Nichtlineare elliptische und parabolische Probleme in unbeschränkten Gebieten

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NICHTLINEARE ELLIPTISCHE UND PARABOLISCHE PROBLEME IN UNBESCHRAENKTEN GEBIETEN

ABHANDLUNG

zur Erlangung des Titels eines
Doktors der Mathematik
der

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vorgelegt von

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ABSTRACT

We discuss problems in unbounded domains Ω in $\mathbb{R}^N$ with sufficiently smooth boundary $\Gamma = \partial\Omega$.

Part I treats elliptic problems for quasilinear operators A of the form

$$(Au)(x) \;=\; \sum_{|\alpha|\leq m} (-1)^{|\alpha|} D^\alpha A_\alpha(x,\delta u(x),D^m u(x)) \qquad x \in \Omega,$$

where the real functions A_α satisfy a variant of the Leray - Lions conditions. First, given a reflexive Banach space V which is a subspace of a Sobolev space, a convex subset K of V and an element $f \in V'$, we obtain the existence of a $u \in K$

with $(Au,v - u) \geq (f,v - u) \quad \forall\, v \in K$.

By the same methods we attack strongly nonlinear variational inequalities. We further get weak solutions to some not koercive boundary value problems, provided we know a weak lower solution ϕ and a weak upper solution ψ with $\phi \leq 0 \leq \psi$ in Ω. Moreover the solution lies between ϕ and ψ.

Part II deals with parabolic boundary value problems

$$\begin{aligned} \frac{\partial u}{\partial t} + Au &= f && \text{in } \Omega\times(0,T) \\ u &= 0 && \text{on } \Gamma\times(0,T) \\ u(0) &= u_0 && \text{in } \Omega. \end{aligned}$$

Here A has properties analogous to those in part I, f and u_0 are given functions and $T > 0$. We get weak solutions in appropriate spaces $\tilde{V}$ with $\tilde{V} \subset L^2(\Omega\times(0,T)) \subset \tilde{V}'$.