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# Can there be a physics of financial markets? Methodological reflections on econophysics

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**Abstract.** We address the question whether there can be a physical science of financial markets. In particular, we examine the argument that, given the reflexivity of financial markets (i.e., the feedback mechanism between expectations and prices), there is a fundamental difference between social and physical systems, which demands a new scientific method. By providing a selective history of the mutual cross-fertilization between physics and economics, we reflect on the methodological differences of how models and theories get constructed in these fields. We argue that the novel conception of financial markets as complex adaptive systems is one of the most important contributions of econophysics and show that this field of research provides the methods, concepts, and tools to scientifically account for reflexivity. We conclude by arguing that a new science of economic and financial systems should not only be physics-based, but needs to integrate findings from other scientific fields, so that a truly multi-disciplinary complex systems science of financial markets can be built.

## 1 Introduction

Driven by the computerization of financial markets and research technologies, econophysics has been introduced in the early 1990s. Econophysics, as a distinct scientific field of research, emerged as a reaction among physicists and some economists to the failure of standard economic theory to realistically model the complex behavior of financial systems. The early developments of the econophysics literature have been focused on studying and extending a set of so-called "stylized facts," many of them previously identified in financial economics, which are defined as robust empirical features that generalize across markets and asset classes. A rich diversity of econophysical models, such as agent-based, evolutionary and minority-game models and model-driven theories of large price fluctuations, bubbles and market crashes have

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evolved over the last two decades. By illuminating the statistical properties of financial return distributions and their underlying social mechanisms, which can generate systemic instabilities, econophysics has since then significantly advanced our understanding of financial markets.

However, a dominant criticism of the quantitative aspirations to build a physical science of financial markets has emerged. This criticism is encapsulated in the claim that "financial modeling is not the physics of markets," as expressed by Emanuel Derman, a prominent former Wall-Street "quant" and theorist [1]. Another well-known practitioner, the hedge-fund manager and philosopher George Soros, who put forward the concept of market reflexivity [2], argued that financial reflexivity – i.e., the positive feedback between expectations and prices that drive market dynamics – demands a new version of the scientific method as physics cannot handle the complexity of reflexive behavior.[1]

In this paper, we provide a methodological reflection on econophysics, which is guided by the fundamental question whether there can be a physics of markets. This higher-level methodological approach allows us to assess the theoretical contributions of econophysics and identify the main challenges in building a new science of economic or financial systems. In particular, by reflecting on the differences and similarities between physical and social systems, respectively physics and economics, we examine whether econophysics is methodologically equipped to approach the problem of reflexivity.

The paper is organized around three parts. Section 2 gives a brief overview of the mutual cross-fertilization between physics and financial economics. This section highlights how physics and economics methodologically diverged – as indicated by the different uses that data and models have with respect to theorizing. It further illustrates the methodological difference by comparing how physics and financial economics approach the "excess volatility puzzle." In Section 3, we consider how econophysics has contributed to an advanced understanding of economic and financial systems. We present here the ideas of financial complexity and causality that flow from econophysics. We consider the introduction of complexity and heterogeneity to financial economics as one of the most important contributions of the relatively new field of econophysics. By rendering idealized assumptions about representative agents and the equilibrium state of financial markets empirically more realistic, econophysics has been able to account for the heterogeneity of economic agents and the out-of-equilibrium and non-linear dynamics that characterize complex financial systems. In Section 4, we analyze how econophysics has provided a novel perspective on complexity and causality, which allows for bottom-up and top-down causation. We show that the econophysics view of complexity and causality allows one to quantitatively approach reflexivity in financial markets. We conclude by arguing that, while econophysics has made considerable progress in, for example, reproducing many of the stylized facts identified in financial data or elucidating the social or behavioral mechanisms underlying market dynamics, it needs to reach beyond physics and integrate the concepts, methods, and tools from other disciplines, so that a new multidisciplinary complex systems science of financial markets can emerge.

## 2 Physics vs. finance

The mutual cross-fertilization between physics and economics has a long history starting well before the emergence of econophysics in the mid 1990s. The history and evolution of economics and physics, from the development of classical and neo-classical

[1] The concept of reflexivity has its origins in sociology and philosophy. See [3–8].

economics to econophysics, is punctuated by various collisions between these two fields. In what follows, we provide a selective overview of the historical interaction between physics and economics and finance, before we consider what distinguishes the development of econophysics from other historical cases of inter-fertilizations between these two fields [9–12].

### 2.1 A brief history of the cross-fertilization between physics and economics

Historically, one of earliest cases of the "physical attraction" [13] of economics was the conceptual influence Isaac Newton's *Philosophiae Naturalis Principia Mathematics* (1687) [14] exerted on Adam Smith's *Inquiry into the Nature and Causes of the Wealth of Nations* (1776) [15]. In particular, the notion of causative force, which was novel at that time, inspired Smith to conceptualize the dynamics of an economic system analogous to Newtonian physics. By the beginning of the $19^{\text{th}}$ century, the notion of social and economical laws, paralleling physical laws, became deeply entrenched in economics.

In his *Essai philosophique sur les probabilités*, Pierre-Simon Laplace, for example, showed in 1812 that certain social phenomena, which appear random, exhibit law-like behavior and some predictability [16]. Amplifying this scientification of social phenomena, Adolphe Quetelet, who studied birth and death rates, crimes and suicides, even coined the term "Social Physics," which aimed at identifying empirical regularities in the asymptotic Normal distributions observed in social data [17]. (See Galam et al. [18] and Galam [19] for a re-discovery and extension using agent-based models and mapping to physical particle models.)

In the second half of the $19^{\text{th}}$ century, the economists Alfred Marshall and Francis Edgeworth imported the concept of equilibrium into economics, drawing on research of Clerk Maxwell and Ludwig Boltzman, who described the macro-equilibrium observed in gases as the result of a multitude of collisions of particles. By analogizing economic activity with the interaction of gas particles, Marshall developed the notion of the equilibrium state of the economy, which still forms the core of current mainstream economics (we come back to this below).

Similarly to a thermodynamic description, which uses a mean-field representation that abstracts from the heterogeneity of the particles and their various microstates, equilibrium theory reduces the rich heterogeneity of economic agents to a single representative agent or firm [20]. Interestingly, the idea of equilibrium, which started to heavily dominate economics in the 1950s and now lies at the heart of the Dynamic Stochastic General Equilibrium (DSGE) models that central banks relay on, was struggling to find acceptance by economists, who had an out-of-equilibrium conception of the economy. However, the idea that economies achieve equilibrium states – i.e., that total demand equals total supply or total consumption equals total output and prices are stable – which was the result of a long maturation process, was subsequently pushed to the methodological extreme in neo-classical economics. Whereas equilibrium in physics is a descriptive feature of physical systems, equilibrium has a normative status in economics; the rich and complex out-of-equilibrium and non-linear dynamics of economic systems are tamed and forced to conform to a set of idealized assumptions and theorems, which are couched in equilibrium terms. In other words, rather than describing how economic or financial systems are, (financial) economics strives to prescribe how they should behave [21,22].

Now, not only has economics been fertilized by physics, but also methods and concepts from economics were transferred to physics. An important historical case, in which the flow of concepts and methods between physics and economics has been reversed, is when the economist and philosopher Vilfredo Pareto described in his *Cours d'Economie Politique* (1897) regularities in income distributions in terms of power laws [23]. As they are closely related to the concepts of scale invariance and universality, power laws have been later widely used in physics and the natural sciences to describe the universal statistical signatures of event sizes such as distributions of city sizes, earthquakes, avalanches, landslides, storms, commercial sales, or war sizes. These distributions are not Gaussian, but "fat-tailed" or sub-exponential distributions. One very important insight, which follows from the nature of fat-tailed distributions of event sizes, which power laws describe, is that the probability of the observation of extreme events is not negligible. Whereas Gaussian distributions attribute negligible likelihood to event sizes that are larger than a few standard deviations from the mean, power law distributions provide more weight to these extreme events, which have been documented in various physical systems [24–26]. As these extreme events often control the long-term behavior and organization of complex systems, power laws became very attractive for physicists. However, despite their appeal, it should also be cautioned that many reported power laws distributions turned out to be of limited validity or spurious [27,28].

Another historically important phase in the historical mutual cross-fertilization between economics and physics was Louis Bachelier's attempt to model the apparent random behavior of stock prices in the Paris stock market. In his PhD thesis *Théorie de la spéculation* (1900) [29], Bachelier, who was a student of Poincaré, developed the mathematical theory of diffusion and solved the parabolic diffusion equation five years before Albert Einstein (1905) established the theory of Brownian motion [30]. The modeling of stock prices as stochastic processes – analogue to the random motion of particles suspended in a gas or liquid – constitutes now one of the fundamental pillars of physics and (financial) economics. Research on fluctuation phenomena in statistical physics, on quantum fluctuation processes in quantum field theory, and on financial time-series in finance is now based on the random walk models and their mathematical sibling, the Wiener process.

The Geometric Random Walk model, which was introduced in economics by Osborn [31] and Samuelson [32], uses the exponential of a standard random walk and now constitutes the theoretical core of the most important theoretical constructs in neo-classical finance and economics, such as Markowitz' portfolio theory [33], the Capital-Asset-Pricing model [34], and the Black-Scholes-Merton option-pricing model [35,36]. The theoretical development of these models, which solidified into what has become known as the neo-classical paradigm, resulted in the complete mathematization of economics and finance. Resting on the fundamental assumptions of equilibrium, rational expectations, and utility maximization, the quantitative aspirations of neo-classical finance and economics have been driven by the belief that is possible to model their theoretical foundations on physics. This "physics envy" [37] was already inherent in Paul Samuelson's *Foundation of Economic Analysis* [38], which provided the theoretical framework of what became neo-classical economics, where he adapted the deductive methodology of thermodynamics for economics. Building on these earlier analogies between physics and economics, the economic and finance literature that followed pushed the mathematization of the field to new extremes. Organized around a few parsimonious postulates and theorems, formalized neo-classical finance and economics often devised analytically rigorous models that sacrifice realism for mathematical elegance. Often, these models have generated predictions that are essentially unfalsifiable. With respect to their validity, the models and theories of neo-classical economics, although inspired by physics in the quest for the quantification of economic phenomena, seemed to have methodologically diverged from what has long been considered the "queen of science." In the remainder of this section, we briefly discuss the different uses data and models have with respect to theory-building in physics, respectively to financial economics. We conclude this Section by briefly presenting the evolution of econophysics.

### 2.2 "Physics Envy"

Although neo-classical economics and finance have been influenced by the methodology of physics of the $19^{\text{th}}$ century and became axiomatized in the second half of the $20^{\text{th}}$ century, there are substantial differences in the way these two disciplines build and test models. While economics privileges models and theoretical principles over empirical confirmation, data come first in physics and models come second [39]. Understanding the methodological difference between physics and (financial) economics, which is manifested in the different ways models are constructed and falsified in these fields, allows one to assess how physics-based approaches can improve scientific models and theories of financial markets.

Whereas economics can be characterized by an axiomatic and non-empirical methodology [40], many physical models are inductively derived from data generated through experiments or simulations.[2] Model construction in economics and finance is often guided by a top-down approach that prioritizes theoretical principles and idealized assumptions, which go into the models. The emphasis on mathematics in neo-classical economics has resulted in highly stylized financial and economic models, which often omit the key characteristics of financial markets. Although many postulates of standard economic theory have been empirically falsified, many economic models, which suffer from their unrealistic assumptions, have not been revised. By contrast, many physical models are generally theoretically more minimal, data-driven, and falsifiable. Furthermore, many physically founded models, which are constructed in econophysics or behavioral finance and are methodologically inspired by statistical physics or biology, are geared towards elucidating the mechanism underlying the phenomenon to be explained or simulated from a collective or many-body perspective. In contrast, economic theory is more interested in how robust individual characteristics of the (representative) decision maker lead to different equilibria.

It is precisely the methodological difference between data-driven and model-based theorizing that allows one to understand the emergence of econophysics and, further, to assess its contributions. In one of the foundational papers of the field, Stanley et al. [41] state that econophysics begins "empirically, with real data that one can analyze in some detail, but without prior models." While it is difficult to agree with Stanley et al.'s implicit assumption of model-free or theory-free observations, econophysics can be nonetheless characterized in large part by the larger emphasis put on data-dependency of its theories and models. However, early econophysics papers were not empirical but more conceptual. Takayasu et al. [42] for example developed the first agent-based model in order to provide an alternative to DSGE models by incorporating agents' heterogeneous characteristics and the role of extended networks. Bouchaud and Sornette [43] were interested in exploring beyond the ideal arbitrage limit in continuous time using Ito calculus to more general situations amenable to the functional integration techniques that have been developed in particle physics and statistical physics.

How the standard approach differs from econophysics can be illustrated by so-called "puzzles" in economic theory. When empirical observations do not conform to the prediction generated by standard models of economics, economists often label these anomalies "puzzles." While the falsification process in physics would dictate to

[2] However, it is important to note that characterizing physics in terms of experimentation is inadequate as some areas of physics, such as string theory, are often experimentally impenetrable. Often, these areas in physics are highly theoretical and difficult if not impossible to confirm empirically, at least in the foreseeable future. Furthermore, over the past decades, economics witnessed the emergence of empirical approaches, such as experimental or behavioral economics.

reject the model on empirical grounds, the theoretical edifices of neo-classical economics are getting improved so as to account for these anomalous economic phenomena. In other words, data must conform to the normative nature of the mathematically parsimonious and elegant models.[3] One of the most famous anomalies in financial economics is the so-called "excess volatility puzzle," which Shiller [45,46] and LeRoy and Porter [47] have unearthed in the 1980s. It refers to the empirical observation that prices fluctuate too much when compared to their fundamental valuation.

It follows from the efficient market hypothesis (EMH) – which states that the trajectory of stock prices instantaneously incorporates all available information – that the observed price $p(t)$ of a share or index is equal to the mathematical expectation, conditional on all available information, of the present value $p^*(t)$ of actual subsequent dividends accruing to that share. As this fundamental value $p^*(t)$ is not known, it has to be forecasted. For EMH, the observed price is equal to the forecasted price, which can be written as

$$p(t) = \mathrm{E_t}[\mathrm{p^*t}], \tag{1}$$

where $\mathrm{E_t}$ refers to the mathematical expectation conditional on all public information available at time $t$. As EMH asserts, only new information on the fundamental value $p^*(t)$, which was not available at the time of the forecast, might result in surprising movements in the stock price. If follows from EMH that

$$p^*(t) = p(t) + \varepsilon(t), \tag{2}$$

where $\varepsilon(t)$is a forecast error, which must be uncorrelated with any information at time $t$ as the forecast would otherwise not be optimal, respectively the market not efficient. However, empirical observations of price behavior show that the volatility of the realized price $p(t)$ is much larger than the volatility of the fundamental price $p^*(t)$. This gives rise to the "excess volatility puzzle" as the empirical observation conflicts with the theoretical predictions of the model (2), since, mathematically, the volatility of $p^*(t)$ obtained as the sum (2) cannot be smaller than the volatility of one of its constituents $p(t)$, given that $\varepsilon(t)$ is uncorrelated with $p(t)$.

However, when viewed through the lenses of the logic of physics, there is no "excess volatility puzzle." As physics operates with the concept of causality, the observed price $p(t)$ can be understood as following from fundamentals. In other words, $p(t)$ should be an approximation of $p^*(t)$. Therefore, expression (2) should be replaced by

$$p(t) = p^*(t) + \varepsilon'(t), \tag{3}$$

and there is no longer a volatility excess paradox. The observed realized volatility of $p(t)$, which is larger than the volatility of $p^*(t)$, provides information on the price-formation process, which is not optimal. The introduction of causality shows that the observed price approximates the fundamental price, up to an error of appreciation of the market. It follows from this that price moves have other causes than fundamental valuations: there exists a noise element in the pricing process, which results in the deviation of the observed price from its fundamentals. However, instead of exploring the mechanisms underlying price fluctuations, most economists do not reject the a priori assumptions of standard economic theory. The difference between (2) and (3) captures this fundamental difference in modeling strategies in economics versus

[3] MacKenzie [44] shows how financial models changed reality. For example, the practical implementation of the Black-Scholes-Merton model by traders at the Chicago Board Options Exchange rendered the model more realistic – until the crash of Oct. 1987 broke its validity, requiring the introduction of the so-called "volatility smile" as a fudge to make it continue working.

physics. In its top-down approach, standard economic theory, such as EMH or rational expectations, dictates that economic reality has to conform to the models because the market is supposed to have had time to absorb all information and converge to its optimal representation, the market price. This economic view assumes that all nonlinearities and reflexive loops of infinite order are taken into account [48]. This is the limit of perfect rationality and infinite effective computing power. In contrast, the bottom-up physics-based approach is more myopic or time-dependent, examining the underlying microscopic system in order to illuminate its aggregate or macroscopic properties through a progressive construction process, which could be called "inductive" [49]. It is not assumed that the investors have digested all available information and have developed optimal strategies accounting for those of their peers in infinite loops of reflexivity.

As the example of the excess volatility puzzle above shows, the bottom-up approach of physics, which introduces the concept of causality and imperfect markets, can substantially advance our understanding of economic phenomena or systems. It is precisely the observation that there are parallels between economic and physical systems and that physics can contribute to their understanding that resulted in the emergence of econophysics as a new and distinct field in the 1990s. We briefly discuss in the next section the history of econophysics and what distinguishes it from other historical cases of cross-fertilizations between physics and economics, before we examine in more detail what we consider the most important theoretical contribution of econophysics.

### 2.3 The emergence of econophysics

The term econophysics – a synthesis of economics and physics – already encodes the theoretical program of the field. Similar to "astrophysics," "geophysics," or "biophysics", econophysics strives to model, predict, and explain economic phenomena by applying tools and concepts from statistical and theoretical physics [50,51]. In this section, we briefly identify the factors that have driven the evolution of econophysics.

Fuelled by the simultaneous computerization of financial markets and academic research, the availability of high-frequency data made the application of physical methods and concepts specifically attractive. One of the most important insights flowing from the last two decades of research in econophysics is that financial markets and their dynamics can be understood as complex adaptive systems, from which a variety of stylized facts emerge. Stylized facts – which represent robust and universal properties that can be identified across data sets from different markets and asset classes – can be understood as emergent properties, which result from the complex and nonlinear interactions of the system's components. Econophysics emerged from attempts to describe these stylized facts, such as volatility clustering, the heavy-tailed nature of return distributions, or the absence of linear correlations between returns [52–54], in terms of statistical frameworks used in physics. This statistical physics approach – which started in the 1960 with Mandelbrot [55–57], Fama [58,59] and Samuelson's [60] re-interpretation of modern portfolio theory using Paretian power laws and stable Lévy distributions [13] [4] – consolidated in the 1990s into a new field, which promised to statistically explain these universal patterns in financial data, which standard economic theory fails to account for. The leptokurticity of financial distributions, which was already unveiled by research in the 1960s following Mandelbrot's foray in the field and subsequent research by Fama himself, Cootner [62] and others, collides

[4] A modern approach generalising Samuelson to the case of distributions of returns with power law tails (not necessary in the Lévy law domain of attraction) was offered by Bouchaud et al. [61].

with standard economic theory, which is methodologically underpinned by Gaussian assumptions. The fundamental pillars of finance theory, such as modern portfolio theory, the Black-Scholes-Merton model of option pricing, CAPM, or value-at-risk, are characterizing the distributions of financial returns within a Gaussian framework. Because of its fundamental assumptions about a Gaussian world where markets are efficient and in equilibrium and economic agents perfectly rational, standard economic theory has invariably failed to account for extreme events in financial systems [63]. Econophysics' promise to model, explain, and predict the non-Gaussian nature of financial systems, their non-linearity and out-of-equilibrium dynamics, accelerated its development during the 1990s.

However, it is important to note that the evolution of econophysics is radically different from the historical cases where physics was infused into (financial) economics. Whereas in the cases we discussed in the previous section, the physical concepts, methods, and tools have been translated into economics – i.e., the physics concepts have been rendered economically meaningful – and integrated into the theoretical framework, econophysics is a new approach to economic or financial systems, which does not incorporate the theoretical foundations of standard economic theory. Econophysics has not simply integrated concepts and methods of statistical physics into the framework of financial economics, but directly applied physics, its concepts and methodologies to economic phenomena, often without building on the standard (neo-classical) theories of financial economics [64]. In other words, econophysics is simply an extension of the physical sciences, which has been motivated by the phenomenological and conceptual similarity between physical and financial systems. Consequently, the fact that econophysicists often filter out standard economic theory, a fact that also concerns the sociological dynamics of these scientific fields, has generated some controversies. While some econophysicists have expressed the desire to replace the theoretical edifice of neo-classical economics with econophysics, some economists claimed that econophysists's ignorance of standard economics theory resulted in a replication of scientific results, which have already been well established in economics [65–67]. It has been argued, for example, that complexity approaches, such as econophysics, are "justifying themselves by how they correspond with already-observed facts, rather than by the new insights they provide" [68]. However, the fact that research in econophysics often strives to identify and explain stylized facts in financial data by reproducing the phenomenon to be explained with computer simulations should be considered a positive contribution. Simulations almost never appear in the standard economic literature[5] as the often non-linear out-of-equilibrium and heterogeneous nature of the phenomena to be modeled cannot be analytically solved by standard economic modeling techniques, which follow the neo-classical dictum of simplicity, tractability and conformity to theoretical principles such as rational expectations with well-defined utility functions. By contrast, simulations have become deeply rooted in the practice of physicists working on economic or financial systems as they allow them to simulate algorithmically the behavior of the interacting components of the system under study. In other words, simulations provide a way to "increase the range of phenomena that are epistemically accessible to us" [72] and which the closed form solutions, demanded by standard economic techniques, fail to model [32]. Whereas standard economic models are in most cases not concerned with underlying causal mechanism emphasizing the transition from the micro-level to the macro-level, adopting the simulation methods used in physics enables econophysicists to build explanatory models of the target system that clarify the self-organization processes at work. By reproducing the basic mechanisms underlying stylized facts, econophysics is able to provide explanations of the various macroscopic patterns and regularities observed in financial data. One can

[5] A notable counter-example is e.g. Schelling's model of segregation, from micro-motives to macro-behaviors [69–71].

consider these generative explanations, which provide micro-specifications of macroscopic patterns, as one of the unique features of econophysics [73].

Now, it is remarkable that econophysics is often methodologically defined. As a cursory scan of some definitions given in relevant papers and textbooks reveals, characterizations of econophysics often appeal to its techniques and methods (such as statistical mechanics, power laws, scale invariance, etc.), rather than its theoretical focus. For example, Mantegna and Stanley [51] write that the "word econophysics describes the present attempts of a number of physicists to model financial and economic systems using paradigms and tools borrowed from theoretical and statistical physics" and the "characteristic difference [from the standard economic approach – TH/DS] is the emphasis that physicists put on the empirical analysis of economic data" (ibid.); Stanley et al. [41] state that the econophysics advances "in the spirit of experimental physics"; and Burda et al. [74] define econophysics as a "a quantitative approach using ideas, models, conceptual and computational methods of statistical physics applied to economic and financial phenomena. "For our part, what we consider the most central contribution of econophysics is that – as Stanley et al. [41] write in their foundational paper – "economic systems are treated as complex systems." Econophysics, in other words, provides a fundamental re-conceptualization of economic systems. The defining feature of econophysics is thus less that it applies the techniques of statistical mechanics to financial systems, but primarily the insight that the dynamics of financial systems are best understood as emergent properties of a complex adaptive system. This complexity approach to financial market is what renders econophysics fundamentally different from standard economic theory. In fact, its roots go back to the complexity approach applied to economics that was promoted by the Santa Fe Institute created in 1984 under the particular push of the Nobel economist K. Arrow together with two Nobel physicists P.W. Anderson and M. Gellman [75]. The Santa Fe Institute has been pushing further the unifying concept of "complex adaptive systems" [76,77], with a strong anchor in biological ecologies and evolutionary selection. In contrast, econophysics is a direct descendant of the more traditional statistical physics and experimental physics approach.

In the next section, we analyze the concept of complexity in econophysics in more detail and explore what novel insights about the dynamics of financial markets flow from it. In particular, we look at how econophysical complexity provides a new understanding of causality, before we conclude by examining whether econophysics can methodologically account for the reflexive behavior that financial markets exhibit.

## 3 Complexity, causality, and reflexivity in econophysics

Viewing physical, biological, or social systems through the lens of complexity has revolutionized many scientific fields over the last decades. The emerging new complexity paradigm, which affected fields as diverse as physics, biology, ecology, or sociology, has furnished explanations to phenomena such as symmetry-breaking, dis-equilibrium, or spontaneous instabilities. Collective or macroscopic phenomena – such as these "critical" phenomena that conflict with the symmetry and equilibrium paradigms, which have dominated physics and biology – are in a bottom-up way understood to be generated by the microscopic interactions of the component parts of a complex system. The aggregate or global states of the system, however, are emergent properties and are not reducible to a particular configuration of the constituents. What is particularly interesting is that these macro-patterns or phenomena can be realized by different systems, i.e., they sometimes exhibit, what physicists call, "universality." Intuitively, economic or financial systems are natural candidates for a complexity approach. In particular, neo-classical economics with its emphasis on equilibrium states, homogeneity of economic agents, and Gaussian distributions, seems especially ripe to transform via the complexity treatment. In the next section, we define a few conceptual properties of complexity and consider briefly how they relate to financial systems, before we examine in more depth how econophysics approaches complexity methodologically.

### 3.1 Complexity

The study of out-of-equilibrium dynamics (e.g. dynamical phase transitions) and of heterogeneous systems (e.g. glasses, rocks) has progressively made popular in physics and then in its sisters branches (geology, biology, etc.) the concept of complex systems and the importance of systemic approaches: systems with a large number of mutually interacting parts, often open to their environment, self-organize their internal structure and their dynamics with novel and sometimes surprising macroscopic "emergent" properties. The complex system approach, which involves seeing interconnections and relationships, i.e., the whole picture as well as the component parts, is nowadays pervasive in the control of engineering devices and business management. It is also plays an increasing role in most of the scientific disciplines, including biology (biological networks, ecology, evolution, origin of life, immunology, neurobiology, molecular biology, etc.), geology (plate-tectonics, earthquakes and volcanoes, erosion and landscapes, climate and weather, environment, etc.), economics and social sciences (including cognition, distributed learning, interacting agents, etc.). There is a growing recognition that progress in most of these disciplines, in many of the pressing issues for our future welfare as well as for the management of our everyday life, will need such a systemic complex system and multidisciplinary approach.

A central property of a complex system is the possible occurrence of coherent large-scale collective behaviors with a very rich structure, resulting from the repeated nonlinear interactions among its constituents: the whole turns out to be much more than the sum of its parts. Recent developments suggest that non-traditional approaches, based on the concepts and methods of statistical and nonlinear physics coupled with ideas and tools from computation intelligence, could provide novel methods into complexity to direct the numerical resolution of more realistic models and the identification of relevant signatures of impending catastrophes. In the following, we identify a few key properties of complex systems that are particularly interesting when one considers financial systems. Most definitions of complexity follow more or less Philip Anderson's classic formulation of complexity, which he gives in his seminal paper "More Is Different" [78]:

> The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear [...].

Financial systems can be considered to qualify as complex systems as they are composed of different levels of complexity that generate new and emergent properties. Hormonally-induced changes in the endocrine system influence, for example, financial risk taking and decision making of individual traders [79] whereas traders are socially affected by imitation and herding dynamics [80]. The resulting global dynamics of markets can in turn trigger collective trader behavior, which can cause financial market crashes [81,82]. Each of these examples can be considered as an emergent property of the underlying level of complexity. Financial systems can thus be characterized as containing a large number of interdependent and mutually interacting microscopic sub-units, which produce non-linear and stochastic dynamics from which novel macroscopic states or properties emerge. The following four properties form the conceptual kernel of complexity relevant for econophysics [82,83]:

- *Non-Linearity*: Whereas in linear models the output is proportional to the cause, non-linearity complicates the relation between output and cause [84,85]. Hence, linear extrapolations from microscopic properties to macroscopic phenomena in non-linear systems are bound to fail. Furthermore, complex nonlinear systems exhibit positive and negative feedback [86]. Negative feedback result when the fluctuations in the output of a system tend to be reduced compared to the disturbances or changes in the input. Under negative feedbacks the system is stable and tends to reverse to the mean. In contrast, positive feedback occurs when a small change in the input of a system or a disturbance of its parameter amplifies into large system-wide perturbations. While negative feedback tends to stabilize the system by forcing it back into its equilibrium state, positive feedback tend to destabilize the system and results in strongly nonlinear oscillations, out-of-equilibrium dynamics, chaotic behavior, or even to transient singular dynamics associated with changes of regimes. In the financial context, positive feedback is interesting as higher (lower) prices, for example, feed back into trader's behavior, thereby accelerating the upward (downward) price trajectory [80] into characteristic transient super-exponential trajectories [87–89].
- *Emergence*: Emergence is a notoriously slippery concept. Minimally, macroscopic properties of a complex system can be characterized as emergent when they are irreducible to the mutually interacting microscopic parts, which generate novel global phenomena on a higher level of complexity. In other words, the aggregate or global phenomena or patterns transcend the dynamics, properties, and configurations of the system's sub-units. Concretely, the knowledge of the microscopic laws does not predict the macro-behavior as many different microscopic laws can lead to the same large-scale behavior while some apparently innocuous changes actually lead to revolutionary alterations at the global level. In that sense, financial markets can exhibit emergent behavior, which is not shared by its constituents [90]. Before crashes, statistical signatures can be identified that result from the increasing global cooperativity and self-organization of markets. A super-exponential accelerating price decorated by log-periodic oscillations reflecting large-scale volatility organization indicates that the market as whole anticipates crashes before its individual parts do [73,91–95].
- *Criticality*: Criticality is another key characteristic of complex systems that can be derived from the microscopic organization and long-range dependence of the system elements. A complex system exhibits criticality when local influences propagate over long distances and it becomes exceedingly sensitive to small perturbations, which can cause massive changes in the overall behavior. A number of extreme events, such as market crashes, have been proposed to belong to the class of critical phenomena. In the case of financial markets, this criticality, which can be observed in the finite-time singular behavior of financial prices before crashes [95], results from the high correlations and cooperation between the system's elements, such as traders, banks, etc. When the market matures towards a so-called "critical point" leading to an unstable phase, a small distortion might trigger a price collapse [86,91,92]. Applying and generalizing the physics of critical phenomena to financial systems has deepened our understanding of market crashes and speculative bubbles, which cannot be explained from the perspective of standard economic theory. When financial systems undergo critical phase transitions, the origin cannot be traced to an exogenous source such as arrival of major news – as post-mortem analyses have revealed – but the critical state often arises endogenously.

– *Qualitative Universality* [96]: The observation that complex systems, under certain conditions, exhibit universality was one of the factors that encouraged physicists in their belief that econophysics could develop as a distinct scientific field. Roughly speaking, universality refers to the fact that physical, biological and social systems have similar properties that can be generalized across many different system classes. Properties of a system are considered universal when they can be described independently of the details of the microscopic organization of its sub-units. While many descriptions of physical systems rely on scale-dependent parameters, scale-invariant descriptions of its behavior – which are often governed by power laws – begin to dominate when complex systems enter a critical phase transition. A specialized and narrower definition of universality is the concept of self-similarity, which in critical phenomena is associated with the notion of universality classes characterized by identical exponents (or fractal dimensions). Studies on herding behavior amongst traders have suggested that financial systems, when they approach a critical point, are self-similar across scales [91,92].

Taken together, these characteristics form the core of the concept of complexity around which most research in econophysics is organized. In some studies, these features are explicitly defined, in others they are part of the theoretical background that is assumed. What is important now is that these definitions of complexity entail a novel conception of causality. This is very relevant as it allows one to tackle the question whether econophysics is equipped with the methods and concepts required to deal with the problem of reflexivity, which is endemic in social systems.

### 3.2 Causality

From the characterization of complex systems in terms of emergence follows that emergent properties are irreducible to or not derivable from the lower levels of complexity. When a complex system exhibits emergence, the higher macro-level, which is the locus of emergent properties, exerts causal influence on the lower micro-level substrate from which these novel features of the system have emerged. This causal propagation of effects from higher to lower levels of complexity is often referred to as "*downward causation*" or "macro-determination" – a term that originated in the context of complex biological systems [97,98]. In physics, it is called "direct cascade" in analogy with the cascade of energy from large to small scales in hydrodynamic turbulence [99]. The existence of such a downward cascade, which collides with the more standard micro to macro cascade, is the main reason for the lack of a generic theory of complex systems. The renormalization group theory has solved essentially the problem of the micro to macro cascade for a restricted class of statistical physics systems [100]. But the occurrence of top-down influences and its interactions with the bottom-up micro-macro cascade make system dynamics and organization much richer and elaborate than we can presently fully fathom. Concretely, financial systems can be considered to exhibit downward causation as the aggregate dynamics of markets can causally influence the microscopic behavior of its individual sub-units [101]. For example, phases of extreme market volatility induce a reaction by traders, which often results in more volatile price behavior [102]. Markets can thus be described through a macro-structure that causally affects all its micro sub-units.

In the context of financial systems, econophysics provides a useful theoretical framework to model, explain, and predict the emerging statistical properties of the aggregate level, which do not exist on the microscopic level. By contrast, standard economic theory is epistemologically and methodologically simply not equipped to fully handle the phenomenon of downward causation. The methodological reason for this failure is the conceptual reductionism that is deeply engrained into standard economic theory. Many current macro-models in standard economics are built on the assumption that the macro is fully reducible to the micro. This reductionist approach in standard economic theory centers on the representative agent framework, which gets rid off the heterogeneity of economic agents [64,103]. In the standard economic literature, there "are no assumptions at the aggregate level, which cannot be justified, by the usual individualistic assumptions. This problem is usually avoided in the macroeconomic literature by assuming that the economy behaves like an individual" [104]. As a consequence of this conceptual reductionism, the methodology of neo-classical economics and finance cannot model the connection between the micro-level with the macro-level beyond the assumption that, roughly speaking, the macro level obeys the same laws as the micro level. By equating the micro-level, which is populated with diverse and heterogeneous economic agents and shaped by their mutual non-linear interactions, to the macro-level, the standard economic framework produces highly unrealistic models, which cannot capture the complex dynamics and evolution of financial markets. In particular, the standard economic approach is ill-suited to explain the occurrence of crises and systemic risks. In the build-up towards the great 2008 crisis in the USA, this blindness was embodied by the claim that the economy has transitioned into a new greater level of functioning, dubbed the "Great Moderation", while in reality this apparently improved performance was bought at the cost of non-sustainable debt and financialization and the build-up of a virtual world increasingly disconnected from the real economic world [105].

The epistemological reason for this breakdown of the standard economic model has to do with the "positive" epistemology of economics, which Milton Friedman introduced in 1953 and that still dominates neo-classical economics and finance today. For Friedman, the ultimate criterion for the scientific validity of a scientific theory lies in the quality of its predictions and not in the realism of its assumptions. Whereas a prediction of a model or hypothesis can be falsified, the underlying theoretical edifice is immune to falsification [106]. It is precisely this conception of a positive scientific methodology that gave rise to the axiomatic and unfalsifiable paradigm of neo-classical economics and finance to which the formation of econophysics reacted.

In the next section, we examine how econophysics approaches and models financial complexity. We then address the more fundamental issue of whether econophysics can methodologically account for reflexivity. The problem of reflexivity in financial markets, in turn, depends on how econophysics can link the micro-level and the macro-level that characterize complex systems. We conclude by discussing briefly the problem of reflexivity and how it relates to the econophysics conception of financial markets as complex systems.

### 3.3 Reflexivity

Given that complex systems, which can be physical, biological or social in nature, are characterized in terms of spontaneous dynamics and properties that emerge from the non-linear interactions and interdependencies of the system's sub-units, a fundamental question arises: what is the source that causes complex systems to behave in this way?

In the context of financial systems, we have seen with the example of the "excess volatility puzzle" that the behavior of asset prices cannot simply be explained in terms of changes in their fundamental values. There is a noise component in the pricing mechanism associated with bounded rationality and out-of-equilibrium processes, which the efficient market hypothesis fails to account for or, rather, hypothesizes away by assuming perfect collective rationality. The question then becomes what is the source of this noise in financial markets. The dynamics of price series cannot be exclusively explained by external "forces" such as the arrival of news, as massive price changes often occur without the presence of any significant piece of information [107,108]. This means that the evolution of financial markets is also subject to endogenous influences, which originate from within the system. Self-organized criticality, and more generally, complex system theory contend that systems with threshold dynamics that are out-of-equilibrium slowly relax through a hierarchy of avalanches of all sizes. Accordingly, extreme events are seen to be endogenous. In economics, endogeneity versus exogeneity has been hotly debated for decades. A prominent example is the theory of Schumpeter on the importance of technological discontinuities in economic history. Schumpeter argued [109] that "evolution is lopsided, discontinuous, disharmonious by nature [...] studded with violent outbursts and catastrophes [...] more like a series of explosions than a gentle, though incessant, transformation." Endogeneity versus exogeneity is also paramount in economic growth theory.

A useful explanation of the endogenous dynamics of markets is rooted in the concept of reflexivity. Popularized by hedge-fund manager and philosopher George Soros, reflexivity captures the fact that, in social or economic systems, expectations of participants influence the evolution of the system, which, in turns, affects the behavior of participants again [2,8]. Soros writes that "the participants' view influence but do not determine the course of events, and the course of events influences but does not determine the participant's view" [8]. The various positive and negative feedbacks, which can be identified in financial system, reflects this financial reflexivity. Reflexive phenomena can thus be characterized by the collision between downward causality and bottom-up aggregation, as discussed above. The microscopic interactions of elements at the lower level (market participants such as traders, hedge funds, regulators, etc.) generate a new macroscopic level of complexity, which, in turn, changes the dynamics and organization of the lower level.

Sharing a widely held view [1,37,110], Soros argues that the fact that financial systems exhibit reflexivity, i.e., that a substantial part of market behavior has a reflexive or endogenous source, demands a scientific method, which is distinct from the tradition of physics. It is argued that, because social systems – the class to which financial markets belong – are fundamentally different from physical system, they are not susceptible to methods borrowed from statistical or theoretical physics. Furthermore, Soros asserts that standard economic theory is "an axiomatic system based on deductive logic, not empirical evidence" [8] and, consequently, it fails to account for the reflexive behavior of financial systems. Based on the argument that one cannot, analogously to physics, identify invariable and universal laws in social systems, Soros then criticizes the "ill-fated attempt by economists to slavishly imitate physics" [8]. As we have shown, the standard economic models, which rest on the assumptions of equilibrium, perfect rationality, and efficient markets, indeed fail to account for the rich reflexive dynamics that exist in financial markets. However, we assert that the problem lies less in the enslavement of financial economics by physics, but in the choices of the physical and mathematical methods and concepts that are getting transferred to economics. As the discussion of the evolution of neo-classical (financial) economics above has shown, economists have been envying or imitating a kind of physics, which was not sufficient to deal with the complexity of financial markets. It was precisely the methodological and conceptual adoption of statistical and out-of-equilibrium physics methods that opened up the possibility to detect the scale-invariant and universal statistical regularities in financial systems that are fundamentally shaped by social and technological forces.

In order to assess whether econophysics has the potential to enlighten the reflexive nature of financial systems, we dissect in the next section how econophysics approaches the complexity of financial markets and address the question whether reflexivity can be quantified.

## 4 Modeling complex financial systems

An understanding of complex systems and of their non-linear dynamics and emergent properties demands the comprehensive modeling of the link between the micro- and macro-levels. However, we can identify in the econophysics literature two methodological currents that deal with these different levels of complexity: agent-based and statistical econophysics. Whereas agent-based approaches aim at modeling the micro-level of financial systems, i.e., the interaction between economic agents, statistical approaches in econophysics are concerned with the macro-level, i.e., the aggregate patterns and phenomena generated by the system's sub-units. These different modeling strategies, which Schinckus [111,112] has identified, are, for example, reflected in two important survey papers by Chakraborti et al. [53,113], which are organized around a review of econophysics research dealing with stylized facts and a survey of research concerning agent-based models. In this section, we clarify the different methodological motivations in modeling complexity in econophysics. We then conclude by evaluating whether econophysics can attack the problem of reflexivity.

### 4.1 Agent-based vs. statistical models in econophysics

The two different strategies of agent-based and statistical modeling approaches in econophysics derive their theoretical foundations both from statistical mechanics and theoretical physics. Agent-based models, which strive to model the microscopic dimension of financial systems, and statistical models, which aim at explaining the macro-regularities or stylized facts that can be identified in financial data, are both data-driven and involve theoretically minimal assumptions. What distinguishes these modeling approaches, however, is their different emphasis on the nature and behavior of economic agents.

Whereas agent-based approaches aim at integrating the learning and adaptive features of market participants, statistical modeling often subdues the individual characteristics of economic agents under the emergent collective organisation. Statistical models often extract stylized facts from past financial time-series by using vast amounts of high-frequency data on prices, volumes, and transactions. The statistical descriptions of these empirical regularities often do not require the specification of the underlying behavioral mechanisms. Obviously, the methodological distinction between the two modeling approaches is not as clear-cut as we present it here. However, in the agent-based literature, we can find research that uses the "order ≡ particle" analogy [113]. Inspired by reaction-diffusion models in physics, Bak et al. [114], for example, simulate price variations on the basis of crowd behavior. The price dynamics, represented as market orders, is mapped onto a model of diffusing and annihilating particles. While this early model has many unrealistic features, the simplicity of the particle-representation inspired richer models based on detailed high-frequency data analyses [115–117]. Often, however, these order models assume so-called "zero-intelligent agents" [118], i.e., economic agents are modeled as particles without any behavioral features. These "particles" obey statistical properties and generate the stylized facts, which are known to exist in financial data, but, they do not have the faculty of anticipation and of forming expectations. When it comes to realism, this atomization of financial dynamics into collisions of unthinking particles can be a drawback of some statistical econophysics models [112]. But it can also be an extraordinary powerful approach for the characterization of the changing risk profiles of financial markets and for predictions [116,117]. This zero-intelligence agent-based model approach thus challenges researchers to identify where higher levels of intelligence might impact the observed structure of financial markets.

Agent-based techniques often compensate for this by enriching their models with adapting, learning, and evolving agents. Similar to the formation of molecules or crystals, these models strive to explain the macro-structures as emerging from the microscopic behavior of a system's heterogeneous components. This strategy results in models of financial markets as adaptive complex systems that evolve in time. Endowing the systems' components with behavioral traits allows for more realistic depictions of market dynamics. However, as LeBaron notes [119], agent-based models, which explain price actions in terms of simple behavioral rules that govern the behavior of market participants, are themselves often exceedingly complex and it can be difficult to isolate the factors responsible for generating the stylized facts. A set of models, which are more faithful in their descriptions of the behavior of real traders and markets, has, for example, introduced behavioral switching mechanisms. These models often divide the population of market participants into two groups – "fundamentalists" and "chartist" or "noise" traders – in order to explain market instabilities, which can be observed in real markets. As these traders can switch between different strategies and states, many empirically observed phenomena such as volatility clustering or herding behavior can be realistically reproduced with this class of agent-based models [120–122]. While statistical models of financial data generate empirically adequate descriptions, this modeling approach needs to be complemented with agent-based modeling. Zhou and Sornette [123] provide, for example, an Ising-model of agent's opinions and how they react to external news. By incorporating behaviorally realistic assumptions, which correspond to evidence in neurobiology and behavioral finance, the model is able to reproduce certain stylized facts that result from crowd behavior (see also the extension of Harras and Sornette [124] to account for the spontaneous emergence of bubbles from over-learning by agents of random news). Put more generally, agent-based models provide micro-foundations to the emergent statistical macro-properties of markets and are able to integrate realistically the heterogeneous features of economic agents, such as the range of preferences, deviations from rationality, and social dynamics such as herding or imitation, which give raise to the stylized facts that statistical approaches seek to reproduce. However, the use of agent-based models is still limited by the difficulties associated with their calibration to empirical data [22].

Given the characterization of reflexivity provided above, it follows that any successful attempt to model the reflexive behavior of market participants must span the micro- as well as the macro-domains of markets. Modeling strategies that only target one level at the expense of the other, seem to inhibit a deeper scientific understanding of reflexivity. Furthermore, it does not do justice to the complexity of financial markets to model solely the connection between their micro- and macro-dimensions. Ultimately, complex systems such as markets are embedded in a wider context or environment; they are part of a nested hierarchy of complexity [125]. In other words, models of reflexivity need to capture additionally the entanglement between exogenous and endogenous causes that influence the global behavior of the system [126]. We now present recent attempts to model reflexivity quantitatively, before we conclude the paper.

### 4.2 Quantifying reflexivity? Endogeneity vs. exogeneity

As it is evident from the discussion above, it is very difficult to scientifically track complex systems, which involve a multitude of mutually and non-linearly interacting parts from which new and surprising phenomena emerge. Complex systems, therefore, demand a new multidisciplinary approach. Nonetheless, over the past two decades, considerable scientific progress has been made by approaching Nature from a complex system science perspective. The complexity of these physical, biological, and social systems has challenged the previous reductionist approach, which consists of decomposing the system into component parts, such that the detailed understanding of the sub-units was believed to generate understanding of the system itself. By contrast, complex systems science approaches the phenomenon through the interactions and links between the sub-units and the system, thereby accounting for positive and negative feedback and downward causation.

Given the complexity of social systems, it has been argued that it is hard to bring them under scientific control [1]. This is due to the fact that the laws or regularities that govern their behavior are difficult to extract. It is also impossible to isolate social and economic systems and to experimentally manipulate them. In particular, the phenomenon of reflexivity often seems “difficult to identify and impossible to quantify” [8]. Ultimately, however, the quantification of the dynamics of financial markets, including their reflexive behavior, is necessary if we want to advance our scientific understanding of financial systems. Only quantification, which allows for the prediction and control of systems, generates a scientific understanding of the target phenomenon. A new science of financial systems, if it were to be effective, needs to be able to quantify the reflexive dynamics that are intrinsic to markets. Before we conclude, we show that is possible to quantitatively disentangle the phenomenon of financial reflexivity by way of the distinction between endogenous and exogenous factors, which contribute to the dynamics of financial markets.

As already mentioned above, the Efficient Market Hypothesis (EMH) assumes that markets almost instantaneously incorporate the flow of information and faithfully reflects it in prices. The “excess volatility puzzle,” amongst other empirical findings, contradicts the main tenet of one of the pillars of standard economic theory. The EMH asserts that, normally, the market efficiently absorbs exogenous shocks and converges towards an equilibrium price, while endogenous processes are absorbed into the price-formation process and disappear as part of the digestion of the exogenous information. Consequently, markets, following the EMH, are only driven by exogenous inputs, and not by endogenous dynamics. However, the reality of financial markets is radically different as price volatility is too high as could be justified by shifts in the underlying fundamental valuations. Furthermore, a variety of studies have refuted the EMH’s assumptions that extreme events in financial systems are induced by the exogenous negative impact of information [80,107,116,117]. Similarly to other complex systems, such as fluctuations in turbulent flows, avalanche dynamics, or earthquakes, financial markets exhibit an endogenous dynamic that is very complex, whereas exogenous forces, which drive the system, are often regular and steady [126,129]. In other words, the behavior of markets is driven less by exogenous events and more by the endogenous dynamics of trading activity itself. The circular loop between price and trading – i.e., past price changes that feed on themselves – results in the erratic deviations from fundamentals, which are otherwise puzzling for standard economic theory but take a natural meaning when accepting the ubiquitous role of endogeneity.

In a series of studies, Sornette and collaborators have introduced measures of the degree of reflexivity or endogeneity in financial systems and built a theoretical framework that allows one to disentangle exogenous from endogenous sources of financial markets crashes [91,130–132]. In these studies, exogeneity refers to the external “forces” that influence the evolution of the system whereas endogeneity captures the self-reinforcing positive feedback processes within the system. Given the limited space, we can provide here only a brief overview of this research to give an intuition of the possible quantification of reflexivity. Combining statistical test of drawdowns distributions (runs of losses) and Log-Periodic Power Law Singular (LPPLS) detection techniques, Johansen and Sornette [133,134] showed that the extreme tails of the distribution belong to a different population than the body, analogue to the different physics that describe distributions in the study of turbulent hydrodynamic flows [135]. They then tested whether a LPPLS structure is present in the price trajectory, which precedes these "outliers," or "Dragon Kings" as Sornette [136] calls them (see also [137]). The emergence of log-periodic power law singular features is a qualifying signature of the endogenous dynamics, which might result in a market crash. As Sornette and co-workers has extensively documented, bubbles manifest themselves in super-exponential power-law accelerations in the price dynamics, which is decorated by log-periodic precursors. In their study, Sornette and Johansen [134] are able to identify two classes of market crashes: exogenously caused crashes that are not preceded by a LPPLS price trajectory and for which an exogenous shock can be identified, and crashes that are triggered endogenously by trading activity. The later are roughly twice as frequent as the former. Viewing crashes in terms of the endogenous dynamics of the market itself has important ramifications for our understanding of extreme event in financial systems. According to the view that most market crashes have an endogenous source – i.e., the increased cooperativity and self-organizing interactions between market participants – exogenous shocks only serve as triggering factors [135]. In other words, identifying proximal causal factors of a crash is often futile – as the extensive literature reflects, which presents diverse and often conflicting evidence about the origins of crashes (see [138]) – as the crash results from the maturation towards an intrinsically unstable phase.

In two more recent studies, our group has provided, for the first time, a quantification of reflexivity that allows us to precisely measure the levels of endogenity in a financial system [131,132]. For this, the so-called self-exciting Hawkes model is calibrated to financial market dynamics. This statistical model was initially developed to model earthquake clustering. As the Hawkes process formalism is able to describe the pattern of foreshocks and aftershocks, which result from the release of accumulated stress between tectonic plates, it is adaptable to financial markets where different regimes of volatility relaxation have been documented. For example, Sornette et al. [139] have shown that the relaxation time of a volatility burst is different after a strong exogenous shock compared with the relaxation of volatility after a peak with no identifiable exogenous source. They suggest that volatility can be understood in terms of response functions of financial agents, which derive from their behavior. By applying the Hawkes process analysis to E-mini S&P 500 futures, Filimonov and Sornette [131] are able to measure the degree of reflexivity as the proportion of price moves due to endogenous interactions to the total number of all price moves, which also include the impact of exogenous news. The self-exciting Hawkes branching process – in which each price changes may lead to an epidemic of other prices changes – allows one to identify different classes of volatility shocks along the separation between endogenous and exogenous dynamics. The Hawkes model has a key parameter, the "branching ratio" "$n$," – i.e., the fraction of endogenous events within the whole price-change population – that enables the direct measurement of the level of endogeneity. Interestingly, this measure reproduces robust behavioral feature of increased herding behavior at short time-scales in times of fear and panic. Filimonov and Sornette's [131] Hawkes process analysis of E-mini S&P 500 futures data from the period from 1998 to 2010 reveals a dramatic increase of endogeneity from 30% to 70–80% of trades triggered by past trades, an effect they attribute to the rise of high-frequency and algorithmic trading.

In a subsequent study on the endogeneity or reflexivity in commodity markets – using a Hawkes self-excited conditional Poisson model on time-series of past price-changes – Filimonov et al. [132] find that more than one out of two price changes is triggered by another price change, indicating a self-reinforcing reflexive mechanism underling the price time-series. Interestingly, Filimonov et al. [132] show that the level of endogeneity does not depend on the intensity of the information about exogenous events, which, as they document, has remained relatively stable over the analyzed period (second half of 20$^{\text{th}}$ century to first decade of 21$^{\text{st}}$ century). They further argue that increased reflexivity leads to a slower convergence of prices towards fundamental values, rendering the price formation process thereby less efficient. The study further shows that high levels of endogeneity or reflexivity result in a larger sensibility of the system to exogenous distortions. Endogenous feedback mechanisms in trading activity can amplify small initial shocks, which might, as it was the case with the May 6, 2010 flash crash, cascade into large crashes.

The studies on the endogenous versus exogenous sources of price volatility indicate that, far from being unquantifiable, reflexive financial phenomena can be disentangled and measured. This research seems to represent a first step towards the full quantification of reflexivity, from which novel insight about trader behavior, the evolution of bubbles, and the emergence of crashes will follow. While it is far from easy to get a quantitative grip on reflexivity [140][6], physics already possesses the concepts, tools, and methods needed to cope with reflexive phenomena. This has been shown in many other scientific fields involving complex systems. In principle, it can be concluded that the problem of reflexivity, as qualitatively described by Soros and many others, does not seem to inhibit the development of a new science of financial markets.

## 5 Conclusion

Historically, the collisions between physics and (financial) economics have resulted in the axiomatization of mainstream economics and gave rise to highly idealized models, which tend to be detached from financial reality. Although we do not deny that standard economic theory has generated deep insights into economic behavior, the neo-classical theoretical edifice with its fundamental pillars of rational expectations, efficiency, and equilibrium, nonetheless failed to deal with non-linear and out-of-equilibrium dynamics of complex financial systems, which are comprised of a myriad of heterogeneous agents interacting with each other. In particular, its failures became apparent to many during and after the great 2008 financial crisis [63,142,143].

Over the last two decades, however, econophysics has substantially advanced the scientific understanding of financial markets. The major contribution of econophysics was, as we have argued above, to view financial markets as complex systems. Describing markets in terms of emergence, scale-invariance, universality, and other properties of complexity, which we conceptually explicated above, allows one to better understand how the microscopic level – which is populated with mutually interacting agents that exhibit a diversity of behavioral traits, heterogeneous preferences and expectations – is linked with the macroscopic level of complexity, on which statistical regularities and patterns, i.e., the so-called stylized facts, can be identified. Econophysics has provided solid empirical foundations for the study of financial systems, which resulted in the falsification of many a priori assumptions of standard economic models. However, at the sociological level, the effectiveness of econophysics in the eyes of the economic profession has been limited due to econophysicists' disregard of standard economic theory and their misplaced aspiration to completely replace it with econophysics without due attention to previous achievements.

Although analogizations between physical and financial systems and extrapolations of concepts and methods of physics to (financial) economics can lead to oversimplified and idealized models of markets, physical concepts and techniques nonetheless provide a useful unifying framework to approach complexity. Contrary to the criticism that financial markets cannot be rendered intelligible by using the

[6] see [132,141] for recent technical developments and improvements.

scientific method of physics, we argued that, by integrating the insights from complex systems science, considerable progress has been made in the modeling of financial market dynamics. In fact, contrary to many claims to the contrary, we have argued that the problem of reflexivity does not demand a new scientific method. As we have documented above, research that applies the Hawkes process model, which was motivated by physics (or more precisely geophysics), to financial systems represents one of the first steps in the quantification of reflexive financial phenomena.

Nevertheless, it is difficult to argue that a new science of financial markets should solely have physics-based foundations. Understood in terms of the research program encoded in the term, econophysics, seems to be unnecessarily restrictive. At present, the most exciting progress seems to be unraveling at the boundaries between finance and the biological, cognitive, and behavioral sciences [144–146]. Recently, there have been many attempts that have started to explore the notion that financial markets are similar to ecologies, populated by species (traders, firms, etc.) that adapt and mutate. Farmer [147], for example, proposed a theory that views markets as ecologies in which – analogous to the evolution of a biological species – better-adapted strategies exploit old-strategies. Hommes [148] reviews the modeling of markets as evolutionary systems in which the survival of different trading strategies can be compared. Lo [149–151] proposed the "adaptive market hypothesis," which characterizes markets less in terms of efficiency, but rather in terms of competition and adaptation. Sornette [22] introduced the "Emerging Intelligence Market Hypothesis", according to which the continuous actions of investors, which are aggregated in the prices, produce a "market intelligence" more powerful than that of most of them. The "collective intelligence" of the market transforms most (but not all) strategies into losing strategies, just providing liquidity and transaction volume. Evolutionary models are able to explain most of the stylized facts documented in the econophysics literature (see [113]). These evolutionary or biological approaches provide another exiting source of inspiration for modeling financial reflexivity.

Concluding, econophysics should not be considered as isolated from other complexity-based approaches in science. What generates the statistical phenomena or patterns, which econophysics dissects with the tools of statistical physics, are fundamentally sociobiological systems. In other words, econophysics needs to reach beyond physics and integrate the concepts, methods, and tools from other disciplines. There can be a physics of financial markets, but the science we envision that helps to understand, diagnose, predict, and control financial markets [152,153] needs to integrate these fields into a complex systems science of financial markets.

## References


1. E. Derman, *Models. Behaving. Badly: Why Confusing Illusion with Reality Can Lead to Disaster, on Wall Street and in Life* (Free Press, New York, 2011)
2. G. Soros, *The Alchemy of Finance: Reading the Mind of the Market* (John Wiley and Sons, Hoboken NJ, 1987)
3. E. Grunberg, F. Modigliani, J. Polit. Econ. **62**, 465 (1954)
4. H.A. Simon, Public. Open. Quart. **18**, 245 (1954)
5. R.C. Buck, Philos. Sci. **30**, 373 (1963)
6. M.K. Vetterling, Philos. Sci. **43**, 278 (1976)
7. E. Grunberg, Am. J. Econ. Sociol. **45**, 475 (1986)
8. G. Soros, J. Econ. Methodol. **20**, 309 (2013)
9. Y.C. Zhang, Physica A **269**, 30 (1999)
10. J.P. Bouchaud, Quant. Finance **1**, 105 (2001)
11. E. Derman, *My Life as a Quant: Reflections on Physics and Finance* (John Wiley and Sons, Hoboken NJ, 2004)

12. J.D. Farmer, T. Lux, J. Econ. Dyn. Control **32**, 1 (2008)
13. F. Jovanovic, C. Schinckus, Hist. Polit. Economy **45**, 443 (2013)
14. I. Newton, *The Principia: Mathematical Principles of Natural Philosophy: A New Translation* (University of California Press, Berkeley, 1999)
15. A. Smith, *An Inquiry into the Nature and Causes of the Wealth of Nations*, edited by R.H. Campbell, A.S. Skinner, W.B. Todd (Oxford University Press, Oxford, 1976)
16. P.S. Laplace, in *Philosophical Essay on Probabilities: Translated from the Fifth French Edition of 1825 with Notes by the Translator*, edited by A.I. Dale (Springer, New York, 1995)
17. A. Quetelet, *A Treatise on Man and the Development of his Faculties*, edited by T. Smibert (Cambridge University Press, Cambridge 2013)
18. S. Galam, Y. Gefen, Y. Shapir, J. Math. Sociol. **9**, 1 (1982)
19. S. Galam, *Sociophysics: A Physicist's Modeling of Psycho-political Phenomena* (Springer, Berlin, 2012)
20. B. Ingrao, I. Israel, *The Invisible Hand: Economic Equilibrium in the History of Science* (MIT Press, Cambridge MA, 1990)
21. J.D. Farmer, J. Geanakoplos, Complexity **14**, 11 (2009)
22. D. Sornette, Rep. Prog. Phys. **77**, 062001 (2014)
23. V. Pareto, in *Oeuvres Complètes de Vilfredo Pareto*, edited by G. Busino (Librairie Droz, Geneva, 1981)
24. B. Mandelbrot, *The Fractal Geometry of Nature* (W.H. Freeman, San Francisco, 1982)
25. P. Bak, *How Nature Works: The Science of Self-organized Criticality* (Copernicus, New York, 1996)
26. D. Sornette, *Critical Phenomena in Natural Sciences, Chaos, Fractals, Self-organization and Disorder: Concepts and Tools*, 2nd edn. (Springer, Berlin, 2004)
27. R. Perline, Stat. Sci. **20**, 68 (2005)
28. A. Clauset, C.R. Shalizi, M.E.J. Newman, SIAM Rev. **51**, 661 (2009)
29. L. Bachelier, in *Louis Bachelier's Theory of Speculation: The Origins of Modern Finance*, edited by M. Davis, A. Etheridge (Princeton University Press, Princeton NJ, 2011)
30. A. Einstein, Ann. Phys. **322**, 549 (1905)
31. M.F.M. Osborne, Oper. Res. **7**, 145 (1959)
32. P.A. Samuelson, Indust. Manag. Rev. **7**, 41 (1965)
33. H. Markowitz, J. Finance **7**, 77 (1952)
34. W.F. Sharpe, J. Finance **19**, 425 (1964)
35. F. Black, M. Scholes, J. Polit. Economy **81**, 637 (1973)
36. R. Merton, Bell J. Econ. **4**, 141 (1973)
37. A.W. Lo, M.T. Mueller, J. Portfolio Manage. **8**, 13 (2010)
38. P.A. Samuelson, *Foundations of Economic Analysis* (Harvard University Press, Cambridge MA, 1947)
39. J.D. Farmer, J. Econ. Methodol. **20**, 377 (2013)
40. Y.C. Zhang, Europhys. News **29**, 51 (1998)
41. H.E. Stanley, L.A.N. Amaral, D. Canning, P. Gopikrishnan, Y. Lee, Y. Liu, Physica A **269**, 156 (1999)
42. H. Takayasu, H. Miura, T. Hirabayashi, K. Hamada, Physica A **184**, 127 (1992)
43. J.P. Bouchaud, D. Sornette, J. Phys. I France **4**, 863 (1994)
44. D. MacKenzie, *An Engine, Not a Camera: How Financial Models Shape Financial Markets* (MIT Press, Cambridge MA, 2006)
45. R.J. Shiller, Am. Econ. Rev. **71**, 421 (1981)
46. R.J. Shiller, *Market Volatility* (MIT Press, Cambridge MA, 1989)
47. S.F. LeRoy, R.D. Porter, Econometrica **49**, 555 (1981)
48. P. Krugman, *The Self-Organizing Economy*, 2nd edn. (Wiley-Blackwell, Oxford, 1996)
49. W.B. Arthur, Am. Econ. Rev. **84**, 406 (1994)
50. J. Voit, *The Statistical Mechanics of Financial Markets* (Springer, Berlin, 2005)

51. R.N. Mantegna, H.E. Stanley, *Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, Cambridge, 2000)
52. R. Cont, Quant. Finance **1**, 223 (2001)
53. A. Chakraborti, I.M. Toke, M. Patriarca, F. Abergel, Quant. Finance **11**, 991 (2011)
54. J.P. Bouchaud, M. Potters, *Theory of Financial Risk and Derivative Pricing: From Statistical Physics to Risk Management*, 2nd edn. (Cambridge University Press, Cambridge, 2003)
55. B. Mandelbrot, Quart. J. Econ. **76**, 57 (1962)
56. B. Mandelbrot, J. Bus. **36**, 394 (1963)
57. B. Mandelbrot, J. Bus. **39**, 242 (1966)
58. E.F. Fama, J. Bus. **36**, 420 (1963)
59. E.F. Fama, Manag. Sci. **11**, 404 (1965)
60. P.A. Samuelson, J. Finan. Quant. Anal. **2**, 107 (1967)
61. J.P. Bouchaud, D. Sornette, C. Walter, J.P. Aguilar, Int. J. Theor. Appl. Finance **1**, 25 (1998)
62. P.H. Cootner, *The Random Character of Stock Market Prices* (MIT Press, Cambridge MA, 1964)
63. D. Colander, M. Goldberg, A. Haas, K. Juselius, A. Kirman, T. Lux, B. Sloth, Crit. Rev. **21**, 249 (2009)
64. J.L. McCauley, *Dynamics of Markets: Econophysics and Finance* (Cambridge University Press, Cambridge, 2004)
65. S. Keen, Physica A **324**, 108 (2003)
66. M. Gallegati, S. Keen, T. Lux, P. Ormerod, Physica A **370**, 1 (2006)
67. P. Ball, Nature **441**, 686 (2006)
68. J.B. Rosser, J. Econ. Perspect. **13**, 169 (1999)
69. T.C. Schelling, Am. Econ. Rev. **59**, 488 (1969)
70. T.C. Schelling, J. Math. Sociol. **1**, 143 (1971)
71. T.C. Schelling, *Micromotives and Macrobehavior* (W.W. Norton, New York, 1978)
72. R. Frigg, J. Reiss, Synthese **169**, 593 (2009)
73. J.M. Epstein, *Generative Social Science: Studies in Agent-Based Computational Modeling* (Princeton University Press, Princeton NJ, 2006)
74. Z. Burda, J. Jerzy, M.A. Nowak, Acta Phys. Pol. B **34**, 87 (2003)
75. K. Arrow, P. Anderson, D. Pines (eds.), *The Economy as an Evolving Complex System* (Addison Wesley, Reading MA, 1988)
76. W.B. Arthur, S. Durlauf, D. Lane (eds.), *The Economy as an Evolving Complex System II* (Addison Wesley, Reading MA 1997)
77. L. Blume, S. Durlauf (eds.), *The Economy as an Evolving Complex System III* (Oxford University Press, New York, 2006)
78. P.W. Anderson, Science **177**, 393 (1972)
79. J.M. Coates, *The Hour Between Dog and Wolf: Risk Taking, Gut Feelings and the Biology of Boom and Bust* (Penguin Press, New York, 2012)
80. D. Sornette, *Why Stock Markets Crash: Critical Events in Complex Financial Systems* (Princeton University Press, Princeton NJ, 2003)
81. T. Kaizoji, D. Sornette, in *Encyclopedia of Quantitative Finance*, edited by R. Cont (John Wiley and Sons, Hoboken NJ, 2010) (long version at `http://arXiv.org/abs/0812.2449`)
82. S. Durlauf, Econ. J. **115**, F225 (2005)
83. W.B. Arthur, in *Complexity and the Economy*, edited by W.B. Arthur (Oxford University Press, New York, 2014)
84. D.A. Hsieh, J. Bus. **62**, 339 (1989)
85. D.A. Hsieh, Financ. Anal. J. **51**, 55 (1995)
86. D. Sornette, P. Cauwels, Rev. Behav. Econ. **2**, 279 (2015)
87. A. Hüsler, D. Sornette, C.H. Hommes, J. Econ. Behav. Organ. **92**, 304 (2013)
88. T. Kaizoji, M. Leiss, A. Saichev, D. Sornette, J. Econ. Behav. Org. **112**, 289 (2015)
89. M. Leiss, H.H. Nax, D. Sornette, J. Econ. Dyn. Control **55**, 1 (2015)

90. D. Sornette, Physica A **284**, 355 (2000)
91. A. Johansen, D. Sornette, O. Ledoit, J. Risk **1**, 5 (1999)
92. A. Johansen, O. Ledoit, D. Sornette, Int. J. Theor. Appl. Finance **3**, 219 (2000)
93. Z.Q. Jiang, W.X. Zhou, D. Sornette, R. Woodard, K. Bastiaensen, P. Cauwels, J. Econ. Behav. Organ. **74**, 149 (2010)
94. D. Sornette, R. Woodard, W. Yan, W.X. Zhou, Physica A **392**, 4417 (2013)
95. D. Sornette, P. Cauwels, J. Risk Manag. Finan. Institutions **8**, 83 (2015)
96. D. Sornette, Phys. World **15**, 50 (2002)
97. D.T. Campbell, in *Studies in the Philosophy of Biology*, edited by F.J. Ayala, T. Dobzhansky (University of California Press, Berkley, 1974)
98. G.F.R. Ellis, T. Roy. Soc. S. Afr. **63**, 69 (2008)
99. U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press, Cambridge, 1996)
100. N. Goldenfeld, *Lectures on Phase Transitions and the Renormalization Group* (Addison-Wesley, Boston, 1992)
101. D. Delli Gatti, E. Gaffeo, M. Gallegati, G. Giulioni, A. Palestrini, *Emergent Macroeconomics: An Agent-Based Approach to Business Fluctuations* (Springer, Milan, 2008)
102. R. Cont, in *Fractals in Engineering*, edited by J. Lévy-Véhel, E. Lutton (Springer, London, 2005)
103. M. Gallegati, in *Econophysics of Markets and Business Networks*, edited by B.K. Chakrabarti, T. Kaizoji, Y. Wang, Y. Fujiwara, S. Sinha, J. Mimkes (Springer, Milan, 2007)
104. W. Hildebrand, A. Kirman, *Equilibrium Analysis: Variations on the Themes by Edgeworth and Walras* (North-Holland, Amsterdam, 1988)
105. D. Sornette, P. Cauwels, Risks **2**, 103 (2014)
106. M. Friedman, *Essays in Positive Economics* (University of Chicago Press, Chicago, 1953)
107. D.M. Cutler, J.M. Poterba, L.H. Summers, J. Portfolio Manage. **15**, 4 (1989)
108. D.M. Cutler, J.M. Poterba, L.H. Summers, Rev. Econ. Stud. **58**, 529 (1991)
109. J.A. Schumpeter, *Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process* (McGraw-Hill, New York, 1939)
110. P. Mirowski, *More Heat than Light: Economics as Social Physics, Physics as Nature's Economics* (Cambridge University Press, Cambridge 1991)
111. C. Schinckus, Quant. Finance **12**, 1189 (2012)
112. C. Schinckus, Contemp. Phys. **54**, 17 (2013)
113. A. Chakraborti, I.M.T. Patriarca, F. Abergel, Quant. Finance **11**, 991 (2011)
114. P. Bak, M. Paczuski, M. Shubik, Physica **A 246**, 430 (1997)
115. S. Mike, J.D. Farmer, J. Econ. Dynam. Control **32**, 1 (2008)
116. Y. Yura, H. Takayasu, D. Sornette, M. Takayasu, Phys. Rev. Lett. **112**, 098703 (2014)
117. Y. Yura, H. Takayasu, D. Sornette, M. Takayasu, Phys. Rev. E **92**, 042811 (2015)
118. J.D. Farmer, P. Patelli, I.I. Zovko, Proc. Natl. Acad. Sci. USA **102**, 2254 (2005)
119. B. LeBaron, in *Handbook of Computational Economics Volume II: Agent-Based Computational Economics*, edited by L. Tesfatsion, K.L. Judd (North-Holland, Amsterdam, 2006)
120. T. Lux, M. Marchesi, Nature **397**, 498 (1999)
121. T. Lux, M. Marchesi, Int. J. Theor. Appl. Finance **3**, 675 (2000)
122. T. Lux, Working Paper No. 1425, University of Kiel, 2008
123. W.X. Zhou, D. Sornette, Eur. Phys. J. B **55**, 175 (2007)
124. G. Harras, D. Sornette, J. Econ. Behav. Organ. **80**, 137 (2011)
125. H.A. Simon, in *Proceedings of the American Philosophical Society* **106**, 467 (1962)
126. D. Sornette, in *Extreme Events in Nature and Society*, edited by S. Albeverio, V. Jentsch, H. Kantz (Springer, Berlin, 2005)
127. A. Joulin, A. Lefevre, D. Grunberg, J.P. Bouchaud, Wilmott Magazine **46** (Sept./Oct. 2008)

128. J.P. Bouchaud, in *Lessons from the 2008 Crisis*, edited by A. Berd (Risk Books, Incisive Media, London, 2011)
129. J.P. Bouchaud, Procedia Comput. Sci. **7**, 22 (2011)
130. A. Johansen, D. Sornette, in *Contemporary Issues in International Finance*, edited by F. Columbus (Nova Science, New York, 2004)
131. V. Filimonov, D. Sornette, Phys. Rev. E **85**, 056108 (2012)
132. V. Filimonov, D. Bicchetti, N. Maystre, D. Sornette, J. Int. Money Finance **42**, 174 (2014)
133. A. Johansen, D. Sornette, J. Risk **4**, 69 (2001)
134. A. Johansen, D. Sornette, Cah. Econ. Bruxelles **53**, 201 (2010)
135. D. Sornette, Phys. Rep. **378**, 1 (2003)
136. D. Sornette, Int. J. Terraspace Sci. Eng. **2**, 1 (2009)
137. D. Sornette, G. Ouillon, Eur. Phys. J. Special Topics **205**, 1 (2012)
138. A.W. Lo, J. Econ. Lit. **50**, 1 (2012)
139. D. Sornette, Y. Malevergne, J.F. Muzy, Risk **16**, 67 (2003)
140. V. Filimonov, D. Sornette, Quant. Finance **15**, 1293 (2015)
141. S. Wheatley, V. Filimonov, D. Sornette, Comput. Stat. Data Analysis **94**, 120 (2016)
142. P. Krugman, Eastern Econ. J. **37**, 307 (2011)
143. D.J. Bezemer, J. Econ. Issues **45**, 1 (2011)
144. C.F. Camerer, G. Loewenstein, M. Rabin (eds.), *Advances in Behavioral Economics* (Princeton University Press, Princeton NJ, 2003)
145. R.J. Shiller, J. Econ. Perspect. **17**, 83 (2003)
146. R.H. Thaler (ed.), *Advances in Behavioral Finance: Volume II* (Princeton University Press, Princeton NJ, 2005)
147. J.D. Farmer, Indust. Corp. Change **11**, 895 (2002)
148. C.H. Hommes, Quant. Finance **1**, 149 (2001)
149. A.W. Lo, J. Portfolio Manag. **30**, 15 (2004)
150. A.W. Lo, J. Invest. Consulting **7**, 21 (2005)
151. A.W. Lo, Financ. Anal. J. **68**, 18 (2012)
152. S. Cincotti, D. Sornette, P. Treleaven, S. Battiston, G. Caldarelli, C. Hommes, A. Kirman, Eur. J. Phys. Special Topics **214**, 361 (2012)
153. H.L.D. de S. Cavalcante, M. Oriá, D. Sornette, E. Ott, D.J. Gauthier, Phys. Rev. Lett. **111**, 198701 (2013)