastic simulation method detailed in Sec-
tion 5.1 is incorporated into the concurrent framework to yield the deformed
equilibrium shapes resulting from the mutual interactions between the com-
pliant structure and the aerodynamic loading. This presents a prerequisite for
designing smooth selective deformation modes able to support the prescribed
distributed loading.

6.1.4 Embeddable Variable Stiffness Bi-Stable Laminates
As already explained, a crucial part of the distributed compliant topology
are the monolithically embedded bi-stable laminates. The two carefully
designed stable configurations (Figure 6.2(b)) impart stiffness variability
on the wing section. Distinct properties in tension add to the substantial
anisotropy provided by the composites, as Figure 6.2(c) illustrates.

The response of the bi-stable components is represented by means of a
user-defined finite element (UEL), detailed in the following section. This
greatly enhances the efficiency and flexibility of the design process, allowing
particular members of almost arbitrary topologies to be selectively replaced
with bi-stable structures to determine the impact on the resulting morphing
behaviour.

### 6.1.4.1 User-Defined Element

The finite element analysis of the shape-adaptive aerofoil with embedded bi-stable laminates can be realised as a full model (Figures 4.4 and 5.8), in which accurate displacement and induced stress fields of the bi-stable elements are obtained from separate cool-down simulations and imported into the wing section model, as implemented in Chapters 4 and 5. The limitations of the corresponding Abaqus® IMPORT option [431], combined with considerable complexity and computational effort, render the method impractical for the purpose of the present, generalised study. A user-defined finite element is therefore developed to represent the axial stiffness behaviour of the bi-stable components when embedded within a larger morphing system. The Abaqus® user subroutine UEL employed to that end allows a general element to be defined by the user and included in the simulations [431, 448]. The approach permits specifying an almost arbitrary number of different UEL types, offering the possibility to incorporate multiple bi-stable laminate designs with distinct stiffness properties into the optimisation process.

Each of the two equilibrium configurations (flexible/curved and stiff/flat) of the bi-stable composites is represented by a dedicated user element type, referred to as a flexible and stiff UEL. The force-displacement curves of both stable states of the laminates clamped in-plane are obtained from numerical displacement-controlled stiffness tests in compression and tension according to the method detailed in Chapter 3. Further, the element is implemented to either accept a particular stiffness curve or a regression equation representing the desired response in a more general manner, as schematically shown in Figure 6.2(b) and (c).

Figure 6.2(a) indicates the conventions adopted for the UEL. The element has a two-dimensional character and lies in the global $x$-$z$ plane of the 3D model (Figure 6.3(a)). As a result, only axial displacements, $u_1$ and $u_2$, are active in the local coordinate system. This assumption follows from preceding studies and the current focus on the longitudinal stiffness response.

The tangent stiffness value at the current increment $n$, $k^n_t$, is approximated as:
Figure 6.2: (a) Schematic of the representation of a variable stiffness bistable laminate as a user-defined finite element. Example force-displacement curves of the stiff and flexible equilibrium configurations of a two-ply layout obtained from FEA for the UEL: (b) in compression, with regression equation (flexible state) and possible linearisation up to the snap-through point (stiff state); (c) in tension, with linear fits.

\[
k^n_t = \frac{F^n - F^{n-1}}{\Delta u_{1,2}^n}
\]

where \(F^n\) and \(F^{n-1}\) denote the interpolated force values at the current and preceding increments, respectively, and \(\Delta u_{1,2}^n\) refers to the relative axial displacement increment during increment \(n\).
The subsequent derivation follows standard methods applied to truss elements [449] and is hence omitted for the sake of brevity. Whereas a significant improvement in computational efficiency is achieved through the implementation of the flexible UEL, the numerical difficulties associated with the simulation of the stiff stable state persist due to the highly non-linear characteristics of the problem. Specifically, the snap-through to the curved shape on exceeding a critical load manifests itself as limit points of the corresponding equilibrium path, as explained in Section 2.2.3. The need for stabilisation thus arises to achieve convergence of an incremental static analysis, accompanied by a high number of solution iterations and increments (see Section 3.3.1.2 for a more comprehensive discussion). A potential alternative would be to use the modified Riks method [431]. This, however, is not pursued herein due to no satisfactory computational set-up being found. By contrast, the static analysis provides a good compromise between accuracy, numerical effort and robustness as regards the solution controls. What is more, the focus of the current approach lies on the selective activation of the deformation modes by virtue of the distinct stiffness characteristics associated with the individual stable states. In this context, the details of the snap-through transition between the equilibrium configurations are not of primary interest and can hence be neglected at the present stage. Accordingly, since the post-snap through condition is known (flexible configuration), the simulation of the stiff stable state is only performed until the snap-through threshold corresponding to the force-displacement response peak (Figure 6.2(b)). The analysis is then interrupted and a message issued indicating the snap-through conditions having been reached. This solution obviates the need for applying a problem-dependent artificial damping to achieve convergence, permitting a standard static analysis, an important aspect in view of automated optimisation implementation.

The performance of the user-defined element is verified through replacing the embedded bi-stable laminates of the prototype design (Figure 4.4) with UELs, as Figure 6.3 illustrates, and repeating selected aeroelastic simulations from Section 5.2. Each end node of an UEL is attached to the corresponding central grid point of the mounting flange by means of a TIE multi-point constraint, which imposes equal displacements on the common global degrees of freedom of the two nodes [431]. Moreover, a kinematic coupling constrains the displacements of the flange nodes contained within the bi-stable element.
width to the motion of the middle point of the flange. This becomes necessary due to the two-dimensional character of the UEL, embedded into a model with a finite third, spanwise dimension (Figure 6.3(a)).

![Prototype design with UELs](image)

**Figure 6.3:** (a) Simplified model of the prototype design with bi-stable components represented by means of UELs. (b), (c) Stiffness curves obtained from FEA for the embedded bi-stable laminates (layout #3 introduced in Section 4.1), used as input to the UELs.

The good agreement between the aero-structural analysis results obtained by means of the full and UEL models, demonstrated in Figure 6.4, confirms the prediction accuracy of this versatile and computationally efficient representation method of the bi-stable laminates.

### 6.1.4.2 Layout and Property Representation

The concurrent framework employs the general two-ply bi-stable element layout shown in Figure 3.1(b). The composites are first analysed with Abaqus/Standard® according to the numerical design improvement strategy elaborated in Chapter 3 to find configurations satisfying the objectives of em-
Figure 6.4: Verification of the user-defined finite element against a full model in weakly coupled static aeroelastic analyses of the prototype design from Section 5.2. (a) Variable stiffness response of the stiff and flexible modes quantified in terms of vertical displacement of the trailing edge for different freestream velocities and an angle of attack of four degrees. No snap-through of the stiff bi-stable laminates under these conditions. (b) Comparison of the drag coefficients for selected aerodynamic points (freestream velocity of 35 m/s) between rigid NACA 0012 and the flexible aerofoil mode.

beddability (in-plane clamping of two opposite edges, and their inclination to the horizontal in each stable shape) and stiffness variability (difference in response between the stable states). Since the bi-stable behaviour ensues from thermally induced stresses, bi-stability loss associated with excessive laminate thickness relative to the edge length has to be considered [366, 406]. The minimum component length (l in Figure 3.1(b)) under the aforementioned two objectives is adopted as 95 mm for the optimisation process (see Section 6.2.1). This would be achievable for instance with a low fibre areal weight unidirectional CFRP prepreg, with a cured ply thickness about 0.1 mm. A constant width of $w = 40$ mm is assumed.

The corresponding force-displacement profiles for input to the UEL within the design and optimisation framework are also obtained from the numerical
methodology of bi-stable component design for a range of laminate lengths and internal layout division ratios \((2x_{us,i}/l, 2x_{us}/l \text{ and } 2x_{s}/l, \text{as defined in Figure 3.1(b)})\) serving full configuration definition. The stiffness properties in tension and compression assume the general form indicated in Figure 6.2(b) and (c), and can be represented by means of regression curves.

Due to the complexity of the problem, the bi-stable laminates are not optimised simultaneously within the integrated framework. Instead, a particular length of a substructural edge (found between the red dot pairs in Figure 6.1) corresponds to stiffness characteristics obtained from curve fitting based on designs analysed in 5 mm increments in \(l\). This implies a specific spatial stacking sequence distribution defined by means of \(x_{us,i}, x_{us} \text{ and } x_{s}\). More precisely, the response of the flexible configuration in compression is expressed as a rational function (Figure 6.2(b)):

\[
F_{R,1}(u_1) = -\frac{p_1|u_1|^2 + p_2|u_1| + p_3}{|u_1| + q_1} \quad u_1 \in [-8,0] \tag{6.2}
\]

where the fit coefficients \(p_1, p_2, p_3, q_1\) vary with laminate length \(l\). Further, a linear stiffness profile of the flat stable shape is assumed until the snap-through point is reached:

\[
F_{R,1}(u_1) = \frac{F_{\text{peak}}}{u_{\text{peak}}} \cdot u_1 \quad u_1 \in [u_{\text{peak}},0] \tag{6.3}
\]

where \(F_{\text{peak}}\) denotes the maximum of the stiffness curve, and \(u_{\text{peak}}\) refers to the corresponding critical compressive displacement. Similarly, the tensile response relies on linear fits to the numerical response curves, as indicated in Figure 6.2(c).

Given a final aerofoil configuration featuring embedded composites of a particular size, this base laminate layout specified by the three variables \(x_{us,i}, x_{us} \text{ and } x_{s}\) can be refined according to particular requirements, as well as material availability and properties.

Details of the bi-stable design developed for the ultimate morphing aerofoil are presented in Section 7.1.
6.2 Design and Optimisation Problem Formulation

6.2.1 Objective Definition

The design of compliant systems features fundamental differences from conventional structures, requiring simultaneous optimisation for low and high deformation resistance. Maximising the desired output displacement implies minimum stiffness, while the ability to perform the intended function requires adequate rigidity [47, 450]. This is reflected in the concurrent approach adopted herein, exploiting two distinctly stiff configurations obtained with bi-stable laminates to realise a globally adaptive aerofoil.

Several studies have already demonstrated that targeted design and optimisation of distributed compliant topologies allow for camber morphing achieved by virtue of the particular internal layout with uniform, linear material properties (see for example [31, 61, 102, 106, 110, 113]). Such arrangements demand actuation energy to work against the structural elasticity for reversible geometry adaptation, in addition to overcoming aerodynamic loads. As previously explained, a potential approach to reduce this strain energy component for augmented actuation efficiency is to introduce variable stiffness elements into the substructure for deliberate activation of desired deformation modes. The pursued stiffness selectivity arises from an on-demand reduction in deformation resistance through snap-through between the equilibrium shapes of the embedded bi-stable components. Therefore, the impact of the stable state switch on the final structural response, or the advantage provided by the proposed stiffness variability, needs quantification and inclusion as objective, being as important as enhanced aerodynamic characteristics of the deformed wing profile.

The design conditions adopted herein are a freestream velocity of 20 m/s and zero incidence. This is to gain first insights into the morphing potential of the present concept and aid comparison between lift increase by virtue of shape adaptation at zero angle of attack versus pitching a conventional, rigid wing section. Applying a constant actuation level allows for conclusions regarding the difference in deformation resistance between the two selective modes of the aerofoil.

The ultimate goal of the optimisation is to obtain a morphing system featuring two dissimilar configurations addressing distinct operational scenarios. First, the flexible aerofoil mode aims at large geometric changes into a high-
lift profile due to reduced actuation demands. This is only possible by virtue of the internally tailored compliance which arises from the low stiffness stable state of the embedded bi-stable components, decreasing the elastic actuation work necessary for reversible system deformation. The second, stiff configuration, targets operation under larger aerodynamic loading while offering a smaller lift variation scope. Indeed, the stiff equilibrium shape adopted by the integrated laminates leads to higher deformation resistance of the wing section. The pursued strong dissimilarity between the two aerofoil modes implies maximising the impact of the embedded bi-stable elements on the global structural response. In other words, the selective stiffness properties of the laminates need to be exploited to the full. Further, equal significance is attached to high aerodynamic efficiency of the ensuing morphed wing profiles. These considerations are reflected in the composition of the fitness function, explained in the following.

According to the aforementioned objectives, two system analyses per individual are performed: first with all the embedded bi-stable laminates in the flexible, and second in the stiff equilibrium state (flexible and stiff aerofoil configuration, respectively). The resulting two contributions, $f_{\text{flexible}}$ and $f_{\text{stiff}}$, form the total fitness function $f$ (6.4a), with an additional term penalising excessively short bi-stable elements, $f_{\text{bp,penalty}}$.

$$f = f_{\text{flexible}} + f_{\text{stiff}} + f_{\text{bp,penalty}}$$ (6.4a)

$$f_{\text{flexible}} = -a_{\text{flexible}} \frac{c_l,\text{flexible}}{c_d,\text{flexible}}$$ (6.4b)

$$f_{\text{stiff}} = a_{\text{stiff,1}} \Delta c_d,\text{stiff} + a_{\text{stiff,2}} c_l,\text{stiff}$$ (6.4c)

The scalars $a_{\text{flexible}}, a_{\text{stiff,1}}, a_{\text{stiff,2}}$ denote weighting factors of the individual components of the fitness function, and $c_l$ and $c_d$ refer to two-dimensional lift and drag coefficients of an aerofoil configuration identified with the respective subscript.

Maximum compliance of the flexible mode of the structure is sought while maintaining an aerodynamically favourable deformed shape, which is expressed as a maximum aerodynamic efficiency term (lift-to-drag ratio $c_l,\text{flexible}/c_d,\text{flexible}$) of the respective fitness function contribution, $f_{\text{flexible}}$ (6.4b). Due to the zero angle of attack, the increase in lift achieved by cam-
bering arising from a prescribed actuation level directly corresponds to the directional compliance, whose high values are pursued. Simultaneously, the shape adaptation of the stiff configuration with identical structural layout should be minimised, for instance at the same design point. This is to ensure that the high-lift deformation mode arises solely from the stiffness selectivity provided by the bi-stable laminates, rather than from the topology as such. The corresponding fitness term (6.4c) is thus comprised of the deviation of the drag coefficient of the stiff mode from a rigid NACA 0012 subjected to identical aerodynamic conditions, $\Delta c_{d,stiff}$, and the achieved lift variation, $c_{l,stiff}$. The physical meaning of the $f_{stiff}$ contribution is that the prescribed actuation level combined with the pressure loading should be unable to significantly deform the stiff configuration, reflecting the desired minimum compliance. Further, a no snap-through requirement is imposed based on estimated critical values in compression to guarantee the existence of the stiff aerofoil mode at the specific operating point. This, however, is not a decisive specification given the bi-stable layouts employed herein, and the considered flight conditions. Indeed, the assumed lowest snap-through loads corresponding to the longest specimens reach about 17 N.

The individual terms of $f_{flexible}$ and $f_{stiff}$, representing the aforementioned objectives for the flexible and stiff aerofoil mode, are combined into a single fitness function by means of linear weighting for the reasons of simplicity and reduced computational effort. The scaling factors are heuristically selected based on the initial experimentation and analysis of preliminary optimisation runs as $a_{flexible} = 1.0/80.0$ and $a_{stiff,1} = 5.0$, $a_{stiff,2} = 1.0$. Aerodynamically efficient designs thus exhibit $f_{flexible}$ close to $-1$, with the contribution $f_{stiff}$ approaching zero.

Furthermore, due to the smallest length constraint associated with the thermally induced bi-stable configurations employed herein, an additional penalty term $f_{bp,pentaly} \in [0,1]$ is introduced in (6.4a). This represents the scaled deviation of the length of each substructural member with the bi-stable characteristics from the minimum threshold (95 mm), normalised with the total of the excessively short elements. The highest penalty of 1 corresponds to a uniform topology without variable stiffness components.

This problem formulation seeks to maximise the difference in structural response between the two configurations of the morphing aerofoil. Applying an equal actuation level permits determining the variation of the global
deformation resistance, and hence the impact of the stable state switch of the embedded laminates. Incorporating physically meaningful objectives in the form of aerodynamic coefficients allows favourable morphed shapes to be generated, while ensuring sufficient stiffness to sustain the prescribed loading conditions.

Since this initial, proof-of-concept stage is expected to provide first insights into the problem, the combinations of different stable states are not considered within one individual for the sake of maintaining a reasonable complexity level. Allowing each laminate to independently adopt a particular equilibrium configuration with the associated flexible or stiff properties would lead to a considerable expansion of the available design space. This would also likely shift the optimisation preference to a larger number of layout members replaced with bi-stable components, leading to a higher variety of stiffness levels or morphing variants. Hence the full potential of the approach could be unleashed.

6.2.2 Optimisation Set-up

The design parametrisation comprises twenty-two optimisation variables, summarised in Table 6.1. Ten parameters define the eight points of the distributed topology shown in Figure 6.1 (chordwise positions of six skin points, two coordinates for each of the remaining two internal points). Each of the resulting eleven edges of the layout constitutes a discrete design variable representing a selection between a uniform material and a bi-stable element in a prescribed equilibrium configuration. The last parameter specifies the position of the right end of the corrugated skin segment (marked with a cross in Figure 6.1). As already stated, its length is fixed at 75 mm.

Due to the characteristic non-linearity, discontinuity and a largely unknown fitness profile of the problem, a mixed-integer optimisation is performed by means of the genetic algorithm (GA) available in MATLAB R2015a. The built-in fitness scaling function is customised to ensure the removal of infeasible designs from the population.

To begin with, initial experimentation with optimisation parameters and fitness function definition using random initial populations is conducted. The computational effort associated with the system analyses, two of which are performed per individual, prevents an exhaustive search of the design space. A single simulation ranges from one up to several minutes, depending on
Table 6.1: Summary of design parameters.

<table>
<thead>
<tr>
<th>Design parameter</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abscissae of substructural points</td>
<td>8</td>
</tr>
<tr>
<td>Ordinates of internal substructural points</td>
<td>2</td>
</tr>
<tr>
<td>Properties of substructural members ∈ {uniform, bi-stable}</td>
<td>11</td>
</tr>
<tr>
<td>Position of right end of corrugation</td>
<td>1</td>
</tr>
</tbody>
</table>

the iteration count to convergence of the aeroelastic scheme with non-linear FEA. On average, a population of 120 requires approximately four to five hours on an Intel Core i7 processor. The substantial computational expense thus leads to the selection of a population size of 120, including thirty-one prescribed individuals adopted from the preliminary studies. Clearly, such a pragmatic approach reduces the population diversity and prevents a thorough exploration of the design space, being likely to only deliver a local optimum. However, it is considered sufficient to gain first insights into the performance of the pursued morphing concept, and the possibility of designing selective deformation modes with bi-stable laminates in the variable stiffness function.

6.3 Conclusions

A concurrent design and optimisation framework is implemented to further explore the morphing strategy combining distributed compliance with stiffness variability. This seeks to take maximum advantage of the available properties of the bi-stable laminates in developing selective morphing modes targeting different flight conditions. The extent of shape adaptation achievable with particular actuation energy is considered, as well as studying the positioning of the bi-stable components embedded in the variable internal layout. The latter aims at maximum impact of the on-demand stiffness modification triggered by the stable state switch on the resulting structural response of the morphing system. The integrated framework thus includes a more complex distributed topology with specific members replaced with bi-stable elements, tailored skin extensibility via a corrugated section, and global actuation controlling the shape adaptation. An aero-structural anal-
ysis method allows physically meaningful objectives to be specified, such as aerodynamic efficiency, while capturing the morphed equilibrium shapes resulting from the mutual interactions between the compliant structure and aerodynamic loads.

The proposed formulation of the optimisation problem concentrates on two selective deformation modes intended for distinct operational scenarios. First, the achievable lift variation, equivalent to directional compliance, is maximised for the flexible configuration in search of reducing the corresponding actuation effort. Second, high deformation resistance of the stiff mode is pursued to permit access to higher loading regimes. Simultaneously considering the impact of the stable state switch on the system response ensures the intentional response dissimilarity between the two configurations, or the desired adaptation selectivity.

As will be seen in the following chapter, the concurrent incorporation of the major contributing aspects leads to a design solution demonstrating the feasibility of combining distributed compliance with stiffness variability for augmented morphing functionality.
This chapter is based on the journal publication:

The current chapter presents the shape-adaptive solution to emerge from the previously introduced multidisciplinary approach. As specified in the optimisation objectives, the morphing system features two selective deformation modes targeting distinct flight conditions. The configurations are evaluated numerically based on the dedicated simulation framework. The results of high fidelity Fluid-Structure Interaction (FSI) analyses follow, seeking to assess the dynamic adequacy of the shape-adaptive wing section as well as detecting potential instabilities. Finally, comparisons to the computationally efficient predictions of the concurrent framework are drawn.
7.1 Final Morphing System

The optimisation concludes after sixty-nine generations due to the average change in objective function values over the preceding fifty generations falling below the prescribed threshold of \(5 \times 10^{-3}\).

Figure 7.1 shows details of the final shape-adaptive system, as well as the side views when undeformed and in the morphed flexible mode. As indicated in a dashed blue line in Figure 7.1(a) and (b), the optimiser chooses to embed only one bi-stable element. This is mostly due to the restrictions on the laminate dimensions arising from the chosen implementation with CFRP (see Section 6.1.4.2).

![Configuration details](image)

(b) Undeformed

(c) Flexible deformed

Figure 7.1: Final individual. (a) Details of the configuration. (b) Undeformed shape. (c) Morphed flexible mode. Bi-stable laminate location marked in a dashed blue line in (a) and (b), its approximate deformed flexible shape overlaid in (c) from a separate compressive stiffness test due to lack of post-processing for UELs.

In the final evaluation of the presented individual, the particular stiffness curves provided in Figure 6.2(b) and (c) are employed, corresponding to the bi-stable component layout summarised in Table 7.1.

Table 7.2 sets out the preliminary material characteristics used for the skin, adjusted from [110]. The properties of the internal structure correspond to laser-sintered polypropylene \((E = 800\, \text{MPa}, \, \nu = 0.3, \, \text{according to [106]})\). Prescribing these is justified by the focus herein lying on the impact of the stiffness variability provided by the bi-stable laminates on the structural
response, rather than providing a detailed design ready for manufacturing.

**Table 7.1:** Geometric parameters of the bi-stable laminate configuration embedded in the final individual. Symbol definitions according to Figure 3.1(b).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( w )</th>
<th>( l )</th>
<th>( x_s )</th>
<th>( x_{us} )</th>
<th>( x_{us,i} )</th>
<th>( BCs )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>Value</td>
<td>40.0</td>
<td>160.0</td>
<td>58.0</td>
<td>48.0</td>
<td>44.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**Table 7.2:** Prescribed properties of aerofoil skin.

<table>
<thead>
<tr>
<th>Component</th>
<th>Stacking Sequence</th>
<th>Thickness [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top skin</td>
<td>([0_{MFC} / 90_{GFRP} / 90_{GFRF} / 90_{GFRP}]_{T})</td>
<td>0.696</td>
</tr>
<tr>
<td>Bottom skin</td>
<td>([90_{CFRP} / 0_{CFRP} / 90_{GFRF}]_{s})</td>
<td>0.772</td>
</tr>
</tbody>
</table>

0: chordwise direction  
f: fabric

The aerodynamic characteristics of the final morphing system at the design point are presented in Figure 7.2. Further, the coefficients predicted using standard MFC properties (PZT-5A fibre [447]) are provided for comparison purposes.

The tailored compliance of the flexible aerofoil mode allows the single crystal MFC actuation to induce a considerable lift variation of \( c_{l,flexible} = 0.69 \), clear from Figure 7.2, accompanied by a downward trailing edge displacement of \( u_{3,TE,flexible} = -21.6 \text{ mm} \). By contrast, the increased global deformation resistance of the stiff configuration greatly limits the shape adaptation achievable with the same actuation level. Specifically, the TE only deflects by \( u_{3,TE,stiff} = -3.8 \text{ mm} \), which yields a global stiffness variability ratio of 5.7 between the two tailored morphing modes. The same value arises from comparison of the individual lift variations, \( c_{l,flexible} / c_{l,stiff} \). Quantifying the impact of the embedded bi-stable components on the structural response, the ratio demonstrates the actuation benefits achieved by virtue of the stiffness selectivity attained through bi-stability. The proposed strategy thus permits surmounting the authority limitations of smart actuation systems through the purposely designed flexible mode. Simultaneously, the second configuration ensures increased deformation resistance, allowing for operation under
higher aerodynamic loading while providing smaller lift variation potential. This will be demonstrated in the following Section 7.2.

![Graph](image.png)

**Figure 7.2:** Aerodynamic characteristics of the flexible and stiff mode of the final morphing system at the prescribed operating point (flying at 20 m/s at zero incidence). Actuation voltage of 1500 V, with single crystal MFCs (PMN-PT fibre) and standard MFCs (PZT-5A fibre). Coefficients of a rigid NACA 0012 pitching between $-4^\circ$ and $8^\circ$ calculated with XFOIL at the same Reynolds number overlaid for comparison.

Employing standard MFC as the available technology, characterised by lower piezoelectric strain and higher stiffness, leads to smaller changes in lift ($c_{l,\text{flexible}} = 0.21$, $u_{3,\text{TE,flexible}} = -6.6$ mm). The global stiffness variability is preserved, with the corresponding ratios of $c_{l,\text{flexible}}/c_{l,\text{stiff}} = 4.4$ and $u_{3,\text{TE,flexible}}/u_{3,\text{TE,stiff}} = 4.3$. Given the relatively large size of the morphing system with a chord length of 500 mm, the results are still promising, proving the feasibility of the strategy. On the other hand, the actual active dimensions of the MFCs are not considered at the current early phase, with reduced active strain expected in real application.

This preliminary study considering the most basic configurations, stiff and flexible, clearly demonstrates the feasibility of exploiting the stiffness variability provided by bi-stable laminates in designing selective deformation modes, including an aerodynamically advantageous high-lift configuration. As already mentioned, allowing for stable state combinations within one individual would greatly expand the available design space, permitting multiple
Due to the limitations associated with the underlying optimisation set-up, especially in terms of the population size, no claim as to the global character of the optimum is made. However, an evident aerodynamic efficiency benefit is still attained compared to pitching, confirmed in Figure 7.2.

In interpreting the results, it is important to remember that XFOIL tends to provide optimistic drag predictions. Further, a slight penalty is expected to result from the corrugated skin segment of the morphing aerofoil [109], modelled as flat shell with specific properties and hence not influencing the aerodynamics.

Despite the reduced actuation effort required to deform the flexible mode of the system, an obvious practical disadvantage is the ensuing decreased structural stiffness. A promising future countermeasure relies on the anisotropic response of the bi-stable components in compression and tension. Specifically, distributing the variable stiffness laminates in pairs of antagonistic orientation allows the compliance of each flexible element undergoing compression to be compensated through its counterpart being forced into the stiff state through the same global morphing motion. This enables the system to autonomously sustain the deformed configuration. Such an approach presents new enhancement possibilities, such as spatially distributed smart actuation, extending the available authority and number of control degrees of freedom. The findings of the present study lay the foundations for this follow-up work.

7.2 High Fidelity FSI Simulations

However favourable for actuation reasons, the reduced stiffness of the flexible mode as implemented at this early stage poses the potential hazard of aeroelastic instabilities. Although the boundary condition of an infinitely rigid D-spar precludes global phenomena, local stability of the compliant design needs further verification. To that end, high fidelity FSI simulations are performed by means of a co-simulation between Abaqus/CFD® and Abaqus/Standard®.

The previously employed structural model generated in Abaqus/Standard® is subjected to an implicit dynamic analysis step. The outer surface of the morphing wing profile is defined as a co-simulation interface permitting data
exchange between the two solvers according to the sequential explicit Gauss-
Seidel coupling scheme, with Abaqus/Standard® leading and Abaqus/CFD®
lagging [431].

The domain generated for the transient incompressible fluid flow analysis
with Abaqus/CFD® extends 6 m in height and 15 m in length. The depth
direction $y$ is constrained to contain only one element to represent the desired
two-dimensional flow condition, corresponding to the spanwise dimension
of the structural model (50 mm, see Figure 7.1(a)). Standard values at sea
level are adopted according to the International Standard Atmosphere (ISA),
and the operating point of a level flight at 20 m/s is maintained. First, a
basic verification of the Computational Fluid Dynamics (CFD) set-up against
available experimental data is performed. Since transition modelling is not available in Abaqus/CFD®, a fully tur-
bulent flow is simulated, using the Spalart-Allmaras model with kinematic eddy viscosity assumed as three times the kinematic viscosity value [452].
This is a crucial consideration when comparing the aerodynamic coefficients,
especially the drag, resulting from the high and low fidelity approaches,
which will be elaborated in the later part of the current section. Nevertheless,
the global nature of the investigation, seeking to detect potential instabilities,
allows secondary importance to be attached to very precise drag predictions.

The FSI co-simulation is preceded by a pure CFD analysis performed for
0.5 s to achieve a converged flow solution as the initial condition. Following
a transient structural response within the first 0.5 s of the co-simulation
ensuing from an instantaneous application of the aerodynamic load, the
MFC actuation is ramped from 0 V to 1500 V between 0.5 s and 0.7 s, and
maintained for the remainder of the analysis time.

Figures 7.3 and 7.4 present the results obtained for the flexible and stiff
configurations, respectively, using both the FSI and the weak static aeroelastic
approach. As stated above, in interpreting the discrepancies obtained, it is
vital to bear in mind the different approaches to transition and turbulence of-
fered by the two tools. In the weak static aeroelastic coupling, free transition
is determined by XFOIL according to the $e^N$ method ($N = 9$) [441].

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1 The initial CFD verifications and FSI studies based on a different configuration were per-
formed within the Master’s thesis of Fasel [451].
The predictions obtained in this manner are labelled as *Static: Transition* in Figures 7.3 and 7.4. To aid comparison with the fully turbulent FSI simulations, the low fidelity analyses are repeated with forced transition at the leading edge of the aerofoil and the results indicated as *Static: Turbulent*. As expected, the ensuing differences become especially clear as regards the drag coefficient. Whereas the transitional low fidelity approach certainly underpredicts the actual value, the fully turbulent assumption is likely to lie on the pessimistic side due to the role of seamless geometry variation in
increasing the laminar flow regime.

![Graphs showing time-displacement and lift/drag coefficients over time for different simulation scenarios.](image)

**Figure 7.4:** Results of FSI simulations of the stiff mode of the final morphing system at the operating point (20 m/s, zero incidence, actuation voltage of 1500 V ramped between 0.5 s and 0.7 s simulation time).

From Figures 7.3 and 7.4, it can be seen that minor oscillations ensue from the instantaneous application of the aerodynamic load at the beginning of the analysis, especially visible in the drag coefficient history due to its low order of magnitude (Figures 7.3(c) and 7.4(c)), decaying as the simulation progresses. These, however, do not present a problem as no rapid changes on the order of 20 m/s are likely to be encountered in real operation. The
oscillations mostly occur in the middle of the corrugated skin portion (see Figure 7.1(a)), an expected phenomenon due to its low bending stiffness. Figure 7.5 provides further insights in this respect, presenting the time history of the vertical displacement of this point for the stiff configuration subjected to increasing velocities. The initial transient response to the instantaneous application of the aerodynamic load, stronger for higher velocities, decays as time progresses, not posing any local instability hazard within the considered flight regime.

Refining the basic corrugated configuration employed herein thus presents a viable means of further improving the dynamic response of the shape-adaptive system. Specifically, increasing the bending stiffness allows for reducing the oscillation magnitude as well as accelerating the final decay. Rather than using the simple circular form, the double corrugation proposed by Previtali et al. [112, 114] could be considered in the design. A straightforward practical solution would be to apply pre-stress when integrating the corrugated panel into the prototype. Incorporating spatially variable distribution of the skin stiffness into the concurrent framework offers yet another enhancement possibility.

![Figure 7.5](image)

**Figure 7.5:** Time history of the vertical displacement of the central node of the corrugated skin portion for the stiff aerofoil mode subjected to increasing freestream velocities of 20 m/s, 25 m/s, and 30 m/s. Zero incidence, actuation voltage of 1500 V ramped between 0.5 s and 0.7 s simulation time.

The very good agreement obtained in terms of the global morphing potential, or the final deformed shape, presented in terms of trailing edge
displacement and lift coefficient, provides a verification of the low fidelity technique used in the optimisation framework. What is more, the response decay demonstrates freedom of local instabilities, a crucial aspect in view of the compliance of the system.

![Diagram](image)

**Figure 7.6:** Results of FSI simulations of the stiff mode of the final morphing system at growing freestream velocities of 20 m/s, 25 m/s, and 30 m/s. Zero incidence, actuation voltage of 1500 V ramped between 0.5 s and 0.7 s simulation time.

As previously stated, the stiff mode targets operation under increased aerodynamic loading, while providing a smaller lift variation range. The
7.3 Conclusions

A shape-adaptive system combining distributed compliance with variable stiffness provided by monolithically embedded bi-stable components is presented and numerically evaluated. This selectively compliant solution, emerging from the dedicated multidisciplinary design and optimisation framework, demonstrates the viability of the proposed strategy in addressing the conflicting requirements of morphing in an actuation-efficient manner. Activating specific deformation modes permits decreasing the associated actuation effort, reducing the amount of the necessary actuation material and hence promising mass savings.

Accordingly, two selective configurations are designed. An aerodynamically smooth high-lift mode presents modest actuation requirements and thus a remedy for the common authority limitations of smart systems. This is only possible by virtue of the internally tailored compliance, or on-demand stiffness reduction through snap-through between the stable states of the embedded bi-stable laminates. Compared to pitching a conventional wing section, the flexible configuration offers clear aerodynamic benefits, demonstrated through numerical simulations. The stiff mode targets operation under higher distributed loading, while offering a smaller lift variation range. A promising ratio of 5.7 is obtained between the morphing capacity of the capabilities of the current configuration in this respect are therefore additionally analysed under freestream velocities of 25 m/s and 30 m/s, with the results shown in Figure 7.6. As before, initial transient response follows the instantaneous application (improbable as a real scenario) of the aerodynamic load, growing with increasing velocity. The oscillations, reflected in the overall aerodynamic coefficients, primarily occur in the corrugated segment (Figure 7.5), decaying as time progresses. Hence the dynamic adequacy of the design is confirmed.

Rather than verifying the dynamic performance of the final morphing system sequentially, as performed herein, the concurrent framework could be further refined to directly incorporate simplified instability prediction means. The implementation by Molinari et al. [110] provides a good example. Hence the corresponding requirements such as the desired higher operating regime of the stiff mode could be directly included in the optimisation process.
two configurations at the design point, emphasising the intentional response dissimilarity. This stiffness selectivity allows the actuation resistance of the compliant substructure to be tailored as required for efficient operation, providing a clear functional enhancement. The results obtained are verified through high fidelity FSI simulations, which demonstrate dynamic adequacy of the design and freedom of instabilities.

The findings of the study open up numerous opportunities for further work, to be detailed in Section 8.2.
CHAPTER 8

Conclusions and Outlook

Morphing systems able to efficiently adjust their characteristics to resolve the conflicting demands of changing operating conditions offer great potential for accomplishing enhanced performance and novel functionalities. The main practical challenge entails combining the desired compliance to realise radical reversible geometry modifications at reduced actuation effort with the need for high stiffness due to functional reasons. This contradiction can be decoupled through the variable stiffness approach, understood as the provision of a controllable, time-varying stiffness range to tune the deformation resistance in operation as required for maximum efficiency. Further, conformal shape adaptation based on distributed compliance constitutes a viable morphing strategy, promising advantages in terms of aerodynamic characteristics, reduced complexity and mass, and function integration.

The main goal of the current research has been to explore the combination of distributed compliance with purely elastic stiffness variability provided by monolithically integrated bi-stable laminates. The findings confirm the feasibility of the proposed approach, demonstrating the ability to design specific, aerodynamically efficient deformation modes satisfying prescribed performance objectives. These tailored configurations are a unique consequence of the internal stiffness selectivity relying on changing the equilibrium shapes of the embedded bi-stable components. The deformation resistance of the distributed topology can thus be reduced as required for augmented actuation
effectiveness, implying mass savings through less actuation material.

### 8.1 Summary of Main Findings and Conclusions

In the first part of the thesis, a numerical methodology is introduced to develop bi-stable laminates acting as variable stiffness components within a larger distributed morphing topology. The critical design aspects include maximised stiffness difference, quantified in axial compression, and retained bi-stability with two opposite edges constrained in plane. Bi-stability assessment relies on two independent criteria, namely the characteristic strain energy profile featuring minima corresponding to the equilibrium shapes, and a displacement-controlled cool-down test. The numerical strategy is applied to two example layouts, two- and three-ply, of given external dimensions, to establish the relevant design sensitivity by means of systematic parametric studies. Tailoring the internal layout of the three-ply design results in modest modifications of the stiffness ratios of the two equilibrium states between two and four, with similarly low sensitivity of the clamping surface inclination. By contrast, the compressive snap-through load of the two-ply specimen varies between approximately 50 N and 190 N, with the corresponding critical displacement reducing from about 1 mm to 0.08 mm. In the same way, considerable sensitivity of the clamping surface angles of each equilibrium configuration ensues, ranging from six to twenty-eight degrees in the flexible, and zero and nine degrees in the stiff state.

Second, a basic proof of concept of the proposed morphing approach is provided numerically and experimentally by means of a NACA 0012 profile whose internal substructure is modified to contain two embedded bi-stable laminates. A simple mechanical test reveals a stiffness ratio of 2.47 between the stiff and flexible modes of the aerofoil: a promising result in view of the thickness of the bi-stable elements only accounting for one fifth of the skin thickness. Experimental testing of the manufactured prototype shows good agreement with numerical predictions, confirming the stiffness variability achieved and validating the simulation methodology.

The purely structural analysis is subsequently extended to include a two-way weak static aeroelastic coupling accounting for the mutual interplay between the compliant system and aerodynamic pressure distribution. This permits capturing the geometrically non-linear, global deformation modes.
ensuing from the internal stiffness modifications through snap-through of the embedded bi-stable laminates under external loading. The global stiffness variability of the concept is confirmed, with larger passive geometry adaptation of the flexible aerofoil mode compared to the stiff configuration. Nevertheless, an aerodynamically unfavourable morphed shape arises in the post-snap-through regime, pointing to the crucial role of a concurrent aero-structural design of desired deformation modes. The characteristics of an improved configuration featuring corrugated skin portions and a modified layout substantiate this observation.

Further, the complexity level is increased to a multidisciplinary design and optimisation framework, seeking to fully exploit the available stiffness variability of the bi-stable laminates in tailoring selective deformation modes to specific performance targets. Selected members of a parametrised distributed topology are intentionally replaced with bi-stable components, while exploring the morphing extent achievable with prescribed global actuation controlling the shape adaptation. The aeroelastic analysis method allows physically meaningful objectives to be specified, including aerodynamic efficiency and estimated snap-through conditions for load-bearing capacity assessment.

The shape-adaptive solution to emerge from this methodology proves the viability of the proposed strategy in addressing the conflicting requirements of morphing in an energy-efficient manner. The system features two selective configurations aimed at distinct operational scenarios. An aerodynamically smooth high-lift mode achieves large geometric changes due to reduced actuation demands. This is only possible by virtue of the internal stiffness re-distribution, or on-demand stiffness reduction through snap-through between the stable states. The stiff mode targets increased distributed loading, while offering a smaller lift variation range. A promising ratio of 5.7 is obtained between the morphing capacity of the two configurations at the design point, emphasising the intentional response dissimilarity. This stiffness selectivity allows the actuation resistance of the compliant topology to be tailored as required for efficient operation, providing a clear functional enhancement. Finally, high fidelity FSI simulations demonstrate the dynamic adequacy of the design.

In conclusion, the results of the present work prove the viability of the devised morphing approach, confirming the expected benefits. The initial
character of the investigation, however, points to the necessity for substantial future work, outlined in the following section. Further, an important limitation of the study is the selected method of obtaining bi-stability through thermally-induced stresses in thin unsymmetric laminates. This leads to practical issues in terms of scalability and the associated manufacturing aspects, and reliability. Specifically, the minimum size constraints to obtain bi-stable structures imply increased ease of tailoring the laminates with growing external dimensions, which stands in opposition to the integration objective within a larger shape-adaptive system. In addition, environmental sensitivity and bi-stability loss with time create a need for more robust multi-stable solutions.

8.2 Suggestions for Future Research

While the present thesis has proved the feasibility and potential of the selective stiffness approach in augmenting the morphing functionality of distributed compliance, more research is needed to proceed to a more mature technology readiness level.

If the concept is to be moved forward, a design solution able to lock and sustain the morphed high-lift shape needs to be developed. Specifically, the simple concept of antagonistic pairs of bi-stable elements could be used, as outlined in Section 7.1. Alternatively, independent topologies could be accommodated in the spanwise direction of the system. This would allow for adjusting the element inclination rather than relying on increased curvature of the flexible state to control the critical displacement of transition to the stiff equilibrium configuration. What is more, considering the combinations of different stable states within one individual would lead to a considerable expansion of the available design space. With each laminate able to independently adopt a particular equilibrium configuration with the associated flexible or stiff properties, a higher variety of stiffness levels or morphing variants would arise. Further, introducing variable spatial distribution of smart actuation in a multifunctional role is an auspicious issue which could be usefully explored in further research. Hence improved actuation authority and more control degrees of freedom over a potentially higher number of deformation modes could be attained. This approach could also be used for snap-through control and vibration suppression, should the latter accompany
the stable state transition phenomenon. Moreover, the stiffness contribution of the monolithically embedded actuators would enhance the overall load-bearing capacity.

A natural progression of this work is to perform experimental investigations, including wind tunnel tests, to characterise the performance of the morphing system and validate the numerical predictions of the concurrent simulation environment. To that end, the conceptual actuation employed herein would need replacing with a simpler, available technology. Alternatively, further optimisation including actuation positioning and spatially variable skin stiffness could be performed to potentially increase the effectiveness of smart systems such as standard MFCs. What is more, the present study has focussed on the stiffness characteristics of the selective deformation modes of the adaptive wing section, neglecting the details of the snap-through transition between the stable states as such. This abrupt phenomenon, however, is envisaged to have non-negligible implications for the dynamic behaviour of the morphing system. Therefore, it would be interesting to experimentally assess these effects.

Another possible area of future research would be to develop enhanced variable stiffness bi-stable components to overcome the geometric and environmental limitations as well as further tailoring the achievable performance. Imaginable solutions include pre-stress, initial curvature, or novel material concepts (platelet reinforcements, programmable matter), moisture-insensitive resin systems. This would also contribute to improved scalability since the minimum dimensions of the thermally induced bi-stable elements employed herein constitute an obstacle in increasing topological complexity while maintaining an overall size of the structure adequate for a typical UAV application.

Extending the morphing aerofoil concept to a three-dimensional wing would be a fruitful area for further work. In this context, greater efforts are needed to provide computationally efficient solutions to dynamic and static aeroelastic instability analysis to incorporate the corresponding constraints into the optimisation process. This is especially important given the considerable compliance of the morphing solutions in question. In addition, the higher operating loads envisaged for the stiff configuration could thus be directly prescribed as an objective, rather than relying on a sequential verification approach.
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