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RAY-BASED IMAGE RECONSTRUCTION IN CONTROLLED-SOURCE SEISMOLOGY WITH AN APPLICATION TO SEISMIC REFLECTION AND REFRACTION DATA IN THE CENTRAL SWISS ALPS

ABHANDLUNG

Zur Erlangung des Titels
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der
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vorgelegt von

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TO
MY BELOVED PARENTS
AND
MY BROTHER KASPAR PHILIPP

HORATIO: O day and night, but this is wondrous strange!

HAMLET: And therefore as a stranger give it welcome.
There are more things in heaven and earth, Horatio,
than are dreamt of in your philosophy.

(Shakespeare, *Hamlet*, Act I, Scene 5)

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PD Dr. Edi Kissling faced the doubtful pleasure of becoming my supervisor after I had supervised myself during the first year. I for my part could neither have wished a more competent and caring supervisor nor a neater friend and room mate. Thank you so much for everything Edi!

"Mid-PhD crises" are quite a common phenomenon; virtually every PhD student has to go through a more or less serious one. To me my crisis seemed to be very existential and the fact that I did not give up is to be attributed to the affection, dedication and encouragement I received from my family, my supervisor Edi Kissling, Professor William Lowrie and from my dear friend Curò Vital.

After already having been a visitor in winter 1986/1987 a generous fellowship of the British Natural Environmental Research Council (NERC) allowed me to spend another four months at the British Institutions' Reflection Profiling Syndicate (BIRPS) in Cambridge, U.K., in summer 1988. I wish to thank all the BIRPers for the wonderful, stimulating and productive time I had. In this context special thanks are due to Dr. Simon Klemperer (now Professor at Stanford University); his straight, energetic and uncompromising approach to science deeply impressed and - hopefully - influenced me. I also thankfully acknowledge the receipt of the BIRPS version of the ray migration program originally developed by Bernard Raynaud.

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ABSTRACT

In this work I have tried to explore the potential of ray theoretical migration and forward modelling techniques for the joint - albeit not synoptic - interpretation of deep seismic reflection and refraction data. Simple analytical considerations show that for the velocities and travel times relevant for deep seismic reflection data migration displacements easily exceed 5 km vertically and 10 km laterally. This implies that virtually every deep seismic reflection profile needs to be migrated and that a minimum profile length of at least some 30 to 50 km is required to allow structural interpretation at greater depths. Estimates of the influence of uncertainties in velocity upon migration show that the error in the average velocity must not exceed 0.2 km/s at Moho depth in order to allow a meaningful comparison of the reflectivity imaged by normal-incidence profiling and the crustal velocity structure inferred from seismic wide-angle data. Conventional migration schemes based on the solution of the scalar wave equation rarely produce satisfying results when applied to deeper crustal data. An extensive review of the corresponding algorithms shows why these methods are highly sensitive to lateral velocity variations as well as to the short, laterally discontinuous reflection segments and high noise levels in conjunction with the high velocities and long travel times characteristic of deep seismic reflection data. These problems can be largely overcome by ray theoretical depth migration of digitised line drawings. As a case study the individual deep seismic reflection profiles of the eastern (ET) and southern (ST) traverses across the Swiss Alps, which were acquired as part of the National Science Foundation Program 20 (NFP20), have been combined along the course of the European Geotraverse (EGT). The resulting reflectivity distribution was simultaneously depth migrated with the contours of the smoothed, laterally consistent velocity field obtained by the reinterpretation of the seismic wide-angle profiles running parallel to the strike of the Alpine arc. This led to an overall excellent agreement between the most prominent reflectivity patterns and the strongest wide-angle reflections, which is considered to be an important criterion for successful migration. Assuming that the result of this migration represents an unbiased acoustic image of the present-day tectonic configuration of the crust below the central Alps low-angle subduction of the lower crust and uppermost mantle of the European plate below the Adriatic promontory of the African plate is clearly depicted. Orogenic crustal thickening is interpreted to arise from the stacking of nappes onto the European upper crust and from wedging of the European and Adriatic middle crusts. At least part of the south-vergent upper crustal thrusting in the Southern Alps can be accounted for by the inferred northward downbending of the Moho and lower crust of the Adriatic plate.

ZUSAMMENFASSUNG

Diese Arbeit beschäftigt sich mit der Anwendung der geometrischen Strahltheorie auf die Migration und Modellierung des Laufzeitverhaltens von krustenseismischen Daten. Das Ziel hierbei ist, durch das Verständnis der theoretischen Beziehungen zwischen Wellen- und Strahltheorie sowie durch die Charakterisierung des zu betrachtenden seismischen Datenmaterials die kombinierte - wiewohl nicht synoptische - Interpretation von reflexions- und refraktionsseismischen Daten zu verbessern. Einfache analytische Betrachtungen des Migrationsvorganges zeigen, dass bei den für die kontinentale Kruste relevanten Laufzeiten und Kompressionswellengeschwindigkeiten der Migrationsweg 5 km in der vertikalen und 10 km in der horizontalen Richtung oft übersteigt. Einerseits bedeutet dies, dass für eine aussagekräftige Interpretation von reflexionsseismischen Daten deren vorgängige Migration eine *conditio sine qua non* darstellt, und andererseits, dass Profillängen von weniger als 30 bis 50 km für krustenseismische Untersuchungen wenig sinnvoll sind. Wiederum ausgehend von analytischen Betrachtungen lässt sich der Einfluss von Fehlern in der Parametrisierung des Geschwindigkeitsfeldes auf den Migrationsweg abschätzen. Dabei ergibt sich, dass für einen aussagekräftigen Vergleich zwischen der aus refraktionsseismischen Daten abgeleiteten Geschwindigkeitsstruktur und der durch die Migration erhaltenen Reflektivitätsverteilung die Unsicherheit in der mittleren Krustengeschwindigkeit einen Wert von 0.2 km/s nicht überschreiten sollte. Die gängigen, vornehmlich von der Erdölindustrie entwickelten Migrationsverfahren basieren auf der numerischen Lösung der skalaren Wellengleichung für die Rand- und Anfangswerte der gemessenen seismischen Daten. Eine eingehende Betrachtung der gebräuchlichsten Migrationsalgorithmen zeigt, dass diese generell sensibel auf laterale Änderungen der Geschwindigkeit sowie Insuffizienzen der Anfangs- und Randbedingungen, wie z.B. geringes Verhältnis von Nutz- zu Störsignal und unvollständige Registrierung des reflektierten Wellenfeldes, reagieren. Hierbei erweist sich, dass mit der Genauigkeit eines Migrationsalgorithmus in Bezug auf die Lösung der skalaren Wellengleichung auch dessen Empfindlichkeit gegenüber den obengenannten Phänomenen zunimmt. Damit lassen sich einerseits die wohlbekannten Probleme bei der Migration von krustenseismischen Reflexionsdaten erklären, und andererseits wird klar, dass wenig Hoffnung besteht, ausgehend von der Wellentheorie Algorithmen zu entwickeln, die diese Probleme grundsätzlich lösen. Die Migration der beobachteten Laufzeiten basierend auf der geometrischen Strahltheorie, einer groben

Hochfrequenzapproximation der Wellentheorie, stellt daher zur Zeit und wohl auch in absehbarer Zukunft für krustenseismische Reflexionsdaten die praktikabelste Lösung dar.

Als Fallstudie wurden die Reflexionsprofile der Ost- und Südtraverse des Nationalen Forschungsprogramms 20 (NFP 20) "Geologische Tiefenstruktur der Schweiz" betrachtet. Die für die Migration benötigte Geschwindigkeitsinformation ergab sich durch eine lateral geglättete Reinterpretation der parallel zum alpinen Streichen verlaufenden Refraktionsprofile. Die Reflexionsprofile wurden entlang des alpinen Segments der Europäischen Geotraverse (EGT) kombiniert und strahlentheoretisch tiefenmigriert. Dies führte zu einer generell guten Übereinstimmung zwischen den dominierenden Reflektivitätsmustern und den über den gesamten Zentralalpenbogen hinweg lateral kontinuierlich verfolgbaren Weitwinkelreflexionen. Das resultierende akustische Bild der Erdkruste unter den Zentralalpen reicht in eine Tiefe von 60 km und weist deutlich auf die etwa 15 Grad nach Süden geneigte Subduktion der unteren Kruste und des obersten Mantels der europäischen unter die afrikanische Platte hin. Die orogene Verdickung der alpinen Kruste lässt sich einerseits durch die Stapelung von Kristallindecken in der Oberkruste der europäischen Platte und andererseits durch ein Ineinanderschieben der Mittelkrusten der europäischen und afrikanischen Platten interpretieren. Das nordvergente Abtauchen der Unterkruste der afrikanischen Platte kann die beobachtete südvergente Verkürzung der Oberkruste in den Südalpen zumindest teilweise erklären.

1. INTRODUCTION

Reflection seismology was developed in the 1920s and by the 1930s was being used on a large scale for oil exploration (cf. Sheriff and Geldart 1983; Robinson 1983). Various geometric transformation schemes to make the picked travel times of the observed reflections more accessible to geological interpretation have been in use since the early beginnings. These transformation techniques were labelled with the pictorial name *migration* as their purpose was to move or *migrate* the apparent positions of the reflections on the recorded seismic time sections to their true position in space. This terminology survived the advent of digital signal processing in the 1960s and early 1970s despite the fact that it became evident that the migration process represents a boundary/initial value problem to the scalar wave equation aiming at extrapolating the observed seismic wave field back through the imaged medium (cf. Claerbout 1970, 1971; Claerbout and Doherty 1972).

The purpose of this work is to illuminate the potential of ray theoretical migration and forward modelling methods for the joint - albeit not synoptic - interpretation of deep seismic reflection and refraction data. As a case study I have chosen the deep seismic reflection profiles across the eastern and southern Swiss Alps, which were shot as part of the Swiss National Science Foundation Program 20 "Geologische Tiefenstruktur der Schweiz" (NFP20) (cf. Schweizerische Arbeitsgruppe für Reflexionsseismik 1988; Frei et al. 1989). The major advantages of this dataset are the well-studied surface geology and tectonics, the possibility to extrapolate gross structures observed at the surface to depths of 10 km due to the eastward axial plunge of the Alpine structural grain (cf. Pfiffner et al. in press) and the unsurpassed wealth of complementary geophysical information, particularly seismic wide-angle profiles (cf. Mueller et al. 1980). The disadvantages of the NFP20 deep seismic reflection data are the commonly low signal to noise ratio (S/N) and their fragmentation into sometimes very short and laterally offset profiles.

As already indicated migration is kind of an unfortunate term not only because it fails to describe the underlying physics of the process but also because it is restricted to the repositioning part *sensu strictu*. Therefore I have decided to use the term *image reconstruction* instead of migration in the title. This is semantically more correct for high-frequency approximations of the wave equation and implicitly avoids the unnatural separation between the critically interdependent processes of velocity estimation and repositioning of the imaged reflectivity.

In my opinion the fundamental difference between a PhD thesis and a corresponding publication in a scientific journal is that the former allows the author not only to document the results of his work but also their theoretical and methodological background. Ideally this makes PhD monographs suitable introductory texts for future generations of PhD or research students working in the same field. Therefore I have written this text in a tutorial and informal style, which hopefully may comfort those who feel somewhat bewildered by the rather formal title of this doctoral dissertation. As part of this policy I have also tried to achieve an acceptable balance between the internal coherency of the text as a whole and loose coupling between the individual chapters and sections in order to ease selective reading. Instead of lengthy phrasings this way of writing almost inevitably leads to the use of commonly accepted abbreviations and idioms, the most important ones of which are explained in Table 1.1.

The text is structured as follows:

- Chapter 2 reviews the most common ways to solve the scalar wave equation for the boundary/initial value problem of the migration of stacked seismic data. The relative merits of the various algorithms are worked out as well as the inadequacy of all these schemes to properly migrate deep seismic reflection data.
- Chapter 3 discusses ray theoretical aspects of migration and the possibilities of this approach to overcome some of the fundamental shortcomings of the previously presented migration schemes based on the wave equation. Using simple analytical considerations the general importance of migration of deep seismic reflection data as well as the necessity of an accurate parametrisation of the velocity field are demonstrated.
- Chapter 4 addresses general problems of velocity estimation on a crustal scale from spatially aliased seismic wide-angle data and the application of the thus derived velocity information to the parametrisation of the velocity field suitable for the migration of deep seismic reflection data. As a practical example a series of seismic wide-angle profiles running roughly parallel to the strike of the Alpine arc are reinterpreted in order to provide a smooth, laterally consistent velocity model across the central Alpine arc, which can subsequently be used for the migration of the eastern and southern segments of the NFP20 deep seismic reflection traverses across the central Alps (Frei et al. 1989).

• Chapter 5 treats the migration of the NFP20 eastern (ET) and southern (ST) traverses as a case study. The individual seismic reflection profiles are combined and the reflectivity of the resulting profile is ray theoretically depth-migrated simultaneously with the contours of the velocity model previously derived from the reinterpreted wide-angle profiles. The tectonic implications of the resulting reflectivity distribution are discussed under the assumption that it represents a truthful acoustic snapshot of the present-day physical state of the crust below the central part of the Alpine arc.

• Chapter 6 discusses fundamental aspects of the migration and interpretation of deep seismic reflection data and relates them to the considered Alpine dataset in order to assess validity of the approach taken and the thus resulting interpretation.

• Chapter 7 is intended to be comprehensive list of the key references on the topics covered in this work.

• The appendices contain user's guides and descriptions of the code of the two major migration programs which I have either substantially modified or newly developed in the course of my PhD work.

TWT Two-way travel time. Time it takes a seismic wave to travel from the source down to the target depth and back up to the receiver.
CMP Common-mid-point. Mid-point between a particular source and receiver. Equivalent but nowadays less common expressions are CDP (common-depth-point) or CRP (common-reflection-point). Sorting data according to CMPs results in so-called CMP gathers. Reflections on CMP gathers exhibit a quasi-hyperbolic dependence of TWT versus spatial offset even if they arise from dipping reflectors.
rms Root-mean-square. The square root of the average of the squares of a series of measurements. In reflection seismology the rms velocity v_{rms} is commonly used to "smooth out" ray bending effects in a vertically stratified velocity structure.
NMO Normal moveout. Approximate description of the TWT versus offset dependence on CMP or shot gathers under the assumption of straight ray paths in a vertically stratified velocity field: $t^2(x) \approx t^2(0) + (x/v_{rms})^2$. NMO-correction aims at removing the effect of NMO, which in the case of a CMP results in flattening the reflection "hyperbolae".
CMP-stack or simply **stack** Summing all NMO-corrected traces of a CMP into one single central trace. Ideally this process collapses the offset domain completely and hence results in a so-called zero-offset section, i.e. a seismic section with coincident source and receiver locations.
Noise Any kind of energy on a seismic section that does not arise from primary reflections or diffractions. Any non-coherent noise is commonly referred to as random noise. Truly random noise in a CMP gather containing n traces is reduced by a factor of $n^{1/2}$ by stacking.
S/N Signal-to-noise ratio.

Table 1.1. Frequently used abbreviations and idioms.

2. WAVE EQUATION MIGRATION

2.1 From Phenomenology to Theory

Most of the popular hand-migration techniques were based on Huygen's principle (cf. Hagedoorn 1954; Robinson 1983), i.e. the subsurface was considered to consist of point diffractors. So by summing along diffraction hyperbolae corresponding to the rms velocity of the overburden laterally coherent assemblages of point diffractors, i. e. reflectors, were moved to their true spatial position, individual diffraction hyperbolae were collapsed to their apexes and randomly distributed point diffractors were cancelled out. Although these manual techniques were painstaking to carry out they were very successful and had a clear and simple phenomenological basis. It was therefore only natural that with the commercial availability of digital computers in the early 1960s the first migration algorithms implemented by the oil industry were so-called "diffraction stack migrations" which - as indicated by their name - just emulated the previous manual techniques. However, though obviously the phenomenology of the migration process was well understood, the theory was not and these early diffraction stack migrations never worked satisfactorily (cf. Robinson 1983). In the early 1970s Jon Claerbout substantially contributed to the solution of the problem of computer migration by formulating the migration process as a boundary/initial value problem to the scalar wave equation (Claerbout 1970, 1971; Claerbout and Doherty 1972). The theoretical clarity and the practical success of this formal approach fruited in the development of a variety of migration algorithms in the following years all of which were explicitly based on the wave equation (e.g. Loewenthal et al. 1976; Claerbout 1976; Larner and Hatton 1990; Schneider 1978; Stolt 1978). Not unexpectedly it was found that the old diffraction stack migration only needed a little mathematical patching up to be a correct solution to the given boundary/initial value problem of the wave equation via the Kirchhoff integral and as such to do as good a job as any of the other methods (Larner and Hatton 1990; Schneider 1978). Common to all these early methods was that they produced an output in two-way travel time (TWT) and that they required a 1-dimensional or at least locally a laterally homogeneous definition of the velocity model. The first is very convenient for the interpreter and the second often is a sufficient approximation. However, little did anyone suspect in the beginning that in the case of strong lateral velocity heterogeneities all these so-called time migration algorithms would break down and that a still deeper look into the theory was required to solve the problem (Gazdag 1980; Judson et al. 1980; Hatton et al. 1981). It was Hubral (1977) who - based on ray theoretical considerations - showed that in the presence of any lateral velocity variations time migration cannot depropagate, i.e. migrate, the observed wave field correctly.

The Exploding Reflector Model

Deep seismic reflection data still are almost exclusively migrated after stack and hence this chapter is restricted to stacked data. Assuming that NMO and stack have correctly collapsed the offset domain the unmigrated stacked section can be looked at as a zero-offset section, in which sources and receivers coincide and the energy travels the same way down to the reflectors and up again. According to Fermat's principle reflections at zero-offset always occur at a right angle to the reflector. Moreover, the zero-offset model assumes that no shear waves and no multiples are generated and consequently that the scalar or acoustic wave equation does model the data correctly.

Since the waves are assumed to travel the same way down and up this model can be further simplified by just considering the upgoing waves. The two-dimensional zero-offset wave field p(x,z=0,t) which we record at the surface can then be imagined to have been generated by a reflector "exploding" at time t=0 (Loewenthal et al. 1976). For this purpose one can either half the observed two-way travel time or half the velocity of the medium through which the wave has travelled. The latter approach is more common and will be pursued in the following.

The scalar wave equation relates the acoustic pressure field p(x,z,t) and the compressional wave velocity v(x,z) as follows:

$$\frac{\partial^2 p(x,z,t)}{\partial t^2} = \frac{v^2(x,z)}{4} \nabla^2 p(x,z,t) = \frac{v^2(x,z)}{4} \left\{ \frac{\partial^2 p(x,z,t)}{\partial x^2} + \frac{\partial^2 p(x,z,t)}{\partial z^2} \right\} \qquad (2.1)$$

The factor four on the right hand side of (2.1) arises from the transformation of v(x,z) to v(x,z)/2 required by the exploding reflector model. Given the seismic zero-offset (i.e. CMP-stacked) section observed at the earth's surface p(x,z=0,t) as the boundary/initial condition we want to solve the scalar wave equation (2.1) for the reflectivity p(x,z,t=0) of the subsurface. This solution corresponds to the *migrated* seismic section. The reverse procedure is called *forward modelling*. Migration and forward modelling are therefore very closely related; whilst migration *depropagates* a wave field observed at the surface in order to unravel the reflectivity of the subsurface, modelling provides an observed wave field by *propagating* it through the reflectivity (Figure 2.1).

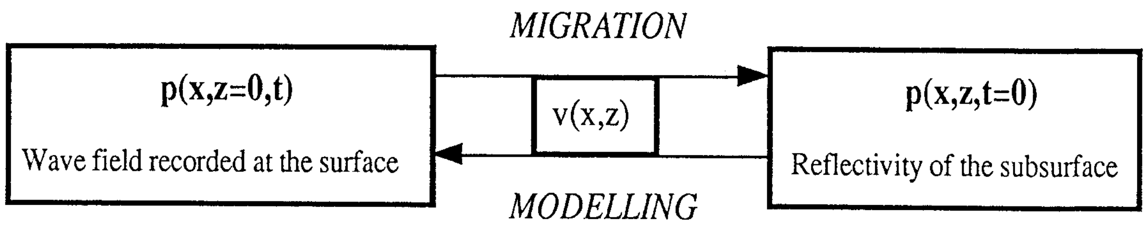

Figure 2.1. The relation between migration and forward modelling of zero-offset seismic data.

In the following several basic methods to solve this boundary/initial value problem are reviewed and their relative merits and drawbacks are discussed. From this discussion it will become evident quite naturally that - even under the best possible circumstances - wave equation migration of deep crustal data is problematic. All the problems are formulated in two dimensions; this has two reasons: first, crustal reflection data are and will be virtually exclusively acquired along profiles (i.e. in two dimensions); second, an extension to the third dimension is in the considered cases straightforward to formulate but painstaking to type. Moreover, there arise formidable conceptual problems in three dimensions, such as the parametrisation of the velocity field, which are well beyond the scope of this text.

2.2 Time Migration with Transform Methods

Phase Shift Migration (Gazdag 1978)

If the velocity is kept constant (v=constant) it is amazing how simple things become. The scalar wave equation has then, as is well known, a set of harmonic plane wave solutions of the form

$$\exp\{i(\omega t-k_x x-k_z z)\} \quad (2.2)$$

constrained by the so-called "dispersion relation"

$$k_x{}^2+k_z{}^2 = \frac{4\omega^2}{v^2} \quad (2.3)$$

Where k_x and k_z are the wave numbers with respect to x and z (positive downward) and ω is the angular frequency. In the following I have adopted the common practice of seismic data processing and not explicitly indicated the integration boundaries whenever they are constrained by the considered data.

Taking the 2-dimensional Fourier transform of p(x,z,t) over x and t yields

$$P(k_x,z,\omega) = \int\int p(x,z,t)\ \exp\{i(\omega t-k_x x-k_z z)\}dxdt \quad (2.4)$$

Considering just upgoing waves (2.4) can be rewritten according to (2.2) and (2.3) as

$$P(k_x,z,\omega) = P(k_x,z=0,\omega)\exp\{ik_z z\} \quad (2.5)$$

To obtain again p(x,z,t) just the 2-dimensional inverse Fourier transform has to be taken

$$p(x,z,t) = \frac{1}{(2\pi)^2}\int\int P(k_x,z=0,\omega)\exp\{i(k_x x+k_z z-\omega t)\}dk_x\ d\omega \quad (2.6)$$

For t=0 (2.6) evidently becomes a migration equation, which is commonly known as phase shift migration (Gazdag 1978)

$$p(x,z,t=0) = \frac{1}{(2\pi)^2}\int\int P(k_x,z=0,\omega)\exp\{i(k_x x+k_z z)\}dk_x\ d\omega \quad (2.7)$$

Although the vertical coordinate in (2.7) is formulated as depth z it is - due to the constant velocity - nothing but scaled travel time and the final output is generally computed as such.

Formula (2.7) is strictly valid only for a constant velocity of the medium. However, for a vertically stratified medium v = v(z) a generalized approximation is possible (Gazdag 1978)

$$P(k_x,z,\omega) = P(k_x,z=0,\omega)\exp\{i\int_0^z k_z(z')dz'\} \tag{2.8}$$

Thus (2.7) becomes

$$p(x,z,t=0) = \frac{1}{(2\pi)^2}\iiiint \exp\{i(k_x - \int_0^z \gamma dt')\}\exp\{i(\omega t' - k_x x')\}p(x',z=0,t')dt'dx'dk_x\, d\omega \tag{2.9}$$

where

$$\gamma = -\frac{k_z v}{2} = \omega\sqrt{1 - \frac{k_x^2 v^2}{4\omega^2}} \tag{2.10}$$

The necessity of a detailed knowledge and specification of the interval velocities v(z) of the medium can be circumvented by expanding the square root

$$\sqrt{1-\frac{k_x^2 v^2}{4\omega^2}} \approx 1 - \frac{k_x^2 v^2}{8\omega^2} \tag{2.11}$$

so that the so-called phase shift term becomes

$$\int_0^z \gamma dt' \approx \omega t - \frac{k_x^2}{8\omega^2} v_{rms}\, t \tag{2.12}$$

which now depends only on the rms velocity. In ray theory this approximation corresponds to a straight ray path approximation in a vertically stratified medium whose inaccuracy increases with increasing deviation of the ray from the vertical, i.e. with increasing dip of the reflector.

Frequency-Wave Number or "F-K" Time Migration (Stolt 1978)

Practice has shown that in the case of a constant velocity medium both accuracy and computational efficiency of formula (2.7) can be enhanced by a change of integration variable from ω to k_z. From the dispersion relation (2.3) we get for upgoing waves

$$\omega = -\frac{k_z v}{2}\sqrt{1 + \frac{k_x^2}{k_z^2}} \qquad (2.13)$$

and thus

$$\frac{d\omega}{dk_z} = -\frac{k_z v^2}{4\omega} \qquad (2.14)$$

With these two relations the constant velocity phase shift migration (2.7) becomes

$$p(x,z,t=0) = r(x,z) = -\frac{v^2}{4(2\pi)^2}\int\int P(k_x,z=0,\omega)\exp\{i(k_x x+k_z z)\}(k_z/\omega)dk_x\, dk_x z \qquad (2.15)$$

This equation is commonly known as frequency-wave number or f-k migration (Stolt 1978) because the same formula results by applying three Fourier transforms (with respect to t,x and z) to the wave equation, which transforms it into an algebraic equation, solving the boundary/initial value problem in the resulting frequency-wave number (f-k) domain and finally transforming the solution back into the time-space domain.

The major difference between the phase shift and the f-k migration is that in the latter the depth domain z is Fourier-transformed as well. The Fourier transform, like all other integral transforms, is a global operator which by definition limits the possibility to adjust to local conditions. As a consequence no velocity variations, not even 1-dimensional ones as in the case of the phase shift migration (2.12) can be explicitly incorporated in the f-k migration formula.

There are, however, several implicit ways to accommodate velocity variations into the f-k migration:

- Migrate the entire section with different velocities and get the final migrated section by "patching together" the correctly migrated parts.