Decentralized Control of Magnetic Rotor Bearing Systems

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Zürich, August 1984 Hannes Bleuler
Dezentrale Regelung von magnetischen Rotorlagerungen

Zusammenfassung:

Die berührungsfreie elektromagnetische Lagerung von Körpern bietet vielfältige und ausserordentliche Vorteile für eine ganze Anzahl von technischen Anwendungen.

Sehr hohe Drehzahlen sind möglich, die Lagerung ist vollständig verschleiss- und wartungsfrei sowie vakuumtauglich, Lagereigenschaften wie Steifigkeit und Rotorpositionierung können über die Regel-Elektronik vorgegeben und im Betrieb angepasst werden.


Weitere Magnetlager Probleme wie z.B. die Auslegung und Konstruktion des Lagers selbst (A. Traxler) werden ebenfalls am Institut für Mechanik durchgeführt. Verschiedene hier gebauten Rotorsysteme belegen die Tauglichkeit der Ergebnisse.

Die Arbeit ist in folgende Teilaufgaben gegliedert:

- Modellierung des Stellgliedes Magnetlager;
- Wahl der Gewichtungsmatrizen des quadratischen Gütekriteriums beziehungsweise der Pole des geregelter Systems; Zusammenhang dieser Wahl mit den mechanischen Lagereigenschaften;
- Modellierung des Systems der Radialbewegungen;
- Herleitung der optimalen Zustandsregelung nach der Riccati Methode; dies ergibt Rückführmatrizen mit 32 Koeffizienten, wie sie in verschiedenen bisherigen Arbeiten angegeben werden.
- Nun wird für diese Regler-Struktur die optimale Wahl der verbleibenden Rückführkoeffizienten angegeben. Dabei gelangt ein Verfahren zur Anwendung, das an einer ETH Promotionsarbeit (Senning, 1979) theoretisch vorgestellt wurde und zum Themenkreis Dezentralisierung grosser Systeme gehört.
- Das Computerprogramm, das zur Berechnung dieser Regelung entwickelt wurde, wird beschrieben.
- Der Beobachter wird ebenfalls dezentralisiert.
- Kreiseleffekte werden mitberücksichtigt.
- Experimentelle Ergebnisse werden vorgestellt.


Die Arbeit soll dazu beitragen, die Lücke zwischen Regeltheorie und praktischer Anwendung zu schliessen.
Abstract:

Active electromagnetic bearings can support a body without any mechanical contact. Such bearings feature outstanding properties for a variety of technical applications, mainly in the fields of vacuum technology, precision machine tools, very fast rotations and space technology. Present applications are realized with "pragmatic" control strategies or with complete state feedback, relying on analog control.

The aim of this thesis is to promote the use of standard microprocessors for this demanding control task (sampling rates in the milliseconds range) by simplifying the structure of the control. A complete state description including gyroscopic effects is used for the subsequent optimization of the feedback parameters with respect to a quadratic performance index. The following aspects are discussed:

- Modelling of magnetic bearing systems with rigid rotor;
- choice of weighting matrices or closed loop system poles and its influence on the mechanical bearing properties;
- the simplified structure applied here consists of decentralizing the control, i.e. dividing it up among several local subsystems (e.g. the physical bearing units). The number of on-line operations to be performed by the control device is thereby reduced by a factor of 8 or more compared to classical state feedback. It is shown that system performance remains satisfactory in spite of this simplification.
- A computer program to determine optimal decentralized feedback is described.
- The Luenberger Observer is also decentralized. It is shown, that decentralization is possible also for strong gyroscopic coupling.

Several systems built at our institute demonstrate the applicability of the results. Some systems are controlled fully digitally by standard microprocessors, leaving open a wide range of possibilities for flexible control. (e.g. rotation about principal axis, smart sensors, interaction of environment and mechanical bearing characteristic, adaptive control).
1 Introduction

1.1 State of the Art

Magnetic bearings can be used to completely support a body by magnetic forces, that is without any mechanical contact. The great technical relevance of such devices, e.g. as rotor bearings, is illustrated in section 1.4, where advantages and applications are presented. Here, a survey of publications will outline the "history" of magnetic bearings. Thus, the specific contribution of this thesis will appear in the appropriate context.

Contact-free suspension of a body with ferro-magnetic forces alone can only be achieved by means of a control-loop since permanent magnet forces by themselves will not lead to a stable equilibrium position. BRAUNBECK /1/ demonstrated in 1939 that only the very weak diamagnetic forces can support a body passively in all six degrees of freedom. Attention therefore turned to actively controlled electro-magnets.

In 1946, BEAMS, YOUNG and MOORE /2/ carried out interesting experiments on the production of high centrifugal fields. Their test rotor was a small solid steel sphere supported
entirely by magnetic forces with the help of a simple control loop.

Space technology then sharply activated research in magnetic bearings. The main interest was on momentum wheels in spacecraft, the primary advantages being low drag torque and unlubricated, wear-free and vacuum-compatible operation. This work was successful and as a result, in the 1970ies many papers were published both in the U.S. /3/ and Europe /4/.

The know-how acquired is now being exploited in industrial domains as well. Examples can be found in /5/ and /6/.

Of the numerous works on magnetic levitation vehicles only the book by JAYAWANT (1981) /7/ will be cited here. It includes a large bibliography of further readings. Our main interest remains directed on rotor bearings.

Extensive investigations on the topic have been carried out at Munich. Frequent reference will be made to the thesis by ULBRICH (1979) /8/, /9/. It applies theory of optimal control successfully to the feedback as well as to the Luenberger observer. The resulting system — a vacuum centrifuge — performs well. It is the predecessor of a similar system built at our institute.

Other publications concentrate on special problems. As an example, one can refer to a VDI-publication by PIETRUSZKA and WAGNER /10/ of the University of Duisburg (1982) on adaptive control handling rotor unbalances.

A recent survey on magnetic bearings has been given by SCHWEITZER /11/ (1983). Further references will be cited later, when specific results are applied. More literature on the topic can be found in the bibliographies of the publications mentioned up to here.
1.2 Motivation of the Present Contribution

Stated in a rather general manner, the aim of this thesis consists of finding a magnetic bearing control which combines structural simplicity with optimal system performance.

This is being done in order to promote the use of the microprocessor for this demanding control task. The versatile type of control then available opens up a wider range of applications for magnetic bearings.

To reach this goal, existing work on magnetic bearings is combined with methods from automatic control theory.

The present applications of magnetic bearing control can be roughly divided into two groups according to a basic structural difference of the control layout:

1) The first group is based on the idea of creating a mechanical bearing stiffness and damping analogous to a conventional bearing. This usually leads to the situation that displacement at each bearing and in each direction is controlled individually.

A typical example of this commonly applied control scheme can be found e.g. in a contribution by OUDIN /12/ with the title "General Comparison between Ball Bearing Wheels and Magnetic Bearing Wheel". As this title indicates, the magnetic bearings are viewed in a classical manner as "local" bearings. Such a control layout structure will therefore be called decentral since control is divided up among several independent controllers.

2) On the other hand, control for the complete magnetic bearings system can be implemented in a single central control. This is meant not only on the physical level, but also in the mathematical system description:

Such control strategies make extensive use of the system theoretical tools available. Most commonly, the system
will be described by state equations, on which control layout is then based. Good results are achieved e.g. by computing "optimal" feedback with the matrix-Riccati equation and providing a Luenberger observer (/8/).

Often the rotor may be modelled as a rigid body, where the magnetic bearing system has to control five degrees of freedom (one in axial direction and four in radial direction). The radial motions then form a system of order eight, which cannot trivially be divided up into independent subsystems as it was done in the decentral approach.

A first aim of this thesis can be described as comparing the two approaches. The question will be treated, how much improvement in system performance the more complex central approach offers compared to the more empirical decentral control structure.

Decentral control will be investigated beyond that comparison, mainly in view of the following situation:

All the examples presented up to now rely on analog electronics for the control implementation. In that case, both layout strategies can be realized with roughly equal hardware requirements.

On the other hand, there is an increasing urge in favor of digital control. The reasons are familiar: Great flexibility to alter the control or add features (e.g. linearizing, adaptive control etc.), robustness with respect to noise and drift. The control task should be performed by relatively cheap standard microprocessors. This enhances the possibilities for larger scale industrial applications of magnetic bearings.

Control of such systems requires relatively short sampling times in the order of milliseconds. A central control layout for the system of order eight as described above cannot be performed fast enough by present standard microprocessors,
unless the control task is simplified. Decentral control represents such a simplified structure.

The second aim of this thesis is therefore formulated as follows: The best possible decentral control is sought, based on the complete state model and optimizing in the same manner as for central feedback. The complete state model accurately describes the gyroscopic effects as well.

The method to compute "optimal decentralized feedback" is taken from the thesis by SENNING /13/ (1979) of the Institute of Automatic Control and Industrial Electronics of the ETH Zürich.

The advantages of optimal central control are thereby conserved as much as possible. These well known advantages consist mainly of the following two points:

1) Good performance of the real system owing to the robustness and low sensitivity of such control strategies.

2) Systematic computation of feedback and observer.

Applications of the results will be presented. It will be demonstrated, that microprocessors can be used for this task, providing cheap and versatile control of magnetic bearing systems.

Naturally decentralized control is but one of a variety of topics on which research in magnetic bearings is of interest. Other subjects - not being treated here - are flexible rotor modelling, sensitivity aspects, touch-down behaviour in the emergency bearings, position and rate sensors etc. Work on some of these problems is currently being done at our institute. Special advantage is taken of a project treated by TRAXLER /14/ on the systematic design of the bearing itself.
1.3 Structure of the Thesis

Chapter 2: As an introduction into control problems of magnetic bearings, a simple axial bearing system of order two is presented. The bearing is described as a linear model (force as function of current and rotor position).

The feedback then derived (minimum energy feedback) is worth some discussion: It provides useful hints as to the choice of weighting matrices and pole locations in the general system. Conclusions valid for the higher order radial bearing system are drawn.

Chapter 3 is on modeling of the system of radial motions. The basic assumptions on the physical rotor model valid throughout the thesis are given. Specifically, the rotor is taken to be a rigid, symmetric body with only small unbalances.

The linearized state equations of order 8 are derived. The different types of interconnections of possible subsystems are described.

Chapter 4 gives numerical examples. The notion of representative numerical examples can be a useful guide as to which theoretical aspects will be considered relevant and which ones will not. Optimal central feedback according to chapter 2 is presented.

Chapter 5 on decentralized optimal control presents the main results of the thesis. The subdivision into subsystems is first described.

An optimal decentralized control is then computed based on methods used in the context of large scale systems /13/.

Such control is implemented without giving up the overall
system model including all its interconnections. The local feedback coefficients are determined in such a way, that the performance of the overall system is optimized.

An algorithm to compute optimal decentral feedback is presented. The computer program developed at our institute is described.

Numerical results are then presented and compared to "empirical" decentral feedback and complete (non decentralized) feedback.

Chapter 6 consists of a short discussion on decentralizing the Luenberger observer or state estimator.

The gyroscopic effects are treated in chapter 7. Numerical examples follow, including plots of eigenvalues in function of rotational speed.

In chapter 8, experimental results are presented. The conclusion sums up the main results of the whole thesis. The resulting control can be regarded as a synthesis of optimal state feedback and more pragmatic decentral control. The advantages from both approaches are conserved.

Microprocessor control of bearing systems has been implemented on the basis of these results. The specific description of the digital control software and hardware however is not subject of this thesis. The reader interested in these problems can refer to other contributions of our institute (MURBACH /15/, MEYER & BLEULER /16/).

The appendix includes some derivations of formulae.

The references are found after the appendix.
1.4 Short Survey of Properties and Applications of Magnetic Bearings

Magnetic bearings feature some outstanding technical properties and they are beginning to be applied in a great variety of different areas. Optimal bearing layout provided, the maximum forces achieved are of the order of roughly 40 through 70 N/cm² of the active bearing sectional area /17/. This area is defined as bearing length multiplied by rotor diameter. The upper bound on the force is due to the magnetic saturation of the bearing material.

The main advantages of magnetic bearings are:

- No lubricant needed
- No wear, no maintenance
- Variable dynamic bearing characteristic
- Very accurate positioning of the rotor axis by the control loop
- Possibility of very high rotational speed
- Small energy dissipation in the bearings

Current applications are in the field of:

- Vacuum techniques /18/
- Machine tools
- Turbo compressors
- Space vehicles (momentum wheels)
- Graphical industry

Future applications of contact-free magnetic rotor bearings may be expected for:

- Auxiliary bearing of a conventionally supported rotor, with the purpose of actively reducing vibrational effects.
- Fly-wheels, e.g. for energy storage
- Textile machines
2 General Aspects on Magnetic Bearing Control

2.1 Bearing Model

Figure 1 shows the basic elements of an active magnetic bearing system in a simplified system with just one degree of freedom.

![Diagram of Magnetic Bearing System]

Fig.1: Main components of the feedback control loop for a simplified contact-free magnetic support. The "plant" consists of the supported body (the rotor). The position sensor produces a signal corresponding to the rotor displacement from nominal position. This signal is fed into the control loop which determines the appropriate input of the power amplifiers.

The magnetic bearing is the actuator of the system. The out-
The input to the control network consists of a prescribed current or voltage for each bearing coil. In both cases, this output signal will be denoted by the letter $u$, as customary in control theory. The signal $u$ is the input to the power amplifier driving the bearing. The input-output characteristic of the amplifier is assumed to be described by a linear, frequency independent transfer function with phase shift zero for the relevant frequency range. The same assumptions are made for the sensors.

The input to the plant consists of the force which the bearings exert on the rotor. Therefore, the bearing model must describe the relationship of bearing force as function of the current or the voltage applied to the bearing coil.

The control concepts derived later on apply as well to the case of semiconductor amplifiers directly controlled by digital pulses. This may lead to a control strategy based on digital signals only, eliminating AD and DA converters. The subsequent bearing description must also be seen in this context.

The transfer function of the bearing itself is composed of two parts defined by their input-output variables:

The first part has voltage as input and current through the coil as output. The second part of the transfer function is the relationship of this current and the resulting bearing force.

Both parts depend on the rotor position in the bearing since the inductance is a function of the geometry. Nevertheless, the first partial transfer function consists primarily of the first order linear element of a simple inductance. An example directly applicable on rotor bearings of a voltage controlled magnet is given in a publication by BREINL /19/ on magnetic levitation vehicles.

If this first part of the transfer function is neglected, a current controlled bearing model is obtained. This is feasi-
ble, if two conditions are met:

First, the time constant of this element has to be an order of magnitude smaller than the mechanical time constants. As a second condition, the power amplifier has to be able to deliver a maximum voltage high enough to ensure that the bearing current can change sufficiently fast.

For many smaller systems, including our application examples, both conditions can be met without excessive hardware requirements. Since the results of this thesis are hardly affected by the choice of voltage or current control, the latter method will be applied from now on. The advantage is reduced system order.

Attention now turns to the second part of the bearing transfer function, the force-current relationship. Measurements as well as calculations yield with good agreement the following quadratic dependency:

$$f(t) = k \cdot \left( \frac{i(t)}{z_0 - z} \right)^2$$

where $k$ is a constant factor characterizing a bearing-rotor combination. The current in the bearing coil is denoted by $i(t)$. The position is separated in a constant part, $z_0$, and a time dependent part $z(t)$. The minus sign with $z$ is because force and deviation have a common positive direction and a decrease in the nominal air gap $z_0$ by a deviation $z$ results in an increase of force $f$. All these variables are shown in figure (2).

Equation (1) is valid with sufficient accuracy as long as three conditions are met:

1) The air gap in the magnetic circuit is so small that stray fields do not have to be considered;
2) The current remains so small that the iron is not saturated.

3) The deviation \( z(t) \) from the zero-position is below ca. 50% of the nominal air gap \( z_0 \).

The value of the constant \( k \) and the range of sufficient validity of equation (1) can be determined by measurement or calculations. Both methods are treated by TRAXLER /14/.

Direct use of the non linear relation (1) in the bearing system model would inhibit the application of a substantial
part of the control engineering tools available. Magnet force as function of current and displacement is therefore expressed in a new equation, which will be more convenient to linearize:

\[ f(t) = k_i(i,z) \cdot i(t) + k_s(i,z) \cdot z(t) \]

The positive factors \( k_i \) and \( k_s \) characterize a bearing-rotor combination in terms of N/Amp and N/mm. These factors of course depend on the point of operation, that is to say the load force, the rotor position and the bearing current. A linear model is obtained when \( k_i \) and \( k_s \) are treated as constants.

In most cases, the load conditions will require that the force \( f(t) \) be applied in positive or negative direction as well. A second magnet must therefore be provided on the opposite side of the rotor (in Fig. 2 on the left side). Only one of the two magnets is active at a time, the sign of the current \( i(t) \) determining which one. Such a pair of magnets will subsequently be referred to as one single actuator. Equation (2) includes this information on the sign of the current \( i(t) \), as opposed to equation (1) which is valid only for a single magnet.

On implementing a bearing system, it may be necessary to account for the dependency of the factors \( k_i \) and \( k_s \) on the point of operation. This can be done in several different ways: Linearizing functions can be provided in the controller (fig. 3a) in order to compensate the nonlinearity partially or completely. When digital control is used, this can be done without any additional hardware, e.g. by providing position dependent feedback coefficients.

In contrast to linearizing on the signal side, the actuator itself can be made as linear as possible (fig. 3b). This is achieved by premagnetizing the magnets with a permanent field or a constant current.
The advantages of this method are simple realization, good system performance and availability based on previous works (SCHWEITZER, LANGE /20/, ULBRICH /8/). The drawback is higher energy consumption of the bearings. Furthermore, premagnetizing yields good results only when applied to bearings consisting of a pair of magnets, one for each sign of the force direction as described above.

In practice, both approaches to linearization can be combined: As an example, the singularity in the parabolic current characteristic (1) near zero can be avoided by premagnetizing both magnets of such a pair with a small constant current. For larger currents, the parabola can be compen-
sated by a square-root function between controller-output and power amplifier.

As a result of these linearizing measures, control layout may be based on equation (2) with constant coefficients $k_S$ and $k_I$. In this case, the overall system contains only linear elements, a fact which greatly simplifies control design. These two coefficients will be the only bearing parameters used in subsequent chapters.

The parameter $k_S$ of equation (2) is typical of magnetic bearings using attractive and not repelling forces. It corresponds to a negative stiffness which is a basic difference to conventional mechanical behaviour. It also sets a limit on reasonable pole locations for the closed loop system, as will be shown below.

2.2 System with One Degree of Freedom

As an example of a system with one degree of freedom, a simple axial (thrust) bearing system is considered. If the rotor is vertical and the maximum downward acceleration of gravity is considered sufficient, only one magnet is needed.

The constant factors $k_S$ and $k_I$ are obtained by expanding (1) in a Taylor series with respect to rotor position and bearing current and taking only the linear coefficients.

The control variable $u$ is defined as the time dependent part of the total bearing current $i(t)$ superposed to a constant part $i_o$ compensating the weight:

$$u(t) = i(t) - i_o \quad \text{where} \quad i_o = \frac{mg}{k_I}$$

With these definitions and the help of (2), the following state equations can be derived with Newton's law of motion for the $z$-coordinate:
Fig. 4: Simple axial bearing system; The z-coordinate of the center of gravity is defined to be zero at nominal rotor position.

\[
m\ddot{z} = k_\text{s} z + k_\text{i} i(t) - mg
\]

(bearing force \quad weight (according to equation (2))

The vectors and matrices are:

\[
x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}, \quad A = \begin{bmatrix} \emptyset & 1 \\ \frac{k_\text{s}}{m} & \emptyset \end{bmatrix}, \quad b = \begin{bmatrix} \emptyset \\ \frac{k_\text{i}}{m} \end{bmatrix}
\]

The open loop system is unstable owing to the force-displacement factor \( k_\text{s} \) acting like a negative stiffness (fig. 5, left)

For this simple system, the system poles are shifted to stable locations by providing a state feedback of the type
(4) \[ u = - ( k_1 \cdot z + k_2 \cdot \dot{z}) = -k' \cdot x \]

By choosing the displacement feedback coefficient \( k_1 \) equal to \( 2 \cdot k_s / k_1 \), the sign of the force-displacement factor is inverted. The natural "speed" of the system is not altered. The poles have been shifted from \( \pm \sqrt{k_s / m} \) to \( \pm i \sqrt{k_s / m} \), corresponding to an undamped mass-spring oscillator (fig. 5, centre).

Fig. 5: Effect of the state feedback coefficients on the pole locations of a system of order two. In this example, the feedback is selected in such a way, that the absolute value of the system poles is kept constant. The natural "speed" of the system is thus conserved.

A damping force (proportional to the displacement speed) is now introduced by the feedback coefficient \( k_2 \).

To avoid overshoot in the step response, the damping constant is selected equal to \( 2 \cdot \sqrt{k_s / m} \). The closed loop poles move from the imaginary axis to a double pole on the negative real axis (fig. 5, right). The damping constant has to be divided by \( k_1 / m \) to yield \( k_2 \). We have thus derived the feedback.
This feedback, derived by conserving the absolute values of the open-loop system, is robust to uncertainties in the system parameters, which is confirmed in applications. It appears to be well suited for a "first try", when no other requirements can be given.

2.3 Optimal Feedback: Choice of Q and R

The simple example of the previous section is now generalized in order to be applied to larger systems.

The closed loop system with feedback (5) has a double pole on the real axis. The unstable pole of the open loop system at $\sqrt{v_k m / k_i}$ has been shifted to a position symmetric with respect to the imaginary axis. It is well known from theory (e.g. SCHWARZ, /21/) that this corresponds to a special optimal feedback, "optimal" meaning that this performance index $J$ is minimized:

$$
(6) \quad J = \lim_{\varepsilon \to 0} \int_0^\infty \left( x' Q x + u' R u \right) dt
$$

The choice of the weighting matrices $Q$ and $R$ is subject to the following well known restrictions:

$R$ has to be positive definite, $Q$ positive semi-definite.
Furthermore, \( Q \) and the system matrix \( A \) have to satisfy the following condition:

\[ Q \text{ can be decomposed without loss of generality into a product } C'^T C. \text{ Matrix } A \text{ and matrix } C \text{ have then to form an observable pair in the control theoretical sense /21/.} \]

Apart from these restrictions, the choice of \( Q \) and \( R \) is free. For general feedback design, the parameter \( \epsilon \) in (6) will not necessarily approach zero.

The restrictions on \( Q \) and \( R \) apparently still leave a considerable freedom of design owing to the large number of parameters in the weighting matrices. This seemingly large freedom of design is obviously not inherent in the physical system. It can therefore be derived that the actual freedom of design is not significantly reduced by appropriate additional restrictions on the choice of \( Q \) and \( R \):

For example, matrix \( R \) can be set equal to identity. For mechanical systems, the following restrictions on \( Q \) are proposed: Displacement speeds and cross products are not weighted unless practical reasons to do so are present. The remaining diagonal elements of \( Q \) are all set equal to one, leaving just the single parameter \( \epsilon \) to play with.

In our application, there is no need for a more sophisticated choice of the weighting matrices. If more freedom of design is needed in terms of pole locations, then pole placement techniques can be applied directly (ACKERMANN /22/).

This section is completed by some remarks on the choice of the remaining design parameter \( \epsilon \):

Minimizing the performance index \( J \) (6) with \( \epsilon \) approaching zero yields the minimum energy feedback since \( J \) then consists mainly of the integral over the square of the control variable \( u \). This integral over time is then proportional to the actuator energy.
In higher order systems, minimum energy feedback can lead to multiple poles of the closed loop system. Such poles can be especially sensitive to system parameter variations. It can therefore be of advantage to split up such multiple poles by increasing ε.

The need to increase ε can also arise from practical requirements. This question is treated in the next section, where the mechanical properties are considered and hints as to the choice of ε can be derived.

2.4 General Feedback Design Considerations for Magnetic Bearings

The resulting mechanical properties of the minimum energy feedback are characterized by a stiffness corresponding to the force-displacement factor k_s of the bearing. The damping force is just large enough to avoid overshoot in the step response. For faster response, this force may be reduced somewhat.

The mechanical stiffness resulting from minimum energy feedback may not be satisfactory for several reasons: The parameter k_s, depending on the point of operation may be too small. The open loop poles may also be too close to the imaginary axis, e.g. owing to fast rotation of the rotor (see chapter 7). In these cases, minimum energy feedback produces system poles too close to the imaginary axis.

Increasing ε in the performance index J (6) moves the poles to the left, raising the stability margin defined as the minimal distance of the system poles from the imaginary axis. This is achieved by larger feedback coefficients. The system response becomes faster, the mechanical bearing stiffness is higher. Of course, the range of operation with unsaturated actuator is decreased accordingly since the maximum bearing force (mentionned in section 1.4) remains constant. The saturation of the actuator will be reached either when the
power amplifier delivers its maximum output current or when the iron of the bearing reaches magnetic saturation, whichever comes first.

It can theoretically be shown, that such a saturation does not necessarily destabilize the system (NUSSBAUM & BAUMGARTNER, /23/). The upper bound on adequate values of $\varepsilon$ does therefore not depend on the saturation of actuators. Sensitivity aspects are more decisive. The question can be put like this: How small may the range of unsaturated operation become?

For digital implementation of the feedback, the stiffness of deadbeat control would be too high. Such control would require that the rotor of the axial bearing example moves back to the nominal position from any initial state in just two sampling periods which is clearly not realistic.

Whether a possible feedback is reasonable or not may be decided by viewing the displacement feedback coefficient as dynamic mechanical stiffness. The actual dynamic bearing stiffness is equal to this stiffness minus the bearing factor $k_s$.

Within the limit of the maximum bearing force, static stiffness to load changes can approach infinity by using an integrating feedback path.

This section on properties of optimal feedback is concluded by an argument against pole placements to the right of the minimum energy solution:

The effect of such a choice of closed loop poles is to slow down the system using increased actuator energy. Even if this is intended, there is a drawback of such system design: The sensitivity of the poles with respect to system parameters can increase drastically. This can be demonstrated by our simple system of equations (3), (4) and (5):

The closed loop poles are equal to $-\sigma \pm \sqrt{\sigma^2 - \omega^2}$, where
\( \sigma = \frac{k_1 k_2}{2 m} \) and \( \omega = \sqrt{\frac{k_i}{1} k_1 - \frac{k_S}{m}} \)

The relative sensitivity \( S^{\omega}_{k_1} \) of the absolute value of the poles \( \omega \) with respect to \( k_1 \) is:

\[
S^{\omega}_{k_1} = \frac{\omega}{k_1} \frac{k_1}{\omega} = \frac{1}{2} \frac{k_1}{m} \frac{k_1}{m} \frac{k_i}{k_1} \frac{k_1}{k_1} \frac{k_S}{m} \frac{k_S}{m}
\]

The absolute value of the poles, \( \omega \), is \( \sqrt{\frac{k_S}{m}} \) for minimum energy feedback. If a pole is placed closer to the origin of the complex plane, e.g. at a distance \( \sqrt{\frac{k_S}{m}} / n \) from the origin, \( k_1 \) has to be set according to the following expression, taken from (7)

\[
k_1 \frac{k_i}{m} = \frac{k_S}{m} (1 + 1/n^2)
\]

This yields a sensitivity equal to \( (n^2 + 1)/2 \). For \( n \) equal to four, for instance, a relative error in \( k_1 \) is amplified already to 8.5 times the relative error in terms of pole location.

For systems with parameters not accurately known, such pole placement is not feasible.
2.5 Summary of the Chapter

The information on a bearing-rotor combination needed for the implementation of a magnetic bearing control can be concentrated in the two factors $k_1$ (force per current) and $k_s$ (force per displacement).

Minimum energy feedback can be considered as a certain "lower limit" for practical pole locations. Poles to the right of it increase sensitivity; poles to the left of it are optimal for performance indices of the type (6) with increasing parameter $\varepsilon$.

The minimum mechanical stiffness from a practical point of view of the bearing system is thus given by the force-displacement parameter of the bearing as defined in section 2.1. In theory, there is no upper bound to this stiffness.

Considerations are given as to the choice of the weighting matrices $Q$ and $R$. They will be applied to the higher order system in the following chapters.
3 Modelling of the Radial Bearing System

3.1 Basic Assumptions, Coordinates and Rotor Geometry

The following six basic assumptions on the mechanical system much simplify the mathematical description. They will be considered valid throughout this thesis.

1) Deviations from zero-position are small compared to the rotor dimensions. This allows decoupling of axial motion from the radial motions.

2) Off-diagonal terms of the matrix of inertia are small compared to the diagonal terms.

3) The rotor is symmetric, i.e. the two principal moments of inertia about the transverse axis are equal.

4) The rotational speed \( \Omega \) changes slowly compared to the other system time constants.

5) Bearing forces in two directions perpendicular to each other are independent.

6) The rotor is modelled as a rigid body.

An analysis of a flexible rotor model considering structural bending modes is given by SALM and SCHWEITZER in \( /24/ \).
Fig 6: Coordinate directions and positions of the radial bearings and sensors. In nominal rotor position, the centre of mass is at the coordinate origin $O$ of the fixed system $x$-$y$-$z$. The value of the distances "$a$" and "$c$" are negative, according to the direction of the $z$-axis.

A fixed cartesian coordinate system $x$-$y$-$z$ (Fig. 6) is used. In the nominal rotor position, the centre of mass is assumed to coincide with the coordinate origin $O$ and the rotation axis of the forced rotation $\Omega$ is then the $z$-axis of the system $x$-$y$-$z$.

The angles $\alpha$ and $\beta$ denote the inclination of the rotor with respect to the system $x$-$y$-$z$. 
The position of the points, where the bearing forces act on the rotor, are given by distances "a" and "b" for the radial bearings with corresponding subscript "a" or "b". Analogous distances are defined for the planes of the radial sensors along the z-axis and denoted by the letters "c" and "d". The sign of the distances a, b, c or d is negative for bearings or sensors placed in negative z-direction from the centre of gravity.

3.2 Equations of Motion

The state equations governing this mechanical system are based on the equations of motion. They can be formulated as Lagranges equations (ULBRICH, /8/). For this simple system, the Newton-Euler equations can be used for a direct derivation (appendix).

The physical parameters are clearly grouped in this representation of the equations of motion:

\[(8) \quad M \ddot{z} + P \dot{z} + S z = B u_f + V s\]

with the following vectors and matrices:

Vector \( z \) consists of the four coordinates used in the Newton Euler equations describing the radial motion:

\[ z = [ \beta, x, -\alpha, y]' \]

\( x(t) \) and \( y(t) \) refer to the radial motion of the centre of gravity. In nominal rotor position, \( z(t) \) is equal to \( \theta \).
The vector $u_f$ contains the bearing forces:

$$u_f = [ f_{ax}, f_{bx}, f_{ay}, f_{by} ]'$$

The first subscript of a force refers to the bearing ("a" or "b"), the 2nd subscript designates the radial direction of the force (x or y).

The harmonic disturbances are introduced by the vector $s$:

$$s = [ \sin \Omega t, \cos \Omega t ]'$$

The mass matrix $M$ is diagonal:

$$M = \begin{bmatrix}
I_x & 0 & 0 & 0 \\
0 & m & 0 & 0 \\
0 & 0 & I_x & 0 \\
0 & 0 & 0 & m
\end{bmatrix}$$

The rotor mass is $m$, the moment of inertia about the x- and y-axis is $I_x$. Since we are dealing with a symmetric rotor containing only small unbalances, $I_y$ equals $I_x$.

The gyroscopic matrix $P$ is proportional to the rotation at speed $\Omega$ and the axial moment of inertia $I_z$. It is the only matrix coupling the subsystems $(x, \beta)$ and $(y, \alpha)$ with each other:
Matrix $S$ describes all position dependent forces except the bearing forces. It includes various influences such as, for instance, a load force applied over a mechanical stiffness. In our centrifuge example (with a vertical rotor), these forces are resulting from gravity as well as from a certain radial stiffness constant of the axial bearing. These influences induce weak pendulous restoring forces. The terms in matrix $S$ become so small compared to the other terms of equation (8) that they may safely be neglected.

Generally matrix $S$ cannot always be neglected. However, it will often be diagonal and therefore not interfere with results derived later on. If not neglectable, it could always easily be included as additional term in all the following derivations; it will be left out for more simplicity.

Matrix $\tilde{B}$ introducing the bearing forces in system (8) consists mainly of the distances $a$ and $b$ (Fig. 6) of the radial bearings from the center of mass. The values of $a$ or $b$ are negative for bearings placed in negative $z$-direction from the center of mass.

$$\tilde{B} = \begin{bmatrix}
a & b & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & a & b \\
0 & 0 & 1 & 1
\end{bmatrix}$$
Matrix \( \bar{V} \) of harmonic disturbances due to mass unbalance contains the products of inertia \( I_{xz} \), \( I_{yz} \) and the offset distance \( e \) of the centre of gravity from the (magnetic) rotor axis. This matrix is proportional to the square of \( \Omega \).

\[
\bar{V} = \begin{bmatrix}
-I_{yz} & I_{xz} \\
0 & me \\
I_{xz} & I_{yz} \\
me & 0
\end{bmatrix} \Omega^2
\]

System (8) can easily be transformed into state equations as they will be used for the control design:

\[
\dot{x} = A\,x + B\,u + V\,s
\]

The state vector \( x \) is defined by \( x' = [z', \dot{z'}] \), as it is usually done for mechanical systems. This yields a system structure in "block companion form" which will be used to compute the characteristic polynomial explicitly (chapter 7).

The input vector \( u \) consists of the four bearing currents, arranged in the same order as the forces in vector \( u_f \) of equation (8). With the help of the force-current-displacement relationship (2) of section 2.1, vector \( u_f \) is replaced by rotor displacements at the bearings and bearing currents.

The rotor displacements in the bearings are expressed in terms of vector \( z \):
The following state equations obtained from (2), (8) and (10) define the matrices $A$ and $B$ of (9):

\[
\begin{bmatrix}
  x_a \\
  x_b \\
  y_a \\
  y_b
\end{bmatrix} = 
\begin{bmatrix}
  a & 1 & 0 & 0 \\
  b & 1 & 0 & 0 \\
  0 & 0 & a & 1 \\
  0 & 0 & b & 1
\end{bmatrix} \begin{bmatrix}
  \alpha \\
  \beta
\end{bmatrix}
\]

with the identity matrix $I$ and the zero matrix $0$ of appropriate dimensions.

Finally, the state equations are transformed into the two state representations most convenient for our purpose: Sensor coordinates $x_S$ and bearing coordinates $x_B$.

In the next two paragraphs, matrices $A$ and $B$ and the state vectors are given for these two state bases. Since matrix $V$ and $B$ are derived in exactly the same way from $\bar{B}$ and $\bar{V}$ respectively, matrix $V$ will be left out here. The transformation formulae are given in the appendix.
3.3 State Equations in Bearing Coordinates

The subscript "B" is used for vectors and matrices in this state base. The main advantage of this representation is its simplicity, mainly the proportionality of the non trivial submatrices in the system matrices $A_B$ and $B_B$. This representation can be used to compute the system matrices "by hand" from the system parameters. The state vector consists of the rotor displacements at the bearings and their derivatives:

$$X_B = [x_a, x_b, y_a, y_b, \dot{x}_a, \dot{x}_b, \dot{y}_a, \dot{y}_b]$$

The elements of vector $u$ are the currents in the control coils of the bearings, arranged in the same order as the displacements in $X_B$.

The transformation from the state vector of equation (9) to the state vector $X_B$ is performed by this equation written with 4x4 submatrices:

$$X_B = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$$

Submatrix $T$ is given in equation (10). If $A$ and $B$ denote the system- and input matrix of equation (9), the corresponding matrices $A_B$ and $B_B$ in bearing coordinates are computed by:

$$A_B = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} A \begin{bmatrix} T^{-1} & 0 \\ 0 & T^{-1} \end{bmatrix} \quad \text{and} \quad B_B = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} B$$

Here matrices $A_B$ and $B_B$ are represented by 2x2 submatrices:
The submatrix $K$ is:

$$
K = \frac{k_s}{m} \begin{bmatrix}
1 + \frac{a^2}{r^2} & 1 + \frac{ab}{r^2} \\
1 + \frac{ab}{r^2} & 1 + \frac{b^2}{r^2}
\end{bmatrix}
$$

with the radius of gyration $r = \sqrt{I_x/m}$ about the $x$-axis and the rotor mass $m$.

Submatrix $G$ contains the gyroscopic terms:

$$
G = \frac{I_z\Omega}{I_x(b-a)} \begin{bmatrix}
a & -a \\
b & -b
\end{bmatrix}
$$

### 3.4 State Equations in Sensor Coordinates

In this section, the subscript "S" marks vectors and matrices of the sensor coordinate description. This representation differs not much from the previous one, when the sensors are not very far away from the bearings.

The state vector $\mathbf{x}_S$ contains the radial displacements in the sensor planes in the same order as vector $\mathbf{x}_B$.
\[ x_S = [ x_c, x_d, y_c, y_d, \dot{x}_c, \dot{x}_d, \dot{y}_c, \dot{y}_d ]' \]

These are the physically available position signals. In order to avoid unnecessary on-line signal processing, this state basis is used on implementing the control.

The transformation matrix to get the system matrices \( A_S \) and \( B_S \) from the matrices \( A \) and \( B \) of equation (9) is block diagonal with the 2x2 blocks now containing the sensor positions expressed by "c" and "d" (See figures 6 and 7, p.25 and 36). The 2x2 block is:

\[
T_S = \begin{bmatrix}
c & 1 \\
d & 1 
\end{bmatrix}
\]

The gyroscopic submatrix, not being dependent on bearing forces, stays exactly the same as in the previous case (12), with the obvious exception of containing now the distances \( c \) and \( d \) of the sensor planes from the centre of gravity, in place of the bearing positions \( a \) and \( b \).

The system matrices in sensor coordinates are not given explicitly as was the case for the bearing coordinates, because the stiffness-submatrix in \( A_S \) gets somewhat messy. It is therefore usually easier, to fill in the system parameters in the bearing coordinate representation and then to transform the system into sensor variables. The transformation from bearing to sensor coordinates is of course done in one step by combining the inverse transformation (10) and the transformation given by \( T_S \).

As a consequence of the structure of the matrices (11) and the transformation matrices, all these calculations can be carried out by performing matrix operations with 2x2 blocks only.
4 Numerical Examples

4.1 On the Choice of Examples

As mentioned in the introduction, the control of several systems has been implemented on the basis of results of this work. We are therefore not compelled to rely on purely academic examples. The systems built include three demonstration rotors of different size and with different linearizing methods. The fourth system is a centrifuge designed for physical experiments in high vacuum (/18/).

Owing to the very unsymmetric bearing-sensor arrangement of the centrifuge, this system is best suited as a general example. The other three rotor systems are closer to symmetry, hence easier to decentralize. It can safely be stated that the methods presented for the general example are easily applied to the simpler symmetric rotors.

The one feature not provided by our experiments is high rotational speed: Our motor produced only 6000 rpm. The examples with higher rotational speed given below do therefore not correspond to experiments actually carried out at our institute. Those cases are numerically simulated examples given to illustrate the gyroscopic effects.

4.2 Precision of Numerical Values

The modelling procedures described in chapter 2 produce a relatively coarse mathematical description of the physical interdependencies. Parameters like $k_S$ and $k_I$ characterize a
magnetic bearing in a direct manner. However, they are difficult to be determined accurately. Furthermore, they depend on the load forces, or, expressed more generally, on the operating point.

In view of these facts, the matrices defined in the previous chapter need no high precision entries. Three significant digits suffice.

4.3 Physical System Parameters for the Centrifuge

As seen in the last two sections of the previous chapter, only ten parameters are needed. The first four belong to the rotor.

- Rotor mass \( m = 9 \) kg
- Radial moment of Inertia \( I_x = 0.616 \) kg m\(^2\)
- Axial moment of Inertia \( I_z = 0.0092 \) kg m\(^2\)
- Rotational speed \( \Omega = 1000 \) rpm or \( 2 \pi \times 1000/60 \) rad/sec

From these values, we obtain the radius of gyration of transverse motion introduced on p.32: \( r = \sqrt{I_x/m} = 262 \) mm.

The next two parameters concern the radial bearings.

- Force-current factor \( k_i = 66 \) N/amp
- Force-displacement factor \( k_s = 80 \) N/mm

The low value of \( k_i \) is a consequence of the relatively wide gap in the magnetic circuit caused by the wall of the vacuum tank.

The next four parameters give the bearing and sensor plane positions measured from the centre of gravity. The planes are perpendicular to the z-axis. These distances, measured from the centre of mass of the rotor and denoted by "a", "b", "c" and "d", are given in figure 7.
Figure 7: Rotor of the vacuum centrifuge. Weight: 9kg, total length ca. 1m; Centre of mass at point 0. The z-coordinates of the bearing and sensor positions are:

- Bearings "a"  
  a = 118 mm
- Bearings "b"  
  b = 346 mm
- Sensors "c"  
  c = 0 mm
- Sensors "d"  
  d = 411 mm

The value of c indicates that sensors are placed in the plane x-y, the plane of the centre of gravity of the rotor.
4.4 System Matrices

The state vector is composed of the four radial displacements and the four displacement rates measured at the two sensor planes "c" and "d". The elements of the state vector $x_S$ are given in section 3.4.

The displacements are scaled in mm, the displacement speeds in dm/sec. This gives a good numerical condition of the system matrices.

The subscript "S" standing for sensor coordinate description will be omitted from now on. Use of any other state base will be mentioned explicitly.

The system matrix $A$ at 1000 rpm has the following entries:

$$A = \begin{bmatrix}
    & & & & 100 & 0 & \\
    & & & & 0 & 100 & \\
    & & & & & & \\
    77 & 100 & & & 0 & 0 & \\
    151 & 274 & & & 0 & 0 & \\
    & & & & & & \\
    151 & 274 & & & 1.56 & -1.56 & \\
\end{bmatrix}$$

As the rotor is elongated, only very fast rotations above 50 000 rpm would produce gyroscopic terms of the same order of magnitude as the other elements of matrix $A$.

The input matrix $B$ is computed according to section 3.4. The elements of the control vector $u$ are the currents through the coils, arranged in the order corresponding to the elements of the state vector:
4.5 Optimal (Central) Feedback at Low Rotational Speed

The weighting matrices $Q$ and $R$ are chosen according to sections 2.3 and 2.4, where matrix $R$ is an identity matrix. In order to avoid multiple poles, the squares of the displacements are weighted with ones. With our scaling of the system, this is still close to the minimum energy feedback.

The vector of control currents $u$ is computed from the state vector by a constant matrix $F$ of feedback coefficients:

$$ u = -F \cdot x $$

The feedback matrices given below are composed of a $4 \times 4$ submatrix of displacement feedback coefficients in amp/mm and a $4 \times 4$ submatrix of displacement velocity feedback coefficients in amp per dm/sec.
Optimal feedback matrix for low rotational speed:

\[
F = \begin{bmatrix}
  2.20 & 0.69 & 0 & 0 & 3.33 & -0.51 & 0 & 0 \\
  0.20 & 2.46 & 0 & 0 & -1.30 & 1.80 & 0 & 0 \\
  0 & 0 & 2.20 & 0.69 & 0 & 0 & 3.33 & -0.51 \\
  0 & 0 & 0.20 & 2.46 & 0 & 0 & -1.30 & 1.80 \\
\end{bmatrix}
\]

Displacement feedback | Velocity feedback

As can be seen from the zeros at the coefficients coupling motions in x-direction to forces in y-direction and vice versa, the layout rotational speed of 1000 rpm is so low in this example, that the effect of the gyroscopic terms on the optimal feedback is not perceivable at reasonable numerical precisions (cf. section 4.2).

The closed loop poles have roughly the absolute values of the open loop poles. Owing to the symmetry of the system, we have double poles at

\[-50.5 \pm 27 \text{ i} \quad \text{and} \quad -195 \pm 70 \text{ i} \quad \text{rad/sec}\]

The mechanical stiffness corresponds about to the bearing parameter \( k_s \) which is 80 N/mm.

Experiments on the real system operated with this feedback confirm what has been stated above. An exact measurement of the actual system poles is difficult for practical reasons, but transient recordings are consistent with the poles given above. Examples of measurements are given in section 8.1. An important feature of such control is its robustness to all the simplifications made in modelling of the system.
4.6 Optimal Central Feedback at High Rotational Speed

The next examples with high rotational speed do not correspond to experiments actually carried out. They are numerical examples reflecting the influence of fast rotation on the system poles.

First, the feedback matrix computed for slow (or zero) rotation is applied to the same system now rotating at 100 000 rpm, that is hundred times faster. The system poles are then at the following locations:

\[-16 \pm 19 \text{i}, -80 \pm 75 \text{i}, -146 \pm 81 \text{i}, -248 \pm 182 \text{i} \text{ rad/sec}\]

The stability margin, defined as the minimum distance of the dominant pole to the imaginary axis, is significantly reduced by a factor three, but the pole locations indicate that the system remains operational.

The optimal control computed from the system matrix A at 100 000 rpm produces the following closed loop poles:

\[-38.5 \pm 6 \text{i}, -51 \pm 80 \text{i}, -170 \pm 111 \text{i}, -183 \pm 41 \text{i} \text{ rad/s}\]

with this feedback matrix:

\[
F = \begin{bmatrix}
1.8 & 0.95 & -0.51 & 0.65 & 2.84 & -0.29 & -0.06 & 0.04 \\
0.9 & 1.02 & -0.38 & -0.86 & -2.25 & 1.28 & -1.11 & 0.04 \\
0.51 & -0.65 & 1.8 & -0.95 & 0.06 & -0.04 & 2.84 & -0.29 \\
-0.38 & 0.86 & 0.9 & 1.82 & 0.11 & -0.04 & -0.25 & 1.28
\end{bmatrix}
\]

This feedback matrix however does not stabilize the system for low rotational speeds for the following reason: The gy-
roscopic coupling terms in the system form an antisymmetric 4x4 submatrix of 2x2 blocks in the lower right quarter of the system matrix A. It is therefore not surprising that the coupling in the feedback matrix also consists of such antisymmetric submatrices, as can be seen above (14).

It follows that the displacement feedback coefficients contain such an antisymmetric stiffness matrix, i.e. a non-conservative term in the mechanical sense. It is well known from theory, that such a term can destabilize a mechanical system (SCHWEITZER /25/).

This hypothesis for the reason of instability is proven by computing the closed loop system poles after removing only this antisymmetric part of the displacement feedback (14): These poles turn out to be stable, demonstrating that the somewhat smaller terms of the block-symmetric parts of (14) compared to (13) are not the cause of the instability.

It follows that "optimal" central feedback as computed above will generally have to be dependent on the rotational speed \( \Omega \) above a certain limit. The value of this limit speed depends on the geometry of the rotor (elongated or disc-like). As can be seen from a comparison of (13) and (14), all 32 feedback coefficients are changing with \( \Omega \).

An adaptive control switching between these two feedback matrices at 100 000 rpm would be a realistic solution. More elegant methods not relying on 64 feedback coefficients will be given in the subsequent chapter.

An apparently simple method to conserve the good performance of the standing rotor controled by (13) at higher \( \Omega \) consists of compensating the gyroscopic terms by the feedback matrix.

This is done by adding block-antisymmetric submatrix \( F_A \) to the speed feedback coefficient submatrix, \( F_A \) being defined as this:
where \( B_2 \) is the non trivial 2x2 submatrix of the input matrix \( B \). It is invertable for any realistic system;

Matrix \( G \) consists of the gyroscopic terms of system matrix \( A \) and is defined according to sections 3.3 and 3.4 as this:

\[
G = \frac{I_x (d-c)}{I_z \Omega} \begin{bmatrix} c & -c \\ d & -d \end{bmatrix}
\]

The resulting feedback produces a closed loop system matrix independent of the rotational speed \( \Omega \). This dependency is now in the eight feedback coefficients introduced by \( F_A \), which is proportional to \( \Omega \).

A constant feedback gain matrix laid out for high \( \Omega \) as described above would produce poles for the standing rotor (feedback of section 4.5) now at the layout rotational speed; the standing rotor on the other hand would have the artificially introduced gyroscopic terms and the corresponding system poles.

Practical application of this kind of feedback is however limited for two main reasons:

The coefficients of \( F_A \) proportional to \( \Omega \) may become much larger than the other feedback coefficients and thus produce large bearing forces which soon saturate the actuators.

The other reason is the bad condition of such a feedback matrix and the resulting sensitivity of the system to inaccurate computation of \( F_A \) (see appendix A3).

The gyroscopic effects are treated more extensively in chapter 7.
5 Decentralization

5.1 Why Decentralization?

The example presented in chapter 4 (vacuum centrifuge) performs well with the "classical" state feedback given there. However, it has been built as a single specimen. As methods are sought for a wider range of magnetic bearing applications, the following facts must be considered.

The control of the centrifuge relies on analog electronics, mainly operational amplifiers. This is possible because no linearizing of the output signals is needed since the bearings are premagnetized with a special coil giving them a good linear characteristic. This will not always be the best solution. A linearizing network as shown in chapter 2 could be required for other systems.

Additional functions are desirable, for instance start-up or touch-down procedures. Changing rotational speed or load conditions can require adaptive control. In many practical applications, an information exchange between the bearing control and other components of a system can be of interest.

All such features are much easier to realize with digital than with analog control. Other important advantages of digital control in industrial applications have been mentioned in section 1.2.

The main difficulty with digital control is the fast sampling rate necessary for magnetic bearing systems. For the centrifuge of chapter 4, the sampling rate should be very
roughly 1 msec.

The value of the sampling time can be obtained by applying the considerations given in ACKERMANN /21/ page 189 and dividing the maximum sampling time thus determined by a "safety factor" of about three. One of the reasons for such a high "safety factor" is that the "natural" speed of the system is tied to the parameters $k_s$ and $k_i$ which depend on the operation point.

This sampling rate is however not designed to control the structural modes of the system. This would require still faster sampling periods. Magnetic bearing control of an elastic rotor is investigated e.g. in /24/. The interest of this work remains on rotors treated as rigid bodies. This is possible for many applications.

The fast sampling rates call for simplifying the feedback matrices given in chapter 4. Every non zero element of those matrices requires a multiplication in the microprocessor.

Decentralization as presented below reduces the number of unnecessary multiplications in the complete state feedback by eliminating a substantial part of the feedback coefficients. The structure of the controller is thus simplified.

Each so called "local control" can be reduced to a minimum of just two multiplications, compared to the 16 or 32 of the central control of the examples of chapter 4.

5.2 Subsystems

The feedback matrix (13) is optimal for a system with neglectable rotational speed $\Omega$. The system then decomposes naturally into two totally uncoupled subsystems, one for motions in the x-z plane, the other one for motions in the y-z plane. All coupling terms in the optimal feedback matrix can be set to zero, since the corresponding coupling terms are
zero in the input matrix $B$ and neglectable in system matrix $A$.

In the feedback matrix, a similar substructure can be discerned also in each subsystem: The $2 \times 2$ submatrices in (13) have dominant diagonal terms. Setting the non diagonal terms of those submatrices equal to zero decouples measurements of sensors $a$ from forces in bearing $b$ and vice versa. Such a local feedback presents a significant reduction in the number of operations to be carried out by the processor.

The term "local" implies the following concept: Radial sensors $a$ and radial bearings $a$ form the subsystem $a$, the other radial sensors and bearings form subsystem $b$. This structuring corresponds to the physical separation of the bearing units. The sensors are each assigned to a bearing unit. (Fig. 8) Sensors $c$ to bearing $a$, Sensors $d$ to $b$)

![Decentralized Control Structure](image)

Fig. 8: Central versus decentral control structure.

If both subdivisions described above are applied at the same time, the system decomposes into four subsystems: Bearing
"a" and x-direction, bearing "a" and y-direction, bearing "b" and x-direction, bearing "b" and y-direction. The possible physical subdivision of the control into smaller local control units with no need of mutual information exchange facilitates use of standard microprocessors.

This special decentralization has been applied often in "empirical" control layouts without microprocessors (e.g. /12/). In the following sections, the same simple feedback structure will be sought, but now considering the complete state model for the control design. The decentral feedback thus derived will be compared to "empirical" decentral control and to central control.

The second decoupling of the feedback matrix does not correspond to a similar structure of the system matrices A and B (12): Some diagonal terms of the 2 by 2 submatrices in A and B are smaller than off-diagonal terms. The reason for this coupling are found by some physical considerations:

A completely decentralized feedback as developed above corresponds in about to a passive classical bearing system with mechanical stiffness and damping, i.e. forces proportional to local displacement or displacement velocity. Coupling terms occur, because a force at one bearing induces accelerations at both sensors.

In such a mechanical system, a deviation at bearing "a" in x-direction will produce a response force only in the corresponding bearing and direction. As known from classical theory of rotating systems, gyro terms cannot cause instability of such a rigid-body system. This is valid in spite of the coupling of the four radial subsystems through the equations of motion.

There is, however, a significant difference in a magnetic bearing system: The signals directly available are not displacements at the bearings themselves, they are displacements at the sensors. In other words, the measurement system introduces an additional coupling of the subsystems "a" and
"b", unless the sensor-signals are preprocessed or the sensors are placed directly at the bearings.

This coupling of the subsystems is responsible for the fact, that the fully decentralized system is not equivalent to a bearing system with a simple mechanical stiffness and damping at each bearing. As a consequence, it cannot be deduced trivially for the general case, that such a decentralized system will always be stabilizable.

Therefore, decoupling of the control will generally force to zero some non trivial feedback coefficients.

Up to now, only two possible divisions into subsystems have been mentioned: Decoupling of the two radial directions from each other and decoupling of the two bearing units. Many quite different subdivisions are conceivable: Separation of rotation and translation (BECKMANN, /26/), modal decomposition etc.

For every such decomposition, the feedback coefficients can be separated into coupling terms and non coupling terms, as long as the appropriate state basis is used. The results of the following sections are applicable to such cases as well.

5.3 Definition of Decentral Feedback

Decentral feedback can now be defined as a feedback avoiding coupling terms from one subsystem to another on the level of the control. The system matrices themselves will by no means be free of coupling, i.e. the overall system description is retained.

It is obvious, that system performance will not improve by decentralizing the control. Compared to optimal central feedback, decentral feedback will have an increased performance index. Generally, the stability margin (the negative real part of the dominant eigenvalue) will become lower.
Even worse, the situation may arise that a system may not be stabilizable by decentral feedback, given a certain division into subsystems.

It is therefore necessary to find methods which yield the best possible feedback coefficients for a given structure of decentralization. The negative effects mentioned above should be as small as possible. As we are dealing with systems of small order, additional off-line effort is justified in order to optimize the on-line solution.

5.4 Optimal Decentral Feedback

Decentral feedback as defined above will be called "optimal", when the quadratic performance index $J$ (6) is minimized.

As in the central case, such optimal feedback will give good results in practical applications. This is not obvious since optimization is reduced to minimizing the scalar performance index $J$. Nevertheless, the results achieved so far justify application of this design strategy.

Systems theory provides optimization strategies for this problem. The urge for investigations on such topics arose of the necessity to simplify control of very large systems, namely in the field of electrical power distribution. A variety of recent works on large scale systems is therefore available; an important part of these concern decentralization.

One such publication, the thesis of SENNING /13/ of the Institute of Automatic Control in Zürich, presents a "Feasibly Decentralized Control" for large interconnected systems. Only interconnection channels necessary for maintaining the stability are retained.

In the present application on much smaller systems, the
structure of the feedback matrix is regarded as given. A method presented in the appendix of /13/ can be applied to find the optimal decentral feedback as defined above. This method, not being restricted to magnetic bearing systems, is described in the next section.

5.5 Necessary Conditions for Optimal Decentralized Feedback

In a first step, the division of the complete system into a set of \( n_s \) subsystems has to be done on the level of the mathematical model. In practice, this means defining vectors \( x_i \) and \( u_i \) belonging to the subsystem with subscript \( i \) (1, \( \ldots \), \( i \), \( \ldots \), \( n_s \)).

The state variables belonging to subsystem \( i \) are selected from the global state vector \( x \) by a rectangular incidence matrix \( C_i \):

\[
x_i = C_i \cdot x
\]

As an illustration, a matrix \( C_i \) is given here for the magnetic bearing system of section 4.4. The second subsystem is formed by displacement and displacement velocity in \( x \)-direction at sensor \( d \), therefore matrix \( C_2 \) takes the form shown:

\[
x_2 = \begin{bmatrix} x_d \\ \dot{x}_d \end{bmatrix} = C_2 \cdot x = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot x
\]

For more convenience, the control variables in vector \( u \) are arranged in such an order that the subvectors \( u_i \) can be defined as follows:
"Decentralized feedback" is now defined as a feedback which produces the local control \( u_1 \) only from the local state \( x_i \). The local decentralized feedback matrix \( D_i \) is defined by the following equation:

\[
(15) \quad u_i = -D_i \cdot x_i
\]

The columns of the global input matrix \( B \) are distributed among the subsystems according to the subdivision of the input vector \( u \). Such a vertical "slice" of matrix \( B \) is written \( B_1 \), the subscript indicating the subsystem it belongs to.

The influence of each local input vector \( u_1 \) on the global state vector \( x \) is now expressed as the product \( B_1 \cdot u_1 \). The influence of all subsystems are added to obtain the closed loop system matrix \( A_0 \) of the overall system:

\[
(16) \quad A_0 x = A x - \sum_{i=1}^{n_s} B_1 u_1 = A x - \sum_{i=1}^{n_s} B_1 D_i C_i x
\]

The transition matrix of \( A_0 \) is denoted by \( \Phi(t) \):

\[
(17) \quad \Phi(t) = \exp \left( A_0 t \right)
\]

The performance index \( J \) to be minimized is defined as:

\[
(18) \quad J = \int_0^\infty \left( x' Q x + u' R u \right) dt
\]
As opposed to (6) in chapter 2, the parameter $e$ is now included in the elements of $Q$. All that has been said on the choice of the weighting matrices $Q$ and $R$ in sections 2.3 and 2.4 can be applied here.

A set of necessary conditions for feedback matrices $D_j$ to minimize the performance index $J$ can now be derived. The following formulae and methods are taken from the appendix of [13].

First, the performance index (18) is expressed as trace of a matrix product $P \cdot X_0$ with the following definitions:

\begin{equation}
P = \int_0^\infty \phi'(t) (Q + \sum_{i=1}^{n_S} C_i D_i R_i D_i C_i) \phi(t) \, dt
\end{equation}

The initial state forms the matrix $X_0$:

\begin{equation}
X_0 = x(t=0) \cdot x'(t=0)
\end{equation}

The performance index $J$ (18) can now be rewritten using the trace operator:

\begin{equation}
J = \text{tr}(P \cdot X_0)
\end{equation}

The integral property of a stable Liapunov equation (see appendix) can be applied to the integral (19). It follows that matrix $P$ must satisfy equation (22), provided the closed loop system matrix $A_o$ is stable:
(22) \( A_0^t P + P A_0 + Q + \sum_{i=1}^{n_s} C_i^t D_i^t R_i D_i C_i = 0 \)

Starting from this equation and using the rules for the derivative of a trace function with respect to a matrix (/13/ and GEERING /27/), the derivative of \( J \) with respect to the unknown feedback matrices \( D_i \) can be expressed as follows (see appendix):

(23) \( \frac{1}{2} \frac{dJ}{dD_i} = R_i D_i C_i X C_i^t - B_i^t P X C_i^t \)

where matrix \( X \) defined as the product \( x(t) \cdot x'(t) \) or

(24) \( X = \int_0^\infty \Phi(t) X_0 \Phi'(t) \, dt \)

satisfies the Lyapunov equation dual to (22):

(25) \( A_0 X + X A_0^t + X_0 = 0 \)

The gradient (23) must vanish for a minimal performance index:

(26) \( R_i D_i C_i X C_i^t = B_i^t P X C_i^t \)

for all subsystems, i.e. for \( i = 1, \ldots, n_s \).

Equations (16), (22), (25) and (26) represent a set of ne-
cessary conditions for an optimal decentralized feedback.

5.6 Initial State dependency of the solution

Optimal central feedback is not depending on the initial state $x(t=0)$. This is not true for optimal decentral feedback, as seen in equation (25).

The dependency on the initial conditions expressed by matrix $X_0$ is well known from similar problems (output feedback /28/, "optimal" observer, e.g. MUELLER /29/ or ULBRICH /8/). It can be settled in the following manner:

Assuming a statistically independent distribution of the initial states leads to a diagonal matrix $X_0$. The only significant parameter is the ratio of the variance of the displacements (first four elements of the diagonal) to the variance of the displacement speeds (last four elements). This ratio depends on the scaling of the system. For a well chosen scaling, it will be near one. In this case the state covariance matrix $X_0$ can be set equal to the identity matrix.

For the examples investigated so far, it can be noted that the resulting feedback matrices $D(i)$ are not very sensitive to the choice of $X_0$. This is also suggested by the structure of equation (26). For practical applications however, it may be advisable to get an estimate of the influence of $X_0$ by comparing numerically two or three examples.

5.7 Two Special Cases

The four necessary conditions just derived can be applied to two special cases, which will lead to interesting possibilities of the computer implementation.

The first restriction imposed on the general case is to con-
sider the situation of only one subsystem, i.e. \( n_S = 1 \). The given system is not subdivided. However, matrix \( C \) has a row dimension smaller or equal to the number of state variables. (The subscript \( i \) of matrices \( B_i, C_i, D_i \) and \( R_i \), now being irrelevant, it is omitted for the discussion of the special cases) The feedback computed according to equation (15) on p.50 with such a matrix \( C \) must be called an output feedback as opposed to a state feedback since only a subset \( x_1 \) of the state vector is available for the feedback.

Equations (16), (22), (25) and (26) with \( n_S = 1 \) nicely reproduce a result published by LEVINE and ATHANS in 1972 /28/ on optimal output feedback. (/13/)

If the additional restriction \( C = \text{Identity} \) is imposed, the situation of familiar linear-quadratic optimal state feedback is given. In this case, equation (26) can be solved for \( D \):

First, matrix \( C \) may now be left out of equation (26). The nxn matrix \( X \) being regular may also be canceled. Equations (26) and (16) may now be rewritten:

\[
\begin{align*}
(27) \quad D &= R^{-1} B' P \\
A_Q &= A - B D
\end{align*}
\]

Equation (22) now takes the form

\[
A_Q' P + PA_Q + Q + D' R D = 0
\]

Matrices \( A_Q \) and \( D \) can easily be eliminated from this equation with the help of the two relations (27) (matrices \( P \) and \( R \) are symmetric). This yields the familiar equation

\[
A' P + PA - PBR^{-1} B' P + Q = 0
\]
which is of course the well known algebraic matrix Riccati equation. The optimal feedback becomes independent of the initial conditions, since equation (25) is no more needed.

5.8 Algorithm to Determine the Optimal Decentral Feedback

An iteration procedure in order to determine the optimal decentral feedback can be derived from the four necessary conditions of optimality. The outline of the procedure was discussed at a lecture on optimization methods by Senning and Maletinsky (post-graduate program on Automatic Control, ETH Zürich, winter 1981). Based on this, the programs described below have been written at our institute.

When applied to well formulated magnetic bearing problems, the procedure converges quickly, i.e. in less than ten iteration steps. In certain cases, numerical relaxation may be necessary. This has never happened with the magnetic bearing examples tried so far. The procedure is outlined as follows:

A set of $n_s$ suboptimal but stable matrices $D_1$ is used in (16), (22) and (25) to compute $X$ and $P$. Then, (26) can be solved for a new set of $D_1$ which are now used for the next iteration step.

The start up of the procedure can be done with any stable state feedback matrix. There is no need at all to start with a decentral feedback. The first step consists of computing stable closed loop system matrices $A_0$ and to solve the dual Liapunov equations.

A convenient start up method is to compute the optimal central feedback first which is done with the classical algebraic Riccati equation. As this equation is a special case of decentralization, this can be done with the same program.

An iteration step as described above is then carried out to compute a first set decentral feedback matrices $D_1$. This
feedback will stabilize the system for most realistic magnetic bearing systems.

If it does not, the iteration cannot proceed since matrix $P$ and the performance index are then not defined. This has occurred for certain cases with large gyroscopic terms. In this case, it can be tried to start the algorithm with an optimal central feedback corresponding to a matrix $Q$ with larger weighting coefficients.

Another possibility is to look for some realistic decentral feedback based on physical considerations. This is often possible with some common sense. When it stabilizes the system, the iteration can be started.

5.9 Short Description of the Computer Programs

The following concept was adopted: The algorithm outlined above is performed by a Fortran subroutine called DECEN. It is designed to run for interactive as well as for batch main programs.

The main program calling DECEN only does data input and output. In the interactive mode of operation, the results can be checked after each iteration step, i.e. after each call to DECEN. In batch operation, DECEN performs iteration steps until one of the several possible termination criteria occurs.

The routine DECEN and some other subroutines needed were written at our institute. They consist of about 600 Fortran statements (including comment). Furthermore, an algorithm to solve the Liapunov equation is needed. We used the Algorithm 432 by BARTELS and STEWART /30/.

This algorithm had two advantages for our application: First, it can solve both dual Liapunov equation in one step. Then, it yields the eigenvalues of the closed loop system,
which can be checked at every iteration step. Other methods to solve Liapunov equations are compared by KOUPTAN and MUELLER in /31/ (1975).

A singular value decomposition routine is also required. It was taken from the well known EISPACK package. The total number of Fortran statements then adds up to roughly 1300 (including comments).

The program was implemented at the CDC Computer of the ETH (batch) as well as on a mini computer at our institute (HP 1000) for the interactive case.

As the possibilities of the batch version are a subset of the possibilities offered by interactive operation, the latter are shortly described:

The routine is first applied to the central case. Any suboptimal but stable central feedback must be given, along with the structural information of having just one subsystem. Matrix C is set equal to identity. The iteration then converges to the optimal central feedback. This can be used as starting value for the computation of decentral feedback.

At every iteration step, the stability and the old and new performance index can be checked. If necessary, relaxation can be activated.

At any stage, the structure of the system can be changed by giving the number of subsystems, the subdivision of vector u and the matrices C_i. The iteration then produces the decentral feedback matrices. The closed loop system poles can be displayed for every iteration step.

For the magnetic bearing systems tested so far, less than ten iteration steps yielded decentral feedback matrices sufficiently close to the optimal solution. The computational work for those systems of order eight involves less than ten second CPU-time on our mini computer.
A more detailed description is to appear as internal report of the institute /32/.

5.10 Numerical Results for a System with Neglectable Gyroscopic Terms

In this section, decentral feedback is applied to the example of chapter 4. Comparison is made mainly on the level of system poles and performance index $J$ (18).

The relevant data of the poles are the stability margin (smallest real part) and the largest ratio of imaginary part to real part (tangent of the pole angle in the complex plane of the poles). This ratio will be denoted by "im/re".

The performance index, being an integral over time of the state and control variables, contains information on the system behaviour in a condensed form.

Functional plots of state variables versus time are not well suited to compare central and decentral feedback, as practically no difference between the two plots can be discerned. Such plots are therefore not presented in this section. They are found in section 8.1 on experimental results.

In figure 9 (p.60), the poles of the example of chapter 4 are given for central and decentral feedback. Two kinds of decentral feedback are used:

The first one can be called "brute force" decentralization. It consists simply of setting the coupling terms of the optimal feedback (13) equal to zero and leaving the other coefficients untouched:
Suboptimal Decentral Feedback for 1000 r.p.m:

\[
\begin{bmatrix}
2.20 & 0 & 0 & 0 & 3.33 & 0 & 0 & 0 \\
0 & 2.46 & 0 & 0 & 1.80 & 0 & 0 & 0 \\
0 & 0 & 2.20 & 0 & 0 & 3.33 & 0 & 0 \\
0 & 0 & 0 & 2.46 & 0 & 0 & 0 & 1.80
\end{bmatrix}
\]

This feedback can also be written in the shorter form of equation (15). Since two of the four subsystems are equal, we need only write the displacement and velocity feedback coefficients for subsystem "1" and subsystem "2":

\[
D_1 = [2.20, 3.33] \quad \text{and} \quad D_2 = [2.46, 1.80]
\]

Subsystem "1" belongs to bearing "a", subsystem 2 to bearing "b".

The poles of the closed loop system with this feedback are at

\[-44.1 + 57.9 \, i, \ -44.9 + 58.8 \, i, \ -48.9 + 0, \ -512 + 0\]

The ratio \( \text{im}/\text{re} \) of the dominant pole is 1.3. The performance index \( J \) (18) is by 12.6 % larger than \( J \) for the optimal central feedback. Summarizing the properties of this feedback, it can be said that the system remains operational with relatively small changes in performance. Computation of the system poles shows a distinct superiority of the optimal central feedback of chapter 4. (See also figure 9)

The second example is the optimal decentral feedback, obtained with sufficient accuracy after six iteration steps and using the optimal central feedback of chapter 4 as starting values for the iteration loop. The local feedback matrices are:

\[
D_1 = [2.04, 3.47] \quad \text{and} \quad D_2 = [3.12, 3.00]
\]

The closed loop system poles are now at
The ratio $\text{im/re}$ of the dominant pole is now $0.93$ while the performance index is $9.4\%$ larger than in the optimal central case.

Figure 9: System poles with different feedbacks for the example of chapter 4 (Centrifuge, $1000$ r.p.m.). The performance index $J$ (18) results with matrix $X_0$ equal to identity and subsequent scaling, $Q$ and $R$ are selected as in chapter 4, i.e. containing only zero's and ones. The largest ratio $\text{imag./real}$ is given for each feedback:

- Optimal central feedback (13) (chapt 4) $\text{im/re} = 0.53$
- "Brute Force" decentralization: $\text{im/re} = 1.30$
- Optimal decentral feedback: $\text{im/re} = 0.93$

As this data demonstrates, optimal decentral feedback brings some improvement of system performance compared to the "brute force" decentralization. Nevertheless, no dramatic difference in system performance can be found among the three examples of fig. 9.

The comparison suggests that good decentral feedback can of-
ten be found with unsophisticated methods of decentralization like setting zero of coupling coefficients or designing the feedback for each bearing locally, without using the complete state model.

On the other hand, there are cases, when such decentral feedback will cause more drastic changes of the system controlled by optimal central feedback. Examples are given in chapter 7 on gyroscopic effects; they may also occur, when the sensors must be placed far from the bearings. The improvement brought by optimal decentral feedback, even if small for the example presented in this section, can become important in such cases.

Furthermore, optimal decentral feedback shows, how much the system performance can be improved just by adjusting the feedback coefficients while retaining the very simple decentral control structure. It can be useful to know such theoretical limits.

It is recalled that the present example concerns a rotor with a rather unsymmetric sensor-bearing arrangement (cf. figure 7). Such an arrangement causes a relatively strong coupling of subsystem "a" with subsystem "b". The results presented here demonstrate that even such a system not naturally suited for decentralization is easily controllable with decentral feedback.
6 Luenberger Observer for Decentral Magnetic Bearing Control

6.1 Determining the State Vector

The results of this chapter concentrate again on magnetic bearings, as opposed to the more general formulation of chapter 5.

Central as well as decentral feedback of magnetic bearing systems make use of the complete state vector \( \mathbf{x}_S \). Direct measurement of position and speed is possible (TRAXLER, Institute of Mechanics, ETHZ; ULBRICH /33/), but usually only position sensors are provided. In that case, control will consist not only of feedback coefficients, but also on some additional signal processing. For continuous control, the most obvious solution is differentiating the position signal, which implies low pass filters.

Both solutions - direct measurement and differentiating - have been applied successfully on bearing systems at the Institute of Mechanics.

The Luenberger observer (or state estimator) has to be considered as a third possibility to get information on the complete state vector. The main advantage compared to differentiation is the reduced sensitivity to random noise.

It cannot generally be decided, which method will be the best for a given application; all possibilities given above have to be considered. Direct measurement and differentiating do not interfere with decentralization at all. The ad-
vantages of decentralization are jeopardized only by the observer since it consists basically of a model of the complete system.

In view of a simple control structure, only the reduced observer is considered here. This name indicates, that only the non measured state variables are estimated, and not, as in the complete observer, the whole state vector.

It will be shown, that such an observer can be at least partly decentralized for magnetic bearings to such an extent that its application will be consistent with decentral feedback.

First, the layout procedure of the reduced observer without decentralization is briefly recalled. It can be found in several textbooks on linear systems control. The notation varies much and it is therefore useful to introduce it here with symbols not conflicting with previous definitions of this thesis. The equations given below and part of the notations are taken from MANSOUR /34/.

6.2 Central Reduced Observer Layout

The system is given by matrices $A$, $B$ and $C$ and the equations

$$\dot{x} = Ax + Bu \quad \text{and} \quad y = Cx$$

with the vector $y$ of the directly available output signals. The system is assumed to be transformed in a state basis where the $k \times n$ matrix $C$ takes the form

$$C = \begin{bmatrix} I \mid 0 \end{bmatrix}$$
with identity matrix "I" and zero matrix "Ø". This transformation is always possible. No transformation is necessary for mechanical systems, where the directly measured displacements form the first part of the state vector and the velocities to be estimated the second part of this vector.

The system matrices A and B are partitioned according to the division of the state vector into measured part and not accessible part as defined by matrix C:

$$A = \begin{bmatrix} A_1 & A_2 \\ - & - \\ A_3 & A_4 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ - \\ B_2 \end{bmatrix}$$

A matrix N of dimension (n-k)xk is now determined in such a way, that the eigenvalues of a matrix W defined as

$$W = A_4 - N \cdot A_2$$

are placed at the desired observer pole locations. These should be placed by a factor of 2 through 3 to the left of the system poles to be controlled. This can be deduced by determining "optimal" observer dynamics according to MUELLER /29/.

The structure of the reduced observer is shown in fig. 10. Matrices U and L are computed according to

$$U = A_3 + W \cdot N - N \cdot A_1 \quad \text{and} \quad L = B_2 - N \cdot B_1$$
Fig. 10: Structure of the reduced non-decentralized observer. Vector $\mathbf{y}$ is of dimension $k$, $k < n$; vector $\hat{\mathbf{q}}$ is the estimated part of the state vector, its dimension is $n-k$. Feedback matrix $\mathbf{F}$ is divided in two parts, $\mathbf{F}_x$ and $\mathbf{F}_v$. For magnetic bearing systems, these parts are displacement feedback and displacement speed feedback.

6.3 Decentralization of the Reduced Observer

A completely decentralized observer would have only diagonal observer matrices $\mathbf{W}$, $\mathbf{U}$, $\mathbf{L}$ and $\mathbf{N}$ (or block diagonal for subsystems with more than one state variable to be estimated).

For systems with low rotational speed, the system submatrix $A_4$ may be neglected. Equation (29) shows, that matrices $\mathbf{W}$ and $\mathbf{N}$ then can be made diagonal by appropriate selection of the estimator poles. The restriction is that the observer dynamics is now described by a diagonal matrix $\mathbf{W}$, i.e. only real estimator poles are possible.
Local feedback matrix: $D_1$

subplot system 1

local observer

$D_1$

Fig. 11: Subdivision of the reduced observer into "local" observers at an example with 2 subsystems. The information exchange between the local observers shown in this drawing is necessary for a general case implementation equivalent to Fig 10. A local estimation of the full state at each local controller involves too extensive on line computations.

For systems with high rotational speed, only one of the two matrices $W$ or $N$ can be made diagonal through the choice of the observer poles, since submatrix $A_4$ now introduces coupling terms.

Further diagonalization is possible for systems with neglectable $A_4$: Matrix $U$ then turns out to be nearly diagonal because the following two conditions are met:

- Matrix $A_1$ is zero
- Matrix $A_2$ is the identity matrix, which implies, according to equation (29), that the product $W \cdot N$ is a diagonal matrix with the squares of the observer poles as elements. These values are dominant compared to submatrix $A_3$. 
The observer matrices N, W and U have therefore been implemented as diagonal matrices on our bearing systems. This is a partially local estimator layout as the coupling of the output signals $u_1$ was retained in this first step of decentralization.

The sensitivity to random noise was clearly decreased by such an observer as compared to the differentiator. No deterioration of system performance could be detected when comparing this partially decentralized observer with a classical "central" state estimator.

As coupling of control is only introduced by matrix L consisting mainly of the non-trivial submatrix of the system input matrix B, only the diagonal elements of this matrix were retained in order to test a fully decentralized local state observer.

Matrix L being block-diagonal, merely the coupling of subsystem "a" with subsystem "b" has to be neglected. Practical experiments demonstrate, that even this coupling may be set equal to zero. This easy decentralizability of the observer can be explained by the comparison shown in the next section.

6.4 Comparison of Differentiator and Decentralized Observer

A subsystem of a fully decentralized bearing system will have an observer of order one since the local state consists of a single displacement (which is measured) and its derivative (to be estimated). The input of the scalar integrator is basically equal to the input of the corresponding integrator in the overall system model plus the observer-specific "correction" variables depending on the observer matrix W.

As stated above, the main coupling of the observer does not come from matrix W, but from the coupling terms of the input matrix B and – to a lesser degree – from the submatrix A3.
The decentralized observer now neglects the contribution of the other subsystems to this input. Figure (12) shows the structure of a fully decentralized local observer. Generally, the dimension of the local observer is more than one. For this magnetic bearing example, the local variables are all scalar.

![Diagram](image)

**Figure 12:** Local decentralized observer of subsystem "i". The coefficients are the diagonal elements of matrices U, L, W and N.

We now compare this observer with a simple differentiator of the type used in analog control: The frequency domain description is given in figure (13) by a transfer function with the Laplace variable s and the corresponding frequency response curve. The layout frequency must be below the frequency \( \omega_0 \). This transfer function is realized in the time domain by the integrator-feedback circuit (fig. 13, right).

Comparison of the decentralized observer and the differentiator shows that the difference consists of the additional input signals from matrix L to the observer integrator of fig. 12. It can therefore be stated that the decentralized observer is an intermediate solution between complete observer and differentiator. If the local input signal is high compared to the input signals of the other subsystems, it behaves as the complete observer. If the local input signal is low and another subsystem has a high input signal, the...
local estimator approaches the differentiator characteristic.

As summary of this chapter, it can be observed that the state estimator layout for magnetic bearing systems is not very delicate. Most if not all coupling parameters may be neglected, yielding a completely decentralized observer pattern. Theoretically, control layout based on such an observer can only be an improvement compared to a differentiator. This is confirmed by applications.

The equivalence of a certain layout of the observer and a "differentiator" is also evident when the control design is based on a sampled (discrete time) system model /16/.
7 Gyroscopic Effects

7.1 Closed Loop Eigenvalues for Fast Rotation

As seen in chapters three and four, the gyroscopic terms couple radial motions in the x-z plane with radial motions in the y-z plane. These coupling terms are proportional to the rotational speed \( \Omega \).

From the point of view of systems theory, such terms would generally have a great impact on decentralizability. It will be shown, that the special properties of rotating systems are responsible for the fact, that these coupling terms do not affect the feasibility of decentralization as much as could be expected.

In this section, the limits of the closed loop system eigenvalues for rotational speed \( \Omega \) approaching infinity are determined. For all rigid body rotor systems of the type of interest here, the gyroscopic effects for increasing \( \Omega \) can be summed up as follows:

Rotor behaviour approaches that of a free gyroscope. Three groups of eigenvalues can be discerned at large \( \Omega \).

The first pair is characterized by imaginary parts increasing proportional to \( \Omega \) in the limit. The real part of this conjugate complex pair approaches a small negative value. These poles are associated with the nutation frequency of a free gyroscope.

The second pair approaches zero at a rate proportional to
\(1/\Omega\) for the imaginary part and proportional to \(1/\Omega^2\) for the real part. This pair corresponds to the precession of a free gyroscope.

The third group of poles consists of two pairs of conjugate complex eigenvalues called pendulous frequencies in rotor dynamics. The locations for large \(\Omega\) approach constant values dependent on the stiffness of the bearings, i.e. the feedback coefficients.

In place of "large \(\Omega\)", the more precise term "large moment of momentum" should be used, as it is always the product \(I_z\Omega\) which is responsible for the gyroscopic and mass-unbalance effects. Since the moment of inertia \(I_z\) is constant, no confusion should arise by using the shorter expression.

An example with large ratio \(I_z/I_x\) has been computed to check the results of this chapter. A momentum wheel with a value \(I_z\) approximately equal to \(2I_x\) serves for this purpose. The only difference to the example presented here is the fact, that the effects described for a certain \(\Omega\) with the centrifuge rotor occur at a rotational speed \(\Omega\) smaller by a factor of about 20 for the momentum wheel example. Otherwise, the momentum wheel has a tendency to be more stable than the elongated rotor, as known from classical rotor dynamics. The centrifuge example is therefore considered a more general example, the only "disadvantage" being that the effects of interest occur at unrealistically high \(\Omega\).

The value of \(\Omega\) for which the gyroscopic effects become dominant is of course also depending on the bearing stiffness.

The results on the eigenvalues for high \(\Omega\) are derived in the following manner:

Equations (8) of chapter three are rewritten now including the stiffness and damping terms introduced by the feedback coefficients. In order to do this, the feedback matrices must first be transformed to the state base of system (9) and then be multiplied by the mass matrix \(M\).
The resulting stiffness and damping terms are denoted by \( s_i \) and \( d_i \). The subscript does not refer to the subsystem. A decentralized feedback is thus included in the following system:

\[
\begin{bmatrix}
I_x & 0 & 0 & 0 \\
0 & m & 0 & 0 \\
0 & 0 & I_x & 0 \\
0 & 0 & 0 & m
\end{bmatrix} z + \begin{bmatrix}
d_4 & d_1 & I_2 \Omega & 0 \\
d_3 & d_4 & 0 & 0 \\
-I_x \Omega & d_4 & d_2 & 0 \\
0 & 0 & d_3 & d_4
\end{bmatrix} z + \begin{bmatrix}
s_4 & s_1 & 0 & 0 \\
s_3 & s_4 & 0 & 0 \\
0 & s_4 & s_2 & 0 \\
0 & 0 & s_3 & s_4
\end{bmatrix} z = 0
\]

The complex notation customary in gyrodynamics theory is now introduced to get a system with second order matrices. A complex coordinate vector \( w \) is defined (e.g. Gasch and Pfuetzner, 1975):

\[
w = \begin{bmatrix}
\beta - i\alpha \\
x + iy
\end{bmatrix}
\]

The system can now be described in the following form:

\[
\begin{bmatrix}
I_x & 0 \\
0 & m
\end{bmatrix} \ddot{w} + \begin{bmatrix}
d_4 + i\Omega I_z & d_2 \\
d_3 & d_4
\end{bmatrix} \dot{w} + \begin{bmatrix}
s_4 & s_2 \\
s_3 & s_4
\end{bmatrix} w = 0
\]

These complex 2x2 matrices are denoted by \( \tilde{M} \), \( \tilde{P} \) and \( \tilde{S} \) respectively. The characteristic equation of the system is given by

\[
\det (\tilde{M} \lambda^2 + \tilde{P} \lambda + \tilde{S}) = 0
\]

From this, the coefficients of the characteristic polynomial can easily be computed analytically.
The complex coefficients are changed to their conjugate complex when the sign of the rotation \( \Omega \) is inverted. Since the system poles are independent of the rotation direction, the conjugate complex of the four complex roots \( \lambda_j \) of this polynomial are also eigenvalues of the closed loop system of order eight.

The limit of the roots of (31) can now be derived for \( \Omega \) large compared to the other coefficients of the characteristic polynomial. The three cases of very small \( \lambda \), finite \( \lambda \) and \( \lambda \) of the same order as \( I_z \Omega \) are treated separately.

1) \( |\lambda| \) near zero

The terms with \( \lambda^4 \), \( \lambda^3 \) and \( \lambda^2 \) are neglected. Equation (31) now reduces to

\[
\lambda_1 = \frac{s_1 s_4 - s_2 s_3}{i \Omega I_z s_4}
\]

This pair of eigenvalues approaches the origin of the complex plane like a parabola open to the left, since the imaginary part decreases as \( 1/\Omega \) and the real part as \( 1/\Omega^2 \).

2) \( |\lambda| \) small

( \( |\lambda| \ll |I_z \Omega| \), but not near zero)

The coefficients with powers four and zero of the polynomial (31) may be neglected in this case. The remaining terms are
divided by $\lambda I_z \Omega$, which yields a quadratic equation:

$$m \lambda^2 + d_4 \lambda + s_4 = 0 \implies \lambda_{2,3} = \frac{-d_4 \pm \sqrt{d_4^2 - 4ms_4}}{2m}$$

These are the eigenvalues which remain independent of $\Omega$ at high rotational speed.

3) $|\lambda|$ large ($|\lambda| \approx |I_z \Omega|$)

Here, only the terms with $\lambda^4$, $\lambda^3$ and $\lambda^2 i \Omega$ are retained. The resulting equation is reduced to a quadratic equation. Only one of the two solutions meets condition 3) and is retained for this limit. It can be shown, that the real part of this solution is small, but negative. The imaginary part is dominant. The solution for this case approaches the value

$$\lambda_4 = \frac{iI_z \Omega}{I_x}$$

This imaginary part is independent of the feedback coefficients.

This behaviour of the eigenvalues for high $\Omega$ is illustrated in fig. 14. The results do not depend on the choice of central or decentral feedback, as the same derivation can be applied for central feedback as well. The only difference to the derivation above would be to include complex feedback coefficients in matrices $\bar{P}$ and $\bar{S}$.

These results allow the conclusion, that decentral feedback is just as good as central feedback for a large product $I_z \Omega$. The dominant eigenvalues are not controllable by feedback coefficients which are always bounded in practical applications.
Fig. 14: Imaginary parts of the eigenvalues as function of rotational speed \( \Omega \). The limits are nutation frequency \( \omega_n \) and precession frequency \( \omega_p \) of the free gyroscope. For high \( \Omega \), only the pendulous modes and the real part of the nutation modes depend on the bearing stiffness (i.e. the feedback).

7.2 Effect of Small Rotor Unbalance

So far, the effect of the harmonic disturbances included in equations (8) and (10) of chapter 3 have not been considered. At low rotational speed and with small unbalances, the neglect was tolerable.

These disturbances are proportional to the square of the rotational speed \( \Omega \). Thus, as \( \Omega \) increases, the effects of unbalance will eventually be so strong, that the feedback will not be able to keep the rotor in nominal position. The bearings will deliver harmonic forces without suppressing rotor vibrations.

For very fast rotation, the harmonic force components exerted by the bearings will have no significant effect on rotor position any more. The rotation then takes place about a principal axis of the rotor.
Bearing control can be designed to adapt to this situation, with the result of having absolutely no vibration forces transmitted to the stator. This is achievable also for slowly changing unbalance of the rotor, an important feature of magnetic bearing application ( /36/, PIETRUSZKA & WAGNER /10/). In that mode of operation, the harmonic disturbance input of equations (8) and (9) vanishes, since the rotation axis of the rotor is now redefined as the mass-geometric principal axis.

For applications at rotational speeds not exceeding a certain limit, the effect of unbalance can be compensated by the feedback, with the aim of forcing the rotation to take place about a given geometric axis in place of the principal axis /10/. In this case, some feedback coefficients will be proportional to the square of $\Omega$.

Therefore, at low $\Omega$, the user can choose between force-free or geometrically fixed operation. Such feedback, given in /10/, is not decentralized. It seems that decentralization is feasible for both modes of operation. No further investigations on this topic have been made at our institute.

7.3 Comparison of Central and Decentral Feedback at Non-Neglectable $\Omega$

So far, decentralization has been proven feasible for very fast (section 7.1) and very slow rotation (section 5.10). No example of a system with $I_2\Omega$ in an intermediate range has been given.

A rotational speed is defined to be in this range, when it is not neglectable for control layout and when it is at the same time small enough, that significant differences between central and decentral feedback can be made out.

The comparison of the two control strategies is carried out as in the previous section, by applying optimal central,
suboptimal decentral and optimal decentral control on the rotor of chapter 4 with 100 000 r.p.m. The gyroscopic terms in matrix A (p.37) are then of the same order of magnitude as the other elements.

The suboptimal decentral feedback matrices are now determined by taking only the diagonal elements of the central feedback matrices computed for this Ω and given on page 40:

Suboptimal Decentral Feedback for 100 000 r.p.m.:

\[ D_a = \begin{bmatrix} 1.80, & 2.84 \end{bmatrix} \quad \text{and} \quad D_b = \begin{bmatrix} 1.82, & 1.28 \end{bmatrix} \]

with the closed loop poles

\[-10 + 25 i, \quad -32 + .2 i, \quad -75 + 125 i, \quad -379 + 56 i\]

The optimal decentral feedback was determined in seven iteration steps:

Optimal Decentral Feedback for 100 000 r.p.m.:

\[ D_a = \begin{bmatrix} 2.10, & 4.13 \end{bmatrix} \quad \text{and} \quad D_b = \begin{bmatrix} 3.49, & 4.47 \end{bmatrix} \]

with the closed loop poles

\[-25 + 24 i, \quad -38 + 2 i, \quad -104 + 99 i, \quad -1146 + 85 i\]

The closed loop poles and the scaled performance index \( J \) (18) are shown in fig. 15.

These results show clearly the improvement of system performance of optimal decentral feedback compared to simple setting zero of coupling feedback coefficients.

As mentioned in chapter 4, the optimal central feedback matrix for 100 000 r.p.m. does not stabilize the system for low rotational speed. It is interesting to note that the optimal decentral feedback does stabilize the system at zero rotational speed. From this point of view, optimal decentral
Fig. 15: Closed loop poles of system (14) (centrifuge, chapter 4), for 100 000 r.p.m. Matrices Q, R and X₀ as in the example of section 5.10. These pole locations show a clear improvement of optimal decentral feedback (marked with "D") as compared to simple elimination of coupling coefficients (poles marked with "x").

The values of "J" are the performance indices scaled to 1.0 for the minimum achievable with central feedback. (Poles marked with "o")

- o Optimal central feedback at 100 000 r.p.m. $J = 1.000$
- x "Brute Force" decentral feedback $J = 2.104$
- d Optimal decentral feedback $J = 1.615$

Feedback for rotor systems is superior even to "optimal" central feedback.

The conclusion that can be drawn from this chapter is that decentralized feedback is feasible also when high gyroscopic coupling occurs.
8 Results and Conclusion

8.1 Experimental Results

Some information on the bearing systems developed at our institute has been given in chapter 4 on numerical examples. There, the data of a vacuum centrifuge is given. The centrifuge (described in /18/) has been operated successfully for two years in Stuttgart.

The same bearings were used for a demonstration rotor with horizontal axis located at the Institute of Mechanics. The rotor of aluminum and iron has a weight of ca. 10 kg. The air gap in the bearings is .7 mm. As a consequence, the dynamic mechanical stiffness can be more than doubled compared to the centrifuge (ca. 160 N/mm). Measurements on this system are shown in fig. 16.

This system with time constants in the range of 60 through 10 msec was first operated with analog control. Digital control with a sampling time of one millisecond could then be realized with a single microprocessor Motorola 6809 and with fully decentralized control structure.

The stiffness could be increased more easily with the digital control, because stiff analog control excites vibrations at the switch-on moment, whereas the program of the digital control can easily be extended by a procedure to lift the rotor gently.

The third system, a demonstration rotor of 15 kg with wide bearing clearances of 8 mm, was planned for digital control
from the beginning. As opposed to the other two systems, this model uses switched power amplifiers, CCD optical sensors and no linearization or premagnetization. This basic layout gives good results and outlines tendencies for future implementations. Further experiments are scheduled with this system, the control program leaving a wide range of possibilities to "play" with. The sampling time is 2.6 msec.

The control layout for these different systems was done according to chapters 2 through 6.

The main feature of the control of those three systems is the robustness to simplifications in the model, to parameter uncertainties and to random noise.

![Diagram](image)

Fig.16: Measured rotor displacement and feedback output signal for a series of step responses of a magnetic bearing system. It can be seen, that there is no significant overshoot of the position signal. A decentralized Luenberger observer is used to produce the velocity signal needed for the damping coefficients. The feedback is analog.
The time domain measurement (fig. 16) shows the system response (displacement $x_a$) to a rectangular input (prescribed rotor position at bearing "a"). It was recorded by a mini computer. The choice of the weighting matrices $Q$ and $R$ according to section 2.4 produces closed loop poles with relatively strong damping, as clearly visible in fig. 16.

The next example of experimental results shows the frequency response of the same system as measured by a fourier analyser with random noise excitation. (Fig. 17)

![Measured and numerically computed frequency response](image)

Fig. 17 Measured frequency response of a magnetic bearing system with analog decentral control. 
Input: Prescribed rotor position 
Output: Measured rotor position
8.2 Conclusions

Owing to their outstanding technical properties, active magnetic bearings meet a growing interest in a variety of industrial applications. The control designs realized up to now can be divided into implementations based on the complete state model on the one hand and more empirical decentral layouts on the other hand.

The wide range of new possibilities offered by digital control is expected to increase the number of applications for magnetic bearings. Therefore, the possibility to apply standard microprocessors for this demanding control task should be given. The fast sampling times required for such systems are easily reached with a decentralized control structure.

This situation leads to the problem investigated in this thesis: How can the methods of control theory be applied to yield the "best possible" decentral feedback? The solution is compared to optimal central state feedback and to "empirical" decentral feedback. The gap between control theory and application is thereby narrowed.

As main result it is shown that decentral feedback of magnetic bearings is possible in most cases without any significant effect on system performance compared to a system controlled by optimal central state feedback.

Optimal decentral feedback is computed by minimizing the familiar quadratic optimization criterion with weighting matrices Q and R. The choice of these matrices is reduced to a single parameter. A region of appropriate pole locations for closed loop systems is derived.

Optimization of decentral feedback can be determined numerically by combining standard matrix operations and a routine to solve the stable Liapunov equations. Computer programs have been written and applied successfully; numerical work is moderate, the iteration procedure usually converges quickly. Details are given in section 5.9
The decentralizability of the Luenberger observer is demonstrated for magnetic bearing systems.

It is shown that the gyroscopic effects do not restrict decentralizability. This is an interesting result, since the gyroscopic terms represent a strong coupling of subsystems.

Some important conclusions valid also for digital control can be drawn:

All the simplifications on the control structure, including the decentralizability of the observer, are applicable for digital implementations as well. This has been demonstrated in practical applications. In the last stage of the implementation, the appropriate design methods for sampled systems can be used.

The clues to adequate pole placement drawn from the simple model of chapter two are also valid for a discrete time description. As derived for analog systems, a reasonable feedback for a "first try" can be obtained with simple calculations by hand on the system of order two.

Finally, it is hoped that a contribution has been made to the question, up to which point theoretical refinement in control engineering is useful and reasonable for this practical application.
Appendix

=*=*=*=*=*

A1 Transformation of the state base:

\[
\begin{align*}
X_{\text{new}} &= T X_{\text{old}} \\
A_{\text{new}} &= T A_{\text{old}} T^{-1} \\
B_{\text{new}} &= T B_{\text{old}} \\
V_{\text{new}} &= T V_{\text{old}} \\
F_{\text{new}} &= F_{\text{old}} T^{-1} \quad \text{Feedback matrix notations} \\
D_{\text{new}} &= D_{\text{old}} \\
Q_{\text{new}} &= (T^{-1})' Q_{\text{old}} T^{-1} \\
X_{0\text{new}} &= T X_{0\text{old}} T' \\
C_{\text{new}} &= C_{\text{old}} T^{-1}
\end{align*}
\]

A2 Derivation of the equations of motion (8) (p.26): 

The 2nd and 4th line of (8) are Newton's law of motion applied to the translational movements of the rotor.

The first and third line follow from Euler's equations formulated in a system fixed to the rotor (axis 1-2-3, leading subscript "R") and subsequent transformation to the fixed system x-y-z (leading subscript "I").

27.7.1984
The transformation matrix is
\[
\begin{bmatrix}
c & -s & 0 \\
s & c & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
where
\[c = \cos \theta t \quad \text{and} \quad s = \sin \theta t\]

The moments of inertia are:
\[
\begin{bmatrix}
I_x & -I_{xy} & -I_{xz} \\
-I_{xy} & I_x & -I_{yz} \\
-I_{xz} & -I_{yz} & I_z
\end{bmatrix}
\]

The angular momentum \(\mathbf{L}\) is expressed in the moving system "\(\mathbf{R}\)"
\[
R_L = I \begin{bmatrix}
\omega_1 \\
\omega_2 \\
\Omega
\end{bmatrix} = \begin{bmatrix}
I_x \omega_1 & -I_{xy} \omega_1 & -I_{xz} \Omega \\
-I_{xy} \omega_1 & I_x \omega_2 & -I_{yz} \Omega \\
-I_{xz} \omega_1 & -I_{yz} \omega_2 & I_z \Omega
\end{bmatrix} \begin{bmatrix}
L_1 \\
L_2 \\
L_3
\end{bmatrix}
\]

The derivative of \(\mathbf{L}\) in the moving system (1-2-3) is formed according to:
\[
\frac{d\mathbf{L}}{dt} = \omega \times \mathbf{L} \quad \text{where} \quad \frac{d\mathbf{L}}{dt} \quad \text{is the derivative in system "R".}
\]

The component in z-direction is omitted from now on. The time derivative of \(\mathbf{L}\) is equal to the external couple (vector \(\mathbf{m}\)):
\[
\frac{d\mathbf{L}}{dt} = \begin{bmatrix}
I_x \dot{\omega}_1 - I_{xy} \dot{\omega}_2 \\
-I_{xy} \dot{\omega}_1 + I_x \dot{\omega}_2
\end{bmatrix} \quad \omega \times \mathbf{L} = \begin{bmatrix}
\omega_2 L_3 - \Omega L_2 \\
\Omega L_1 - \omega_1 L_3
\end{bmatrix}
\]

\[
\frac{d\mathbf{L}}{dt} = \begin{bmatrix}
I_x \dot{\omega}_1 - I_{xy} \dot{\omega}_2 + \Omega I_x \omega_4 - I_{xz} \omega_1 + (I_z - I_x) \Omega \omega_1 \omega_2 + I_y \omega_1^2 - I_{yz} \omega_2^2 \\
-I_{xy} \dot{\omega}_1 + I_x \dot{\omega}_2 - \Omega I_{xy} \omega_2 + I_y \omega_2^2 + (I_x - I_z) \Omega \omega_1 \omega_4 + I_{xy} \omega_1^2 - I_{xz} \omega_2^2
\end{bmatrix}
\]

\[
= \begin{bmatrix}
m_1 \\
m_2 \\
\vdots
\end{bmatrix} = \mathbf{R} \mathbf{m}
\]
The last equation is now transformed back into the inertial system (x-y-z):

\[
I_m = \begin{bmatrix} m_x \\ m_y \end{bmatrix} = \begin{bmatrix} cm_1 - sm_2 \\ sm_1 + cm_2 \end{bmatrix}
\]

With this, the couple in x-direction can be expressed as:

\[
m_x = I_x(c_1 - s_2) - I_{xy}(c_2 - s_1) + \Omega I_{xy}(c_1 + s_2) - \\
I_{xz}(c_1 s_2 + s_1^2 - s_2^2) - I_{yz}(c_2^2 - s_2^2 + s_1 s_2) + \\
(I_z - I_x)(c_1 s_2 + s_1)
\]

Recalling that the inertial tensor is assumed to contain only very small cross products of inertia $I_{xy}$, $I_{yz}$ and $I_{xz}$ this equation yields the third equation of system (8):

\[
m_x = I_x \ddot{\alpha} + I_z \Omega \dot{\beta} + I_{xz} \frac{1}{\Omega} \sin \Omega t - I_{yz} \frac{1}{\Omega} \cos \Omega t
\]

with the following kinematic relations:

\[
\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} c_1 - s_2 \\ s_1 + c_2 \end{bmatrix} (\alpha \text{ and } \beta \text{ see fig.6 p.25})
\]

\[
\ddot{\alpha} = c_1 - s_2 + \Omega(-s_1 - c_2)
\]
A3 On the sensitivity of a feedback compensating the gyroscopic effects: (p.42)

The control vector \( \mathbf{u} \) is determined according to

\[
\mathbf{u} = \mathbf{F} \cdot \mathbf{x} + [0 : \mathbf{F}_A] \cdot \mathbf{x}
\]

where \( \mathbf{F} \cdot \mathbf{x} \) is the control of the standing rotor and
[0 : \mathbf{F}_A] \cdot \mathbf{x} with the 4x4 submatrix 0 is the compensation of the gyroscopic terms. When the elements of \( \mathbf{F}_A \) (which are proportional to \( \mathbf{\Omega} \)) become large compared to the feedback coefficients of matrix \( \mathbf{F} \), a small relative error of \( \mathbf{F}_A \) will have a significant effect on the closed loop system poles determined by matrix \( \mathbf{F} \).

A4 Integral property of the stable Liapunov Equation: (p.51)

Let \( \mathbf{A} \) and \( \mathbf{B} \) be stable \( nxn \) matrices. (Stable: Real parts of all eigenvalues negative). Matrices \( \mathbf{C} \) and \( \mathbf{X} \) have also dimension \( nxn \).

The solution of the Liapunov Equation

\[
\mathbf{A} \mathbf{X} + \mathbf{X} \mathbf{B} = -\mathbf{C}
\]

is then equal to the following integral:

\[
\mathbf{X} = \int_{0}^{\infty} \dot{\mathbf{\Phi}}(t) \cdot \mathbf{C} \cdot \Gamma(t) \, dt
\]

where the fundamental matrices \( \dot{\mathbf{\Phi}}(t) \) and \( \Gamma(t) \) of \( \mathbf{A} \) and \( \mathbf{B} \) are defined by

\[
\dot{\mathbf{\Phi}}(t) = \mathbf{A} \mathbf{\Phi}(t) \quad \text{and} \quad \dot{\Gamma}(t) = \Gamma(t) \mathbf{B}
\]

\( \mathbf{\Phi}(0) = \text{Identity} \quad \text{and} \quad \Gamma(0) = \text{Identity} \)
A5 Derivative of the Performance Index $J$ with respect to the feedback matrices $D_i$ (p. 52):

Equation (23) on page 52 is derived starting from (22), which is rewritten here:

\[
(22) \quad A_0' P + P A_0 + Q + \sum_{j=1}^{n_s} C_i' D_i' R_i D_j C_j = 0
\]

In this equation, $D_i$ is substituted by the expression $(D_i + dD_i)$. This causes a perturbation $dP$ of matrix $P$. Note that only a single term of the summation is substituted at a time. The whole procedure applies to each subsystem "$i" for $i$ from 1 through $n_s$.

\[
(A_o + B_i dD_i C_i)' (P+dP) + (P+dP) (A_o + B_i dD_i C_i) +
\]

\[
Q + C_i'(D_i+dD_i)' R_i (D_i+dD_i) C_i + \sum_{j \neq i} C_i' D_i' R_j D_j C_j
\]

Reordering the terms, applying (22) and neglecting 2nd order terms yields:

\[
A_o' dP + dP A_o + C_i' dD_i' (B_i' P + R_i D_i C_i) +
\]

\[
(C'D'R + P B_i) dD C = 0
\]

Applying the integral property (A4) to this Lyapunov equation gives

\[
dP = \int_0^\infty \Phi'(t) (C_i' dD_i (B_i' P + R_i D_i C_i)) + \text{(sym.)} \Phi(t) \, dt
\]
The derivative \( \frac{dj}{dD_i} \) of \( J = \text{tr}(PX_o) \) is therefore expressed as

\[
\frac{\text{tr}(dPX_o)}{dD_i} = 2 \frac{\text{tr}\left[ \left( \int_0^\infty \Phi_i' C_i' dD_i'(R_i D_i C_i + B_i P) \Phi_i dt \right) X_o \right]}{dD_i}
\]

The following lemma on the derivative of a trace function with respect to a matrix \( D \) of mutually independent entries will now be applied:

\[
\frac{\text{tr}(dD'G)}{dD} = G \quad \text{for nxm matrices } D \text{ and } G
\]

For further details on this lemma see Senning /13/ (appendix) or Geering /27/.

Recalling that \( \text{tr}(A B) = \text{tr}(B A) \), the trace expression above is transformed to

\[
2 \frac{\text{tr}\left( dD_i'(R_i D_i C_i + B_i P) \cdot \int_0^\infty (\Phi_i X_o \Phi_i') dt \right. C_i' \left. \right)}{dD_i}
\]

The integral above is denoted by matrix \( X \) according to equation (24) on page 52. The derivative of the performance index \( J \) is therefore expressed as

\[
(23) \quad \frac{1}{2} \frac{dj}{dD_i} = (R_i D_i C_i + B_i P) X C_i' \quad \text{for all } i \in (1, \ldots, n_o)
\]

This completes the derivation of equation (23).
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Curriculum:

Born in Rabat (Morroco) on February 19th 1954, I attended the French school up to the Lycee in Kenitra from 1958 to 1965.

My family moved to Zürich in 1965. From 1966 to 1973 I went to the Kantonales Realgymnasium Zürichberg. In 1971/1972, I had the opportunity to spend a year in an American family as an AFS scholarship student and to obtain a High School diploma before my Matura at the gymnasium in Zürich in 1973.

My subsequent studies at the electrical engineering department of the ETH Zürich from 1973 up to 1978 were only interrupted by the military service. I graduated in 1978. The diploma project on large scale systems was carried out at the Institute of Prof. Mansour.

Since 1979 I have been working as an assistant at the Institute of Mechanics of the ETH, being involved with mechanical topics as well as control theory and application.