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Monte Carlo Approach to the Flow Problems of Turbomachinery

THESIS

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7. Concluding Remarks

The random walk technique, representing a kind of Monte Carlo method, which is used here for solving the equations governing the flow in turbomachinery, offers several advantages over the other relevant methods.

i) The value of the stream function at any particular point could be obtained without solving the equation in the whole region.

ii) If only the non-homogeneous term changes the new values of ψ can be determined rapidly from the former values of ψ by changing only the corrections since the walk remains unchanged.

iii) Provided the position of the boundaries remains unchanged and only the values at the boundary change, the new solution could be immediately obtained.

iv) The method is particularly well adapted to the problem under consideration because the velocity distribution is required only on the blade surface and thus the calculation of the stream function at a few points lying in the vicinity of blades is sufficient. The effort required for the solution of this problem by the relaxation method is quite large.

v) It would be stated without explanation that during the course of solution by the random walk technique the effects of various parameters could be better judged and the physical picture of the phenomena becomes clearer in the depth of mathematical complexity. After little experience with this method several interesting features become apparent and the method offers full scope for applying individuality in improving or quickening the procedure.

vi) The procedure is quite simple and does not require any special mathematical ability on the part of the computer whereas engineering understanding of the problem is often of help.

About 40 hours are required, by the method developed here, for calculating the velocity distribution around a blade in cascade for an incompressible flow. The computational time increases to about 70 hours for the corresponding problem in a compressible flow for three inlet Mach numbers. The determination of the velocity distribution around the impeller blade of a radial flow compressor for three different shroud forms of impeller requires approximately 100 hours of computation by this method.

The computational time can be greatly reduced if a digital computer can be used to simulate the random walk. For this purpose, the random walk technique, developed in this paper, has to be somewhat modified. This modification and its various consequences are discussed in Appendix C where a sample FORTRAN programme for "IBM 1620 Data Processing System" is also given.

From the comparison of the results obtained by this method with those of tests or other theories, it is found that the accuracy of the method is quite sufficient for treating the incompressible flow in cascades and Poisson equation. In the case of a compressible flow through axial cascade the results of the Monte Carlo method cannot be compared directly with the test results which are found to be considerably affected by the non-potential character of the flow. Unfortunately, exact solutions of compressible potential flow through axial cascades are not available at present for comparison with the results of Monte Carlo method. The results of the Monte Carlo method are compared with the results of the linearised theory. Both the theories show exactly the same trend regarding the effect of compressibility on the pressure distribution around the profile. It is found that the adverse pressure gradient on the suction surface increases with Mach number. Further, for all compressor cascades, the blade loading increases with Mach number although the extent of this influence differs widely for various cascade configurations. The increase in blade loading with Mach number is more predominant for cascade configuration with small stagger angle. For analysing the flow conditions in impellers of centrifugal compressors having arbitrary blade shape, this technique appears, to be the only one to solve, with reasonable effort and accuracy, the equations which include the effect of both the compressibility of the flow medium and the rotation of the impeller. In view of the simplicity of this method it could find application in industrial practice.

Although many details regarding the rigorous mathematical establishment of the method, systematical error investigation due to the first order approximation for compressibility, due to the number of trials, length of unit step etc. are not dealt here and thus the method, as Forsythe [24] would put it, is evolved more as an art rather as science based on intuition and analogy; it appears to be promising in view of the fact that the results obtained by the application of this method are realistic and consistent with experience and should serve as a good basis for predicting relative merit of various flow-components used in turbomachinery.

8. Zusammenfassung

Nach einer kurzen Einführung in die Monte Carlo Methode und einem Ueberblick über deren geschichtliche Entwicklung wurde der "Random Walk Process", welcher eine spezielle Form der obengenannten Methode darstellt, entwickelt. Er dient zur Lösung von Gleichungen, die bei Strömungsproblemen in Turbomaschinen auftreten. Diese Gleichungen sind unter der vereinfachenden Annahme von Reibungs- und Wirbelfreiheit formuliert. An Hand von konkreten Beispielen inkompressibler und kompressibler Strömung durch Axial- und Radialgitter wurde die Zweckmässigkeit und Genauigkeit des hergeleiteten Verfahrens studiert. Nach dieser Methode gerechnete Geschwindigkeitsverteilungen geben Aufschluss über die Einflüsse der Kompressibilität in Axialverdichtergittern sowie über die Auswirkung von Laufradrotation und Kompressibilität des Mediums in Radialverdichter-Laufrädern. Die vorliegende numerische Methode ist einfach in ihrer Anwendung, sodass keine spezielle Ausrüstung benötigt wird. Sie liefert mit zeitlich vernünftigem Rechenaufwand zufriedenstellende Resultate und dürfte für die Praxis ein brauchbares Werkzeug darstellen.