A hybrid technique for the estimation of strong ground motion in sedimentary basins

Author(s): Fäh, Donat

Publication Date: 1992

Permanent Link: https://doi.org/10.3929/ethz-a-000668669

Rights / License: In Copyright - Non-Commercial Use Permitted
A hybrid technique for the estimation of strong ground motion in sedimentary basins

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZURICH
for the degree of
Doctor of Natural Sciences

presented by
DONAT JAKOB FÄH
Dipl. Natw. ETH
born February 9, 1960
in Rorschacherberg (SG)
Place of origin: Kaltbrunn (SG)

accepted on the recommendation of
Prof. Dr. St. Mueller, examiner
Prof. Dr. G. F. Panza, co-examiner

1992
Contents

Zusammenfassung .................................................. 1
Abstract .............................................................. 4
1. Introduction ...................................................... 6

PART I: Theory

2. Mode summation method ........................................ 13
   2.1. Introduction .............................................. 13
   2.2. Computation of eigenvalues for SH waves ............... 15
   2.3. Computation of eigenfunctions .......................... 20
   2.4. Computation of eigenvalues and eigenfunctions
        for P-SV waves ........................................ 21
   2.5. Group velocities ........................................ 22
   2.6. Attenuation due to anelasticity ........................ 23
   2.7. Response to buried sources .............................. 24
   2.8. Examples of computation in the frequency domain .... 28
       2.8.1. Phase velocities .................................. 28
       2.8.2. Group velocities ................................. 30
       2.8.3. Quality factor Qx ................................. 32
       2.8.4. Energy integral .................................. 32

3. Finite difference method ...................................... 35
   3.1. Introduction ............................................ 35
   3.2. Finite difference scheme for SH waves ................. 36
       3.2.1. Grid dispersion of the finite difference scheme
            for SH waves ....................................... 39
   3.3. Finite difference scheme for P-SV waves ............... 43
       3.3.1. Grid dispersion of the finite difference scheme
            for P-SV waves ..................................... 45
   3.4. Intrinsic attenuation ................................... 50
       3.4.1. Basic theory ..................................... 51
       3.4.2. The rheological model of the generalized Maxwell body. 54
       3.4.3. Body-wave dispersion and quality factor Q ........ 56

4. The hybrid technique .......................................... 64
   4.1. Coupling of the mode summation method to the finite
        difference technique .................................. 64
   4.2. Geometry and geometrical spreading ..................... 67
   4.3. Change of grid spacing ................................ 67
   4.4. Artificial boundaries .................................. 69
5. Comparing and testing of computational techniques .......................... 76
  5.1. Comparison between modal summation and the finite
difference technique for one-dimensional structural models.  76
  5.2. The 2D mode summation technique .................................... 78
  5.3. Comparison between results obtained with the hybrid
technique and the 2D mode summation method ...................... 80

PART II: Examples of the application of the hybrid technique

Introduction to the seismic response of sedimentary basins .......... 88

6. An example of numerical modelling for sites close to the
seismic source: Friuli area .................................................. 91
  6.1. Numerical modelling for a one-dimensional structural
model .............................................................................. 93
  6.2. Numerical modelling for two-dimensional structural
models ............................................................................ 96
  6.3. Influence of the dipping interface between bedrock and
sediments at the edge of the sedimentary basin .................. 109
  6.4. Lateral resonance effects and excitation of local
surface waves .................................................................. 115
  6.5. Comparison of computed ground motion with observations .. 117
  6.6. Sensitivity of the signals to changes of the quality factor
and of the shear wave velocity of the sediments ............... 123
  6.7. Some properties of the local surface waves derived from
the 2D mode summation method ....................................... 126
  6.8. The limits of the one-dimensional modelling with the
Haskell technique for seismic hazard studies ................. 130
  6.9. Fourier spectra and response spectra ............................ 133

7. An example of numerical modelling for intermediate distances
from the seismic source: the city of Rome .............................. 141
  7.1. The January 13, 1915 Fucino earthquake ....................... 142
  7.2. Numerical modelling for a two-dimensional
structural model ................................................................ 145
  7.3. Properties of strong ground motion in comparison with
the observed distribution of damage ............................... 149
  7.4. Results in the frequency domain ................................. 157
8. An example of numerical modelling far from the seismic source: Mexico City .............................................. 162
  8.1. Geological and geotechnical information in Mexico City ...... 163
  8.2. Damage distribution in Mexico City ................................ 166
  8.3. The observed signals in Mexico City .............................. 166
  8.4. A structural model for Central Mexico ............................ 170
  8.5. Source characteristics of the 1985 Michoacan earthquake ...... 170
  8.6. Numerical modelling for Mexico City ............................. 174
      8.6.1. One-dimensional structural models ......................... 175
      8.6.2. The cross-section Chapultepec-Penón ...................... 178
          8.6.2.1. Effects of the deep sediments ...................... 179
          8.6.2.2. Effects of a clay layer ............................ 187
      8.6.3. Comparison between the synthetic signals and the observed ground motion .......... 191

9. Summary and Conclusions ........................................... 195

References ........................................................................ 204

Acknowledgments ............................................................ 214

Curriculum Vitae ............................................................ 215

Appendix A: Phase attenuation for Rayleigh waves .................. 216

Appendix B: Finite difference scheme for P-SV waves and its stability condition ........................................ 218

Appendix C: Intrinsic attenuation ........................................ 221
      C.1. Introducing the intrinsic attenuation into the wave equation for SH waves .............. 221
      C.2. Introducing the intrinsic attenuation into the elastic wave equation .................. 222
ZUSAMMENFASSUNG


Am Beispiel des Friaul Bebens wird spezielles Gewicht auf drei Faktoren gelegt: einerseits auf die in Sedimentbecken erzeugten Wellentypen, andererseits auf die Unterschiede zwischen dem SH und P-SV Fall und schliesslich auf den Vergleich zwischen den synthetischen


Im Fall des Michoacan Bebens ist es möglich, Eigenschaften der in Mexico City beobachteten Seismogramme mit Hilfe numerischer Simulationen zu erklären. Für ausgesuchte Modelle können die Dauer, der Frequenzinhalt und die relativen Amplituden der beobachteten Bodenbewegungen gut reproduziert werden.

ABSTRACT

A computational hybrid technique is presented to estimate ground motion in two-dimensional, laterally heterogeneous media, with special emphasis given to sedimentary basins. The technique combines modal summation and the finite difference method to describe wave propagation in realistic two-dimensional structures that are anelastic. It can take into account the source, path and local soil effects to calculate the local wavefield due to a seismic event. Propagation of the waves from source position to the sedimentary basin is computed by the modal summation method for P-SV and SH waves, with the treatment of P-SV waves being based on Schwab's (1970) optimization of Knopoff's (1964b) method, and the handling of SH waves being based on Haskell's (1953) formulation; these computations include the "mode-follower" procedure and structure minimization described by Panza and Suhadolc (1987). Explicit finite difference schemes are then used to simulate the propagation of seismic waves in the sedimentary basin. These schemes are based on the formulation of Korn and Stöckl (1982) for SH waves, and on the velocity-stress, finite difference method for P-SV waves (Virieux, 1986). The schemes are stable for materials with high, as well as normal values of Poisson's ratio. Intrinsic attenuation in soft sediments is an important process and is taken into account in the computations to prevent serious errors in the estimation of seismic hazards. In the mode summation method, anelasticity is included by means of the variational method, while in the finite difference computations, it is included by using a method based on the rheological model of the generalized Maxwell body.

To define the limits and possibilities of this hybrid technique, three computational examples are presented: one for sites that are far from the source (Mexico City during the September 19, 1985, Michoacan earthquake), one at an intermediate distance (Rome during the January 13, 1915, Fucino earthquake), and one for sites close to the source (a sedimentary basin in the Friuli region during the September 11, 1976, Friuli aftershock at 16h35m04s). Special emphasis is given to understanding the different features of ground motion in sedimentary basins. The most important effects that can be observed are the excitation of local surface waves at lateral heterogeneities, the focusing effects, and local resonances. In one-dimensional structural models, local surface waves are never generated, and therefore, such models are not suitable for the prediction of seismic ground motion in sedimentary basins.

In the numerical simulation of accelerograms for the September 11, 1976, Friuli earthquake, the coda of the transverse component is mainly
composed of the local, fundamental-mode Love wave, whereas the P-SV wavefield shows dominant contributions of a high-frequency crustal wave and diffracted waves that originate at the edge of the sedimentary basin. A parametric study is performed that demonstrates the sensitivity of computed ground motion to small changes in the structural model close to the receiver.

For the January 13, 1915, Fucino (Italy) earthquake, a comparison is performed between the observed distribution of damage in Rome, and certain quantities related to the computed ground motion. These quantities are those commonly used for engineering purposes, e.g. the peak ground acceleration, the maximum response of a simple oscillator, and the so-called “total energy of ground motion” which is related to the Arias Intensity. The “total energy of ground motion” is in good agreement with the observed distribution of damage and turns out to give a good representation of the ground motion.

For the 1985 Michoacan earthquake, the synthetic signals explain the major characteristics of the observed ground motion in Mexico City. For selected structural models, the relative amplitudes, durations, and frequency content of the different components of the synthetic signals agree well with the observations.

In general, a good representation of two-dimensional effects is given by the computation of spectral ratios between the signals obtained (1) for the two-dimensional model of the sedimentary basin, and (2) for a reference model in which the upper few hundreds meters of the basin are replaced by the one-dimensional structure present in the model used to describe the wave propagation from the source to the basin. Both signals have in common the source and the path, so that from the spectral ratios the effects of the sedimentary basin are clearly exhibited. The spectral ratios allow the identification of frequency bands in which amplification effects occur, and also the sites at which amplification effects occur. An important result of our study is that the presence of a near-surface layer of rigid material is not sufficient to classify a location as a “hard-rock site” when the rigid material has a sedimentary complex below it. This is because the underlying sedimentary complex causes amplifications due to resonances. Within sedimentary basins, incident energy in certain frequency bands can also be shifted from the vertical, into the radial component of motion. This phenomenon is very localized, both in frequency and space, and closely neighboring sites can be characterized by very large differences in the seismic response, even if the lateral variations of local soil conditions are relatively smooth.
1. Introduction

Over the past few decades there has been a rapid growth in the world's population, and a tendency for this growth to be concentrated in urban areas. This growth of the cities, in general, is accompanied by the concentration of population in sedimentary valleys where it is easy to build. Such valleys are the zones most vulnerable to earthquakes, due to the presence of soft sediments. The 1985 Michoacan, Mexico event, the 1988 Spitak earthquake, and the 1989 Loma Prieta earthquakes are recent reminders that local ground conditions can have a strong influence on the damage in urban areas. It is therefore of crucial interest to estimate seismic ground motion in such areas, before an earthquake occurs. This requires knowledge of both the sub-soil structures and the possible causative seismic sources, along with the availability of computational techniques that permit us to map the expected ground motion and the seismic hazard. Special developments will be necessary to improve current-day seismic programs, which in general do not include two- and three-dimensional effects in sedimentary basins, and which underestimate the hazard on unconsolidated soils.

Several methods for estimating the seismic shaking potential are currently in use, but their reliability and accuracy are not well known and have not been rigorously tested. Moreover, the costs of their application are not well determined. For these reasons, in the last few years several working groups have instituted international programs to pursue these questions. One of these projects is the IASPEI/IAEE (International Association of Seismology and Physics of the Earth's Interior/ International Association of Earthquake Engineering) Joint Working Group. The purpose of this group is to coordinate the establishment of an international series of test areas (for example the Turkey Flat test area and the SMART-1 array in Taiwan) to provide a database for comparing, testing and improving the available methods. Also, in 1990 the United Nations began a ten-year project: the International Decade for Natural Disaster Reduction, to reduce the vulnerability of high-risk areas.

Some of the promising methods for the estimation of shaking potential involve strong-motion simulations. These methods play an important role in complementing traditional empirical approaches, such as making estimates based on analyses of accelerograms recorded in different areas of the world. Strong-motion simulations can provide synthetic signals for areas where recordings are absent, and can thus give estimates of the dependence of strong ground motion on different
parameters: the characteristics of the seismic source, the path, and local soil conditions. Time histories can be generated whose wave composition, duration, and frequency content reflect these characteristics. These computational methods have several advantages over instrumental methods, which consist of placing strong motion instruments in the area of interest. The main advantage is that there is no need to wait for an earthquake to obtain the recordings. In many seismic zones, strong earthquakes occur rarely. Experience in the "Benevento Seismic Risk Project", for example, has shown only seven usable records in nine months of recording. Moreover, in general the recorded events do not have sufficient magnitudes to allow estimations of expected, strong ground motion. There are mainly weak motion records available, which in their frequency content can scarcely represent a strong event. An additional problem is the density of instruments in a strong motion array; only a few cities have sufficiently dense networks, that cover most types of local soil conditions. Examples of sufficiently dense arrays are those in Mexico City and San Francisco.

Numerical simulation should always be compared with observed, strong ground motion to establish validity of the numerical results. Once established, the site effects can be tested numerically regarding sensitivity to the earthquake characteristics (azimuth, depth, magnitude, source mechanism, etc.). This step is important if the results are to be used for micro-zoning studies, since a zonation should be insensitive to all parameters of the source and path.

All computational methods that are proposed in the literature for calculating site effects, can be used to estimate shaking potential. However, it is necessary to justify the validity of the selected method, relative to whether the structure of the site under consideration falls within the domain of the method. Far away from lateral heterogeneities such as edges of sedimentary basins, a local structure can sometimes be approximated by a horizontally-layered structural model (e.g. Swanger and Boore, 1978). This allows the application of techniques such as the generalized ray method (Heaton and Helmberger, 1977), the reflectivity method (Fuchs, 1968; Fuchs and Müller, 1971) or the mode summation method (Thomson, 1950; Haskell, 1953; Knopoff, 1964b; Panza, 1985). These techniques are widely used for zonation studies since they are computationally cheap and can give an estimate of ground motion on a regional scale. They fail, however, to predict ground motion at sites close to lateral heterogeneities such as edges of sedimentary basins, where focusing of waves, excitation of local surface waves, and two-, or three-dimensional resonance effects can become important (e.g. Aki and
Larner, 1970; Boore et. al., 1971; Trifunac, 1971; Bard and Bouchon, 1980a; 1980b; Sánchez-Sesma et al., 1988b). This requires more general techniques--those which can handle the more complex lateral heterogeneities. Most of these last techniques have thus far only been applied to two-dimensional structural models because of the excessive computer-resource requirements of the three-dimensional case.

For many applications, as in seismic reflection or refraction studies, it is sufficient to treat wave propagation in laterally heterogeneous structures with asymptotic forms for high frequency. These are termed 'ray methods' (Hong and Helmberger, 1978; Cerveny, 1985), and need little computer time; however, they can only be applied to smoothly varying media, in which characteristic dimensions of the inhomogeneities are considerably larger than the prevailing wavelengths. The ray methods also fail in the vicinity of some surfaces, lines or points. To evaluate high-frequency synthetic seismograms which overcome partially or fully such difficulties, several techniques have been proposed, e.g. the WKBJ method (Chapman, 1978), the Maslov asymptotic theory (Chapman and Drummond, 1982), the Kirchhoff-Helmholtz methods (e.g. Frazer and Sen, 1985), and Gaussian beams (Cerveny et al., 1982). These approaches, also, are restricted to media with inhomogeneity dimensions that are considerably larger than the prevailing wavelengths, and therefore, are only of limited use for zonation studies in sedimentary basins.

Techniques that are not limited to high-frequency asymptotics are the integral equation methods (Sánchez-Sesma and Esquivel, 1979), 2D modal summation (Levshin, 1985; Vaccari et al., 1989a) which will be outlined in this thesis, and the Rayleigh-Ansatz method (Aki and Larner, 1970; Bard and Bouchon, 1980a; 1980b) which is also known as the Aki-Larner technique. The last technique has been widely used to study wave propagation in sedimentary basins (e.g. Bard and Bouchon, 1980a; 1980b; Bard and Gariel, 1986). It is efficient in computation, but is unable to treat very steep boundaries. The discrete-wavenumber, boundary-element method (e.g. Kawase, 1988) overcomes also this difficulty. The last two methods can be used for zonation studies, but they are restricted to simple, two-dimensional geometries of the sedimentary basin.

Computational techniques that are based on an approximate mathematical method for solving the formal representation of the problem are the finite difference method (Alterman and Karal, 1968; Boore, 1972), the pseudo-spectral method (Gazdag, 1981; Kosloff and Baysal, 1982), and the finite element method (Lysmer and Drake, 1972). These approaches are very powerful in terms of flexibility of modelling:
they allow treatment of wave propagation in very complex structures. Their limitations are the computer CPU time and memory that they require. Computer memory limitations necessitate the introduction of artificial boundaries, which restricts spatial extent of the structural model. This is one of the most severe problems with these methods. Further, and again due to computer memory limitations, often the source cannot be included in the structural model. This occurs when the epicenter is too far from the site of interest. In such cases, the incoming wavefield is approximated by a plane polarized body-wave.

The computation of synthetic seismograms for media with localized heterogeneous areas can be performed using hybrid techniques which combine methods such as modal summation, the reflectivity method or ray methods, with techniques such as the finite difference method or the finite element method (Shtivelman, 1984; 1985; Vidale, 1987; Berg van den, 1984; Kummer et al., 1987; Suetsugu, 1989; Emmerich, 1989). In this approach, only a small part of the medium under study is laterally heterogeneous, and therefore, the finite difference and finite element methods alone are very inefficient due to their need for large CPU time and computer memory. In hybrid computational schemes the finite difference method or finite element method is applied only in the laterally heterogeneous part of the medium, whereas the laterally homogeneous part is treated with methods such as modal summation, the reflectivity method and ray methods. Therefore, such computations are very efficient.

The development and testing of such a hybrid method is the basis of this thesis. The technique combines modal summation and the finite difference method, and it can be used to study wave propagation in sedimentary basins. Each of the two techniques is applied in that part of the structural model where it works most efficiently: the finite difference method in the laterally heterogeneous part of the structural model which contains the sedimentary basin, and modal summation is applied to simulate wave propagation from source position to the sedimentary basin of interest. The advantage of this hybrid technique, in comparison with other computational methods, is that it allows to take into consideration source, path, and local soil effects. This allows us to calculate the local wavefield from a seismic event, both for small (a few kilometers) and large (a few hundreds of kilometers) epicentral distances. An extended source can be modelled easily by a sum of point sources, appropriately distributed in space. This allows the modelling of active faults which are so close to the site that their geometrical extension must be taken into account. The path from source position to the sedimentary basin can be
well approximated by a structure composed of a sequence of flat, homogeneous layers. Along this path modal summation allows the treatment of many layers which can account for low-velocity zones and fine details of the crustal section under consideration. This allows the simulation of a realistic incident wavefield, which is used as input in the finite difference computations. The finite difference method permits the modelling of complicated and rapidly varying velocity structures in the final part of the propagation path. Such strong lateral heterogeneity is a characteristic of sedimentary basins, which in general consist of very complicated subsurface topography and velocity distribution.

The first part of the thesis is dedicated to the theory of seismic wave propagation in one-, and two-dimensional structural models. In the case of a one-dimensional layered structure, the mode summation method is a powerful tool for computing broadband synthetic seismograms. The theory for SH waves—the Love wave case—is presented in Chapter 2. There, the case of P-SV waves is only briefly outlined as well. The phase velocity spectrum, group velocity spectrum, quality factor, and the energy integral for a layered structural model are discussed in detail. These frequency spectra are the necessary quantities for computing synthetic signals in the time domain.

Modal summation method is not applicable to local structures, which in general do not have plane-layer characteristics. Since high-frequency seismograms are very sensitive to lateral heterogeneities, the influence of these heterogeneities should be included in the numerical modelling. This in turn, requires the use of at least two-dimensional models to take into account different tectonic settings and site effects. A powerful technique for simulating wave propagation in two-dimensional media is the finite difference method, which is introduced in Chapter 3. Intrinsic attenuation in near-surface sediments is an important process and is taken into account in the computations to prevent serious errors in the estimations of seismic hazards. Emmerich and Korn (1987) proposed a method for incorporating attenuation into time-domain computations of SH-wave propagation. In this thesis, their method is implemented for the SH case and is here generalized to the P-SV case. This approach is based on the rheological model of the generalized Maxwell body and allows us to approximate the viscoelastic modulus by a low-order rational function of frequency. This approximation of the viscoelastic modulus can account for a constant quality factor over a certain frequency band. Replacement of all elastic moduli by viscoelastic ones, and transformation of the stress-strain relation into the time-domain, yields a formulation which can be handled with a finite difference algorithm.
In Chapter 4, the hybrid technique is introduced for SH and P-SV wave propagation. We discuss the problems of the coupling the mode summation method with the finite difference technique. Special emphasis is given to the problem with geometrical spreading caused by the use of two-dimensional models, and to the artificial boundaries in the finite difference method. Different types of artificial boundaries are tested and compared.

In Chapter 5, the results obtained with the hybrid technique are compared with the results from the 2D mode summation method, applied to structural models with a vertical discontinuity. The accuracy of both techniques, and the difference in results, are discussed in detail.

In the second part of the thesis, numerical simulations are presented and compared with observed, strong ground motion and with distributions of damage from recent earthquakes. The first example, presented in Chapter 6, is the numerical simulation of the September 11, 1976, Friuli (Italy) aftershock at 16h35m04s. This is an application of the hybrid method close to a source. Special attention is paid to the types of waves generated inside sedimentary basins, to the differences between SH and P-SV wave propagation, and to the comparison between computed and recorded ground motion. Results obtained with different modelling techniques lead to several consequences for seismic hazard studies of sedimentary basins. The methods used are (1) the “Haskell technique”, which—in seismic engineering—excludes a realistic source, and the amplification in the sedimentary cover is studied using an incident, plane polarized body-wave; and (2) the mode summation and hybrid techniques, which include a realistic source model. Special emphasis is given to differences in the results obtained for one-dimensional models and two-dimensional models, and to the variability of ground motion for different realistic, two-dimensional models of the sedimentary basin in the Friuli region.

The second example, presented in Chapter 7, is the numerical simulation of the January 13, 1915, Fucino (Italy) earthquake. This event caused structural damage in the city of Rome. Since the distribution of damage in Rome is well documented, a comparison is performed between this observed distribution and some quantities that are related to the ground motion, and that are commonly used for engineering purposes. These include the peak ground acceleration, the maximum response of a simple oscillator, and the so-called “total energy of ground motion” which is related to the Arias Intensity. These quantities can be used for a rough zonation in the city of Rome.
The third example, presented in Chapter 8, is the well-known case of the September 19, 1985, Michoacan earthquake ($M_s=8.1$), which together with its aftershocks, produced the worst earthquake damage in the history of Mexico. The extensive damage can be attributed to the characteristics of the sediments in the valley of Mexico City. Owing to the distance of the epicenter from Mexico City, finite difference or finite element methods cannot be used alone to determine the effects in this urban area: computer memory and CPU time requirements are simply too large. On the other hand, methods such as modal summation, that are restricted to very simple, two-dimensional geometries, cannot deal with the complex geological situation in Mexico City. The hybrid technique offers a solution to this problem. The Michoacan earthquake was one of the most carefully monitored earthquakes affecting a large metropolitan area. Therefore, it is of particular interest to compare observed and computed ground motion. Unfortunately, the mechanical properties of the sediments and the geology in Mexico City are poorly known, and the structural models used by different authors exhibit much variability. Therefore, a sensitivity study of the computed ground motion is performed with respect to certain parameters of the structural model and the source: the source mechanism, the geometrical, and the mechanical properties of the sedimentary basin in Mexico City.
2. Mode summation method

2.1. Introduction

The summation of modes is a technique for simulating the propagation of seismic waves. It is nearly free of significant approximations in one-dimensional structural models—models that vary only with depth—and can be extended to the two-, and three-dimensional cases. By means of this method, we can compute very realistic signals for the relatively simple, one-dimensional case. The equation of motion for a homogeneous, isotropic elastic medium is

\[
\rho \begin{pmatrix}
  u_{tt} \\
  v_{tt} \\
  w_{tt}
\end{pmatrix} = (\lambda + 2\mu) \text{ grad div } \begin{pmatrix}
  u \\
  v \\
  w
\end{pmatrix} - \mu \text{ rot rot } \begin{pmatrix}
  u \\
  v \\
  w
\end{pmatrix},
\]

where \(u, v\) and \(w\) are the displacements in the \(x, y\) and \(z\)-direction. The \(x\) axis is taken parallel to the free surface, with positive sense in the direction of propagation; the \(z\) axis is directed downward into the medium; \(\lambda\) and \(\mu\) are the Lamé parameters, and \(\rho\) is the density. In this equation body forces are neglected.

The structural model of interest is composed of a sequence of homogeneous layers as shown in Figure 2.1. This allows the separation of Equation (2.1) into two independent problems: the propagation of SH (Love) waves, which have particle motion in the \(y\)-direction, and the propagation of P-SV (Rayleigh) waves, with particle motion in the \(xz\) plane. Modal summation makes use of the fact that in a layered medium, the equation of motion (2.1) can be solved exactly. The waves are decomposed into either those propagating upward and downward in some layers, or into horizontally propagating waves which either decay and grow exponentially with depth in the other layers. In the half-space that terminates the structural model at depth, horizontal propagation describes the wave motion and the coefficient of the exponentially increasing wave must vanish. The problem is then reduced to fulfilling all boundary conditions at the interfaces which separate the layers. This leads to an eigenvalue problem in which the eigenvalues (phase velocities) and eigenfunctions (displacement-depth and stress-depth functions) are to be determined. For a flat, layered-earth model, a numerical scheme for dealing with this problem was first proposed by Thomson (1950) and was
later corrected and expanded by Haskell (1953). For the Rayleigh-wave case, Knopoff (1964b) proposed a modification of the initial scheme which avoids the precision loss that is intrinsic to the original formulation. This approach finally permitted the automatic computation of broad-band synthetic seismograms for P-SV waves (e.g. Panza, 1985), which are complete in a given frequency-phase velocity window.

The algorithm developed by Panza (1985) and Panza and Suhadolc (1987) for P-SV waves was later expanded to SH waves. This has been illustrated by Florsch et al. (1991). Loss-of-precision does not occur in the SH-wave computations; most of the other features of the computations for P-SV waves and SH waves are equivalent, though much simpler for SH waves. A first order approximation is used for the introduction of anelasticity into the computations. This is based on variational methods (Takeuchi and Saito, 1972; Schwab and Knopoff, 1972), and includes Futterman's (1962) results concerning dispersive, body-wave velocity in a linearly anelastic medium. This approach allows us to consider anelastic media characterized by Q values as low as about 20. By comparison with the results obtained from the exact method (Schwab, 1988; Schwab and Knopoff, 1971; 1972; 1973), the attenuation effects obtained from the variational technique can be estimated to have errors of up to 6-20 percent.

**Figure 2.1.** Coordinate system and geometry for the layered structural model. Numbers without parentheses denote layers, while numbers in parentheses denote interfaces.
The seismic source is introduced by using the Ben-Menahem and Harkrider (1964) formalism. In these expressions, the first-term approximation to cylindrical Hankel functions is used which gives the displacements in the far field. Calculation of synthetic seismograms are then accurate to at least three significant figures, as long as the distance to the source is greater than the wavelength (Panza et al., 1973). The seismograms computed in this way contain all the body waves whose phase velocities are smaller than the S-wave velocity of the half-space that terminates the structural model.

In this chapter, the mode summation method is introduced. We first treat the case of Love waves. The case of P-SV waves is only briefly outlined as well. The phase velocity spectrum, group velocity spectrum, quality factor, and the energy integral for a layered structural model are discussed in detail. These frequency spectra are the necessary quantities for computing synthetic signals in the time domain.

### 2.2. Computation of eigenvalues for SH waves

For the multimode eigenvalue computations we make use of the notation employed by Schwab and Knopoff (1972). The density-depth and velocity-depth distributions in the earth are approximated by a structure composed of a sequence of flat, homogeneous layers. In the mth layer, the components of displacement and stress are given by

\[ u = w = 0 \]

\[ \sigma_{xx} = \sigma_{zz} = \sigma_{yy} = \sigma_{xz} = 0 \]

\[ v(z) = \begin{bmatrix} v_m' e^{-ikz r} \beta_m + v_m'' e^{ikz r} \beta_m \\ \frac{\partial v_m'}{\partial z} \end{bmatrix} e^{i(\omega t - kx)} \]

\[ \sigma_{yz}(z) = \sigma_{z}(z) = \mu_m \left[ \frac{\partial v_m}{\partial z} \right] \]

\[ = ik\mu_m r \beta_m \left[ -v_m' e^{ikz r} \beta_m + v_m'' e^{ikz r} \beta_m \right] e^{i(\omega t - kx)} \]

\[ \sigma_{xy}(z) = \mu_m \left[ \frac{\partial v_m}{\partial x} \right] = -ik\mu_m v(z) \]

where \( k \) is the horizontal wavenumber, \( \mu_m \) is the Lamé constant, \( v_m' \) and \( v_m'' \) are the frequency-dependent layer constants in the mth layer, and \( \sigma_{ij} \) are the stress components. For the definition of the quantity \( r\beta_m \), see
Equation (2.4). Neglecting the factor exp(i(\omega t-kx)), continuity of displacement and stress at the m\textsuperscript{th} interface leads to the relation

\[
\begin{bmatrix}
  v_m \\
  (\sigma_z)_m
\end{bmatrix} = a_m
\begin{bmatrix}
  v_{m-1} \\
  (\sigma_z)_{m-1}
\end{bmatrix},
\]

where \( a_m \) is the 'layer matrix' for the m\textsuperscript{th} layer, \( v_i \) is the displacement and \( (\sigma_z)_i \) the stress at interface i. The dispersion function for the layered structure can be defined as the product of the modified layer matrices \( b_m \) (Schwab and Knopoff, 1972)

\[
F_L(\omega, c) = b_n b_{n-1} b_{n-2} \cdots b_1
\]

(2.2)

where \( n \) is the number of layers, including the lower half-space, and the modified scheme of matrix multiplication that follows (2.4) is used to evaluate the above product. The dispersion function is a scalar quantity and has the symbolic matrix representation \([1x2][2x2][2x2][2x1]\); it is formed from left to right, so that each new layer (or interface) is represented by a \([1x2][2x2]\) multiplication resulting in a new \([1x2]\) matrix with which the process is continued. In Equation (2.2) \( b_n \) is given by

\[
\begin{align*}
  b_n &= (s, -1) \text{ if the half-space is solid} \\
  b_n &= (0, -1) \text{ if the half-space is liquid} \\
  b_n &= (1, 0) \text{ if the half-space is rigid};
\end{align*}
\]

and the quantity \( s \) is defined in (2.4).

The general mathematical formulation for surface-wave propagation can be written with two types of waves in the solid half-space: one with amplitude increasing exponentially with depth, and one with amplitude decreasing exponentially with depth. To avoid infinite values of the solution at infinite depth, the coefficient of the exponentially increasing wave in the half-space must vanish. If the half-space is taken to be liquid, the deepest interface is at the analog of the mantle-core boundary. Introduction of a rigid lower half-space results in the locked-mode approach (Harvey, 1981); then, the half-space becomes a perfect reflector, and at certain frequencies the depth of penetration of the eigenfunction changes when the depth of the rigid boundary is varied.
The modified layer-matrices $b_m (0 < m < n)$ are given by

$$b_m = \begin{cases} 
\begin{bmatrix}
\cos Q_m & \sin Q_m \\
\mu_m r_{\beta_m} & \mu_m r_{\beta_m}
\end{bmatrix} & \text{if } c > \beta_m \\
\begin{bmatrix}
cosh Q_m^* & \sinh Q_m^* \\
-\mu_m r_{\beta_m}^* & \cosh Q_m^*
\end{bmatrix} & \text{if } c < \beta_m
\end{cases}$$

$$b_m = \begin{bmatrix}
1 & \frac{\omega d_m}{\mu_m c} \\
0 & 1
\end{bmatrix} \quad \text{if } c = \beta_m$$

In expressions (2.3), in the $m$th layer, $\mu_m = \rho_m (\beta_m)^2$, $\beta_m$ is the S-wave velocity, $\rho_m$ is the density, $d_m$ is the thickness of the layer, $\omega$ is the angular frequency, and $c$ is the phase velocity. An asterisk denotes the imaginary part of a quantity, $\xi^* = \text{Im}(\xi)$; in our particular problem, it is used with purely imaginary quantities so that $r_{\beta_m} = i r_{\beta_m}^*$. Moreover,

$$r_{\beta_m} = \sqrt{\left( \frac{c}{\beta_m} \right)^2 - 1}$$

$$Q_m = \frac{\omega r_{\beta_m} d_m}{c} = k r_{\beta_m} d_m$$

$$Q_m^* = \frac{\omega r_{\beta_m}^* d_m}{c} = k r_{\beta_m}^* d_m$$

$$s = -\mu_n \sqrt{1 - \left( \frac{c}{\beta_n} \right)^2}.$$
The modified matrix product of \( b_m \) and \( b_{m-1} \) in Equation (2.2) is defined as

\[
[b_m \ b_{m-1}]_{jp} = \begin{cases} 
(b_m)_{jl} \ (b_{m-1})_{lp} & \text{if } (j+p) \text{ is even} \\
(-1)^{j+1} (b_m)_{jl} \ (b_{m-1})_{lp} & \text{if } (j+p) \text{ is odd.}
\end{cases}
\]

Equations (2.3) and (2.4) are the representation of the Thomson-Haskell formulation for Love waves. The only difference from Knopoff's method is in the matrix representations of the dispersion function (Schwab, 1970). The values of the dispersion functions are the same, except for a possible sign reversal at some values of \( n \). Both algorithms also have the same program speed.

Seeking eigenvalues—seeking the phase velocity that corresponds to a given angular frequency—requires the determination of roots of the dispersion function. This can be done by root bracketing and root refining, according to the procedure described by Schwab and Knopoff (1972). This complete procedure is only necessary at the beginning of each mode. For all other points, the phase velocity can be estimated by cubic extrapolation and a root-refining procedure in the \( F-c \) plane (Panza, 1985; Panza and Suhadolc, 1987).

Two kinds of overflow problems can occur. The first appears, when \( Q_m^* \) has a large absolute value. In this case, calculation of \( \cosh(Q_m^*) \) and \( \sinh(Q_m^*) \) are prevented by using the approximations

\[
\cosh(Q_m^*) \approx -\sinh(Q_m^*) \approx \frac{1}{2} \exp(-Q_m^*),
\]

where \( Q_m^* \) always has negative values. In the matrix product (2.2), \( \exp(-Q_m^*) \) can be factored out and replaced by unity, since only the roots of the dispersion function are of interest. This operation also saves computation time. In analogy with the P-SV case (Schwab et al., 1984), we call this the "single-layer" overflow control.

The second kind of overflow generally appears when the entire matrix product (2.2) is computed with too many layers retained in the computations, or with a phase velocity that is too far from the root. Here a normalization procedure is used to prevent overflow: in the \([1x2]\) matrix (resulting from each \([1x2][2x2]\) product formed from the left in (2.2)), both elements are divided by the greatest absolute value of the matrix elements (Schwab and Knopoff, 1972). Again by analogy with the P-SV case (Schwab et al., 1984), this is termed "multi-layer" overflow control.
To handle realistic structural models of the earth, the computational scheme must permit the use of a large number of layers in order to model all possible gradients in the physical properties. Such gradients can be approximated by a sequence of thin, homogeneous layers. To optimize efficiency in computations with such structures, both a mode follower and a structure minimization procedure are required. These are described by Panza and Suhadolc (1987). Structural minimization is relevant to the avoidance of computations in those parts of the structure where the eigenfunctions effectively vanish. It consists of an algorithm that temporarily retains only the upper part of the structure, where the eigenfunctions do not vanish. This permits a very high accuracy in the calculation of eigenfunctions, and also saves computer time. To find the minimum of the eigenfunction, below the location of which the structure can be neglected in the computations, the function

$$E_m = \rho_m \left( \frac{v_m}{v_0} \right)^2$$

(2.5)

is used, where \(v_0\) is the displacement at the free surface of the structure. The maximum depth of penetration of the mode (and frequency) being considered, corresponds to the deepest minimum of \(E_m\). The layers below that minimum can be discarded, and the parameters of the uppermost of them are then used to define the terminating half-space.

At high frequencies, modes are very close to one another. This creates problems in following an individual mode in the phase velocity-frequency space, and in distinguishing it from the neighboring modes. As mentioned before, the root-bracketting is only necessary at the beginning of each mode. For all other points, the phase velocity is estimated by cubic extrapolation and a root-refining procedure in the F-c plane. Thus, a test is required to determine whether a root corresponds to the mode of interest. The mode follower provides an efficient means for this test, based on the fact that for a given mode the sign of \(\partial F/\partial c\) is constant. An increase in mode number by one, causes the sign to change, and this identifies an unwanted jump to a neighboring mode.
2.3. Computation of eigenfunctions

With the geometry shown in Figure 2.1, the computation of the eigenfunctions at the layer interfaces can be performed as follows (see, for example, Florsch et al., 1991):

\[
\begin{bmatrix}
  v_m \\
  (\sigma_z)_m
\end{bmatrix} =
\begin{bmatrix}
  \cos Q_m & \frac{\sin Q_m}{k\mu_m r_{\beta_m}} \\
  -k\mu_m r_{\beta_m} \sin Q_m & \cos Q_m
\end{bmatrix}
\begin{bmatrix}
  v_{m-1} \\
  (\sigma_z)_{m-1}
\end{bmatrix}
\begin{cases}
  \text{if } c > \beta_m \\
  \text{if } c = \beta_m
\end{cases}
\] (2.6)

\[
\begin{bmatrix}
  v_m \\
  (\sigma_z)_m
\end{bmatrix} =
\begin{bmatrix}
  \cosh Q^*_m & \frac{\sinh Q^*_m}{k\mu_m r^*_{\beta_m}} \\
  k\mu_m r^*_{\beta_m} \sinh Q^*_m & \cosh Q^*_m
\end{bmatrix}
\begin{bmatrix}
  v_{m-1} \\
  (\sigma_z)_{m-1}
\end{bmatrix}
\begin{cases}
  \text{if } c > \beta_m \\
  \text{if } c = \beta_m
\end{cases}
\] (2.6)

\[
\begin{bmatrix}
  v_m \\
  (\sigma_z)_m
\end{bmatrix} =
\begin{bmatrix}
  1 & \frac{d_m}{\mu_m} \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  v_{m-1} \\
  (\sigma_z)_{m-1}
\end{bmatrix}
\begin{cases}
  \text{if } c = \beta_m
\end{cases}
\]

where \(v_m\) is the displacement and \((\sigma_z)_m\) the stress at interface \(m\). Once the dispersion or eigenvalue problem has been solved by seeking roots of the dispersion function, the problem is reduced to the determination of the displacement and stress components at the interfaces. Using the initial values at the free surface: \((v_0,(\sigma_z)_0)=(1,0)\), as our starting point, the stress and displacement at each new interface is computed by a [2x2][2x1] multiplication (Equation (2.6)), resulting in a new [2x1] matrix with which the process is continued.

If one attempts to use layers that are many wavelengths thick, "single-layer" overflow will occur in eigenfunction computations because here the factorization above cannot be used. However, by splitting these thick layers into a series of thin ones, this problem can be avoided (Schwab et al., 1984). Splitting of the layers simply increases the number of layers used in the entire program, and is the most desirable solution since the density and body-wave velocity in these thinner layers can be varied to obtain a better approximation to the actual structure.
2.4. Computation of eigenvalues and eigenfunctions for P-SV waves

The Rayleigh-wave case is formally analogous to that for SH waves. However, in computations of Rayleigh-wave dispersion on a sequence of homogeneous layers, it is well known that the original Thomson-Haskell technique contains a loss-of-precision problem. Knopoff (1964b) has given the solution to this problem. As outlined in the previous sections, the Thomson-Haskell technique formulates the surface-wave dispersion function by constructing layer matrices which relate the components of motion at one interface to those at the next. The product of these layer matrices then relates the components of motion at the deepest interface to those at the free surface, and this layer matrix product is used to construct the dispersion function. Knopoff's technique begins with the immediate construction of the dispersion function in its full determinantal form, and then decomposes the determinant into a product of interface matrices, which are derived from submatrices of the determinant. Each of these interface submatrices relates the components of motion in the layer on one side of the interface to those in the layer on the other side.

In the Rayleigh-wave case, all underflow/overflow problems can be controlled by simple modifications of those methods discussed above. The dispersion function analogous to (2.2) is a scalar quantity having the symbolic matrix representation $[I_{6x6}] [6x6] ... [6x6][6x1]$, and is formed from left to right, so that each new interface is represented by a $[1x6][6x6]$ multiplication resulting in a new $[1x6]$ matrix with which the process is continued. "Multi-layer" overflow can be eliminated by a normalization procedure (Schwab and Knopoff, 1970). This involves the determination of the maximum amplitude of the six elements resulting from each matrix multiplication, and the division of these elements by this positive constant. These modifications do not affect the accuracy of the calculated dispersion (Schwab et al., 1984). If, to normalization, we add the previous procedure for handling "single-layer" overflow, all overflow/underflow problems can be avoided in these computations with P-SV waves.

The computation of displacement-depth and stress-depth functions (eigenfunctions) was studied by Schwab et al. (1984) to extremely high frequencies--up to 10'000 Hz. Once again, the original Rayleigh-wave formulation (Haskell, 1953) encounters loss-of-precision difficulties in the eigenfunction evaluation. The use of Knopoff's method completely solves this aspect of the problem with precision loss (Schwab et al., 1984). All underflow/overflow problems can be controlled easily by the same general means as were described for SH waves. Here again, these means do not
affect the accuracy of the computed eigenfunctions. For more details see Schwab and Knopoff (1972), Schwab et al. (1984) and Panza (1985).

For these P-SV wave computations, an additional loss-of-precision problem can occur in the determination of the energy integral (Schwab et al., 1984). Along with eigenvalues and eigenfunctions, this integral is necessary for the formation of synthetic seismograms and it will be introduced in the formalism in Section 2.7. If the layer thickness exceeds by too much the extent of a vertical lobe of the displacement-depth function, precision loss in the energy-integral computations can become noticeable. The simplest solution to this problem is to split a layer into a sequence of thinner layers whenever the vertical extent of an eigenfunction lobe becomes less than the layer thickness. A formal treatment of the breakup of such layers is given by Schwab et al. (1984).

### 2.5. Group velocities

The group velocities, once the phase velocities have been determined, are computed using

\[ u = \frac{c}{1 - \frac{dc}{d\omega} \omega} \quad . \tag{2.7} \]

The derivative \(dc/d\omega\) is obtained from implicit function theory:

\[ \frac{dc}{d\omega} = - \frac{\left( \frac{\partial F}{\partial \omega} \right)}{\left( \frac{\partial F}{\partial c} \right)} \omega \quad . \tag{2.8} \]

For details concerning group velocity computations see Schwab and Knopoff (1972) and Florsch et al. (1991).
2.6. Attenuation due to anelasticity

The treatment of anelasticity requires, for causality reasons, the introduction of body-wave dispersion (Futterman, 1962). In a medium with constant $Q$, the body-wave phase-velocities can be expressed as

$$A_1(\omega) = \frac{A_1(\omega_0)}{1 + \frac{2}{\pi} A_1(\omega_0) A_2(\omega_0) \ln \left( \frac{\omega_0}{\omega} \right)}$$

$$B_1(\omega) = \frac{B_1(\omega_0)}{1 + \frac{2}{\pi} B_1(\omega_0) B_2(\omega_0) \ln \left( \frac{\omega_0}{\omega} \right)}$$

The layer index $m$ is omitted. $A_1(\omega_0)$ and $A_2(\omega_0)$ are the P-wave phase velocity and the P-wave phase attenuation, at the reference angular frequency $\omega_0$. $B_1(\omega_0)$ and $B_2(\omega_0)$ are the comparable S-wave parameters. The quantities $A_1$ and $A_2$ are related to the complex body-wave velocity $\alpha$, and $B_1$ and $B_2$ are related to the complex body-wave velocity $\beta$ by

$$\frac{1}{\alpha} = \frac{1}{A_1} - iA_2$$

$$\frac{1}{\beta} = \frac{1}{B_1} - iB_2$$

(Schwab and Knopoff, 1972). In the computation we have chosen the reference angular frequency $\omega_0=2\pi$ radians. For more details concerning intrinsic attenuation, see Chapter 3.

In anelastic media the surface-wave phase velocity $c$ must be also expressed as a complex quantity

$$\frac{1}{c} = \frac{1}{C_1} - iC_2$$

with $C_1$ being the actual (physical) phase velocity and $C_2$, the phase attenuation. The quantity $C_2$ can be estimated by using the variational technique (Takeuchi and Saito, 1972; Aki and Richards, 1980).
This approximate phase attenuation for Love waves is given by

\[ C_{2L} = \frac{\int_0^\infty \mu B_1 B_2 \left( \frac{\sigma_z^2 + v^2}{\mu^2 k^2} \right) dz}{c \int_0^\infty \mu v^2 dz} \quad (2.12) \]

This integral can be calculated from exact formal expressions, since formal representations are known for the eigenfunctions. The comparable expression for the phase attenuation \( C_{2R} \) of Rayleigh waves is given in Appendix A. As the variational technique is an approximate method, the resulting \( Q \) values will be in error; comparison with the exact computational results allows us to estimate this error as about 6-20 percent. These errors arise mainly from the use of perfectly-elastic layers, and therefore real eigenfunctions, to compute the phase attenuation. The exact mathematical treatment of intrinsic attenuation is described by Schwab and Knopoff (1971; 1972; 1973).

Recently Day et al. (1989) showed the limits of the variational technique for the locked-mode approximation: terminating the structural model with a rigid or liquid half-space. When dealing with low \( Q \) values, they showed that an error in amplitude of up to 100 percent can occur. The error increases when \( Q \) undergoes large variations with depth. Introducing a solid half-space into the model, and using the structure minimization procedure, will prevent this kind of error, since this does not require the introduction of an unrealistic boundary at the top of the terminating half-space.

2.7. Response to buried sources

To include the seismic source in the computations, the formulation of Harkrider (1970) and Ben-Menahem and Harkrider (1964) is used. In Figure 2.2 the fault model of an earthquake, and the coordinate system associated with the free surface is shown. The angle \( \theta \) is that between the strike of the fault and the line connecting epicenter and station, \( \lambda \) is the rake angle, \( \delta \) the dip angle, \( h \) is the source depth and \( s \) represents the direction of the displacement dislocation of the hanging wall relative to the foot wall. If the source function is assumed to have a step-function
time-dependence in the jump in displacement across the fault, and the normal component of stress is continuous across the fault, then the equivalent force replacement in an unfaulted medium is the usual point-source double-couple, without moment and with step-function time-dependence (e.g. Burridge and Knopoff, 1964).

![Figure 2.2. Source geometry and coordinate system associated with the free surface. The angle $\theta$ is that between the strike of the fault and the epicenter-station line, $\delta$ is the dip, $\lambda$ the rake, and $h$ is the source depth.](image)

The response to a double couple is obtained from the superposition of two perpendicular point-force couples; these couples being obtained from the singlet response by differentiating the latter in the direction of the moment arm, which is normal to the fault plane. If we assume that the receiver is at a sufficient distance from the epicentre, we can consider only the first term of the Hankel function expansions (Panza et al., 1973). This will give us the displacement in the far field. The time series $v(t)$, $u(t)$ and $w(t)$ are then given by the inverse Fourier transform of the sum of the single modes

$$v(t) = \int \left[ \sum_j (V_L)_j \right] e^{i\omega t} \, d\omega$$

$$u(t) = \int \left[ \sum_j (U_R)_j \right] e^{i\omega t} \, d\omega$$

$$w(t) = \int \left[ \sum_j (W_R)_j \right] e^{i\omega t} \, d\omega$$

where $j$ is the mode number, which is omitted in what follows. The quantities $V_L$, $U_R$ and $W_R$ are the depth-dependences of the
eigenfunctions at angular frequency $\omega$. The Fourier time transform of the Love-wave displacement ($V_L$) and Rayleigh-wave displacements (radial component $U_R$, vertical component $W_R$) for each mode are given by

$$V_L = |\hat{n}| |R(\omega)| e^{i\Phi} e^{-\frac{i3\pi}{4} \sqrt{k_L} \chi_L(\theta, h)} A_L \frac{e^{-i\omega \frac{r}{C_{1L}}}}{\sqrt{2\pi r}} e^{-\omega r C_{2L}}$$ (2.13)

$$U_R = |\hat{n}| |R(\omega)| e^{i\Phi} e^{-\frac{i3\pi}{4} \sqrt{k_R} \chi_R(\theta, h)} A_R \frac{e^{-i\omega \frac{r}{C_{1R}}}}{\sqrt{2\pi r}} e^{-\omega r C_{2R}}$$

$$W_R = (\varepsilon(z))^{-1} \frac{1}{2} e^{-\frac{i\pi}{2}} U_R$$

where $r$ is the source-receiver distance, $R(\omega)$ is the Fourier transform of the source time function, $\Phi_0 = \arg(R(\omega))$ is the source apparent initial phase and $\varepsilon(z) = -u^*(z)/w(z)$ is the ellipticity at the depth $z$ of the receiver. The subscripts $L$ and $R$ denote Love waves and Rayleigh waves, respectively. The quantity $|\hat{n}|$ is the absolute value of the normal vector to the plane of motion, with units of length. The factors $A_L$ and $A_R$ are given by

$$A_L = \frac{1}{2c_L u_L I_{1L}} \quad \text{and} \quad A_R = \frac{1}{2c_R u_R I_{1R}}$$

where $c_L$ and $u_L$ are the phase and group velocities of the Love-wave mode, and $c_R$ and $u_R$ are the comparable velocities of the Rayleigh-wave mode. $I_{1L}$ and $I_{1R}$ are the energy integrals, which are given by

$$I_{1L} = \int_0^\infty \rho \left( \frac{v(z)}{v_0} \right)^2 dz$$

$$I_{1R} = \int_0^\infty \rho \left( \frac{w(z)}{w_0} \right)^2 + \left( \frac{u^*(z)}{w_0} \right)^2 dz$$

These integrals can be calculated from exact formulations since formal expressions are known for the eigenfunctions. For details concerning the
energy integrals, see Schwab et al. (1984), Panza (1985) and Florsch et al. (1991). Anelasticity is expressed by the terms

$$e^{-\omega r C_{2L}} \quad \text{and} \quad e^{-\omega r C_{2R}}$$

For a double-couple point-source, \(\chi_L\) and \(\chi_R\) are given by

\[
\chi_L(\theta, h) = i (d_{1L} \sin \theta + d_{2L} \cos \theta) + d_{3L} \sin 2\theta + d_{4L} \cos 2\theta \\
\chi_R(\theta, h) = d_{0R} + i (d_{1R} \sin \theta + d_{2R} \cos \theta) + d_{3R} \sin 2\theta + d_{4R} \cos 2\theta
\]

where

\[
d_{0R} = \frac{1}{2} B(h) \sin \lambda \sin 2\delta \\
d_{1R} = -C(h) \sin \lambda \cos 2\delta \\
d_{2R} = -C(h) \cos \lambda \cos 2\delta \\
d_{3R} = A(h) \cos \lambda \sin \delta \\
d_{4R} = -\frac{1}{2} A(h) \sin \lambda \sin 2\delta
\]

The source geometry and the coordinate system associated with the free surface is given in Figure 2.2. \(A(h)\), \(B(h)\), \(C(h)\), \(G(h)\) and \(V(h)\) depend on the values of the eigenfunctions at the hypocenter and at the receiver

\[
A(h) = -\frac{u^*(h)}{w_0} \frac{w(z)}{w_0} \\
B(h) = \left( \frac{\beta^2(h)}{\alpha^2(h)} \right) u^*(h) \frac{w(z)}{w_0} - \frac{\sigma_{zz}^*(h)}{\rho(h) \alpha^2(h)} \frac{2}{\dot{w}_0 / c} \frac{w(z)}{w_0} \\
C(h) = -\frac{1}{\mu(h)} \frac{\sigma_{xz}^*(h)}{\dot{w}_0 / c} \frac{w(z)}{w_0} \\
G(h) = \frac{1}{\mu(h)} \frac{\sigma_{yz}^*(h)}{\dot{v}_0 / c} \frac{v(z)}{v_0} \\
V(h) = \frac{\dot{v}(h)}{\dot{v}_0} \frac{v(z)}{v_0}
\]

(e.g. Ben-Menahem and Singh, 1981), where \(v_0\) and \(w_0\) are the values of the eigenfunctions at the free surface, and \(v(z)\) and \(w(z)\) are the values of the eigenfunctions at the receiver depth \(z\). On a horizontal plane: for the Love-wave case, \(\sigma_{yz}(h)\) is the tangential stress at the depth of the source; for Rayleigh-waves, \(\sigma_{zz}(h)\) is the normal stress and \(\sigma_{xz}(h)\) the tangential
stress at the source depth. Equations (2.13) are valid for a receiver at any depth \( z \). This variability in our formulation is necessary for the hybrid technique (see Chapter 4), which requires the computation of the time series for an array of receivers at many depths.

### 2.8. Examples of computation in the frequency domain

The layered model in Table 2.1 represents an average structure of the Friuli seismic area in the Southern Pre-Alps, close to the May 6, 1976, Friuli earthquake. Since the Rayleigh-wave and Love-wave spectra show essentially the same characteristics, only the spectra for Love waves are shown here. For the same structure, the spectra for Rayleigh waves can be found in Panza and Suhadolc (1987).

<table>
<thead>
<tr>
<th>Thickness (km)</th>
<th>Density (g/cm(^3))</th>
<th>P-wave velocity (km/s)</th>
<th>S-wave velocity (km/s)</th>
<th>( Q_\beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>2.00</td>
<td>1.50</td>
<td>0.60</td>
<td>20</td>
</tr>
<tr>
<td>0.06</td>
<td>2.30</td>
<td>3.50</td>
<td>1.80</td>
<td>30</td>
</tr>
<tr>
<td>0.20</td>
<td>2.40</td>
<td>4.50</td>
<td>2.50</td>
<td>100</td>
</tr>
<tr>
<td>0.70</td>
<td>2.40</td>
<td>5.00</td>
<td>2.90</td>
<td>200</td>
</tr>
<tr>
<td>2.00</td>
<td>2.60</td>
<td>6.00</td>
<td>3.30</td>
<td>400</td>
</tr>
<tr>
<td>3.50</td>
<td>2.60</td>
<td>6.20</td>
<td>3.45</td>
<td>400</td>
</tr>
<tr>
<td>4.50</td>
<td>2.60</td>
<td>6.00</td>
<td>3.35</td>
<td>100</td>
</tr>
<tr>
<td>10.00</td>
<td>2.60</td>
<td>5.50</td>
<td>3.30</td>
<td>50</td>
</tr>
<tr>
<td>3.50</td>
<td>2.60</td>
<td>6.00</td>
<td>3.50</td>
<td>100</td>
</tr>
<tr>
<td>2.50</td>
<td>2.75</td>
<td>6.50</td>
<td>3.75</td>
<td>400</td>
</tr>
<tr>
<td>2.50</td>
<td>2.80</td>
<td>7.00</td>
<td>3.85</td>
<td>400</td>
</tr>
<tr>
<td>7.50</td>
<td>2.80</td>
<td>6.50</td>
<td>3.75</td>
<td>100</td>
</tr>
<tr>
<td>4.00</td>
<td>2.85</td>
<td>7.00</td>
<td>3.85</td>
<td>200</td>
</tr>
<tr>
<td>3.00</td>
<td>3.20</td>
<td>7.50</td>
<td>4.25</td>
<td>400</td>
</tr>
<tr>
<td>1.50</td>
<td>3.40</td>
<td>8.00</td>
<td>4.50</td>
<td>400</td>
</tr>
<tr>
<td>( \infty )</td>
<td>3.45</td>
<td>8.20</td>
<td>4.65</td>
<td>400</td>
</tr>
</tbody>
</table>

**Table 2.1.** Structure FRIUL\(7\)A. Parameter \( Q_\alpha \) is taken as 2.5 \( Q_\beta \).

#### 2.8.1. Phase velocities

The dispersion curves for the first 154 Love modes are shown in Figure 2.3. For S-wave velocities less than 3.35 km/s the modes are well separated. This velocity corresponds to the S-wave velocity in the upper part of the crustal low-velocity zone (LVZ). Modes situated in the part of the spectrum below 3.35 km/s therefore sample the part of the crust above the uppermost LVZ.
Figure 2.3. (a) Love-wave dispersion curves for structural model FRIUL7A. Mode numbering: 0 for the fundamental mode, 1 for the first higher mode, 2 for the second higher mode, and so on up to 153. (b) Enlarged portion (modes 6-153) of (a) showing the effect of low-velocity waveguides.
In the part of the spectrum with higher phase velocities the dispersion curves are packed together. An enlarged portion of this part is presented in Figure 2.3b. Since two LVZ are present in the structural model, areas are seen where the higher Love-wave modes decompose into families of low-velocity channel waves and families of waves propagating in the upper crust. They appear in the dispersion curves as an apparent continuity of the phase velocities between adjacent modes. This mode-to-mode continuation leads to the identification of a family of waves. Each member of a wave family begins with one of the Love-wave modes and contains segments of all successive higher modes. They have almost continuous phase velocities, broken only at the points of near-osculation. The segments of members of the family of upper-crustal waves form curves that appear to be continuous, and which sometimes seem to intersect the more horizontal trending family of the channel-wave curves. A member of the family of upper-crustal waves can be identified at a frequency of about 4 Hz in the phase velocity range 3.35-3.45 km/s.

Another type of apparent continuity, of the phase velocities for adjacent modes, is related to structural layering (for example at a phase velocity of about 4.25 km/s). Such parts of the spectrum represent refracted waves at strong contrasts in the elastic parameters. These refracted waves are characterized by phase velocities which tend to become constant with increasing frequency.

### 2.8.2. Group velocities

The group velocity spectrum is presented in Figure 2.4. Due to the complexity of the pattern, it has been divided into two parts. Modes with group velocities less than about 2.8 km/s correspond to waves propagating in the low-velocity sediments. In the part of the spectrum where group velocities are in the interval 2.8-3.2 km/s, several higher modes form stationary phases. The stationary phases correspond to families of waves propagating in the upper crust and are characterized by the same type of mode-to-mode continuation as in the phase velocity curves. They can be interpreted as the high-frequency equivalent of Lg phases (Schwab and Knopoff, 1972; Knopoff et al., 1973; Panza and Calcagnile, 1975), which are propagating in the upper part of the continental crust.

The flat portions of group velocity curves, which are formed by a large number of higher modes at about 3.35 km/s (Figure 2.4a) and 3.75 km/s (Figure 2.4b), correspond to waves propagating in the upper and lower channels. One of the most attractive aspects of the phase and group
velocity spectra presented here is the possibility of identifying particular arrivals, such as refracted waves, waves propagating in the sediments, Lg waves, or the fundamental mode.

Figure 2.4. Love-wave group velocities for structure FRIUL7A. The spectrum is divided into two parts: (a) Love modes 0-30, and (b) Love modes 31-153.
2.8.3. Quality factor $Q_x$

The phase attenuation $C_2$ of the SH modes is related to the quality factor $Q_x$ by the approximate relation

$$1/Q_x = 2 C_1 C_2,$$

(Schwab and Knopoff, 1972). The quality factor is presented in Figure 2.5. Its values are very low in the sedimentary layers. Modes that propagate mainly in these layers are therefore characterized by low $Q_x$ values (Figure 2.5a). This is the case for the first few modes, especially the fundamental and first higher mode. The effect of layering on $Q_\beta$ can be observed for several adjacent modes that have almost constant $Q_x$, for example $Q_x$ close to 65. The resulting $Q_x$ value is close to the $Q_\beta$ of that layer, which is mainly sampled by the eigenfunction.

2.8.4. Energy integral

The energy integral serves as an inverse estimate of the contribution of the different modes to the surface displacement. In general, neglecting the influence of source depth on the excitation of different modes, small values of the energy integral correspond to large surface displacements. Over the entire frequency range, the fundamental mode has the lowest values of the energy integral $I_{1\alpha}$ (Figure 2.6). For a shallow source, the fundamental mode generally dominates the surface displacement.

The mode-to-mode continuations in the lower part of the energy integral curves (Figure 2.6a) correspond to the high frequency equivalent of $L_g$ waves. The low values of the energy integral indicate that these waves give rise to significant amplitudes at the surface. By definition, most of the energy of channel waves is concentrated within the channel; therefore, the energy integrals of these families, seen in the upper part of Figure 2.6a, take higher values than those for upper-crustal waves. For a given member of this family, the maximum displacement in the low-velocity zone becomes larger relative to the displacement at the free surface, with increasing frequency. Thus, the energy integral of this member is characterized by values that increase with frequency. This can be seen in the general pattern of the upper part of Figure 2.6a.
Figure 2.5. Love-wave quality factor $Q_x$ for structure FRIUL7A. The spectrum is divided into two parts: (a) Love modes 0-30, and (b) Love modes 31-153.
Figure 2.6. Love-wave energy integral $I_{1L}$ for structure FRIUL7A. The spectrum is divided into two parts: (a) Love modes 0-30, and (b) Love modes 31-153.
3. Finite difference method

3.1. Introduction

The finite difference method is one of the techniques for computing wave propagation in two- or three-dimensional media (e.g. Alterman and Karal, 1968; Boore, 1972; Kelly et al.; 1976; Aki and Richards, 1980). The differential equations are replaced by a set of recursive or explicit finite difference equations. The space is discretized by a regular grid. At each point of the grid the wavefield is updated in subsequent small time steps.

Two techniques can be used to compute the space derivatives in the differential equations. The first is the so-called Fourier or pseudo-spectral method, introduced by Gazdag (1981) and by Kosloff and Baysal (1982). In this method, the space derivatives are computed in the wavenumber space, in which differentiation becomes equivalent to a multiplication by \( ik \), where \( i \) is the imaginary unit and \( k \) is the wavenumber. For each time step, two Fourier-transformations are required for each line and row of the finite difference grid. For accurate modelling, only a few grid points per wavelength (four or five grid points) are required; and this allows a three-dimensional modelling of wave propagation (e.g. Johnson, 1984; Reshef et al., 1988a; 1988b). The second technique used to obtain the space derivatives is an approximation with finite differences (e.g. Alterman and Loewenthal, 1970; Boore, 1972; Korn and Stöckl, 1982; Virieux, 1986). This technique is called the explicit finite difference method, and it requires ten or more grid points per wavelength (e.g. Alford et al., 1974). This limits the maximum size of the structural model, but allows the modelling of complicated and rapidly varying velocity structures, as it is required when dealing with sedimentary basins. In our computations, the explicit finite difference method is used.

The discretization introduces two effects that have to be taken care of. The first is aliasing in time and space, which can be avoided by applying a low-pass filter to the incident wavefield. The second effect is the dependence of the accuracy of the finite difference method on the sampling interval of the seismic wavefield in space and time. Thus, the spatial accuracy depends on how many grid points there are per seismic wavelength. At short wavelengths, the phase velocity of body waves becomes a function of frequency, a phenomenon known as grid dispersion. Accuracy can be improved by a finer grid in space or by passing from a low-order finite difference scheme to a high-order scheme (e.g. Bayliss et al., 1986; Levander, 1985; 1988).
The biggest advantage of the finite difference method is the fact that it is not restricted to velocity variations occurring over distances similar to the seismic wavelength. This is the regime where the scattering effects can be most severe. Another advantage of the finite difference method is the fact that it is easy to program; many problems can be solved with only minor alterations. A convenient feature is the possibility of making snapshots of the wavefield. This is an excellent tool for illustrating the types of waves generated by the heterogeneities of a structure. The limitations of the technique are the computer speed, cost of CPU time, and available memory. Limited computer memory requires the introduction of artificial boundaries limiting the finite difference grid in space. This is one of the most severe problems in finite difference methods and it is discussed in Chapter 4.

In this chapter, the explicit finite difference techniques for SH- and P-SV-wave propagation in two-dimensional media will be introduced and discussed. Anelastic effects are included in the numerical algorithms to provide a realistic modelling of wave propagation in sedimentary basins.

### 3.2. Finite difference scheme for SH waves

In SH-wave computation, an explicit finite difference scheme is used which is based on the formulation of Korn and Stöckl (1982). In this section, the finite difference approximation is first formulated for the equations of motion in homogeneous, elastic media, and it is then extended to a formulation for two-dimensional, heterogeneous media. The starting point is the equation of motion for SH waves which are propagating in the xz plane of a homogeneous medium:

\[
\rho \frac{\partial^2 v}{\partial t^2} = \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right],
\]

where \( \rho \) is the density, \( \mu \) is the shear modulus and \( v \) is the SH-displacement. Equation (3.1) can be approximated by a finite difference equation with a truncation error of second-order (Boore, 1970; Aki and Richard, 1980). The time step of integration is \( \Delta t \), and the space is discretized by a regular grid (see Figure 3.1).
Thus,
\[
\frac{v_{m,n}^{t+\Delta t} - 2v_{m,n}^{t} + v_{m,n}^{t-\Delta t}}{\Delta t^2} = \mu \frac{v_{m+1,n}^{t} - 2v_{m,n}^{t} + v_{m-1,n}^{t}}{\Delta x^2} + \mu \frac{v_{m,n+1}^{t} - 2v_{m,n}^{t} + v_{m,n-1}^{t}}{\Delta z^2}.
\]

(3.2)
The following notation is used: \(v_{m,n}^{t+\Delta t} = v(m\Delta x, n\Delta z, t + \Delta t)\)

The displacement and the material properties are defined on the regular grid shown in Figure 3.1. To pass from a homogeneous to a heterogeneous medium, the conditions for the continuity of stress and displacement have to be fulfilled at any interface. First, the continuity conditions will be applied to only one interface at depth \(z=a\) (Figure 3.1). The generalization to a heterogeneous medium will then be straightforward. The boundary conditions at the interface at \(z=a\) are as follows

\[
\mu^{(1)} \frac{\partial}{\partial z} v^{(1)}|_{z=a} = \mu^{(2)} \frac{\partial}{\partial z} v^{(2)}|_{z=a}.
\]

(3.3)

The superscripts (1) and (2) denote media 1 and 2. The interface is located between the two grid lines at \(N\) and \(N+1\). A line of fictitious points is then added on either side of this interface (Figure 3.1); the quantities defined at the fictitious points are denoted by a tilde. These fictitious points are introduced to fulfill the boundary conditions, but they do not enter into the final finite-difference scheme. The boundary conditions (3.3) are approximated as follows: for the stress by central differences with respect to the interface, and for the displacement by a linear interpolation.

\[
\mu^{(1)} \frac{v_{m,N+1}^{t} - v_{m,N}^{t}}{\Delta z} = \mu^{(2)} \frac{v_{m,N+1}^{t} - v_{m,N}^{t}}{\Delta z}.
\]

(3.4)

\[
\frac{1}{2}(v_{m,N+1}^{t} + v_{m,N}^{t}) = \frac{1}{2}(v_{m,N+1}^{t} + v_{m,N}^{t}).
\]
Equations (3.4) can be solved for the unknown fictitious points:

$$
\varphi^t_{m,N} = \frac{2\varphi^t_{m,N} + (G - 1) \varphi^t_{m,N+1}}{G + 1}
$$

$$
\varphi^t_{m,N+1} = \frac{2\varphi^t_{m,N+1} + (G^{-1} - 1) \varphi^t_{m,N}}{G^{-1} + 1}; \text{ with } G = \frac{\mu^{(2)}}{\mu^{(1)}}
$$

where $G$ is the so-called weight parameter. We consider now a completely heterogeneous medium with density $\rho(x,z)$ and shear modulus $\mu(x,z)$. We assume that each grid point $(m,n)$ is located in the centre of a homogeneous rectangular region. Between two neighboring points, there is an interface at which the continuities of stress and displacement have to be satisfied. For a heterogeneous medium, the finite difference formulation (3.2) can be written as follows

$$
\rho_{m,n} \frac{\varphi^t_{m,n} + \Delta t - 2\varphi^t_{m,n} + \varphi^t_{m,n} - \Delta t}{\Delta t^2} = \mu_{m,n} \frac{\varphi^t_{m+1,n} - 2\varphi^t_{m,n} + \varphi^t_{m-1,n}}{\Delta x^2} + \mu_{m,n} \frac{\varphi^t_{m,n+1} - 2\varphi^t_{m,n} + \varphi^t_{m,n-1}}{\Delta z^2}
$$

Figure 3.1. Discretization of the medium for the case of SH wave propagation. The concept of fictitious grid points is discussed in the text.
Fictitious points are used for all neighboring points of the point at position \( (m,n) \). The displacements at the fictitious points can be expressed in terms of the displacements at real points by inserting Equations (3.5). Equation (3.6) then becomes

\[
\begin{align*}
v_{m,n}^{t+\Delta t} &= -v_{m,n}^{t-\Delta t} + 2v_{m,n}^t + 2\frac{\mu_{m,n}}{\rho_{m,n}} \left( \frac{\Delta t}{\Delta x} \right)^2 \left[ G_1 \left( v_{m+1,n}^t - v_{m,n}^t \right) \\
&+ G_2 \left( v_{m-1,n}^t - v_{m,n}^t \right) \right] \\
&+ 2\frac{\mu_{m,n}}{\rho_{m,n}} \left( \frac{\Delta t}{\Delta z} \right)^2 \left[ G_3 \left( v_{m,n+1}^t - v_{m,n}^t \right) \\
&+ G_4 \left( v_{m,n-1}^t - v_{m,n}^t \right) \right]
\end{align*}
\] (3.7)

with

\[
G_1 = \frac{\mu_{m+1,n}}{\mu_{m,n} + \mu_{m+1,n}}, \quad G_2 = \frac{\mu_{m-1,n}}{\mu_{m,n} + \mu_{m-1,n}}, \\
G_3 = \frac{\mu_{m,n+1}}{\mu_{m,n} + \mu_{m,n+1}}, \quad G_4 = \frac{\mu_{m,n-1}}{\mu_{m,n} + \mu_{m,n-1}}.
\]

This is a heterogeneous formulation, in which the elastic constants may vary from grid point to grid point. The truncation error of the numerical scheme (3.7) has leading terms proportional to \((\Delta x)^2\) and \((\Delta z)^2\) (e.g. Boore, 1970; Aki and Richards, 1980). A standard stability analysis of the scheme (O'Brien at al., 1950) indicates that the finite difference equation (3.7) is stable if

\[
\beta_{\text{max}} \Delta t \sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta z)^2}} \leq 1,
\] (3.8)

where \(\beta_{\text{max}}\) is the maximum shear wave velocity in the structural model. This equation (3.8) is valid for a heterogeneous medium. Its derivation is not presented here and can be found in Aki and Richards (1980).

### 3.2.1. Grid dispersion of the finite difference scheme for SH waves

The phase velocity of body waves in elastic media can become a function of frequency if there are only a few grid points per wavelength (e.g. Alford et al., 1974). In this section, this numerical error of the SH finite difference scheme will be derived for a homogeneous medium. From these calculations, the maximum error can be estimated for the case of a
heterogeneous medium. We consider plane S-wave propagation in a regular grid ($\Delta x=\Delta z$):

$$\tilde{u}(x, z, \omega) = \tilde{u}_0(\omega) e^{i(k_x \cos \theta + k_z \sin \theta - \omega t)} ,$$

where $\theta$ is the angle of the plane wave with respect to the x axis, $k$ is the wavenumber and $\omega$ is the angular frequency. If this is substituted into the second-order finite difference approximations of the operators $\partial^2/\partial x^2$, $\partial^2/\partial z^2$ and $\partial^2/\partial t^2$, they can be written in the frequency-wavenumber space as (e.g. Alford et al., 1974; Kindelan et al., 1990)

$$D^2(k_x) = -\frac{4}{\Delta x^2} \sin^2 \left( k_x \frac{\Delta x}{2} \right) = -\frac{4}{\Delta x^2} \sin^2 \left( k \frac{\Delta x}{2} \cos \theta \right)$$

$$D^2(k_z) = -\frac{4}{\Delta x^2} \sin^2 \left( k_z \frac{\Delta x}{2} \right) = -\frac{4}{\Delta x^2} \sin^2 \left( k \frac{\Delta x}{2} \sin \theta \right) \quad (3.9)$$

$$D^2(\omega) = -\frac{4}{\Delta t^2} \sin^2 \left( \frac{\omega \Delta t}{2} \right) .$$

The equation of motion for shear-wave propagation in a homogeneous medium is given by Equation (3.1) and can then be written in the wavenumber-frequency space as follows

$$D^2(\omega) = \beta^2 \left( D^2(k_x) + D^2(k_z) \right) , \text{ with } \beta = \sqrt{\frac{\mu}{\rho}} , \quad (3.10)$$

where $\beta$ is the shear wave velocity. Substituting $D^2(\omega)$, $D^2(k_x)$ and $D^2(k_z)$ with the expressions (3.9), we obtain

$$\frac{1}{\beta^2 (\Delta t)^2} \sin^2 \left( \frac{\omega \Delta t}{2} \right) = \frac{1}{(\Delta x)^2} \left( \sin^2 \left( k \frac{\Delta x}{2} \cos \theta \right) + \sin^2 \left( k \frac{\Delta x}{2} \sin \theta \right) \right) .$$

The phase velocity $c$, which is the velocity in the discretized medium, is given by $c=\omega/k$, and therefore,

$$c = \frac{2}{k \Delta t} \arcsin \left[ \frac{\beta \Delta t}{\Delta x} \sqrt{\sin^2 \left( \frac{\pi \cos \theta}{N} \right) + \sin^2 \left( \frac{\pi \sin \theta}{N} \right)} \right] . \quad (3.11)$$

Defining the nondimensional ratio $q$ between the phase velocity $c$ and the body-wave velocity as a function of the number of grid points per wavelength $N=\lambda/\Delta x$, Equation (3.11) becomes

$$q = \frac{c}{\beta} = \frac{N \Delta x}{\pi \beta \Delta t} \arcsin \left[ \frac{\beta \Delta t}{\Delta x} \sqrt{\sin^2 \left( \frac{\pi \cos \theta}{N} \right) + \sin^2 \left( \frac{\pi \sin \theta}{N} \right)} \right] .$$
The quantity $q$ describes the error of the phase velocity with respect to the (physical) body-wave velocity. This quantity is always less than 1, so that it decreases for small wavelengths and approaches 1 for large $N$. The biggest error is obtained for waves propagating in the x- or z-direction ($\theta=0^\circ$ and $\theta=90^\circ$).

If the time step $\Delta t$ is too large, the finite difference equation (3.7) becomes numerically unstable. For a homogeneous medium, the stability condition for the explicit second-order scheme is given by Equation (3.8). This relation is used to define the quantity $\gamma$. In a homogeneous medium and for a regular grid ($\Delta x=\Delta z$), Equation (3.8) becomes

$$\gamma = \sqrt{\frac{2}{\beta}} \frac{\Delta t}{\Delta x} \leq 1.$$  

The quantity $\gamma$ can be used as parameter to control the error in the phase velocity introduced by the discretization in space and time. The quantity $q$ can now be written as

$$q = \frac{\sqrt{2} N}{\pi \gamma} \arcsin \left[ \frac{\gamma}{\sqrt{2}} \sqrt{\sin^2 \left( \frac{\pi \cos \theta}{N} \right) + \sin^2 \left( \frac{\pi \sin \theta}{N} \right)} \right].$$  

Equation (3.12) illustrates that $q$ becomes independent on the physical phase velocity $\beta$, and it depends only on the number of grid points per wavelength. The dependence of $q$ on $H=1/N$ is presented in Figure 3.2 for two different angles $\theta$ ($0^\circ$ and $45^\circ$). The angle $\theta=0^\circ$ corresponds to the largest error in phase velocity. It can be seen that the grid spacing must be less than one tenth of the wavelength to obtain an error less than one percent. In other words, we need about ten points per wavelength for accurate modelling. In Figure 3.3, the error in the phase velocity is shown for different values of the quantity $\gamma$ ($\theta=0^\circ$). It turns out that $\gamma$ should be as close as possible to 1 to minimize the error. The use of values near the stability limit is also desirable from the viewpoint of minimizing computation time since small values of the quantity $\gamma$ correspond to a small time step of integration $\Delta t$.

In heterogeneous media, the smallest value of the shear wave velocity determines the grid spacing $\Delta x$, so that the error is limited to acceptable values. The largest value of the shear wave velocity determines the time step of integration $\Delta t$, which is chosen as close as possible to the stability limit. In heterogeneous media, the numerical error is not constant at all places in the structural model. It is smallest in the regions
which have the largest shear wave velocity, and it is largest in the regions with the lowest shear wave velocity (small $\gamma$ in Figure 3.3).

**Figure 3.2.** Error in the phase velocity for plane SH-wave propagation in a homogeneous medium, with a parameter $\gamma=0.95$. The error is given in the form of the nondimensional ratio $q$ between the phase velocity $c$ and the body-wave velocity $\beta$. The time derivative and space derivatives are approximated by second-order finite difference operators. $H$ is the inverse of the number of grid points per wavelength $N$. The quantity $q$ is computed for two angles $\theta (0^\circ$ and $45^\circ$).

**Figure 3.3.** As Figure 3.2, but using different values of the parameter $\gamma$ and for a fixed angle $\theta=0^\circ$. 
3.3. Finite difference scheme for P-SV waves

The two-dimensional P-SV finite difference scheme is based on the Madariaga-Virieux staggered grid formulation. Madariaga (1976) developed this scheme to model an expanding circular crack in an elastic medium. Virieux (1984; 1986) adapted it to the general forward modelling of SH and P-SV waves. The first-order hyperbolic system of equations of motion for P-SV waves and of the constitutive equations are as follows

\[
\begin{align*}
\frac{\partial \dot{u}}{\partial t} & = \frac{1}{\rho} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} \right) \\
\frac{\partial \dot{w}}{\partial t} & = \frac{1}{\rho} \left( \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} \right) \\
\frac{\partial \sigma_{xx}}{\partial t} + \frac{\partial \sigma_{zz}}{\partial t} & = (2\lambda + 2\mu) \left( \frac{\partial \dot{u}}{\partial x} + \frac{\partial \dot{w}}{\partial z} \right) \\
\frac{\partial \sigma_{zz}}{\partial t} - \frac{\partial \sigma_{xx}}{\partial t} & = 2\mu \left( \frac{\partial \dot{w}}{\partial z} - \frac{\partial \dot{u}}{\partial x} \right) \\
\frac{\partial \sigma_{xz}}{\partial t} & = \mu \left( \frac{\partial \dot{u}}{\partial z} + \frac{\partial \dot{w}}{\partial x} \right)
\end{align*}
\] (3.13)

where \( \dot{u} \) and \( \dot{w} \) are the horizontal and vertical velocity components, \( \rho \) is the density, and \( \lambda \) and \( \mu \) are the Lamé coefficients. The normal stresses are combined linearly to reduce the necessary computer memory requirements when anelasticity is introduced into the algorithm (see Section 3.4). The set of equations (3.13) contains no space and time derivatives of the material properties. Therefore, internal interfaces are represented simply by changes in elastic parameters and density on the staggered grid (Figure 3.4). The fact that the stress-strain relations are separated from the equations of motions allows anelasticity and anisotropy to be easily introduced into the algorithm.

The \( L \)th order approximation to the derivative operator with respect to \( x \) is given by

\[
\frac{\partial f}{\partial x} = \sum_{l=1}^{L/2} d_{2l-1} \frac{f \left[ x + (2l-1)\frac{\Delta x}{2} \right] - f \left[ x - (2l-1)\frac{\Delta x}{2} \right]}{\Delta x}
\] (3.14)

where \( L \) is the order of the operator (\( L=2,4,6,\ldots \)), and \( \Delta x \) is the spatial grid spacing. The coefficients \( d_{2l-1} \) can be chosen to design an optimum differentiator for a required relative error on the group velocity. Holberg
(1987) proposed a method which is based on maximizing the frequency band for which the error of the numerical scheme is bounded by a specific value. An alternative construction of the optimum operators was proposed by Kindelan et al. (1990). They simplified the computation of Holberg's operators and presented a closed form for the fourth-order operator.

In our computations, the error is not bounded by a specific value. Then the weights $d_{2i-1}$ are called the conventional, staggered finite-difference weights. They have also been used by Virieux (1986) for the second-order approximation ($L=2$ and $d_1=1$) and by Levander (1988) for the fourth-order approximation of the space differentiators ($L=4$, $d_1=9/8$ and $d_3=-1/24$). In these schemes, the components of the velocity and stress are defined at discrete locations on a staggered grid in space as shown in Figure 3.4. The horizontal velocity component and the density are defined at locations $(m, n+1/2)$, while the vertical velocity component and the density are defined at locations $(m+1/2, n)$. The normal stresses and Lamé coefficients are defined at locations $(m+1/2, n+1/2)$, while the tangential stress and rigidity $\mu$ are defined at locations $(m, n)$. The grid is also staggered in time. The velocities are computed at time $t$ from the stress components at time $t-\Delta t/2$, and the stresses are computed at time $t+\Delta t/2$ from the velocities at time $t$. The second- and fourth-order finite-difference schemes to the set of equations (3.13) are given by Virieux (1986) and Levander (1988). Grid dispersion and numerical error for these schemes are small in comparison with other finite differences schemes and are not very sensitive to the Poisson's ratio (Virieux, 1986; Stephen, 1988).

![Figure 3.4. Discretization of the medium on a staggered grid.](image-url)
Since the velocities at the surface of the models are usually smaller than at depth, the resulting numerical error increases with decreasing depth. Therefore, a fourth-order approximation to the spatial differential operators is used for the upper part of the structural model. This gives the possibility to reduce the spatial sampling required to accurately model wave propagation, or alternatively, it improves the accuracy of the solution for a given mesh size. For a given accuracy, it saves computer memory and computation time. The finite difference operator in time is always of second order, since a fourth-order approximation would require too much computer memory.

### 3.3.1. Grid dispersion of the finite difference scheme for P-SV waves

In the frequency space, the derivative operator \( \partial / \partial x \) is replaced by a multiplication with \( ik_x \), where \( i \) is the imaginary unit and \( k_x \) is the wavenumber. If we consider plane, harmonic wave propagation in a regular grid, the \( L^{th} \)-order finite difference approximation to the operator \( \partial / \partial x \) (Equation 3.14) can be written as (e.g. Alford et al., 1974; Kindelan et al., 1990)

\[
D(k_x) = \sum_{l=1}^{L} \text{id} \frac{\sin \left[ \frac{(2l-1)k_x \Delta x}{2} \right]}{\Delta x / 2},
\]

(3.15)

For analysing the stability and the numerical error, we assume a homogeneous medium. We first consider plane S-wave or P-wave propagation and a second-order finite difference scheme. Then, the problem is reduced to the acoustic two-dimensional wave equation which describes pure shear-wave or pure dilatational-wave propagation. Analogous to Equation (3.11), the phase velocity \( c \), which is the velocity in the discretized grid, is now given by

\[
c = \frac{2}{k \, \Delta t} \arcsin \left[ \frac{c_0 \, \Delta t}{\Delta x} \sqrt{\sin^2 \left( \frac{k \, \Delta x}{2} \cos \theta \right) + \sin^2 \left( \frac{k \, \Delta x}{2} \sin \theta \right)} \right],
\]

(3.16)

where \( c_0 \) is the (physical) phase velocity corresponding to the shear wave velocity \( \beta \) or to the dilatational wave velocity \( \alpha \) in the homogeneous medium. The phase velocity \( c \) depends only on the body-wave velocity \( c_0 \), both for pure shear-wave and compressional-wave propagation. Defining
the nondimensional ratio \( q = c / c_0 \) as a function of the number of grid points per wavelength \( N = \lambda / \Delta x \), Equation (3.16) becomes

\[
q = \frac{N \Delta x}{\pi c_0 \Delta t} \arcsin \left[ \frac{c_0 \Delta t}{\Delta x} \sqrt{\sin^2 \left( \frac{\pi \cos \theta}{N} \right) + \sin^2 \left( \frac{\pi \sin \theta}{N} \right)} \right]. \tag{3.17}
\]

If the time step \( \Delta t \) is too large, the difference approximation in Equation (3.14) gets numerically unstable. For a homogeneous medium, the stability condition for the explicit second-order scheme is (Virieux, 1986)

\[
\alpha \frac{\Delta t}{\sqrt{\Delta x^2 + (\Delta z)^2}} \leq 1. \tag{3.18}
\]

For the derivation of Equation (3.18), see Appendix B. The S-wave velocity does not enter into Equation (3.18). The quantity \( \gamma \), which controls the numerical error, is now defined for a regular grid as follows

\[
\gamma = \sqrt{2} \frac{\alpha \Delta t}{\Delta x} \leq 1. \tag{3.19}
\]

The quantity \( q \) for pure P-wave propagation \( q_p \) and for pure shear-wave propagation \( q_s \) can then be written as

\[
q_p = \frac{\sqrt{2} N}{\pi \gamma} \arcsin \left[ \frac{\gamma}{\sqrt{2}} \sqrt{\sin^2 \left( \frac{\pi \cos \theta}{N} \right) + \sin^2 \left( \frac{\pi \sin \theta}{N} \right)} \right],
\]

\[
q_s = \frac{\sqrt{2} N \alpha}{\pi \beta \gamma} \arcsin \left[ \frac{\beta \gamma}{\sqrt{2} \alpha} \sqrt{\sin^2 \left( \frac{\pi \cos \theta}{N} \right) + \sin^2 \left( \frac{\pi \sin \theta}{N} \right)} \right]. \tag{3.20}
\]

The quantities \( q_p \) and \( q_s \) represent the deviations from the respective theoretical (physical) phase velocities, and they are shown in Figure 3.5. The quantity \( q_p \) is computed for two different angles of \( \theta \). Once the quantities \( \gamma \) and \( N \) are fixed, the quantity \( q_p \) is determined, while the computations of \( q_s \) require the knowledge of the ratio between the phase velocities of shear waves and compressional waves, i.e. \( q_s \) depends on the Poisson's ratio. In Figure 3.5, the quantity \( q_s \) is shown for an angle \( \theta = 0^\circ \), which corresponds to the largest error in phase velocity, and for different Poisson's ratios. Since the S-wave velocity is always smaller than the P-wave velocity, the error for S-waves is always larger than the error for P-waves. To achieve reasonable accuracy for second-order finite-difference schemes, ten grid points per wavelength are required to limit the grid dispersion and the numerical error to a few percent (see Figure 3.5).
This staggered grid scheme is stable for all Poisson’s ratios, while \( q_s \) becomes infinite in most of the standard finite difference schemes (Stephen, 1988). In liquids, the set of equations (3.13) reduces to the acoustic wave equation and is also numerically stable. This can be seen from the behavior of \( q_s \) (Figure 3.5) which does not degenerate as the Poisson’s ratio approaches 0.5.

![Figure 3.5. Error in the phase velocity for plane P- and S-wave propagation in a homogeneous medium, with a parameter \( \gamma = 0.95 \). The error is given in the form of the nondimensional ratios \( q_p \) and \( q_s \) between the phase velocity \( c \) and the body-wave velocity. The time and space derivatives are approximated by a second-order operator. \( H \) is the inverse of the number of grid points per wavelength \( N \). The quantity \( q_p \) is computed for two different angles \( \theta \) of the plane wave with respect to the x-axis, whereas \( q_s \) is shown for different Poisson’s ratios, fixing the angle \( \theta \) to 0°.](image)

The same concept can be applied to study the error and stability conditions of the fourth-order space and second-order time, finite-difference scheme. This scheme is applied in the surface part of the model, and it allows a
larger grid spacing without an increase in numerical error. The response of the fourth-order space operator to a plane harmonic wave is

\[ D(k_x) = \frac{2i}{\Delta x} \cdot \sin \left( \frac{k \Delta x}{2} \cos \theta \right) + \frac{2i}{\Delta x} \cdot \sin \left( 3 \frac{k \Delta x}{2} \cos \theta \right) \]  

(3.21)

\[ D(k_x) \] has the same form as \( D(k_x) \), while the time derivative remains of second-order. Introducing \( D(k_x) \), \( D(k_z) \) and \( D(\omega) \) into Equation (3.10), under the assumption of a homogeneous medium, one gets

\[ q = \frac{N \Delta x}{\pi c_0 \Delta t} \cdot \arcsin \left( \frac{c_0 \Delta t}{\Delta x} \sqrt{a_1 + a_2 + a_3} \right) \]  

(3.22)

with

\[ a_1 = d_1^2 \left( \sin^2 \left( \frac{\pi \cos \theta}{N} \right) + \sin^2 \left( \frac{-\pi \sin \theta}{N} \right) \right) \]

\[ a_2 = d_3^2 \left( \sin^2 \left( \frac{3\pi \cos \theta}{N} \right) + \sin^2 \left( \frac{3\pi \sin \theta}{N} \right) \right) \]

\[ a_3 = 2d_1 d_3 \left( \sin \left( \frac{\pi \cos \theta}{N} \right) \sin \left( \frac{3\pi \cos \theta}{N} \right) + \sin \left( \frac{\pi \sin \theta}{N} \right) \sin \left( \frac{3\pi \sin \theta}{N} \right) \right) \].

The stability condition is given by

\[ \alpha \Delta t \left( |d_1| + |d_3| \right) \sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta z)^2}} \leq 1 \]  

(3.23)

For the derivation of Equation (3.23), see Appendix B. Then, the parameter \( \gamma \) for a regular grid is

\[ \gamma = \sqrt{2} \alpha \left( |d_1| + |d_3| \right) \frac{\Delta t}{\Delta x} \leq 1 \]  

Introducing \( \gamma \) into Equation (3.22), \( q_p \) and \( q_s \) can be written as

\[ q_p = \frac{\sqrt{2} N \left( |d_1| + |d_3| \right)}{\pi \gamma} \arcsin \left( \frac{\gamma}{\sqrt{2 \left( |d_1| + |d_3| \right)}} \sqrt{a_1 + a_2 + a_3} \right) \]

\[ q_s = \frac{\sqrt{2} N \alpha \left( |d_1| + |d_3| \right)}{\pi \beta \gamma} \arcsin \left( \frac{\gamma \beta}{\sqrt{2 \alpha \left( |d_1| + |d_3| \right)}} \sqrt{a_1 + a_2 + a_3} \right) \]  

(3.24)

The quantities \( q_p \) and \( q_s \) are shown in Figure 3.6. The spatial sampling requires at least 5 grid points per wavelength to keep the grid dispersion
and the numerical error within acceptable values. The error is reduced with respect to the second-order finite difference scheme. For the general discussion of the numerical error, including both P- and S-wave propagation, see Levander (1988).

**Figure 3.6.** Error in phase velocity for plane P- and S-waves, with the parameter $\gamma=0.95$. The error is given in the form of the nondimensional ratios $q_P$ and $q_S$ between the phase velocity $c$ and the body-wave velocity. The time derivative $\partial/\partial t$ is approximated by a second-order operator and the space derivatives $\partial/\partial x$ and $\partial/\partial z$ are approximated by a fourth-order operator. $H$ is the inverse of the number of grid points per wavelength $N$. The quantity $q_P$ is computed for two different angles $\theta$ of the plane wave with respect to the x-axis, whereas $q_S$ is shown for different Poisson's ratios, fixing the angle $\theta$ to $0^\circ$. 
3.4. Intrinsic attenuation

There is a lot of literature on theoretical and experimental studies of anelasticity. One of the most important questions discussed, is the physical mechanism for anelasticity. An interesting review has been given by Stacey et al. (1975, corrected by Savage (1976)) covering some questions about the non-linearity of the model for anelasticity, and the frequency dependence of body-wave velocity. From a microscopic point of view, there are different physical mechanisms which are important for anelasticity: for example, liquid saturation in rocks (Toksoz and Johnston, 1981), and scattering mechanisms in dry rocks due to porosity, to crystalline inhomogeneities and to partial lubrication of the interfaces between crystals by gas molecules (Schwab, 1988). On the macroscopic level scattering attenuation cannot be separated from anelastic effects (Wennerberg and Frankel, 1989). These two physical processes can be interpreted by identical equations. A frequency-dependent intrinsic attenuation is related to a distribution of relaxation mechanisms in exactly the same manner as the frequency dependence of scattering attenuation is related to the distribution of the size of the scatterers. Scattering attenuation may also be frequency independent, as demonstrated by numerical calculations of Frankel and Clayton (1986) for self-similar media.

In time-domain algorithms for wave propagation, such as in finite difference schemes, there are few studies that incorporate intrinsic absorption. This has its reason in the fact that absorption enters into the differential equations in the form of a convolution operator. An approximation of this convolution can be done by adding a damping term to the wave equation (Kosloff and Kosloff, 1986; Cerjan et al., 1985; Levander, 1985; Lysmer and Kuhlemeyer, 1969; Korn and Stöckl, 1982) or by introducing complex velocities (Faccioli and Tagliani, 1989). These algorithms cannot simulate realistic attenuation in the whole seismic frequency range (Carcione, 1987), but they have been successfully applied to reduce unwanted reflections from the artificial boundaries, which limit the finite difference grid in space. An alternative technique for introducing absorption is to convolve the finite difference synthetics of the elastic case with a time-varying Q-operator (Boore et al., 1971). This method yields good results, but is not appropriate for a space-dependent Q, and if the P-wave quality factor $Q_\alpha$ is not equal to the shear-wave quality factor $Q_\beta$ (Zahradnik et al., 1990).
Day and Minster (1984) proposed a method to incorporate attenuation into finite difference methods by approximation of the viscoelastic modulus with a low-order rational function of frequency. In this method, the convolution operator is replaced by a system of first-order differential equations, and the quality factor $Q$ can be approximated in a certain frequency band. Good results can be obtained by using the rheological model of the standard linear solid (Carcione, 1987) or the rheological model of the generalized Maxwell body (Emmerich and Korn, 1987). The technique which is based on the generalized Maxwell body was introduced into finite difference schemes for SH waves by Emmerich and Korn (1987). In this last part of the chapter, I intend to outline the method and extend it to the case of P-SV waves.

### 3.4.1. Basic theory

In perfectly linear elastic solids, according to Hooke's law, stress is proportional to strain. The mechanical energy is stored without dissipation. In viscous liquids, in accordance with Newton's law, the stress is proportional to the rate of strain. In this case the energy is completely dissipated. A combination of the mechanical properties of elastic solids and of viscous liquids can be used for modelling the earth's properties. Thus, in viscoelastic solids, the mechanical energy is partly stored and partly dissipated. Not all of the energy can be recovered, as is the case in purely elastic solids. For a one-dimensional solid, the relation between stress $\sigma$ and strain $\varepsilon$ in linear viscoelasticity can be written in the frequency domain as

$$\sigma(\omega) = M(\omega) \cdot \varepsilon(\omega) \quad ,$$

(3.25)

where $M(\omega)$ is the complex viscoelastic modulus. It describes the elastic and anelastic properties of the material. The symbol $M$ denotes the Fourier transform of the related time domain variable $M$. For infinitesimal deformations, the relation between the strain $\varepsilon$ and the deformation $u$ is given as follows

$$\varepsilon = \frac{\partial u}{\partial x} \quad .$$

Since the finite difference algorithms work in the time domain, Equation (3.25) has to be Fourier-transformed.

$$\sigma(t) = M(t) * \varepsilon(t) \quad (3.26)$$
The stress is the result of the convolution of the viscoelastic modulus $M$ with the strain $\varepsilon$. It is now useful to introduce the relaxation function $R(t)$, which is the response to a unit step in strain $H(t)$ (Figure 3.7). From Equation (3.26), with $\sigma(t) = R(t)$, we can write

$$R(t) = M(t) \ast H(t)$$

The viscoelastic modulus is then equal to the first time derivative of the relaxation function $R(t)$, and $R(t)$ can be written as (Emmerich and Korn, 1987)

$$R(t) = M_R + \delta M \int_0^\infty r(\omega) e^{-t\omega} d\omega$$

(3.27)

The mechanical system described by (3.27) has a rheology characterized by multiple relaxation mechanisms, with the relaxation frequencies $\omega$ and the relaxation spectrum $r(\omega)$, which is normalized

$$\int_0^\infty r(\omega) d\omega = 1.$$

The relaxed modulus $M_R$ and the unrelaxed modulus $M_U$ are defined as follows

$$M_R = \lim_{t \to \infty} R(t) \text{; and } M_U = \lim_{t \to 0} R(t) = M_R + \delta M.$$

The unrelaxed modulus $M_U$ provides the proportionality between stress and strain at that time at which the stress is applied, and before the
material starts to relax to some new configuration. For an isotropic elastic medium, $M_U$ and $M_R$ are equal and can be related to the shear modulus $\mu$ for transverse waves and to $\lambda+2\mu$ for longitudinal waves. Introducing $R(t)$ into the Boltzmann's after-effect equation (e.g. Christensen, 1982) given by

$$\sigma(t) = M_U \epsilon(t) + \int_{-\infty}^{t} \epsilon(\tau) R(t-\tau) \, d\tau,$$

the stress-strain relation takes the following form:

$$\sigma(t) = M_U \epsilon(t) - \delta M \int_{-\infty}^{t} d\tau \int_{0}^{\infty} r(\omega) e^{-(t-\tau)\omega} \epsilon(\tau) \, d\omega \quad (3.28)$$

In general the stress at a certain time $t$ depends on the strain at the same place and in the entire past. This is called the memory of the system. A linear elastic solid has no memory, because the stress depends only on the strain at the same time and place.

In numerical calculations, Equations (3.26) and (3.28) are difficult to handle because an integration at each space point and time step requires a large amount of computer time and memory. However, in the frequency domain, the modulus $M$ can be approximated as a rational function of $i\omega$:

$$M \approx M_n(\omega) = \frac{P_n(i\omega)}{Q_n(i\omega)}$$

with $P_n(i\omega) = \sum_{j=0}^{n} p_j (i\omega)^j$ and $Q_n(i\omega) = \sum_{j=0}^{n} q_j (i\omega)^j$.

Equation (3.25) can be rewritten as

$$\sigma(\omega) Q_n(i\omega) = P_n(\omega) \epsilon(\omega).$$

By Fourier-transformation one obtains:

$$\sum_{j=0}^{n} q_j \frac{d^j}{dt^j} \sigma(t) = \sum_{j=0}^{n} p_j \frac{d^j}{dt^j} \epsilon(t). \quad (3.30)$$

Equation (3.26) is now transformed into a convergent series of differential operators of increasing order and with constant coefficients. This equation can be written as a system of $n$ ordinary differential equations of first order, and can be solved by a finite difference algorithm (Emmerich and Korn, 1987). The order $n$ has to be limited due to the limitations in computer time and memory. Therefore, the approximation of the viscoelastic modulus is only possible in a certain frequency band.
3.4.2. The rheological model of the generalized Maxwell body

Equation (3.30) can now be related to viscoelastic media, where the standard linear solid is the most general linear equation in stress, strain and their first time derivatives. It is represented by the following stress-strain relation:

\[ \sigma + \tau_0 \dot{\sigma} = M_R (\varepsilon + \tau_\varepsilon \varepsilon) , \]

where \( \tau_0 \) is the stress relaxation time under constant strain and \( \tau_\varepsilon \) is the strain relaxation time under constant stress. A generalization of this type of the rheological model has been applied by Liu et al. (1976) who considered the superposition of 12 standard linear solids. They approximated a constant intrinsic attenuation \( Q \) over more than four decades of frequency.

An alternative to a superposition of standard linear solids is the generalized Maxwell body (Emmerich and Korn, 1987), shown in Figure 3.8. For the single Maxwell body the time derivative of the strain can be expressed as a function of the stress and its time derivative. The viscoelastic modulus for the generalized Maxwell body (Figure 3.8) can be written as a low-order rational function of frequency, with an expansion into partial fractions:

\[ M_n (i\omega) = M_R + \sum_{j=1}^{n} a_j \delta M \frac{i\omega}{i\omega + \omega_j} \]  

(3.31)

Each part of the sum represents a classical Maxwell body with viscosity \( a_j \delta M / \omega_j \) and elastic modulus \( a_j \delta M \) (Christensen, 1982). The relaxed modulus \( M_R \) is given by an additional elastic element. The frequencies \( \omega_j \) are called the relaxation frequencies of the Maxwell body. The modulus \( M_n \) is a causal system since \( M_n \) is analytic in the plane where \( i\omega \) is negative (Emmerich, 1986).

Equation (3.31) can be written as

\[ M_n (i\omega) = M_U - \delta M + \sum_{j=1}^{n} a_j \delta M \frac{i\omega}{i\omega + \omega_j} . \]

Without loss of generality, we can assume that \( \sum_{j=1}^{n} a_j = 1 \),

and \( \delta M \) can be written as

\[ \delta M = \sum_{j=1}^{n} a_j \delta M \frac{i\omega + \omega_j}{i\omega + \omega_j} . \]
The complex viscoelastic modulus in the frequency domain therefore becomes

\[ M_n(i\omega) = M_U - \sum_{j=1}^{n} a_j \delta \frac{\omega_j}{i\omega + \omega_j} \]  

(3.32)

Combining Equation (3.32) and (3.26), the stress-strain relation in the time-domain is given by

\[ \sigma(t) = M_U \left( \varepsilon(t) - \sum_{j=1}^{n} \Phi_j(t) \right) \]  

(3.33a)

(Emmerich and Korn, 1987), where each of the variables \( \Phi_j \) satisfies a differential equation of first order

\[ \dot{\Phi}_j(t) + \omega_j \Phi_j(t) = \frac{a_j \omega_j \delta M}{M_U} \varepsilon(t) \]  

(3.33b)

The total stress is the difference between the elastic term \( M_U \varepsilon(t) \) and the anelastic term given by the sum of the \( \Phi_j \). This stress-strain relation will be used in the computations, and it can be handled with a finite difference algorithm.
3.4.3. Body-wave dispersion and quality factor Q

The rheological model of the generalized Maxwell body will now be used to relate the viscoelastic modulus to the quality factor Q. For the finite difference computations, intrinsic attenuation will be restricted to a frequency independent quality factor. This is reasonable because a variety of different materials has to be covered with our viscoelastic model.

First we will outline the general formalism for treating anelasticity in wave propagation problems. We shall assume a plane wave $u(x,t)$ propagating in the positive $x$-direction. The wavefront is in $x=0$ at the time $t=0$. Thus,

$$u(x=0,t) = 0 \quad \text{for} \quad t < 0 .$$

With the convention of the linear wave theory, the wavefield can be expanded in plane wave components. Without loss of generality, only the propagation of one single plane wave can then be considered:

$$\bar{u}(x, \omega) = \bar{u}(0, \omega) \cdot e^{i(kx - \omega t)} ,$$

where $\omega$ and $k$ are real and describe the propagation without attenuation. Attenuation can be introduced by the factor $\exp[-\alpha x]$ with a real attenuation $\alpha(\omega)$:

$$\bar{u}(x, \omega) = \bar{u}(0, \omega) e^{-\alpha x} \cdot e^{i(kx - \omega t)} = \bar{u}(0, \omega) e^{i(Kx - \omega t)} , \quad (3.34)$$

where $K$ is the complex wavenumber, which can be written as

$$K = \frac{\omega}{c(\omega)} + i\alpha(\omega) . \quad (3.35)$$

The quantity $c(\omega)$ is the phase velocity which--for causality reasons and as consequence of the linear attenuating medium--has to be frequency dependent (e.g. Lomnitz, 1957; Futterman, 1962; Knopoff, 1964a). This is termed the body-wave dispersion. The quality factor $Q(\omega)$ and the attenuation $\alpha(\omega)$ are related by the following equation (Futterman, 1962):

$$Q(\omega) = \frac{\omega}{2c(\omega)\alpha(\omega)} \quad (3.36)$$

This approximation holds for $Q>>1$. An alternative expression for the wave slowness, respectively for Equation (3.35), was proposed by Schwab
and Knopoff (1972). The complex slowness, for example for shear waves, is given by

$$\frac{1}{\beta} = \frac{1}{B_1} + iB_2$$  \hspace{1cm} (3.37)

where $B_1$ is the frequency-dependent phase velocity $c(\omega)$ and $B_2$ is the phase attenuation, which is equal to the damping factor $\alpha$ divided by the angular frequency $\omega$. The sign of the imaginary part is opposite to the convention used by Schwab and Knopoff (1972). This depends on the sign of the propagation term in Equation (3.34). In a medium with constant intrinsic attenuation, causality requires a frequency-dependent phase velocity. One possible approximation of the phase velocity, which fulfils causality, has been proposed by Futterman (1962). In a medium with constant, frequency independent $Q_\beta$, the phase velocity $B_1$ can be expressed by

$$B_1(\omega) = \frac{B_1(\omega_0)}{1 - \frac{2}{\pi} B_1(\omega_0) B_2(\omega_0) \ln \frac{\omega}{\omega_0}}$$ \hspace{1cm} (3.38)

where $B_1(\omega_0)$ and $B_2(\omega_0)$ are the S-wave phase velocity and the S-wave phase attenuation at the reference angular frequency $\omega_0$. $B_2$ can now be expressed by the following approximation

$$B_2 = \frac{1}{2 B_1(\omega) Q_\beta} \approx \frac{1}{2 B_1(\omega_0) Q_\beta}$$ \hspace{1cm} (3.39)

where $Q_\beta$ is the quality factor for shear waves. The empirical relations (3.38) and (3.39) have been applied in our modal summation (Panza, 1985; Florsch et al., 1991) and in the Haskell technique (Haskell, 1960). The reference angular frequency has been chosen as $2\pi$ s$^{-1}$ radians. Introducing (3.38) and (3.39) into Equation (3.37) leads us to the following expression for the complex velocity $\beta(\omega)$

$$\beta(\omega) = \frac{B_1(\omega_0)}{1 - \frac{1}{\pi Q_\beta} \ln \frac{\omega}{\omega_0} + \frac{i}{2Q_\beta}}$$ \hspace{1cm} (3.40)

Schwab (1988) showed, that this Futterman's type of model for anelasticity can be a reasonable approximation to the development of physical models
for microscopic scattering mechanisms. The same formalism can also be applied to compressional-wave propagation.

Another possibility to fulfill causality has been proposed by Azimi et al. (1968). They define the attenuation $\alpha(\omega)$ by the following equation

$$\alpha(\omega) = \frac{\alpha_0 \omega}{1 + \alpha_1 \omega},$$

where $\alpha_0$ and $\alpha_1$ are two constants. This approximation makes $Q$ effectively constant over the seismic frequency band. For pure shear-wave propagation, the complex velocity, with first-order corrections in $Q_\beta$ is given by (e.g. Aki and Richards, 1980; Kennett, 1983)

$$\beta(\omega) = B_1(\omega_0) \left( 1 + \frac{1}{\pi Q_\beta} \ln \frac{\omega}{\omega_0} - \frac{i}{2Q_\beta} \right).$$

This approximation has also been applied in the Haskell technique for a comparison with the approximation (3.40). For the body-wave dispersion of compressional waves see Kennett (1983).

The difference of Azimi's law and Futterman's law has its origin in the choice of whether to use the relaxed or the unrelaxed modulus as the "non-dispersive" modulus. Futterman (1962) chooses his non-dispersive behavior at zero frequency, whereas Azimi et al. (1968) choose it at infinite frequency. Recalling that for $|x| < 1$ $(1 + x)^{-1} = 1 - x$,

expressions (3.41) and (3.40) are practically equal.

Synthetic signals are very sensitive to the quality factor, and an important effect of intrinsic attenuation is the dependence of the seismic velocities on frequency. This is shown in the numerical examples in Figure 3.9, where different wavelets are compared which have passed a layer of 2.0 km thickness ($\rho=2.0 \text{ g/cm}^3$ and $\beta=1.0 \text{ km/s}$). The linear elastic case is compared with the case of constant intrinsic attenuation ($Q_\beta=20$), defined by Azimi's law (Equation (3.41)) and by Futterman's law (Equation (3.40)), for two reference angular frequencies $\omega_0$ ($2\pi \text{ s}^{-1}$ and $200\pi \text{ s}^{-1}$ radians). The signals have been computed with the Haskell method. The two attenuation laws produce no difference in the signal shape if the dominating frequency of the pulse is close to the reference frequency of the attenuation laws (here, the case of $\omega_0=2\pi \text{ s}^{-1}$ radians). In the other case ($\omega_0=200\pi \text{ s}^{-1}$ radians), small differences occur in the peak amplitudes of
the wavelets. This is due to the fact that the series expansion which makes (3.40) equal to (3.41) is not valid anymore. Moreover, the definition of the reference frequency strongly influences the phase velocity of a wavelet in a medium with a low quality factor, and it leads to differences in amplitudes and arrival times for different reference frequencies.

Figure 3.9. Comparison between different pulse shapes for waves in elastic and anelastic media. Body-wave dispersion is defined by Azimi’s law (1968) and by Futterman’s law (1962). The reference angular frequency is set equal (a) to $2\pi$ s$^{-1}$ radians and (b) to $200\pi$ s$^{-1}$ radians.

The quality factor $Q$ is related to the viscoelastic modulus $M$ by the following equation (e.g. Futterman, 1962):

$$\frac{1}{Q(\omega)} = \frac{\text{Im } M(\omega)}{\text{Re } M(\omega)}$$

(3.42)

In order to fulfil causality, the real and imaginary part of the viscoelastic modulus must obey the Kramers-Krönig relation. In a linear theory, this relation determines the real part of a complex filter-function from its
values of the imaginary part summed over the entire range of frequencies. Consequently, the viscoelastic modulus is uniquely determined by a given attenuation law $Q(\omega)$. Introducing the viscoelastic modulus of the generalized Maxwell body (Equation (3.32)) into Equation (3.42), gives the following relation

$$Q^{-1}(\omega) = \frac{\text{Im}(M(\omega))}{\text{Re}(M(\omega))} = \frac{\sum_{j=1}^{n} a_j \frac{\delta M}{\omega_j}}{\frac{\omega}{\omega_j}} \frac{1}{1 + \left(\frac{\omega}{\omega_j}\right)^2} - \frac{M}{M_U}. \quad (3.43)$$

An arbitrary $Q(\omega)$-law can be approximated with the rheological model of the generalized Maxwell body, by using appropriately the parameters $n$, $\delta M$, $a_j$ and $\omega_j$ in Equation (3.43). The weight factors $a_j \delta M/M_U$ are determined by fitting numerically a $Q(\omega)$-curve to Equation (3.43), using the least squares technique and the Gaussian elimination method (e.g. Fröberg, 1985). The number $n$ of Maxwell bodies and the relaxation frequencies $\omega_j$ can be chosen to approximate the $Q(\omega)$-law over the frequency band of interest. To make this choice in an optimized way, we first discuss an approximation of Equation (3.43). By assuming $\delta M \ll M_U$, the intrinsic attenuation is taken as a second-order effect and the denominator of (3.43) reduces to $M_U$. This is equivalent to $M_U \approx M_R$. Equation (3.43) becomes

$$Q^{-1}(\omega) = \frac{\delta M}{M_U} \sum_{j=1}^{n} a_j \frac{\omega}{\omega_j} \frac{1}{1 + \left(\frac{\omega}{\omega_j}\right)^2}. \quad (3.44)$$

This equation shows that $Q^{-1}$ is approximately the sum of $n$ Debye functions with maxima $a_j \delta M/2M_U$ located at the relaxation frequencies $\omega_j$ (Emmerich, 1986). The Debye functions are symmetric on a logarithmic scale and have a half-width of 1.444 decades. Therefore, the relaxation frequencies $\omega_j$ should be chosen equidistant in the logarithmic scale. The smaller the distance between adjacent relaxation frequencies the better is the approximation of the attenuation law. Good results can be obtained by setting the distance in logarithmic scale between two adjacent frequencies to 0.8. For practical computations it is crucial to keep the number of classical Maxwell bodies as low as possible, because each of them will give a differential equation of the form (3.33b). The high-frequency parts of a wavetrain will undergo more energy loss for a certain travel distance
than do the low-frequency parts, and therefore, the approximation to the Q-law should be better at high frequencies. The largest relaxation frequency is chosen to be the upper frequency limit of the computation (from 1 Hz up to 10 Hz). The number n of Maxwell bodies has been chosen in this work to be 4 in the SH case and 3 in the P-SV case in order to optimize both accuracy and computation time.

The numerical curve fitting is performed for a set of frequencies \( \omega_i \) for which \( Q(\omega_i) \) is known. A good choice for the \( \omega_i \) is to use the relaxation frequencies \( \omega_j \), and for symmetry reasons, the intermediate frequencies between two adjacent relaxation frequencies. This is a set of \( 2n-1 \) equations.

The implementation of intrinsic attenuation for SH-wave and P-SV-wave propagation is now straightforward. The stress-strain relation derived for the rheological model of the generalized Maxwell body is introduced into the equation of motion. For the case of SH waves one complex modulus is necessary to describe shear-wave propagation, whereas for P-SV waves two complex moduli are necessary, which describe the dilatational and shear behavior of the medium. The numerical schemes are given in Appendix C. In the seismic frequency band, \( Q \) can be taken to be frequency independent or only slowly varying with frequency (Liu et al., 1976). Therefore, for all numerical examples, the quality factor \( Q \) has been chosen frequency independent. This choice is not a restriction of the method itself.

The numerical scheme has been tested for SH-wave propagation. The structural test-model is a half-space, defined by the shear wave velocity \( \beta_0 = 1.0 \text{ km/s} \), density \( \rho = 2.0 \text{ g/cm}^3 \), and a constant quality factor \( Q_\beta = 20 \). The incident waves are sinusoidal, plane polarized shear waves which are propagating in the positive x-direction. By measuring the amplitudes of these waves at two observation points \( x_1 \) and \( x_2 \), the quality factor can be redetermined from the amplitudes \( A(x,t) \):

\[
Q(\omega) = \frac{\omega (x_2 - x_1)}{2\beta_0 \ln \left( \frac{\max [A(x_1, t)]}{\max [A(x_2, t)]} \right)}
\]

This equation can be derived from Equations (3.34) and (3.36). The quality factor has been redetermined for some discrete frequencies and can be compared with the simulated quality factor of the structural model. The
The result of this comparison is shown in Figure 3.10. The comparison indicates that our numerical scheme gives very satisfactory results. The quality factor is effectively constant within 5% error and over the range 0.04 to 10 Hz. We can expect, that the relative error in the approximation of the quality factor decreases with increasing Q values.

Such tests with sinusoidal, plane polarized body-waves are difficult to perform in the case of P-SV waves. This has its reason in the reflection properties of the plane P- or S-waves at the free surface (e.g. Aki and Richards, 1980). It is not possible to generate a single body-wave, because P- or S-waves interact with the free surface and lead to the formation not only of the other type of body wave but also of surface waves. The same problems occur at the artificial boundaries. A test of the finite difference scheme for P-SV waves can only be performed by comparison with the results from other computational techniques, as it is shown in Chapter 5.

![Figure 3.10](image)

**Figure 3.10.** Inverse of the quality factor Q of the generalized Maxwell body as a function of frequency (continuous line). Q⁻¹ values determined from numerical experiments are shown by stars.

The phase velocity $B_1$ in a medium, characterized by the rheological model of the generalized Maxwell body, can be determined from Equation (3.32) and takes the following form:

$$\frac{1}{B_1(\omega)} = \text{Re} \sqrt{\frac{\rho}{M}} \approx \text{Re} \left[ \frac{\rho}{M_U - \sum_{j=1}^{n} a_j \delta M \frac{\omega_j}{i\omega + \omega_j}} \right].$$ (3.46)
The phase velocity \( B_1(\omega) \) obtained with our rheological model can now be compared with the body-wave dispersion obtained with Azimi's law and with Futterman's law (Figure 3.11). The shear wave velocity in the medium is 1.0 km/s, and the theoretical quality factor is taken to be 20. The number \( n \) of Maxwell bodies is set to 4, with a distance of 0.8 in logarithmic scale between two adjacent relaxation frequencies. The largest relaxation frequency is chosen to be 10 Hz. In the frequency range from 0.04 to 10 Hz, limited by the smallest and largest relaxation frequency, the phase-velocity curve of the generalized Maxwell body has a frequency dependence like Azimi's law. The same conclusion was drawn by Liu et al. (1976) for the superposition of standard linear solids. Considering a continuous superposition of standard linear solids, specified by a density function, they demonstrated that the phase velocity has a linear dependence on \( \ln(\omega) \) like Azimi's law. For the entire seismic frequency band, mechanical models of rheology lead to a dependence in the angular frequency like in Azimi's law, whereas Futterman's law cannot be connected with these mechanical models. The good agreement of the rheological model of the generalized Maxwell body with other laws of attenuation confirms its reliability in numerical simulation of wave propagation in anelastic media.

**Figure 3.11** Comparison between the body-wave dispersion obtained for the generalized Maxwell body, Futterman’s law and Azimi’s law. For each of the two laws, two curves are shown for the reference angular frequency equal to \( 20\pi \) s\(^{-1} \) and equal to \( 200\pi \) s\(^{-1} \) radians, respectively.
4. The hybrid technique

4.1. Coupling of the mode summation method to the finite difference technique

The hybrid technique combines modal summation and the finite difference method, and it can be used to study wave propagation in sedimentary basins. Each of the two techniques is applied in that part of the structural model where it works most efficiently: the finite difference method in the laterally heterogeneous part of the structural model which contains the sedimentary basin (see Figure 4.1), and modal summation is applied to simulate wave propagation from source position to the sedimentary basin of interest. The advantage of this hybrid technique, in comparison with other computational methods, is that it allows to take into consideration source, path, and local soil effects. This hybrid approach allows us to calculate the local wavefield from a seismic event, both for small (a few kilometers) and large (a few hundreds of kilometers) epicentral distances. The technique combines the advantages of both modal summation and finite difference technique. The use of the mode summation method allows us to include an extended source, which can be modelled by a sum of point sources appropriately distributed in space. This allows the simulation of a realistic rupture process on the fault. The path from source position to the sedimentary basin can be approximated by a structure composed of flat, homogeneous layers. Modal summation then allows the treatment of many layers which can take into consideration low-velocity zones and fine details of the crustal section under consideration. The finite difference method, applied to treat wave propagation in the sedimentary basin, permits the modelling of wave propagation in complicated and rapidly varying velocity structures, as is required when dealing with sedimentary basins. The coupling of the two methods is carried out by introducing the resulting time series obtained with modal summation into the finite difference computations. In the SH computations the displacements are used as input in the finite difference calculations, whereas in the P-SV case the input consists of the velocity time series, in relation to the specific finite difference techniques used for SH and P-SV waves.

Following Alterman and Karal (1968), the finite difference algorithm can be designed in such a way that the two vertical grid lines, where the wave field is introduced, are transparent to any reflection from the laterally heterogeneous part of the model. For SH waves, this can be realized in the following way: at each step of integration in time, the finite
difference algorithm uses the present and the past displacement to compute the future displacement (see Section 3.2). During each time step, energy is inserted at the grid points on the grid line $S_1$ (Figure 4.2) by the incoming wavefield, corresponding to the analytical solution obtained with the mode summation method, and by the backscattered waves from the laterally heterogeneous part of the model. Assuming $v_{m,n}(t)$ to be the displacement at the time $t$ of the point $(m,n)$ on the grid line $S_1$ (Figure 4.2), it can be written as the superposition of two wavefields

$$v_{m,n}(t) = v'_{m,n}(t) + r_{m,n}(t),$$

where $v'_{m,n}(t)$ is the contribution of the analytical solution and $r_{m,n}(t)$ is due to the backscattered waves. The displacements $v_{m+1,n}(t+\Delta t)$ at the position $m+1$ on the grid line $S_2$ can be computed with the usual finite difference scheme.

**Figure 4.1.** Geometry of the problem.
We know the contribution of the analytical solution $v'_{m+1,n}(t+\Delta t)$ on the grid line $S_2$, and therefore, the backscattered wave can be separated:

$$r_{m+1,n}(t+\Delta t)= v_{m+1,n}(t+\Delta t) - v'_{m+1,n}(t+\Delta t)$$

This residual $r_{m+1,n}(t+\Delta t)$ is used as input into the finite difference calculation on the fictitious plane $A$ (Figure 4.2), where only the backscattered wavefield is present. The residual $r_{m,n}(t+2\Delta t)$, because it has been separated from the incident wavefield, can be calculated with the usual finite difference scheme. This residual is then added to the contribution $v'_{m,n}(t+2\Delta t)$ of the analytical solution, and this gives the total displacement $v_{m,n}(t+2\Delta t)$ at the time $t+2\Delta t$ on the grid line $S_1$. In this way, the backscattered waves pass the grid lines $S_1$ and $S_2$ without producing any reflections. What is required is the knowledge of the analytical solution on the two adjacent vertical grid lines $S_1$ and $S_2$ for each time step. For P-SV waves, the same numerical scheme can be used for the two velocity components.

**Figure 4.2** Grid configuration and geometry which is used to make the vertical grid lines $S_1$ and $S_2$ transparent to backscattered waves.

For problems involving an explosive source within the finite difference grid, the method of Alterman and Karal (1968) can also be used. The algorithm can be designed so that the source region is transparent to all waves. For explosive sources, this is very simple due to the separation of
the stress-strain relation from the equation of motion in the stress-velocity finite-difference scheme for P-SV waves (Virieux, 1986). The stresses $\sigma_{xx}$ and $\sigma_{zz}$ are defined at the same grid points (see Figure 3.4). By adding equal incremental amplitudes to $\sigma_{xx}$ and $\sigma_{zz}$ at the point source, the given source excitation can be simulated and the source region is transparent. As the velocities are not computed at the source, infinite amplitudes are avoided.

4.2. Geometry and geometrical spreading

Computations with the finite difference technique are restricted here to two-dimensional structural models. Thus, geometrical spreading is not included in the algorithm. On the other hand, the mode summation method accounts for geometrical spreading and includes also the radiation pattern of the seismic source. The coupling of a two-dimensional and a three-dimensional geometry creates some unrealistic features, even if a cylindrical coordinate system is chosen for the finite difference computations. In the interpretation of seismograms, this difficulty with geometrical spreading must be kept in mind. Computed seismograms, which are not corrected for the effect of geometrical spreading, can be taken as a conservative point of view, which holds in the application of the hybrid method for micro-zonation studies.

A first-order correction of geometrical spreading can be achieved by multiplication of the seismogram amplitudes by a factor $(r_0/r_1)^{1/2}$, in which $r_0$ is the distance from source position to the vertical grid lines where the wavefield is introduced into the finite difference computations, and $r_1$ is the source-receiver distance. This gives correct amplitudes only for direct waves, and only if we assume a cylindrical symmetry of the structural model with respect to the source.

4.3. Change of grid spacing

The structural model described by the finite difference grid must extend to great depths. This guarantees the completeness of the signals even at large distances from the vertical grid lines where the incoming wavefield is introduced into the finite difference computations. In general, the seismic velocities of the structural model increase with depth. Therefore, the grid spacing in the $z$ direction can be increased, and no problems are induced by the numerical error. This change in grid spacing is applied at
a certain depth, as shown in Figure 4.3 for the case of P-SV waves. The finite difference algorithm is the same as that applied to a regular grid, except for the vertical velocity component \( \dot{w} \) and the tangential stress \( \sigma_{xz} \) at the position \( N_{FB} \). The differences with respect to the usual finite difference algorithm are the space derivatives \( \partial / \partial z \) of the horizontal velocity component \( \dot{u} \) and of the normal stress \( \sigma_{zz} \) on the irregular grid. One can describe these quantities, here for example \( \dot{u} \), by a quadratic function of the \( z \)-coordinate:

\[
\dot{u}(z) = a_1 z^2 + a_2 z + a_3
\]

where \( a_1 \), \( a_2 \) and \( a_3 \) are determined from the known values of \( \dot{u} \) at one grid point in the region with large grid spacing and at two grid points in the region with small grid spacing. The space derivatives are computed as the derivative of the quadratic function at position \( N_{FB} \). This approximation corresponds to an expansion of the quantity \( \dot{u} \) in a Taylor series.

---

**Figure 4.3.** Staggered, irregular finite difference grid for the case of P-SV waves.
The accuracy of this scheme can be increased by increasing the order of the above function. This has been done for the case of SH waves. For symmetry reasons, five grid points are used: the point itself \((m, N_{FB})\) for which the derivative has to be computed, two grid points in the region with small grid spacing, and two points in the region with large grid spacing. This allows the approximation of the displacements on the grid line \((m, N_{FB})\) by a cubic function. The leading term of the error of the finite difference approximation is then of the order \((\Delta x^2)^2\), which is the same as that for the numerical scheme which is applied in the region with the regular grid.

4.4. Artificial boundaries

Since computer memory and CPU time are limited, the finite difference grid has to be restricted within artificial boundaries. These boundaries generate reflections that contaminate the solution, and therefore, the artificial boundaries have to be made transparent or absorbing. This is a severe problem in finite difference computations. In recent years, several methods have been proposed to overcome these problems (e.g. Smith, 1974; Clayton and Engquist, 1977; Reynolds, 1978; Korn and Stöckl, 1982; Cerjan et al., 1985). Each of these methods provides an interesting solution, but none can reduce all the reflections from the artificial boundaries. In this section, an overview of the different techniques will be given. Their main advantages will be pointed out, and it will be shown that a combination of different absorbing boundary conditions gives good results.

Smith (1974) introduced a method to eliminate the reflections from a plane artificial boundary. For SH waves, the wave propagation problem is solved twice. In the first run, the artificial boundary is assumed to be free (Neumann condition). Therefore, the tangential shear stress is zero at the boundary, and an incident plane wave is reflected completely with a reflection coefficient equal to 1. In the second run, the boundary is chosen to be rigid (Dirichlet condition). The displacement at the boundary is zero, and therefore, the reflection coefficient is equal to -1. The sum of the two solutions contains no reflections from the boundary. Taking into consideration two artificial boundaries, the problem must be solved for each combination of the Dirichlet and Neumann conditions at the two boundaries. Thus, there are four different problems which have to be solved, and the solutions have to be summed up. In general, for SH waves and \(n\) boundary planes, \(2^n\) problems must be solved. This procedure, of course, requires a large amount of computer time. A further
disadvantage of Smith's method is that wave cancellation works only for a limited number of reflections. Let us assume two artificial boundaries which are parallel and of infinite dimension, as shown in Figure 4.4. A plane wavelet with its wavefront parallel to the boundaries propagates towards B. After the reflection at B, the wavelet propagates in the opposite direction. The sign of the amplitudes changes in correspondence to the boundary conditions at B. To avoid reflections at the boundaries A and B, we have to sum the solutions of all possible combinations of free and rigid boundary conditions at A and B. Then for the sum, the wavelet disappears at the boundaries. In the case of four reflections, the sum of the four independent solutions is not zero anymore (see Figure 4.4). Therefore, the artificial boundaries are transparent only up to three reflections. At the fourth reflection, the original wavelet reappears at the artificial boundary A and then disappears once again at the boundary B, at the fifth reflection, and so on.

Figure 4.4. Results for each combination of Dirichlet and Neumann conditions at the two artificial boundaries A and B. The sign of the amplitudes is given after each reflection. The original wavelet is assumed to have positive amplitudes. The sum of the four independent solutions is given, and a zero implies that the four solutions compensate each other.

The same method can be applied to eliminate P-SV waves at the artificial boundaries. Starting with one boundary only, the wave propagation problem again has to be solved for two different boundary conditions.
Choosing the artificial boundary perpendicular to the x-axis, we can assign the following boundary conditions at the artificial boundary

\[ \dot{u} = 0 \quad \text{and} \quad \sigma_{xz} = 0 \]  

(4.1a)

in the first run, and

\[ \dot{w} = 0 \quad \text{and} \quad \sigma_{xx} = 0 \]  

(4.1b)

in the second run, where \( \dot{u} \) and \( \dot{w} \) are the velocity components in the x- and z-direction. For both sets of boundary conditions and the case of incident P-waves, there are no reflected SV-waves. The reflection coefficient is 1 for the first condition and -1 for the second. The sum of the two solutions is free from waves reflected at the artificial boundary. The same can be stated for incident SV-waves, where no conversion to P-waves can occur at the artificial boundary. Since this type of artificial boundary is stable at high frequencies even for long durations of the ground motion, it can also be applied to n boundaries. Also for P-SV waves, Smith's method works only for a limited number of reflections at the artificial boundaries.

Another method for introducing nonreflecting boundaries was proposed by Clayton and Engquist (1977). The artificial boundary is chosen to be perpendicular to the x-axis. The method is based on the one-way or paraxial wave equation which replaces the wave equation in the boundary region. In this equation, energy is only permitted to propagate in a limited angle range. The paraxial approximation is used which models only energy moving outward from the interior of the grid towards the artificial boundary. The main features of this technique will be indicated by starting with the acoustic wave equation, which in a homogeneous medium is given by

\[ \frac{\partial^2 v}{\partial t^2} = \beta^2 \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right) . \]  

(4.2a)

This equation describes SH-wave propagation. The quantity \( \beta \) is the shear wave velocity and \( v \) is the transverse displacement. In the frequency-wavenumber space Equation (4.2a) becomes

\[ \omega^2 = \beta^2 \left( k_x^2 + k_z^2 \right) . \]

Thus,

\[ k_x = \pm \frac{\omega}{\beta} \sqrt{1 - \frac{\beta^2 k_z^2}{\omega^2}} . \]  

(4.2b)
For small $\beta k_x/\omega$, the square root in Equation (4.2b) can be expanded in series:

$$\frac{\beta k_x}{\omega} = 1 - \frac{1}{2} \left( \frac{\beta k_z}{\omega} \right)^2 + \ldots$$  \hfill (4.2c)

Therefore, the approximation to the first order yields

$$\frac{\beta k_x}{\omega} = 1 ,$$

which by inverse Fourier transform gives

$$\frac{1}{\beta} \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} = 0 .$$  \hfill (4.3a)

The second-order approximation leads to

$$\frac{1}{\beta} \frac{\partial^2 v}{\partial t^2} + \frac{\partial^2 v}{\partial t \partial x} - \frac{\beta}{2} \frac{\partial^2 v}{\partial z^2} = 0 .$$  \hfill (4.3b)

The energy is permitted to propagate in a certain angle range around the x-direction, but not in the opposite direction. Equations (4.3a) and (4.3b) can be solved with a finite difference algorithm. In practical computations, the second-order paraxial approximation (4.3b) of the wave equation is used. The discretization of this equation in time and space is given by Clayton and Engquist (1977).

In the case of P-SV wave propagation, the same concept can be applied using the approximate radiation condition (A1) proposed by Clayton and Engquist (1977):

$$\frac{\partial u}{\partial x} + \frac{1}{\alpha} \frac{\partial u}{\partial t} = 0$$
$$\frac{\partial w}{\partial x} + \frac{1}{\beta} \frac{\partial w}{\partial t} = 0$$  \hfill (4.4)

These are first-order paraxial approximations of the wave equations for the velocity components. If the artificial boundary is the one on the right side of the structural model (Figure 4.1), perpendicular to the x-axis, the range of solutions is restricted to those waves that are travelling within a cone around the x-axis. For stability reasons at sharp elastic interfaces, the finite difference scheme proposed by Emerman and Stephen (1983) is used to solve Equations (4.4) at the artificial boundaries. This approach is computationally cheap and simple to apply. The effectiveness to reduce reflections degrades for plane waves which impinge upon the boundary at...
large incidence angles. For incidence angles less than 60°, the reflection coefficient is less than 0.1, increasing to perfect reflection at 90°. Problems occur at grid points where an interface of the structural model intersects an artificial boundary. At such points, diffractions occur due to the violation of the condition of homogeneous media; a cylindrical wave with small amplitudes is generated, which propagates backwards into the structural model.

A third scheme for constructing a nonreflecting artificial boundary is the gradual reduction of amplitudes with damping elements (Lysmer and Kuhlemeyer, 1969) or anelastic properties (Cerjan et al., 1985; Korn and Stöckl, 1982; Reynolds, 1978). In P-SV computations, damping elements are introduced at the artificial boundaries (Lysmer and Kuhlemeyer, 1969). The implementation of this scheme is easy because the stress-strain relations are well separated in the set of coupled differential equations (Virieux, 1986). This method corresponds to a situation in which the stress boundary is supported on infinitesimal dashpots oriented normal and tangential to the boundary. For the artificial boundary at the bottom of the finite difference model, the absorbing condition is given by

\[ \sigma_{zz} = -\rho \alpha \dot{w} \]
\[ \sigma_{xz} = -\rho \beta \dot{u} \]  

This method cannot give perfect absorption over the whole range of incidence angles, but it provides a stable absorber for harmonic elastic waves. When interfaces intersect the boundary, interface waves with complicated particle motions must be absorbed as well as body waves. Many schemes for P-SV waves can break down, especially at solid-liquid interfaces (Stephen, 1988). A combination of the viscous boundary condition (4.5) in the stress space and the radiation condition (4.4) in the velocity space provides stability even for long duration of ground motions.

The last technique we made use of is the gradual reduction of the waves with a region of high absorption. One possible technique includes a dissipation factor \( \exp(-\chi t) \). In the equation of motion for SH waves, this can be described by two additional terms (Korn and Stöckl, 1982):

\[ \frac{\partial^2 v}{\partial t^2} + 2\chi \frac{\partial v}{\partial t} + \chi^2 v = \beta^2 \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right) \]

The suppression of unwanted reflections is now achieved by including a zone with nonvanishing \( \chi \) along the boundaries of the grid. This zone has
to be large enough so that waves passing through it twice become negligible. \( \chi \) has to be space dependent with a gradient increasing linearly to avoid additional reflections from the region that separates the elastic part from the part with damping. With this method, the amplitude of the incoming wavefield is very well attenuated as long as the zone including damping is larger than the dominant wavelength.

As dissipation by constant intrinsic attenuation \( Q \) is included in the finite difference scheme (see Chapter 3), it can also be used to reduce the unwanted reflections. Like in the previous case, \( Q \) has to be space dependent so that \( Q \) is decreasing linearly towards the artificial boundary. The gradient should not be too steep to avoid reflections. Its adjustment to a certain structural model and a frequency band of interest is a matter of experience and can be controlled after each computation.

For certain structural models, the wavelengths of the incident wavefield can be larger than the size of the finite difference model. This happens for sites far from the seismic source, which induces considerable amount of energy at lower frequencies, and for structural models with low-velocity sediments which require a small grid spacing (see the example of Mexico City in Chapter 8). The gradient of the quality factor close to the artificial boundary can be too strong for the low-frequency part of the wavefield, producing reflections of the outgoing waves in the zone of high absorption (see Figure 4.1). Since the approximation of the quality factor is frequency band limited (see Chapter 3), the lower frequency limit can be adjusted in such a way that the low-frequency part of the wavefield undergoes no intrinsic attenuation in the model, and therefore, does not feel the zone of increased attenuation. Another more elegant method is to increase the grid spacing in the zone of high absorption. This can be done at a certain grid position (Section 4.3) or by making the grid spacing space-dependent so that grid spacing is increasing linearly towards the artificial boundary. This technique has been applied in the computations shown in Chapter 8.

As none of the described methods can reduce all the reflections from the artificial boundaries, a combination adapted to the structural model is used. Paraxial approximation works well at the boundary limiting the structural model at depth. The two vertical boundaries at each side of the grid are chosen in correspondence of the reflection properties in the layered model and the type of waves occurring during the calculation. As long as all waves impinge upon the boundaries with a small incidence angle, paraxial approximation in combination with a dissipative region yields good results. In the case of strong reflections from the artificial
boundaries due to waves with a large angle of incidence, the method proposed by Smith (1974) reduces this contamination almost perfectly. Smith's method is only applied at the right boundary (Figure 4.1), whereas the left boundary remains with the paraxial approximation and a dissipative region. With this technique, the contamination first appears at the right boundary having passed through the model two times. The disadvantage of Smith's boundary conditions is an increase in computer time by a factor of two. In some cases, Smith's boundary condition is too time-consuming to be applied routinely (see the example of Mexico City in Chapter 8). Then, the paraxial wave equation is applied at the right artificial boundary (Figure 4.1), in combination with a zone of high absorption in which the grid spacing is increased versus the boundary.
5. Comparing and testing of computational techniques

5.1. Comparison between modal summation and the finite difference technique for one-dimensional structural models

Before going into two-dimensional computations, we must first compare the results of modal summation and the finite difference technique for the simple case of a one-dimensional structural model. This comparison is necessary each time the hybrid technique is applied in a new region. The comparison allows us to establish control over the accuracy of the finite difference part of computations, relative to: (1) the correct discretization of the structural model in space, (2) the efficiency of the absorbing, artificial boundaries, (3) the presence of all phases in the seismograms, and (4) the treatment of anelasticity. The comparison is performed for the same layered structural model which describes the path from source position to the region where the finite difference method is applied. Without geometrical spreading in either of the two methods, the testing is fully defined for its purpose of putting specific error bounds on the finite difference technique.

The first cause of differences between our reference, modal summation results and those from finite difference computations, is a grid spacing in the finite difference model that is too large. Considering the layered structure of the modal summation computations as the reference model, then each horizontal, physical interface is well defined. In opposition, finite difference computations assume that each grid point is located in the center of a homogeneous rectangular region, so that interfaces are located between two neighboring points. Depending upon the grid spacing selected, the location of the physical interfaces can be shifted by as much as half the grid spacing. This changes the finite difference model with respect to the reference model, and therefore, can give rise to changes in the dispersion characteristics of surface waves, and to differences in signal amplitudes. The only means for improving accuracy relative to this problem is to increase the fineness of the finite difference grid in space.

The second cause of differences between the reference results and those from our finite difference computations is the need for artificial boundaries. These form the border of the finite difference grid, and generate spurious reflections of the waves impinging upon them from the interior of the grid. The simplest solution to this problem is to improve upon the specification of the regions of high absorption close to the border.
(see Section 4.4 for details), and to increase the grid spacing towards the border. The optimization of grid spacing close to the artificial boundaries is a matter of experience and can be controlled after each computation by making snapshots of the displacement wavefield. These snapshots allow the identification of reflections in the regions of high absorption.

The third cause of differences between the results of the two techniques is insufficient depth of the structural model described by the finite difference grid. When this insufficiency obtains, the signals are incomplete for receivers at large distances from the vertical grid lines where the incident wavefield is introduced into the finite difference computations. To deal with this problem, the lower artificial boundary of the finite difference grid is simply placed at greater depth. (It should be noted that this problem of signal incompleteness cannot always be resolved. An example of this is discussed in Chapter 7 for the Fucino earthquake.)

A fourth cause of differences between the results of the two techniques is the treatment of anelasticity. In the mode summation method, anelasticity is included by means of the variational method, while in the finite difference computations, it is included by using a method based on the rheological model of the generalized Maxwell body. Both methods for treating anelasticity are approximations. If we use structural models with surficial low-velocity sediments, the synthetic signals become very sensitive to the quality factor in these sediments. An important effect of intrinsic anelasticity is the dependence of the seismic velocities on frequency, i.e. body-wave dispersion. In the two techniques, the reference frequency for seismic velocity is defined in different ways. The definition of this reference frequency can strongly influence the phase velocity of a wavelet which propagates in a medium with a low quality factor, and can create differences in the results obtained with the two techniques. (An instructive computational example will be given in Figure 6.26 of Section 6.7.). The only means for improving accuracy relative to this problem is to define the seismic velocities in both techniques at the same reference frequency. For the general discussion of anelasticity, the reader is referred to Section 3.4.

With the results from modal summation used as our reference, comparison with the finite difference results yields accuracy estimates for our finite difference computations. By optimizing the finite difference approach with the means described above, amplitude errors can be reduced to the order of 2 to 5 percent. They are due to the remaining
problem of discretization in the finite difference computations, and the different means of treating anelasticity in the two techniques.

5.2. The 2D mode summation technique

With some approximations, the 2D mode summation technique (Vaccari et al., 1989a; Vaccari, 1990) allows us to treat wave propagation in structures with vertical discontinuities. In the simplest case, one vertical discontinuity separates two quarter-spaces, which are in welded contact (see Figure 5.1). Such a contact is defined by continuity of stress and displacement across the interface. A surface wave is propagating through quarter-space 1 in the positive x-direction. Without loss of generality it can be assumed that this incident wavefield contains just one mode of Rayleigh or Love waves. In this context only normal incidence at the vertical interface is considered, which is not a limitation of the method itself, but will give rise only to the same type of surface wave (Love or Rayleigh wave) at the vertical interface.

**Figure 5.1.** Schematic representation of the structural model used in the 2D mode summation method. Two quarter-spaces are in welded contact. They are composed of n and r layers, respectively, each defined by the compressional wave velocity, the shear wave velocity, the density, the quality factor, and the thickness.
The incident surface wave is reflected and transmitted at the vertical interface. These secondary waves are composed of surface waves and body waves, including also diffractions at the corners. This problem of surface wave reflections and transmissions can not be solved exactly due to the body waves generated at the vertical contact. The necessary approximation for the solution has been proposed by Alsop (Alsop, 1966; Gregersen and Alsop, 1974). This method assumes that the intensity of body waves generated at the vertical interface is small compared to that of the normal modes. Therefore, diffracted waves arising at the corners are neglected. This is a good approximation for models with a small contrast in the elastic parameters characterizing the quarter-spaces (Gregersen and Alsop, 1974). The continuity of stress and displacement at the vertical contact allows now the determination of the reflection and transmission coefficients of normal modes. The starting point of the procedure is the stress-displacement system of the incoming mode. The incident wave is decomposed into homogeneous waves, propagating upward and downward in the layers, and into inhomogeneous waves which fall off or increase exponentially with depth. The problem is now reduced to the problem of reflection and transmission of plane waves. For every single section in the vertical interface in Figure 5.1, the problem is solved as if the section would be infinite. Applying Snell's law and the continuity of stress and displacement leads to the transmission and reflection coefficients for every section, and therefore, to the reflected and transmitted waves. This system of homogeneous and inhomogeneous waves can exactly satisfy the boundary conditions on the vertical interface. It does not satisfy the boundary conditions at the free surface and at the horizontal interfaces, which excludes the diffracted waves arising at the corners.

The medium with the receiver (Figure 5.1) is now characterized by a set of normal modes which are used to represent the transmitted waves. This is done using the orthogonalization formula for normal modes given originally by Herrera (1964) and used by Alsop (1966), Gregersen and Alsop (1974), and Vaccari et al. (1989a). If the contrast in elastic parameters between the two quarter-spaces is strong, the body waves can no longer be neglected. A test of the adequacy of our approximation can be carried out by evaluating the energy flux to the left and right of the vertical interface (e.g. Gregersen and Alsop, 1974; Levshin, 1985; Vaccari et al., 1989a). A detailed review of the problems involving a vertical contact and their possible solutions can be found in the book edited by Keilis-Borok (1989). The development of the complete algorithms for Love and Rayleigh waves is given by Gregersen and Alsop (1974), and Vaccari et al. (1989a).
5.3. Comparison between results obtained with the hybrid technique and the 2D mode summation method

For structural models with two layered quarter-spaces, we can perform a comparison of the results from the hybrid technique and those from the 2D modal summation. This comparison will be performed for a structural model with a small contrast in the elastic parameters characterizing the two quarter-spaces. This choice of model minimizes the generation of diffracted waves at the corners—the waves that we neglect in 2D modal summation. (A large contrast in elastic parameters would generate diffracted waves giving a major contribution to the total wavefield; therefore, in this case a comparison between the results of the two techniques would be very difficult. An example of the importance of diffractions in sedimentary basins is given in Section 6.7.) For the hybrid technique used in this comparison, the propagation of waves from source position to the vertical interface is treated with modal summation. The propagation of waves across this vertical interface and through the second quarter-space is then handled with the finite difference technique. An important disadvantage of the hybrid technique is the fact that geometrical spreading cannot be treated in the finite difference computations in an exact manner. In the finite difference computations, a correction of geometrical spreading can be achieved by multiplication of the seismogram amplitudes by a factor \((r_0/r_1)^{1/2}\), in which \(r_0\) is the distance from source position to the vertical grid lines where the wavefield is introduced into the finite difference computations, and \(r_1\) is the source-receiver distance.

In the 2D mode summation method, the wave propagation problem is always studied in terms of normal modes. If we have a small contrast in the elastic parameters characterizing the quarter-spaces, for a given incident mode it is generally sufficient to consider only the contribution of the outgoing modes with mode numbers close to that of the incident mode (Vaccari, 1990). This reduces considerably the requirements of computer time and memory, since only a limited number of mode-to-mode coupling coefficients need to be computed. In the examples which are shown below, the number of outgoing modes for a given incident mode is chosen to be five. With the finite difference method, on the other hand, all outgoing modes and also diffractions are automatically present in the seismograms that are computed.

For our comparisons a well-tested structure is used, whose contrast in the elastic parameters characterizing the two quarter-spaces is small. These quarter-spaces correspond to two different lithospheric models, C
and P (Table 5.1), originally considered by Patton (1980) and subsequently used by Levshin (1985), Fäh et al. (1989) and Vaccari et al. (1989b). Model C corresponds to the continental Gutenberg model and the model P has been suggested for the Pamir region by Patton (1980).

<table>
<thead>
<tr>
<th>thickness (km)</th>
<th>( \rho_i ) (g/cm(^3))</th>
<th>( \alpha_i ) (km/s)</th>
<th>( \beta_i ) (km/s)</th>
<th>thickness (km)</th>
<th>( \rho_j ) (g/cm(^3))</th>
<th>( \alpha_j ) (km/s)</th>
<th>( \beta_j ) (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>2.74</td>
<td>6.14</td>
<td>3.55</td>
<td>4</td>
<td>2.41</td>
<td>4.41</td>
<td>2.55</td>
</tr>
<tr>
<td>19</td>
<td>3.00</td>
<td>6.58</td>
<td>3.80</td>
<td>26</td>
<td>2.66</td>
<td>5.58</td>
<td>3.18</td>
</tr>
<tr>
<td>12</td>
<td>3.32</td>
<td>8.20</td>
<td>4.65</td>
<td>30</td>
<td>2.90</td>
<td>6.50</td>
<td>3.76</td>
</tr>
<tr>
<td>10</td>
<td>3.34</td>
<td>8.17</td>
<td>4.62</td>
<td>20</td>
<td>3.36</td>
<td>8.10</td>
<td>4.51</td>
</tr>
<tr>
<td>10</td>
<td>3.35</td>
<td>8.14</td>
<td>4.57</td>
<td>further on as in model C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3.36</td>
<td>8.10</td>
<td>4.51</td>
<td>25</td>
<td>3.39</td>
<td>7.93</td>
<td>4.37</td>
</tr>
<tr>
<td>25</td>
<td>3.41</td>
<td>7.85</td>
<td>4.35</td>
<td>25</td>
<td>3.43</td>
<td>7.89</td>
<td>4.36</td>
</tr>
<tr>
<td>25</td>
<td>3.46</td>
<td>7.98</td>
<td>4.38</td>
<td>25</td>
<td>3.48</td>
<td>8.10</td>
<td>4.42</td>
</tr>
<tr>
<td>25</td>
<td>3.50</td>
<td>8.21</td>
<td>4.46</td>
<td>50</td>
<td>3.53</td>
<td>8.38</td>
<td>4.54</td>
</tr>
<tr>
<td>50</td>
<td>3.58</td>
<td>8.62</td>
<td>4.68</td>
<td>50</td>
<td>3.62</td>
<td>8.87</td>
<td>4.85</td>
</tr>
<tr>
<td>( \infty )</td>
<td>3.69</td>
<td>9.15</td>
<td>5.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.1.** Elastic parameters for the structures C and P, originally considered by Patton (1980). The quality factors for P- and S-waves are \( Q_\alpha = 750 \) and \( Q_\beta = 300 \) for both quarter-spaces.

The maximum frequency in the computations is chosen to be 1 Hz. A point source is placed in the quarter-space corresponding to the model C, as is shown in Figure 5.2. The duration of the source is taken to be 0 s. The double-couple point source is 100 km away from the vertical boundary. The depth of the source is taken to be 10 km, the dip is 60° and the rake is 270°. The angle between the strike of the fault and the epicenter-station line is 40° for the Love-wave case and 90° for Rayleigh waves. The seismic moment of the source is taken to be 1 dyne cm. Nine receivers are placed in quarter-space P, the first one 10 km from the vertical interface (Figure 5.2).
The fundamental mode and the first higher mode have been considered as incoming wavefields in quarter-space C. We first treat the case of an incoming, fundamental-mode Love wave. The displacement time series of the transmitted Love waves are shown in Figure 5.3. By comparing the results obtained with the hybrid technique, with those from the 2D mode summation technique (for five outgoing modes), it can be seen that the results practically coincide. There are differences of the order of a few percent in the maximum amplitude, which originate in the differences between the two computational techniques, and in the use of only five outgoing modes in our 2D modal summation. The contribution of the outgoing fundamental mode to the total wavefield is the most important part of the transmitted wavefield.

The displacement time series for the incidence of the first higher mode Love waves are shown in Figure 5.4. Here, the energy transfer from first higher mode in the model C to first higher mode in model P is much less important than it was for fundamental mode to fundamental mode transfer in Figure 5.3. The shapes of the wavelets are almost coincident for the two techniques, and the difference in amplitudes is of the order of a few percent. This indicates that more than five outgoing modes should be used in the 2D mode summation method. (Results obtained subsequent to this presentation show that by using more than five outgoing modes, the differences in the Love-wave results become considerably smaller.)
Figure 5.3. Displacement time series (transverse component) obtained for the geometry shown in Figure 5.2 and an incident, fundamental-mode Love wave. Results with the 2D mode summation technique are shown for five outgoing modes and for the outgoing fundamental mode only, and are compared with results obtained with the hybrid technique. The distance to the source is given in units of km. All seismograms are normalized to the same amplitude, given in units of cm on the upper part of the figure.

The comparison of the two techniques has also been carried out for Rayleigh waves, again using an incident fundamental, and first higher mode. In Figures 5.5 and 5.6 the vertical velocity time series are shown. The coincidence of the results for Rayleigh waves is better than it was for Love waves, both in the signal shape and maximum amplitude. This indicates that for this structural model, the use of five outgoing Rayleigh modes in the 2D mode summation method is sufficient to obtain the complete wavefield. The results show the consistency of the results obtained with the hybrid technique and the mode summation technique, even though both techniques handle the problem of wave propagation across a vertical interface with some approximations.
Figure 5.4. Displacement time series (transverse component) obtained for the geometry shown in Figure 5.2 and an incident, first higher mode Love wave. Results with the 2D mode summation technique are shown for five outgoing modes and for the outgoing first higher mode only, and are compared with results obtained with the hybrid technique. The distance to the source is given in units of km. All seismograms are normalized to the same amplitude, given in units of cm on the upper part of the figure.

This section's comparison of the two computational techniques also establishes a basis of reference for later applications to more complicated structures: since 2D modal summation does not treat diffracted waves, we can use the comparison between the results of 2D modal summation and those of the hybrid technique to recognize diffracted waves in sedimentary basins (Section 6.7). For this reason, in this section we have considered first a structural model where such diffracted waves are not important (Levshin, 1985; Vaccari et al., 1989a); and here, the results obtained with the hybrid technique and 2D modal summation are consistent. This provides a firm basis for then treating wave propagation in sedimentary basins, where diffracted waves will give a major contribution to the total wavefield due to their generation by strong contrasts in elastic parameters, e.g. at the interface between bedrock and sediments. The comparison of our results of the finite difference technique, with those from 2D modal summation, will then allow us to identify diffracted waves. The most important approximation in the numerical examples shown above is the consideration of a fixed number of outgoing modes from a given incident mode. As we have demonstrated above for Love
waves, this can lead to incomplete signals in the computations with 2D modal summation. A general rule for the determination of the required number of outgoing modes that must be used cannot be given because this number depends upon the incident mode, the structural model under consideration, and whether Love waves or Rayleigh waves are being studied. Therefore, in the examples where we will make use of the 2D mode summation method as a basis of reference for identifying diffracted waves in the hybrid technique’s result, the number of outgoing modes will not be restricted.

**Figure 5.5.** Velocity time series (vertical component) obtained for the geometry shown in Figure 5.2 and an incident, fundamental-mode Rayleigh wave. The left column of seismograms shows the results obtained with the hybrid technique. Results with the 2D mode summation technique are shown on the right, for five outgoing modes. The distance to the source is given in units of km. All seismograms are normalized to the same amplitude, given in units of cm/s on the upper right part of the figure.
Figure 5.6. Velocity time series (vertical component) obtained for the geometry shown in Figure 5.2 and an incident, first higher mode Rayleigh wave. The left column of seismograms shows the results obtained with the hybrid technique. Results with the 2D mode summation technique are shown on the right, for five outgoing modes. The distance to the source is given in units of km. All seismograms are normalized to the same amplitude, given in units of cm/s on the upper right part of the figure.

The one-dimensional experiment in Section 5.1 allows us to consider the modal summation result as a reference; a comparison between modal summation and the finite difference technique then establishes accuracy bounds on the results from the hybrid technique. From the one-dimensional experiment we can estimate the lower bound of error in the
hybrid technique to be 2 to 5 percent. This error can only be obtained if the discretization in space is optimized, and if the finite difference grid is deep enough to guarantee the completeness of the signals. This lower error bound for the hybrid method can be expected for any laterally heterogeneous structure, since the finite difference technique treats vertical interfaces in exactly the same way as the horizontal ones. Such relatively high accuracy allows the application of the hybrid technique to the study of wave propagation in sedimentary basins.
PART II: Examples of the application of the hybrid technique

Introduction to the seismic response of sedimentary basins

Local soil conditions and irregular geological structures can significantly affect the characteristics of ground motion during earthquakes. Despite the difficulties in modelling such effects, they should not be ignored in the assessment of seismic hazard or in zonation studies. In general, the structural models used in theoretical studies of the seismic response of sedimentary basins are chosen to be very simple. They usually consist of a homogeneous half-space with sedimentary basins modelled by overlying sequences of layers. The velocity contrast between the basins and the half-space is chosen to be quite large in order to emphasize the main effects of the basins. The seismic response of such structures is then calculated, rather schematically, by assuming a plane incident wavefield.

Studies of this sort have shown the well-known amplification of the incident plane waves, which occurs when a seismic wave travels through an interface from a medium with relatively high rigidity, into a medium of lower rigidity. Mechanical resonances of the sedimentary basin can occur (Haskell, 1960; 1962). For vertically incident waves, the critical frequencies of resonance are $f_n = (2n+1)\beta/4h$, where $\beta$ is the shear wave velocity in the basin and $h$ is its thickness. An irregular interface between bedrock and sediments can cause a focusing of wave energy (e.g. Aki and Larner, 1970; Boore et. al., 1971; Sánchez-Sesma et al., 1988a), and can excite local surface waves (e.g. Trifunac, 1971; Bard and Bouchon, 1980a; 1980b; Harmsen and Harding, 1981). These local surface waves can be excited not only by body waves but also by the incidence of surface waves (Drake, 1980). Bard and Bouchon (1985) demonstrated numerically that the occurrence of local surface waves is determined primarily by the depth of the basin and the contrast between the shear wave velocities of the basin and the bedrock. When the wavelengths of the incident waves are comparable to the depth of the basin, a local surface wave can have larger amplitudes than the direct signal. It can be reflected at the edges of the basin, since the contrast in elastic parameters between the sediments in the basin and the underlying bedrock in general is high. This leads to a long duration of the ground motion in the basin. Studies that introduce vertical stratification of the sediments (Bard and Gariel, 1986), with a large vertical velocity gradient, show that the qualitative behavior does not change with respect to a homogeneous basin. The observed amplifications can attain very large values due to low shear wave velocities at the surface. Localized amplification of the signals can often be related to
lateral irregularities in subsurface topography (e.g. Jackson, 1971; Campillo et al., 1990). Smooth variations of near-surface structure can induce a large level of differential motion, so that adjacent observation points show different amplitudes and signal durations. These effects are limited to receiver positions just above the irregularities.

These findings, which are mostly from numerical experiments, can be related to observed distributions of damage from earthquakes (e.g. Gutenberg, 1957; Poceski, 1969; Fäh, 1985; Sánchez-Sesma et al., 1988b). Intensity maps for great and intermediate size earthquakes often show larger intensity values inside sedimentary basins, which cannot be explained by the radiation pattern of the seismic source (Suhadolc et al., 1988; Panza et al., 1991). This phenomenon in sedimentary basins has been long studied (see Gutenberg's 1957, now-standard review) due to the high concentration of population in such areas. In recent years, some large earthquakes have produced extensive destruction in sedimentary basins; for example, in Mexico City during the 1985 Michoacan earthquake (see special issues of Earthquake Spectra Vol.4, N.3 and 4, 1988; Vol.5, N.1, 1989), and in the city of Leninakan during the 1988 Spitak earthquake (see special issue of Earthquake Spectra, special supplement, Aug. 1989).

For many earthquakes, it is observed that a significant increase in damage often takes place close to lateral heterogeneities. An example of this is the damage distribution in Skopje during the Skopje, Yugoslavia earthquake of July 26, 1963 (Poceski, 1969). Skopje is located approximately 8 km from the epicenter. The greatest damage is observed in two zones of the city. The first is located along a belt which is characterized by an abrupt change of the thickness of the sedimentary layer. The second zone is at the edge of the sedimentary basin, closest to the source, where the interface between bedrock and sediments is dipping smoothly. In the regions where the thickness of sedimentary cover varies only slowly, the damage was small. A remarkable fact is that the badly damaged area was not limited to the regions where the sediments are thick, but also extended into sites with shallow alluvium. Another example of localized effects is found in the damage statistics for buried utility pipes after the Miyagiken-Oki earthquake of June 12, 1978 (Kubo and Isoyama, 1980). An increase in the number of failures in these pipes was observed near the cut-and-fill boundary of a newly developed area. The cause of the damage was associated with failures of the ground.

Other phenomena related to the presence of sedimentary basins are the resonance effects inside sedimentary basins in the Garm region (King
and Tucker, 1984), and the local surface waves at the edge of the Osaka plain (e.g. Liu and Heaton, 1984; Horike, 1988). For the latter region, Horike (1988) numerically reproduced some of the recorded seismograms using the Aki-Larner technique.

In the following three chapters, theoretical results for sedimentary basins are presented which allow us the very realistic modelling of observations. These results will define the limits and possibilities of the hybrid technique. They will show how powerful the hybrid technique is in terms of flexibility of the structure and source modelling. The first example simulates an event in the Friuli seismic region and is an application of the hybrid method for sites close to the seismic source. Special attention is paid to the types of waves generated inside sedimentary basins. The second example is a numerical simulation of the January 13, 1915, Fucino earthquake. This event caused structural damage in the city of Rome. Since the distribution of damage in the city is well documented, a comparison between the observed damage and some characteristics of the computed ground motion is performed. The third example simulates the well-known case of Mexico City, where the seismic source is far away from the sedimentary basin.

One aspect which is not included in our discussion, is the influence of surface topography on ground motion. This approximation can be justified for sites inside sedimentary basins, where topographic features are in general small. However, topography can become important at the edges of sedimentary basins, especially in mountainous regions. For example, topographic effects were observed during the Chile Earthquake of March 3, 1985, where the major damage was limited to a series of ridges (Çelebi, 1987) composed of alluvial deposits and decomposed granite. Studies concerning topographic effects show that experimental and theoretical results are consistent only on a qualitative basis (Geli et al., 1988). An example of a numerical study is the one by Boore (1972), who found that for SH-wave incidence, amplitudes on the crest of a mountain are larger than elsewhere; along the flanks, amplitudes could either be amplified or damped depending on the geometry of the model, the position of the receiver, and on the dominant wavelengths of the incident field. These results were confirmed by Davis and West (1973), who performed a series of measurements on the crest, and at the base of different mountains. Observed ground motion recorded on three different mountains was considerably larger than that recorded at the base. This amplification is frequency dependent, and is different for each mountain.
6. An example of numerical modelling for sites close to the seismic source: Friuli area

The Friuli region is located at a complex junction between two large-scale structures: the Alpine system, oriented E-W, and the Dinaric system, oriented NW-SE (Anderson and Jackson, 1987). The September 11, 1976 Friuli aftershock (16h35m04s), which is discussed in this chapter, has been recorded by a few accelerographic stations (CNEN-ENEL, 1977). Records from one of the nearest stations—the three-component records at Station Buia—will be considered and compared with theoretical computations. The uncorrected accelerograms recorded at station Buia are shown in Figure 6.1a.

The area where the station of Buia is placed is characterized by terrigenous sediments (Flysch), widely outcropping at Monte Buia (Figure 6.2). They are covered locally by a thin quaternary layer, forming a sedimentary basin, and overlap a carbonatic mesozoic sequence. The surface sediments are incoherent, of glacial, alluvial, and lacustrine origin (Barnaba, 1978). They form the so-called Amphitheater of the Tagliamento river. The thicknesses of the quaternary sediments are well-known (Giorgetti and Stefanini, 1989) and locally can reach 100 m. At station Buia they reach a thickness of about 57 m. A general overview of the region is given in Figure 6.2.

For numerical modelling, we have to define the geometry of the sedimentary basin and the seismic velocities of the sediments. The depths to bedrock in the sedimentary basin are taken from the work published by Giorgetti and Stefanini (1989). For the definition of the seismic P-wave velocities of the quaternary sediments, results from refraction measurements in the region of Madonna di Buia (Giorgetti, 1976) and different parts of the Amphitheater (Martinis et al., 1976) were taken (see Table 6.1). The variability of shear wave velocities is known from cross-hole measurements in the Tarcento region (Brambati et al., 1980). Material properties of the quaternary sediments vary between $\rho = 1.8 \text{ g/cm}^3$, $\alpha = 1.3 \text{ km/s}$ and $\beta = 0.5 \text{ km/s}$ near the surface, and $\rho = 2.0 \text{ g/cm}^3$, $\alpha = 1.8 \text{ km/s}$ and $\beta = 0.8 \text{ km/s}$ at larger depths (Table 6.1).
Figure 6.1. Comparison of the observed ground acceleration at station Buia for the September 11, Friuli, $M_L=5.7$ aftershock ($16^h35^m04^s$) with the results obtained from waveform fitting with the mode summation technique for a layered, anelastic structural model:

A) Observed, uncorrected accelerograms, after Gaussian filtering, with a cutoff frequency of 6.5 Hz. The zero of the time axes does not coincide with the origin time. The amplitudes are given in cm s$^{-2}$.

B) Synthetic accelerograms from six point sources located at the same depth, 7.1 km, and the same distance, 15 km (Mao et al., 1990; Florsch et al., 1991). The strike, the angle between the strike of the fault and the epicenter-station line, the dip, and the rake are 225°, 19°, 28°, and 115°, respectively. The six point sources have different weights and time shifts (1.0, 0.6, 0.6, 0.5, 0.5, and 0 s, 0.77 s, 1.13 s, 1.37 s, 1.9 s, 2.18 s). Weight 1 corresponds to a source with seismic moment of 1 dyne cm. The seismic moment-release in time is shown at the bottom; the maximum value of $6.0 \times 10^{24}$ dyne cm is obtained from the comparison between synthetic and observed signals.
<table>
<thead>
<tr>
<th>Type</th>
<th>$\alpha$ (km/s)</th>
<th>$\beta$ (km/s)</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flysch</td>
<td>2.3 - 5.3</td>
<td>2.2 - 2.8</td>
<td>0.9(1)</td>
<td>Martinis et al., 1976</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Giorgetti, 1976</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Brambati et al., 1980</td>
</tr>
<tr>
<td>Limestone</td>
<td>2.4 - 3.8</td>
<td>2.2 - 2.8</td>
<td>0.9(1)</td>
<td>Martinis et al., 1976</td>
</tr>
<tr>
<td>Moraines</td>
<td>2.4 - 3.8</td>
<td>2.2 - 2.8</td>
<td>0.9(1)</td>
<td>Martinis et al., 1976</td>
</tr>
<tr>
<td>Clay</td>
<td>1.6 - 1.8</td>
<td>1.3 - 1.8</td>
<td>0.9(1)</td>
<td>Martinis et al., 1976</td>
</tr>
<tr>
<td>top soils</td>
<td>0.35 - 0.58</td>
<td>0.31 - 0.34(2)</td>
<td>1.8 - 1.95</td>
<td>Brambati et al., 1980</td>
</tr>
</tbody>
</table>

Table 6.1. Material properties published by different authors for the Friuli area.
(1) Cross-hole measurements at 10 m depth.
(2) In the upper 10 to 20 meters a velocity gradient was observed, with shear wave velocities varying between 0.3 and 0.6 km/s.

6.1. Numerical modelling for a one-dimensional structural model

Assuming a one-dimensional, layered, anelastic structural model for the Friuli seismic region, Panza and Suhadolc (1987) have shown that the observed signals at station Buia for the September 11, 1976 earthquake cannot be explained by only a single point source. They used the mode summation technique to reproduce the observed vertical component, by trial-and-error varying of a set of source parameters. A good fit was obtained with three point sources having different weights and time shifts, but the same focal depth, mechanism, and duration. The same conclusion was drawn by Mao et al. (1990), by modelling all three recorded components (Figure 6.1b), and combining SH and P-SV waves. In the last study, the source is approximated by a sum of six point sources, still modelling three different rupturing episodes. The data were low-pass filtered with a cut-off frequency of 6.5 Hz. The parameters varied in the process were the number of point sources, their origin time, and the weights of the individual sources. The source-receiver distance, the source depth, the strike, dip and rake were varied, but kept constant for all subevents. All these parameters were adjusted until a satisfactory waveform fit was obtained, both in the time and in the frequency domain. This fitting was limited to frequencies below 6.5 Hz.
Figure 6.2. Overview of the Friuli seismic region, outlining the presence of the quaternary basin, the ISC epicenter determination of the Sept. 11, Friuli 1976 aftershock (16h35m04s), and the position of station Buia. The solid lines indicate the cross-sections, for which 2D modelling has been performed.

The layered P-, and S-wave velocity model used in that study is representative of the Friuli area (model FRIUL7W in Table 6.2). It was obtained by a damped, least-square inversion of arrival time data from local earthquakes (Mao and Suhadolc, 1987; Mao et al., 1990). The structure is similar to structural model FRIUL7A (Section 2.8). The main differences in the two structures are the depth and shape of the upper low-velocity zone, the thickness of the sedimentary cover, and the quality factors. In model FRIUL7W, the thickness of the surficial sediments correspond to their thickness at station Buia (Giorgetti and Stefanini, 1989). The quality factor Q has been adjusted during the process of waveform fitting (Mao et al., 1990).

In all cases of waveform fitting, the orientation of the sources agrees well with previously published results by Slejko and Renner (1984), who interpreted the event as one of thrust on a very shallow, NW dipping plane. Several point sources with different weights and time shifts are required to fit the observed signals (Figure 6.1). The vertical component can be well reproduced, whereas the synthetic NS-component has
excessively large amplitudes in the coda. The duration of the observed EW-component cannot be explained by this set of point sources. The difficulty in reproducing the observed horizontal components of motion with a one-dimensional structural model arises from the fact that local lateral heterogeneities are the main influence on these components. On the other hand, the vertical component of motion is much less affected by local site conditions and can be modelled quite well with one-dimensional structural models.

<table>
<thead>
<tr>
<th>Thickness (km)</th>
<th>Density (g/cm$^3$)</th>
<th>P-wave velocity (km/s)</th>
<th>S-wave velocity (km/s)</th>
<th>Q$_\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.057</td>
<td>2.00</td>
<td>1.50</td>
<td>0.60</td>
<td>20</td>
</tr>
<tr>
<td>0.043</td>
<td>2.30</td>
<td>3.50</td>
<td>1.80</td>
<td>20</td>
</tr>
<tr>
<td>0.20</td>
<td>2.40</td>
<td>4.50</td>
<td>2.50</td>
<td>50</td>
</tr>
<tr>
<td>0.70</td>
<td>2.40</td>
<td>5.55</td>
<td>3.05</td>
<td>100</td>
</tr>
<tr>
<td>2.00</td>
<td>2.60</td>
<td>5.88</td>
<td>3.24</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>5.70</td>
<td>3.14</td>
<td>50</td>
</tr>
<tr>
<td>0.20</td>
<td>2.60</td>
<td>5.65</td>
<td>3.10</td>
<td>50</td>
</tr>
<tr>
<td>0.20</td>
<td>2.60</td>
<td>5.60</td>
<td>3.06</td>
<td>50</td>
</tr>
<tr>
<td>1.00</td>
<td>2.60</td>
<td>5.57</td>
<td>3.03</td>
<td>50</td>
</tr>
<tr>
<td>0.50</td>
<td>2.60</td>
<td>5.55</td>
<td>3.02</td>
<td>50</td>
</tr>
<tr>
<td>1.00</td>
<td>2.60</td>
<td>5.57</td>
<td>3.03</td>
<td>50</td>
</tr>
<tr>
<td>0.20</td>
<td>2.60</td>
<td>5.60</td>
<td>3.06</td>
<td>50</td>
</tr>
<tr>
<td>0.20</td>
<td>2.60</td>
<td>5.65</td>
<td>3.10</td>
<td>50</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>5.70</td>
<td>3.14</td>
<td>50</td>
</tr>
<tr>
<td>4.50</td>
<td>2.60</td>
<td>5.88</td>
<td>3.25</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>6.10</td>
<td>3.40</td>
<td>200</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>6.20</td>
<td>3.50</td>
<td>200</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>6.30</td>
<td>3.60</td>
<td>200</td>
</tr>
<tr>
<td>0.70</td>
<td>2.60</td>
<td>6.45</td>
<td>3.75</td>
<td>200</td>
</tr>
<tr>
<td>2.50</td>
<td>2.60</td>
<td>6.47</td>
<td>3.77</td>
<td>200</td>
</tr>
<tr>
<td>5.00</td>
<td>2.60</td>
<td>6.50</td>
<td>3.80</td>
<td>200</td>
</tr>
<tr>
<td>5.00</td>
<td>2.60</td>
<td>6.55</td>
<td>3.82</td>
<td>200</td>
</tr>
<tr>
<td>1.00</td>
<td>2.75</td>
<td>6.55</td>
<td>3.82</td>
<td>200</td>
</tr>
<tr>
<td>2.00</td>
<td>2.75</td>
<td>7.00</td>
<td>3.85</td>
<td>200</td>
</tr>
<tr>
<td>2.00</td>
<td>2.80</td>
<td>7.00</td>
<td>3.85</td>
<td>200</td>
</tr>
<tr>
<td>7.50</td>
<td>2.80</td>
<td>6.50</td>
<td>3.75</td>
<td>100</td>
</tr>
<tr>
<td>4.00</td>
<td>2.85</td>
<td>7.00</td>
<td>3.85</td>
<td>200</td>
</tr>
<tr>
<td>3.00</td>
<td>3.20</td>
<td>7.50</td>
<td>4.25</td>
<td>400</td>
</tr>
<tr>
<td>1.50</td>
<td>3.40</td>
<td>8.00</td>
<td>4.50</td>
<td>400</td>
</tr>
<tr>
<td>9.00</td>
<td>3.45</td>
<td>8.20</td>
<td>4.65</td>
<td>400</td>
</tr>
</tbody>
</table>

Table 6.2. Structural model FRIUL7W, representative of the Friuli (Italy) area, and obtained by damped, least-square inversion of arrival time data (Mao and Suhadolc, 1987; Mao et al., 1990). $Q_\alpha = 2.5 Q_\beta$. 

Q$_\alpha$ = 2.5 Q$_\beta$. 


6.2. Numerical modelling for two-dimensional structural models

The presence of a sedimentary cover of several tens of meters implies that the sedimentary basin can have a large influence on wave propagation. Its effect is studied with the hybrid approach and represents an application of this technique for sites close to the source and for frequencies from 0.5 Hz to 6.5 Hz. The lower frequency limit is chosen in order for it to be possible to neglect the higher-order cylindrical Hankel functions in the source expression (Panza et al., 1973). The higher frequency limit is introduced because of the chosen grid spacing selected for our finite difference computations (see Chapter 3). These frequency limits do not influence our results, since the dominant energy of the observed signals is contained in this frequency band.

![Distance to the source [km]]

![Source](Layered Structure)

Figure 6.3. Geometry of the models used in the hybrid method. The number of grid points in the finite difference computations is 750x300. The mesh size is 10 m by 10 m in the upper part of the model and 10 m by 30 m in the lower part, resulting in a model size of 7.5 by 7.0 km. The distances between the observation points and the source are of the order of 8 to 16 km.
We are restricted to two-dimensional models, but the real structure—mainly the depth of the sediments and their material properties—varies in all horizontal directions. In 2D modelling, this can be accounted for partly by studying the wave propagation along different cross-sections. Their positions are shown in Figure 6.2. The propagation path of the seismic waves in these sections is not perpendicular to the strike of the transition from bedrock to sediments. Energy transfer from SH to P-SV waves and vice versa at this contact has to be expected, but cannot be taken into account in this work due to the restriction to two-dimensional structural models. A general overview of the model size and geometry used in the computations is shown in Figure 6.3.

**Figure 6.4.** Transverse, radial, and vertical acceleration obtained at the free surface for a point source 8.0 km distant. Each signal is decomposed in different sets of modes (upper three traces for each component: modes 31-143, modes 16-30, modes 0-15). All amplitudes correspond to a source with seismic moment of 1 dyne cm. The signals are normalized. The peak acceleration is in units of cm s\(^{-2}\).
The layered one-dimensional model, describing the propagation of waves from source position to the sedimentary basin, is chosen to be model FRIUL7W (Table 6.2) without the sedimentary cover of 57 m thickness. The source is always placed in the plane of the cross-section, having the mechanism and parameters of the first source in the set of sources taken by Mao et al. (1990) (source-depth 7.1 km, angle between the strike of the fault and the epicenter-station line 19°, dip 28°, rake 115°, and source duration 0.6 s). The incident wavefield is obtained by adding all modes present in a given frequency - phase velocity band (Chapter 2). In our case, the wavefield exhibit little dispersion, as can be seen in Figure 6.4. The energy contribution of the higher modes is important. The major part of the energy is concentrated in shear waves. They have different angles of incidence with respect to the sedimentary layer, varying from about 30° to almost vertical incidence.

The first example of computations with the hybrid technique is the comparison between waves propagating in the one-dimensional layered structural model, and the two-dimensional structure corresponding to cross-section A (Figure 6.2). The velocity models are shown in Figure 6.5. The main features of cross-section A are a series of overthrusts and two sedimentary basins intersected by the outcropping of Flysch at Monte di Buia. The accelerograms for an array of receivers at the free surface for the two structures are shown in Figures 6.6 and 6.7. In both cases, the signals have been computed with the hybrid technique. In the case of the one-dimensional layered structure (Figure 6.6), the main difference between the signals along the array is the decrease in amplitude due to geometrical spreading and attenuation.

Heterogeneities with size comparable to the wavelength of the incident wavefield generate significant spatial variations of the ground motion (Figure 6.7). These heterogeneities have the same effect as would have fictitious sources placed at the location of the heterogeneities themselves. The closer they are to the observation point the more important are the effects. Even for close stations it is possible to observe big differences in shape, duration, and frequency content of the signals.
Figure 6.5. 1D structural model and 2D structural model corresponding to cross-section A, shown in Figure 6.2. Only the part near to the surface is shown, where the 2D model deviates from the layered structural model.
<table>
<thead>
<tr>
<th>Transverse component</th>
<th>Radial component</th>
<th>Vertical component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0e-23</td>
<td>2.0e-23</td>
<td>1.0e-23</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Figure 6.6.** Acceleration time series for P-SV and SH waves at an array of receivers, for the layered model shown in Figure 6.5. All amplitudes correspond to a source with a seismic moment of 1 dyne cm. The signals are normalized. The peak acceleration is indicated in units of cm s\(^{-2}\). The distance to the source for each seismogram is given in units of km. The time scale is shifted by 2 seconds from the origin time (0 s in the figure is really 2 s origin time).
At the first station (9.8 km from the source) the sediments are shallow. Therefore, the signals differ only slightly in amplitude and shape from those in the layered, one-dimensional structure. The high-frequency part of the P-SV wave energy is concentrated in the radial components. When the sediments become deeper the energy is redistributed by diffractions at the beginning of the basin, and by multiple reflections in the dipping sedimentary layer. The multiply reflected body waves can dominate the
shape of the signals and, after a short propagation path, form local surface waves. They are the dominant phase for P-SV waves and have much smaller amplitudes for SH waves. Local Love waves are slower and attenuate faster than the local Rayleigh waves.

The amplitudes of the reflected waves at the right edge of the first basin are small. They interfere with local surface waves excited by the direct body waves at the right edge of the sedimentary basin. The amplitudes of these waves are also very small (see Figure 6.7). Resonance effects of the surface layer dominate the shape of the signals at all stations inside the sedimentary basin. The predominant periods of the signal at each station are related to the thickness of the sedimentary layer. A good example of a strong local resonance can be seen on the transverse component (at 11.8 km from the source) with an arrival time of about 3 s and with dominant frequencies around 5 Hz (see Figure 6.7). This special type of resonance at the edges of the sedimentary basins can often be observed in theoretical computations (Fäh et al., 1990). They can probably be related to the effects reported for earthquakes, where severe damage has been observed at the edges of sedimentary basins, e.g. the concentration of damage observed during the Skopje, Yugoslavia earthquake of 1963 (Poceski, 1969).

On the outcrop, the signals have less high-frequency content and the amplitudes are smaller than elsewhere. In the case of SH-wave propagation in the 2D structure, the first phases of the signals are the same as those obtained for the layered structure (Figure 6.6). This is not the case for P-SV-wave propagation.

Local surface waves are also excited in the second sedimentary basin (located after the outcrop with respect to the source), but now with greater amplitudes on the transverse component. These local surface waves are mainly formed by trapped S-waves interfering constructively for frequencies around 2 Hz, which is close to the first mechanical resonance frequency of the sedimentary layer. A characteristic of these local surface waves is their strong dispersion, which can be explained by the group velocity curve shown in Figure 6.8 for the corresponding layered model FRIUL7W. From the study of Drake (1980) one should expect not only the fundamental mode to be present in the sedimentary basin, but also some of the higher modes. This might also be expected from the group velocity curves, especially for Rayleigh waves, where the modes are much closer to each other than they are for Love waves. This problem will be discussed in more detail in Section 6.7.
Figure 6.8. Group velocity curves of the fundamental and the first 3 higher modes of Love and Rayleigh waves for the structural model FRIUL7W.

The geometry of the overthrusts shown in cross-section A is taken from the work of Barnaba (1978). The influence of the overthrusts are small due to the small impedance contrast between the two bedrock layers. The shape of the observed signal mainly depends upon the sedimentary basin. Consequently, the study will now be restricted to the influence of the sedimentary basin on ground motion.

The computations have been repeated for cross-sections A, B and C, without the overthrusts, and for a model with two quarter-spaces (cross-section D) in welded contact. The cross-sections are shown in Figure 6.9. The structural model in cross-section B represents an average model for the region, whereas cross-section C represents the part of the Amphitheater with deep sediments. Cross-section D is chosen to be very simple, having only a sharp vertical discontinuity between two layered quarter-spaces: the first corresponding to layered structure FRIUL7W without the sedimentary layer, the second to FRIUL7W with the sedimentary layer. The thickness of the sedimentary layer is equal to the thickness (57 m) of the quaternary sediments at station Buia.
Figure 6.9. 2D models corresponding to cross-sections A, B and C in Figure 6.2, and to a model with two quarter-spaces in welded contact (cross-section D). Only the part near the surface is shown, where the models deviate from one another.

The acceleration time series for P-SV and SH waves are shown in Figures 6.10, 6.11, 6.12, and 6.13 for arrays of receivers at the surface, and for the different cross-sections. There are three major effects which are caused by the presence of the sedimentary cover: (1) the excitation of surface waves at the edges of the sedimentary basin, (2) resonances due to the subsurface topography of the bedrock, and (3) the excitation of very dispersed local surface waves, which have their peak energy at about 2 Hz. These effects will be discussed in the following three sections. We
will focus on the physical processes occurring inside the sedimentary basin and on the comparison between observed ground motion at station Buia and the theoretical results.

<table>
<thead>
<tr>
<th>Transverse component</th>
<th>Radial component</th>
<th>Vertical component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9.00 \times 10^{-23}$</td>
<td>$1.0 \times 10^{2}$</td>
<td>$9.00 \times 10^{-23}$</td>
</tr>
<tr>
<td>$1.5 \times 10^{4}$</td>
<td>$1.5 \times 10^{4}$</td>
<td>$1.5 \times 10^{4}$</td>
</tr>
<tr>
<td>$1.5 \times 10^{4}$</td>
<td>$1.5 \times 10^{4}$</td>
<td>$1.5 \times 10^{4}$</td>
</tr>
<tr>
<td>$1.5 \times 10^{4}$</td>
<td>$1.5 \times 10^{4}$</td>
<td>$1.5 \times 10^{4}$</td>
</tr>
<tr>
<td>$1.4 \times 10^{2}$</td>
<td>$1.4 \times 10^{2}$</td>
<td>$1.4 \times 10^{2}$</td>
</tr>
<tr>
<td>$1.4 \times 10^{2}$</td>
<td>$1.4 \times 10^{2}$</td>
<td>$1.4 \times 10^{2}$</td>
</tr>
<tr>
<td>$1.3 \times 10^{2}$</td>
<td>$1.3 \times 10^{2}$</td>
<td>$1.3 \times 10^{2}$</td>
</tr>
<tr>
<td>$1.3 \times 10^{2}$</td>
<td>$1.3 \times 10^{2}$</td>
<td>$1.3 \times 10^{2}$</td>
</tr>
<tr>
<td>$1.2 \times 10^{2}$</td>
<td>$1.2 \times 10^{2}$</td>
<td>$1.2 \times 10^{2}$</td>
</tr>
<tr>
<td>$1.2 \times 10^{2}$</td>
<td>$1.2 \times 10^{2}$</td>
<td>$1.2 \times 10^{2}$</td>
</tr>
<tr>
<td>$L_{1}$</td>
<td>$R_{2}$</td>
<td>$R_{1}$</td>
</tr>
<tr>
<td>$1.1 \times 10^{2}$</td>
<td>$1.1 \times 10^{2}$</td>
<td>$1.1 \times 10^{2}$</td>
</tr>
<tr>
<td>$1.1 \times 10^{2}$</td>
<td>$1.1 \times 10^{2}$</td>
<td>$1.1 \times 10^{2}$</td>
</tr>
<tr>
<td>$1.1 \times 10^{2}$</td>
<td>$1.1 \times 10^{2}$</td>
<td>$1.1 \times 10^{2}$</td>
</tr>
<tr>
<td>$1.8 \times 10^{2}$</td>
<td>$1.8 \times 10^{2}$</td>
<td>$1.8 \times 10^{2}$</td>
</tr>
<tr>
<td>$1.8 \times 10^{2}$</td>
<td>$1.8 \times 10^{2}$</td>
<td>$1.8 \times 10^{2}$</td>
</tr>
<tr>
<td>$9.8$</td>
<td>$9.8$</td>
<td>$9.8$</td>
</tr>
</tbody>
</table>

Figure 6.10. Acceleration time series for P-SV and SH waves at an array of receivers along cross-section A. All amplitudes correspond to a source with seismic moment of 1 dyne cm. The signals are normalized. Peak acceleration is indicated in units of cm s$^{-2}$. The time-scale is shifted by 2 seconds from the origin time (0 s in the figure is really 2 s origin time).
Figure 6.11. The same as in Figure 6.10, but for cross-section B shown in Figure 6.9.
Figure 6.12. The same as in Figure 6.10, but for cross-section C shown in Figure 6.9.
Figure 6.13. The same as in Figure 6.10, but for cross-section D shown in Figure 6.9.
6.3. Influence of the dipping interface between bedrock and sediments at the edge of the sedimentary basin

The problem of a dipping interface between bedrock and sediments has been discussed by Dravinski (1983) using the boundary integral technique, Sánchez-Sesma et al. (1988a) using geometrical ray theory, and by Bard and Bouchon (1980a; 1980b) using the Aki-Larner method. The discussion given here is restricted to the different models of the sedimentary basin in the Friuli seismic region. The slopes of the bedrock-sediment interface in the different basins, used in numerical modelling, are shown in Figure 6.14. Notice that the inclination of the slopes is not constant. It varies between 3° and 6°. There are only small variations between the models A, B and C. For such smooth basins, one might expect that lateral effects are small and the response is similar to that of a flat layer of similar thickness. This is not the case, as is shown in Figures 6.10, 6.11, 6.12, and 6.13.

The case of SH waves (transverse component):

As can be seen from Figures 6.10, 6.11, 6.12 and 6.13, multiple reflections of SH waves can generate a local surface wave (phase L₁) forming the coda of the signals. They are excited as soon as the frequency exceeds the fundamental Haskell resonance frequency (Haskell 1960, 1962) for the sedimentary basin. With increasing depth of the sediments, and after a short propagation path, these surface waves separate from the direct wavetrain. The amplitudes can be comparable to the amplitude of the first arrival (see Figure 6.12). The zone in the sedimentary basin, within which these local surface waves are of considerable amplitude, depends on the waves' frequency content and the quality factor of the sediments. The lower the frequencies, the longer the propagation path. The frequency content of local surface waves, at a given distance from the edge of the basin, depends on the inclination of the bedrock-sediment interface and the thickness of the sediments. For shallow basins (cross-sections A and D), low frequencies are not excited. Since the interface in cross-section C is dipping more steeply than in the other cross-sections, local surface waves of a given frequency content are excited closer to the edge of the sedimentary basin than in the other cross-sections.

In all cases, the group velocity of local surface wave L₁ is of the order of 0.40 to 0.60 km/s. Considering the group velocity curves in Figure 6.8, these waves can be interpreted as the fundamental mode with frequencies
close to or higher than the minimum in the group velocity curve. These local surface waves can be reflected inside the basin (for example, $L_{11}$ in Figure 6.11, which corresponds to the reflected local Love wave $L_1$), where the sediments become shallow, or at the end of the sedimentary basin itself. In cross-section D, there is an excitation of local surface waves corresponding to the fundamental Love mode with frequencies near those of the minimum in the group velocity curve. This wavetrain is strongly dispersed and never separates from the direct wavetrain. The lowest velocity corresponds to the group velocity of the fundamental mode at its minimum (0.40 km/s). Since this wavetype is mainly excited in cross-section D, we can conclude that its existence requires a constant depth of the sedimentary basin. This particular wavetype will be discussed in Section 6.5.

![Figure 6.14. Slopes of the interfaces between bedrock and sediments for cross-sections A, B and C, and the model with two quarter-spaces in welded contact (cross-section D). The slopes are shown in digitized form as used for the discrete, finite difference grid.](image)

**The case of P-SV waves:**

Also in the P-SV case, a dipping layer at the edge of a sedimentary basin gives rise to multiple reflections of body waves and the excitation of local surface waves (phases $R_1$ and $R_2$ in Figures 6.10, 6.11, 6.12 and 6.13). At the bedrock-sediment interface and at the free surface, energy is converted from S- to P-waves, and vice versa. (For a discussion of reflection and transmission coefficients, see Aki and Richards, 1980.) The amplitudes of the transmitted P-waves can be quite large if the incident S-
wave arrives at the free surface with an angle of incidence that is close to the critical angle. In this case, the path of the transmitted P-wave leads along the free surface, and we can speak of a P-headwave, also referred to as an SP-wave (Bouchon, 1978). This can be a prominent phase on the seismogram for near-source records (Kawasaki et al. 1973; 1975). Its energy is concentrated in the sediments and consequently attenuates very rapidly.

![Diagram](image)

**Figure 6.15.** a) Reflection and transmission angles due to S-wave incidence on the free surface for a model with a layer over a half-space. The layer has a compressional wave velocity, \( \alpha_1 \), which is smaller than the shear wave velocity of the half-space, \( \beta_2 \). The horizontal slowness of the incident wave is preserved on reflection and transmission:

\[
\sin \varphi_i = \frac{\beta_1}{\beta_2} \cdot \sin \varphi_0 \quad \text{and} \quad \sin \varphi_j = \frac{\alpha_1}{\beta_2} \cdot \sin \varphi_0
\]

Therefore if \( \beta_2 \) is greater than \( \alpha_1 \), the S-wave never reaches the free surface with the critical angle.

b) Effect of a dipping layer on the wave propagation. SP indicates S-to-P transmission at the free surface. The angle \( \varphi_{in} \) of the \( n^{th} \) multiple reflection at the free surface is given by

\[
\varphi_{in} = \varphi_{i0} + 2n \cdot \delta
\]
In the structural models we have used, the P-wave velocity in the sedimentary layer is lower than the shear wave velocity in the bedrock. In those parts of the model where the sediments are horizontally layered, only an interface wave at the sediment-bedrock interface can exist, but no P-headwave at the free surface. The reason for this is that in the horizontally layered part of the model, the S-waves always arrive at the free surface with an angle of incidence smaller than the critical angle, as is shown in Figure 6.15a.

At the edge of the sedimentary basin where the interface is dipping, a large amount of S-wave energy can be converted to P-waves. Figure 6.15b shows how the angle of incidence of S-waves can increase. More and more energy can be converted into P-waves. When the critical angle is reached, the reflection and transmission coefficients become frequency dependent and the body waves form local surface waves. These wave conversions at dipping interfaces depend on the type of incident wavefield, the inclination and length of the interface between bedrock and sediments, and the impedance contrast.

In the case of structural models A, B and C, about one third of the incident S-wave energy is converted into P-wave energy. The distribution and propagation of P- and S-wave energy can be studied by making snapshots of the wavefield. The divergence and curl of the particle velocity are shown in Figure 6.16 for the portion of model B, where the bedrock-sediment interface is dipping. The divergence is nonzero where there is compressional-wave energy, and the curl is nonzero for shear-wave energy. Snapshots of the curl and divergence allow the study of S-to-P wave conversions at the edge of the sedimentary basin, and the identification of local surface waves inside the sedimentary basin. The curl and the divergence are first-order spatial derivatives, and for that reason increase the amplitudes of waves with short wavelengths. The snapshots in Figure 6.16 are given at several instances of time and they clearly show the different types of waves. In the first two snapshots of the curl, taken at 4.0 s and 4.5 s origin time (upper left part of the figure), and for distances larger than 9.4 km from the source, the first S-wave arrivals can be seen. They are propagating upward and downward in the sedimentary layer. In the corresponding snapshots of the divergence (upper two snapshots on the right) for distances larger than 10 km from the source, we can observe the P-waves that are generated by the incident S-waves. These transmitted P-waves are propagating faster than the S-waves. Their wavefront is almost vertical and their path leads along the free surface; and we can speak of a P-headwave.
Figure 6.16. Snapshots of the divergence and curl of the P-SV velocity wavefield inside the sedimentary basin. The part of cross-section B with the dipping interface at the edge of the sedimentary basin is shown. The first snapshots at the top are taken at 4.0 s (origin time). The time step between snapshots is 0.5 s. The amplitudes are normalized for the curl and the divergence. The white and dark areas show the concentration of energy. Due to smaller Q values for shear waves than compressional waves, the amplitudes of the former waves attenuate faster than those of the latter.
The local surface waves are the dominant part of the wavefield inside the sedimentary basin. They correspond to the phase $R_2$ in Figure 6.11. They are characterized by vertically-oriented dark and white areas in Figure 6.16. Local surface waves have both compressional-wave and shear-wave energy, so that we can identify them in the snapshots of the curl and of the divergence. Divergence and curl of the wavefield do not allow to study the distribution of the total energy of local surface waves. This requires the computation of the total amplitude of P-SV waves. The total amplitude of the velocity (Figure 6.17) shows that the energy of the local surface waves is concentrated near the free surface. The amplitudes decay rapidly below the sedimentary layer.

The interpretation of the different phases in the P-SV seismograms is much more difficult than it was for SH waves. The group velocity curves are packed together (see Figure 6.8). Therefore, several modes are excited at the edge of the sedimentary basin. Certain modes, with frequencies higher than that of the group velocity minimum, can separate from the direct signal. Two distinct phases can be seen in the
seismograms (Figures 6.10, 6.11, 6.12, and 6.13): the fundamental mode (phase R1) which is prominent on the vertical component, and the first higher modes (phase R2). Since the depth of the sedimentary cover does not remain constant, the group velocity of the local surface waves can vary and mode conversions can occur within sedimentary basins. Reflections of the local Rayleigh waves inside the sedimentary basin do not appear as clearly as in the SH case. Their amplitudes are small in comparison with the amplitudes of the primary waves (see, for example, R22 in Figure 6.11).

As was observed in the SH case, there is also excitation of local Rayleigh waves with frequencies lower than that of the group velocity minimum. They are strongly dispersed and never separate from the direct wavetrain. Here also, it should be expected that not only the fundamental mode is locally excited, but also some of the higher modes. An effect of particular interest is the regular pattern of interferences in the coda of the radial component in cross-sections D (Figure 6.13), at distances beyond 11 km from the source. A superposition of different wavetypes causes a regular variation of the signal durations by constructive and destructive interferences. These waves will be discussed in more detail in Section 6.7.

6.4. Lateral resonance effects and excitation of local surface waves

The second set of major effects occurring inside sedimentary basins are resonances. These occur in those parts of the basins with smooth variations of the geometry of interfaces between bedrock and sediments. The resonances originate from superposition of forward propagating local surface waves, with their reflections within sub-basins of the sedimentary cover. Examples are seen for the two sub-basins in cross-section B, especially for the receivers at 9.8 km and at 11.8 km distance from the source (Figure 6.11). The resonance is very strong for SH waves, but does not appear so clearly in the P-SV case. The duration and amplitude can become very large, as is visible in the signal at 11.8 km. Snapshots of the displacement field of SH waves in this part of the model show that the energy is mainly concentrated in the sedimentary layer (Figure 6.18). The effect of this subsurface irregularity is visible only in the receivers just above it.
Figure 6.18. Snapshots of the amplitudes of the SH-wave displacement-field inside the sedimentary basin. The part of cross-section B with the sub-basin at 12 km from the source is shown. Positive amplitudes of the displacement are shown in white, and negative amplitudes in black. The amplitudes are normalized to the same value. The time given for each snapshot is shifted by 2 seconds from the origin time (0 s in the figure is really 2 s origin time).

The third major effect, occurring inside sedimentary basins, is the excitation of a strongly dispersed local surface wave by lateral heterogeneities. This wave can be observed in all cross-sections at distances larger than 13 km from the source. A snapshot of the SH
wavefield (Figure 6.19) in cross-section B shows the distribution of the energy in the sedimentary basin and the bedrock. The energy is concentrated near the free surface and the amplitudes decay exponentially with increasing depth, clearly proving that local surface waves are excited. The interpretation of these waves as surface waves is supported by the strong dispersion, which is in agreement with the group velocity curves for the first few Love and Rayleigh modes shown in Figure 6.8. These local surface waves are excited for both SH-, and P-SV-wave incidence.

Figure 6.19. Snapshot of the amplitudes of the SH-wave displacement-field inside the sedimentary basin (at 8 s origin time). The part of cross-section B is shown where distances from the source are greater than 13 km. Positive amplitudes of the displacement are shown in white, and negative amplitudes in black. The amplitudes are normalized to the same value.

6.5. Comparison of computed ground motion with observations

At a distance of 15 km from the source, the thickness of the sedimentary cover is the same for all models (cross-sections A, B, C and D). This position corresponds to the location of station Buia. A comparison is shown in Figure 6.20 between the transverse component of ground motion computed for the four different cross-sections of the basin, the synthetic
signal obtained with the mode summation method for a layered one-dimensional structural model, and the observed transverse acceleration at station Buia. The synthetic signals have been computed for two different shear wave velocities of the sediments (600 m/s and 700 m/s).

Figure 6.20. Comparison between the recorded transverse component of acceleration and synthetic signals, obtained by application of different techniques and models. The synthetic signals have been computed for two different shear wave velocities of the sediments (600 m/s for the seismograms on the left side and 700 m/s for the seismograms on the right side). The source-receiver distance is 15 km. The time-scale is shifted by 2 s from the origin time, i.e. t=0 s in the figure corresponds to t=2 s origin time. All amplitudes of the synthetic signals correspond to a source with a seismic moment of 1 dyne cm. The synthetic signals are normalized to the same value. The peak acceleration is indicated in units of cm s\(^{-2}\). Notation:

OBS: Observed transverse acceleration, recorded at station Buia. The seismogram is aligned to agree with the synthetic signals.

MOD: Synthetic seismogram obtained with the mode summation technique for structural model FRIUL7W.

ND2, ND4: Synthetic seismogram obtained with the hybrid method for model D with two layered quarter-spaces.

NC2, NC4: Synthetic seismogram obtained with the hybrid method for the model corresponding to cross-section C in Figure 6.2.

NB2, NB4: Synthetic seismogram obtained with the hybrid method for the model corresponding to cross-section B in Figure 6.2.

NA2, NA3: Synthetic seismogram obtained with the hybrid method for the model corresponding to cross-section A in Figure 6.2.
At this distance, the local Love waves—in the text, also referred to as the 2 Hz surface waves—never separate from the direct wavetrain (see transverse components in Figures 6.10, 6.11, 6.12, and 6.13). From the group velocity curves (Figure 6.8) and the strong dispersion of the coda which has a dominant frequency around 2 Hz, one can conclude that this is mainly fundamental-mode energy. Owing to its strong attenuation, this mode does not propagate over large distances. The fundamental-mode Love wave can be excited at the edge of the sedimentary basin or at lateral heterogeneities within the sedimentary basin. For model D, formed with just two quarter-spaces, lateral heterogeneities inside the basin are absent; thus at larger distances from the edge of the basin, the local Love waves are strongly attenuated (ND2 and ND4 in Figure 6.20). In the case where the interface between the bedrock and the sediments approaches the free surface (cross-section B and C), or in the case of an outcropping of the bedrock (cross-section A), the local 2 Hz surface waves are generated by these heterogeneities. For cross-sections B and C, several phases with local surface waves, excited at different heterogeneities, can interfere.

The first part of the observed transverse component resembles the first part of the signals obtained for model D with its two quarter-spaces, and for the one-dimensional model. This can be interpreted as the Lg phase. In the coda of the observed signal, however, there is evidence for local surface waves generated by the sedimentary basin. The synthetic local surface waves, obtained for cross-sections A, B and C, have amplitudes that are too large in comparison with the observed signals. To reproduce the observed transverse component, the heterogeneities inside the sedimentary basins, responsible for the excitation of these local surface waves, would have to be different from those used in the numerical modelling. The heterogeneities are either too close to station Buia or the bedrock-sediment interfaces approach the free surface too much. Good agreement with the observed transverse acceleration can be obtained for the simplest model, D. On the other hand, the rather large amplitudes of the surface waves in the observed signals can be explained if we assume an excitation of local surface waves within the sedimentary basin.

The radial component of ground motion, computed for the four different cross-sections of the basin, the synthetic signal obtained with the mode summation method, and the observed radial acceleration at station Buia are shown in Figure 6.21. The attenuations of the local Love and Rayleigh waves are different (Figure 6.22). For fundamental-mode Rayleigh waves, the quality factors are bigger around the dominant frequency (2 Hz) of the signal than they are for the fundamental-mode
Love wave. Therefore, the local Love waves attenuate faster than the local Rayleigh waves.

Figure 6.21. The same as in Figure 6.20, but comparing the recorded radial component of acceleration with synthetic signals obtained by applying different techniques and models. The synthetic signals have been computed for two different shear wave velocities of the sediments (600 m/s for the seismograms on the left side and 700 m/s for the seismograms on the right side).

For Rayleigh waves the situation with the excitation of local surface waves is not as clear as it is for Love waves. In the P-SV case there are also three sources of local surface waves: the edge of the sedimentary basin, the places where the bedrock-sediment interface approaches the free surface, and the outcropping of the bedrock. From the signals obtained for cross-sections A, B and C, it can be concluded that: the stronger the lateral heterogeneity within the basin is, the greater are the amplitudes of the local Rayleigh waves. On the other hand, these lateral heterogeneities can reflect most of the local Rayleigh waves coming from
the edge of the sedimentary basin closest to the seismic source; this is observed for cross-sections A, B and C.

The comparison of the synthetic signals with the observed radial component shows good agreement between that observation and the signal obtained for cross-section C. Due to the small amplitudes of the coda in the observed radial component, it can be concluded that the local surface waves have travelled through the deeper parts of the sedimentary basin and that the lateral heterogeneity within the basin has reflected the local surface waves from the first edge of the basin. To reproduce the observed signal, the lateral heterogeneity within the basin cannot be strong; a strong heterogeneity, in fact, would excite large-amplitude local surface waves inside the basin, and these are not observed experimentally. No agreement can be found between the observed radial acceleration and the synthetic signal obtained with modal summation for the one-dimensional, layered structural model.

\textbf{Figure 6.22.} Quality factor $Q$ for the fundamental and the first 3 higher modes of Love and Rayleigh waves for structural model FRIUL7W.
Comparison between the observed vertical component of ground motion and synthetic signals is shown in Figure 6.23. For the lower-frequency part (below 4 Hz) of the signals, good agreement is obtained between all vertical components. This similarity of the vertical components of motion is a fact that was also observed at different sites in Mexico City during the 1985 Michoacan earthquake (Campillo et al., 1988); and therefore, on the basis of our calculations also, it can be considered a quite general property. The high-frequency ringing in the synthetic signals for models A, B and C, are the remainders of the resonance effects in the shallow part of the sedimentary cover. This does not occur in the observed signal; and this difference to our theoretical results indicates that in numerical
modelling, the shallow parts of the sedimentary cover are chosen to be too close to the observation point.

Keeping in mind the restriction of the structural models to the available geotechnical and geological information, to two-dimensional geometries, and to a rather simple seismic source, the synthetic signals can explain the major characteristics of the observed, strong ground motion at station Buia. The relative amplitudes of the different components of the synthetic signals agree in a convincing way with the observed signals. Assuming a point source, the long duration of the signals can be explained by local surface waves, even though an approximation of the magnitude $M_l=5.7$ earthquake with a point source might not seem realistic. Therefore, it can be concluded that the complexity of the source proposed by Mao et al. (1990) is only partly real, and could partly be an artifact of his assumption of a laterally homogeneous structure.

6.6. Sensitivity of the signals to changes of the quality factor and of the shear wave velocity of the sediments

In the previous section, we have seen that waveforms of synthetic signals are sensitive to small changes in the subsurface topography. We now want to focus on the influence of the local surface-soil properties on strong ground motion, i.e. the influence of shear wave velocity and quality factor in the sediments. For our computations, average structural model B has been chosen. The results obtained for different shear wave velocities are shown in Figure 6.24, where the three components of motion are compared with the records at Station Buia.

The horizontal components of motion are very sensitive to small changes in shear wave velocity, whereas the vertical component in its low-frequency part (below 4 Hz), as we might expect, shows smaller differences. The lower the shear wave velocity of the sediments, the larger the amplitudes of the local surface waves, and the stronger their dispersion. The effects are more evident in the case of SH waves due to the dominant contribution of the fundamental-mode Love wave. Low shear-wave velocities reduce the high-frequency content of the synthetics due to increased spatial attenuation. The maximum amplitude of the first body-wave pulse changes only slightly.
Figure 6.24. Comparison between the recorded components of acceleration (OBS) and synthetic signals obtained for cross-section B. The synthetic signals have been computed for four different shear-wave velocities of the sediments (0.5 km/s, 0.6 km/s, 0.7 km/s and 0.8 km/s, as indicated for each seismogram). The source-receiver distance is 15 km. All amplitudes of the synthetic signals correspond to a source with a seismic moment of 1 dyne cm. The synthetic signals are normalized to the same value. The peak acceleration is given in units of cm s\(^{-2}\).
The influence of the quality factor on ground motion is shown in Figure 6.25. The synthetic signals have been computed for three different quality factors $Q_\beta$ of the sediments ($Q_\beta=5$, $Q_\beta=10$, and $Q_\beta=20$, as indicated for each seismogram).

The influence of the quality factor on ground motion is shown in Figure 6.25. The synthetic signals have been computed for three different quality factors $Q_\beta$ of the sediments, by keeping the shear wave velocity fixed at 0.6 km/s. The amplitudes of the local surface waves are more and more attenuated as the quality factor decreases. Since the heterogeneity
which is mainly responsible for the local surface waves is about 1 km from the receiver, a quality factor of 5 is sufficiently low to damp out the local surface waves almost completely. What remains in this case, is the one-dimensional response of the local structure just below the receiver, and the duration of the signals become similar to that of the synthetics obtained with the 1D mode summation method (label MOD in Figures 6.20, 6.21, and 6.23).

6.7. Some properties of the local surface waves derived from the 2D mode summation method

In the finite difference computations, the interpretation of the coda of the signals is rather difficult for a site like station Buia. The heterogeneities exciting the local surface waves are close to the observation point and different modes have about the same arrival time. To overcome this disadvantage of the finite difference method, the computation for model D (Figure 6.9) is performed also with the 2D mode summation method (Vaccari et al., 1989a). Special attention is paid to determining which of the modes is excited inside the sedimentary basin.

In Figure 6.26, the transverse component of acceleration is considered. The receiver is 15 km from the source. The lower two traces are signals computed with the finite difference method, whereas the uppermost trace is a signal obtained with the 2D mode summation method, which shows the contribution of the fundamental Love mode only. For arrival times later than 8 s, the coda is almost completely formed by the fundamental-mode Love wave.

As consequence of our linearly attenuating medium the phase velocity is frequency dependent. This is called the body-wave dispersion. In the case of a detailed knowledge of the soil properties (seismic velocities at the reference frequency and quality factor), the effect of the body-wave dispersion must be introduced into the computations correctly. Even if we never expect to have accurate information about this for our numerical modelling, it is worthwhile to give an example of the influence of the reference frequency on the computed ground motion. Recalling the results from Chapter 3, the approximation of a constant quality factor in the finite difference computations is based on the rheological model of the generalized Maxwell body. The unrelaxed moduli, and therefore the seismic velocities, are generally defined in the high-frequency limit. To get an idea of how the seismic velocities depend on frequency, see Figure 3.11 in Chapter 3. In the 2D mode summation method, the
reference frequency for the moduli can be chosen arbitrarily and has been fixed at 1 Hz.

The two lower traces in Figure 6.26 are the result of the finite difference computation for two different reference frequencies (at which the seismic velocities are defined). The effect of the different reference frequencies can be seen by comparing the uppermost and lowest traces in Figure 6.26. The amplitudes of the two codas are comparable, but the wavetrains are out of phase. Defining the reference frequency in the finite difference computations at 1 Hz—as it is in the modal-summation computations—leads to the seismogram shown in the center of Figure 6.26 (label CORR). Its coda is now in phase with the wavetrain obtained with the 2D mode summation method. There is some difference in the amplitudes of the signals due (1) to the different approximations for the intrinsic attenuation in the two methods, and (2) to the limitation to the fundamental mode in the 2D mode summation technique.

Figure 6.26. Transverse acceleration 15 km from the source. The structural model corresponds to the two quarter-spaces in welded contact (model D). The upper trace shows the contribution of the fundamental-mode Love wave to the signal, obtained with the 2D mode summation method and for a reference frequency of 1 Hz. The two lower traces are the complete signals for two different reference frequencies at which the seismic velocities are defined (middle trace at 1 Hz; lower trace in the high-frequency limit—about 15 Hz).

Some results for the P-SV case are shown in Figure 6.27 for the radial (left column), and vertical component of acceleration (right column). The lower traces in the figure (label FD) show the result obtained with the finite difference method. These are the complete signals, containing all modes as well as diffracted waves. We first consider the contribution of the transmitted fundamental (label FUND in Figure 6.27), and first higher (label 1 in Figure 6.27) mode in cross-section D by summing the contributions of the first 35 incident modes. The restriction to 35 incident modes is reasonable since we are only interested in the coda of the
wavetrain characterized by its relatively low frequencies (around 2 Hz), and group velocities smaller than 3.0 km/s. The results obtained for P-SV waves are quite different than those for the SH case; the contributions of the fundamental and first higher mode to the signal are very small. They cannot explain the amplitudes of the coda that is obtained in the finite difference computation.

The group velocity curve for Rayleigh waves in Figure 6.8 shows that close to the frequency of 2 Hz there is an apparent continuity of the group velocity between adjacent modes. This mode-to-mode continuation leads to an almost continuous group velocity, broken only at the point of near-oscillations. It begins with the first higher mode and contains segments of all successive higher modes. This wave can be interpreted as a high-frequency crustal wave (referred to further on as a HFC-wave). In fact, this HFC-wave can explain a part of the signal’s coda. The upper traces in Figures 6.28 and 6.29 show the transmitted wavefield due to the incident, first 35 modes. The contributions of the closest, eleven outgoing modes to each incident mode have been summed. The results obtained with the 2D mode summation method are compared with the result obtained with the finite difference technique (lower traces in the figures).
All signals are low-pass filtered (5-pole Butterworth filter with corner frequency at 4 Hz) to allow us to discuss the differences in the low-frequency part of the signals. Now, the codas of the signals obtained with 2D modal summation coincide in some parts with the seismograms obtained with the finite difference method. The region of coincidence corresponds to the HFC-wave. Differences exists in the first four seconds of the signals. These are caused primarily by the limited number of modes used in the 2D mode summation method. Another difference appears in the amplitudes of the last part of the coda. (Anelasticity, as the origin of this discrepancy, has been excluded by performing a comparison of the two techniques for the one-dimensional, layered model FRIUL7W.)

**Figure 6.28.** Radial (left column) and vertical (right column) acceleration for a receiver placed 15 km from the source. The structural model corresponds to model D with two quarter-spaces in welded contact. The traces show the result of the finite difference computation (label FD), and the contribution of the first 35 incident modes to the total wavefield (label R/MD: radial component; label V/MD: vertical component).

**Figure 6.29.** The same as in Figure 6.28, but for a receiver placed 14.2 km from the source.
Recalling the observation of the regular pattern of constructive and destructive interference in the codas of the radial components in Figure 6.13, one concludes that in finite difference calculations diffracted waves contribute significantly to the coda, whereas these waves cannot be synthetized with the 2D modal summation. The diffractions have their origin at the edge of the sedimentary basin. On the radial component of motion the diffracted waves interfere constructively with the HFC-waves, as can be seen at a distance of 15 km from the source (left lower trace in Figure 6.28); or destructively, as can be seen at a distance of 14.2 km from the source (left lower trace in Figure 6.29). The vertical component of motion does not show this pattern of interference, since the HFC-waves are polarized in the radial direction.

In this section it has been shown that in sedimentary basins, the behavior of SH waves and P-SV waves are different. The coda of SH waves is essentially made up of the fundamental-mode Love wave, while in the P-SV case, higher Rayleigh-wave modes give a significant contribution in forming the HFC-wave described above. Each higher Rayleigh-wave mode has small surface amplitudes, but their interference generates large amplitudes in the coda of the signals. The P-SV wavefield shows dominant contributions of diffracted waves that originate at the edge of the sedimentary basin.

6.8. The limits of the one-dimensional modelling with the Haskell technique for seismic hazard studies

In engineering seismology, the modelling of seismic ground motion is often based on the Haskell method (Haskell, 1960, 1962), in which the subsurface structure just below the site is assumed to be flat. When the soil layers are horizontal, with few or no lateral irregularities, the application of such one-dimensional models may be good enough to account for site effects (e.g. Joyner et al., 1976; Seale and Archuleta, 1989). As we have seen in the previous examples, such irregularities cause focusing of waves, resonances, and local surface waves; these produce significant variations in the predicted ground response, relative to the response for horizontally stratified soils. It is therefore of interest in engineering seismology to compare the observed signals at station Buia with synthetic seismograms calculated with (1) the Haskell method, (2) the mode summation technique, and (3) the hybrid technique which can account for lateral variations of the subsurface topography. The results for SH-wave propagation are shown in Figure 6.30.
Figure 6.30. Comparison between the recorded transverse component of acceleration and synthetic signals obtained by application of different techniques and models. The source-receiver distance is 15 km. The time scale is shifted by 2 s from the origin time (0 s in the figure is really 2 s origin time). Notation:

OBS: Observed transverse acceleration recorded at station Buia. The seismogram is aligned to agree with the synthetic signals.

INC: Synthetic seismogram obtained for the layered model FRIUL7W without sedimentary cover (see text).

HAS: Synthetic seismogram obtained with the Haskell method for a model with a single sedimentary layer, 57 m thick, and an S-wave angle of incidence of 15°.

MOD: Synthetic seismogram obtained with the mode summation technique for structural model FRIUL7W.

ND4: Synthetic seismogram obtained with the hybrid method for the model with two layered quarter-spaces.

NB4: Synthetic seismogram obtained with the hybrid method for the model corresponding to cross-section B in Figure 6.2.

The Haskell technique—in seismic engineering—excludes a realistic source, and the amplification in the sedimentary cover is studied using an incident, plane polarized body-wave which is propagating from the underlying rock formation through the soil deposit. In general this incident wave is chosen to be a recorded seismogram observed at a hard-rock site. For our computation, the incident wave is taken to be the synthetic signal at the site itself, which is obtained for the layered model FRIUL7W without sediments, using modal summation. This synthetic signal can be viewed as the superposition of incident and reflected waves, but the Haskell method requires the use of only the incident wave. Therefore, the amplitudes of the incident wave (label INC in Figure 6.30) are taken as half of the synthetic signal. The angle of incidence of the plane wave has been varied between 30° and vertical incidence (0°) with no significant variation of the results obtained. This is a consequence of the
strong impedance contrast between bedrock and the sediments, which leads to an almost vertical incidence of the waves at the free surface. In Figure 6.30 only the result for a 15° angle of incidence is shown.

The comparison between the observed signal, the results from the theoretical 2D modelling, and the approximate method of Haskell shows that the synthetic seismograms, obtained with the Haskell method (label HAS), cannot reproduce the waveform and duration of the observed signal. This can be explained by the fact that local surface waves cannot be excited in a laterally-homogeneous, layered structural model. The seismograms obtained with the mode summation method (label MOD), and Haskell’s method, are similar to one another. Consequently, the approximation of the incident wave by a plane shear wave is in this example good. However, if the incoming wavefield is not a plane body wave the results can differ dramatically. When the lateral heterogeneity is far from the observation point, as it is in the model with two layered quarter-spaces (label ND4), the local surface waves are strongly attenuated. Therefore the signals are similar to the one obtained with the Haskell method. (“Far” is used here in the sense that the distance of the observation point from the heterogeneity is much greater than the dimension of the heterogeneity.)

These results have important implications for seismic hazard studies. If we consider sites that are far from strong lateral heterogeneities, these heterogeneities have almost no influence on the maximum acceleration of SH waves. Therefore, for SH waves, in many sites it is a good approximation to take into consideration only the local stratigraphy below the site itself. This result is only partly true for P-SV waves (see Figures 6.21 and 6.23), where the maximum acceleration can vary up to 50 percent for different geometries. Close to the lateral heterogeneities and in places where lateral resonances can occur, such conclusions are not valid anymore, both for SH and P-SV waves.

It is a fact that not only the maximum acceleration is responsible for destructive effects, but also, the duration and frequency content of the signals. Regarding these parameters, techniques such as 1D modal summation and Haskell’s cannot account for the influences of sedimentary basins; therefore, techniques should be used which can treat two-, or three-dimensional structures.
6.9. Fourier spectra and response spectra

The three components of the ground motion recorded or computed at a specific site provide a complete description of the earthquake action on any construction at that site. The most important features of the seismic signals in producing structural damage are their amplitudes, duration, and frequency content. The frequency content of a signal is best exhibited in the form of a Fourier spectrum, or a response spectrum. The Fourier spectrum represents the actual frequency content of the ground motion, whereas the response spectrum is a measure of the response of a simple oscillator to the ground motion.

The time signals that have been analysed are shown in Figures 6.20, 6.21, and 6.23, where the synthetics are in the left columns. The local structures just below the site are identical for all structural models used in the computations. Since the source is also the same in all computations, the differences in the spectra obtained for the different models have to be attributed to the effects of the (non-local) structural model. The Fourier spectra for the transverse, radial and vertical accelerations are shown in Figure 6.31.

The overall trend of the spectra for the synthetic signals is similar, and is determined by the duration of the seismic source (0.6 s), which is a fixed parameter in our numerical modelling. The general shape of the observed spectra is well reproduced by all synthetic signals if we ignore the lack of energy in certain frequency bands of the synthetic signals--this lack being caused by the duration of the point source. The shape of the Fourier spectra in a given frequency band depends mostly on the local surface waves that are excited, their relation to the geometrical dimensions of the lateral heterogeneities, and on the distance of the receiver from the lateral heterogeneity. The more distant the receiver, the smoother the spectrum. The complexity of a spectrum depends, therefore, not only on the complexity of the rupture process of the seismic source, but also on the complexity and geometry of the lateral heterogeneities close to the observation point. The complexity of the observed acceleration spectra in the high-frequency range (above 1 Hz) can be explained by the assumed properties of the sedimentary cover; in the lower-frequency range (below 1 Hz), spectral complexity of the observed acceleration could be caused either by large-scale heterogeneities that have not been considered in the numerical modelling, or by the seismic source.
Spectra like the ones shown in Figure 6.31 are not a very good description of the effects caused by the sedimentary basin. A better representation is given by spectral ratios that include a reference signal. In general, a recorded seismogram observed at a hard-rock site is chosen as the reference signal. For our computation, two reference signals have been
chosen at the site itself. They are both obtained for layered model FRIUL7W: one has the sedimentary cover (label MOD), and the other does not (label LAY). The spectral ratios for the transverse components, for both reference signals, are shown in Figure 6.32. The first type of spectral ratio can be used to describe the two-dimensional effects, while the second illustrates the contrast between the basin and a hard-rock site.

The plot of the spectral ratio (label MOD/LAY in Figure 6.32) that corresponds to the two layered models (MOD and LAY) shows an amplification for the model with the sedimentary cover in the frequency range from 1 to 4 Hz. This amplification can also be observed in all results obtained for the two-dimensional models. This effect indicates that the assumed duration of the seismic source—0.6 s—is not completely justified, since the shape of the spectrum is affected by the presence of the sedimentary basin. We justify selecting a specific duration, as an attempt to minimize the number of parameters used in the modelling. This has also been done by Mao et al. (1990), who determined the duration of the seismic source from the shape of the observed, vertical acceleration spectrum at Station Buia (upper part of Figure 6.33). The spectral ratio between the vertical components for the models with and without the sedimentary cover (lower part of Figure 6.33), shows that the sedimentary cover amplifies the incident wavefield in the frequency range from 1.0 Hz to 2.5 Hz, which is similar to the effect introduced by a finite duration of the seismic source.

The spectral ratios based on the reference signal obtained for the one-dimensional model of the sedimentary basin, are shown on the right of Figure 6.32. The amplification effect between 1 and 4 Hz is now removed. For the same structural model, the general shape of the ratios based on the two reference signals does not change significantly. The spectral-ratio plots are characterized by constructive and destructive interference. The excitation of local surface waves is responsible for the two main peaks, whose frequencies are close to the Haskell resonance frequencies (about 2 and 5 Hz). Destructive interference occurs at frequencies between those of the main peaks.
Figure 6.32. Spectral ratios for transverse accelerations computed with different structural models. The label of each curve indicates the structural model (for more details see Figure 6.20). The spectral ratios for the reference model with the sedimentary cover are at the right; the ratios for the reference model without sediments are shown on the left.
Figure 6.33. The Fourier spectrum of observed vertical accelerations is shown in the upper part of the figure, and is compared with the normalized Fourier transform of the source time dependence corresponding to a 0.6 s duration of the source (dotted line). In the lower part of the figure, the spectral-ratio plot for the two synthetic vertical components (MOD/LAY) is shown.

Quantities illustrating strong ground motion, such as the maximum amplitude, the duration, and the Fourier spectrum, provide only a limited description of the ground motion; they certainly do not quantify its damage-producing potential. A better quantity is the response spectrum of the earthquake ground motion (e.g. see Biot, 1941). The spectra are computed from recorded ground motion and correspond to the maximum response of a simple oscillator by varying both its natural period and damping coefficient. These spectra were introduced by Biot (1941) and applied by Nigam and Jennings (1969).
A common means of representing the response spectrum is the so-called tripartite logarithmic plot, from which the displacement, pseudo-velocity, and pseudo-acceleration spectra can be read simultaneously (Figures 6.34, 6.35, and 6.36). The resulting curves provide a description of the frequency characteristics of the ground motion and give the response of a simple structure to an earthquake. This technique represents an approach intermediate between a design based on static loads, and a complete integration of the equations of motion for complex structures.

In Figures 6.34, 6.35, and 6.36, three response spectra to the transverse components are shown. They correspond to signals obtained for the observed transverse component, and to signals computed with layered model FRIUL7W and two-dimensional model B. Different realistic structural models can lead to significantly different response
spectra. The comparison between the last two response spectra (Figures 6.35 and 6.36) illustrates the difference introduced by a two-dimensional structural model. For the two-dimensional model, there are peak values of the acceleration in the response spectrum that are more than 50 percent larger than the corresponding peak values for the one-dimensional model. These peaks are caused by local surface waves. By assuming large damping of the oscillator, the two response spectra become similar, both in their shape and peak values. The response spectrum of the observed, transverse ground motion (Figure 6.34) has a similar shape as the synthetic spectrum obtained for a two-dimensional model. From the response spectra, the seismic moment can be estimated to about $5.0 \times 10^{24}$ dyne cm, which is in good agreement with the result obtained by Mao et al. (1990). With information obtained from the numerical modelling for different realistic models, it is possible to construct average response spectra for a given site. To establish their validity, they should always be compared with observations.

Figure 6.35. The same as in Figure 6.34 for the synthetic transverse component of motion obtained with the one-dimensional model of the sedimentary basin (structure FRIUL7W). For the source a seismic moment of $10^{24}$ dyne cm has been assumed.
Figure 6.36. The same as in Figure 6.35 for the synthetic transverse component of motion obtained with the two-dimensional model B.
7. An example of numerical modelling for intermediate distances from the seismic source: the city of Rome

One benefit of the important historical role of Rome during the last two thousand years is the large quantity of descriptions of earthquakes that have been felt in the city (Molin et al., 1986; Basili et al., 1987). The most important seismogenetic zones (Figure 7.1) which can cause structural damage in Rome are the Colli Albani (observed maximum intensity in Rome MCS VI-VII), the Central Apennines (observed maximum intensity in Rome MCS VII-VIII) and the Tyrrhenian Sea (observed maximum intensity in Rome MCS V-VI). There is also local seismic activity in the area of Rome (observed maximum intensity in Rome MCS VII). The modification of local geotechnical properties in the last 50 years, due to digging, water pumping, and man-made fill, may also increase current intensities beyond those observed earlier even if the earthquake magnitudes do not exceed historical ones.

Figure 7.1. Approximate epicenter location of the January 13, 1915 earthquake in the Fucino valley (dotted area). The dashed line indicates the cross-section with which the numerical modelling has been performed.
An important event, which caused structural damage in the city of Rome, was the January 13, 1915 Fucino earthquake (Figure 7.1). Since the distribution of damage in the city is well documented, a series of different numerical simulations has been performed. The results are used for comparison between the observed distribution of damage, and certain characteristics of the computed ground acceleration: the peak ground acceleration (PGA), maximum of the response spectra (POR), and the total energy (W) of ground motion.

7.1. The January 13, 1915 Fucino earthquake

The January 13, 1915 Fucino earthquake (Figure 7.1) is one of the strongest events that occurred in Italy during this century. This event, with maximum Intensity XI on the MCS scale, caused damage in the city of Rome (Ambrosini et al., 1986). The damage distribution is shown in Figure 7.2.

The epicenter is localized about 85 km east of Rome (Figure 7.1). The source parameters are given by Gasparini et al. (1985). The strike of the fault--N62°W--coincides well with the geological lineaments of the area and with the observed direction of the main fault (Serva et al., 1986). The source depth is 8 km, the angle between the strike of the fault and the epicenter-station line is 38°, the fault dip 39°, and the rake (with respect to the strike) 172°. The duration of the source is taken to be 0 s. Ward and Valensise (1989), by using geodetic data, estimated the seismic moment ($M_0$) of this event to be $10^{26}$ dyne cm. This value is valid for frequencies below 0.1 Hz, which are much lower than the dominant frequency content of the signals computed to model the effects of the Fucino event in Rome. Consequently, using the wide-band spectrum-scaling law proposed by Gusev (1983), the seismic moment for an average dominant frequency of 2 Hz is estimated as about $10^{24}$ dyne cm. For this value of $M_0$, and at a distance of about 85 km from the seismic source, the maximum horizontal acceleration reaches values of the order of 40-60 cm s$^{-2}$. This is in good agreement with the values estimated from an empirical relation between maximum, horizontal peak ground acceleration, epicentral distance, and seismic moment, proposed by Sabetta and Pugliese (1987) for Italian earthquakes.

The one-dimensional model describing the path from the source to Rome is shown in Table 7.1 (Iodice, 1991). The P-wave velocities in the crust are based on seismic refraction measurements along the profile Latina-Pescara (Nicolich, 1981). The S-wave velocities are chosen by
assuming that $v_S^2 = v_p^2/3$. The Moho depth—28 km—is in good agreement with other published results (Nicolich, 1989; Suhadolc and Panza, 1989). The part of the upper mantle corresponds to the model proposed by Vaccari et al. (1990) for the Irpinia, Italy, area, and is in agreement with surface-waves dispersion measurements (Panza et al., 1980).

**Figure 7.2.** Damage distribution in Rome caused by the January 13, 1915 Fucino earthquake (after Ambrosini et al., 1986), thickness of alluvial sediments (given in meters), and lithology (Ventriglia, 1971; Funiciello et al., 1987; Feroci et al., 1990). Three types of damage are distinguished: slight damage (cracking of plaster, the downfall of small pieces of mouldings), intermediate damage (between slight and heavy damage), and heavy damage (deep and diffuse damage of indoor and outdoor walls, downfall of large parts of mouldings and of chimneys). The dashed line indicates the position of the cross-section, for which numerical modelling has been performed. The distribution of damage within the area limited by the two dotted lines has been projected on the cross-section to construct the histograms shown in Figures 7.7, 7.8, and 7.9.
<table>
<thead>
<tr>
<th>Thickness (km)</th>
<th>Density (g/cm³)</th>
<th>P-wave velocity (km/s)</th>
<th>S-wave velocity (km/s)</th>
<th>Qβ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>2.24</td>
<td>2.40</td>
<td>1.29</td>
<td>30</td>
</tr>
<tr>
<td>0.10</td>
<td>2.25</td>
<td>2.50</td>
<td>1.30</td>
<td>30</td>
</tr>
<tr>
<td>0.10</td>
<td>2.25</td>
<td>2.80</td>
<td>1.50</td>
<td>30</td>
</tr>
<tr>
<td>0.10</td>
<td>2.30</td>
<td>4.00</td>
<td>2.31</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.40</td>
<td>4.10</td>
<td>2.37</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.40</td>
<td>4.20</td>
<td>2.43</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.50</td>
<td>4.30</td>
<td>2.48</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.50</td>
<td>4.40</td>
<td>2.54</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.50</td>
<td>4.50</td>
<td>2.60</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.50</td>
<td>4.60</td>
<td>2.65</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>4.70</td>
<td>2.70</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>4.80</td>
<td>2.77</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>4.90</td>
<td>2.83</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>5.00</td>
<td>2.88</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>5.20</td>
<td>3.00</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>5.40</td>
<td>3.10</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.60</td>
<td>5.60</td>
<td>3.20</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.80</td>
<td>5.70</td>
<td>3.30</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>6.00</td>
<td>3.46</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>5.89</td>
<td>3.40</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>5.80</td>
<td>3.35</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>5.71</td>
<td>3.30</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>5.54</td>
<td>3.20</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>5.40</td>
<td>3.12</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>5.49</td>
<td>3.17</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>5.63</td>
<td>3.25</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>5.71</td>
<td>3.30</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>5.89</td>
<td>3.40</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>6.06</td>
<td>3.50</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>2.85</td>
<td>6.20</td>
<td>3.58</td>
<td>300</td>
</tr>
<tr>
<td>1.70</td>
<td>2.88</td>
<td>6.50</td>
<td>3.75</td>
<td>300</td>
</tr>
<tr>
<td>1.70</td>
<td>2.90</td>
<td>6.70</td>
<td>3.87</td>
<td>300</td>
</tr>
<tr>
<td>2.40</td>
<td>2.95</td>
<td>7.00</td>
<td>4.04</td>
<td>300</td>
</tr>
<tr>
<td>4.75</td>
<td>3.35</td>
<td>7.90</td>
<td>4.56</td>
<td>300</td>
</tr>
<tr>
<td>4.75</td>
<td>3.35</td>
<td>7.92</td>
<td>4.57</td>
<td>300</td>
</tr>
<tr>
<td>4.75</td>
<td>3.35</td>
<td>7.94</td>
<td>4.58</td>
<td>300</td>
</tr>
<tr>
<td>4.75</td>
<td>3.35</td>
<td>7.96</td>
<td>4.59</td>
<td>300</td>
</tr>
<tr>
<td>4.75</td>
<td>3.35</td>
<td>7.98</td>
<td>4.60</td>
<td>300</td>
</tr>
<tr>
<td>4.75</td>
<td>3.35</td>
<td>8.00</td>
<td>4.62</td>
<td>300</td>
</tr>
<tr>
<td>4.75</td>
<td>3.35</td>
<td>8.02</td>
<td>4.63</td>
<td>300</td>
</tr>
<tr>
<td>4.75</td>
<td>3.37</td>
<td>8.04</td>
<td>4.64</td>
<td>300</td>
</tr>
<tr>
<td>4.75</td>
<td>3.38</td>
<td>8.06</td>
<td>4.65</td>
<td>300</td>
</tr>
<tr>
<td>∞</td>
<td>3.39</td>
<td>8.08</td>
<td>4.66</td>
<td>300</td>
</tr>
</tbody>
</table>

Table 7.1. Structural model ROMA5B, representative of the path from the epicenter of the January 13, 1915 Fucino earthquake to the city of Rome (Iodice, 1991). $Q_\alpha = 2.2 Q_\beta$. 
7.2. Numerical modelling for a two-dimensional structural model

A two-dimensional structural model for Rome has been proposed by Iodice (1991), and is shown in Figure 7.3. It is based on the available geological and geotechnical information (Ventriglia, 1971; Funiciello et al., 1987; Boschi et al., 1989; Feroci et al., 1990; Carboni et al., 1991). Alluvial sediments can be found in two areas, the river beds of the Aniene and Tiber. The ancient river bed of the Tiber (referred as Paleotiber) is composed by clays, sands and gravels (Siciliano). This sedimentary complex is covered by volcanic rocks which have their origin in the volcanic activity in the Pleistocene (Ventriglia, 1971). The volcanic rocks have greater wave velocities than the underlying sediments (Siciliano), which therefore defines a low-velocity zone. The surficial layer (thickness of 10 m) consists of compacted fill, and of the foundations of man-made structures. The transition from the alluvial sediments in the Tiber river bed to the compacted clay in the west is characterized by a very high impedance contrast. In Figure 7.3, the compacted clay is referred as bedrock.

The structural model is too large to be treated in a single computer run with the finite difference method. For this reason, the model had to be divided into two parts (cross-section “Paleotiber”, and cross-section “Tiber” in Figure 7.3), that overlap for a length of 4.0 km. This procedure is justified since the features of the seismograms are determined mainly by the local soil conditions, and control of continuity from one cross-section to the other can be made in the overlapping portions of the two cross-sections. For both cross-sections, the same one-dimensional model (Table 7.1) describing the path from the source position to the two-dimensional structure is used. The high-frequency limit has been chosen to be 4 Hz, which is in agreement with the experience derived from strong motions generated by magnitude 5-to-6 earthquakes recorded in Italy (e.g. CNEN-ENEL, 1977; Vaccari et al., 1990). Using a point-source approximation, to these earthquakes a source duration between 0.4 and 0.6 s is assigned. This leads to a strong reduction of the energy for frequencies above 3-5 Hz (Aki and Richards, 1980).
Figure 7.3. Two-dimensional model (uppermost cross-section) corresponding to the dashed line shown in Figure 7.2. Only the part near the surface is shown, where the 2D model deviates from the horizontally-layered structural model. For practical reasons, the numerical modelling has been carried out for two parts separately: the cross-section "Paleotiber" (middle cross-section), and the cross-section "Tiber" (lowest cross-section).
Figure 7.4. Acceleration time series for SH waves and P-SV waves at an array of receivers over cross-section "Paleotiber" which is shown in Figure 7.3. All amplitudes are related to a point source with seismic moment of $10^{24}$ dyne cm. For each component, the signals are normalized to the same peak acceleration. This peak acceleration is indicated in units of cm s$^{-2}$. The time scale is shifted by 20 s from the origin time (0 s in the figure is really 20 s origin time).
Figure 7.5. The same as in Figure 7.4 for an array of receivers over cross-section "Tiber" which is shown in Figure 7.3.
The acceleration time series obtained for arrays of receivers over cross-sections "Paleotiber" and "Tiber" are shown in Figures 7.4 and 7.5. The main features of the time series are two peaks which can be related to the Sg and the Lg phases. In general, due to the presence of low-velocity sediments at the surface, the radial component is two-to-three times larger than the vertical component. The transverse component is about half the size of the radial component. This is due to the SH and P-SV radiation patterns of the source. There are several characteristics of ground motion that can be related to the local structures. The strongest effects are caused by the presence of alluvial sediments in the river beds of the Tiber and Aniene. Signals have larger amplitudes and durations due to the low impedance of the alluvial sediments, resonance effects, and the excitation of local surface waves. The effects are the strongest in the deeper alluvial basin of the Tiber river.

Minimum accelerations can be observed for receivers placed above the volcanic layer. This layer acts as a shield, reflecting part of the incoming energy; the thicker the layer, the smaller the observed surface accelerations. The underlying layer (Siciliano) can function as a waveguide, and at places where this waveguide and the overlying volcanic layer are thinning, focusing effects occur and the trapped energy reaches the surface (receivers R21, R22, and R23). This leads to large amplitudes of the ground motion with respect to the values obtained in the area with a thick volcanic layer.

7.3. Properties of strong ground motion in comparison with the observed distribution of damage

The synthetic seismograms shown in Section 7.2 can be used to compute some ground-motion related quantities—commonly used in seismic engineering—that are reasonably well correlated with structural damage. These quantities are (1) the peak ground acceleration (PGA), (2) the peak acceleration of the response of a simple undamped oscillator to the ground motion (peak oscillator response, POR), which corresponds to the maximum acceleration of the response spectrum, and (3) a quantity W, defined in Equations 7.1 and 7.2.
An energy-related quantity, known as the Husid plot, is defined as

\[ w(t) = \int_{0}^{t} [\ddot{x}(\tau)]^2 \, d\tau, \quad (7.1) \]

where \( x \) is the ground displacement. The function \( w(t) \) is monotonically increasing. The so-called "total energy of ground motion" or "total energy of acceleration" \( W \) (Jennings, 1983), which is related to the Arias Intensity, is defined as

\[ W = \lim_{t \to \infty} w(t). \quad (7.2) \]

We make use of the acceleration time series obtained for the two cross-sections "Paleotiber" and "Tiber". In the two sections, the signals at the same distance from the source differ due to the different propagation paths. The abrupt change from the one-dimensional model, to the model "Tiber", can lead to local surface waves and diffractions. In fact, at the beginning of the zone of overlapping, the acceleration time series obtained for the two cross-sections differ strongly (Figures 7.4 and 7.5). Inside the zone of overlapping, the signals become more and more similar, as the distance from the abrupt discontinuity in the cross-section "Tiber" increases. In the central part of the zone of overlapping, the signals are similar, as is shown for the receiver R25 in Figure 7.6. The differences are due to dispersion effects in the ancient, Tiber river bed, and to diffractions at the vertical interface in cross-section "Tiber".

The critical point is to combine the information obtained from the two cross-sections. Therefore, PGA, POR, and \( W \) have been computed for the signals obtained for the two cross-sections "Paleotiber" and "Tiber". In the central part of the zone of overlapping (81.5-83.0 km from the source), the values obtained for the two cross-sections differ only by some percent, where the maximum difference in PGA for the horizontal components of motion, is up to 30 percent; in POR, up to 35 percent, and in the quantity \( W \), up to 30 percent. If the transition from one cross-section to the other is performed, independently for each horizontal component and each quantity (PGA, POR, or \( W \)), when the difference reaches its minimum, the errors introduced by the use of two distinct cross-sections are much smaller than the variations in PGA, POR, and \( W \) over the entire array. Following this procedure, we can extract from our signals a set of values of PGA, POR, and \( W \), which can be compared with the distribution of damage from the Fucino earthquake.
Figure 7.6. Comparison between the signals obtained at the same distance from the source for the two cross-sections “Paleotiber” and “Tiber”. The transverse (on the left), radial, and vertical (on the right) accelerations are shown. The lower traces are related to the cross-section “Paleotiber”, the upper traces, to the cross-section “Tiber”. The epicentral distance is 82.8 km. The source has a seismic moment of $10^{24}$ dyne cm. The signals are normalized. The peak acceleration is in units of cm s$^{-2}$. At this distance from the source, the differences in PGA and W are 8 percent and 24 percent for the transverse components, 17 percent and 16 percent for the radial components, and 5 percent and 12 percent for the vertical components.

The distribution of damage from the Fucino earthquake (Figure 7.2) has been projected on the cross-section used in the numerical modelling. Only those points of the distribution have been used which are situated in an area where the geometry of the structure does not differ too much from the geometry of the two-dimensional cross-section. The limits of this area are indicated in Figure 7.2. Each damaged building has been projected on the cross-section. The criteria for this projection are the depth of the sedimentary cover and the lithology at the site. The points on the cross-section have then been summed up for regular segments. The resulting histogram is shown in Figures 7.7, 7.8, and 7.9. These damage statistics should be interpreted only in a qualitative manner, as neither the type of buildings nor the density of urbanization has been taken into account. Moreover, the shape of the histogram depends on the choice of the segments' size. Not influenced from this choice are the two peaks at the edges of the alluvial basin of the Tiber, and the concentration of the heavy and intermediate damage in this basin. The approximate limits of the area of urbanization in 1916 are indicated by two arrows in the figures.
The comparison between the observed damage in the city of Rome, with the spatial distribution of PGA, POR and W for the transverse component of motion, is shown in Figure 7.7. Also shown are some results obtained with the one-dimensional layered structure. For these 1D models, the waveform does not change significantly along the array of receivers. Peak ground acceleration (PGA) increases with increasing distance from the source (Figure 7.7a), and this fact can be defined as the regional trend of PGA. This behavior of the PGA is due to the incoming of postcritically reflected shear waves from the Moho for epicentral distances larger than 80 km (Campillo and Bouchon, 1983; Suhadolc and Chiaruttini, 1985).

The structural model described by the finite difference grid does not include structural features extending to large depths. This causes incompleteness of the signals at large distances from the vertical grid lines, where the incoming wavefield is introduced into the finite difference computations. As a consequence, for the last few receivers, the PGA value obtained with the hybrid method is systematically smaller than that from the mode summation method (Figure 7.7a). The curves obtained for a 2D structural model also contain this loss of energy at the end of the model. The difference between the results obtained with 1D modal summation, and the 1D finite difference method, provides us with an easy way to correct the 2D results. The corrections are shown in Figures 7.7a and 7.7c.

For the two-dimensional structural model, high PGA are observed where the impedance of the surficial sediments is small (Figure 7.7a); PGA values are low where the volcanic layer is thick. Relative peaks of the curve are located at the beginning of the alluvial valley of the Tiber and within the alluvial valley of the Aniene. The peak oscillator response (Figure 7.7b) generally shows the same trend as the curve for the PGA. There are four peaks in the POR curve: at the edges of the Tiber basin, within the alluvial valley of the Aniene, and a broad peak at the end of the Siciliano low-velocity zone. The peaks are even stronger for W, the total energy curve (Figure 7.7c). There is good agreement between the observed damage and the total energy, or the POR curve of the transverse component of motion. This result, therefore, confirms the validity of using W as a reliable parameter for strong ground motion in seismic hazard studies. Since the lateral heterogeneities included here have small influence on the maximum acceleration of the SH-wave signals, the correlation between the damage and the PGA curve, even if qualitatively the same, is less impressive.
Figure 7.7a,b. (a) Peak ground acceleration (PGA), and (b) peak oscillator response (POR), obtained for the one-dimensional model given in Table 7.1, and the two-dimensional model of Rome shown in Figure 7.3. They are compared with the damage distribution caused by the January 13, 1915 Fucino earthquake. The thin solid line indicates the two-dimensional results corrected for the loss of signal due to the vertical limitation of the used finite difference grid.
figure 7.7c. Total energy (W) of the transverse acceleration, obtained for the one-dimensional model given in Table 7.1, and the two-dimensional model of Rome shown in Figure 7.3.

Similar conclusions can be drawn for PGA and W of acceleration for the radial component (Figure 7.8). There is a strong peak at the edge of the ancient Tiber bed which is not observed for the SH waves. This difference is due to the different frequency content of SH and P-SV components of motion, and to different physical processes, e.g. the S- to P-wave conversion for P-SV waves at strong impedance contrasts.

The source effect and regional trend in PGA and W can be removed by calculating the ratios between the values obtained for the two-dimensional model and those obtained for the one-dimensional model. The ratios computed for the SH wavefield (Love waves), the P-SV wavefield (Rayleigh waves) and the radial component of motion are shown in Figure 7.9. The curves corresponding to the radial component are similar to the curves of the P-SV wavefield since the former component of motion is largely dominant.
Figure 7.8. Peak ground acceleration and total energy of the radial acceleration, obtained for the one-dimensional model given in Table 7.1, and the two-dimensional model of Rome shown in Figure 7.3. They are compared with the damage distribution caused by the January 13, 1915 Fucino earthquake.
Figure 7.9. Ratios of the peak ground accelerations, $A_{2D}/A_{1D}$, and of the total energy of acceleration, $W_{2D}/W_{1D}$, between the results obtained for the two-dimensional, and one-dimensional models. They are compared with the damage distribution caused by the January 13, 1915 Fucino earthquake.
The representation of the results given in Figure 7.9 can be used directly for zonation purposes with respect to the Apennines seismogenetic area. The results show local site effects with respect to a layered structural model (bedrock). Four zones can be distinguished: (1) the bedrock, which is chosen as the reference for the zonation; (2) zone above the volcanic layer (the thicker this layer, the smaller the accelerations and seismic energies observed at the surface); (3) zones inside the sedimentary basins where large accelerations and seismic energies are observed at the surface; and (4) the edges of the sedimentary basins, which are characterized by the greatest accelerations and seismic energies.

These four zones can also be distinguished in the observed damage distribution from the Fucino earthquake. Since the correlation is very good between the damage statistics and the ground-motion related quantities, PGA and W, it is possible to extend the zonation to the entire area of Rome, thus providing a basis for prediction of expected damage from a future event similar to the one studied.

7.4. Results in the frequency domain

A good representation of 2D effects is given by the computation of spectral ratios between the 2D and 1D signals from the same site. The result for the transverse component is shown in Figure 7.10, where these ratios are illustrated as a function of frequency and spatial location along the section. The darker an area, the stronger the amplifications that characterize the two-dimensional model with respect to its 1D analog. The greatest amplification is observed at the western edge of the sedimentary basin of the Tiber river (87 km from the source), for frequencies around 2 Hz. The maximum amplification with respect to the one-dimensional model is of the order of 5-6. This factor is due to the combination of resonance effects and the excitation of local surface waves.

The general distribution of the shaded areas can be related to the geometry of the structural model. An amplification over almost the entire frequency band in Figure 7.10 is seen outside the Paleotiber basin (82-87 km from the source). The strongest effects are seen in the deep alluvial sediments of the Tiber basin. Some amplification effects can be seen in the Aniene basin also, for frequencies above 2 Hz. For frequencies above 0.8 Hz, in the Paleotiber basin the volcanic layer acts as a shield reflecting part of the incoming energy, and the values of the spectral ratios are smaller than 1. The underlying sedimentary complex (Siciliano) causes amplifications (spectral ratios between 4 and 5) due to resonances. These
Figure 7.10. Spectral ratios for transverse accelerations over the entire cross-section. The reference signal is the one obtained for the one-dimensional layered model ROMA5B.

are most pronounced at frequencies around 0.4 Hz, where the fundamental resonance of this low-velocity channel is excited. In this zone of the Paleotiber, in the frequency band 1.5-2.0 Hz, there is also evidence for the excitation of some higher modes of resonance. The resonance of the Siciliano layer around 0.4 Hz is easily recognized in the spectral ratios, but due to the very small wave energy below 1 Hz, it cannot be seen in the time-domain signals. Therefore, the presence of a near-surface layer of rigid material is not sufficient to classify a site as a "hard-rock site". A correct zonation requires knowledge of both the thickness of the surficial layer and of the deeper parts of the structure, down to the real bedrock. This is especially important in volcanic areas, where lava flows often cover alluvial basins.
At distances of the order of 82-83 km from the source, between the Paleotiber and Tiber basins, where the waveguide and overlying volcanic cover are thinning, focusing effects occur and the trapped energy reaches the surface. This leads to amplifications over almost the entire frequency band being considered, and values of the spectral ratios are of the order of 2.

When considering the radial component of motion, the situation is more complicated (Figure 7.11). The sedimentary cover causes polarization of P-SV waves into the radial component. This polarization then gives rise to a series of scattered dark zones--high amplifications--for frequencies higher than 1 Hz; in the frequency band 0.8-1.0 Hz, the polarization and amplification effect is present along almost the entire section. The amplification at 0.8-1.0 Hz leads to a maximum value of the spectral ratio of 25.0 at 80.0-80.5 km from the source. For frequencies above 1 Hz, the polarization effects are highly localized, both in frequency and distance from the source, so that at neighboring sites we can have...
large differences in the seismic response. Therefore, the polarization (amplification) in the radial component of motion can explain the very localized damage that is often observed.

Spectral ratios can also be computed for the total P-SV wavefield by considering both the radial, and vertical component. The result of this combination is shown in Figure 7.12. Since the amplitudes of the radial component are about double than those of the vertical component, the result is similar to the spectral ratios obtained for the radial component alone. A shift of energy from the vertical to the radial component of motion disappears in this representation, as long as there is no amplification of the incident waves. This occurs in the frequency band 0.8-1.0 Hz, at which a polarization was present for the spectral ratios of the radial component along almost the entire cross-section.

An amplification can be observed in and around the river bed of the Aniene at frequencies of the order of 1 Hz. This amplification was less emphasized in the spectral ratios of the radial component, and therefore mostly affects the vertical component of motion. At frequencies above 2 Hz, the features of the spectral ratios are rather complex. They are caused by constructive and destructive interference of the incident waves.

The example shown in this chapter illustrates the use of 2D modelling for seismic hazard studies in urban areas. Local resonance effects, frequency-dependent polarization of the P-SV wavefield into the radial component of motion, and the excitation of local surface waves can cause large amplifications of the incident wavefield and long durations of the ground motion. Synthetic seismograms can be used to estimate the maximum accelerations and total energies. Spectral ratios allow the identification of frequency bands in which amplification effects occur, and also the sites at which amplification effects occur.
Figure 7.12. The same as Figure 7.10, for the total P-SV wavefield.
8. An example of numerical modelling far from the seismic source: Mexico City

Mexico City has experienced extensive damage in the recent past due to strong earthquakes with hypocenters in the Mexican subduction zone. The Michoacan earthquake of September 19, 1985 (Ms=8.1), together with its aftershocks, produced the worst earthquake damage in the history of Mexico. More than 10,000 people died in Mexico City, 300,000 people were rendered homeless, and about 1000 buildings were destroyed (Beck and Hall, 1986). Although the epicenter of the earthquake was close to the Pacific coastline (Figure 8.1), damage at coastal sites was relatively small. The reason for this is that most of the populated areas near the coast are sited on hard bedrock. In contrast, Mexico City, which is about 400 km from the epicenter, suffered extensive damage. This can be attributed to the geotechnical characteristics of the sediments in the valley of Mexico City.

The numerical modelling of the Michoacan earthquake, and the effects of this earthquake in Mexico City, are presented in this chapter. This modelling exhibits some interesting numerical problems that arise from the large distance of Mexico City from the seismic source. This distance causes a long duration of the seismic signals in Mexico City, and the presence there of sediments with very low shear-wave velocities forces a small grid spacing in the finite difference computations. The computer memory required to contain a model with an extension of 400 km--from source position to Mexico City--makes it impossible to model this event and its effects in Mexico City with finite difference methods alone. On the other hand, modal summation which can deal with such distances, is limited to quite simple 2D geometries and cannot successfully model the complex geological situation in Mexico City.

As the Michoacan earthquake was one of the most precisely monitored events affecting a large metropolitan area, it is of particular interest to compare observed and computed strong ground motion. Unfortunately, the mechanical properties of the sediments and geology in Mexico City, and the source of the Michoacan earthquake, are poorly known. Also, the structural models used by different authors exhibit much variability. Therefore, a sensitivity study of the computed ground motion is performed with respect to certain parameters of the structural models and the source.
Figure 8.1. Map of central Mexico (after Eissler et al., 1986) showing the approximate aftershock areas of subduction events since 1950, with M>7 (Astiz and Kanamori, 1984; Eissler et al., 1986). The dashed line outlines the aftershock area of the 1932 Jalisco event (Singh et al., 1985). The shaded areas correspond to the earthquake rupture zones of the three subevents of the main shock of the Michoacan 1985 earthquake found by Houston and Kanamori (1986), and the two stars indicate the epicenters of the main shock and the largest aftershock.

8.1. Geological and geotechnical information in Mexico City

Before the Pleistocene, the region of Mexico City was a river valley (e.g. Sánchez-Sesma et al., 1988b). The valley was closed due to volcanic activity at the end of the Pliocene, which built up the Sierra Chichinautzin. The valley was then transformed into a sedimentary basin during the early Quaternary. Many lakes were present inside this basin, maintained by melting water from the mountains around the valley. Due to lack of a natural drainage, the lakes progressively dried and were filled with eruptive products and clayey lacustrine sediments.

From the geotechnical point of view, the valley of Mexico City can be divided into three zones (Figure 8.2A): the hill zone, transition zone, and lake-bed zone. The hill zone is formed by alluvial and glacial deposits, and
by lava flows. The transition zone is mainly composed of sandy and silty layers of alluvial origin. The surficial layers in the lake-bed zone consist mainly of clays. These deposits are poorly consolidated, with high water content and very low rigidity. The geometrical characteristics and mechanical properties are quite well-known from different borehole, and laboratory tests. The surficial layer, as shown in Figure 8.2C (after Suarez et al., 1987), varies between 10 and 70 m in thickness, where this thickness increases regularly towards the east. The topmost layer is composed of compacted fill, and of the foundations for man-made structures (Chávez-Garcia and Bard, 1990). It is more resistant than the clay, and its thickness can be up to 10 m.

The clay layer is situated over the so-called "deep sediments" found below 10-70 m. These deeper deposits reach depths down to 700 m, where the uncertainty of the thickness of the deep sediments may be as large as a few hundred meters (e.g. Bard et al., 1988). The mechanical characteristics of the deep sediments are very poorly known; the topography of the bedrock interface is shown in Figure 8.2D, having been estimated from boreholes and gravimetric data (Suarez et al., 1987). There are three outcrops of the basement: at Chapultepec, Peñon, and Cerro de la Estrella, and there are three ancient canyons.

<table>
<thead>
<tr>
<th>Type of material</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>$\alpha$ (km/s)</th>
<th>$\beta$ (km/s)</th>
<th>$\nu$</th>
<th>$Q_\alpha$</th>
<th>$Q_\beta$</th>
<th>Author</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fill deposits</td>
<td>1.7</td>
<td>1.5</td>
<td>0.20</td>
<td>0.491</td>
<td>100</td>
<td>50</td>
<td>1*</td>
</tr>
<tr>
<td>Clay</td>
<td>1.3</td>
<td>1.5</td>
<td>0.08</td>
<td>0.499</td>
<td>50</td>
<td>25</td>
<td>1*</td>
</tr>
<tr>
<td>Clay</td>
<td>1.5</td>
<td>0.08</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
<td>2*</td>
</tr>
<tr>
<td>Deep sediments</td>
<td>2.0</td>
<td></td>
<td>0.10</td>
<td>0.333</td>
<td>100</td>
<td>50</td>
<td>1*</td>
</tr>
<tr>
<td>Deep sediments</td>
<td>2.0</td>
<td></td>
<td>0.4-1.0</td>
<td></td>
<td></td>
<td></td>
<td>2*</td>
</tr>
<tr>
<td>Deep sediments</td>
<td>2.0</td>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td>3*</td>
</tr>
<tr>
<td>Deep sediments</td>
<td>2.0</td>
<td></td>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
<td>4*</td>
</tr>
<tr>
<td>Bedrock</td>
<td>2.0</td>
<td></td>
<td>1.00</td>
<td>0.333</td>
<td></td>
<td></td>
<td>5*</td>
</tr>
<tr>
<td>Bedrock</td>
<td>2.67</td>
<td></td>
<td>2.0-2.8</td>
<td></td>
<td></td>
<td></td>
<td>2*</td>
</tr>
<tr>
<td>Bedrock</td>
<td>2.1</td>
<td></td>
<td>2.00</td>
<td></td>
<td></td>
<td></td>
<td>3*</td>
</tr>
<tr>
<td>Bedrock</td>
<td>2.50</td>
<td></td>
<td>0.333</td>
<td></td>
<td></td>
<td></td>
<td>4*</td>
</tr>
</tbody>
</table>

Table 8.1. Mechanical properties of the surficial layers, deep sediments, and bedrock, proposed by different authors: 1*) Chávez-Garcia and Bard, 1990; 2*) Bard et al., 1988; 3*) Campillo et al., 1988; 4*) Sánchez Sesma et al., 1988b; 5*) Kawase and Aki, 1989.
Figure 8.2. Maps of the area of Mexico City. On each map the location of the strong motion accelerometric stations is shown; these stations were operating during the 1985 Michoacan event. The location of stations CU01 and CUIP is the same. A) Subdivision of the valley of Mexico City, following geotechnical criteria, into the hill zone, transition zone, and lake-bed zone. B) Damage distribution produced by the 1985 Michoacan earthquake. Crosses stand for the zones with building collapse and severe structural damage (Meli and Miranda, 1986). C) Thickness contours of the lacustrine clay (Suarez et al., 1987). D) Thickness contours of the "deep sediments" (Suarez et al., 1987).
The mechanical properties of the different materials in the area of Mexico City have been investigated by many authors (e.g. Zeevaert, 1964; Suarez et al., 1987). These properties exhibit a great variability, especially the surficial clay. The values used for numerical modelling by different authors are shown in Table 8.1. Large values of Poisson's ratio \( v \) characterize the fill deposits and the clay. Some mechanical properties of the surficial layers, based on unpublished geotechnical data, have been proposed by Chávez-Garcia and Bard (1990). For the deeper sediments, the mechanical properties are poorly known.

8.2. Damage distribution in Mexico City

After the 1985 Michoacan earthquake, the most severe damage was reported within the lake-bed zone of Mexico City, approximately between the bedrock outcrops of Chapultepec and Peñon. The distribution of damage is shown in Figure 8.2B, without distinction for the type of damaged buildings. About the same distribution of damage was observed after the 1957 Acapulco Earthquake (Ms=7.5), which occurred in Southern Guerrero (Figure 8.1), and after the 1979 earthquake in the Petatlan area in the Northern Guerrero gap. Damage is obviously related to the characteristics of the sedimentary cover: it is concentrated where the clay is 20 to 50 m thick, and the deep sediments, between 200 and 400 m thick.

8.3. The observed signals in Mexico City

During the Michoacan earthquake, a strong motion network was operating in the valley of Mexico City (Mena et al. 1986). The positions of the stations are shown in Figure 8.2. Some of these were located in the lake-bed zone (SCT1, CDAO, CDAF), some in the hill zone (TACY CUIP, CU01, CUMV), and one in the transition zone (SXVI). The observed accelerations and displacements are shown in Figures 8.3 and 8.4. The records are corrected for the instrumental response and have been convolved with a high-pass Ormsby filter whose largest low-frequency cutoff is 0.10 Hz for the stations outside the lake-bed zone, and 0.07 Hz for those inside the lake-bed zone (Mena et al., 1986). These frequency limits do not influence our conclusions since the synthetic signals have dominant energy above these frequency limits.
Figure 8.3. Horizontal and vertical components of acceleration recorded in the valley of Mexico City (Mena et al., 1986). The NS-component of motion is denoted by A/NS, the EW-component by A/EW, and the vertical component by A/UP. The amplitudes of all signals are normalized to the same values, given in cm s$^{-2}$. 
Figure 8.4. Same as in Figure 8.3 for the displacements. The amplitudes of all signals are normalized to the same values, given in cm.
Absolute time references are absent in the recorded signals. To estimate the relative times to an arbitrarily chosen zero-time, the signals have been shifted so that the long-period part of the vertical displacements are in phase (Campillo et al., 1988). This is justified since the long-period vertical displacements (Figure 8.4, label D/UP) have nearly identical waveforms and amplitudes at all stations. The time shifts have been given by Bard et al. (1988): CDAF (2.00 s), CDAO (35.75 s), SCT1 (26.00 s), TACY (40.00 s), CUIP (3.00 s) and CU01 (6.50 s). The time shifts for the stations SXVI and CUMV are not given in the literature, and are here determined to be 4.50 s and 5.00 s, respectively.

The differences between the records in the lake-bed zone, and the records in the hill zone, reflect the differences at the site of the instruments. In the lake-bed zone (stations SCT1, CDAO and CDAF) the horizontal components of motion are the dominant ones. They have a larger high-frequency content than the corresponding vertical, or horizontal components recorded outside the lake-bed zone. The maximum acceleration in the lake-bed zone reaches values of the order of 0.2 g, which is quite a large value considering the distance of 400 km from the source. On solid rocks, the peak horizontal acceleration is only 0.03 g. The spectral peaks of acceleration are the interval between 0.2 Hz and 0.5 Hz (periods between 2 and 5 s) (Singh et al., 1988).

By studying the 1D response of the surficial clay layer with the Haskell method, it is possible to reproduce the differences in amplitude between records from the different areas of Mexico City (Romo and Jaime, 1986; Sánchez-Sesma et al., 1988b; Kawase and Aki, 1989). One of the unexplained aspects of the ground motion is its duration inside the lake-bed zone, which reaches values of up to 180 s (see the record at station CDAO, with its almost rectangular envelope). Possible explanations for the long signal durations can be found in two-dimensional effects: resonances or the generation of local surface waves (e.g. Sánchez-Sesma et al., 1988b; Bard et al., 1988), or in gravity effects (Del Valle-Toledo, 1986; Lomnitz, 1990). As we will see in the following sections, conceptually simple modelling of the source, path, and local soil effects, based on models that have already been published, are also capable of explaining the long duration of observed signals.
8.4. A structural model for Central Mexico

The flat-layered structure, describing the path from the seismic source to the valley of Mexico City, is shown in Table 8.2. This model has been proposed by Campillo et al. (1989), and was deduced directly from refraction measurements in the Oaxaca, Southern Mexico region (Valdes et al., 1986). The structural model is rather simple, reflecting the resolving power of the available data. The depth of the Moho is about 45 km, and the upper five kilometers are composed of relative low-velocity material. The structural model does not include a realistic model of the upper mantle, and assumes an average value there for the density and the seismic velocities. The influence of this simplification is very small and will be discussed in Section 8.6.1.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (km)</th>
<th>( \rho ) (g/cm(^3))</th>
<th>( \alpha ) (km/s)</th>
<th>( \beta ) (km/s)</th>
<th>( Q_\alpha )</th>
<th>( Q_\beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.0</td>
<td>2.67</td>
<td>4.30</td>
<td>2.53</td>
<td>800</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
<td>2.77</td>
<td>5.70</td>
<td>3.30</td>
<td>800</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>15.0</td>
<td>3.09</td>
<td>6.80</td>
<td>4.03</td>
<td>800</td>
<td>500</td>
</tr>
<tr>
<td>4</td>
<td>15.0</td>
<td>3.09</td>
<td>7.00</td>
<td>4.10</td>
<td>800</td>
<td>500</td>
</tr>
<tr>
<td>5</td>
<td>( \infty )</td>
<td>3.30</td>
<td>8.20</td>
<td>4.82</td>
<td>800</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 8.2. Crustal model for the path from the source in the Michoacan subduction zone, to Mexico City.

8.5. Source characteristics of the 1985 Michoacan earthquake

The epicenter of the earthquake was close to the Pacific coastline (Figure 8.1). It is a typical intraplate subduction event, with a small dip angle and striking parallel to the Central-America trench. The rupture started in the north, and propagated through the zone of the 1981 Playa Azul earthquake with a low moment release (Eissler et al., 1986; Houston and Kanamori, 1986). It then triggered an asperity in the south. Least-squares inversion of the Michoacan earthquake records, for the source time function, yielded three source pulses (Houston and Kanamori, 1986). The first two subevents have similar seismic moments (1.1-1.7\( \times \)10\(^{28}\) dyne cm) (Eissler et al., 1986) and durations (about 15 s); the
moment of the third is about 20 percent that of the first subevent, and has a duration of about 10 s. The two large events can be seen in the near-field strong motion records. The arrival time of the second event is shifted by 26 s, with respect to the first, and the location is about 80 km to the south-east of that for the first event. The third event occurred 21 s after the second one, and its position is about 40 km seawards of the second event (Figure 8.1).

The Michoacan earthquake is characterized by high spectral amplitudes for periods between 2 and 5 s. These short-period arrivals can be interpreted as Lg waves, which in terms of normal modes correspond to the first 15 to 20 higher modes (Panza and Calcagnile, 1975). Campillo et al. (1989) attribute the enhanced energy in the 2-to-5-second period range to the irregularity of the rupture propagation. They suggested that the rupture developed as a smooth crack towards the ocean. A fact that supports the interpretation of Campillo et al. (1989) is that the 2-to-5-second motion is also found in the teleseismic P-waveforms of the broadband records from digital stations of different networks (Zirbes et al., 1985).

For most of the computations presented in this chapter, an average source model for the 1985 Michoacan earthquake has been chosen. We restrict on a simple point source with a duration of 0 s. This allows us to study the behavior of the waves in the entire frequency band from 0.01 to 1 Hz, with no a priori assumption about the frequency content of the source. The focal mechanism (Figure 8.1) corresponds to the one proposed by Campillo et al. (1989), based on the results of Houston and Kanamori (1986) and Riedesel et al. (1986). The distance from the source to the valley of Mexico City is 400 km, the angle between the strike of the fault and the epicenter-station line is 220° (further on, referred to as the strike-receiver angle), the source depth is 10 km, the dip 15°, and the rake is 76°.

Computed seismograms are very sensitive to certain parameters of the source, e.g. the depth of the hypocenter, duration, and strike-receiver angle. Therefore, a sensitivity study of the computed ground motion is performed with respect to these parameters. The dependence of the displacements on different strike-receiver angles is shown in Figure 8.5A. For different strike-receiver angles, the shapes of the signals are similar. The maximum amplitude of the signals change for different strike-receiver angles due to the SH and P-SV radiation patterns of the source. The smallest amplitudes for Love and Rayleigh waves are observed at two different angles. These angles differ by 90°; Rayleigh waves have the smallest amplitudes for a strike-receiver angle of about
170°, and Love waves have the smallest amplitudes for an angle of about 260°. For a strike-receiver angle of 220°, the amplitudes of the three components of motion are comparable. During the Michoacan earthquake, this was also observed for locations outside the lake-bed zone (Figure 8.4), and therefore, it is reasonable to use this strike-receiver angle (220°) in the numerical modelling. For the expected earthquake in the Guerrero gap, this sensitivity study also provides a basis for prediction of displacement in Mexico City: by assuming the source mechanism of the Michoacan earthquake, one should expect Rayleigh waves to be dominant over Love waves.

Point sources at different depths are considered in Figure 8.5B. A depth of 5 km corresponds to the depth of the Pacific Ocean just off the Mexican coastline, and is therefore the lower limit for the source. As depth decreases to this limit, the maximum amplitude of displacement increases. For a source depth of 5 km the fundamental-mode Rayleigh and Love waves are the dominant phases on the seismograms. For this shallow source, the computed displacements show the long-period fundamental Rayleigh-wave mode superimposed on the Lg phase (2-to-5-second period), as was observed during the Michoacan earthquake (Figure 8.4); for a source depth of 10 km, however, the Lg waves and the long-period fundamental-mode Rayleigh wave have different arrival times. Since Campillo et al. (1989) used an average source depth of 10 km, they concluded that the Lg waves were emitted some time after the beginning of the rupture process. This interpretation of the records is not unique, as can be seen immediately from the group velocity curves (Figure 8.6). The fundamental-mode Rayleigh waves, at frequencies 0.05 to 0.1 Hz, can have group velocities the same as those of Lg waves at higher frequencies (first higher mode in the frequency range 0.2 to 1.0 Hz).

The displacement time series for three different durations of the seismic source are shown in Figure 8.5C. A duration of 15 s corresponds to the duration proposed by Houston and Kanamori (1986) for the two first subevents of the earthquake. The effect on the seismograms at relatively small distances from the source is obvious, since the time duration of the source acts as a low-pass filter; it leads to a strong reduction of the energy for higher frequencies. A similar effect, on the frequency content of the signals, can be obtained by introducing a source of appropriate finite extension in space.
Figure 8.5. Variation of the displacements with changes in the strike-receiver angle, the hypocentral depth, and the duration of the seismic source. The transverse (left column), radial (middle column) and vertical displacements (right column) correspond to a source with seismic moment of 1 dyne cm. The signals are normalized. The peak displacement is in units of cm. The three parts of the figure correspond to: A) the dependence of the displacements on strike-receiver angle (180°, 200°, 220°, 240° and 260°); B) the dependence of the displacements on hypocentral depth (5 km, 10 km and 15 km); and C) the dependence of the displacements on source time duration (0 s, 3 s and 15 s).
Figure 8.6. Group velocity curves for Love and Rayleigh waves for the structural model shown in Table 8.2.

8.6. Numerical modelling for Mexico City

One-dimensional structural models for the Mexico City valley will be considered first, with the purpose of studying the one-dimensional response of the sedimentary cover. The two-dimensional models that will then be considered, describe the cross-section Chapultepec-Peñon of the pre-Chichinautzin basin (Figure 8.2). This cross-section is of particular interest since it intersects the area where extensive damage occurred in the strong earthquakes of 1957, 1979, and 1985. This area has been the subject of several theoretical studies (e.g. Sánchez-Sesma et al., 1988b; Bard et al., 1988; Kawase and Aki, 1989). The last part of this section is dedicated to the comparison between observed and synthetic seismograms. Due to the uncertainties in the models, and the variations among them, only qualitative estimates of the strong ground motion in Mexico City can be given. Nevertheless, this modelling allows us to understand the main physical effects which occur in the Mexico City valley as a consequence of seismic waves generated by strong earthquakes.
8.6.1. One-dimensional structural models

The one-dimensional, layered, anelastic structural model is given in Table 8.2. This model accounts for path effects from source position to the sedimentary basin in Mexico City. Due to the large distance of Mexico City from the source, and the relatively low velocities of waves in the upper crust, the signals are strongly dispersed (Figure 8.7A). The simple, depth-limited structural model has only 15 modes in the frequency-phase velocity band being considered; the energy at frequencies below 0.2 Hz is limited to the fundamental mode, both for Love and Rayleigh waves. The assumption of a very simple upper mantle can be justified by comparison with computations of the displacement time series (Figure 8.7B) for a model which contains a more realistic upper mantle (taken from the continental model proposed by Patton (1980): Table 5.1 in Chapter 5). We find that the differences between the displacements obtained for models with and without a realistic upper mantle are insignificant.

In a second step of the one-dimensional modelling, the sedimentary cover that is present in the valley of Mexico City is included in our structural model. Two models are proposed. In the first, a surficial sedimentary layer of 400 m thickness replaces the upper 400 m of the model shown in Table 8.2. This layer represents the deep sediments. The second model is obtained from the previous one by replacing the upper 55 m of the deep sediments with a surficial clay layer. The resulting 1D structure represents a one-dimensional model for the lake-bed zone. The values of the densities, body-wave velocities and quality factors of the layers are given in the figure captions of Figures 8.8 and 8.9, where the displacements obtained for both models are shown. The seismic source remains the same for all results shown in Figures 8.7, 8.8, and 8.9.

By passing from a model without a sedimentary layer (Figure 8.7), to a model with sediments (Figures 8.8 and 8.9), the arrival times and dispersion characteristics of the surface waves change. This is explained by the group velocity curves that correspond to our model of the lake-bed zone (Figure 8.10). Group velocities can decrease to values as low as 0.08 km/s. The main effect of the sedimentary cover on the fundamental mode is a strong dispersion, caused by the low-velocity surficial layer(s). The amplitudes of fundamental-mode Love and Rayleigh waves remain approximately the same for all three structural models, whereas for the high-frequency part of the wavefield, the amplitudes, frequency content, and arrival times are different. The amplitudes are larger for the models with a sedimentary cover. Due to the low shear-wave velocity of the clay layer, the high-frequency part (0.2-0.5 Hz) of the waves is concentrated in
the horizontal components of motion (Figure 8.9). The vertical components of motion obtained for the different structural models are similar.

![Diagram](image)

**Figure 8.7.** Transverse (left column), radial (middle column), and vertical displacements (right column) obtained for a point source at a distance of 400 km. All amplitudes correspond to a source with seismic moment of 1 dyne cm. The signals are normalized. The peak displacement is in units of cm. The results from two computations are shown: A) for the structural model shown in Table 8.2, each component is decomposed into different sets of modes (upper three traces for each component: modes 6-14, modes 1-5, fundamental modes FUND); B) for the structural model shown in Table 8.2, and here assuming a realistic model of the upper mantle (see text), each component is decomposed into different sets of modes (upper three traces for each component: modes 6-111, modes 1-5, fundamental modes FUND).

The seismograms obtained for one-dimensional models of the Mexico City valley are of limited value for the interpretation of the observations in Mexico City; since in fact, the surficial sedimentary layers are present only in the region surrounding Mexico City. In our results, this leads to the unrealistically strong dispersion of the fundamental modes. On the
other hand, these simple models show that the sedimentary cover produces the observed concentration of high-frequency waves (0.2-0.5 Hz) in the horizontal components. These models also explain the difference in amplitudes for receivers inside and outside the lake-bed zone, and the almost unchanged form and amplitude of the vertical displacements.

Figure 8.8. The same as in Figure 8.7A, but replacing the first 400 m of the layered model (Table 8.2) with a surficial sedimentary layer, which represents the deep sediments (400 m thickness, $\rho=2.0$ g/cm$^3$, $\alpha=2.0$ km/s, $\beta=0.6$ km/s, $Q_\alpha=100$, and $Q_\beta=50$).

Figure 8.9. The same as in Figure 8.8, but now the model also contains a surficial clay layer (55 m thickness, $\rho=1.3$ g/cm$^3$, $\alpha=1.5$ km/s, $\beta=0.08$ km/s, $Q_\alpha=50$, and $Q_\beta=25$), which replaces the first 55 m of the sedimentary layer described in the caption of Figure 8.8.
8.6.2. The cross-section Chapultepec-Peñon

Locations of building collapses due to the 1985 Michoacan earthquake were essentially limited to the lake-bed zone between the Chapultepec and Peñon outcrops (Figure 8.2B). Since the lake-bed zone is filled by the deep sediments and the clay layer, it is of interest to isolate the influence of these two layers on the ground motion. We first consider only the deep sediments. Due to uncertainties in the structural parameters, we consider different maximum thicknesses (400 m, 700 m) and two extreme values of the wave velocities (0.5 km/s, 1.0 km/s) of the sediments. In a further step, a low-velocity surficial clay layer is added to the structural model; this layer replaces the first tens of meters of the deep sediments. For all computations, the source is the same point source with a duration of 0 s (hypocenter depth 10 km, dip=15°, rake=76°, angle strike-
receiver=220°). The one-dimensional structural model, without the sedimentary and clay layer, describing the path from source position to the valley of Mexico City is given in Table 8.2. This is the model used in the first portion of our hybrid computations.

8.6.2.1. Effects of the deep sediments

The two-dimensional structural model corresponding to the cross-section Chapultepec-Peñon is shown in Figure 8.11. It is based on geological information and the mechanical properties of the sedimentary cover in Mexico City. The sedimentary basin is characterized by constant wave velocities and a simple shape. The mesh size used in the finite difference grid is 25 m by 25 m for SH waves and 50 m by 50 m for P-SV waves. The distances between the observation points and the source is in the range 400 to 412 km.

![Distance to the source [km]](image)

**Figure 8.11.** Two-dimensional model corresponding to the cross-section Chapultepec-Peñon. Only the part of the structure near the surface is shown, where the 2D model deviates from the plane-layered structural model.
Figure 8.12. Displacement time series for P-SV and SH waves at an array of receivers over cross-section Chapultepec-Peñon. All amplitudes correspond to a source with seismic moment of 1 dyne cm. The signals are normalized. The peak displacement is indicated in units of cm. The time scale is shifted by 90 s from the origin time (0 s in the figure is really 90 s origin time).
Figure 8.13. The same as in Figure 8.12, for the acceleration time series. The peak acceleration is indicated in units of cm s\(^{-2}\).
The displacement and acceleration time series, corresponding to the receivers in Figure 8.11, are shown in Figures 8.12 and 8.13. There is great spatial variability of ground motion within the sedimentary basin. This effect is important for the high-frequency part of the wavefield. Two adjacent observation points can have completely different signal shapes; this phenomenon is more evident in the acceleration time series (Figure 8.13). At the western edge of the sedimentary basin (402 km from the source), the transverse acceleration dominates that for P-SV waves; upon passing into the central area of the basin, the radial component has the largest amplitudes.

Due to the long duration of the incident wavefield, a superposition of different types of waves occurs. Therefore, it is difficult to distinguish different phases on the seismograms. In comparison with the results obtained for the 1D model (Figure 8.7), the most important effect is the longer duration of the ground motion—up to 40 s at some sites. This effect is mainly caused by local surface waves and their reflections at the edges of the sedimentary basin. In agreement with the observations in Mexico City during the Michoacan earthquake, our computational results show that the fundamental Rayleigh mode passes the sedimentary basin without significant change in amplitude and shape. The reason of this is the large wavelengths of the fundamental mode with respect to the spatial dimensions of the sedimentary basin.

The uncertainty in ground motion predicted from numerical modelling can now be quantified by a parametric study. As the deep sediments are known very poorly, different velocity models and thicknesses are considered. The first variation assumes a relatively high shear-wave velocity for the deep sediments ($\beta=1.0$ km/s). The second variation is characterized by a gradient in the material properties (shear wave velocities vary from 0.25 km/s at the surface to 0.6 km/s at the interface between sediments and bedrock). The displacement time series for the two models are shown in Figures 8.14 and 8.15, respectively. Due to the large spatial variability of the acceleration time series, only the displacement time series are shown.

The model containing high-velocity sediments is used to estimate the minimum effects caused by the deep sediments (Figure 8.14). Apart from small differences in amplitude, there are no variations between signals outside and inside the sedimentary basin. The difference in amplitudes can be explained by the impedance contrast between bedrock and sediments. There is no significant excitation of local surface waves at the edges of the basin.
Figure 8.14. The same as in Figure 8.12 for the displacement time series, assuming relatively high wave velocities for the deep sediments: $\alpha=2.0 \text{ km/s}$, $\beta=1.0 \text{ km/s}$, $\rho=2.0 \text{ g/cm}^3$. 
Figure 8.15. The same as in Figure 8.12 for the displacement time series, assuming gradients in the material properties of the deep sediments: \( \alpha = 1.3-1.5 \text{ km/s}, \beta = 0.25-0.6 \text{ km/s}, \rho = 1.8 \text{ g/cm}^3, Q_{\alpha} = 50-100, \text{ and } Q_{\beta} = 25-50. \)
The model containing a gradient in the material properties is used to estimate the maximum possible effects caused by the deep sediments (Figure 8.15). The effects are a strong amplification and the lateral propagation of local surface waves excited at the edges of the sedimentary basin. These effects are much larger than those observed in the previous examples of two-dimensional models, especially relative to the generation of local surface waves.

The maximum thickness of the deep sediments is increased in the last example. The geometry of the model is shown in Figure 8.16, while the material properties are the same as in Figure 8.11. This deep basin causes the excitation of long-period local surface waves, which dominate the signal shape and increase its duration (Figure 8.17). Qualitatively, the results are the same as those discussed for the shallow sedimentary basin; however, different arrivals are responsible for the excitation of local surface waves, since these surface waves are excited in different frequency bands. For the deep basin, local surface waves are mainly excited by the fundamental modes, while in the shallow basin, higher modes are responsible for the generation of these waves. This leads to the different signal shapes for the two structural models.

![Distance to the source [km]](image)

**Figure 8.16.** Two-dimensional model corresponding to cross-section Chapultepec-Peñon. The maximum depth of the sediments is chosen to be 700 m. Only the part of the structure near the surface is shown, where the 2D model deviates from the plane-layered structural model.
Figure 8.17. Displacement time series for P-SV and SH waves at an array of receivers over cross-section Chapultepec-Peñón. All amplitudes correspond to a source with seismic moment of 1 dyne cm. The signals are normalized. The peak displacement is indicated in units of cm. The time scale is shifted by 90 s from the origin time (0 s in the figure is really 90 s origin time).
8.6.2.2. Effects of a clay layer

A surficial clay layer is now added to the structural model of the deep sediments (Figure 8.18). As indicated by the little information available about the soil properties in Mexico City, the clay thickness increases regularly towards the east. Due to the low shear-wave velocity of the clay ($\beta=0.08$ km/s), the mesh size used in the finite difference grid is 10 m by 10 m for SH waves and 20 m by 20 m for P-SV waves. With this structural model, we reach the CPU time limits of the finite difference technique: for this example, about 24 hours on an IBM 3090/300 computer (with vector facility).

![Distance to the source [km]](image)

**Figure 8.18.** Two-dimensional model corresponding to cross-section Chapultepec-Peñon, with a clay layer at the surface. Only the part of the structure near the surface is shown, where the 2D model deviates from the 1D layered structural model.

The displacement time series are shown in Figure 8.19. (The displacement time series in Figures 8.12 and 8.19 are normalized to different values.) At the beginning of the sedimentary basin, ground motion is determined mainly by the one-dimensional response of the uppermost clay layer (see also Sánchez-Sesma et al., 1988b). At the end of the basin, both for incident Love and Rayleigh waves, there is evidence of local surface waves which propagate in the clay layer. Local resonance
effects lead to long duration of the signals, and to large amplitudes; this phenomenon is most pronounced on the radial component, at distances of 409-410 km from the source. The vertical components of our synthetics, for frequencies below 0.5 Hz, are not significantly changed by the presence of the clay layer; this agrees with the recordings taken in Mexico City during the Michoacan earthquake.

A good representation of the amplification effects in sedimentary basins is given by the computation of spectral ratios between the signals obtained for the two-dimensional model (Figure 8.18), and the signals obtained for the one-dimensional model of the path (Table 8.2). The results for the transverse and the radial components are shown in Figure 8.20, where these ratios are illustrated as a function of frequency, and location along the section. The darker an area, the stronger the amplifications that characterize the two-dimensional model with respect to the 1D case. The general distribution of the shaded areas can be related to the geometry of the structural model. Both the clay layer and the deeper sediments have an influence on ground motion observed at the surface. Their effects can be well separated, as is illustrated in the lower part of Figure 8.20. The greatest amplifications are caused by the clay layer, for frequencies corresponding to the fundamental mode of resonance for this layer. The maximum amplification with respect to the one-dimensional model is of the order of 30-50. This amplification factor coincides with the observed factors of up to 50 for the 1985 earthquake. It also agrees with numerical experiments performed by Bard et al. (1988) who computed amplification factors of up to 20 for vertically incident SH waves. At higher frequencies (above 0.8 Hz), there is also evidence for the excitation of the first higher mode of resonance for the clay layer. The step-like character of the spectral ratios clearly reflects the steps of the model in the finite difference computations, which is a characteristic of the algorithmic process.

By interactions with the clay layer, the deep sediments cause spectral ratios of up to 30. At distances between 403 and 408 km from the source, and for frequencies above 0.5 Hz, the spectral ratios for the SH case are characterized by a regular pattern of constructive and destructive interference caused by the deep sediments. When considering the radial component of motion, more complicated features are seen (central part of Figure 8.20). The sedimentary cover causes the polarization of P-SV waves into the radial component. This polarization gives rise to a series of scattered dark lines. For example, around 0.5 Hz the polarization effect is present at distances between 403 and 408 km, and spectral ratios reach values of up to 35.
Figure 8.19. The same as in Figure 8.12 for the displacement time series, for cross-section Chapultepec-Peñon shown in Figure 8.18.
Figure 8.20. Spectral ratios for the transverse and radial components of motion over the entire cross-section.
8.6.3. Comparison between the synthetic signals and the observed ground motion

Due to the variability of strong ground motion within the sedimentary basin, the poorly-known structure of the sedimentary basin in Mexico City, the simple source model, and the fact that none of the accelerometric stations is located on the cross-section studied, it is difficult to compare synthetic signals directly with observed strong ground motion. On observed records, an oscillation with a period of 2 to 5 s is superimposed on the fundamental modes (Figure 8.4). This feature has been interpreted as a source effect (Campillo et al., 1989) and is not present on the synthetic signals. On synthetic signals, the two wavetrains are separated in time due to the simple source model (point source with 0 s duration) used in the modelling. For the synthetic signals (Figure 8.19), the maximum duration outside the sedimentary basin is about 90 s for P-SV waves and 60 s for SH motion. Within the sedimentary basin, the maximum duration is 150 s in the P-SV case, and 120 s for SH waves. If we assume a seismic source that is composed of three subevents, as proposed by Houston and Kanamori (1986), the durations increase by about 45 s (Figure 8.21B). The durations are then in good agreement with observations in the hill zone, and in the lake-bed zone. Some of the computed horizontal components of motion (Label L5.5 or R9.5 in Figure 8.21B) are very similar to the observed signals at station CDAO. On the vertical component, the duration of the fundamental mode is too long with respect to the observations, and this is caused by the relatively low velocities in the uppermost layer in the layered model (Table 8.2). Therefore, the upper five kilometers of the crustal model used in our computations are not realistic, and the values for the wave velocities should be higher.

The frequency content of the horizontal components of the computed ground motion (Figure 8.19) agrees well with the observations, while the synthetic vertical displacements have too much energy for frequencies above 0.5 Hz. This can be accounted for in several ways, e.g. by the quality factor in the layered model (Table 8.2), or by the choice of a point source with 0 s duration. Assuming lower quality factors in the layered model (Label ATT in Figure 8.22), or a source with finite duration (Label DUR in Figure 8.22) will reduce the high-frequency content of the synthetic signals.
Figure 8.21. Transverse (left column), radial (middle column), and vertical displacements (right column) obtained at three receivers within the sedimentary basin: at the western edge of the basin, 401.5 km from the source; in the center of the basin, 405.5 km from the source; and at the eastern edge of the basin, 409.5 km from the source. The results of two computations are shown: A) Synthetic displacements due to one point source. B) Synthetic displacements due to three point sources, all located at a depth of 10 km and the same distance. The strike-receiver angle, dip and rake are 220°, 15°, 76°, respectively. The three point sources have different weights and time shifts (1.0, 1.0, 0.2 and 0 s, 26 s, 47 s). Weight 1 corresponds to a source with seismic moment of 1 dyne cm.

Even though none of the accelerometric stations is located on the cross-section studied, the spectral ratios obtained for the horizontal components of the synthetic seismograms can be very similar to the spectral ratios obtained from observations, as is shown in Figure 8.23. The maximum amplification factors, the shape of the spectral ratios, and the frequency bands at which amplification occurs, can be explained by our numerical modelling.

The model that we have used for the Mexico City area can explain the difference in amplitudes for receivers located inside and outside the lake-bed zone. The ratio between the computed, horizontal peak ground displacements (or accelerations) inside and outside the lake-bed zone reaches values of the order of 5 to 7; about the same ratio is obtained for the observed ground motion.
The area of severe structural damage in Mexico City is characterized by increasing thicknesses of the deep sediments and the clay layer. Such geometries favor the excitation of local surface waves, which could be responsible for the observed distribution of damage. The large impedance contrast between the clay at the surface, and the deep sediments, causes strong resonance effects in the clay layer. The computed seismograms (Figure 8.19) have large amplitudes and are characterized by almost monochromatic wavetrains of long duration. Such shaking can cause a nonlinear response of the soils, or a building collapse, if the dominant frequency of the motion is close to the resonant frequencies of the building itself.
Figure 8.23. Smoothed spectral ratios obtained for the synthetic seismograms (a) at 404 km and (b) at 409.5 km from the source, in comparison with the spectral ratios (c) of the horizontal components recorded at station SCT1 with respect to station TACY, and (d) of the horizontal components recorded at station CDAO with respect to station TACY.
9. Summary and Conclusions

Numerical simulations play an important role in the estimation of strong ground motion. They can provide synthetic signals for areas where recordings are absent and are therefore very useful for engineering design of earthquake-resistant structures. In recent years many computational techniques have been proposed to estimate ground motion at a specific site. The methods commonly used are one-, and two-dimensional techniques; three-dimensional studies are too expensive to be applied routinely. The standard one-dimensional methods that are generally applied in zonation studies, are those developed by Thomson and Haskell, and by Seed (Program Shake). These techniques are very cheap and they provide the first few resonance frequencies (fundamental and harmonics) of unconsolidated soil layers; and, the results show that the strongest effects usually occur at the fundamental frequency. Relative to the response of a reference, bedrock model, one-dimensional techniques yield estimates of the wave amplification caused by unconsolidated surficial sediments overlying the bedrock. However, such techniques fail to predict the ground motion close to lateral heterogeneities or when the soil layers have a non-planar geometry. For realistic structures where lateral heterogeneities and sloping layers are common, these departures from lateral homogeneity can cause effects that dominate ground motion: the excitation of local surface waves, focusing and defocusing, and resonances. Thus the treatment of realistic structures requires at least two-dimensional techniques to estimate ground motion.

For ease of reference we divide the methods for handling wave propagation in two-dimensional media into a numerical class of methods, where the computational algorithm is based on an approximate mathematical method for solving the formal representation of the problem; and into an analytical class of methods, where the computational algorithm is based on an exact formal solution. The numerical class includes the finite difference method (Alterman and Karal, 1968; Boore, 1972), the pseudo-spectral method (Gazdag, 1981; Kosloff and Baysal, 1982), and the finite element method (Lysmer and Drake, 1972); while the analytical class includes the integral equation methods (Sánchez-Sesma and Esquivel, 1979), the 2D mode summation technique (Levshin, 1985; Vaccari et al., 1989a), the Rayleigh-Ansatz method (Aki and Larner, 1970; Bard and Bouchon, 1980a, 1980b), and the discrete-wavenumber, boundary-element method (Kawase, 1988). The finite difference method, the pseudo-spectral method, and the finite element method are capable of treating very complex structures; however,
they are restricted in the size of the models, which they can handle by computer memory limitations. Often, the source cannot be included in the structural model, because its distance from the site of interest is too large; and the incoming wavefield is approximated by a plane polarized body wave. The advantage of the analytical class of methods, such as 2D mode summation or the Rayleigh-Ansatz method, is the possibility of treating realistic source models and extended structural models. These methods can be applied effectively to simple two-dimensional geometries of sedimentary basins.

To include both a realistic source model and a complex structural model of the sedimentary basin, a hybrid method has been developed that combines modal summation and the finite difference technique. The propagation of waves from the source position to the sedimentary basin is treated with the mode summation method for a plane layered structure. In our modal summation the treatment of P-SV waves is based on Schwab's (1970) optimization of Knopoff's (1964b) method, and the handling of SH waves is based on Haskell's (1953) formulation; these computations include the "mode-follower" procedure and structure minimization described by Panza and Suhadolc (1987). Explicit finite difference schemes are then used to simulate the propagation of seismic waves in the sedimentary basin. These schemes are based on the formulation of Korn and Stöckl (1982) for SH waves, and on the velocity-stress, finite difference method for P-SV waves (Virieux, 1986). This hybrid method is particularly suitable to estimate ground motion in sedimentary basins of any complexity, and it allows us to take into account the source, path, and local site effects, even when dealing with path lengths of a few hundreds kilometers. This simultaneous consideration of source, path and local site effects is an innovation, and it is the main advantage of the hybrid method with respect to other techniques.

The hybrid technique combines the advantages of both modal summation and finite difference technique:

1. The use of the mode summation method allows us to include an extended source, which can be modelled by a sum of point sources appropriately distributed in space. This allows the simulation of a realistic rupture process on the fault.

2. The path from source position to the sedimentary basin can be approximated by a structure composed of flat, homogeneous layers. Modal summation then allows the treatment of many layers which can take into consideration low-velocity zones and fine details of the crustal section under consideration.
3. The seismograms computed by modal summation contain all the body waves and surface waves, whose phase velocities are smaller than the S-wave velocity of the half-space that terminates the structural model at depth.

4. The finite difference method, applied to treat wave propagation in the sedimentary basin, permits the modelling of wave propagation in complicated and rapidly varying velocity structures, as is required when dealing with sedimentary basins. The algorithms can handle structural models containing a solid-liquid interface, and materials with normal, as well as the high values of Poisson's ratio that are often present in near-surface sediments.

Intrinsic attenuation in soft sediments is an important process and is taken into account in the computations to prevent serious errors in the estimates of seismic hazards. In our mode summation method, anelasticity is included by means of the variational method, while in the finite difference computations, anelasticity is included by using a method based on the rheological model of the generalized Maxwell body. Emmerich and Korn (1987) developed the latter method for the case of SH-wave propagation. In this thesis, their method has been implemented for the SH case, and has then been generalized to the P-SV case. This approach allows us to approximate the viscoelastic modulus by a low-order, rational function of frequency. This approximation of the viscoelastic modulus can account for a constant quality factor over a certain frequency band. Replacement of all elastic moduli by viscoelastic ones, and transformation of the stress-strain relation into the time-domain, yields a formulation which can be handled with a finite difference algorithm. It is recognized that the synthetic signals are very sensitive to small variations of the quality factor in the sediments, and an important effect of intrinsic anelasticity is the dependence of the seismic velocities on frequency, i.e. body-wave dispersion. For small quality factors, the frequency at which the seismic velocities are described becomes important: with a quality factor of 20, for example, the seismic velocities differ by as much as 5 percent over one decade of frequency. This consequently influences the dispersion characteristics of local surface waves, as is illustrated in one of our numerical experiments.

The finite difference method has the disadvantage that limitations of computer memory require the introduction of artificial boundaries, which form the border of the finite difference grid in space. These boundaries are a severe problem in finite difference methods, since they can generate spurious reflections of the waves impinging upon them from the interior of the grid. In this study, several methods for the prevention of these
reflections have been tested and compared. As a result we have developed an efficient and computationally cheap technique for reducing these reflections. This technique combines Smith's (1974) method with the paraxial wave equation (Clayton and Engquist, 1977) applied at the border of the finite difference grid. Close to the border, regions of high absorption are introduced; and, the grid spacing is increased towards the border, which allows the absorption of waves with wavelengths large compared to the size of our model of the sedimentary basin.

In a set of numerical experiments carried out to establish error bounds for the hybrid computations, the results obtained with the hybrid technique have been tested against the results from the 2D mode summation method. The tests employ layered structures separated by a vertical discontinuity. First of all, the comparisons show the consistency of the two techniques. There is some difference in the maximum amplitudes of the signals due to (1) the different approximations with which the intrinsic attenuation is handled in the two methods, (2) the discretization of the finite difference model in space, and (3) the limitation to a rather low number of modes (five outgoing modes) in the 2D modal summation. A preliminary, one-dimensional experiment allows us to consider the modal summation result as a reference; a comparison between modal summation and the finite difference technique then places accuracy bounds on the results from the hybrid technique. An important disadvantage of this technique is the fact that geometrical spreading cannot be treated in the finite difference computations in an exact manner; despite this problem, from the one-dimensional experiment we can estimate the lower bound of error in the hybrid technique to be 2 to 5 percent. This error can only be obtained if the discretization in space is optimized, and if the finite difference grid is deep enough to guarantee the completeness of the signals. This lower error bound for the hybrid method can be expected for any laterally heterogeneous structure, and this allows the application of the hybrid technique to the study of wave propagation in sedimentary basins with relatively high accuracy.

To define the limits and possibilities of the hybrid technique, three computational examples have been presented: one for sites that are far from the source (Mexico City during the September 19, 1985 Michoacan earthquake), one at an intermediate distance (Rome during the January 13, 1915 Fucino earthquake), and one for sites close to the source (a sedimentary basin in the Friuli region during the September 11, 1976 aftershock at 16h35m04s). These examples show how powerful the hybrid technique is in terms of flexibility of the structure and source modelling. The limits of the technique are imposed by the need for relatively large
amounts of CPU time and computer memory in the finite difference computations. One major problem of such numerical simulations is the enormous number of parameters which have to be entered as input. The choice of these parameters is based on all available seismological, geological and geotechnical information for the area under consideration. Our numerical simulations predict the seismic response at a specific site, only if the mechanical parameters (density, velocities, damping, etc.) and geometrical parameters (such as thickness) are reasonably well-known. Due to the restriction of the hybrid method to two-dimensional models, and the limited knowledge about the structure and the seismic source, it is in general not possible to define one single structural model, but a class of models. Since waveforms are sensitive to small changes of the source parameters, and of the structural model close to the receiver, it is necessary to perform parametric studies which will provide a complete database for the ground motion that is to be expected at sites of interest.

In our numerical examples, special emphasis has been given to the analysis of the nature of ground motion in sedimentary basins. The examples have shown the well-known amplification of the incident waves, which occurs when a seismic wave travels through an interface from a medium with relatively high rigidity, into a medium of lower rigidity. An irregular interface between bedrock and sediments can cause a focusing of wave energy. This focusing of waves is limited to observation points just above the irregularities, and it can induce large differences in motion at neighboring sites; thus, observation points that are close to one another can exhibit different amplitudes and signal durations. Sloping interfaces, such as the transitions from bedrock to sediments, can generate local surface waves in the sedimentary layers. These local waves can dominate the seismic signal, especially close to the transition zone. Their energy propagates mainly in the sedimentary layer. For both SH and P-SV waves, strong resonances can occur in parts of the basins with smooth variations of the geometry of the interface between bedrock and sediments. The resonances originate in the superposition of forward propagating local surface waves, and their reflections at lateral heterogeneities. Subsurface soil formations, with elastic-wave velocities that are lower than those of the overlying material, can also trap energy. This energy can be scattered to the surface by lateral heterogeneities.

Considering more specifically the aspects of ground motion modelling related to seismic hazard studies, one of the main conclusions is that one-dimensional structural models are not suitable for the prediction of seismic ground motion in sedimentary basins. This is because local surface waves cannot be generated in such models. For
hazard assessment, amplification effects in sedimentary basins can be viewed best in the form of spectral ratios between the signals obtained (1) for the two-dimensional model of the sedimentary basin, and (2) for a reference model in which the upper few hundreds meters of the basin are replaced by the one-dimensional structure present in the model used to describe the wave propagation from the source to the basin. Both signals have in common the source and the path, so that from the spectral ratios the effects of the sedimentary basin are clearly exhibited. These spectral ratios allow the identification of frequency bands in which amplification effects occur, and also the sites at which amplification effects occur. The amplifications are strongly associated with the geotechnical and geometrical properties of the sedimentary cover.

In general, the character of SH waves and P-SV waves in sedimentary basins is different. This is due to the different frequency content of the two classes of waves, and different physical processes, e.g. the S- to P-wave conversion for P-SV waves at strong impedance contrasts. This difference in SH- and P-SV-wave propagation in sedimentary basins has been demonstrated for the numerical simulation of the accelerograms for the September 11, 1976 Friuli earthquake. Within the sedimentary basin, the coda of the transverse component is mainly composed of the local, fundamental-mode Love wave, whereas the P-SV wavefield shows dominant contributions of a high-frequency crustal wave. Each higher Rayleigh-wave mode has surface amplitudes that are small, but their interference leads to the large amplitudes that are observed in the coda of the signals. In some structural models, diffracted P-SV waves, that originate at the edge of sedimentary basin, characterize the coda of the signals.

The Michoacan earthquake of September 19, 1985 (Ms=8.1), together with its aftershocks, produced the worst earthquake damage in the history of Mexico City. The reasons for the damage can be found in the special geological conditions of the valley of Mexico City. This valley consists mainly of two layers: the so-called deep sediments, and a surficial clay layer which is present in the lake-bed zone. The latter layer is poorly consolidated and has high water content. By studying the one-dimensional response of these two layers with the mode summation method, it was possible to reproduce the difference in amplitudes between records for receivers inside and outside the lake-bed zone. These simple models show that the sedimentary cover produces the concentration of high-frequency waves on the horizontal components of motion. One aspect that cannot be explained with one-dimensional models is the long duration of ground motion inside the lake-bed zone. This requires the use
of two-dimensional models for the sedimentary basin. For such models, the ground motion is mainly controlled by the response of the uppermost clay layer. Local surface waves and resonance effects can explain the long duration of the signals inside the lake-bed zone. Small variations of the geometry of the uppermost clay layer lead to very different ground motion. Even in closely neighboring stations it is possible to observe large differences in the shape, duration, and frequency content of the signals. Due to the long duration of the incident wavefield, the ground motion inside the sedimentary basin is very complex, and single wave types are difficult to identify. Spectral ratios that are computed for sites inside the sedimentary basin show that the clay layer, by interaction with the deep sediments, causes amplifications of the order of 30 to 50. This amplification factor agrees with those observed in the lake-bed zone. Within the sedimentary basin, incident energy in certain frequency bands is shifted from the vertical, into the radial component of motion. The amplification that accompanies this effect can give rise to amplification factors of up to 35 for the radial component of motion. The vertical components of our synthetics, for frequencies below 0.5 Hz, are not significantly changed by the presence of the clay layer; this agrees with the recordings taken in Mexico City during the Michoacan earthquake.

An important test of our numerical results is the comparison between the synthetic signals and the observed ground motion. The synthetic signals explain the major characteristics of the observations, as is shown for the example of Mexico City and the sedimentary basin in the Friuli area. For selected structural models, the relative amplitudes, durations, and frequency content of the different components of the synthetics agree well with the recorded data. In general, there is considerable variability of synthetic ground motion for different, realistic two-dimensional models of a sedimentary basin. Theoretical studies show that waveforms and frequency content of computed seismograms are sensitive to small changes in the source parameters, the layered structural model, the subsurface topography of the sedimentary basin, and the velocity and quality factor of the sediments. This has also been noted experimentally for records from different sites in existing strong-motion arrays. What remains constant for different, realistic structural models are the physical processes that occur within sedimentary basins, e.g. the excitation of local surface waves and resonance effects. The frequency content and dispersion characteristics of the waves induced by these processes are clearly related to the depth of the sediments, the steepness and irregularity of the sediment-bedrock interface, and the seismic velocities.
In absence of instrumental data, a numerical simulation of the January 13, 1915 Fucino (Italy) earthquake has been compared with the observed distribution of damage in Rome. This distribution has been compared with certain quantities related to the computed ground motion; these are quantities commonly used for engineering purposes: the peak ground acceleration, the maximum response of a simple oscillator, and the so-called “total energy of ground motion” which is related to the Arias Intensity. The damage distribution in Rome shows essentially that the damage is concentrated at the edges of the alluvial basin of the Tiber river; the heavy and intermediate damage appears in that basin. The same distribution of damage can be expected on the basis of our numerical simulations of this event. The highest values of the ground motion related quantities are observed at the edges of this alluvial basin. Strong amplification effects can be observed in the river beds of the Tiber and Aniene. The signals are characterized by large amplitudes and long durations due to (1) the low impedance of the alluvial sediments, (2) resonance effects and (3) the excitation of local surface waves. Minimum values of the ground motion related quantities can be observed for receivers placed above the volcanic layer which is present in the ancient river bed of the Tiber. This layer acts as a shield, reflecting part of the incoming energy. The thicker the layer, the smaller the observed surface amplitudes of the signals. The best agreement with the observed distribution of damage is obtained for the “total energy of ground motion”. This quantity and the related Arias Intensity turn out to be good representations of ground motion for hazard assessment. From the computation of spectral ratios, it has been recognized that the presence of a near-surface volcanic layer of rigid material is not sufficient to classify a location as a “hard-rock site”, since the existence of an underlying sedimentary complex can cause amplifications due to resonances. A correct zonation requires knowledge of both the thickness of the surficial layer and of the deeper parts of the structure, down to real bedrock. This is especially important in volcanic areas, where lava flows often cover alluvial basins. The presence of sediments causes the shift of energy in certain frequency bands from the vertical, into the radial component of motion. This phenomenon is very localized, both in frequency and space, and closely neighboring sites can be characterized by very large differences in the seismic response, even if the lateral variations of local soil conditions are relatively smooth.

The examples considered in our study have shown the important influence of lateral heterogeneities on ground motion. The hybrid technique makes it possible (1) to study local effects even at large distances (hundreds of kilometers) from the source, (2) to include highly realistic
modelling of the source, and (3) of the propagation path. This technique can assist in the interpretation and prediction of ground motion at a given site. This new approach can be applied routinely in micro-zonation studies, and it provides realistic estimates of ground motion for two-dimensional, anelastic models. The resulting synthetic seismograms can be used to complete existing databases of recorded ground motion. Such databases would then serve to establish estimates of the maximum accelerations and total energies of ground motion to be expected at given sites. This would be a more adequate technique than making estimates based on analyses of accelerograms recorded in different areas of the world, or of synthetic signals generated by random processes.

The extension of the hybrid technique to three-dimensional models is straightforward and depends mostly on the computer resources, which are made available by the present-day, parallel computers like the Connection Machine. The treatment of the non-linear response of certain materials could be a future extension of the algorithms presented here, since such behavior can only be studied with time-domain methods, e.g. the finite difference technique. Surface topography is not yet included in our numerical scheme, but can also be implemented in future, improved algorithms. In this thesis, the hybrid technique has been applied specifically to the study of sedimentary basins, but of course it can also be used to simulate wave propagation through complex lateral heterogeneities of any kind.
References


Suetsugu, D., 1989. Lower mantle high-velocity zone beneath the Kurils as inferred from P-wave travel time and amplitude data. J. Phys. Earth, 37, 265-295.


Acknowledgments

For direction, and assistance in the completion of this thesis I would like to thank a number of people who have contributed to this research, and who have supported me in other ways.

The skills of four fine scientists have been fundamental to my scientific development. Without the help of Prof. Stephan Mueller and Prof. Giuliano Panza the completion of this thesis would not have been possible. They created a stimulating environment for seismological research both by their enthusiastic encouragement, and by giving me valuable advice in the course of my research. With Prof. Panza I enjoyed many discussions, and gained much insight from his suggestions and great knowledge of seismology. I would also like to express my gratitude to Dr. Fred Schwab for his careful review of this thesis, and his advice and help in bringing the thesis into final form. Working with him has been a pleasure, and I benefited much from his impressive knowledge of mathematical and numerical techniques for treating problems involving the propagation of seismic waves. I am very grateful to Prof. Peter Suhadolc who always supported my research and guided me whenever he could. His understanding has been of crucial importance. Lastly, I wish to thank all of the other colleagues and scientists who have helped me in various ways, and who have made these years of research enjoyable.

This study has been supported by funds or facilities of the "Ente Nazionale Energie Alternative", the "Ministero della Università e della Ricerca Scientifica e Tecnologica" and the European Community. I want to thank the "Janggen-Pöhn Stiftung", the "Holderbank Stiftung", the Swiss Federal Institute of Technology and the Canton Zurich for their scholarship support. Most of the computations in this research were carried out on the IBM3090E computer at the ENEA INFO BOL Computer Center.
Curriculum Vitae

February 9, 1960 born in Rorschacherberg (SG), Switzerland
father: Willi Fäh
mother: Lydia Fäh née Hagmann

1975-1979 High School at the Kantonsschule Heerbrugg, CH.
1979 High School diploma type C (Natural Sciences).

1985 Degree in Geophysics (M.Sc.). The main subjects of study were theoretical geophysics, general geophysics, theoretical physics, solid state physics and electronics. Diploma thesis in "Regional Microzonation in Switzerland", within the research group of the Swiss Seismological Service.

1985-1987 Research fellowship at the Laboratory of Biomechanics of the Swiss Federal Institute of Technology in Zurich. The special field of research was wave propagation in human bones.

Since 1987 Ph.D. studies in the field of seismic strong motion synthetics at the Institute of Geodesy and Geophysics of the University of Trieste (Italy), and at the Institute of Geophysics of the Swiss Federal Institute of Technology.
Appendix A: Phase attenuation for Rayleigh waves

In anelastic media the surface-wave phase-velocity $c$ must be expressed
as a complex quantity:

$$\frac{1}{c} = \frac{1}{C_1} - \imath C_2 .$$

The quantity $C_1$ is the (physical) phase velocity, and $C_2$ is the phase
attenuation. This attenuation can be estimated by using the variational
technique (Takeuchi and Saito, 1972; Aki and Richards, 1980). The phase
attenuation $C_{2R}$ for Rayleigh waves is given by

$$C_{2R} = \frac{\text{Im}(I_4)}{2\omega I_3 k} ,$$

where $k$ is the wave number in the case of perfectly-elastic structures
and

$$I_3 = \int_0^\infty \left\{ \left[ \frac{\lambda}{\lambda + 2\mu} \right] y_3^2 + \frac{1}{k} \left( y_1 y_4 - \frac{\lambda}{\lambda + 2\mu} y_2 y_3 \right) \right\} dz$$

$$I_4 = \int_0^\infty \left\{ \delta(\lambda + 2\mu) \left[ \frac{1}{(\lambda + 2\mu)^2} \left( y_2^2 + 2k\lambda y_2 y_3 \right) + k^2 \left( 1 + \frac{\lambda^2}{(\lambda + 2\mu)^2} y_3^2 \right) \right] \right. $$

$$+ \delta\mu \frac{1}{\mu^2} y_4^2 - \delta\lambda \left[ \frac{2k}{(\lambda + 2\mu)} \left( y_2 y_3 + k\lambda y_3^2 \right) \right] \left\} dz ,$$

where

$$y_1 = \frac{w(z)}{w_0} , \quad iy_3 = \frac{u(z)}{w_0} , \quad y_2 = \frac{\sigma_{zz}(z)}{w_0} \quad \text{and} \quad iy_4 = \frac{\sigma_{xz}(z)}{w_0} .$$

The quantity $\sigma_{zz}(z)$ is the normal stress and $\sigma_{xz}(z)$ is the tangential stress,
and $\delta\mu$, $\delta\lambda$ and $\delta(\lambda+2\mu)$ are given as follows

$$\delta\mu = \rho \left( \beta_1^2 - \beta_2^2 - \beta^2 \right) + 2i\rho\beta_1 \beta_2$$

$$\delta\lambda = \rho \left[ \alpha_1^2 - \alpha_2^2 - \alpha^2 \right] - 2 \left( \beta_1^2 - \beta_2^2 - \beta^2 \right)$$

$$+ \rho 2 \left( \alpha_1 \alpha_2 - 2\beta_1 \beta_2 \right)$$

$$\delta(\lambda + 2\mu) = \rho \left( \alpha_1^2 - \alpha_2^2 - \alpha^2 \right) + i 2\rho\alpha_1 \alpha_2$$
In these expressions $\alpha$ and $\beta$ are the compressional and shear-wave velocities in the case of perfectly-elastic structures, and

$$\rho (\beta_1 + i\beta_2)^2 = \mu + \delta\mu$$

$$\rho (\alpha_1 + i\alpha_2)^2 = (\lambda + 2\mu) + \delta(\lambda + 2\mu).$$

The integrals $I_3$ and $I_4$ can be computed analytically from the layer constants (Schwab et al., 1984).
Appendix B: Finite difference scheme for P-SV waves and its stability condition

The set of equations (3.13) can be approximated by a central finite-difference scheme. The time and space derivatives are defined as follows

\[
\frac{\partial f}{\partial x} = \sum_{l=1}^{L} \frac{f[x + (2l - 1)\frac{\Delta x}{2}] - f[x - (2l - 1)\frac{\Delta x}{2}]}{\Delta x}.
\]  

(B.1)

In practical computations, second- and fourth-order approximations (L=2,4) are used. Here, the time derivative \(\partial/\partial t\) is approximated by a second-order operator, and the space derivatives \(\partial/\partial x\) and \(\partial/\partial z\) are approximated by second- or fourth-order operators. To find the stability condition of the above approximation, one can consider an initial error and see if the error grows with time. We assume harmonic disturbances of the following form

\[
E(\hat{u}) = A \cdot \exp\{-i\omega p At + ik_x m \Delta x + ik_z n \Delta z\}
\]

\[
E(\hat{w}) = B \cdot \exp\{-i\omega p At + ik_x m \Delta x + ik_z n \Delta z\}
\]

\[
E(\sigma_{zz} + \sigma_{xx}) = C_1 \cdot \exp\{-i\omega p At + ik_x m \Delta x + ik_z n \Delta z\}
\]

\[
E(\sigma_{zz} - \sigma_{xx}) = C_2 \cdot \exp\{-i\omega p At + ik_x m \Delta x + ik_z n \Delta z\}
\]

\[
E(\sigma_{xz}) = C_3 \cdot \exp\{-i\omega p At + ik_x m \Delta x + ik_z n \Delta z\}
\]

(B.2)

The space derivatives \(\partial/\partial x\) and \(\partial/\partial z\) are first approximated by the second-order operators \(D_x\) and \(D_z\) (Equation (3.15)). For real and fixed \(k\), one has to find the time dependence of the error in the form of the relation \(\omega(k)\). The error satisfies the wave equations (3.13):

\[
A \sin\left(\frac{\omega At}{2}\right) = \frac{At}{\rho} \left(\frac{1}{2\Delta x} (C_1 - C_2) \sin\frac{k_x \Delta x}{2} + \frac{1}{\Delta z} C_3 \sin\frac{k_z \Delta z}{2}\right)
\]

\[
B \sin\left(\frac{\omega At}{2}\right) = \frac{At}{\rho} \left(\frac{1}{2\Delta z} (C_1 + C_2) \sin\frac{k_z \Delta z}{2} + \frac{1}{\Delta x} C_3 \sin\frac{k_x \Delta x}{2}\right)
\]

\[
C_1 \sin\left(\frac{\omega At}{2}\right) = At (2\lambda + 2\mu) \left(\frac{B}{\Delta z} \sin\frac{k_z \Delta z}{2} + \frac{A}{\Delta x} \sin\frac{k_x \Delta x}{2}\right)
\]

\[
C_2 \sin\left(\frac{\omega At}{2}\right) = At \cdot 2\mu \left(\frac{B}{\Delta z} \sin\frac{k_z \Delta z}{2} - \frac{A}{\Delta x} \sin\frac{k_x \Delta x}{2}\right)
\]

\[
C_3 \sin\left(\frac{\omega At}{2}\right) = At \mu \left(\frac{B}{\Delta x} \sin\frac{k_x \Delta x}{2} + \frac{A}{\Delta z} \sin\frac{k_z \Delta z}{2}\right)
\]

(B.3)
Eliminating $A$, $B$, $C_1$, $C_2$ and $C_3$, we get

\[
\begin{align*}
\left( \frac{1}{\Delta t^2} \sin^2 \frac{\omega \Delta t}{2} - \alpha^2 \right) \left( \frac{1}{\Delta x^2} \sin^2 \frac{k_x \Delta x}{2} + \frac{1}{\Delta z^2} \sin^2 \frac{k_z \Delta z}{2} \right) & = 0 \\
\left( \frac{1}{\Delta t^2} \sin^2 \frac{\omega \Delta t}{2} - \beta^2 \right) \left( \frac{1}{\Delta x^2} \sin^2 \frac{k_x \Delta x}{2} + \frac{1}{\Delta z^2} \sin^2 \frac{k_z \Delta z}{2} \right) & = 0
\end{align*}
\]

where $\alpha$ and $\beta$ are the P-wave and S-wave velocity. We end up with two possible solutions, which describe pure P-wave and pure S-wave propagation:

\[
\begin{align*}
\frac{1}{\Delta t^2} \sin^2 \frac{\omega \Delta t}{2} & = \alpha^2 \left( \frac{1}{\Delta x^2} \sin^2 \frac{k_x \Delta x}{2} + \frac{1}{\Delta z^2} \sin^2 \frac{k_z \Delta z}{2} \right) \\
\frac{1}{\Delta t^2} \sin^2 \frac{\omega \Delta t}{2} & = \beta^2 \left( \frac{1}{\Delta x^2} \sin^2 \frac{k_x \Delta x}{2} + \frac{1}{\Delta z^2} \sin^2 \frac{k_z \Delta z}{2} \right)
\end{align*}
\]

If $\beta \Delta t \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta z^2}} \leq 1$ and $\alpha \Delta t \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta z^2}} \leq 1$, then $\sin \frac{\omega \Delta t}{2}$ is always less than or equal to 1. Consequently, $\omega$ is real and the error will not grow with time. The condition for P-wave propagation is more restrictive, and therefore, the condition for stability of the second-order finite difference scheme is the following

\[
\Delta t \leq \frac{1}{\alpha_{\text{max}}} \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta z^2}}, \quad (B.4)
\]

where $\alpha_{\text{max}}$ is the maximum P-wave velocity used in the structural model.
The stability condition for fourth-order finite difference operators of the space derivatives and a second-order operator of the time derivative can be determined by assuming disturbances of the form (B.2). This results in the following equation

\[
\left( \frac{1}{\Delta t^2} \sin^2 \frac{\omega \Delta t}{2} - \alpha^2 \right) \left[ \frac{1}{\Delta x^2} \left( d_1 \sin \frac{k_x \Delta x}{2} + d_3 \sin \frac{3k_x \Delta x}{2} \right)^2 \\
+ \frac{1}{\Delta z^2} \left( d_1 \sin \frac{k_z \Delta z}{2} + d_3 \sin \frac{3k_z \Delta z}{2} \right)^2 \right] = 0.
\]

The condition for P-wave propagation is more restrictive for the definition of the stability condition. Hence, \( \omega \) is real for all \( k_x \) and \( k_z \), if

\[
\Delta t \leq \frac{1}{\alpha_{\text{max}} \left( |d_1| + |d_3| \right) \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta z^2}}}.
\]

Equation (B.5) is the stability condition for the fourth-order, finite difference scheme which is used in the near-surface part of the structural models.
Appendix C: Intrinsic attenuation

C.1. Introducing the intrinsic attenuation into the wave equation for SH waves

In Section 3.4, a formalism has been developed to describe the anelastic behaviour of a one-dimensional medium. The stress-strain relation derived for the rheological model of the generalized Maxwell body will now be introduced into the equation of motion for SH-wave propagation in two-dimensional media. The equations will have a form which can be handled with a finite difference scheme (Emmerich and Korn, 1987). The wave equation of motion in a two-dimensional medium is given by

$$\rho \frac{\partial^2 v}{\partial t^2} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial z},$$

(C.1)

where $\rho$ is the density and $v$ is the displacement in the $y$ direction. In two dimensions the viscoelastic stress-strain relation is given for each stress component $\sigma_{lm}$ as follows

$$\sigma_{lm} = 2M_U \left[ \epsilon_{lm} - \sum_{j=1}^{n} \Phi_{jm} \right].$$

(C.2)

The strain is defined as

$$\epsilon_{lm} = \frac{1}{2} \left( \frac{\partial v}{\partial x_l} + \frac{\partial v}{\partial x_m} \right).$$

Each element $\Phi_{jm}$ satisfies a first-order differential equation of the form (3.33b). The parameters $\delta M, a_j$ and $\omega_j$ are the same for $\sigma_{xy}$ and $\sigma_{zy}$, which means that the attenuation is independent from the direction of propagation. We consider a homogeneous medium ($M_U=\mu=\text{const}$). Inserting Equation (C.2) into (C.1) yields the following set of differential equations

$$\frac{\partial^2 v}{\partial t^2} = \beta^2 \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} - \sum_{j=1}^{n} \Gamma_j \right],$$

(C.3a)

with

$$\dot{\Gamma}_j + \omega_j \Gamma_j = -\frac{\omega_j a_j}{M_U} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right).$$

(C.3b)
The quantities $\Gamma_j$ are defined as

$$\Gamma_j = 2\left( \frac{\partial \Phi_{jxy}}{\partial x} + \frac{\partial \Phi_{jzy}}{\partial z} \right),$$

where the dot indicates time-derivative. This set of equations can be handled with a finite difference algorithm. For the second-order derivatives a centered finite-difference scheme is used. Equation (C.3b) can be discretized in time by writing $\Gamma_j$ in the following form

$$\Gamma_j(t) = \frac{1}{2} \left( \Gamma_j(t + \frac{\Delta t}{2}) + \Gamma_j(t - \frac{\Delta t}{2}) \right).$$

Therefore, Equations (C.3a) and (C.3b) can be discretized in a first step by

$$\frac{\partial^2 V}{\partial t^2} = \beta^2 \left[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} - \frac{1}{2} \sum_{j=1}^{n} \left( \Gamma_j(t + \frac{\Delta t}{2}) + \Gamma_j(t - \frac{\Delta t}{2}) \right) \right]$$

(C.4)

$$\Gamma_j(t + \frac{\Delta t}{2}) = \frac{2 - \omega_j \Delta t}{2 + \omega_j \Delta t} \Gamma_j(t - \frac{\Delta t}{2}) + \frac{2 \omega_j \Delta t a_j \delta M}{(2 + \omega_j \Delta t) M_U} \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} \right).$$

The generalization of the set of equations (C.4) to a inhomogeneous medium, and the complete discretization in time and space can be accomplished by the method proposed by Korn and Stöckl (1982) as described in Section 3.2. The error of the viscoelastic scheme is of second order in $\Delta t$. Therefore, it does not require smaller discretization intervals in time and space with respect to the scheme for elastic media (Emmerich and Korn, 1987).

C.2. Introducing the intrinsic attenuation into the elastic wave equation

The implementation of intrinsic attenuation for P-SV wave propagation is now straightforward. The stress-strain relation derived for the rheological model of the generalized Maxwell body will be introduced into the equation of motion for P-SV-wave propagation in two-dimensional media. Two complex moduli are necessary which describe the dilatational and shear behaviour of the medium.
The elastic wave equation in a two-dimensional medium can be written as the following set of differential equations (Virieux, 1986)

\[ \frac{\partial u}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} \right) \]  
\[ \frac{\partial w}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} \right) \]  
\[ \frac{\partial \sigma_{xx}}{\partial t} + \frac{\partial \sigma_{zz}}{\partial t} = (2\lambda + 2\mu) \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \]  
\[ \frac{\partial \sigma_{zz}}{\partial t} - \frac{\partial \sigma_{xx}}{\partial t} = 2\mu \left( \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right) \]  
\[ \frac{\partial \sigma_{xz}}{\partial t} = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \]

where \( u \) and \( w \) are the horizontal and vertical velocity components, \( \rho \) is the density, and \( \lambda \) and \( \mu \) are the Lamé coefficients. The normal stresses are combined linearly to reduce the computer memory space required to store the variables, when anelasticity is introduced into the algorithm. Dissipative terms will be introduced into the stress-strain relations (C.5c), (C.5d) and (C.5e). For an isotropic medium, these terms depend on \( Q_\alpha \) and \( Q_\beta \), the intrinsic attenuation for dilatational and shear waves. The approximations of the intrinsic attenuation with the rheological model of the generalized Maxwell body introduced into Equations (C.5d) and (C.5e) have the same form as those for SH waves, because the stress-strain relations depend only on the shear modulus. Introducing intrinsic attenuation into Equation (C.5d), the latter takes the form of Equations (3.33):

\[ \frac{\partial \sigma_{zz}}{\partial t} - \frac{\partial \sigma_{xx}}{\partial t} = 2\mu \left( \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right) - 2 \sum_{j=1}^{n} \phi_j(t) \]

\[ \ddot{\phi}_j(t) + \omega_j^2 \dot{\phi}_j(t) = \omega_j a_j \delta M_2 \left( \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right) \]

The parameters \( a_j \) and \( \delta M_2 \) are determined by a curve fitting to the \( Q_\beta=\text{const} \), as described for Equation (3.44). Defining

\[ \Gamma_{j2} = \dot{\phi}_j \]

with the subscript 2 for the second stress-strain relation (C.5d), the equations take the following form
\[
\frac{\partial \sigma_{zz}}{\partial t} - \frac{\partial \sigma_{xx}}{\partial t} = 2\mu \left( \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right) - 2 \sum_{j=1}^{n} \Gamma_{j2}(t) \tag{C.6a}
\]

\[
\dot{\Gamma}_{j2}(t) + \omega_{j} \Gamma_{j2}(t) = \omega_{j} a_{j2} \delta M_{2} \left( \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right) \tag{C.6b}
\]

The same procedure can be performed for the stress-strain relation (C.5e), resulting in the following set of equations

\[
\frac{\partial \sigma_{xz}}{\partial t} = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) - \sum_{j=1}^{n} \Gamma_{j3}(t) \tag{C.7a}
\]

\[
\dot{\Gamma}_{j3}(t) + \omega_{j} \Gamma_{j3}(t) = \omega_{j} a_{j3} \delta M_{3} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \tag{C.7b}
\]

Since Equations (C.6) and (C.7) are solved for different grid positions in the staggered grid, as defined in Section 3.3, the values \(a_{j3} \) and \( \delta M_{3} \) can be different from the values \(a_{j2} \) and \( \delta M_{2} \) (see Figure C.1). Equation (C.5c) is related to both \( Q_{\alpha} \) and \( Q_{\beta} \). Thus, the introduction of the attenuation into the stress-strain relation leads to the following set of equations

\[
\frac{\partial \sigma_{zz}}{\partial t} + \frac{\partial \sigma_{xx}}{\partial t} = (2\lambda + 2\mu) \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right) - 2 \sum_{j=1}^{n} \dot{\theta}_{j}(t) + 2 \sum_{j=1}^{n} \Phi_{j}(t) \tag{C.8}
\]

\[
\ddot{\Phi}_{j1}(t) + \omega_{j} \Phi_{j1}(t) = \omega_{j} a_{j2} \delta M_{2} \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right) \tag{C.8}
\]

\[
\ddot{\theta}_{j}(t) + \omega_{j} \dot{\theta}_{j}(t) = \omega_{j} a_{j1} \delta M_{1} \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right) \tag{C.8}
\]

The differential equation for \( \Phi_{j1} \) describes the attenuation of shear waves and the differential equation for \( \theta_{j} \) describes the attenuation of dilatational waves. The values of the \( a_{j2} \) and \( \delta M_{2} \) have been determined by fitting a curve to \( Q_{\beta} \). The same can be done for dilatational waves and \( Q_{\alpha} \), in order to determine the values of \( a_{j1} \) and \( \delta M_{1} \) through the relation

\[
Q_{\alpha}^{-1}(\omega) = \frac{\sum_{j=1}^{n} a_{j1} \delta M_{1} \frac{\omega}{\omega_{j}}}{\left( \lambda + 2\mu \right) - \sum_{j=1}^{n} a_{j1} \delta M_{1} \frac{1}{1 + (\omega/\omega_{j})^{2}}} \tag{C.8}
\]
Defining
\[ \Gamma_{j1} = \dot{\theta}_j - \Phi_{j1}, \]

the system of equations can be written as follows
\[
\frac{\partial \sigma_{zz}}{\partial t} + \frac{\partial \sigma_{xx}}{\partial t} = (2\lambda + 2\mu) \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right) - 2 \sum_{j=1}^{n} \Gamma_{j1}(t) \quad (C.9a)
\]
\[
\dot{\Gamma}_{j1}(t) + \omega_j \Gamma_{j1}(t) = \dot{\omega}_j (a_{j1} \delta M - a_{j2} \delta M_2) \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right). \quad (C.9b)
\]

For the parameter \( \Gamma_{jk} \) (k=1,2,3), all sets of equations (C.6b), (C.7b) and (C.9b) can be discretized in time, by writing \( \Gamma_{jk} \) in the following form
\[
\Gamma_{jk}(t) = \frac{1}{2} \left( \Gamma_{jk}(t + \frac{\Delta t}{2}) + \Gamma_{jk}(t - \frac{\Delta t}{2}) \right).
\]

A second order discretization of Equations (C.6b), (C.7b) and (C.9b) yields
\[
\Gamma_{j1}(t + \frac{\Delta t}{2}) = \frac{2 - \omega_i \Delta t}{2 + \omega_i \Delta t} \Gamma_{j1}(t - \frac{\Delta t}{2}) + \frac{2 \omega_i \Delta t (a_{j1} \delta M - a_{j2} \delta M_2)}{2 + \omega_i \Delta t} \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right)
\]
\[
\Gamma_{j2}(t + \frac{\Delta t}{2}) = \frac{2 - \omega_i \Delta t}{2 + \omega_i \Delta t} \Gamma_{j2}(t - \frac{\Delta t}{2}) + \frac{2 \omega_i \Delta t a_{j2} \delta M_2}{2 + \omega_i \Delta t} \left( \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right)
\]
\[
\Gamma_{j3}(t + \frac{\Delta t}{2}) = \frac{2 - \omega_i \Delta t}{2 + \omega_i \Delta t} \Gamma_{j3}(t - \frac{\Delta t}{2}) + \frac{2 \omega_i \Delta t a_{j3} \delta M}{2 + \omega_i \Delta t} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)
\]

and the stress-strain relations are
\[
\frac{\partial \sigma_{zz}}{\partial t} + \frac{\partial \sigma_{xx}}{\partial t} = (2\lambda + 2\mu) \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right) - \sum_{j=1}^{n} \left( \Gamma_{j1}(t + \frac{\Delta t}{2}) + \Gamma_{j1}(t - \frac{\Delta t}{2}) \right)
\]
\[
\frac{\partial \sigma_{zz}}{\partial t} - \frac{\partial \sigma_{xx}}{\partial t} = 2\mu \left( \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right) - \sum_{j=1}^{n} \left( \Gamma_{j2}(t + \frac{\Delta t}{2}) + \Gamma_{j2}(t - \frac{\Delta t}{2}) \right)
\]
\[
\frac{\partial \sigma_{xz}}{\partial t} = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) - \frac{1}{2} \sum_{j=1}^{n} \left( \Gamma_{j3}(t + \frac{\Delta t}{2}) + \Gamma_{j3}(t - \frac{\Delta t}{2}) \right).
\]

For the complete discretization in time and space, the numerical scheme proposed by Virieux (1986) can be applied. The positions of the variables in
the staggered grid are shown in Figure C.1. The error of the viscoelastic scheme is of second order in $\Delta t$ (see Section 3.3.1). Therefore, it does not require smaller discretization intervals in time and space with respect to the scheme for elastic media.

![Diagram](image)

**Figure C.1.** Discretization of the medium on a staggered grid, indicating also the grid positions of the quality factors $Q_\alpha$ and $Q_\beta$. 