ZUR SPIEGELUNGSGEOMETRIE DER ISOMETRIEGRUPPE DES 
n-DIMENSIONALEN REELLEN EUKLIDISCHEN RAUMES

ABHANDLUNG

zur Erlangung des Titels eines

DOKTORS DER MATHEMATIK

der

EIDGENÖSSISCHEN TECHNISCHEN HOCHSCHULE ZÜRICH

vorgelegt von

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ABSTRACT:

THE GROUP OF CONGRUENCE TRANSFORMATIONS IN THE n-DIMENSIONAL REAL EUCLIDEAN SPACE GENERATED BY INVOLUTIONS

The congruence transformations (isometries or rigid motions) of the n-dimensional real euclidean space are represented by orthogonal matrices and vectors. In the same manner we give the symmetries (reflections in hyperplanes). The product of two symmetries represents a rotation or a translation. On the other hand every congruence transformation may be represented by a product of rotations, a translation or a symmetry. Thus it can be represented by a product of at most n+1 symmetries (see [4] and [20]), most of which are perpendicular to each other (canonical representations). But a given congruence transformation can also be represented by a minimal product of reflections in subspaces (general isometric involutions) of any fixed dimension greater than 0, whenever the parity of the transformation is even or equals the parity of the generating involution. An answer is provided specifying how many factors must have such minimal products.

A synthetical representation of the 4-dimensional space by a descriptive geometry enables us to make a generalisation about the synthetical investigations of the group of two- and three-dimensional congruence transformations (see [6] and [7]). A second synthetical proof is given of the existence of the canonical representations in four dimensions as an example for the general n-dimensional case.