Optimales Rangieren nach Turnieren

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ABHANDLUNG
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Summary

The thesis grew out of the attempt to show the optimality (in a decision theoretical sense) of a ranking procedure after knockout tournaments which J. A. Hartigan proposed 1966 in his paper "Probability completion of a knockout tournament" (Ann. Math. Statist. 37, 495 - 503). (He ranks the players according to the mean of the ranking vectors r = (r₁, r₂, ..., rₙ) consistent with the acyclic tournament outcome, where rᵢ = k means, that player number i has been assigned to the k-th place in the chosen ranking.)

The mathematical model therefore required a more rigorous definition of things like "game", "tournament plan" and "outcome of a tournament" than it has been done so far. Using graph theoretic language, a "tournament plan" \( \varphi \) is interpreted as a mapping of the set \( T \) of all complete digraphs T (with the n players as vertices) into the set \( D \) of all digraphs D of order n, such that a) \( \varphi(T) \) \( \in T \) and b) \( \varphi(T) \) \( \in T^* \) \( \Rightarrow \varphi(T^*) = \varphi(T) \) for all \( T, T^* \in T \). The image \( \varphi(T) \) of \( \varphi \) applied on T is called a "tournament outcome of \( \varphi ""). A plan \( \varphi \) is called "equivalent" to the given plan \( \varphi^* \), if there exists a permutation of the players (vertices) which transforms all outcomes \( \varphi(T), T \in T \), into the outcomes \( \varphi(T^*), T^* \in T \). A plan \( \varphi \) is called "simple" if all possible outcomes of \( \varphi \) are isomorphic; they then are acyclic too. (E. g. symmetric knockout plans on 2ⁿ players are equivalent and simple.) There exist relations between the number of different plans equivalent to a simple plan \( \varphi \) and the number of orders consistent with an outcome of \( \varphi \).

The model describing the ranking problem is the following: One assumes a) that out of a class K of equivalent plans with only acyclic outcomes a plan \( \varphi \) is chosen at random, b) that the relative "strength" of the players is given by an unknown ranking vector r and finally c) that the outcome \( F = \varphi(T_r) \), where \( T_r \) is the transitive complete digraph representing r, can be observed.

The decision problem then is determined by:

- \( S_n := \{ r \} \), space of all ranking vectors r: parameter space,
- \( \varphi := \{ F = \varphi(T_r); \varphi \in K, r \in S_n \} \), space of the outcomes: sample space,
S_n =: \{d\} : decision space and the loss function L(d,r), d,r ∈ S_n, with L : S_n × S_n → R_1^+ ∪ \{0\}.

Given a prior distribution p := (p_r ; r ∈ S_n) and an outcome F with R(F) the set of all ranking vectors consistent with F, a ranking vector d* is a Bayes solution iff

\[ \sum_{r \in R(F)} L(d^*,r) p_r \leq \sum_{r \in R(F)} L(d,r) p_r \quad \text{for all} \quad d \in S_n. \]

Making two obvious assumptions on L concerning monotony and invariance, one can show that d* ∈ R(F). If L furthermore is separable i.e., L : (d,r) → \sum_{i=1}^n f(d_i,r_i) then there exist "Branch and Bound" procedures for the search of d* and finally if the matrix (f) is of a special form, d* can be found by means of a generalisation of Hartigan's procedure (e.g. if f(i,j) := v(i)(i - j), v monotonically increasing, then d* is the "Hartigan solution" itself.)

A class of loss functions, different from the separable one, leads to a generalisation and unification of procedures which are already well known in order to rank the players in Round Robin tournaments as e.g. the "row sum"-procedure and Slater's principle of "minimum inconsistencies".

Finally an approach is made to find an optimum "seeding" of the players (i.e. an optimum choice of a plan \( P \) out of a given class \( K \)) if prior information on the players' strength is available and a ranking procedure is prescribed.