Torsion decomposition of finite CW-complexes

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TORSION DECOMPOSITION OF FINITE CW-COMPLEXES

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CHAPTER IV: TORSION DECOMPOSITION

28) Theorem There exists a function $\sigma : |\mathcal{C}_f| \longrightarrow N^+$ from the set of spaces which have the same homotopy type as a finite CW-complex to the set of non-negative integers such that, whenever $X \in |\mathcal{C}_f|$ there exists a map

$$f : S^{p_1} \vee \ldots \vee S^{p_k} \longrightarrow T \qquad T \text{ torsion}$$

such that

$$\Sigma^{\sigma(X)} X \simeq T_f \sqcup C(S^{p_1} \vee \ldots \vee S^{p_k})$$

The sum of spheres is uniquely determined by $\sigma(X)$ and the infinite cyclic part of the reduced integral homology of X. On the other hand, T is not even determined up to homotopy type.

Proof: It suffices to carry out the induction step announced in 17), observing the special choices made in 19) and 23) as well as their consequences for the top-space mentioned in 27).

a) In case the top-space is a sum of spheres only, the desired form is already present, since the base space is torsion by 19)

b) If the top-space contains a pseudo-projective space in its sum the result follows from Theorem 7 and Theorem 14, combining the base space, which is torsion, with the torsion part of the top-space to a new torsion space.

This argument only applies after the initial situation has been sufficiently often suspended.

From the homology-properties of the relative CW-complex (C_f, T) it follows immediately that the infinite cyclic part of the reduced integral homology together with the number $\sigma(X)$ determines the sum of spheres uniquely.

It remains to give an example which will show that the torsion space is not determined up to homotopy type:

$$S^{n+1} \simeq (S^n{}_k \sqcup CS^n)_{\bar{i}} \sqcup CS^n \simeq *_* \sqcup CS^n \qquad \text{for any } k,\ n \geq 2$$

where $\bar{i}$ is, as usually, the inclusion of the base space into a mapping cone.