INTRODUCTION

Exploitation of marble quarries in the Apuan Alps (Italy) requires a careful assessment of the stability of high and steep faces, created and continuously modified by the excavation activity. Spacing of discontinuities in the areas selected for quarries is generally wide. In fact, excavated blocks must be suitable to be employed as ornamental stones. Despite the generally massive structure of the rock mass, the presence of discontinuity sets of high persistency makes the assessment and control of safety conditions particularly difficult.

Stability analyses must be continuously updated as excavation advances and in situ surveys give new data on the geo-structural conditions. For this reason ordinary limit equilibrium methods (LEM) and stress analysis methods are both widely utilized. LEM methods present the advantage of easily identifying an overall “factor of safety”, but need a priori hypotheses on the failure mechanism. On the contrary, stress analysis methods allow to investigate the failure mechanism more thoroughly, but require specific and usually more complex calculations to obtain a suitable “measure” of stability.

In the context of FEM applications, Zienkiewicz et al. [1] proposed to evaluate the factor of safety utilizing the stresses deriving from finite element analyses. This procedure was refined by Naylor [2] and called “enhanced limit” method. In order to assess the safety conditions of slopes many other Authors [3, 4, 5, 6 and 7] have performed stress analyses applying the “shear strength reduction method”, i.e. a progressive reduction of the strength parameters until failure is eventually reached. The method is based on iterative techniques and can require long computational times.

The estimate of a global stability factor of the slope has to be added to the typical advantages of stress analysis methods. In the special case of blocky rock masses the Distinct Element Method (DEM) represents a powerful tool for analyzing complex failure mechanisms, such as instabilities due to the combined sliding and rotation of blocks.

In this note the stability of a plane section of a typical face of a marble quarry is analyzed by DEM modelling. Spacing and persistency of major joint...
sets have been explicitly accounted for. Staged analyses representing the future excavations have been carried out. The differences in the stress severity at the various excavation stages have been highlighted by calculating non-conventional stability indexes.

2. SITE DESCRIPTION

The quarry is located on the north-western slope of Monte Bettogli which belongs to the Apuan Alps mountain range (Carrara, Italy). The area of the quarry is within the southern zone of the Apuan Metamorphic Complex. The rock mass experienced a regional metamorphism in the green-schist facies during the tectonic phase (Lower Lias) which gave rise to Appenines.

The quarried rock is marble, sold as an ornamental stone called ‘Marmo Bianco’.

The area of the quarry is a very deep and long gully reshaped by vertical or stepped cuts, reaching a height of 180 m. The excavation activity develops between 540 - 665 m a.s.l. height. At present excavation is localized at the bottom of the gully, while the natural slope remains untouched in the upper part (Figs. 1 and 2). Benches have a typical height of 8 m (Fig. 1).

One of the excavated faces, having northwards dip, is particularly high. Section A in Figure 2 is drawn perpendicularly to this face.

3. GEOTECHNICAL INVESTIGATIONS

3.1. Rock Material

The rock material is slightly grayish-white coloured, its texture is crystalline and massive, with homogeneous grain size (medium crystal size of 0.5 mm). It is also characterized by a slight chromatic variation (brownish to dark grayish), called “macchia”, associated to schistosity.

The index properties of the rock material exhibit quite homogeneous characteristics. Table 1 provides the mean values of the main geotechnical properties of the material.

Table 1. Principal characteristics of the rock material: dry density ($\rho_d$), P- and S-wave velocities ($V_p$ and $V_s$), uniaxial strength ($\sigma_f$), secant Young modulus ($E_{s,50}$).

<table>
<thead>
<tr>
<th>$\rho_d$ (Mg/m$^3$)</th>
<th>$V_p$ (km/s)</th>
<th>$V_s$ (km/s)</th>
<th>$\sigma_f$ (MPa)</th>
<th>$E_{s,50}$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.71</td>
<td>6.04</td>
<td>3.25</td>
<td>86.7</td>
<td>70.4</td>
</tr>
</tbody>
</table>

Triaxial and uniaxial tests carried out on 13 specimens allowed to determine the strength envelope, which appears to be only slightly non-linear. A cohesion of 18 MPa and a friction angle of 44° were obtained. Tensile strength determined by Brazilian tests is 8.9 MPa.
3.2. Description of Discontinuities

Quantitative analysis of fracturing was performed through statistical surveys carried out in different locations within the quarry area.

On a large scale, discontinuities appear as wavy, very irregular and frequently stepped. In the upper zone discontinuities are generally less inclined, in comparison to the bottom.

Joint orientation was measured on variably oriented faces, belonging either to artificial cuts or to the natural slope. The traditional compass survey, as well as a terrestrial laser scanner survey, were utilized. The good correspondence between the two sets of data have assessed the capability of the terrestrial laser scanner to provide high-quality structural surveys of the discontinuity orientations [8].

The poles of planes are represented in Figure 3 on an equal-area stereographic projection. From the spatial distribution of the poles four sets of discontinuities were recognized (Table 2).

Table 2. Mean orientations of the discontinuities sets.

<table>
<thead>
<tr>
<th>Sci</th>
<th>Ja</th>
<th>Jb</th>
<th>Jc</th>
</tr>
</thead>
<tbody>
<tr>
<td>dip direction (°)</td>
<td>221</td>
<td>319</td>
<td>99</td>
</tr>
<tr>
<td>dip (°)</td>
<td>67</td>
<td>78</td>
<td>48</td>
</tr>
</tbody>
</table>

In Figure 3 it is evident that most of the measured poles belong to the sets of discontinuities Ja and Sci, while Jb is the less frequent set.

In the area of section A the schistosity set (Sci) has a dip direction towards the rock mass (Fig. 3). The joint set Ja appears in the whole quarry area and it is by far the most important one, due to its higher persistency. Discontinuities of the Ja set also facilitate the detachment of marble blocks during excavation activity. The set Jc is less recurrent in the zone. In the section A the traces of the families Jc and Ja overlap (Fig. 3).

The sets of discontinuities Sci and Ja display spacings from 0.3 m to values greater than 3 m, while the Jc set varies from 1 m to values greater than 10 m. A great decreasing of spacing from the bottom to the top of the slope can be observed, so that the quarry area is characterized by a wide spacing, up to values of 4 m.

The areal extent of discontinuities varies for the different sets of discontinuities. The discontinuity trace lengths on the surface of exposures widely vary from 1 m (Sci and Jc sets) up to 20 m (Ja set).

Aperture can be considered tight and rarely some discontinuities of the Ja set show an aperture up to moderately wide (5 mm).

Discontinuities of the Ja and Jc sets often present patinas due to oxidation and occasionally a pedogenic filling.

Generally all the discontinuities are not very rough and undulated. Roughness parameter JRC [9] from profilometer measurements, determined on 50 mm cylindrical specimens, provided a very wide distribution, from 2 to 8. Roughness was also determined from back-calculation of the shear strength (8 tests on the same previous specimens), obtaining an overall roughness parameter equal to 5.9.

Tilt tests were performed on two groups of specimens, having length of 50 mm (5 tests) and 260 mm (4 tests), respectively. The smaller specimens confirmed the profilometer results, while the larger specimens provided lower values of JRC: mean value equal to 4.7, instead of 6.9 from the profilometer.

With the profilometer a single measurement was performed at the field-scale (length of 0.90 m). The JRC derived from the Z2 parameter [10] is equal to 4.8 or 3.7, respectively when a base length of 0.9 m or 0.1 m is utilized. The increase in JRC as the base length increases could be ascribed to waviness.
Normal stiffness of discontinuities was investigated through compression tests on core specimens, obtaining values in the range 20-80 GPa/m.

Laboratory shear tests on a smooth sawed surface gave a basic friction angle of 32.2°. Cyclic shear tests at increasing normal stress provided an ‘ultimate’ friction angle of 37°.

Extensive series of Schmidt hammer tests, carried out both on fresh sawed surfaces and on natural discontinuities, indicated a slight alteration of the joint walls.

4. STABILITY ANALYSES BY DISTINCT ELEMENT METHOD

Stability analyses were focused on a vertical cross section through the central zone of the quarry (Section A in Fig. 2). This plane section (strike N22°E) cuts the northwards face of the gully almost perpendicularly, where the excavation activity has since created a very steep profile as high as 180 m. The analyzed sector is therefore likely to represent one of the most critical situations for the ongoing exploitation of the quarry.

The basic idea of the performed study is first of all to obtain a sufficiently realistic model for the blocky rock mass, validated against the current excavation phase, and then use it as a predictive tool of the mechanical response for future excavation plans.

As illustrated in the following, the geo-structural setting of the North face is deemed to be suitable to a plain strain approach. Specifically, the DEM code UDEC [11] was utilized.

Recourse to deformable blocks, instead of rigid blocks, in the context of stability-oriented analyses, is essentially motivated by the better accuracy achieved in the representation of joint contact forces and rock bridges. Nevertheless, modelling of large scale problems in fractured rocks mass, as in the present case, implies a strong idealization of the effective blocky structure.

The rock material was assumed to be linear elastic, bulk and shear modulus 61 and 27 GPa, respectively, unit volume weight 27 kN/m³.

Initial stress state was simulated through the excavation of the natural slope preceding the current geometry; the unknown profile of the original natural slope was assumed symmetrical to the slope on the opposite side, not subjected to quarry activity (Fig. 4).

4.1. Representation of Discontinuities and Rock Bridges

The sets of discontinuities Ja and Sci were the only ones represented in the model (Figs. 3 and 4). In fact, the Jb discontinuities have a mean strike almost parallel to the Section A, therefore they are supposed to have a minor influence on slope stability. Indeed, the presence of Jb joints tends to reduce lateral restraint on blocks, possibly sliding on Ja, and thus makes the adopted 2D modelling more realistic.

The Jc set, rarely observed in the North face, was not considered.

Discontinuities of the Sci set were created by random generation: mean values and standard deviations (uniform probability distribution) adopted for dip angle and spacing are reported in Table 3. Instead geometrical properties of the Ja set were assumed deterministic.

Table 3. Geometrical data of joint sets utilized for rock blocks generation (elevation z is referred to the model coordinate system).

| Set  | dip (°) | spacing (m) | elevation | mean | st. dev.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ja</td>
<td>65</td>
<td></td>
<td>z &lt; 70 m</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>z &gt; 70 m</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>Sci</td>
<td>77</td>
<td>5</td>
<td>z &lt; -20 m</td>
<td>8</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>z &gt; -20 m</td>
<td>4</td>
<td>0.5</td>
</tr>
</tbody>
</table>
For each joint set, different spacings were assumed in the lower and upper part of the model (Table 3). For two reasons: to simulate the growing degree of fracturing at increasing elevation and, moreover, to economize in number of blocks where less accuracy is needed.

Two different approaches were adopted to represent non-persistent discontinuities: the Sci set was represented as joints of fixed length (30 m) separated by stretches of intact rock (3.3 m); the Ja set were at first generated as fully persistent joints, then rock bridges were introduced by giving appropriate properties to short segments of joint (“fictitious joints”).

The mechanical behaviour of the joints was represented by the Barton-Bandis model [12], calibrated on the basis of laboratory testing; strength and stiffness properties were assumed equal for both Ja and Sci joint sets (Table 4).

### Table 4. Input data for Barton-Bandis joint model in UDEC.

<table>
<thead>
<tr>
<th>K_{nj}</th>
<th>K_{sj}</th>
<th>JRC₀</th>
<th>JCS₀</th>
<th>φᵣ</th>
<th>l₀</th>
<th>lₙ</th>
<th>σₜ</th>
<th>(GPa/m)</th>
<th>(GPa/m)</th>
<th>(-)</th>
<th>(MPa)</th>
<th>(m)</th>
<th>(m)</th>
<th>(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>16</td>
<td>6</td>
<td>75</td>
<td>30</td>
<td>0.1</td>
<td>20</td>
<td>87</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In our analyses, the generalized B-B model [12] has been kept as simple as possible. Considering that progressive excavation generally produces monotonic loading (or unloading) of the critical discontinuities in the upper part of the slope, cyclic damaging effects and dilation were disregarded.

The large value of field-scale discontinuity length (lₙ = 20 m) if compared to the average size of blocks in the model stems from the following considerations: i) massive blocks spanning the entire height (H > 8m) of sawed surfaces are frequently obtained; ii) block sizes in the 10-20 m range can thus be considered a realistic estimate; iii) the reduced block size (4 m) in the model can eventually be considered a compromise between a realistic estimate and an “ubiquitous joint” approach, which represents a safe-side estimate.

The rock bridges along the discontinuities of the Ja set, represented by fictitious joint segments, were modelled by the Mohr-Coulomb elastic ideal-plastic behaviour. Strength parameters obtained from laboratory testing (see sub-section 3.1) were: cohesion c = 18 MPa, friction angle φ = 44° and tensile strength σₜ = 8.9 MPa. This is deemed a reasonable approach, taking into account that each bridge is actually modelled by a few lumped springs, which can therefore represent only the average stress state.

Estimate of stiffness parameters for rock bridges represents a more difficult task. Considering that the overall behaviour of an ideal “contact element”, composed of a “rock joint” plus a “rock bridge”, (Fig. 5) is governed by the relative stiffness of the components, a 1/1000 ratio for K_{nj}/K_{sb} and K_{sj}/K_{sb}, normal and shear stiffness ratio respectively, was simply assumed. As known, both the normal and shear stiffness of joints, K_{nj} and K_{sj}, increase with normal stress in the B-B model. The 1/1000 ratio refers to a normal stress of 1 MPa, which can be considered a typical value for the joints in the critical zones of the slope.

![Fig. 5. Scheme of non-persistent discontinuity and relevant parameters: Lₐ, length of continuous joint; Lₖ and Sₖ, length and spacing of rock bridges.](image)

4.2. Preliminary Analyses

Modelling was developed in two phases: first, the influence of different rock bridge geometries on the stability of the current quarry face was analyzed, then the future excavation phases were simulated, according to the exploitation plan of the quarry (Fig. 6).

Preliminary analyses refer to the current situation. Both fully persistent Ja discontinuities and different patterns of rock bridges were considered.

![Fig. 6. Initial profile of the slope, current situation and future excavation phases.](image)
In the first case, the sliding failure of a large wedge in the upper part of the face mass was obtained: the sliding surface belongs to the Ja set. The response of the model with fully persistent Ja discontinuities clearly disagrees with the current stability.

Nine geometries of rock bridges were thereafter considered, each of them characterised by different value of the total length \( L \) and persistency ratio \( p \)

\[
L = L_b + L_j \quad \quad p = \frac{L_j}{L} \quad (1)
\]

\( L_j \) and \( L_b \) (Fig. 5) are lengths of segment real joint and rock bridge respectively; Table 5 shows the values of the adopted geometrical parameters.

Table 5. Different patterns of joint and rock bridge considered in DEM modelling.

<table>
<thead>
<tr>
<th>( L ) (m)</th>
<th>( p ) (%)</th>
<th>( L_j ) (m)</th>
<th>( L_b ) (m)</th>
<th>( S_b ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>45</td>
<td>5</td>
<td>47.5</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>35</td>
<td>15</td>
<td>42.5</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>27</td>
<td>3</td>
<td>28.5</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>24</td>
<td>6</td>
<td>27.0</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>21</td>
<td>9</td>
<td>25.5</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>13.5</td>
<td>1.5</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>12.0</td>
<td>3.0</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>10.5</td>
<td>4.5</td>
<td>12.7</td>
<td></td>
</tr>
</tbody>
</table>

Only three geometries of rock bridges assured the equilibrium of the existing face, i.e. those corresponding to \( L = 15 \) m and \( p = 70 \), 80 and 90 %. Figure 7 shows the displacement vectors in the upper part of the slope for the case \( L = 15 \) m and \( p = 70 \) %.

For the stability analysis of the planned excavation phases (see sub-section 4.4) the most critical one, corresponding to the minimum persistency ratio (\( p = 70 \) %), was considered.

Figures 8 and 9 show the collapse mode of the slope for the case \( L = 50 \) m and \( p = 90 \). Two failure mechanisms were triggered: a simple sliding failure of a shallow wedge (Fig. 8) and a more complex mechanism (Fig. 9), which involves both the Ja and Sci discontinuity sets. This last mechanism starts with the downwards movement of a central wedge of blocks at the top of the model, which pushes the blocks on the two sides outwards and induces tensile failures in some joints.
4.3. Excavation Phases and Stability Indexes

The planned sequence of excavations (1, 2 and 3) was simulated in UDEC by successively deleting groups of blocks (Fig. 6).

The conventional strength reduction factor and two non-conventional parameters were used to compare the stability conditions of the different phases of excavation.

The additional “stability indexes” herein considered are the following:
- percentage of “active” contacts compared to the total number of contacts;
- mean value of the ratio between the shear displacement corresponding to peak strength and the current shear displacement for each “active” contact.

For rock bridges, the “active” contacts are all those that are subjected to compressive stress or to a tensile stress lower than the tensile strength. For rock joints, the “active” contacts are those subjected to compressive stresses (tensile strength of joints is equal to zero).

These parameters can be useful to compare the rock mass stability of different excavation phases. They cannot give an absolute but only a relative estimate of the safety level.

The traditional strength reduction factor was also used to estimate the overall ‘stability level’ of the rock mass. Several analyses were carried out gradually reducing, by the same factor FR, the cohesion and tensile strength of rock bridges until a failure mechanism in the model was triggered; whereas the other strength parameters, considered less critical for the analysis, were kept fixed.

4.4. Final Analyses

Equilibrium conditions and very limited deformations were obtained for each of the future excavation phases.

Both the rock bridges and discontinuities resulted generally far from the peak strength conditions, apart from small block fragments of negligible importance, as indicated by the mean value of the shear displacements ratio $\delta_{\text{spic}}/\delta_{\text{smob}}$ (Table 6 and Fig. 10).

Table 6. Stability indexes and strength reduction factor FR for the current (0) and future (1, 2, 3) excavation steps.

| Phase | Rock joint | | Rock bridge | |
|-------|------------|------------|-------------|
|       | Active contacts $\delta_{\text{spic}}/\delta_{\text{smob}}$ | Active contacts $\delta_{\text{spic}}/\delta_{\text{smob}}$ | FR |
| 0     | 74 (%) 431 | 98 (%) 84.5 | 2.11 |
| 1     | 77 (%) 581 | 96 (%) 115.3 | 2.67 |
| 2     | 75 (%) 153 | 97 (%) 94.5  | 1.74 |
| 3     | 70 (%) 130 | 97 (%) 79.5  | 1.43 |

Also the number of tensile and shear failures of rock bridges is practically negligible in each of the excavation phases, as indicated by the active contacts percentage (Table 6 and Fig. 10), almost equal to 100 %. On the contrary, a consistent loss of joint contacts (60 %) occurs, owing to the null value of the tensile strength of joints (Table 6 and Fig. 10).

Fig. 10. Stability indexes and strength reduction factor FR as a function of the excavation steps.
All the non-conventional parameters indicate that the third excavation phase is the most critical for the overall stability of the slope (Fig. 10), whereas the first planned excavation step appears safer than the current situation.

Table 6 and Figure 10 also show the maximum strength reduction factors FR corresponding to the limit equilibrium conditions of each excavation phase. The obtained values of the parameter FR indicate that the safety level of the rock mass decreases after the first excavation phase, with a trend quite similar to that of the δs_pic/δs_mob ratio, although the decrease in FR is less marked.

5. CONCLUSIONS
The application of the distinct element method to the stability analysis of the rock slope required a detailed characterization of the geometry of joints and rock bridges.

Yet, a strong idealization of the real structure of the rock mass was necessary, as customary for this kind of modelling, and therefore an extensive sensitivity analysis had to be performed to calibrate the model.

Numerical investigations highlighted also the advantages in the use of fictitious joints for modelling the rock bridges.

Although computationally expensive, the strength reduction method is to be preferred in order to identify the failure mechanisms and obtain an absolute measure of safety.

It was shown that in problems of staged excavations also other parameters can be very useful as a relative measure of the safety level in the different stages. In fact, the trend of the proposed non-conventional parameters as the excavation advances is quite similar to the trend of the strength reduction factor.

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REFERENCES