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# Communication Theory and Coding for Channels with Intersymbol Interference

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# Abstract

This thesis is primarily concerned with the analysis of discrete-time communication channels with intersymbol interference (ISI) and additive white Gaussian noise (AWGN).

Complex random variables and processes are useful for describing the baseband equivalent of bandpass communication channels with memory and - as shown in this thesis - for deriving the capacity of ISI channels with a complex unit-sample response and complex AWGN. The complex noise process in the equivalent baseband channel usually exhibits certain symmetries in the autocovariance and crosscovariance function of its real and imaginary part. We show that in order to achieve capacity the information-carrying process must obey the same covariance symmetries.

It is shown that the 'covariance' of complex random variables and processes, when defined consistently with the corresponding notion for real random variables, is specified by the conventional (complex) covariance and a quantity called the *pseudo-covariance*. A characterization of uncorrelatedness and wide-sense stationarity in terms of covariance and pseudo-covariance is given. The above covariance symmetries correspond to a vanishing pseudo-covariance. Complex random variables and processes with a zero pseudo-covariance are called *proper*. It is shown that properness is preserved under affine transformations and that the complex-multivariate Gaussian density assumes a natural form only for proper random variables. The maximum-entropy theorem is generalized to the complex-multivariate case; the differential entropy of a complex random vector with a fixed correlation matrix is shown to be maximum if and only if the random vector is proper, Gaussian and zero-mean. The notion of *circular stationarity* is introduced. For proper complex random variables, a discrete Fourier transform correspondence is derived that relates circular stationarity in the time domain to uncorrelatedness

in the frequency domain.

As an application of the theory, the capacity of an ISI channel with complex inputs, a finite complex unit-sample response, and proper complex AWGN is determined. This derivation is considerably simpler than an earlier derivation by Hirt and Massey for the *real* ISI channel with AWGN, whose capacity is obtained as a by-product of the results for the complex channel.

Motivated by the fact that the capacity of an ISI channel with AWGN does not depend on the phase response of the channel filter, we introduce a transfer-function equivalence class, whose members are equal up to an allpass factor. It is shown that the mutual information between a finite-length input block and the relevant channel outputs is the same for all filters in such an equivalence class, regardless of the input probability distribution. This result allows a simpler proof of a lower bound on the information rate for independent, identically distributed inputs due to Shamai, Ozarow, and Wyner.

The state transitions of a finite-state, time-invariant trellis encoder can be described by a directed graph with exactly  $K$  branches leaving every node, called a  $K$ -ary state-transition diagram (STD). The problem of finding all isomorphism classes of  $K$ -ary STD's with certain topological constraints is investigated.  $K$ -ary STD's are constructed recursively from so-called partial  $K$ -ary STD's. Necessary and sufficient conditions are derived for when a partial  $K$ -ary STD is extendable to a (complete) strongly connected  $K$ -ary STD. We consider  $K$ -ary STD's with *maximum detour memory*, i.e., with the property that the shortest detour in the associated trellis has maximum length. Such STD's are shown to be strongly connected and to have the same number of branches ending at every state. All non-isomorphic binary STD's with maximum detour memory and  $N = 2^M$  states,  $M \leq 4$ , are determined. These binary STD's can be used, e.g., for the construction of matched spectral-null codes for partial-response channels.

The performance of trellis-coded data transmission over channels with finite ISI and AWGN is analyzed by investigating the trellis encoder that results from the cascade of the channel encoder (the outer encoder) with the subsequent channel filter (the inner encoder). Such a composite trellis encoder usually has a non-uniform distance spectrum. A well-known upper bound on bit error probability for convolutional encoders and maximum-likelihood decoding is generalized to trellis encoders with a non-uniform distance spectrum. The generalized upper bound involves an *average distance spectrum*, which can be obtained by using a modified

Viterbi algorithm. As an application, some bipolar trellis encoders for the dicode channel are compared. It is shown that, for trellis encoders with a non-uniform distance spectrum, the average number of bit errors over all detours at free distance can be much smaller than one and is therefore a more important parameter than in the case of a uniform distance spectrum.

**Keywords.** Intersymbol interference, proper complex random variables, capacity of ISI channel, information rate for i.i.d. inputs, equivalent ISI channels, strongly connected state-transition diagram, non-isomorphic state-transition diagrams, detour memory, trellis-coded ISI channel, steady-state encoder, average distance spectrum.