Dissertation No. 4010

Theory of Kepler Motion:
The General Pertubed Two Body Problem

A Dissertation submitted

to the

SWISS FEDERAL INSTITUTE OF TECHNOLOGY

ZURICH

For the degree of
Doctor of Mathematics

presented by

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Basel
Buchdruckerei Birkhäuser AG
1968

Furthermore, differentiation of (7b) yields

$$\left.\begin{aligned}\frac{dt}{ds} &= -\delta_1' B1 + \delta_2' B2 + \delta_0' + \frac{1}{\omega^2} - \frac{2\,\omega\,\omega'}{\omega^4}\\ &- \delta_1 \frac{\cos\omega s}{\omega^3}(\omega + s\,\omega') + 3\,\delta_1 \frac{\omega\,\omega'}{\omega^5}\sin\omega s\\ &+ \delta_2 \frac{\sin\omega s}{\omega^3}(\omega + s\,\omega') - 2\,\delta_2 \frac{\omega\,\omega'}{\omega^4}(1 - \cos\omega s)\,.\end{aligned}\right\} \tag{7c}$$

But $t = \int_0^s t'\,ds = \int_0^s r(s)\,ds$; thus, equating (7c) and (7a) one obtains the expression for δ_0':

$$\delta_0' = \delta_1' \frac{1}{\omega^2} B1 - \delta_2' B2 + \omega\,\omega' \left\{\frac{2s}{\omega^4} + \delta_1\left(\frac{s}{\omega^4}\cos\omega s - \frac{3}{\omega^4} B1\right) + \delta_2\left(\frac{2}{\omega^2} B2 - \frac{s}{\omega^4} B1\right)\right\},$$

or

$$\delta_0' = r\,C3\,B2 - C1\,(\delta_1 B5 + 2\,\delta_3 B3 + \delta_2 B2^2)\,, \tag{8}$$

with a new special function

$$B5 = \frac{1}{\omega^2}(3\,B3 - B1\,B2)\,,$$

which is also well defined for all values for s and ω^2 including $\omega^2 = 0$.

Acknowledgment

The author is very deeply indebted to Professor Dr. E. STIEFEL for this constant support and unlimited encouragement during these investigations.

He is also very grateful to Professor Dr. H. RUTISHAUSER for the kind interest he has shown for this study.

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Zusammenfassung

Im ersten Kapitel wird die regularisierte Theorie der Kepler-Bewegung dargestellt. Dies geschieht in einer Form, die eine regularisierte dreidimensionale Störungsrechnung des Zweikörperproblems zulässt; dabei werden neue, sogenannte natürliche Elemente eingeführt. Die so erhaltenen Differentialgleichungen für diese natürlichen Elemente werden dann anhand von durchgerechneten Beispielen eines durch einen dritten Körper gestörten Zweikörperproblems getestet und diskutiert.

(Received: August 1, 1967.)