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The Secondary Flow in Curved Pipes

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Chapter I. General Considerations

§ 1. Introduction

The goal of every engineer is to always improve the efficiency and performance of the machines and equipment he designs. In order to do so it is important to understand the phenomena and processes associated with these machines. As a result of this understanding, the sources of the losses and inefficiencies can more easily be found, and subsequently their magnitudes and effects evaluated.

It has been known for some time that when fluid with a non-uniform velocity flows in a curved pipe, there arise losses which are greater than those for the corresponding length of straight pipe. Therefore, there must be a phenomenon present in the flow in a bend, which is not present in a straight pipe, from which these losses originate. Another consequence of this phenomenon is that it distorts the shape of the axial-velocity distribution as the flow moves around the bend of the pipe. To obtain a knowledge, about this flow in a curved pipe, is desirable and important from both the scientific and practical view-points. It was, thus, the purpose of this investigation to obtain an understanding of the flow processes involved, and subsequently to develop a theory for predicting these processes. Recognizing the undisputed value of having a physical comprehension of a problem, the investigation was carried out using, where possible, a close connection between physics and theory.

Applications of the results of an investigation of the flow in curved pipes are rather apparent. For example, the airplane, which today shows, indeed, a great improvement in performance and efficiency over those of the previous decade, still offers opportunities for improvements. Its overall efficiency is, of course, determined by the efficiencies of its various components, which with the complexity of the present-day airplane are numerous. Herein lie the possibilities of an application of this information. It can be used to good advantage in the design of the many flow passages used to conduct air from one place to another within the airplane, for example the air intakes to the engines. In addition, the advent of the turbo-jet and rocket engines, as applied to airplanes, has opened many fields where the knowledge about flow in curved passages and pipes is a necessity if these machines are to be designed for maximum efficiency. To be sure, the information can be applied to stationary machines as well, the airplane being simply one example.

In the following work a study is made of the fluid flow in a curved pipe. Discussed first from a physical point of view, a basis for the origin of the phenomenon which arises is suggested. The results of experiments on the flow in curved pipes, which were conducted at the Institut für Aerodynamik der E.T.H., Zürich, support this physical argument, thus indicating that the deductions are sound. Drawing on the above information, a first order theory is developed to predict the flow phenomenon in a curved pipe whose cross-

sectional dimensions are small compared to its radius of curvature and whose angle of bend is small. As such, the theoretical study is a study of the initial phases of the secondary flow phenomenon in a curved pipe. An application of this theory is then made, the results of which are compared with the results of the above mentioned experiments.

§ 2. Definition of Secondary Flow

Many basic fluid-motion studies are made by assuming that the motion is irrotational. Irrotational motion is defined by that motion for which the curl of the velocity vector is zero. The curl of the velocity vector is called the vorticity, which is defined physically as twice the angular velocity of the fluid particle. Irrotational motion is equivalent to or constitutes motion free of vorticity. Since the curl of every gradient is identically zero, it is possible, when irrotational motion exists, to write the velocity vector as the gradient of a scalar function. This scalar function is called the potential function and the motion that exists under these conditions is called potential flow. The necessary as well as the sufficient condition that must be satisfied in order to have a potential flow is that the flow be irrotational. Restricted to the consideration of an incompressible fluid, the differential equation satisfied by the potential function is Laplace's equation. From this one scalar potential function the three mutually perpendicular velocity components of a general flow problem can be found and thus an extreme simplification has been achieved. Much attention in the past has been given to the problem of finding solutions of Laplace's equation as applied to fluid motion. The general problem of solving Laplace's equation, which constitutes Potential Theory, is the subject of a great literature. The irrotational assumption is also equivalent to saying that the stagnation pressure is the same along all streamlines, in fact throughout the whole flow field.

In practice, however, it often happens that the stagnation pressure is not the same along all streamlines, but varies slightly from one streamline to another. This then means the flow is not irrotational but only approximately so, and hence the definition of a scalar potential function for the flow is impossible. The difference between the potential flow and this approximately potential flow in the normal plane is called secondary flow. Consequently the secondary flow must contain vorticity. This vorticity, in turn, constitutes a motion of the fluid particles, which is not necessarily restricted to involve motion of the fluid particles along the streamlines of the potential flow, but can define other streamlines. If this difference, which describes the secondary flow, is small, then it is conceivable that the velocities and, subsequently, the losses of this secondary flow are correspondingly small.

§ 3. The Secondary Flow Phenomenon in Curved Pipes

According to the definition of secondary flow there arises a vorticity in the flow field. Since the vorticity is defined mathematically as being equal to the curl of the velocity vector, this vorticity must arise from a velocity gradient. It is clear that a non-uniform axial-velocity distribution in a straight pipe will produce vorticity in the flow due to the non-uniform distribution. If this flow