Doctoral Thesis

Congruence and existence of differentiable maps

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CONGRUENCE AND EXISTENCE
OF DIFFERENTIABLE MAPS

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INTRODUCTION

A fundamental theorem for submanifolds of spaces of constant curvature is the congruence and existence theorem which states that there exists up to isometries exactly one submanifold with prescribed first fundamental form and prescribed normal bundle with fibre metric, normal connection, and second fundamental form, satisfying the conditions of Gauss, Codazzi, and Ricci (For the local version see Eisenhart [5, p. 212], for the global version see Bishop and Crittenden [1, p. 202] and Szczarba [16], summarized by Chen [2, pp. 48-49]).

We give a comprehensive generalization of this congruence and existence theorem together with a direct geometric proof. Our general theorem concerns arbitrary differentiable maps $f$ of a manifold $M$ into an arbitrary riemannian manifold $N$ or, more generally, into a manifold $N$ which has a $G$-structure and a compatible connection $\nabla$. It is formulated by means of the pullback $f^*TN$ with the induced $G$-structure, the induced connection $\nabla^f$, and the differential $f_*$ considered as a map of $TM$ into $f^*TN$ (If $f$ is an immersion and $N$ is a riemannian manifold, the pullback $f^*TN$ splits into the tangent and normal bundle, and $\nabla^f$ splits into the tangential and normal connection and the second fundamental form).

The general congruence theorem (see section 4) states that if $f$ and $g$ are differentiable maps of $M$ into $N$ (where $M$ is connected), and if there exists a vector bundle isomorphism of $f^*TN$ into $g^*TN$ which preserves the $G$-structure and sends $\nabla^f$ to $\nabla^g$ and $f_*$ to $g_*$, then there exists, under a certain assumption, a congruence transformation of $N$ which sends $f$ to $g$. 