Applicability of the streamer breakdown criterion to inhomogenous gas gaps

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APPLICABILITY OF THE STREAMER BREAKDOWN CRITERION TO INHOMOGENEOUS GAS GAPS

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ABSTRACT

It is generally accepted that the simplified streamer breakdown criterion, \( \int \alpha \, dx = K \), can be applied to calculate the inception or breakdown voltage of gas gaps under various pressures and electric field distribution. A successful calculation however depends on three parameters; 1. a good information of electric field distribution in the gas gaps, 2. an accurate relationship between the effective ionisation coefficient (\( \bar{\alpha} \)), gas density and the electric field, and 3. the value of \( K \) which can be called a streamer constant or a generalized feed-back coefficient.

Accurate field calculation is possible today and accurate values of effective ionisation coefficients are available for the gases investigated. But the value of \( K \) still has some uncertainty. Therefore, the dependency of \( K \) on the discharge conditions is studied in this investigation.

Based on calculations and accurate experiments, it has been found that \( K \) has a magnitude of about 10 and not 18 as generally believed in the past. Such high value of \( K = 18 \) can essentially be attributed to erroneous effective ionisation coefficients.

Applicability of \( \bar{\alpha}/p = f(E/p) \) for synthetic air to atmospheric air has been demonstrated by analyzing the originally proposed streamer breakdown criterion of Meek and Raether for uniform field geometry, and by the calculation of breakdown voltages for sphere-gaps with quite different non-uniform field geometry. Based on the simplified streamer breakdown criterion, this relationship for synthetic air can be applied successfully to determine both the inception and breakdown voltages for atmospheric air.
Es ist allgemein anerkannt, dass das vereinfachte Streamer-Durchschlagskriterium, \( \int \alpha \, dx = K \), dazu verwendet werden kann, die Einsatz- oder Durchschlagsspannung von Gasentladungsstrecken für verschiedene Drucke und Geometrien zu berechnen. Eine erfolgreiche Berechnung hängt von folgenden drei Parametern ab: 1. der genauen Kenntnis des Verlaufs des elektrischen Feldes in der Entladungsstrecke, 2. eines genau bekannten Zusammenhangs zwischen dem effektiven Ionisationskoeffizienten (\( \alpha \)), der Gasdichte und dem elektrischen Feld, und 3. dem Wert \( K \), der Streamer-Konstante oder verallgemeinerter Rückwirkungskoeffizient genannt wird.

Während präzise Feldberechnungsprogramme und exakte Werte für den effektiven Ionisationskoeffizienten der untersuchten Gase heute verfügbar sind, ist der Wert von \( K \) immer noch mit einiger Unsicherheit behaftet. Daher wird in der vorliegenden Arbeit die Abhängigkeit von \( K \) von den Entladungsbedingungen untersucht.

Auf Grund von numerischen Berechnungen und hochpräzisen Experimenten wurde festgestellt, dass \( K \) für die gasige Luft und SF\(_6\) einen Wert von ungefähr 10 und nicht von 18, wie bis anhin angenommen, besitzt. Höhere Werte von \( K = 18 \) können im wesentlichen auf fehlerhafte effektive Ionisationskoeffizienten zurückgeführt werden.

Durch die Analyse des ursprünglich von Meek und Raether für Homogenfeldanordnungen vorgeschlagenen Streamer-Durchschlagskriteriums sowie durch die numerische Berechnung der Durchschlagsspannungen von Kugelfunkenstrecken mit verschiedenen inhomogenen Feldgeometrien konnte die Anwendbarkeit von \( \alpha/p = f(E/p) \) für synthetische Luft auf atmosphärische Luft bestätigt werden. Auf dem vereinfachten Streamer-Durchschlagskriterium basierend, kann diese Beziehung für synthetische Luft erfolgreich für die Bestimmung von Einsatz- und Durchschlagsspannungen von atmosphärischer Luft verwendet werden.
1. INTRODUCTION

1.1 General

New electrical transmission systems are regularly built due to either the increasing demand of electrical power or necessary replacement of old systems. The system voltages of these systems are also often increased to deliver a larger amount of electrical power over a distance and to reduce the transmission loss.

In rural areas, where there is plenty of space for the construction of such system, the main insulating media is still atmospheric air. The advantages are obvious: atmospheric air is a good insulation medium and it is available at no cost. In urban areas where the space is limited and the cost of land is high, engineers are forced to design a system which is compact, efficient, economic, reliable, and harmless. This situation resulted in the development of gas insulated system, GIS.

From the beginning of this new technology, GIS has been filled with sulphur hexafluoride, SF₆, which possesses all dielectric, chemical, and physical properties required for an insulating gas. Since more than a decade, there have been also numerous comparative studies of an alternative gas or gas mixtures. When considering the overall properties, however, SF₆ is still superior to those gases.

Many system equipments are nowadays filled with gases, mainly with SF₆, e.g. transformers, circuit breakers and cables. Apart from its light weight and non-flammability, one main property of gas insulated equipment surpassing solid or liquid insulated equipments is that its insulation is self-restoring. Gases return to its original dielectric properties after the fault is cleared.

To design an equipment, one of the most valuable tools for engineers is the calculation method, e.g. analytical or numerical. With this method, electrical, thermal, and mechanical properties of the equipment can be evaluated more efficiently, faster, and at lower cost than by experimental methods. However, the successful calculation is also dependent on the method selected and the precise values of the parameters used which are required to complete the computation.

It is generally assumed that the breakdown voltage, $U_b$, for gas insulated electrode configurations providing uniform or slightly non-uniform fields and the discharge inception voltage, $U_i$, in non-uniform fields can be determined by using the streamer
breakdown criterion. For an electronegative gas (or attaching gas), e.g. air or SF₆, where an electron can be attached to a gas molecule to form a negative ion, the streamer breakdown criterion can be mathematically expressed as

\[ \int_0^{x_c} \alpha(x) dx = K \quad (1.1) \]

where

- \( x_c \) = Critical avalanche length.
- \( \alpha(x) \) = Effective ionization coefficient. (See section 2.1 for more details)
- \( K \) = Streamer constant.

Before using the streamer breakdown criterion to determine \( U_n \), one must know 1. The electric field line, \( E(x) \), that provides the largest value of the integral on the left hand side of eq. 1.1. 2. \( \alpha(x) \), which is generally a function of \( E(x) \) and the gas density, \( N \), (or the gas pressure, \( p \), at constant temperature, \( T \)), i.e. \( \alpha/p = f(E/p) \) at constant \( T \). and 3. The magnitude of the "streamer constant", \( K \).

1.2 The problem

Nowadays, the electric field in an electrode configuration can be precisely calculated with the aid of a field calculation program. The dependency \( \alpha/p = f(E/p) \) can be accurately measured with new experimental techniques.

However, there is still some uncertainty about the magnitude of \( K \) since eq. 1.1 was introduced about 50 years ago. There are many indications that \( K \) may be a constant or may depend on the discharge conditions, e.g. field non-uniformities, pressure, type of gases, or voltage polarity.

To demonstrate this problem in some detail, figure 1.1 shows the calculated pressure reduced breakdown field strength (\( E_u/p \)) as a function of the product of pressure and gap distance, \( d \), for a plane-to-plane electrode where the fields are uniform. For a coaxial sphere electrode, where the field are non-uniform, the pressure reduced inception field strength (\( E_i/p \)) is plotted as a function of the product of pressure and inner sphere radius, \( r \), with SF₆ as an insulating media. The calculations are carried out by using eq. 1.1 together with \( \alpha/p = f(E/p) \) from Aschwanden (1984, 1985) and two constant values of \( K = 18 \) as often applied, and 10.5 as suggested by Pedersen (1975 and 1984 respectively).

We can see that the value of \( K \) has only a small influence on the calculated \( E_u/p \)-values at high \( pd \) (or \( pr \))-products in the uniform field but has a large effect on \( E_i/p \) in the non-uniform field.
Figure 1.1 Effect of $K$ on the calculated dielectric strength of SF$_6$ for the case of uniform and non-uniform fields.

In earlier investigations $K$ has been intensively studied mostly in the uniform or slightly non-uniform field configuration with breakdown experiments. Every breakdown will, however, roughen the electrode surface if the breakdown experiments have not been carried out under very well controlled conditions to reduce the effects of breakdown currents. It is well known today that surface roughness can lower the experimental breakdown voltage. Thus, experimental studies of $K$ in configurations as used for figure 1.1 are very difficult, because a small error in the measured breakdown voltage at high $pd$ product can result in a large deviation in the determination of the magnitude of $K$.

Additionally, engineering practices relate mainly in non-uniform field configurations. Therefore, the knowledge of $K$ is of high importance for the successful determination of the discharge inception voltage as the effect of $K$ on the calculation is larger for non-uniform fields than for uniform fields in the total range of $pd$ (or $pr$) products, as shown in figure 1.1.

Therefore, the aim of this investigation is to apply the streamer breakdown criterion in the non-uniform field gap and to determine whether or not the value of $K$ depends on the non-uniformity, pressure, gas, and polarity.
1.3 Organization of this investigation

The details of the investigation will be given in the following section: Section 2 reviews the breakdown criteria, i.e Townsend's and streamer breakdown criteria, which are generally accepted as a useful tools to predict the $U_s$ or $U_i$ of a gas gap. The discussion, especially on the streamer breakdown criterion, is also given in this section. Section 3 describes a procedure to calculate $U_i$ and discusses the effect of some parameters on the calculated results. Section 4 describes the equipment used in this investigation, the procedure to perform the experiments and finally explains the experimental method to determine $U_i$. Section 5 provides the results of the experiments with interpretations and discussions. In section 6 the value of $K$ for non-uniform field in dry air, determined in section 5, is used to calculate the breakdown voltages of all standard sphere gaps according to IEC-52: 1960. Lastly, the conclusions are drawn in a summary section.
2. HISTORICAL REVIEW AND PRESENT INVESTIGATION

This section will shortly review the two most successful breakdown criteria which are Townsend’s breakdown criterion and the streamer breakdown criterion. Physical mechanisms of both criteria and some definitions, which will be used throughout this work, are explained.

The application and modification of both criteria in uniform and non-uniform fields are reviewed. The problems to quantify the magnitude of $K$ are stated and a possible experiment to quantify $K$ accurately, concerning this investigation is outlined.

2.1 Breakdown criteria review

Nowadays, two breakdown criteria in gas gaps are generally accepted: Townsend’s breakdown criterion and the streamer breakdown criterion. Both criteria were originally developed to predict the breakdown voltage in a uniform field gas gap. However, they have been modified to predict the discharge inception or breakdown voltage for electrode systems providing non-uniform fields as engineering practice relates mainly to such configurations.

Both criteria are based on the occurrence of a critical avalanche that causes the instability in the gap, i.e. a generation process in the case of Townsend’s breakdown mechanism or an avalanche-to-streamer transformation in the case of the streamer breakdown mechanism.

The following section will describe both breakdown criteria in detail.

2.1.1 Breakdown mechanisms in the uniform field gap

When the DC voltage, irradiated by proper means, across an uniform field gas gap is increased very slowly, a small current can be measured with an instrument of high sensitivity. This current increases with increasing voltage until it reaches nearly a constant value known as saturation current (or photo current, $I_0$), provided that a constant level of irradiation is applied. Saturation occurs because all charged particles produced by the irradiation are collected by the electrodes with negligible loss to recombination or diffusion.
Some further increase in voltage causes a change in the current. This increase results from the increasing ionisation due to collisions between the accelerated electrons and the gas molecules. Ionisation by electron collision takes place when the amount of kinetic energy gained by electrons reaches a sufficiently high level, i.e. the kinetic energy of the accelerated electron is greater than the ionisation energy of the gas molecule.

The number of electrons or positive ions produced per unit length in this type of ionisation is known as ionisation coefficient by electron collision, $\alpha$, or Townsend's first ionisation coefficient. At constant temperature, $\alpha$ is dependent on $E$ and $p$ and the relationship of $\alpha/p = f(E/p)$ is unique for most types of gases.

For $n_0$ electrons, produced by radiation starting from the cathode, $n$ electrons are expected at the anode, with $n$ given by

$$n = n_0 e^{\alpha d}$$  \hspace{1cm} \text{(2.1)}$$

where

$$d = \text{gap spacing}.$$  

If only one electron would successfully start at the cathode, $e^{\alpha d}$ electrons will arrive at the anode. This process is called electron avalanche due to the fast increase in the electron number.

In terms of steady-state current, eq. 2.1 can be written as

$$I = I_0 e^{\alpha d}$$  \hspace{1cm} \text{(2.2)}$$

This current $I$ is called a non-self-sustaining current as it (nearly) disappears if the irradiation source is switched off.

An additional voltage increase may cause an over-exponential increase in current and finally result in breakdown. The cause of this eventually more than exponential increase is due to the occurrence of various secondary processes which take place either within the gas or at the cathode. When breakdown occurs, the current is essentially only limited by the external circuit and is not any more dependent on any external irradiation source. For this reason this current is called self-sustained current.

Townsend believed that the main secondary process would be due to the collision between positive ions and gas molecules. However, this explanation was very soon abandoned due to the fact that: 1) positive ions cannot efficiently ionise the gas at the magnitude of electric field at which breakdown occurs; 2) cathode material was shown to alter the breakdown threshold.
Alternative mechanisms, that can be used to explain this over-exponential current, are based on electrons liberated from the cathode. Two main processes are mentioned:

a) *Incidence of positive ions on the cathode:* In the electron avalanche process, positive ions are created at the same rate as new electrons. Positive ions will drift along the line of electric field to the cathode. When they reach the cathode, each one will have a certain probability to liberate an active secondary electron, in addition to the electron it requires for neutralizing itself. This probability will be designated as \( \gamma_i \). It varies with cathode materials and conditions.

b) *Cathode emission due to the incidence of photons:* In the course of electron avalanche formation, electrons will not only ionise gas molecules but also excite gas molecules and produce photons. The number of excited molecules is in any case larger than the number of ionised molecules due to the fact that, in general less electron energy is required to excite than to ionise molecules. A fraction of photons will reach the cathode (gas also may absorb photons) and release photo electrons with some efficiency. The probability that the impact of these photons liberate photo electrons will be designated as \( \gamma_p \).

Apart from the last mentioned secondary process at the cathode, secondary electrons can also be generated due to *photo ionisation* within the gas (Dutton et al., 1953; Penney and Voshall, 1962; Teich, 1967; Nasser, pp. 236-240, 1971; Kuffel and Zaengl, p. 341, 1984). The probability that photons ionise gas molecules will be designated as \( \gamma_p \).

These different secondary processes may be active at the same time in producing new electrons. It is customary to express these probabilities as a single generalized secondary ionisation coefficient, \( \gamma \), i.e. \( \gamma = \gamma_i + \gamma_p + \gamma_r + \ldots \). \( \gamma \) is often referred to as *Townsend's secondary ionisation coefficient* (Nasser, p. 240, 1971).

The new or secondary electrons from the secondary processes will form a larger electron avalanche. This process will repeat with increasing electron multiplication until breakdown is known to occur, i.e. the current is only limited by the external circuit. This mechanism is called *Townsend's breakdown mechanism* to honour Townsend. (Actually, only \( \gamma \) can be directly attributed to Townsend, but nevertheless his pioneering work laid the basic foundation for all these considerations.) However, if the number of secondary electrons is less than the number of initial electrons, \( n_0 \), the current will decrease from one generation to the other until the effect of feedback cease. Please also note, that Townsend's mechanism neglects the effect of space charge that is left behind by the preceding electron avalanche formation.
When the secondary processes are considered, the current equation in the form of eq. 2.2 can be written as (Sanders, 1933; Llewellyn Jones, pp. 5-8, 1956; Kuffel and Zaengl, pp. 342-343, 1984)

\[
\frac{I}{I_0} = \frac{n}{n_0} = \frac{e^{\alpha d}}{1 - \gamma (e^{\alpha d} - 1)}
\]

At the breakdown threshold, where current becomes independent of the external irradiation source,

\[
\gamma (e^{\alpha d} - 1) \geq 1
\]

which means that each one of \(n_0\) initial electrons must produce at least one successor by some means. Eq. 2.4 is also known as Townsend's breakdown criterion and can be used to predict the breakdown voltage in the uniform field gap.

Even if many of gaseous breakdown processes can be explained by using the Townsend's breakdown mechanism, particularly at high values of \(pd\), it cannot be used. Some of those breakdown phenomena in homogeneous fields are:

a) formative time lag of breakdown for high \(pd\)-values becomes much shorter than that are predicted by using Townsend's mechanism;

b) the spark channel was found to be both branched and zigzagged; (Nasser, p. 253, 1971)

and in addition for the case of non-uniform field: (Meek, 1940a)

b) breakdown voltage is not dependent on cathode material, e.g. with atmospheric air.

c) the cathode has no effect on positive point breakdown in long air gaps, e.g. with atmospheric air.

The difficulty to explain these phenomena by using Townsend's breakdown mechanism has led to the development of the streamer breakdown mechanism. In contrast to Townsend's breakdown mechanism which usually neglects the space charge effect, the streamer breakdown mechanism is based on the space charge effect.

In a uniform field, an electron avalanche develops from cathode to anode. The streamer criterion assumes that an electron avalanche reaches a critical state when the radial field produced by the positive ions has a magnitude comparable to the external applied electric field (Meek, 1940a) or the number of electrons in the avalanche head attains a certain value (Raether, 1964).

Meek (1942) described the breakdown process in terms of streamer formation in the gap as
Let us consider a plane-parallel gap of length $d$ cm across which a potential gradient $X$ volts per cm is applied. An electron leaving the cathode creates new electrons at the rate of $\alpha$ per cm advance in the field direction, so that when it has moved a distance $x$ cm the number of electrons created is $e^\alpha$. An equal number of $e^\alpha$ positive ions are left behind by the electron group, and owing to their low mobility they may be regarded as virtually stationary in comparison with the much-faster electrons. This cumulative process is appropriately termed an electron avalanche. At the instant when the head of the avalanche reaches the anode, and the electrons are swept to the latter, the positive ions remain in a nearly conical channel whose base is at the anode. Owing to the exponential character of the ionization in the electron avalanche the bulk of the positive ions are generated in the region close to the anode. This positive-ion concentration gives rise to a field not only in the direction of the avalanche but also in the direction radial to the axis, i.e. perpendicular to the external field. Thus photo-electrons produced in the gas in the immediate vicinity of the avalanche will not only be accelerated in the direction of the external applied field but will also be drawn into the stem of the avalanche. For a sufficiently high concentration of positive ions it is suggested that photo-electrons may be attracted from the surrounding space in sufficient numbers to originate a self-propagating streamer, which will then develop from anode to cathode to form a conducting filament bridging the electrodes and so cause breakdown of the gap. The criterion for the transition from an avalanche into a positive streamer has been stated by the present author as follows: A streamer will develop when the radial field about the positive space-charge in an electron avalanche attains a value of the order of the external applied field.

He also stated that the exact equality of the radial electric field strength, $E_r$, and the external applied field, $E$, need not to be insisted upon, and it is possible that the requisite criterion could be satisfied when $E_r$ is only 50 % of $E$. He derived the equation of breakdown based on the space charge density assuming that this positive space charge is concentrated in a spherical volume of radius $r_\infty$, equal to that of the avalanche.

The electric field $E_r$ at the surface of a spherical volume of uniformly distributed space charge is given by (note that the following derivation is based on Meek's paper (1940a), e.g. in CGS unit)

$$E_r = \frac{Q}{r_\infty^2} = \frac{4}{3} \pi r_\infty^3 Nq \cdot \frac{1}{r_\infty^2} = \frac{4}{3} \pi r_\infty^2 Nq$$

2.5
where

\[ E_r = \text{radial electric field at the surface of the sphere, V/cm} \]
\[ Q = \text{electrical charge, esu} \]
\[ N = \text{positive ions density, cm}^{-3} \]
\[ q = \text{electron charge} = 4.8 \cdot 10^{-10} \text{ esu} \]
\[ r_{sc} = \text{sphere radius, cm} \]

The positive ions formed at the tip, when the avalanche has progressed a distance \( x \) cm, is \( n_0 e^{\alpha x} \), if the avalanche started from cathode with \( n_0 \) primary electrons. In a distance \( dx \) the number of ions created is therefore \( \alpha \cdot n_0 e^{\alpha x} \cdot dx \). These ions are \textit{assumed} to be concentrated in a cylindrical volume of radius \( r_{sc} \) and of length \( dx \) so that

\[
N = \frac{\alpha \cdot n_0 e^{\alpha x} \cdot dx}{\pi r_{sc}^2 \cdot dx} = \frac{\alpha \cdot n_0 e^{\alpha x}}{\pi r_{sc}^2} \quad 2.6
\]

If \textit{only one} electron created this avalanche, eq. 2.5 can be written as

\[
E_r = \frac{4q \alpha e^{\alpha x}}{3 r_{sc}} \quad 2.7
\]

The radius \( r_{sc} \) is related to the distance \( x \) of avalanche travel by means of the diffusion equation and according to Raether

\[
r_{sc}^2 = 2D t = 2Dx/v \quad 2.8
\]

\[ D = \text{diffusion coefficient, cm}^2/\text{s} \]
\[ t = \text{travelling time from cathode to distance} \ x, \text{ s} \]
\[ v = \text{velocity of avalanche of electron, cm/s} \]

Since

\[
v = bE \quad 2.9
\]

where

\[ b = \text{mobility, cm}^2/(\text{Vs}) \]

by substituting eq. 2.8 and 2.9 into 2.7, we have

\[
E_r = \frac{4q \alpha e^{\alpha x}}{3 \sqrt{\frac{2D}{bE}}} \quad 2.10
\]

Both \( D \) and \( b \) are function of \( E/p \) and Meek approximated this two values with the expression
\[
\frac{D}{b} = \frac{P c_t^2}{F e^2}
\]

where
- \( P \) = atmospheric pressure = 760 Torr
- \( F \) = Faraday constant = 2.892\cdot10^{14} \text{ esu mol}^{-1}
- \( c \) = speed of thermal agitation of electron = 1.2\cdot10^7 \text{ cm/s}
- \( c_t \) = RMS velocity of the electron in the applied field, cm/s

The value of \( c_t \) is given by

\[
c_t^2 = 1.33 \frac{q E \lambda}{m \sqrt{f}}
\]

where
- \( \lambda \) = mean free path of the electron at gas pressure \( p \) Torr, cm
- \( m \) = mass of the electron = 9.1\cdot10^{-28} \text{ g}
- \( f \) = a factor which depends on the fractional loss of energy per collision

If \( \lambda_0 \) is the mean free path of an electron at atmospheric pressure, i.e. 760 Torr, the value of \( c_t \) at pressure \( p \) Torr will be, i.e. \( \lambda_0 p = \lambda_p \)

\[
c_t^2 = 1010 \frac{q E \lambda_0}{m p \sqrt{f}}
\]

The value of \( \lambda_0 \) and \( f \) depend on \( E/p \) and is not known for the high value of \( E/p \). Meek calculated, therefore, the value of \( f \) from Raether experiments. Raether found that the speed of electron avalanche was 1.25\cdot10^7 \text{ cm/s at } E/p \text{ equal to } 41 \text{ V/cm-Torr.}

From the relationship of

\[
b = \frac{\nu}{E} = 0.815 \frac{q \lambda}{m c_t}
\]

and eq. 2.12, the result of \( \lambda_0 \sqrt{f} \) is 5.7\cdot10^4. Note that eq. 2.14 is valid only for ions, but it is assumed to be applicable also for electrons. The mean free path \( \lambda_0 \) is taken as 3.6\cdot10^{-5}. Meek got, therefore, \( f = 0.025 \) and \( c_t = 2.01\cdot10^7 \sqrt{E/p} \). By substituting eq. 2.11 into 2.10, we have

\[
E_r = \frac{4 q a e^a x}{3 \sqrt{2 p c_t^2}}
\]

\[
= 5.56 \cdot 10^{-7} \frac{a e^a x}{\sqrt{x/p}}
\]
Note that Meek got the factor of 5.28 \times 10^{-7} instead of 5.56 \times 10^{-7} which may result from different value of Faraday constant in the equation. The unit of eq. 2.15 is in CGS unit where, \( E_r \) in V/cm, \( \alpha \) in cm\(^{-1} \), \( x \) in cm and \( p \) in Torr. The result of this equation in MKS unit where \( E_r \) in kV/mm, \( \alpha \) in mm\(^{-1} \), \( x \) in mm and \( p \) in bar is

\[
E_r = 4.815 \cdot 10^{-8} \frac{\alpha e^{\alpha x}}{\sqrt{x/p}} \tag{2.16}
\]

Now for \( x = d \), the gap length for a plane parallel gap, application of the criterion, that \( E_r = E \), gives

\[
E = 4.815 \cdot 10^{-8} \frac{\alpha e^{\alpha d}}{\sqrt{d/p}} \tag{2.17}
\]

This equation can be used to determine the breakdown voltage of uniform field gap by using iteration method. Meek calculated the \( U_b \) of 10 mm gap in air at atmospheric pressure, i.e. 1.013 bar. Eq. 2.16 satisfied for \( E/p = 3.18 \text{ kV/mm-bar} \ (\alpha = 1.86) \), i.e. \( E = 3.22 \text{ kV/mm} \), which he claimed that it is in close agreement with the observed value of 3.15 kV/mm. Note that Meek has not given the source of \( \alpha \) values though throughout his literature he referred to those from Sanders (1933).

Pedersen (1967a) said that the term \( \exp(\alpha d) \) is the dominating factor and can be treated as a constant with the value of about \( 10^8 \) for air at atmospheric pressure. Eq. 2.17 can thus be written as

\[
\alpha d = K = 18 \tag{2.18}
\]

A full discussion on this statement of Pedersen and on Meek's streamer breakdown criterion will be given later in section 5.6.

In the case of Raether (pp. 126 - 127, 1964), he described streamer formation in the gap as

"The avalanche develops in the normal way until the carrier number leads to a space charge concentration and thus to an additional space charge field in the region of the head of the avalanche,... This field leads at carrier numbers of \(~10^6 - 10^8\),......, to a reduction of ionization efficiency. In later stages at carrier numbers of the order of \( 10^8 \) - however still in the region where \( \alpha_{\text{eff}} < \alpha_0 \) (as has been observed with the optical method) - the anode directed streamer starts. (Such a high carrier number has been reached in air, because an overvoltage of about 10 per cent had been used.) This additional space charge field is regarded as the reason for the transformation of the avalanche into the anode directed plasma streamer."
The empirical condition for streamer formation, according to Raether, as above mentioned was

$$\alpha x_c = 20$$  \hspace{1cm} 2.19

Raether (p. 131, 1964) said clearly that other experiments in different gases verified that this equation is of general validity at usual conditions: $d \sim 1 \text{ cm}$ and $p \sim 10^2 \text{ Torr}$. It is to be wondered why later investigators still used this equation to determine breakdown voltage at large gap spacing and/or at the different pressure range.

The above expression can also be derived mathematically in the following way. Let assume that all the electrons of an avalanche are contained in a spherical volume at the tip of the avalanche. The number of electron is $e^{\alpha x}$ where $x$ is the avalanche length in the uniform field $E$. At the surface of the sphere when the avalanche reached its critical size, the space charge field is

$$E_r = \frac{q e^{\alpha x}}{4 \pi \varepsilon_0 r_{xc}^2}$$  \hspace{1cm} 2.20

where

- $E_r =$ space charge field at the sphere surface, V/cm
- $q =$ electron charge $= 1.6 \cdot 10^{-19} \text{ C}$
- $\varepsilon_0 =$ permittivity of free space $= 8.85 \cdot 10^{-14} \text{ F/cm}$
- $r_{xc} =$ radius of the avalanche tip, cm

Note that $E_r$ of Raether differs from Meek, eq. 2.5. The major difference is that eq. 2.20 is based on number of electron while eq. 2.5 is based on density of positive ion and both are determined from different assumption. The radius of avalanche tip is related to diffusion coefficient as

$$r_{xc}^2 = 4D t = 4 \frac{D x_c}{b E}$$  \hspace{1cm} 2.21

Note also that the radius of avalanche tip in eq. 2.21 according to Raether is different from Meek in eq. 2.8. Raether used

$$D = \frac{\lambda c_{sh}}{3}$$  \hspace{1cm} 2.22

and

$$q U_{sh} = \frac{1}{2} m c_{sh}^2$$  \hspace{1cm} 2.23
where
\[ \lambda = \text{mean free path, cm} \]
\[ c_{th} = \text{thermal velocity of the carriers, cm/s} \]
\[ U_{th} = \text{electron energy, V} \]
\[ m = \text{electron mass} = 9.1 \cdot 10^{-28} \text{ g} \]

to find \( r_{xc} \) in equation 2.21. Note that \( D \) in eq. 2.22 is derived from the Einstein equation, which is for ions. Raether extended the Einstein equation to the case of electrons under the assumption that it holds good also for electrons. From eq. 2.22 and the relation \( b = \frac{q\lambda}{mc_{th}} \) (Raether, p. 57, 1964), we have

\[
\frac{D}{b} = \frac{mc_{th}^2}{3q} \tag{2.24}
\]

and from eq. 2.23

\[
\frac{D}{b} = \frac{2}{3} U_{th} \tag{2.25}
\]
or

\[
r_{xc}^2 = \frac{8}{3} U_{th} \frac{x_c}{E} \tag{2.26}
\]

Substituting this equation into eq. 2.20, we have

\[
E_r = \frac{q e^{\alpha x_c}}{4\pi \varepsilon_0 \frac{8}{3} U_{th} x_c} E \tag{2.27}
\]

According to Raether, if we assume very roughly that the streamer starts, if the space charge field \( E_r \) equals to the applied field \( E \) and select \( U_{th} = 1.5 \text{ V} \), one have

\[
e^{\alpha x_c} = \frac{4\pi \varepsilon_0 \frac{8}{q} U_{th} x_c}{3} \]

\[
= 3 \cdot 10^7 x_c \tag{2.28}
\]

However, Raether used a factor of \( 10^8 \) instead of \( 3 \cdot 10^7 \). Raether also said that

... the above equation represents also a condition for streamer breakdown. It is not suitable on account of its approximative character to calculate breakdown voltage from it, but it gives the possibility of saying that an avalanche which grows to amplification of~10^8 will very probably start a streamer breakdown. In this concept \( x_c = d \) gives the smallest
value of $\alpha$ (or of the applied field) to produce streamer breakdown; one should therefore expect a streamer mechanism at static voltage if equation 5.30 is fulfilled with $x_c = d$ (gap distance).

Eq. 5.30 in the case of Raether is eq. 2.28 above. Eq. 2.28 can, therefore, be written as

$$e^{\alpha d} = 3 \cdot 10^7 d$$

when $\alpha$ in cm$^{-1}$ and $d$ in cm.

A discussion on Raether's streamer breakdown criterion will be given later in section 5.6.

From eq. 2.18 and 2.19, we can roughly speak that the avalanche to streamer transformation occurs when

$$\alpha d = K = 18 - 20$$

Many gases, such as O$_2$ and SF$_6$, have the ability to capture electrons by the dissociative electron attachment process to form negative ions, which lead to a reduction of electrons in the avalanche. These gases are known as electronegative gases (or attaching gases). It will be called strongly negative if it has a high ability to capture the electrons.

Air is also an electronegative gas because it contains about 20% of O$_2$. It is a weak electronegative gas because its ability to capture electrons is not very high.

The attachment process relatively increases the breakdown voltage of some strong electronegative gases, e.g. Bhalla and Craggs (1962) found that $U_b$ of SF$_6$ in a uniform field was 2.8 times higher than $U_b$ of air at 5 bar-mm. However, electrons can also be detached from the negative ions by photo or collisional detachment processes. The detached electrons can, therefore, act in the same way as secondary electrons in secondary processes.

In the past, since the gas discharge theory was developed by Townsend, the presence of the attachment process was not always recognized, because the measurements of $\alpha/p = f(E/p)$ in electronegative gases, especially in air, were performed at a high value of $E/p$ where the electron energy is high, i.e. positive ion formation dominates. Only later, it was observed (Harrison and Geballe, 1953) that the In ($I/I_0$) against $d$ curves (see eq. 2.2) show a non-linear relation, at small values of $E/p$, which is caused by the occurrence of attachment process.

Townsend's breakdown criterion, eq. 2.4, taking the effect of electron attachment into account, can be then written as

$$\frac{\alpha \gamma}{\alpha - \eta} \left[ e^{(\alpha - \eta) d} - 1 \right] = 1$$
or

\[
\frac{\alpha \gamma}{\bar{\alpha}} [e^{\bar{\alpha}d} - 1] = 1 \quad 2.33
\]

where

\( \eta \) = attachment coefficient
\( \bar{\alpha} \) = number of electrons attached to gas molecule per unit length
\( \alpha \) = effective ionisation coefficient = \( \alpha - \eta \)

And the streamer breakdown criterion, eq. 2.31, can be written as

\[
(\alpha - \eta)d = K \quad 2.34
\]

### 2.1.2 Breakdown criterion in non-uniform fields

The electron multiplication process in non-uniform fields is based on the variation of the electric field with the position in the gap space, i.e. \( E = f(x) \) where the x axis coincides with the field direction. The electron multiplication \( e^{\alpha d} \) of uniform fields is thus replaced by \( \exp(\int_a^b \alpha \, dx) \) in a non-uniform field.

For a highly stressed cathode, the integral limit \( a \) is the position at the electrode surface, i.e. \( x = 0 \), and \( b \) is a position in space where the ionisation by collision is reduced to zero. For a highly stressed anode, the reverse is true.

Since the integral \( \int_a^b \alpha \, dx \) is independent of the field direction and also of the avalanche size, it is convenient to write this integral for non-attaching gases as

\[
\int_0^{x_c} \alpha \, dx
\]

where

\( x_c \) = critical avalanche length
\( x = 0 \)

Hence, in an attaching gas, the streamer breakdown criterion can be written as

\[
\int_0^{x_c} (\alpha - \eta) \, dx \geq K \quad 2.35
\]

and Townsend's breakdown criterion can be written as (Pedersen, 1970)

\[
\gamma \int_0^d e^{\int_0^b (\alpha - \eta) \, dx} \alpha \, dx = 1 \quad 2.36
\]
Please note that for a highly stressed anode in a highly non-uniform field, eq. 2.36 may lose its validity if \( \gamma \) is related only to the secondary process at the cathode alone. The electric field near the cathode under this circumstance may be low such that \( (\alpha - \eta) \) is negative. This means that an electron avalanche cannot start from the cathode, i.e. the secondary process at the cathode cannot lead to new generation of avalanches (Pedersen, 1970). However, breakdown or the inception of a discharge is still possible if the secondary processes in the gas, such as photo ionisation, are considered.

### 2.2 Applications of Townsend's and the streamer breakdown criteria review

Application of both Townsend's and the streamer breakdown criterion to predict \( U_b \) in uniform fields and \( U_t \) or \( U_b \) in non-uniform fields are briefly reviewed in this section. The modifications of the right hand side of the original criteria by each investigator are presented and the disadvantages of those modifications are pointed out.

In the following subsection, \( d \) will always refer to the gap distance between electrodes in uniform or slightly non-uniform fields and \( r \) will refer to the inner conductor radius for coaxial cylinder geometry, the inner sphere radius for coaxial sphere geometry, the radius of rod tip for the rod to plane geometries, and the sphere radius for sphere to sphere and sphere to plane geometries.

#### 2.2.1 Townsend’s breakdown criterion in uniform fields

In non-attaching gases (and also, in the past, for air when the investigators have not yet been aware of the attachment process), Townsend’s breakdown criterion for uniform fields, eq. 2.4, can be rearranged and written as

\[
\alpha d = \ln \left( \frac{1}{\gamma} + 1 \right)
\]

and for an attaching gas, eq. 2.33, as

\[
(\alpha - \eta)d = \ln \left( \frac{1}{\gamma} \frac{(\alpha - \eta)}{\alpha} + 1 \right)
\]

When W.O. Schumann (Chapter 3, 1923) calculated the breakdown field strength of atmospheric air based on breakdown experiments, which have been available at his time, he concluded that the right hand side of eq. 2.37 could be assumed to be a constant \( K \). (Note that Schumann used the secondary process that was described by Townsend, i.e. due to the collision between positive ions and gas molecules.)
This can be done because eq 2.37 is somewhat insensitive to the value of $\gamma$ at high pressure (Meek, 1940a) and the logarithm of $\gamma$ changes much more slowly than the absolute change in $\gamma$.

Equation 2.37, also known as Schumann's condition if the right hand side is constant, is therefore formally identical to eq. 2.31, i.e. the streamer breakdown criterion.

Honda (1965) investigated Schumann's condition in atmospheric air at various gap lengths ($2.5 < E/p < 3 \text{ kV/bar}\cdot\text{mm}$). He calculated $K$ from his experiment and using values of $\alpha/p = f(E/p)$ which were similar to those of Sanders (1933). His calculation showed that $K$ varied in the range between 17 to 50. By using $K = 20$ in the calculation, however, the calculated results deviated from the experiment by less than 10%. He modified Schumann's condition by replacing $d$ with $x_0 = f(d)$ to make the calculation results agree with experiments while he kept $K$ constant, i.e. $\alpha \cdot x_0 = 20$.

Heymann (1965) also showed that the right hand side of eq. 2.38 may depend on $pd$ in air. He used intermediate values of $(\alpha - \eta)/p = f(E/p)$ from those of Harrison and Geballe (1953) and Prasad and Craggs (1960) to calculate the right hand side of eq. 2.38 based on experiments of Prasad and Craggs, Kuffel, and Köhrmann (references are given in Heymann, 1965). The results showed that the magnitude of the right hand side of eq. 2.38 varies linearly with $pd$ in the range between 7 - 15, for $1 < pd < 20 \text{ bar}\cdot\text{mm}$. He then suggested a constant value of 10 in uniform field for the purpose of determining an empirical relation of breakdown in air as has been done by Schumann.

### 2.2.2 Townsend's breakdown criterion in non-uniform fields

In 1941, Ver Plank modified Townsend's breakdown criterion to predict inception voltage, $U_i$, or breakdown voltage, $U_b$, of the coaxial cylinder and concentric sphere geometry in air. In his modification, he assumed that breakdown is only initiated by the secondary processes at the cathode, i.e. by positive ion bombardment and photo emission at the cathode. Only positive ion bombardment will be discussed in this section. The photo emission, which leads to the same result as positive ion bombardment according to Ver Plank, can be found in his literature.

Ver Plank explained that the probability that positive ions striking the cathode will cause an electron to be released is a function of the kind of surface and the energy of the impinging ions, i.e. $E/p$ at the cathode. The probability that a released electron would escape diffusion back to cathode or not become attached to a neutral molecule and thus is able to start an avalanche would also be dependent on $E/p$. Therefore, the number $N$ of positive ions which must strike the cathode to assure the initiation of one secondary avalanche is a function of $E/p$ at the cathode and is independent
of the electric field non-uniformities. The condition for breakdown is that the number of positive ions left by the previous avalanche must be equal or exceed the number \( N \) which is

\[
\int_0^s e^\alpha ds \geq N \tag{2.39a}
\]

or

\[
\int_0^s \alpha ds \geq \ln(N) \tag{2.39b}
\]

where

\[ S = \text{gap spacing} \]

for non-uniform field geometry. The integration is taken from the cathode, \( s = 0 \), along the line of electric field to the anode, \( s = S \). However, for non-uniform field gaps, if \( E/p \) becomes less than about 1.5 kV/bar-mm (according to Ver Plank) before the anode is reached, the upper limit may be taken at the point where \( E/p = 1.5 \) kV/bar-mm because \( \alpha = 0 \) for \( E/p < 1.5 \) kV/bar-mm. Note that \( \alpha/p = f(E/p) \) used by Ver Plank is never negative. He also realized that there is an attachment process in air, but he neglected its effect on breakdown.

According to Ver Plank, eq. 2.39a implies that the breakdown occurs when \( \exp(\int_0^s \alpha ds) \), the number of electrons in an avalanche which has crossed the gap, equals or exceeds \( N \). This statement of Ver Plank makes eq. 2.39a exactly the same as the Raether’s streamer breakdown criterion.

Since \( N \) is a function of \( E/p \) and independent of gap geometry, \( N \) can be found from uniform field breakdown measurements, i.e., \( \ln(N) = \alpha S = f(E/p)pS \). With the data of \( \alpha/p = f(E/p) \) from Townsend, Wheatley, Paavola, Masch, and Sanders (see reference in Van Plank, 1941), he could calculate \( \ln(N) \) as a function of \( E/p \) at the cathode in the range of \( 1 < E/p < 100 \) kV/mm-bar. Those results showed that \( \ln(N) \) varied in the range between 3 - 40. He said that when positive ion bombardment is the predominant secondary mechanism, the reciprocal of \( N \) is identical to \( \gamma \).

The value of \( \ln(N) \) increases sharply with \( E/p \) beyond \( E/p \) lower than 4 kV/bar-mm but decreases slowly with \( E/p \) as \( E/p \) becomes greater than 30 kV/bar-mm. In the range of \( 4 < E/p < 30 \) kV/mm-bar, however, \( \ln(N) \) varies only 9 - 6. The variation of \( \ln(N) \) outside the range of \( 4 < E/p < 30 \) kV/mm-bar can be attributed to the discrepancy of \( \alpha/p = f(E/p) \) which was used by Ver Plank (1941) in the calculation of \( \ln(N) \). Such a tendency of \( \ln(N) \) with \( E/p \) was also found in 1974 by Zaengl and Nyffenegger (Zaengl and Nyffenegger used \( K \) instead of \( \ln(N) \)).
At low $E/p$ near the point where $\alpha$ approaches zero, $\alpha$ varies rapidly with the field strength. Erroneous $\alpha$-values will, therefore, have a large effect on the calculation of $\ln(N)$. At high $E/p$, where only a few measurements of $\alpha/p = f(E/p)$ has been made, we know that $\alpha$-values have inherent errors because the electron swarm may not attain a steady state when it left the cathode (Dutton, 1975).

Now, the condition for breakdown according to VerPlank in eq. 2.39b can be written as

$$\int_0^s \alpha ds = f(E/p)_{at\ cathode} \quad 2.40$$

VerPlank applied this modified equation to calculate the voltage threshold of coaxial cylinder and coaxial sphere configurations (negative inner conductor or sphere, respectively), the results of which agreed very well with the experimental values.

Waters and Stark (1975) studied positive discharges in atmospheric air with coaxial cylinder electrode. They observed that a positive corona discharge may occur in two forms: Streamer discharge and glow discharge. They also found in their investigations that the glow discharge could be established for all conductor radii, $6 < r < 32$ mm, with irradiation from a $\beta$-source.

They used a simplified form of Townsend’s breakdown criterion in non-uniform fields

$$\int_{r_0}^r (\alpha - \eta)dr = \ln\left(\frac{1}{\gamma} + 1\right) \quad 2.41$$

where

- $r$ = radius of the inner conductor
- $r_0$ = position in space where $(\alpha - \eta) = 0$, measured from the center of the inner conductor

To calculate $U_c$. The results of calculation using a constant value of $\gamma = 10^4$, i.e. $\ln\left(\frac{1}{\gamma} + 1\right) = 9.2$, and values of $(\alpha - \eta)/p = f(E/p)$ from Harrison and Geballe (1953) agreed well with the experimental results in the range $6 < pr < 30$ bar-mm. Again, eq. 2.41 is formally identical to eq. 2.35, i.e. the streamer breakdown criterion.

2.2.3 Streamer breakdown criterion in uniform and slightly non-uniform fields

Meek used his streamer breakdown criterion, eq. 2.17, to determine the breakdown voltage in air at high $pd$-values down to about 2 bar-mm. For lower values of $pd$,
his calculations showed some deviations from the experimental values. This deviation was explained by a reduction in photo ionisation efficiency in the gas and by secondary processes at the cathode.

Honda (1965), apart from analyzing the Schumann condition, analyzed Meek's streamer criterion, eq. 2.17. From his experiment, he concluded that in uniform fields the electron avalanche could proceed to only a fraction of gap length which was equal to the avalanche length, \( x_c \), when it transformed to streamer. He showed that \( x_c \) was very close to \( x_0 \) (see section 2.2.1) in the range of gaps between 3 to 20 cm and concluded that \( x_0 \approx x_c \).

Honda concluded that if the breakdown is based on Townsend's mechanism then Schumann's condition, i.e. \( \alpha \delta = K \), was true. If \( \alpha \delta \) was large the breakdown is of streamer type, then Schumann's condition would still be valid if \( \delta \) was replaced by \( x_c \), i.e. \( \alpha \cdot x_0 = \alpha \cdot x_c = K \).

Pedersen (1967a; 1967b) modified the streamer breakdown criterion of eq. 2.17, because it was not possible to include any realistic quantities depending on photo ionisation which might be responsible for the formation of the streamer. His modified equation for air was of the form

\[
\alpha_{x_c} e^{\int_0^{x_c} \alpha dx} = G \{ x_c, p, f(E_x), \mu, \%H_2O, \ldots \}
\]  \quad 2.42

where

\[
G = \text{unknown function of } x_c, p, \text{ field distribution } f(E_x), \text{ photo ionisation in gas } \mu, \text{ humidity } \%H_2O, \text{ and other possible variables.}
\]

\[
\alpha_{x_c} = \text{Townsend's first ionisation coefficient at the avalanche head.}
\]

However, he neglected all other variables except \( x_c \) and \( p \) because these two would be the dominating variables. He wrote eq. 2.42 for air at atmospheric pressure as

\[
\ln(\alpha_{x_c}) + \int_0^{x_c} \alpha dx = g(x_c)
\]  \quad 2.43

He claimed that the numerical value of \( g(x_c) \) can be found from uniform field experiments for which \( x_c = d \) and \( \alpha \) is constant, i.e. the left hand side of eq. 2.43 could be determined from \( E_b \) and \( \alpha/p = f(E/p) \). He accepted, however, that if \( g(x_c) \) is calculated from the uniform field experiments, the field distortion caused by space charge in the avalanche head is neglected.
Pedersen applied eq. 2.43 to calculate the breakdown voltage of a standard sphere gap (only for 25 cm sphere diameter and a few gap spacings). The calculated results, by using \( \alpha/p = f(E/p) \) from Sanders (1933), agreed very well with the values tabulated in IEC-52:1960 (The discussion of his calculations will be given in section 6).

One can note that eq. 2.43 can be rewritten as

\[
\int_0^{x_c} \alpha \, dx = g(x_c) - \ln(\alpha_{x_c}) = H(E)_{\text{at avalanche head}} \quad 2.44
\]

The function \( g \) is related to \( x_c \), which can in turn relate to \( E \), and \( \alpha \)-value is also related to \( E \). We can conclude that both terms, i.e. function \( g(x_c) - \ln(\alpha_{x_c}) \), are actually a function of \( E \) which can be determined together from the uniform field breakdown experiments in air at atmospheric pressure (Pedersen determined only \( g \)). One can immediately notice that eq. 2.44 has the same form as eq. 2.40 of Ver Plank (1941). This modified streamer breakdown criterion of Pedersen is, therefore, similar to that of Ver Plank (1941) even though physical processes assumed are different.

In 1970, Pedersen modified eq. 2.43 to determine the impulse breakdown voltage of sphere gaps in SF\(_6\). He reasoned that eq. 2.43 should show a polarity effect in moderate non-uniform fields because the variation of \((\alpha - \eta)\) with electric field in SF\(_6\) is very high. Note that \( \ln(\alpha_{x_c}) \), in eq. 2.43, will change with polarity because the avalanche develops into the low field region when the highly stressed electrode is a cathode; but if this electrode is anode it develops into the high field region. The experiment of Howard (1957), however, showed that there was little difference between the positive and negative impulse breakdown voltage values, i.e. no polarity effect.

In this modification, Pedersen took therefore photo ionisation into account and derived the equation

\[
\frac{1}{2} \ln(\alpha_{x_c}) + \int_0^{x_c} (\alpha - \eta) \, dx = f(x, p) \quad 2.45a
\]

The function \( f(x, p) \) must now be evaluated from the impulse breakdown in highly uniform field. Such data had, however, not been available at his time. Pedersen reasoned that, at threshold, the integral term in eq. 2.45a was dominating and would attain an almost constant value independent of \( x_c \) and \( p \). The condition for streamer breakdown in SF\(_6\) was therefore fulfilled if
\[ \int_0^\infty (\alpha - \eta) \, dx = K \]  \hspace{1cm} 2.45b

which is exactly the same as eq. 2.35. He proposed a value of \( K = 18 \) and said that this equation could be interpreted that breakdown occurs when the total number of electrons in an avalanche attained a certain value, \( N_\sigma \).

Pedersen applied eq. 2.45b to calculate the impulse breakdown voltage between sphere gaps in SF\(_6\) in the pressure range between 1 to 4 bar. His calculation results are in good agreement with the experimental results of Howard (1957).

In 1971, Takuma pointed out that even if eq. 2.43 would give a very small error in the calculation, the validity of eq. 2.43 could not be confirmed, due to the following arguments:

1. Generally, the measured values of \( (\alpha - \eta)/p = f(E/p) \) are erroneous and show significant scatter. But he accepted that eq. 2.35 gives surprisingly accurate values of \( U_t \) in spite of the errors in the \( (\alpha - \eta)/p = f(E/p) \) measurements.

2. The integral term in eq. 2.43 is very large, therefore the first term on the left hand side and the function \( g \) will not have a large effect on the sparking voltage.

To confirm his arguments, he used the streamer breakdown criterion, eq. 2.35, to calculate the sparking voltage of standard sphere gaps according to IEC-52:1960, for all standard sphere diameters but for only a limited number of gap spacings. He used \( K = f(d) \) as

\[ K = 13.0 + \log(d) \]  \hspace{1cm} 2.46

which was derived from the uniform field breakdown experiments. His calculation results are in good agreement with IEC-52:1960. One can note, however, when Takuma calculated \( f(\alpha - \eta)dx; \)

1. He used the electric field of two spheres in space, without stem and ground.

2. He approximated the curve of \( (\alpha - \eta)/p = f(E/p) \) which was a quadratic function by a piece-wise straight line.

In 1981, Malik showed the dependence of \( K \) on \( pd \) for the uniform field breakdown in air, \( N_2 \), and SF\(_6\). He pointed out that at high pressure, \( pd \approx 10 \) bar-mm, the calculated value of \( K \) could be affected by \( (\alpha - \eta)/p = f(E/p) \), i.e. using different relations of \( (\alpha - \eta)/p = f(E/p) \) in the calculation. If we had an exact knowledge of \( (\alpha - \eta)/p = f(E/p) \), the measuring errors of breakdown voltage would still affect the value of \( K \).
Again, Pedersen et al. (1984) analyzed the streamer breakdown criterion, eq. 2.35. He showed that $K \neq \ln(N_a)$. He proposed that $K$ is 10.5 in SF$_6$. He used an indirect method to calculate $K$ from the uniform field breakdown data (Boyd and Crichton, 1972) with the knowledge of $(\alpha - \eta)/p = f(E/p)$. He said that this $K$ value of SF$_6$ might not be pertinent to other strongly electronegative gases and that for a weak or non-attaching gas $K$ need not be a constant value.

Recently, Zaengl et al. (1991) investigated breakdown voltage in nearly uniform field in synthetic air in the range of $1 < pd < 12$ bar-mm. Comparison of the calculation results by using the streamer breakdown criterion, with a constant value of $K = 9.15$, agreed within $\pm 1\%$ with the experimental results.

2.3 Present investigation

Tables 2.1 and 2.2 summarize the previous investigations that have been reviewed in section 2.2. Over a long period of time, both Townsend's and the streamer breakdown criterion have been modified in various ways. The modifications were made on the right hand side, $K$, of both breakdown criterion, e.g. eq. 2.35 and 2.38, to take into account the effect of $p$, $d$ and/or $r$. The $pd$- (or $pr$-) values were in the range of 0.01 to 1000 bar-mm for air and 0.01 to 400 bar-mm for SF$_6$.

For application of the streamer breakdown criterion, we can note that only gap geometries of uniform field and slightly non-uniform field types have been investigated.

To determine the value of $K$ in any gas, we have to know $(\alpha - \eta)/p = f(E/p)$ very accurately. As it can be seen from Tables 2.1 and 2.2, most researchers used different sources for their $(\alpha - \eta)/p$-values together with their own experimental data or even data from other sources. Only Pedersen et al. (1984), for SF$_6$, and Zaengl et al. (1991), for synthetic air, used their own values of $(\alpha - \eta)/p = f(E/p)$ and their own experimental breakdown data. Surprisingly, for both gases the value of $K$ seemed to be constant with a value of about 10.

Experiments to quantify $K$ with uniform field geometry are possible but an accurate value of $K$ is difficult to obtain, especially for SF$_6$. This can be shown by taking the data of $U_b$ in SF$_6$ from Boyd and Crichton (1972) as an example. In the range $0.5 < pd < 8$ bar-mm ($9.6 > E/p > 8.9$ kV/mm-bar) $U_b$ could be expressed as

$$U_b = A + B \cdot pd$$

where

$A = 0.376$ kV

$B = 8.845$ kV/mm-bar

Note that $p$ is the gas pressure at 20 °C.
Table 2.1  Summary of the literature reviewed in section 2.2.1 and 2.2.2 on Townsend’s breakdown criterion.

<table>
<thead>
<tr>
<th>Literature</th>
<th>$K$</th>
<th>$pr, pd$ (bar-mm)</th>
<th>$\alpha/p$ or $\bar{\alpha}/p = f(E/p)$ from</th>
<th>Gas</th>
<th>Geometry</th>
<th>Effect from</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schumann (1923)</td>
<td>-</td>
<td>Atmosphere</td>
<td>-</td>
<td>Air</td>
<td>Uniform field</td>
<td>-</td>
</tr>
<tr>
<td>Ver Plank (1941)</td>
<td>3 - 40</td>
<td>0.01 $&lt; pr &lt; 1000$</td>
<td>Townsend, Wheatley, Paavola, Masch, Sanders</td>
<td>Air</td>
<td>Coaxial cylinder</td>
<td>E/p at cathode</td>
</tr>
<tr>
<td>Heymann (1965)</td>
<td>7 - 15</td>
<td>1 $&lt; pd &lt; 20$</td>
<td>Prasad &amp; Craggs, Harrison &amp; Geballe</td>
<td>Air</td>
<td>Uniform field</td>
<td>$pd$</td>
</tr>
<tr>
<td>Honda (1965)</td>
<td>17 - 50</td>
<td>10 $&lt; pd &lt; 200$</td>
<td>similar to Sanders</td>
<td>Air</td>
<td>Uniform field</td>
<td>$d$</td>
</tr>
<tr>
<td>Waters &amp; Stark (1975)</td>
<td>9.2</td>
<td>6 $&lt; pr &lt; 30$</td>
<td>Harrison &amp; Geballe</td>
<td>Air</td>
<td>Coaxial cylinder</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2.2  Summary of the literature reviewed in section 2.2.3 on the streamer breakdown criterion.

<table>
<thead>
<tr>
<th>Literature</th>
<th>$K$</th>
<th>$pr, pd$ (bar-mm)</th>
<th>$\alpha/p$ or $\bar{\alpha}/p = f(E/p)$ from</th>
<th>Gas</th>
<th>Geometry</th>
<th>Effect from</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meek (1940a)</td>
<td>20?</td>
<td>1.3 $&lt; pd &lt; 130$</td>
<td>Sanders</td>
<td>Air</td>
<td>Uniform field</td>
<td>-</td>
</tr>
<tr>
<td>Raether (1964)</td>
<td>18 + ln($d$)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Uniform field</td>
<td>-</td>
</tr>
<tr>
<td>Pedersen (1967)</td>
<td>?</td>
<td>5 $&lt; pd &lt; 10$</td>
<td>Sanders</td>
<td>Air</td>
<td>Sphere gap</td>
<td>$g(x_s) = -\ln(\alpha_s)$</td>
</tr>
<tr>
<td>Pedersen (1970)</td>
<td>18</td>
<td>5 $&lt; pd &lt; 40$</td>
<td>Bhalla and Craggs</td>
<td>SF$_6$</td>
<td>Sphere gap</td>
<td>-</td>
</tr>
<tr>
<td>Takuma (1971)</td>
<td>13 + log($d$)</td>
<td>2 $&lt; pd &lt; 400$</td>
<td>Geballe</td>
<td>Air</td>
<td>Sphere gap</td>
<td>$d$</td>
</tr>
<tr>
<td>Malik (1981)</td>
<td>1 - 70</td>
<td>0.1 $&lt; pd &lt; 200$</td>
<td>?</td>
<td>Air</td>
<td>Uniform field</td>
<td>$pd$</td>
</tr>
<tr>
<td>Malik (1981)</td>
<td>12 - 20</td>
<td>0.01 $&lt; pd &lt; 10$</td>
<td>?</td>
<td>SF$_6$</td>
<td>Uniform field</td>
<td>$pd$</td>
</tr>
<tr>
<td>Pedersen et al. (1984)</td>
<td>10.5</td>
<td>0.5 $&lt; pd &lt; 8$</td>
<td>Boyd &amp; Crichton, Teich</td>
<td>SF$_6$</td>
<td>Uniform field</td>
<td>-</td>
</tr>
<tr>
<td>Zaengl et al. (1991)</td>
<td>9.15</td>
<td>1 $&lt; pd &lt; 12$</td>
<td>Friedrich</td>
<td>Synthetic air</td>
<td>Sphere-Sphere</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 2.1 shows the effect of \((\alpha - \eta)/p = f(E/p)\) on the \(K\) values which have been calculated by using \((\alpha - \eta)/p = f(E/p)\) from Aschwanden (1985) and Boyd and Crichton (1971). We can see that for \(pd\)-values larger than 4.7 bar·mm, \(K\)-values calculated from Aschwanden's data become negative which is meaningless. The negative values of \(K\) occur because \((\alpha - \eta)/p = f(E/p)\) from Aschwanden has a \(E_c/p\) higher than \(E_u/p\) from the experiment of Boyd and Crichton. This also happened with Pedersen (1984) when he used \((\alpha - \eta)/p = f(E/p)\) from Boyd and Crichton (1971) and Teich (1981) to evaluate \(K\) from Cigré's (Dakin et al., 1974) uniform field breakdown data in SF₆.

Figure 2.1 The influence of \((\alpha - \eta)/p = f(E/p)\) on the value of \(K\), based on the uniform field breakdown experiment in SF₆ from Boyd and Crichton (1972).

One can see the strong interaction between the physical data of a gas, \((\alpha - \eta)/p = f(E/p)\), and the breakdown experiments to quantify \(K\). Therefore, we need a very precise knowledge of \((\alpha - \eta)/p = f(E/p)\) and very accurate breakdown experiments. The main factor that influences the accuracy of breakdown voltage measurements is the quality of the electric field in the uniform field gaps. Many publications discussed such an effect of field non-uniformity on the determination of breakdown voltages, especially in electronegative gases (Harrison, 1967; Pearson and Harrison, 1969; Pedersen et al., 1971; McAllister, 1984).
Up to now, for non-uniform fields, many successful applications of the streamer breakdown criterion to determine \( U_b \) (or \( U_i \)) are based on \( K = 18 \) (Nitta and Shibuya, 1970; Bortnik and Cooke, 1972; Cooke, 1975; Hazel and Kuffel, 1976; Somerville and Tedford, 1979; Li and Kuffel, 1989). However, these applications were performed at high pressures in a \( pr \)-range where \( K \) has only little effect on \( U_b \) (or \( U_i \)), and were mostly performed in SF\(_6\).

Considering the difficulty to determine an accurate value of \( K \) based on experiments in uniform or slightly non-uniform field described previously, and also considering the fact that engineering practices are mainly related to non-uniform field gaps, the present investigation is restricted to non-uniform fields at low \( pr \)-range where the effect of \( K \) becomes large (see figure 1.1 in section 1.2). This investigation is carried out both in air and SF\(_6\) because of their importance in engineering applications.
3. INCEPTION VOLTAGE CALCULATION

The following information is required for the determination of \( U_i \) in non-uniform field geometry by using the streamer breakdown criterion, as given in eq. 2.35

1. The precise information of electric field strength in the electrode geometry, \( E(x) \), along the line that provides the largest value of \( \int E \, dx \).

2. The accurate relationship of \( \alpha/p = f(E/p) \) for the gas under consideration.

3. The knowledge of \( K \).

This section will describe the procedure to determine \( U_i \) in a so called Sphere-Pot geometry which represents a non-uniform field in this investigation. The influence of electric field calculation errors and different sets of \( \alpha/p = f(E/p) \) on the calculation of \( U_i \) are analyzed. The influence of experimental errors on the determination of \( K \)-values are also evaluated.

3.1 Electric field calculation

Many electric field calculation programs are available today for both two and three dimensional problems. The various methods used are, finite difference method, finite element method, boundary element method, charge simulation method, and surface charge simulation method. Very likely, the most accurate electric field calculation program is the high speed surface charge simulation method, HSSSM, a program that was developed by Sato (1991). This program provides excellent accuracy with reasonable calculation time for quasi-two-dimensional problems. Cigré (Parraud, 1992) selected this program as a reference to evaluate the performance of other field calculation programs. This investigation also selected HSSSM to calculate the electric field in various electrode geometries.

The real accuracy of numerical field calculation programs to calculate electric fields in practical geometries is very difficult to predict. Therefore, the accuracy of HSSSM is demonstrated by using a simple geometry with well known analytical expressions for its electric field distribution. A comparison in coaxial sphere- and coaxial cylinder-geometries will be presented.
The electric fields in both geometries are calculated with the HSSSM program version 5.02 - 386 (Sato, 1991). The inner electrodes have a radius, r, of 2.5 mm, and gap spacings, g, of 10, 12.5, 15, 17.5 and 20 mm. The outer electrode radius is equal to r+g. The number of electric field points are varied between 900 to 1700; the larger the gap spacing the more points are located. The electric fields calculated by HSSSM are compared with the analytical calculation (see Appendix A and B for the analytical formula of coaxial sphere and coaxial cylinder respectively). Differences are expressed in percent as

\[
\% \text{ Error} = \frac{(E(x)_{\text{HSSSM}} - E(x)_{\text{Analytical}})}{E(x)_{\text{Analytical}}} \cdot 100
\]

The location of the field point, for which the error is calculated is also expressed in percent of gap spacing as

\[
\% x = \frac{x}{g} \cdot 100
\]

where

\[x = \text{Distance from the inner electrode surface}\]

The results are shown in figure 3.1 and 3.2 for coaxial sphere and coaxial cylinder geometry respectively. Both figures show only 200 maximum error points that occur in the gap even if the calculated points are varied between 900 to 1700.

![Figure 3.1 Percentage error of electric field calculated by HSSSM compared to those determined analytically in coaxial sphere geometry, r = 2.5 mm.](image-url)
Figure 3.2 Percentage error of electric field calculated by HSSSM compared to those determined analytically in coaxial cylinder geometry, $r = 2.5$ mm.

The maximum error in figure 3.1 and 3.2 is less than 0.04 %. For $r = 5$ and 10 mm, the maximum error is less than 0.02 % and 0.01 % respectively, though the results are not shown here.

The non-uniform field gap as used in this investigation is represented by a Sphere-Pot geometry, figure 3.3. The electric field line $E(x)$ providing the largest value of $\int \sigma dx$ is located on the axis of symmetry. The electric field points between 900 to 1700 points were calculated on this line by using HSSSM, the larger the gap spacing the more points were assigned. However, 100 points were always located in the region of 0.1 mm from the sphere surface to increase the accuracy.

The calculated values were fitted to a polynomial of order 14 by least-square procedure, which was available as a commercial product (Borland International, 1987), to give an integrateable equation. The equation are expressed as

$$E(x) = U \cdot \sum_{n=0}^{14} a_n x^n$$

where

$U = 1$ was the voltage assigned to the sphere in the HSSSM

$a_n$ = polynomial coefficients of order $n$

$x$ = axial distance from the sphere surface.
Figure 3.3 Sphere-Pot geometry.

The coefficients $a_n$ of all $r$-$g$ combination in the investigation are shown in Table C.1 - C.3 in Appendix C.

The validity of this least-square fitting procedure is verified by comparing the calculated results of eq. 3.1 with the results calculated by HSSSM at the same points. The comparison showed that the maximum deviation was less than 0.8% for all $r$-$g$ combination. (Details are shown in Table C.1 - C.3 in Appendix C.)

The voltage across the gap can be calculated by integrating eq. 3.1 over the gap spacing. The results in Table 3.1 show that the maximum deviation is less than 0.008% when compared with the voltage assigned to the sphere in the HSSSM program, i.e. $U = 1$.

The field factor, which indicates the non-uniformity of the electrode system is defined as

$$FF = \frac{E_{\text{max}}}{E_{\text{avg}}}$$

where

- $E_{\text{max}}$ = the maximum electric field strength at the sphere surface
- $E_{\text{avg}}$ = defined as $U/g$
- $U$ = applied voltage corresponding to $E_{\text{max}}$

In the Sphere-Pot geometry, the field factor was varied between 3.5 - 7.8 as shown in Table 3.2.
Table 3.1  Percentage deviation between the voltage that was assigned to HSSSM program and the integration results from eq. 3.1.

<table>
<thead>
<tr>
<th>r, mm</th>
<th>Percentage deviation at gap spacing g of, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>2.5</td>
<td>-1.2E-4</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.2  Field factor of Sphere-Pot geometry.

<table>
<thead>
<tr>
<th>r, mm</th>
<th>Field factor at gap spacing g of, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>2.5</td>
<td>4.2</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
</tr>
</tbody>
</table>

3.2 Effective ionisation coefficient

Measurements of ionisation coefficients (Townsend, 1901) are as old as the Townsend's breakdown criterion itself. The measurement is based either on the current growth method or on time resolved methods.

In the current growth method, the $I/I_0$ (see eq. 2.2) is observed as a function of various gap spacings $d$ in uniform fields with $E/p$ as a parameter (See e.g. Nasser, 1971 for more information). The plot of $I/I_0$ against $d$ in semi-logarithmic paper will show a straight line with a slope of $\alpha$ in non-electronegative gases. In electronegative gases, e.g. air and SF$_6$, at low $E/p$, this curve will bend towards the $d$-axis, signalling that attachment has occurred in the gas (Harrison and Geballe, 1953; Boyd and Crichton, 1971).

In time resolved methods, $\alpha$ and $\eta$ are evaluated from the shape of current pulses which are produced by an electron avalanche, i.e. the current of the carriers across the gap during the transit time of the electrons and ions (see e.g. Raether, chapter 2, 1964; Teich, 1978; for more information).

Both ionisation and attachment coefficients can also be calculated. The drawback of those methods, however, is the necessity to assume many cross section parameters to match the calculation results to those from the experiment (Saelee and Lucas, 1977; Kline et al., 1978; Lucas, 1978; Dincer and Raju, 1984).
Some of the ionisation coefficient measurements in air and SF$_6$, frequently referred to in the literature, will shortly be presented.

**Humid or dry air**

Sanders (1933) measured $\alpha$ with the current growth method in dry air at pressures between 1 - 380 Torr in the range of $1.5 < E/p < 12 \text{kV/mm-bar}$. He found that in the range $E/p > 3 \text{kV/mm-bar}$, the current obeyed eq. 2.3. At $E/p$ lower than 3 kV/mm-bar, the current did not obey eq. 2.3 which suggested the appearance of attachment processes (note that no knowledge about electron attachment was available at this time).

In 1953, Harrison and Geballe modified the Townsend’s equation to include the attachment coefficient $\eta$. With the current growth method and the least square fitting procedure, they could evaluate $\alpha$ and $\eta$ in dry air (atmospheric air passes through a liquid nitrogen trap) separately at pressures between 40 - 80 Torr in the range $1.7 < E/p < 5 \text{kV/mm-bar}$.

Prasad and Craggs (1960) measured $\alpha$ and $\eta$ with the current growth method in humid air in the range $2.3 < E/p < 3 \text{kV/mm-bar}$, at pressures between 150 - 300 Torr. Lakshminarasimha and Lucas (1977) measured $\alpha$ and $\eta$ in dry air with the current growth method in the range $0.4 < E/p < 34.9 \text{kV/mm-bar}$, whose data agreed well with the data from Rao and Raju (1971) and Moruzzi and Price (1974).

**Synthetic air**

Recently, Friedrich (p. 190, 1992, see also Zaengl et al., 1991) measured $\alpha$ in synthetic air (80 % N$_2$: 20 % O$_2$, both N$_2$ and O$_2$ had a purity of 99.999 %) with the time resolved method in the range $2.5 < E/p < 7.9 \text{kV/mm-bar}$. His data are in good agreement with those of Verhaart (1982) who also measured $\alpha$ with the time resolved method in air with some water content, at least in the region of $(E/p)_c$.

**SF$_6$**

Bhalla and Craggs (1962) measured $\alpha$ and $\eta$ in SF$_6$ using the current growth method at pressures between 5 - 200 Torr in the range $6.8 < E/p < 12 \text{kV/mm-bar}$ and found that it had a very large attachment coefficient. The breakdown voltage in a uniform field was 2.8 times higher than in dry air at about 5 bar-mm. They also measured uniform field breakdown voltages and plotted $(E/p)_b$ against $pd$. They found that as $pd$ increases the value of $(E/p)_b$ falls rapidly at first, then gradually and eventually approaches a limiting value below which breakdown does not occur until $pd$ is increased beyond a value which increases with pressure. (Note that the decrease in $(E/p)_b$ beyond the limiting value may result from the influence of surface roughness, see Zaengl, 1988.) This limiting value of $(E/p)_c$...
Boyd and Crichton (1971) measured $\bar{\alpha}$ in SF$_6$ with the current growth method at pressures up to 400 Torr in the range $8.6 < E/p < 15$ kV/mm-bar. In 1984, Pedersen used this effective ionisation coefficient data to evaluate $K$. Pedersen proposed that $K$ in SF$_6$ is 10.5.

Recently, Aschwanden (1984; pp. 186-195, 1985) measured $\bar{\alpha}$ in SF$_6$ with the time resolved method in the range $8.2 < E/p < 100$ kV/mm-bar. His data are in good agreement with the data of Teich and Branston (1974) in the region of $(E/p)_c$.

Graphs of effective ionisation coefficients in air (and synthetic air) and SF$_6$ are shown in figure 3.4 and 3.5 respectively. From these figures it can be seen that measured values with the current growth method for both air and SF$_6$ show some disagreement while results based on the time resolved method agree at least in some regions. The discrepancy may partly be attributed to the purity of gases in the earlier measurements. With the current growth method the results are strongly effected by the stability of the irradiation source used to produce a constant value of $I_0$ (Nasser, 1971).

The data from Friedrich (1992) and Rao and Raju, which are given in Dutton's publication (1975), were used in this investigation for dry air because of the measurement accuracy and the state of the art technique that Friedrich used and the high values of $E/p$ that Rao and Raju used for their measurement. In the range $2.588 < E/p < 7.943$ kV/mm-bar, the relationship of $\bar{\alpha}/p = f(E/p)$ from Friedrich can be expressed as

$$\frac{\bar{\alpha}}{p} = C \left[ \frac{E}{p} - \frac{E}{(E/p)_M} \right]^2 - A$$

where

- $C = 1.6053$ mm-bar/kV$^2$
- $(E/p)_M = 2.165$ kV/mm-bar
- $A = 0.2873$ 1/mm-bar

From this equation, we get $(E/p)_e = 2.588$ kV/mm-bar. In the range $7.943 < E/p < 14$ kV/mm-bar, the relationship of $\bar{\alpha}/p = f(E/p)$ from Rao and Raju are fitted to a linear equation as

$$\frac{\bar{\alpha}}{p} = \frac{C_1 E}{p} - A_1$$

where

- $C_1 = 1.72$ mm-bar/kV
- $A_1 = 0.5$ 1/mm-bar

In the range $7.943 < E/p < 14$ kV/mm-bar, the relationship of $\bar{\alpha}/p = f(E/p)$ from Rao and Raju are fitted to a linear equation as
a) For $2 < E/p < 5$

b) For $2 < E/p < 14$

Figure 3.4 $\alpha/p = f(E/p)$ in air.
a) For $8 < E/p < 12$

b) For $8 < E/p < 22$

Figure 3.5 $\bar{\alpha}/p = f(E/p)$ in SF$_6$. 
where
\[ C_1 = 16.7766 \text{ 1/kV} \]
\[ A_1 = 80.0006 \text{ 1/mm-bar} \]

In this investigation we used two relationships of \( \bar{\alpha}/p = f(E/p) \) for dry air, one from synthetic air and another from dry air, because the gas composition of synthetic air is nearly the same as atmospheric dry air with negligible water content. The following sections will, therefore, refer to these two relationships of \( \bar{\alpha}/p = f(E/p) \) simply as dry air. The applicability of the \( \bar{\alpha}/p = f(E/p) \) from synthetic air to atmospheric air is shown in section 5.6 and section 6.

The data of Aschwanden (1984, 1985) are used for SF\(_6\) because it covers the entire range of \( pr \) in this investigation, and his data are in good agreement with the data of Teich and Branston (1974). Nevertheless, the relationship of \( \bar{\alpha}/p = f(E/p) \) still requires two sets of equations to cover the whole \( pr \)-range. In the range where \( 8.9246 < E/p < 12.36 \text{ kV/mm-bar} \), this relationship can be expressed as

\[
\frac{\bar{\alpha}}{p} = C \left[ \frac{E}{p} - \left( \frac{E}{p} \right)_0 \right]
\]

where
\[ C = 27.9 \text{ 1/kV} \]
\[ (E/p)_0 = 8.9246 \text{ kV/mm-bar} \]

In the range \( 12.36 < E/p < 21 \text{ kV/mm-bar} \), this relationship can be expressed as

\[
\frac{\bar{\alpha}}{p} = C_1 \frac{E}{p} - A_1
\]

where
\[ C_1 = 22.359 \text{ 1/kV} \]
\[ A_1 = 180.171 \text{ 1/mm-bar} \]

### 3.3 Inception voltage calculation

In sections 3.1 and 3.2, we obtained numerical data of the electric field line along the axis of symmetry and \( \bar{\alpha}/p = f(E/p) \) for the calculation of \( U_i \), by using the streamer breakdown criterion. The factor \( K \) in the streamer breakdown criterion, is a parameter that must be known prior to the calculation of \( U_i \). \( K \) may be a constant or a function of discharge conditions as described in section 2. For the calculation, \( K \) will be first assumed as a constant equal to 9.15 in air and 10.5 in SF\(_6\) as reported by Zaengl et al. (1991) and Pedersen (1975, 1984) respectively.

A computer program was written to carry out the calculation, which is described in Appendix D. The program is based on Turbo Pascal version 5.5 (Borland International, 1988) to run on IBM-AT personal computer. A flow chart of the program is shown in figure 3.6.
Calculate both sides of streamer breakdown criterion
Eq. D10 for SF$_6$
Eq. D15 for air

Calculate $U_i$, Eq. D3
Calculate $(E_{\text{max}}/p)$, Eq. D2

Figure 3.6 Flow chart to calculate $U_i$ in dry air and SF$_6$ in Sphere-Pot geometry.
Note: Appendix D shows details of each equation.
Figure 3.7  Percentage deviation of the calculated $U_i$ of Sphere-Pot geometry and uniform field in dry air by using $K = 18$ with reference to $K = 9.15$, $r = 5$ mm and $g = 20$ mm.

Figure 3.8  Percentage deviation of the calculated $U_i$ of Sphere-Pot geometry and uniform field in SF$_6$ by using $K = 18$ with reference to $K = 10.5$, $r = 5$ mm and $g = 20$ mm.
Calculated $U_r$-values based on $K = 9.15$ for dry air and $10.5$ for SF$_6$ are compared with $U_r$-values calculated with $K = 18$ to show the effect of $K$ on the results of the calculations. The results are shown in figures 3.7 and 3.8 for air and SF$_6$, respectively, for the Sphere-Pot geometry as well as for a uniform field gap. The comparison is expressed in percent as

$$\% U_r \text{ (for air)} = \left( \frac{U_{i,K=18} - U_{i,K=9.15}}{U_{i,K=9.15}} \right) \cdot 100$$

$$\% U_r \text{ (for SF}_6\text{)} = \left( \frac{U_{i,K=18} - U_{i,K=10.5}}{U_{i,K=10.5}} \right) \cdot 100$$

It can be seen that the influence of $K$ is larger in non-uniform fields than in uniform fields, especially for the strong electronegative gas like SF$_6$. In both dry air and SF$_6$, $pr$ or $pd$ has larger effects on the percentage deviation as it is reduced.

Figures 3.7 and 3.8 also show that experiments to evaluate $K$ with uniform fields would be as effective as with non-uniform field for dry air but not for SF$_6$. Thus, to cope with both air and SF$_6$, the experiments have been performed with non-uniform fields, i.e. the Sphere-Pot geometry (see figure 3.3).

### 3.4 Sensitivity analysis

As a good knowledge of the electric field in the electrode geometry, as well as the knowledge of $\alpha$ and $K$ are important in the calculation of $U_i$ by the streamer breakdown criterion, the influence of the first two parameters on $U_i$ will be presented in sections 3.4.1 and 3.4.2 respectively. On the other hand, to evaluate $K$ from the experiments we need accurately measured values of $U_i$. Section 3.4.3 will, therefore, present the influence of experimental errors on the determination of $K$.

#### 3.4.1 Influence of the electric field on the calculated $U_i$

The accuracy of HSSSM to calculate the electric field is already demonstrated in section 3.1. The influence of this small inaccuracy together with the validity of the polynomial equation, eq. 3.1, which represents the electric field, on the calculated $U_i$ is, however, not yet verified. This influence will be shown again by using the coaxial sphere and coaxial cylinder geometry.

The resulting electric fields of both coaxial sphere and coaxial cylinder, which were calculated with HSSSM in section 3.1, are fitted to a polynomial of order 14 with the same fitting program as in the case of Sphere-Pot geometry. The $U_i$ are
determined as described in section 3.3 together with \( \bar{\omega}/p = f(E/p) \) of eq. 3.3 and 3.4 for dry air and eq. 3.5 and 3.6 for SF\(_6\). \( K \) is assumed to be 9.15 in dry air and 10.5 in SF\(_6\).

The results of calculated \( U_i \) as compared to those determined analytically are shown in figure 3.9 and 3.10 for coaxial sphere in dry air and coaxial cylinder in SF\(_6\) respectively. The results are expressed in percentage deviation as

\[
\% \text{ Error} = \frac{(U_{i, \text{Analytical}} - U_{i, \text{HSSM}})}{U_{i, \text{HSSM}}} \times 100
\]

The error is plotted against \( pr \) in the range between 0.2 to 20 bar-mm. The maximum error from figures 3.9 and 3.10 is less than 0.02 \% (when \( r = 5 \) and 10 mm, the maximum error is less than 0.01).

This result suggests that the procedure to calculate \( U_i \) as described in section 3.1 to 3.3 can be used with an accuracy lower than 1 \%.

### 3.4.2 Influence of \( \bar{\omega}/p = f(E/p) \) on the calculated \( U_i \)

The influence of \( \bar{\omega}/p = f(E/p) \) on the calculated \( U_i \) will be shown by using Sphere-Pot geometry. \( K \) is again assumed to be 9.15 in dry air and 10.5 in SF\(_6\). This influence is shown by comparing the resulting \( U_i \) which are calculated by using only eq. 3.3 as well as both 3.3 and 3.4 for dry air. In SF\(_6\), the resulting \( U_i \) which are calculated by using just eq. 3.5 and as well as 3.5 and 3.6 are compared.

The deviations of \( U_i \) are expressed in percent as

\[
\% \text{ Deviation, for air} = \frac{(U_{i, \text{eq 3.3}} - U_{i, \text{eq 3.3 and eq 3.4}})}{U_{i, \text{eq 3.3 and eq 3.4}}} \times 100
\]

\[
\% \text{ Deviation, for SF}_6 = \frac{(U_{i, \text{eq 3.5}} - U_{i, \text{eq 3.5 and eq 3.6}})}{U_{i, \text{eq 3.5 and eq 3.6}}} \times 100
\]

These are plotted against \( pr \) in the range between 0.2 to 20 bar-mm as shown in figures 3.11 and 3.12 for dry air and SF\(_6\) respectively.

In air, the deviation is less than 5 \% and the point where the deviation becomes greater than zero occurs at about \( pr = 1 \) bar-mm. In SF\(_6\), the deviation is less than 2.5 \% and the point where the deviation becomes greater than zero occurs also at about \( pr = 1 \) bar-mm. The negative deviation, when \( pr \) is less than 1 bar-mm, indicates that \( \bar{\omega}/p \) of eq. 3.3 is higher than the one from eq. 3.4 in dry air and eq. 3.5 is higher than the one from eq. 3.6 in SF\(_6\) at the same \( E/p \).
Figure 3.9  Percentage error of $U_i$ calculated by the procedure described in section 3.1 - 3.3 compared to those determined analytically in coaxial sphere geometry in dry air, $r = 2.5$ mm.

Figure 3.10  Percentage error of $U_i$ calculated by the procedure described in section 3.1 - 3.3 compared to those determined analytically in coaxial cylinder geometry in SF$_6$, $r = 2.5$ mm.
Figure 3.11  Percentage deviation of $U_1$ calculated by using only eq. 3.3 and both eq. 3.3 and 3.4 for Sphere-Pot geometry in dry air.

Figure 3.12  Percentage deviation of $U_1$ calculated by using only eq. 3.5 and both eq. 3.5 and 3.6 for Sphere-Pot geometry in $\text{SF}_6$. 
3.4.3 Influence of experimental errors on the determination of $K$

In this investigation, the Sphere-Pot geometry was used. The accuracy of the experiment with Sphere-Pot geometry to determine $U_i$ should, therefore, be known beforehand as it also indicates the uncertainty of the $K$-values which are calculated from this measured $U_r$-values. For example, if we want to determine $K$ with an accuracy of ± 3 % we must design an experiment that can measure $U_i$ with an accuracy of some ± ε %.

The problem arises when we attempt to determine this uncertainty because the measured $U_r$-values and the dependency of $K$ on discharge conditions are still not known. It can, however, be done by assuming that $K$ is a constant and by calculating $U_i$ based on this constant value of $K$ (It can also be done if we know the dependency of $K$ on discharge parameters ). The $U_r$-values which are measured by the experiment with no error, $U_i, \text{err}=0$, will be assumed to be equal to these calculated $U_r$-values.

Now, if we assume that the experiment can only be carried out with the accuracy of ε %, we will get $(1 + \frac{\varepsilon}{100})U_i, \text{err}=0 = U_i, \text{err}=\varepsilon$. From this $U_i, \text{err}=\varepsilon$ we can evaluate the value of $K$ which will be called $K_{\text{err}=\varepsilon}$. Therefore, the deviation of $K_{\text{err}=\varepsilon}$ from the $K$-value that we assumed to be a constant can be expressed as a function of experimental accuracy.

In this section, the relationships of $\frac{\alpha}{p} = f(\frac{E}{p})$ from eq. 3.3 and 3.4 are used for dry air and those from eq. 3.5 and 3.6 are used for SF$_6$ in the calculation of $U_i, \text{err}=0$. $K$ is assumed to be 9.15 for dry air and 10.5 for SF$_6$. The deviation of $K_{\text{err}=\varepsilon}$ from this constant value are expressed in percent, for dry air, as

$$\%K = \left(\frac{K_{\text{err}=\varepsilon} - 9.15}{9.15}\right) \cdot 100$$

and for SF$_6$ as

$$\%K = \left(\frac{K_{\text{err}=\varepsilon} - 10.5}{10.5}\right) \cdot 100$$

The influence of experimental errors with Sphere-Pot geometry on the determination of $K$ are shown in figures 3.13 and 3.14 for dry air and SF$_6$ respectively. It can be seen that, at a specific experimental accuracy, % $K$ increases with increasing $pr$ and the percentage error of $K$ in SF$_6$ is about 3 times higher than in air.

From these figures, it can be seen that the experiment with Sphere-Pot geometry must have an accuracy of less than 2 % in dry air while the accuracy must be less than 0.5 % in SF$_6$ to evaluate $K$ with an uncertainty of about 10 % in the range $0.2 < pr < 20$ bar-mm.
Figure 3.13  Percentage error of $K$ that were evaluated from experiments with given errors compared to $K = 9.15$ in dry air, $r = 5$ mm, $g = 25$ mm.

Figure 3.14  Percentage error of $K$ that were evaluated from experiments with given errors compared to $K = 10.5$ in SF$_6$, $r = 5$ mm, $g = 25$ mm.
4. EXPERIMENTAL EQUIPMENT AND PROCEDURE

This section describes the equipment and procedures to determine $U_i$ with high accuracy. The experiments are based on the knowledge one has on the sensitivity of the evaluation of $K$ due to possible experimental error or uncertainty which is described in section 3.4.3.

4.1 Test Equipment

As already established in section 3, the non-uniform field is represented by Sphere-Pot electrode arrangement, which is basically a sphere-plane electrode configuration, as shown in figure 3.3.

This electrode arrangement was selected because a variation of the degree of the field non-uniformity can be easily achieved by varying the gap spacing between the sphere and the bottom of the pot or by varying the sphere radius. The electric field inside the pot is also exactly defined because the pot walls act as shields against the influence of external potentials. A commercial stainless steel ball bearing, which has an extremely high accuracy of sphere radius and smoothness, was used. This provides a high quality electric field in the vicinity of this sphere up to the plane at the bottom of the pot.

The ball bearing has a diameter accuracy of about 10 $\mu$m and a form tolerance of 1 $\mu$m. The surface roughness, $\xi$, is less than 1 $\mu$m. This surface roughness has no effect on the inception voltage $U_i$ as the maximum gas pressure in this investigation is less than 2 bar. $U_i$ will be influenced only when the product $p\xi$ is greater than 40 bar-$\mu$m in SF$_6$ and 400 bar-$\mu$m in air (Pedersen et al., 1975; Berger, 1976; Zaengl, 1988).

The pot has a diameter of 185 mm and is 200 mm in height. The spheres in this investigation had radii of 2.5, 5, and 10 mm. The gap spacing was varied between 10 to 70 mm, the $r$-$g$ combination together with the field factor are shown in Table 3.2 of section 3.1. The gap spacing can be measured with an accuracy of $\pm$ 10 $\mu$m.
4.1.1 Test vessel

The Sphere-Pot electrode is placed *up-side-down* in a pressurized vessel where the high voltage is applied to the pot as shown in figure 4.1, to facilitate UV irradiation and to avoid accumulation of dust at the bottom of the pot. The pressurized vessel itself consists of two glass chambers, 300 mm diameter and 500 mm height. The flanges at the middle of the vessel are covered with an aluminium toroidal ring to avoid any partial discharges in this region. All flanges are made of stainless steel. At the lower end there is an inlet for pressurizing or evacuating the vessel. This vessel has a maximum pressure limit of about 2 bar.

The lower end of the vessel is grounded. The stem of the sphere, which is isolated from the bottom end of the vessel, is connected via a coaxial cable with a current measuring instrument.

4.1.2 Electrical and control circuits

The experiments were carried out by using DC voltage of both polarities. The high voltage circuit is shown in figure 4.2.

High voltage between 0 - 100 kV of both polarities were generated by a portable DC high voltage power supply. The ripple factor is less than 0.05 % at voltages higher than 4 kV. This ripple is further reduced by using a low-pass filter R1-C1 and R2-C2 respectively.

The high DC voltage is measured by using a 200 MΩ resistor voltage divider which has a voltage ratio of 997:1. The low voltage from the high voltage divider was measured by using an electrometer, Keithley: Model 617. The voltage measuring system was calibrated against a reference high voltage divider which has an uncertainty of 0.01 %. The calibration results show that the measuring system can be characterized by an accuracy of less than 0.2 % for voltages of less than 5 kV and 0.1 % for voltages higher than 5 kV.

The voltage divider is also influenced by temperature and humidity. For a temperature range of 10 - 50 °C and a relative humidity between 20 to 80 %, the measured temperature coefficient of the high voltage part is about 30 ppm/°C. But the temperature during the tests varied only between 21 - 23 °C.

The test vessel receives the test voltage via R2 and C2, which is also used as a coupling capacitor. This resistor R2 provides a means to reduce the high frequency coupling between the high voltage supply and the test section and also limits the current when a breakdown occurs in the gap. However, in this investigation we avoided the breakdown of the gap as far as possible as the breakdown destroys the smoothness of the sphere surface.
Figure 4.1  Pressurized vessel with *Sphere-Pot* electrode arrangement placed inside.
Figure 4.2  High voltage test circuit (see also figure 4.3).
Any discharge phenomena in the gap is detected by using the following three methods:

1. By using a Partial Discharge, PD, detector with the aid of the coupling capacitor (C2) and a measuring impedance to detect the discharge current pulses. The PD detector is a narrow band detector manufactured by Tettex. The center frequency was set to 200 kHz with 30 kHz bandwidth. The sensitivity of this PD detector is about 1 pC.

2. By measuring the average current that passes through the gap with an electrometer. The electrometer can measure DC currents in the fA-range.

3. By detecting discharge current signals with a digital storage oscilloscope, DSO, which has a bandwidth of 100 MHz.

The reason for using several methods in this investigation will be explained later in section 4.3.

Figure 4.3 shows the control and measuring circuits for the high voltage circuit in figure 4.2. A personal computer, PC, is used as a command center. This PC is equipped with a GPIB card to communicate with the electrometer and DSO and a data input/output card, I/O card, to communicate with the DC power supply and the pulse extender. A program was written in Turbo Pascal: Version 5.5, to control all the communications.

The DC power supply can be controlled by using an external DC control voltage of 0 - 6 V to generate the high voltage output between 0 - 100 kV. This control voltage is generated by a 12 bit D/A converter circuit on the I/O card. The I/O card can however generate only a DC voltage from 0 - 5 V in 4096 steps. The high voltage output from the DC power supply can therefore be generated in single steps of approximately 20 V.

This voltage step can further be decreased by inserting an attenuator between the I/O card and the control circuit of the power supply. The attenuator can reduce the input voltage at the ratio of 5:1, 2:1, and 1:1. The high voltage output can thus be increased by steps of 4 V, 10 V, or 20 V. In this manner, a linearly increasing or a step-wise increasing high voltage can be produced as shown in figure 4.4. The voltage increase can be achieved automatically by the program or manually via the computer keyboard.

Every time the program instructs the I/O card to increase the high voltage output, it also monitors the signal from the pulse extender. The pulse extender will generate a single square pulse of a duration that can be selected between 1 ms to 1 s, when it receives a signal from the PD detector. The signal from the PD detector is the partial discharge signal in the test gap which is detected, processed and amplified by the PD detector.
Figure 4.3 Control and measuring circuit.
Figure 4.4 A linearly rising or a step-wise rising high voltage waveform can be generated.

When the I/O card detects a signal from the pulse extender, the program keeps the voltage constant and then reads the voltage data from the electrometer via the GPIB and instructs the I/O card to decrease the voltage to zero at a rate of about 2 kV/s.

This procedure will be repeated when a waiting time of about 30 s is reached or the program will stop the measurement sequence when the procedure is already repeated a specific number of times.

Another way to stop the voltage increase is to interrupt the program by the keyboard. This method is useful when $U_i$ is determined by measuring the average current with the electrometer or by detecting the current signal with the DSO.

Two methods are used to detect a current signal with the DSO:

1. By detecting signals directly across its input impedance, 50 Ω or 1 MΩ.

2. By detecting the signal from the electrometer amplifier output. In this method, the signal is connected to the electrometer which has a very high sensitivity. The electrometer has a built-in current-to-voltage converter which will convert and amplify the current signal to a voltage signal that can be displayed on the DSO. The output signal from the electrometer is interfered by power frequency, however, this power frequency can be attenuated by using a 50 Hz notch filter.

The waveform on the DSO can be read by the computer and can be stored on the storage media for later processing.
4.1.3 Gas filling and pressure measurement

The gas system is shown schematically in figure 4.5. Both sphere and pot electrodes are thoroughly cleaned with alcohol and the remaining dust on the surface is removed by using pressurized air before they are installed in the test vessel. The gap is zero adjusted before the vessel is evacuated down to about 0.05 mbar for at least 3 hours. The gas under test was compressed air and commercial grade SF₆, which were both stored in high pressure bottles. Gas was slowly introduced to the test vessel via a 2 μm dust filter to 1 bar to flush the test vessel. After 30 minutes the test vessel was evacuated down to 0.05 mbar again for 30 minutes. The flushing process is repeated 3 times before the real measurement started.

The pressure was measured by using an analog differential pressure or a vacuum gauge. Both have been calibrated against a standard gauge and had an accuracy of 0.1 %. The environmental pressure is measured with a mercury barometer which has an accuracy of ±0.25 mbar. All pressure measurements are corrected to 20 °C.

4.1.4 UV irradiation

UV irradiation is provided by placing a pencil-UV lamp inside the test vessel at about 60 cm below the pot bottom. The UV lamp has its main radiation spectrum at about 250 nm, i.e. the photon energy of about 5 eV or about 10⁻¹⁸ J, and the intensity of about 10 μW/cm² at the distance of 50 cm. The photons that incident on the pot bottom are judged to be 10¹⁵ photons/cm²-s.

The photon energy of 5 eV exceeds the work function, φ, of most metals (Weissler, p. 346, 1956), e.g. φ = 3.0 - 4.4 eV for aluminium and φ = 3.9 - 4.7 eV for iron. As the probability that photons can release photo electrons from stainless steel was not available, it was assumed to be 100 times less than that of aluminium, i.e. about 10⁻⁵. The photo electron will, therefore, leave the pot bottom at the rate of about 100 electrons/cm²·μs.

Apart from direct irradiation of the pot bottom, some fraction of UV light will be reflected from the pot bottom or the pot walls to the sphere surface. It is assumed that the amount of photons that fall on the sphere surface is only 10 % of those falling on the pot bottom. The photo electrons will, therefore, leave the sphere surface at the rate of just about 10 electrons/cm²·μs.
Figure 4.5 Gas system.

Vacuum gauge
WIKA, No.1111570, -1.0 to 0 kPa/cm², Class 0.1 %

Pressure gauge
WIKA, No.1126384, -1 to 5 bar, Class 0.1 %

Vacuum pump
PFEIFFER, TYPE DUO 012-A
No. 4112 PK D23 305

Atmosphere
SS-4TF-FILTER 20um, N2/RO

Gas
SS-4TF-FILTER 20um, N2/RO

Test Vessel < 2 bar

Vacuum pump

Liquid Nitrogen + Zeolite
4.2 Test procedure

The gas is slowly introduced to the test vessel until the desired pressure is reached at each sphere radius and gap spacing setting. The experiments were carried out at various pressures starting from low pressure. More gas was introduced to the test vessel for higher pressures without evacuating the test vessel.

The voltage was raised to 97% of the predetermined inception voltage, at a rate of rise as shown in Table 4.1, at each pressure. After that, the voltage was increased in steps, also shown in Table 4.1, until $U_i$, the inception point, was found. There was a pause between each step of 2 s and 10 s in the experiment with and without UV irradiation respectively. After $U_i$ was determined, the voltage was decreased to zero at a rate of 2 kV/s. A delay time of 30 s was introduced before the next voltage application.

Table 4.1 Rate of rise of test voltage in this investigation.

<table>
<thead>
<tr>
<th>Predetermine $U_i$</th>
<th>Rate of rise, kV/s</th>
<th>Voltage step, V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_i &lt; 10$ kV</td>
<td>0.4</td>
<td>4</td>
</tr>
<tr>
<td>$10 &lt; U_i &lt; 20$ kV</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>$U_i &gt; 20$ kV</td>
<td>2</td>
<td>20</td>
</tr>
</tbody>
</table>

$U_i$ was determined about 20 times at each pressure with the method that will be described later in section 4.3.

For a new gap spacing setting, the test vessel is evacuated to about 0.05 mbar for 30 minutes before the procedure described in this section is repeated.
4.3 Inception voltage determination

Many methods can be used to determine the inception voltage, $U_i$, of a gas gap. A simple electrical method is based on the measurement of the average current with a highly sensitive ammeter or electrometer, but as discharge phenomena are often impulsive by nature, an oscilloscope or a PD detector is usually used to detect the current signal. Other methods may detect the sound by using ultrasonic microphones, photon pulses by using a photo multiplier or simply light by visual observation.

Discharge inception can appear in many forms depending on electrode geometry, pressure, gas, applied voltage waveform, voltage polarity and the presence of radiation. Discharge currents (corona) can also change its shape with the applied voltage level, in the region between the inception point until breakdown occurs.

For example, in atmospheric air, positive discharge phenomena may start with the occurrence of burst pulses, followed by streamer pulses at high voltages and a glow discharge at still higher voltage. However, at some conditions all the three threshold levels may lie close together. It has never been confirmed that burst pulse threshold is lower than that of pre-onset streamers (Loeb, p. 99 and p. 115, 1965). It is probable that streamers initiate most burst pulses but the space charge of the burst pulse prohibits further streamer formation until the voltage is increased.

For negative discharges in air, the discharge phenomena starts with the appearance of Trichel pulses which will increase its frequency as the voltage is increased. Further increase in voltage may result in glow discharge or breakdown.

In this investigation, $U_i$ is defined in the same way as stated by Loeb in his book (Loeb, p. 325, 1965):

"The true threshold is the sudden increase of current and the appearance of the first pulses, streamers, or bursts, signaling transition from a field intensified current to a self-sustaining one."

Discharge phenomena are always activated by an avalanche. In air, the appearance of burst pulses or pre-onset streamers for positive corona and Trichel pulses for negative corona begin when an avalanche reaches the size of $10^4 - 10^5$ electrons (Loeb, p. 69, 108, 134, 321 and 334, 1965). It is thus logical to determine $U_i$ by using Townsend’s or the streamer breakdown criteria. However, both criteria only tell us that a single avalanche reaches a state where the instability in the gap occurred but not the type of phenomenon that is going to occur. Inception voltage in this investigation is thus determined from the first phenomenon that can be detected.

The phenomena in air and SF$_6$ were studied before the real $U_i$ measurement were performed, which will be presented in section 4.3.1 and 4.3.2. These studies were
also carried out with and without UV irradiation. No attempts were made to investigate the effect of UV intensity and its wave length. The experimental results are only subdivided in situations with and without UV irradiation.

According to the up-side-down arrangement of the Sphere-Pot geometry, the sphere will have the voltage polarity opposite to the applied voltage. To avoid confusion, the polarities as indicated will always refer to sphere polarities which is anode (positive) or cathode (negative).

4.3.1 Atmospheric and dry air

Negative sphere

When the sphere is a cathode, the current signal near $U_t$ in both dry and atmospheric air with and without UV irradiation are similar as shown in figures 4.6 and 4.7. Both figures show the current signal at 400 mbar with $r = 5$ mm and $g = 30$ mm. The signals are measured directly with the DSO via a 50 Ω coaxial cable. The input mode of the DSO was DC 50 Ω.

![Current scale: 4 mA/div. Time scale: 100 ns/div.](image)

Figure 4.6 The current signal at $U_t$ when the sphere is cathode in atmospheric air, with and without UV irradiation. The multiple traces were recorded at different voltage between 12.77 - 12.96 kV. $r = 5$ mm and $g = 30$ mm.
Figure 4.7 The current signal at $U_i$ when the sphere is cathode in dry air, with and without UV irradiation. The multiple traces were recorded at different voltage between 12.74 - 12.90 kV. $r = 5$ mm and $g = 30$ mm.

These current signals can also be detected by using a PD detector, whose charge magnitude is in the order of some thousand pC. When the DSO is replaced by an electrometer, at the voltage below $U_i$ the average current is less than 1 nA and abruptly increases to tens of nA at $U_i$. It means that all of the three methods can be used to determine $U_i$ effectively in both atmospheric and dry air when the sphere is a cathode.

The current signal are in the form of Trichel pulses. The frequency of the pulses is not stable at $U_i$. Only at voltages of about 0.5% higher than $U_i$, the pulse repetition rate becomes stable.

The fast increase of the wave front results from many repetitive avalanches (Loeb, pp. 333 - 347, 1965). The first avalanche may consist of only $10^5$ electrons. In the course of its development, it can create a sufficient amount of photons which in turn liberate the photo electrons from the cathode surface by photo emission processes (at this instance the instability of gas gap occurs). These photo electrons start the second avalanche which is bigger than the first avalanche (or many avalanches which have the same size as the first avalanche?). More photo electrons are generated and the third avalanche is initiated and so on. This repetitive process will, however, be terminated due to the effect of negative space charge. At this state, the ionisation reaches its maximum.
The efficiency to create photoelectrons depends on several parameters, e.g. pressure and the location where photons are generated. The lower the pressure, the further the avalanche penetrates into the gap and the photons are generated at a larger distance from the cathode. In the case of \( r = 5 \text{ mm} \) and \( g = 30 \text{ mm} \), results of calculation show that the avalanche travelled only 2 mm into the gap at 900 mbar and about 4.5 mm at 100 mbar before the repetitive process occurs. Therefore, it takes more time to reach full ionisation at low pressure than at high pressure, i.e. the slope of the wave front rises slower at low pressure than at high pressure. A comparison of current signals at pressures of 100 mbar and 900 mbar is shown in figure 4.8.

Moreover, at high gas pressures, the \( E/p \) value is low, therefore, the excitation occurs nearer to the cathode which results in higher photo emission efficiency. Therefore, complete ionisation will be reached in a fewer number of repetitive processes at high gas pressures. The exact time for complete ionisation is difficult to calculate because the exact number of repetitions is not known and the knowledge of photo emission efficiency is limited. However, a rough estimate can be done as follows:

![Current signal at \( U_i \) when the sphere is cathode in dry air without UV irradiation at a pressure of 100 and 900 mbar. \( r = 5 \text{ mm} \) and \( g = 30 \text{ mm} \).]
Assume that the first avalanche consists of $10^5$ electrons and the photo emission efficiency is $5 \cdot 10^{-5}$. Then photons will liberate 5 new electrons to start a new avalanche. In 5 cycles it will reach the size of $5^5 \cdot 10^5$ or about $3 \cdot 10^8$ electrons. If we assume that the average drift velocity of electrons is $5 \cdot 10^5$ m/sec, the time required to travel 2 mm into the gap (at 900 mbar) is 4 ns. The total time required for complete ionisation to be developed is then equal to 20 ns.

In figure 4.8, this total time is about 50 ns and there are about $15 \cdot 10^8$ electrons in the discharge. Even if the measured ionisation size is 5 times larger than the estimation and the measured time is 3 times higher than the estimation, both results are roughly of the same order of magnitude.

During the time of avalanche development, the positive ions are also moving to the cathode. By bombarding the cathode surface, new electrons are generated which in turn start new avalanches. The avalanche process is now started by ion bombardment rather than photo emission. However, the electrons are attached rapidly to form negative ions and the avalanche is thus quenched. The higher the pressure the faster is the quenching as can be seen in figure 4.8.

The pulse starts again when the negative ions move away from the cathode. The ion current is low and can hardly be seen from the current signal, especially at high pressures. The current signal in figure 4.9 taken at 100 mbar, may show this ion current but this indication is not certain because of its low amplitude.

![Figure 4.9](image_url)  
**Figure 4.9** The current signal at $U_1$ when the sphere is cathode in dry air with UV irradiation at a pressure of 100 mbar. $r = 5$ mm and $g = 30$ mm.
Positive sphere
In the case when the sphere is anode, the current signals at about $U_i$ in both dry and atmospheric air with UV irradiation are shown in figures 4.10 and 4.11. Both figures show the current signal with $r = 5 \text{ mm}$ and $g = 30 \text{ mm}$. The signals were measured directly with the DSO via a $50\Omega$ coaxial cable, but the input mode of DSO was DC $1 \text{ M}\Omega$. The current signals were taken at $U_i$ except for 100 mbar which was taken at a voltage slightly above $U_i$.

By replacing the DSO with an electrometer, the average current continuously increases with the applied voltage at pressures below about 100 mbar at $U_i$. However, the average current abruptly increases from less than 1 nA to few nA at pressures above 100 mbar at $U_i$.

The PD detector cannot be used to detect these pulses at low pressures due to their long duration and small pulse amplitude. However, it can be used when the voltage is slightly higher than $U_i$ at pressures between 400 - 900 mbar and at $U_i$ at pressure above 900 mbar. At voltages much higher than $U_i$, the PD detector is useless, which will be demonstrated later.

Figure 4.10 The current signal at $U_i$ when the sphere is anode in atmospheric air, with UV irradiation at pressures of 100, 200, 400, and 900 mbar. $r = 5 \text{ mm}$ and $g = 30 \text{ mm}$.
The current signal has the form of burst pulses, which overlap to form big pulses. The duration of each pulse is reduced as $p$ is increased. Finally, at low pressure, it appears as a continuous current with fluctuations. When the voltage is increased above $U_i$, as shown in figure 4.12 and 4.13 at a pressure of 400 mbar for atmospheric air and dry air respectively, the current signal has gradually changed into a continuous current with fluctuations. The higher the voltage above the inception point, the smaller is the fluctuation and thus it is difficult to detect the discharge current by using a PD detector.

Without UV irradiation, the current signals appear as pulses of very long duration as shown in figure 4.14. It seems that the first avalanche suddenly cause the discharge phenomena to enter into a glow discharge regime. The average current measured with an electrometer, shows less than 1 nA below $U_i$ and abruptly increases to tens of nA at $U_i$. However, the PD detector shows no response to these pulses.

The inception voltage $U_i$ for the case when the sphere is anode (in atmospheric and dry air) with and without UV irradiation is thus best determined by measuring the average current with an electrometer. $U_i$ is the point where the average current exceeds 1 nA.

Figure 4.11 The current signal at $U_i$ when the sphere is anode in dry air, with UV irradiation at pressures of 100, 200, 400, and 900 mbar. $r = 5$ mm and $g = 30$ mm.
Figure 4.12 The current signal at $U_p$, 0.5% above $U_n$, and 1% above $U_i$ when the sphere is anode in atmospheric air, with UV irradiation at a pressure of 400 mbar. $r = 5$ mm and $g = 30$ mm.

Figure 4.13 The current signal at $U_p$, 0.5% above $U_n$, and 1% above $U_i$ when the sphere is anode in dry air, with UV irradiation at a pressure of 400 mbar. $r = 5$ mm and $g = 30$ mm.
According to Loeb (pp. 482-483, 1956; Loeb, pp. 53-54, 1965), the formation of burst pulses occurs when the avalanche reaches a certain size. The ionisation process is strong near the anode surface so the electrons are quickly swept into the anode. A large number of positive ions are positioned in this region and move slowly away from the anode.

Photons which can ionise gas molecules are also generated in the ionisation process. Electrons from photo ionisation and from detachment can generate a new avalanche which may occur after or may merge with the first avalanche. This process can be repeated several times before a large number of positive ions are accumulated and move away from the anode to a certain distance. The positive ions increase the electric field on the cathode side but decrease the electric field on the anode side.

At voltages not too high above $U_n$, the effect of the space charge generated by positive ions, and the statistical nature of the photo ionisation process, will interrupt the discharge process. The burst pulse starts again only when the positive ions are cleared from the anode and a starting electron is available in the gas gap at the right location.

At higher overvoltage, the burst pulses change into a glow discharge. According to Hermstein, the transition to a glow discharge is due to the effect of the space charge of negative ions (Loeb, pp. 90-94, 1965).
Loeb agreed with Hermstein's explanation for a long time, but lately he doubted this explanation and suggested that the glow discharge are burst pulses that merge in time and space and cause a steady current (Loeb, 1969). Loeb's explanation was supported, based on the calculation, by Abdel-Salam (1976). However, El-Debeiky (1976) considered that the burst-to-glow discharge transition might result from both explanations of Hermstein and Loeb.

If we accept the explanation of Loeb, the stability of glow discharge is thus dependent on the availability of new starting electrons. New electrons will not only come from photo ionisation and detachment but also from the effect of UV irradiation. UV irradiation can cause both photo ionisation in air and photo emission from cathode (Waters and Stark, 1973).

In this investigation, the discharge process is not completely independent of cathode effects. Calculations show that the avalanche starts far from the anode at low pressures (up to 40% of gap spacing). In the case of UV irradiation, the photo electrons from the cathode can survive to be attached to the gas molecules and drift to the anode more effectively at lower pressures. This probably is the reason why the discharge pulse duration becomes shorter as the pressure increases.

4.3.2 Sulphurhexafluoride, SF₆

In SF₆ without UV irradiation it was not possible to observe any discharge phenomena even at voltages as high as 30% above the calculated $U₁$-values (see figures E.4 to E.6 in Appendix E). A further increase in voltage might result in breakdown of the gap which we have avoided as it destroys the sphere smoothness.

However, with UV irradiation the discharge phenomena appeared at quite distinct voltage levels, but is totally different from those in air.

Inception phenomena has been studied with $r$ = 5 mm and $g$ = 40 mm at various pressures. Calculated values of $U₁$ and $x_c$ (also expressed in percent of gap spacing) for this $r$-g setting, at different pressures, are shown in Table 4.2.

The current signal detected by using the DSO, in DC 1 MΩ input mode, are shown in figure 4.15 and 4.16 for positive and negative spheres respectively. The voltage was increased at a rate shown in Table 4.1, which corresponds to the calculated $U₁$. The $U₁$ point, however, cannot be indicated in the figures because the current increases with increasing voltage above the calculated $U₁$. When the applied voltage reached the value that was indicated along each trace, it was held constant for some time before it was decreased to zero. The voltage levels indicated in the figure are expressed in percent of the calculated $U₁$ as
where

\[ U = \text{applied voltage.} \]

\[ U_i = \text{calculated inception voltage.} \]

**Table 4.2** Calculated \( U \)-values and \( x_c \) at different pressures for \( r = 5 \text{ mm} \) and \( g = 40 \text{ mm} \), by using the streamer breakdown criterion with \( K = 10.5 \).

<table>
<thead>
<tr>
<th>Pressure, mbar</th>
<th>( U_i ), kV</th>
<th>( x_c ), %</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>3.99</td>
<td>17</td>
</tr>
<tr>
<td>100</td>
<td>7.68</td>
<td>11</td>
</tr>
<tr>
<td>300</td>
<td>18.77</td>
<td>8</td>
</tr>
<tr>
<td>500</td>
<td>29.30</td>
<td>6</td>
</tr>
<tr>
<td>800</td>
<td>44.65</td>
<td>5</td>
</tr>
</tbody>
</table>

Current scale : 3 \( \mu \text{A}/\text{div.} \).  
Time scale : 5 s/div.

**Figure 4.15** The current signal above \( U \) when the sphere is anode in SF\(_6\), with UV irradiation at different pressures. \( r = 5 \text{ mm} \) and \( g = 40 \text{ mm} \).
Figure 4.16 The current signal above $U_i$ when the sphere is cathode in SF$_6$, with UV irradiation at different pressures. $r = 5$ mm and $g = 40$ mm.

**Positive sphere**

In figure 4.15, the discharges are always superimposed on the continuous current, beginning from small overvoltage above the calculated $U_i$. The higher the pressure, the larger are the pulse amplitudes. This phenomenon was also observed by Farish et al. (1979). They indicated that three corona modes could occur namely a glow, a filamentary corona and a leader-type discharge depending on the gas pressure. They explained that the filamentary corona was the restriking of discharges and could also be superimposed on glow discharge. The wide fluctuation in the interval between discharges are a result of statistical time-lag effects.

For positive sphere, photo electrons from the cathode must first travel through low field regions, which is nearly 90% of the gap spacing at pressures above 100 mbar, to reach the position where they can start the ionisation process. They are thus unlikely to survive the attachment process with SF$_6$ molecules. Under these conditions photo detachment is negligible when compared to the collisional detachment process (Van Brunt and Misakian, 1983).
At high pressures, where $E/p$ is low, the collision detachment probability, which depends on the ion energy (i.e. $E/p$), is also low (O'Neill and Craggs, 1973; Wiegart, 1985). The detachment coefficient also has a tendency to decrease for increasing pressure as clearly shown by Hansen et al. (1973). The discharge current becomes more pulsating as the pressure increases due to the lack of starting electrons.

By nature, discharge pulses are of very short duration (Albiez, pp. 130 - 134, 1992). Multiple discharges can, however, occur and can superimpose on each other to appear as a more prolonged pulse. Numerous discharges can therefore appear as a continuous current.

Figure 4.17 shows the current pulse as observed by DSO with DC 50 $\Omega$ input mode. This figure was taken at 500 mbar and at a voltage of 10% above the calculated $U_i$ at $r = 5$ mm and $g = 40$ mm. It can be seen that this pulse has a very short duration.

However, this type of pulse cannot be observed at the calculated $U_i$ with the DSO in DC 50 $\Omega$ input mode due to its low amplitude.

![Current pulse graph](image)

**Figure 4.17** The current pulse at 10% above $U_i$ when the sphere is anode in SF$_6$, with UV irradiation at 500 mbar. $r = 5$ mm and $g = 40$ mm.
Negative sphere

Figure 4.16 shows the current signal when the sphere is cathode. Here, the current is obviously more continuous with only small fluctuations superimposed, except at 300 mbar when high amplitude discharges occur. The explanation for small fluctuations is that a large amount of starting electrons are available. The starting electrons are generated at the cathode by photo emission and by detachment which is high near the sphere surface. The electrons can initiate an avalanche (and may cause the Trichel pulse type mechanism), but without sufficient ionisation to produce a measurable electron current or Trichel pulse (Gardiner, 1978). The discharge transforms directly into a glow discharge. The negative glow discharge results from negative pulses produced in large numbers merging in time and space in the same manner as done by burst pulses in a positive discharge (El-Debeiky, 1976). Due to the large number per unit time and the small amplitudes of this negative pulses, it is difficult to discriminate it from the continuous current at the voltage around the inception point. Van Brunt and Misakian (1982) also observed that a steady DC-current was always preceding the negative corona pulse activity in their experiment with large point electrode in point-plane arrangement.

The above mechanism is confirmed when a voltage above the calculated $U_i$ was applied to the electrodes and the UV irradiation was abruptly stopped as shown in figures 4.18 and 4.19 for the cases when the spheres were anode and cathode respectively. The applied voltages are expressed in percent of the calculated $U_i$ and are indicated along each trace. The point where the UV irradiation was stopped is indicated by the symbol 'O'.

When the sphere is anode, the current suddenly drops to zero when the UV irradiation was terminated, indicating the lack of starting electrons to initiate the discharge process.

However, when the sphere was cathode, discharges still occur for some time before the current drops to zero. This indicates that starting electrons still appear near the cathode to initiate the discharge even though the UV irradiation is terminated. Those starting electrons are likely to be emitted from the cathode surface by positive ion bombardment.

The occasional discharge which occurs after that may result from the collisional detachment in the gas gap. It is probable to occur because new gaseous products are produced in the discharge, but will gradually decay to reform SF$_6$ as indicated by Bastien et al. (1985) in their measurement of photo absorption in SF$_6$. Furthermore, the applied voltage is much large than $U_i$. The critical volume is large and thus the probability that a starting electron can initiate the discharge is also high.
Figure 4.18 The current signal above $U_i$ when the sphere is anode in SF$_6$ at different pressures, after the UV irradiation was stopped. $r = 5$ mm and $g = 40$ mm.

Figure 4.19 The current signal above $U_i$ when the sphere is cathode in SF$_6$ at different pressures, after the UV irradiation was stopped. $r = 5$ mm and $g = 40$ mm.
At the same percentage of overvoltage, the critical volume increases as the pressure is reduced. (This is confirmed by the calculation in coaxial sphere electrode arrangement, see section 5.5.)

The phenomena of chemical degradation in SF₆ can also be observed by the change in discharge current as shown in figures 4.20 and 4.21. In figure 4.20, the current signal was taken at 40 mbar when the sphere was cathode, a rising voltage was applied to the sphere until it reached the point indicated in the figure and was then held constant. The current increases to nearly 2 times the current at this point.

In figure 4.21, the DSO was replaced by an electrometer to measure the average current at the pressure of 500 mbar and at the same overvoltage as in figure 4.20. The electrometer reads the average current every 330 ms and stores this value in the memory. However, the capacity of this memory is only enough for 100 readings. The upper curve was thus taken at about 1 minute after the lower curve. The phenomena are similar to figure 4.20, currents build themselves and become constant.

In the absence of UV irradiation and the previous discharge process, it is not surprising that at overvoltages of about 30 - 40 % above the calculated $U$, no discharge occurs.

**Figure 4.20** The current builds itself up when the sphere is cathode with UV irradiation in SF₆ at 40 mbar. $r = 5 \text{ mm}$ and $g = 40 \text{ mm}$. 
Figure 4.21 The average current gradually increases while the applied voltage was kept constant. The sphere was cathode with UV irradiation in SF$_6$ at 500 mbar. $r = 5$ mm and $g = 40$ mm.

Average currents have also been measured at various pressures by using an electrometer of both polarities, for the case of $r = 5$ and $g = 40$ mm, as shown in figures 4.22 and 4.23. In contrast to the case where the sphere was anode in air when the current suddenly increased to more than one order of magnitude at a overvoltage of less than 1% above $U_i$, both sphere polarities in SF$_6$ showed a continuous changing in average current. At pressure below 100 mbar, the average current changed from 1 nA to 10 nA when the voltage was raised 10% above the calculated $U_i$. For pressures higher than 100 mbar, it required 3 to 8% overvoltage above the calculated $U_i$ to cause an average current change from 1 nA to 10 nA. The higher the pressure, lower is the overvoltage required.

The determination of the inception point by measuring the average current is thus too insensitive. This method to identify $U_i$ was therefore not used.

Even at 10% overvoltage, the PD detector practically shows no response when the sphere was cathode in the pressure range up to 800 mbar except when there occurred some large discharge pulses superimposed on the continuous current which rarely happened. When the sphere is anode, PD detector can detect the discharge phenomena at high pressures with small overvoltage, but at low pressures it requires a large overvoltage.
Figure 4.22 The average current measured by using an electrometer at different voltages and pressures, the sphere was anode with UV irradiation in SF$_6$. $r = 5$ mm and $g = 40$ mm.

Figure 4.23 The average current measured by using an electrometer at different voltages and pressures, the sphere was cathode with UV irradiation in SF$_6$. $r = 5$ mm and $g = 40$ mm.
The determination of the inception point by using the DSO, the electrometer, or the PD detector alone is thus insensitive. The most effective method was, therefore, to use the built-in current-to-voltage converter of the electrometer in combination with the DSO.

The built-in current-to-voltage converter of the electrometer has a very high sensitivity. It amplifies the current signal and converts that signal into a voltage of opposite polarity of the current signal which can be displayed on an oscilloscope. However, there was always some power frequency interference which obviously came from the electrometer itself. This power frequency interference could be attenuated by using a notch filter between the electrometer and the DSO.

Figure 4.24 shows this signal at zero voltage from the built-in current-to-voltage converter without using a notch filter. The pulses superimposed on the power frequency signal are not the discharge pulse. They are pulses from the UV lamp that provided the irradiation when it is operated in AC mode (there were no such pulses in DC mode of operation). The pulse is intentionally introduced to serve two purposes: 1. To use as a trigger pulse for the DSO. 2. To indicate that the subtraction process, which will be described later, is correct.

Figure 4.24 Signal at zero voltage from the electrometer as displayed on the DSO.
The signal in figure 4.24 was stored in one of the internal memories of the DSO as a reference signal. Continuous signals arriving at the DSO are subtracted from this reference signal before being displayed, as shown in the upper trace of figure 4.25. This signal is difficult to interpret because of the high frequency interference. Thus it is processed by a low pass filter software (built-in software of the DSO) which has a cut-off frequency of about 3 kHz. The result is shown in the lower trace of figure 4.25. The process of subtraction and low pass filtering as described above are performed in real time by the DSO.

![Graph](image)

**Figure 4.25** The resulting signals of figure 4.24 at zero voltage after performing the subtraction and the low pass filtering.

As can be seen, the signal at zero voltage shows only a small noise and is thus easy to be interpreted. The first UV pulse, which is used as the trigger signal, is completely eliminated. This suggests that the procedure described previously is effective. Even though the second UV pulse still appears at point 'A' in figure 4.25, it has no effect on the interpretation because we know the exact location of this pulse on the DSO screen.

Figure 4.26 shows the result when the signal from the electrometer, at a voltage slightly above the voltage that was decided to be the inception point, was subtracted from the reference signal in figure 4.24 and processed by the low pass filter. It can be seen that the signals have drastically changed compared to the signals at zero applied voltage, even though the second UV pulse still appears.
Current scale : 1 nA/div. Time scale : 5 ms/div.

Figure 4.26 The resulting signals at the voltage slightly above the voltage that was decided to be the inception point, after performing the subtraction and the low pass filtering.

Figure 4.27 shows the signals at values above and below $U_i$. It clearly shows that the $U_i$ point can be determined with an uncertainty of about ±1%.

$U_i$ points in SF$_6$ for both sphere polarities with UV irradiation have thus been determined by observing the major change in these signals, with the aid of the electrometer and the DSO.

4.4 Effect of the rate of rise of the voltage

In SF$_6$ with UV irradiation, $U_i$ for both voltage polarities are indicated by observing a change in the current signal with the aid of a DSO and an electrometer as explained before. The applied voltage was thus increased in steps, as shown in Table 4.1, with a time delay between each step long enough to observe the phenomena and making a decision. The time delay of 2 s is decided to be the minimum required time.

This time delay was also applied to investigations in dry air with UV irradiation where $U_i$, in the case when the sphere is anode. The discharge inception voltage, $U_i$, was identified by observing the current level exceeding 1 nA and in the case where the sphere was cathode, by detecting a discharge pulse with the PD detector.
Current scale: 1 nA/div. Time scale: 5 ms/div.

Figure 4.27 The signals above and below $U_p$ being detected by using the electrometer and the DSO in combination.

Without UV irradiation in dry air, because of the lack of initial electrons to initiate a critical avalanche, effects of the rate of rise of the voltage have to be expected.

This effect was investigated with $r = 5$ mm and $g = 25$ mm. $U_i$ at various pressures have been determined at different rates of rise, i.e. voltage steps with 2 s and 10 s time delay between each step and linearly rising voltages with slopes of 100 V/s and 300 V/s, for both sphere polarities.

$U_i$ was determined about 10 times at each pressure. The mean values of $U_i$ are expressed in percent compared to the calculated $U_p$, based on $K = 9.15$, as

$$\% U_i = \frac{(U_{i, \text{experiment}} - U_{i, \text{calculation}})}{U_{i, \text{calculation}}} \cdot 100$$

and are plotted against $pr$ in bar-mm, see figure 4.28.

This figure shows that the relative $U_i$-values increase somewhat with the rate of rise. $\% U_i$ from the linearly rising voltages are about 2 % higher than for steps with a delay time.

Even if the mean value of $U_i$ from a step with 2 s and 10 s delay time is comparable, what is not shown in figure 4.28 is the dispersion of the data. Generally, the higher the rate of rise of voltage the higher the dispersion.
Thus, the experiments without UV irradiation in dry air are performed by increasing the applied voltage in steps with 10 s delay time between each step. The dispersion of $U_i$ in this case is less than about ±0.5 % of the mean value.

Figure 4.28 The effect of rate of rise of voltage on $U_i$ when the sphere is cathode and anode in dry air without UV irradiation, $r = 5$ mm and $g = 25$ mm.
5. RESULTS AND DISCUSSION

This section discusses the experimental results of inception voltage measurements in a Sphere-Pot arrangement, in both dry air and SF₆, which are measured with the method described in the previous section. A comparison between the measured and calculated inception voltage ($U_t$)-values were used, to analyze the dependency of $K$, which can be called the streamer constant or generalized feed-back coefficient, on the discharge conditions. Possible explanations of such a tendency of $K$ for both gases are discussed.

5.1 Experimental results

The experiments have been carried out under conditions as shown in Table 5.1. Given geometrical data can be identified by figure 3.3.

Table 5.1  Conditions under which $U_r$-values are measured.

<table>
<thead>
<tr>
<th>Gas</th>
<th>UV irradiation</th>
<th>$r$, mm</th>
<th>$g$, mm</th>
<th>$p$, bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry air No</td>
<td>5</td>
<td>20, 25, 30, 35, 40</td>
<td>0.15 - 1.0</td>
<td></td>
</tr>
<tr>
<td>Dry air Yes</td>
<td>5</td>
<td>20, 25, 30, 35, 40</td>
<td>0.04 - 2.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>30, 40, 50, 60, 70</td>
<td>0.04 - 1.0</td>
<td></td>
</tr>
<tr>
<td>SF₆  Yes</td>
<td>2.5</td>
<td>10, 12.5, 15, 17.5, 20</td>
<td>0.08 - 1.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>20, 25, 30, 35, 40</td>
<td>0.04 - 0.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>30, 40, 50, 60, 70</td>
<td>0.04 - 0.3</td>
<td></td>
</tr>
</tbody>
</table>

Examples of a typical measured $U_r$-dependencies are shown in figure 5.1 for both dry air and SF₆. This figure shows the measured $U_r$-values for both sphere polarities, $r = 5$ mm and $g = 40$ mm, under UV irradiation at various pressures.
Figure 5.1 Results of the measured $U_r$-values in dry air and SF$_6$ in the presence of UV irradiation. \( r = 5 \) mm and \( g = 40 \) mm.

At each pressure, about 20 $U_r$-values were measured. There are, however, only small deviations between individual $U_r$-values. Each point in figure 5.1 represents the arithmetic mean, $\bar{U}_i$, of the 20 measured $U_r$-values and is, therefore, plotted without a standard deviation bar for the sake of clarity. The standard deviation of each point in figure 5.1 is shown separately in figure 5.2. This standard deviation is expressed in percent of $\bar{U}_i$ as

$$\%\sigma = \frac{\sqrt{\sum_{n=1}^{20} (U_{i,n} - U_i)^2}}{20 - 1} \cdot 100$$

where

- $U_{i,n}$ = individually measured $U_r$-values
- $\bar{U}_i = \left( \sum_{n=1}^{20} U_{i,n} \right) / 20$
As can be seen from figure 5.2, the standard deviation is so small that further statistical evaluation is not necessary. Generally speaking, in all the measurement of $U_\text{r}, \% \sigma$ is less than 0.07 % for dry air and less than 0.4 % for SF$_6$. Hereafter, the arithmetic mean of 20 measured $U_\text{r}$-values will therefore be simply referred to as the measured $U_\text{r}$-values.

![Figure 5.2 Standard deviation of each $U_\text{r}$ point of figure 5.1.](image)

From figure 5.1, it can easily be seen that the sphere polarity does not have any effect in SF$_6$ while it shows some influence in dry air, for the electrode configuration and the pressure range displayed in the figure. As will be seen later, the influence of the sphere polarities appears much larger in dry air than in SF$_6$ if the measured $U_\text{r}$-values are expressed as the percentage deviation from a selected reference $U_\text{r}$-values. The reference $U_\text{r}$-values are from the calculations which have been described in section 3. The calculation results will, therefore, be presented in the next sub section. The comparison of the measured and calculated $U_\text{r}$-values will be discussed in section 5.3.
5.2 Calculation results

The calculated values of $U_r$ for Sphere-Pot geometry are based on the streamer breakdown criterion, eq. 2.35, and on $\alpha/p = f(E/p)$ from Friedrich (1992) and Rao and Raju (given in Dutton's publication, 1975) for dry air and from Aschwanden (1985) for SF$_6$. In the calculation, $K$ is assumed to be a constant equal to 9.15 and 10.5 for dry air and SF$_6$ respectively. The results for the range of $pr$ between 0.2 to 20 bar-mm are shown in Appendix E. An example of the comparison between measured and calculated $U_r$-values by using absolute values is shown in Figure 5.3. The solid line shows the calculated $U_r$-values and the symbols show the measured $U_r$-values. It can be recognized that the experimental results agree quite well with the calculation in SF$_6$, but are a little bit lower than the calculation for dry air. This figure also indicates that the assumption that $K$ is constant and about 9.15 in dry air and 10.5 for SF$_6$ is acceptable.

Before proceeding to any further comparison between the calculated and measured $U_r$-values, the calculated avalanche length, $x_c$, will be presented due to its usefulness in the subsequent discussion.

Figures 5.4 and 5.5 show $x_c$ divided by the sphere radius, $r$, in function of $pr$ in dry air and SF$_6$ respectively. These two figures are a variation of similarity law plot which state that $(E/p)$ for a similar electrode system is only a function of $pr$. For example, for the coaxial sphere geometry, $(E/p)$ is only a function of $pr$, as long as $x_c$ is smaller than gap distance, as shown in eq. A.12 in Appendix A and $x_c/r$ is in turn only a function of $(E/p)$. Therefore, $x_c/r$ is only a function of $pr$. At a specific $r$ and $p$, $x_c$ is, therefore, independent of the radius of the outer sphere in the case of coaxial spheres.

From figure 5.4 and 5.5, it can be seen that the similarity law is not exactly fulfilled for our Sphere-Pot geometry. At low $pr$-values, there are small deviations from the similarity law which is due to the influence of gap spacing, $g$, as the field distribution is somewhat changed with $g$. Note that pot diameter has nothing to do with this discrepancy from the similarity law because the calculation with infinite pot diameter also show the same tendency.

From these two figures, one can see that $x_c$ in dry air is approximately at least 3 times larger than in SF$_6$. For example, when $r = 5$ mm and $g = 40$ mm at about 1 bar-mm, the avalanche travels to about 10% of gap length in dry air before it reaches the critical size while it travels to about 3% of gap length in SF$_6$. In other words, the avalanche reaches its critical size in the region close to the highly stressed electrode in SF$_6$.
Figure 5.3 Comparison between the absolute value of measured and calculated $U_r$-values in dry air and SF$_6$ in the presence of UV irradiation.

a) $r = 5 \text{ mm}$ and $g = 40 \text{ mm}$

b) $r = 10 \text{ mm}$ and $g = 50 \text{ mm}$
Figure 5.4 Calculated $x_e$ of Sphere-Pot geometry in dry air.

Figure 5.5 Calculated $x_e$ of Sphere-Pot geometry in SF$_6$. 
5.3 Comparison between measured and calculated $U_r$-values.

The results of comparison are expressed in percentage deviation between measured and calculated $U_r$-values as

$$\% U_i = \frac{(U_{\text{measured}} - U_{\text{calculated}})}{U_{\text{calculated}}} \cdot 100$$

They are plotted as a function of $pr$.

Dry air

The experimental results without UV irradiation are shown in figure 5.6. These results show that the degree of electric field inhomogeneities, $FF$, has only a slight effect on $\% U_i$, while the sphere polarity has a large influence. $U_r$-values with positive sphere are lower than with negative throughout the pressure range in this experiment. However, the polarity effect becomes smaller as the pressure is increased.

For the dependence on gas pressure, the negative sphere seems to be insensitive to a change in gas pressure while the positive sphere strongly depends on gas pressure.

With UV irradiation, figure 5.7, the results have the same tendency as without UV irradiation. The sphere radius has obviously a small influence on $\% U_i$.

![Figure 5.6 Experimental results in dry air without UV irradiation, $r = 5$ mm.](image-url)
Figure 5.7 Experimental results in dry air with UV irradiation.
Figure 5.8 Comparison of the experimental results obtained with and without UV irradiation in dry air, $r = 5$ mm.

Figure 5.8 shows a comparison of the results from figure 5.6 and 5.7, for $r = 5$ mm. It can be seen that there is some effect of irradiation on $U_i$. However, this effect is very small.

$SF_6$

Experiment in $SF_6$ without UV irradiation could not be performed as no inception voltage could be detected even at voltages as high as 30% above the calculated values. Consistent inception voltage could only be measured with UV irradiation.

The results, figure 5.9 and 5.10, show a completely different tendency of $\% U_i$ with $pr$ when compared to those of dry air. In $SF_6$ the sphere polarity, field inhomogeneities, and $r$ seem to have only a slight effect. Only $p$ has a marked effect on $\% U_i$. 
Figure 5.9 Experimental results in SF₆; positive sphere.
Figure 5.10 Experimental results in SF₆; negative sphere.
5.4 Discussion on the experimental results in dry air

Earlier investigations (Loeb, 1948; Llewellyn Jones, 1956; Loeb, pp. 541 - 550, 1965; Waters et al., 1972; Hackam and Raju, 1973; Raju and Hackam, 1973) also encountered results similar to what we see in this investigation; namely: negative $U_i$ is higher than positive one, the variation of $U_i$ with pressure, the occurrence of glow discharge instead of burst pulses for positive sphere and the effect of UV irradiation. The discrepancy of the experimental results from the calculation can be attributed to the source of primary electrons which initiate the critical avalanche and the feedback processes which are responsible for the variation of $K$ as well as for the appearance of the phenomena that we consider to be the point of discharge inception.

One can also place criticism on the method to determine $U_i$ in this investigation because two methods are used for different sphere polarity. For negative sphere, $U_i$ is well defined as Trichel pulse appear quite pronounced which can easily be detected by using a PD detector. For positive sphere, $U_i$ is not well defined as it is the point where the discharge current exceeds 1 nA. However, the experimental results of the positive sphere with and without UV irradiation, figure 5.8, show the same tendency and the difference in $U_i$ is also small. Hence, it is unlikely that the method to determine $U_i$ is responsible for such a tendency; furthermore, at $U_i$ without UV irradiation, where the current exceeds 1 nA, the discharge current also appears quite sudden if it is observed by using the DSO, figure 4.14.

The streamer breakdown criterion, which we discussed earlier, is based on the assumption that a critical avalanche will transform to a streamer when the amount of electrons in the critical avalanche head reaches a certain value or when the radial electric field at the avalanche head is comparable to the external electric field. The latter assumption is related to the first in that the electric field at the avalanche head is also dependent on the density of electrons and ions in the avalanche head. The streamer breakdown criterion can thus be simply regarded as in the former case. This criterion can be expressed (presented here again for further reference) as

$$\int_0^x \frac{\alpha}{dx} = \ln(N_{cr}) = K$$  \hspace{1cm} (5.1)

Firstly, we can note that eq. 5.1 provides no polarity effect. Secondly, the criterion assumes that the primary (also known as initial or starting) electron is at the right place, i.e. at the electrode surface for negative highly stress electrode or within the gas-gap for positive electrode, to initiate the critical avalanche. Thirdly, the criterion assumes that the critical avalanche is initiated with only one primary electron.

If the above conditions are fulfilled, there should be no polarity effect on the experimental measured $U_i$-values. Any discrepancy that might result is due to the physical phenomena in gas-gap.
As mentioned earlier, the source of primary electrons which initiate the critical avalanche and the feedback processes which are responsible for the variation of $K$ can cause this discrepancy because it relates directly to eq. 5.1. The feedback processes, which are responsible for the appearance of the phenomena that can be detected, are also important because it is the indication that eq. 5.1 is already fulfilled.

**Pressure and polarity effect**

From figures 5.6 - 5.8, one can note that there is no pressure dependence when the sphere is negative. In this case, the critical avalanche is initiated by the primary electron from the surface or in the region near the surface of the sphere. There should be only a small difference in the calculated and the experimental values of $U_t$, because we assume that the critical number of electron is constant, about 9400 electrons for $K = 9.15$, and the physical explanation also demand a certain number of electrons in the avalanche to activate the feedback process at the cathode surface, see section 4.3.1.

However, we can expect some deviation if the cathode material or condition of the cathode surface is changed, e.g. by some oxide layer.

For positive sphere, a deviation of 10 % can be observed at $pr$ about 0.2 bar-mm. However, the deviation decreases as $pr$ is increased. The critical avalanche in this case is initiated by an initial electron from a suitable place in the gas. The critical avalanche will start the feedback process in the gas as described in section 4.3.1. The starting electrons may come from many sources in this case. It may come from detachment from negative ions, photo ionisation or from the cathode which is irradiated by UV, especially at low $p$.

The detachment is high at low $p$ since the detachment rate increases with $E/p$ (O'Neill and Craggs, 1973). The critical avalanche can, therefore, be initiated with *more than one* electron in contrast to the streamer breakdown criterion which assumes that the critical avalanche is started from *only one* electron. More starting electrons will directly cause a reduction of $K$. This is a plausible explanation of the tendency that $\%U_t$ is reduced with $p$ for positive sphere.

The critical avalanche will initiate the feedback process which occur in gas, the phenomena which is known as glow discharge. Loeb (1969) explained that the occurrence of the glow discharge is a result of many burst pulses which merge in time and space. If we accept this explanation, the feedback process will depend on the arrival of new electrons in the gas to initiate those burst pulses. The source of new electrons is again the detachment process and also photo ionisation in the gas. The lower the pressure the higher are the number of new electrons which are detached, and the discharge current becomes more continuous, as shown in figure 4.11.
UV irradiation effect

The effect of UV irradiation in dry air is very small. The $U_i$-values in the case of UV irradiation are slightly lower than without UV irradiation for both sphere polarities.

This effect of UV irradiation on $U_i$ is not difficult to explain because the irradiation increases the number of primary electrons at the surface or in the region near the surface of the negative sphere. Without irradiation, one must wait until a free electron passes through the region near the surface where it can initiate the critical avalanche. It is unlikely that, without irradiation, the field emission will contribute to the production of primary electrons from the sphere surface. This is because the highest electric field strength in this investigation is about 8 kV/mm while the field emission will have an effect only when the electric field strength, measured in vacuum, is greater than 10 kV/mm (Llewellyn Jones and Morgan, 1953; Cox and Williams, 1977).

For the positive sphere, apart from the primary electrons that come from the detachment process, additional electrons are also produced by the cathode if it is irradiated by UV. The chance that photoelectrons, released from the plane electrode by UV irradiation, will not be attached to gas molecules is quite high at low $p$ as the critical avalanche start not too far away from the plane as shown in figure 5.4.

Electric field inhomogeneities and sphere radius

There is only a small influence of electric field inhomogeneities and sphere radius on the inception voltage in dry air. The influence of $r$ and thus electric field inhomogeneities to $U_i$ can be explained by using the concept of the initial volume (also known as critical volume). The initial volume is a region where any electron in this region can successfully initiate a critical avalanche. Generally, this initial volume is increased with the applied voltage above $U_i$. If the percent overvoltage above $U_i$ is kept constant, it increases with the radius of curvature of the high stress electrode. The variation of the initial volume with $p$ is small only for negative high stress electrode while it changes drastically for positive high stress electrode.

The calculation of the initial volume for Sphere-Pot geometry is difficult and time consuming. However, the calculation is quite simple for a geometry such as coaxial sphere as shown in figure 5.11.

The tendency of the initial volume as shown in figure 5.11 can be expected to be similar to the Sphere-Pot geometry. However, it should be kept in mind that the initial volumes of Sphere-Pot geometry are far less than those of coaxial sphere.
arrangement. For example, at the exact \( U \) level, the initial volume of coaxial sphere is reduced to the surface area while that of the Sphere-Pot geometry is reduced to a point.

![Figure 5.11 Initial volume of coaxial spheres in dry air at 0.5% above the calculated \( U \). \( r \) is the inner sphere radius.](image)

As the initial volume is increased with increasing sphere radius; \( U \) may be reduced because the chance that an electron (or a higher number of electrons) can initiate a critical avalanche is higher. However, at a specific \( r \) and \( p \), the initial volume will not change with field inhomogeneity due to the similarity law. The ideal value of \( U \) should thus not change with field inhomogeneities, for both sphere polarities. As can be seen in figure 5.6 - 5.8, the real value of \( U \) are in agreement with the calculation.

The concept of initial volume can also be used to explain why with positive sphere the discharge current is more continuous as the overvoltage is increased, figure 4.13. The increase in the initial volume will increase the production of new electrons in the feedback process.
5.5 Discussion on the experimental results in SF₆

At $U_i$ in SF₆, the discharge current is continuous with small fluctuations. Therefore, glow discharge conditions, for both sphere polarities are obviously provided. The positive and negative ions cloud form a barrier which effectively shields the sphere. This effect has been known for a long time and is termed corona stabilization. (Works and Dakin, 1953; Hazel and Kuffel, 1976; Cookson and Wootton, 1978; Farish et al., 1979; Van Brunt and Misakian, 1982; Korasli and Farish, 1982) Corona stabilization raises the breakdown voltage of gas gaps significantly. The region of occurrence of corona stabilization is determined by the gas pressure, field non-uniformity and the polarity of the high stress electrode (Sangkasaad, 1976).

However, the dependency of $\% U_i$ with $p$ as reported in this investigation cannot be found in other literature. This might be a result of the different methods used to experimentally determine $U_i$. For example, Cookson and Wootton (1978) identified $U_i$ as the point where the discharge current exceeds 1 μA while Van Brunt and Misakian (1982) defined it as the point where the count rate of PD pulses is greater than 0.1 count/s (they also found that there is a dc current before their criterion is reached for negative corona).

It may well be, that the tendency found in this investigation can be attributed to the method of quantifying $U_i$, as $U_i$ was defined as the point where some change in discharge current is observed. The change in discharge current is smaller as the pressure is reduced. However, there are some other possible explanations to this tendency which will be as follows.

**Pressure and polarity effect**

From figures 5.9 and 5.10, one can see that pressure has a strong influence on the $\% U_i$ while the polarity has only a slight effect.

As in air, the critical avalanche in SF₆ is also initiated by a primary electron from the surface or in the region near the surface of the negative sphere but for a positive sphere it is initiated from a position in the gap. The major difference to air is that the ionisation zone in SF₆ is much closer to the sphere because $x_c$ of SF₆ is only about 1/3, or less, than in air, see section 5.2.

The source of primary electrons, for negative sphere, is photo emission and collisional detachment but for a positive sphere it may only be collisional detachment. Photo detachment can be neglected in SF₆ as shown by Van Brunt and Misakian (1983). For positive sphere, electrons emitted from the negative plane electrode can hardly survive the attachment to gas molecules because the ionizing zone is close to the sphere; i.e. they can only be attached and detached again near the ionizing zone.
Detachment is high near the sphere because the rate of detachment increases with $E/p$ (O'Neill and Craggs, 1973; Wiegart, 1985). The primary electron thus has a higher probability to appear near the sphere than in the gas gap. If the sphere is cathode, the primary electron can also come from photo emission, and $U_t$ is therefore about 2% lower than in the case of positive sphere. However, the polarity effect should not be large because the ionizing zone is close to the sphere so that the electric field is not much different at the positions where the critical avalanche is initiated. For SF$_6$, the ratio of the electric field strength at the point where the critical avalanche is initiated (for negative and positive sphere respectively) is only 1 to 2, or less depending on $p$, see figure E.7 in Appendix E. (In air the ratio is about 1 to 5, see figure E.3 in Appendix E.)

The critical avalanche starts the feedback process which causes the transition to a glow discharge. The glow discharge is maintained by the generation of new electrons which come from the same source as the primary electron. The resulting discharge current is therefore expected to have a lower fluctuation for the negative than for the positive sphere, figure 4.15 and 4.16, as the source of new electrons is more effective in the former case.

The small decrease of $\%U_t$ with $p$ cannot be explained by referring to the detachment rate as this was done in dry air because the detachment rate is higher at low $p$. If one tries to explain this dependency by using the detachment rate, the tendency of $\%U_t$ with $p$ will be opposite to those in figures 5.9 and 5.10.

Apart from the possibility, that the method of determining $U_t$ could be responsible for the effect, another source of error might be the effective ionisation coefficient which is used to calculate $U_t$. It seems that $\bar{\alpha}/p = f(E/p)$ at high $E/p$ value is prone to error because there are large differences of $\bar{\alpha}/p = f(E/p)$ from various measurements (Mailer and Naidu, 1976; Aschwanden, p. 194, 1985).

In this investigation, the calculations are based on two relationships of $\bar{\alpha}/p = f(E/p)$. The comparison of the calculated $U_t$ based on only one or based on both equations, figure 3.12, shows that the major deviation occurs when the pressure is lower than 1 bar-mm which is roughly at the same magnitude where $\%U_t$ starts to increase, figures 5.9b and 5.10b. If $pr$ is larger than 1 bar-mm, $U_t$ seems to be insensitive to the variation of $p$. $\%U_t$ is also small, $\pm 1.5\%$, except for 10 mm negative sphere.

**Electric field inhomogeneities and sphere radius**

From figures 5.9a and 5.10a, we can see that $\%U_t$ is not so sensitive to the electric field inhomogeneities but somewhat depend on the sphere radius for both sphere polarities. The value of $\%U_t$ decrease with increasing sphere radius and this decreasing is more pronounced for negative sphere than positive sphere.
The influence of electric field inhomogeneities and \( r \) on the measured \( U_r \)-values can be explained by using the concept of initial volume as in the case of dry air. The calculation of the initial volume in \( \text{SF}_6 \), again for coaxial spheres, is shown in figure 5.12 (see Van Brunt and Misakian, 1982, for the calculation of the effect of overvoltage on the positive initial volume in the case of conical point to plane gaps in \( \text{SF}_6 \)).

![Initial volume of coaxial sphere in SF₆ at 0.5 % above the calculated \( U_r \), \( r \) is the inner sphere radius.](image)

Comparing figure 5.12 to figure 5.11, the positive initial volume in \( \text{SF}_6 \) is a factor 10 lower than in dry air. However, it is nearly the same for negative initial volume.

Electric field inhomogeneities, at a specific \( r \) and \( p \), have no effect on the initial volume. Therefore, \( \% U_r \) hardly changes with electric field inhomogeneities for both sphere polarities. There should also be some effect of \( r \), however it will be small because \( x_i \) is small, as is also the initial volume. The effect of \( r \) on \( U_i \) will be clearly shown when there is a large change in \( r \) which causes a large change on initial volume. For example at \( pr = 1 \) bar-mm, in the case of coaxial spheres, the positive volume changes by 1000 mm\(^3\) when \( r \) changes from 5 to 10 mm while it changes only by 90 mm\(^3\) when \( r \) change from 2.5 to 5 mm. It is also true for the negative initial volume, however, the effect of \( r \) here is by far low.
The explanation should also hold for the case of Sphere-Pot geometry, even though the effect of $r$ is less than in the case of coaxial spheres. Therefore, $\% U_i$ is hardly effected when $r$ changes from 2.5 to 5 mm, but have some noticeable effect when $r$ increases to 10 mm, figure 5.9a and 5.10a, because the change of initial volume directly influences the generation of primary electrons and also the feedback process.

The variation of initial volume with $p$ depends, again, on sphere polarity, i.e. the negative initial volume is insensitive to $p$ while the positive initial volume is reduced drastically as $p$ is increased, see figure 5.12. For the positive sphere, it is to be expected that as $p$ is increased, it will be more difficult to maintain the glow discharge which results in a more fluctuating current as shown in figure 4.15. The effect of $p$ on the appearance of the discharge current for negative sphere is only small as shown in figure 4.16.

5.6 Discussion on $K$

The magnitude of $K$ for uniform fields in SF$_6$, which is about 10, was obtained about 10 years ago by Pedersen et al. (1984). While $K$ for nearly uniform fields in synthetic air, which is about 9, was only recently obtained by Zaengl et al. (1991). Both values are only about half of the magnitude of $K$ that was believed to be effective in the past, i.e. $K = 18 - 20$.

The results in this investigation show that the magnitude of $K$, for non-uniform fields, is also not about 18 - 20 as believed up to now. Its magnitude is, again, only about half for both dry air and SF$_6$.

Nevertheless, it is necessary to discuss finally the influence of $K$ on $U_i$ in a more general form. As $U_i$ becomes quite sensitive on $K$ for low pressures $p$ or products $pr$ respectively, it should be discussed which magnitudes of different $K$-values would alter $U_i$, especially for low values of $p$ or $pr$. The discussion must be based on the individual gas, as the influences are gas-specific.

**Dry air**

Figures 5.6 - 5.8 show that a constant $K$-value can be well assumed to be correct for negative sphere. However, for positive spheres $K$ obviously depends on $p$, figure 5.7a, at least if $p \leq 1$ bar.

Figure 5.13 demonstrates, which value of $K$ would be necessary to calculate the inception voltage as measured. The calculated $U_r$-values based on different values of $K$ are compared to the $U_r$-values that were calculated based on $K = 9.15$. They are expressed in percent as
\[
\% U_i = \frac{(U_{i,K=x} - U_{i,K=9.15})}{U_{i,K=9.15}} \times 100
\]

where
\[
U_{i,K=x} = U_i \text{ calculated based on } K \text{ equal to the value in figure 5.13.}
\]
\[
U_{i,K=9.15} = U_i \text{ calculated based on } K \text{ equal to 9.15.}
\]

Figure 5.13  Effect of \(K\) on the calculated \(U_i\) in dry air when compared to those calculated with \(K = 9.15\).

For negative sphere, figure 5.13 shows that the experimental results will be in agreement with the calculations if we reduce \(K\) from 9.15 to about 8 in the calculation. It is therefore believed that there is no need to change the value of \(K = 9.15\) in the calculation because the experimental \(U_i\)-values deviate from the calculated ones by only 3\%, which can be considered small, at least if the influence of the UV irradiation is taken into account.

For positive sphere, the measured \(U_i\)-deviation of about 10\% at about \(p = 0.3 \text{ bar}\cdot\text{mm}\) can be accounted for by a reduction of \(K\) from 9.15 to about 7 in the calculation. This reduction is plausible because if the critical avalanche starts with 9 primary electrons, instead of only one electron as assume by streamer breakdown criterion, the critical number of 9400, for \(K = 9.15\), is still achieved.
On the other hand, if we use the same number as in the case of nearly uniform field, i.e. $K = 9.15$, we should expect a deviation of about 10 % at $pr = 0.3$ bar-mm. However, the deviation disappears for atmospheric pressure.

$SF_6$

Figures 5.9 - 5.10 show that $K$-values for both polarity is insensitive to most discharge conditions, except the variation of $p$.

As in dry air, figure 5.14 demonstrates, which value of $K$ would be necessary to calculate the inception voltage as measured. The calculated $U_r$-values based on different values of $K$ are compared to the $U_r$-values that were calculated based on $K = 10.5$. They are expressed in percent as

$$\% U_i = \frac{(U_{i,K=x} - U_{i,K=10.5})}{U_{i,K=10.5}} \cdot 100$$

where

$U_{i,K=x} = U_i$ calculated based on $K$ equal to the value in figure 5.14.

$U_{i,K=10.5} = U_i$ calculated based on $K$ equal to 10.5.

From figure 5.14, we can see that a value of $K$ between 9 to 12 is needed in the calculation to bring the experimental results in agreement with the calculation, except for the negative sphere with $r = 10$ mm. Therefore, $K$ can well be assumed to be constant equal to 10.5 as in the case of uniform field.

By using $K = 10.5$, the measured $U_r$-values deviated from the calculation only about up to 4 %. This difference can be considered small in engineering applications.

Meek's streamer breakdown criterion

According to eq. 2.16, Meek’s streamer breakdown criterion, for uniform field breakdown in air, can be expressed as

$$E_r = \frac{\alpha e^{ax}}{\sqrt{x/p}}$$

or

$$e^{ax} = \frac{1}{\kappa} \frac{E_r \sqrt{x/p}}{\alpha}$$
Figure 5.14 Effect of $K$ on the calculated $U_t$ in SF$_6$ when compared to those calculated with $K = 10.5$.

where

$E_r =$ radial electric field at the avalanche head, kV/mm

$\alpha =$ Townsend's first ionisation coefficient, 1/mm

$x =$ avalanche length, mm

$p =$ pressure, bar

$K =$ constant for air = $4.82 \times 10^{-8}$ kV/(mm/bar)$^{0.5}$

Generally, $E_r$ is taken equal to the applied electric field, $E$, in the calculation of breakdown voltage, as Meek (1940) showed that deviations between measured breakdown values and calculated ones using $E_r/E$ between 0.1 to 1 is obviously not large. $x$ is also taken equal to the gap length, $d$, in the calculation.

A comparison between calculated and measured $U_b$-values is shown in the first row of Table 5.2. The measured values (the humidity not known), are taken from Table 9.1 page 262 of Nasser (1971). Eq. 5.2a is calculated based on the relationship of $\alpha/p = f(E/p)$ from Sanders (1933) which can be expressed as

$$\alpha/p = A \cdot [E/p - B]^2$$  \hspace{1cm} 5.3
where

\[ A = 1.6 \text{ mm-bar/kV}^2 \]
\[ B = 2.1 \text{ kV/mm-bar} \]

The comparison result is expressed in percent as

\[
\% \text{ deviation} = \frac{(U_{b, \text{calculated}} - U_{b, \text{measured}})}{U_{b, \text{measured}}} \times 100
\]

Table 5.2 Comparison between calculated and measured \( U_b \) at various gap spacing, \( d \), in air at atmospheric pressure, i.e. 1.013 bar. (For Meek’s streamer breakdown criterion.)

<table>
<thead>
<tr>
<th>( \kappa )</th>
<th>( \alpha/\rho )</th>
<th>% deviation at gap spacing of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5 mm</td>
</tr>
<tr>
<td>Meek, eq. 5.2a</td>
<td>4.8 \times 10^{-8}</td>
<td>Sanders</td>
</tr>
<tr>
<td>Pedersen, ( \alpha d = 18 )</td>
<td>-</td>
<td>Sanders</td>
</tr>
<tr>
<td>Meek, eq. 5.2a</td>
<td>3.6 \times 10^{-7}</td>
<td>Sanders</td>
</tr>
<tr>
<td>Meek, eq. 5.2a</td>
<td>5.6 \times 10^{-5}</td>
<td>Friedrich</td>
</tr>
<tr>
<td>Zaengl, ( \alpha d = 9.15 )</td>
<td>-</td>
<td>Friedrich</td>
</tr>
</tbody>
</table>

Measured \( U_b \), kV (Nasser, 1971) | 17.1 | 31.6 | 73.0 | 265.0 | 510.0 |

Although the deviation of breakdown field strength, or breakdown voltage, between the calculation by using eq. 5.2a and experimental results are not large, one should not regard this equation as most satisfactory. The reason being the various assumption made, see section 2.1.1, especially those made in order to determine the charge density and the tip radius of the avalanche. One can notice that the assumption made to determine the tip radius of the avalanche is directly appears in the factor \( \kappa \). The radius cannot be accounted for on the basis of thermal diffusion alone (Nasser, 1971). Photo ionisation and other unknown processes may have a greater effect than the thermal diffusion itself.

According to Pedersen (1976a), the exponent term in eq. 5.2b is the dominating factor and can be treated as a constant with the value of about \( 10^8 \), i.e. \( \alpha d = 18 \), for air at atmospheric pressure. This statement of Pedersen is not exactly true as the exponent term in eq. 5.2b varies in a wide range as shown in the upper curve of
The values of $\alpha d$ are calculated from $\alpha/p = f(E/p)$ of Sanders (1933). One can see that in the range of $d$ between 5 - 200 mm, $\alpha d$-values varies between 17.5 - 22.5, i.e. $\exp(\alpha d)$ varies between $10^7$ - $10^{10}$, at atmospheric air pressure.

However, by using the simplified streamer breakdown criterion, i.e. $\alpha d = K = 18$, in the calculation, the calculated $U_b$ agree quite well with the results achieved with Meek's streamer breakdown criterion, eq. 5.2a, as shown in the second row of Table 5.2.

As stated earlier, the factor $\kappa$ has inherent uncertainty because of the various assumptions in the derivation of eq. 5.2a. One can however, determine this factor $\kappa$ more accurately by using the experimental result because $\alpha/p = f(E/p)$ is known and the measured $U_b$ at a specific gap distance and air pressure is also known.

For example, $U_b$ of 10 mm uniform field gap is 31.6 kV at atmospheric air pressure. The only question is which relationship of $\alpha/p = f(E/p)$ will be used?

First, we considered the data from Sanders (1933), eq. 5.3, which is more than half century old. The new value of $\kappa$ from the $U_e$-value and $\alpha$ from Sanders is $3.6 \times 10^{-7}$ kV·(mm/bar)$^{0.5}$. The calculated $U_b$ from this new $\kappa$ by using eq. 5.2a is...
shown in the third row of Table 5.2. These comparisons show that the calculation with \( \kappa = 3.6 \times 10^{-7} \text{kV} \cdot (\text{mm}/\text{bar})^{0.5} \) is no better than the calculation with \( \kappa = 4.8 \times 10^{-8} \text{kV} \cdot (\text{mm}/\text{bar})^{0.5} \) or with \( \alpha d = K = 18 \).

Now, if we use the data of \( \tilde{\alpha}/p = f(E/p) \) from Friedrich (1992), eq. 3.3, assuming that this data can be applied to atmospheric air, we have the factor \( \kappa = 5.6 \times 10^{-5} \text{kV} \cdot (\text{mm}/\text{bar})^{0.5} \). The calculation result of \( U_b \) using this \( \kappa \), together with eq. 5.2a and \( \alpha/p = f(E/p) \) from Friedrich (1992) is shown in the fourth row of Table 5.2. One can immediately see that the calculation results agree very well with the measured \( U_s \)-values, with the exception at 200 mm gap spacing.

Note that when Meek's streamer breakdown criterion, eq. 5.2a is applied to calculate the breakdown voltage in an attaching gas, the term \( \alpha e^{\alpha x} \) must be replaced by \( \alpha e^{(\alpha - \eta) x} \). The data from Friedrich has, however, no separate value of \( \alpha \) and \( \eta \). The term \( \alpha e^{\alpha x} \) in eq. 5.2a is, therefore, replaced by \( (\alpha - \eta)e^{(\alpha - \eta) x} \) in the calculation using data from Friedrich. This approximation will only slightly affect the calculation results as the exponent term is the dominating factor.

The value of \( \alpha d \), calculated with \( \kappa = 5.6 \times 10^{-5} \text{kV} \cdot (\text{mm}/\text{bar})^{0.5} \) and Friedrich data by using eq. 5.2a is shown in the lower curve of figure 5.15. We can see that \( \alpha d \) varies between 11 - 16.

As Zaengl et al. (1991) suggested the value of \( \tilde{\alpha}/d = K = 9.15 \) for the calculation of \( U_b \) of uniform field gap in synthetic air, we can also apply this simplified streamer breakdown criterion to calculate \( U_b \) in dry air. The result of calculation is shown in the last row of Table 5.2. The results agree quite well with the measured values and thus they can be said to be generally better than the calculation results in rows 1 to 3.

**Raether's streamer breakdown criterion**

Raether's streamer breakdown criterion, for uniform field breakdown in air, can be expressed as

\[
\alpha d = \ln(\psi d)
\]

where

\[
\psi = \text{constant for dry air} = 3 \times 10^6 \text{1/mm}
\]

This factor \( \psi \) has also inherently uncertainty as it was determined based on various assumptions, similar to that of Meek. The comparison between the calculated and the measured \( U_b \) is shown in the first row of Table 5.2. The calculation is based on the relation ship of \( \alpha/p = f(E/p) \) from Sanders (1933), eq. 5.3.
The new values of \( \psi \), determined using the same procedure as described earlier in the determination of \( \kappa \) of Meek (based on \( U_b = 31.6 \text{kV} \) at \( d = 10 \text{ mm} \) and \( p = 1.013 \text{ bar} \)), is \( 2.07 \times 10^6 \text{ 1/mm} \) when using \( \alpha/p = f(E/p) \) from Sanders (1933) and is \( 1.5 \times 10^4 \text{ 1/mm} \) when using \( \alpha/p = f(E/p) \) from Friedrich. The comparison with the measured \( U_b \)-values with these new \( \psi \) values for eq. 5.4 are shown in the second and third rows of Table 5.3 for the relationship of \( \alpha \) from Sanders (1933) and \( \alpha \) from Friedrich (1992) respectively. The results in Table 5.3 show the same tendency as Table 5.2.

Table 5.3 Comparison between calculated and measured \( U_b \) at various gap spacing, \( d \), in air at atmospheric pressure, i.e. 1.013 bar. (For Raether’s streamer breakdown criterion.)

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>( \alpha/p )</th>
<th>% deviation at gap spacing of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5 mm</td>
</tr>
<tr>
<td>Raether, eq. 5.4</td>
<td>3 \times 10^6</td>
<td>Sanders</td>
</tr>
<tr>
<td>Raether, eq. 5.4</td>
<td>2.1 \times 10^6</td>
<td>Sanders</td>
</tr>
<tr>
<td>Raether, eq. 5.4</td>
<td>1.5 \times 10^4</td>
<td>Friedrich</td>
</tr>
</tbody>
</table>

Measured \( U_b, \text{kV} \) (Nasser, 1971) | 17.1 | 31.6 | 73.0 | 265.0 | 510.0 |

From the discussion of Meek’s and Raether’s streamer breakdown criteria, we can conclude that

1. The value of factor \( \kappa \) in eq. 5.2a should be \( 5.6 \times 10^{-5} \text{kV} \cdot (\text{mm/bar})^{0.5} \) instead of \( 4.8 \times 10^{-8} \text{kV} \cdot (\text{mm/bar})^{0.4} \) and the value of \( \psi \) in eq. 5.4 should be \( 1.5 \times 10^4 \text{ 1/mm} \) instead of \( 3 \times 10^6 \text{ 1/mm} \).

2. Considering the calculation results in the fourth row of Table 5.2 and third of Table 5.3, it indicates that \( \tilde{\alpha}/p = f(E/p) \) from Friedrich (1992) for synthetic air can also be used for atmospheric dry air. Therefore, the uniform field breakdown voltage can be calculated more accurately by using eq. 5.2a (with the factor \( \kappa = 5.6 \times 10^{-5} \text{kV} \cdot (\text{mm/bar})^{0.5} \)) or eq. 5.4 (with the factor \( \psi = 1.5 \times 10^4 \text{ 1/mm} \)). The calculation of \( U_b \) by using the simplified streamer breakdown criterion, \( \tilde{\alpha}d = K = 9.15 \), also gives the result that agree quite well with the measured values.

3. The value of \( \alpha/p = f(E/p) \) of Sanders for dry air are obviously not very accurate.

As already discussed in section 2, the (simplified) streamer breakdown criterion eq. 2.35 is usually applied to calculate \( U_b \) of uniform or nearly uniform field electrode configurations. In last section, experiments have been performed to investigate the magnitude of \( K \) for non-uniform field configurations in dry air. The streamer breakdown criterion was applied to calculate \( U_i \) for synthetic air (which has nearly the same gas composition as dry air), for which \( \alpha/p = f(E/p) \) is known. A comparison is made between the measured and calculated \( U_i \).

It was shown that \( K \) for pressures around atmospheric pressure can be assumed to be 9.15. This magnitude was also confirmed by direct breakdown measurements for nearly uniform gaps by Zaengl et al. (1991).

Up to now, the possible applications of \( \alpha/p = f(E/p) \) of synthetic air (eq. 3.3) on atmospheric air has not yet been established for non-uniform field geometry, although the application of this \( \alpha \)-data to uniform field is satisfactory (see section 5.6). Final confirmation of its applicability to atmospheric air would therefore be necessary.

The confirmation may be established by comparing \( U_b \) from experiments in atmospheric air to those from calculations. Since experimental data of 'high quality' are available in IEC-52: 1960, i.e. Recommendations for voltage measurement by means of sphere-gaps (one sphere earthed), it was decided to apply this data and not to perform additional experiments.

This section will, therefore, apply the calculation procedure described in section 3 to determine \( U_b \) of standard sphere-gaps according to IEC-52: 1960, and compare the results with the values that are tabulated in this standard.

6.1 General

The breakdown voltage, \( U_b \), of sphere-gaps has been investigated since the beginning of this century because sphere-gaps are used in high voltage testing as a
standard for high voltage measurement. Nowadays, with the biggest standardized sphere diameter, i.e. 200 cm diameter of each sphere, according to the international standard, IEC-52: 1960, voltages up to about 2 MV can be measured. Sphere diameters bigger than this size are unlikely to be built because of difficulties in the manufacturing process, the cost, and the large size of space within a high voltage laboratory that is required. The most important reason is that there are other methods to measure such high voltage levels with more convenience and accuracy, e.g. by high voltage dividers.

However, sphere-gaps of small size with a diameter of less than about 100 cm can be included in high voltage laboratories without affecting construction costs. It can be used as a reliable measuring device for calibration process and as a checking device against other measuring equipments to ensure that nothing serious is wrong with those more sophisticated equipment.

The exact accuracy of the breakdown voltages given in IEC-52: 1960 is not known. However, IEC-52: 1960 states that the given values have an uncertainty of ± 3 % for AC, positive impulse and negative impulse voltage and ± 5 % for both polarities of DC voltage. This accuracy, applies only when the measurement procedure and the sphere-gap arrangement is conformed to IEC-52 recommendation.

Recently, in 1992, a revision of this IEC was proposed. (This revision is under the responsibility of IEC TC 42 WG 12, Convenor: Prof. G. Pesavento, Italy). The objectives of this revision can be summarized as follows:

1. to specify more clearly on the matter of irradiation,
2. to include the measurement of switching impulses, and
3. to test the applicability of air-gaps for checking the linearity of other measuring systems.

Apart from the above aims, the revision also indicates that the limited accuracy of the table values are assumed to be somewhat questionable.

To perform a full scale experiment to ensure the breakdown values given in this standard is time-consuming as there are about 320 values of breakdown voltage for standard sphere diameters, D, at various gap spacings, S, which are tabulated (U_b in Table I of IEC-52: 1960 are shown in Table F.1 in Appendix F).

One possible approach to shorten the experiment time is to perform the experiment only in the ranges that are suspected. The suspected values in turn may be identified with the aid of the calculation. Various attempts have been made to calculate the breakdown voltage of sphere-gaps. They are shortly reviewed and discussed in the following sub-sections.

Note that in the following sub-sections the international standard IEC-52: 1960 will be simply referred to as 'IEC'.
6.2 Previous attempts to calculate the breakdown voltage of sphere-gaps

The calculation of $U_b$ of sphere-gaps may have started 80 years ago by Peek (1913). Peek did extensive work to demonstrate that sphere-gaps can be used as a reliable high voltage measuring device. He also succeeded in calculating breakdown voltages for some sphere-gaps based on his semi-empirical approach. Peek gave an empirical formula which stated that the breakdown of sphere-gaps occurs when $E_{\text{max}}$ at the surface of the high voltage sphere attained a value given by

$$E_{\text{max}} = A + \frac{B}{\sqrt{D}} \quad 6.1$$

where

- $A, B =$ Constant
- $D =$ Sphere diameter

Therefore, $E_{\text{max}}$ should be independent of gap spacing.

Peek's formula could, however, not be applied to determine $U_b$ of small gap spacings. Toepler (see Pedersen, 1967b) suggested that in this case $U_b$ can be determined by

$$U_b = X \cdot S + Y \cdot \sqrt{S} \quad 6.2$$

where

- $X, Y =$ Constant
- $S =$ Gap spacing

Hence, $U_b$ is independent of sphere diameter.

Two empirical formulas are required to determine $U_b$ of sphere-gaps at a specific diameter. Toepler suggested that this requirement comes from the transition between two breakdown mechanisms.

Meek (1940b) explained that in a small gap spacing the electron avalanche travels across the entire gap while in a large gap spacing a critical electron avalanche may be developed only within some part of the gaps as the electric field is so non-uniform and ionisation cannot take place in the mid-gap region. He applied the streamer breakdown criterion to calculate $U_b$ of the symmetrical sphere-gaps and found, that the results were in agreement within 3% to those measured by Hüter (1936). However, Meek used $\alpha/p = f(E/p)$ for dry air from Sanders (1933) in his calculations while the experimental data from Hüter refer to atmospheric air.

As described in section 2.2.3, Pedersen (1966, 1967a) modified the streamer breakdown criterion to calculate $U_b$ of sphere-gaps. He used $\alpha/p = f(E/p)$ in dry air from Sanders (1933) to calculate $U_b$ by using eq. 2.43. However, he represented
the standard sphere-gaps arrangement according to IEC by using two - spheres, one sphere earthed, without stem in space to calculate the electric field along the axis of symmetry. The field calculation is based on numerical method.

This step forward could be expected as up to this time electric field calculation programs were still in the state of development and were not widely used due to the lack of computer to run those programs within acceptable time. Therefore, Pedersen (1967a) calculated only a few points of gap spacing for 25 cm sphere diameter.

Another reason why Pedersen (1967a) calculated only a few points may be attributed to the difficulty to determine \( g(x_c) \) at large gap spacing, see eq. 2.43. The function \( g(x_c) \) is determined from uniform field breakdown experiments. As a specific electrode diameter can only be used for a certain range of gap spacings (to ensure a high quality uniform field in the gap) electrodes with a diameter of 40 - 50 cm are required for 10 cm gap spacing. One can expect that it is quite difficult to manufacture and to set up the electrode in the proper arrangement in order to produce a high quality uniform field. Surprisingly, however, the calculation results of Pedersen (1967a) were in very good agreement with the values given in IEC.

Takuma (1971) modified the streamer breakdown criterion to calculate \( U_b \) of standard sphere-gaps. However, the calculation procedure of Takuma is questionable, as described previously in section 2.2.3.

Recently, Nishikori et al. (1993) used the modified streamer breakdown criterion (Pedersen, 1967a) and \( \alpha/p = f(E/p) \) in dry air from Sanders (1933) to calculate \( U_b \) of all standard sphere diameters but only for gap spacings of less than 20 cm. Though the numerical electric field calculations are obviously quite accurate, the calculation results seem to have large deviations from IEC at large gap spacing, see figure 9 in his publication. The large deviations may result from the difficulty in determining \( g(x_c) \) as in the case of Pedersen (1967a). It may also come from the fact that Nishikori neglected the influence of nearby objects in the electric field calculation and that he compared his results with positive impulse voltage values that are given in Table II of IEC.

From the above examples of previous attempts to calculate \( U_b \) of sphere-gaps, one can note that (with the exception of Peek (1913) and Toeppler, given in Pedersen, 1967b):

- all of the calculations are based on a streamer breakdown criterion;
- always ionisation coefficients, \( \alpha/p = f(E/p) \), for dry air from Sanders (1933) has been used; and
- no attention has been paid to the effect of humidity even if this effect on the \( U_b \) of sphere-gaps has been known since a long time (Meek, 1946).
6.3 Electric field calculation

For sphere-gaps with one sphere earthed, a basic factor influencing $U_b$ are the surroundings, i.e. the potential of nearby earthed objects (walls, etc.). IEC-52 takes such effects into account by clearance values "A" and "B". "A" is defined as the distance of the sparking point (P) of the high-voltage sphere to "floor level", and "B" as the radius of a virtual sphere, the center of which is at P, where no external structures are allowed to be placed inside except the objects as indicated in figure 2 on page 36 of IEC. As the electric field calculation must simulate these boundary conditions, the spherical space is simulated by a cylindrical "pot" of zero potential.

In Table 6.1, the value of "A" and "B" are reproduced from the table on page 13 of IEC. "A" is a function of $D$ and "B" is a function of $S$.

Table 6.1 Clearance around the spheres (from the table in clause 2.5 of IEC)

<table>
<thead>
<tr>
<th>Sphere diameter $D$, cm</th>
<th>Minimum value of &quot;A&quot;</th>
<th>Maximum value of &quot;A&quot;</th>
<th>Minimum value of &quot;B&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>up to 6.25</td>
<td>7 $D$</td>
<td>9 $D$</td>
<td>14 $S$</td>
</tr>
<tr>
<td>10 - 15</td>
<td>6 $D$</td>
<td>8 $D$</td>
<td>12 $S$</td>
</tr>
<tr>
<td>25</td>
<td>5 $D$</td>
<td>7 $D$</td>
<td>10 $S$</td>
</tr>
<tr>
<td>50</td>
<td>4 $D$</td>
<td>6 $D$</td>
<td>8 $S$</td>
</tr>
<tr>
<td>75</td>
<td>4 $D$</td>
<td>6 $D$</td>
<td>8 $S$</td>
</tr>
<tr>
<td>100</td>
<td>3.5 $D$</td>
<td>5 $D$</td>
<td>7 $S$</td>
</tr>
<tr>
<td>150</td>
<td>3 $D$</td>
<td>4 $D$</td>
<td>6 $S$</td>
</tr>
<tr>
<td>200</td>
<td>3 $D$</td>
<td>4 $D$</td>
<td>6 $S$</td>
</tr>
</tbody>
</table>

"A" is chosen to be a constant value for a given sphere diameter from column 2 (i.e. minimum values required by IEC) in the electric field calculation because the high voltage sphere is usually fixed, such that the distance from the sparking point at high voltage sphere to laboratory floor is constant. Only the gap spacing is adjusted by moving the earthed sphere up and down with the aid of a driving mechanism which is mounted near the laboratory floor. This description is only applied to the sphere-gaps that are arranged in a vertical configuration. For horizontal configuration, "A" is seldom adjusted because the high voltage and earthed sphere are usually fixed on the same frame (see figure 2 on page 37 of IEC).

The value of "B" can be chosen from Table 6.1 for the calculation. In practice, however, sphere-gaps are usually erected at a predetermined location and only the gap spacing is adjusted in the breakdown measurement. It is unlikely that the investigators 'moved' the location of sphere-gaps each time they changed the gap
spacing to meet the requirement of IEC, i.e. "B" value. A suitable value of "B" is
, therefore, related to the recommended value of the maximum gap spacing, \( S_{\text{max}} \)
for a given sphere diameter, i.e. \( S_{\text{max}} = D/2 \). This makes the value of "B" exactly
equal to the minimum recommended value of "A" for a given sphere diameter. This
configuration is shown in figure 6.1 and will be called 'small surrounding'.

![Figure 6.1](image)

**Figure 6.1** Configuration that is used to represent the actual standard sphere-gaps
according to IEC, 'small surrounding'.

IEC states clearly that the tabulated \( U_p \)-values were measured under conditions
where sphere-gaps were erected in an open laboratory with not more than one wall
at distance "B" and the other walls at a greater distance. Thus, different values
would be obtained if the sphere-gaps is placed in a cylindrical container of radius
"B", except when the gap spacing is small. In the simulation, to account for nearby
objects, "B" is chosen to be 2 times of "A", for a given sphere diameter. This
configuration is shown in figure 6.2 and will be called 'large surrounding'.

Note that both values of "B" for small and large surrounding do not violate the
requirements of IEC, as can be seen in Table 6.1.

The electric field line along the axis of symmetry was calculated by HSSSM. Along
this line, 1414 points have been selected for each combination of sphere diameter
and gap spacing according to IEC. These points have been unequally distributed
as shown in figure 6.3.
Figure 6.2  Configuration that is used to represent the actual standard sphere-gaps according to IEC, 'large surrounding'.

Figure 6.4 shows an example of the field distribution for a gap with $D = 10$ cm and different $S$ in normalized scale. Note that fields become quite inhomogeneous with increasing $S/D$ values.

The computed values of the electric field at 1414 points are fitted to two different polynomial equations of order 13. The first equation is used in the calculations to simulate the electric field between the high voltage sphere and the point where the electric field is minimal (will be indicated later). To simulate the electric field between the earthed sphere and this minimum field point, a second equation is used. Both polynomial equations can be expressed as

\[ E_1(x) = U \sum_{n=0}^{13} a_n \cdot x^n \] \hspace{1cm} 6.3

\[ E_2(x) = U \sum_{n=0}^{13} b_n \cdot x^n \] \hspace{1cm} 6.4

where

- $U$ = applied voltage in kV
- $a_n$ = polynomial coefficient of the first equation, for high voltage sphere
- $b_n$ = polynomial coefficient of the second equation, for earthed sphere
- $x$ = distance from the high voltage sphere in eq. 6.3, or
to earthed sphere in eq. 6.4
Figure 6.3 The electric field points along axis of symmetry that were calculated by using HSSSM.
Figure 6.4 Example of electric field along the axis of symmetry in the case of large surrounding, $D = 10$ cm. X scale is the distance from the high voltage sphere normalized to the gap spacing. Y scale is the electric field normalized to the maximum electric field which is at the sparking point of the high voltage sphere. $E_{\text{max}}$ is the field strength at the sparking point, i.e. $x = 0$.

6.4 Breakdown voltage calculation

The breakdown voltages were calculated by using the simplified form of the streamer breakdown criterion together with the knowledge of the electric field from the previous clause, $\overline{\alpha}/\rho = f(E/\rho)$ in synthetic air from Friedrich (1992), eq. 3.3, and the magnitude of $K = 9.15$ from section 5.

The calculated breakdown voltage is based on the standard conditions according to IEC, i.e. $p = 1.013$ bar and $T = 20$ °C. The simplified form of the streamer breakdown criterion can be written as (see also eq. 2.35)

$$\int_0^x \overline{\alpha} dx = K$$

6.5
Application of the calculation method described in section 3, to calculate $U_b$ at gap spacings from 0.05 to 100 cm will lead to new problems due to the large range of completely different field distributions. The field distributions vary from nearly homogeneous distributions at small gap spacing to quite inhomogeneous distribution at large gap spacing, see figure 6.4. The electric fields are also not continuously decreasing from the high voltage sphere to the earthed sphere. It is more or less symmetrical, depending on gap spacing at a specific sphere diameter, due to the nearly symmetrical arrangement of sphere-gaps.

At a specific sphere diameter, various situations can occur in the calculation. Detailed explanations will be given below.

**Situation I - "small S/D-values"**

Figure 6.5 shows the electric field distribution along the axis of symmetry when $S/D$-values are small (represented approximately for the case of $D = 10$ cm and $S = 0.5$ cm). The electric field for this small $S/D$-values are quite uniform and symmetrical, i.e. the electric field at earthed sphere $E_w$ has nearly the same magnitude as $E_{\text{max}}$.

![Figure 6.5 Situation I - "small S/D-values" that occurs when calculating $U_b$ of sphere-gaps with the streamer breakdown criterion.](image)

As the voltage across the sphere-gaps is increased, the avalanche will develop deeper into the gap, i.e. $x_c$ increases. At a specific applied voltage, $x_c$ will reach a point where the electric field is minimum, $E_{\text{min}}$, and equal to $(E/p)_{\text{cr}}p$, where the effective ionisation coefficient at this point is equal to zero. The avalanche
can develop further toward the earthed sphere because the electric field beyond
this point is again higher than \((E/p)_{ap}.p\). The integration limits of the left hand
side in eq. 6.5 are taken from the high voltage to the earthed sphere, i.e. 0 to \(S\).

The total amount of electrons in the avalanche is still less than that required for
the transformation to a streamer. For the specific case in figure 6.5 it is only
5% of the required value. The voltage across the sphere-gaps must be further
increased to satisfy the requirement of the streamer breakdown criterion. At
breakdown, \(E_{min}\) will be larger than \((E/p)_{ap}.p\) as shown in figure 6.5.

This means, that a critical avalanche can be transformed to a breakdown streamer
within the full width of the gas gap. There should also be no dependency on the
polarity of the applied voltage as high fields exist on the high voltage and on the
earthed sphere and both can be cathode. From IEC, it is well known that even
for positive impulse voltages the breakdown voltages are not higher than those
of DC or AC voltages, provided the ratio \(S/D\) is small.

**Situation II - "intermediate \(S/D\)-values"
**

As \(S/D\)-values increase, the electric field distribution becomes less symmetrical.
Figure 6.6 shows the electric field along the axis of symmetry (represented
approximately for the case of \(D = 10 \text{ cm}\) and \(S = 2.4 \text{ cm}\)).

![Figure 6.6](image)

**Figure 6.6** Situation II - "intermediate \(S/D\)-values" that occurs when calculat-
ing \(U_b\) of sphere-gaps with the streamer breakdown criterion.

This situation begins in the same way as situation I until a point where \(E_{min} =
(E/p)_{ap}.p\) is reached. The avalanche develops from the high voltage sphere further
to the earthed sphere and the integration limits of the left hand side in eq. 6.5 are also taken from 0 to $S$.

However, the amount of electrons in the avalanche is larger than those required by the streamer breakdown criterion, i.e. the left hand side is larger than the right hand side in eq. 6.5. The breakdown is, therefore, assumed to occur. Situation IV, which will be described later, will be introduced to deal with this oversatisfied situation.

Note that for a given $D$, there is only one gap spacing where this situation occurs and the streamer breakdown criterion is exactly fulfilled. Of course, this $S/D$-values is not tabulated in IEC (for 10 cm sphere diameter, this specific gap spacing lie between 2.2 - 2.4 cm).

**Situation III - "large S/D-values"**

Figure 6.7 shows the electric field distribution along the axis of symmetry when $S/D$-values are large (represented approximately for the case of $D = 10$ cm and $S = 5$ cm). The electric field is completely unsymmetrical and quite inhomogeneous. In this situation, the avalanche develops into the gap and transforms to a streamer when its length is less than the point where electric field is minimal, i.e. $x_c < x(E_{\min})$ in figure 6.7. In the calculation, eq. 6.5 is fulfilled when the integration limits are taken from 0 to only $x_c$.

Note that this situation takes only one side, the one with the high voltage sphere, of the integration into account.

![Diagram of electric field distribution](image)

**Figure 6.7** Situation III - "large S/D-values" that occurs when calculating $U_b$ of sphere-gaps with the streamer breakdown criterion.
Situation IV - "space charge effect"

This situation is assumed to occur at the intermediate range of \( S/D \)-values as in situation II. The range of occurrence is, however, wider than situation II. Figure 6.8 shows the electric field distribution along the axis of symmetry in this situation.

This situation is introduced because space charge effects will be responsible for the breakdown of gaps. As the primary electron avalanche develops from the high voltage sphere in the direction of the field, the local electric field at the avalanche head increases the geometrical electric field due to the space charge produced. Excitation also occurs, which is more frequent than ionisation.

![Diagram](https://via.placeholder.com/150)

**Figure 6.8** Situation IV - "space charge effect" that are used when calculating \( U_b \) of sphere-gaps with the streamer breakdown criterion.

For the effect of space charge, when the electron avalanche reaches the point \( x_c \) (see figure 6.8) where the geometrical field is equal to \( (E/p)_\alpha \cdot p \) and \( \bar{\alpha} = 0 \), the avalanche may keep its size and neither increase nor decrease, as the local electric field is higher than \( (E/p)_\alpha \cdot p \), i.e. the local \( \bar{\alpha} \) at avalanche head is still greater than zero. Therefore, it can be assumed that the electron number in the avalanche is \( N_1 \) when it reaches \( x_c \). Then the avalanche moves further into the gap and will pass the region where \( \bar{\alpha} \leq 0 \). It will lose some electrons due to the attachment process in gas but it will also gain electrons due to the space charge field. Thus, it will travel to the point \( x_c' \) where \( \bar{\alpha} = 0 \), and beyond this point where \( \bar{\alpha} \) is greater than zero. The avalanche then starts to increase its size again and finally will reach a critical size where the avalanche-to-streamer transformation can take place. This means, that the ionisation process can take place in both high field regions, see photo in figure 6.9. Note that photo ionisation can also take place.
at the same time in the gas which will lead to a much more complicated phenomena.

A quantitative determination of the space charge effects which are assumed to occur in mid-gap ($\alpha \leq 0$) is not possible. To simplify this problem, one may assume that the avalanche has a size of $N_1$ at $x_c$ and it will travel through the low field region, where $\alpha \leq 0$, without effectively losing electrons. When it reaches $x_c'$, it will increase its size again until it transforms to a streamer and leads to breakdown. The mathematical expression can, therefore, be written as

$$\int_0^{x_c} \overline{\alpha}_1 \, dx + \int_{x_c'}^s \overline{\alpha}_2 \, dx = K$$

where

$$\overline{\alpha}_1 = \int \frac{1}{E_1 / p} \, dx$$
$$\overline{\alpha}_2 = \int \frac{1}{E_2 / p} \, dx$$

and note that $\exp\left(\int_0^{x_c} \overline{\alpha}_1 \, dx\right)$ is equal to $N_1$.

Figure 6.9 Photo of two streamers developing from high voltage and earthed sphere at the same time.

Detailed calculations of these various situations are given in Appendix G. However, the order of situation is changed for the sake of easiness in mathematical derivation.
6.5 Results of $U_b$ calculation

The calculated results of $U_b$-values are tabulated in Table F.2 of Appendix F for the calculation without taking space charge effect into account and in Table F.3 of Appendix F for the case where space charge effect was taken into account. These two types of calculation will be referred to as 'w/o-SC' and 'w-SC' respectively.

As the calculated $U_b$ cover a range up to nearly 1.9 MV and as the difference between the calculated and tabulated $U_b$-values cannot be clearly shown within the plot of absolute values, the comparison will be expressed as percentage deviation as

\[
\% U_b = \frac{(U_{b,\text{Calculation}} - U_{b,\text{ECTTable1}})}{U_{b,\text{ECTTable1}}} \times 100
\]

They are plotted against gap spacing, $S$, which are expressed in percent of the relevant sphere diameter, $D$. Therefore

\[
\% \text{ gap spacing} = \frac{S}{D} \times 100
\]

A value of 50% gap spacing is, therefore, the limit for which sphere-gaps of diameter $D$ should be used.

Typical calculated results of 'w/o-SC' and 'w-SC' are shown in figure 6.10 for the case of large surrounding with $D = 50$ cm. It can be seen that the calculation results of 'w-SC' are more uniform than 'w/o-SC'. Various situations which occur in each type of calculation are:

'w/o-SC': For a given $D$, at small $S/D$-values the field distribution is nearly homogeneous, the calculation starts from situation I where $x_c = S$ and $E_{\text{min}}$ is always larger than $(E/p)_{\alpha} \cdot p$.

At intermediate $S/D$-values $E_{\text{min}}$ becomes equal to $(E/p)_{\alpha} \cdot p$ and $U_b$ may still be calculated from $x_c = S$. However, the streamer breakdown criterion is over satisfied which is situation II.

At large $S/D$, the streamer criterion is easily fulfilled in the high-field region near the high voltage electrode which falls into situation III.

Note that in this case of 'w/o-SC', the electric field $E_{\text{h}}$ may or may not be lower than $(E/p)_{\alpha} \cdot p$.

'w-SC': This type of calculation begins in the same way as 'w/o-SC' where $S/D$-values are small. In the intermediate range of $S/D$ where $E_{\text{min}}$ becomes smaller than $(E/p)_{\alpha} \cdot p$, we assume that the space charge effect and photo ionisation are active, i.e. situation IV.

For large $S/D$-values the calculation are accomplished in the same way as 'w/o-SC' which is situation III. This is because the electric field of the earthed sphere, $E_{\text{h}}$, is lower than $(E/p)_{\alpha} \cdot p$. 
Figure 6.10 Comparison between two types of calculation, i.e. 'w/o-SC' and 'w-SC', for the case of large surrounding with $D = 50$ cm.

The complete results of calculated $U_h$ for large surrounding are shown in figure 6.11 and 6.12 for the case of 'w/o-SC' and 'w-SC' respectively.

Apart from the effect of the calculation type on $U_h$, various influences such as surrounding, the magnitude of $K$ and humidity are demonstrated. A summary of the parameters that influence the calculated $U_h$ are shown in Table 6.2 together with the figure number.

Table 6.2 Various influences on the calculated $U_h$-values

<table>
<thead>
<tr>
<th>Influence</th>
<th>Calculation type</th>
<th>$K$</th>
<th>Surrounding</th>
<th>Humidity</th>
<th>Figure No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surrounding $K$</td>
<td>w/o-SC</td>
<td>9.15</td>
<td>-</td>
<td>no correction</td>
<td>6.13</td>
</tr>
<tr>
<td>Calculation type</td>
<td>w/o-SC</td>
<td>-</td>
<td>large</td>
<td>no correction</td>
<td>6.14-6.15</td>
</tr>
<tr>
<td>Humidity</td>
<td>w-SC</td>
<td>9.15</td>
<td>large</td>
<td>with correction</td>
<td>6.16-6.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.18</td>
</tr>
</tbody>
</table>
Figure 6.11 Calculated results of $U_b$ in large surrounding, without taking the space charge effect into account, 'w/o-SC'.

Figure 6.12 Calculated results of $U_b$ in large surrounding, when taking the space charge effect into account, 'w-SC'.
6.6 Discussion

Influence of the surrounding

The nearby earthed objects will increase $E_{\text{max}}$ at the sparking point of the high voltage sphere. Figure 6.13 shows the comparison between $U_b$ which are calculated by using small surrounding simulation, see figure 6.1, to those which are calculated by using large surrounding simulation, see figure 6.2.

In the small surrounding, an enhancement of $E_{\text{max}}$ occurs. The influence on $U_b$ is, however, less than 1.5% even at large $S/D$-values and negligible up to $S/D$-values up to about 0.2.

![Graph showing the comparison between small and large surrounding simulation](image)

**Figure 6.13** The deviation of breakdown voltage from IEC Table I, comparison between small and large surrounding simulation. Calculated by using $K = 9.15$, 'w/o-SC' and $D = 75, 100, 150$ and $200$ cm.
Influence of $K$

Up to now, all results are based on $K = 9.15$. Therefore the influence of usual values for $K$ on the results are of interest.

The comparison of $U_b$, which are calculated by using $K = 9.15$ and $K = 18$ are shown in figure 6.14 and 6.15. It can be seen that deviations from IEC as high as 25% will occur when $K = 18$ is used for calculation. In contrast, the suggested value of $K = 9.15$ show reasonable deviations.

The lower $K$-value of 9.15 is, therefore, confirmed again by the calculation results in this section.

Figure 6.14 The deviation of breakdown voltage from IEC Table I, comparison between $K = 9.15$ and $K = 18$. Calculated by using large surrounding, 'w/o-SC' and $D = 2.5, 5, 6.25, 10, 12.5$ and 15 cm.
Figure 6.15 The deviation of breakdown voltage from IEC Table I, comparison between \( K = 9.15 \) and \( K = 18 \). Calculated by using large surrounding, 'w/o-SC' and \( D = 25, 50, 75, 100, 150 \) and 200 cm.

**Influence of space charge effect**

The comparison of \( U_b \) which are calculated without taking space charge effect into account to those when taking space charge effect into account are shown in figure 6.16 - 6.17. The results of 'w-SC' are more homogeneous than 'w/o-SC'.

The more homogeneous results come from the assumption that space charge can play a major roll in the breakdown of the gap. This situation takes into consideration both the electric fields in the region of the high voltage and the earthed sphere in the evaluation of breakdown voltage. A comparison of eq. G.15 and G.11 shows that the right hand side of equation G.15 is always higher than the right hand side of eq. G.11 at the same voltage, i.e. the streamer breakdown criterion is satisfied at lower voltage.
Figure 6.16 The deviation of breakdown voltage from IEC Table I, comparison between 'w-SC' and 'w/o-SC'. Calculated by using large surrounding, \( K = 9.15 \) and \( D = 12.5, 15, 25 \) and 50 cm.

Figure 6.17 The deviation of breakdown voltage from IEC Table I, comparison between 'w-SC' and 'w/o-SC'. Calculated by using large surrounding, \( K = 9.15 \) and \( D = 75, 100, 150 \) and 200 cm.
Influence of the humidity

So far, the breakdown voltage of sphere-gaps according to IEC were calculated by using the streamer criterion together with the relationship of $\alpha/p = f(E/p)$ in synthetic air. With $K = 9.15$ and assuming that space charge effects can occur, the deviations of the calculated results and the value from Table I of IEC were within $+1.5\%$ to $-5.5\%$, see figure 6.12, except for $D = 200$ cm.

Within IEC-52, the influence of humidity on the breakdown voltage is actually taken into account, although no need to correct humidity effect on breakdown voltage is suggested. This assumption is due to the fact, that the IEC-values are based on measurements for atmospheric air under usual conditions, for which humidity is present. It is, however, widely accepted and known, that the humidity has a systematic influence on the breakdown voltage. As humidity, $h$, of $11\,\text{g/m}^3$ is "normal" for atmospheric air and $h$ increases $U_b$ also for sphere-gaps by $0.2 - 0.3\%/\text{g/m}^3$ (Allen and Hughes, 1994; Kuffel and Zaengl, pp. 103 - 105, 1984), we can assume that at $11\,\text{g/m}^3$ the $U_b$ is going to increase about $2.5\%$, i.e. about $0.25\%/\text{g/m}^3$ (Allen, 1985).

To take this influence of humidity into account, the calculated results from figure 6.12 are multiplied by 1.025 as shown in figure 6.18. Now, the deviations from IEC in this case are within $-3\%$ to $4\%$, except for $D = 200$ cm.

![Figure 6.18](image)

**Figure 6.18** The deviation of breakdown voltage from IEC Table I, when the influence of humidity is taken into account. Calculated by using large surrounding, 'w-SC', $K = 9.15$ and all $S/D$-values.
These comparison indicates that the relationship of $\tilde{\alpha}/p = f(\mathcal{E}/p)$ in synthetic air from Friedrich (1992) can actually be used for the calculation of breakdown voltage in atmospheric air, if proper measures are taken to correct the humidity effect.

Additional remarks on the tabulated values in IEC

Comparisons between the calculated $U_b$ and the tabulated values in IEC Table I have been made. The general tendency of deviation is more or less similar for all $D$, except for $D = 200$ cm and for $S/D$ larger than 0.25. The breakdown voltages, tabulated in IEC, in this range are very likely to be too low. The breakdown voltage of the 200 cm sphere-gap should, therefore, have a special consideration in the current revision of IEC-52:1960.

The second remark arises from a critical view to the individual results, as e.g. displayed in figure 6.19. The region designated with A, B and C, where the deviations show curve segments, are indicated as regions where IEC values are suspected to be interpolated. These suspected interpolations occur more often at large sphere diameters than at small diameter. When these regions are plotted by its absolute values, it is be a perfect straight line as shown in figure 6.20, for region B, while those from the calculation are curved as forecasted by theory. The quality of calculation is thus also confirmed by this effect.

The third remark arises when the tendency of $%U_b$ in figure 6.18 is considered. The calculated $U_b$ in this section is based on the assumption that the starting electron is available at the right place and time to initiate the critical avalanche. This means that the calculated $U_b$ is the minimal breakdown voltage that can occur. When it is compared with the measured value, it should show only the negative deviation and positive deviations should not occur, see figure 6.18.

Positive deviations, where the calculated $U_b$ is higher than the measured $U_b$, may be attributed to imperfections. Imperfections can come from the sphere itself, the sphere-gap arrangement, and also from small particles, i.e. dust. One can expect that imperfections increase with increasing $D$. The deviations of $U_b$ in figure 6.18 become more and more positive indeed with increasing sphere diameter.

The calculated breakdown voltage cannot be applied to Table II of IEC (i.e. positive impulse breakdown voltages) due to the well known fact that the 50 % breakdown in this case is influenced by the effect of statistical time lag caused by the absence of primary electrons (Meek, 1946).
Figure 6.19  The deviation of breakdown voltage from IEC Table I, the regions where the tabulated values in IEC are suspected to be interpolated. The example at $D = 50$ and $150$ cm, calculated by using large surrounding, 'w-SC', $K = 9.15$.

Figure 6.20  Absolute value of breakdown voltage in the region B in figure 6.19.
SUMMARY

The applicability of the simplified streamer breakdown criterion, i.e. $\int \bar{\alpha} dx = K$, to determine the inception voltage of non-uniform field gas gaps has been investigated. The dependency of the magnitude of $K$ on the discharge conditions, e.g. applied voltage polarity, field non-uniformity, gas pressure, has been studied as the magnitude of $K$ still has some uncertainty.

In the past, the magnitude of $K \approx 18$ was typically assumed for many gases. However, recently, it has been shown that $K$ has the magnitude of about 10 for synthetic air and SF$_6$ (Pedersen, 1984; Zaengl et al., 1991).

Such a high value of $K = 18$ is obviously the result of an inaccurate relationship of $\bar{\alpha}/p = f(E/p)$ which was used in the derivation and modification of the streamer breakdown criterion. For example in dry air, see figure 3.4 a), one can note that at equal $E/p$-magnitude the values of $\bar{\alpha}/p$ from Sanders (1933) are higher than those of Harrison and Geballe (1953) and $\bar{\alpha}/p$ for synthetic air from Friedrich (1992) are lowest. However, it is well known that water content in air tends to increase attachment coefficients which in turn decrease the value of $\bar{\alpha}$. The value of $\bar{\alpha}/p = f(E/p)$ of Friedrich should be the highest as the synthetic air in his experiments had almost no water content. One can then conclude that such a high value of $\bar{\alpha}/p = f(E/p)$ of Sanders (1933) and Harrison and Geballe (1953) results from the error in their measurements.

This observation is also true for SF$_6$. Extensive work in the past, e.g. Nitta and Shibuya (1970), Bortnik and Cooke (1972), Hazel and Kuffel (1976), are based on $K = 18$ and $\bar{\alpha}/p = f(E/p)$ from Bhalla and Craggs (1962). The value of $\bar{\alpha}/p = f(E/p)$ from Bhalla and Craggs are higher than today's accepted values of Boyd and Crichton (1971) or Aschwanden (1984), see figure 3.5 a).

One can immediately see that, if we use high values of $\bar{\alpha}/p = f(E/p)$ in the calculations, we also need a high value of $K$ to make the calculated results match with the experiments, and vice versa.
In this investigation, $K$ values are studied by using a very accurate electric field calculation program, the best value of $\alpha/p = f(E/p)$ and an accurate experimental procedure. The experiments were carried out by using; 1. the Sphere-Pot configuration as most engineering applications relate to a non-uniform electric field distribution, 2. dry air and SF$_6$ as these two gases are widely used as an insulating medium, and 3. in the range of the product of pressure and sphere radius, i.e. $pr$, between 0.1 - 10 bar-mm as the magnitude of $K$ is not very sensitive at high $pr$.

It has been found that $K$ has a magnitude of about 10 and not 18 as generally believed. Its value slightly depends on the discharge conditions. Within the conditions investigated, $K$ can be treated as a constant equal to 9.15 for dry air and 10.5 for SF$_6$. With such constant values, the calculated inception voltages agree quite well with experimental results for both gases, except for the positive highly stressed electrode in dry air. In this case, $K$ will change with $pr$. However, with $K$ equal to 9.15, the deviation between the calculated and the experimental results is not very large at low $pr$ and it decreases with increasing $pr$.

The applicability of the relationship of $\alpha/p = f(E/p)$ for synthetic air to atmospheric dry air has also been confirmed. It can be applied successfully to predict the uniform field breakdown voltage of atmospheric dry air based on Meek's or Raether's streamer breakdown criteria or the simplified streamer breakdown criterion. It can also be applied to predict the non-uniform field breakdown voltage in normal atmospheric air by using the simplified streamer breakdown criterion, e.g. the breakdown voltages of sphere-gaps according to IEC-52: 1960, if the effect of the humidity is taken into account.

The simplified streamer breakdown criterion can, therefore, be used to predict both the inception or the breakdown voltage of non-uniform field gas gaps.
ZUSAMMENFASSUNG

In der vorliegenden Arbeit wurde die Anwendbarkeit des vereinfachten Streamer-Durchschlagskriteriums, d.h. $\int \alpha dx = K$, auf die Bestimmung der Einsatzspannung von Gasentladungsstrecken mit inhomogener Feldgeometrie untersucht. Da die absolute Größe der Streamer-Konstante $K$ immer noch mit einiger Unsicherheit behaftet ist, wurde dazu die Abhängigkeit von $K$ von den Entladungsbedingungen, wie z.B. der Polarität der angelegten Spannung, der Inhomogenität des elektrischen Feldes und des Gasdruckes experimentell ermittelt.

Bis anhin wurde typischerweise ein Wert von $K = 18$ für die meisten Gase angenommen. Erst kürzlich wurde gezeigt, dass für synthetische Luft und SF$_6$ $K$ einen Wert von ca. 10 annimmt (Pedersen, 1984; Zaengl et al., 1991).

Der bisher angenommene hohe Wert von $K = 18$ ist offensichtlich das Resultat einer ungenauen Funktion $\alpha/p = f(E/p)$, die für die Ableitung und Modifikation des Streamer-Durchschlagskriteriums verwendet wurde. So kann zum Beispiel nachgewiesen werden (Fig. 3.4 a), dass für trockene Luft die Werte von $\alpha/p$ von Sanders (1933) bei gleichem $E/p$ grösser sind als diejenigen von Harrison und Geballe (1953), die wiederum grösser sind als die Werte von $\alpha/p$, die Friedrich für synthetische Luft gefunden hat. Unter Berücksichtigung der Tatsache, dass der Anlagerungskoeffizient mit steigendem Wassergehalt der Luft zunimmt, was zu einer Reduktion von $\alpha$ führt, sollten die Werte von $\alpha/p = f(E/p)$ von Friedrich die höchsten sein, da die synthetische Luft in seinen Experimenten praktisch kein Wasser enthielt. Daraus kann man schliessen, dass die hohen Werte von $\alpha/p = f(E/p)$ von Sanders (1933) und Harrison und Geballe (1953) auf Grund von Messfehlern zustande gekommen sind.

Daraus ist unmittelbar ersichtlich, dass man ein grosses $K$ benötigt, wenn man in numerischen Berechnungen hohe Werte für $\alpha/p = f(E/p)$ verwendet hat, um Berechnungen und Experimente in Einklang zu bringen (und umgekehrt).

Die hier etablierten Werte für $K$ wurden unter Verwendung eines sehr genauen elektrischen Feldberechnungsprogramms, den sichersten Werten für $\alpha/p = f(E/p)$ sowie durch sehr exakte Experimente ermittelt. Die Experimente wurden unter folgenden Bedingungen durchgeführt; 1. es wurde eine Kugel-Topf-Anordnung verwendet, da die meisten praktischen Anwendungen auf inhomogener Feldgeometrie beruhen, 2. es wurden trockene Luft und SF$_6$ untersucht, da diese zwei Gase in der Hochspannungs-Isoliertechnik weit verbreitet sind, und 3. wurde in einem Bereich des Produktes von Druck und Kugelradius, d.h. $pr$, zwischen 0.1 und 10 bar-mm gemessen, da sich der Betrag von $K$ bei hohen Werten von $pr$ nicht stark ändert.


Das vereinfachte Streamer-Durchschlagskriterium kann deshalb sowohl zur Berechnung der Einsatz- als auch der Durchschlagsspannung von Gasentladungsstrecken mit inhomogener Feldgeometrie verwendet werden.
APPENDIX A. INCEPTION VOLTAGE DETERMINATION IN COAXIAL SPHERE GEOMETRY

A.1 Basic equations

The electric field at radius $r$ can be represented by

$$E(r) = \frac{U r_1 r_2}{(r_2 - r_1) r^2}$$

and, with $E_{\text{max}} = E(r_1)$

$$E(r) = \frac{E_{\text{max}} r_1^2}{r^2}$$

At inception voltage $U_i$, $E/p = (E/p)_a$ occurs at $x = x_c$. Then

$$\left( \frac{E_{\text{max}}}{p} \right) = \frac{u_i}{p} \frac{r_2}{(r_2 - r_1) r_1}$$

$$\left( \frac{E}{p} \right)_{\alpha} = \frac{(E_{\text{max}}/p) r_1^2}{(r_1 + x_c)^2}$$

For SF$_6$ in the region $8.9246 < E/p < 12.36$ kV/mm-bar, the relationship of $\alpha/p = f(E/p)$ can be represented by

$$\frac{\alpha}{p} = C \left[ \frac{E}{p} - \left( \frac{E}{p} \right)_{\alpha} \right]$$

where $C = 27.9$ kV and $(E/p)_{\alpha} = 8.9246$ kV/mm-bar. In the range $12.36 < E/p < 21$ kV/mm-bar, this relationship can be expressed as

$$\frac{\alpha}{p} = C_1 \frac{E}{p} - A_1$$

where $C_1 = 22.359$ kV and $A_1 = 180.171$ 1/mm-bar. The transition point from eq. A.5 to A.6 in SF$_6$ occurs at $(E/p)_{\text{SF}_6} = 12.36$ kV/mm-bar.
For dry air in the region $2.588 < \frac{E}{p} < 7.943 \text{ kV/mm-bar}$, the relationship of $\frac{\alpha}{p} = f(\frac{E}{p})$ can be represented by

$$\frac{\alpha}{p} = C\left[\frac{E}{p} - \left(\frac{E}{p}\right)_w\right]^2 - A \quad \text{A.7}$$

where $C = 1.6053 \text{ mm-bar/kV}^2$, $(\frac{E}{p})_w = 2.165 \text{ kV/mm-bar}$, $A = 0.2873 \text{ 1/mm-bar}$, and $(\frac{E}{p})_\alpha = 2.588 \text{ kV/mm-bar}$. In the range $7.943 < \frac{E}{p} < 14 \text{ kV/mm-bar}$, this relationship can be expressed as

$$\frac{\alpha}{p} = C_1\frac{E}{p} - A_1 \quad \text{A.8}$$

where $C_1 = 16.7766 \text{ 1/kV}$ and $A_1 = 80.0006 \text{ 1/mm-bar}$. The transition point from eq. A.7 to A.8 in dry air occurs at $(\frac{E}{p})_{air} = 7.943 \text{ kV/mm-bar}$

### A.2 SF₆ in the range $8.9246 < \frac{E}{p} < 12.36 \text{ kV/mm-bar}$

From the streamer breakdown criterion and the relationship of $\frac{\alpha}{p} = f(\frac{E}{p})$

$$\int_{r_1}^{r_1+x_c} \left(\frac{E}{p}\right) dr - \left(\frac{E}{p}\right)_{x_c} = \frac{K}{pC} \quad \text{A.9}$$

Considering only the first term on the left hand side

$$\int_{r_1}^{r_1+x_c} \left(\frac{E}{p}\right) dr = \int_{r_1}^{r_1+x_c} \left(\frac{E_{max}}{p}\right) \frac{r_1^2}{r^2} dr$$

$$= \left(\frac{E_{max}}{p}\right)_{r_1} \left[\frac{1}{r_1+x_c} - \frac{1}{r_1}\right]$$

$$= \left(\frac{E_{max}}{p}\right)_{r_1} - \left(\frac{E_{max}}{p}\right)_{r_1} \frac{r_1}{r_1+x_c} \quad \text{A.10}$$

Substituting eq. A.10 into A.9, we get

$$\frac{K}{pC} = \left(\frac{E_{max}}{p}\right)_{r_1} - \left(\frac{E_{max}}{p}\right)_{r_1} \frac{r_1}{r_1+x_c} \left(\frac{E}{p}\right)_{x_c} \quad \text{A.11}$$

or

$$\left(\frac{E_{max}}{p}\right) = \left[\sqrt{\left(\frac{E}{p}\right)_{x_c}} + \sqrt{\frac{K}{C \cdot p r_1}}\right]^2 \quad \text{A.12}$$

$U$, can be easily calculated by using eq. A.3.
A.3 SF₆ in the range 12.36 < \( \frac{E_{\text{max}}}{p} < 21 \) kV/mm·bar

This situation occurs when \( \frac{E_{\text{max}}}{p} \) from section A.2 is greater than \( (E/p)_{SF6} \). From the streamer breakdown criterion and two relationship of \( \alpha = \frac{E}{p} \times \frac{E}{p} \)

\[
\frac{K}{p} = C_1 \int_{r_1}^{r_1+x_1} \left( \frac{E}{p} \right)^2 dr - A_1 x_1 + C \int_{r_1+x_1}^{r_1+x_c} \left( \frac{E}{p} \right)^2 dr - C \left( \frac{E}{p} \right)_{\sigma} (x_c - x_1) \quad A.13
\]

where \( x_1 \) is the point that \( E/p = (E/p)_{SF6} \). The integration result is

\[
\frac{K}{p} = C_1 \int_{r_1}^{r_1+x_1} \left( \frac{E_{\text{max}}}{p} \right)^2 \frac{r_2^2}{r_2} dr - A_1 \cdot x_1 + C \cdot \int_{r_1+x_1}^{r_1+x_c} \left( \frac{E_{\text{max}}}{p} \right)^2 \frac{r_2^2}{r_2^2} dr - C \cdot \left( \frac{E}{p} \right)_{\sigma} (x_c - x_1)
\]

\[
= -C_1 \cdot \left( \frac{E_{\text{max}}}{p} \right) r_1 \left( \frac{r_1}{r_1 + x_1} - 1 \right) - A_1 \cdot x_1 - C \cdot \left( \frac{E}{p} \right)_{\sigma} (x_c - x_1) \quad A.14
\]

There are three variables in the eq. A.14, \( x_c, x_1, \) and \( (E_{\text{max}}/p) \). However, eq. A.14 can be computed by varying \( x_c \) alone because at a specific voltage \( U \),

\[
x_c = r_1 \cdot \sqrt{\left( \frac{E_{\text{max}}}{p} \right)_{\sigma}} - r_1 \quad A.15
\]

\[
x_1 = r_1 \cdot \sqrt{\left( \frac{E_{\text{max}}}{p} \right)_{SF6}} - r_1 \quad A.16
\]

The point where both sides of the eq. A.14 are equal will be reached, provided that \( x_c < (r_2 - r_1) \). Substituting \( x_c \) into eq. A.4, \( E_{\text{max}}/p \) can be determined. \( U \) can also be determined with the aid of eq. A.3.

A.4 Air in the range 2.588 < \( \frac{E_{\text{max}}}{p} < 7.943 \) kV/mm·bar

From the streamer breakdown criterion and the relationship of \( \alpha = f(E/p) \)

\[
\frac{K}{pC} = \int_{r_1}^{r_1+x_c} \left( \frac{E}{p} \right)^2 dr - 2 \left( \frac{E}{p} \right) \cdot \int_{r_1}^{r_1+x_c} \left( \frac{E}{p} \right)^2 dr + \left( \frac{E}{p} \right)^2 x_c - \frac{A}{C} x_c \quad A.17
\]
The integration result will give

\[
\frac{K}{pC} = \int_{r_1}^{r_1+x_c} \left( \frac{E_{\text{max}}}{p} \right)^2 \frac{r_1^4}{r^4} dr - 2 \left( \frac{E}{p} \right)_M \int_{r_1}^{r_1+x_c} \left( \frac{E_{\text{max}}}{p} \right)^2 \frac{r_1^2}{r^2} dr + \left( \frac{E}{p} \right)_M^2 x_c - \frac{A}{C} \cdot x_c
\]

\[= - \frac{1}{3} \left( \frac{E_{\text{max}}}{p} \right)^2 r_1^3 \left( \frac{r_1^3}{(r_1+x_c)^3} - 1 \right) + 2 \left( \frac{E}{p} \right)_M \left( \frac{E_{\text{max}}}{p} \right) r_1 \left( \frac{r_1}{(r_1+x_c)} - 1 \right) + \left( \frac{E}{p} \right)_M^2 x_c - \frac{A}{C} \cdot x_c \quad \text{A.18}
\]

Eq. A.18 can be evaluated by using the relationship of \( x_c \) and \( (E_{\text{max}}/p) \) in eq. A.15 and by increasing \( x_c \) from zero, the point where both side of the eq. A.18 are equal will be reached, provided, that \( x_c < (r_2 - r_1) \). Substituting \( x_c \) into eq. A.4, \( E_{\text{max}}/p \) can be determined. \( U_i \) can also be determined with the aid of eq. A.3.

A.5 Air in the range \( 7.943 < E_{\text{max}}/p < 14 \, \text{kV/mm-bar} \)

This situation occurs when \( E_{\text{max}}/p \) from section A.4 is greater than \( (E/p)_{\text{air}} \). From the streamer breakdown criterion and two relationship of \( u/p = f(E/p) \)

\[
\frac{K}{p} = C_1 \int_{r_1}^{r_1+x_i} \left( \frac{E}{p} \right) dr - A_1 x_1 + C \int_{r_1}^{r_1+x_c} \left( \frac{E}{p} \right)^2 dr - 2C \left( \frac{E}{p} \right)_M \int_{r_1}^{r_1+x_1} \left( \frac{E}{p} \right) dr + C \left( \frac{E}{p} \right)_M^2 (x_c - x_i) - A(x_c - x_1) \quad \text{A.19}
\]

where \( x_i \) is the point that \( E/p = (E/p)_{\text{air}} \). The integration result is

\[
\frac{K}{p} = C_1 \int_{r_1}^{r_1+x_i} \left( \frac{E_{\text{max}}}{p} \right)^2 r_1^4 \frac{r_1^2}{r^2} dr - A_1 x_1 + C \int_{r_1}^{r_1+x_c} C \left( \frac{E_{\text{max}}}{p} \right)^2 r_1^4 \frac{r_1^2}{r^2} dr - 2C \left( \frac{E}{p} \right)_M \int_{r_1}^{r_1+x_1} \left( \frac{E_{\text{max}}}{p} \right)^2 \frac{r_1^2}{r^2} dr + C \left( \frac{E}{p} \right)_M^2 (x_c - x_i) - A(x_c - x_1)
\]
\[ \frac{K}{p} = - C_1 \left( \frac{E_{\text{max}}}{p} \right) r_1 \left( \frac{r_1}{(r_1+x_c)} - 1 \right) - A_1 x_1 - \]

\[ \frac{1}{3} C \left( \frac{E_{\text{max}}}{p} \right)^2 r_1 \left( \frac{r_1^3}{(r_1+x_c)^3} - \frac{r_1^3}{(r_1+x_1)^3} \right) + \]

\[ 2C \left( \frac{E}{p} \right) \left( \frac{E_{\text{max}}}{p} \right) r_1 \left( \frac{r_1}{(r_1+x_c)} - \frac{r_1}{(r_1+x_1)} \right) + \]

\[ C \left( \frac{E}{p} \right)^2 (x_c - x_1) - A (x_c - x_1) \quad \text{A.20} \]

There are three variables in the eq. A.20, \( x_c, x_1 \), and \( (E_{\text{max}}/p) \). However, eq. A.20 can be computed by varying \( x_c \) alone because of the relationship between \( x_c \) and \( (E_{\text{max}}/p) \) in eq. A.15 and between \( x_1 \) and \( (E_{\text{max}}/p) \) which is

\[ x_1 = r_1 \sqrt{\frac{E_{\text{max}}}{p_{\text{air}}}} - r_1 \quad \text{A.21} \]

The point where both side of the eq. A.20 are equal will be reached, provided, that \( x_c < (r_2 - r_1) \). Substituting \( x_c \) into eq. A.4, \( E_{\text{max}}/p \) can be determined. \( U_1 \) can also be determined with the aid of eq. A.3.
APPENDIX B. INCEPTION VOLTAGE DETERMINATION IN COAXIAL CYLINDER GEOMETRY

B.1 Basic equations

The electric field at radius \( r \) can be represented by

\[
E(r) = \frac{U}{r \cdot \ln\left(\frac{r_2}{r_1}\right)} \tag{B.1}
\]

and, with \( E_{\text{max}} = E(r_j) \)

\[
E(r) = \frac{E_{\text{max}}r_1}{r} \tag{B.2}
\]

At inception voltage \( U_0, \frac{E}{p} = (\frac{E}{p})_\alpha \) occurs at \( x = x_c \). Then

\[
\left( \frac{E_{\text{max}}}{p} \right) = \frac{(\frac{\mu_1}{p})}{r_1 \cdot \ln\left(\frac{r_2}{r_1}\right)} \tag{B.3}
\]

\[
\left( \frac{E}{p} \right)_\alpha = \frac{(\frac{E_{\text{max}}}{p})r_1}{(r_1 + x_c)} \tag{B.4}
\]

For SF\(_6\) in the region \( 8.9246 < \frac{E}{p} < 12.36 \) kV/mm-bar, the relationship of \( \frac{\alpha}{p} = f(\frac{E}{p}) \) can be represented by

\[
\frac{\alpha}{p} = C \left[ \frac{E}{p} - \left( \frac{E}{p} \right)_\alpha \right] \tag{B.5}
\]

where \( C = 27.9 \) 1/kV and \( (\frac{E}{p})_\alpha = 8.9246 \) kV/mm-bar. In the range \( 12.36 < \frac{E}{p} < 21 \) kV/mm-bar, this relationship can be expressed as

\[
\frac{\alpha}{p} = C_1 \frac{E}{p} - A_1 \tag{B.6}
\]

where \( C_1 = 22.359 \) 1/kV and \( A_1 = 180.171 \) 1/mm-bar. The transition point from eq. B.5 to B.6 in SF\(_6\) occurs at \( (\frac{E}{p})_{\text{SF6}} = 12.36 \) kV/mm-bar.
For dry air in the region $2.588 < \frac{E}{p} < 7.943 \text{ kV/mm-bar}$, the relationship of $\frac{\alpha}{p} = f(\frac{E}{p})$ can be represented by

\[
\frac{\alpha}{p} = C \left[ \frac{E}{p} - \left( \frac{E}{p} \right)_{M} \right]^{2} - A \quad \text{B.7}
\]

where $C = 1.6053 \text{ mm-bar/kV}^2$, $(\frac{E}{p})_{M} = 2.165 \text{ kV/mm-bar}$, $A = 0.2873 \text{ l/mm-bar}$, and $(\frac{E}{p})_\alpha = 2.588 \text{ kV/mm-bar}$. In the range $7.943 < \frac{E}{p} < 14 \text{ kV/mm-bar}$, this relationship can be expressed as

\[
\frac{\alpha}{p} = C_1 \frac{E}{p} - A_1 \quad \text{B.8}
\]

where $C_1 = 16.7766 \text{ 1/kV}$ and $A_1 = 80.0006 \text{ 1/mm-bar}$. The transition point from eq. B.7 to B.8 in dry air occurs at $(\frac{E}{p})_{\text{tr}} = 7.943 \text{ kV/mm-bar}$

**B.2 SF$_6$ in the range $8.9246 < \frac{E_{\text{max}}}{p} < 12.36 \text{ kV/mm-bar}$**

From the streamer breakdown criterion and the relationship of $\frac{\alpha}{p} = f(\frac{E}{p})$

\[
\int_{r_1}^{r_1+x_c} \left( \frac{E}{p} \right) dr - \left( \frac{E}{p} \right)_{\alpha} x_c = \frac{K}{pC} \quad \text{B.9}
\]

Considering only the first term on the left hand side

\[
\int_{r_1}^{r_1+x_c} \left( \frac{E}{p} \right) dr = \int_{r_1}^{r_1+x_c} \left( \frac{E_{\text{max}}}{p} \right) r_1 dr
\]

\[
= \left( \frac{E_{\text{max}}}{p} \right) r_1 \ln \left( \frac{r_1 + x_c}{r_1} \right) \quad \text{B.10}
\]

Substituting eq. B.10 into B.9, we get

\[
\frac{K}{pC} = \left( \frac{E_{\text{max}}}{p} \right) r_1 \ln \left( \frac{r_1 + x_c}{r_1} \right) - \left( \frac{E}{p} \right)_{\alpha} x_c \quad \text{B.11}
\]

There are two variables in the eq. B.11, $x_c$ and $(\frac{E_{\text{max}}}{p})$. However, eq. B.11 can be computed by varying $x_c$ alone because of the relationship of $x_c$ and $(\frac{E_{\text{max}}}{p})$ in eq. B.4. Eq. B.11 can also be expressed as a function of $(\frac{E_{\text{max}}}{p})$, as

\[
\frac{K}{pC} = r_1 \left[ \left( \frac{E_{\text{max}}}{p} \right) \ln \left( \frac{E_{\text{max}}}{p} \right) - \left( \frac{E}{p} \right)_{\alpha} \right] \quad \text{B.12}
\]

$U_i$ can be easily calculated by using eq. B.3.
B.3 SF₆ in the range 12.36 < E_max/p < 21 kV/mm·bar

This situation occurs when E_max/p from section B.2 is greater than (E/p)$_{SF6}$. From the streamer breakdown criterion and two relationships of $\alpha/p = f(E/p)$

\[
\frac{K}{p} = C_1 \int_{r_1}^{r_1+x_1} \left( \frac{E}{p} \right) dr - A_1 x_1 + C \int_{r_1}^{r_1+x_1} \left( \frac{E}{p} \right) dr - C \left( \frac{E}{p} \right)_{cr} (x_c - x_1) \tag{B.13}
\]

where $x_1$ is the point at which $E/p = (E/p)_{SF6}$. The integration results in,

\[
\frac{K}{p} = C_1 \left( \frac{E_{max}}{p} \right) r_1 \ln \left( \frac{r_1 + x_1}{r_1} \right) - A_1 x_1 + C \left( \frac{E_{max}}{p} \right) r_1 \ln \left( \frac{r_1 + x_c}{r_1 + x_1} \right) - C \left( \frac{E}{p} \right)_{cr} (x_c - x_1) \tag{B.14}
\]

There are three variables in the eq. B.14, $x_c$, $x_1$, and (E_max/p). However, eq. B.14 can be computed by varying $x_c$ alone because at a specific voltage $U$,

\[
x_c = \frac{\left( \frac{E_{max}}{p} \right) r_1}{\left( \frac{E}{p} \right)_{cr}} - r_1 \tag{B.15}
\]

\[
x_1 = \frac{\left( \frac{E_{max}}{p} \right) r_1}{\left( \frac{E}{p} \right)_{SF6}} - r_1 \tag{B.16}
\]

The point where both sides of the eq. B.14 are equal will be reached, provide that $x_c < (r_2 - r_1)$. Substituting $x_c$ into eq. B.4, $E_{max}/p$ can be determined. $U_i$ can also be determined with the aid of eq. B.3.
**B.4 Air in the range \(2.588 < E_{\text{max}}/p < 7.943 \text{ kV/mm-bar}\)**

From the streamer breakdown criterion and the relationship of \(\bar{\omega}/p = f(E/p)\)

\[
\frac{K}{pC} = \int_{r_1}^{r_1+x_1} \left( \frac{E}{p} \right)^2 dr - 2 \left( \frac{E}{p} \right)_M \int_{r_1}^{r_1+x_1} \left( \frac{E}{p} \right)^2 dr + \left( \frac{E}{p} \right)^2 x_e - \frac{A}{C} x_e \quad \text{B.17}
\]

Integrating,

\[
\frac{K}{pC} = \int_{r_1}^{r_1+x_1} \left( \frac{E_{\text{max}}}{p} \right)^2 r^2 dr - 2 \left( \frac{E}{p} \right)_M r_1 \ln \left( \frac{r_1+x_c}{r_1} \right) + \left( \frac{E}{p} \right)^2 x_e - \frac{A}{C} x_e
\]

\[
= - \left( \frac{E_{\text{max}}}{p} \right)^2 \left( \frac{r_1}{r_1+x_c} - 1 \right) - 2 \left( \frac{E}{p} \right)_M \left( \frac{E_{\text{max}}}{p} \right) r_1 \ln \left( \frac{E_{\text{max}}}{p} \right) + \left( \frac{E}{p} \right)^2 x_e - \frac{A}{C} x_e
\]

Eq. B.18 can be evaluated by increasing \((E_{\text{max}}/p)\) from \((E/p)_\text{cr}\) until both sides of eq. B.18 are equal. By substituting \(E_{\text{max}}/p\) into eq. B.3, \(U_1\) can be determined.

**B.5 Air in the range \(7.943 < E_{\text{max}}/p < 14 \text{ kV/mm-bar}\)**

This situation occurs when \(E_{\text{max}}/p\) from section B.4 is greater than \((E/p)_\text{cr}\). From the streamer breakdown criterion and two relationship of \(\bar{\omega}/p = f(E/p)\)

\[
\frac{K}{p} = C_1 \int_{r_1}^{r_1+x_1} \left( \frac{E}{p} \right)^2 dr - A_1 x_1 + C \int_{r_1}^{r_1+x_1} \left( \frac{E}{p} \right)^2 dr - 2C \left( \frac{E}{p} \right)_M \int_{r_1}^{r_1+x_1} \left( \frac{E}{p} \right)^2 dr + C \left( \frac{E}{p} \right)^2 (x_e-x_1) - A(x_e-x_1) \quad \text{B.19}
\]

where \(x_1\) is the point that \(E/p = (E/p)_\text{cr}\). Integrating,
\[
\frac{K}{p} = \int_{r_1}^{r_{1+x_1}} C_1 \left(\frac{E_{\max}}{p}\right) \frac{r_1}{r} \, dr - A_1 x_1 + \int_{r_1+x_1}^{r_{1+x_2}} C \left(\frac{E_{\max}}{p}\right) \frac{r_2}{r} \, dr
\]
\[
- \int_{r_1+x_1}^{r_{1+x_2}} 2C \left(\frac{E}{p}\right) \left(\frac{E_{\max}}{p}\right) \frac{r_1}{r} \, dr + C \left(\frac{E}{p}\right)^2 (x_c - x_1) - A(x_c - x_1)
\]
\[
= C_1 \left(\frac{E_{\max}}{p}\right) r_1 \ln \left(\frac{r_1 + x_1}{r_1}\right) - A_1 x_1 - C \left(\frac{E_{\max}}{p}\right) r_1 \left(\frac{r_1}{(r_1 + x_1)} - \frac{r_1}{(r_1 + x_c)}\right)
- 2C \left(\frac{E}{p}\right)_M \left(\frac{E_{\max}}{p}\right) r_1 \ln \left(\frac{r_1 + x_c}{r_1 + x_1}\right) + C \left(\frac{E}{p}\right)^2_M (x_c - x_1) - A(x_c - x_1)
\]
\[
= C_1 \left(\frac{E_{\max}}{p}\right) r_1 \ln \left(\frac{\frac{E_{\max}}{p}}{\frac{E}{p}_{\text{air}}}\right) - A_1 \left(\frac{\frac{E_{\max}}{p}}{\frac{E}{p}_{\text{air}}} - 1\right) -
C \left(\frac{E_{\max}}{p}\right)^2 r_1 \left(\frac{\frac{E}{p}_{\text{cr}}}{\frac{E_{\max}}{p}} - \frac{\frac{E}{p}_{\text{air}}}{\frac{E_{\max}}{p}}\right) - 2C \left(\frac{E}{p}\right)_M \left(\frac{E_{\max}}{p}\right) r_1 \ln \left(\frac{\frac{E}{p}_{\text{air}}}{\frac{E}{p}_{\text{cr}}}\right) +
C \left(\frac{E}{p}\right)_M^2 \left(\frac{\frac{E_{\max}}{p}}{\frac{E}{p}_{\text{cr}}} - \frac{\frac{E_{\max}}{p}}{\frac{E}{p}_{\text{air}}}\right) - A \left(\frac{\frac{E}{p}_{\text{cr}}}{\frac{E}{p}_{\text{air}}} - \frac{\frac{E}{p}_{\text{cr}}}{\frac{E}{p}_{\text{air}}}\right) \tag{B.20}
\]

Eq. B.20 can be evaluated by increasing \((E_{\max}/p)\) from \((E/p)_{\text{air}}\) until both sides of the eq. B.20 are equal. By substituting \(E_{\max}/p\) into eq. B.3, \(U_i\) can be determined.
# APPENDIX C. POLYNOMIAL COEFFICIENTS OF SPHERE-POT GEOMETRY

Table C.1 The polynomial coefficients of Sphere-Pot geometry: \( r = 2.5 \) mm and pot radius = 92.5 mm.

<table>
<thead>
<tr>
<th>( g ) (mm)</th>
<th>Maximum Err, %</th>
<th>Polynomial coefficient until order 14&lt;sup&gt;th&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.05610&lt;sup&gt;4&lt;/sup&gt; %</td>
<td>( a_0, a_1, a_2 )</td>
</tr>
<tr>
<td>12.5</td>
<td>7.31710&lt;sup&gt;2&lt;/sup&gt; %</td>
<td>4.0396743951E-01</td>
</tr>
<tr>
<td>15.0</td>
<td>1.74110&lt;sup&gt;1&lt;/sup&gt; %</td>
<td>3.9044012820E-01</td>
</tr>
<tr>
<td>17.5</td>
<td>3.54910&lt;sup&gt;1&lt;/sup&gt; %</td>
<td>3.8051640995E-01</td>
</tr>
<tr>
<td>20.0</td>
<td>6.78110&lt;sup&gt;4&lt;/sup&gt; %</td>
<td>3.7291052095E-01</td>
</tr>
</tbody>
</table>
Table C.2 The polynomial coefficients of Sphere-Pot geometry: \( r = 5 \) mm and pot radius = 92.5 mm.

<table>
<thead>
<tr>
<th>Gap, mm</th>
<th>Maximum Err, %</th>
<th>Polynomial coefficient until order 14*</th>
</tr>
</thead>
<tbody>
<tr>
<td>g = 20 mm</td>
<td>Max. Err = 2.169 \times 10^2 %</td>
<td>( a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14} )</td>
</tr>
<tr>
<td>Max. Err</td>
<td>2.155535608E-01</td>
<td>-3.0249285613E-06</td>
</tr>
<tr>
<td>g = 25 mm</td>
<td>Max. Err = 5.937 \times 10^2 %</td>
<td>5.591464367E-10</td>
</tr>
<tr>
<td>Max. Err</td>
<td>2.067845588E00</td>
<td>-1.1050439246E-04</td>
</tr>
<tr>
<td>g = 30 mm</td>
<td>Max. Err = 1.019 \times 10^3 %</td>
<td>8.811187714E-11</td>
</tr>
<tr>
<td>Max. Err</td>
<td>2.010590673E-03</td>
<td>-1.1775852954E-04</td>
</tr>
<tr>
<td>g = 35 mm</td>
<td>Max. Err = 3.674 \times 10^3 %</td>
<td>7.85909613E-12</td>
</tr>
<tr>
<td>Max. Err</td>
<td>1.971000617E-05</td>
<td>-1.1250709656E-03</td>
</tr>
<tr>
<td>g = 40 mm</td>
<td>Max. Err = 7.502 \times 10^3 %</td>
<td>3.654213276E-11</td>
</tr>
</tbody>
</table>

Table C.3 The polynomial coefficients of Sphere-Pot geometry: \( r = 10 \) mm and pot radius = 92.5 mm.

<table>
<thead>
<tr>
<th>Gap, mm</th>
<th>Maximum Err, %</th>
<th>Polynomial coefficient until order 14*</th>
</tr>
</thead>
<tbody>
<tr>
<td>g = 30 mm</td>
<td>Max. Err = 1.329 \times 10^2 %</td>
<td>( a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14} )</td>
</tr>
<tr>
<td>Max. Err</td>
<td>1.173911972E01</td>
<td>-2.350756109E-02</td>
</tr>
<tr>
<td>g = 40 mm</td>
<td>Max. Err = 1.681 \times 10^3 %</td>
<td>1.167278673E11</td>
</tr>
<tr>
<td>Max. Err</td>
<td>1.112033330E03</td>
<td>-2.224484814E-02</td>
</tr>
<tr>
<td>g = 50 mm</td>
<td>Max. Err = 5.678 \times 10^3 %</td>
<td>3.318157752E-13</td>
</tr>
<tr>
<td>Max. Err</td>
<td>1.079743766E01</td>
<td>-2.157260754E-02</td>
</tr>
<tr>
<td>g = 60 mm</td>
<td>Max. Err = 1.768 \times 10^4 %</td>
<td>1.171505835E05</td>
</tr>
<tr>
<td>Max. Err</td>
<td>1.067165429E00</td>
<td>-2.117283796E-02</td>
</tr>
<tr>
<td>g = 70 mm</td>
<td>Max. Err = 4.607 \times 10^4 %</td>
<td>1.051283005E16</td>
</tr>
<tr>
<td>Max. Err</td>
<td>4.322437755E-14</td>
<td>-4.620694358E-16</td>
</tr>
</tbody>
</table>
APPENDIX D. INCEPTION VOLTAGE DETERMINATION IN SPHERE-POT GEOMETRY

D.1 Basic equations
The electric field along the axis of symmetry in sphere-pot geometry can be represented by a polynomial equation of order 14 as,

\[ E(x) = U \cdot \sum_{n=0}^{14} a_n x^n \]  \hspace{1cm} D.1

At inception voltage \( U_i \), \( \frac{E}{p} = (\frac{E}{p})_a \) occurs at \( x = x_c \). Then

\[ \left( \frac{E_{\text{max}}}{p} \right) = \frac{U_i}{a_0} \]  \hspace{1cm} D.2

and

\[ \left( \frac{E}{p} \right)_{\text{cr}} = \frac{U_i}{a_0} \cdot \sum_{n=0}^{14} (a_n x_c^n) = \left( \frac{E_{\text{max}}}{p} \right) \cdot \frac{1}{a_0} \cdot \sum_{n=0}^{14} (a_n x_c^n) \]  \hspace{1cm} D.3

For SF₆ in the region \( 8.9246 < \frac{E}{p} < 12.36 \text{ kV/mm-bar} \), the relationship of \( \frac{\alpha}{p} = f(\frac{E}{p}) \) can be represented by

\[ \frac{\alpha}{p} = C \left[ \frac{E}{p} - \left( \frac{E}{p} \right)_{\text{cr}} \right] \]  \hspace{1cm} D.4

where \( C = 27.9 \text{ 1/kV} \) and \( (\frac{E}{p})_a = 8.9246 \text{ kV/mm-bar} \). In the range \( 12.36 < \frac{E}{p} < 21 \text{ kV/mm-bar} \), this relationship can be expressed as

\[ \frac{\alpha}{p} = C_1 \frac{E}{p} - A_1 \]  \hspace{1cm} D.5

where \( C_1 = 22.359 \text{ 1/kV} \) and \( A_1 = 180.171 \text{ 1/mm-bar} \). The transition point from eq. D.4 to D.5 in SF₆ occurs at \( (\frac{E}{p})_{\text{SF₆}} = 12.36 \text{ kV/mm-bar} \).

For dry air in the region \( 2.588 < \frac{E}{p} < 7.943 \text{ kV/mm-bar} \), the relationship of \( \frac{\alpha}{p} = f(\frac{E}{p}) \) can be represented by
\[ \frac{\bar{\alpha}}{p} = C \left[ \frac{E}{p} - \left( \frac{E}{p} \right)_{cr} \right]^2 - A \]  

where \( C = 1.6053 \text{ mm-bar/kV}^2 \), \( (E/p)_{cr} = 2.165 \text{ kV/mm-bar} \), \( A = 0.2873 \text{ /mm-bar} \), and \( (E/p)_{cr} = 2.588 \text{ kV/mm-bar} \). In the range \( 7.943 < E/p < 14 \text{ kV/mm-bar} \), this relationship can be expressed as

\[ \frac{\bar{\alpha}}{p} = C \frac{E}{p} - A_1 \]  

where \( C = 16.7766 \text{ 1/kV} \) and \( A_1 = 80.0006 \text{ 1/mm-bar} \). The transition point from eq. D.6 to D.7 in dry air occurs at \( (E/p)_{air} = 7.943 \text{ kV/mm-bar} \)

**D.2 SF₆ in the range 8.9246 < \( E_{max}/p < 12.36 \text{ kV/mm-bar} \)**

From the streamer breakdown criterion and the relationship of \( \bar{\alpha}/p = f(E/p) \)

\[ \int_{0}^{x_c} \left( \frac{E}{p} \right) dx - \left( \frac{E}{p} \right)_{cr} = \frac{K}{pC} \]  

Considering only the first term on the left hand side

\[ \int_{0}^{x_c} \left( \frac{E}{p} \right) dx = \int_{0}^{x_c} \frac{U}{p} \cdot \sum_{n=0}^{14} (a_n x^n) dx \]

\[ = \frac{U}{p} \cdot \sum_{n=0}^{14} \int_{0}^{x_c} a_n x^n dx \]

\[ = \frac{U}{p} \cdot \sum_{n=0}^{14} \left[ \frac{a_n x^{n+1}}{n+1} \right]_{0}^{x_c} \]

\[ = \frac{U}{p} \cdot \sum_{n=0}^{14} \left[ \frac{a_n x^{n+1}}{n+1} x_c^n \right] \]

At inception voltage \( U_i \)

\[ \int_{0}^{x_c} \left( \frac{E}{p} \right) dx = \left( \frac{E}{p} \right)_{cr} \cdot x_c \cdot \frac{\sum_{n=0}^{14} (a_n x^n)}{\sum_{n=0}^{14} (a_n x^n)} \]
Substituting eq. D.9 into eq. D.8 and integrating,

\[ \frac{K}{pC} = \left( \frac{E}{p} \right)_{\sigma} \cdot x_c \cdot \sum_{n=0}^{14} \left( \frac{a_n}{n+1} x_c^n \right) - \left( \frac{E}{p} \right)_{\sigma} \cdot x_c \]

\[ = \left( \frac{E}{p} \right)_{\sigma} \cdot x_c \cdot \left[ \sum_{n=0}^{14} \left( \frac{a_n}{n+1} x_c^n \right) - \sum_{n=0}^{14} (a_n x_c^n) \right] \]

D.10

By increasing \( x_c \) from zero, the point where both sides of eq. D.10 are equal will be reached, provided that \( x_c < g \). Substituting this \( x_c \) into eq. D.3, the inception voltage \( U_i \) can be determined. \( E_{\text{max}}/p \) can also be determined with the aid of eq. D.2.

**D.3 SF₆ in the range 12.36 < \( E_{\text{max}}/p \) < 21 kV/mm-bar**

This situation occurs when \( E_{\text{max}}/p \) from section D.2 is greater than \( (E/p)_{SF₆} \). From the streamer breakdown criterion and two relationship of \( \alpha/p = f(E/p) \)

\[ \frac{K}{p} = C_1 \int_0^{x_c} \left( \frac{E}{p} \right) dx - A_1 x_1 + C \int_{x_1}^{x_c} \left( \frac{E}{p} \right) dx - C \left( \frac{E}{p} \right)_{\sigma} (x_c - x_1) \]

D.11

where \( x_1 \) is the point that \( E/p = (E/p)_{SF₆} \). The integration results in

\[ \frac{K}{p} = C_1 \frac{U}{p} \cdot x_1 \cdot \sum_{n=0}^{14} \left( \frac{a_n}{n+1} x_1^n \right) - A_1 x_1 + C \frac{U}{p} \cdot x_c \cdot \sum_{n=0}^{14} \left( \frac{a_n}{n+1} x_c^n \right) - C \left( \frac{E}{p} \right)_{\sigma} (x_c - x_1) \]

\[ = (C_1 - C) \cdot \frac{U}{p} \cdot x_1 \cdot \sum_{n=0}^{14} \left( \frac{a_n}{n+1} x_1^n \right) + C \frac{U}{p} \cdot x_c \cdot \sum_{n=0}^{14} \left( \frac{a_n}{n+1} x_c^n \right) + \left( C \left( \frac{E}{p} \right)_{\sigma} - A_1 \right) \cdot x_1 - C \left( \frac{E}{p} \right)_{\sigma} x_c \]
\[ \frac{K}{p} = (C_1 - C) \cdot \left( \frac{E}{p} \right)_{cr} \cdot \frac{\sum_{n=0}^{14} \left( \frac{a_n}{n+1} x_n^n \right)}{\sum_{n=0}^{14} (a_n x_n^n)} + C \cdot \left( \frac{E}{p} \right)_{cr} \cdot x_e \cdot \frac{\sum_{n=0}^{14} \left( \frac{a_n}{n+1} x_n^n \right)}{\sum_{n=0}^{14} (a_n x_n^n)} + \left( C \left( \frac{E}{p} \right)_{cr} - A_1 \right) \cdot x_1 - C \left( \frac{E}{p} \right)_{cr} x_c \]  

D.12

There are two variables in eq. D.12, \( x_c \) and \( x_i \). By increasing \( x_c \) from zero until \((E_{max}/p)\) in eq. D.3 greater than \((E/p)_{SF6}\), \( U/p \) can be calculated by using eq. D.2. Eq. D.1 can be used to evaluate \( x_1 \).

The point where both sides of eq. D.12 are equal will be reached, provided that \( x_c < g \). Substituting \( x_c \) into eq. D.3, the inception voltage \( U_i \) can be determined. \( E_{max}/p \) can also be determined with the aid of eq. D.2.

D.4 Air in the range 2.588 < \( E_{max}/p \) < 7.943 kV/mm-bar

From the streamer breakdown criterion and the relationship of \( \omega/p = f(E/p) \)

\[ \frac{K}{pC} = \int_0^{x_c} \left( \frac{E}{p} \right)^2 dx - 2 \left( \frac{E}{p} \right)_M \int_0^{x_c} \left( \frac{E}{p} \right) dx + \left( \frac{E}{p} \right)_{cr} x_c - \frac{A}{C} x_c \]  

D.13

Considering only the first term on the right hand side

\[ \int_0^{x_c} \left( \frac{E}{p} \right)^2 dx = \left( \frac{U}{p} \right)^2 \cdot \sum_{i=0}^{14} \sum_{j=0}^{14} \left( a_i a_j \int_0^{x_c} x^{i+j} dx \right) \]

\[ = \left( \frac{U}{p} \right)^2 \cdot \sum_{i=0}^{14} \sum_{j=0}^{14} \left( a_i a_j \frac{x^{i+j+1}}{i+j+1} \right)_0 \]

\[ = \left( \frac{U}{p} \right)^2 \cdot x_c \cdot \sum_{i=0}^{14} \sum_{j=0}^{14} \left( \frac{a_i a_j x^{i+j}}{i+j+1} \right) \]

At inception voltage \( U_i \)

\[ \int_0^{x_c} \left( \frac{E}{p} \right)^2 dx = \left( \frac{E}{p} \right)_{cr}^2 \cdot x_c \cdot \sum_{i=0}^{14} \sum_{j=0}^{14} \left( \frac{a_i a_j x^{i+j}}{i+j+1} \right) \]

\[ = \left( \frac{E}{p} \right)_{cr}^2 \cdot x_c \cdot \sum_{i=0}^{14} \sum_{j=0}^{14} \left( \frac{a_i a_j x^{i+j}}{i+j+1} \right) \]  

D.14
Substituting eq. D.14 and D.9 into D.13, and integrating,

\[
\frac{K}{pC} = \left(\frac{E}{p}\right)\sigma \cdot x_0 \cdot \sum_{i=0}^{14} \sum_{j=0}^{14} \frac{E_{C_{i,j}}}{(a_{i,j}x_{C_{i,j}})} - 2 \cdot \left(\frac{E}{p}\right)\sigma \cdot x_1 \cdot \sum_{i=0}^{14} \frac{E_{C_{i,j}}}{(a_{i,j}x_{C_{i,j}})} + \left(\frac{E}{p}\right)\sigma \cdot x_t - \frac{A}{C} \cdot x_c
\]

\[
= x_0 \left[ \left(\frac{E}{p}\right)\sigma \cdot \sum_{i=0}^{14} \sum_{j=0}^{14} \frac{E_{C_{i,j}}}{(a_{i,j}x_{C_{i,j}})} - 2 \cdot \left(\frac{E}{p}\right)\sigma \cdot \sum_{i=0}^{14} \frac{E_{C_{i,j}}}{(a_{i,j}x_{C_{i,j}})} + \left(\frac{E}{p}\right)\sigma \cdot \frac{A}{C} \right]
\]

D.15

By increasing \( x_c \) from zero, the point where both sides of the eq. D.15 are equal will be reached, provided that \( x_c < g \). Substituting this \( x_c \) into eq. D.3, the inception voltage \( U_t \) can be determined. \( E_{\text{max}}/p \) can also be determined with the aid of eq. D.2.

D.5 Air in the range 7.943 < \( E_{\text{max}}/p < 14 \) kV/mm-bar

This situation occurs when \( E_{\text{max}}/p \) from section D.4 is greater than \( (E/p)_{\text{air}} \). From the streamer breakdown criterion and two relationship of \( \sigma/p = f(E/p) \)

\[
\frac{K}{p} = C_1 \cdot \int_0^{x_1} \left(\frac{E}{p}\right)\sigma \cdot dx - A_1 x_1 + C \cdot \int_{x_1}^{x_2} \left(\frac{E}{p}\right)\sigma \cdot dx - 2C \cdot \left(\frac{E}{p}\right)_\sigma \cdot \int_{x_1}^{x_2} \left(\frac{E}{p}\right)\sigma \cdot dx +
\]

\[
C \cdot \left(\frac{E}{p}\right)_\sigma (x_c - x_1) - A(x_c - x_1)
\]

D.16

where \( x_1 \) is the point that \( E/p = (E/p)_{\text{air}} \). The integration results in

\[
\frac{K}{p} = C \left[ \left(\frac{E}{p}\right)\sigma \cdot \frac{\sum_{i=0}^{14} \frac{E_{C_{i,j}}}{(a_{i,j}x_{C_{i,j}})}}{x_1 \cdot \sum(a_{i,j}x_{C_{i,j}})} - \frac{A_1}{C_1} x_1 \right] +
\]

\[
C \left[ \left(\frac{E}{p}\right)\sigma \cdot \sum_{i=0}^{14} \frac{E_{C_{i,j}}}{(a_{i,j}x_{C_{i,j}})} - 2 \cdot \left(\frac{E}{p}\right)_\sigma \cdot \frac{E_{C_{i,j}}}{(a_{i,j}x_{C_{i,j}})} + \left(\frac{E}{p}\right)_\sigma x_1 - \frac{A}{C} x_c \right] -
\]

\[
C \left[ \left(\frac{E}{p}\right)_\sigma \cdot \sum_{i=0}^{14} \frac{E_{C_{i,j}}}{(a_{i,j}x_{C_{i,j}})} - 2 \cdot \left(\frac{E}{p}\right)_\sigma \cdot \frac{E_{C_{i,j}}}{(a_{i,j}x_{C_{i,j}})} + \left(\frac{E}{p}\right)_\sigma x_1 - \frac{A}{C} x_1 \right]
\]

D.17

There are two variables in the eq. D.17, \( x_c \) and \( x_1 \). By increasing \( x_c \) from zero until \( (E_{\text{max}}/p) \) in eq. D.3 greater than \( (E/p)_{\text{air}} \), \( U/p \) can be calculated by using eq. D.2. Eq. D.1 can be used to evaluate \( x_1 \).

The point where both side of the eq. D.17 are equal will be reached, provided that \( x_c < g \). Substituting \( x_c \) into eq. D.3, the inception voltage \( U_t \) can be determined. \( E_{\text{max}}/p \) can also be determined with the aid of eq. D.2.
APPENDIX E. CALCULATION RESULTS OF $U_i$ IN SPHERE-POT GEOMETRY

The calculation results in this section are based on the calculation procedure described in Appendix D. The magnitude of $K$ is assumed to be 9.15 for dry air and 10.5 for SF$_6$.

Figure E.1 Calculated $U_i$ of sphere-pot geometry in dry air, $r = 5$ mm.
Figure E.2 Calculated $U_1$ of sphere-pot geometry in dry air, $r = 10$ mm.

Figure E.3 Calculated $E_1/p$ of sphere-pot geometry in dry air.
Figure E.4 Calculated $U_1$ of sphere-pot geometry in SF$_6$, $r = 2.5$ mm.

Figure E.5 Calculated $U_1$ of sphere-pot geometry in SF$_6$, $r = 5$ mm.
Figure E.6 Calculated $U_i$ of sphere-pot geometry in SF$_6$, $r = 10$ mm.

Figure E.7 Calculated $E_{ij}/p$ of sphere-pot geometry in SF$_6$. 
APPENDIX F  BREAKDOWN VOLTAGE OF SPHERE-GAPS

Table F.1 Breakdown voltage of sphere-gap, reproduced from Table I of IEC-52: 1960.

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^ See discussion in section 6.6
Table F.1 Breakdown voltage of sphere-gap, reproduced from Table I of IEC-52: 1960 (continued)

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See discussion in section 6.6
Table F.2 The calculated breakdown voltages of sphere-gaps: without taking space charge effect into account, i.e. 'w/o-SC'.

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NOTE: \( K = 9.15, \bar{\alpha}/p = f(E/p) \) from Friedrich, large surrounding, no humidity correction.
Table F.2  The calculated breakdown voltages of sphere-gaps: without taking space charge effect into account, i.e. 'w/o-SC'. (continued)

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NOTE:  \( K = 9.15, \ \bar{\omega}/p = f(E/p) \) from Friedrich, large surrounding, no humidity correction.
Table F.3 The calculated breakdown voltages of sphere-gaps: when taking space charge effect into account, i.e. 'w-SC'.

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NOTE: $K = 9.15$, $\bar{\sigma}/p = f(E/p)$ from Friedrich, large surrounding, no humidity correction.
Table F.3 The calculated breakdown voltages of sphere-gaps: when taking space charge effect into account, i.e. 'w-SC'. (continued)

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NOTE: \( K = 9.15, \bar{\alpha}/p = f(E/p) \) from Friedrich, large surrounding, no humidity correction.
Table F.4

Representation of the results of Table F.3 by polynomial of 6th order: Fitted from the calculated $U_b$ data in Table F.3. The calculated $U_b$ from these polynomial equations deviate from the value given in Table F.3 by less than 0.5%.

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For each sphere diameter

$$U_b = \sum_{n=0}^{6} a_n g^n$$

where

$U_b$ = breakdown voltage in kV

g = gap spacing in cm
APPENDIX G. BREAKDOWN VOLTAGE DETERMINATION IN SPHERE-GAPS GEOMETRY

G.1 Basic equations

The electric field along the axis of symmetry in sphere-gaps geometry (see figure 6.1 and 6.2) are represented by using two polynomial equations of order 13. They can be written as

\[ E_1(x) = U \sum_{n=0}^{13} a_n x^n \]  
\[ E_2(x) = U \sum_{n=0}^{13} b_n x^n \]

where

- \( U \) = applied voltage in kV
- \( a_n \) = polynomial coefficients of the first equation, for high voltage sphere
- \( b_n \) = polynomial coefficients of the second equation, for earthed sphere

Eq. G.1 represents the electric field distribution between the high voltage sphere and the point where electric field in the gap is minimum. \( x \) in this equation is the distance from high voltage sphere.

For the electric field distribution between earthed sphere and the minimum field point, eq. G.2 is used. \( x \) in the equation is the distance from earthed sphere.

If only the high voltage sphere is taken into account, at breakdown voltage \( U_b \), \( E_1/p \) = \( (E/p)_{av} \) occurs at \( x = x_c \). Then

\[ \left( \frac{E_{\text{max}}}{p} \right) = \frac{U_b}{a_0} \frac{1}{p} \]

G.3

and

\[ \left( \frac{E}{p} \right)_{av} = \frac{U_b}{p} \cdot \sum_{n=0}^{13} (a_n x_c^n) = \left( \frac{E_{\text{max}}}{p} \right) \cdot \frac{1}{a_0} \cdot \sum_{n=0}^{13} (a_n x_c^n) \]

G.4

The relationship of \( \alpha/p = f(E/p) \) in synthetic air from Friedrich (1992), eq. 3.3, can be represented by
\[ \frac{\alpha}{p} = C \left[ \frac{E}{p} - \left( \frac{E}{p} \right)_M \right]^2 - A \]

where

\[ C = 1.6053 \text{ mm-bar/kV}^2 \]

\[ (E/p)_M = 2.165 \text{ kV/mm-bar} \]

\[ A = 0.2873 \text{ 1/bar-mm} \]

The point where \( \alpha = 0 \) occurs at \( (E/p)_\alpha = 2.588 \text{ kV/(mm-bar)} \).

**G.2 Calculation without taking space charge effect into account**

From the streamer breakdown criterion and the relationship of \( \alpha/p = f(E/p) \)

\[ \frac{K}{pC} = \int_0^{x_c} \left( \frac{E_1}{p} \right)^2 dx - 2 \left( \frac{E}{p} \right)_M \int_0^{x_c} \left( \frac{E_1}{p} \right) dx + \left( \frac{E}{p} \right)_M^2 x_c - \frac{A}{C} x_c \]

which represents the calculations in the region of high voltage sphere. Firstly, we consider only the second integral term on the right hand side of this equation.

\[ \int_0^{x_c} \left( \frac{E_1}{p} \right) dx = \int_0^{x_c} \frac{U}{p} \sum_{i=0}^{13} (a_i x^i) dx \]

\[ = \frac{U}{p} \sum_{i=0}^{13} \left( \int_0^{x_c} a_i x^i dx \right) \]

\[ = \frac{U}{p} \sum_{i=0}^{13} \left[ \frac{a_i}{i+1} x_c^{i+1} \right]_0^{x_c} \]

\[ = \frac{U}{p} \sum_{i=0}^{13} \left( \frac{a_i}{i+1} x_c^{i+1} \right) \]

At breakdown voltage \( U_b \)

\[ \int_0^{x_c} \left( \frac{E_1}{p} \right) dx = \left( \frac{E}{p} \right)_\alpha \sum_{i=0}^{13} \left( \frac{a_i}{i+1} x_c^{i+1} \right) \]

\[ \sum_{i=0}^{13} (a_i x_c^i) \]

For the first integral term on the right hand side of eq. G.6
\[ \int_{0}^{x_{c}} \left( \frac{E_{1}}{p} \right)^{2} \, dx = \left( \frac{U}{p} \right)^{2} \cdot \sum_{i=0}^{13} \sum_{j=0}^{13} \left( a_{i}a_{j} \int_{0}^{x_{c}} \frac{x_{i+j}^{i+j+1}}{i+j+1} \right) \]
\[ = \left( \frac{U}{p} \right)^{2} \cdot \sum_{i=0}^{13} \sum_{j=0}^{13} \left( \frac{a_{i}a_{j}x_{c}^{i+j+1}}{i+j+1} \right) \]
\[ = \left( \frac{U}{p} \right)^{2} \cdot \sum_{i=0}^{13} \sum_{j=0}^{13} \left( a_{i}a_{j}x_{c}^{i+j+1} \right) \]

At breakdown voltage \( U_{b} \)
\[ \int_{0}^{x_{c}} \left( \frac{E_{1}}{p} \right)^{2} \, dx = \left( \frac{E}{p} \right)^{2} \cdot \sum_{i=0}^{13} \sum_{j=0}^{13} \left( \frac{a_{i}a_{j}x_{c}^{i+j+1}}{i+j+1} \right) \]
\[ = \left( \frac{E}{p} \right)^{2} \cdot \sum_{i=0}^{13} \sum_{j=0}^{13} \left( a_{i}a_{j}x_{c}^{i+j+1} \right) \]

By substituting eq. G.8 and G.10 into eq. G.6, the integration will result in
\[ \frac{K}{pC} = \left( \frac{E}{p} \right)^{2} \cdot \sum_{i=0}^{13} \sum_{j=0}^{13} \left( \frac{a_{i}a_{j}x_{c}^{i+j+1}}{i+j+1} \right) - 2 \cdot \left( \frac{E}{p} \right) \cdot \left( \frac{E}{p} \right) \cdot \sum_{i=0}^{13} \left( \frac{a_{i}x_{c}^{i+1}}{i+j+1} \right) + \left( \frac{E}{p} \right) \cdot x_{c} - \frac{A}{C} \cdot x_{c} \]

From this equation, \( x_{c} \) can be numerically evaluated and thus \( U_{b} \) from eq. G.4.

The numerical evaluation is done by an iterative procedure starting with a small value of \( x_{c} \) and comparing the right hand side of eq. G.11 with \( K/pC \). Iteration is stopped if the difference between the left and right hand side is smaller than 0.01 %.

This calculation procedure can be applied only when \( x_{c} \) is less than \( x(E_{\text{min}}) \), see figure 6.7, which is the situation III where \( S/D \)-values are large.

When \( S/D \)-values are decreased, its value will finally fall into the intermediate range where \( x_{c} \) reaches the point where \( E_{\text{min}} = (E/p)_{\infty} p \) (see figure 6.6). The streamer breakdown criterion, therefore, can be rewritten as,
\[ \frac{K}{pC} = \int_0^{\xi} \left( \frac{E_1}{p} \right)^2 dx - 2 \left( \frac{E}{p} \right) \int_0^{\xi} \left( \frac{E_1}{p} \right) dx + \frac{E^2}{p} \xi - \frac{A}{C} \xi + \]  
\[ \int_0^{\xi} \left( \frac{E_2}{p} \right)^2 dx - 2 \left( \frac{E}{p} \right) \int_0^{\xi} \left( \frac{E_2}{p} \right) dx + \]  
\[ \left( \frac{E}{p} \right)_M \left( S - \xi \right) - \frac{A}{C} \left( S - \xi \right) \]  
G.12

By substituting the corresponding polynomial equation, eq. G.7 and G.9 into this equation, the integration will result in  
\[ \frac{K}{pC} = \left( \frac{U}{p} \right)^2 \sum_{i=0}^{13} \sum_{j=0}^{i+j+1} a_i a_{j} \left( E \right) \left( \frac{U}{p} \right) \sum_{i=0}^{13} \frac{a_{i+j+1}}{i+j+1} + \frac{E^2}{p} \left( \frac{U}{p} \right)_M \]  
\[ x_0 \cdot \frac{A}{C} + \left( \frac{U}{p} \right)^2 \sum_{i=0}^{13} \sum_{j=0}^{i+j+1} b_i b_j \left( S - \xi \right)^{i+j+1} - 2 \left( \frac{E}{p} \right) \left( \frac{U}{p} \right)_M \sum_{i=0}^{13} \frac{b_i \left( S - \xi \right)^{i+j+1}}{i+j+1} + \]  
\[ \left( S - \xi \right) \left( \frac{E}{p} \right)_M - \left( S - \xi \right) \cdot \frac{A}{C} \]  
G.13

\( \xi \) in this equation is equal to \( x(E_{\text{min}}) \) which is known in advance by the field calculation. Eq. G.13 can thus be satisfied by varying \( U \) up to \( U_b \), starting with \( U \) such that \( E_{\text{min}} \) is equal to \( (E/p)_{cr} \cdot p \). At this point, if the right hand side of eq. G.13 is greater than or equal to the left hand side, the calculation will fall into situation II.

As \( S/D \)-values are further decreased, i.e. the electric field distribution is more uniform, the right hand side of eq. G.13 will become less than the left hand side. Further increasing of \( U \) is required to fulfill the streamer breakdown criterion. The calculation falls, therefore, into situation I (see figure 6.5).

G.3 Calculation when taking space charge effect into account

For the intermediate \( S/D \)-values which takes the space charge effect into account (situation IV) the streamer breakdown criterion can be written as, (see figure 6.9)
\[
\frac{K}{pC} = \int_0^s \left( \frac{E_1}{p} \right)^2 dx - 2 \left( \frac{E}{p} \right)_M \int_0^s \left( \frac{E_1}{p} \right) dx + \left( \frac{E}{p} \right)_M^2 x_c - \frac{A}{C} x_c + \\
\int_0^s - x_c' \left( \frac{E_2}{p} \right)^2 dx - 2 \left( \frac{E}{p} \right)_M \int_0^s - x_c' \left( \frac{E_2}{p} \right) dx + \\
\left( \frac{E}{p} \right)_M^2 (S - x_c') - \frac{A}{C} (S - x_c') \tag{G.14}
\]

By substituting the corresponding polynomial equation, eq. G.8 and G.10 into this equation, the integration will result in

\[
\frac{K}{pC} = \left( \frac{E}{p} \right)_M^2 \sum_{i=0}^{13} \sum_{j=0}^{13} a_i x_c^{i+j} - 2 \left( \frac{E}{p} \right)_M \left( \frac{E}{p} \right)_M^{i+1} + x_c' \left( \frac{E}{p} \right)_M^2 x_c - \frac{A}{C} + \\
\left( \frac{E}{p} \right)_M^2 \sum_{i=0}^{13} \sum_{j=0}^{13} b_i b_j (S - x_c')^{i+j} - 2 \left( \frac{E}{p} \right)_M \left( \frac{E}{p} \right)_M^{i+1} + \\
(S - x_c') \cdot \left( \frac{E}{p} \right)_M^2 - (S - x_c') \cdot \frac{A}{C} \tag{G.15}
\]

The numerical evaluation is made by an iterative procedure starting with a small value of \(x_c\). \(x_c'\) can be calculated because at the same applied voltage

\[
\left( \frac{E}{p} \right)_M = \frac{U_b}{p} \sum_{n=0}^{13} (a_n x_c^n) = \frac{U_b}{p} \sum_{n=0}^{13} (b_n (S - x_c')^n) \tag{G.16}
\]

or

\[
\sum_{n=0}^{13} (a_n x_c^n) = \sum_{n=0}^{13} (b_n (S - x_c')^n) \tag{G.17}
\]

It is also done by iterative procedure starting with a small value of \(x'_c\). The required precision of \(x'_c\), however, is very high. The successful calculation is, therefore, decided to be the point where the difference between both side of eq. G.17 is less than 10^{-10} \%. Note that \(x'_c\) will only be calculated when \(E_{b_i}\) is greater than \((E/p)_a p\).

The right hand side of eq. G.15 can be compared with \(K/pC\) by inserting the value of \(x'_c\) and the selected \(x_c\) into the right hand side of eq. G.15. Iteration is stopped if the difference between the left and right hand side is smaller than 0.01 \%.
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