Numerical Modelling and Capacity Design of Earthquake-Resistant Reinforced Concrete Walls

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PREFACE

Reinforced concrete structural walls are very efficient elements for protecting buildings against excessive early damage and against collapse under earthquake actions. Structural walls designed by the world-wide increasingly used capacity design method exhibit a very ductile behaviour. The plastic hinge region chosen for the development of a suitable mechanism has to be detailed for the design ductility according to well known rules.

Complementary to the design procedures there is a need for tools for the analysis of the nonlinear behaviour of structural walls in the time domain. The plastic hinge region must be modelled in such a way that a realistic simulation of the hysteretic behaviour is possible. Furthermore, the local ductility demand in the plastic hinge should be extracted for the purpose of comparison with the design ductility demand assumed at the beginning of the design process.

In this report, which is based on a doctoral thesis, two kinds of model for simulating the nonlinear behaviour of plastic hinge regions in the time domain are developed: A macro model and a micro model. The macro model consists of four nonlinear springs connected by rigid beams. The micro model is a smeared crack finite element approach. In particular, the macro model proved to be capable of realistically simulating the overall earthquake behaviour of structural walls.

The results of the analysis including among others the ductility demand, the moment resistance demand, and the shear resistance demand, may be directly obtained and compared to the assumptions of the design process. Hence, using the modern computer models and tools developed in this report remarkable progress not only in the technique of time history analysis of reinforced concrete wall structures but also for practical earthquake design purposes has been made.

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Prof. Hugo Bachmann
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Gratefulness is also expressed to the foundation Stiftung für wissenschaftliche, systematiche Forschungen auf dem Gebiet des Beton- und Eisenbetonbaus des Vereins Schweizerischer Zement-, Kalk-, und Gipsfabrikanten (VSZKGF) and to the Swiss Federal Institute of Technology (ETH) for the generous support of the work presented in this report.
Reinforced concrete structural walls constitute an important unit for the resistance of buildings against seismic action. In order to successfully design structures against earthquakes, it is therefore of interest to develop a numerical model which simulates the typical behaviour of these units. This report is concerned with numerical models intended to be used in analysis of complete buildings, with focus on capacity designed multi storey buildings.

A major part of the report is devoted to the development of a new macro model which simulates the highly nonlinear behaviour of structural walls based upon relatively simple kinematics and physical behaviour. The formulation of a macro element is presented.

As a complement to the macro model, a micro model is derived with which it is attempted to treat the behaviour of the different material components of a structural wall in a relatively detailed manner, yet also based upon physical observations.

The models are implemented into a general finite element code and extensive tests are presented including comparisons with experimental data.

An important part of the report deals with the capacity design of structural walls. Performance checks are carried out on capacity designed walls by means of the newly developed macro model.

It is shown that the dynamic curvature demand in the plastic hinge may be different than suggested in the existing capacity design procedure, when varied over different wall aspect ratios. It is further shown that during nonlinear time history analysis flexural yielding may frequently take place in the upper storeys of the wall which are intended to remain elastic, when the existing capacity design procedures are used. It is also shown that the dynamic shear forces may be larger than anticipated by existing capacity design assumptions.

An improved distribution of flexural strength over the height of the wall is proposed, which clearly reduces the risk of unintended yielding in the upper storeys.

**Keywords:** capacity design; ductility; dynamic structural analysis; earthquake-resistant structures; finite elements; flexural strength; hysteresis; reinforced concrete; shear strength; standards; stiffness; strength; structural analysis; structural design; walls
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CHAPTER ONE

INTRODUCTION

1. General

In many buildings, reinforced concrete structural walls provide an important part of the resistance against lateral actions, such as wind and earthquake. In multi-storey buildings, R/C structural walls may often be the only possible means to achieve sufficient lateral resistance. Tall structural walls act to a large degree as cantilever beams, and the lateral resistance they offer is mainly of a flexural nature. The term "shear walls", which is also commonly used, may therefore be somewhat misleading in that it gives the impression of major shear behaviour. The term "structural wall" does not lead to such misinterpretations and will hence be used throughout this report.

Should, despite a careful structural design, a severe action lead to the failure of a structural wall, a flexural failure is strongly desirable. This is due to the fact that flexural failures occur less suddenly than shear failures, and provide a better means of avoiding a structural collapse, as well as better rebuilding possibilities. Furthermore, by consistent and carefully performed structural detailing, a structural wall may stretch flexurally in a controllable way far into the nonlinear range without failing.

In the early days of reinforced concrete structural walls were mainly designed and analysed as "wide" columns. During the 1950's increasing interest in the behaviour of reinforced concrete structural walls developed. Experimental studies focused mainly on the shear behaviour [TO55] or on the axial load carrying behaviour [Lars59].

Increased interest in the behaviour of tall structural walls subjected to lateral action emerged in the 1960's in connection with more widely spread seismic design provisions, mainly developed in California. The design was essentially still dominated by the assumption of "elastic" behaviour. The finite element method, emerging at this time, made computer simulations of the behaviour of tall structural walls increasingly feasible, although they were essentially limited to linear programs and to research purposes. Nonlinear computer analysis of reinforced concrete structures had just started [Nils68].
In the 1970's the inclusion of the nonlinear behaviour of reinforced concrete in design gained increased international interest. As a continuation of the extensive findings on reinforced concrete published by Park and Paulay in [PP75], substantial advances in the area of seismic design of reinforced concrete structures were achieved by the introduction of the "Capacity Design Method" developed in New Zealand during the last two decades by Paulay et al, see [PBM90] and [PP92]. Further summaries are given in [BP90] and [MP90]. This method deals with the design of reinforced concrete structures so as to achieve controllable ductile behaviour. The method has been confirmed by a large number of experiments, mainly carried out at the University of Canterbury in Christchurch, New Zealand.

The preferable ductile behaviour is achieved by selection of plastic hinge zones, careful structural detailing of these, and protection of the rest of the structure against yielding. The plastic hinge zone of structural walls is usually located at the base of the wall, while the rest of the wall is intended to act essentially elastically, although it may become cracked.

Figure 1.1 shows a typical multi-storey capacity designed building with dimensions length $L$, depth $D$, and height $H$, in which structural walls resist horizontal actions. The plastic hinge of each wall is located at the base and stretches over the length $L_p$, taken as the largest of $H/6$ or the wall length $L_w$.

![Diagram of a typical multi-storey building with structural walls for the resistance of horizontal actions](image)

*Figure 1.1 Typical multi-storey building with structural walls for the resistance of horizontal actions*
**INTRODUCTION**

Confinement hoops, tightly spaced in plastic hinge zone

V L_y, i

a) Cross section with protruding boundary elements

Z.

b) Rectangular cross section

Figure 1.2 Typical wall cross sections for capacity designed walls

Figure 1.2 shows two typical cross sections of capacity designed multi-storey structural walls. Flexural reinforcement bars are concentrated at the ends and are confined by hoops which are tightly spaced in the plastic hinge zone. Sometimes protruding boundary elements may be necessary in order to resist large normal forces in combination with bending moments. Vertical reinforcement in the "web" of the wall is usually placed as economically as possible, often according to minimum rules. Detailed recommendations for flexural-, shear- and confining-reinforcement placement are provided in [PBM90].

In the modern structural design process, where materials are used efficiently and for which the demands for a safe and reliable end result are increasing, numerical tools for the analysis of the structural behaviour constitute a very important aid in the verification of the performance of the designed structure. For complex multi-storey buildings a rational design is usually no longer possible without the aid of computer programs. Also, when consistent and rational design methods, such as the above mentioned Capacity Design Method, are used, it is of great value to verify the nonlinear performance by means of a computer program.

For R/C structural walls, however, there is as of yet, among the extensive commercial and university software and despite numerous promising attempts, hardly any simple and readily available numerical model designed specifically to simulate the nonlinear behaviour of structural walls under seismic action.
CHAPTER ONE

1.2 Objectives and Limitations

The study presented in this report should be seen as a further attempt to clarify the most important features of the nonlinear behaviour of structural walls by means of numerical modelling. Emphasis is placed on performance control of capacity designed walls.

In order to accomplish this, the work must be organised so as to fulfil a number of objectives set out at the beginning of the study. The major objectives of this report were set out as follows:

1. Based upon an overview of the state of the art of major numerical wall models to select a model worthy of further development, intended for simulating multi-storey wall buildings with simple and regular wall geometry. This model should in its final state be capable of analysing various structural properties relevant to the capacity design method.

2. For the control of the behaviour of this model during dynamic analyses as well as a complement to this model in cases of involved and irregular geometry etc., a second model should be developed, describing the material behaviour of the constituents in a relatively detailed manner. The second model should not in the first place be designed for control of capacity design performance.

3. To perform a detailed and transparent development of these models into workable numerical tools for nonlinear structural wall analysis. Emphasis should be placed on simple solutions, for both models, particularly when no proof of the superiority of complex solutions exists.

4. To check the reliability of the developed models in a number of numerical examples including comparisons with experimental data, and identify which model parameters are important and which are not. Further, to compare the responses of the two models for dynamic behaviour of multi-storey buildings.

5. To review the structural performance of capacity designed buildings by means of the model developed for this purpose, and consider it in relation to basic criteria specified at the design phase.
6. If applicable, to suggest some general improvements in the capacity design procedure for structural walls.

7. Finally, to give recommendations for further research.

In order to perform this study in an efficient manner a number of limitations have to be imposed on the extent of the work. The major limitations of this report were set out as follows:

1. Mainly the global behaviour of structural walls is of interest. In-depth analysis of local phenomena, other than the degree necessary for the model development, is not carried out.

2. Except for gravity loading, this report focuses on earthquake action only. Other actions such as wind, creep, shrinkage, temperature, environmental effects, etc., are not considered in this study.

3. The modelling in this report focuses on the nonlinear behaviour of walls from the outset. Therefore, there will be essentially no discussion of linear dynamic analysis results.

4. The study limits itself to the behaviour of structural walls and essentially does not deal with problems associated with the connection of walls to other structural elements such as beams and columns.

5. The work does not enter into the problems of major irregularities of walls: openings in walls, coupled walls, as well as three-dimensional configurations such as lift shafts, etc., although the capability of modelling such cases should be regarded in an overall evaluation of a model.

6. Interaction between superstructure and soil is beyond the scope of this study, although it is known to have an important influence on the structural performance in some situations, e.g. soft soil.
1.3 Scope of Report

The organisation of the report is as follows:

After this introductory first chapter, the second chapter is devoted to a review of different numerical models for the simulation of nonlinear behaviour of structural walls. The existing models are divided into major categories, and for each category a brief background is given. Advantages and disadvantages of each model category are briefly discussed. The second chapter concludes with a selection of two numerical model types to be further developed in this report.

Chapter three deals with the development of a numerical model intended for the simulation of multi-storey capacity designed walls. This model is classified in chapter two as a macro model. An efficient and transparent macro model is developed in stages, including the derivation of kinematic relations, flexural and shear behaviour, as well as axial load effect. Cross sectional characteristics such as ductility ratio are implemented in the model.

A micro model is developed in chapter four. After identification of the most important phenomena to be regarded in a micro model, the crack behaviour of the concrete is developed. Then the steel behaviour as well as the concrete-steel interaction is included in the model.

Numerical testing of the developed models is presented in chapter five. These tests include comparison with experimental data, as well as a check on the performance of a capacity designed multi-storey building. In chapter six some aspects of the capacity design method in relation to numerical results are discussed. The report concludes with chapter seven, containing a summary, conclusions and recommendations for future research.

The developed numerical models were implemented into a general finite element code as "user elements" and "user material", respectively. In the Appendix of this report there is a short users' manual, describing the use of these models.
CHAPTER TWO

REVIEW OF NUMERICAL MODELS

2.1 Introduction

There are many different models for the numerical modelling of structural walls found in the literature. As a background to the subsequent chapters of this report some of the more important numerical models are presented in this chapter. The different models are briefly described and their advantages and limitations mentioned. For a more comprehensive study of any particular model the reader is referred to the appropriate reference. The discussion of the different models is here seen mainly relation to the analysis of a global building structure, subjected to gravity loading as well as earthquake forces applied statically or dynamically.

The numerical modelling of structural walls may, from a structural engineering point of view, be classified in two major model levels: macro models and micro models. The former attempt to model the overall behaviour of a structural wall cross section over a certain height, while the latter base the behaviour upon the constitutive laws of the mechanics of solids. Models which may be placed between the two major categories may be referred to as meso models. A good discussion on the two major model categories is provided by Vulcano and Bertero in [VB87]. It may be especially noted that the definitions used here should not be confused with similar definitions used in the field of fracture mechanics.

After a description of the different numerical models and their applicability to various structural analysis situations this chapter concludes with a discussion of a suitable model for further development in this report.

2.2 Macro Models

The category "Macro Model" is here understood as a numerical model which attempts to incorporate the entire behaviour of a major region of a structural wall, such as a storey height or part thereof, including the wall's constituents such as the concrete, the reinforcing steel, and the interaction effects between concrete and steel.
In the literature several different macro models are found for structural walls. However, it appears possible to divide the more important and frequently used models into three types, which will be discussed separately in the following.

2.2.1 Beam element models

The simplest numerical model for a structural wall consists of beam elements, with six degrees of freedom per element. The wall in this case is regarded as a deep column. This is a very commonly used concept, and in some analysis situations it may provide a model which is sufficiently realistic. If the vertical deformations at the wall edges due to flexure are considered unimportant or are assumed to be small, the entire wall model for one storey may consist of a single beam element.

For the prototype wall seen in figure 2.1 the beam element modelling is shown in figure 2.2. For walls with a considerable horizontal length, as well as in the case of an interaction with a structural frame, it may be necessary to consider the vertical edge deformations. A simple solution including this effect has been suggested by adding hori-
horizontal rigid beams on either side of the vertical beam \([\text{BASC84}]\), as shown in figure 2.3, and thereby obtain vertical deformations at the wall edges.

The advantages of these beam models consist of the uncomplicated modelling, and sometimes possibilities to check the results by frame analogy hand calculations. Few degrees of freedom is another advantage, especially in dynamic analysis.

The limitations are mainly due to the inability to describe the wall's behaviour along its cross section. The vertical deformations at each edge of the wall are not considered if there are no horizontal rigid beams. Even with these rigid beams the strain distribution will not be realistically modelled, since the shift of the neutral axis, which is typical for a wall when flexural cracking and subsequent yielding occurs, cannot be reproduced. This is especially noticeable under flexure at the tensile edge where the large tensile strains are not considered by the model.
2.2.2 Truss element models

The next macro model is the truss model, of which different versions are presented. Typically, a truss model as shown in figure 2.4, consists of two vertical truss elements, and at least one diagonal truss element. These are connected by a rigid horizontal beam.

Truss models like this have been used e.g. by Vallenas et al [VBP79], and by Hiraishi in [ACISP84]. The diagonal truss is supposed to model the concrete "compression strut" which forms under lateral force. This behaviour may be reproduced quite well, however, under force reversal it is necessary to use a diagonal truss in the opposite diagonal direction. Furthermore, the reproduction of behaviour under various moment/shear applications seems problematic, as well as the realistic modelling of deformations due to gravity load and lateral force, each by itself, and combined.

For static monotonic force application, and for a small gravity load, the model may provide useful results, if carefully calibrated. However, its use appears to be limited to rather squat walls, where a compression strut of this nature actually develops. Further, the versatility of the model may be limited compared to other models, and dynamic analysis does not appear feasible.

2.2.3 Multiple spring element models

The third macro model is the multiple spring element model, which originated in the early 1980's within the framework of the US-Japan Cooperative Earthquake Research Program [ACISP84]. The model was intended for the wall modelling in the analytic prediction of the experimental tests on a full-scale seven storey R/C structure, carried out at the Building Research Institute in Tsukuba. For elevation and plan of the test specimen, see figures 2.5a and 2.5b.

The first numerical model of this type, suggested by Kabeyasawa et al, was used for the modelling of single storey wall sections as seen in figure 2.5c, and is shown in figure 2.6. It comprises three vertical springs, one rotational spring, and one horizontal spring, which are all connected by rigid beams. The nonlinear behaviour of the seven storey test structure could be simulated quite well.

Generally, important characteristics of nonlinear structural wall behaviour, such as large tensile strains, shifting of the neutral axis, as well as significant shear deformations, can be simulated adequately by models based on this approach.
Some major limitations of the model are: the rigid beams imply that plane cross sections remain plane which is a poor assumption for deep beams and walls, but less critical for very tall and slender walls, the study of which is the main objective here. Experimentally obtained strain distributions of a slender wall specimen [WK85] and even
of a relatively squat specimen [VBP79] show that in mainly flexural modes the cross sections remain close to plane even far into the nonlinear range. Furthermore, the model is not capable of taking into account a bending moment gradient along its element height, and it does not provide much information on localised damage, such as crack direction. Nevertheless, the model appeared to give reasonable agreement with some experimental data.

Refinements of this original model have been attempted by some authors, and is dealt with more fully in chapter three.

2.3 Micro Models

The category of Micro Models is based upon the mechanics of solids, and comprises the wide field of the modelling of constitutive relations, and their implementation in continuum elements. In the case of structural walls, with the usual approximations, this may be performed by applying the plane stress relations of the materials and by implementation in membrane elements, as shown in figure 2.7

During the early research of nonlinear concrete behaviour in the late 1960's two major approaches for modelling cracking of concrete evolved: the discrete approach and the smeared approach. It has been found that the smeared crack approach lent itself more efficiently to modelling the behaviour of reinforced concrete with its interaction effects between reinforcement and concrete and well distributed cracks of moderate crack width. On the other hand, the discrete crack approach, pioneered by Ngo and Scordelis [NS67] and Nilson [Nils68] was found to be well-suited to unreinforced structures such as concrete dams, where a few cracks with wide openings play a significant role in the changed structural behaviour. For the discrete approach the problem of mesh updating has been treated among others by Skrikerud [Skri82]. These findings essentially still hold today, although the discussion on this topic continues.

The smeared crack approach was introduced by Rashid [Rash68] for the analysis of concrete pressure vessels. The first attempts at nonlinear analysis of structural walls by the smeared crack approach date back to around 1970, [Cerv70, Fran70]. An application to the global analysis of tall structural walls by membrane elements was given by Moazzami and Bertero in 1987 [MB87], by their modelling of the seven-storey wall of the concrete test structure at Tsukuba [ACISP84] for monotonic load conditions. Bolander and Wight [BW91] analysed a ten-storey concrete building with several structural walls under monotonic loading. During the seventies and eighties, further efforts went
Figure 2.7  Micro modelling of cracking zone in wall panel by finite elements; different crack model approaches

into modelling the behaviour of cracked concrete and the interaction between steel and concrete. However, relatively few attempts have been made to develop a simple micro model which exhibits reasonable global behaviour under seismic action.

Clear advantages of the micro model are its versatility and ability to give information on localised behaviour. The more elaborate model generation and higher numerical effort compared to macro models however, is a clear drawback. For multi-storey buildings with several structural walls a comprehensive dynamic simulation may not be feasible. Another limitation may be the lack of interest shown by design engineers in the rather involved formulations often presented in many reports as well as the lack of global results, resulting in a limited use in practice.

Some of the commercially-available finite element codes provide material behaviour described as e.g. "concrete behaviour" to be used with existing library elements. A problem seems to be that this is usually only intended for monotonic loading, and thus may not be of use in detailed earthquake analysis comprising cyclic or dynamic behaviour. Furthermore, the freedom to modify this concrete behaviour by the user is usually limited.

2.4 Meso Models

A category of models which may be placed between the macro models and the micro models is presented by e.g. Meskouris et al [MKHH91], and will be denoted as meso models. In this reference as a complement to detailed micro models, simplified two
dimensional wall models are presented. The justification for these models was mainly given with regard to computational efficiency in comparison to the more detailed models.

The meso models consist of two dimensional membrane elements, with simple bilinear material models. Explicit crack formulation is not taken into account by orthotropic material expressions, but instead simplified hysteretic rules are used to account for cracking and yielding.

Thus, meso models, although they are implemented in continuous elements, use simplified material behaviour which belongs more to the macro models. The results of these models may be of varying quality. In cases where the overall behaviour is only slightly nonlinear these models may represent a good compromise between performance and computational effort.

2.5 Choice of Models for Development

The numerical models described above all offer particular advantages and disadvantages for given analysis tasks. It appears difficult to find a model which displays only or mainly advantages. The opportunity to compare the results of some different numerical models applied to the same problem should be helpful for complex analyses.

In this report the analysis of the global behaviour of tall slender structural walls under seismic excitation is of primary interest. Thus, the models on the macro-level appear suitable for this purpose, since they function to a large degree in a global manner. Of the three types of macro models described previously, the multiple spring model generally appears to be the most promising and capable of simulating the main characteristics of nonlinear wall behaviour.

Its predominant global behaviour, as well as relatively easily defined cross sectional output quantities, make it suitable for the analysis of capacity designed walls. Consequently, the multiple spring model was selected for further investigation in this report, and from now on is referred to as the "macro model". The other model types on the macro-level are treated in this report.

The reliability of the selected macro model may be checked for static behaviour against experimental data. However, since the nonlinear dynamic response of multi-storey wall buildings cannot be reproduced experimentally reliably (on a large scale), it is necessary to complement the multiple spring model by a different numerical model, in order to have a comparison basis for dynamic problems.
Furthermore, for some cases of global wall analysis, mainly with irregularities of various kinds, the micro-level model seems preferable. When a more detailed analysis of a particular region of a wall is of interest, a micro model also seems to be favourable. In order to fulfil these purposes an attempt to develop a relatively simple and transparent micro model for structural walls will be within the framework of this report. This micro model should be based on the smeared crack approach, which appears to be the most suitable for the modelling of uniformly reinforced concrete structures such as structural walls.

Although the meso models may be useful in some cases, especially as a complement to micro models in the same analysis, e.g. for regions which behave less non linearly, the more clearly defined macro and micro models seem to cover the essential points of view in the discussion of wall models. Therefore, meso models will not be treated in this report.

The development of a macro model is presented in chapter three, and the development of a micro model in chapter four. Separate reliability tests as well as comparison of their respective dynamic behaviour is performed in chapter five. The actual performance check of a multi-storey capacity designed wall is performed mainly by the use of the macro model, and is partly based on results in chapter five, but discussed in more detail in chapter six.

Based upon findings from the nonlinear time history analyses using the macro model, some modifications in the capacity design procedure for structural walls are also presented in chapter six. A general view of the selected models and analysis objectives with these models is presented in graphical form in figure 2.8. Much emphasis will be placed on the area of performance testing of capacity designed wall structures, as well as the discussion on improvement of the design procedure.
CHAPTER THREE

MACRO MODEL

3.1 Introduction

This chapter is devoted to the development of a numerical model for the simulation of the overall nonlinear behaviour of multi-storey structural walls subjected to seismic action. The model works with nonlinear springs and belongs to the category of macro models. Of the three types of model belonging to the category of macro models discussed in chapter two the type based on nonlinear springs connected by rigid beams was found to be the most suitable for the simulation of structural walls. In this chapter, a model of this type is developed into a functioning numerical tool for use in the subsequent chapters of this report.

A model of this type was originally suggested by Kabeyasawa et al [KSOA82], and was shown in figure 2.6. This model type was used during the US-Japan cooperative earthquake research program during the 1980's, and its primary objective was to provide a simple tool for the nonlinear analysis of multi-storey structural walls subjected to earthquake actions. Some further developments of this model type have been made by other authors in the late 1980's until recently and are briefly discussed in the following sections.

In this chapter the geometric considerations are discussed first based upon a short review of previous work by different authors, and a simplified and efficient model geometry is suggested. Based on kinematic relations the basic model properties for elastic behaviour are derived, and based on the well known material behaviour of concrete and steel, combined with observations of physical behaviour, the nonlinear flexural properties are developed.

Axial and shear behaviour are treated separately. Simple and efficient hysteretic rules are developed largely based upon empirical observations. Finally, the formulation of the stiffness matrix of a macro element is presented.
3.2 Model Configuration

The original model by Kabeyasawa et al, shown again in figure 3.1, consisted of five nonlinear springs, connected by rigid beams. The springs were made up as follows: two vertical outer springs, representing the axial behaviour of the boundary columns, one central vertical spring, representing the axial behaviour of the web, one central horizontal spring representing the shear behaviour of the wall section, and finally one central rotational spring, intended to represent the flexural behaviour of the web. The three central springs were located at the base of the element, or near the base.

Each one of the seven storeys of the full scale test specimen in Tsukuba, presented in chapter two, was modelled by a set of springs to form an element used as a storey model as shown in figure 3.1. In later developments models based in this type have been used in examples in which each storey was discretised into more than one element for better accuracy, see e.g. [BWL92a].

![Diagram of the original macro model by Kabeyasawa et al.](image)

Figure 3.1 Original macro model by Kabeyasawa et al [KSOA82]

The original model by Kabeyasawa et al was essentially used in numerical analyses for the prediction of the static cyclic and pseudo dynamic tests of the full scale wall in Tsukuba, and the scale models in the US-Japan cooperative research program.

The attempt to separately model flexural and axial behaviour in this manner led to compatibility problems, discussed by Vulcano and Bertero [VB87] and Linde [Lind88], [Lind89]. These difficulties arise mainly when flexural and axial properties are assigned to the rotational and vertical springs, respectively, as suggested for the original model, since these assigned properties base on the independent behaviour of the web and the
boundary columns. Attempts were made in [VB87] to correct this problem by assigning softening stiffness properties to the rotational spring. However, this solution was not fully reliable or efficient. It also seems difficult to explain the softening stiffness physically.

Furthermore, it was attempted in [VB87] to model the outer vertical springs by a spring assembly which simulates the physical behaviour of cracking and yielding. This was represented by a parallel and in-series spring assembly, as seen in figure 3.2a. The single spring on top is intended for uncracked concrete while the parallel springs below model cracked concrete and steel, respectively. The steel spring follows a bilinear curve, and the concrete crack-spring either seizes to act (cracked state) or takes up action (closing of crack). This interesting approach may be seen as an attempt to kinematically model the physical behaviour rather than employing hysteretic models for the composite material of reinforced concrete.

![Diagram of spring assemblies](image)

**Figure 3.2 Suggestions for improved macro models**

Vulcano et al [VBC88] and Fajfar and Fischinger [FF91] then replaced the rotational spring by several additional vertical springs to simulate the axial behaviour of the web. This method was able to simulate the gradual yielding of the vertical reinforcement more smoothly, but it consists of more components and thus leads to a more complicated model. Generally, refinements lead in the direction of micro models.
Attempts have also been made to develop simple and clear kinematic formulations of the macro model. In [Lind88], [Lind89] simpler geometry was suggested and tested for static loading.

In this study this concept will be continued and a model will be developed, which is based on a geometric spring arrangement as shown in figure 3.3. The idea behind this arrangement is to omit the central rotational spring and to perform a derivation of the properties for the remaining three vertical springs so as to satisfy both axial and flexural behaviour. The horizontal spring, modelling shear behaviour, continues its function, making a total of four nonlinear springs connected by rigid beams, as seen in figure 3.3. The model thus fulfils the necessary and sufficient spring arrangement in order to simulate the most important kinematic wall behaviour.

The idea behind the arrangement in figure 3.3 is thus to achieve simplicity by using as few springs as possible. The flexural behaviour which is treated in detail for elastic behaviour in the next section, can be made to simulate typical wall flexural behaviour quite accurately with only the two outer vertical springs, in combination with the third central vertical spring. Since the beams connecting the nonlinear springs are flexurally rigid, the kinematic possibilities are essentially the same as for models with more complicated spring configurations, and thereby this model is able to provide an efficient result, with the nonlinear behaviour derived properly.

Figure 3.3 Suggested simplified macro model, based on model in [Lind89]
3.3 Elastic Flexural Behaviour

For the wall model which was shown in figure 3.3, we derive here realistic properties for the elastic flexural behaviour. Axial behaviour, inelastic flexural behaviour and shear behaviour will all be treated in subsequent sections of this chapter.

For the flexural behaviour we have the two outer vertical springs at our disposal, and let them simulate the flexural behaviour of the entire wall section, i.e. the web of the wall, and boundary columns (if present), together giving a certain moment of inertia and area.

We consider first the elastic behaviour of the wall model. This is preferably accomplished by comparing the kinematic relations of a simple real wall with those of the model. For this purpose, we consider the cantilever structural wall as shown in figure 3.4a, which we refer to as the "real wall", and which obeys simple elastic theory for beams. Under pure flexure the model in figure 3.4b would have to simulate the uniform curvature.

![Diagram of wall rotation](image)

Figure 3.4 Kinematics of wall rotation for uniform moment distribution

By expressing the rotation and displacement at the top of the wall for the two walls in figures 3.4a and b, we can derive some simple model parameters. The real wall has the height \( h \), length \( l \), cross sectional area \( A \), moment of inertia \( I \), and Young's modulus \( E \).
For the model we may then, by prescribing the same rotation and displacement as the real wall and assuming the distance between outer springs to be \( l \), obtain the area of the outer springs, \( A_s \), as well as the location for the centre of rotation, \( h_c \). For this purpose, we lock the horizontal spring of the model, so as to obtain only flexural deformations. For figure 3.4a we obtain

\[
\theta = \frac{Mh}{EI} \tag{3.1}
\]

\[
\delta_h = \frac{Mh^2}{2EI} \tag{3.2}
\]

and for figure 3.2b, with \( F_s = \frac{M}{l} \), \( \theta = \frac{2\delta_s}{l} \), and \( \delta_s = \frac{Fhs}{A_sE} = \frac{Mh}{lA_sE} \), we get

\[
\theta = \frac{2Mh}{l^2A_sE} \tag{3.3}
\]

\[
\delta_h = \theta h_c = \frac{Mhh_c}{l^2A_sE} \tag{3.4}
\]

By setting (3.1) equal to (3.3), and (3.2) to (3.4) we obtain the model properties

\[
A_s = \frac{2l}{l^2} \tag{3.5}
\]

\[
h_c = \frac{h}{2} \tag{3.6}
\]

representing the outer spring area and the relative centre of wall rotation, respectively. The elastic stiffness \( K_{as} \) of the outer spring is then given by

\[
K_{as} = \frac{A_sE}{h} \tag{3.7}
\]

By replacing the moment \( M \) with a shear force \( V \), acting at the top of the wall, and using the model properties derived above, as seen in figure 3.5 we obtain a check on the elastic flexural behaviour for shear force with moment gradient.
Figure 3.5 Kinematics of wall rotation for shear force with moment gradient

We thus maintain the properties derived through equations 3.1 to 3.7, and from figure 3.5 we obtain for the real wall by elastic theory

\[ \theta = \frac{Vh^2}{2EI} \]  
(3.8)

\[ \delta_h = \frac{Vh^3}{3EI} \]  
(3.9)

and for the model

\[ F_s = \frac{V h_c}{l} = \frac{Vh}{2l} \]  
(3.10)

\[ \delta_v = \frac{F h}{A_s E} = \frac{Vh^2l}{4EI} \]  
(3.11)

\[ \theta = \frac{2\delta_v}{l} = \frac{Vh^2}{2EI} \]  
(3.12)

\[ \delta_h = \theta h_c = \frac{Vh^3}{4EI} \]  
(3.13)
Comparison of equation 3.8 to 3.12 shows agreement for rotation, but the deflection given by elastic theory (equation 3.9) differs from the model's deflection (equation 3.13). This is due to the fact that the model derived for pure flexure is not able to accurately account for a moment gradient over the height. This deficiency may, however, be reduced by discretization using several elements, [Lind88], [Lind89].

Another possibility would be to derive the quantities $A_x$ and $h_c$ using the case with a shear force $V$ only, acting on top of the wall, as in figure 3.5. The values for rotation and deflection at the top would remain according to equations 3.8 and 3.9 for elastic theory. For the model we obtain

$$F_x = \frac{Vh_c}{l}$$

(3.14)

$$\delta_r = \frac{F_x h}{A_x E} = \frac{Vh_c}{l A_x E}$$

(3.15)

$$\theta = \frac{2\delta_r}{l} = \frac{2Vhh_c}{l^2 A_x E}$$

(3.16)

$$\delta_h = \theta h_c = \frac{2Vhh_c^2}{l^2 A_x E}$$

(3.17)

Equating the expression 3.8 to 3.16 and 3.9 to 3.17, we obtain the model properties

$$A_x = \frac{8l}{3l^2}$$

(3.18)

$$h_c = \frac{2h}{3}$$

(3.19)

This shows that for the case of shear force only with moment gradient, the centre of relative rotation would be located lower ($h/3$ from the base) than for the case of pure flexure ($h/2$ from the base). This means that in a general case where we always have moment and shear, the centre of relative rotation $h_c$, and the area of the outer springs $A_x$ depend on the ratio of moment to shear.

Since in reality we always have both moment and shear acting at a wall section, a possibility would be to combine the results from the two extreme cases. The simplest method of combination would be to take the average of the above two cases, which would give
A more sophisticated way would be to combine the model properties according to expected ratio between moment and shear force, or possibly according to the instantaneous ratio. Equations 3.20 and 3.21 then take the following form

\[ A_s = a \frac{l}{l^2} + b \frac{4l}{3l^2} \]  
(3.22)

\[ h_c = a \frac{h}{4} + b \frac{h}{3} \]  
(3.23)

\[ a + b = 1 \]  
(3.24)

However, since the range between the two extreme cases is already quite narrow, the effort does not appear justified in relation to other more essential modelling considerations to be described.

Since the above assumptions are only valid in the elastic range, and we are going to examine structural walls which are mainly acting in a flexural state, we concentrate on the derivation based on a moment acting at the top, as shown above. Discretization would reduce the displacement error for pure shear action, as discussed in [Lind88]. Nonlinear considerations will, as we will see, be more important, than the analytical differences discussed above.

### 3.4 Elastic Behaviour under Normal Force

The spring stiffnesses of the two outer springs, derived in section 3.3, do not account for the entire axial behaviour alone. A third spring stiffness, that of the central vertical spring, is needed to accomplish this goal. In figure 3.6, the real wall and the model are each subjected to a normal force \( N \), assumed to cause uniformly distributed compressive axial deformation. For the elastic theory we obtain

\[ \delta_a = \frac{Nh}{EA_w} \]  
(3.25)
a) Elastic theory

![Diagram of elastic theory](image)

b) Model

![Diagram of model](image)

Figure 3.6 Behaviour under normal force

and for the model we correspondingly obtain

\[
\delta_a = \frac{N}{2K_{se} + K_{ea}} = \frac{N}{\frac{2EA_w}{h} + K_{ea}}
\]  

(3.26)

where \(K_{ea}\) is the elastic compressive spring stiffness for the central vertical spring. By introduction of the ratio \(\alpha\) between one outer spring and the cross sectional wall area, i.e. \(\alpha = A_s / A_w\), the elastic spring stiffness for the central vertical spring may be written

\[
K_{ea} = \frac{EA_w}{h} \left(1 - \frac{2\alpha}{h}\right)
\]

(3.27)

Together with the outer springs this spring accounts for the axial behaviour in compression and thereby the complete elastic axial behaviour of the model is determined.

3.5 Nonlinear Flexural Behaviour

To determine of the model properties beyond the linear elastic region it is necessary to study the physical behaviour of the cross section of a real wall. For this purpose we establish a moment-curvature relation, by means of a computer program, dividing a wall cross section into a finite number of fibres, see figure 3.7. In each fibre, the concrete and vertical steel obey their own constitutive relations. These are shown in figures 3.8 and 3.9.
For the concrete we use the relation given by Kent and Park [KP71] for the compressive behaviour, stated as follows

\[ \begin{align*}
\{ \varepsilon_c \leq 0.002 \} & \rightarrow f_c = f'_c \left[ \frac{2\varepsilon_c}{0.002} - \left( \frac{\varepsilon_c}{0.002} \right)^2 \right] \\
\{ \varepsilon_c \geq 0.002 \} & \rightarrow f_c = f'_c \left[ 1 - Z(\varepsilon_c - 0.002) \right]
\end{align*} \]  

(3.28)  

(3.29)  

\[
Z = \frac{0.5}{\varepsilon_{50u} - \varepsilon_{50h} - 0.002}
\]  

(3.30)
where $\varepsilon_{50u}$ and $\varepsilon_{50h}$ represent the strain at 50% strength on the descending branches of the unconfined and confined (hooped) concrete, seen in Figure 3.7. The following relations determine $\varepsilon_{50u}$ and $\varepsilon_{50h}$

$$\varepsilon_{50u} = \frac{3 + 0.002f'_c}{f'_c - 1000}$$  \hfill (3.31)

$$\varepsilon_{50h} = 0.75\rho_s \sqrt{\frac{b_h}{s_h}}$$  \hfill (3.32)

where $\rho_s$ is the confinement ratio, $b_h$ is the width of the hoops, and $s_h$ is the spacing of the hoops, and equation 3.28 assuming $f'_c$ in psi. No difference in strength is assumed for confined and unconfined concrete, although an increase of perhaps 10% was found by some researchers. Cracking was assumed for each fibre having reached the strain 0.0001.

For the reinforcing steel a bilinear elastic (i.e. linearly strain hardening) model was used, shown in figure 3.9. The hardening ratio may be chosen by the user. In the fibre model, the web steel area is smeared out and each fibre of the web obtains a steel area which is proportional to the fibre area. For the boundary elements a more refined modelling is possible so that essentially each single bar may be allocated to the proper fibre.

![Figure 3.9 Steel material model](image)
The computer program for cross sectional curvature behaviour employs an incremental iterative procedure described as follows. Any gravity load is applied at first as a normal force, resulting in a uniform compressive strain, assuming elastic behaviour. From this state the compressive edge is subjected to an incremental compressive strain. The tensile edge is thereby undergoing a trial incremental tensile strain, set to a fraction of the compressive increment. Between the two edges, a linear strain distribution is assumed.

For each fibre the concrete and steel both produce a force, calculated as their respective area within the fibre times the stress which is based on the strain in that particular fibre according to the material models of figures 3.8 and 3.9.

The fibre forces are accumulated for the entire cross section giving a resulting normal force. The initial normal force $N$ acting on the cross section is compared to the resulting force and a residual force results from subtraction. Vertical equilibrium is checked as follows

$$\sum_i F_i^c + \sum_i F_i^t - N \leq F_r$$

(3.33)

where the first and second terms represent the concrete and steel fibre forces respectively, the third term the normal force, and the right hand side the force residual. Summation is performed over all $n$ fibres. The fibre forces are obtained as

$$F_i^c = A_i^c \sigma_i$$

(3.34)

$$F_i^t = A_i^t \sigma_i$$

(3.35)

If this residual is larger than a preselected value, the tensile strain and thus the location of the neutral axis is adjusted, and a new iteration is performed, with the compressive strain kept fixed, until a sufficiently small residual is obtained.

In addition to summing up the fibre vertical forces, these forces are also multiplied with the fibre distance to the centroid (location of the normal force), giving a resulting internal bending moment, which is acting on the cross section due to the selected strain distribution. The internal bending moment is obtained as

$$M = \sum_i F_i^c x_i + \sum_i F_i^t x_i$$

(3.36)

where $x_i$ is the distance from the fibre to the centroid, as seen in figure 3.7.
The theoretical curve — simplified trilinear curve, fitted to theoretical curve

\[ \frac{M}{l} = \frac{F_s}{F_y} \]

\[ F_s = \frac{M}{l} \]

Figure 3.10 Nonlinear flexural behaviour for wall cross section

The curvature of the cross section is also recorded at the end of the iterative procedure when vertical equilibrium is reached, taken as the difference of the edge strains divided by the wall length. The tensile edge strain is increased by new increments, and in this manner we obtain a moment curvature relation of the wall cross section which is extended into the nonlinear range, seen in figure 3.10a.

This moment curvature relation is intended for guidance when modelling the nonlinear flexural behaviour by assigning nonlinear properties to the outer springs. The moment is transformed into a couple (two spring forces) by dividing the moment by the distance \( l \) between the springs. This distance should not be chosen significantly smaller than the length of the prototype in order to obtain realistic dynamic behaviour. As was stated in section 3.3, we have

\[ F_s = \frac{M}{l} \]  

It is then possible to fit a bilinear or trilinear curve for the spring behaviour to the resulting nonlinear force relation, see Figure 3.10b. This represents the skeleton curve, which is followed for pure monotonic loading, and to which the behaviour returns upon unloading and reloading as discussed subsequently. For a trilinear simplification, the two break points would represent flexural cracking and yielding, respectively [Lind88], [Lind89].
We will here simplify the approach further by assuming that the wall is to some degree cracked (flexurally) at the beginning of the analysis. Thus due to zero normal force we have a cracked stiffness, which is set as a fraction of the elastic (uncracked) stiffness. Factors of cracked to uncracked stiffness ranging from 0.5 to 0.8 give reasonable agreement with experimental data approximately resulting in the same ratio of cracked to uncracked flexural stiffness. This procedure is more pragmatic than attempting to obtain the cracked stiffness directly from a moment curvature relation as in [Lind89], and may be justified due to the fact that essentially the moment curvature relation only gives the behaviour at a flexural crack.

In reality, flexural cracks only occur at some spacing, with uncracked sections in between where the concrete has a stiffening effect. The cracked stiffness is kept until flexural yielding occurs. The yield force level may be extracted from the transformed moment curvature relation. It should be noted here that the yield curvature is determined from the procedure discussed above by using a preselected cracked stiffness until yielding, rather than attempting to read the yield curvature from the moment curvature relation. To account for some strain hardening of the steel and stiffening effect of concrete between cracks, the yield stiffness is set to some fraction of the uncracked stiffness chosen by the user. Values ranging from a fraction of one percent to a few percent were tested, and generally a value of around one percent was found to be reasonable. The suggested skeleton curve for the outer vertical springs will thus be as shown in figure 3.11.
Small stiffness around $10^{-6}K_{cs}$ for numerical purposes

Elastic stiffness in compression, $K_{cs}$

Figure 3.12 Behaviour of central vertical spring

The central vertical spring is active in compression only, and consequently when the moment acting on the cross section is sufficiently large to produce a zero strain at the centroid the stiffness of this spring is set to zero. A small value is assigned to the tensile stiffness in order to avoid numerical difficulties. The behaviour of the central vertical spring is shown in figure 3.12.

The above discussion implies that the two outer springs alone govern the behaviour at large curvatures, and that any initial normal force will be transferred through that outer spring which is in a compressive state. The moment transferred by a cross section with a certain effective normal force $N$, is then equal to the moment of a cross section with zero normal force plus the normal force times the distance between outer spring and centroid, i.e.

$$M^N = M^0 + \frac{NI}{2}$$ (3.38)

where $M^N$ denotes moment for a cross section with normal force $N$, and $M^0$ denotes moment for a cross section with zero normal force. This is an approximation which implies that the line of action of the compressive force is close to one edge, which is actually found to be the case in walls with low to moderate normal force. With this approximation the influence of the normal force on the position of the neutral axis and hence on the size of the compression area of the cracked cross section is neglected. This is justified for walls with low to moderate normal force.

Generally the above mentioned expression "low to moderate" normal forces may be understood in such a way that no significant compressive yielding may occur, if the macro model is to give reasonable accuracy. The effect of higher normal forces may for
this purpose be counteracted by protruding boundary elements, in order to limit compressive nonlinearities, and the model will function well.

With the above the monotonic behaviour of the wall model is established. We have treated the skeleton curve for the outer springs, i.e. elastic compressive stiffness, cracked tensile stiffness, and yield stiffness, as seen in figure 3.11. We will now determine the unloading and reloading behaviour used for cyclic loading.

Although we attempt to explain the unloading and reloading behaviour physically, as was done for monotonic behaviour, experimental data which pertains exactly to the outer vertical spring discussed here is not available. However, certain data is available from the full scale seven-storey wall specimen tested at Tsukuba [ACISP84], in the form of boundary element elongation versus base shear. If we assume that for pure static cyclic testing, the base shear is proportional to the spring force in the outer vertical spring, the test results from Tsukuba may serve in the development of a reasonable hysteretic model, describing the force-displacement relationship during cyclic behaviour. In figure 3.13 the boundary element elongation over the first storey versus base shear for the full-scale seven-storey wall example is shown.

The original model by Kabeyasawa et al [KSOA82] used a relatively complicated hysteretic model for the outer springs, which is shown in figure 3.14.
CHAPTER THREE

Figure 3.14 Original hysteretic model for outer springs by Kabeyasawa et al [KSOA82]

Figure 3.15 Hysteretic model for outer springs by Yanes [Yane82]

Figure 3.16 Modified hysteretic model for outer springs by Vulcano & Bertero [VB87]
Other authors modified the hysteretic models for the outer springs and proposals for modifications are shown in figures 3.15 to 3.18. For notation and detailed discussions on these models see the appropriate references.

All the models shown hitherto were mainly used in static cyclic numerical applications, where the prescribed deformations were obtained from some test program, such as the full-scale seven-storey wall structure tested in Tsukuba [ACISP84] or other reduced scale models of the same wall tested at different laboratories within the framework of the same research program. The full scale wall structure was tested by the SPD method (Single degree of freedom Pseudo Dynamic test) which resulted in floor displacement histories which were subsequently used in numerical analyses.
The macro model developed in this report is to be used with a general finite element code, the dynamic analysis being performed by direct integration methods using general ground motion histories as input. Therefore, the requirements on the hysteretic model used with this macro model are simplicity and reliability for global results.

Simplicity is needed mainly due to the already heavy computational demands by the method of analysis. It is not desirable to combine a very complicated hysteretic model having many different stiffness branches with this analysis method. Generally, every change of stiffness will cause residual forces requiring equilibrium iteration which consumes costly computational time. The hysteretic models suggested by some authors, and shown above, often display several stiffness branches in the same direction, and the location of stiffness break points are determined by involved expressions. In [Lind89] it was shown that relatively simple hysteretic models may in general give at least as reliable a global result with regard to test data. Essentially there should only be a stiffness change when it may be justified physically or is clearly necessary from experimental data. Generally, the fewer the stiffness changes of the model the better.

Despite the simplicity requirement discussed above a good hysteretic model must still be reliable compared to experimental data. It may, therefore, not be possible to simplify it more than to a certain degree. Otherwise, some of the characteristic behaviour of structural walls may be lost or impaired.

We propose here a hysteretic model for the outer springs, seen in figure 3.19, which major characteristics are simplicity and reasonable agreement with experimental results. It is based on the model in figure 3.17, and is made more efficient as follows. The stiffness break point in the tensile region of the skeleton curve is moved to the origin, i.e. it is assumed that the wall is already slightly cracked. This assumption is quite realistic for existing buildings, since slight cracking does usually exist a number of years after construction due to wind and other actions such as environment and possibly traffic-induced or seismic ground motion. In this way some input data for the tensile crack point may be omitted and the numerical behaviour may be more efficient.

Secondly, the proposed model displays a modified reloading behaviour as seen from figure 3.19b. All reloading occurs directly towards the maximum tensile displacement reached. In this way a purely trilinear loading and reloading model is obtained including the elastic compressive branch where cracks are closed. Previous models such as the one of figure 3.17 displayed four reloading branches which is rather a lot. The cyclic behaviour using this simple and effective hysteretic model is shown in chapter five to be
The proposed model in figure 3.19 displays the force in one of the outer vertical springs versus the axial deformation in that spring. The loading or "skeleton" curve consists of three basic stiffnesses. It is modelled with an elastic compressive stiffness $K_e$ which is used when the wall is first subjected to gravity load in terms of normal force, and later during cyclic behaviour when one edge (one outer spring) gets into a compressive state. In tension the simplification is used that the cross section is considered to be cracked to a certain extent prior to the analysis as discussed above, thus giving a cracked stiffness $K_{cr}$ already for the skeleton curve. Finally the yield stiffness $K_y$ is taken as a fraction of around one percent of the elastic stiffness.

Behaviour prior to yielding is shown in the figure on the left (small amplitude cycles). In tension unloading always occurs in a direction towards a point on the elastic compressive branch representing crack closure after compressive yielding of the flexural reinforcement steel.

The figure on the right displays the behaviour upon yielding (large amplitude cycles). The unloading from yielding occurs with the stiffness $K_u$, parallel to the unloading stiffness at the yielding point as seen in the figure ("elastic" unloading stiffness). From there the cross section yields in compression (reinforcing bars bridging open cracks yield) and the unloading moves to a point on the compressive elastic branch, chosen by the
Figure 3.20 Hysteretic rule numbers for suggested model

user, which represents the point where cracks are closing. This point was found to be dependent on the normal force, and is set to the yield force multiplied by a factor $\alpha_{cl}$. A larger normal force tends to require a larger negative force level for crack closure in order to obtain realistic global hysteretic behaviour, compared to experimental data. A force level in the range of $-F_y$ was found realistic for small normal forces.

For high normal forces, as found in the case of many storeys and high gravity loads, the crack closure force level tends to increase to e.g. $-4F_y$ for the case of eight storeys (normal force on the wall at first storey equal to 4 MN). This is explained by the fact that a large part of the internal moment is made up of the axial force (passing through the compressive zone located near the edge). This part may account for more than half of the internal moment in multi-storey walls. The tensile forces (flexural reinforcing bars mainly at the tensile edge) only contribute a smaller part of the total internal moment.

Reloading occurs towards a point on the yield branch of the skeleton curve where the maximum displacement was reached earlier, as seen in the figure. For partial reloading and unloading the stiffness is proposed shown in the figure, with stiffness change at zero force level. In order to obtain a clearer overview of the hysteretic rules, it is useful to assign a number to each rule, or branch, in figure 3.19. These numbers are shown in figure 3.20 and are briefly described as follows:
Rule 1. Elastic compression.
At the beginning of the analysis the element is expected to be at rule 1, due to the effect of gravity load. Due to the assumption of some cracking prior to the analysis no tensile elastic stiffness is considered, and when reaching tensile stress, the rule changes to rule 2.

Rule 2. Cracked tensile stiffness.
Is entered from rule 1. Models the cracked tensile stiffness without prior yielding. Unloading leads to rule 4. Further tensile force leads to rule 3 representing yielding.

Is entered from rule 2 if no prior yielding. If prior yielding occurred, it is entered from rule 7. Unloading leads to rule 5.

Rule 4. Unloading from cracking.
Is entered from rule 2. Below force level $-\alpha_{cl}F_y$ it leads to rule 1. At forces above entrance level, it leads back to rule 2.

Rule 5. Unloading from yielding.
Is entered from rule 3. Below zero force level it leads to rule 6. At force levels above entrance level, it leads back to rule 3.

Rule 6. Unloading at negative force level.
Is entered from rule 6. If reloading has occurred which did not lead to further yielding, it may be entered from rule 9 at zero force level. At force levels below $-\alpha_{cl}F_y$ it leads to rule 8. If reloading occurs, it leads to rule 7.

Rule 7. Reloading.
Is entered from rule 8, or from rule 7. Takes direction towards maximum displacement reached where it leads to rule 3. If unloading occurs, it leads to rule 9.

Is entered from rule 6. Force levels above $-\alpha_{cl}F_y$ lead to rule 7.

Is entered from rule 7. If force level is below zero, it leads to rule 6. If force level is above entrance level, it leads back to rule 7.
Figure 3.21 Influence of outer vertical spring stiffness on global flexural behaviour

The model suggested here for the hysteretic behaviour of the outer vertical spring was found to result in an overall flexural behaviour for the macro model which appeared reasonable compared to experimental data. It should be noted that the appearance of the hysteretic loops of this spring is in itself of more limited value, although some comparison with experiments could be made of its deformation versus some outer force such as the base shear. The vertical spring is otherwise essentially artificial in order to reproduce the entire flexural behaviour.

Concerning the resulting overall flexural hysteretic behaviour, the original vertical spring hysteresis model, seen in figure 3.14, was found to give thin loops sometimes. Partly this appears to be due to the unloading stiffness which is dependent on the maximum displacement reached. This has been observed to a small degree from some experiments, although usually not as pronounced as suggested in the original model. The unloading stiffness of the model proposed in this report was deliberately chosen to be constant, partly due to observed behaviour and partly for the sake of simplicity. The variable location of some of the stiffness break points in the original model was deliberately abandoned, which did not impair the overall behaviour. The other hysteretic rules were also deliberately chosen to be as simple as possible, but as comprehensive as necessary to obtain a reasonable simulation of flexural wall behaviour in general.

Before turning to the behaviour in shear some remarks are made regarding the relation between the "local" behaviour of the flexural springs described here, and the "global"
In order to show this relation, a simple example is shown in figure 3.21, displaying flexural behaviour in the form of a moment versus curvature plot, obtained as a numerical result by the macro model. A wall cross section with a yield moment of 3.0 MNm is subjected to pure bending. As a reference, a completely elastic behaviour is shown by the solid line. For this reference the tensile stiffness of the outer springs was set equal to the uncracked compressive stiffness, and no yielding was allowed for. Two comparison solutions were obtained, each with a value for cracked and yielded stiffness of the outer vertical springs. The first solution set the spring stiffnesses as follows: cracked: 80% of uncracked, yielded: 2% of uncracked. The second solution had the stiffnesses: cracked: 50% of uncracked, yielded: 1% of uncracked.

The global flexural behaviour resulting from these solutions may be measured in figure 3.21, in the form of flexural stiffness defined as moment divided by curvature. Normalising the elastic flexural stiffness of the reference solution to unity, we obtain for the first comparison solution a global flexural cracked stiffness of around 89% and global flexural yield stiffness of around 4%. For the second solution we obtain corresponding values of around 65% and 2%.

These values may be confirmed by solving this relation analytically. In figure 3.22 a wall model is shown consisting of two springs with different stiffnesses, separated from each other at a distance $l$. We only deal here with the case of two active springs, assuming a deformed state such that the central vertical spring has "yielded", or, alternatively, that we have a completely elastic compressive state, in which the central spring does not contribute to the flexural stiffness. For the discussion of the wall section moment of
inertia we work here with the areas of the outer springs, rather than with the spring
stiffnesses. Since the spring is an artificial creation its area is proportional to its stiffness
(height \(h\) and concrete modulus \(E\) remain unchanged). We define an idealised spring area
\(K_e\), taken equal to the elastic spring stiffness \(A_s E / h\) assuming unit element height, and
unit E-modulus, see equation 3.7. Each time the stiffness is changed, the spring area \(A_s\)
is changed, since the other two factors in the stiffness expression are constant. The
moment of inertia about the strong axis of the completely elastic model, \(I_e\), may then,
with use of Steiner's rule, be written

\[
I_e = K_e \left( \frac{l}{2} \right)^2 + K_e \left( \frac{l}{2} \right)^2 = K_e \frac{l^2}{2}
\]  

Modifying one of the two idealised spring areas to the cracked value \(K_{cr}\), we obtain a
moved centroid, located from the right edge a distance \(l_c\), where

\[
l_c = \frac{K_{cr} l}{K_{cr} + K_e}
\]  

and the corresponding global cracked moment of inertia

\[
I_{cr} = K_{cr} (l - l_c)^2 + K_e (l_c)^2
\]  

It is now possible to define factors of reduced global moment of inertia divided by
elastic global moment of inertia, i.e., for cracked idealised area, and thereby for cracked
stiffness

\[
\alpha_{cr} = \frac{I_{cr}}{I_e}
\]  

and correspondingly for yield stiffness

\[
\alpha_{y} = \frac{I_{y}}{I_e}
\]  

the latter basing on expressions (3.40) and (3.41) with the cracked spring stiffness \(K_{cr}\)
substituted by the yield spring stiffness \(K_{y}\).
Inserting the spring stiffness values of the numerical example shown in figure 3.21, as a fraction of the to unity normalised elastic spring stiffness, into expressions 3.40 to 3.43 we obtain for the first comparison solution

- Cracked stiffness, \( \alpha_{cr} = 0.8 \): \( l_c = 0.444l \) and \( \alpha_{cr}^f = 0.889 \)
- Yield stiffness, \( \alpha_y = 0.02 \): \( l_c = 0.0196l \) and \( \alpha_y^f = 0.039 \)

And for the second comparison solution

- Cracked stiffness, \( \alpha_{cr} = 0.5 \): \( l_c = 0.333l \) and \( \alpha_{cr}^f = 0.667 \)
- Yield stiffness, \( \alpha_y = 0.01 \): \( l_c = 0.0099l \) and \( \alpha_y^f = 0.020 \)

All of which show good agreement with the numerical result. As a rule of thumb it is possible to state that for local (spring) yield stiffness in the typical range of a few percent, the global flexural stiffness will be roughly twice that of the yield spring stiffness. As for the local (spring) cracked stiffness, one may estimate the global stiffness to be roughly 10 to 25 percent higher than the spring stiffness, with the higher value applying to lower \( \alpha_{cr} \) values.

A continuous relation between the local (spring) and the global (flexural) stiffness is shown in figure 3.23. On the horizontal axis is shown the reduction of the spring stiffness (local), and the corresponding global flexural stiffness reduction is plotted against the vertical axis. The region of the lower left corner of the figure typically represents the yield behaviour, whereas the rest of the figure essentially represents cracked behaviour. The upper right corner with coordinates \((1.0, 1.0)\) represents elastic uncracked behaviour.
3.6 Shear Behaviour

The behaviour in shear is completely simulated by the horizontal spring. In accordance with earlier discussion, we derive here the properties of this spring by comparing model behaviour to that of a real wall obeying elastic theory, as seen in figure 3.24.

Figure 3.24 Shear behaviour

For elastic behaviour we obtain by elastic theory the horizontal deflection of the real wall as follows

$$\delta_{he} = \frac{\kappa V h}{G A_w}$$

(3.44)

where $G$ is the modulus of rigidity, and $\kappa$ is the form factor in shear, which is geometry-dependent and often ranges from 1.2 to 1.3. Tomii and Osaki [TO55] give the following empirical value of $\kappa$

$$\kappa = \frac{0.75(1+u)(1-u^2(1-v))}{[1-u^3(1-v)]}$$

(3.45)

where the geometrical parameters $u$ and $v$ are shown in figure 3.25. The elastic shear stiffness $K_{se}$ for a section of height $h$ is then given by

$$K_{se} = \frac{G A_w}{h \kappa}$$

(3.46)
This stiffness is used until shear cracking occurs, whereupon which a cracked stiffness is employed. An empirically-based ratio $\alpha_s$ of cracked to uncracked shear stiffness was suggested by Otani et al [ACISP84] as follows

\[
\alpha_s = \frac{0.46\rho_w}{f_c} + 0.14
\]  

(3.47)

where $\rho_w$ is the horizontal reinforcement ratio. Equation 3.47 gives for typical reinforcement ratios a factor $\alpha_s$ in the range 0.15 to 0.20. The parameter $\alpha_s$ was derived for wall cross sections of different geometry ranging from rectangular walls ($t = b$) to walls with pronounced protruding boundary elements.

The cracking force in shear, $V_c$, may be estimated quite simply by

\[
V_c = A_{wc} f_c
\]  

(3.48)

where $A_{wc}$ is the cross sectional area of the wall, inclined at about 45 degrees, and $f_c$ is the shear strength of the concrete. It should be noted that (3.48) gives a very simplified result, without any regard to normal force, geometry, reinforcement, etc. A more comprehensive empirical relation was suggested by Hiraishi [ACISP84] as

\[
V_c = K_t \mu p f_t
\]  

(3.49)

where $K_t$ is a form factor given by
\[ K_f = \frac{1 + 1.05t/b}{1 + 1.77t/b} \]  
(3.50)

where \( t \) is the thickness of the web, \( l_p \) is the length of the web, and \( f_r \) is an equivalent shear stress given as

\[ f_r = C_t \sqrt{1 + \frac{\sigma_0}{C_t}} \]  
(3.51)

where \( \sigma_0 \) is axial stress and \( C_t \) is given by

\[ C_t = 2.2 \sqrt{f_r} - 0.00546 f_r^{0.456} \rho_{wh}^{0.075} \]  
(3.52)

Equations 3.49 to 3.52 are taken directly from the source and use units kg and cm.

For a simplest possible estimation of the ultimate shear capacity we may add the concrete contribution of (3.48) to the horizontal steel contribution of the bars with area \( A_u \) crossing the inclined surface, as suggested in [Lind88]

\[ V_u = V_c + V_f = A_u f_r + A_u f_p \]  
(3.53)

A more realistic estimation is provided by the empirical expression of Hirosawa [Hiro75]

\[ V_u = \left[ \frac{0.0679 \rho_z^{0.22} (f_r + 17.6)}{\sqrt{M/\eta L} + 0.12} + 0.845 \sqrt{f_{wp} \rho_{wh} + 0.1 \sigma_0} \right] b_s f \]  
(3.54)

where the result is given in MPa, and the constituents are explained as follows

\[ \rho_z = \text{effective tensile reinforcement ratio in percent} \]

\[ \rho_z = \frac{100 A_z}{b_s (L - h/2)} \]  
(3.55)

\( A_z \) = area of vertical reinforcement in tension side boundary element,

\( M/\eta L \) = shear span to depth ratio,

\( f_{wp} \) = yield strength of horizontal wall reinforcement
\[ \rho_{wh} = \text{effective horizontal wall reinforcement ratio} \]

\[ \rho_{wh} = \frac{A_{se}}{b_s s} \]  \hspace{1cm} (3.56)

\[ s = \text{vertical spacing of horizontal wall reinforcement} \]

\[ \sigma_0 = \text{average axial stress over entire wall cross section area} \]

\[ b_w = \text{average width of wall section} \]

\[ j, L \text{ and } h \text{ are geometrical parameters of which} \]

\[ j = \frac{7}{8} \left( L - \frac{h}{2} \right) \]  \hspace{1cm} (3.57)

and for \( L \) and \( h \), see figure 3.24

Since the yielding and ultimate strength levels in shear are relatively close, the expressions (3.53) or (3.54) may be used as the yield level in a trilinear hysteretic model, as was done in [Lind89], followed by a yield shear stiffness taken as a small fraction of the uncracked shear stiffness. However, relatively little is known about this behaviour generally, and since shear failures often occur in a brittle manner, this region of behaviour is clearly to be avoided, and will not be treated in the model developed here. Design recommendations for avoidance of shear failures are provided in the capacity design method, see, e.g. [PBM90] or [PP92].

Concerning the cyclic behaviour less weight is put here on modelling the unloading behaviour for large shear deformations, due to the abovementioned reason. An experimentally obtained curve is still presented in figure 3.26, obtained from cyclic tests on a three-storey wall specimen [VBP79] tested at the University of California, Berkeley. It shows horizontal shear deformation at the third floor versus base shear. Since these shear deformations are relatively large, the typical pinching behaviour may be seen during unloading, discussed further in [VBP79]. For smaller cycles it may be seen from the figure that the pinching is much less pronounced. It should be further noted that the test specimen was not capacity designed and the test programme did not correspond to actions expected for a capacity designed structural wall.

The original model by Kabeyasawa et al [KSOA82] employed a trilinear origin-oriented hysteresis model for the shear behaviour, which was modelled by the shear spring alone. The loading curve is made up of a trilinear skeleton curve, consisting of an
elastic part, a cracked part, and a yield part. The unloading curves are always directed towards the origin. The reloading follows the latest unloading path until the skeleton curve is reached. This trilinear model is shown in figure 3.27.

As may be seen from figure 3.27, the origin-oriented model does not give a particularly realistic simulation of typical shear behaviour for large deformations. The loops are too thin in the vicinity of the origin compared to experimental results, and, furthermore, the unloading path should not be directly origin-oriented. Rather it should produce plastic deformations and possibly, in addition, exhibit some of the pinching (slip) behaviour typical for shear.

On the other hand, with design according to the capacity design method (nonlinear behaviour predominantly of flexural nature, and careful detailing) we do not enter into regions of excessive shear deformation. Much of the pinching disappears and is of less importance to model. For the analysis of general buildings, i.e. including buildings which are not capacity designed, it may be necessary however to regard the possibility of strong nonlinear shear behaviour of walls.

A modified shear model was proposed by Fajfar et al, and exhibits elasto-plastic slip behaviour. This model is seen in figure 3.28, with elastic, fully-plastic behaviour on the left, and elastic, linear strain hardening on the right. In a different proposal, Colotti [Colo93] recently attempted to calibrate the shear spring by means of a shear panel model.
An improved hysteretic model was proposed in [Lind89] which is shown in figure 3.29. The model attempts to closely simulate the experimental behaviour of concrete under shear. This, however, is a relatively complex model employing several unloading stiffnesses.
The studies in this report mainly deal with structural walls which behave predominantly in a flexural mode. The models developed in this report, furthermore, mainly serve as numerical tools together with the capacity design method, in which extensive shear deformations are avoided.

Based on these assumptions it is considered sufficient to use a bilinear model for the horizontal spring for the studies in this report. Thus we obtain a bilinear shear behaviour, with an elastic stiffness, and a cracked stiffness in each direction. The bilinear hysteretic model is shown in figure 3.30. This model is numerically efficient due to its relatively few stiffness changes, and the shear behaviour which is of interest for capacity designed walls is modelled quite realistically.
For the cracking force of the bilinear model we use equation (3.49) or alternatively (3.48), and for the cracked stiffness equation (3.47), as was intended for the trilinear models. The ultimate force of (3.48) may be readily calculated and used as a limit on the cracked path of the bilinear model, which should not be exceeded during numerical analysis.

3.7 Element Formulation

The macro model developed in the previous sections of this chapter is to be used in a general finite element code, see [Abaq89] and [Abaq91]. All the physical properties developed in the previous sections, comprising flexural behaviour, axial behaviour and shear behaviour, are introduced into a general wall element, intended to simulate completely typical wall behaviour over a chosen height of a structural wall. This element will be referred to a macro element.

The generally formulated element equation of motion (3.58) contains three terms on the left hand side, representing internal element forces due to: stiffness, damping, and inertia.

In order to distinguish between internal and external element properties we use lower case letters for terms related to the macro element, and upper case letters for spring properties within the macro element. Vectors and matrices will be denoted by bold and
scalar properties by normal letters. The summed element forces may then be written as follows

\[ f_e + f_d + f_i = f_i \]  

(3.58)

where the terms are given by

\[ f_e = ku \]  

(3.58a)

\[ f_d = c \dot{u} \]  

(3.58b)

\[ f_i = m \ddot{u} \]  

(3.58c)

\[ f_i = a_4(t)m \]  

(3.58d)

where \( k, c, \) and \( m \) represent the element stiffness, damping, and mass matrices and \( u, \dot{u}, \) and \( \ddot{u} \) represent the element displacement, velocity and acceleration vectors. The right hand side expression represents the dynamic force acting on the element's degrees of freedom, where \( a_4(t) \) represents the general ground acceleration.

In this section we concentrate on the formulation of the element stiffness matrix \( K_e \). In chapter five the modelling of the other two internal forces is discussed.

The nonlinear stiffness properties described in the preceding sections are adopted to the springs of the macro model shown in figure 3.3. The introduction of this macro model into a usable finite element was performed in three different versions.

The first and simplest version involved the separate assembly of the Springs which were modelled as 2-degree of freedom (d.o.f.) elements, and further by the assembly of the stiff beams modelled as 6-d.o.f. elements. This version was mainly used for initial kinematic verification tests, involving one or a few storeys.

The second version adopted is to numerically assemble the Springs and the stiff beams into one "macro element". In this option a number of undesirable degrees of freedom appeared, some of which may be relatively easily condensed out by static condensation. Apart from that, the macro element in this version is relatively versatile, and allows for connections of girders and joining walls at mid-height of the web.

The third and final formulation involves a derivation of a macro element based on linear kinematic relations. In this way the flexural stiffness of the rigid beams may be omitted and a stiffness matrix may be obtained which has a closed form. Thus, an element is obtained which essentially acts as a membrane element, see figure 3.31, and
MACRO MODEL

which can be readily used for the numerical modelling of multi-storey structural walls.

On the left in figure 3.31 is shown the macro element which was programmed and tested in this report. It has ten d.o.f.'s of which two are rotational for connection to frame girders. The stiffness properties of this element expressed as the stiffness terms corresponding to the degrees of freedom, are based upon fictitious springs arranged according to the previously described nonlinear spring model (macro model), shown on the left in figure 3.31.

The resulting macro element may be used essentially as a membrane element, seen on the right in figure 3.31, but offers the convenient connection possibility for girders at the top. The ten d.o.f. macro element as implemented may be used with a few input data only pertaining to the wall cross section, reinforcement and yield moment. None of the internal spring properties need to be given as input.

By expressing the internal d.o.f.'s in the spring model (e.g. at the intersection of the rigid beams and at the central springs) in terms of the outer ten d.o.f.'s which are displayed in figure 3.31, we omit excessive terms which would normally not be connected to any other element.

In order to derive the stiffness terms, we apply the direct stiffness method, setting the displacement of each d.o.f. equal to unity, and at the same time keeping all the other displacements to zero. The forces at all d.o.f.'s needed to maintain a state of equilibrium are the stiffness terms in the column of the stiffness matrix, which correspond to the
d.o.f. for which the displacement was set to unity. We thus have $k_{ij}$, the column of stiffness terms due to the displacement $u_j$, set to unity.

We begin the derivation of the stiffness matrix, considering only the eight translational d.o.f.'s. The two rotational d.o.f.'s are treated separately later. In order not to change the final d.o.f. numbering both rotational d.o.f.'s are shown from the beginning.

We denote the stiffness of the horizontal spring by $K_s$, the stiffness of the left vertical outer spring by $K_1$, the right outer spring by $K_2$, the central vertical spring by $K_3$, and the axial stiffness of the horizontal stiff beams by $K_A$.

The beams connecting the springs, which were previously described as "rigid", are for the numerical formulation considered as follows. All the internal beams are flexurally rigid. The vertical beams, in the centre of the element, are axially rigid. Since a complete frame-wall interaction was a goal of the element capability, although such an application is not treated in this report, horizontal degrees of freedom at each corner of the macro element are needed. The horizontal beams, which have a horizontal degree of freedom at each end, are provided with a large axial stiffness $K_A$, which numerically is chosen so as to physically represent the elastic behaviour of the tributary part of the wall.

As an example we set the displacement of the d.o.f. number one equal to unity, i.e. $u_1 = 1$, as shown in figure 3.32. The stiffness terms arise from the internal spring forces of the disassembled macro element, shown in figures 3.33a-c.

![Figure 3.32 Deformed state at $u_1 = 1$](image)
Figure 3.33a Internal forces acting on lower assembly due to horizontal spring force, $u_1 = 1$

Figure 3.33b Axial forces acting on lower assembly, $u_1 = 1$

Figure 3.33c Internal forces acting on upper assembly, $u_1 = 1$

Thus for $u_1 = 1$, we obtain the stiffness terms

$$k_{11} = K_A + \frac{K_s}{4}$$
$$k_{21} = \frac{K_s c}{2 l}$$
$$k_{31} = -K_A + \frac{K_s}{4}$$
$$k_{41} = -\frac{K_s c}{2 l}$$
$$k_{51} = -\frac{K_s}{4}$$

$$k_{61} = \frac{K_s}{2} \left( \frac{h-c}{l} \right)$$
$$k_{71} = 0$$
$$k_{81} = -\frac{K_s}{4}$$
$$k_{91} = \frac{K_s}{2} \left( \frac{h-c}{l} \right)$$
$$k_{10,1} = 0$$

(3.59)
It is also shown how the stiffness terms for the case when \( u_2 = 1 \) are obtained, since this involves the vertical springs. The deformed state is shown in figure 3.34, and the spring forces and reaction forces for the case when \( u_2 = 1 \) are shown in figures 3.35a-h.

\[ u_2 = 1 \]

*Figure 3.34 Deformed state at \( u_2 = 1 \)*

\[ K_i(1) \]

*Figure 3.35a Internal spring forces acting on lower assembly, \( u_2 = 1 \)*

\[ K_i(1) \]

*Figure 3.35b Spring force from left outer spring and its reaction on lower assembly, \( u_2 = 1 \)*

\[ K_i(1) \]

*Figure 3.35c Spring force from central vertical spring and its reactions on lower assembly, \( u_2 = 1 \)*
Figure 3.35d Spring force from horizontal spring and its reactions on lower assembly, \( u_2 = 1 \)

Figure 3.35e Internal spring forces acting on upper assembly, \( u_2 = 1 \)

Figure 3.35f Spring force from left outer spring and its reaction on upper assembly, \( u_2 = 1 \)
Figure 3.35g Spring force from central vertical spring and its reactions, on upper assembly, \( u_2 = 1 \)

Figure 3.35h Spring force from horizontal spring and its reactions on upper assembly, \( u_2 = 1 \)

We thus obtain the stiffness terms in the column corresponding to \( u_2 = 1 \) as follows, where it should be noted that several terms are created by superposition of the cases shown in figures 3.35a-h.

\[

k_{12} = \frac{K_3 c}{2 l} \\
k_{22} = K_1 + \frac{K_3}{4} + K_s \left( \frac{c}{l} \right)^2 \\
k_{32} = \frac{K_s c}{2 l} \\
k_{42} = \frac{K_3}{4} - K_s \left( \frac{c}{l} \right)^2 \\
k_{52} = -\frac{K_s c}{2 l} \\
k_{62} = -K_1 - \frac{K_3}{4} + K_s \left( \frac{c}{l} \right) \left( \frac{h-c}{l} \right) \\
k_{72} = 0 \\
k_{82} = -\frac{K_s c}{2 l} \\
k_{92} = -\frac{K_3}{4} - K_s \left( \frac{c}{l} \right) \left( \frac{h-c}{l} \right) \\
k_{10,2} = 0
\]  

(3.60)
All other stiffness terms for horizontal and vertical degrees of freedom may then be obtained in a similar way as for the two cases shown. Each column was derived by setting the corresponding displacement equal to zero, and the relation

\[ k_{ij} = k_{ji} \]  \hspace{1cm} (3.61)

which applies due to symmetry of the stiffness matrix, was used rather as control.

The two rotational degrees of freedom, seen in figure 3.31, still need to be treated. We employ an internal flexural beam connected only between the d.o.f.'s 7 and 10. This internal beam has a total of four d.o.f.'s, and is assembled into the degrees of freedom 6, 7 and 9, 10. The properties of the internal beam are only relevant for the connection to joining girders in the analysis of frame-wall structures. These properties may be calibrated to simulate the elastic connection into the wall by girders. In this report, only isolated walls were studied and no such calibration was performed. Isolated walls may be modelled numerically and analysed satisfactorily without the internal beam. It should be noted that there would also be other possibilities of modelling the elastic girder connection, e.g. by rotational springs or other arrangements.

For the sake of completeness the general formulation of the internal beam properties and the inclusion if its stiffness terms in the macro element are treated briefly. The internal beam is shown in figure 3.36 with its d.o.f.'s, as they refer to the macro element of figure 3.31. The stiffness matrix, \( k_{ib} \), for the internal beam of figure 3.36 is given in equation 3.62. No derivation is given here, but it may be found in any structural matrix text book.

\[
k_{ib} = \begin{bmatrix}
\frac{12EI}{l^3} & \frac{6EI}{l^2} & \frac{12EI}{l^2} & \frac{6EI}{l^2} \\
\frac{4EI}{l} & \frac{6EI}{l^2} & \frac{2EI}{l^2} & \frac{2EI}{l^2} \\
\frac{12EI}{l^3} & \frac{6EI}{l^2} & \frac{12EI}{l^2} & \frac{6EI}{l^2} \\
\text{Sym} & \frac{4EI}{l} & \frac{4EI}{l} & \frac{4EI}{l} 
\end{bmatrix}
\]  \hspace{1cm} (3.62)
The blown up stiffness matrix of the internal beam, i.e. (3.62) assembled into the macro element d.o.f. numbers of figure 3.31, will then look as follows

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{12EI}{l^2} & \frac{6EI}{l^2} & 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 & \frac{6EI}{l^2} & \frac{2EI}{l} & 0 & 0 & 0 \\
\frac{4EI}{l} & 0 & \frac{6EI}{l^2} & \frac{2EI}{l} & 0 & 0 & 0 & \frac{4EI}{l} & \frac{6EI}{l} & 0 & 0 & 0 \\
\frac{12EI}{l^3} & \frac{6EI}{l^2} & \frac{4EI}{l} & \frac{6EI}{l} & \frac{4EI}{l} & \frac{12EI}{l^3} & \frac{6EI}{l^2} & \frac{4EI}{l} & \frac{6EI}{l} & \frac{12EI}{l^3} & \frac{6EI}{l^2}
\end{bmatrix}
\]

\textit{Blown up stiffness matrix for internal beam of macro element}

The blown up stiffness matrix for the macro element with only translational d.o.f.'s is shown on the next page, followed by the complete stiffness matrix including the contributions from the internal beam with rotational d.o.f.'s.
Blown up stiffness matrix for lower and upper assemblies of macro element comprising translational degrees of freedom
Complete stiffness matrix of macro element including rotational degrees of freedom
3.8 Ductility demand

The monitoring of the ductility of certain preselected critical zones of a structure is one of the most important observations to be made during nonlinear analysis. The designer is then able to compare this behaviour with the assumptions on which the design was based. Especially in plastic hinge zones, where flexural yielding is intended to take place, the demand for curvature ductility needs to be known, since the design of the wall section and the structural detailing must allow for this demand. More elaborate discussion on this topic is provided by Paulay et al in [PBM90].

The above described Macro model will readily lend itself to the computation of ductility values during nonlinear time history analyses. In order to discuss the ductility measurement we will need to make some definitions as follows. Rotation \( \theta \), in general, is given by

\[
\theta = \frac{\delta_v - \delta_w}{l}
\]

where \( \delta_v \) and \( \delta_w \) are the vertical displacements of the outer springs. We define the wall rotation at which yielding begins in one of the outer springs as the yield rotation, see figure 3.37. The rotation for which the left outer spring just yields, i.e. at the onset of yielding, is

\[
\theta_y = \frac{\delta_{vy} - \delta_{wr}}{l}
\]

![Figure 3.37 Ductility measures](image_url)

a) Rotational ductility  
b) Curvature ductility

Figure 3.37 Ductility measures
For rotations larger than the yield rotation the ratio of $\theta$ to $\theta_y$ is defined as the rotational ductility.

$$\mu_\theta = \frac{\theta}{\theta_y}$$

(3.65)

By taking the element height $h$ into account we can convert the vertical displacements to strains and the rotation into curvature. For an element we then obtain

$$\phi = \frac{\theta}{h}$$

(3.66)

and conversely we may define the curvature ductility as

$$\mu_\phi = \frac{\phi}{\phi_y}$$

(3.67)

whereby the equation 3.66 may also be obtained as

$$\phi = \frac{\varepsilon_t - \varepsilon_y}{l}$$

(3.68)

Curvature ductility according to expression 3.67 is implemented in the Macro model described in sections 3.2-3.6. Ductility for left and right curvature is monitored separately for each increment, and an absolute value of the largest ductility reached so far is stored.

In chapter six, ductility demands as described above, obtained from nonlinear time history analysis, is discussed in relation to the capacity design method.
CHAPTER FOUR

MICRO MODEL

4.1 Introduction

Since structural walls mainly act in their own plane, they lend themselves readily to the application of the plane stress relation implemented in membrane elements. This allows for separate modelling of the constituents such as concrete, reinforcing steel, as well as the their interaction, so that a composite element is formed. The use of continuum elements makes micro models much more versatile than a macro model, although the separate treatment of the constituents is more elaborate.

The nonlinear global behaviour of reinforced concrete structural units such as structural walls is mainly due to the tensile behaviour of the concrete, and the yielding of the reinforcement. A simple and reliable numerical model should therefore first of all attempt to reasonably simulate these two phenomena.

As mentioned in chapter two, there are among micro models two major approaches for modelling the cracking of concrete; the discrete approach, and the smeared approach, of which the latter has proven to be promising for the modelling of reinforced concrete structures. The smeared approach is based on the assumption that no single physical crack is simulated by the numerical model, but the overall cracking within an area, e.g. an area associated with an element or with an integration point, is simulated as fictitious smeared cracks. The early studies of the 1970's commonly used tension cut-off, which essentially involves setting the stiffness matrix to zero when reaching the tensile strength, and balancing the released concrete stress on the right hand side, see e.g. [Cerv70], and [Chen82].

The performance of the smeared crack models have improved with the introduction of tension softening upon reaching the tensile strength. The tension softening approach was pioneered by Hillerborg et al [HMP76] who generalised its use by relating the tension softening to the fracture energy. Other improvements, such as the introduction of rotating cracks (following direction of principal stress) or fixed cracks with variable shear retention upon the formation of cracks have contributed to making the smeared crack approach a competitive and efficient tool for the modelling of reinforced concrete structures.
Based upon the above mentioned this chapter is devoted to the development of a micro model, as a complement to the macro model of chapter three. The micro model is based on a composite element, see figure 4.1, with separate treatment of the concrete employing the smeared crack approach, the reinforcement steel, and the interaction effects.

The material models developed here could essentially be used with any plane stress element. In this study, the isoparametric plane stress membrane elements from the software [Abaq91] were used for the implementation of the material models developed in this chapter. In the numerical examples presented in chapter five the four noded element CPS4 with 2x2 integration points is used, see figure 4.2a. Other usable elements including the eight noded CPS8 seen in figure 4.2b are briefly introduced in Appendix D.

The numerical integration is carried out according to standard integration procedures and the element stiffness matrix $k$ takes the form

$$k = \int_{V} B^T DB dV$$

(4.1)

where $D$ is the material modulus matrix, $B$ is the strain-displacement matrix, and $V$ is the element volume, see e.g. [Zien77] or [Bath82]. The procedures for element integration were provided by the software, and will not be discussed in any further detail in this study, see [Abaq89] and [Abaq91].

We will rather concentrate on the development of the material model expressed by the incremental modulus matrix $D$ which was coded by the user material option, and is given as follows

Figure 4.1 Composite model of reinforced concrete element
\[ d\sigma = Dd\varepsilon \]  
\[ D = D_c + D_s + D_{ia} \]

where \( d \) denotes incremental stress and strain. The different constituents contribute to the total modulus stiffness \( D \) as follows:

where \( D_c \) represents concrete, \( D_s \) represents steel, and finally \( D_{ia} \) represents the interaction effects. The behaviour of materials and different physical phenomena which are relevant to the overall behaviour are subsequently treated in the following sections.

### 4.2 Material Behaviour of Concrete

This section is devoted to the development of a useful concrete model, based upon the smeared fictitious crack approach discussed above. Since the main objective of the report is to focus on the global behaviour of walls, we limit the modelling of the concrete on the micro level to some of the most important phenomena needed for this purpose, such as the behaviour upon crack formation, the so-called tension softening, the unloading behaviour and effects associated with the cracks.

For the compressive behaviour of concrete many material models have been proposed, which may mainly be divided into linear and nonlinear elastic models, and models based
on plasticity. An review of such models is given e.g. in [Chen82]. The behaviour in the compressive region is here assumed to be elastic, which may be acceptable for many structural wall designs, if during an analysis control is kept of the compressive stresses, and there is reassurance that they ideally do not exceed around 70 % of the compressive strength, to which level linearity is commonly observed, and that no stress in excess of the compressive strength occurs. It is also known that local compressive nonlinearity mostly affects the global responses in an insignificant way.

In the following presentation we will now develop the concrete contribution to the modulus matrix. For elastic uncracked concrete, the material is treated as fully isotropic, and the incremental equilibrium equation (4.2) is given by

$$
\begin{bmatrix}
    d\sigma_x \\
    d\sigma_y \\
    d\sigma_{xy}
\end{bmatrix}
= \frac{E_c}{1-V^2}
\begin{bmatrix}
    1 & \nu & 0 \\
    \nu & 1 & 0 \\
    0 & 0 & \frac{1-\nu}{2}
\end{bmatrix}
\begin{bmatrix}
    de_x \\
    de_y \\
    de_{xy}
\end{bmatrix}
$$

(4.4)

where $E_c$ represents Young's modulus for elastic small strain concrete, and $\nu$ is the Poisson's ratio for concrete. Although concrete displays a small nonlinearity in tension before reaching the tensile strength, the relation (4.4) is commonly used in tension until cracking occurs, and will be done so in this study as well.

Upon reaching a certain tensile strength the relation (4.4) must be changed. A simple solution to this problem was presented in the early study by Cervenka [Cerv70]. It was assumed that upon reaching a principal stress corresponding to a given crack strength, the material would crack in a direction perpendicular to the principal tensile stress direction. This cracking was assumed to be smeared out in several parallel layers, and furthermore the assumption was made that no shear stress could be transferred between these layers. The physical interpretation of this model is shown in figure 4.3.

The cracked formulation of (4.4) for cracks in one direction was given by the following expression [Cerv70]

$$
D = E_c CC^T
$$

(4.5)
where $E_c$ is the uncracked Young's modulus of concrete and $C$ is a transformation matrix written as

$$
C = \begin{bmatrix}
\cos^2 \phi \\
\sin^2 \phi \\
\cos \phi \sin \phi
\end{bmatrix}
$$

(4.6)

where $\phi$ denotes the angle between the global x-axis and the crack plane, as seen in figure 4.3. This formulation has been used in many studies, and has provided reasonable results for some simple problems. For cracks in a second direction, perpendicular to the first, the modulus matrix $D$ was usually simply set equal to zero [Chen82].

A simplification of relation (4.5) and (4.6) is the lack of shear transfer between the crack planes. This fact may pose numerical problems due to a lack of shear stiffness. In simple examples of monotonically loaded structures with uniform geometry and evenly distributed reinforcement, this deficiency was however usually remedied by supporting neighbouring elements and the stiffness of orthogonally modelled reinforcement. The issue of shear transfer between crack planes will be treated subsequently in this chapter.

Another simplification is that the transfer from the uncracked state as in relation (4.3) was commonly performed directly to the cracked state as in (4.4) by cutting off the tensile stress, so-called "tension cut-off". The released forces originating from the concrete stresses have to be balanced using some procedure. A simple commonly used method was to apply these forces as external loads in the subsequent increment.
Experimentally, however, it has been shown that upon reaching a certain peak stress, the concrete displays a softening behaviour, which has been carefully studied by among others Reinhardt [Rein84], and Cornelissen et al [CHR86]. Figure 4.4 shows the typical softening behaviour observed for unreinforced concrete in tension. This softening behaviour is explained by the formation of micro cracks within a zone, referred to as the process zone.

Various simplified models have been suggested to numerically simulate this softening behaviour. Apart from the commonly used linear or bilinear models, more elaborate multilinear models were proposed by Gustavsson [Gust85]. Rule and Rowlands [RR92] also proposed a nonlinear softening model.

To simulate the tensile behaviour, we use a model here, which assumes linear elastic behaviour until the tensile strength is reached, followed by a linear softening branch, as seen in the figure 4.5. When the largest principal stress reaches the tensile strength of concrete, the material becomes orthotropic. In the direction normal to the crack, tensile stresses can still be carried by the concrete, referred to as the tension softening effect. The physical interpretation of this effect is that micro cracks form in a so-called process zone, in which normal stresses across these cracks can be transferred. The descending slope of the stress-strain relation in the softening region is determined by the tension softening factor $k_{ts}$, see figure 4.5.
Figure 4.5 Linear tension softening model for concrete assuming smeared cracks over the length $L$

Here we use a tensile strength $f_t$ calculated as $E_c\varepsilon_{cr}$, where $\varepsilon_{cr}$ is the cracking strain set to 0.0001, a value commonly observed in tests. This value, as well as $E_c$, may to some degree be dependent on the strain rate, see e.g. [CB89], [Chap87], which however is not treated here.

The softening factor may be related to the fracture energy $G_f$ as follows. The area under the curve representing normal stress as a function of crack width represents the fracture energy. Since we always have an area associated with an integration point, we want to make sure that for this area, the fracture energy would correspond to the same area in a real structure. According to [CPE90] it is then possible to normalise the tension softening factor with respect to the mesh as follows

$$\frac{1}{-k_nE_c} = \frac{1}{E_c} \left(1 - \frac{\lambda}{L}\right)$$  \hspace{1cm} (4.7)

where $L$ represents a length associated with the integration point, and $\lambda$ is given by

$$\lambda = \frac{2E_cG_f}{f_t^2}$$  \hspace{1cm} (4.8)

where $G_f$ is the fracture energy, representing the area under the stress versus process zone width relation for a fictitious crack, see figure 4.6, assumed to have behaviour corresponding to observed experimental data.
Figure 4.6 Fracture energy as area under stress versus process zone width [CPE90]

\[ G_f = \int_0^w \sigma dw \] (4.9)

The length \( L \) is here assumed to be one of the sides of a rectangular integration area which is close to quadratic. For other integration areas and element shapes, an equivalent length may be calculated as shown by Dahlblom and Ottosen in [DO90].

The tensile strength as discussed above is further influenced by the transverse stress, and much well known experimental data has been presented on the biaxial concrete behaviour, see e.g. [Kupf69].

Numerically, many relatively involved bell-shaped failure functions have been established, some of the most well-known by Ottosen, see e.g. review in [Chen82]. These numerical models deal to a large degree with the compressive behaviour, and how it is affected by the biaxial influence.

Since we know that the major contribution to the global nonlinear behaviour arises from the local tensile phenomena, and since we deal with structures designed in such a way that crushing or other severe compressive nonlinearities will usually not play a major role, we are able to essentially focus our interest on the tension/tension region and to some extent consider the effect in the tension/compression regions.

Several biaxial failure tests have been performed for concrete, see e.g. Kupfer [Kupf69]. We will use a simplified cracking criterion shown in figure 4.7, together with Kupfer's test. For each principal stress direction cracking is assumed to occur when the
tensile strength is reached. A linearly decreasing cracking criterion in the tension/compression zones was also tested but did not change the global behaviour.

In the local coordinate system, we obtain the equilibrium equation for cracked concrete

$$d\sigma = D_t d\varepsilon$$

(4.10)

written in matrix form

$$
\begin{bmatrix}
    d\sigma_1 \\
    d\sigma_2 \\
    d\sigma_{12}
\end{bmatrix} =
\begin{bmatrix}
    -k_\varepsilon E\varepsilon & 0 & 0 \\
    0 & E\varepsilon & 0 \\
    0 & 0 & \beta G
\end{bmatrix}
\begin{bmatrix}
    d\varepsilon_1 \\
    d\varepsilon_2 \\
    d\varepsilon_{12}
\end{bmatrix}
$$

(4.11)

where $D_t$ is the modulus matrix in the local coordinate system, $\beta$ is the shear retention factor, discussed subsequently. The coupling terms $D_{12} = D_{21}$ have been suggested by some authors as a small value involving the square root of $D_{11}$ and $D_{22}$. However since the physical meaning of this value is not fully clear we set these terms equal to zero. The local modulus matrix in equation (4.10) may be transformed to the global coordinate system as follows

$$D_t = QD_t Q^T$$

(4.12)
where $D_g$ is the modulus matrix in the global coordinate system, and $Q$ is a transformation matrix, shown in (4.13), containing $\phi$ taken as the angle between the global x-axis and the normal to the crack plane.

$$Q = \begin{bmatrix}
\cos^2 \phi & \sin^2 \phi & -\sin 2\phi \\
\sin^2 \phi & \cos^2 \phi & \sin 2\phi \\
\frac{\sin 2\phi}{\sqrt{2}} & \frac{-\sin 2\phi}{\sqrt{2}} & \cos 2\phi
\end{bmatrix}$$

(4.13)

When $e_1$ exceeds the value $e_0$, see figure 4.5, the tension softening does not act any more and the cracks have opened up (zero normal stress). For this case we obtain the relation

$$\begin{bmatrix}
da_1 \\
da_2 \\
da_{12}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 \\
0 & E_c & 0 \\
0 & 0 & \beta G
\end{bmatrix} \begin{bmatrix}
de_1 \\
de_2 \\
de_{12}
\end{bmatrix}$$

(4.14)

Unloading behaviour has not been treated very much in the literature, compared to monotonic behaviour. There exist, however, a number of experimental results from uniaxial loading of concrete specimens with alternating tensile and compressive stresses. Figure 4.8a shows the stress-displacement behaviour of a such a test carried out by Reinhardt and Cornelissen [RC80]. It is seen that the unloading follows a somewhat different path than the reloading path. It is also seen that high compressive stiffness during unloading, i.e. the closure of cracks, is reached further out in the tensile displacement region for cycles which have reached large maximum displacements. This crack closure also tends to take place at a compressive stress which appears to increase for cycles with large maximum displacements. In figure 4.8b a proposed numerical model by Curbach [Curb87] is shown, which attempts to simulate the behaviour from the test shown in figure 4.8a. The skeleton curve is modelled well in figure 4.8b as well as the compressive stress at crack closure, but the residual tensile strain at crack closure is not modelled. Admittedly, this proposal has the advantage of exhibiting fully isotropic behaviour between the cycles.

Some other proposals for numerical models of cyclic behaviour may be found in the literature. Often, linear unloading is assumed from the softening branch back to the origin, upon which the material again is considered fully isotropic. Gylltoft [Gyll83] pro-
posed a multilinear model for unloading behaviour. Relatively complex focal point models, attempting to closely follow experimental data, are further evaluated in [Gyl89].

We here propose a simple model for cyclic behaviour which takes into account residual strain at crack closure. This model is given in figure 4.9. It consists of linear branches only, for simplicity, and in this way is simpler than e.g. Curbach's model.

Unloading from the tension softening region occurs linearly in a direction towards a point where the cracks close, at a stress level $-\sigma_{cc} = -k_{cc}f_0$, and a residual strain $\varepsilon_{cc}$ equal to around 20 percent of the peak strain $\varepsilon_p$, reached before the unloading. By setting the factor $k_{cc}$ to 0.5, reasonable correspondence to experimental data was achieved. The variation of the stress level at crack closure appears to be much smaller, as seen in the test result from figure 4.8a, than the variation of the residual displacement (strain) at crack closure. Therefore, it is believed that the simple model presented here is reasonable in this regard.

The fact that we do not allow for crack closure already at zero stress level, as is often assumed in theory, may physically be explained to be due to crushing and spalling of aggregate on the crack faces causing gradual build-up of compressive stress during the closure procedure.

After crack closure further unloading occurs elastically. Reloading follows the same path back to $\varepsilon_p$. Thereby, we deliberately neglect the energy dissipation taking place during cyclic action between crack closure and the strain from which the unloading occurred, since in earthquake-related problems we mainly deal with growing cracks and
cycles beyond the concrete tensile capacity.

Unloading from open cracks, i.e. from strains larger than $\varepsilon_0$, occurs with zero stress until $\varepsilon_0$, from where it continues to $\varepsilon_{cc} = 0.2\varepsilon_0$, and then elastically as described above. Once the opened crack state has been reached for an integration point, no tensile stress will occur in that region during subsequent cycles.

Should the stress in the second principal direction exceed the tensile strength, a tension softening region will also develop in this direction:

\[
\begin{bmatrix}
  d\sigma_1 \\
  d\sigma_2 \\
  d\sigma_{12}
\end{bmatrix} =
\begin{bmatrix}
  0 & 0 & 0 \\
  0 & -k_\sigma E_c & 0 \\
  0 & 0 & \beta G
\end{bmatrix}
\begin{bmatrix}
  d\varepsilon_1 \\
  d\varepsilon_2 \\
  d\varepsilon_{12}
\end{bmatrix}
\]  

(4.15)

Due to shear retention there are still shear stresses acting along the first opened crack direction, and this may cause a deviation of the second principal tensile stress from the direction orthogonal to the first one. Due to this fact, non-orthogonal crack modelling was proposed by deBorst and Nauta [deBN85]. An alternative to using non-orthogonal crack modelling is to allow for cracking in the orthogonal direction and to some degree correct its effect with respect to the shear stresses acting, i.e. begin the cracking earlier than would be the case based only upon orthogonal stress. A further possibility would be to
keep a orthogonal system, but to let it rotate continuously with the current principal
directions was shown by Rots [Rots88]. Although a progressive crack development does
change its direction at the crack tip, the concept of continuously rotating the entire existing
cracks seems problematic from a physical point of view, although it has sometimes
proven to yield good agreement with experimental data. The modelling presented here
will be kept within the framework of fixed crack directions, and attempt to realistically
model the shear effects within these fixed cracks.

4.3 Aggregate Interlock

In cracks undergoing opening, as well as in opened cracks, there is some shear trans¬
ferred, commonly referred to as "aggregate interlock". Physically, this effect consists of
several contributing parts; cohesion, friction between the uneven crack surfaces, and
misfit due to offset surfaces. Dowel action arising from reinforcement bars crossing the
crack is sometimes included in this context, but is treated here separately under the section
"Interaction Effects". General discussions of the phenomena associated with aggregate
interlock are presented in work by Pruijssers [Prui88] and Walraven [Walr80] among
others.

Numerically, the combined effects of aggregate interlock, have been taken into account
by reducing the shear modulus $G$ by an amount $\beta$, referred to as the shear retention
factor. In the early finite element analyses this factor was set to an arbitrary constant value
between zero and one, sometimes 0.5 as in [ZPO74], which often gave a somewhat too
stiff solution with higher $\beta$ values. Attempts to set $\beta$ equal to zero, i.e. no shear transfer
across cracks, as in [CG71] provided reasonable solutions, but may pose numerical
difficulties in the nonlinear range. Glemberg and Samuelsson [GS83] proposed a shear
retention factor proportional to the ratio of the softening modulus and the uncracked
Young's modulus, so as to obtain a moderating effect on the equilibrium equations.
Kolmar [Kolm85] presents a variety of different proposals for representing the shear
retention. A physical interpretation of shear transfer across cracks is shown in figure
4.10.

Generally, with a low shear retention factor of around 0.01 to 0.05, good agreement
with experimental data may be obtained. A better solution is to let $\beta$ vary with the crack
opening strain, i.e. Mode I behaviour. Rots [Rots84] derived expressions for the shear
retention factor as a function of the opening strain. The first of these expressions is based
upon experimental work by Paulay and Loeber [PL84], and takes the form
Figure 4.10 Shear transfer across cracks

\[ \beta = \frac{1}{1 + 4447 \varepsilon_n} \]  
(4.16)

The second expression is based upon work by Bazant and Gambarova [BG80] and reads as follows

\[ \beta = \frac{1}{4762 \varepsilon_n} - \frac{1}{1346 \sqrt{\varepsilon_n}} \]  
(4.17)

where \( \varepsilon_n \) is the strain in the direction normal to the crack. The curves of equations (4.16) and (4.17) are shown in figure 4.11.

Some interest has been directed towards experimental study on a mixed mode (Mode I and Mode II, i.e. opening mode and shearing mode) behaviour, see e.g. Hassanzadeh [Hass91], [Hass92], Bazant and Gambarova [BG84], Pruijssers [Prui88], and Nooru-Mohamed [Noor92]. It has been observed that some degree of path dependency between the two modes exists, but it is generally concluded that further research is necessary. An expression for the shear retention taking into account the relation between normal and shear strain as well as the aggregate size was suggested Pruijssers [Prui85], where the ratio \( \varepsilon_n / \gamma = 3 \) gave a curve similar to the formula by [Rots84]. Higher ratios gave lower retention for a given normal strain. The expression reads

\[ \beta = \frac{1}{P \varepsilon_n + 1} \]  
(4.18)

with

\[ P = \frac{2500}{D_{\text{max}}^{0.14} \left(0.76 - 0.16 \frac{\varepsilon_n}{\gamma} \left(1 - \exp \left(-6 \frac{\gamma}{\varepsilon_n} \right) \right)\right)} \]  
(4.19)
where $D_{\text{max}}$ is the diameter of the largest aggregate. For some constant ratios of $\varepsilon_n/\gamma$, the expression is plotted for $D_{\text{max}}$ equal to 19 mm, in figure 4.12.

Here we mainly use a pure Mode I model, and choose the expression derived by Rots [Rots84], based on the work by Paulay and Loeber, due to its simplicity. However, the influence of the other proposals presented here on the global behaviour will be briefly discussed in chapter five.
It appears clear that during cyclic behaviour, such as is the case during earthquake action, the shear retention factor should gradually decrease with the number of cycles, due to grinding effects. It is, however, unclear how this decrease would affect the relation 4.16, which was established for monotonic behaviour. Furthermore, the unloading shear modulus has been observed from experiments to be higher than the loading modulus. This would imply that for unloading, one should not follow the same path back as indicated by the curve by Rots, seen in figure 4.11, but a different curve which would take some convex shape. This effect was taken into account by Skrikerud [Skri82] in his model for element spütting, whose results, however, may not be directly applicable to the problem of the smeared crack formulation for reinforced concrete. This is an area where more research is needed.

We use equation 4.16 therefore in an unaltered form for monotonic as well as for cyclic behaviour. For the closed crack branch with elastic compressive stiffness, however we employ here almost full shear retention, or rather, a factor of 90% of the undamaged isotropic value. The slight reduction allows for grinding damage on the crack faces, occurring during cyclic behaviour. A more refined solution would suggest this value to drop a few percent at each cycle.

4.4 Material Behaviour of Reinforcement Steel

The reinforcement bars may be modelled according to two basic types, either by the discrete model, or by the smeared model, see figure 4.13. For discrete modelling the mesh size may be made to correspond to the distance between the reinforcing bars, which may however often give very elaborate mesh models. The other alternative is to model the reinforcement within an element lumped in such a manner that a number of equivalent reinforcing bars are connected between adjacent nodes. In either case the reinforcing bars are modelled as truss elements which are assembled separately from the concrete element, as shown in figure 4.13b. The discrete modelling of reinforcing was used much in early studies of nonlinear concrete models.

The extension of lumping reinforcement within an element will result in the smeared reinforcement model, shown in figure 4.13c, in which the reinforcing bars create a continuous layer for each direction of bars, and enter as uniaxial contributions into the material modulus matrix.

Smeared modelling is more convenient from a user's point of view, since mainly the reinforcement ratio and steel properties of each direction need to be entered. The additional fact that interaction effects may be treated more efficiently with the smeared
Modelling approach, gives us reason to implement a smeared reinforcement model. Since we assume bars only horizontally and vertically, the transformation to the global coordinate system is omitted. The smeared model has proved to give good agreement with experimental data, as long as the reinforcement bars have been evenly distributed and the wall geometry has not changed suddenly [Chen82]. The occurrence of few bars with large diameters has been shown to be better modelled by the discrete method.

Dörr [Dörr80] suggested a combination of both methods arguing that uniformly reinforced parts should be modelled by the smeared method, and in addition, single heavier bars should be modelled by the discrete method which would also allow for some
nonlinear bond slip springs to be applied, as studied for walls in [Dörr80] and for reinforced concrete frames by Glemberg [Glem84].

Since in this study we mainly focus on the global behaviour of well-detailed and uniformly reinforced walls, the smeared reinforcement model is used for the modelling of all reinforcement. Different reinforcement ratios vertically and horizontally however are generally applied, and the ratio may be altered between different elements in the same mesh.

The reinforcement contribution $D_s$ to the modulus matrix is, for bars in horizontal and vertical direction only, obtained as

$$D_s = \begin{bmatrix} \rho_x E_{sx} & 0 & 0 \\ 0 & \rho_y E_{sy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(4.20)

where $\rho$ denotes the reinforcement ratio. It is noted that $D_s$ contains no coupling terms, and the smeared reinforcement acts as trusses in its own direction. The elastic-linearly hardening model used for the reinforcement steel is seen in figure 4.14.

![Figure 4.14 Elastic-linearly hardening model for reinforcement](image)

The hardening ratio $\alpha_y$ may be given by the user. For cyclic behaviour unloading occurs with elastic stiffness and compressive yield stress and hardening are assumed to be the same as for the tensile region. In both the tensile and the compressive regions, the skeleton curve, i.e. the strain hardening branch, will be followed during yielding at all cycles, which more or less corresponds to kinematic hardening. This behaviour shows
reasonable agreement with experimental data for reinforcement bars subjected to cyclic action, as opposed to the sometimes applied kinematic hardening concept, where yielding at all cycles occurs at a preselected yield stress.

4.5 Interaction between Concrete and Steel

The interaction between reinforcement steel and concrete may be divided into two main categories: a stiffening effect under tension of concrete between cracks involving bond behaviour, and secondly, dowel action of reinforcement across opened cracks.

The first category, known as the "tension stiffening" effect, is due to the fact that when cracking occurs, the concrete located between the cracks still acts under tensile stress and thus gives a stiffness contribution. The physical behaviour of this phenomenon was treated early among others by Bachmann [Bach67]. Numerical attempts to treat tension stiffening were also made by Cervenka et al [CPE90], Dinges [Ding85], Gupta and Maestrini [GM90], and Kollegger [Koll88]. Numerically, this effect, has been simulated mainly in three ways; either by modifying the stress-strain curve for the reinforcement steel, or by modifying the behaviour of the concrete, or lastly, by modelling the effect as a separate fictitious material. The last way of modelling is the most complex one and involves a description of the effect along the rebars, and then a transformation to the cracked local coordinate system.

The tension stiffening effect in the direction of reinforcing bars is shown in figure 4.15. Upon formation of cracks across the reinforcement, the interaction begins. As tension softening of the concrete has terminated, the interaction will be constant at a stress $\sigma_t$ of about $0.4 f_t$ until the steel starts to yield, whereupon the tension stiffening effect will cease to act and the interaction stress drops to zero. This interaction effect may be transformed to the cracked concrete coordinate system, and will then modify the stress-strain relation of the concrete as shown in figure 4.16, i.e. for any given strain the tension stiffening allows the concrete to carry a higher tensile stress than it would otherwise.

In order to simplify our modelling the stepwise behaviour of figure 4.16 may be smoothed, as shown in figure 4.17. Some researchers attempted a nonlinear tension stiffening path with a long tail, for example Hayami et al [HMM91], Ohomori et al [OTIKW89], and Rothe and König [RK88].

It has been shown experimentally that tension stiffening acts mainly in the direction of the reinforcement, and the best way would be to separately treat the effect numerically or add the effect to the reinforcement modulus matrix. However, it is also found that the ten-
Tension stiffening effect mainly acts during the first few cycles during cyclic action. Thereafter, when cracks have been opened and closed a few times, the tensile stresses, carried by the concrete between the cracks, become small and do not contribute significantly to the overall stiffness any more. In earthquake engineering problems it is quite important to realistically model the behaviour after tension stiffening no longer acts.

For the purposes in this report it appears adequate therefore to have the option of including the effect in a limited way for the first cycles, and it will be performed by modifying the tensile behaviour of unreinforced concrete (figure 4.5) by superimposing the additional stress carrying capacity of figures 4.15, as shown in figure 4.16.
Figure 4.17 Smoothed tension stiffening model with linearly descending branch

We will use a smoothed modification for the tension stiffening by applying a linear branch from the point where the tensile strength is reached to the termination of the tension stiffening at a the strain $\varepsilon_{ots}$. The strain $\varepsilon_{ots}$ would in fact always vary somewhat, due to the direction of cracks in relation to reinforcement directions.

By superimposing the tension stiffening effect on the tension softening, we achieve the advantage of a less abrupt loss of stress, and largely avoid the previously mentioned considerations concerned with the fracture energy and its mesh dependence, compared to the problems of unreinforced concrete. We are now able to treat the tension stiffening to some extent as a material parameter, with some care taken concerning the reinforcement ratios and directions. Dinges [Ding85] conservatively suggested that for a smoothed tension stiffening effect superimposed on the concrete, the stiffening would cease to act at $\varepsilon_{ots}$ taken as $10\varepsilon_{cr}$. Based on the assumption of uniform reinforcement in both directions and reinforcement ratios which do not differ considerably between the directions we will use $\varepsilon_{ots}$ ranging from Dinges' suggestion of $10\varepsilon_{cr}$ up to about $20\varepsilon_{cr}$, which is in the vicinity of $\varepsilon_{y}$ for steel and thus is to be regarded as an upper limit. For simplicity, we also use the same unloading behaviour from the tension stiffening zone as for the softening zone discussed earlier.

A qualitative idea of the resulting composite modulus for an integration point is given in figure 4.18. The effect of the fictitious unreinforced concrete in addition to the reinforcement is seen in the figure as well as the smoothed tension stiffening acting until the onset of yielding. This figure is only correct if the reinforcing runs orthogonal to crack planes, otherwise the tension stiffening as implemented in this model will not cease...
exactly at the onset of yielding. It should further be noticed that the resulting modulus at the tension stiffening phase will also depend much on the area of reinforcement. If the reinforcement ratio is low, the resulting modulus will be low and may even become negative. It may occur, at reinforcement ratios which are around or below the minimum reinforcing according to the SIA standard 162 (0.20 % for structural walls), that some element may not exhibit positive-definite behaviour during a few increments. Usually, the neighbouring elements have a supporting action, and numerical difficulties at tension stiffening were generally not encountered.

The effect of tension stiffening on the reinforcing steel will also be briefly discussed in the following. The reinforcing bars which bridge a crack must carry the stresses not only of the steel area but also the force resulting from the tensile stresses in the uncracked concrete sections. This causes the stress in the reinforcing bar to be higher over the crack than in the uncracked section, see detailed discussion [Bach67]. For a smeared reinforcing model it would be necessary to allow for this effect by adding tensile stress to the smeared reinforcing bars, and leave the yield level at its correct state. This may also be further simplified and regarded as an increase of the Young’s modulus for the reinforcing due to the tension stiffening.

The modification of the reinforcement modulus is shown qualitatively in figure 4.19.

Additional steel force across cracks due to tension stiffening over area $A_c$

$$F_u = \sigma_u A_c$$

(4.21)

may be transformed into additional steel stress $f_u$
(4.22)

which after a build up phase should be added parallel to the original stress strain relation of steel, as seen in figure 4.19a. For numerical modelling, the simplified approach, given in figure 4.19b, may be adequate. Here, the effect of the tension stiffening, seen in figure 4.19a, is replaced by an increased Young's modulus for the steel. The increased modulus may be approximated as

\[ E_{\text{r,}\varepsilon} = \frac{f_y + f_u}{\varepsilon_y} \]  

(4.23)

A simple calculation example with concrete tensile strength of 3.0 MPa, tension stiffening stress taken as 40% of this value, i.e. 1.2 MPa, and assuming a reinforcement ratio of one percent, an original Young's modulus of 200000 MPa and a yield strength of 500 MPa gives the following modification.

\[ f_u = \frac{\sigma_u}{\rho} = \frac{1.2}{0.01} = 120 \text{ MPa} \]

\[ \varepsilon_y = \frac{f_y}{E_y} = \frac{500}{200000} = 0.0025 \]
From expression (4.22) it is seen that the effect becomes larger for smaller reinforcement ratios. This is physically feasible in so far as the tensile stresses must be carried across the cracks by means of a smaller reinforcement area. At smaller bar diameters the bond relation between bar and concrete may influence the simplification described here.

In earthquake engineering problems involving cyclic behaviour cracks open up and close repeatedly, and after a few cycles essentially no tensile stresses are carried through the concrete, and the tension stiffening effect no longer occurs. Since the Young's modulus of the reinforcement may be given as an input parameter of the implemented user material model (see Appendix B) nothing in particular is prescribed in this question. This is rather a problem-related question: for the best possible numerical simulation of a monotonic experiment the modulus should probably be corrected to some degree. However, for an earthquake simulation it should preferably be left unchanged to better model the cycles with damaged concrete. An advanced option would be to include the possibility of modified Young's modulus for a few initial cycles and reset its value to the unmodified level.

We will lastly turn briefly to the second interaction effect mentioned. Across opened cracks the tensile reinforcement will cause dowel action, which may be modelled numerically by a dowel modulus matrix, added to the concrete modulus matrix. Since the effect of this action is still being researched it is currently modelled as a numerical fraction of the concrete modulus \( G \), and to some extent related to the reinforcement ratio of each direction. Although it is assumed that the dowel action also affects the modulus stiffness terms \( d_{11} \), \( d_{13} \), and \( d_{31} \), the entire dowel action is located at \( d_{33} \) in this model. In some examples a numerical dowel modulus stiffness corresponding to between 2% and 5% \( G \), delivered reasonable results. For the sake of simplicity the dowel contribution for the micro model in this report is added into the local system, and together with the concrete contribution rotated to the global system.

Hereby, the description of the various contributions to the composite modulus of the micro model is completed. The micro model is subjected to a number of tests in the following chapter involving comparison with experimental data.
CHAPTER FIVE

NUMERICAL EXAMPLES

5.1 Implementation

The models developed in chapters three and four were implemented in the Abaqus software [Abaq91], which is a general purpose finite element code. The reasons for the selection of this software are discussed by Wenk and Bachmann [WB90] and by Wenk [Wenk90].

The macro model of chapter three was coded by means of the User Element option. With this option an arbitrary element behaviour may be described by the user's own Fortran code contained in a subroutine, which is connected to the other parts of the software via a user element interface. This interface essentially delivers displacements for each increment, and the user element subroutine returns the stiffness matrix and the right hand side (force) vector for each increment. Each of the nonlinear springs was coded as a user element for initial test purposes. For a wall section as shown in figure 3.3, the closed form stiffness matrix of the macro element developed in section 3.7 was also coded as a user element.

Likewise, the micro model of chapter four was coded by means of the User Material option. This option is similar to the user element option, except that the material behaviour only needs to be described by the user at an integration point. Standard library elements may then be used, such as various plane stress elements with an arbitrary number of integration points. The interface delivers the strain increments, and the user subroutine supplies the material modulus matrix and the right hand side (stress) vector. For both the user element and user material options state variables are used to store quantities like accumulated stresses etc., which need to be accessed for correct formulation in subsequent increments. These state variables are updated in the user subroutine. For description of the state variables, see [WLB93].

For dynamic problems the structural mass was modelled by point mass elements at floor levels, with the mass acting only in a preselected direction, chosen here to be the horizontal one.

An introduction to the use of the models developed in chapters three and four is given in the form of a users' manual for the macro model in Appendix A, and for the micro model in Appendix B.
5.2 Analysis Procedure

5.2.1 General

The used software [Abaq91] provides analysis procedures divided into steps. Each step deals with a separate analysis, such as static monotonic, eigenvalue, or dynamic analysis. Fixed or automatic time increment length may be selected within each step. Several steps may be executed in a sequence, such as e.g. a static analysis for gravity loads followed by a dynamic analysis for earthquake action. The stress and strain states at the end of a preceding step are kept as initial values for the following step.

The analyses in this report were generally carried out as follows: At first, gravity loads were applied at floor levels during a static monotonic analysis step. During this step, user supplied behaviour was assumed to remain within the elastic range. Following the gravity load step, a nonlinear analysis is carried out by applying a lateral action to the tested structure. This nonlinear analysis may either consist of a monotonically applied static load in one step, or a cyclic static load applied in several steps. Furthermore, it may consist of a dynamic analysis, with a ground motion as input, carried out in one step.

5.2.2 Time integration

The solution procedures follow standard methods described in [Abaq89]. For the solution of nonlinear dynamic problems direct time integration methods are generally used, which satisfy the equations of motion at discrete times, here denoted \( n \).

For the time integration the Hilber-Hughes-Taylor \( \alpha \)-scheme is used, see e.g. [HHT77], [Hugh87], or [Abaq89] which is an extension of the original Hilber \( \alpha \)-method [Hilb76]. Only a brief introduction to the integration scheme is given here. The integration scheme is based on the Newmark family of equations, which for time \( n+1 \) may be written as

\[
M\dddot{\mathbf{U}}_{n+1} + C\dot{\mathbf{U}}_{n+1} + K\mathbf{U}_{n+1} = \mathbf{F}_{n+1} \tag{5.1}
\]

\[
\mathbf{U}_{n+1} = \mathbf{U}_n + \Delta t \dot{\mathbf{U}}_n + \frac{\Delta t^2}{2} [(1-2\beta)\dddot{\mathbf{U}}_n + 2\beta \dddot{\mathbf{U}}_{n+1}] \tag{5.2}
\]

\[
\dot{\mathbf{U}}_{n+1} = \dot{\mathbf{U}}_n + \Delta t [(1-\gamma)\dddot{\mathbf{U}}_n + \gamma \dddot{\mathbf{U}}_{n+1}] \tag{5.3}
\]
where $M$, $C$, and $K$ represent the global mass-, damping-, and stiffness matrices, respectively, and $\ddot{U}$, $\dot{U}$, and $U$ represent the global acceleration-, velocity-, and displacement vectors, respectively. Lastly, $F$ represents the externally applied force, generally varying with time.

The solution is based on a predictor-corrector procedure. The predictors are defined as

$$\ddot{U}_{n+1} = \dot{U}_n + \Delta t \ddot{U}_n + \frac{\Delta t^2}{2} (1 - 2\beta) \dddot{U}_n$$

(5.4)

$$\dddot{U}_{n+1} = \dddot{U}_n + (1 - \gamma) \Delta t \dddot{U}_n$$

(5.5)

With known values for displacement and velocity at time $n = 0$ the process may be started by obtaining the acceleration $\dddot{U}_0$ as

$$M \dddot{U}_0 = F - C \dot{U}_0 - KU_0$$

(5.6)

or specifying it directly. Then $\dddot{U}_{n+1}$ is determined as

$$\left(M + \gamma \Delta t C + \beta \Delta t^2 K\right) \dddot{U}_{n+1} = F_{n+1} - C \dddot{U}_{n+1} - K \dddot{U}_{n+1}$$

(5.7)

where the factors $\beta$ and $\gamma$ are chosen by the user, see further e.g. [Hugh87]. From (5.7) it may be noted that $\beta = 0$ permits explicit integration, (uncoupled system of equations provided that $C$ and $M$ are diagonal), otherwise implicit integration is carried out. The corrected displacements and velocities are obtained as

$$U_{n+1} = \dddot{U}_{n+1} + \beta \Delta t^2 \dddot{U}_{n+1}$$

(5.8)

$$\dot{U}_{n+1} = \dddot{U}_{n+1} + \gamma \Delta t \dddot{U}_{n+1}$$

(5.9)

The $\alpha$-scheme introduces some algorithmic damping without deteriorating accuracy by retaining relations (5.2) and (5.3) and modifying (5.1) as follows

$$M \dddot{U}_{n+1} + (1 - \alpha) C \dot{U}_{n+1} - \alpha C \dddot{U}_n + (1 + \alpha) K U_{n+1} - \alpha K U_n = F(t_{n+1})$$

(5.10)

where $(t_{n+1}) = t_{n+1} + \alpha \Delta t$. In [HHT77] the following choices for the parameters are recommended: $\alpha \in [-\frac{1}{4}, 0]$, $\beta = (1 - \alpha)^2 / 4$ and $\gamma = (1 - 2\alpha) / 2$. 
Low values for $\alpha$ such as -0.05 were found to be favourable according to among others [Abaq89], giving very slight algorithmic damping at lower modes, and is the (changeable) default value in [Abaq91].

Since the problems are generally nonlinear, the above equations are solved for incremental unknowns, which are then accumulated. Fixed and automatically changing increment lengths are available in [Abaq91], the former being chosen for all problems solved in this report.

5.2.3 Residual forces

For nonlinear problems, the internal element forces, which result upon the solution of an increment, are generally not in equilibrium with the applied external force. Thereby, unbalanced residual forces develop. Different procedures exist to deal with these residual forces. Only a short discussion of this problem is given here.

Purely incremental procedures [Yane82] do not correct for the residual forces, and thereby generally require very small increments in order not to deteriorate the solution.

The event to event procedure [Lind88], [SB90] essentially performs linear solutions until a nonlinearity occurs. This is in practice performed by scaling the solution to reach the nonlinearity, and is best suited for problems with a few sharp stiffness changes. The procedure is competitive only for relatively small systems of equations.

The incremental-iterative procedures [Abaq89] are the most effective for arbitrary and large systems. They may be divided into full and modified Newton-Raphson procedures. Whereas the former generally update the internal forces by recalculating the element matrices, the latter adjust the residuals by applying equilibrating forces on the unbalanced nodes.

For the problems computed in this report the full Newton-Raphson procedure was used. Typical tolerance values were selected in the range of one percent of maximum element forces. Within each increment iterations are performed (update of stiffness matrix during iterations) until the preselected tolerances are met. Based upon the stiffness (modulus) matrix obtained from user elements during the assembling at the beginning of each increment, predicted element forces (stresses) are computed, using the obtained incremental displacements. Residual forces are computed as the difference of the predicted and calculated element forces at the end of the increment. If the residual forces exceed the tolerances, iterations are performed, updating the predicted values, until the tolerances are met.
5.2.4 Damping

For dynamic analysis the hysteretic behaviour accounts for the major energy dissipation. Since the algorithmic damping was very slight in the frequency range of interest it was decided to provide some additional viscous damping. The major reason for this is that viscous damping may be better controlled physically than the algorithmic damping.

This additional damping is provided for by the introduction of a Rayleigh damping matrix $C$, consisting of one mass proportional part and one stiffness proportional part.

$$C = \alpha_1 K + \alpha_2 M \quad (5.11)$$

The mass and stiffness coefficients were determined in the well-known manner by selecting a desired damping ratio at two frequencies, see e.g. [Bath82], whereby the lowest frequency is chosen well below the elastic first mode, to account for the decreasing eigenfrequency due to yielding during the time history analysis. The second frequency is taken from an upper mode above which the viscous damping will be important.

For the examples with the eight-storey structural wall building, presented later in this chapter, around two percent viscous damping was selected at around half the frequency of the elastic first mode and at the second mode, giving $\alpha_1$ as 0.0009 and $\alpha_2$ as 0.12.

5.2.5 Ground motion

The software generally permits ground motion to be applied in three perpendicular directions simultaneously. In the examples studied in this report, which were simplified into planar analysis, horizontal ground motion in one direction only was applied. Time history ground accelerations were used as input.

One of these consists of an artificially generated ground acceleration, compatible to the elastic design response spectrum of the Swiss standard SIA 160 [SIA160] for five percent damping and medium stiff soil.

The other ground acceleration consists of the recorded N-S component of the 1976 Friuli earthquake, recorded at Tolmezzo.

Both ground accelerations are shown in section 5.4.2 and are discussed further in connection with the numerical examples.
5.3 Selection of Numerical Examples

Due to the nonlinear behaviour of structural walls, the only method to check the reliability of a numerical model is by comparison with experimental data. Therefore we carry out some elementary comparisons between the models developed in chapters three and four and experimental data from a shear wall specimen. The selection and description of this specimen is presented below in section 5.3.1.

In addition to the reliability check it is desirable to use the developed models in a performance check on the behaviour of a capacity designed multi-storey building. Such a check is performed in this chapter as well, and the building is presented in section 5.3.2.

5.3.1 Test specimen

A large number of tests on structural wall specimens of different scale have been carried out. A good review is given by Abrams [Abra91]. It is, however, difficult to find tests carried out as a realistic simulation of earthquake action on tall slender walls. It is desirable to use a test specimen as close to full scale as possible in order to avoid scale effects such as the wrong amount of concrete cracking, difficulty in reproducing reinforcing and concrete aggregate properly, etc. For dynamic tests the scale effects on the frequency is an added factor.

A large part of the tests with full-scale or close to full scale specimens, documented in the literature, concern "squat shear walls", i.e. walls with an aspect ratio (height to length) smaller than two. (In [PBM90] walls with aspect ratios below three are regarded as squat walls).

In several other tests the specimen was assumed to be part of a tall wall, representing the lower stories, and subjected to a high vertical force simulating gravity load from upper stories as well as a large shear force simulating the accumulated inertia forces arising from the fictitious upper stories. However, the overturning moment from the upper stories, which may be of a considerable magnitude for multi-storey buildings, is usually not included in the test.

Among tests with specimens assumed to be part of a wall, the test series of Vallenas, Bertero, and Popov [VBP79] included an attempt to simulate the overturning moment as an action on the specimen. This test series also provided extensive test data which lends itself to comparisons with the numerical models developed in this report.
The tests comprised the experimental study of two three-storey wall specimens in the scale of 1:3. One wall had protruding boundary elements and the other was rectangular in cross section. Both specimens were subjected to gravity loads, monotonic and cyclic lateral loads combined with monotonic and cyclic vertical loads.

Some of the results from this experimental study were also used for reliability checks in previous studies on numerical wall models such as in [VB87], and in [Lind89]. We use here the wall with boundary elements for the purpose of checking the reliability of the
numerical models developed in this report. Elevation and section of the test specimen are shown in figure 5.1. The specimen was fabricated as a 1:3 scale model of the three lowest storeys of the structural wall of a ten-storey building, seen in figure 5.2. The section forces of the wall at the fourth floor level were transferred to the scale model as a shear force (shear force applied to the top of the model) and a bending moment (applied as a vertical force couple, i.e. two vertical forces applied at the fourth floor with opposite sign, and coupled to the shear force as shown in figure 5.2). It should be noted that although usually better than omitting the bending moment entirely, this is also a simplification of the relation between the section forces which assumes that during a time history analysis the bending moment is proportional to the shear force, which is generally not the case. The basic mechanical properties of the test specimen are given in Table 5.1.
Concrete compressive strength | 34.8 MPa  
Young's modulus for small strain concrete | 27 900 MPa  
Concrete tensile strength | 3.48 MPa  
Concrete tensile strain | 0.0001  
Reinforcement yield strength, boundary element | 444 MPa  
Reinforcement yield strength, web | 507 MPa  
Young's modulus, all bars | 211 000 MPa  
Strain hardening, all bars | about 1% of elastic modulus

Table 5.1 Mechanical properties of 1:3 scale test specimen [VBP79]

5.3.2 Capacity designed multi-storey wall building

The objective with the numerical models, after the reliability tests are satisfactory, is to use the models in the performance check of capacity designed buildings. We use here an eight-storey building, which essentially corresponds to the one used in [BWL92b].

The eight-storey office building, which is located in Switzerland, in seismic zone 3b according to the Swiss Standard [SIA160], is horizontally stabilised by structural walls, and slender gravity load dominated columns carry the vertical loads not carried by the wall. Plan and elevation of the building are shown in figures 5.3 and 5.4.

The eight-storey building was designed according to the capacity design method [PBM90], and we here give only a brief presentation and the data relevant to the numerical tests performed later in this chapter. More comprehensive information is given in [BWL92b].

The lateral design forces were obtained by the static equivalent force method, based on an elastic design spectrum, assuming seismic zone 3b and medium stiff ground according to [SIA160]. Force reduction was performed according to two global displacement ductility levels. Overstrength was accounted for by reducing the design forces by a factor $C_d$ equal to 0.65. For basic design definitions see Appendix F.

The two ductility levels, employed a global displacement ductility $\mu_A = 3$, and $\mu_A = 5$, respectively, the former known as restricted ductility, and the latter as full ductility. The fundamental period of vibration used to obtain the spectral acceleration value was found to be 1.38 Hz according to the simplified code formula [SIA160] which does not consider wall dimensions. A computer evaluation involving the wall only resulted in 0.74 Hz (elastic), and 0.67 Hz (cracked) which is obviously somewhat on the low side.
Figure 5.3 Plan of capacity designed eight-storey wall building [BWL92b]

Figure 5.4 Elevation of capacity designed eight-storey wall building [BWL92b]
NUMERICAL EXAMPLES

With the slight addition of girders and nonstructural elements 0.90 Hz was obtained as in [BWL92b] which was also used in this study as a realistic design value between the two extremes. With a storey mass of 3.08 MN (2.78 MN for roof) the results of the static equivalent force calculation were obtained as \( M_e = 16.1 \text{ MN} \) bending moment at the base and \( V_e = 0.72 \text{ MN} \) base shear for the restricted ductility design, and \( M_e = 9.7 \text{ MNm} \) and \( V_e = 0.43 \text{ MN} \) for the full ductility design, respectively. Using a resistance factor \( \gamma_R \) equal to 1.2, the corresponding flexural demands are obtained as \( M_i = 16.1(1.2) = 19.3 \text{ MNm} \) for the restricted ductility design, and \( M_i = 9.7(1.2) = 11.6 \text{ MNm} \) for the full ductility design.

The material specifications were set according to the SIA Standard 162 [SIA162] as follows. Concrete for walls: concrete B40/30 (design strength \( f_c = 19.5 \text{ MPa} \)), Reinforcing steel: S500 (design strength \( f_y = 460 \text{ MPa} \)). Since this building is analysed by nonlinear time history analysis, the most realistic nonlinear behaviour is desirable (no reduction in the input ground motion applied). Therefore, overstrength is accounted for in the time history analysis by adjusting the strengths to their effective mean values as follows: \( f_y = 550 \text{ MPa} \), \( f_c = 30 \text{ MPa} \).

The wall cross section, shown in figures 5.5 and 5.6, is rectangular and 300 mm thick by 6.0 m in length for both ductility levels. The buckling criterion according [PP92] was found to be in order. The vertical reinforcement ratios were set as follows: For the restricted ductility: 0.26% distributed uniformly over the cross section, except at the

\[ \begin{align*}
6 \text{ D20} & \quad \text{D10/200} & \quad \text{Sym} \\
500 & \quad L_w/2 = 3000
\end{align*} \]

Figure 5.5 Wall cross section from first storey of restricted ductility design, \( \mu_\Delta = 3 \)

\[ \begin{align*}
6 \text{ D12} & \quad \text{D10/250} & \quad \text{Sym} \\
500 & \quad L_w/2 = 3000
\end{align*} \]

Figure 5.6 Wall cross section from first storey of full ductility design, \( \mu_\Delta = 5 \)
ends, where there is a confined zone of 500 mm length with 1.2 % reinforcement. For the full ductility design these values were altered to 0.21% and 0.45 %, respectively. The horizontal reinforcement has a ratio of 0.20 % over the height of the plastic hinge and the remaining wall. The resulting design strengths at the base of the wall are as follows: $M_R = 19.4$ MNm bending moment and $V_R = 2.7$ MN base shear for the restricted ductility design, and $M_R = 15.8$ MNm bending moment and also $V_R = 2.7$ MN base shear for the full ductility design. By assuming an overstrength factor for reinforcement steel $\lambda_o$ of 1.2 we obtain the two flexural overstrengths as follows:

Restricted ductility: $\Phi_{o,w} = 1.2 \frac{19.4}{16.1} = 1.45$

Full ductility: $\Phi_{o,w} = 1.2 \frac{15.8}{9.7} = 1.95$

These values are to be used for the shear demand calculation which, using a dynamic magnification factor of

$\omega = 1.3 + \frac{n}{30} = 1.3 + \frac{8}{30} = 1.57$

where $n$ is the number of storeys, and inserted in the formula for shear demand [PBM90]

$V_w = \omega \Phi_{o,w} V_E$

gives shear demands of $1.41(1.57)0.72 = 1.59$ MN for restricted ductility, and $1.95(1.57)0.43 = 1.32$ MN for full ductility, respectively. With the concrete contribution to the shear capacity of

$V_c = \nu_e b_w d = 0.91(0.3)0.8(6.0) = 1.31$ MN

where the concrete shear stress is taken as

$\nu_e = 0.6 \sqrt{\frac{P_u}{A_t}} = 0.6 \sqrt{\frac{4.15}{0.3(6.0) = 0.91$ MPa}

the remaining steel contribution would be rather small. However, the minimum requirement for horizontal reinforcing ratio of 0.20 % [SIA162], and assuming bar D10 in each face spaced at 250 mm, gives a steel contribution of
\[ V_s = A_s f_s \frac{d}{s} = 2 \left( \frac{\pi 0.01^2}{4} \right) 460 \frac{0.8(6.0)}{0.25} = 1.39 \text{ MN} \]

which together with the concrete contribution results in a shear capacity of 2.7 MN, exceeding the demand for both ductility levels. Even with an assumed material overstrength of 1.4 rather than 1.2, the demands are clearly exceeded. This larger effective shear capacity may, however, be necessary as we will see later. Reinforcing curtailment over the height of the wall was performed according to [PBM90]. Hereby the introduction to the design of the eight-storey wall is completed.

Due to the building symmetry, and with assumption of horizontal ground motion perpendicular to the building in the direction of the walls, only half the building needs to be analyzed, laterally stabilised by one structural wall. The total mass used in the analysis for the single wall corresponds to half the building mass.

### 5.4 Macro Model Results

#### 5.4.1 Comparison with experimental results

In order to check the reliability of the macro model developed in chapter three a number of tests are performed. These tests are carried out as a comparison of the analysis results obtained from the numerical model with the results obtained from experimental tests. It is clear that an exact agreement cannot be obtained with a relatively simple model like the one developed here. However, attention should rather be directed towards the ability of the numerical model to simulate the major kinematic phenomena of structural walls subjected to earthquake action.

In order to facilitate the clarity of the following reliability tests, two major principles were followed. The first is the establishment of a clear and relatively simple test example which still represents a reasonably realistic behaviour of a structural wall, as well as a clear and simple numerical modelling of this example. This example will be followed throughout the reliability tests. The second major principle is that of enhancing the clarity of the numerical tests presented, by essentially varying one parameter only at a time, and keeping all other parameters fixed.
5.4.1.1 Monotonic behaviour

One of the test specimens was subjected to a static monotonic test. After application of gravity loads by two vertical forces of 0.434 MN at each end of the fourth floor of the specimen, a shear force $V$ applied at the fourth floor was monotonically increased till failure occurred at a shear force slightly above 1.0 MN. Some unloading and reloading was performed. In addition to the shear force, the aforementioned vertical force couple was also applied and increased according to the prescribed ratio of 0.644 $V$, as shown in figure 5.2c. The results from this test are suited to checking the influence of basic properties of the numerical model.

As for all finite element models the chosen element mesh will affect the obtained solution. For the macro model, we test this effect here by discretising the three-storey test specimen described in section 5.3.1 into three different meshes. Figure 5.7 shows the test specimen with its load pattern and the three meshes for the macro model developed in chapter three. Macro elements formulated as described in section 3.7 are used. The first storey is discretised into a different number of macro elements for each mesh as seen from the figure.

![Figure 5.7 Macro model meshes](image-url)
With wall geometry and material data from section 5.3.1, the input was prepared for the macro model, described in Appendix A, resulting in a basic set of input values kept for all examples and shown in table 5.2.

<table>
<thead>
<tr>
<th>Cross sectional area, ( A )</th>
<th>0.321 m(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross sectional moment of inertia, ( I )</td>
<td>0.193 m(^4)</td>
</tr>
<tr>
<td>Young's modulus for small strain concrete, ( E_c )</td>
<td>27900 MPa</td>
</tr>
<tr>
<td>Ratio of cracked to uncracked stiffness for vertical spring, ( \alpha_{cr} )</td>
<td>0.5</td>
</tr>
<tr>
<td>Ratio of yielded to uncracked stiffness for vertical spring, ( \alpha_y )</td>
<td>0.01</td>
</tr>
<tr>
<td>Yield moment, ( M_y ), (selection shown in Appendix C)</td>
<td>3.0 MNm</td>
</tr>
<tr>
<td>Shear force at shear cracking, ( V_c ), acc. to expr. (3.49)</td>
<td>0.57 MN</td>
</tr>
<tr>
<td>Ratio of cracked to uncracked shear stiffness, ( \alpha_s )</td>
<td>0.18</td>
</tr>
</tbody>
</table>

*Table 5.2 Basic input values for Macro model test examples*

Figure 5.8 shows the experimentally obtained horizontal displacement at the fourth floor versus base shear for monotonic load. Some bond slip of vertical bars in the foundation footing of the test specimen occurred, causing an additional 10 to 15 % horizontal displacement included in figure 5.8, which should be kept in mind when comparing with numerical results. For the numerical models only the mesh is changed and all other properties are kept constant.

In the following numerical examples some of the parameters of the basic set are varied, however only when specifically mentioned, and are otherwise reset to the original value.

For the investigation of the mesh effect a monotonic shear force is applied at the fourth floor, increased to 1.0 MN, according to the pattern shown in figure 5.7, including the vertical forces which are coupled to the shear force. The numerical application of force was performed in increments of 0.02 MN, giving a total of 50 increments.

The result is the fourth floor displacement shown in figure 5.9 for the three meshes. The fixed end rotation which occurred accidentally at the base of the test specimen due to slip of vertical bars in the foundation is not accounted for in the numerical modelling.

It is seen that the mesh A, with the first storey discretised into only one macro element, shows a softer stiffness, which is visible already at around 0.15 MN base shear, where flexural cracking occurs, again visible upon yielding. This is due to the fact that the smaller stiffnesses (cracking and yielding) occur over a larger part of the structure than in the meshes with finer discretization. The comparisons made here do not exceed a shear force of 1.0 MN or a fourth floor displacement of 40 mm. Exceeding these values would not be realistic at this floor height with a trilinear model.
Figure 5.8 Fourth floor monotonic horizontal displacement versus base shear, obtained from experiment [VBP79]

Figure 5.9 Mesh effect, fourth floor horizontal displacement
In the subsequent tests, where the effect of other parameters are studied, mesh A is used in order to demonstrate the other respective effects more clearly.

The next basic parameters to check are obtained by studying the input properties. There it is seen that the ratios between cracked and uncracked concrete, as well as the ratio between yielded stiffness to uncracked stiffness of the outer vertical springs appear. Some suggested values for these both ratios are mentioned in chapter three as well as in Appendix A. However, the influence on the global behaviour, when they are varied, should be briefly shown.

The first parameter tested is the ratio of cracked to uncracked stiffness of the outer vertical springs, $\alpha_{cr}$. This ratio represents the relation between the two stiffnesses $K_{cr}$ and $K_e$, seen in figure 3.19. Varying this ratio from 0.2 over 0.5 to 0.8, by using the mesh A of figure 5.2, and applying the same monotonic force as for the mesh test, the result is shown in figure 5.10.

It is seen from figure 5.10 that the choice of $\alpha_{cr} = 0.2$ appears to give the best correspondence to the test (figure 5.8) until yielding occurs. However, one must keep in mind that 10 to 15 % of the horizontal displacement values of the test are caused by bond
slip in the foundation. Furthermore, some cyclic testing appears to have been preceding the monotonic test, (as may be seen in figure 5.8) which may have caused some flexural cracking, although unintentionally, thus making the tested cracked stiffness smaller. These circumstances together with a somewhat uneven concrete quality of the test specimen [VBP79], suggest that a value for $\alpha_{cr}$ of around 0.5 (giving a global flexural stiffness of around 70% of elastic, according to figure 3.22) will still be optimum, if the physical test environment is taken into account.

Next, the influence of the ratio of yielded stiffness to uncracked stiffness of the outer vertical springs, $\alpha_y$, is tested. This ratio represents the relation between the stiffnesses $K_y$ and $K_e$ in figure 3.19. Values of 0.005, 0.01, and 0.02 are tested, with the ratio $\alpha_{cr}$ kept constant at 0.5 again. The result is shown in figure 5.11.

It may be seen from both figures 5.10 and 5.11, that the ductility obtained is dependent on the chosen cracking stiffness (figure 5.10) and on the chosen yield stiffness (figure 5.11). Both the displacement ductility, which may be directly seen in the figures for the roof level horizontal displacement, and the curvature ductility are generally dependent on the choice of these two parameters. Although the obtained displacement ductility varies with a factor of almost two between a chosen stiffness reduction due to flexural cracking from 0.8 to 0.2, it is also seen that between the factors 0.8 and 0.5 only a small difference in ductility will occur. However, for the choice of the yield stiffness
as a fraction of uncracked stiffness, ranging from 0.005 to 0.02, we also obtain a displacement ductility difference of about two. For the normal ranges of the parameters of cracking stiffness and yield stiffness, it appears that the yield stiffness will have the most important influence on the ductility.

Concerning ductility we are in the first place interested in the demand of curvature ductility, which we may obtain as the largest ductility reached during a time history analysis. This quantity will be used for the proper design of the wall cross section at that location in order to ensure that it can take this amount of curvature without failure. In the coming section dealing with a capacity designed eight-storey wall structure, the curvature ductility will be examined and discussed, whereby the influence of the yield stiffness also will be shown.

In the same manner as for the outer vertical springs, the influence of the cracking ratio for the horizontal spring will be shown. The factor $\alpha_s$, describing the ratio of the cracked shear stiffness to the uncracked shear stiffness, which was discussed in section 3.6, is varied from the recommended minimum value 0.14 over 0.18 to 0.25, and the result is shown in figure 5.12.

The next parameter to be varied is the location of the relative centre of rotation as discussed in section 3.3, and denoted by $h_c$. The purely elastic derivations in that section suggested values of this ratio of around $2h/3$ to $h/2$, where $h$ is the height of the macro element, and $h_c$ was the distance to the top of the element.
We will here test this parameter, expressed by the value $c$, which is obtained as

$$c = \frac{h - h_s}{h}$$

i.e. the value to be the distance of the centre of the relative rotation to the bottom of the element related to the element height. According to the above the suggested values would give $c$ in the range of $0.33h$ to $0.5h$. By taking yielding into account, which usually starts from the bottom of the element, this value would be lower, perhaps close to zero. We will here show the influence of this ratio by assigning it the values $0$, $0.2h$, and $0.4h$. The result is shown in figure 5.13. When the centre of relative rotation is placed low, the flexural yielding will start earlier provided the bending moment is increasing towards the base of the wall, which is normally the case. This effect may also be seen in figure 5.13.

Thereby the reliability tests for monotonic behaviour are concluded, and our attention now turns to the behaviour under cyclic loading.

5.4.1.2 Cyclic behaviour

For a similar test specimen to the one used for the reliability tests for monotonic behaviour, some test data for cyclic tests is also available, which we use to check the reliability of our numerical model under cyclic loading.
Figure 5.14 Control displacement history for wall test specimen [VBP79]

Figure 5.15 Fourth floor horizontal displacement versus base shear, experiment [VBP79]

The test specimen was subjected to statically increasing horizontal load, coupled with the vertical force couple as shown previously in this chapter. This load set was then reversed in a number of cycles with prescribed maximum horizontal displacement at the fourth floor. This control displacement history is shown in figure 5.14.
The fourth floor displacement versus the base shear from the experimental test is shown in figure 5.15. For the numerical model the response to cyclic static loading is shown in figure 5.16, but with the number of cycles reduced to two for clarity, with fourth floor maximum horizontal displacements of around 20 mm and 40 mm. Since repeated cycles of the same maximum displacement did not produce any significant stiff-
ness or strength degradation, the results from these two cycles are suitable for comparison of experiment and analysis.

In order to obtain an idea of how the outer vertical springs work during cyclic behaviour, the spring force versus spring elongation is shown in figures 5.17 and 5.18 for the same cyclic test for the outer vertical springs of the first storey macro element of mesh A.
In figure 5.16 a change of stiffness may be noted on the reloading branch at a force level of around 0.70 MN (e.g. seen in the upper right part of the plot). This is due to the change from yielding in compression of the right edge of the wall (for the model: the right spring seen in figure 5.17) to closed cracks i.e. compressive elastic behaviour. This stiffness change only takes place when yielding in tension has occurred at this side prior to yielding in compression.

We will also need to examine the behaviour of the horizontal spring during the cyclic test. This spring, which models the shear behaviour, employs the relatively simple bilinear origin oriented hysteretic model, whose justification was discussed in chapter three. Spring force versus spring elongation during the cyclic test is shown for the first storey macro element in figure 5.19.

In order to complete the study of the kinematic behaviour the spring force versus elongation for the central vertical spring of the first storey macro element is shown in figure 5.20. As seen in the figure this spring is active only in compression. The small stiffness observed in tension was attributed to numerical causes.

The global cyclic behaviour as shown in figure 5.16 is characterised by the shape of the hysteresis loops, as defined in figure 3.18. It is of importance that the shape of these loops does not deviate too much from the experimentally obtained one, since in dynamic
problems the area within these loops provides a measure of the dissipated energy for the cycle in question, referred to as the hysteretic damping. The most important characteristics of the hysteretic shape are the "fatness" and "pinching" of the loops. Among others Saatioglu [Saat91] provides an extensive discussion on this topic.
In the model presented in chapter three of this report both these characteristics of the hysteretic shape are mainly determined by the factor $\alpha_{ci}$, described in chapter three as a fraction of the tensile yield force level, at which flexural cracks are closing on the compressive branch during unloading. As a default value, this factor was chosen as 1.0, meaning that crack closure occurs at a compressive force equal to the yield force. In order to study the influence of this factor, $\alpha_{ci}$ was varied to 0.8, and 1.2. The results for $\alpha_{ci}$
equal to 0.8 are shown in figures 5.21 and 5.22. Figure 5.21 shows the fourth floor horizontal displacement versus base shear, and figure 5.22 shows the left vertical spring behaviour. Corresponding results for $\alpha_c$ equal to 1.2 are shown in figures 5.23 and 5.24.

It is seen from figure 5.21 that a more pinched shape of the hysteretic loops is obtained with smaller values for $\alpha_c$. The loops also tend to become somewhat thinner around the origin. Less pinching is obtained in figure 5.23 (only during the last cycle, seen in the upper right corner). In this case crack closure was barely reached for $\alpha_c=1.2$.

No particular attempt is be made to adjust the input properties so as to obtain an optimum agreement with the experimental data shown in this example. Such a set of properties would not necessarily deliver good results in other comparisons. Rather, the general effect of some important properties were shown, with experimental results from the lower stories of a structural wall as a basis.

It was seen that the ratio of yield stiffness to uncracked stiffness is important for the nonlinear behaviour, as well as to some degree the ratio of cracked to uncracked stiffness, which however is only important in the first one or two cycles. Both ratios must be obtained mainly empirically. The cracked shear behaviour proved to be of lesser importance for a wall of this configuration.

For the cyclic behaviour the points where flexural cracks close largely determined the shape of the global hysteretic loops.

Reasonable agreement could be obtained between experimental data and analysis for both static monotonic and static cyclic behaviour, and this mostly with the chosen default set of input parameters. Only the level of shear cracking appears to be somewhat lower for the experiment (around 0.40 to 0.45 MN compared to 0.57 MN for analysis), which did not influence the good overall agreement significantly.

Thereby, the reliability tests for the macro model are completed and the focus will be directed towards the performance of capacity designed buildings by use of this model.

5.4.2 Multi-storey wall

The following numerical tests should be seen as dynamic performance tests of the structural behaviour of a capacity designed structural wall building subjected to seismic action. This action is numerically applied as a ground motion history.

The structural wall of the eight-storey building presented in section 5.3.2 is used here for the test. This wall was designed to resist all the horizontal action of the building, and
the slight moment resisting effect of the gravity load dominated columns is neglected in the analysis. We will perform analyses of the walls for the both designs corresponding to restricted and full ductility. The geometry is the same for both designs, the difference being found in the wall reinforcement.

Figure 5.25 Numerical model of capacity designed eight-storey wall

One mesh only will be used for each design. The mesh corresponds to the one used by Bachmann et al in [BWL92b]. With this mesh, the four lowest eigenfrequencies for the elastic (slightly cracked) wall without stiffening beams and nonstructural elements were found to be $f_1 = 0.67$ Hz, $f_2 = 4.0$ Hz, $f_3 = 9.8$ Hz, $f_4 = 16.7$ Hz. Eigenfrequencies for different states of damage are found in Appendix E. The mesh is shown in figure 5.25, where it may be seen that the plastic hinge, stretching over the height $L_p$ taken as $L_w = 6.0$ m, is discretised into three macro elements.

The ground motion in form of an artificially generated acceleration history, compatible to the SIA design spectrum [SIA160] for seismic zone 3b and for medium stiff ground is shown in figure 5.26. In figure 5.26a is shown the time history of the ground acceleration of 10 seconds length. The strong motion phase lasts about seven seconds. In figure 5.26b is shown the design spectrum according to the SIA 160 for medium stiff ground and for five percent damping.
The input properties for the macro model, as described in Appendix A, are summarized in Table 5.3.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross sectional area, $A$</td>
<td>1.8 m$^2$</td>
</tr>
<tr>
<td>Cross sectional moment of inertia, $I$</td>
<td>5.4 m$^4$</td>
</tr>
<tr>
<td>Young's modulus for small strain concrete, $E_c$</td>
<td>34000 MPa</td>
</tr>
<tr>
<td>Ratio of cracked to uncracked stiffness for vertical spring, $\alpha_{cr}$</td>
<td>0.7</td>
</tr>
<tr>
<td>Ratio of yielded to uncracked stiffness for vertical spring, $\alpha_y$</td>
<td>0.01</td>
</tr>
<tr>
<td>Yield moment, $M_y$, plastic hinge, (acc. to Appendix C), ($\mu_A = 3$)</td>
<td>7.9 MNm</td>
</tr>
<tr>
<td>Yield moment, $M_y$, plastic hinge, (acc. to Appendix C), ($\mu_A = 5$)</td>
<td>4.6 MNm</td>
</tr>
<tr>
<td>Shear force at shear cracking, $V_c$, acc. to expr. (3.49)</td>
<td>3.0 MN</td>
</tr>
<tr>
<td>Ratio of cracked to uncracked shear stiffness, $\alpha_s$</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 5.3 Macro model input properties for eight-storey wall

It is evident that there exists no experimental data to compare the following results with, but instead one must consider the results in view of the intentions which were set at the design phase. We divide up the treatment of capacity designed building so that at first a number of numerical results are shown for the selected building. This is performed in the subsequent and final part of this chapter. Then, in chapter six, a number of more fundamental questions regarding the capacity design of walls are treated, partially based on results of the examples tested in this chapter.

We directly deal with the nonlinear dynamic behaviour of the capacity designed eight-storey wall. The following numerical results base on a time history analysis, performed by direct time integration, as mentioned in section 5.2. The input is given by the 10 second ground acceleration history of figure 5.26. Computations were performed for 12 seconds. The time integration utilises time increments of 0.01 s, and a residual force tolerance of 1/100 of the maximum element forces. A maximum of six equilibrium iterations per increment were allowed, whereby two to three were normally found to be sufficient. Slight Rayleigh damping of 2% (at 0.5 Hz and 5 Hz) was prescribed.

The first results from the nonlinear time history analysis to be displayed is the horizontal displacements of the roof level (ninth floor), shown for both design ductility levels in figure 5.27. For comparison purposes, a linear elastic solution is also plotted.

The major difference with respect to the elastic linear solution lies in the yielding which occurs for both capacity designed walls after around seven seconds.
It may be seen in the figure that the difference between both ductility levels is not particularly large. This may be explained by the fact that the full ductility design gives a softer system, for which the first mode is excited less after damage occurs in the plastic hinge. The global displacement ductilities set at the design phase cannot be verified direct-
ly from figure 5.27, since the roof displacement at the onset of yielding at the base, was found to be close to zero, or even negative due to the influence of higher modes, which would give infinite displacement ductility values.

In order to estimate the influence of the time increment length, the roof displacement of the full ductility design is shown in figure 5.28 for increment lengths of 0.005 s, 0.01 s,
and for 0.02 s. It is not possible to distinguish between the three solutions until after about eight seconds, where slight differences occur. No essential difference in maximum displacement can be observed. The solutions for the two smaller increments appear closer together indicating convergence.

The nonlinear behaviour at the base of the wall may be displayed through the overturning moment and shear quantities. We start with the overturning moment at the base,
shown versus time in figure 5.29 for the restricted ductility design and in figure 5.30 for the full ductility design. The flexural demand at the wall base $M_r$ taken as $\gamma_k M_E$ (where $\gamma_k$ is the resistance factor of 1.2 and $M_E$ is the moment at the base from the equivalent static force calculation) is shown in the figures. With the roof displacement known, we
may here estimate the P-Δ effect to be about 0.1 meters (displacement) times 2 MN (gravity load for upper stories) = 0.2 MNm, i.e. conservatively estimated as around one percent of the base moment, which may be neglected.
The curvature ductilities as a function of the time are given in figure 5.31 and 5.32 for the lowermost element of the plastic hinge, each plot separately showing right and left curvature. The plots of figures 5.31 and 5.32 separately show the left and right curvature demand versus time.

The ductility is defined as based on the maximum curvature which has been reached at any given occasion. These values were obtained using the default yield stiffness factor $\alpha_y$ equal to 0.01 for the outer vertical springs. It is clear that the yield stiffness has some influence on the maximum curvature ductility reached. This influence is seen in figure 5.32, where the results of $\alpha_y$ taken as 0.02 are shown. Maximum values from left and right are compared. It is seen from figure 5.32 that higher yield stiffness has a clear reducing effect.

In order to get a better view of how the bending moment and curvature relate, and distribute over the lower part of the wall, three moment vs. curvature plots are displayed for each ductility design. Figure 5.33 shows for the restricted ductility design the moment curvature relation in three plots which describe this relation at (starting with the plot on the bottom of the page): the lowermost element of the plastic hinge, the uppermost element of the plastic hinge (third element from below), and the lowermost element of the elastic region (fourth element from below).

In figure 5.34, the corresponding quantities are shown for the full ductility design. It is seen from figures 5.33 and 5.34 that the curvature clearly decreases towards the top of the plastic hinge, however, the bending moment decreases to a lesser degree, since the plastic hinge is still in a yielded state at its upper end. It is also seen that some yielding takes place at the lower end of the elastic region, especially for the full ductility design. The implications of this are treated more fully in chapter six.

Since a relationship between chosen global displacement ductility and expected curvature ductility in the plastic hinge is proposed in [PBM90] based upon a static analytical example it would be possible to compare this proposal to the curvature ductility obtained from the time history analysis in order to check the agreement. It should be noted that the plots only displayed the curvature ductility for the lowermost element of the plastic hinge, while [PBM90] defines it over the entire plastic hinge height. Due to the discretization of the plastic hinge into three elements, the element closest to the base exhibits a larger curvature ductility than a value suggested as an average value assuming uniformly distributed curvature ductility. This matter is also discussed further in chapter six.
Lowermost element of elastic region

Uppermost element of plastic hinge

Lowermost element of plastic hinge

Figure 5.33 Moment versus curvature relations, restricted ductility design
Lowermost element of elastic region

Uppermost element of plastic hinge

Lowermost element of plastic hinge

Figure 5.34 Moment versus curvature relations, full ductility design
In order to follow the nonlinear spring behaviour during the time history analysis, the spring force versus the spring elongation is shown for the left vertical spring of the lower- and uppermost elements of the plastic hinge as well as for the lowermost element of the elastic region, in analogy with figures 5.33 and 5.34.

Figure 5.35 shows the spring behaviour for the restricted ductility design, and figure 5.36 for the full ductility design. In these two figures it is more clearly seen than in figures 5.33 and 5.34 that there is indeed some yielding in the region intended to remain elastic.

For both the restricted and the full ductility design, the point where cracks close on the unloading branch, was taken as four times the yield force in tension, i.e. $\alpha_{cl} = 4$, giving a closure force level somewhat below and above the effective normal force acting on the wall, for the restricted and full ductility level respectively.

From figures 5.33, 5.34, as well from 5.35 and 5.36, it may be seen that the unloading will be follow a flatter curve for the full ductility design than for the restricted ductility design.

Physically, this may be thought of as reasonable in so far as the compressive yielding in unloading should be somewhat more pronounced for the full ductility design due to a smaller vertical reinforcement area over which the compressive forces may be transferred as well as due to the larger plastification that has occurred in tension before unloading. This leads to a somewhat more "pinched" shape of hysteretic loops of the moment vs. curvature for the full ductility design, as may be clearly seen e.g. in the plot for the lowermost element in the plastic hinge of figure 5.34.

The shear force at the wall base is displayed versus time in figure 5.37 for the restricted ductility design and in figure 5.38 for the full ductility design. In the plots, the shear force demand at the wall base $V_w$, calculated as $V_w = \omega_\omega \Phi_{o,w} V_E$ (where $\Phi_{o,w}$ is the overstrength factor, $\omega_\omega$ is the dynamic magnification factor, and $V_E$ is the shear force at the wall base from the equivalent force calculation, see section 5.3.2) is shown as a dotted line. The shear capacity $V_K$, determined as the sum of concrete and steel contribution based on the capacity design method, see section 5.3.2, is also indicated in the plots.

It is seen that the capacity is much larger than the equivalent static force. This is mainly due to the large wall length (6.0 m horizontally) in combination with the minimum requirement for horizontal reinforcement of 0.20 % which together with the concrete contribution give around 2.7 MN capacity, see section 5.3.2. For a wall with less length this capacity would be smaller, although the minimum requirements would be fulfilled. It may be seen that for both the restricted and full ductility designs, an effective shear force
Figure 5.35  Left vertical spring behaviour, restricted ductility design
Figure 5.36  Left vertical spring behaviour, full ductility design
is reached which come close to the capacity (2.3 MN for restricted ductility and 2.1 MN for full ductility). Due to conservative estimates on the concrete contribution it is assumed that the wall would suffer extensive shear cracking during the simulated earthquake, but it would not fail in shear.

Since the time history results so far are based on an artificially generated ground motion, it would be of interest to examine the performance of the same building subjected
to a recorded ground motion. For this purpose we repeat some of the computations, using a N-S component of the ground motion recorded at Tolmezzo, during the 1976 Friuli earthquake. The ground motion is shown in figure 5.39. The record is of 20 seconds length, with a strong motion phase of seven seconds, starting at around 3 seconds.

It should be noted that the maximum ground acceleration is around 3.6 m/s², i.e. a factor 2.3 higher than the ground acceleration compatible to the SIA design spectrum. The same walls as used previously are subjected to the recorded ground motion, without being redesigned. This was done intentionally so as to get a performance check for the two designs, when subjected to a realistic ground motion with significantly higher acceleration levels.

As a first plot, the roof displacement versus time is shown in figure 5.40. A maximum displacement of around 0.13 m for the full ductility design gives an average inter storey drift of 0.13/32.0 = 0.004, i.e. 0.4 percent, compared to around 0.3 percent for the SIA compatible ground motion input, both being relatively moderate values.

Rather than repeating all the previous plots, we limit the results to some critical quantities, of which the base shear increased most dramatically. The base shear versus time is shown in figures 5.41 and 5.42 for both designs respectively.
Figure 5.40 Horizontal roof displacements

It is seen that for both designs, an effective base shear of around 2.7 MN is reached, which means that the shear capacity is fully utilised, and depending on how conservative the shear capacity estimation really is, that a shear failure may not be excluded.

It should be mentioned here again that the shear behaviour of the macro model does not simulate shear failure, since this is partly not desirable and partly difficult to obtain realistically in a simple numerical model. Rather, cracked shear stiffness is used, and the shear capacity should be estimated by the user. Realistic modelling is obtained by the model until the region where the shear capacity is reached. Some thought should rather be given here to the question about how realistic is the estimated shear capacity.

For the estimation of the concrete contribution to the 2.7 MN capacity, a full normal force of 4.15 MN (including dead and live load tributary to the wall) was used, without multiplying it by the resistance factor $\gamma_r$ equal to 1.2, as is sometimes done in capacity design examples in the literature (or dividing by a strength reduction factor $\phi$ taken as 0.9) and which may be rather unconservative.

If, however, we further reduce the effective normal force acting on the wall to 90 percent of its value, this results in a concrete shear strength of 0.86 MPa rather than 0.91 MPa, and a total shear capacity of 2.56 MN rather than 2.7 MN, indicating that a shear failure is quite likely for both the ductility designs, when subjected to a ground motion like the one used here and a normal force slightly smaller than that assumed.
The curvature ductility demand obtained for the recorded input was also clearly larger than for the SIA input. The curvature ductility demand for the lowermost element in the plastic hinge is shown versus time for both designs in figures 5.43 and 5.44, respectively. For the restricted ductility design, initial yielding in right curvature occurred during reloading from yielding in left curvature which resulted in a large curvature ductility. More pinched hysteretic behaviour for the full ductility design delayed the
yielding during reloading and thus limited the resulting curvature ductility.

The results displayed from the eight-storey building were intended to give a basic over-view of the analysis capacity of the numerical model for such problems, and to serve as a background for the discussion on the capacity design method in chapter six. Thereby, the tests of the macro model are concluded and the rest of this chapter is devoted to some basic tests of the micro model developed in chapter four.
5.5 Micro Model Results

In this section the micro model which was developed in chapter four is subjected some reliability tests. For the sake of simplicity we will use the same test specimen as for the macro model, i.e. the three-storey specimen tested by Vallenas et al in 1979.

We will mainly limit the numerical tests shown here to reliability tests, since the establishment of meshes as well as the numerical effort are both quite considerable when this model is used to simulate tall multi-storey structural walls subjected to ground motion. A few results from numerical simulations of the eight-storey building will complement the reliability tests.

5.5.1 Comparison with experimental results

The specimen and force application are the same as described for the macro model. A basic set of model properties was established from which deviations are not made unless specifically stated, in accordance with the macro model. The properties are summarised in table 5.4.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus for concrete</td>
<td>27900 MPa</td>
</tr>
<tr>
<td>Poisson's ratio for concrete</td>
<td>0.20</td>
</tr>
<tr>
<td>Cracking strain</td>
<td>0.0001</td>
</tr>
<tr>
<td>Young's modulus for steel</td>
<td>211000 MPa</td>
</tr>
<tr>
<td>Yield strength for boundary element reinforcement</td>
<td>444 MPa</td>
</tr>
<tr>
<td>Yield strength for web reinforcement</td>
<td>507 MPa</td>
</tr>
<tr>
<td>Strain at seized tension stiffening</td>
<td>0.002</td>
</tr>
<tr>
<td>Strain hardening ratio for steel in terms of elastic modulus</td>
<td>0.01</td>
</tr>
</tbody>
</table>

*Table 5.4 Input properties for Micro model test examples*

The hear retention upon cracking is modelled according to [Rots84]

Three meshes are tested, shown in figure 5.45. Mesh A consists of 28 elements regarded as a minimum mesh, mesh B has 60 elements, and finally, mesh C has 170 elements. Forces were discretised and applied at as follows. The shear force was applied at all nodes of the uppermost elements. The vertical forces (gravity and vertical force couple) were applied at the four nodes of the uppermost corner elements.
The fourth floor horizontal displacement versus base shear is shown in figure 5.46 for all three meshes. It is seen from figure 5.46 that the results of meshes B and C are closer, indicating convergence. For the subsequent plots mesh B will be used, as a compromise between accuracy and numerical effort.
The influence of the concrete tensile strength is shown in figure 5.47, using mesh B. This strength is varied from 2.5 MPa to 3.5 MPa (default value: 2.8 MPa). It is seen that the influence is mainly visible in the pre-yielding region, where the higher tensile strength gives a somewhat higher global strength. It should be noted that the micro model essentially simulates uncracked tensile behaviour, via its skeleton curve, as opposed to the macro model, where cracked flexural behaviour was assumed at the start of the analysis, whereas the shear behaviour was assumed elastic.

The next parameter to be analysed is the influence of the strain hardening ratio for the reinforcement, shown in figure 5.48. The ratio of the yield modulus to the elastic steel modulus is here varied from 0.005, over 0.01 to 0.02. The global effect is essentially proportional to the set parameter, as may be seen from the figure. In the micro model this ratio represents more directly than in the macro model the slope of the yielded branch for the reinforcement only.

Furthermore, the influence of the tension stiffening effect is shown in figure 5.49. The strain at which the tension stiffening stops acting, i.e. where zero tensile stress is reached for the concrete, is varied from 0.001 over 0.002 (approximately at steel yielding) to 0.003. For the low value of 0.001, the typical temporary yielding followed by a stiffness increase may be observed. It is due to the temporary negative modulus in the tension stiffening phase, appearing simultaneously over several integration points, followed by a post-tension stiffening phase with zero modulus, which is stiffer than the negative modulus, compare figure 4.14. The value of 0.002 appears to give a reasonable result.
The next quantity to be analysed is the shear retention. The default shear retention used for this micro model is the expression by [Rots84] based upon experimental work by Paulay and Loeber, see equation (4.16). We will here compare this expression with the expression by Bazant and Gambarova [BG84] which is of a similar nature, see equation (4.17), and with the expression by Pruijssers [Prui85], equation (4.18).
The influence on the global behaviour is shown in figure 5.50. It is seen that the first two expressions are relatively close ([Rots84] and [BG84]), whereas the expression of [Prui85] gives a stiffer global behaviour. This is due to the fact that in this expression small ratios between normal strain and shear strain give a larger retention factor for a given normal strain. Since there is much bending (i.e. normal tensile stress) in the computed example this effect shows up clearly. Further, a maximum aggregate diameter of 19 mm was assumed, which may be slightly too large for the 1:3 scale specimen. Reducing this to 12 mm gave a somewhat lower stiffness, still above the curve for [BG84] however. It may be concluded that the simplest of the expressions, the one by [Rots84] appears to give reasonable results compared to experimental data in this example.

Some attempts with a constant shear retention factor showed that factors of 0.5 and more gave a much too stiff solution, and lower factors gave considerable equilibrium iteration problems due to the sudden drop in shear stress when passing the tensile strength. Therefore, the gradually decreasing nonlinear functions, as the ones shown in the plot, seem to be preferable.

We will now turn to the influence of the Young's modulus of the reinforcement steel, discussed at the end of chapter four. Modifying the Young's modulus allows a better simulation of the additional stresses which must be carried by the reinforcement bars across cracks. As discussed in chapter four the additional stresses in the bars arise from tensile stress carried in the uncracked concrete sections.
It was shown in expressions 4.21 to 4.23 how a possible modification of the steel modulus may be performed. In the previous reliability tests, using the example with the three-storey test specimen, no modification of the steel modulus was made.

In order to study the influence on the global behaviour of such a modification we here perform a modification of the steel moduli relevant to the three-storey specimen, according to the suggestion in chapter four. The necessary material data is given below in Table 5.4.

<table>
<thead>
<tr>
<th>Material property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete tensile strength ( f_t )</td>
<td>2.8 MPa</td>
</tr>
<tr>
<td>Concrete stress between cracks (tension stiffening stress)</td>
<td>( 0.4f_t = 1.2 ) MPa</td>
</tr>
<tr>
<td>Unmodified Young's modulus ( E_s ), for all reinforcing steel</td>
<td>211000 MPa</td>
</tr>
<tr>
<td>Yield strength: boundary elements/web</td>
<td>444 MPa / 507 MPa</td>
</tr>
<tr>
<td>Yield strain: boundary elements/web</td>
<td>0.0021 / 0.0024</td>
</tr>
<tr>
<td>Vertical reinforcement ratio: boundary elements/web</td>
<td>3.50% / 0.73%</td>
</tr>
<tr>
<td>Horizontal reinforcement ratio: boundary elements/web</td>
<td>0.73% / 0.73%</td>
</tr>
</tbody>
</table>

Table 5.4 Material data for check of tension stiffening

Since the horizontal steel, as will be seen later in this chapter, does not reach a stress in the vicinity of the yield strength, we perform here a simplified modification of steel moduli, as follows: In the boundary element horizontal and vertical steel is modified based on the vertical reinforcement ratio (overestimates the modification for the horizontal steel). In the web both vertical and horizontal steel is correctly modified based on the vertical reinforcement ratio of the web, which is the same as the horizontal ratio.

Modification of the steel modulus according to expressions 4.20-23 gives the modified steel moduli, as follows:

**Boundary element**:

\[
\sigma_u = \frac{\epsilon_y}{\rho} = \frac{1.12}{0.035} = 32 \text{ MPa}
\]

\[
E_{s,u} = \frac{f_y + f_u}{\epsilon_y} = \frac{444 + 32}{0.0021} = 226700 \text{ MPa}
\]

**Web**:

\[
\sigma_u = \frac{\epsilon_y}{\rho} = \frac{1.12}{0.074} = 151 \text{ MPa}
\]

\[
E_{s,u} = \frac{f_y + f_u}{\epsilon_y} = \frac{444 + 151}{0.0024} = 274200 \text{ MPa}
\]
Thus we obtained an increase for the Young's modulus for the steel in the order of $\frac{226700}{211000}$ equal to 1.074, i.e. 7.4 percent for the boundary elements, and $\frac{274200}{211000}$ equal to 1.300, i.e. 30 percent for the web.

The global influence of this modification is shown in figure 5.51. It is seen that the effect gives a slight increase in stiffness in the cracking phase and a few percent additional strength. Inspection of figure 5.8 suggests that the influence of the modified steel moduli achieves a result which is a little closer to the experimental data.

It can however, not be stated that this influence is of significant importance in relation to the influence of other parameters of the micro model which were studied previously in this chapter.

Some of the internal behaviour of the micro model is presented here to conclude the monotonic loading study.

The behaviour of the concrete and the steel at the left edge of the base level is shown in figures 5.52 applying to mesh B. The lower outer integration point of the edge element closest to the base was used to obtain the plots. In 5.52a the concrete stress vs. strain, normal to the cracked plane is displayed. In 5.52b the shear stress vs. shear strain across cracks is displayed. In 5.52c and 5.52d the steel stress versus strain is displayed for the
**Numerical Examples**

- **Normal strain**
  - 4.0
  - . . .
  - . . .
  - . . .
  - . . .

- **Shear strain**
  - 0.000 0.002 0.004 0.006 0.008

- **a) Concrete stress vs. strain in direction to crack plane**
- **b) Concrete shear stress vs. strain across cracks**

- **c) Vertical steel stress vs. strain**
- **d) Horizontal steel stress vs. strain**

*Figure 5.52 Internal stress quantities of micro model, left edge at base*

Vertical and horizontal bars, respectively.

A global pattern of the concrete cracking is shown in figure 5.53 for three different shear force levels up until 1.0 MN. In 5.53a the crack pattern from the prototype after completion of test is shown. In the plots 5.53b-d the crack pattern from the micro model is shown using element CPS4. The numerically obtained crack patterns, which are shown...
for mesh B, show reasonable agreement with the experimentally obtained crack pattern in the final phase.

Thereby, the reliability tests are concluded for the micro model, and the attention will be directed directly to the dynamic behaviour of this model, which incorporates the cyclic behaviour.
5.5.2 Multi-storey wall

A limited number of the results from the eight-storey wall shown for the macro model, will here be repeated for the micro model. Since the nonlinear time history analysis using a micro model, simulating multi-storey walls, is very comprehensive regarding modelling as well as numerical effort, only a limited comparison of results is given here. In a separate subsequent section some results will be directly compared between the macro and micro models.

The micro model mesh for the eight-storey wall is shown in figure 5.54, consisting of 68 elements. Gravity loads were distributed to floor nodes, and floor masses were lumped into two edge nodes at every floor. Free vibration data is given in Appendix E.

![Figure 5.54 Micro model mesh for eight-storey wall](image)

**Figure 5.54 Micro model mesh for eight-storey wall**

The edge elements of 0.5 m length correspond to the confined end zones of the walls containing flexural reinforcement. For the restricted ductility design the edge elements over the plastic hinge (four elements vertically) were given 1.2 percent vertical reinforcement, and for the full ductility design 0.5 percent. The artificially generated ground motion compatible to the SIA spectrum only will be used here. A time increment of 0.01 seconds only will be used. Rayleigh damping of 2% (at 0.5 and 5 Hz) was used.

The horizontal roof displacement versus time is displayed in figure 5.55 for both designs. The finding from the macro model results, that the difference in roof displacement between the two designs is not as large as expected, is here confirmed by the micro model.
Many quantities such as cross sectional forces and curvature ductility are not defined for the micro model in a manner comparable to the macro model and will therefore be left out of the discussion. The roof displacements will suffice as a small demonstration of the nonlinear dynamic capabilities of the micro model, and the last section of this chapter is devoted to a brief comparison between the two models.

5.6 Comparison between Macro Model and Micro Model

In this section, some comparisons will be presented between results obtained by the macro and micro models. Although it would be possible to perform a relatively extensive comparison between various result quantities for both models, this presentation will be limited to a few plots displaying the displacement of the eight-storey wall at roof level.

For restricted ductility design, the comparison is shown in figure 5.56. It is seen that there is a reasonable agreement between macro and micro models. During the last few seconds of the time history analysis the macro model exhibits a slightly longer period and somewhat smaller amplitudes.

The corresponding results for the full ductility design are seen in figure 5.57. The same observations as for the restricted ductility design hold true. For the last three seconds, however, the micro model clearly shows larger amplitudes, indicating more pro-
nounced yielding than the macro model. The somewhat thinner shape of the hysteresis loops of the micro model may be responsible for the evident less damping in the post yielding phase. The reason for this originates to the concrete hysteretic behaviour as shown in figure 4.9 combined with the reinforcement hysteretic behaviour of figure 4.14. Of these two the reinforcement is responsible for the major part of the hysteretic damping, which altogether is somewhat insufficient. A further explanation is that the macro model
keeps the yield stiffness set at the beginning, also for deep excursions into the nonlinear range, which limits the amplitudes somewhat more than is the case for the micro model for which no comparable specific yield stiffness may be set.

A finer overall mesh for the micro model was also tested for the eight-storey wall, but did not change the results significantly. The same holds for a finer discretised plastic hinge region only.

The fact that both models display a reasonable agreement in these highly nonlinear problems, except for at the end of the time history for the full ductility design, may be seen as a further proof of the reliability of each model. This is especially valid for problems which do not have experimental data to check against.

Having discussed mainly the numerical aspects of the two models, this chapter will conclude with a small overall comparison of the two models, whereby their respective advantages and drawbacks will be underscored, and the ideal use of each model will be identified.

The macro model was from the beginning of this study designed to use the same parameters as in a design process, that is sectional forces, curvature in the plastic hinge, etc. Thus it may be readily used in the modelling and nonlinear dynamic analysis of global building structures for direct comparison of the mentioned parameters. Although of a relatively simple mechanical nature, it is capable of simulating the global behaviour of structural walls subjected to earthquake action. The typical structural wall behaviour characteristics such as large tensile strains, pronounced shifting of the neutral axis, and shear deformation may all be simulated relatively well by this model, especially compared to other macro models. It is furthermore capable of simulating the typical hysteretic behaviour under load reversals relatively well.

In the present implementation, as described in the Appendix of this report, several walls within one building may be modelled as long as they run in the same direction (essentially two dimensional analysis). Although not described within the framework of this report, a minor test was performed on a modified macro model prepared for three dimensional analysis (still with planar wall action) with satisfactory results. A full three dimensional version is planned, and three dimensional cores such as lift shafts should be possible to model readily.

In the present version the macro model is capable of providing all desired cross sectional quantities, such as section forces and curvature ductility. All these quantities may be extracted for any element along the height of the wall. These cross sectional quantities are very valuable in the confirmation of the dynamic performance in a design
NUMERICAL EXAMPLES

process and for research purposes. These quantities, which are obtained readily, constitute one of the major advantages of the macro model compared to the micro model in the context of comparison of design parameters. Another clear advantage of the macro model is its generally smaller computational effort. Deliberately, no direct CPU time comparisons etc., were performed since the models are essentially too different for that reason. Another important aspect of this matter is the very rapid computer development, which will make this advantage of the macro model smaller and perhaps less important in the future. However, presently a reasonable micro model will typically involve at least twice to three times the number of degrees of freedom, and this fact was still very noticeable during the solution of the nonlinear time history analyses of the multi-storey building.

The micro model, as developed in this report, mainly fulfilled the purpose of a comparison model for the macro model. This comparison should be seen more as an existing tool to work with and to be available for complicated problems. In this report its final use mainly suffices for some nonlinear time history comparisons with the macro model for a case where no experimental data is available. Thus, when each model fulfilled its comparison with experimental static tests satisfactory and showed agreement in dynamic analyses, it appears likely that their respective behaviour also in this case may be quite realistic.

The micro model as presented in this study does not contain new development to the same extent as the macro model, which is in accordance with its primary use as a comparison and reference model. Most of the development steps may in their essence be found in the literature. However, in the present form it constitutes a clear and logical development, and shows how certain phenomena may be simplified into an efficient model. Particular strengths of this model are the possibility to look in detail at local areas of a structural wall, and examine local damage, such as cracking, crack directions, and yielding of reinforcement. These local damage analyses may be performed much more accurately with the micro model than with the macro model. In the present implementation the micro model may be connected with a graphic post-processing software, developed by Mr. Thomas G. Wenk of the Institute of Structural Engineering of the ETH in Zurich in cooperation with the author. Some of the initial capabilities of this graphic software are seen in figures 5.53b-d, currently comprising magnified deformed shape and crack directions at any time increment. The yielding of reinforcement in different directions and a more refined crack indication revealing closure and reopening, are to be implemented. These damage analysis capabilities make the micro model clearly superior to the macro model in certain analysis cases, such as the analysis of damaged existing wall structures.
Another case where the micro model is superior is when the wall geometry is irregular, exhibiting openings for windows and doors, which is quite often the case. The geometric discretization may be performed readily with the micro model, since it uses membrane elements, whereas the macro model may be almost impossible to use.

Some clear drawbacks with the micro model are its relatively complex constitution. It basically consists of a modulus matrix, which is made up of different contributions as described in chapter five. This modulus matrix forms a part of the stiffness matrix formulation, together with a displacement-strain transformation matrix. In addition, a local-global transformation matrix is generally needed for the orthotropic concrete behaviour upon cracking. The major drawbacks due to this complexity lie partly in the code and model development: it is generally difficult for other programmers to continue developing or improve a model due to its complexity. Secondly, these models need a more refined discretization to work well. Thereby more effort is needed to generate the element mesh, and after that clearly more numerical effort is needed during the solution. This is, as already mentioned above, especially noticeable for extensive dynamic time history solutions. With rapidly increasing computing power, and thereby decreasing CPU time, the drawback of the extensive computational effort of the micro model will become less important in the future. More powerful mesh generators and post-processing software will also reduce the drawbacks of the micro model.

As for the cross sectional quantities available for the macro model, these may not be produced readily using the micro model. Some possibilities theoretically exist of creating e.g. base shear and base overturning moment by the summation of the proper reaction forces, and handling by a graphical post-processor. However, the ready availability of all cross sectional quantities at any level of the wall as obtained by the macro model may generally not be available. However, special post processing could theoretically also provide that.

For checking the dynamic performance in a design process involving multi-storey walls, the macro model presently offers the most convenient and efficient tool, which also has still some development potential for three dimensional analysis. The micro model currently represents a more complicated tool, on the other hand suitable for some special analyses, as mentioned above, and possesses considerable development potential in view of the rapidly growing computing power.

By these comparisons, the chapter on numerical examples is concluded. A discussion of a number of design problems within the framework of the capacity design method, in regard to numerical results, will be presented in the subsequent chapter.
CHAPTER SIX
CAPACITY DESIGN CONSIDERATIONS

6.1 Introduction

Among the efforts to achieve effective resistance against seismic actions on reinforced concrete buildings, the capacity design method [PBM90] has become particularly successful. The method is systematic and oriented towards practising engineers. Relatively comprehensive recommendations are provided for the structural detailing.

A major reason for the development of the numerical models described in the previous chapters, is the need for a tool with which one can verify the seismic performance of a structure and relate it to the performance expectations set out at the design phase.

Based on the numerical models presented in the previous chapters, this chapter deals with questions regarding the performance of capacity designed structures. The interest focuses on multi-storey building structures in which the lateral resistance against seismic action is provided by structural walls. These walls should act essentially as cantilevers and preferably the nonlinear behaviour should be mainly of a flexural nature. The capacity design procedure is not described here, but parts of it are briefly introduced as far as relevant in the context of the present studies. For a complete presentation of the capacity design method the reader is referred to [PBM90] or [PP92].

Since the fundamental idea of the capacity designed structural wall is the definition of a plastic hinge zone located at the base of the wall, and the protection of the rest of the wall against yielding, it is of major importance to assess the magnitude of flexural strength which may develop in the plastic hinge. During severe seismic action the moment at this strength, referred to as flexural overstrength moment, is transferred to the remaining elastic part of the wall. In order to safeguard against yielding in this part, it is thus of importance to know the level of possible flexural overstrength of the plastic hinge. The elastic region must be designed in such a way that at a bending moment which acts in the plastic hinge zone, corresponding to the flexural overstrength, it must still not yield. Another important reason for knowing the flexural overstrength is its effect on the magnitude of the shear force in the plastic hinge zone and in the elastic region [PBM90].
For a chosen global displacement ductility level, and a selected wall geometry, it is furthermore necessary to estimate the level of curvature ductility demand which will develop in the plastic hinge zone. This is due to the fact that proper detailing in this zone is based on correctly estimated deformations.

Following the curvature ductility demand a brief investigation is carried out of the energy distribution in wall that develops during a nonlinear time history analysis. The results of this energy balance will help to explain the different dynamic curvature ductility demands compared to static calculations.

Then, we deal with the question of the distribution of flexural strength demand during seismic action, and the proper flexural design. The curtailment of the flexural reinforcing is an important aspect in order to avoid the forming of plastic hinges at other locations than the intended one at the wall base.

Lastly, the dynamic base shear demand is discussed, based on results from nonlinear time history analyses. Comparisons are made to recommendations in the design literature.

The five aspects introduced here: flexural overstrength, curvature ductility demand, energy balance, the distribution of dynamic flexural strength demand, and dynamic shear demand will be treated in the following five sections of this chapter.

6.2 Flexural Overstrength

In the plastic hinge zone of a structural wall, large deformations may develop during seismic action. These deformations may be considerably larger than those at which the design strength is determined, (usually at 0.35% strain at compressive edge). At the tensile edge strain of several percent may develop. Thereby, strength reserves of the material are mobilised, referred to as material overstrength. Consequently, section forces will develop, which are correspondingly larger than the section design strengths.

In the capacity design literature [PBM90], suggestions for sectional overstrengths are presented to be used for structural walls and for beams in frames, based on material overstrength and assumptions regarding the section deformation mainly for an R/C beam. For nonlinear time history analysis which attempts to achieve a realistic simulation of the structural behaviour under seismic action, it is important to calibrate the input parameters. One of the most important parameters is the one which accounts for the behaviour upon yielding and thereby includes the overstrength.
For this reason, we perform here a small numerical study of the overstrength as it may develop for relevant wall cross sections. The numerical procedure designed to compute flexural behaviour of R/C cross sections which was introduced in chapter three in connection with the calibration of the nonlinear flexural behaviour of the macro model is used here. The same material models, including the concrete behaviour by Kent and Park and the idealised bilinear steel behaviour shown in chapter three, will be used. The strain at which the concrete compressive strength is reached, however, is modified from 0.002 (as in equation 3.28) to 0.003 in order to model the concrete used more realistically. The wall cross section from the eight-storey building of chapter five is used as a calculation example. The cross sections for the restricted and for the full displacement ductility design are given again in figure 6.1. and 6.2. The following set of input data was used:

\[
E_c = 33000 \text{ MPa}, \quad f_c' = 19.5 \text{ MPa}, \quad E_s = 210000 \text{ MPa}, \quad f_y = 460 \text{ MPa}, \\
\varepsilon_{50h} = 0.0054 \text{ (confined concrete)}, \quad \varepsilon_{50u} = 0.0034 \text{ (unconfined concrete)} \\
\text{Strain hardening stiffness for steel: } 0.7 \% \text{ of } E_s \\
\text{Strain at concrete compressive strength: } 0.003 \\
\text{Axial force: } 4.15 \text{ MN} \\
\text{Number of fibres: } 106 \\
\text{Equilibrium tolerance: } 0.05 \text{ MN}
\]
Three computations were performed. The first one utilised the design values for strength as stated above, but no strain hardening. The second computation used effective mean values for steel, i.e. the yield strength for the steel was adjusted to $f_y = 550$ MPa, and in addition the strain hardening was taken as $0.7 \% \, E_y$. For the third computation, the concrete compressive strength was additionally adjusted to an effective mean value of $30$ MPa. Thus a gradual move was performed from the typical design calculation values towards a behaviour as realistic as possible. The resulting moment versus curvature relations are shown in figures 6.3 to 6.6.

Figures 6.3 and 6.4 are based on restricted displacement ductility and figures 6.5 and 6.6 are based on full displacement ductility. In figure 6.3 the effective normal force of $4.15$ MN was included in the computation and thus contributes significantly to the internal moment. From figure 6.3 it is seen that an increase of flexural strength from the design values to effective mean values for yield strength is around 10 percent, and effective mean value for the concrete compressive strength gives additionally around seven percent. Thus a total cross sectional flexural overstrength of 17 percent is obtained, where the concrete still accounts for almost half of that amount. Without the normal force, we obtain as expected roughly 20 percent overstrength at small curvatures when using effective mean yield strength (corresponding to the relation between the yield strengths: $550/460$), and a few percent more at higher curvatures due to the strain hardening. The concrete only contributes with an additional one to two percent, indicating that the concrete strength has a smaller effect when the normal force is removed. Figure 6.4 shows further that with zero normal force the moment dropped over 40 % compared to the curves including the normal force of $4.15$ MN.

For the full ductility design figure 6.5 shows the case with normal force. Effective mean yield strength adds around 7 percent, and effective mean concrete strength additionally around eight percent. Thus the concrete accounts for over the half of the total flexural overstrength of 15 percent. Figure 6.6 shows that for zero normal force a similar behaviour as for the restricted ductility design is found.

We obtain here relatively low overstrength values compared with suggestions from e.g. [PBM90] which are intended for use with beams and walls based on calculations of beams. In these suggestions values in the range of 18 to 41 % percent are given for sectional overstrength, based upon material tensile overstrength (for 2% and 4% tensile strain, respectively) and considering the effect of the steel only.
Figure 6.3 Moment vs. curvature relation incl. normal force, restricted ductility design

Figure 6.4 Moment vs. curvature relation with zero normal force, restricted ductility design

It was here shown that mainly due to the effect of the normal force the relative effect of the steel is much smaller, and in addition that the effect of concrete is of the same order as that of the steel. Thus, for walls with axial force the total flexural overstrength is lower than anticipated for beams, and the concrete accounts for a significant part of the overstrength.
The conclusions that may be drawn from the numerical studies on the two wall cross sections can be stated as follows.

For beams and for structural walls with no or small normal force there is usually sufficient concrete area to accommodate the compressive force without excessive softening or crushing of the concrete. This allows for large tensile strains and forces. The large steel strains account for overstrength behaviour as suggested in [PBM90].
However, for walls with considerable normal force, the compressive strain exceeds the point of concrete strength much sooner, and a maximum compressive force has developed and cannot be exceeded any more. The neutral axis has reached its position farthest from the centroid, and will tend to move back towards the centroid. The tensile strains developed are only as large as needed for the steel forces to balance the bending part of the compressive force. These limited steel strains account for a less dominant effect of steel in the total overstrength. The concrete, which is the limiting factor in this case, accounts for a relatively large part of the total overstrength.

There are several aspects where the flexural overstrength plays an important role. In [PBM90] as well as many other references it is emphasised that the shear forces which develop in a structural wall are dependent on the moments which develop. And if larger moments develop (flexural overstrength) this will have the consequence of larger shear forces. Accordingly, the flexural overstrength is considered in the capacity design procedure in the calculation of the demand of shear force. When it was found in the above numerical studies that for walls with high normal force, the flexural overstrength will be smaller than anticipated with simpler hand calculations according [PBM90], one may state that the hand calculation suggestion is a conservative estimate, i.e. on the safe side. It is, however, of importance to obtain an estimate of how conservative the suggestion is, and that may be achieved with use of the above used computer simulation. Perhaps more economical and realistic suggestions may rise from the more exact procedure used above.

The next aspect where the flexural overstrength is important is the flexural design of the wall cross section in the region where the plastic hinge ends and the elastic region starts. The internal moment developed in the upper part of the plastic hinge will be transferred to lowest part of the elastic region. Since this region should remain elastic it is important to have as good an estimate as possible of what moment is transferred. As will be seen later in this chapter, it may be necessary to place additional longitudinal reinforcing bars in this region in order to fulfil this requirement.

6.3 Local and Global Ductility Demand

In order to adequately detail a plastic hinge zone it is necessary to assess the amount of deformation this zone may undergo. The curvature ductility demand in the plastic hinge zone of a structural wall is particularly important in two aspects. Firstly, this measure gives an estimate of the deformations for which the reinforcement detailing must be performed. Secondly, it will influence the amount of shear force the wall has to withstand.
Both experimental and analytical investigations [PBM90] have been performed in order to suggest the curvature ductility demand for a structural wall. The experimental investigations comprised static model tests on wall members and have shown that for increasing wall slenderness, the curvature ductility demand will increase. Analytical studies of a cantilever with elastic behaviour except at the hinge, and subjected to a static point load at the free end give similar results, see figure 6.7. For each chosen global displacement ductility level a shaded area is given rather than a line. The shaded area covers different assumptions concerning the length of the plastic hinge. The upper edge of the shaded areas represents yielding assumed over a hinge length taken as half the horizontal wall length, whereas the lower edge assumes yielding over a length taken equal to the horizontal wall length. It should also be mentioned that in the analytical study in [PBM90] it was assumed that uniform yielding takes place over this assumed plastic hinge area, and totally elastic behaviour is assumed over the rest of the wall.

It is clear that the static analysis considers the dynamic behaviour of the first mode only, and the effects of higher modes are disregarded. With the numerical model of chapter three a dynamic analysis series is performed in order to investigate the dynamic curvature ductility demand.
Curvature Ductility Demand

Figure 6.8 Dynamic curvature ductility demand, as function of wall aspect ratio, and global displacement ductility

The same eight-storey structure as used in chapter five will here be used as reference. The aspect ratio of the reference wall is 32m / 6m = 5.33. For a number of further walls, the aspect ratio was adjusted by changing the wall length. The reinforcement of the comparison walls was adjusted to give proper flexural strength. Changed fundamental frequency of vibration was considered for all comparison walls giving different demands from the static equivalent force calculation. For the more slender walls minimum reinforcement requirements was mostly governing.

The plastic hinge was discretised into three macro elements, with a total length of 6m (taken from the reference wall), and this arrangement was kept for mesh consistency for all tested walls, although strictly not entirely correct for the two most slender walls. However, this measure is rather to be regarded as a "construction" length with detailing allowing for major yielding, whereas the extension of the effective yielding will be determined within this area by the numerical analysis. A clear difference between the numerical analyses and the analytical analyses of figure 6.7 is thus that for the analytical analyses a clear division was made concerning the yielding. It was assumed uniform and only within the plastic hinge length. The rest of the wall was assumed to be entirely elastic. In the numerical analyses, however, yielding proceeded along the plastic hinge height, and possibly above it to the extent that was determined during the nonlinear time history analyses. No particular attempts were made to suppress yielding right above the
plastic hinge, other than that the flexural design was entirely performed according to the recommendations in [PBM90] with curtailment of the flexural bars along the height of the wall according to the recommended linearly decreasing line discussed in section 5.4.2 of [PBM90] (shown also in figure 6.13 in this report) according to which no yielding should be allowed to take place.

The 10 second SIA compatible ground motion input as discussed in chapter five was used, and nonlinear time history analyses were performed under the same premises as discussed in chapter five.

Figure 6.8 shows the dynamic curvature ductility demand obtained from the numerical comparisons. The values are taken from the element closest to the wall base. Yield stiffness ratios \( \alpha_y \) for the outer flexural spring of the macro elements, as discussed in section 3.5 were taken as 0.01 and 0.03. The higher yield stiffness gives lower curvature ductility demand, as was shown in chapter five, section 5.4.2. In figure 6.8, each of the two chosen design ductility levels is thus represented by a shaded area where, for each aspect ratio, the upper obtained curvature ductility demand arises from the lower yield stiffness ratio.

In the figure, the first natural frequency of the calculated walls is also displayed. A tendency to lower curvature ductility demand for higher aspect ratio is seen from the figure. The tendency appears to be more pronounced for higher design ductility level and at higher aspect ratios. This difference compared to figure 6.7 is mainly explained from the fact that the results in figure 6.8 are extracted from nonlinear time history analyses, where direct time integration was employed, thus containing all higher modes. For more flexible walls, i.e. with high aspect ratios, more energy is dissipated over the upper storeys where major cracking and some yielding takes place, and the relative concentration of rotation at the plastic hinge is decreased. Secondly, only the lower third of the plastic hinge is regarded, giving higher curvature ductility values.

Normal forces due to gravity load for eight floors giving a total of 4.15 MN on the wall cross section at the ground floor were employed throughout the analyses shown in figure 6.8. Since the results of static calculations in figure 6.7 apparently did not consider the effect of normal force, the same numerical examples of figure 6.8 were repeated with smaller gravity loads, down to zero load, without changing the results significantly.

It should further be noted that the shown global displacement ductility level in figure 6.8 only pertains to the design phase, i.e. it is not measured during the time history analysis, and cannot be defined easily, as discussed in section 5.4.2. Briefly stated, the reason is that, due to influence of higher modes, the roof level may not be displaced in the
Figure 6.9 Global displacement behaviour for monotonic static loading, used to obtain a displacement ductility from time history analysis

The same direction as the curvature at the wall base. This gives possibilities for positive or negative global displacement ductility if based upon roof displacement at the instance when yielding begins at the wall base. Essentially, infinite global ductility values may result in this manner.

One possibility of obtaining a reasonable global displacement ductility based upon results from the time history analysis, is described as follows. The maximum roof level displacement obtained from the time history analysis is extracted. Then, a monotonic static analysis is performed on the same wall, using an inverted triangular equivalent static force pattern, applied incrementally. The increments are added until statically the same roof level displacement is reached as was obtained for the time history analysis. In the static analysis the onset of yielding at the wall base is kept as a reference, which is then set in relation to the maximum roof level displacement. The principle is illustrated in figure 6.9.

The resulting global displacement ductility, based on the maximum roof level displacement from the time history analysis will here be referred to as the dynamic displacement ductility, \( \mu_{\Delta \phi m} \). It will thus be obtained as

\[
\mu_{\Delta \phi m} = \frac{\delta_{\text{max}}}{\delta_y}
\]  

(6.1)
The assumed global displacement ductility values at the design phase, will generally not be reached exactly by this method. For the eight-storey wall the global ductilities obtained using the above method were found to be:

For assumed restricted ductility ($\mu_\Delta = 3$): $\mu_\Delta^{dyn} = 2.9$

For assumed full ductility ($\mu_\Delta = 5$): $\mu_\Delta^{dyn} = 5.3$

In this example the correspondence between the ductility values obtained by this method to the chosen global displacement ductilities is thus relatively good. A number of other methods of determining global displacement ductility from time history analyses are discussed in [Wenk93].

6.4 Energy Dissipation

In section 6.3 a comparison between walls of different aspect ratios was made regarding the curvature ductility demand. It was found that for high aspect ratios, i.e. for flexible walls, the curvature ductility demand of the plastic hinge zone turned out to be smaller than expected from static analysis of elasto-plastic cantilevers. This fact is explained by the assumption that more energy is dissipated over the upper storeys for flexible walls.

We study here the energy dissipation during time history analysis of two of the previously used walls, in order to get an answer to these assumptions. Thereby some energy terms are introduced as follows.

There are two types of strain energy. The first is the elastic strain energy $E_{se}$, (recoverable strain) which for an element may be written

$$E_{se} = \frac{1}{2} u^T k_e u \quad (6.2)$$

where the $u$ is the element displacement vector, and $k_e$ is the elastic element stiffness matrix.

The second strain energy type is the inelastic strain energy $E_{sia}$ (irrecoverable strain) also known as the hysteretic energy, which may be written

$$E_{sia} = \frac{1}{2} u^T k_{iu} u \quad (6.3)$$

where $k_{iu}$ is the inelastic element stiffness matrix.
Two more energy types are defined as the viscous energy, $E_v$, which may be obtained by integrating the viscous effect $P_v$,

$$P_v = \frac{1}{2} \dot{u}^T c \dot{u}$$  \hspace{1cm} (6.4)$$

$$E_v = \int P_v dt$$  \hspace{1cm} (6.5)$$

where $\dot{u}$ is the velocity vector and $c$ is the damping matrix, and lastly the kinetic energy $E_k$, defined as

$$E_k = \frac{1}{2} \dot{u}^T m \dot{u}$$  \hspace{1cm} (6.6)$$

where $m$ is the mass matrix. The total energy $E_t$ transmitted to a structure by a ground motion, may then be defined as

$$E_t = E_{tot} + E_{pl} + E_v + E_k$$  \hspace{1cm} (6.7)$$

We are here especially interested in dissipated energy, and of the above energy expressions, two contribute to the energy dissipation, namely the inelastic energy, and the viscous energy, together accounting for the dissipated energy $E_d$
We mainly focus here on how the dissipated energy is divided between the plastic hinge area and the rest of the wall, referred to as the elastic region. For this purpose the eight-storey wall of chapter five was selected, with an aspect ratio of 32.0/6.0 = 5.33. For comparison purposes, the flexible wall with aspect ratio of 32.0/4.0 = 8.0, which was used in section 6.3, figure 6.8, is chosen. Both selected walls were designed for a restricted global displacement ductility (\( \mu_\Delta = 3 \)).

In order to limit the amount of output data, and for clarity, we display here only the most relevant data. The total dissipated energy, consisting of the contributions from all inelastic strain as well as the slight viscous Rayleigh damping of around two percent, is displayed as accumulated dissipated energy versus time. The contribution of the plastic hinge elements will be displayed separately so that its relation to the total dissipated energy may be estimated.

Figure 6.10 shows the dissipated energy for the wall with aspect ratio 5.33. The total energy transmitted to the structure is shown by the solid line. The difference between the total energy and the total dissipated energy is made up of elastic strain energy and kinetic energy. Figure 6.11 shows the same quantities for the wall with aspect ratio equal to 8.0.

\[
E_d = E_{\text{el}} + E_v
\]  

(6.8)
It is seen that for the slender wall the amount of total dissipated energy is increased, and this in absolute terms and in relation to the total energy. This may be partly explained by lesser elastic behaviour overall, and by lower first periods.

Secondly, one may observe that the plastic hinge accounts for clearly less of the total dissipated energy in the flexible wall. The relative increase in dissipation of the upper storeys is related to more flexural cracking, and partial yielding.

Based on the stiffer wall, the decrease of the contribution of the plastic hinge to the total dissipated energy may be estimated from figures 6.10 and 6.11 as from 35% to around 24%, which is a relative decrease of roughly a third.

It was possible to track the increased nonlinear behaviour of the upper storeys by means of the macro model. Some results in this regard were already presented in chapter five. However, in the subsequent section of this chapter, this behaviour will be dealt with more in detail. Suggestions on how to encounter the problems associated with outspoken nonlinear behaviour of the upper storeys will then also be discussed.

6.5 Flexural Strength

6.5.1 Implication of numerical results

Once the plastic hinge zone is detailed, the question of the flexural strength for the rest of the structural wall rises. Based upon assumptions of an equivalent static force, a suggestion has been made [PBM90] concerning the distribution of flexural strength from the plastic hinge zone to the top of the wall. This suggestion is visualised in figure 6.12. The essential features of this suggestion are as follows:

The flexural strength is kept constant for the entire plastic hinge zone, stretching a length \( L_p \) upwards from the base of the wall. Values for \( L_p \) are usually taken as the horizontal length of the wall \( L_w \), or a fraction thereof. Above this zone, a linear decrease of the flexural strength is suggested stretching from the upper end of the plastic hinge zone to the top of the wall, or until a flexural strength is reached which corresponds to minimum reinforcement requirements.

It is further assumed that the distribution of the effective bending moment acting over the wall height has a shape similar to that of a cantilever subjected to a lateral static force with inverted triangular distribution.
With the ability to perform nonlinear time history analysis, using the wall elements developed in chapter three, we now examine the effective moment distribution acting on a wall subjected to ground motion. The bending moment distribution is shown here for the wall used in chapter five. This wall may be regarded as relatively stiff. In figure 6.13 the distribution for this wall is shown to the left, and in addition the distribution for a more slender wall of the same height shown to the right. The fundamental natural frequency without any nonstructural elements or frame stiffening is 0.67 Hz for the stiff wall and 0.40 Hz for the flexible wall. Both walls were designed for restricted ductility. The aspect ratios are 5.33 and 8.0 respectively. The most relevant flexural design quantities are summarized for both walls as follows.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$r_a = 5.33$</th>
<th>$r_a = 8.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental frequency (Abaqus)</td>
<td>0.67 Hz</td>
<td>0.40 Hz</td>
</tr>
<tr>
<td>Moment at wall base from equivalent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Static force calculation $M_E$</td>
<td>16.1 MNm</td>
<td>8.9 MNm</td>
</tr>
<tr>
<td>Moment demand $M_I = \gamma_R M_E$</td>
<td>19.3 MNm</td>
<td>10.6 MNm</td>
</tr>
<tr>
<td>Design strength $M_R$</td>
<td>19.4 MNm</td>
<td>11.1 MNm</td>
</tr>
</tbody>
</table>

*Figure 6.12 Distribution of flexural strength, proposed by [PBM90]*
Figure 6.13 Effective moment distributions extracted from nonlinear time history analysis for eight-storey wall [BW.L92b] with aspect ratio of 5.33 and $f_1=0.67\ Hz$ (left), and for a more slender eight-storey wall with aspect ratio of 8.0 and $f_1=0.40\ Hz$ (right).

It may be clearly seen that for the more flexible wall (moment distribution shown to the right), the higher modes have more influence, i.e. higher moments are obtained at mid height of the structure. It should again be pointed out that the moments at a particular storey are extracted as the maximum moment reached at that storey during the time history analysis, and thus the values for different storeys are generally not obtained at the same time.

In order to further explain the tendency shown in figure 6.13 we examine eigenfrequencies of the two walls, seen in the response spectrum of the ground motion input, displayed in figure 6.14.

From figure 6.14 it is seen that the first mode of the flexible wall has a small spectral value compared to the stiff wall. Relatively seen, the higher modes will therefore be important for the behaviour of the flexible wall. As the nonlinearities occur during the time history analysis, all the eigenfrequencies will generally move towards the left in figure 6.14, i.e. the system becomes more flexible when damage occurs.
It is clear that the shape of the ground motion response spectrum plays an important role, and thus the relation between the structures' eigenfrequencies, and the shape of the input spectrum will determine to what extent the higher modes will be important or not. As a consequence, it is possible to state that the relation between the structural frequencies and the shape of the input spectrum will determine the distribution of bending moment over the height of the structure. The principle may be illustrated as in figure 6.15.

It is thus clear that the flexural strength distribution should be dependent on the above relationship. It is, however, difficult to obtain a general relationship including ground motion, structural eigenfrequencies, and moment distribution.

Figure 6.15 Influence on moment distribution of relation between wall eigenfrequencies and ground motion
Therefore, it may be reasonable to establish strength distribution criteria for a given ground motion, e.g. the design ground motion according to the SIA. A suggestion, based upon the above findings, for a solution to the flexural strength distribution is described in the following.

6.5.2 Suggested flexural strength design

A suggestion will here be presented as how to avoid flexural yielding in the elastic region, and thereby fulfil one of the basic goals of the capacity design. The suggestion is based on results obtained by numerical analyses with the macro model developed in this report. These results indicated that if the flexural strength was reduced over the height of a multi-storey structural wall, yielding would generally occur over the region intended to remain elastic. We will here base the design suggestion on the assumption that the overstrength moment at the lowermost end of the plastic hinge (i.e. at the wall base) may develop as well at the uppermost end of the plastic hinge, and transfer to the immediately bordering elastic region.

This simplification is based on the findings from the numerical modelling in this report, which show that although the curvature drops over the height of the plastic hinge, the moment only decreases slightly. This is due to the nonlinear moment curvature relationship, discussed earlier in this chapter.

Qualitatively, the distribution of curvature and moment over the height of the plastic hinge of a multi-storey wall, discretised into three macro elements, may be illustrated as shown in figure 6.16. In figure 6.16a, a distribution of curvature along the plastic hinge height is shown, and in figure 6.16b the moment distribution. The corresponding moment curvature relation is given in figure 6.16c. The numerical results from the macro model studies indicated that some yielding always occurred over the uppermost element during time history analysis.

This has the consequence that although the curvature of the upper end of the plastic hinge may only be about half or a third of the one at the wall base, the moment is only a little smaller. This effect is illustrated clearly in figure 6.16c. Since the difference between the moment at the wall base $M_1$ and the moment of the uppermost element of the plastic hinge $M_3$ is rather small it appears reasonable to neglect this difference for design purposes, especially since it is very difficult to estimate without extensive nonlinear dynamic analysis. The assumption that $M_1$ should be transferable to the elastic region is thus only slightly conservative.
Figure 6.16 Qualitative moment-curvature distribution over plastic hinge for multi-storey structural wall, discretised into three macro elements

Figure 6.17 shows a suggestion on how to distribute flexural strength in order to avoid undesired yielding. The strength over the plastic hinge height is obtained according to known capacity design principles [PBM90], and is denoted with $R_p$ in the figure.

Above the plastic hinge zone, the wall must behave elastically, i.e. no major yielding is permitted. Directly above the plastic hinge zone the flexural overstrength which may develop in this zone is transferred to the elastic region.

The elastic region constitutes the rest of the wall and has the length $L_e$ in the figure. The strength in the elastic region bordering to the plastic hinge zone is denoted $R_e$ in the figure. This strength must be such that it can take the overstrength from the plastic hinge.
zone without yielding, i.e. its tensile strain must not exceed about 0.0025. This means that there must be an increase of flexural reinforcement right above the plastic hinge zone.

A general way to state the required constant strength of the elastic region denoted $R_e$ in figure 6.17 is in terms of the available strength in the plastic hinge denoted $R_p$ in figure 6.17. We thus obtain

$$R_e = \lambda_e R_p$$  \hspace{1cm} (6.9)

where $\lambda_e$ is the overstrength factor for reinforcement steel which is usually taken as 1.2.

Depending on what the effective bending moment distribution looks like, this strength must be kept constant over a distance $L_{ec}$, seen in the figure. According to the findings earlier in this chapter, this length is dependent on the eigenfrequencies of the wall in relation to the spectrum of the ground motion. For a given ground motion spectrum, we may thus be able generally to find that the length $L_{ec}$ may be short for a stiff wall, but must be longer the more flexible the wall becomes. In figure 6.17 the strength distribution for a typical stiff wall is shown to the left, and that for a typical flexible wall to the right.

Above the height $L_{ec}$, a linear decrease of flexural strength is suggested, by curtailing the vertical bars. If a minimum strength due to minimum reinforcement requirements from the code is met, this minimum strength must be kept constant to the top of the wall.

It is necessary to establish an estimate of the length of constant flexural strength. As we have already seen, this length will be a fraction $\alpha_c$ of the total elastic length, i.e.

$$\alpha_c = L_{ec}/L_e.$$

\hspace{1cm} (6.10)
Theoretically, this fraction will vary between zero and one. However, these extreme values will seldom be reached in practice. Assuming a ground motion input compatible with the SIA design spectrum, it should be possible to establish a relation between the length of constant flexural strength and the eigenfrequencies (now being related to SIA spectrum). It would be desirable to establish this relation in such a manner that the modes immediately higher than the fundamental mode will govern the result in addition to the first mode. However, since this procedure would be quite involved for design purposes, and since our goal is to establish a design guideline which may be readily usable we will attempt to use only the fundamental mode and calibrate our relation so that it agrees with the results from nonlinear time history analysis.

Thus the desired relation would have as an input the fundamental eigenfrequency of the wall, and as output the fraction of the elastic wall length which has to be designed for a constant flexural strength (corresponding to the overstrength in the plastic hinge zone). The effective moment distribution over the height of the wall in figure 6.13 implies that the fraction of constant strength be proportional to the inverse of the fundamental frequency, i.e. proportional to the fundamental period $T_1$. We thus have $\alpha_c$ proportional to $T_1$. We here suggest $\alpha_c$ be taken as $0.2 T_1$, i.e.

$$\alpha_c = 0.2T_1$$ \hspace{1cm} (6.11)

$$\alpha_c = 0.2 \frac{1}{f_1}$$ \hspace{1cm} (6.12)

It is clear that this simple suggestion cannot be regarded as generally valid for frequency regions which have not been tested in this study. For very flexible walls the suggestion would give too high ratios. For such cases some more refined method should be used, e.g. a direct inspection of the effective moment demand.

6.5.3 Numerical example

The flexural design suggested above will here be illustrated by a numerical example which is also discussed in [BL93] involving two walls with different fundamental eigenfrequencies due to different aspect ratios. The two eight-storey walls of figure 6.13 with aspect ratios of 5.33 and 8.0 respectively will be used. Both walls were designed for a global displacement ductility factor $\mu_A$ equal to three.

Using expression (6.12) for the two walls of figure 6.13 we obtain
Wall with aspect ratio 5.33: $\alpha_c = 0.2 \times 1/0.67 = 0.3$

Wall with aspect ratio 8.00: $\alpha_c = 0.2 \times 1/0.40 = 0.5$

Placing these strengths over the effective moment distributions gives a reasonable coverage of the demand, as illustrated in figure 6.18 for the stiff wall with aspect ratio $r_a = 5.33$. For the demand the maxima of negative and positive effective moments are taken from figure 6.13.

The curtailment is here just schematically performed to the top of the wall. A simple curtailment possibility is to evaluate the strength at the roof level (i.e. with no normal force) with nominal minimum reinforcement ratio, and curtail the flexural reinforcement linearly from the region of constant elastic strength towards the roof level until intersecting the level from which the nominal minimum reinforcing is used.

The demand from the nonlinear time history analysis of the same wall is shown in figure 6.19 together with the recommended demand from the capacity design method [PBM90] which was shown in figure 6.12. It is seen that above the plastic hinge zone, the demand from the nonlinear time history analysis exceeds the demand recommended in [PBM90] at two locations, where plastifications may occur. The numerical results confirmed this by indicating limited but clear yielding.

For the more flexible wall with the aspect ratio $r_a = 8.0$ and the fundamental eigen-frequency $f_1 = 0.40$ Hz, the outcome of the design approach suggested here is shown in figure 6.20. The plastic hinge length $L_p$ was taken as 5.3 m (from entire height $H$ divided by six) which is governing since it is larger than the wall length $L_y = 4.0$ m.

The necessity of the suggested flexural strength distribution becomes obvious in figure 6.21. If the earlier approach suggested in [PBM90] with a linear strength decrease directly above the plastic hinge zone would have been used here, one or several plastic hinges would certainly have developed over the mid and upper storeys, which would violate the fundamental ideas of the capacity design method. The demand from the nonlinear time history analysis is compared to the recommended demand according to the existing capacity design method [PBM90] in figure 6.21. In this figure, the shaded areas represent moment demand which is not covered if the recommendations of [PBM90] are followed. In these areas stretching over no less than four storeys, more or less uncontrolled plastification will take place, which could be verified by numerical analyses.

Two basic difficulties are thus identified with the earlier approach of [PBM90]. The first is the fact that the flexural overstrength may develop over large parts of the plastic hinge and may thus transfer a moment of that size to the region intended to remain elastic. This means that it is necessary to provide for an increased strength at the beginning of the
Design strength with nominal minimum reinforcement and zero normal force

Proposed design strength in elastic region

Overstrength $M_o$

Design strength $M_R \geq \gamma R M_E$

Demand from nonlinear time history analysis

Moment from equivalent static force calculation $M_E$

Demand from capacity design rules according to [PBM90], [PP92]

Demand from nonlinear time history analysis taking into account steel strain hardening

Moment from equivalent static force calculation $M_E$

Design strength $M_R \geq \gamma R M_E$

$L_p = 6.0\, \text{m}$

$M$ (MNm)

$f_1 = 0.67\, \text{Hz}$

Figure 6.18 Suggested strength distribution for eight-storey wall, aspect ratio $r_a = 5.33$

Figure 6.19 Comparison of moment demand from nonlinear time history analysis to demand recommended in the capacity design method [PBM90], aspect ratio $r_a = 5.33$
Figure 6.20 Suggested strength distribution for eight-storey wall, aspect ratio $r_a = 8.0$

Figure 6.21 Comparison of moment demand from nonlinear time history analysis to demand recommended in the capacity design method [PBM90], aspect ratio $r_a = 8.0$
elastic region. The increase must be such that the yield moment of that cross section corresponds to the overstrength moment of the plastic hinge. This is a phenomenon which essentially has to be separated from the behaviour of upper storeys, and it is not accounted for in the earlier approach.

The next difficulty pertains to the tacit assumption that the moment demand will follow a curve similar to the one shown in figure 6.12. It is mentioned that higher modes may alter this curve, but apparently the assumption is that the influence of these will never be larger than a straight line with a linear decrease following the design curve of the same figure. As we have seen, the higher modes may influence the total moment curve for flexible walls so much that a linear decrease of strength directly above the plastic hinge is not enough.

The aspect ratio of the wall does not enter into the discussion on what the flexural demand distribution will look like. This pertains to the Swiss code [SIA160] as well as to the capacity design method [PBM90]. Due to this fact it may be worth reflecting over the fact that the expected demand as seen in figure 6.12 is representative for typical structural walls as they are designed in countries with severe seismicity such as New Zealand; considerably stiffer, and for a given number of storeys, with smaller aspect ratios than would be typical for Europe. The demand in figure 6.12 will more typically reflect a dominant first mode behaviour as expected from stiff walls. The conclusion would be that the demand suggested in figure 6.12 may be adequate for New Zealand practice, but may not generally be projected unchanged to typical European practice.

The impact of the above results on the design of the flexural reinforcement is discussed here briefly for the wall with aspect ratio of 5.33. The flexural strength in the plastic hinge region was computed at a strain at the compressive edge equal to 0.0035 and amounts to 19.4 MNm. This value was computed using the ordinary assumption of design strength of \( f_y = 460 \text{ MPa} \) and no strain hardening. Using the overstrength factor \( \lambda_o \) we may obtain the proposed necessary flexural strength for the elastic region above the plastic hinge as

\[
R_e = \lambda_o R_p = 1.2(19.4) = 23.3 \text{ MNm}
\]

According to the above it is required that the wall in the elastic region reaches a flexural strength of 23.3 MNm. This can be achieved by adding a number of flexural bars D20 at the confined zones at the ends. Adding four D20 at each end as shown in figure 6.23 gives an elastic strength of about 22.5 MNm. By comparison, the cross section of the plastic hinge zone is shown in figure 6.22. The 22.5 MNm are not quite enough to guarantee that no yielding will occur since the overstrength which may be transferred was
calculated to be 23.3 MNm. However, this difference of around three percent may be tolerable.

The cross section strengthened with four additional bars D20 at each end will in this example be necessary to safeguard against uncontrolled development of plastic hinges in the mid and upper storeys. It is proposed that the additional flexural reinforcement bars be arranged in a U-shaped loop in order to practically allow for enough bond. Figure 24 shows the transition region between the plastic hinge and the strengthened elastic region in elevation (left) and the vertical section A-A (right) which is indicated in plan in figure 23. Each added bar may be continued upwards on the other side of the U-turn, making up the opposite bar, or may be spliced at the other side of the turn, using the splice length depending on bar diameter prescribed in the code. As seen in the vertical section of figure 24 (right) it is important to place horizontal bars (no less than bar D10) at the inner corners of the U-shape.

It should noted that by the arrangement of the reinforcement proposed here, a better opportunity to determine the geometrical extension of the plastic hinge zone is given compared to previous more vague assumptions such as e.g. a sixth of the total height of the wall. The extension may now e.g. be chosen to be exactly equal to the height of the first storey, which may be advantageous of a construction point of view.
It is difficult to suggest a good rule of thumb for the increased flexural strength to be used generally, based on the relatively few computations of this report. Since we observe that the bending moment at the upper end of the plastic hinge generally appears to be slightly smaller than at the base, the proposed value of 20% is probably somewhat on the conservative side. However, since we do not know the impact of e.g. different aspect ratios in this respect, a value of around 20% still appears justifiable at this time.

Findings on the need for a slightly increased flexural strength right above the plastic hinge were also made by Haas [Haas93] in his study on a four-storey capacity designed wall with an aspect ratio of 2.7 and a fundamental frequency of about 2.0 Hz, using among others the model developed in chapter three of this report.

It is clear that although the suggested design approach worked well for the two examples shown here, it is not shown how this approach would work generally for other examples. It would be desirable with an extensive parametric study partly in order to obtain the effective moment distribution and partly in order to calibrate design parameters.
6.6 Shear Behaviour

The dynamic shear forces developed at the base of a multi-storey structural wall during severe seismic action display the following characteristics:

1) The sign of the cross sectional shear force changes more rapidly than the sign of the bending moment.

2) The magnitude of the cross sectional shear forces may be considerably larger than the shear force obtained only from the equivalent static force method.

Both these phenomena have been verified in several experimental studies, [ES89], [SE92]. The physical explanation for the first characteristic which also has to do with the second one, may be attributed to the several higher modes which are contributing simultaneously to the deformations over the height of the wall, which result in a shear force brought down to the base which changes its sign more rapidly.

This is due to the fact that the few lowest modes have the bulk of their inertia forces concentrated in the upper storeys and thereby relatively long lever arms to the base of the wall. The moments caused by these modes will take periods belonging to these modes when plotted versus time. However, the higher modes have the bulk of their inertia forces located at mid and lower storeys giving relatively small moment contributions at the base but considerable shear contributions.

The effect may be illustrated by the eight-storey wall used in chapter five and previously in this chapter by transforming the overturning moment at the wall base from the time history analysis (figure 5.20) into the frequency domain. Figure 6.25 shows a Fourier spectrum of the overturning moment at the wall base, for the SIA ground motion input, and restricted ductility design. It is seen that the first mode (visible at around 0.5 Hz obtained from the entire 12 s, i.e. including the yielding phase) dominates. Figure 6.26 shows a Fourier spectrum of the base shear of the same wall, displaying a considerable contribution from mode two (at 4 Hz), mode 3 (around 9 Hz), and even something from mode 4 (around 15 Hz).

The second characteristic is partly due to the developed flexural overstrength at the base of the wall, and partly to the effect of low centre of gravity of the horizontal inertia forces of higher modes just discussed, giving a resulting centre of inertia forces which may be located considerably lower than anticipated by the inverted triangular force distribution of the equivalent static force method.
The focus will here be directed on the shear behaviour of the eight-storey wall presented in chapter five, and already used in the tests in that chapter and to some degree for the previous discussion on flexural behaviour on this chapter.
The base shear obtained during the nonlinear time history analysis as the cross-sectional shear force of the macro element in the plastic hinge placed closest to the base, was displayed versus time in figures 5.37 and 5.38. For the restricted as well as for the full ductility design, the maximum base shear occurs at around 3.8 seconds, and the magnitude amounts to about 2.3 MN and 2.1 MN for the restricted and full ductility designs, respectively.

We will discuss here the numerical result in relation to the design criteria set for shear force by the capacity design method. The higher dynamic shear forces are recognised in the formula for design shear in [PBM90] according to the following

\[ V_w = \omega_v \Phi_{o,v} V_E \]  \hspace{1cm} (6.13)

where \( V_w \) is the design shear force, \( \omega_v \) is the dynamic amplification factor, \( \Phi_{o,v} \) is the flexural overstrength factor, and \( V_E \) is the shear force from the equivalent static force calculation. The formula (6.13) originally arises from recommendations in the New Zealand Standard NZS 3101, see [NZS3101].

The flexural overstrength was treated earlier in this chapter, and for the dynamic amplification factor, [NZS3101] suggests the following expressions

**Buildings with up until six storeys:**

\[ \omega_v = 0.9 + \frac{n}{10} \]  \hspace{1cm} (6.14)

**Building with more than six storeys:**

\[ \omega_v = 1.3 + \frac{n}{30} \leq 1.8 \]  \hspace{1cm} (6.15)

In our example, formula (6.15) would apply with \( n = 8 \), giving \( \omega_v = 1.57 \). Starting with the restricted ductility design, the shear force \( V_E \) from the inverted triangular force distribution of the equivalent static force calculation in our example was obtained as 0.74 MN. The flexural overstrength developing in walls was in section 6.2 determined to be in the range of 15 to 17 percent, depending on chosen displacement ductility.

An attempt to confirm the quantities of the formula (6.13) shows that we know the left hand side from the time history analysis, and on the right hand side we know \( V_E \) and we have estimates of the overstrength reached according to the numerical model. In this manner, the dynamic magnification factor may be determined and compared to the proposals (6.14) and (6.15).

At first we may establish a total shear magnification factor \( m_s \), defined as the product of the flexural overstrength factor and the dynamic amplification factor
\[ m_s = \omega_s \Phi_{o,w} \]  

(6.16)

For our example we obtain \( m_s \) equal to \( 2.3/0.74 = 3.11 \). Values in this range may be found in several of the examples presented in [SE92]. Inspection of the actually reached overstrength moment during the time history analysis (around 22.5 MNm, from figure 5.19) gives a numerically obtained cross sectional flexural overstrength factor with respect to available strength as \( 22.5/19.4 = 1.16 \), showing good agreement with the estimations made in this chapter. Here, however, we would rather use the overstrength factor with respect to the equivalent static moment of 16.1 Nm, which gives \( \Phi_{o,w} = 22.5/16.1 = 1.40 \).

Using expression (6.16) we may now solve for \( \omega_s \) and obtain \( \omega_s = 3.11/1.40 = 2.22 \). This value is clearly higher than the limiter 1.8 for a large number of storeys given by expression (6.15), and in the first place it is much higher than the obtained value of 1.57 of the same expression using the correct number of storeys equal to eight. The difference is of the magnitude \( 2.22/1.57 = 1.41 \) i.e. 41 percent.

Repeating the procedure for the full ductility design we obtain \( V_E \) as \( 0.43 \) MN, which gives \( m_s \) equal to \( 2.1/0.43 = 4.88 \). From figure 5.20 the maximum bending moment reached for full ductility is about 19.0 MNm, which gives the flexural overstrength with respect to the equivalent static moment as \( 19.0/9.7 = 1.96 \). Thus \( \omega_s \) may be solved as \( 4.88/1.96 = 2.49 \). By comparison with the suggested magnification this value is \( 2.49/1.57 = 1.58 \), i.e. 58 percent higher.

The actual shear capacity of around 2.7 MN for both designs is only due to minimum reinforcing requirements of the code (which are also applied in the capacity design method), and would for example not have been reached if the wall cross section had a smaller area. One could actually conclude that in the present examples, the minimum reinforcement requirement is more conservative than the absolute shear capacity requirement of expression (6.13).

Although only two numerical examples of the shear behaviour were presented here, the results indicate, together with several experimental studies carried out, that the shear forces obtained at the wall base during nonlinear dynamic behaviour reaches large values which exceed even those anticipated using conservative design formulae.

Keintzel [Kein88a], [Kein88b] performed elaborate numerical studies on the dynamic shear force demand for multi-storey walls, using a beam element model (as described in section 2.2.1 in this report) to simulate the wall. During these studies, it was found that the higher modes contributed considerably to the increased shear forces.
In contrast to the flexural design, it will not be attempted here to derive an improved hand calculation capacity design procedure for shear. Further results from nonlinear time history analyses of various walls using a numerical wall model such as the macro model presented in this report would be needed first. Merely a few words will be said as to what might be the result concerning the hand calculation formula (6.12) for shear demand upon completion of such additional analyses.

One possibility would be to further increase the value $\omega_r$ for dynamic magnification by modifying the expressions (6.14) and (6.15). A further refined method would give the dynamic amplification factor directly in relation to the natural frequencies of the wall, and in relation to the ground motion characteristics.

Partly, the risk that expression (6.13) is on the unconservative side is reduced by the fact that the included flexural overstrength is somewhat overestimated, and partially makes up for the underestimation of the dynamic magnification factor. The latter appears to be suggested with the high flexural overstrength factor included in the expression for the total shear, resulting overall in a fairly realistic estimate of the dynamic shear forces. However, as indicated in this section as well as in the first section of this chapter, the constituents forming the total dynamic shear force display a different relation to each other, and this should be the subject of further research and considered in future design formulae.
SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The objective of the study presented in this report is the development of numerical models for the simulation of the behaviour of multi-storey reinforced concrete structural walls in buildings subjected to earthquake action, with particular emphasis on capacity designed walls. After an introduction and a review of previous work in this field, two models working in a considerably different manner were selected for further development.

The first of these, the so-called macro model, deals directly with the cross sectional behaviour of a structural wall by means of nonlinear springs. An efficient and transparent version of this type of model is derived, with emphasis on fulfilling simple kinematic conditions. Global hysteretic rules are derived largely based on a knowledge of the basic physical behaviour complemented with empirical observations. A closed mathematical form of the stiffness matrix of a macro element is derived.

The second model, referred to as the micro model, is based on the mechanics of solids and on nonlinear material models. The development of this type of model was carried out by accommodating the most essential features of reinforced concrete behaviour. As for the macro model, the development utilised basic physical behaviour and empirical observations. Different contributions to the composite material modulus matrix were derived in a clear manner.

Both these models were programmed and implemented in an existing general finite element code. The simple use of the two models is facilitated by the user element and user material option. The models developed here are user-friendly, with input of data according to specifications in the Appendix of this report.

Both models had to undergo a series of tests, which served the purpose of checking the reliability against experimental data from static tests and gave an estimate of parameter influence. Since no suitable experimental data was available for the dynamic behaviour of multi-storey walls, the micro model had to serve as comparison for the macro model.

After a chapter devoted to reliability tests and tests on the performance of a capacity designed building, a further chapter deals with some specific problems of the capacity design method in view of the numerical models developed here.

An eight-storey capacity designed wall is modelled by macro elements and analysed dynamically. Special attention is paid to the dynamic curvature ductility demand in the
plastic hinge, the dynamic bending moment and shear force demand. An improved distribution of the flexural strength over the height of the wall is proposed.

Conclusions

In the introduction of this report it was concluded that it is difficult to find a numerical model for a structural wall which is ideal for all analysis tasks.

For the performance check of multi-storey walls of capacity designed buildings, the macro model was found especially useful. This is due to its

- realistic hysteretic behaviour,
- capability of monitoring cross sectional quantities, especially such as section forces and curvature ductility, and
- limited numerical effort.

The reliability tests of the macro model were performed using experimental data from static tests. It was concluded that the macro model was capable of simulating the most important aspects of the static tests without any major parameter adjustments.

However, for dynamic behaviour no suitable experimental data exists, and in order to have a comparison basis for the macro model for such problems it was concluded that a micro model should be developed, which should also serve as a complimentary model for cases when detailed analysis is needed and for irregular geometries.

The macro model developed in this report is first of all relatively simple to comprehend. Extensive knowledge of the mechanics of solids is not necessary for its understanding or for its basic use. It consists of only the necessary number of spring members for a full description of the basic kinematic cases. The model proved to be an efficient tool especially for the case of cyclic or dynamic behaviour. This is attributed to the direct way its relatively few components influence the global behaviour. Furthermore, the effect of parameter modifications on the global behaviour can be easily followed in the macro model.

The model, as implemented and described in the Appendix, is also user-friendly. Compared to the micro model its much lower number of degrees of freedom may be important during extensive dynamic computations involving large models. While providing a reasonable global behaviour, however, the macro model does not provide much information on localised damage, such as crack directions and yielding. The macro model
also inherently assumes a regular wall geometry, a symmetric cross section, and cannot easily handle openings in a wall.

For the micro model developed in this report it is concluded that it is be possible to obtain a clear and simple material model for uniformly reinforced structural walls. It was found that it is usually adequate to consider the most important phenomena of reinforced concrete in order to obtain reasonable results. It was further shown that it is possible to divide the contributions to the material modulus matrix into different parts. The interaction effects between concrete and reinforcement may be modelled in a transparent manner, which may be generalised into different levels. The major advantage of the micro model is its capability to give relatively detailed information of local damage, such as direction of cracks and yielding of reinforcement. Furthermore, an irregularity or opening in a wall may be easily modelled. Some of the model's more important drawbacks are its general inability to monitor cross sectional quantities, such as section forces and curvature ductility, and its high computational demands compared to the macro model, which is especially important during solution of dynamic problems. Lastly, in order not to use the micro model as a black box, the user ought to be familiar with the nonlinear behaviour of materials.

The performance tests on the example of a capacity designed building indicated that a reasonable structural behaviour may be achieved for nonlinear dynamic analysis, using the capacity design recommendations in the current version. However, in some respects the time history analysis indicated that particular care in the design must be taken. The dynamic curvature ductility demand in the plastic hinge as a function of the wall aspect ratio and the displacement ductility, differs from the existing suggestions in the capacity design method.

The distribution of flexural strength is one problematic area, in which the current design recommendations do not always provide a conservative solution. This is especially true in the case of slender walls. It was shown by an energy study that a slender wall dissipates more energy in the upper storeys compared to the plastic hinge than is the case for a stiffer wall. The briefly presented proposal on how to improve the distribution of flexural reinforcement appeared to work well for a two chosen examples, but should be tested more generally.

The maximum base shear obtained from the time history analyses also considerably exceeded the anticipated dynamic shear force given in the capacity design recommendations.
Recommendations for Future Research

The models developed and tested in this report, have only been presented in their basic forms, which enabled simple comparisons with experimental data and an estimation of the behaviour of a regular cantilever wall. The purpose of the numerical models, seen in a wider perspective, to serve as a tool for structural design and for the analysis of existing large structures, could not be dealt with within the scope of this report.

Therefore, it is recommended that modifications and further development of the models be performed so as to enable the modelling of more complicated walls found in real structures, with features such as;

- connections to frames,
- coupling beams, (coupled walls)
- and walls building up three dimensional cores, such as stairways and lift shafts

Regarding the flexural strength distribution of capacity designed walls, an extensive parametric study on improved design parameters should be carried out. Attempts should be made to try to confirm the findings on the effective moment distribution by further experimental and numerical studies. Nonlinear time history analyses, in particular, should be carried out on designs with improved flexural strength distribution.

The larger magnification of base shear obtained during time history analysis should be confirmed by an additional parametric study and by simple experiments which easily allow the shear of the wall to be extracted as opposed to frequently performed complex frame-wall tests.
ZUSAMMENFASSUNG, SCHLUSSFOLGERUNGEN UND AUSBlick

Zusammenfassung


Sowohl mit dem Makro- als auch mit dem Mikromodell wurden numerische Testserien durchgeführt, die dazu dienten, die Zuverlässigkeit der Modelle aufgrund von Daten statischer Versuche zu überprüfen sowie die Einflüsse der Modellparameter abzuschätzen. Da keine geeigneten Versuchsdaten von dynamischen Beanspruchungen mehrstöckiger Tragwände zur Verfügung standen, dienten die Ergebnisse dynamischer Berechnungen mit dem Mikromodell als Vergleichsbasis für das Makromodell.

Nach einem Kapitel, das den Tests zur Ueberprüfung der Zuverlässigkeit numerischer Modelle sowie den Tests bezüglich des Verhaltens eines kapazitätsbemessene Gebäudes
gewidmet ist, wird in einem weiteren Kapitel auf einige spezielle Probleme der Be-
messung mit der Kapazitätsmethode eingegangen. Diese werden anhand der hier ent-
wickelten numerischen Modelle erläutert.


**Schlussfolgerungen**

In der Einführung wurde bereits darauf hingewiesen, dass es schwierig ist, ein für alle Rechenaufgaben ideales Modell für Tragwände zu bilden.

Zur Überprüfung des Verhaltens kapazitätsbemessener Tragwände wird das Makro-
modell als besonders geeignet erachtet, und zwar aufgrund
- seines realistischen Hystereseverhaltens und
- seiner Fähigkeit, Querschnittsgrössen, insbesondere Schnittgrössen und
Krümmungsduktilität, wiederzugeben, sowie
- wegen seines begrenzten Rechenaufwandes.

Die Zuverlässigkeitsüberprüfungen des Makromodelles wurden aufgrund von Ver-
suchsdaten statischer Versuche durchgeführt. Es wurde festgestellt, dass das Makro-
modell fähig ist, die wichtigsten Aspekte der statischen Versuche ohne wesentliche Para-
metermodifikationen zu simulieren.

Weil vom dynamischen Verhalten mehrstöckiger Tragwände keine geeigneten Ver-
suchsdaten vorliegen, wurde ein Mikromodell entwickelt, das als Vergleichsbasis bei der Beurteilung dynamischer Berechnungen dienen soll, und das auch als zusätzliches Modell für detaillierte Berechnungen und unregelmässige Geometrie verwendet werden kann.

Das Makromodell, dessen Entwicklung hier beschrieben wird, ist in erster Linie gut verständlich. Vertiefte Kenntnisse der Mechanik kontinuierlicher Medien sind weder für das Verstehen des Modells noch für dessen Anwendung notwendig. Das Makromodell besteht aus der absolut notwendigen Anzahl Elemente, die für eine umfassende Beschrei-
bung der grundlegenden kinematischen Bedingungen notwendig sind. Das Modell er-
weist sich als effizientes Werkzeug, insbesondere im Fall zyklischen und dynamischen Verhaltens. Der Grund dafür ist, dass die vergleichsweise wenigen Elemente das globale Verhalten direkt beeinflussen. Im weiteren ist der Einfluss von Parametermodifikationen auf das globale Verhalten meist gut vorhersehbar.
Das Makromodell, wie es implementiert und im Anhang A beschrieben wurde, ist weiter besonders benützerfreundlich. Bezogen auf das Mikromodell ist die normalerweise beträchtlich kleinere Anzahl Freiheitsgrade ausschlaggebend bei umfassenden dynamischen Rechenaufgaben, die sich bei der Modellierung grosser Systeme stellen. Während das Makromodell ein realistisches globales Verhalten zeigt, ist es dagegen nicht in der Lage, genaue Informationen über lokalisierte Schäden wie Rissrichtungen und lokales Fliessen der Bewehrung zu ermitteln. Das Makromodell baut im weiteren auf Voraussetzungen wie gleichmässige Wandgeometrie und symmetrischer Wandquerschnitt auf und ist demzufolge auch nicht in der Lage, Abweichungen, wie z.B. Öffnungen, zu erfassen.


Einige der schwerwiegenden Nachteile des Mikromodellies liegen darin, dass es i.a. keine Querschnittsgrössen wie Schnittkräfte und Krümmungsdüktilität wiedergibt, und dass es meistens verhältnismässig rechenintensiv ist, was insbesondere bei der Lösung umfangreicher dynamischer Probleme wesentlich ist. Im weiteren gilt auch, dass wenn das Mikromodell nicht als eine Black Box benützt werden soll, so muss der Anwender gewisse grundlegende Kenntnisse der nichtlinearen und orthotropen Mechanik besitzen.

Die durchgeführten Ueberprüfungen bezüglich des Verhaltens kapazitätsbemessener Tragwerke deuten darauf hin, dass während einer nichlinearen dynamischen Berechnung mit den heutigen Empfehlungen der Methode der Kapazitätsbemessung ein gutmütiges Strukturverhalten erreichbar ist.

Die Ergebnisse der Zeitverlau bserechnungen deuten allerdings darauf hin, dass hinsichtlich der Bemessung noch einige weitergehende Uberlegungen anzustellen sind. Der dynamische Krümmungsdüktilitätsbedarf im plastischen Gelenk, als Funktion der Wand schlankheit und der gewählten Verschiebedüktilität, unterscheidet sich von den bisherigen Angaben der Methode der Kapazitätsbemessung. Die bisher übliche Verteilung von Biegekapazität kann problematisch sein, und die heutigen Bemessungsempfehlungen
ermöglichen nicht immer eine Lösung auf der sicheren Seite. Dies ist insbesondere der Fall bei ausgeprägt schlanken Tragwänden. Durch eine Energiestudie wurde gezeigt, dass in einer flexiblen Wand in den oberen Stockwerken im Vergleich zum plastischen Gelenk mehr Energie dissipiert wird als in einer gedrungenen und daher steifen Wand. Der kurz beschriebene Vorschlag zur einer verbesserten Verteilung der Biegebewehrung scheint für das gezeigte Beispiel gut zu funktionieren, sollte aber noch genereller getestet werden.

Im weiteren ist zu erwähnen, dass die aus der Zeitverläufsrechnung resultierende maximale Querkraft am Wandfuss die in den Empfehlungen der Methode der Kapazitätsbemessung angegebenen Werte deutlich übersteigt.

**Ausblick**


Es wird deshalb empfohlen, die notwendigen Modifikationen vorzunehmen und eine weitere Entwicklung dieser Modelle durchzuführen, so dass das Modellieren von komplizierteren Tragwänden, wie sie in reellen Gebäuden gefunden werden, künftig realisierbar wird. Besonders gedacht wird an Tragwände

- mit Koppelung zu Rahmensystemen,
- mit Koppelungsriegeln (gekoppelte Tragwände),
- ausgebildet als dreidimensionale Kerne, wie z.B. Treppenhäuser und Liftschächte.


Die grössere Amplifikation der Querkraft, die aus den Zeitverläufs berechnungen resultierte, sollte durch eine weitere Parameterstudie überprüft werden. Ebenso sollten dynamische Versuche durchgeführt werden, die es erlauben, die Querkraft an der Wand zu messen, was bei den häufig vorkommenden komplizierten gemischten Wand-Rahmen Versuchskörpern nicht möglich ist.
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**Abbreviations:**

ACI: American Concrete Institute  
ASCE: American Society of Civil Engineers  
EERC: Earthquake Engineering Research Center (University of California)  
IABSE: International Association for Bridge and Structural Engineering  
RILEM: Réunion Int. des Labs. d'Essais et de Rech. sur les Matériaux et les Constr.  
SIA: Schweizerischer Ingenieur- und Architekten-Verein  
UCB: University of California, Berkeley  
WCEE: World Conference on Earthquake Engineering
NOTATION

Greek Upper Case

Φ_w Flexural overstrength factor
Θ Rotation
Θ_y Yield rotation

Greek Lower Case

α Ratio of outer vertical spring area and wall area, parameter in time integration
α_c Fraction of length of constant elastic strength to length of elastic region
α_o Fraction of yield force of outer vertical spring for which flexural cracks are closing
α_cr Ratio of cracked to elastic stiffness of outer vertical spring
α_cr* Ratio of global flexural stiffness reduction of cracked cross section
α_s Ratio of cracked to uncracked shear stiffness
α_y Ratio of yielded to elastic stiffness of outer vertical spring
α_y* Ratio of global flexural stiffness reduction of yielded cross section
β Shear retention factor, factor in time integration
δ_a Axial displacement
δ_h Horizontal displacement
δ_he Shear displacement
δ_s Shear displacement
δ_sc Shear displacement at shear cracking
δ_v Vertical displacement
δ_vl Vertical displacement at left element edge
δ_vr Vertical displacement at right element edge
δ_y Yield displacement
ε Strain
ε_0 Concrete tensile strain at zero stress
ε_0ts Tensile strain at zero stress in tension stiffening model
ε_50u Softening compressive strain for unconfined concrete at 50 % strength
ε_50h Softening compressive strain for confined concrete at 50 % strength
ε_c Concrete strain
\( \varepsilon_{cr} \) - Concrete cracking strain
\( \varepsilon_l \) - Vertical strain at left element edge
\( \varepsilon_{ly} \) - Vertical yield strain at left element edge
\( \varepsilon_n \) - Strain normal to crack direction
\( \varepsilon_r \) - Vertical strain at right element edge
\( \varepsilon_s \) - Steel strain
\( \varepsilon_y \) - Yield strain
\( \phi \) - Curvature, angle between local and global coordinate system
\( \phi_y \) - Yield curvature
\( \gamma \) - Shear strain, factor in time integration
\( \gamma_R \) - Resistance factor
\( \kappa \) - Form factor in shear
\( \lambda \) - Mesh correction factor
\( \lambda_o \) - Overstrength factor for reinforcement steel
\( \mu_{\Delta} \) - Global displacement ductility
\( \mu_{\phi} \) - Curvature ductility
\( \mu_{\theta} \) - Rotational ductility
\( \omega_{\nu} \) - Dynamic magnification factor
\( \rho \) - Reinforcement ratio
\( \rho_x \) - Reinforcement ratio in x-direction
\( \rho_y \) - Reinforcement ratio in y-direction
\( \sigma \) - Stress
\( \sigma_0 \) - Axial stress
\( \sigma_f \) - Fibre concrete stress
\( \sigma_s \) - Fibre steel stress
\( \rho_s \) - Confinement ratio
\( \rho_t \) - Tensile reinforcement ratio
\( \sigma_{ts} \) - Concrete tensile stress between cracks (tension stiffening stress)
\( \rho_w \) - Horizontal reinforcement ratio
\( \rho_{wh} \) - Horizontal reinforcement ratio of web

**Latin Upper Case**

\( A_c \) - Concrete area
\( A_g \) - Cross sectional gross area
\( A_f \) - Fibre concrete area
NOTATION

$A_f$  Fibre steel area
$A_s$  Reinforcing steel area
$A_{si}$ Reinforcing steel area crossing inclined surface
$A_v$  Shear reinforcement area
$A_w$  Wall cross sectional area
$A_{wi}$ Wall cross sectional area inclined at 45 degrees
$A_{ws}$ Horizontal reinforcement area of web
$B$  Strain-displacement matrix
$C$  Global damping matrix, transformation matrix
$C_d$ Overstrength reduction factor
$C_t$ Factor for equivalent shear stress
$D$  Material modulus matrix
$D_c$ Concrete modulus matrix
$D_g$ Modulus matrix in global coordinate system
$D_{ia}$ Interaction modulus matrix
$D_l$ Modulus matrix in local coordinate system
$D_{max}$ Largest aggregate diameter
$D_s$ Steel modulus matrix
$E$  Young's modulus
$E_c$ Young's modulus for elastic small strain concrete
$E_d$ Dissipated energy
$E_k$ Kinetic energy
$E_{se}$ Elastic strain energy
$E_{sle}$ Inelastic (irrecoverable) strain energy
$E_{st}$ Increased Young's modulus for reinforcement steel due to tension stiffening
$E_{sx}$ Young's modulus for reinforcement steel in x-direction
$E_{sy}$ Young's modulus for reinforcement steel in y-direction
$E_t$ Total energy
$E_{ts}$ Young's modulus for concrete between cracks in opening phase
$E_v$ Viscous energy
$F$  Force
$F$  Global force vector
$F_c$ Cracking force of outer vertical spring
$F_{cs}$ Spring force of central vertical spring
$F_{fc}$ Fibre concrete force
$F_f$ Fibre steel force
$F_r$ Residual force
\( F_s \) Spring force of outer vertical spring
\( F_{ts} \) Steel force across cracks due to tension stiffening
\( F_y \) Yielding force of outer vertical spring
\( G \) Modulus of rigidity
\( G_f \) Fracture energy
\( H \) Building height
\( H_w \) Wall height
\( I \) Moment of inertia
\( I_{cr} \) Moment of inertia of cracked cross section
\( I_e \) Moment of inertia of elastic cross section
\( I_y \) Moment of inertia about strong axis
\( K \) Global stiffness matrix
\( K_1 \) Stiffness of left vertical outer spring
\( K_2 \) Stiffness of right vertical outer spring
\( K_3 \) Stiffness of central vertical spring
\( K_A \) Axial stiffness of horizontal beam
\( K_c \) Elastic compressive stiffness of outer vertical spring
\( K_{cs} \) Elastic spring stiffness of central vertical spring
\( K_t \) Form factor for shear cracking
\( K_S \) Horizontal spring stiffness
\( K_{se} \) Elastic spring stiffness of outer vertical spring
\( K_u \) Unloading stiffness of outer vertical spring
\( K_{ve} \) Elastic stiffness of horizontal (shear) spring
\( K_y \) Yield stiffness of outer vertical spring
\( L \) Building length, length of integration area
\( L_e \) Length of elastic region
\( L_{ec} \) Length of region of constant elastic strength
\( L_p \) Length of plastic hinge zone
\( L_w \) Wall length
\( M \) Bending moment
\( M \) Global mass matrix
\( M_E \) Bending moment from static equivalent force
\( M_i \) Bending moment demand
\( M_R \) Bending moment strength
\( M_y \) Yield moment
\( M^0 \) Bending moment resistance of cross section without normal force
\( M^N \) Bending moment resistance of cross section including normal force
\begin{itemize}
\item \textbf{\textit{N}} Normal force
\item \textbf{\textit{P}} Shear retention coefficient
\item \textbf{\textit{P}_u} Effective design normal force on wall section for calculation of shear stress
\item \textbf{\textit{P}_v} Viscous effect
\item \textbf{\textit{Q}} Transformation matrix
\item \textbf{\textit{R}_e} Bending moment resistance in constant elastic region
\item \textbf{\textit{R}_m} Nominal minimum bending moment resistance with zero normal force
\item \textbf{\textit{R}_p} Bending moment resistance in plastic hinge zone
\item \textbf{\textit{T}_i} Period of mode i
\item \textbf{\textit{U}} Global displacement vector
\item \textbf{\textit{\dot{U}}} Global velocity vector
\item \textbf{\textit{\ddot{U}}} Global acceleration vector
\item \textbf{\textit{V}} Shear force, element volume
\item \textbf{\textit{V}_c} Cracking force in shear, concrete contribution to shear capacity
\item \textbf{\textit{V}_E} Shear force from static equivalent force
\item \textbf{\textit{V}_R} Shear strength
\item \textbf{\textit{V}_s} Steel contribution to shear capacity
\item \textbf{\textit{V}_u} Ultimate shear capacity
\item \textbf{\textit{V}_w} Shear demand
\item \textbf{\textit{Z}} Softening modulus of confined concrete
\end{itemize}

\textbf{Latin Lower Case}

\begin{itemize}
\item \textbf{\textit{ag(t)}} Ground acceleration history
\item \textbf{\textit{b}_c} Average width of wall cross section
\item \textbf{\textit{b}_n} Width of confinement hoops
\item \textbf{\textit{b}_w} Wall thickness
\item \textbf{\textit{c}} Lower distance to centre of relative rotation
\item \textbf{\textit{c}} Element damping matrix
\item \textbf{\textit{d}} Static height
\item \textbf{\textit{f}_c} Concrete stress
\item \textbf{\textit{f}_c'}} Compressive concrete design strength
\item \textbf{\textit{f}_d} Damping force
\item \textbf{\textit{f}_g} Dynamic force due to ground motion
\item \textbf{\textit{f}_i} Eigenfrequency of mode i
\item \textbf{\textit{f}_i} Inertia force
\( f_s \)  Equivalent shear stress

\( f_s \)  Stiffness force

\( f_t \)  Concrete tensile strength

\( f_{st} \)  Steel stress due to tension stiffening

\( f_u \)  Ultimate strength of steel

\( f_v \)  Shear strength of concrete

\( f_{w_y} \)  Yield strength of horizontal wall reinforcement

\( f_y \)  Design yield strength of steel

\( h \)  Macro element height

\( h_c \)  Upper distance to centre of relative rotation

\( k \)  Element stiffness matrix

\( k_e \)  Elastic element stiffness matrix

\( k_{ib} \)  Element stiffness matrix of internal beam

\( k_{ie} \)  Inelastic element stiffness matrix

\( k_{ts} \)  Concrete tension softening factor

\( l \)  Macro element length

\( l_c \)  Centroidal distance

\( l_w \)  Web length

\( m \)  Element mass matrix

\( m_s \)  Shear magnification factor

\( n \)  Number of storeys

\( r_a \)  Wall aspect ratio

\( s \)  Spacing of reinforcement bars

\( s_h \)  Spacing of confinement hoops

\( t \)  Web thickness

\( u \)  Geometrical parameter for shear form factor

\( u \)  Element displacement vector

\( \dot{u} \)  Element velocity vector

\( \ddot{u} \)  Element acceleration vector

\( u_i \)  Degree of freedom No. \( i \)

\( v \)  Geometrical parameter for shear form factor

\( v_c \)  Concrete shear stress

\( w \)  Concrete crack width

\( w_0 \)  Concrete crack width at zero tensile stress

\( x_i \)  Fibre centroidal distance
The macro model developed in Chapter three was coded as a "User Element" [Abaq91], which may be used essentially in the same manner as the library elements. The function, the input properties, as well as some useful output quantities of this user element are briefly described in this appendix.

Only the features which are specific for the user element will be described here. A small example of a complete input file is given at the end of Appendix A. For the complete use of the software, the reader is referred to the Abaqus manual [Abaq91].

Function

The user element is coded as a subroutine on a separate file of source code, and describes the nonlinear behaviour of the element according to the discussion in chapter three. In each increment, the subroutine performs an update of the stiffness properties and the resulting element forces. These are delivered to the program via a user element subroutine interface, which has a fixed format [Abaq91].

In each increment, the element displacements are fed back into the subroutine by the program. In order to maintain certain properties from one increment to another, a vector of solution dependent "state variables" is used, which may be changed by the subroutine and which is stored until later increments. Since the user element subroutine is written as a separate Fortran file, it has to be connected to the program during execution, and this is made by including the following line in the input file:

*USER SUBROUTINES, INPUT=15

The user element has to be specified just like the library elements. The simplest user element type describing the macro model is the U30 with four corner nodes only. It is specified by including the following line in the input file (one line of specifications, two lines of d.o.f. numbers):
*USER ELEMENT, TYPE=U30, NODES=4, COORDINATES=2, PROPERTIES=8, VARIABLES=45
1, 2
3, 1, 2, 6

Except what was stated above, the first line specifies that two coordinates are required per node, the number of input properties is equal to eight, and the number of solution-dependent state variables is equal to 45. The two last lines prescribe the d.o.f. numbers available for the nodes; for the first node the first and second degrees of freedom are available, which remains the same until the third node, from which the sixth degree of freedom is also available. It should here be noted that the coordinates of the nodes represent the location of the end points of the flexural springs of the macro model, see chapter three.

Input properties

The eight input properties prescribed in the line of element specification are given on a line in free format and are described as follows (the units given are recommended but any other units consistent with other input data may be used):

1) Cross sectional area of entire wall section (m²)
2) Moment of inertia (about strong axis) of entire wall section (m⁴)
3) Young's modulus of uncracked concrete (MPa)
4) Cracking factor for stiffness of flexural springs in tension, equal to ratio of compressive to tensile stiffness, (useful range: 0.4 to 0.8)
5) Yielding factor, equal to ratio of yielded to compressive stiffness (useful range: 0.01 to 0.03)
6) Bending moment at flexural yielding of cross section with zero axial load, for calculation procedure see chapter three (MNm)
7) Shear force at the onset of shear cracking, for calculation see chapter three (MN)
8) Cracking factor in shear, equal to ratio of cracked to uncracked shear stiffness, for calculation see chapter three, (useful range: 0.10 to 0.30).

A wall element with a cross sectional area of 1.8 m², a moment of inertia of 5.4 m⁴, a Young's modulus of 33000 MPa, cracking and yielding factors in flexure of 0.7 and 0.03 respectively, a yield moment of 6.5 MNm with no axial load, and an uncracked shear force capacity of 2.8 MN and a cracked shear stiffness of 16% of uncracked, will
thus require the following user element property set written as follows on two lines on the input file, assuming that the element belongs to the element set WALL:

*UEL PROPERTY, ELSET=WALL
1.8, 5.4, 33000., 0.7, 0.03, 6.5, 2.8, 0.16

Based upon the information contained in input properties and the coordinates of the nodes, i.e. the element geometry, essentially all other relevant element properties are computed automatically in the user element subroutine.

Output quantities

For plotting purposes, a number of output quantities are made available among the solution-dependent state variables, which may be useful in the evaluation of the nonlinear behaviour of the user element. These are written on an element file during execution, for each increment, if within the calculation step, see [Abaq91], the following two lines are included:

*EL FILE, ELSET=ELOUT, FREQUENCY=2
SDV

which assumes that the element(s) of interest for output are included in the element set "ELOUT" (default: all elements), and that information is desired for every second increment (default: every increment).

On a "post file", see [Abaq91], plot commands may be given and these will in the case of a user element often contain one or more of the state variables of this element. These are denoted SDVnn, where nn is the number if the specific state variable desired. For instance, the shear force of the user element is stored in state variable 22, which will be denoted SDV22. As a brief example, the input sequence on a post file for a plot of the shear force vs. time will here be given on three lines:

*HISTORY
TIME           BASE SHEAR
SDV22, , SHEAR-EL1, 1, 1, ,

The third line specifies, after the state variable number, a scaling factor which is set to one as default (if left blank as here), then a label for the plot, then the element number
For closer details and options, see [Abaq91].

Some useful state variables for output purposes for user element type 30 are listed in the following:

<table>
<thead>
<tr>
<th>SDV22</th>
<th>Base shear (MN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDV23</td>
<td>Shear deformation over element height (m)</td>
</tr>
<tr>
<td>SDV28</td>
<td>Rotational ductility, left curvature</td>
</tr>
<tr>
<td>SDV29</td>
<td>Rotational ductility, right curvature</td>
</tr>
<tr>
<td>SDV30</td>
<td>Interstorey (element) drift, % of story height</td>
</tr>
<tr>
<td>SDV34</td>
<td>Bending moment (MNm)</td>
</tr>
<tr>
<td>SDV35</td>
<td>Curvature</td>
</tr>
</tbody>
</table>

**Example of Input File**

In order to facilitate the understanding of the use of the macro model, implemented by means of a user element, a small example of a structural wall modelled by two elements is presented here. The small number of elements is deliberately chosen so as to create an example as simple as possible. The side view of the wall is shown in figure A.1.

The following basic wall properties are assumed: Cross sectional area: 1.8 m², cross sectional uncracked moment of inertia: 5.5 m⁴, Young's modulus of concrete: 33000 MPa, flexural cracking factor: 0.7, flexural yielding factor: 0.03, bending moment at zero axial load: 6.5 MNm, shear force at cracking: 2.8 MN, cracked shear stiffness ratio: 0.16. It is further assumed that a gravity load is applied to the wall at the top as two
concentrated loads each of 0.5 MN, and then a static horizontal force is applied to the wall from the left at the top. The central distance between the two flexural spring is chosen as 5.5 m. Furthermore, it is assumed that residual force tolerances are set as 0.05 MN for forces and 0.1 MNm for moments.

At first, the input file for the structural analysis is given below. On the next page, a post file is shown for the same example containing commands for plotting the second step of the horizontal top displacement vs. time, the shear force of element one versus top displacement, and the moment versus curvature of element one.

*HEADING
Example of two story wall
*USER SUBROUTINES, INPUT=15

*NODE
  1, 0., 0.
  2, 5.5, 0.
  3, 0., 4.
  4, 5.5, 4.
  5, 0., 8.
  6, 5.5, 8.

*USER ELEMENT, TYPE=U30, NODES=4, COORDINATES=2, PROPERTIES=8,
VARIABLES=45
  1, 2
  3, 1, 2, 6

*UEL PROPERTY, ELSET=WALL
  1.8, 5.4, 33000., 0.7, 0.03, 6.5, 2.8, 0.16

*ELEMENT, TYPE=U30, ELSET=WALL
  1, 1, 2, 3, 4
  2, 3, 4, 5, 6

*BOUNDARY
  1, ENCASTRE
  2, ENCASTRE

*STEP, LINEAR
*STATIC
*CLOAD
  5, 2, -0.5
  6, 2, -0.5

*END STEP
*STEP, INC=20
*STATIC, PTOL=0.05, MTOL=0.1, DIRECT
0.05, 1.0
*CLOAD
5, 1, 1.0
*NODE FILE
U
*EL FILE, FREQUENCY=1
SDV
*END STEP

Post file for use with element and node data resulting from calculation with above input file:

*POST FILE
Example of two story wall
*HISTORY, BSTEP=2
TIME     DISPL (m)
U1, , TOP-DISPL, 6, 1, ,
*VARIABLE-VARIABLE, BSTEP=2
DISPL (m)     SHEAR FORCE (MN)
U1, , TOP-DISPL, 6, 1, ,
SDV22, , SHEAR-EL1, 1, 1, ,
*VARIABLE-VARIABLE, BSTEP=2
CURVATURE     MOMENT (MNm)
SDV35, , CURVATURE, 1, 1, ,
SDV34, , MOMENT, 1, 1, ,
APPENDIX B

USER MATERIAL INPUT DESCRIPTION

The micro model developed in Chapter four was coded as a "User Material" [Abaq91], which may be used essentially in the same manner as the library materials. The function, the input properties, as well as some useful output quantities of this user material are briefly described in this appendix.

Only the features which are specific for the user material will be described here. A small example of a complete input file is given at the end of Appendix B. For the complete use of the software the reader is referred to the Abaqus manual [Abaq91].

Function

The user material is coded as a subroutine in a separate file of source code, and describes the nonlinear behaviour of the material at every integration point according to the discussion in chapter four. In each increment the subroutine performs an update of the material modulus properties and the resulting stresses. These are delivered to the program via a user material subroutine interface, which has a fixed format [Abaq91].

In each increment, the strains for each integration point are fed back into the subroutine by the program. In order to maintain certain properties from one increment to another, a vector of solution-dependent "state variables" is used, which may be changed by the subroutine and which is stored until later increments. Since the user material subroutine is written as a separate Fortran file, it has to be connected to the program during execution, and this is made by including the following line in the input file:

*USER SUBROUTINES, INPUT=15

The user material has to be specified just like the library materials. The user material is to be used in connection with some of the available membrane elements, such as the CPS4, or the CPS8, see the Abaqus manual [Abaq91]. The user material has to be named in the same way as for library materials. In the material block the user material is specified as follows:
followed by a line with the eight input constants, described below in the section Input properties. The abovementioned solution-dependent state variables which are 23 for the simplest user material model are then specified in the input file as follows:

*DEPVAR
23

Input properties

The eight input properties prescribed as constants in the line of material specification are given on a line in free format and are described as follows (the units given are recommended but any other units consistent with other input data may be used):

1) Young's modulus of uncracked concrete (MPa), (useful range: 20000 to 40000)
2) Poisson's ratio (useful range: 0.15 to 0.20)
3) Strain at onset of cracking (useful range: 0.00007 to 0.00015)
4) Strain at end of tension stiffening (useful range: 0.0010 to 0.0030)
5) E-modulus for reinforcement steel (MPa) (useful range: 180000 to 220000)
6) Yield strength of steel, without regard to tension stiffening (MPa) (useful range: 300 to 600)
7) Reinforcement ratio in horizontal direction (useful range: 0.002 to 0.05)
8) Reinforcement ratio in vertical direction (useful range: 0.002 to 0.05)

A material with a concrete E-modulus of 33000 MPa, Poisson's ratio of 0.2, cracking strain of 0.0001, strain at end of tension stiffening of 0.002, E-modulus for steel of 200000 MPa, reinforcement ratios of 0.01 and 0.02 in the horizontal and vertical directions respectively, will thus require the following user material property set written as follows on one line on the input file:

33000., 0.2, 0.0001, 0.002, 200000., 570., 0.01, 0.02
Output quantities

For plotting purposes, a number of output quantities are made available among the solution-dependent state variables, which may be useful in the evaluation of the nonlinear behaviour of an element employing the user material. These are written on an element file during execution, for each integration point in the element, and for each increment, if within the calculation step, see [Abaq91], the following two lines are included:

*EL FILE, ELSET=ELOUT, FREQUENCY=2
SDV

which assumes that the element(s) of interest for output are included in the element set "ELOUT" (default: all elements), and that information is desired for every second increment (default: every one increment).

In a "post file", see [Abaq91], plot commands may be given and these will in the case of a user element often contain one or more of the state variables of this element. In the post file these are denoted SDVnn where nn is the number if the specific state variable desired. For instance the stress normal to the first crack direction is stored in state variable 9, which will be denoted SDV9. As a brief example, the input sequence on a post file for a plot of the shear force vs. time will here be given on three lines:

*HISTORY
TIME BASE SHEAR
SDV9, , SHEAR-EL1, 1, 1, ,

The third line specifies, after the state variable number, a scaling factor which is set to one as default (if left blank as here), then a label for the plot, then the element number (El. No. one in this case) and integration point number. For closer details and options, see [Abaq91].

Some useful state variables for output purposes for the user material are finally listed in the following:
Example of Input File

In order to facilitate the understanding of the use of the macro model, implemented by means of a user element, a small example of a structural wall modelled by two elements will here be presented. The small number of elements is deliberately chosen so as to create an example as simple as possible. The side view of the wall is seen in figure B.1.

![Diagram of wall prototype and model with element and node numbers]

_Wall prototype  Model with element and node numbers_  
*Figure B.1 Structural wall modelled by six library membrane elements of type CPS4, with material behaviour specified according to the USER MATERIAL option*

The following basic wall properties are assumed: Wall thickness 0.3 m (uniform), Young's modulus of concrete: 33000 MPa, Poisson's ratio of 0.2, cracking strain of 0.0001, strain at end of tension stiffening of 0.002, E-modulus for steel of 200000 MPa,
reinforcement ratios of 0.01 and 0.02 in the horizontal and vertical directions respectively. Furthermore, it is assumed that residual force tolerances are set to 0.05 MN for forces.

The input file for the structural analysis is given below. On the next page, a post file is shown for the same example containing commands for plotting of the second step of the horizontal top displacements vs. the base shear, and the stress normal to the first crack direction of element one, integration point one.

*HEADING
Example of two story wall
*USER SUBROUTINES, INPUT=15
*NODE
1, 0., 0.
2, 3., 0.
3, 6., 4.
4, 0., 2.67
5, 3., 2.67
6, 6., 2.67
7, 0., 5.33
8, 3., 5.33
9, 6., 5.33
10, 0., 8.
11, 3.8.
12, 6., 8.
*ELEMENT, TYPE=CPS4
1, 1, 2, 5, 4
2, 2, 3, 6, 5
3, 4, 5, 8, 7
4, 5, 6, 9, 8
5, 7, 8, 11, 10
6, 8, 9, 12, 11
*SOLID SECTION, MATERIAL=WALL
0.3
*MATERIAL, NAME=WALL
*USER MATERIAL, CONSTANTS=8
33000., 0.2, 0.0001, 0.002, 200000., 570., 0.01, 0.02
APPENDIX B

*DEPVAR
23

*BOUNDARY
1, ENCASTRE
2, ENCASTRE
3, ENCASTRE

*STEP, LINEAR

*STATIC

*CLOAD
10, 2, -0.5
12, 2, -0.5

*END STEP

*STEP, INC=20

*STATIC, PTOL=0.05, MTOL=0.1, DIRECT
0.05, 1.0

*CLOAD
10, 1, 1.0

*NODE FILE
U, CF

*EL FILE, FREQUENCY=1
SDV

*END STEP

Post file for use with element and node data resulting from calculation with above input file:

*POST FILE
Example of two story wall

*VARIABLE-VARIABLE, BSTEP=2
DISPL (m) SHEAR FORCE (MN)
U1,, TOP-DISPL, 12, 1,,
CF1,, SHEAR FORCE, 10, 1,,

*VARIABLE-VARIABLE, BSTEP=2
N STRAIN N STRESS
SDV6,, STRAIN, 1, 1,,
SDV9,, STRESS, 1, 1,,
APPENDIX C

YIELD MOMENT FOR MACRO MODEL

The selection of the yield moment for the macro model was described in chapter three, section 3.5. In this Appendix, the selection of the yield moment for the three storey test specimen, presented in chapter five, will be demonstrated, followed by comments on the eight storey capacity designed wall.

A moment curvature relation was established for the wall cross section, according to the procedure described in chapter three, section 3.5. The results of this procedure apply rather to the behaviour in a flexural crack. Over a certain height of the wall there are, however, large parts of uncracked concrete. Therefore, the average flexural stiffness is considerably higher than implied by the moment curvature results.

In figure C.1 the moment curvature relation for the wall cross section of the test specimen used in chapter five is given. The test specimen, which is a 1:3 scale model of the three lowest stories in a ten storey building, was presented in section 5.3, and used for the reliability tests of the macro model in section 5.4.1, as well as in subsequent reliability tests for the micro model.

In figure C.1 the moment curvature relation as obtained from the computer program described in section 3.5 is displayed. This result is mainly representative for the behaviour across a crack. The cracking in tension may clearly be seen at a moment of around 0.5 MNm. From there the program exhibits a relatively soft behaviour due to the fact that essentially all tensile forces over the cross section now must be carried by the reinforcement. The yielding behaviour is seen to develop gradually, and the onset of yielding is from the curve estimated to occur at around 2.8 MNm. The gradual yielding gives a rounded shape of the curve which continues until around 3.2 to 3.3 MNm where the behaviour becomes more straight, which means that the strain hardening alone dominates the behaviour, rather than the yielding of additional bars.

The hysteretic model for the outer vertical links of the macro model assume a trilinear skeleton curve, as shown in chapter three. In addition, for the uncracked stiffness of the compressive branch a cracked tensile branch was assumed. This cracked stiffness will prevail until yielding occurs. This yielding is for the model assumed to simulate the yielding of the entire cross section in a representative manner. This means that a yield level must be found from the nonlinear cracked behaviour in figure 6.1, which may be used in a trilinear model.
5.0 -i—r-

[Graph 1]

Figure C.1 Selection of yield moment for 1:3 scale test specimen

[Graph 2]

Figure C.2 Flexural behaviour including effect of axial load

In figure C.1 a yield level of around 3.0 MNm was found to be representative, as a single level, representing yielding for the entire cross section. The selected model behaviour is denoted in the figure by a dotted line. The cracked stiffness is taken as around 70 percent of the elastic value until the selected yield level. Thereafter a fraction of yielded to uncracked vertical spring stiffness is taken in the range of a few percent as
discussed in chapter three. For the test specimen a model yield stiffness of around one percent was found to give reasonable agreement with experimental data.

The model behaviour with the full axial load of 0.87 MN, as was applied on the test specimen, is seen in figure C.2. The approximation of the model behaviour according to expression 3.38 may here be checked against the result of the cross sectional behaviour. Relatively good agreement may be observed.

It should be noted that figures C.1 and C.2 only apply to the skeleton curve of a the hysteretic model of chapter three. As soon as load reversal has taken place, the relations of these figures are no longer relevant, and the flexural behaviour is governed by unloading and reloading rules as presented in chapter three.

For the eight storey capacity designed wall, presented in section 5.3, and used for the nonlinear dynamic problem of chapter five, the procedure of determining the yield moment will not be repeated since it follows the same pattern as described above. From figure 6.4 and 6.6, the moment curvature relation for the cross sections of the limited and full ductility design may be seen. From there, the yield levels of 6.6 and 3.8 MNm, respectively were read, based on the curve with design values for material strength. Increasing these values by 20 % in order to arrive at effective mean values gives 7.9 MNm and 4.6 MNm, respectively.

Inspection of figures 6.4 and 6.6 for the curves of yield strength of 550 MPa, strain hardening of 0.7% of elastic modulus, and concrete compressive strength of 30 MPa, suggests good agreement, and would have given about 7.9 and 4.6 MNm as well. The procedure was repeated for the storeys of the elastic region.
APPENDIX D

ELEMENTS FOR MICRO MODEL

A short overview of elements available for the micro model developed in chapter four is presented in this Appendix. The elements shown here belong to the Abaqus element library, [Abaq91].

The presentation here is limited to the plane-stress family of elements, to be used in pure two-dimensional problems. The membrane elements could theoretically also be used for the micro model, in the case of three-dimensional analysis.

Figure D.1 Plane stress elements for use with micro model of chapter four
Figure D.1 shows the six available plane-stress elements. In the figure the dots denote nodes with two in-plane degrees of freedom each and the crosses denote integration points. In the element designation in the figure an R denotes reduced integration, see e.g. [Abaq89].

For structural walls the triangular elements may rather be useful in the case of irregularities or mesh refinement between rectangular elements. Normally the rectangular elements will be used, and for the nonlinear models developed in this report, the two elements CPS4 and CPS8 proved to be preferable. The elements with reduced integration CPS4R and CPS8R necessitated finer meshes in order to avoid zero energy modes which may occur in problems with dominant flexural action, and they are thus not recommended for typical structural wall analysis.

A comparison was made between the elements CPS4 and CPS8 for the nonlinear static monotonic behaviour. The three-storey 1:3 scale structural wall test specimen presented in chapter three is used. The meshes A and B, presented in chapter five, were used for this comparison. In figure D.2 is shown the used mesh-element combinations. The same number of elements used in mesh A with CPS4 (left) and with CPS8 (right). Virtually the same number of degrees of freedom are used in mesh B with CPS4 (centre) and in mesh A with CPS8 (right).

![Figure D.2 Meshes for element comparison: Mesh A with CPS4 (left), mesh B with CPS4 (centre), mesh A with CPS8 (right)](image)

Figure D.3 shows the numerical result of the element comparison. The base shear is plotted versus the horizontal fourth floor (free end) displacement. It is seen that solution of mesh A with CPS8 approaches the solution of mesh B with CPS4.

It should finally be noted that due to the formulation of the micro model as described in chapter four, the same integration scheme is used for both the concrete and the reinforcement. Different integration for these materials within a certain element is suggested by some authors.
Figure D.3 Base shear versus fourth floor horizontal displacement: comparison between element CPS4 and CPS8
APPENDIX E

FREQUENCY STUDY OF DAMAGED STATES

In order to display the frequency behaviour of the gradually damaged eight storey wall used in the examples on the nonlinear dynamic behaviour in chapters five and six, a few free vibration studies were performed. The results in form of eigenfrequencies are given below.

The macro model allowed for the possibility of changing the stiffness of the internal springs of chosen macro elements. This feature was used in order to simulate free vibration behaviour in various damaged states. Four different states of damage were simulated in this manner. The first state, referred to as completely elastic state, simulates the behaviour for which all four Springs of all macro elements have an elastic stiffness. The second state, the cracked state, is achieved by letting the vertical outer spring of each macro element develop a cracked stiffness. In this case a cracking factor $\alpha_{cr}$ of 0.7 was used, as described in chapter three. The third state simulates moderate flexural yielding in the three plastic hinge elements (see mesh, figure 5.20). This is achieved by setting the stiffness of the outer vertical Springs at one side of the plastic hinge elements equal to yielded stiffness. The yield factor used here was taken as 0.01. The fourth state is intended to simulate a damaged state in which major yielding occurred, and is obtained by letting the central vertical spring of the plastic hinge elements develop yielded stiffness (i.e. close to zero). The eigenfrequencies for the eight first modes, which are all horizontal modes, are given on the following page.

For the micro model the elastic free vibration behaviour was obtained by prescribing high tensile strength and high yield limit. Since cracking is a procedure which starts gradually and needs built-up stress states to form properly it was not found feasible to perform a meaningful cracked state study by means of the micro model. The same applies essentially to a yielded state. The elastic state only will therefore be displayed for the eight lowest modes by means of the micro model. This state may be compared to the elastic state obtained by the macro model. The comparison of the two elastic states is shown in table E.1. The damaged states, simulated by the macro model, are shown in table E.2.

From table E.1 it is seen that the frequencies for the lower modes are in a fairly good agreement with both models. The first mode is about 6% lower for the macro model. Somewhat refined meshes for the micro model delivered about the same lowest frequencies as for the macro model.
Table E.1 Eigenfrequencies for elastic behaviour

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>f (Hz) Macro Model</th>
<th>f (Hz) Micro Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.74</td>
<td>0.79</td>
</tr>
<tr>
<td>2</td>
<td>4.25</td>
<td>4.60</td>
</tr>
<tr>
<td>3</td>
<td>10.4</td>
<td>11.3</td>
</tr>
<tr>
<td>4</td>
<td>17.4</td>
<td>18.4</td>
</tr>
<tr>
<td>5</td>
<td>24.3</td>
<td>23.3</td>
</tr>
<tr>
<td>6</td>
<td>29.9</td>
<td>24.5</td>
</tr>
<tr>
<td>7</td>
<td>34.2</td>
<td>29.1</td>
</tr>
<tr>
<td>8</td>
<td>37.0</td>
<td>32.3</td>
</tr>
</tbody>
</table>

Table E.2 Eigenfrequencies (Hz) obtained by macro model for eight storey wall of chapter five, for different states of damage

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Cracked state</th>
<th>Moderate Yielding</th>
<th>Major Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.67</td>
<td>0.54</td>
<td>0.19</td>
</tr>
<tr>
<td>2</td>
<td>3.96</td>
<td>3.90</td>
<td>2.97</td>
</tr>
<tr>
<td>3</td>
<td>9.79</td>
<td>9.90</td>
<td>7.16</td>
</tr>
<tr>
<td>4</td>
<td>16.7</td>
<td>16.7</td>
<td>13.9</td>
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<td>5</td>
<td>23.5</td>
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<td>6</td>
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<td>7</td>
<td>33.6</td>
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<td>8</td>
<td>36.5</td>
<td>36.9</td>
<td>36.8</td>
</tr>
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</table>

As for the damaged states, obtained by the macro model, the tendency may be clearly seen that the lowest modes shift towards smaller frequencies while the higher modes are unaffected. It should be kept in mind that these states were achieved by setting certain stiffnesses to fixed values. During the time history analyses, however, the stiffnesses are constantly changing. The damage state behaviour suggested here is therefore only a simplification. The "major damage" state may not be applicable to any free vibration occurring e.g. after a strong ground motion, since certain parts of the structure will then alternatingly describe unloading behaviour with typical larger stiffnesses compared to yielding in loading. Therefore, this state should rather be seen as a hypothetical lower bound on "free vibration", as a complement to the upper bound of elastic behaviour. It
should be noted that during time history analysis, cracked flexural stiffness is assumed already at the beginning of the analysis, as described in chapter three.

As a comparison to the different states of damage estimated by the free vibration data, the behaviour during the time history analysis may be displayed in the frequency region. In figures E.1 to E.3 the Fourier spectrum of the base shear of the limited ductility design of the eight storey wall of chapter five is shown. The three plots represent three different time segments. Figure E.1 shows the spectrum for the time segment 0 to 4 seconds, which is largely dominated by elastic/cracked behaviour. Figure E.2 shows the spectrum for the time segment 4 to 8 seconds, which contains substantial yielding behaviour. Figure E.3, finally, shows the spectrum obtained for the time segment 8 to 12 seconds, which is largely dominated by free vibration behaviour.

The movement of the lower eigenfrequencies may be seen from the spectra, and shows relatively good agreement with the suggestions from free vibration studies of damaged states. For 0 to 4 seconds the first three frequencies may be clearly seen in figure E.1 as: 0.7 Hz, 4.3 Hz, and 9.5 Hz. These values should essentially be comparable to a mixture of the free vibration values for elastic and cracked states. It is seen that this is largely the case.

For time 4 to 8 seconds the figure E.2 shows the three first frequencies as: 0.5 Hz, 4.1 Hz, and 9.2 Hz. These values seem to correspond largely to the "moderate yielding" state of free vibration. Lastly, for time 8 to 12 seconds, we have essentially a damaged system in free vibration. The first mode dominates the damping out of the vibrations. It is here seen that the time history analysis ended at a point at which the system was in a

![Figure E.1 Base shear spectrum of eight storey wall, time segment 0 to 4 seconds, characterised as elastic behaviour](image-url)
Figure E.2 Base shear spectrum of eight storey wall, time segment 4 to 8 seconds, characterised as moderate yielding behaviour

Figure E.3 Base shear spectrum of eight storey wall, time segment 8 to 12 seconds, characterised mainly as free vibration

damaged state with the lowest frequencies as follows: 0.5 Hz, 3.8 Hz and around 9 Hz. These values are placed in between the "moderate yielding" and "major damage" states of the free vibration tests, but are only a little lower than the moderate yielding state.

This confirms the earlier mentioned statement, that the major damage state should normally only be regarded as a lower bound. In the here shown case, the wall may be
regarded as being capable of withstanding the ground motion suggested here with moderate yielding, and some local major yielding.

It should be kept in mind that the level of the ground acceleration will affect the extent degree the damage will reach. The findings of this study are therefore limited to the input data used here and assumptions of modelling.
APPENDIX F

DESIGN DEFINITIONS

In the presentation of chapters five and six, a number of definitions mainly related to the capacity design of structural walls are used. These definitions are discussed in detail in the design references, such as [PBM90]. A comprehensive discussion of these definitions could in general not be made in this study.

In order to facilitate the understanding of the definitions used in this study, a summary is provided in this Appendix. The summary is given as a listing, where expressions and relations between the various definitions are stated briefly. The definitions listed here are essentially limited to the subject of structural walls, rather than to the wider area of general capacity design.

Equivalent Static Force Method according to the Swiss Standard SIA 160

The earthquake action may be modelled by an equivalent static force. Section 4 19 of the Swiss Standard SIA 160 treats this modelling in detail, see [SIA 160]. Further explanations of the clauses of the Standard SIA 160 are given in [Bach89]. Only the most important definitions pertaining to the equivalent static force calculation are given below.

The fundamental frequency for buildings where the horizontal earthquake forces are carried by structural walls and cores or diagonal bracing, may according to Clause 4 19 64 of the SIA 160 be estimated as follows

\[ f_0 = 13C_s \frac{\sqrt{l}}{h} \]

where

- \( f_0 \) : fundamental frequency of the building in Hz
- \( C_s \) : ground coefficient:
  - stiff ground : \( C_s = 0.9 \) to \( 1.1 \)
  - medium-stiff ground : \( C_s = 0.7 \) to \( 0.9 \)
- \( l \) : dimension in m of the building in the direction of vibration under consideration
- \( h \) : height in m of the building measured from the level of embedment
The static equivalent force according to Clause 4 19 506 of the SIA 160 is defined as follows

\[ Q_{\text{acc}} = \frac{a_h}{g} C_k \left( C_m + \sum \psi_{\text{acc}} Q_j \right) \]

where

- \( Q_{\text{acc}} \): total horizontal equivalent force
- \( a_h \): horizontal acceleration in accordance with Clause 4 19 62 and figure 27 of the SIA 160 (or figure 5.26b of this report) as function of the fundamental frequency of the structure which may be obtained according to Clause 4 19 64
- \( g \): acceleration due to gravity
- \( C_k \): construction coefficient in accordance with Clause 4 19 71 which takes into account a reduction of the equivalent force due to plastic deformations and the use of design values. It is calculated as

\[ C_k = \frac{1}{K} C_d \]

where

- \( K \): deformation coefficient according to Clause 4 19 72, taking into account the influence of plastic deformations. According to the SIA 160, the deformation coefficient depends on structural classes (Clause 4 19 74) for which structural wall buildings obtain a deformation coefficient of 2.0, 1.7, and 1.3 for structural classes SCI, SCII, and SCIII, respectively. The capacity design method [PBM90] allows a replacement of the deformation value with a global displacement ductility factor \( \mu_\Delta \) which may be chosen as follows. Restricted ductility: \( \mu_\Delta = 3 \), full ductility: \( \mu_\Delta = 5 \).

- \( C_d \): design coefficient according to Clause 4 19 73, which takes into account the difference between design values (minimum values of strength, reduction of the resistance factor, reduction of failure strain) and probable values in earthquake conditions. The design coefficient is taken as \( C_d = 0.65 \).

The design coefficient can be seen as an overstrength reduction factor [Bach91], [Bach93], obtained as
\[ C_d = \frac{1}{\phi_o} = \frac{1}{1.5} = 0.65 \]

where \( \phi_o = \lambda_o \gamma_R \approx 1.25 \cdot 1.2 \approx 1.5 \)

\[ G_m \] : mean value of the self-weight of the load-bearing structure, according to Clause 3.22 of the SIA 160

\[ \sum \psi_{acc} Q_r \] : sum of the accompanying actions occurring simultaneously with the earthquake in accordance with Clause 3.25.3 of the SIA 160

\[ \psi_{acc} \] : load factor for accompanying action to earthquake as predominant action, usually taken as 0.3, according to Clause 3.24 of the SIA 160

\[ Q_r \] : representative value of action, according to Clause 3.24 of the SIA 160

For buildings, the horizontal force shall be distributed over the height of the structure in the following manner, according to Clause 4.19.508

\[ Q_{acc,i} = Q_{acc} \frac{\left(G_m + \sum \psi_{acc} Q_r\right)_i h_i}{\sum_{i=1}^n \left(G_m + \sum \psi_{acc} Q_r\right)_i h_i} \]

where

\[ Q_{acc,i} \] : component of the horizontal equivalent force \( Q_{acc} \) at height \( h_i \)

\[ \left(G_m + \sum \psi_{acc} Q_r\right)_i \] : vertical action at storey \( i \)

\( h_i \) : height measured from the level of embedment, as discussed in Clause 4.19.507

**Cross sectional Actions**

\[ M_E \] : bending moment resulting from horizontal equivalent earthquake force components \( Q_{acc,i} \), distributed according to Clause 4.19.508, described above, of the equivalent static force calculation (in the case of moment redistribution in systems of structural walls, \( M_E \) is the redistributed moment)
Figure F.1 Levels of flexural strength

\[ \frac{M}{M_E} = \lambda M_R = \Phi_{\text{v,we}} M_E \]

\[ M_R \geq \gamma R M_E \]

\[ \gamma R M_E \]

\[ M_E \]

\[ N_G \] : effective normal force acting on the structural wall, due to permanent gravity loads tributary to wall. Calculated as \( (G_m + \sum \psi_{\text{acc}} Q_r) \) acting on the area tributary to the wall.

\[ V_E \] : shear force resulting from horizontal equivalent earthquake force components \( Q_{\text{sec},i} \), distributed according to Clause 4 19 508, described above, of the equivalent static force calculation

**Cross sectional strength**

\( \gamma R \) : resistance factor according to SIA 162, usually taken as 1.2. accounts for:
- Deviation of real structural system compared to calculated
- Simplifications and inaccuracies of the strength model
- Cross sectional inaccuracies

\( M_l \) : flexural demand, calculated as \( \gamma R M_E \) and valid for a normal force \( N_G \) (note: not \( \gamma R N_G \))

\( M_R \) : available flexural strength, calculated with design value (nominal) material strengths. Including effect of normal force \( N_G \) (note: not \( \gamma R N_G \)). Must be equal to, or exceed the flexural demand.
Design Definitions

\( M_o \) : flexural overstrength, calculated as \( \lambda_o M_R \) or as \( \Phi_{o,w} M_E \)
where \( \lambda_o \) is the overstrength factor for reinforcement steel, generally calculated as shown under section "Material strength" given below, usually obtained as 1.2 or more.

\( \Phi_{o,w} \) : overstrength factor for wall cross sectional, calculated as \( M_o / M_E \)

\( V_w \) : shear demand (equal to design shear force), calculated as \( \omega V_E \)

\( \omega \) : dynamic magnification factor for shear, calculated as
\[
\begin{align*}
\omega &= 0.9 + \frac{n}{10} \\
\omega &= 1.3 + \frac{n}{30} \leq 1.8
\end{align*}
\]
- for buildings up to six stories
- for buildings over six stories

\( V_R \) : shear strength at design level, calculated with design material strengths as \( V_R = V_c + V_s \)

\( V_c \) : concrete contribution to shear strength, expressed as \( V_c = \nu_c b_w d \)
where \( \nu_c \) is the concrete shear stress given as
\[
\nu_c = 0.6 \frac{P_u}{A_g}
\]
where \( P_u \) is the effective normal force acting on the cross section and \( A_g \) is the gross area, \( b_w \) is the wall thickness, and \( d \) may be taken as 0.8 times the wall length \( L_w \).

\( V_s \) : reinforcement contribution to shear strength, expressed as \( V_s = A_s f_y \frac{d}{s} \)
where \( A_s \) is the area of shear reinforcement (horizontal) bars in both faces, \( f_y \) is the design yield strength, \( d \) is the distance described above, and \( s \) is the vertical spacing between the bars.

Material strength

\( f_y \) : design value (nominal value) for steel yield strength (design strength)

\( f_{o,y} \) : overstrength value for steel yield strength, calculated as \( \lambda_o f_y \)
Figure F.2 Levels of material strength

\[ f_{o,y} = \lambda_o f_y \]
\[ f_{m,y} = \phi_m f_y \]

\[ \lambda_o \] : overstrength factor for reinforcement steel, generally calculated as \[ \lambda_o = \frac{f_{o,y}}{f_y} \]
usually obtained as \( \sim 1.2 \) or more

\[ \varepsilon(f_{o,y}) \] : steel strain at overstrength, usually taken in the range of 2% to 4%

\[ f_{m,y} \] : effective mean value for steel yield strength

\[ \phi_m \] : material strength reduction factor for steel, calculated as \[ \phi_m = \frac{f_{m,y}}{f_y} \]
usually obtained as \( \sim 1.2 \)

\[ f_{cw,min} \] : minimum value for test-cube compressive concrete strength according to SIA 162 (3 23)

\[ f_c \] : design value (nominal value) for compressive concrete strength, (design strength) obtained as \( 0.65 f_{cw,min} \)