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Publication Date:
1980

Permanent Link:
https://doi.org/10.3929/ethz-a-000214946

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Thèse No 6634

SUR LE GROUPE DE BRAUER D'UN ANNEAU DE POLYNOMES EN CARACTERISTIQUE P ET LA THEORIE DES INVARIANTS

présentée à
1'ECOLE POLYTECHNIQUE FEDERALE ZURICH

pour l'obtention
du titre de Docteur ès sciences mathématiques

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1980
Abstract.

The simplest commutative ring of prime characteristic $p$ whose Brauer group is not well known is the polynomial ring in two indeterminates $R = \mathbb{F}_q[X,Y]$ over a finite field with $q = p^r$ elements. We know that the Brauer group $\text{Br}(R)$ is an infinite countable direct sum of copies of $\mathbb{Z}/(p^\infty)$ (Knus-Ojanguren-Saltman, Lecture Notes in Mathematics 549). Our work contributes to understand the structure of $\text{Br}(R)$ and in particular it determines completely the subgroup $\text{Br}_p(R)$ of elements of order $p$ if $q = p > 2$. The natural action of the linear group $G = \text{SL}(2,q)$ on $R$ induces an action of $G$ on the Brauer group $\text{Br}(R)$.

The study of this action was suggested by Amitsur.

We first show that the group $\text{Br}_p(R)$ has a natural structure of $\mathbb{F}_q[G]$-module. Then we construct an injective equivariant map from $R$ to $\text{Br}_p(R)$. Therefore the representations of $G$ in $R$ give representations of $G$ in $\text{Br}_p(R)$. In particular, we obtain that $\text{Br}_p(\mathbb{F}_q[X,Y])$ contains all kinds of finite dimensional $\mathbb{F}_q[G]$-modules. Then if characteristic $p > 2$, we give explicitly an isomorphism of $\text{Br}_p(\mathbb{F}_q[X,Y])$ with $p^2 - 1$ copies of the additive group of $\mathbb{F}_q[X,Y]$.

The study of the action of $\text{SL}(2,q)$ on $\text{Br}_p(R)$ leads us to the invariant theory of $R$. For the characteristic $p \neq 0$, we found only partial results in the literature. We therefore construct the rings of invariants $k[X,Y]^G$ for all finite subgroups $G$ of $\text{SL}(2,k)$, $k$ a field of characteristic $p > 2$, and we obtain necessary and sufficient conditions for them to be polynomial rings.

Finally, we study the homomorphism from $\text{Br}(R^G)$ to $\text{Br}(R)^G$ induced by the inclusion of $R^G$ in $R$. If $S$ is the field of fractions of $R$, then the map from $\text{Br}(S^G)_p$ to $\text{Br}(S)^G_p$ is surjective for all subgroups $G$ of $\text{GL}(2,q)$. If $p$ does not divide the order of $G$, this is even an isomorphism. The study of the map from $\text{Br}(R^G)$ to $\text{Br}(R)^G$ is more difficult since
the extensions $R/R^G$ are not Galois. We prove that the map from $\text{Br}(R^G)$ to $H^0(R/R^G, \text{Br})$ in Amitsur cohomology is surjective if $R^G$ is regular. On the other hand, if $G$ is a subgroup of $\text{SL}(2,q)$ whose order is divisible by $p$, then the map from $\text{Br}(R^G)$ to $\text{Br}(R)^G$ is not surjective. The cokernel of this map contains even a copy of $R^G$. For fields, we give also a sufficient condition for the map from $\text{Br}(.)^G$ to $\text{Br}(.)^G$ to be surjective. For example, the map from $\text{Br}(F_q(X))^G$ to $\text{Br}(F_q(X))^G$ is surjective for all perfect subgroups $G$ of $\text{PGL}(2,q)$. 