Doctoral Thesis

Quantum flux creep in high-Tc superconductors

Author(s):
Aupke, Klaus

Publication Date:
1995

Permanent Link:
https://doi.org/10.3929/ethz-a-001513585

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Quantum Flux Creep
in High-\(T_c\) Superconductors

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY
ZURICH

for the degree of
Doctor of Natural Sciences

presented by

KLAUS AUPKE
Dipl. Phys. ETH
born on the 28th of September 1964
citizen of Germany

accepted on the recommendation of

Prof. Dr. A. C. Mota, examiner
Prof. Dr. J. Blatter, co-examiner

1995
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Abstract

According to the classical models of flux creep in type-II superconductors which are based on thermal activation of vortices over the pinning barriers, the rates of magnetic relaxation should vanish with decreasing temperature. Recently measurable temperature independent vortex creep rates for $T \to 0$ have been observed in the Chevrel phase superconductors, high-$T_c$ superconductors, organic, and heavy fermion superconductors. These observations have been interpreted as evidence for quantum tunneling of vortices through the pinning barriers. The theory of quantum collective creep has been developed by Blatter, Geshkenbein, and Vinokur within the framework of weak collective pinning theory. In the limit of low and intermediate fields parallel to the $c$-axis and strong dissipation, the quantum creep rate for $T \to 0$ is determined by the saddle point solution of the effective Euclidean action

$$\frac{\partial \ln M}{\partial \ln t} \simeq -\frac{\hbar}{S_{E}^{\text{eff,}c}}$$

with

$$\frac{S_{E}^{\text{eff,}c}}{\hbar} = \frac{\hbar}{e^2} \frac{L_c}{\rho_n}$$

where $\rho_n$ is the normal state resistivity and $L_c^c$ is the collective pinning length given by

$$L_c^c \simeq \xi \left( \frac{j_0}{j_c} \right)^{1/2}$$
Abstract

Here $\xi$ is the in-plane coherence length, $j_c$ the in-plane critical current density, and $j_0$ the depairing current density, $\epsilon = \sqrt{m/M} < 1$ the anisotropy parameter, with $m$ and $M$ the effective electronic masses within the superconducting layers and perpendicular to them, respectively. In the case of layered superconductors, in which the collective pinning length $L_c^\xi$ is smaller than the interlayer distance $d$, $L_c^\xi$ has to be replaced by $d$ so that in this case

$$\frac{S_n^{\text{eff},c}}{\hbar} = \frac{\hbar}{e^2 \rho_n} \frac{d}{e^2 \rho_n}$$

In order to study the dependence of quantum creep on various parameters, such as the anisotropy as well as the influence of temperature, we have chosen to investigate the high-temperature superconductors $Y_1Ba_2Cu_3O_7$ and $Bi_2Sr_2CaCu_2O_x$ in the temperature range $10 \text{mK} \leq T \leq 20 \text{K}$. These superconductors have favorable parameters for the observation of quantum tunneling, namely short coherence length, high normal state resistivity, and strong anisotropy. With the theoretical expressions one obtains quantum creep rates at $T \to 0$ of approximately 0.5% for $Y_1Ba_2Cu_3O_7$ and about 5% for $Bi_2Sr_2CaCu_2O_x$. We have performed measurements of the relaxation of the remanent magnetization after cycling the specimens in an external field of $H_i = 2250 \text{Oe}$. The measured quantum creep rates at $T \to 0$ are approximately 0.15% for $Y_1Ba_2Cu_3O_7$ and 1.7% for $Bi_2Sr_2CaCu_2O_x$. Considering the experimental uncertainties and the approximations made in the theory, the agreement between theory and experiment is satisfactory. In particular, since $Bi_2Sr_2CaCu_2O_x$ is much more anisotropic than $Y_1Ba_2Cu_3O_7$, we have confirmed experimentally that strong anisotropy leads to a considerable enhancement of the quantum creep rate.

For the crossover temperature from the quantum regime to the regime of thermal activation, the quantum collective creep theory predicts

$$T_{qc} \simeq \frac{U_c}{k_B} \frac{\hbar}{S_n^{\text{eff}}(T = 0)}$$

which leads to $T_{qc} \simeq 1.5 \text{K}$ for $Y_1Ba_2Cu_3O_7$ and $T_{qc} \simeq 4 \text{K}$ for $Bi_2Sr_2CaCu_2O_x$. This is also in satisfactory agreement with the experimental observations.
Finally we have investigated quantum creep in a $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_7$ crystal with columnar defects, which were introduced by 580-MeV Sn-ion irradiation at a dose of $1.5 \times 10^{11}$ ions/cm$^2$. This kind of defects has been reported to considerably increase flux pinning at high temperatures and fields. At low temperatures however, we observe that in the configuration with the external field parallel to the c-axis and to the tracks, the quantum creep rate for $T \rightarrow 0$ is higher by roughly a factor of two in the irradiated specimen than in the unirradiated one. In a model where the columnar defect is viewed as a sharp square-well potential with radius bigger than $\xi$, it has recently been shown by Morais Smith, Caldeira, and Blatter that the result for the quantum creep rate at $T \rightarrow 0$ obtained in the theory of quantum collective creep is also applicable to the case of superconductors with columnar defects. According to this result, the quantum creep rate for $T \rightarrow 0$ should be proportional to the square root of the critical current density $j_c$. Other parameters as $\rho_n$ and $\xi$ are not strongly affected by the irradiation of the used dose. Since the critical current density is enhanced by a factor of approximately four due to the irradiation, we find that our experimental results are in agreement with the $\sqrt{j_c}$ dependence of the quantum creep rate found in the theory of quantum collective creep and in the work of Morais Smith et al.
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Kurzfassung

Gemäß klassischen Modellen für Flußkriechen in Supraleitern II. Art, die auf thermischer Aktivierung der Vortizes über die Pinningbarrieren basieren, müßte die magnetische Relaxationsrate mit abnehmender Temperatur verschwinden. In den letzten Jahren sind jedoch messbare temperaturunabhängige Vortexkriechraten für $T \rightarrow 0$ in Chevrel-Phase-Supraleitern, Hochtemperatur-Supraleitern, organischen und Schwer-Fermionen-Supraleitern beobachtet worden. Diese Resultate wurden als Nachweis für quantenmechanisches Tunneln von Vortizes durch die Pinningbarrieren interpretiert. Die „Quantum Collective Creep“-Theorie wurde von Blatter, Geshkenbein und Vinokur im Rahmen der Theorie des schwachen kollektiven Pinnings entwickelt. Im Grenzfall schwacher und mittlerer magnetischer Felder parallel zur $c$-Achse und starker Dissipation ist die Quantenkriechrate für $T \rightarrow 0$ durch die Sattelpunktsslösung der effektiven euklidischen Wirkung bestimmt

$$\frac{\partial \ln M}{\partial \ln t} \approx - \frac{\hbar}{S_{E}^{\text{eff},c}}$$

mit

$$\frac{S_{E}^{\text{eff},c}}{\hbar} = \frac{\hbar}{e^{2}} \frac{L_{c}^{c}}{\rho_{n}}$$

wobei $\rho_{n}$ der spezifische Widerstand im Normalzustand und $L_{c}^{c}$ die kollektive Pinninglänge sind. $L_{c}^{c}$ ist gegeben durch

$$L_{c}^{c} \approx \xi \left( \frac{j_{0}}{j_{c}} \right)^{1/2}$$
Hierbei sind ξ die Kohärenzlänge in den Ebenen, \( j_c \) die kritische Stromdichte in den Ebenen und \( j_0 \) die Entpaarungsstromdichte, \( ε = \sqrt{m/M} < 1 \) der Anisotropieparameter mit den effektiven elektronischen Massen \( m \) und \( M \) in den supraleitenden Ebenen beziehungsweise senkrecht dazu. Im Fall von geschichteten Supraleitern, in denen die kollektive Pinninglänge \( L_c^e \) kleiner ist als der Schichtabstand \( d \), muß \( L_c^e \) durch \( d \) ersetzt werden, so daß in diesem Fall

\[
\frac{S_{E_{\text{eff},c}}^e}{h} = \frac{\hbar d}{e^2 \rho_n}
\]

Um die Abhängigkeit des Quantenkriechens von verschiedenen Parametern wie der Anisotropie und den Einfluß der Temperatur zu erforschen, haben wir die Hochtemperatur-Supraleiter \( Y_1Ba_2Cu_3O_7 \) und \( Bi_2Sr_2CaCu_2O_x \) im Temperaturbereich \( 10 \text{ mK} \leq T \leq 20 \text{ K} \) untersucht. Diese Supraleiter haben günstige Parameter für die Beobachtung dieses Phänomens, nämlich kleine Kohärenzlänge, hohen spezifischen Widerstand im Normalzustand und große Anisotropie. Mit den theoretischen Resultaten erhält man Quantenkriechraten für \( T \to 0 \) von ungefähr 0,5% für \( Y_1Ba_2Cu_3O_7 \) und etwa 5% für \( Bi_2Sr_2CaCu_2O_x \). Wir haben Messungen der Relaxation der remanenten Magnetisierung durchgeführt, nachdem die Proben in einem äußeren Feld von \( H_i = 2250 \text{ Oe} \) zyklisch wurden. Die gemessenen Quantenkriechraten für \( T \to 0 \) sind ungefähr 0,15% für \( Y_1Ba_2Cu_3O_7 \) und 1,7% für \( Bi_2Sr_2CaCu_2O_x \). In Anbetracht der experimentellen Unsicherheiten und der in der Theorie gemachten Approximationen ist die Übereinstimmung zwischen Theorie und Experiment zufriedenstellend. Da \( Bi_2Sr_2CaCu_2O_x \) sehr viel anisotroper ist als \( Y_1Ba_2Cu_3O_7 \), haben wir insbesondere experimentell bestätigt, daß eine große Anisotropie zu einer beträchtlichen Erhöhung der Quantenkriechrate führt.

Für die Übergangstemperatur vom Quantenbereich in den Bereich der thermischen Aktivierung sagt die "Quantum Collective Creep"-Theorie

\[
T_{qc} \simeq \frac{U_c}{k_B} \frac{\hbar}{S_{E_{\text{eff}}}^e(T = 0)}
\]

voraus. Dies führt zu \( T_{qc} \simeq 1,5 \text{ K} \) für \( Y_1Ba_2Cu_3O_7 \) und \( T_{qc} \simeq 4 \text{ K} \) für \( Bi_2Sr_2CaCu_2O_x \).
Dies ist ebenfalls von der Größenordnung in zufriedenstellender Übereinstimmung mit dem Experiment.

Schließlich haben wir Quantenflußkriechen in einem $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_7$-Kristall mit „Columnar Defects“ untersucht, die durch 580-MeV-Zinn-Ionenbestrahlung einer Dosis von $1,5 \times 10^{11}$ Ionen/cm$^2$ erzeugt wurden. In Proben mit dieser Art von Defekten ist von einer beträchtlichen Erhöhung des Pinnings bei hohen Temperaturen und hohen Feldern berichtet worden. Bei tiefen Temperaturen beobachten wir jedoch, daß in der Konfiguration mit dem äußeren Feld parallel zur c-Achse und zu den Defekten die Quantenkriechrate für $T \to 0$ in der bestrahlten Probe um ungefähr einen Faktor zwei größer ist als in der unbestrahlten. In einem Modell, in dem ein „Columnar Defect“ durch ein scharfes Rechteckpotential dargestellt wird und der Radius größer ist als die Kohärenzlänge, ist kürzlich von Morais Smith, Caldeira und Blatter gezeigt worden, daß die Resultate für die Quantenkriechrate für $T \to 0$, die in der Theorie des „Quantum Collective Creep“ gefunden worden sind, auch auf den Fall von Supraleitern mit „Columnar Defects“ anwendbar sind. Gemäß diesen Resultaten ist die Quantenkriechrate für $T \to 0$ proportional zur Wurzel aus der kritischen Stromdichte $j_c$. Andere Parameter wie $\rho_n$ und $\xi$ werden nicht wesentlich von der Bestrahlung der verwendeten Dosis beeinflußt. Da die kritische Stromdichte durch die Bestrahlung um ungefähr einen Faktor vier erhöht wurde, finden wir, daß unsere experimentellen Resultate gut mit der in der „Quantum Collective Creep“-Theorie und von Morais Smith et al. gefundenen $\sqrt{j_c}$-Abhängigkeit der Quantenkriechrate übereinstimmen.
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Chapter 1

Introduction

Like in no other superconductor, almost the whole phase diagram of high-temperature superconductors corresponds to the vortex state. In the vortex state the superconductor can sustain a dissipation-free current only if the vortices are prevented from moving by pinning centers. Besides the static, quenched disorder, which provides pinning, there are two types of dynamical disorder in the system: thermal and quantum fluctuations. One of the most important phenomena caused by the competition between quenched disorder and fluctuations is flux creep. Creep in the classical type-II superconductors has been successfully described by considering only thermal fluctuations [1, 2], where vortices jump over the pinning barriers by thermal activation leading to relaxation rates which vanish with decreasing temperature. The first experimental observation of relaxation rates, which do not extrapolate to zero for $T \to 0$, has been made by Mitin in a Chevrel phase superconductor [3]. The first observations of non-vanishing flux creep at millikelvin temperatures have been made by Mota et al. on high-$T_c$ superconductors, heavy fermion, and organic superconductors [4–6]. This has been interpreted as quantum tunneling of the vortices through the pinning barriers, and the theory of quantum collective creep, which is in satisfactory agreement with most of the experimental findings, has been developed
by Blatter, Geshkenbein, and Vinokur [7, 8]. According to this theory, the quantum creep rate is enhanced in superconductors with short coherence length, high normal state resistivity, and strong anisotropy. Therefore, high-$T_c$ superconductors are ideal candidates to study the phenomenon of quantum creep in detail, whereas in classical type-II superconductors with their low normal state resistivity, large coherence length, and small anisotropy, quantum creep is smaller by several orders of magnitude. We have therefore chosen to investigate quantum creep in the high-$T_c$ superconductors $Y_1Ba_2Cu_3O_7$ and $Bi_2Sr_2CaCu_2O_x$.

We have performed measurements of the relaxation of the remanent magnetization after zero field cooling and cycling the specimens in an external field of $H_i = 2250$ Oe. At low temperatures, the decays deviate only slightly from a logarithmic-in-time law. The measured quantum creep rates as well as the crossover temperature separating the quantum regime from the regime of thermal activation are in satisfactory agreement with the values calculated with the results of the quantum creep theory. In particular, the expectation has been confirmed that the strong anisotropy in $Bi_2Sr_2CaCu_2O_x$ as compared to $Y_1Ba_2Cu_3O_7$ leads to a considerable enhancement of quantum creep.

Regarding technical applications, it is important to optimize pinning in the high-$T_c$ superconductors. Very promising results have been obtained by introducing columnar defects into the material. Columnar defects are extended cylindrical tracks of damaged, non-superconducting material with a diameter of approximately the size of the vortex core. These tracks can be generated by high-energy heavy-ion irradiation. Particularly at high temperatures and fields, this kind of defects have been reported to result in a considerable enhancement of pinning and an enlargement of the irreversibility region in the $H$-$T$ plane. We have investigated quantum creep in a $Y_1Ba_2Cu_3O_7$ crystal with columnar defects which were introduced by 580-MeV tin-ion irradiation at a dose of $1.5 \times 10^{11}$ ions/cm$^2$ corresponding to a matching field of $B_\phi = 3$ T. In the configuration with the external field parallel to the c-axis and to the tracks, we find that the quantum creep rate for $T \to 0$ is by roughly a factor of two higher in the irradiated specimen than in the unirradiated one.
Chapter 1. Introduction

This thesis is organized as follows: Chapter 2 summarizes the results of the quantum collective creep theory for anisotropic and layered superconductors for low and moderate magnetic fields in the limits of strong dissipation as well as in the superclean limit, where the Hall term of vortex motion becomes dominant. Furthermore, recent results on quantum creep in superconductors with columnar defects are described. In chapter 3 the experimental arrangements used in the presented investigations are described. In chapter 4 we present magnetic relaxation measurements in single crystals of Y$_1$Ba$_2$Cu$_3$O$_7$ and Bi$_2$Sr$_2$CaCu$_2$O$_x$ and a single crystal of Y$_1$Ba$_2$Cu$_3$O$_7$ with columnar defects down to millikelvin temperatures. In chapter 5 the data are discussed and compared with theoretical results. Chapter 6 gives a summary and conclusions of the presented work.
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Chapter 2

Theory

A theory for tunneling of vortices in bulk superconductors has been developed for the case of strong dissipation by Blatter, Geshkenbein, and Vinokur [7] within the framework of collective pinning theory [9]. The effects of anisotropy, layering and finite temperature have been studied by Blatter, and Geshkenbein in the low and intermediate field regime, where single vortex collective pinning is relevant [8]. Feigel'man, Geshkenbein, Larkin, and Levit [10] pointed out that high-temperature superconductors may belong to the class of very clean materials, in which case quantum tunneling of vortices is shown to be governed by the Hall term in the equation of motion of vortices. For the case of superconductors with columnar defects, the quantum creep rate has been derived by Vinokur [11], by Radzihovsky [12], and by Morais Smith, Caldeira, and Blatter [13]. In this theoretical overview, we will mainly follow the references given above as well as the work of Blatter et al. [14].
2.1 Creep in Anisotropic Materials

The new oxide superconductors as well as the organic superconductors are layered compounds. The transport properties are roughly uniaxial with a large anisotropy between the directions perpendicular and parallel to the layers and essentially isotropic behaviour within the layers. For not too large anisotropy, a description in terms of a continuous Ginzburg-Landau or London theory is applicable. On the other hand, for very large anisotropy the discreteness of the structure becomes relevant and a description in terms of a set of weakly coupled superconducting layers is more appropriate. Such a description is provided by the discrete Lawrence-Doniach model [15]. Often materials, where an anisotropic continuous Ginzburg-Landau or London free-energy functional gives an accurate description of the physics, are referred to as "anisotropic superconductors" and the term "layered superconductors" is used to refer to materials, which have to be described by the discrete Lawrence-Doniach model. In general, a continuous anisotropic description is applicable to YBCO over a large temperature regime, whereas the more strongly layered Bi and Tl compounds belong to the class of materials, for which a Lawrence-Doniach model is more appropriate. The layered organic superconductors of the BEDT-TTF family are other candidates requiring a discrete description.

We first consider the case of a uniaxially anisotropic superconductor characterized by the mass anisotropy ratio \( \epsilon^2 = \frac{m}{M} < 1 \), where \( m \) and \( M \) are the effective electronic masses within the superconducting layers and perpendicular to them, respectively. In the following, we adopt the convention used for the oxide superconductors, where the superconducting CuO planes are the \( ab \)-planes and the direction perpendicular to the layers is the \( c \)-axis.

A magnetic field \( H \) enclosing an angle \( \vartheta_H \) with the \( ab \)-plane is applied to the sample. In anisotropic superconductors, the direction of the external field \( \vartheta_H \) in general deviates from the direction \( \vartheta \) of the vortices. Here \( \vartheta \) is again measured with respect to the \( ab \)-plane. An additional complication occurs in strongly layered superconductors, where the vortices are locked to the \( ab \)-plane below a critical
2.1 Creep in Anisotropic Materials

Figure 2.1: Chosen coordinate systems: The \( z \)-axis of a first coordinate system is aligned parallel to the crystallographic \( c \)-axis and the \( xy \)-plane contains the vortex direction. The \( z' \)-axis of the second, rotated coordinate system points along the vortex direction and has the \( x' \)-axis in common with the first system, \( x' = x \).

angle \( \phi \) [16]. Furthermore, in irreversible superconductors the vortices go into a metastable state rather than to a stable configuration. The internal angle \( \phi \) then depends on the condition under which the critical state was formed. For the phenomenon of creep however, it turns out that the relaxation process, except for the very small angular regime \( \phi < \varepsilon \) in strongly layered superconductors, is independent of the angle \( \phi \) between the vortices and the crystallographic \( ab \)-plane. Blatter et al. therefore chose to express all the quantities by the internal variable \( \phi \). A coordinate system is chosen where the \( z \)-axis is aligned parallel to the crystallographic \( c \)-axis and the \( yz \)-plane contains the vortex direction (see Fig. 2.1). In general, the creep process may depend not only on the angle \( \phi \), but also on the direction of motion of the vortex. Therefore we introduce a second, rotated coordinate system with \( z' \) pointing along the vortex direction and a common \( x \)-axis, \( x = x' \). Vortex motion parallel to the planes, i.e. along \( x = x' \), is denoted “in-plane” motion. Motion within the plane containing the vortex and the crystal \( c \)-axis, i.e. motion along \( y' \) is called “out-of-plane”.

2.1.1 Classical Creep

Vortices entering the sample will minimize their energy with respect to the weak random pinning potential $U_{\text{pin}}$. For not too large fields, the interaction between the vortices is small compared with the single vortex tilt elastic energy and the interaction of the vortex with the pinning centers, so that we can study the single-vortex free energy

$$\mathcal{F}[u] = \int dz' \left[ \frac{\varepsilon_{||}(\vartheta)}{2} (\partial_{z'} u_x)^2 + \frac{\varepsilon_{\perp}(\vartheta)}{2} (\partial_{z'} u_y)^2 + U_{\text{pin}}(z', u) \right]$$  \hspace{1cm} (2.1)$$

The two-dimensional field $u(z, t)$ is the displacement of the vortex from its equilibrium position and $\varepsilon_{||}(\vartheta)$ and $\varepsilon_{\perp}(\vartheta)$ are the elasticities for in-plane and out-of-plane motion given by

$$\varepsilon_{\perp}(\vartheta) \simeq \frac{\varepsilon_0 \xi^2}{\varepsilon_3}, \quad \varepsilon_{||}(\vartheta) \simeq \frac{\varepsilon_0 \xi^2}{\varepsilon_3^3}$$

with the angle dependent anisotropy parameter

$$\varepsilon_3 = \epsilon^2 \cos \vartheta + \sin^2 \vartheta$$

and

$$\varepsilon_0 = \left[ \frac{\Phi_0}{4\pi\lambda} \right]^2$$

where $\Phi_0 = \frac{hc}{2e}$ is the flux quantum and $\lambda$ is the London penetration depth in the $ab$-plane.

Within weak collective pinning theory [9], vortex segments of length $L_c$ are pinned independently. Each segment $L_c$ is subject to the competition between elastic tilt energy and the pinning energy, so that the individual pinning forces add up only randomly within the collective pinning volume $V_c \propto L_c$. On the other hand, the net pinning forces of the segments add up linearly, resulting in a finite pinning force density. The collective pinning length $L_c$ is determined by minimization of the free energy (2.1). The solution minimizing (2.1) can be obtained using dimensional estimates. Within the collective-pinning volume

$$V_c \simeq \varepsilon_3 \xi^2 L_c$$
the elastic energy competes with the pinning energy and the equality between the two energies determines the collective pinning length $L_c$, where $\xi$ denotes the planar coherence length.

For the in-plane-relaxation mode, the relevant length scale is $\xi$. The elastic energy $E_{el}^{||}$ and the pinning energy $U_c^{||}$ are given by

$$E_{el}^{||} = \frac{\varepsilon_0^{||}(\vartheta) \xi^2}{L_c^{||}}, \quad U_c^{||} = (f_{pin}^2 n_d L_c^{||} \xi^2 \epsilon_0)^{1/2} \xi$$

Here $n_d$ denotes the defect density and $f_{pin}$ is the individual mean pinning force. Solving for $L_c^{||}$ yields

$$L_c^{||} = \left[ \frac{\varepsilon_0^{||} \xi^2 \epsilon_0^4}{W_a^2 c_0^3} \right]^{1/2} \sim \frac{L_c}{\epsilon_0}$$  \hspace{1cm} (2.2)

Here we have introduced the mean-squared random force density $W = f_{pin}^2 n_d (\xi/a)^2$ with $a = \sqrt{\Phi_0/B}$ the mean vortex separation. $L_c^{||}$ is the collective-pinning length for a field applied parallel to the $c$-axis of the crystal.

For the out-of-plane-relaxation mode, the relevant length scale is $\epsilon_0 \xi$. The elastic energy $E_{el}^{\perp}$ and the pinning energy $U_c^{\perp}$ are given by

$$E_{el}^{\perp} = \frac{\varepsilon_0^{\perp}(\vartheta) (\epsilon_0 \xi)^2}{L_c^{\perp}}, \quad U_c^{\perp} = \left[ \left( f_{pin}^{\perp} \frac{\epsilon_0}{\epsilon_0} \right)^2 n_d L_c^{\perp} \xi^2 \epsilon_0 \right]^{1/2} \epsilon_0 \xi$$

Solving for $L_c^{\perp}$ yields

$$L_c^{\perp} \sim L_c^{||} \sim \frac{L_c}{\epsilon_0}$$  \hspace{1cm} (2.3)

The vortex relaxes to the pinning potential by choosing the mode characterized by the smaller collective-pinning length. Since $L_c^{\perp} \sim L_c^{||}$, the relaxation involves both in-plane and out-of-plane motions. Putting these results for the collective-pinning lengths back into the respective pinning energies one obtains

$$U_c^{\perp} \sim U_c^{||} \sim U_c^{\perp} \simeq (Wa^2 L_c^{\perp})^{1/2} \xi \simeq \epsilon_0 \xi^2 / L_c^{\perp}$$
Thus the collective-pinning energy is independent of the angle $\theta$ between the field and the $ab$-plane and independent of the direction of motion of the vortices.

In order to express the collective-pinning length by experimentally accessible quantities, we relate $L_c^e$ for a field $H \parallel c$ to the corresponding critical current density $j_c$ in the $ab$-plane: The critical current density $j_c$ is determined by the equality between the driving Lorentz force $j_c \Phi_0 L_c^e / c$ on a segment of length $L_c^e$ and the pinning force $U_c^e / \xi$. Using the definition of the depairing current density $j_0 = e \Phi_0 / 12 \sqrt{3} \pi \lambda^2 \xi$, we obtain

$$L_c^e \simeq \xi \left( \frac{j_0}{j_c} \right)^{1/2}$$

and therefore for the collective pinning energy

$$U_c^e \simeq \epsilon_0 \xi \left( \frac{j_c}{j_0} \right)^{1/2}$$

Minimizing the free energy for the single-vortex configuration (2.1), we have obtained the energy scale for pinning, the collective-pinning energy. Equating this energy to the product of the Lorentz force and the relevant length scale for the pinning potential, we have obtained the critical current density. However, the creep rate is determined by the saddle point solution of the free-energy functional. In general, the elasticity involved in the relaxation of the vortex to the pins may differ from the elasticity involved in the hop, leading to an increase in the length $L_h$ of the hopping segment with respect to the collective-pinning length $L_c$. In anisotropic superconductors the elastic energy density is identical for the two cases of in-plane and out-of-plane relaxation. Therefore, the elastic energy density involved in the hop always agrees with the one involved in the relaxation process. Consequently, the minimum and the saddle point solution for the free energy agree with each other, $L_h = L_c$, and the typical activation energy for creep is $U_c^e$, independent of the angle $\theta$.

The range of applicability of the obtained results is restricted to the regime of single vortex pinning. As the external field is increased, the vortex separation decreases and the interaction between neighboring vortices becomes increasingly
important. Comparing the elastic energy due to tilt of an individual vortex to the interaction energy with the nearest neighbour, Blatter and Geshkenbein obtain the condition

\[ B \lesssim \alpha \frac{j_c}{j_0} H_{c2}(\vartheta) \]

with \( \alpha \) a numerical of the order of 10. Using typical parameters for \( j_c/j_0 \) and \( H_{c2}(\vartheta) \) for the oxide superconductors, they estimate that the single-vortex pinning regime extends up to fields of the order of 10 T.

### 2.1.2 Quantum Creep

#### Zero Dissipation Limit

The Lagrangian generating the classical equation of motion for the vortex is given by

\[ \mathcal{L}[u] = \int dz \left[ \frac{M^\parallel(\vartheta)}{2} (\partial_t u_x)^2 + \frac{M^\perp(\vartheta)}{2} (\partial_t u_y)^2 \right] - \mathcal{F}[u] \quad (2.6) \]

The vortex masses \( M^\parallel(\vartheta) \) and \( M^\perp(\vartheta) \) have to be determined by calculating the kinetic energy of a moving vortex or by studying the inertial response of a vortex to an external force. Using different kinds of time-dependent Ginzburg-Landau theories, corresponding calculations for the isotropic case have been carried out by Suhl [17] and by Kupriyanov and Likarhcrev [18] with similar results. Such an approach is strictly valid only for a gapless superconductor [19] and the applicability of the results to a superconductor with a finite gap is unclear. However, Blatter et al. have given a qualitative estimate of the vortex mass involving general arguments which should hold for a finite-gap superconductor at low temperatures as well. In their theory of quantum collective creep, these estimates are given mainly in order to show that the massive term is not important, so that the exact values of the estimates do not influence the accuracy of their final results. The basic idea of the estimate is that the electronic contribution to the vortex mass is due to the local
change in dispersion within the vortex core. The number of electrons experiencing
this change is $2\pi^2 N(\epsilon_F) \delta \epsilon$. Here $\delta \epsilon \simeq h v_F / \pi \xi$ is the change in energy due to the
confinement of quasiparticles to the vortex core, $N(\epsilon_F) = k_F m_e / \hbar^2 \pi^2$ is the density
of states at the Fermi level, $m_e$ denotes the effective electronic mass and $k_F$ and $v_F$
are the Fermi wave vector and the Fermi velocity, respectively. The effective mass
of the electrons confined to the core will be modified by an amount $m_e \delta \epsilon / \epsilon_F$. The
result for the vortex mass per unit length is $M_v = (2/\pi^3) m_e k_F$, which agrees with
the results of Suhl and of Kupriyanov and Likhararev. Finally, Blatter et al. obtain
the vortex masses $M^\parallel(\vartheta)$ and $M^\perp(\vartheta)$ for the anisotropic case by generalizing Suhl's
analysis to the anisotropic case

$$M^\parallel(\vartheta) = \epsilon_0 M_v \quad , \quad M^\perp(\vartheta) = \frac{M_v}{\epsilon_0}$$

where $M_v$ is the vortex mass per unit length for the case of the magnetic field
applied parallel to the c-axis given by $M_v = (2/\pi^3) m_e K_F$. Here $m_e$ denotes the
planar electronic mass and $K_F$ is the Fermi wave vector along the c-axis.

Besides the electronic contribution, a second term $M_{em}$ of electromagnetic ori¬
contributes to the vortex mass. Typically the electromagnetic contribution $M_{em}$
to the vortex mass is negligible compared to the electronic contribution. However,
for the case of a Josephson vortex in a layered superconductor $M_{em}$ can become
large. For this contribution Blatter and Geshkenbein obtain $M_{em} \simeq \frac{\epsilon \epsilon_d \hbar^2}{2 \pi d^2 \epsilon^2}$ where $\epsilon_d$ is the dielectric constant and $d$ is the distance between two neighboring layers.

Using typical material parameters, they find that the ratio $M_{em}/\epsilon M_v$ is of order
unity, so that in most cases also for the Josephson vortices the electronic mass $\epsilon M_v$
can be used.

Whereas the thermally activated creep is determined by the saddle point so¬
olution of the free-energy functional (2.1), it has been shown [20, 21] that for a 1D
string the tunneling rate is determined by the saddle point solution of the Euclidean
action of the vortex

$$S_B[u] = \int dt \left\{ \int dx' \left[ \frac{M^\parallel(\vartheta)}{2} (\partial_x u_x)^2 + \frac{M^\perp(\vartheta)}{2} (\partial_y u_y)^2 \right] + F[u] \right\}$$

(2.7)
2.1 Creep in Anisotropic Materials

and

\[
\frac{\partial \ln M}{\partial \ln t} \simeq \frac{\hbar}{S_{E, \text{saddle point}}}
\]  

(2.8)

Thus the quantum problem corresponds to the \((n+1)\)-dimensional generalization of the \(n\)-dimensional classical problem, with \(n=1\) for a string and \(n=3\) for the problem of moving vortex bundles. The additional dimension becoming relevant for quantum motion is time, because quantum mechanically, energy conservation is violated in a virtual process, and the amplitude of the process decays exponentially with the size of the time interval during which the energy conservation is violated.

For determining the saddle point solution of the Euclidean action, Blatter et al. used again the method of dimensional estimates. The tunneling segment is given by the same length \(L_h\) as in the classical problem. The estimate for the characteristic time \(t_c\) is obtained by equating the kinetic and elastic energy densities in (2.7). For the in-plane relaxation mode, the kinetic energy density and the elastic energy density are given by

\[
M^\parallel(\theta) \left( \frac{\xi}{t_c^\parallel} \right)^2, \quad \epsilon_e^\parallel(\theta) \left( \frac{\xi}{L_c^\parallel} \right)^2
\]

respectively. Inserting the result (2.2) for \(L_c^\parallel\) and solving for \(t_c^\parallel\) we obtain

\[
t_c^\parallel \simeq \left( \frac{M_v}{\epsilon_0} \right)^{1/2} \frac{L_c^\parallel}{\epsilon} = t_c^M
\]

where \(t_c^M\) is the tunneling time for a vortex aligned with the \(c\)-axis. Finally, the Euclidean action of the saddle point is

\[
\frac{S_E^\parallel}{\hbar} \simeq \frac{t_c^\parallel}{\hbar} U_c^\parallel \simeq \epsilon_0^2 \left( \frac{\epsilon_0 M_v}{\hbar^2} \right)^{1/2} \simeq \epsilon_0^2 K_F k_F \left( \frac{\ell}{\xi_0} \right)^{1/2} \simeq \frac{S_c^E}{\hbar}
\]

where \(S_E^\parallel\) is the action for a vortex parallel to the \(c\)-axis and \(k_F\) and \(K_F\) denote the Fermi wave vectors in the \(ab\)-plane and along the \(c\)-axis, respectively. For a clean superconductor, the mean free path \(\ell\) has to be substituted by the coherence length \(\xi_0\). The analogous calculation for the out-of-plane relaxation mode with the
respective expressions for mass and elasticity result in a tunneling time

\[ t_{c}^{\perp} \approx t_{c}^{M} \]

and

\[ \frac{S_{E}^{\perp}}{\hbar} \approx \frac{S_{E}^{M}}{\hbar} \approx \frac{S_{E}^{c}}{\hbar} \]

Thus for an anisotropic superconductor, Blatter et al. obtain the simple result that the Euclidean action of the saddle point solution and consequently the quantum relaxation rate are independent of the angle \( \vartheta \) and of the direction of motion.

**Finite Dissipation**

The above results apply to the limit of vanishing dissipation. Usually, macroscopic quantum tunneling is a dissipative process as the macroscopic variable is coupled to environmental degrees of freedom. Generalizing a solution obtained by Caldeira and Leggett [22] for macroscopic quantum tunneling in a SQUID to the case of a tunneling vortex, Blatter et al. derived an effective Euclidean action \( S_{E}^{\text{eff}} \) for the macroscopic variable coupled dissipatively to the environment. The environment is accounted for by adding a term

\[
\int dt dt' dz' \left\{ \frac{\eta^{\parallel}(\vartheta)}{4\pi} \left( \frac{u_{x}(z', t) - u_{x}(z', t')}{t - t'} \right)^{2} + \frac{\eta^{\perp}(\vartheta)}{4\pi} \left( \frac{u_{y}(z', t) - u_{y}(z', t')}{t - t'} \right)^{2} \right\}
\]

to the Euclidean action (2.7). Here it has been assumed that the dissipation is ohmic. The viscous drag coefficients are given by

\[
\eta^{\parallel}(\vartheta) = \epsilon_{\vartheta} \eta , \quad \eta^{\perp}(\vartheta) = \frac{\eta}{\epsilon_{\vartheta}}
\]

where

\[
\eta = \frac{\Phi_{0}^{2}}{2\pi c^{2} \xi^{2} \rho_{n}}
\]

is the viscous drag coefficient for a vortex aligned with the c-axis. In the limit of large damping they obtain
leading to the final expression for the effective action for anisotropic superconductors

\[ \frac{S_{\text{E}}}{\hbar} \simeq \frac{S_{\text{E}}}{\hbar} \simeq \frac{t_{\text{E}}^{\parallel}}{\hbar} \simeq \frac{\eta_{\text{E}}^{2} L_{\text{E}}}{\hbar} \simeq \frac{\hbar \epsilon_{\text{E}}}{\epsilon^{2} \rho_{n}} \left( \frac{j_{0}}{j_{c}} \right)^{1/2} = \frac{S_{\text{E}}^{\text{eff,c}}}{\hbar} \]

independent of the angle \( \vartheta \) and the direction of motion.

**Thermal Enhancement of Tunneling**

The temperature dependence of the tunneling process can be obtained by first considering the case where the tunneling object is a point-like object, for which Grabert, Weiss, and Hänggi \cite{23} have shown that the finite-temperature corrections are mainly determined by the dissipation mechanism alone. They obtain for the zero dissipation limit

\[ \Delta S_{\text{E}}(T) = S_{\text{E}}(T) - S_{\text{E}}(0) \simeq -S_{\text{E}}(0) \exp \left( -\frac{\hbar}{k_{B} T t_{t}} \right) , \quad t_{t} \simeq 1/\omega_{0} \]

and for strong ohmic damping

\[ \Delta S_{\text{E}}^{\text{eff}}(T) = S_{\text{E}}^{\text{eff}}(T) - S_{\text{E}}^{\text{eff}}(0) \simeq -S_{\text{E}}^{\text{eff}}(0) \left( \frac{k_{B} T t_{t}}{\hbar} \right)^{2} , \quad t_{t} \simeq \frac{\eta}{M \omega_{0}^{2}} \]

where \( S_{\text{E}}^{\text{eff}}(0) \) is the (effective) Euclidean action for \( T=0 \), \( \omega_{0} \) denotes the frequency of small oscillations around the metastable minimum, and \( \eta(\omega) \) is the frequency dependent damping coefficient. Blatter et al. have shown that the tunneling problem for the vortex can be reduced to the tunneling of a point-like object for the case of a cubic model potential. Thus using the results of Grabert et al., they obtain the finite-temperature correction for zero dissipation

\[ \Delta S_{\text{E}}(T) \simeq -S_{\text{E}}(0) \exp \left( -\frac{\hbar}{k_{B} T t_{\text{E}}} \right) \]

(2.10)
and for strong damping

\[ \Delta S_E^{\text{eff}}(T) \simeq -S_E^{\text{eff}}(0) \left( \frac{k_B T}{\hbar} \right)^2 \]  

(2.11)

where the appropriate tunneling times \( t_c \) have to be inserted.

The crossover temperature \( T_{qc} \) from the quantum to the classical regime of motion is defined by the condition

\[ \frac{\Delta S_E^{\text{eff}}(T)}{S_E^{\text{eff}}(0)} \simeq 1. \]

\( T_{qc} \) thus depends on the tunneling time \( t_c \)

\[ k_B T_{qc} \simeq \frac{\hbar}{t_c} \]

Using \( S_E^{\text{eff}}(T = 0) = t_c U_c \) we obtain the relation

\[ \frac{S_E^{\text{eff}}(T = 0)}{\hbar} \simeq \frac{U_c}{k_B T_{qc}} \]  

(2.12)

between the zero-temperature (effective) Euclidean action \( S_E^{\text{eff}}(0) \), the classical activation energy \( U_c \), and the crossover temperature \( T_{qc} \).

2.2 Creep in Layered Materials

The above discussion is based on a continuous anisotropic Ginzburg-Landau theory which is applicable for superconductors with not too large anisotropy. As mentioned above, for very anisotropic materials a description in terms of the discrete Lawrence-Doniach model is appropriate. In such materials a vortex line is not a simple rectilinear object but rather is viewed as an array of two-dimensional pancake vortices threading the individual layers and interconnected by interplanar Josephson-type vortices (see Fig. 2.2). The pancake vortices can be pinned against motion within the \( ab \)-plane, the characteristic length scale for pinning being the coherence length \( \xi \). The individual pinning force of one small defect acting on a pancake vortex, \( f_{\text{pin}} \), is identical to the one acting on an Abrikosov vortex pointing along the \( c \)-axis. Josephson vortices on the other hand are intrinsically pinned against motion parallel to the \( c \)-axis, however, the pinning with respect to motion along the \( ab \)-plane
2.3 Hall Tunneling of Vortices

Figure 2.2: Single flux-line structure in a strongly anisotropic superconductor which has to be described in terms of a set of weakly coupled superconducting layers. The vortex line can be viewed as an array of two-dimensional pancake vortices threading the individual layers and interconnected by interplanar Josephson-type vortices.

is very weak. The pinning force acting on a Josephson vortex is suppressed by a factor $(\xi/\Lambda)^3$ as compared to the pinning force acting on a pancake vortex, where $\Lambda = d/\epsilon$. Studying the pinning properties of a single vortex in a layered superconductor, Blatter and Geshkenbein find several different regimes depending on the tilt angle $\vartheta$ between the vortices and the layers. However, they find that the results for the layered superconductors agree with the corresponding quantities obtained for the anisotropic superconductors in the regime $\vartheta > \epsilon$. Considering the fact that $\epsilon$ is a very small number in layered superconductors, this region $\vartheta > \epsilon$ is the most important one. A difference in the creep behaviour between anisotropic and layered materials is restricted to small angles $\vartheta < \epsilon$. 
2.3 Hall Tunneling of Vortices

In a recent paper Feigel'man, Geshkenbein, Larkin, and Levit [10] argue that the assumption of strong dissipation in the quantum collective creep theory might not be fulfilled. The criteria distinguishing between dissipative and non-dissipative vortex motion is cleanness. The equation of vortex motion in the flux flow regime which has been derived by Kopnin and Kravtsov [24] is given by

\[ \eta \mathbf{v}_L + \alpha [\mathbf{v}_L \times \mathbf{n}] = \frac{\Phi_0}{c} [\mathbf{j} \times \mathbf{n}] \]  

(2.13)

Here \( j \) is the transport current density, \( \mathbf{v}_L \) is the velocity of the vortex line, \( \mathbf{n} \) is the unit vector along the vortex, \( \eta \) is the viscous drag coefficient and \( \alpha \) the Hall coefficient. Viscous flow is characterized by the condition \( \eta \gg \alpha \), whereas the opposite limiting case \( \alpha \gg \eta \) corresponds to non-dissipative motion. The transport expressions \( \eta \) and \( \alpha \) take the form

\[ \eta = \frac{\Phi_0}{c} \rho_s \frac{\omega_0 \tau_r}{1 + \omega_0^2 \tau_r^2} \]

\[ \alpha = \frac{\Phi_0}{c} \rho_s \frac{\omega_0^2 \tau_r^2}{1 + \omega_0^2 \tau_r^2} \]

where \( \rho_s = 2e|\psi|^2 \) denotes the charge density of the superconducting condensate, \( \omega_0 \) is the characteristic frequency given by the level separation of the quasiparticle states bound to the vortex core, and \( \tau_r \) is the scattering relaxation time. The condition \( \alpha \gg \eta \) is therefore equivalent to \( \omega_0 \tau_r \gg 1 \). \( \omega_0 \) can be expressed as \( \Delta/\varepsilon_F \) where \( \Delta \) is the energy gap and \( \varepsilon_F \) is the Fermi energy. Using \( \tau_r = \ell/v_F \), where \( \ell \) is the mean free path and \( v_F \) is the Fermi velocity, and the BCS-expression \( \xi = \hbar v_F/\pi \Delta \), the condition \( \omega_0 \tau_r \gg 1 \) and therefore \( \alpha \gg \eta \) can finally be expressed as

\[ \ell \gg \xi \varepsilon_F/\Delta \]  

(2.14)

This condition is the condition of the superclean limit. It is much stronger than the usual condition for the clean limit \( \ell \gg \xi \) and is practically never realized in conventional superconductors, where typically \( \varepsilon_F/\Delta \sim 10^3 \). However, high-\( T_c \) superconductors have much larger values of \( \Delta \) with \( \varepsilon_F/\Delta \sim 10 \) leading to \( \xi \varepsilon_F/\Delta \sim 10 - 20 \text{ nm} \).
Extrapolation of the normal state resistivity to zero temperature gives mean free paths of about $\ell \sim 70 \text{ nm}$. So, Feigel'man et al. conclude that HTSCs might indeed be in the "superclean limit" $\ell \gg \xi \varepsilon_F / \Delta$. In this limit they obtain effective Euclidean actions of

$$\frac{S^\text{eff}}{\hbar} \simeq \frac{\alpha}{\hbar} \xi^2 \simeq \pi n_s^{(2)} \xi^2 \quad (2.15)$$

for the 2D case and

$$\frac{S^\text{eff}}{\hbar} \simeq \frac{\alpha}{\hbar} \xi^2 L_c \simeq n_s \xi^2 L_c \quad (2.16)$$

for the 3D case. Here $n_s^{(2)}$ denotes the superfluid density per one superconducting layer and $n_s$ is the 3D superfluid density. It is interesting to notice that in the result (2.15) for the 2D case $\xi^2$ is the area enclosed by the trajectory of the vortex and in (2.16) for the 3D case $\xi^2 L_c$ is the volume enclosed by the trajectory of the vortex. Since these expressions are multiplied by the respective superfluid densities, the Euclidean actions $S/\hbar$ in the two cases are given by the number of particles enclosed by the trajectory of the vortex.

An important question is of course how to distinguish between the different types of tunneling. The estimation given by Feigel'man et al. based on equation (2.14) is of course very rough, because the used parameters as the energy gap, the Fermi energy, the coherence length, and the mean free path are not easily accessible quantities and therefore rather poorly known. Choosing only slightly different parameters and combining them, one can equally well show that $\omega \tau \simeq 1$, i.e. that the viscous drag coefficient and the Hall coefficient are of the same order. An important qualitative difference between the two types of tunneling however is predicted concerning the low-temperature correction to the $T \to 0$ limit of the quantum creep rate. Whereas in the dissipative case this correction is proportional to $T^2$, it is exponentially small, i.e. proportional to $\exp ( - T_0 / T )$, in the Hall case.
2.4 Quantum Creep in High-$T_c$ Superconductors with Columnar Defects

The vortex dynamics in high-$T_c$ superconductors with correlated disorder, such as twin boundaries and artificial columnar defects resulting from irradiation of the sample with high-energetic heavy-ions, has been treated theoretically by Nelson and Vinokur [25] and by Blatter et al. [14]. Vortex creep from columnar defects at very low temperatures and low fields has been treated by Vinokur [11], by Radzihovsky [12], and by Morais Smith, Caldeira, and Blatter [13]. A columnar defect is modeled by a cylindrical cavity with radius $r_0$. The tunneling rate is determined by the saddle point trajectory of the Euclidean action

$$S_E = \int \frac{d\omega}{2\pi} \frac{dq}{2\pi} \left[ \frac{1}{2} \left( |\omega| \eta + \epsilon_t q^2 \right) |u(q,\omega)|^2 + V_B(q,u) \right]$$

where $V_B$ represents the pinning potential and $\epsilon_t = \epsilon^2 \epsilon_0$ is the line tension of the vortex. The liberation of a vortex line from its rod takes place via thermal activation of a finite segment of length $L$ and a transverse displacement $u$ out of the pinning well, creating a half-loop excitation of the string. The energy barrier for creating a half loop of length $L$ is given by

$$U = \epsilon_r \cdot L$$

where $\epsilon_r$ is the depth of pinning potential. The tunneling time $t_t$ is determined by balancing the kinetic energy term against the elastic term in (2.17) which yields

$$t_t \simeq \frac{\eta L^2}{\epsilon_t}$$

The saddle point solution of the Euclidean action then follows as

$$\frac{S}{\hbar} \simeq \frac{t_t U}{\hbar} \simeq \frac{\eta \epsilon_r L^3}{\hbar \epsilon_t}$$

The optimal ratio between $L$ and $u$ is determined by the equality of the elastic energy and the pinning energy

$$\epsilon_t \frac{u^2}{L^2} = \epsilon_r$$
so that the geometrical shape of the half-loop is determined by
\[
\frac{u}{L} = \sqrt{\frac{\varepsilon_r}{\varepsilon_l}}
\]
The Euclidean action then reads
\[
\frac{S}{\hbar} = \frac{\eta \varepsilon_r L^3}{\hbar \varepsilon_l} = \frac{\hbar}{\hbar} \frac{\varepsilon_l}{\varepsilon_l} \left( \frac{u}{\xi} \right)^3 \sqrt{\frac{\varepsilon_0}{\varepsilon_r}}
\]
where \( \varepsilon_l = \varepsilon^2 \varepsilon_0 \) and \( \eta_l = \phi_0 H_{c2}/\rho_n c^2 \simeq h^2/e^2 \rho_n \xi^2 \) have been used.

The next step is to express the pinning potential in terms of the critical current density \( j_c \). At \( j_c \), the Lorentz force and the pinning force are equal.

\[
j_c \Phi_0 \frac{c}{u} = \frac{\varepsilon_r}{u}
\]
Using \( j_0 \simeq \frac{c \varepsilon_0}{\xi \Phi_0} \), one obtains
\[
\frac{\varepsilon_0}{\varepsilon_r} = \frac{\xi}{u} \frac{j_0}{j_c}
\]
so that finally
\[
\frac{S}{\hbar} = \frac{\hbar}{\varepsilon^2 \rho_n} \left( \frac{u}{\xi} \right)^{2.5} \frac{j_0}{j_c}
\]
In order to determine which length scale we have to use for \( u \), we begin by considering first a sharp cylindrical square-well potential with a radius \( r_0 > \xi \)
\[
U_{\text{pin}}(x) = \begin{cases} 
-\varepsilon_r & |x| > r_0 \\
0 & \text{else} 
\end{cases}
\]
The pinning energy \( E_{\text{pin}}(x) \) experienced by a vortex is given by
\[
E_{\text{pin}}(x) = \int dx' U_{\text{pin}}(x') p(x' - x)
\]
where \( p(x) \) is a form function which for the large \( \kappa \)-limit is given by
\[
p(x) = 1 - \frac{x^2}{x^2 + 2\xi^2}
\]
The resulting functions are plotted in Fig. 2.3 for the case \( r_0 = 5 \xi \). The pinning energy is smeared out on the scale \( \xi \) of the width of the vortex core. Therefore, the relevant length scale for the pinning force is given by \( \xi \). If now a current density \( j \) is
applied, the pinning potential is tilted. In Fig. 2.4 the resulting effective potentials are shown for different current densities between $j = 0$ and $j = j_c$, where $j_c$ is reached when the border of the resulting potential well is horizontal. As can be seen in the lower part of Fig. 2.4, the scale of the distance the vortex has to travel under the barrier for the case $j < j_c$ is also given by $\xi$. For the case of a sharp potential
2.4 Quantum Creep in High-Tc Superconductors with Columnar Defects

with \( r_0 > \xi \) and \( j \lesssim j_c \) one therefore obtains the final result [13]

\[
\frac{S}{\hbar} = \frac{\hbar}{e^2 \rho_n} \sqrt{\frac{j_0}{j_c}}
\]

which is identical to the one obtained for the case of weak collective pinning (2.9).

If instead of \( r_0 > \xi \) we have \( r_0 < \xi \), then still \( \xi \) remains the relevant length scale because the vortex cannot resolve smaller lengths than \( \xi \) and the result is the same as above. If now \( r_0 > \xi \) but instead of a sharp potential we choose a shallow potential for \( U_{pin} \), the relevant length scale is \( r_0 \) and not \( \xi \). In this model we obtain

\[
\frac{S_E}{\hbar} \approx \frac{\hbar}{e^2 \rho_n} \frac{\xi}{\xi} \left( \frac{r_0}{\xi} \right)^{2.5} \sqrt{\frac{j_0}{j_c}}
\]  

(2.18)
which is essentially the result published by Radzihovsky [12]. Vinokur has obtained still another result

\[ \frac{S_E}{\hbar} \approx \varepsilon \frac{\eta r_0^2}{\hbar} \approx \frac{\hbar}{c^2 \rho_n} \left( \frac{r_0}{\xi} \right)^3 \]  

(2.19)

In order to reproduce this result, we have to choose \( r_0 \) as the scale for the tunneling segment and \( \xi \) as the scale for the pinning force. It seems however unphysical to choose different scales for these two quantities. In conclusion both, the result of Morais Smith et al. and Radzihovsky’s result are consistent and differ in the chosen model for the columnar defect. In practice, \( \xi \) and \( r_0 \) are of the same order of magnitude, in fact, in the sample investigated in this work, \( r_0 \) is by a factor of roughly 2 bigger than \( \xi \). Therefore, within the accuracy of the theoretical method of dimensional estimates, the difference between \( r_0 \) and \( \xi \) is unimportant. Important is however that both theories predict a square root dependence of the quantum creep rate at \( T \to 0 \) on the critical current density.

The thermally activated creep is determined by \( T/U \). Retaining the model where \( u = \xi \) we obtain

\[ U = \varepsilon r \cdot L = \varepsilon \varepsilon_0 \xi \sqrt{\frac{j_c}{j_0}} \]  

(2.20)

which again is identical to the one obtained for the case of weak collective pinning. Thus, the thermally activated creep rate is proportional to \( 1/\sqrt{j_c} \). Finally, the crossover temperature which is given by \( T_{ac} = \frac{U}{T S_E(0)} \) is therefore proportional to \( j_c \). In summary, for increasing current density it is found that the quantum creep rate for \( T \to 0 \) is enhanced proportionally to \( \sqrt{j_c} \), the thermally activated rate is reduced as \( 1/\sqrt{j_c} \) and the crossover temperature is increased linearly with \( j_c \).

The temperature dependence of the relaxation rate is sketched in Fig. 2.5 in a double logarithmic scale for four different current densities. The continuous line corresponds to some chosen critical current density \( j_c \), the dotted line to a higher current density \( 2j_c \), the dashed line to \( 4j_c \), and the dotted/dashed line to a current density \( 8j_c \). As described before, it can be seen in the graph that for increasing critical current density the quantum creep rate increases, the thermally activated rate
2.4 Quantum Creep in High-$T_c$ Superconductors with Columnar Defects

Figure 2.5: Qualitative temperature dependence of the relaxation rate for different critical current densities. For increasing current density the quantum creep rate for $T \to 0$ is enhanced proportionally to $\sqrt{j_c}$, the thermally activated rate is reduced as $1/\sqrt{j_c}$, and the crossover temperature is increased linearly with $j_c$.

decreases and the crossover temperature increases. For different current densities, the relaxation rates as a function of temperature cross each other because of the inverse current density dependence in the quantum and in the thermal regime.

Furthermore, it is interesting to notice that it is possible to move from the quantum regime to the thermal regime not only by changing the temperature at constant critical density but also by changing the critical current density at a given temperature. For example for the lowest critical current density $j_c$ in Fig. 2.5, the relaxation at $T = 2K$ is mainly thermally activated. By increasing the critical current density from $j_c$ to $2j_c$, the creep rate is reduced. By further increasing $j_c$ to $4j_c$ and $8j_c$, the relaxation rate is increased again and the process is mainly governed by quantum motion. The behaviour of the creep rate as a function of the
critical current density at different temperatures is sketched in Fig. 2.6. Here the arbitrary units for $j_c$ are chosen such that $j_c = 1$ corresponds to $j_c$ in Fig. 2.5, $j_c = 2$ corresponds to $2j_c$ and so on. As discussed before, at $T = 2\,\text{K}$ the relaxation rate decreases with increasing $j_c$ at low current densities where the relaxation process is mainly thermally activated and increases with increasing $j_c$ for high current density where the relaxation process is mainly due to quantum motion. For low temperature, e.g. at $T = 0.01\,\text{K}$, the creep rate increases with current density for the whole considered current density range and the main process is quantum motion in the whole considered range.
Chapter 3

Experimental Arrangement

3.1 Low-Temperature Measuring System

The cryostat used for the low-temperature measurements is a $^3$He-$^4$He dilution refrigerator from S.H.E. [26]. The cryostat and the measuring system have been described in detail in the references [27–31] so that only a short out-line of the most important features of this system will be given here.

The experimental cell consists of a custom-made extension of the mixing chamber of which a cross-section is shown in Fig. 3.1. The extension completely consists of the epoxy resin Stycast 1266 [32]. The lower part consists of three cylinders (only two of them are shown in the cross-section), in each of which a sample can be positioned on top of removable plugs. An important feature of this set-up is the fact that the samples are in direct contact with the liquid $^3$He-$^4$He mixture, so that optimal thermal contact is guaranteed.

Each cylinder has its own field coil for measurements under variable dc magnetic fields up to $H \approx 0.25$ T, a primary coil for applying ac fields in a frequency range
from 16 Hz to 160 Hz and amplitudes from 6.6 nT to 3.3 μT, and a secondary coil (also called pick-up coil) in form of a gradiometer. All coils are made of NbTi wires. A cryoperm shield is placed inside the helium dewar and reduces the earth field to less than 0.2 μT in the experimental space. No superconducting shield is used, so that the fields can be changed without changing the temperature. The gradiometer is inductively coupled to a multi-function rf-SQUID [33] through a superconducting loop (dc flux transformer) which consists of the gradiometer, a signal coil, and the interconnecting leads. Any flux variation in the secondary coils will induce a current in the superconducting loop in order to maintain the total flux constant. This current generates a flux change at the signal coil which is detected by the
3.2 High-Temperature Measuring System

A Cerium Magnesium Nitrate (CMN) thermometer and a calibrated germanium resistor [34] are tightly attached to the body of the mixing chamber. The calibration of the germanium resistor extents from 0.3 K to 6 K. Temperatures below 0.3 K are determined from measurements of the magnetic susceptibility $\chi$ of CMN, which obeys the Curie-Weiss law $\chi = C/T + \chi_0$, where $C$ is the Curie constant and $\chi_0$ is the temperature independent Van Vleck susceptibility [35]. The CMN thermometer is calibrated against equilibrium temperature measurements with the germanium resistor between 0.3 K and 1 K for every cooldown.

3.2 High-Temperature Measuring System

The cryostat TS90 used for the measurements above 4.2 K has been designed and constructed in our lab. A detailed description is given in the references [36–38]. A vacuum chamber is immersed into a liquid $^4$He-bath, which is at 4.2 K. A cross-section of the experimental cell, which is located at the bottom of the vacuum chamber, is shown in Fig. 3.2. The coil system is very similar to the one in the dilution refrigerator with three concentric superconducting NbTi-coils: A field coil for applying dc fields, a primary coil for applying ac fields, and a secondary coil in form of a gradiometer which is inductively coupled to an rf-SQUID. Also in this set-up, the earth field has been reduced by cryoperm shields and no superconducting shield has been used. The sample is positioned in the upper coil of the gradiometer by means of a top-loading and centered insert.

The temperature regulation in this cryostat is mainly governed by two heat flows: Heat is transferred from the room temperature environment through the insert down to the sample. On the other hand, heat is transported away from the insert and the sample to the helium bath through a $^4$He-exchange gas in the vacuum chamber. By regulating the pressure of the gas, the equilibrium temperature at the
Figure 3.2: Cross-section of the experimental cell in the high-temperature cryostat TS90.
sample can be chosen between 4.2 K and 80 K. An additional electrical heater is used for fine regulation and stabilization of the temperature as well as for obtaining temperatures above 80 K.
Leer - Vide - Empty
Chapter 4

Experimental Results

4.1 Sample Descriptions

$Y_1Ba_2Cu_3O_7$ with and without columnar defects

The schematic structure of the unit cell of the fully oxygenated $Y_1Ba_2Cu_3O_7$ compound is shown in Fig. 4.1. Two closely spaced CuO$_2$ planes are separated by a plane of Y atoms. The sets of immediately adjacent CuO$_2$ double planes are separated from each other by a BaO plane, a set of CuO chains extending indefinitely in the direction of the $b$-axis and by a second BaO plane. The crystal structure is orthorhombic with the lattice parameters [41]: $a = 3.82\,\text{Å}$, $b = 3.89\,\text{Å}$ and $c = 11.68\,\text{Å}$. The investigated $Y_1Ba_2Cu_3O_7$ single crystals have been provided by L. Krusin-Elbaum at IBM Yorktown Heights and have been grown using a flux-melt technique which is described in Ref. [42]. We are going to discuss measurements on an as-grown crystal and on a crystal with columnar defects introduced by heavy-ion irradiation. The dimensions of the samples are approximately $0.7 \times 1.8 \times 0.03\,\text{mm}^3$ for the unirradiated crystal and $1 \times 1.7 \times 0.025\,\text{mm}^3$ for the irradiated crystal with the crystallographic $c$-axis parallel to the shortest dimension. The latter one has
been irradiated with 580-MeV $^{116}$Sn$^{30+}$ ions with the incident beam at 2° to the $c$-axis [40 and references therein]. This irradiation produces permanent damage effects in $Y_1Ba_2Cu_3O_7$ in form of linear tracks with diameters of approximately 50 Å aligned with the initial beam direction. The sample has been exposed to an irradiation dose of $1.5 \times 10^{11}$ ions/cm$^2$. This corresponds to a matching field of $B_\Phi = 3$ T, which is defined as the magnetic field that would thread the the sample, if each columnar defect were occupied by one vortex carrying one flux quantum $\Phi_0$. The transition temperatures are approximately $T_c \simeq 94.5$ K for the unirradiated crystal.
Figure 4.2: ac susceptibility of the Y$_1$Ba$_2$Cu$_3$O$_7$ single crystal. The arbitrary units have been chosen to be zero for the normal state and −1 for the complete diamagnetic signal.

Figure 4.3: ac susceptibility of the Y$_1$Ba$_2$Cu$_3$O$_7$ single crystal with columnar defects with a matching field of $B_0 = 3$ T.
Figure 4.4: Temperature dependence of the critical current density in the ab-plane for an applied field of $H = 1\, T$ with $H$ parallel to the c-axis [39] for $Y_1Ba_2Cu_3O_7$ crystals from the same batch as the ones investigated in this work. The squares represent an unirradiated crystal, the circles an irradiated crystal with a matching field of $B_\Phi = 3\, T$.

Figure 4.5: Field dependence of the critical current density in the ab-plane at a temperature of $T = 5\, K$ for $H$ parallel to the c-axis [40] for $Y_1Ba_2Cu_3O_7$ crystals from the same batch as the ones investigated in this work. The symbols are the same as in Fig. 4.4.
4.1 Sample Descriptions

and $T_c \approx 95 \text{K}$ for the irradiated one and have been determined by measurements of the ac susceptibility shown in Figs. 4.2 and 4.3. As can be seen from the graphs the transitions are very sharp with a transition width, defined by a 10% criterion, $\Delta T_c < 0.5 \text{K}$ for both specimens. The critical current density $j_c$ has been determined from magnetization measurements by Civale et al. [40] and Thompson et al. [39] for samples from the same batch as the ones investigated in this work. The temperature dependence of $j_c$ at a field of $H = 1 \text{T}$ for an unirradiated and an irradiated crystal with a matching field of $B_\phi = 3 \text{T}$ is shown in Fig. 4.4 and the field dependence at $T = 5 \text{K}$ is shown in Fig. 4.5. For all investigated fields and temperatures, $j_c$ is considerably enhanced due to the presence of the columnar defects. In particular at 5 K and zero field, $j_c$ is about a factor of 4 higher in the irradiated crystal than in the unirradiated one.

$\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$

The $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ compound is characterized by two closely spaced $\text{CuO}_2$ planes which are separated by a plane of Ca atoms (see Fig. 4.6). The sets of adjacent $\text{CuO}_2$ double planes are separated by a SrO plane, two BiO planes, and another SrO plane. The four BiO and SrO planes form an isolation layer and act as an electric charge reservoir. The crystal structure is orthorhombic with the lattice parameters [43]: $a = 5.41 \text{Å}$, $b = 5.44 \text{Å}$, and $c = 30.8 \text{Å}$. The distance between the superconducting layers is thus $d = 15.4 \text{Å}$.

The investigated $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ single crystal has been provided by V. N. Zavaritsky and N. V. Zavaritsky. The preparation and characterization of such crystals are described in Ref. [44]. The dimensions of the sample are roughly $1 \times 1 \times 0.05 \text{mm}^3$ with the shortest dimension parallel to the $c$-axis. A photograph of the sample is shown in Fig. 4.7. The transition temperature has been determined by the measurement of the ac susceptibility shown in Fig 4.8. The onset of the transition is at $T = 95.5 \text{K}$, the width defined by a 10% criterion is $\Delta T_c \approx 5 \text{K}$. 

In this work the most important measurements were decay measurements of the remanent magnetization after zero field cooling the sample to the desired temperature and then sweeping an externally applied field to a maximum value $H_f$ and back to zero. Typically the field was swept up in about 5 seconds, kept constant for 1–2 s, and reduced to zero in 0.5–1 s. After about 1 second from the reduction of the field the flux changes at the SQUID due to the relaxation of the magnetization of the sample were recorded. Typically the relaxation is recorded for about $10^4$ seconds.
4.2 Experimental Procedure

Figure 4.7: Top view photograph of the investigated Bi$_2$Sr$_2$CaCu$_2$O$_x$ single crystal.

Figure 4.8: ac susceptibility of the Bi$_2$Sr$_2$CaCu$_2$O$_x$ single crystal. The arbitrary units have been chosen to be zero for the normal state and -1 for the complete diamagnetic signal.
In the high-temperature cryostat TS90, flux changes can be recorded while heating the sample up to $T_c$. The total remanent magnetization at the beginning of the measurement of the sample is thus given by the flux difference recorded during the decay measurement plus the flux being released from the sample during the warm-up to $T_c$.

4.3 Magnetic Relaxation in a Single Crystal of $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_7$

4.3.1 High-Temperature Measurements ($T \geq 4.2\,\text{K}$)

Remanent Magnetization as a Function of Temperature and Field

In Figs. 4.9 and 4.10 the remanent magnetization $M_{\text{rem}}$ is shown as a function of temperature and field. Fig. 4.9 shows the temperature dependence of $M_{\text{rem}}$ for a given field of $H_i = 2250\,\text{Oe}$. Above approximately $10\,\text{K}$, $M_{\text{rem}}$ increases with increasing temperature, reaches a maximum at around $30\,\text{K}$ and decreases then with increasing temperature. Fig. 4.10 shows the cycling field dependence of $M_{\text{rem}}$ at a constant temperature of $T = 4.2\,\text{K}$. The remanent magnetization increases strongly with increasing cycling field.

This behaviour can be qualitatively described with the Bean model [45] for the critical state. In the critical state, the Lorentz force acting on the vortices is balanced by the pinning force and a change of magnetic field leads to a flux density gradient which implies the capability of sustaining a lossless macroscopic current density, the so-called critical current density $j_c(T)$. Bean assumed a critical current density which is independent of the local magnetic field. Qualitative pictures of the flux density distribution of the remanent magnetization in an infinitely extended slab of thickness $d$ parallel to the external field are shown in Fig. 4.11. Here we neglect $H_{c1}$ for simplicity. The pictures (A) to (C) in the first line represent the flux distribution of the remanent magnetization for three different cycling fields $H_i$. 
4.3 Magnetic Relaxation in a Single Crystal of Y$_1$Ba$_2$Cu$_3$O$_7$

Figure 4.9: Remanent magnetization of the Y$_1$Ba$_2$Cu$_3$O$_7$ single crystal as a function of temperature for a cycling field of $H_i = 2250$ Oe.

Figure 4.10: Remanent magnetization of the Y$_1$Ba$_2$Cu$_3$O$_7$ single crystal as a function of the cycling field at a temperature of $T = 4.2$ K.
Figure 4.11: Qualitative pictures of the flux density distribution of the remanent magnetization in an infinitely extended slab of thickness $d$ parallel to the external field. The pictures (A) to (C) represent the flux distribution of the remanent magnetization for three different cycling fields $H_i$, with $H_i$ increasing from (A) to (C). The pictures (a) to (c) represent the flux distribution of the remanent magnetization for three different temperatures, with temperatures increasing (i.e. $j_c(T)$ decreasing) from (a) to (c), after applying the same cycling field in the three cases. The situations (a) and (A) will be referred to as the undercritical state, the situations (b) and (B) as the partially critical state and (c) and (C) as the fully critical state. For the case of a slab, the remanent magnetization averaged over the sample
4.3 Magnetic Relaxation in a Single Crystal of $Y_1Ba_2Cu_3O_7$

The average remanent magnetization for the two models are shown in Figs. 4.12 and 4.13 as a function of $H_i$ for a constant critical current density and as a function of the critical current density for a given cycling field. For both models, the qualitative behaviour is very similar. The remanent magnetization increases as a function of the cycling field $H_i$ for a constant critical current density until $H_i$ reaches the value of $2H^*$, where $H^* = j_c d/2$ for the case of the slab and $H^* = j_c R$.
Figure 4.12: Remanent magnetization as a function of the cycling field $H_i$ for constant critical current density $j_c$ for the two geometries of a slab and of a cylinder. Here $H^\# = j_c d/2$ for the slab and $H^\# = j_c R$ for the cylinder.

Figure 4.13: Remanent magnetization as a function of the critical current density $j_c$ for constant cycling field $H_i$ for the two geometries of a slab and a cylinder. Here $\tilde{j}_c = H_i/d$ for the slab and $\tilde{j}_c = H_i/2R$ for the cylinder.
for the case of the cylinder. $H^*$ is the field at which for increasing cycling field $H_i$ it passes from the undercritical state to the partially critical state. The remanent magnetization as a function of the critical current density for a given cycling field $H_i$ increases for small $j_c$ linearly with $j_c$ until $j_c = j_c^*$, where $j_c^* = H/d$ for the slab and $j_c^* = H/2R$ for the cylinder. $j_c$ is the critical current density at which for increasing $j_c$ the sample passes from the fully critical state to the partially critical state. At $2j_c^*$, it passes from the partially critical state to the fully critical state. Between $j_c^*$ and $2j_c^*$, i.e. in the partially critical state, $M_{\text{rem}}$ reaches a maximum, and for $j_c > j_c^*$, $M_{\text{rem}}(j_c)$ decreases monotonically with increasing $j_c$.

In spite of the simplifying assumptions, the experimental data of $M_{\text{rem}}$ as a function of $H_i$ and of $T$ can be qualitatively interpreted by a simple Bean model. First of all the fact, that $M_{\text{rem}}$ is increasing as a function of field up to the highest applied cycling field of $H_i = 2700 \text{ Oe}$ at $T = 4.2 \text{ K}$, implies that the sample is not in the fully critical state in the observed field range at this temperature. In fact, using the Bean model for a cylinder with radius $R \approx 0.5 \text{ mm}$ and using $j_c(H \approx 2T, T \approx 5 \text{ K}) \approx 10^6 \text{ A/cm}^2$ for the critical current density (see Fig. 4.5) and a demagnetization factor of $D \approx 0.95$, we estimate the cycling field at which for $T \approx 5 \text{ K}$ the sample would pass from the undercritical state to the partially critical state as $H_i \approx 0.3 \text{ T}$ and the field at which the sample would be fully critical as $H_i \approx 0.6 \text{ T}$, which is bigger than the used field of $H_i \approx 0.2 \text{ T}$. The temperature dependence of the remanent magnetization in Fig. 4.9 fits in the model in the following way: At low temperatures, the sample is in the undercritical state. For increasing temperature, $j_c$ decreases (see Fig. 4.4) and therefore $M_{\text{rem}}$ increases (see Fig. 4.13). For increasing temperature, the sample passes over to the partially critical state between approximately 20 K and 30 K and it is in the fully critical state at higher temperatures, where $M_{\text{rem}}$ decreases with decreasing $j_c$ and increasing $T$. Using the same value for $D$ and the critical current densities given in Fig. 4.4, we expect from the Bean model the crossover from the undercritical state at $T \approx 20 \text{ K}$ and the crossover from the partially to the fully critical state at $T \approx 35 \text{ K}$. These estimations are of course very rough, but they give
the right order of magnitude, which is in satisfactory qualitative agreement with the measured data. A model which takes a correction of demagnetization effects due to flux penetration into account is proposed and discussed in appendix A.

Relaxation of the Remanent Magnetization

The relaxation of the remanent magnetization has been investigated in the high-temperature measuring system TS90 for temperatures between 4.2 K and 18.5 K. In this temperature range, the decay curves deviate slightly from the logarithmic-in-time behaviour as described by the classical flux creep theory of Anderson [1]. Therefore, the slope \( \frac{\partial M_{\text{rem}}(t)}{\partial \ln t} \) of the decay curve in a logarithmic scale depends slightly on the chosen fitting interval. We have therefore chosen to determine the slope always in the time window from 10 to 100 seconds. The normalized rates

\[
S = \frac{1}{M_{\text{rem}}(t = t_0)} \frac{\partial M_{\text{rem}}(t)}{\partial \ln t}, \quad t_0 \approx 10 \text{s}
\]
are shown in Fig. 4.14. In this temperature range, the data show a linear dependence on temperature with $S \approx 0.5\%$ at 4.2 K and a slope of about 0.1%/K.

### 4.3.2 Experimental Error Due to Incomplete Flux Penetration

As mentioned in section 4.3.1, the sample is not in the fully critical state at temperatures below approximately 40 K with the used cycling field of $H_i = 2250\text{ Oe}$. As a result, a certain part of the trapped vortices is exposed to a flux density gradient and hence a Lorentz force, pointing to the inside instead of to the outside of the sample. Those vortices relax to the inside of the sample and for relaxation rates of typically 1% will not leave it within the experimentally observed time window. Whereas the movement of those vortices during the relaxation is not detected by the measuring system, they are in fact counted in the total remanent magnetization, because all the vortices have to leave the sample and are therefore detected during warming up to $T_c$. The measured normalized relaxation rate is therefore smaller than the rate $S_F$ that would be obtained in a relaxation from the fully critical state. As proposed by Pollini et al. [46], the incomplete flux penetration can be accounted for by considering in the normalization only those vortices subjected to an outward pointing flux density gradient

$$S_{out} = \frac{1}{M_{rem,out}(t = t_0)} \frac{\partial M_{rem}(t)}{\partial \ln t}$$

For the case of an infinitely extended slab, the correction factor $r = \frac{S_F}{S}$ is 2 for the undercritical state ((a) and (A) in Fig. 4.11), 1 for the fully critical state ((c) and (C) in Fig. 4.11), and a function of $H_i$, varying between 2 and 1 in the partially critical state. For a general geometry, the situation is more complicated or impossible to calculate. It seems however plausible that the correction factor must be within 1 and 2, with $r = 1$ for the fully critical state and $r$ attaining a maximal, $H_i$ independent value in the undercritical state.

These assumptions have been tested experimentally in the diploma thesis of M. Brändli [47] by measuring the cycling field dependence of the normalized decay
Chapter 4. Experimental Results

Figure 4.15: Remanent magnetization of an $Y_1Ba_2Cu_3O_7$ crystal with columnar defects with a matching field of $B_\phi = 5 \, T$ as a function of temperature for a cycling field of $H_i = 2250 \, Oe$ [47].

Figure 4.16: Remanent magnetization of an $Y_1Ba_2Cu_3O_7$ crystal with columnar defects with a matching field of $B_\phi = 5 \, T$ as a function of the cycling field at a temperature of $T = 80 \, K$ [47].
rate. In this work, the investigated sample was a Y$_1$Ba$_2$Cu$_3$O$_7$ crystal with columnar defects with a matching field of $B_\phi = 5$ T. This crystal is similar to the Y$_1$Ba$_2$Cu$_3$O$_7$ crystals described in section 4.1. It is however a different specimen and it has been irradiated at a higher dose. In this crystal both, the temperature dependence of the remanent magnetization with a cycling field of $H_i = 2250$ Oe, shown in Fig. 4.15, and the cycling field dependence at a constant temperature of $T = 80$ K, shown in Fig. 4.16, show that the sample is in the fully critical state for $H_i > 2000$ Oe and in the undercritical state for $H_i \lesssim 500$ Oe. In Fig. 4.17 we show the cycling field dependence of the normalized relaxation rate. The important result is first that, although the remanent magnetization changes by a factor of 10, the relaxation rate increases only by a factor of 1.3. Second, the normalized rate is approximately field independent in a field regime where the sample is in the undercritical state, increases for higher cycling fields and is again constant in the fully critical state. This experiment proves, that the normalized relaxation rate does not significantly depend on the state of flux penetration, in particular it is field independent in the
undercritical as well as in the fully critical state and the rates are underestimated in the undercritical state by a factor smaller than 1.5.

4.3.3 Low-Temperature Measurements ($7 \, \text{mK} \leq T \leq 4.2 \, \text{K}$)

The measurements of the relaxation of the remanent magnetization for the unirradiated $Y_1\text{Ba}_2\text{Cu}_3\text{O}_7$ crystal have been continued in the dilution refrigerator from $T = 4.2 \, \text{K}$ down to $T = 7 \, \text{mK}$ for the cycling field of $H_i = 2250 \, \text{Oe}$. Some examples of measured decay curves are shown in Fig. 4.18. Since also for the low temperatures the deviation from a logarithmic-in-time law were small, the slope of

![Diagram of relaxation curves](image)

Figure 4.18: Relaxation curves $M_{\text{rem}}(t)$ after cycling the unirradiated and the irradiated ($B_\Phi = 3 \, \text{T}$) $Y_1\text{Ba}_2\text{Cu}_3\text{O}_7$ crystal in a magnetic field of $H_i = 2250 \, \text{Oe}$. Each decay curve has been normalized by the first measured point and they have then been displaced on the same linear vertical scale for clarity.
4.3 Magnetic Relaxation in a Single Crystal of $Y_1Ba_2Cu_3O_7$

Figure 4.19: Normalized relaxation rates $S = \frac{L}{M_{\text{rem}}(t=t_0)} \frac{\partial M_{\text{rem}}(t)}{\partial H(t)}$ as a function of temperature for the $Y_1Ba_2Cu_3O_7$ crystal in a double logarithmic scale. In the inset the data are shown in a linear scale.

The decay curves could again be determined from fits in the time interval from 10 to 100 seconds. In the dilution refrigerator however it is not possible to determine the total remanent magnetization, because the sample cannot be heated above $T_c$ independently of the gradiometer which is made of NbTi with a critical temperature of about 9 K. Knowing however the remanent magnetization as a function of temperature between 4.2 K and 18.5 K, it is possible to make an extrapolation to lower temperatures. Keeping in mind that the important parameter determining $M_{\text{rem}}$ is $j_c$, we do not expect a strong temperature dependence of $j_c$ below 4.2 K. This expectation is supported by considering the temperature dependence of $j_c$ above 5 K shown in Fig. 4.4. Furthermore, the remanent magnetization shown in Fig. 4.9 is practically temperature independent below 10 K. It seems therefore reasonable to choose a constant extrapolation of $M_{\text{rem}}(T)$ to low temperatures. In order to
use this extrapolation for normalizing the decay rates obtained in the dilution refrigerator, a conversion factor had to be used between the two measuring systems, because the absolute values measured at the SQUIDs depend on the gradiometer and flux transformer geometry, which are different in both systems. We determined this conversion factor by comparing the not normalized decay rates at $T = 4.2 \text{ K}$ and $H_i = 2250 \text{ Oe}$, which can be measured in both systems. The final result for the normalized relaxation rates is shown in Fig. 4.19 in a double logarithmic and in a linear scale. The rate decreases to a good approximation linearly with decreasing temperature and reaches a temperature independent limiting value below 700 mK. The quantum creep rate is approximately 0.15% at $T \rightarrow 0$.

For comparison we show in Fig. 4.20 our previous results on granular $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_8$ [48,49]. The sample consisted of randomly oriented grains with diameters $\varnothing \leq 32 \mu\text{m}$. The powder was not pressed so that the grains can be considered as practically isolated from each other. The data show the normalized relaxation rate of the remanent magnetization after cycling the field to a maximum value of

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.20.png}
\caption{Normalized relaxation rates $S = \frac{1}{M_{\text{rem}}(t=\text{to})} \frac{\partial M_{\text{rem}}(t)}{\partial \ln t}$ as a function of temperature for granular $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_8$ [48,49] in a double logarithmic scale.}
\end{figure}
Figure 4.21: Normalized relaxation rates $S = \frac{1}{M(t-t_0)} \frac{\partial M(t)}{\partial \ln t}$ as a function of temperature for $Y_1Ba_2Cu_3O_{7-\delta}$ taken with a Hall probe technique by other groups: Fruchter et al. [50] at $H = 0.2\, T (\Delta)$ and $H = 1.7\, T (\triangledown)$ in a field-on configuration with $H \parallel c$, Seidler et al. [51] at $H = 2\, T (\diamond)$ in a field-on configuration with $H \parallel c$, and Uji et al. [52], who measured the relaxation of the remanent magnetization in field-off after cycling the field parallel to the c-axis ($\circ$) and perpendicular to c ($\bullet$).

$H_i = 340\, Oe$. As for the case of the single crystal described in this section, the powdered sample of $Y_1Ba_2Cu_3O_8$ was in the undercritical state. Therefore, the measured rates were corrected by multiplying them by a factor of 2. Correcting the data for the single crystal of $Y_1Ba_2Cu_3O_7$ by the same factor, we obtain a quantum creep rate for $T \to 0$ of 0.3%. This compares well with the $T \to 0$ value of 0.4% for $Y_1Ba_2Cu_3O_8$ powder shown in Fig. 4.20.
Our results are also in fairly good agreement with other groups' results [50–52] on crystals of $Y_1Ba_2Cu_3O_7$. In Fig. 4.21 we show the data of Fruchter et al. [50], Uji et al. [52], and Seidler et al. [51], who all used a Hall probe technique for their low-temperature measurements and obtained quantum creep rates between 0.7% and 1.2%.

4.4 Magnetic Relaxation in a Single Crystal of $Y_1Ba_2Cu_3O_7$ with Columnar Defects

Remanent Magnetization as a Function of Temperature and Field

As in the case of the unirradiated crystal, an external field was applied parallel to the $c$-axis and to the columnar tracks and then reduced to zero. The obtained total remanent magnetization is shown in the temperature range from 4.2 K to 20 K for a cycling field of $H_i = 2250$ Oe in Fig. 4.22 and as a function of field at $T = 4.2$ K in Fig. 4.23. Both, the temperature and the field dependence indicate that the sample is in the undercritical state at $T = 4.2$ K and the maximal cycling field. First of all, $M_{\text{rem}}$ is increasing as a function of field up to the highest field of $H_i = 2250$ Oe at $T = 4.2$ K. Using, as for the unirradiated sample in section 4.3.1, the Bean model for a cylinder with radius $R \approx 0.5$ mm and using $j_c(H \approx 2T, T \approx 5$ K) $\approx 5 \times 10^6$ A/cm² for the critical current density (see Fig. 4.5) and a demagnetization factor of $D \approx 0.95$, we estimate the cycling field at which for $T \approx 5$ K the sample would pass from the undercritical state to the partially critical state as $H_i \approx 1.5$ T and the field at which the sample would be fully critical as $H_i \approx 3$ T, which is above the used field of $H_i \approx 0.2$ T. Since $j_c$ decreases with increasing temperature (see Fig. 4.4), the fact that $M_{\text{rem}}$ increases with increasing temperature is consistent with the conclusion that the sample is in the undercritical state for the used cycling field at the investigated temperatures. From the Bean model and the data for the critical current density as function of temperature (Fig. 4.4), we estimate that for the used cycling field of $H_i = 0.2$ T the sample would be in the undercritical
4.4 Magnetic Relaxation in Y$_1$Ba$_2$Cu$_3$O$_7$ with Columnar Defects

Figure 4.22: Remanent magnetization of an Y$_1$Ba$_2$Cu$_3$O$_7$ crystal with columnar defects with a matching field of $B_\phi = 3\, T$ as a function of temperature for a cycling field of $H_i = 2250\, \text{Oe}$. The line is a fit to the data of the form $\exp(a \times T + \text{const})$.

Figure 4.23: Remanent magnetization of an Y$_1$Ba$_2$Cu$_3$O$_7$ crystal with columnar defects with a matching field of $B_\phi = 3\, T$ as a function of the cycling field at a temperature of $T = 4.2\, K$. 
state up to temperatures of the order of 70 K. Finally, the temperature dependence of the remanent magnetization, shown in Fig. 4.22, as well as the temperature dependence of $j_c$, shown in Fig. 4.4, imply a weak temperature dependence of $j_c$ and therefore $M_{\text{rem}}$ below 4.2 K and suggest an exponential extrapolation for $M_{\text{rem}}$ to low temperatures.

Relaxation of the Remanent Magnetization

The relaxation of the remanent magnetization has been investigated in the high-temperature cryostat for temperatures between 4.2 K and 18.5 K and in the dilution refrigerator between 7 mK and 4.2 K. Typical decay curves are shown in Fig. 4.18. In the whole investigated temperature range, deviations from the logarithmic-in-time law are small so that the determination of the decay rate is only weakly affected by the choice of the time window. As in the case of the unirradiated crystal, we chose the fitting interval from 10 to 100 seconds. For the normalization of the low-temperature decays we extrapolated the remanent magnetization with an exponential law of the form $\exp(a \times T + \text{const})$. The resulting normalized decay rate as a function of temperature is shown in Fig. 4.24 together with the results obtained for the unirradiated crystal. As for the unirradiated crystal, the relaxation rate of the remanent magnetization of the crystal with columnar defects is to the precision of our measuring procedure temperature independent below approximately 1 K. For higher temperatures, the rate increases with temperature. Compared to the rates for the unirradiated crystal, the relaxation rate is reduced in the crystal with columnar defects above roughly 10 K. This was to be expected since the columnar defects are expected to improve pinning. Below 4.2 K however, we observe that the relaxation rate is enhanced by a factor of about 2.5 due to the presence of the columnar defects.

The question arises to which extent the absolute values of both measurements can be compared. As discussed in section 4.3.2, a source for experimental errors is the fact, that the investigated crystals were not in the fully critical state at $H_i = 2250$ Oe. However, we have shown experimentally, that the experimental error
in the normalized relaxation rate due to the incomplete flux penetration is of the order of 30%. More precisely, the normalized relaxation rate is independent of the cycling field in the undercritical state and in the fully critical state, and the rate is underestimated by a factor of approximately 1.3 in the undercritical state. Now, both investigated crystals, the unirradiated and the irradiated ($B_\Phi = 3$ T) one, were in the undercritical state for the low-temperature measurements. Furthermore, they are very similar in size and shape, so that the correction factor for the normalized relaxation rate due to the incomplete flux penetration should roughly cancel out. We estimate the relative experimental error between both measurements due to incomplete flux penetration to be at the most 30%. Another possible source of
experimental uncertainty is of course the fact, that the total remanent magnetization used for the normalization of the decay rates could not be measured for temperatures below 4.2 K, but had to be extrapolated from the respective measurements above 4.2 K. Considering the weak temperature dependencies of $j_c$ and $M_{\text{rem}}$ above 4.2 K, the extrapolations were in both cases rather well defined. We tried however other extrapolations as well, but in all cases the result was, that the quantum creep rate of the irradiated crystal was well above the rate of the unirradiated crystal. Finally, the measurements had to be taken on different crystals. In order to rule out a possible sample dependence, an experiment on the same sample before and after irradiation has to be carried out. This was not yet possible during this work, but it is planned to confirm this interesting observation in the future.

4.5 Magnetic Relaxation in a Single Crystal of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$

Remanent Magnetization as a Function of Temperature and Field

The remanent magnetization has been determined as a function of temperature for three different cycling fields (shown in Fig. 4.25) and for $T = 4.2$ K as a function of field (shown in Fig. 4.26). The data for fields larger than 2500 Oe have been taken in another cryostat [53], which is very similar to the TS90 but higher fields can been applied. The field dependence of the remanent magnetization at 4.2 K shows that the sample is in the fully critical state for fields larger than approximately 3500 Oe. The temperature dependence shows an increase of the remanent magnetization with decreasing temperature for the higher fields $H_i = 1800$ Oe and $H_i = 2250$ Oe above 12 K and practically constant values below 10 K for all fields. A constant extrapolation of the remanent magnetization for $T < 4.2$ K is therefore reasonable.
4.5 Magnetic Relaxation in a Single Crystal of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.25}
\caption{Remanent magnetization of the $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ single crystal as a function of temperature for three different cycling fields.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.26}
\caption{Remanent magnetization of the $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ single crystal as a function of the cycling field at $T = 4.2\,\text{K}$.}
\end{figure}
Figure 4.27: Relaxation curves $M_{\text{rem}}(t)$ after cycling the Bi$_2$Sr$_2$CaCu$_2$O$_x$ crystal in a magnetic field of $H_i = 880$ Oe. The curves are displaced on the linear vertical scale for clarity.

Figure 4.28: Relaxation curves $M_{\text{rem}}(t)$ after cycling the Bi$_2$Sr$_2$CaCu$_2$O$_x$ crystal in a magnetic field of $H_i = 2250$ Oe. As in Fig. 4.27, the curves are displaced on the linear vertical scale for clarity, the scale in this Fig. is however not the same as in Fig. 4.27.
4.5 Magnetic Relaxation in a Single Crystal of Bi$_2$Sr$_2$CaCu$_2$O$_x$

Figure 4.29: Normalized relaxation rates $S = \frac{1}{M_{\text{rem}}(t=t_e)} \frac{\partial M_{\text{rem}}(t)}{\partial \ln t}$ as a function of temperature for the Bi$_2$Sr$_2$CaCu$_2$O$_x$ crystal. The empty circles are measurements with a cycling field of $H_i = 2250$ Oe, the full cycles are measurements with a cycling field of $H_i = 880$ Oe. The inset shows the same data in a double-logarithmic scale [54,55].

Relaxation of the Remanent Magnetization

The decay of the remanent magnetization has been recorded as a function of temperature for the two cycling fields $H_i = 880$ Oe and $H_i = 2250$ Oe. Some examples of the measured decay curves are shown in Fig. 4.27 for a cycling field of $H_i = 880$ Oe and in Fig. 4.28 for a cycling field of $H_i = 2250$ Oe. For clarity the curves have been displaced on the vertical linear scales. The form of the decay curves was almost logarithmic in the observed time window from approximately 1 to $10^4$ seconds at low temperatures, but for higher temperature the decays slowed down at long times for temperatures above approximately 10 K. For that reason, we have determined
Figure 4.30: Normalized relaxation rates of the irreversible magnetization in Bi$_2$Sr$_2$CaCu$_2$O$_x$ crystals with and without columnar defects measured by Prost et al. [56]. Circles: as grown crystal; squares: irradiated crystal; empty: $H = 0.2$ T; filled: $H = 0.5$ T.

The rate by fitting a line to the decay in a logarithmic scale in the time interval 4 to 100 seconds. The resulting normalized decay rates are shown in Fig. 4.29. In spite of the fact that the sample is in the undercritical state for $H_t = 880$ Oe and probably in the partially critical state for $H_t = 2250$ Oe, we obtain a cycling field independent relaxation rate, which indicates that the state of flux penetration does not significantly influence the obtained relaxation rates.

Our results are also in good agreement with measurements on Bi$_2$Sr$_2$CaCu$_2$O$_8$ by Prost et al. [56] shown in Fig. 4.30. They obtain a quantum creep rate of $-\frac{1}{M_{irr}} \frac{dM}{dt} \ln t \approx 2\%$. 
Chapter 5

Discussion

5.1 Quantum Creep in Anisotropic and Layered High-\(T_c\) Superconductors

The experimental findings can be compared with theoretical results. As described in chapter 2, the quantum creep rates in anisotropic and layered superconductors have been derived in the theory of quantum collective creep [7,8,14]. In the limit of low and intermediate fields and strong dissipation, the theoretically derived quantum creep rate is determined by the saddle point solution of the effective Euclidean action given by

\[
\frac{S_{\text{eff,c}}}{\hbar} = \frac{\hbar}{e^2 \rho_n} \frac{L_c^c}{\hbar}
\]

with the collective pinning length \(L_c^c\) given by

\[
L_c^c \simeq \xi \left( \frac{j_0}{j_c} \right)^{1/2}
\]

These results are valid for anisotropic superconductors independent of the angle between the applied field and the crystal orientation. In layered superconductors
the same results are valid, if the angle \( \vartheta \) between the applied field and the axis perpendicular to the layers is bigger than the anisotropy parameter.

For the anisotropic superconductor \( \text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_7 \), the London penetration depth in the superconducting planes at \( T \to 0 \) is typically \( \lambda_L \simeq 1400 \text{ Å} \) and the planar coherence length at \( T \to 0 \) is approximately \( \xi \simeq 16 \text{ Å} \) so that from \( j_0 = \frac{1}{12\sqrt{3} \pi^2} \frac{e}{\lambda^2 \xi} \), the depairing current density can be estimated as \( j_0 \simeq 3 \times 10^8 \text{ A/cm}^2 \). From measurements by Thompson et al. [39] (see Fig. 4.4) we use \( j_c = 2 \times 10^6 \text{ A/cm}^2 \) for the critical current density. Using furthermore \( \epsilon = 1/5 \) for the anisotropy parameter and \( \rho_n = 10 \mu\Omega\text{cm} \) for the normal state resistivity at \( T \to 0 \), one obtains a collective pinning length of \( L_c \simeq 40 \text{ Å} \) and a Euclidean action of \( S_{\text{eff}}^{\text{theor.}} \simeq 160 \hbar \) or

\[
\left| \frac{\partial \ln M}{\partial \ln t} \right|_{\text{theor.}} \approx 0.6 \%, \quad \text{at } T \to 0
\]

Our experimental result is

\[
\left| \frac{\partial \ln M}{\partial \ln t} \right|_{\text{exp.}} \approx 0.15 \%, \quad \text{at } T \to 0
\]

As discussed in the section 4.3, our measurements have been done in the undercritical state so that our results underestimate the quantum creep rate, that would be obtained from the fully critical configuration, by a factor between 1 and 2. Assuming the maximal possible correction factor of 2 we obtain 0.3%. Considering the approximations made in the theory and the uncertainty in the parameters entering the theoretical formula on the one hand and the experimental uncertainties on the other hand we find a satisfactory agreement between experiment and theory.

Another point of comparison with theory is the crossover temperature from the regime of quantum motion to the regime of thermal activation. Experimentally we obtain a crossover temperature of the order of 1K. In order to compare this with the theoretical result (2.12), we estimate the height of the activation barrier from the slope of the normalized relaxation rate versus temperature between 4.2K and 15K as \( U_c \simeq 900\text{K} \). Combining this with the measured quantum creep rate of 0.15% we obtain with (2.12) theoretically a crossover at \( T_{qc} \simeq 1.5\text{K} \) which agrees well with the experimental data. Due to the incomplete flux penetration we have
underestimated the quantum creep rate by a factor of up to 2. On the other hand
the activation barrier has been overestimated by roughly the same factor, because
in the temperature range from 4.2 K to 15 K, where \( U_c \) has been determined, we still
expect the sample to be in the undercritical state. Therefore, in the determination
of \( T_{qc} \) a correction factor due to the incomplete flux penetration does not have to be
taken into account because it appears twice in the formula for \( T_{qc} \) and cancels out.

\( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x \) is considered as a layered superconductor and in our measure¬
ments the magnetic field was applied perpendicular to the \( ab \)-plane, so that the same
formulae can be used for calculating the quantum creep rate as in the anisotropic
case. Using typical parameters we obtain a collective pinning length of \( L_c^e \simeq 3 - 7 \AA \).
In this material the layer separation is \( d = 15 \AA \), so that the condition \( L_c^e < d \) is
fulfilled. In this case, for low magnetic fields pointing along the \( c \)-axis, the pancake
vortices are pinned individually and the length of the tunneling segment is given by
\( d \) and not by \( L_c^e \) so that expression (2.9) has to be replaced by

\[
\frac{\gamma_{e, c}^{\text{eff}}}{\hbar} = \frac{h}{e^2 \rho_n} \frac{d}{L_c^e}
\]

Using \( \rho_n = 30 \mu\Omega cm \) for the normal state resistivity at \( T \rightarrow 0 \) we obtain

\[
\left| \frac{\partial \ln M}{\partial \ln t} \right|_{\text{theor.}} \simeq 5\% , \quad \text{at} \quad T \rightarrow 0
\]

This is in satisfactory agreement with the experimental value of

\[
\left| \frac{\partial \ln M}{\partial \ln t} \right|_{\text{exp.}} \simeq 1.7\% , \quad \text{at} \quad T \rightarrow 0
\]

In order to calculate the theoretically predicted crossover temperature \( T_{qc} \) with ex¬
pression (2.12) we estimate the pinning energy from the slope of the linearly increas¬
ing rate above \( T = 5 \text{K} \) as \( U_c \approx 240 \text{K} \). This results in a crossover temperature of
\( T_{qc} \approx 4 \text{K} \) which again is in satisfactory agreement with the experimental data.

In conclusion, both the experimentally determined quantum creep rate at
\( T \rightarrow 0 \) and the crossover temperature are in satisfactory agreement with the theoret¬
ically obtained values from the quantum collective creep theory. In particular, the
expectation has been confirmed that, due to the large anisotropy in \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x \)
as compared to Y$_1$Ba$_2$Cu$_3$O$_7$, the quantum creep rate is considerably enhanced in Bi$_2$Sr$_2$CaCu$_2$O$_x$.

### 5.2 Quantum Creep in High-$T_c$ Superconductors with Columnar Defects

Columnar defects have attracted a lot of recent interest, because this kind of defect is particularly effective in improving pinning in high-$T_c$ superconductors. Defects with this geometry confine long sections of the vortex core, while simultaneously destroying a minimal volume fraction of the superconducting material itself. In particular at high temperatures and fields, it has been shown experimentally that the pinning produced by columnar defects is much greater than that produced by random point defects, and causes a considerable enlargement of the irreversibility region in the $H$-$T$ plane [40]. Quantum creep in high-$T_c$ superconductors with columnar defects has been treated theoretically by Vinokur [11], by Radzihovsky [12], and by Morais Smith, Caldeira, and Blatter [13]. As discussed in section 2.4, their theoretical results are

\[
\frac{S_E}{\hbar} \approx \frac{\hbar}{e^2 \rho_n} \left( \frac{r_0}{\xi} \right)^3
\]

as obtained by Vinokur [11]

\[
\frac{S_E}{\hbar} \approx \frac{\hbar}{e^2 \rho_n} \left( \frac{r_0}{\xi} \right)^{2.5} \sqrt{\frac{j_0}{j_c}}
\]

as obtained by Radzihovsky [12]

\[
\frac{S_E}{\hbar} = \frac{\hbar}{e^2 \rho_n} \sqrt{\frac{j_0}{j_c}}
\]  

(2.18)

as obtained by Morais Smith et al. [13]. The differences are due to different model potentials for the columnar defects.

Using $\rho_n = 10 \mu\Omega$cm for the normal state resistivity at $T \to 0$, $\xi = 16$ Å for the planar coherence length at $T \to 0$, $j_0 \simeq 3 \times 10^8$ A/cm$^2$ for the depairing current
density, $j_c = 10^7 \text{A/cm}^2$ for the critical current density, and $r_0 = 25 \text{Å}$ for the radius of the columnar defects we obtain quantum creep rates at $T \to 0$ of approximately 1.3% with the result of Morais Smith et al., 2% with Vinokur's result and 0.5% with Radzihovsky's formula. It is however important to keep in mind to which extent the theoretical formulae are able to give quantitative predictions. First of all, the formulae have been derived by using dimensional estimates, in which numericals which usually combine to a factor of order of unity are dropped. Second, the parameters entering the formulae as $\xi$, $r_0$, $\rho_n$, and $j_0$ are not precisely known. Considering also the experimental uncertainties, we conclude that Vinokur's theoretical result seems to give a too high value, but the theoretical results of Morais Smith et al. and of Radzihovsky are in satisfactory agreement with the experimentally measured quantum creep rate of $\left| \frac{\partial \ln M}{\partial \ln t} \right| \approx 0.3\%$ for $T \to 0$.

In order to compare the results of the case with columnar defects with the one without columnar defects, it is interesting to notice that the result of Morais Smith et al. and Radzihovsky's result for the quantum creep rate at $T \to 0$ contain a dependence on the square root of the critical current density. While Radzihovsky's result however is only valid for the case of a sample with columnar defects, the result of Morais Smith et al. is applicable to both cases. According to Bourgault et al. [57], who have investigated the dependence of the normal state resistivity $\rho_n$ on the density of columnar tracks in $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_7$ introduced by 3.5-GeV xenon irradiation slightly above $T_c$, $\rho_n$ is increased by not more than 10% after irradiating the sample to a dose which corresponds to $B_\phi = 3 \text{T}$. Since we also expect $\xi$ to be not strongly affected by the irradiation of the used dose, the result of Morais Smith et al. implies that the quantum creep rate at $T \to 0$ should be proportional to $\sqrt{j_c}$. From Fig. 4.5 we see that at $T = 5 \text{K}$ and zero field, the critical current density is by approximately a factor of four higher in the irradiated crystal as compared to the unirradiated one. Therefore our experimental result that the quantum creep rate in the irradiated crystal is bigger by roughly a factor of two as compared to the unirradiated crystal is in agreement with the $\sqrt{j_c}$ dependence of the quantum creep rate found by Morais Smith et al. Furthermore, also the observed reduction
of the relaxation rate in the thermally activated regime is in qualitative agreement with the theoretical results. In the model where the columnar defects are treated as sharp square-well potentials, the theoretical expression for thermally activated creep (2.20) is the same as the one obtained in the collective creep theory (2.5), which is valid for the unirradiated case. According to this theoretical expression, the thermally activated rate contrary to the quantum creep rate should be reduced with increasing critical current density. Furthermore, since for increasing critical current density the quantum creep rate is enhanced and the thermally activated creep rate is reduced, the creep rates as a function of temperature for two different critical current densities have to cross each other. This crossing is experimentally observed (see Fig. 4.24) and agrees with the described theoretical model. Finally, from the theoretical analysis it is expected that the crossover temperature from the quantum to the thermally activated regime should be proportional to \( j_c \). This can neither be confirmed nor disproved on the basis of our experimental results, from which one can only conclude that \( T_{qc} \) is of the same order of magnitude for both cases. However, the experimental uncertainties and the somewhat arbitrary definition of \( T_{qc} \) do not allow a more precise statement about \( T_{qc} \) than the statement about the order of magnitude.

In this context we would like to compare our results to a similar work in Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_x\) crystals by Prost et al. [56]. They have investigated magnetic relaxation effects before and after irradiating the same sample with 5.3-GeV Pb-ions with a fluence of \( 10^{11} \) ions/cm\(^2\), which corresponds to a matching field of \( B_\phi = 2 \) T. They have found a large reduction of the relaxation rate after the introduction of columnar defects at high temperatures, but only a small difference in the quantum creep rates at low temperatures (Fig. 4.30). From a similar theoretical analysis as the one presented in section 2.4, Morais Smith et al. [13] have derived that for a layered superconductor with columnar defects the quantum creep rate is given by \( S_{qu} \approx \frac{e^2 \rho_n}{\hbar d} \left( \frac{\xi}{u} \right)^2 \) and the thermally activated creep rate is \( S_{th} \approx \frac{T j_0 \xi}{e_0 d j_c u} \), where \( u \) is the relevant transversal displacement of the pancake vortex. Assuming also in Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_x\) that a columnar defect can be modeled by a sharp square-well
potential, one obtains that this length scale is given by $\xi$. In this case the formulae reduce to the respective results of the quantum collective creep theory for a layered superconductor with $L_c^2 < d$, namely $S_{\text{cu}} \approx \frac{e^2 \rho_n}{h d}$ for the quantum creep rate and $S_{\text{th}} \approx \frac{T \rho_n}{\varepsilon_0 d j_c}$ for the thermally activated creep rate. Assuming that also in Bi$_2$Sr$_2$CaCu$_2$O$_x$ the main effect on the material parameters caused by the irradiation is an increase in $j_c$, we see that as for Y$_1$Ba$_2$Cu$_3$O$_7$ the thermally activated creep rate is expected to be decreased after irradiation. Contrary to Y$_1$Ba$_2$Cu$_3$O$_7$ however, the theoretical expression for the quantum creep rate in Bi$_2$Sr$_2$CaCu$_2$O$_x$ does not contain a dependence on $j_c$. Therefore the experimental observation that in Bi$_2$Sr$_2$CaCu$_2$O$_x$ the quantum creep rate is not changed by irradiation is also in agreement with theory. Finally, the crossing of the relaxation rates as a function of temperature in Y$_1$Ba$_2$Cu$_3$O$_7$ with and without columnar defects is neither experimentally observed nor theoretically expected in Bi$_2$Sr$_2$CaCu$_2$O$_x$ because the quantum creep rate is approximately the same and the thermally activated creep rate is reduced after irradiation.

5.3 Thermal Enhancement of Quantum Creep

Quantum tunneling of vortices in superconductors has been treated mainly in the two limits of strong dissipation and in the superclean limit (see chapter 2). In the dissipative limit the viscous drag coefficient is the important parameter and in the superclean limit the Hall term is dominant in the equation of vortex motion (2.13). In order to decide which limit is appropriate for quantum creep in high-$T_c$ superconductors, the temperature dependence of the creep rate, which is different for the two limits, has to be investigated. Below the crossover temperature $T_{qc}$, the theoretical predictions for the temperature dependence of the (effective) Euclidean action are (2.10, 2.11)

$$\Delta S_E^{(\text{eff})}(T) = S_E^{(\text{eff})}(T) - S_E^{(\text{eff})}(0) \approx -S_E^{(\text{eff})}(0) f(T)$$
Figure 5.1: Fits to the normalized relaxation rates for the $Y_1Ba_2Cu_3O_7$ crystal. The continuous curve corresponds to the law for the superclean limit fitted to the experimental data up to 1.5K. The dotted and the dashed curves are fits with the law for strong dissipation, where the dotted curve is a fit to the data up to 700 mK and the dashed curve is a fit up to 1.5K. In the inset the fits are shown in a linear scale.

with

$$f(T) = \left( \frac{k_B t_c}{\hbar} T \right)^2$$

for strong dissipation and

$$f(T) = \exp \left( -\frac{1}{\frac{k_B t_c}{\hbar} T} \right)$$

for the superclean limit. With the identification (2.8) of $\frac{\partial \ln M}{\partial \ln t} \approx -\frac{\hbar}{S_E^{(eff)}(T)}$, the respective predictions for the temperature dependencies of the relaxation rates are

$$\frac{\partial \ln M}{\partial \ln t}(T) = \frac{\partial \ln M}{\partial \ln t}(T \to 0) \frac{1}{1 - f(T)}$$
In Figure 5.1 we show fits of these laws to the experimental data for the Y$_1$Ba$_2$Cu$_3$O$_7$ single crystal. For all fits, the measured value of the relaxation rate at 7 mK has been used for the limit $\frac{\partial \ln M}{\partial \ln \tau}(T \to 0)$ and the tunneling time $t_c$ has been used as the only fitting parameter. Since the theoretical predictions are only valid for low temperatures, i.e. $T \leq T_{qc}$, the strong dissipation law has been fitted to the data points up to 700 mK in one fit and up to 1.5 K in a second fit. The fits for the clean limit were coincident for the data points up to 700 mK and up to 1.5 K, so only the latter is shown. In Figure 5.2 we show fits to the experimental
data for the Bi$_2$Sr$_2$CaCu$_2$O$_x$ single crystal. Also for these data two fits are shown for the dissipative law, one fit up to 2.5 K and a second up to 4.2 K. Only one fit, fitted up to 4.2 K, is shown for the clean limit law, because the fits for this law were essentially coincident for different fitting intervals.

It has to be concluded, that it is very difficult to differentiate clearly between the two laws. First of all, there are only few experimental data in the crossover temperature regime. Second, the differences between the two laws below $T_{qc}$ are very small as can be seen in the Figs. 5.1 and 5.2, so that, even if there were more data, the difference would be of the order of 10% which exceeds the experimental precision. It is however interesting to notice that we obtain for both kinds of fits similar values for the tunneling time $t_c$, namely $t_c \approx 3 - 6 \times 10^{-12}$ s for Y$_1$Ba$_2$Cu$_3$O$_7$ and $t_c \approx 1 - 1.5 \times 10^{-12}$ s for Bi$_2$Sr$_2$CaCu$_2$O$_x$.

In this context we would like to mention a recent experimental work by Seidler et al. [58], who have developed a new "bithermal" magnetization technique. With this method, they claim to have discovered a linear temperature contribution to the low-temperature semiclassical action for vortex tunneling in single crystals of Y$_1$Ba$_2$Cu$_3$O$_{7-x}$. They find a linear slope of $dS_E/dT = -5.2 \hbar/K$ in the temperature interval $50 \text{ mK} \leq T \leq 750 \text{ mK}$. The basic idea of their technique is to change the temperature at a certain time during the decay from a given initial temperature, e.g. 100 mK, to a final temperature, e.g. 300 mK. They observe an acceleration of the decay at the time of temperature change, if the final temperature is higher than the initial temperature and a respective deceleration in the opposite case. Seidler et al. interpret their data as evidence for a temperature dependence of the Euclidean action at low temperatures.

We would like to point out that a small temperature dependence of the critical current density would result in the same experimentally observable consequences, even if the Euclidean action is temperature independent. We illustrate this remark by giving two example decays "A" and "B" (see Fig. 5.3). Decay "A" is assumed to be at a lower temperature, say $T = 100$ mK, and decay "B" at a higher temperature, say
Figure 5.3: Illustration of the decay of the field profile in an infinitely extended slab for two temperatures 0.1 K and 0.3 K. For explanations see text.

$T = 300 \text{ mK}$. We assume first, that decay “A” have a lower critical current density than decay “B” and second that the normalized decay rate be the same, i.e. the effective Euclidean action be the same, for the two decays. We start the observation of the decay after 400 sec. For the sake of clarity in the graphical illustration we assume that the current density be twice as large at $T = 100 \text{ mK}$ than at $T = 300 \text{ mK}$ after 400 sec. The irreversible magnetization corresponds to the shaded area in Fig. 5.3. Here we assume for simplicity a Bean profile for an extended slab. The irreversible magnetization, which in the critical state is proportional to the current density, is then also a factor 2 higher in decay “A” than in decay “B”. Using the second assumption of the equality of the normalized decay rates, we find that the not normalized rate also is by factor of 2 higher in decay “A” than in decay “B”, say for example $dM/dt = -100 \text{ Oe/sec}$ in decay “A” and $dM/dt = -50 \text{ Oe/sec}$ in decay “B”. Both decays will evolve as a function of time, but the values of the irreversible magnetization as well as the not normalized decay rate will stay by the same factor.
of two higher in decay “A” as compared to decay “B”, because the normalized rates are the same.

Let us now investigate what would be experimentally observed in a bithermal experiment. If the initial temperature is 100 mK and the temperature is changed to 300 mK after 4'000 sec, the sample is in the same state as if it had been cycled at 300 mK and as if it had decayed only for 400 sec. In this situation the decay of the irreversible magnetization would accelerate, in the illustrated example from 10 Oe/sec to 50 Oe/sec. If now in the opposite case the sample is cycled at an initial temperature of 300 mK and the temperature is changed after 4'000 sec to the final temperature of 100 mK, the sample is in the same state as if it had been cycled at 100 mK and as if it had decayed already for 40'000 sec. Therefore the decay would decelerate, in the illustrated example from 5 Oe/sec to 1 Oe/sec.

Clearly the numbers in this Gedankenexperiment are exaggerated. In particular, the temperature dependence of $j_c$ has been overestimated. But the example works also with smaller numbers and it shows that the experimental observation of accelerating or decelerating decays after temperature changes could also be due to a small temperature dependence of the critical current density at low temperatures.

5.4 Self-Heating versus Quantum Tunneling

In a recent paper Gerber and Franse [59] reported on measurements of the magnetic relaxation and the accompanying power dissipation in BiSrCaCuO crystals at temperatures between 0.5 and 4.2 K. They argue that the non-vanishing relaxation rate in bulk superconductors at low temperatures is not an evidence of quantum tunneling of vortices but is governed by self-heating of the superconductor caused by the motion of vortices.

At a temperature of $T = 0.5 \text{K}$ they measured a power dissipation of $P \approx 0.5 \text{nW/mm}^3$ some thousand seconds after applying a field of $H = 1 \text{T}$ to the sample.
5.4 Self-Heating versus Quantum Tunneling

In a simplified model, they assumed that all the power is released in the center of the sample. In order to calculate the temperature difference $\Delta T$ between the center and the surface of the sample they used the formula

$$\Delta T = \frac{P}{kI}$$

(5.1)

where $k$ is the thermal conductivity of the sample and $l$ is the distance from the heating source to the surface. Referring to measurements by Sparn et al. [60] and taking $l = 0.5$ mm, Gerber and Pranse calculated

$$\Delta T \approx 12 \mu K \quad \text{for} \quad T_s = 1 K$$
$$\Delta T = 12 mK \quad \text{for} \quad T_s = 100 mK$$
$$\Delta T = 12 K \quad \text{for} \quad T_s = 10 mK$$

where $T_s$ is the temperature at the surface of the crystal. These calculations depend of course very much on the values of the thermal conductivity and of the applicability of equation (5.1). Gerber et al. explicitly referred to the paper by Sparn et al. [60] and used in their calculations the phonon contribution, namely

$$k = bT^3, \quad \text{with} \quad b = 2500 \mu W/K^4 cm$$

(5.2)

In this very paper however, the main point is that the thermal conductivity of Bi$_2$Sr$_2$CaCu$_2$O$_8$ does not follow a cubic temperature dependence due to phonons but rather it has an additional linear contribution

$$k = aT + bT^3$$

(5.3)

with $a = 63 \mu W/K^2 cm$ and $b = 2500 \mu W/K^4 cm$. In Fig. 5.4, the measurements of the thermal conductivity in the $ab$-plane of a Bi$_2$Sr$_2$CaCu$_2$O$_8$ single crystal are shown together with the fitted function (5.3) and the pure cubic and linear terms in comparison [60]. Taking the purely cubic function (5.2) instead of the correct function (5.3) or, what seems to be the most obvious choice, the real measured data, Gerber and Franse underestimate the thermal conductivity at 100 mK by almost an order of magnitude. Taking the measured data instead of the wrong function (5.2) we obtain

$$\Delta T = 12 \mu K \quad \text{for} \quad T_s = 1 K$$
$$\Delta T = 3 mK \quad \text{for} \quad T_s = 100 mK$$

(5.4)
The most spectacular estimation which Gerber and Franse obtained for $T_s = 10\,\text{mK}$ is very questionable, because for these low-temperature experimental measurements do not even exist in the quoted reference. If one would like to make an extrapolation one order of magnitude below the lowest measured point, it seems more reasonable to take the expression that fits the existing data, i.e. (5.2), and not (5.3). With this expression we obtain a thermal conductivity of $0.6\,\text{nW/Kcm}$ instead of $2.5\,\text{pW/Kcm}$ which leads to

$$\Delta T = 57\,\text{mK} \quad \text{for} \quad T_s = 10\,\text{mK}$$

instead of $\Delta T = 12\,\text{K}$ estimated by Gerber and Franse.
5.5 Quantum Creep in Other Types of Superconductors

In comments on Gerber's and Franse's paper Fruchter et al. [61] and Griessen et al. [62] pointed out that the formula (5.1) for calculating $\Delta T$ is by far too rough an approximation and they propose more precise expressions, which again reduce the calculated temperature difference by more than an order of magnitude at low temperatures. Furthermore, it is obvious that the heating power transferred to the sample depends on the strength of the applied field and is therefore much reduced by working at fields below 0.2 T as we used in our experiments as compared to 1 T applied by Gerber et al.

The estimates given in (5.4) are therefore clearly upper limits. Hence at least down to temperatures of 100 mK, where the heating is at the most 3 mK, self-heating is definitely not an obstacle for the observation of quantum tunneling of vortices. If, as Gerber and Franse claim, self-heating and not quantum tunneling of vortices were responsible for the observed non-vanishing relaxation rate at low temperature in high-temperature superconductors, the saturation of the creep rate as a function of temperature should occur at temperatures of the order of at the most 100 mK. Contrary to this scenario the crossover is experimentally observed at approximately 1 K, which is in good agreement with predictions of the theory of quantum collective creep (see section 5.1).

5.5 Quantum Creep in Other Types of Superconductors

Measurable rates of vortex motion by quantum tunneling have been observed not only in the high-$T_c$ oxides, but also in the organic, the heavy fermion, and the Chevrel phase superconductors.

The first observation of non-vanishing normalized rates for $T \to 0$ was made by Mitin [3] in measurements on the Chevrel phase superconductor $\text{Pb}_{1.2}\text{Mo}_{6.4}\text{S}_8$. His results for the normalized relaxation rates of the frozen field $H_i$ in tubes of polycrystalline $\text{Pb}_{1.2}\text{Mo}_{6.4}\text{S}_8$ are shown in Fig. 5.5. Extrapolating the data to $T = 0$, one obtains a rate of 0.3%. Based on this results he proposed that quantum tunneling...
of vortices through the potential barriers is responsible for flux motion at very low temperatures. In order to compare Mitin’s results with the QCC theory, we calculate from a typical value of the upper critical field $H_{c2} \approx 50$ T \cite{63} the coherence length as $\xi = 25$ Å. From the effective mass ratio $m^*/m \approx 9.6$ and the carrier density $n \approx 7 \times 10^{22}$ cm$^{-3}$ in sputtered PbMo$_6$S$_8$ \cite{64} it follows $\lambda_L \approx 600$ Å for the London penetration depth, so that the depairing current density can be estimated as $j_0 \approx 10^9$ A/cm$^2$. Using $j_c \approx 10^5$ A/cm$^2$ for the critical current density \cite{3} and $\rho_n \approx 0.5$ mΩcm \cite{65}, we calculate from formula (2.9) a rate of 0.5%, which agrees well with the experimental results.

The organic superconductors have several features in common with the high-$T_c$ oxides. Both classes of superconductors have restricted dimensionality, low carrier density, high normal state resistivity, short coherence length, and weak pinning. Strong temperature independent motion of vortices has been observed in the quasi-two-dimensional organic superconductor $(\text{BEDT-TTF})_2\text{Cu}(\text{NCS})_2$ \cite{48,49,66}.

Figure 5.5: Normalized rates of the decay of the frozen field $H_i$ in tubes of polycrystalline Pb$_{1.2}$Mo$_{0.4}$S$_8$ measured by Mitin \cite{3}. 

![Graph showing normalized rates of the decay of the frozen field $H_i$ in tubes of polycrystalline Pb$_{1.2}$Mo$_{0.4}$S$_8$ measured by Mitin.]
5.5 Quantum Creep in Other Types of Superconductors

Figure 5.6: Normalized relaxation rates $S = \frac{1}{M_{\text{rem}}(t=to)} \frac{\partial M_{\text{rem}}(t)}{\partial \ln t}$ of (BEDT-TTF)$_2$Cu(NCS)$_2$ as a function of temperature in a double logarithmic scale [48, 49, 66].

This material becomes superconducting at $T_c \simeq 10.4$ K and the anisotropy parameter can be estimated as $\epsilon = \sqrt{m/M} = 1/20$. The low-temperature relaxation rates of the remanent magnetization in a single crystal of (BEDT-TTF)$_2$Cu(NCS)$_2$ are shown in Fig. 5.6 in a double logarithmic scale for the two field orientations $H \parallel bc$ and $H \perp bc$. The lines in Fig. 5.6 are fits given by

$$\frac{\partial \ln M}{\partial \ln t} = 0.006 + 0.008 T^2; \quad H \parallel bc$$

$$\frac{\partial \ln M}{\partial \ln t} = 0.002 + 0.0005 T^2; \quad H \perp bc$$

From the high-temperature data the activation energy has been estimated as $U_c \simeq 38$ meV for $H \parallel bc$ and $U_c \simeq 2 - 4$ meV for $H \perp bc$. The corresponding crossover temperatures are $T_{qc}^\parallel \simeq 0.8$ K and $T_{qc}^\perp \simeq 0.3$ K and they are indicated by arrows in Fig. 5.6. In order to calculate the theoretical value of the creep rates, expression (2.9) can be used for $H \perp bc$. Using $\rho_n = 100 \mu\Omega cm$, $\xi_{bc} = 150 \AA$, $\epsilon = 1/20$,
Chapter 5. Discussion

1.0

Figure 5.7: Relaxation curves [31] of the remanent magnetization in a UPt$_3$ single crystal after cycling the sample in a field of $H_i = 330$ Oe at the following temperatures: $\sim 10$ mK ($+$), 209 mK ($\times$), 350 mK ($\circ$), 396 mK ($\diamond$), 415 mK ($\triangledown$), 435 mK ($\blacktriangle$).

$j_c = 10^3$ A/cm$^2$, and $j_0 \approx 2 \times 10^6$ A/cm$^2$, one obtains

$$\left| \frac{\partial \ln M}{\partial \ln t} \right|_{\text{theor.}} \approx 0.007$$

The measured value for $H \perp bc$ is

$$\left| \frac{\partial \ln M}{\partial \ln t} \right|_{\text{exp.}} \approx 0.006$$

in good agreement with the theoretical estimations. The $T \to 0$ relaxation rate for $H \parallel bc$ is smaller by a factor of three than the $T \to 0$ rate for $H \perp bc$. A reduction of the rate for $H \parallel bc$ with respect to the case $H \perp bc$ in strongly layered superconductors is in agreement with expectations of the quantum collective creep theory. However, this effect has been predicted only for very good alignment between the field and the planes, which should of the order of 0.1°. In the performed experiment however it was not possible to determine the alignment to that accuracy so that the agreement with experiment and theory in this point has to be interpreted with care.
In spite of their low transition temperatures, strong relaxation effects with creep rates which are comparable to the rates in the high-temperature superconductors are observed in the vortex state of the heavy fermion superconductors CeCu$_2$Si$_2$ [46], UPt$_3$ [31,67,68], and UBe$_{13}$ [67]. In particular the dynamics of the vortex system in UPt$_3$ shows unconventional behaviour. The unconventional (H-T-p) phase diagram of UPt$_3$, which is based on specific heat measurements as well as measurements of various other physical quantities, exhibits at least three distinct superconducting phases with two transitions at $T^+_c$ and $T^-_c$ at $H = 0$ and $p = 0$. It has been observed [67,68] that the flux dynamics at low magnetic fields show a distinction between the relaxation of bulk vortices and those close to the surface. For very low fields, where the dynamics of only vortices very close to the surface is probed, the decays of the remanent magnetization are strongly non-logarithmic down to the lowest measuring temperature of $T \sim 10$ mK and almost temperature independent.

Figure 5.8: Decay fractions $[M_{\text{rem}}(0) - M_{\text{rem}}(10^4 s)]/M_{\text{rem}}(0)$ (+) and $[M_{\text{rem}}(0) - M_{\text{rem}}(\infty)]/M_{\text{rem}}(0)$ (*) as a function of temperature for relaxation curves of the remanent magnetization in a UPt$_3$ single crystal after cycling the sample in a field of $H_i = 330$ Oe [31].
below 350 mK. In Fig. 5.7 typical relaxation curves are shown after the sample has been cycled up to a maximum external field of $H_i = 33$ Oe applied perpendicularly to the crystallographic $c$-axis. The decays can be fitted with a stretched exponential law of the form $M_{\text{rem}}(t) - M_{\text{rem}}(\infty) = [M_{\text{rem}}(0) - M_{\text{rem}}(\infty)] \exp \left[\left(-t/\tau\right)^\beta\right]$ [31]. At temperatures as low as approximately 10 mK, the remanent magnetization decays to approximately half of its value in about $1.8 \times 10^5$ s. Values of the quantities $[M_{\text{rem}}(0) - M_{\text{rem}}(10^4 \text{ s})]/M_{\text{rem}}(0)$ of the initial remanent magnetization which decays in the first $10^4$ s, the fitted fraction $[M_{\text{rem}}(0) - M_{\text{rem}}(\infty)]/M_{\text{rem}}(0)$ which decays until the saturation of the relaxation process, and the fitted stretched exponent $\beta$ are shown in Figs. 5.8 and 5.9. With increasing cycling fields, the decays show a non-zero logarithmic rate at short times (see Fig. 5.10) which is not observed at very low fields and which can be attributed to the decay of the vortices in the bulk. The decay of the bulk vortices is thermally activated. On the other hand,
5.5 Quantum Creep in Other Types of Superconductors

Figure 5.10: Relaxation curve of remanent magnetization for the UPt$_3$ crystal at $T = 450$ mK after a cycling field $H_i = 680$ Oe. In the insert, a similar curve for a smaller cycling field $H_i = 3.4$ Oe. The remanent magnetization $M_{\text{rem}}$ is given in the same arbitrary units for both decays (flux quanta $\Phi_0$ at the sample).

The strong, temperature independent decay rates in the low-field regime cannot be explained with the quantum collective creep theory. Using $\lambda_L \approx 3600$ Å for the London penetration depth and $\xi_{\text{BCS}} = 200$ Å for the BCS-coherence length [69], we obtain a value of $j_0 \approx 10^7$ A/cm$^2$ for the depairing current density. Neglecting the small anisotropy [70] and using $j_c \approx 5 \times 10^3$ A/cm$^2$ for the critical current density at $T = 0$ [31] and $\rho_n \approx 4 \mu\Omega$cm [69], we calculate with (2.9) an effective Euclidean action $S_E^{\text{eff}}|_{\text{theor.}}$ of the order of $10^5 \hbar$ or a creep rate of the order of $10^{-3}$ %.

So this material exhibits in the low-field regime a temperature independent decay which is by several orders of magnitude higher than the value theoretically obtained from the theory of QCC. The anomalous, giant creep of vortices close to the surface may have its origin in the unconventional nature of the order parameter in UPt$_3$. It has been suggested by Sigrist et al. [71] that one way of probing an unconventional superconductor is by investigating surface effects. A similar behavior as in UPt$_3$
Figure 5.11: Normalized rates $S = \frac{M\text{rem}^{-1}(t_0) |\partial M\text{rem}/\partial \ln t|}{\partial t}$ of decays of the remanent magnetization after cycling a CeCu$_2$Si$_2$ single crystal in an external field of $H_i = 40.5$ Oe. The rates have been corrected by taking incomplete flux penetration into account where necessary \cite{46}.

is also observed in UBe$_{13}$ \cite{67}, but the stretched exponential contribution to the relaxation of the remanent magnetization in UBe$_{13}$ is at least a factor of 10 weaker than in UPt$_3$ at all temperatures.

The anomalous flux dynamics observed in UPt$_3$ and UBe$_{13}$ is not found in the heavy fermion superconductor CeCu$_2$Si$_2$ \cite{46}. Measurements of flux creep in single crystal as well as polycrystalline specimens show that the decays follow a power law of the type $M(t) \sim t^{-\alpha}$ for $1 \text{s} \leq t \leq 10^5 \text{s}$, where the exponent $\alpha$ ranges between 0.015 and 0.06. For short times the relaxation data can be approximated by a logarithmic time dependence. The so obtained normalized decay rates are shown in Fig. 5.11. The data have been corrected taking the incomplete flux penetration into account. Calculating the theoretical value for the quantum creep rate according to the theory of QCC, one obtains approximately $3 \times 10^{-4}$, which is smaller than the measured value by more than one order of magnitude.
In summary, a good agreement between experimental results and the results of the QCC theory is found for the high-$T_c$ superconductors, the organics, and the Chevrel phase superconductors. In the heavy-fermion superconductors, experimental findings and theoretical results do not agree. Whereas the disagreement in CeCu$_2$Si$_2$ is limited to roughly one order of magnitude in the creep rate, the decay laws in UPt$_3$ are unusual and the strength of the decays at low temperatures is by several orders of magnitude higher than the value obtained from the QCC theory, which points to explanations based on the unconventional nature of the order parameter in this material.
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Chapter 6

Summary and Conclusions

Classical theories of flux creep in irreversible type-II superconductors are based on the picture of thermally activated jumps of vortices over the pinning barriers and they predict relaxation rates which should vanish with decreasing temperature. The first experimental observation of relaxation rates, which do not extrapolate to zero for \( T \to 0 \), has been made by Mitin in a Chevrel phase superconductor. He interpreted his results as evidence for quantum tunneling of the vortices through the barriers. The first observations of non-vanishing flux creep at millikelvin temperatures has been made by Mota et al. on high-\( T_c \) superconductors. In order to understand these results theoretically, the theory of quantum collective creep has been developed by Blatter, Geshkenbein, and Vinokur within the framework of weak collective pinning theory. In the limit of low and intermediate fields parallel to the \( c \)-axis and strong dissipation, the quantum creep rate for \( T \to 0 \) is determined by the saddle point solution of the effective Euclidean action

\[
\frac{\partial \ln M}{\partial \ln t} \sim -\frac{\hbar}{S^\text{eff,}\kappa}
\]

with

\[
\frac{S^\text{eff,}\kappa}{\hbar} = \frac{\hbar L_c}{e^2 \rho_n}
\]
where \( \rho_n \) is the normal state resistivity and \( L^c_c \) is the collective pinning length given by

\[
L_c^c \simeq \xi \left( \frac{j_0}{j_c} \right)^{1/2}
\]

Here \( \epsilon \) is the anisotropy parameter, \( \xi \) the in-plane coherence length, \( j_c \) the in-plane critical current density, and \( j_0 \) the depairing current density. In the case of layered superconductors, in which the collective pinning length \( L_c^c \) is smaller than the interlayer distance \( d \), \( L_c^c \) has to be replaced by \( d \) so that in this case

\[
\frac{S_{E, c}^{\text{eff.}}}{\hbar} = \frac{\hbar}{e^2 \rho_n} \frac{d}{\epsilon^2}
\]

In order to study the dependence of quantum creep on various parameters such as the anisotropy as well as the influence of temperature, we have chosen to investigate the high-temperature superconductors \( Y_1Ba_2Cu_3O_7 \) and \( Bi_2Sr_2CaCu_2O_x \). These materials are characterized by short coherence lengths, high normal state resistivities, and strong anisotropies, which are favorable parameters for the observation of quantum creep. In particular, the most important difference between these two materials is that \( Bi_2Sr_2CaCu_2O_x \) is much more anisotropic than \( Y_1Ba_2Cu_3O_7 \), so that quantum creep is expected to be enhanced in \( Bi_2Sr_2CaCu_2O_x \) with respect to \( Y_1Ba_2Cu_3O_7 \). With the theoretical expressions one obtains quantum creep rates at \( T \to 0 \) of approximately 0.5% for \( Y_1Ba_2Cu_3O_7 \) and about 5% for \( Bi_2Sr_2CaCu_2O_x \).

We have performed measurements of the relaxation of the remanent magnetization after zero field cooling and cycling the specimens in an external field of \( H_i = 2250 \text{Oe} \). At low temperatures, the decays deviate only slightly from a logarithmic-in-time law. The measured quantum creep rates at \( T \to 0 \) are approximately 0.15% for \( Y_1Ba_2Cu_3O_7 \) and 1.7% for \( Bi_2Sr_2CaCu_2O_x \). Considering the experimental uncertainties and the approximations made in the theory, the agreement between theory and experiment is satisfactory. In particular, the expectation has been confirmed that the strong anisotropy in \( Bi_2Sr_2CaCu_2O_x \) as compared to \( Y_1Ba_2Cu_3O_7 \) leads to a considerable enhancement of quantum creep.
Chapter 6. Summary and Conclusions

A further point of comparison between theory and experiment is the crossover temperature from the quantum regime to the regime of thermal activation. The result of the quantum collective creep theory is

\[ T_{qc} \approx \frac{U_c}{k_B S_B(T = 0)} \frac{h}{\xi} \]

where \( U_c \) is the pinning potential, which can be determined from the high-temperature relaxation data. This expression leads to \( T_{qc} \approx 1.5 \text{ K} \) for \( \text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_7 \) and \( T_{qc} \approx 4 \text{ K} \) for \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x \). This is also in satisfactory agreement with the experimental observations.

Regarding technical applications, it is important to optimize pinning in the high-\( T_c \) superconductors. Very promising results have been obtained by introducing columnar defects into the material. Columnar defects are extended cylindrical tracks of damaged, non-superconducting material with a diameter of approximately the size of the vortex core. They can be generated by high-energy heavy-ion irradiation. Particularly at high temperatures and fields, this kind of defects has been reported to result in a considerable enhancement of pinning and an enlargement of the irreversibility region in the \( H-T \) plane. We have investigated quantum creep in a \( \text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_7 \) crystal with columnar defects which were introduced by \( 580 \text{-MeV} \) tin-ion irradiation at a dose of \( 1.5 \times 10^{11} \text{ions/cm}^2 \) corresponding to a matching field of \( B_{\phi} = 3 \text{ T} \). In the configuration with the external field parallel to the \( c \)-axis and to the tracks, we find that the quantum creep rate for \( T \to 0 \) is by roughly a factor of two higher in the irradiated specimen than in the unirradiated one. On the theoretical side it has recently been shown by Morais Smith, Caldeira, and Blatter that the result for the quantum creep rate at \( T \to 0 \) obtained in the theory of quantum collective creep is also applicable to the case of superconductors with columnar defects, if a columnar defect is modeled by a sharp square-well potential and the radius of the columnar defect is larger than the coherence length. According to this result the quantum creep rate for \( T \to 0 \) should be proportional to the square root of the critical current density \( j_c \). Other parameters as \( \rho_n \) and \( \xi \) are not strongly affected by the irradiation of the used dose. Since the critical current density is enhanced by
roughly a factor of four in the irradiated specimen as compared to the unirradiated one, the enhancement of the quantum creep rate at $T \to 0$ by roughly a factor of two in the irradiated specimen as compared to the unirradiated one is in agreement with the $\sqrt{J_c}$ dependence of the quantum creep rate found in the theory of quantum collective creep and in the work of Morais Smith et al.
Appendix A

A Model for Demagnetization Effects in the Critical State

In order to interpret the magnetization in irreversible type-II superconductors, the simple Bean model is often used. Particularly for simple geometries as an infinitely extended slab or cylinder parallel to the externally applied field, the model gives simple results. For example, the remanent magnetization after cycling a virgin sample in an externally applied field up to a maximal value $H_i$ and back to zero is given by the equations (4.1) and (4.2) for the cases of an infinitely extended slab or cylinder parallel to the externally applied field. In many experiments with high-$T_c$ superconductors however, the samples are flat disks or platelets and the external field is applied perpendicularly to the large surface, so that strong demagnetization effects have to be taken into account. The simplest way of doing this is to correct the externally applied field by a factor $1/(1 - D)$, where $D$ is the demagnetization factor which is given by the geometrical shape of the sample. The reasoning for this correction is the following: For a homogeneously magnetized ellipsoid with one axis of revolution parallel to the externally applied field $H^o$, it can be shown that the
effective field $H^{\text{eff}}$ in the sample is given by
\[ H^{\text{eff}} = H^a - D M^{\text{eff}} \]  
(A.1)

where $M^{\text{eff}}$ is the magnetization of the sample. In a superconductor in the Meissner state we have $B^{\text{eff}} = 0$, so that \( 0 = \mu_0 B^{\text{eff}} = H^{\text{eff}} + M^{\text{eff}} \). In this case one obtains
\[ H^{\text{eff}} = \frac{H^a}{1 - D} \]

However in the case where vortices enter the sample, this formula cannot be applied anymore because $B^{\text{eff}} = 0$ is no longer fulfilled and $M^{\text{eff}}$ in equation (A.1) is not simply given by $-H^{\text{eff}}$ but has to be determined differently. Furthermore in the presence of pinning, the magnetization in the sample is not homogeneous and the demagnetization effects cannot be treated by a simple demagnetization factor anymore. The goal of this appendix is to find an expression for the effective field $H^{\text{eff}}$ as a function of the externally applied field in the Bean critical state for the case where an increasing external field is applied to the virgin sample. We propose for this case a simple model for demagnetization effects which takes flux penetration into account. Once $H^{\text{eff}}(H^*)$ has been found, the remanent magnetization can be calculated as a function of the externally applied cycling field using equations (4.1) or (4.2).

We use the Bean model for an infinitely extended slab parallel to the effective field, neglecting $H_{c1}$ and any field dependence of the critical current density $j_c$. In order to estimate the demagnetization factor, we assume that the sample can be approximated by a flat rotational ellipsoid perpendicular to the applied field. For this geometry with the field along the shortest dimension, the demagnetization factor is approximately given by
\[ D = 1 - \frac{\pi t}{2d} \]  
(A.2)

where $t$ is the thickness and $d$ is the diameter of the sample [72]. In the upper part of Fig. A.1, flux profiles are shown for the three cases: (1a) $H^{\text{eff}} < H^\#$, (2a) $H^{\text{eff}} = H^\#$, and (3a) $H^{\text{eff}} > H^\#$ with $H^\# := d j_c / 2$.

We first consider the case of the not fully penetrated state (see Fig. A.1, (1a)). The field profile can be considered as composed of a completely diamagnetic inner
Appendix A. A Model for Demagnetization Effects in the Critical State

Figure A.1: Upper part: Field profiles in the sample for the cases (1a) $H_{\text{eff}} < H^{\#}$, (2a) $H_{\text{eff}} = H^{\#}$, and (3a) $H_{\text{eff}} > H^{\#}$, where $H^{\#} := \alpha_{\text{j}}/2$. Lower part: Proposed model profiles for the respective cases. For explanations see text.

part and an outer part with a width of the penetration depth $H_{\text{eff}}/\alpha_{\text{j}}$ with a reduced diamagnetic response. In order to estimate the effective field in this configuration, we propose to approximate this situation by a sample with complete diamagnetic response, but with a reduced effective width $d$. The reduction of the width is taken to be half the penetration depth on both sides of the slab, i.e. $d = d - H_{\text{eff}}/\alpha_{\text{j}}$ in total. In Fig. A.1, (1b) the proposed profile model is sketched for this situation. In this case we can again use equation (A.1) with $M = -H_{\text{eff}}$ and $D = 1 - \frac{\pi t}{2 d}$. Solving for $H_{\text{eff}}$ yields
Appendix A. A Model for Demagnetization Effects in the Critical State

Figure A.2: Effective magnetic field $H^{\text{eff}}$ as a function of the externally applied field $H^a$ for a diameter to thickness ratio of $d/t = 20$. Both fields are given in units of $H^\# = d/j_c/2$.

\[ H^{\text{eff}} = \frac{1}{\frac{\pi t}{2 d} + \frac{H^a}{H^\#}} \]  

(A.3)

The sample is in the not fully penetrated state until $H^{\text{eff}} = H^\#$. At this field the flux fronts meet in the middle of the sample (see Fig. A.1, (2a)). The magnetization in the proposed model profile is then $M = -H^\#$ and the effective thickness is $d/2$ (see Fig. A.1, (2b)). Equating $H^{\text{eff}}$ given in equation (A.3) with $H^\#$, we obtain for the applied field the condition $H^a = \frac{\pi}{2} t j_c$.

In the fully penetrated state finally, the value as well as the distribution of the magnetization are assumed to be independent of the applied field. We therefore have $M = -H^\#$ and, in analogy to the case of first full penetration, an effective thickness of $d/2$. We then obtain directly from equation (A.1)
Appendix A. A Model for Demagnetization Effects in the Critical State

Figure A.3: Enhancement factor between the effective magnetic field $H^{\text{eff}}$ and the externally applied field $H^a$ as a function of $H^a$ for a diameter to thickness ratio of $d/t = 20$.

$$H^{\text{eff}} = H^a + \left(1 - \frac{t}{d}\right) H^\#$$  \hspace{1cm} (A.4)

In summary, the effective field in the sample as a function of the applied field is given by

$$H^{\text{eff}} = \begin{cases} 
\frac{1}{\pi \frac{t}{d}} H^a & H^a < \frac{\pi}{2} t_jc \\
\frac{\pi}{2} \frac{H^a}{d} + \frac{H^\#}{H^\#} & H^a > \frac{\pi}{2} t_jc \\
H^a + \left(1 - \frac{t}{d}\right) H^\# & \end{cases}$$

In Fig. A.2 this function is shown in units of $H^\#$ for the case of a diameter to thickness ratio of $d/t = 20$. The effective field $H^{\text{eff}}$ reaches $H^\#$ relatively fast. More precisely, $H^{\text{eff}}$ reaches $H^\#$ when $H^a = \frac{\pi}{2} t_jc = \frac{t}{d} H^\#$, which for the chosen example is $H^a \approx 0.17 H^\#$. On the other hand, the effective field reaches the value...
Figure A.4: Theoretically calculated remanent magnetization as a function of temperature. For the critical current densities, the values given in Fig. 4.4 have been used, for the cycling field $H_i = 2250 \text{ Oe}$, for the width $d = 0.6 \text{ mm}$, and for the thickness $t = 30 \mu \text{m}$. The circles ($\circ$) display the results of the new model which takes a correction of demagnetization effects into account. For comparison also the results of a Bean model for a slab without demagnetization effects are shown ($\Diamond$).

$2H^a$ only when $H^a$ is of the order of $H^\#$ itself. This is due to the fact that, once the fully penetrated state is reached, the field produced by the magnetization does not increase with increasing applied field. The field produced by the magnetization then contributes only with a constant offset to the total effective field. In Fig. A.3 the relative enhancement factor between the effective field $H^{\text{eff}}$ and the externally applied field $H^a$ is shown. At low fields the enhancement factor is maximal. The value of approximately 13 is consistent with the field enhancement $1/(1 - D)$ with $D = 1 - \frac{\pi t}{2 d}$ for the full diamagnetic response in the Meissner state for the chosen diameter to thickness ratio. The increasing field enhancement decreases rather rapidly for increasing applied field. For $H^a \approx H^\#$ it is of the order of only 2 and on further increasing $H^a$ it approaches 1, which is equivalent to the case without demagnetization effects.
In order to test the validity of this model, we calculate for the unirradiated \(Y_1Ba_2Cu_3O_7\) single crystal described in section 4.1 the remanent magnetization from the critical current density as a function of temperature shown in Fig. 4.4 for an applied cycling field of \(H_i = 2250\) Oe, a width \(d = 0.6\) mm, and a thickness of \(t = 30\) \(\mu\)m. The resulting theoretical remanent magnetization is shown in Fig. A.4. For comparison, we also show the remanent magnetization as a function of temperature obtained from the Bean model for a slab without taking any demagnetizing effects into account, i.e. \(D = 0\). The calculated data are in qualitative agreement with the measured data shown in Fig. 4.9. Both curves increase with temperature for low temperatures, have a maximum at a temperature of about 30 K and decrease with increasing temperature for higher temperatures. The relatively sharp cusp in the theoretically calculated data at \(T \approx 28\) K might be somewhat unphysical and due to the rough treatment in the model at the change of the regime from the not fully penetrated to the fully penetrated state.
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Acknowledgements

My most profound gratitude goes to Prof. Dr. Ana Celia Mota for giving me the opportunity to do this Ph.D. work in her group. Her scientific support and her great experience in low-temperature physics and superconductivity were vital for the realization of this work.

I am very grateful to Prof. Dr. G. Blatter, Dr. C. Morais Smith, and Dr. V. B. Geshkenbein for fruitful discussions on the theoretical aspects of this thesis.

My special thanks to my colleagues and friends Marco Nideröst, Roberto Frassanito, Andreas Amann, Piero Visani, Tiziano Teruzzi, Andrea Pollini, and Giovanni Juri for the extraordinary atmosphere in the group. From all of them I learned a lot in an infinite amount of discussions about the different projects in the group, technical and computer problems, and physics in general. They also contributed a lot to the measurements presented in this work. A particular thanks goes to Andreas for working with me on the measurements on Bi$_2$Sr$_2$CaCu$_2$O$_x$ during his diploma work and for helping me to use \LaTeX for writing this thesis.

I would also like to thank all the diplomants, who did their diploma works in our group, in particular Björn Lundqvist and Matti Brändli. Matti made important contributions to the measurements on Y$_1$Ba$_2$Cu$_3$O$_7$ with columnar defects presented here.
For their competent technical assistance I wish to thank Paul Caminada and Hansruedi Aeschbach.

I acknowledge Dr. L. Krusin-Elbaum for providing the $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_7$ crystals and Dr. V. N. Zavaritsky for providing the $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ crystal.

This work was supported by the "Schweizerischer Nationalfonds zur Förderung der wissenschaftlichen Forschung".
Curriculum Vitae

1964 Born in Herford (Germany)

1971–1975 Primary school in Herford


1984–1985 Military Service


1987–1988 Study of Physics at the ETH Zürich as "Hörer in Fortbildung" (1988: entrance examination as an ordinary student at the ETH Zürich)


since 1991 Research and teaching assistant in the group of Prof. A. C. Mota
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List of Publications

1. A. C. Mota, G. Juri, P. Visani, A. Pollini, T. Teruzzi, K. Aupke, and B. Hilti
   Physica C 185-189, 343 (1991)
   “Flux Motion by Quantum Tunneling”

2. A. C. Mota, G. Juri, A. Pollini, K. Aupke, T. Teruzzi, P. Visani, and B. Hilti
   “Quantum and Classical Creep in High-\(T_c\) and Organic Superconductors”

3. K. Aupke, T. Teruzzi, P. Visani, A. Amann, A. C. Mota, and V. N. Zavaritsky
   “Quantum Creep in a \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x\) Single Crystal”

4. A. C. Mota, P. Visani, A. Pollini, and K. Aupke
   Physica B 197, 95 (1994)
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5. A. C. Mota, K. Aupke, A. Amann, T. Teruzzi, A. Pollini, and P. Visani
   J. Low Temp. Phys. 95, 377 (1994)
   “Quantum Tunneling of Vortices in Cuprate and Heavy Fermion Superconductors”
6. P. Visani, A. C. Mota, K. Aupke, A. Amann, and B. Lundqvist  
Proceedings of the XXIXth Rencontre de Moriond: “Coulomb and Interference  
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“Mesoscopic Effects in Proximity Induced Superconducting Cylinders of Copper  
and Silver”

7. A. Amann, P. Visani, K. Aupke, A. C. Mota, M. B. Maple, Y. Dalichaouch, P.  
E. Armstrong, Z. Fisk, and A. V. Mitin  
“Unconventional Flux Dynamics in the Heavy Fermion Superconductors UPt₃  
and UBe₁₃”

8. A. Amann, P. Visani, K. Aupke, A. C. Mota, M. B. Maple, Y. Dalichaouch, P.  
E. Armstrong, and Z. Fisk  
submitted to Europhysics Letters  
“Unconventional Vortex Dynamics in the Low-Field Superconducting Phases of  
UPt₃”